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DEMOGRAPHICS, WEALTH, AND GLOBAL IMBALANCES IN THE TWENTY-FIRST CENTURY

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Demographics, Wealth, and Global Imbalances in the Twenty-First Century
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ABSTRACT

We use a sufficient statistic approach to quantify the general equilibrium effects of population aging on wealth accumulation, expected asset returns, and global imbalances. Combining population forecasts with household survey data from 25 countries, we measure the compositional effect of aging: how a changing age distribution affects wealth-to-GDP, holding the age profiles of wealth and labor income fixed. In a baseline overlapping generations model this statistic, in conjunction with cross-sectional information and two standard macro parameters, pins down general equilibrium outcomes. Since the compositional effect is positive, large, and heterogeneous across countries, our model predicts that population aging will increase wealth-to- GDP ratios, lower asset returns, and widen global imbalances through the twenty-first century. These conclusions extend to a richer model in which bequests, individual savings, and the tax-and-transfer system all respond to demographic change.

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1 Introduction

The world is experiencing rapid demographic change. The share of the world population above 50 years of age has increased from 15% to 25% since the 1950s, and it is expected to rise further to 40% by the end of the twenty-first century (figure 1, panel A). There is a widespread view that this aging process has been an important driver of three key macroeconomic trends to date. According to this view, an aging population saves more, helping to explain why wealth-to-GDP ratios have risen and average rates of return have fallen (panels B and C).[1] Insofar as this mechanism is heterogeneous across countries, it can further explain the rise of global imbalances (panel D).

Beyond this qualitative consensus lies substantial disagreement about magnitudes. For instance, structural estimates of the effect of demographics on interest rates over the 1970–2015 period range from a moderate decline of less than 1 percentage point (pp, Gagnon, Johannsen and López-Salido 2021) to a large decline of over 3 pp (Eggertsson, Mehrotra and Robbins 2019).[2] Turning to predictions for the future, economists are starkly divided about the direction of the effect. Some structural models predict falling interest rates going forward (e.g. Gagnon et al. 2021, Papetti 2021a). At the same time, an influential hypothesis argues, based on the dissaving of the elderly, that aging will eventually push savings rates down and interest rates back up. This argument, popular in the 1990s as the "asset market meltdown" hypothesis (Poterba 2001, Abel 2001), was recently revived under the name "great demographic reversal" (Goodhart and Pradhan 2020). It is central to Larry Summers's recent view that interest rates will be persistently high going forward (as quoted in Rubin 2023):

> Once people have aged and they're retiring, then they draw down their savings and spend. And so I think we're making a transition from more saving because of aging, to less saving because aging has happened.

Our paper refutes this argument and shows that, instead, demographics will continue to push strongly in the same direction, leading to falling rates of return and rising wealth-to-GDP ratios. The key to our results is the *compositional effect* of an aging population: the direct impact of the changing age distribution on log wealth-to-GDP, holding the age

[1] We focus primarily on the expected return on total wealth, which we proxy historically by calculating the average return on total wealth, excluding changes in asset valuations. We will often refer to this measure as the "interest rate"; it has been declining since the 1950s. As is well known, safe rates of return have also fallen, though their fall is most pronounced since the 1980s. Appendix A provides details on the data, and appendix D.3 a model in which demographic change influences the spread between risky and safe returns.

[2] Appendix G presents a selective summary of findings in the literature and shows how to interpret them through the lens of this paper's framework.

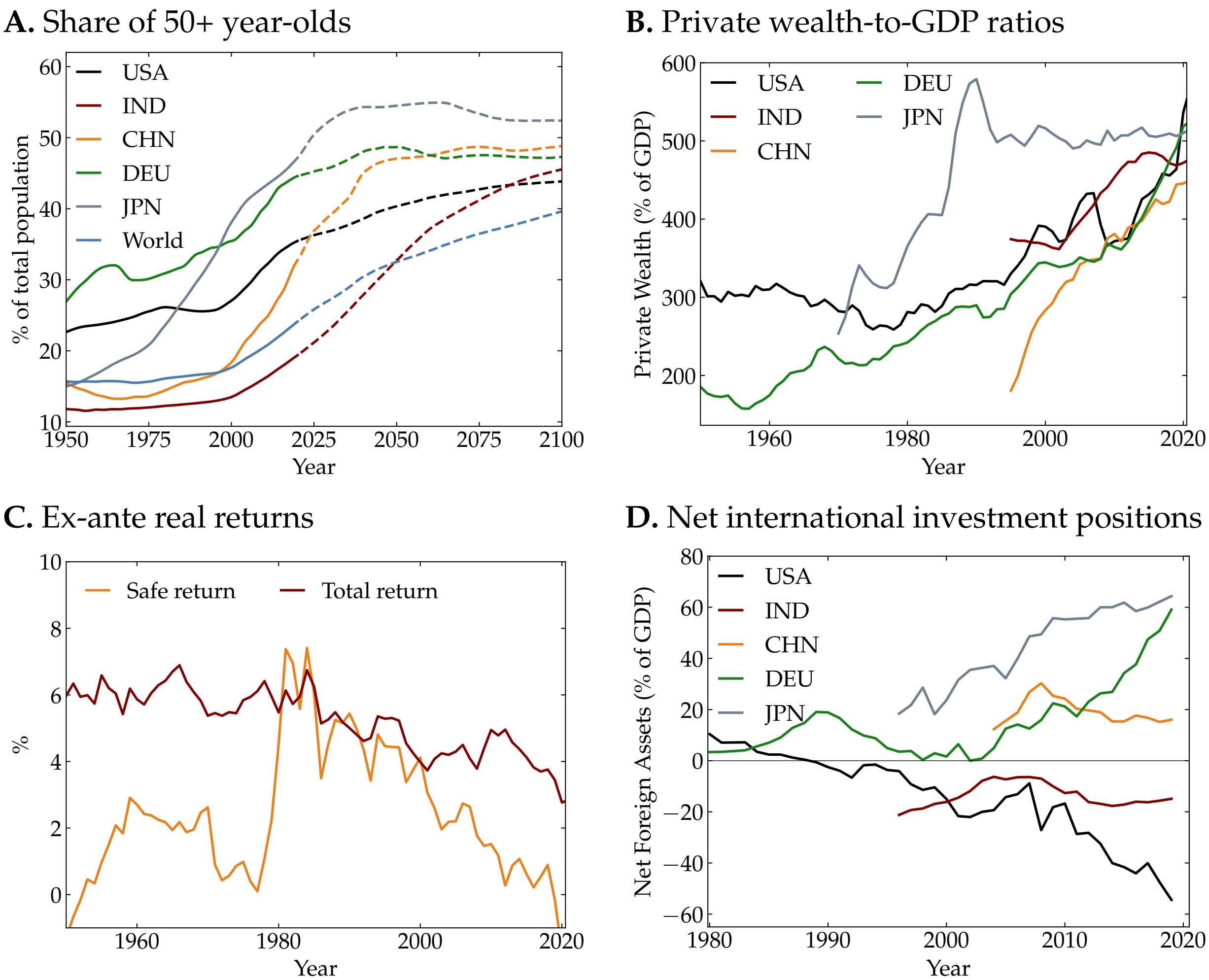

Figure 1: Demographics, wealth, interest rates and global imbalances

Notes: Panel A presents the share of 50+ year-olds in the five largest economies by GDP and the world as a whole (source: 2019 UN World Population Prospects, the projection is the central scenario). Panel B presents private wealth-to-GDP ratios (source: World Inequality Database). Panel C presents a measure of the US total return on wealth (orange line) and of the US safe rate of return (red line). Details on the construction of these series are in appendix A. Panel D presents net international investment positions normalized by GDP (source: IMF and Penn World Tables).

profiles of wealth and labor income fixed. In a baseline overlapping generations (OLG) model, this is a sufficient statistic for the actual change in wealth-to-GDP for a small open economy. Further, for a world economy, the compositional effect—when aggregated across countries, and combined with elasticities of asset supply and demand that we obtain with other sufficient statistic formulas—fully pins down the general equilibrium effect on wealth-to-GDP, asset returns, and global imbalances.

We measure the compositional effect by combining population forecasts with household survey data from 25 countries over the period 2016–2100. We find that it is positive and large everywhere, but also heterogeneous, ranging from 17 log points in Sweden to 56 in India, with a global wealth-weighted average of 32 log points. Behind these numbers lies a key feature of age-wealth profiles: the old hold much more wealth than the young, and on average do not dissave much as they age. This feature, which is remarkably robust across countries and time, interacts with large and heterogeneous increases in the old-age share to produce our compositional effects.

Since the average effect is positive and large, our model predicts that there will be no great demographic reversal: through the twenty-first century, population aging will continue to push down global rates of return, with our central estimate being -1.07pp, and push up global wealth-to-GDP, with our central estimate being a 8.9 log point increase (from 456% to 498% of world GDP). Since the effect is heterogeneous across countries, our model also predicts large global imbalances. For instance, we find that India's net foreign asset position will grow until it reaches 179% of GDP in 2100, while the United States's and Germany's net foreign asset positions will decline to absorb this asset demand.

Our sufficient statistic framework offers a transparent way to compute the effect of a changing age distribution on key macroeconomic variables. General equilibrium outcomes can be obtained with limited information: other than the data required to compute the compositional effect, we only need data on macro aggregates and assumptions on two standard parameters, the elasticity of intertemporal substitution and the elasticity of substitution between capital and labor. Our framework also clarifies a key limitation of the great demographic reversal hypothesis, which focuses on the decline in one flow (savings) when another (investment) is also declining due to demographic change. In contrast, the compositional effect on stocks (rising wealth-to-GDP) unambiguously implies a falling rate of return.

Our baseline model allows for a broad range of savings motives, but rules out some mechanisms through which population aging can affect behavior. To evaluate how much these can matter, we numerically simulate a richer model in which bequests, individual savings, and the tax-and-transfer system all respond to demographic change. We find that the results are always the same qualitatively, and that with one exception—extreme fiscal adjustments that fall entirely either on tax increases or benefit cuts—they are also close quantitatively to those we obtain directly from our sufficient statistic methodology.

Existing literature has followed two broad approaches, which our paper combines, to quantify the impact of demographic change on macroeconomic outcomes. The first is reduced-form. One branch of this literature, following Mankiw and Weil (1989) and Poterba (2001), computes the effect of a changing age distribution over fixed asset profiles.[3] Another branch, following Cutler, Poterba, Sheiner and Summers (1990) and the "demographic dividend" literature (Bloom, Canning and Sevilla 2003), computes the effect of changing age distributions over fixed income profiles.[4] These "shift-share" calculations are very intuitive, but are not tied to specific general equilibrium counterfactuals. We show that a ratio of two such shift-shares is the driver of equilibrium outcomes in a fully specified OLG model.

The alternative approach is structural, relying on quantitative general equilibrium OLG models. This tradition, which originated in Auerbach and Kotlikoff (1987), has tackled effects of demographics on aggregate wealth accumulation,[5] asset returns,[6] and international capital flows.[7] Our contribution here is to trace quantitative results back to primitive elasticities, and to the calibration moments that are relevant for the counterfactual of interest. One benefit of this approach is that it can identify the source of conflicting estimates: for instance, the compositional effect in Gagnon et al. (2021) is about the same as in the data, while that in Eggertsson et al. (2019) is about triple that in the data.

In this paper, we focus on the causal effect of projected demographic change in the twenty-first century. We do not explain the underlying sources of this change; instead, we take demographic projections as given. We also rule out some indirect effects of aging, such as changes in total factor productivity or market structure, which are difficult for us to quantify.[8] Although our baseline exercise holds government debt-to-GDP policy fixed, we show how rising government debt can mitigate or even undo the effect of demographic change on real interest rates, while increasing the effect on wealth-to-GDP.

The compositional effects we identify are large in the past as well as in the future. This suggests that demographic change has been a key historical driver of macro trends.

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[3]There is also a tradition that computes the effect of changing age distributions over fixed age profiles of savings *rates* (Summers and Carroll 1987, Auerbach and Kotlikoff 1990, Bosworth, Burtless and Sabelhaus 1991, Mian, Straub and Sufi 2021). We relate to this literature and discuss its limitations in section 5. Jaimovich and Siu (2009) also explore the effect of changing age distributions on business cycle volatility.

[4]This accounting-based approach to aging has been systematized in the literature and practice of National Transfer Accounts (see e.g. Lee and Mason 2011).

[5]E.g., İmrohoroğlu, İmrohoroğlu and Joines (1995), Kotlikoff, Smetters and Walliser (1999), De Nardi, Imrohoroğlu and Sargent (2001), and Kitao (2014).

[6]E.g., Abel (2003), Geanakoplos, Magill and Quinzii (2004), Carvalho, Ferrero and Nechio (2016), Eggertsson et al. (2019), Lisack, Sajedi and Thwaites (2021), Jones (2023), Papetti (2021a), Rachel and Summers (2019), Kopecky and Taylor (2022), Antunes and Ercolani (2020), Gagnon et al. (2021), and Peruffo and Platzer (2024).

[7]E.g. Henriksen (2002), Börsch-Supan, Ludwig and Winter (2006), Domeij and Flodén (2006), Attanasio, Kitao and Violante (2006), Attanasio, Kitao and Violante (2007), Krueger and Ludwig (2007), Backus, Cooley and Henriksen (2014), Papetti (2021b), Bonfatti, İmrohoroğlu and Kitao (2022), and Sposi (2022).

[8]For the effects of demographics on TFP, see the debate between Maestas, Mullen and Powell (2023) and Acemoglu and Restrepo (2017) for the effect of aging, and Jones (2022) for the effect of slower population growth. For models in which demographics can affect markups via either the structure of consumer demand or firm entry incentives, see Bornstein (2021) vs. Peters and Walsh (2022).

Indeed, according to our model, it can explain roughly half the decline in interest rates since 1950, as well as a smaller share of the rise in wealth-to-GDP. To fully rationalize both time series, substantial upward shifts in both asset demand and supply are needed. We calculate that demographics was a primary driver of the shift in asset demand, accounting for 30% of the overall increase in our central case; the remainder is consistent with a large literature documenting other forces that pushed up asset demand and supply.[9]

The paper proceeds as follows. In section 2, we describe our baseline model, define the compositional effect, and prove our main sufficient statistic results. In section 3, we turn to measurement, documenting compositional effects across countries and calculating their general equilibrium implications. In section 4, we extend the baseline model to capture additional macroeconomic effects of population aging and show that the results from section 3 are a close fit in nearly all cases. Finally, in section 5 we show why the great demographic reversal hypothesis's focus on savings rates is misleading.

2 The compositional effect of demographics

In this section, we set up a benchmark life-cycle model with overlapping generations to study the effects of demographic change. We derive two main theoretical results. First, in a small open economy, demographic change only affects macroeconomic aggregates by changing the age composition of the population. Given a demographic projection, these *compositional effects* can be calculated using data from a single cross section. Second, in an integrated world economy, the long-run effects of demographic change on wealth accumulation, interest rates, and global imbalances can be obtained by simply combining these compositional effects with macroeconomic aggregates, other cross-sectional statistics, and assumptions about two primitive elasticities.

2.1 Environment

Our environment is a world economy with overlapping generations of heterogeneous individuals. Time is discrete and runs from $t = 0$ to ∞, agents have perfect foresight, and capital markets are integrated. All assets share the same global return; other variables and parameters can vary across countries c.[10] We drop indices c unless there is risk of ambiguity.

[9] These forces include, on the demand side, falling TFP growth and rising inequality, and on the supply side, the rise of automation, intangible capital, housing, and markups. See, for instance, McGrattan and Prescott (2010), Rognlie (2015), Rachel and Smith (2017), Auclert and Rognlie (2018), Eggertsson et al. (2019), Straub (2019), Eggertsson, Robbins and Wold (2021), and Moll, Rachel and Restrepo (2022).

[10] Appendix D.3 considers an extension that features a spread between safe and risky asset returns.

Individuals. At each time t, a country has a population $N_t = \sum_j N_{jt}$ growing at rate $1+n_t \equiv N_t/N_{t-1}$, with N_{jt} being the number of individuals of age j. Each individual faces an exogenous probability ϕ_j of surviving from age j to age $j+1$, so the probability of surviving from birth to age j is $\Phi_j \equiv \prod_{k=0}^{j-1} \phi_k$. The maximal lifespan is J, so that $\phi_J = 0$. We assume that this survival profile is constant over time, and that there is no migration (both assumptions are relaxed in section 4). Hence, the age distribution, $\pi_{jt} \equiv \frac{N_{jt}}{N_t}$, only varies over time due to changes in fertility and convergence dynamics.[11]

Individuals supply labor exogenously, face idiosyncratic income risk, and can partially self-insure and smooth income over their life cycle by saving in an actuarially fair annuity, which has a purchase price equal to the survival probability ϕ_j, and which pays out $(1+r_t)$ in case of survival.[12] Their effective labor supply is $\ell(z_j)$, where z_j is a stochastic process, and unless stated otherwise, all individual variables at age j are a function of the whole history of the idiosyncratic shocks z_j, which we denote z^j. The stochastic process z_j can be different across countries, and is arbitrary subject to being the same across cohorts, meaning that over time, individuals of the same age have identical distributions across histories regardless of their birth year.

Individuals with birth year k choose sequences of consumption $\mathfrak{c}_{jt}$ and annuities $\mathfrak{a}_{j+1,t+1}$ for all ages $j = 0, \ldots, J$ (with $t = j+k$) to solve the utility maximization problem

$$
\begin{aligned}
&\max_{\{\mathfrak{c}_{jt}, \mathfrak{a}_{j+1,t+1}\}} \mathbb{E}_k \left[\sum_{j=0}^{J} \beta_j \Phi_j \frac{\mathfrak{c}_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right] \\
\text{s.t.} \quad & \mathfrak{c}_{jt} + \phi_j \mathfrak{a}_{j+1,t+1} \leq w_t \left((1-\tau)\ell_j(z_j) + tr(z^j) \right) + (1+r_t)\mathfrak{a}_{jt} \\
& \mathfrak{a}_{j+1,t+1} \geq -\bar{a} Z_t,
\end{aligned} \tag{1}
$$

where $\bar{a}$ is a borrowing constraint, w_t is the real wage per efficiency unit of labor at time t, r_t is the return on wealth, τ is the labor tax rate, and $tr(z^j)$ denotes transfers from the government, including wage-indexed social insurance and retirement transfers, for agents of age j with a history z^j.[13] The utility weight at age j is $\beta_j \Phi_j$, combining the survival probability Φ_j and an arbitrary age-specific utility shifter β_j. Deviations from exponential discounting ($\beta_j = \beta^j$ for some β) stand in for age-dependent factors that affect

[11] Convergence dynamics for demographics are sometimes called "momentum". Appendix B.1 shows that, in most countries, fertility and momentum together account for the majority of population aging during their demographic transitions.

[12] One implication of this setup is that mortality does not show up in the Euler equation, since the utility shifter coming from survival is canceled by the price of the annuity.

[13] We use $\mathfrak{c}_{jt}$ and $\mathfrak{a}_{jt}$ to denote consumption and annuities at the individual level, and will reserve ordinary letters for the cross-sectional averages $c_{jt} \equiv \mathbb{E}\mathfrak{c}_{jt}$ and $a_{jt} \equiv \mathbb{E}\mathfrak{a}_{jt}$ by age.

the marginal utility of consumption, such as health status or the presence of children. Hence, this model can capture many of the factors that the literature considers essential to understand savings: agents save for life-cycle reasons, for self-insurance reasons, to cover future health costs, and to provide for their children.[14]

The total wealth held by individuals of age j is the product of N_{jt} and the average wealth at age j, $a_{jt} \equiv \mathbb{E}\mathfrak{a}_{jt}$. Aggregate (private) wealth W_t is the sum across age groups:

$$W_t \equiv \sum_{j=0}^{J} N_{jt} a_{jt}. \tag{2}$$

Production. There is a single good used for private consumption, government consumption, and investment. Final output Y_t of this good is produced competitively from physical capital K_t and effective labor input L_t according to an aggregate production function F

$$Y_t = F(K_t, Z_t L_t),$$

where $Z_t \equiv Z_0(1+\gamma)^t$ captures labor-augmenting technological progress. We assume that F has constant returns to scale and diminishing returns to each factor. Effective labor input L_t is a standard linear aggregator

$$L_t = \sum_{j=0}^{J} N_{jt} \bar{\ell}_j, \tag{3}$$

where $\bar{\ell}_j$ denotes average effective labor input per person of age j, capturing variation in experience and hours of work over the life cycle. Capital has a law of motion $K_{t+1} = (1-\delta)K_t + I_t$ where I_t is aggregate investment, and factor prices equal marginal products. The net rental rate of capital is $r_t = F_K\left(K_t/(Z_t L_t), 1\right) - \delta$, and the wage per efficiency unit of labor is $w_t = Z_t F_L\left(K_t/(Z_t L_t), 1\right)$.

We write $g_t \equiv Y_t/Y_{t-1} - 1$ for the growth rate of the economy. If r_t is constant and labor supply L_t grows at a constant rate n, then $g_t = (1+\gamma)(1+n) - 1$. Otherwise, g_t also reflects changes in capital intensity and in the labor force growth rate.

Government. The government purchases G_t goods, maintains a constant tax rate on labor income τ, gives individuals state-contingent transfers $tr(z^j)$ indexed to current wages w_t, and finances itself using a risk-free bond with real interest rate r_t. It faces the flow

[14] We assume that children live with one of their parents, whose consumption at age j includes that of the children they care for. Formally, we set $\beta_j = \ell(z_j) = tr(z_j) = 0$ when $j \leq J^w$, for a J^w that denotes the start of working life independent from parents. Given this assumption, children do not consume or accumulate assets until age J^w.

budget constraint

$$G_t + w_t \sum_{j=0}^{J} N_{jt} \mathbb{E} tr_j + (1 + r_t) B_t = \tau w_t \sum_{j=0}^{J} N_{jt} \bar{\ell}_j + B_{t+1}, \tag{4}$$

where a positive B_t denotes government borrowing. When demographic change disturbs the balance of aggregate tax receipts and expenditures, the government adjusts G_t to ensure that the debt-to-output ratio $\frac{B}{Y}$ remains fixed.

Equilibrium. Given demographics, government policy, an initial distribution of assets, and initial levels of bonds and capital across countries such that $F_K - \delta$ is equal to r_0 in each country, an *equilibrium* is a sequence of returns $\{r_t\}$ and country-level allocations such that, in each country, individuals optimize, firms optimize, and asset demand from individuals equals asset supply from firms and governments,

$$\sum_c W_t^c = \sum_c (K_t^c + B_t^c).$$

Dividing by world GDP Y_t, the above expression can be written as

$$\sum_c \frac{Y_t^c}{Y_t} \frac{W_t^c}{Y_t^c} = \sum_c \frac{Y_t^c}{Y_t} \left[\frac{K_t^c}{Y_t^c} + \frac{B_t^c}{Y_t^c} \right]. \tag{5}$$

Defining a country's net foreign asset position as the excess of wealth over capital and bonds, $NFA_t^c \equiv W_t^c - (K_t^c + B_t^c)$, (5) states that the average NFA-to-GDP ratio is zero, when countries are weighted by their GDP.

2.2 A small economy aging alone

We first study a small open economy undergoing demographic change, while all other countries have constant demographic parameters. In this case, the economy faces a global rate of return r which is exogenous and fixed—exogenous because the economy is small, and fixed since all other countries have fixed demography. This can be seen as the limit case when the economy has an arbitrarily small world GDP weight $\frac{Y_t^c}{Y_t}$, so that its demand and supply of assets do not affect the world equilibrium condition (5).[15] By studying this case, we can analyze how demographics affect macroeconomic aggregates *directly*,

[15] To obtain a fixed interest rate, we assume that all other countries $c' \neq c$ are in demographic steady state given a set of mortality profiles ϕ_j^c and a common growth rate of newborns n, where the constant growth rate ensures that countries preserve their relative size over time.

independent of any effects operating through equilibrium adjustments in returns r_t.

Focusing on wealth, our key finding is that demographic change does not affect the distribution of assets within age groups, only the distribution of people across age groups. Intuitively, the economy converges to a "balanced growth path by age", where each age group has a stable asset distribution growing at the same rate as technology.

Lemma 1. *In a small open economy facing a fixed interest rate r, the distribution of normalized asset holdings by age $\mathfrak{a}_{jt}/Z_t$ converges to an invariant distribution that depends on j and r, but not on the economy's age distribution. If normalized asset holdings start at this invariant distribution, there exists a time-invariant function $a_j(r)$ such that average asset holdings by age satisfy*

$$\frac{a_{jt}}{Z_t} = a_j(r), \quad \forall t. \tag{6}$$

Proof. See appendix B.2. □

The lemma follows since demographic change does not affect the parameters of individuals' life-cycle problems, once these problems are normalized by productivity. Hence, normalized savings decisions are identical as a function of age, income state, and asset holdings. Thus, beyond the initial cohorts, all cohorts have the same distribution of normalized assets. Further, if initial cohorts start at the invariant distribution, which we assume from now on, the result holds for each t, not only in the limit.

Given lemma 1, aggregate wealth per person satisfies

$$\frac{W_t}{N_t} = \sum_j \pi_{jt} a_{jt} = (1+\gamma)^t \sum_j \pi_{jt} a_{j0} \tag{7}$$

Wealth per person changes with the age composition π_{jt} of the population, and otherwise grows at the technological growth rate $1+\gamma$.

We next derive output per person. A constant global r implies a constant ratio of capital to effective labor $k(r)$, defined by $F_K(k(r), 1) = r + \delta$. Aggregate output is then $Y_t = Z_t L_t F(k(r), 1)$, where, from (3), aggregate effective labor is $L_t = N_t \sum_j \pi_{jt} \bar{\ell}_j$. Hence

$$\frac{Y_t}{N_t} = Z_t F(k(r), 1) \sum_j \pi_{jt} \bar{\ell}_j = \frac{F(k(r), 1)}{F_L(k(r), 1)} (1+\gamma)^t \sum_j \pi_{jt} h_{j0} \tag{8}$$

where $h_{j0} = Z_0 F_L \bar{\ell}_j = w_0 \bar{\ell}_j$ is equal to average labor earnings of individuals of age j, and we have used the fact that the initial wage is $w_0 = Z_0 F_L(k(r), 1)$.

Taking the ratio of (7) and (8), we find that W_t/Y_t is proportional to the ratio of $\sum_j \pi_{jt} a_{j0}$ and $\sum_j \pi_{jt} h_{j0}$. The following proposition summarizes this result.

Proposition 1. *Consider a small open economy facing a constant r, with asset holdings starting at the invariant distribution associated with r. Then, wealth-to-GDP satisfies*

$$\frac{W_t}{Y_t} \propto \frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}} \qquad \forall t, \tag{9}$$

where $h_{j0} \equiv w_0 \bar{\ell}_j$ is average pre-tax labor income by age, and $a_{j0} \equiv \mathbb{E}\mathfrak{a}_{j0}$ is average asset holdings by age, in year 0. The change in the wealth-to-GDP ratio is the same as the change in net foreign asset-to-GDP ratio: $W_t/Y_t - W_0/Y_0 = NFA_t/Y_t - NFA_0/Y_0$.

The proposition implies that all changes in W_t/Y_t reflect the changing age composition π_{jt} of the population, given fixed age profiles a_{j0} and h_{j0}. Equation (9) implies that the log change in wealth to GDP between year 0 and year t is given by

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_0}{Y_0}\right) = \log\left(\frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum \pi_{j0} a_{j0}}{\sum \pi_{j0} h_{j0}}\right) \equiv \Delta_t^{comp}. \tag{10}$$

A key feature of (10) is that Δ_t^{comp} can be calculated from demographic projections and cross-sectional data alone, with demographic projections providing π_{jt} and cross-sectional data providing a_{j0} and h_{j0}. We call Δ_t^{comp} the *compositional effect* of aging on W_t/Y_t. Proposition 1 shows that, for a small open economy, this equals the log change in W_t/Y_t. The next section shows that Δ_t^{comp} also plays a key role in the integrated world economy.

2.3 Many countries aging together

Next, we analyze a world economy going through demographic change before converging to a balanced growth path where age structures and relative sizes of countries are stable. While the previous analysis assumed countries faced a constant interest rate $r_t = r$, interest rates now adjust period-by-period to clear the world asset market.

To analyze this case, we consider a first-order approximation of the world asset market clearing condition (5)

$$\sum_c \frac{W_0^c}{W_0} \Delta \log\left(\frac{W_t^c}{Y_t^c}\right) = \sum_c \frac{W_0^c}{W_0} \Delta \log\left(\frac{K_t^c + B_t^c}{Y_t^c}\right), \tag{11}$$

where $\Delta \log\left(\frac{W_t^c}{Y_t^c}\right) \equiv \log\left(\frac{W_t^c}{Y_t^c}\right) - \log\left(\frac{W_t^0}{Y_t^0}\right)$ and $\Delta \log\left(\frac{K_t^c + B_t^c}{Y_t^c}\right) \equiv \log\left(\frac{K_t^c + B_t^c}{Y_t^c}\right) - \log\left(\frac{K_0^c + B_0^c}{Y_0^c}\right)$ represent changes in log asset demand and supply relative to period 0.[16]

[16] In this first-order approximation, the term involving $\Delta \frac{W_t^c}{W_t}$ drops out since we assume that initial net

Absent any changes in interest rates, household asset profiles do not change, and the left of (11) is $\bar{\Delta}_t^{comp} \equiv \sum_c \frac{W_0^c}{W_0} \Delta_t^{comp,c}$ by proposition 1. In this sense, the compositional effect summarizes the full demographic "shock" to the world equilibrium. This shock causes a disequilibrium in the world asset market that must be resolved by adjustments in r_t.

Interest rate changes work through both asset supply and demand. For asset supply, the capital-output ratio only depends on contemporaneous r_t, implying $\Delta \log \left(\frac{K_t^c + B_t^c}{Y_t^c} \right) \simeq (r_t - r_0)\epsilon^{s,c}$, where $\epsilon^{s,c} \equiv -\frac{\partial \log((K^c + B^c)/Y^c)}{\partial r} = \frac{\eta}{r_0 + \delta} \frac{K_0^c}{W_0^c}$ is the semielasticity of asset supply, and η is the elasticity of substitution between capital and labor.[17] For asset demand, adjustment is more complex, potentially depending on the full sequence of $\{r_t\}$. However, long-run asset demand $\frac{W^c}{Y^c}$ only depends on the long-run interest rate r^{LR} and demographics. Writing $\epsilon^{d,c} \equiv \frac{\partial \log(W^c/Y^c)}{\partial r}$ for the semielasticity of this terminal asset demand evaluated at r_0,[18] the long-run interest rate change satisfies

$$\bar{\Delta}_{LR}^{comp} + \bar{\epsilon}^d \cdot (r_{LR} - r_0) \simeq -\bar{\epsilon}^s \cdot (r_{LR} - r_0), \tag{12}$$

where bars denote averages across countries using initial wealth shares $\omega^c \equiv W_0^c / W_0$. The following proposition summarizes our results and the $r_{LR} - r_0$ implied by (12).

Proposition 2. *Suppose that asset holdings start at the invariant distribution given r_0, that initial net foreign asset positions are zero, and that governments keep debt-to-GDP ratios constant. Then, the long-run change in the rate of return is, to first order,*

$$r_{LR} - r_0 \;\; \simeq \;\; -\frac{1}{\bar{\epsilon}^d + \bar{\epsilon}^s} \bar{\Delta}_{LR}^{comp}, \tag{13}$$

where $\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \frac{\bar{K}_0}{\bar{W}_0}$ is the average semielasticity of asset supply to r, and $\bar{\epsilon}^d$ is the average semielasticity of individual asset holdings to r. The wealth-weighted average log change in the wealth-to-GDP ratio is given by

$$\overline{\Delta_{LR} \log \left(\frac{W}{Y} \right)} \simeq \frac{\bar{\epsilon}^s}{\bar{\epsilon}^s + \bar{\epsilon}^d} \bar{\Delta}_{LR}^{comp} \tag{14}$$

foreign asset positions $W_0^c - K_0^c - B_0^c$ are zero. In appendix D.1, we show how the formulas change in the case where initial NFAs are non-zero.

[17] This expression obtains due to our baseline assumption that B/Y is constant, as well as due to the absence of rents. If B/Y responds to r, then this adds an additional term to $\epsilon^{s,c}$. If fully capitalized rents are part of wealth then their value is proportional to $1/(r - g)$. As we show in appendix D.2, this adds a further term to $\epsilon^{s,c}$, and also adds a direct effect of population growth on asset supply. However, we abstract from endogenous adjustment of human capital as in Ludwig, Schelkle and Vogel (2012).

[18] Formally, $\epsilon^{d,c}$ is the derivative with respect to r of the balanced growth level of $\log W/Y$ in a small open economy with exogenous r, evaluated at the long-run steady-state age distribution and the initial r_0. This includes both the direct individual asset accumulation response to r, and the indirect response from the effect of r on wages. We discuss $\epsilon^{d,c}$ further in the next section.

Proof. See appendix B.3. □

The proposition shows how the excess asset demand from the compositional effect $\bar{\Delta}_{LR}^{comp}$ is absorbed. If $\bar{\epsilon}^s + \bar{\epsilon}^d$ is large, r falls little, because capital and assets are very sensitive to r. If $\frac{\bar{\epsilon}^s}{\bar{\epsilon}^s+\bar{\epsilon}^d}$ is large, wealth rises a lot, because a large share of the adjustment occurs through an increase in the capital stock rather than through a reduction in asset accumulation.

Beyond interest rates and wealth levels, our framework also speaks to global imbalances. To see why, note first that absent an adjustment in r, the net foreign asset position (NFA) of a country would increase one-for-one with its compositional effect. In equilibrium, r must fall to ensure that NFAs are zero on average, so the adjustment in r has to reduce average NFAs by the average compositional effect. Hence, the change in a country's NFA is determined by the difference between its compositional effect and the average compositional effect, subject to an additional adjustment when countries have heterogeneous semielasticities of asset demand and supply. The following proposition summarizes this argument.

Proposition 3. *Given the conditions of proposition 2, the long-run change in country c's net foreign asset position NFA^c satisfies*

$$\log\left(1+\frac{\Delta_{LR}NFA^c/Y^c}{W_0^c/Y_0^c}\right) \simeq \Delta_{LR}^{comp,c} - \overline{\Delta}_{LR}^{comp} + \left(\epsilon^{d,c}+\epsilon^{s,c}-\left(\bar{\epsilon}^d+\bar{\epsilon}^s\right)\right)(r_{LR}-r_0) \quad (15)$$

Proof. See appendix B.3. □

Finally, since we have no direct way to predict the effect of demographics on long-run government debt targets, propositions 2 and 3 both assume a benchmark where each country keeps long-run debt-to-GDP constant. Appendix B.4 discusses alternative settings where debt-to-GDP changes in response to demographics. Two special cases stand out: when each country increases its debt-to-GDP target by the amount of its compositional effect, and when each country increases debt-to-GDP by the *average* world compositional effect. In the first case, there is no change in interest rates or net foreign assets, and each country's wealth increases by exactly its compositional effect. In the second case, the same conclusions hold for interest rates and world wealth, but net foreign assets in each country increase by the difference between its compositional effect and the global average, leaving the global imbalances predicted by proposition 3 intact.[19]

[19]This second case can be viewed as the limit of a specification where we make long-term debt-to-GDP highly responsive to interest rates, taking $\partial(B^c/Y^c)/\partial r$ uniformly to $-\infty$ across all countries.

2.4 The asset demand semielasticity ϵ^d

Propositions 2 and 3 show that compositional effects determine aggregate outcomes given asset supply and demand semielasticities ϵ^s and ϵ^d.[20] As we have noted, the asset supply semielasticity ϵ^s is only a function of observables and of the elasticity of substitution between labor and capital η.

The asset demand semielasticity ϵ^d is more challenging to obtain. As noted by Saez and Stantcheva (2018), there is a "paucity of empirical estimates" for how long-run asset accumulation responds to changes in the rate of return.[21] Remarkably, however, in a special case of our model, it is possible to express ϵ^d only in terms of macroeconomic aggregates, the observed age profiles of assets and consumption, and the elasticities σ and η. The latter are standard macro parameters that have been studied by an extensive empirical literature.

This special case is a version of our model without income risk or borrowing constraints. To build intuition, we start by specializing further to the case where technology is Cobb-Douglas and $r = g$ in the initial steady state. Then, our results take the simple form:

$$\epsilon^d = \sigma \underbrace{\frac{C}{(1+g)W} \frac{\mathrm{Var}Age_c}{1+r}}_{\equiv \epsilon^d_{substitution}} + \underbrace{\frac{\mathbb{E}Age_c - \mathbb{E}Age_a}{1+r}}_{\equiv \epsilon^d_{income}}. \tag{16}$$

Here, Age_a and Age_c are random variables that capture how asset holdings and consumption are distributed across different ages. The random variables range over ages j, with probabilities proportional to assets and consumption at each age.[22] Thus, $\mathrm{Var}Age_c$ is large when consumption is spread out across different ages, and $\mathbb{E}Age_c - \mathbb{E}Age_a$ is positive if consumption, on average, occurs at higher ages than asset holdings do.

In appendix B.5, we derive equation (16), connecting it to the broader logic of life-cycle problems and the cross-sectional outcomes that they produce. The substitution effect $\sigma\epsilon^d_{substitution}$ scales with the elasticity of intertemporal substitution σ, and is proportional to $\mathrm{Var}Age_c$ since there is more scope for intertemporal substitution if consumption is more spread out over the life cycle. The income effect ϵ^d_{income} reflects the fact that a higher r

[20] In this section we drop the country superscripts c for convenience. Subscripts c denote consumption.

[21] An elasticity of this kind is important in a variety of contexts, including for capital taxation (Feldstein 1978, Saez and Stantcheva 2018), the response of interest rates to automation (Moll et al. 2022), and the welfare implications of increasing the public debt (Aguiar, Amador and Arellano 2024). See section 3.2 for a discussion of empirical estimates.

[22] Formally, we define the probability mass of Age_a at each age j to be $\pi_j a_j / A$, the share of assets in the cross section held by people of age j, and likewise for Age_c. For the case $g = 0$, this is equivalent to defining the mass as the share of assets held at age j across the life cycle, but with the cross-sectional definition our result holds more generally.

increases total income. The size of the increase is proportional to total wealth W and accrues at an average age of $\mathbb{E}Age_a$, and it is used to increase consumption by a uniform proportion across all ages, implying that the rise in consumption occurs at an average age of $\mathbb{E}Age_c$. Aggregate wealth increases if $\mathbb{E}Age_a$ is lower than $\mathbb{E}Age_c$, because then, on average, the extra interest income is saved before it is consumed.

For the more general case, there are two complications. First, when technology is not Cobb-Douglas, the labor share changes with r, introducing a new term. Second, our previous result relied on current values being the same as present values normalized by growth, which is no longer true when $r \neq g$. We have the following proposition.

Proposition 4. *Consider a small open economy with a steady-state population distribution π. If individuals face no income risk or borrowing constraints, the long-run semielasticity of the steady-state W/Y to the rate of return is given by*

$$\epsilon^d \equiv \frac{\partial \log W/Y}{\partial r} = \sigma \epsilon^d_{substitution} + \epsilon^d_{income} + (\eta - 1)\epsilon^d_{laborshare}. \tag{17}$$

When $r = g$, $\epsilon^d_{substitution}$ and ϵ^d_{income} are given by (16); when $r \neq g$ they are explicit functions of π_j, c_j and a_j given in the appendix. In either case, $\epsilon^d_{laborshare} \equiv \frac{(1-s_L)/s_L}{r+\delta}$ with $s_L \equiv \frac{wL}{Y}$.

Proof. See appendix B.5. □

Proposition 4 provides, to our knowledge, the first expression for the semielasticity of aggregate asset demand in a rich quantitative model as a function of measurable sufficient statistics. Earlier work has instead relied on numerical simulations (e.g. Summers 1981, Evans 1983, Cagetti 2001, Aguiar et al. 2024). While the literature has pointed out that this elasticity can be affected by idiosyncratic income uncertainty, we show in section 4 that our formula still provides a close approximation in that context. Further, the results of proposition 4 are continuous at $r = g$, so that for small $r - g$, (16) is a good approximation to the actual $\epsilon^d_{substitution}$ and ϵ^d_{income}.[23]

3 Measurement and implications

This section uses the framework provided by propositions 1–4 to quantify the impact of demographics on macroeconomic aggregates. First, we combine demographic projections with representative household surveys to measure the compositional effect Δ_t^{comp} in 25 countries. Second, we use information on age profiles of consumption and wealth together with assumptions on η and σ to calculate the semielasticities of asset supply

[23] $\epsilon^d_{laborshare}$ tends to be small enough that for η close to 1, its contribution is insignificant.

and demand to interest rates. Finally, we combine these results to forecast interest rates, wealth levels, and global imbalances until the end of the twenty-first century.

3.1 The compositional effect

Implementation. We take age distributions π_{jt} from the historical data and future projections of the 2019 United Nations World Population Prospects (reported in 5-year age buckets). For these projections, we consider three different scenarios, corresponding to the UN's baseline projection as well as their "high" and "low" fertility scenarios.[24]

For the age profiles of labor income and wealth, we use representative household surveys. We use labor income data from the Luxembourg Income Study (LIS), which provides harmonized labor surveys for a wide range of countries; we use wealth data from a collection of wealth surveys such as the US Survey of Consumer Finances (SCF) and the European Household Finance and Consumption Survey (HFCS). Our exercise starts in 2016 and the surveys are from this year whenever possible; otherwise, we use the closest available year. See appendix table A.1 for a complete list of data sources and survey years.

For labor income, h_{j0} is the 2016 average pretax labor income of individuals of age j. We calculate it by dividing the total labor income earned by individuals of age j—including wages, salaries, bonuses, fringe benefits, and self-employment income before social security and labor income taxes—by the number of individuals of age j.

For assets, a_{j0} is the 2016 average individual net worth of individuals of age j. We measure it as total assets net of liabilities, with housing[25] and defined contribution pension wealth included as assets, and mortgages included as liabilities. For the United States, we also add age-specific estimates of the funded component of the empirically important private defined benefit (DB) pension plans (Sabelhaus and Volz 2019). We map household wealth from the surveys to individuals by splitting wealth equally across the head of household, the spouse, and any other household members who are at least as old as the head.[26]

[24] Note that in addition to fertility, these age distributions reflect changing mortality, which is outside the formal scope of section 2 and matters for both compositional and non-compositional reasons. We use the full projected age distributions to calculate the compositional effect here, and will show in the richer model of section 4 that the non-compositional effects of changing mortality are relatively small.

[25] Barring any direct feedback from the age distribution to the household problem via, say, land prices, proposition 1 remains true even if households partly accumulate assets in the form of housing. However, the presence of land does have general equilibrium implications by affecting the elasticity of asset supply, as we show in appendix D.2.

[26] Appendix C.3 shows that the results are robust to using different splitting rules, or to constructing income and wealth at the household level, and combining this with demographic projections for the age distribution of the heads of households.

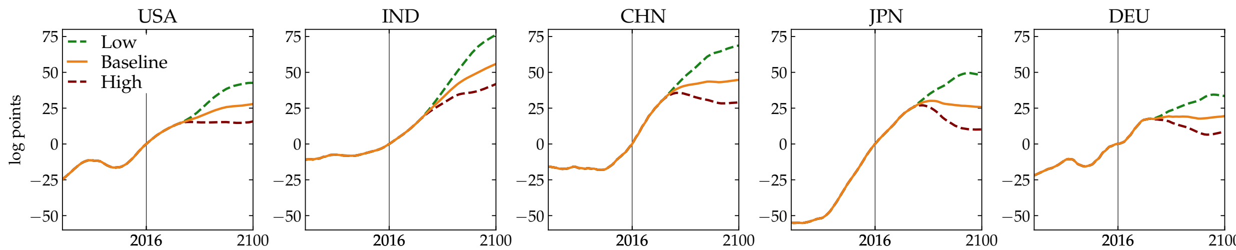

Figure 2: Compositional effect of demographics, 1950 to 2100

Notes: This figure depicts the evolution of the compositional effect of demographic change on wealth-to-GDP, calculated using equation (10) for t =1950 to 2100, reported in log points (100 log). The base year is 2016 (vertical line). The solid orange line corresponds to the medium fertility scenario from the UN, the dashed green line to the low fertility scenario, and the dashed red line to the high fertility scenario.

We use these demographic projections and age profiles of asset and labor income to construct the compositional effect from 1950 to 2100 for the twenty-five countries in our sample. Recall that in the model, Δ^c_{comp} corresponds to the log change in W/Y that a country would experience if it were a small open economy facing a fixed r.

Results. The results from this calculation are displayed in figure 2 for five major countries, with additional countries in figure A.3. Between 1950 and 2016, the compositional effect is positive everywhere, equal to 23 log points on average, and 25 in the United States. To provide context for these magnitudes, we note that the actual log change in W/Y over this period—which reflects other forces as well as general equilibrium adjustments—was 96 log points for the average country with available data, and 32 in the US (see table A.4).

Looking ahead from 2016 to 2100, the effect remains positive, is even larger on average, and is heterogeneous across countries, ranging from 17 log points in Sweden to 45 in China and 56 in India, with 28 log points in the United States. In the high fertility scenario, the effect is reduced by a younger population: it is brought down to 29 log points in China and to 16 in the United States; in contrast, the low fertility scenario sees even sharper aging, and the effect swells to 69 log points in China and 43 in the United States.

Figure 3 provides more detail on the heterogeneity across countries, with solid bars displaying the compositional effect to 2100 for the main population scenario. In principle, this cross-country heterogeneity could reflect either differences in demographic evolution or differences in the age profiles of assets and labor income. While both matter, the former is the main factor: countries with large effects are those whose demographic transitions are later and faster. The transparent bars in figure 3 illustrate this point by recomputing compositional effects while counterfactually assuming that all countries have the same

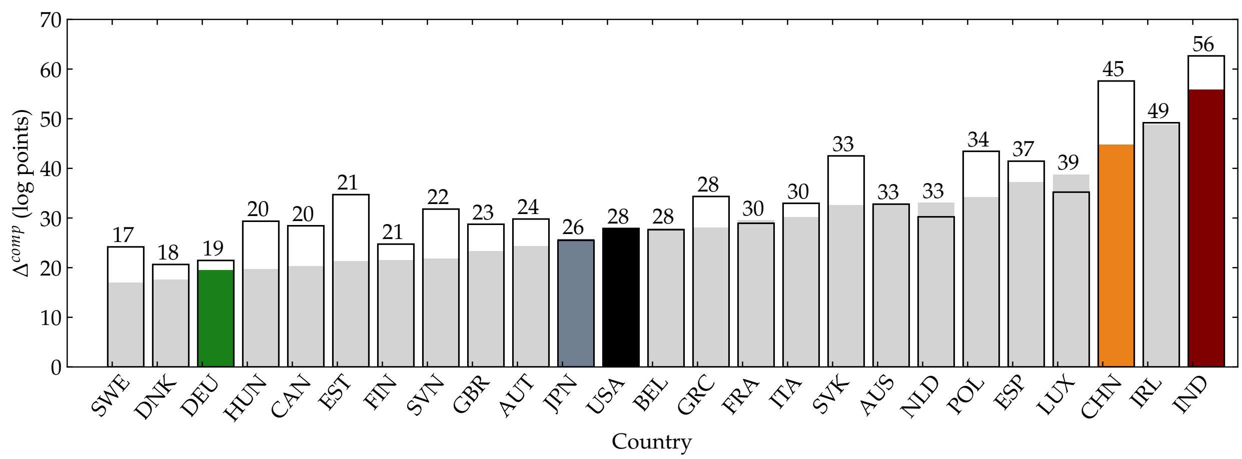

Figure 3: Compositional effect and contribution from demographics alone, 2016-2100

Notes: The solid bars show, for each country, the compositional effect on wealth-to-GDP between 2016 and 2100 in pp, calculated using equation (10), and also reported on top of the bars. These values correspond to the end point of Figure 2. The transparent bars calculate Δ^{comp} using the US age profiles a_{j0} and h_{j0}, but country-specific age distributions π_{jt}.

asset and income profile as the United States. While the levels are slightly higher, meaning that US age profiles imply bigger compositional effects, the cross-country heterogeneity is similar.[27] In appendix C.4, we consider a related exercise, recalculating the average wealth-weighted compositional effect $\bar{\Delta}^{comp}$ when we randomly reshuffle age-wealth and age-labor profiles across countries, holding fixed their demographic projections. The resulting distribution of $\bar{\Delta}^{comp}$ is tight and centered close to the actual $\bar{\Delta}^{comp}$ we calculate in the data.

Unpacking the compositional effect: the case of the United States. The compositional effect reflects the interaction between population aging and the shapes of the wealth and income profiles. To help explain the magnitudes that we find, we study the case of the United States in greater detail.

The main mechanisms are summarized in figure 4. The grey bars show the evolution of the population distribution, starting young in 1950 and growing progressively older over time. In the figure, this population evolution is superimposed with the 2016 profiles of assets and labor income, with panel A illustrating how demographic change pushes up assets by moving individuals into high asset ages, and panel B illustrating how demographic change first pushes up aggregate labor income as the baby boomers reach middle age—the so-called "demographic dividend" (Bloom et al., 2003)—and later pushes down

[27] By contrast, appendix figure A.5 shows that countries tend to experience similar compositional effects if they are all assumed to experience US demographics.

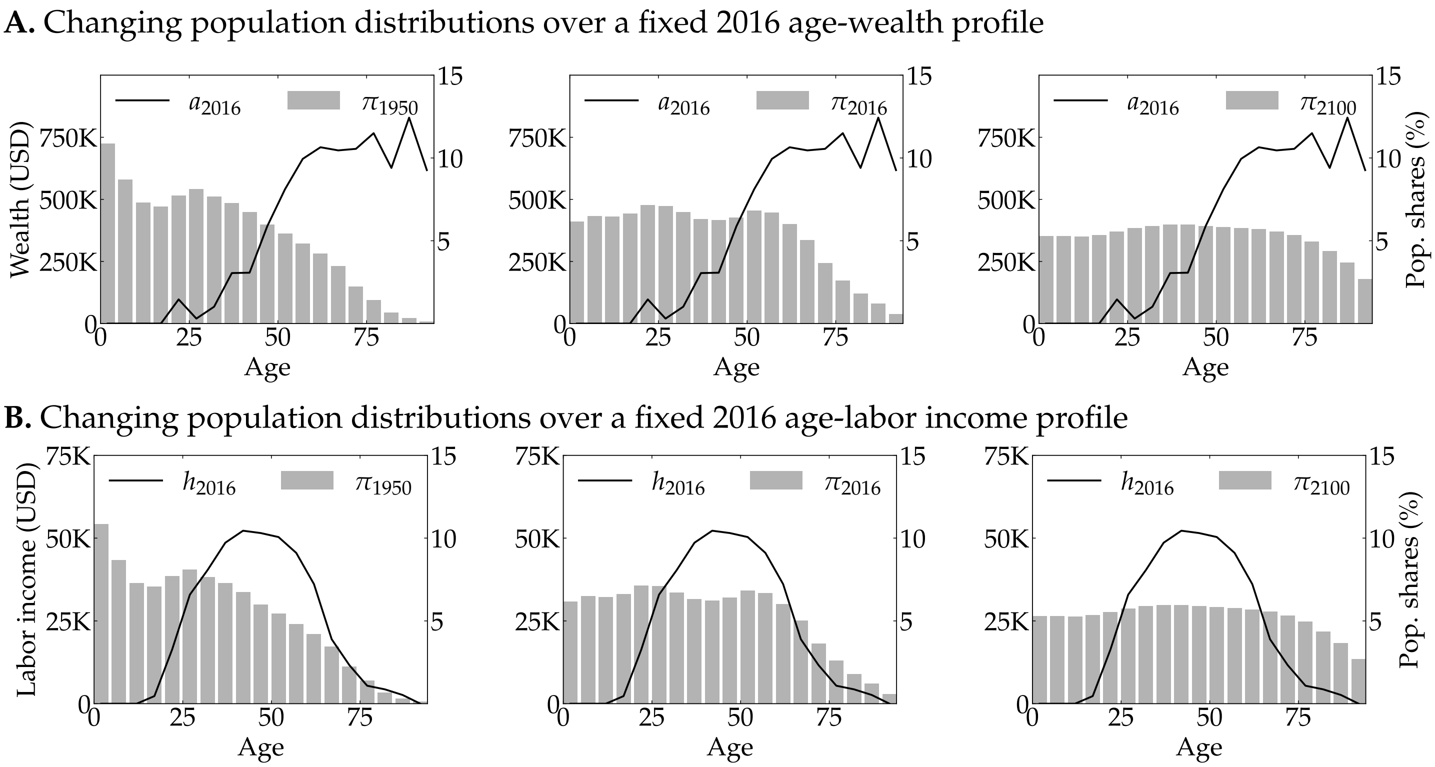

Figure 4: US age-wealth and labor income profiles with population age distributions

Notes: The solid lines in Panel A show the 2016 US age-wealth profiles from the SCF, expressed in current USD. The solid lines in panel B show the 2016 age-income profile from the LIS (CPS), expressed in current USD. Bars represent age distributions: 1950 age distribution in the left panels, 2016 age distribution in the middle panels, and 2100 age distribution in the right panels.

aggregate labor income as more individuals reach old age.

The total compositional effect in (10) can be expressed in terms of a shift in assets and labor supply:

$$\Delta_t^{comp} = \underbrace{\log\left(\frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{j0} a_{j0}}\right)}_{\Delta_t^{comp,a}} + \underbrace{\left(-\log\left(\frac{\sum_j \pi_{jt} h_{j0}}{\sum \pi_{j0} h_{j0}}\right)\right)}_{\Delta_t^{comp,h}}. \tag{18}$$

$\Delta_t^{comp,a}$ is positive if the share of people in high asset ages increases, and $\Delta_t^{comp,h}$ is positive if the share of people in high labor income ages *decreases*. Since old people hold relatively more assets and work relatively less, aging eventually makes both terms positive.

Figure 5 displays the evolution of $\Delta_t^{comp,a}$ and $\Delta_t^{comp,h}$. Panel A shows that $\Delta_t^{comp,a}$ is positive throughout the sample period. The trend flattens towards the end of the 21st century as aging becomes concentrated in very old ages where asset accumulation ceases. However, the trend never reverses, due to the well-known fact that asset decumulation

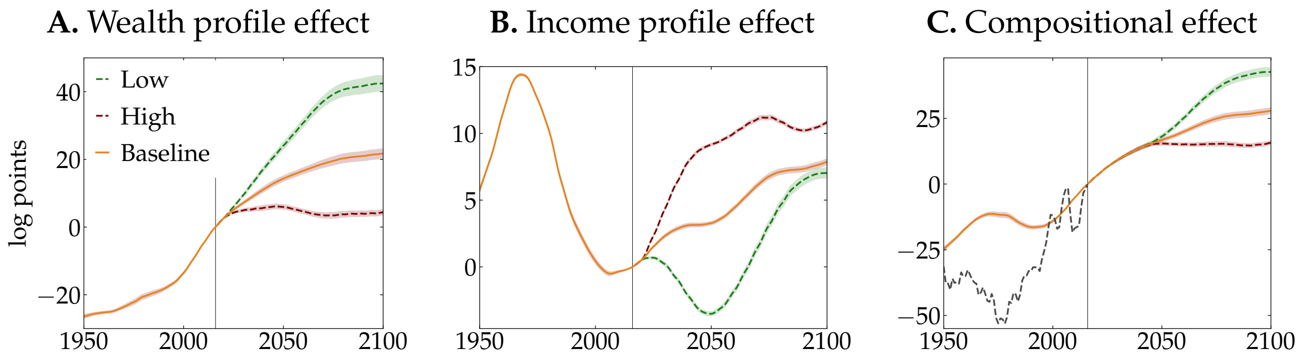

Figure 5: Compositional effects for W and Y: United States 1950-2100

Notes: This figure shows the evolution of the two terms in equation (18). Panel A presents the contribution from the wealth profile, $\Delta_t^{comp,a}$. Panel B presents the contribution from the labor income profile, $\Delta_t^{comp,h}$. Panel C presents the overall compositional effect Δ_t^{comp} from equation (10), which is equal to the sum of panel A and panel B by equation, overlaid with historical data from the WID. In all graphs, the solid orange line corresponds to the baseline fertility scenario, the dashed green to the low fertility scenario, and the dashed red line to the high fertility scenario of the 2019 UN World Population Prospects. A bootstrapped 95% confidence interval is computed by resampling observations 10,000 times with replacement.

in old age is quite limited. A large literature has debated the extent to which this limited decumulation reflects life-cycle forces, late-in-life-risks, or bequest motives (see e.g. Abel 2001, Ameriks and Zeldes 2004, De Nardi, French and Jones 2010, De Nardi, French, Jones and McGee 2023). Our model allows for these forces in a reduced-form way by allowing β_j to vary arbitrarily with age.[28]

Panel B shows $\Delta_t^{comp,h}$ falling between 1970 and 2010 and then increasing throughout the rest of the 21st century, eventually contributing 8 log points to the compositional effect. This non-monotonic pattern is due to the so-called "demographic dividend". The literature on this topic has shown that, as the population distribution moves across the hump-shaped profile of labor earnings, there is initially an output increase followed by a decline (Bloom, Canning and Sevilla 2003; Cutler et al. 1990). Our findings complement this literature by connecting the output effect of demographics to an inverted effect on the wealth-to-GDP ratio. Quantitatively, this effect contributes a quarter of the full increase in Δ_t^{comp} for the United States between 2016 and 2100.

Our results relate to earlier findings by Poterba (2001), who used a shift-share analysis with population projections until 2050 and data from the 1983–1995 waves of the SCF to conclude that $\Delta_t^{comp,a}$ (which he called "projected asset demand") would be stable beyond 2020. He used this result to argue that an asset market meltdown was unlikely. In con-

[28]A high β_j in old age implies that individuals must continue to hold assets in old age to finance high late-in-life expenditures, which we interpret as a reduced form for late-in-life health costs or bequests. When we formally model bequests in section 4, we find that the compositional effect remains dominant.

trast to Poterba, we find a substantial increase in $\Delta_t^{comp,a}$ throughout the remainder of the twenty-first century, reflecting our use of later SCF waves, and, more importantly, population projections with narrower age bins. In addition, Poterba's analysis abstracted from the labor supply term $\Delta_t^{comp,h}$, which we find is not trivial.

For other countries, the logic behind Δ^{comp} is broadly similar to that for the United States. In an online appendix,[29] we reproduce Figures 4 and 5 for all twenty-five countries in our sample. While each country has its own peculiarity—for instance, the timing of the demographic dividend is very uneven—in all of them, aging pushes individuals into higher-asset, lower-income age groups after 2050.

Robustness to base year and construction of age profiles. Our calculations use a single cross section of asset and labor income profiles. This is consistent with the model, where age profiles are stable over time and grow at a constant rate γ. Given this feature, any cross section will imply the same compositional effect, and cross-sectional estimates of a_{j0} and h_{j0} will agree with estimates of age effects from a time-age-cohort decomposition of repeated cross sections, provided that growth loads on time rather than on cohort effects.

In practice, this property is not satisfied exactly: the age profiles of wealth and labor income vary in shape over time. In particular, one worry is that 2016 profiles are not be representative due to transient one-time factors (say, large capital gains since 1980, as in Bauluz and Meyer 2024).

Appendix C.3 examines the robustness of our results to using different base years for labor income and asset profiles. Focusing on the US, we use twelve waves of the LIS (since 1976) for labor income and 21 waves of the SCF (since 1958) for wealth.[30] Calculating Δ^{comp} between 2016 and 2100 for all 252 possible combinations, we find that numbers are similar for all post-1989 waves, and somewhat smaller—between 1/2 and 2/3 as big—for earlier SCF waves, consistent with capital gains having increased elderly wealth in later periods. We also show that our results are robust to using age profiles from a time-age-cohort decomposition as well as to different methods of allocating household wealth to individuals.

3.2 Asset supply and demand semielasticities

We now turn to calculating the semielasticities of asset supply and demand using the formulas in proposition 2 and 4.

[29] Available at http://web.stanford.edu/~aauclert/demowealth21_country_appendix.pdf

[30] Prior to 1989, we use the SCF+ data developed by Kuhn et al. (2020), a reweighted and harmonized version of SCF designed to to maximize compatibility with post-1989 data.

Asset supply semielasticity $\bar{\epsilon}^s$. The global asset supply semielasticity captures the response of the capital-output ratio to the required rate of return. Proposition 2 provides a closed-form solution for this semielasticity,

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \frac{\bar{K}_0}{\bar{W}_0}, \tag{19}$$

showing that $\bar{\epsilon}^s$ is proportional to the initial global capital-wealth ratio $\frac{\bar{K}_0}{\bar{W}_0}$, the inverse of the user cost of capital $r_0 + \delta$, and the elasticity of substitution between capital and labor η. To ensure that asset supply equals total wealth, we define the global value of capital as total wealth net of government bonds, implying $\frac{\bar{K}_0}{\bar{W}_0} = 0.79$.[31] We obtain $r_0 + \delta = 9.5\%$ when calibrating r_0 using the method from appendix A, and taking average depreciation δ of private fixed assets from the BEA Fixed Asset Accounts (see the calibration in section 4 for details). Given these numbers, $\bar{\epsilon}^s$ is between 4.1 and 12.4 for η in a plausible range from 0.5 to 1.5; it is equal to 8.3 with a Cobb-Douglas aggregate production function ($\eta = 1$).

Asset demand semielasticity $\bar{\epsilon}^d$. The semielasticity $\bar{\epsilon}^d$ captures how much global asset accumulation responds to changes in r in the long-run. Proposition 4 expresses $\epsilon^{d,c}$ in each country c as a function of cross-sectional observables, the elasticity of intertemporal substitution (EIS) σ, and capital-labor substitution η. With common σ and η across countries, we obtain

$$\bar{\epsilon}^d = \sigma \cdot \bar{\epsilon}^d_{substitution} + \bar{\epsilon}^d_{income} + (\eta - 1) \cdot \bar{\epsilon}^d_{laborshare}, \tag{20}$$

where bars denote cross-country averages weighted by initial wealth levels.

Implementing the formulas for $\epsilon^{d,c}$ in proposition 4, we obtain $\bar{\epsilon}^d_{substitution} = 43.7$, $\bar{\epsilon}^d_{income} = -0.6$ and $\bar{\epsilon}^d_{laborshare} = 5.8$. The dominant and positive substitution effect means that $\bar{\epsilon}^d$ is positive unless σ is extremely low.[32] When σ has a reasonable value of 0.5 and the production function is Cobb-Douglas, $\bar{\epsilon}^d = 21.2$. This means that a decrease in r by one percentage point would reduce households' desired wealth relative to GDP by around 21%.

The formulas in proposition 4 rely on the distribution of consumption and wealth across different age groups j. We construct these distributions by weighting the wealth

[31] Implicitly, the assumption is that any deviation between measured assets and wealth reflects undermeasured capital. Another way to explain such differences is to have land or capitalized markups. This case is discussed in appendix section D.2.

[32] For example, with a Cobb-Douglas production function, $\bar{\epsilon}^d$ is positive as long as σ exceeds 0.014.

and consumption profiles by the long-run (2100) population distribution. The wealth profiles by age are obtained as in section 3, and the consumption profiles are backed out from the household budget constraint (1) given the age profiles of wealth and income.[33]

To get a sense for the drivers of income and substitution effects, equation (16) is helpful.[34] The income effect is approximately the gap between average ages of consumption and asset holdings. Since consumption occurs slightly earlier in life than asset accumulation, there is a small negative income effect. For the substitution effect, we multiply C/W (roughly 1/8) by the variance of the age of consumption. If consumption is distributed uniformly from ages 20 to 85, this variance is approximately $(85-20)^2/12 \approx 352$. This back-of-the-envelope calculation suggests $\bar{\epsilon}^d_{substitution} \approx (85-20)^2/(12 \cdot 8) = 44$, close to our actual result of 43.7.

Comparison to existing empirical estimates. Several studies have examined how asset accumulation responds to capital taxes. Moll et al. (2022) survey this literature and find that ϵ^d ranges from 1.25 to 35.[35] This range aligns with equation (20) for reasonable σ and η values. No empirical estimates are negative, in line with our finding that substitution effects dominate income effects.

3.3 General equilibrium implications

We now combine our compositional effects and semielasticities with propositions 2 and 3 to obtain long-run general equilibrium changes. Here, we define the long run as 2100.

The rate of return and wealth-to-GDP ratios. Proposition 2 shows that long-run changes in the rate of return and average wealth levels are functions of $\bar{\Delta}^{comp}_{LR}$, $\bar{\epsilon}^s$, and $\bar{\epsilon}^d$.

We calculate $\bar{\Delta}^{comp}_{LR} \equiv \sum_c \omega^c \Delta^{comp,c}_{2100} = 31.7$ log points by taking each country's compositional effects until 2100 from section 3.1, averaged using 2016 wealth levels. Equations (19) and (20) in section 3.2 express $\bar{\epsilon}^s$ and $\bar{\epsilon}^d$ in terms of capital-labor substitutability η and the elasticity of intertemporal substitution σ. Our central estimate uses canonical values

[33] We use this indirect procedure, rather than consumption surveys directly, since the latter tend to be less comprehensive than wealth surveys and are not available for all countries in our study. The implied distributions are presented in appendix figure A.7. Note that this procedure effectively counts any bequests as part of consumption.

[34] Our exact implementation uses slightly different equations allowing for $r \neq g$, but in practice $r - g$ is sufficiently small to make equation (16) a good quantitative guide.

[35] This literature includes Kleven and Schultz (2014), Zoutman (2018), Jakobsen, Jakobsen, Kleven and Zucman (2020), and Brülhart, Gruber, Krapf and Schmidheiny (2022). While Moll et al. (2022) focus on $\partial \log W/\partial r$, this should equal our $\epsilon^d = \partial \log(W/Y)/\partial r$ since the underlying micro experiments in this literature (mostly wealth taxes) likely did not create differential changes in Y across groups.

Table 1: Change in world interest rate and wealth-to-GDP

	A. $r_{LR} - r_0$			**B.** $\overline{\Delta_{LR} \log\left(\frac{W}{Y}\right)}$		
		σ			σ	
η	0.25	0.50	1.00	0.25	0.50	1.00
0.60	-2.45	-1.33	-0.69	12.2	6.6	3.4
1.00	-1.71	**-1.07**	-0.62	14.1	**8.9**	5.1
1.25	-1.43	-0.96	-0.58	14.8	9.9	6.0

Notes: This table presents predictions for the change in the total return on wealth (r) and the wealth-weighted log wealth-to-GDP (W/Y) between 2016 ($t = 0$) and 2100 ($t = LR$) using our sufficient statistic methodology. Columns vary the assumption on the elasticity of intertemporal substitution σ, rows vary the assumption on the elasticity of capital-labor substitution η. Central estimates are in bold. r is expressed in percentage points, and wealth in percent ($100 \cdot \log$).

of $\eta = 1$ and $\sigma = 0.5$. Given the uncertainty surrounding the value of these parameters, however, we also consider a collection of lower and higher values. For the EIS, we consider a low value of $\sigma = 0.25$ and a high value of $\sigma = 1$, spanning the range typically considered in the macroeconomics literature (e.g. Havránek 2015). For capital-labor substitution, we consider a low value $\eta = 0.6$ taken from Oberfield and Raval (2021), and a high value $\eta = 1.25$ taken from Karabarbounis and Neiman (2014).

Table 1 presents our results. The left-hand panel shows the changes in the rate of return, calculated using equation (13), while the right-hand panel shows the average change in log wealth-to-GDP in percent, calculated using equation (14).

We find that the equilibrium return r unambiguously falls in response to demographic change, refuting the "great demographic reversal" hypothesis (Goodhart and Pradhan, 2020). This result follows because $\bar{\Delta}_{LR}^{comp}$ and $\bar{\epsilon}^s + \bar{\epsilon}^d$ are both positive for any plausible combination of σ and η. Intuitively, the compositional effect increases net asset demand, and if $\bar{\epsilon}^s + \bar{\epsilon}^d > 0$, then a fall in r is required to equalize the world's supply and demand of assets.[36] In our central scenario, r falls by $31.7/(21.2 + 8.3) = 1.07$ percentage points by the end of the twenty-first century; the fall is larger when σ or η are small, since this limits the responsiveness of asset supply and demand to falling returns.[37]

For our central scenario, average wealth-to-GDP increases by 8.9 log points (corresponding to an increase from 456% to 498% of world GDP). While substantial, this in-

[36]In section 5, we explain why thinking of equilibrium in terms of flows rather than stocks can lead one to miss this conclusion.

[37]Using numerical simulations, Papetti (2021a) presents similar comparative statics. Appendix G.2 shows that the functional form implied by our sufficient statistic formulas fit his results very well. Our calculations also assume that all countries have a zero initial NFA; appendix D.1 relaxes this assumption and finds quantitatively similar results.

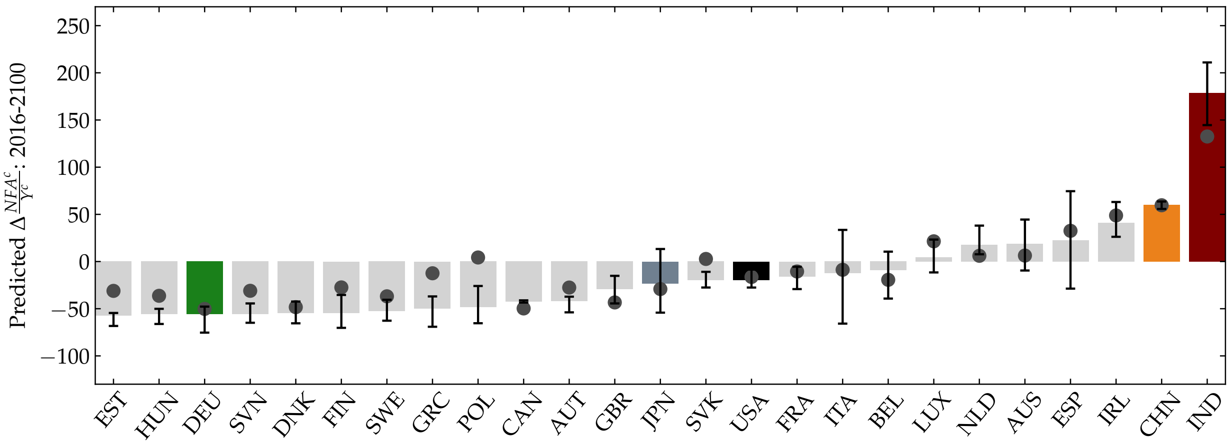

Figure 6: Long-run NFAs under alternative assumptions for σ and η

Notes: This figure presents predictions for $NFAs$ using our sufficient statistic methodology. The solid bars report $\Delta_{LR} NFA^c / Y^c$ calculated by applying equation (15), assuming $\sigma = 0.5$ and $\eta = 1$. The confidence intervals correspond to the maximum and the minimum value obtained from this formula across all possible combinations of σ and η considered in Table 1. The dots correspond to the demeaned compositional effect, $\Delta_{LR}^{comp,c} - \overline{\Delta}_{LR}^{comp}$, the first term in equation (15), which is independent of σ and η.

crease is smaller than the average compositional effect of 31.7 log points, since the equilibrium response from the compositional effect is multiplied by a factor of $\bar{\epsilon}^s / (\bar{\epsilon}^s + \bar{\epsilon}^d) \simeq 1/3$, which is the share of adjustment occurring through increases in investment rather than through reductions in asset accumulation. Intuitively, whenever $\bar{\epsilon}^d > 0$, the general equilibrium response is smaller than the compositional effect, since households accumulate fewer assets as interest rates fall. Wealth responses are larger when investment is elastic relative to accumulation; that is, when η is large relative to σ.

Our finding of sizable but not radical increases in wealth-to-GDP ratios lies between the predictions by Piketty (2014) and Krusell and Smith (2015): Piketty argues that a lower population growth rate will push up W/Y dramatically,[38] while Krusell and Smith argue that the prediction of constant W/Y from the representative-agent model is more consistent with empirical responses of savings rates to changes in the growth rate.

Robustness to risky assets and rents. The results presented so far have assumed the existence of a single, global world interest rate. In appendix D.3, we relax this assumption, instead letting households invest in both "safe" and "risky" assets with different returns, with government bonds making up the supply of world safe assets and capital

[38] Piketty (2014) uses the identity $W/Y = s/g$ with a stable savings rate s. Given a long-run fall in g from 1.5% to 1% induced by slower population growth, this implies a log increase in W/Y of $100 \cdot \log(1.5) = 40$ log points, far larger than our 8.9 log points.

the supply of world risky assets. We show that, for a range of plausible values of the elasticity of the risky asset share to the risk premium, the effect on the average interest rate remains quantitatively close to the findings in table 1. In this extended model, demographic change pushes up the risk premium, as older households reduce the risk in their portfolios and push up the net demand for safe assets (similar to Kopecky and Taylor 2022). This effect, however, is of only moderate size, with the risk premium rising by 0.37pp in our central case, and it is largely orthogonal to the effect on the average return, which now falls by 1.12pp rather than 1.07pp.

Our results also assume that capital is the only privately supplied asset, with no role for capitalized rents. Appendix D.2 allows for rents originating with either markups or land, but finds little change in our central predictions. While capitalizing rents raises the semielasticity of asset supply to returns, this is roughly offset by lower population growth reducing asset supply through a lower value of future rents.

Global imbalances. Next, we turn to the evolution of net foreign asset positions. Figure 6 shows the changes between 2016 and 2100 predicted by the formula in proposition 3. The bars display the main results, which feature a large divergence in NFA positions, with India experiencing an increase of 179 percentage points, China an increase of 60 percentage points, and Germany a decrease of 56 percentage points.

The divergence in NFAs mainly reflects the substantial heterogeneity in compositional effects found in section 3.1. By proposition 3, this heterogeneity affects global imbalances through the demeaned compositional effects $\Delta_{LR}^{comp,c} - \bar{\Delta}_{LR}^{comp}$, whose direct implications for NFAs (assuming no heterogeneity in ϵ^s and ϵ^d) are plotted as circles in figure 6. The demeaned compositional effects broadly mirror the predicted changes in NFAs.

Compared to the results on r and W/Y, the results on global imbalances are less sensitive to the value of the elasticities η and σ. As proposition 3 shows, semielasticities only affect global imbalances insofar as they differ across countries. Since changing η and σ primarily moves semielasticities in parallel across countries, they have a relatively limited effect on the differences across countries. In the figure, the confidence bands show the minimum and the maximum prediction as η and σ parameters are varied in the range considered above. With a few exceptions, these bands are quite tight.

The importance of demeaned compositional effects suggests a *dynamic* projection for

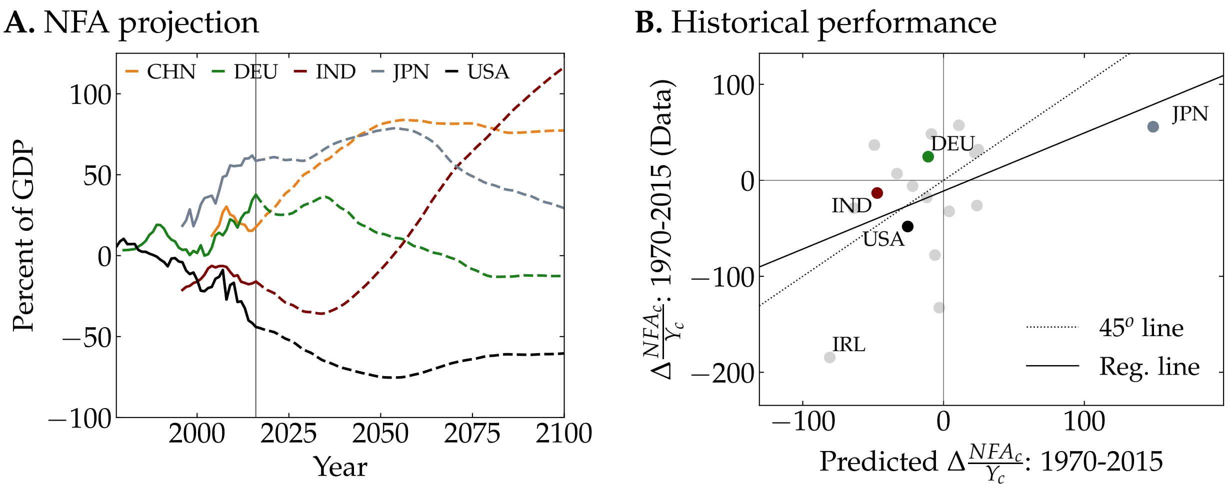

Figure 7: Using the demeaned compositional effect to project NFAs

Notes: Panel A projects NFA-to-GDP ratios between 2016 and 2100 from demeaned compositional effects, using equation (21). Solid lines are historical developments from figure 1. Panel B shows the projection from the demeaned compositional effect (x-axis) against the actual change in NFA between 1970 and 2015 (y-axis) for the 18 countries for which the data is available (source: Lane and Milesi-Ferretti 2017). The dotted line is a 45^o line, the solid line is the ordinary least squares regression line, with slope 0.6.

NFAs that simply uses the demeaned compositional effect at each point in time:[39]

$$\Delta\frac{NFA_t^c}{Y_t^c} \simeq \frac{W_0^c}{Y_0^c}\left(e^{\left(\Delta_t^{comp,c}-\bar{\Delta}_t^{comp}\right)}-1\right) \tag{21}$$

Panel A of figure 7 implements this calculation. The solid lines show global imbalances until today for the five large economies discussed in the introduction, and the dashed lines show the projections from equation (21). In the next few decades, we expect to see a widening of existing global imbalances: China's net foreign assets will rise substantially, while those of the US will decline. Although these trends flatten mid-century, the second half of the 21st century features a conspicuous rise in India's net foreign assets, offset partly by a decline in Germany and Japan, whose demographic transitions at that point are nearly complete. These results arise from the heterogeneity in compositional effects that we documented in section 3.1, with China and India having very large $\Delta_t^{comp,c}$ relative to the world average.

[39] Appendix figure A.8 instead applies equation (15) at each point, taking into account the interest rate adjustment and the heterogeneity in elasticities across countries.

3.4 Historical fit

We now explore how well our framework explains historical patterns in returns, wealth, and NFAs. To do so, we conduct a historical exercise that is symmetric to the forward-looking exercise in the previous section: we take the formulas for changes from the base year 2016 to the "long run" of 2100, but replace 2100 age distributions with the age distributions at time t, for each t from 2016 going back to 1950. Below is a high-level summary, with appendix C.6 providing details.

For r and W/Y, figure A.9 shows that in our central case with $\sigma = 0.5$ and $\eta = 1$, demographics explain half of the historical interest rate decline (112 of 215 basis points) and 15% of the wealth increase (7 of 47 log points) since 1950.[40] With $\sigma = 0.25$ and $\eta = 1.25$, these shares rise to 80% and 22% respectively.

These findings suggest an important role for demographics but also for other forces, especially in determining W/Y. This pattern is consistent with a literature stressing positive shifts in both asset demand (falling productivity growth, inequality) and asset supply (automation, debt, markups) during this era,[41] which all tended to raise W/Y but had offsetting effects on r. Implementing a simple demand-supply accounting framework, we infer that there have been positive shifts in both asset demand and supply, with demographics accounting for 30% of the asset demand shift in our central case.

Next, we compare the NFAs predicted from equation (21) with observed historical changes. Panel B in figure 7 shows the raw correlation between predicted and actual changes in NFAs between 1970 and 2015. For such a simple exercise, the two line up quite well: the regression coefficient is 0.6 and statistically significant, with the theoretically predicted slope of '1' inside the confidence interval.

Given the small number of observations and relatively limited variation in the explanatory variable, however, the correlation is sensitive to two prominent outliers (Japan and Ireland). Further, non-demographic forces have also influenced NFAs historically: in addition to the movements in asset supply and demand already mentioned, there were valuation effects from fluctuations in exchange rates and relative stock market performance.

With these concerns in mind, table A.7 does several variations on the analysis, which appendix C.6 discusses in detail. With alternative specifications and controls, we sometimes lose statistical significance, but almost all point estimates are positive, and '1' always remains inside the confidence interval. This is especially notable in light of the

[40] For wealth, we focus on the subset of countries with available wealth data in 1950: US, UK, France, Germany, the Netherlands, Spain, and Sweden.

[41] See the references in footnote 9.

allocation puzzle (see e.g. Gourinchas and Jeanne 2013), where neoclassical theory fails to predict even the sign of the relationship between productivity growth and NFAs.

4 The compositional effect in a quantitative model

In our sufficient statistic analysis so far, we predicted equilibrium outcomes from a small set of parameters and data moments. The underlying model in section 2 was rich in some respects, but it also abstracted from a number of forces that the literature has found to be important to explain savings: bequest motives, changing mortality, and changes in government taxes, transfers, and retirement policy.

In this section, we extend the baseline model to incorporate these features. We simulate the model numerically, and study how well the sufficient statistic analysis holds up. We find that it remains an excellent guide for predicting changes in rates of return, both qualitatively and quantitatively. The main exception is when the fiscal adjustment in response to an aging population is one-sided, falling either entirely on tax increases or benefit cuts.

4.1 Extending the model

The basic setup is the same as in section 2. Below we outline the main new features, and provide details in appendix E.1.

We continue to omit the country superscript c unless there is a risk of ambiguity. For the production sector, we now assume that F is a CES production function with elasticity η. We make two modifications to the specification of demographics: survival rates ϕ_{jt} can vary over time, and there is an exogenous number of migrants M_{jt} of age j at time t.

To allow for a longer working life, we introduce a time-varying retirement policy. We also introduce bequests governed by non-homothetic preferences, which help explain asset inequality and the limited decumulation of assets at old ages. We remove annuity markets given their limited share in aggregate wealth; individuals now self-insure against mortality risks, with assets remaining at death given as bequests. Last, we assume that there is intergenerational transmission of ability. These are all standard features of quantitative OLG models (e.g. De Nardi 2004). We obtain the following individual problem:

$$\max \mathbb{E}_k \sum_{j=0}^{J} \beta_j \Phi_{jt} \left[\frac{\mathfrak{c}_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \Upsilon Z_t^{\nu-\frac{1}{\sigma}} \left(1-\phi_{jt}\right) \frac{\left(\mathfrak{a}_{jt}\right)^{1-\nu}}{1-\nu} \right] \tag{22}$$

$$\text{s.t.} \quad \mathfrak{c}_{jt} + \mathfrak{a}_{j+1,t+1} \leq w_t \left((1-\tau_t)\ell_{jt}(z_j)(1-\rho_{jt}) + tr_{jt}(z_j) \right) + (1+r_t)[\mathfrak{a}_{j,t} + b^r_{jt}(z_j)] \tag{23}$$

$$\mathfrak{a}_{j+1,t+1} \geq -\bar{a} Z_t.$$

Compared to the setup in section 2, the second term in the utility function captures preferences for bequests. Bequest preferences have curvature $\nu \leq \frac{1}{\sigma}$ to allow for non-homotheticity, and are scaled with a constant Υ, mortality risk $1-\phi_{jt}$, and a term $Z_t^{\nu-\frac{1}{\sigma}}$ that makes this non-homotheticity consistent with balanced growth. In the budget constraint, $b^r_{jt}(z_j)$ denotes bequests received. The factor $\rho_{jt} \in [0,1]$ denotes retirement policy, and specifies how much labor individuals of age j are allowed to supply at time t.

The individual state z_j consists of a permanent component θ, which is Markov across generations, and a transient component ε_j, which is Markov across years, both normalized to have mean 1. Total labor supply is the product of these two components and a deterministic age profile: $\ell_{jt}(z_j) = \theta\varepsilon_j\bar{\ell}_j$. Bequests received $b^r_{jt}(z_j)$ are obtained from pooling all bequests from parents of each type θ, distributing them across ages j in proportion to a fixed factor F_j, and across types θ' in proportion to the intergenerational transition matrix of types $\Pi(\theta'|\theta)$.

For government policy, we assume that transfers reflect the social security system and are given by $tr_{jt}(z_j) = \rho_{jt}\theta d_t$, where d_t denotes the time-varying replacement rate. The government policy consists of a sequence of retirement policies $\{\rho_{jt}\}$ and a fiscal rule that targets an eventually converging sequence of government debt $\{\frac{B_t}{Y_t}\}$, where the debt sequence is obtained by dynamically adjusting replacement rates d_t, taxes τ_t and consumption G_t.

4.2 Asset demand and supply in the extended model

Unlike in the baseline model of section 2, demographic change in our extended model affects individual asset accumulation and labor supply even for a fixed r by generating variation over time in received bequests $b^r_{jt}(\theta)$, survival rates ϕ_{jt}, tax and benefit policy $\{\tau_t, d_t\}$, and retirement policy ρ_{jt}. These changes create non-compositional effects on the wealth-to-GDP ratio, and imply that propositions 2 and 3 no longer hold, since these propositions relied on the compositional effect summarizing all effects of demographics.

However, the asset demand and supply framework underpinning these propositions still applies to the extended model, provided that we replace the compositional effect $\Delta_t^{comp,c}$ with the more general notion of a "small open economy" effect $\Delta_t^{soe,c}$. This effect captures the full shift in the asset demand curve at a fixed interest rate, with $\Delta^{soe} - \Delta^{comp} \neq 0$ indicating non-compositional effects on asset demand.

Proposition 5. *If the wealth holdings of agents start in a steady-state distribution given r_0 and π_0^c, then proposition 2 and 3 hold in the extended model, with $\Delta^{comp,c}$ replaced by $\Delta^{soe,c}$, where $\Delta_t^{soe,c}$ is defined as the change in the wealth-to-GDP ratio between 0 and t in a small open economy equilibrium with a constant rate of return r_0.*

Proof. See appendix E.2. □

Proposition 5 provides a general framework for interpreting the effects of demographics. In appendix G.1, we use this framework to analyze the findings in Eggertsson et al. (2019) (EMR) and Gagnon et al. (2021) (GJLS), two recent papers that find very different effects of demographics on the real interest rate from 1970 to 2015. EMR's higher estimate is explained primarily by a compositional effect that is much larger than we calculate in the data, driven both by a steeper age-wealth profile and by a larger change in the age distribution.[42]

Next, we calibrate our model and interpret the results through the lens of proposition 5.

4.3 Calibration

We calibrate a world economy consisting of the same 25 economies studied in section 3. To obtain parameters for each country, we calibrate a steady-state version of our model to 2016 data. Starting from this steady state, we then simulate the model from 2016 onward given demographic projections.

Steady-state calibration procedure Appendix E.3 spells out the steady-state version of our model, which for the most part is standard.[43] The main calibration parameters and

[42] EMR's lower semielasticities, especially a low ϵ^s, also play some role.

[43] The main non-standard element is a counterfactual flow of migrants, which we introduce to ensure that the steady state implied by the 2016 birth and death rates can exactly match the observed age distribution in 2016. This method is similar to the one used in Penn Wharton Budget Model (2019), and is one way to address a generic problem in the calibration of steady-state demographic models, which is that observed mortality and population shares are generally inconsistent with a stationary population distribution. This adjustment is only needed in the steady state: to simulate the dynamics after 2016, we use the migration flows given in demographic projections.

results are displayed in table 2. For parameters that are common across countries, the "All" column displays the world value. Country-specific parameters have a c-superscript, and the US values are displayed for illustration. Below we summarize the main elements of the calibration, with some supplemental information in appendix E.4.

The real rate of return r is the 2016 value from figure 1 in the introduction, with the calculation described in appendix A. For the wealth-to-GDP ratio W^c/Y^c, we use the same data as in section 3. We use data from the IMF to obtain country-specific debt levels B^c/Y^c and net foreign asset positions NFA^c/Y^c, adjusted to ensure that $\sum_c NFA^c = 0$. The capital-output ratio is obtained residually as $K^c/Y^c = W^c/Y^c - B^c/Y^c - NFA^c/Y^c$.[44]

On the production side, we set the elasticity of substitution between labor and capital to unity, $\eta = 1$. Countries have a common labor-augmenting growth rate γ calibrated to the average growth rate in output per effective labor unit Y_t^c/L_t^c between 2000 and 2016. The common depreciation rate is calibrated from average depreciation of US private fixed assets in 2016, obtained from the BEA fixed asset accounts. Given these parameters, we obtain the investment to output ratio and the labor share in each country from K^c/Y^c, as well as the country-specific growth rate $g^c \equiv (1+n^c)(1+\gamma) - 1$.

For government, we calibrate retirement policy ρ_j^c to match the decline of labor income after the retirement age $J^{r,c}$. Formally, we set $\rho_j^c = 0$ for $j < J^{r,c}$ and $\rho_j^c = 1 - h_j^c/h_{J^{r,c}-1}^c$ for $j \geq J^{r,c}$, where h_j^c is gross labor income as measured in section 3. The retirement age is obtained from the OECD, adjusted to ensure that labor income is not lower than benefit income.[45] We define the income tax rate τ using OECD data on the average tax wedge on personal earnings. Transfers capture the social security system, and satisfy $tr^c(z_j) = \rho_j^c \theta \bar{d}^c$, where we calibrate the replacement rate d^c by targeting country-specific benefit-to-GDP ratios net of taxes from the OECD Social Expenditure Database. Government consumption G^c/Y^c is set to satisfy the government budget constraint, given B^c/Y^c.

For the income process, the deterministic component of labor supply $(1-\rho_j^c)\ell_j^c$ is set proportionally to h_j^c. For the idiosyncratic term z, the log transient component follows an AR(1) process over the life-cycle, and the log permanent component follows an AR(1) process across generations. The parameters of these processes are taken from Auclert and Rognlie (2018) and De Nardi (2004). We assume that the distribution of bequests received across ages F_j is common across countries, and we match it to the age distribution of

[44] Note that the implied K/Y for the US is high relative to standard measures of capital stock. As mentioned in section 3.2, our methodology implicitly assumes that unmeasured capital accounts for this gap. See appendix D.2 for an alternative assumption using markups.

[45] Our main source is the OECD's data on "effective age of labor market exit" from the OECD Pensions at a Glance guide. In seven countries, the age provided by the OECD implies that labor market exit happens after the age at which aggregate labor income falls below implied benefit income. In those cases, we define the latter age as the date of labor market exit. See appendix E.4 for details.

Table 2: Calibration parameters

Parameter	Description	US	All	Source
Demographics				
J^w, J	Initial and terminal ages		$20, 95$	
n^c	Population growth rate	0.5%		UN World Population Prospects
π_j^c	Population distribution			—
ϕ_j^c	Survival probabilities			—
Returns and assets				
r	Real return on wealth		3.8%	Appendix A
W^c/Y^c	Total wealth over GDP	440%		WID
B^c/Y^c	Debt over GDP	107%		IMF
NFA^c/Y^c	Net foreign assets	−36%		IMF
K^c/Y^c	Capital over GDP	369%		$\frac{W^c}{Y^c} - \frac{B^c}{Y^c} - \frac{NFA^c}{Y^c}$
Production side				
I^c/Y^c	Investment over GDP	30%		$\frac{K^c}{Y^c}(\delta + g^c)$
$1 - s_L^c$	Capital share	35%		$(r+\delta)\frac{K^c}{Y^c}$
δ	Depreciation rate		5.70%	$\frac{\delta K}{K}$ (BEA private fixed asset tables)
γ	Technology growth		1.99%	Average 2000-16 growth in $\frac{Y_t}{\sum N_{jt} h_{j0}}$
η	K/L elasticity of subst.		1	Standard
Government policy				
$J^{r,c}$	Retirement age	62		See text
G^c/Y^c	Gov. cons. over GDP	13%		Government budget
$\bar{d}^c$	Social security benefits	92%		Benefits-to-GDP from OECD
τ^c	Labor tax rate	32%		Balanced total budget
Income process				
χ_ϵ	Idiosyncratic persistence		0.91	Auclert and Rognlie (2018)
v_ϵ	Idiosyncratic std. dev.		0.91	Auclert and Rognlie (2018)
χ_θ	Intergenerational persist.		0.677	De Nardi (2004)
v_θ	Intergenerational std. dev.		0.61	De Nardi (2004)
$\underline{a}$	Borrowing limit		0	
Preferences				
σ	EIS		0.5	Standard
$\bar{\beta}^c$	Discount factor	0.9655		See text
ξ^c	Discount factor curvature	0.00071		See text
Υ^c	Bequests scaling factor		78.5	See text
ν	Bequest curvature		1.92	See text

bequests received in the U.S. Survey of Consumer Finances.

The remaining parameters are the elasticity of intertemporal substitution σ, the time preference profile β_j, and the weight and curvature on bequests (Υ, ν). We assume that parameters σ, Υ, and ν are common across countries, and set σ to 0.5, in line with the central case of section 3. To match country-specific age-wealth profiles, we allow the level shifters β_j to vary across countries according to a quadratic formula, $\log \beta_j^c = -j \times \log \bar{\beta}^c + \xi^c (j-40)^2$, where $\xi^c = 0$ corresponds to exponential discounting. To discipline the common ν, we calibrate it jointly with $\bar{\beta}^{US}$, ξ^{US}, and Υ to minimize the squared distance to the US profile of wealth by age and the distribution of bequests, subject to the constraint of precisely matching the US aggregate wealth to GDP ratio.[46] For all other countries, we set $\bar{\beta}^c$ and ξ^c to fit the profile of wealth by age, again subject to the constraint of exactly matching the wealth-to-GDP ratio.

Appendix E.4 presents our calibration outcomes. The model fits the data's labor and wealth profiles well. This implies in particular that the model's long-run compositional effects $\Delta^{comp,c}$ are always within a couple of log points of that of the data (see table A.12).

4.4 Simulations and results

The steady-state calibration pins down the individual parameters, the production parameters, and the initial state of all economies. To study the effect of demographic change, we feed in paths for demographic variables from the UN World Population Prospects for 2016 to 2100, assuming a smooth transition to a long-run world demographic steady state from 2100 onwards.[47] We are interested in how wealth levels, rates of return, and net foreign asset positions evolve, and how this evolution relates to our findings from section 3.

Formally, we assume that the world economy has reached a stationary equilibrium in 2300 and we solve for the transition between 2016 and 2300. We hold preferences and the aggregate production function constant, but let government policy instruments change over time as aging creates fiscal shortfalls that need to be compensated. In our main specification, we assume that the retirement schedule ρ_{jt} in all countries shifts upward by one month per year over the first 60 years of the simulation (in line with CBO's projection for the US), and that the government operates a fiscal rule that keeps the debt-to-output ratio constant by relying equally on tax increases, benefit cuts, and government consumption reductions.

[46] Appendix E.4 reports the bequest distribution against the data from Hurd and Smith (2002).

[47] See appendix E.5 for details about our assumptions for the demographic transition after 2100.

Table 3: Extended model vs. baseline sufficient statistic results: 2016–2100

	Δr	$\overline{\Delta \log \frac{W}{Y}}$	$\bar{\Delta}^{comp}$	$\bar{\Delta}^{soe}$	$\bar{\epsilon}^{d}$	$\bar{\epsilon}^{s}$
Extended model	-1.24	12.3	32.2	33.6	19.7	8.3
Sufficient statistic analysis	-1.07	8.9	31.7	31.7	21.2	8.3
From sufficient statistic to extended model						
+ Drop annuities, add bequests	-1.16	12.8	32.1	32.1	17.1	8.3
+ Adjust bequests received	-1.36	15.2	32.1	42.2	23.2	8.3
+ Add income risk	-1.41	15.5	32.2	38.3	18.2	8.3
+ Change perceived mortality	-1.43	14.6	32.2	40.5	20.7	8.3
+ Increase retirement age	-1.25	12.8	32.2	35.0	20.4	8.3
+ Change taxes and transfers (= extended)	-1.24	12.3	32.2	33.6	19.7	8.3
Alternative fiscal rules						
Only lower expenditures	-1.25	12.8	32.2	35.0	20.4	8.3
Only higher taxes	-0.96	8.3	32.2	23.2	17.4	8.3
Only lower benefits	-1.49	15.4	32.2	42.7	20.7	8.3

Notes: Δr, $\overline{\Delta \log \frac{W}{Y}}$, $\bar{\Delta}^{comp}$, and $\bar{\Delta}^{soe}$ denote the changes in the model simulation between 2016 and 2100, with Δr reported in percentage points and the other three reported in percent ($100 \cdot \log$).

Table 3 reports the simulation results for Δr and $\overline{\Delta \log W/Y}$, together with the corresponding average compositional effect $\bar{\Delta}^{comp}$, the average small open economy effect $\bar{\Delta}^{soe}$, and the average asset demand and supply semielasticities $\bar{\epsilon}^{d}$ and $\bar{\epsilon}^{s}$.[48] We present the results from our extended model on the first line and the sufficient statistic analysis for the same σ and η on the second line as a point of comparison.

Changes in r. The interest rate decline in the extended model is $\Delta r = -1.24$pp, mildly larger than -1.07pp in the sufficient statistic analysis. The larger decline is partly due to the first-order approximation from proposition 5 under-predicting the true change: it gives $\Delta r = -\frac{\bar{\Delta}^{soe}}{\bar{\epsilon}^{d}+\bar{\epsilon}^{s}} = -1.20$pp. The remaining difference reflects the combined effect of $\bar{\Delta}^{soe}$ being slightly larger than the data compositional effect $\bar{\Delta}^{comp}$ (33.6 vs. 31.7), and the extended model having a slightly lower $\bar{\epsilon}^{d}$ (19.7 vs. 21.2).[49]

The subsequent lines decompose the extended model results in terms of successive

[48] Here, $\bar{\Delta}^{comp}$ is calculated as in section 3, and we construct $\bar{\Delta}^{soe}$ by simulating the model for each country given a fixed r_0. For each country, the semielasticities $\epsilon^{d,c}$ and $\epsilon^{s,c}$ are obtained by perturbing r at a small open economy steady state constructed with 2100 demographics, and calculating the effect on steady-state W/Y and K/Y.

[49] The $\bar{\epsilon}^{s}$ is identical since it is a function of external parameters and moments that are targeted in the calibration. The extended model's $\bar{\Delta}^{soe}$ is higher in part because the model has a slightly higher $\bar{\Delta}^{comp}$ than in the data (32.2 vs. 31.7); the two numbers are close because the model targets a least-squares fit to empirical age-wealth profiles, but not identical because the model does not precisely hit those profiles.

modifications of the baseline model that underlies the sufficient statistic results. The first four modifications all make the decline in r larger, while the last two make it smaller.

We first modify the baseline model minimally by removing annuities and introducing bequest utility as in (22), with bequests given at death.[50] Still, all household problem inputs remain fixed at 2016 levels during the transition: bequest amounts stay constant despite rising old-to-young ratios; government maintains fixed taxes, transfers, and retirement ages; and households' savings decisions ignore falling mortality rates.[51] Under these assumptions, $\bar{\Delta}^{comp} = \bar{\Delta}^{soe}$, as assets profiles remain constant for fixed interest rates.

The next line adjusts bequests received to match bequests given throughout the transition, not just in steady state. This significantly raises $\bar{\Delta}^{soe}$ through a "bequest concentration" effect, as bequests are distributed among fewer young people. This change also increases the $\bar{\epsilon}^d$, as savings responses to r persist across generations. Adding income risk on the next line reduces both $\bar{\Delta}^{soe}$ and, more substantially, $\bar{\epsilon}^d$. This occurs because income risk induces a buffer-stock behavior that shortens horizons, leading to faster spend-downs of inheritances and smaller responses to r. These three changes combined result in rates falling more steeply than in the sufficient statistic analysis: -1.41pp versus -1.07pp.

The next step is to have individuals perceive the true declining path of mortality when making their savings decisions, rather than having them assume a fixed 2016 mortality profile. This increases $\bar{\Delta}^{soe}$, since individuals save more for the now-higher probability of old age, but also increases $\bar{\epsilon}^d$, since a longer horizon expands the scope for intertemporal substitution. The net effect is a slightly larger fall in r, at -1.43pp rather than -1.41pp.

The small effect of mortality might be surprising, especially in light of standard life-cycle models, which often find that mortality declines are the primary demographic driver of falling interest rates (Carvalho et al., 2016, Bielecki, Brzoza-Brzezina and Kolasa, 2020, Papetti, 2021a). In part, this is because the mortality effect documented in those papers generally includes the effect of lower mortality increasing the share of old people, which is compositional and already included in our results. It is also, however, because our model has a relatively muted response of savings to longevity.

The standard life-cycle story on mortality and savings is that assets are held to finance consumption during old age—and when old age becomes longer, people hold more assets. But life-cycle models struggle to match the limited decumulation of assets in old age (De Nardi, French and Jones 2016), and when we calibrate our model to hit the empirical

[50] This change includes adopting the extended model's calibration approach, using bequest utility parameters and a quadratic log discount factor path instead of arbitrary age-by-age discount factors as in the baseline.

[51] Formally, we assume that the government confiscates a share of bequests to keep age- and type-specific bequest receipts constant over the transition, adjusting government consumption to balance its budget.

wealth-by-age profile, the calibration leans heavily on bequest motives.[52] Even after the mortality decline, people still die and leave bequests with probability one, and pushing that date later in life does not dramatically change the incentive to save. In appendix E.6, we show that perceived mortality would play a far larger role in a life-cycle model that lacks a bequest motive and does not target the full age-wealth profile.

The next adjustment is a 5-year increase in the retirement age, which pushes out the age at which households stop working, and instead receive benefits, by five years. This shrinks the decline in r substantially, from -1.43pp to -1.25pp. This is because later retirement attenuates the decline in labor supply from an aging population, causing the denominator of wealth-to-GDP to fall by less, so that $\bar{\Delta}^{soe}$ is smaller.

Finally, we let the government raise taxes and cut retirement benefits to close the deficits from population aging, rather than just cutting G, giving us the full extended model. This last step does little to Δr. However, this is not because fiscal adjustment is irrelevant—indeed, a one-sided adjustment can matter a great deal. We show this in the final three lines of the table, where we modify the extended model by assuming that the government closes deficits along the transition entirely through lower consumption G, higher taxes τ or cuts to social security $\bar{d}$, rather than relying on all three equally. The first case is identical to the model two lines above, prior to adjusting taxes and transfers. When adjustment takes place only through higher taxes, however, workers have fewer resources to save, and $\bar{\Delta}^{soe}$ is far smaller, leading to a smaller Δr. When adjustment takes place only through lower social security benefits, workers are forced to save more for retirement, and the opposite happens.[53]

Changes in W/Y. Consistent with the formula $\overline{\Delta \log \frac{W}{Y}} \approx -\bar{\epsilon}^s \Delta r$ from proposition 5, the variation in $\overline{\Delta \log \frac{W}{Y}}$ across the rows of table 3 closely parallels the variation in Δr, showing that the two are driven by similar mechanisms. Similarly, the larger Δr in the model also translates into a larger $\overline{\Delta \log \frac{W}{Y}}$, explaining a third of the difference between the model and sufficient statistic analysis. Most of the remaining difference comes from there being a negative correlation between projected population growth and initial NFAs. Appendix D.1 augments the sufficient statistic analysis to allow for initial NFA positions, and explains how this correlation increases $\overline{\Delta \log \frac{W}{Y}}$, contributing an extra 1.4 log points.

[52] These bequest motives play a similar role to medical expenditures required near the end of life, also emphasized by a large literature including De Nardi et al. (2010).

[53] The importance of fiscal adjustment choices for macroeconomic outcomes has been discussed in the pension reform literature (see, for example, Feldstein 1974, Auerbach and Kotlikoff 1987, and Kitao 2014).

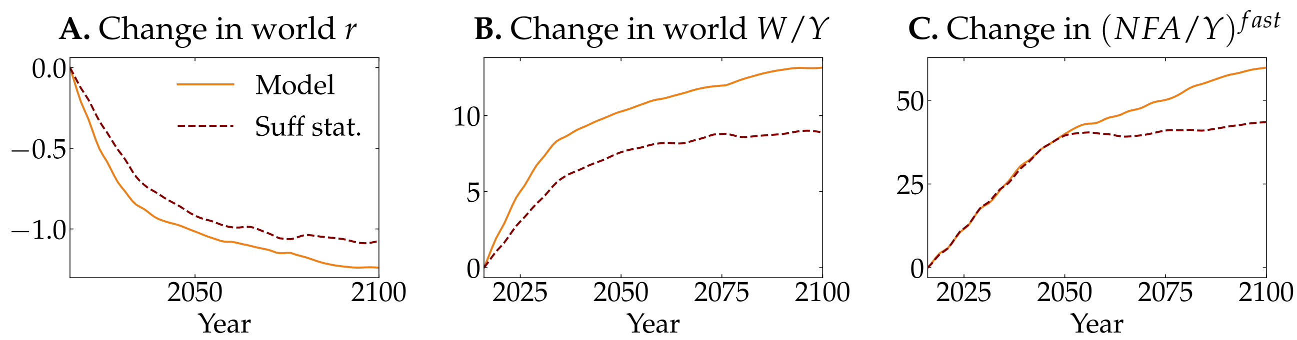

Figure 8: Transition dynamics for rates of return, wealth & NFAs in fast-aging countries

Notes: This figure presents the model change in world interest rate and wealth-to-GDP between 2016 and 2100. Solid lines correspond to simulations from our extended model, dashed lines to the sufficient statistic formulas $\Delta r = -\frac{\bar{\Delta}_t^{comp}}{\bar{\epsilon}^d + \bar{\epsilon}^s}$ and $\frac{W_0}{Y_0}\overline{\Delta \log W/Y} = \frac{W_0}{Y_0}\frac{\bar{\epsilon}^s}{\bar{\epsilon}^d + \bar{\epsilon}^s}\bar{\Delta}_t^{comp}$. The rate of return r is in pp, $\log W/Y$ is in log points, and NFA in percent of GDP.

Transition dynamics and changes to net foreign asset positions. We display the full transition paths of world r and W/Y in figure 8. To test how well the long-run sufficient statistic formulas in propositions 2–3 work at different horizons, we apply them at each date t through 2100, combining the time-varying compositional effects $\bar{\Delta}_t^{comp}$ with the semielasticities $\bar{\epsilon}^d$ and $\bar{\epsilon}^s$, with $\bar{\epsilon}^d$ recalculated using the age distribution at t.

For r and W/Y, the fit is similar throughout the transition: the change in world r is consistently a bit smaller in the sufficient statistic analysis than in the full model, and the change in average W/Y is consistently smaller by a larger proportion, in line with table 3.

Next, panel C illustrates the dynamics of global imbalances by plotting the change in the combined NFA/Y of "fast-aging" countries, defined as those with an above-median compositional effect. The sufficient statistic line is based on the demeaned compositional effect $\Delta_t^{comp,c} - \bar{\Delta}_t^{comp}$, as in figure 7. We see that it captures the first half of the transition in the extended model very well, but it falls behind the model in the later 21st century. Appendix figure A.14 further plots the relationship between compositional and extended model changes in NFA until 2100 across all countries, showing that NFA predictions based on Δ^{comp} are strongly correlated to model NFAs, and that predictions based on demeaned Δ^{soe}, which accounts for non-compositional effects (but not heterogeneity in ϵ^s and ϵ^d), are almost identical to model NFAs.[54]

[54]The extended model's larger global imbalances in the second half of the 21st century arise from a large non-compositional effect $\Delta^{soe} - \Delta^{comp}$ in India, which ages rapidly during that period.

5 Demographic change and savings rates

So far, we have analyzed demographics through the lens of stocks: wealth, capital, and net foreign asset positions. An alternative perspective is to focus on flows: savings, investment, and the current account.

The flow perspective has a long tradition in the literature on aging.[55] One key observation in this literature is that the savings rate is hump-shaped in age, so that as the population continues to age, the aggregate savings rate eventually declines. Observers have made various macroeconomic predictions based on this effect: that aging will raise interest rates (see the Summers quote in the introduction or Lane 2020), decrease standards of living by impairing capital accumulation (Bloom, Canning and Fink 2010), or exert inflationary pressure as the number of consumers increases relative to the number of producers (Goodhart and Pradhan 2020). Using a shift-share on savings, Mian et al. (2021) argued that demographics were unimportant for interest rate trends in post-war US.

These predictions are not borne out in our analysis. Instead, we find that aging unambiguously lowers the real interest rate, thereby increasing capital intensity and output.[56] A lower real interest rate also implies *less* inflationary pressure in any standard model in which this pressure is captured by the natural interest rate.[57]

To unpack this apparent contradiction, we return to our baseline model of section 2. We first show that this model also predicts a negative effect of aging on savings rates going forward, in line with the literature discussed above. To do this, we note that the aggregate net private savings rate in a small open economy satisfies

$$\frac{S_t}{Y_t} \propto \frac{\sum_j \pi_{jt} s_{j0}}{\sum_j \pi_{jt} h_{j0}}, \tag{24}$$

where $s_{jt} = \phi_j a_{j+1,t+1} - a_{j,t}$ is average net personal savings by age at date 0, which equals total net income from capital, labor, and transfers minus consumption (see appendix F for a proof). Equation (24) shows that holding r constant, changes in the aggregate savings rate are purely determined by compositional forces, just like with wealth-to-GDP.[58] Figure

[55]See, e.g., Summers and Carroll (1987), Auerbach and Kotlikoff (1990), Bosworth et al. (1991), Higgins (1998), Lane (2020), and Mian et al. (2021).

[56]GDP per person may still decline overall if the workforce composition effect overwhelms capital deepening, but this is a separate channel that does not go through savings, as is clear from equation (8).

[57]That is, in a version of our model with nominal rigidities, if monetary policy does not fully accommodate the natural rate decline by lowering the intercept of its Taylor rule, actual inflation will decline.

[58]While we could in principle perform this calculation using measured savings rates by age, we prefer instead to calculate (24) using cross-sectional profiles of assets and income alone. This avoids the amplifi-

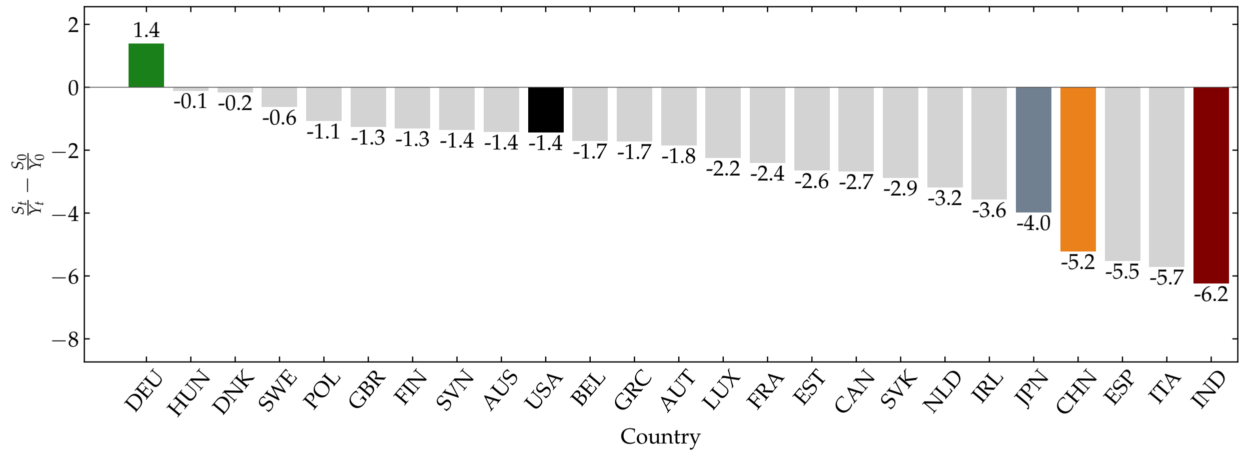

Figure 9: Compositional effect on savings, 2016-2100

Notes: Each bar shows the value of the implied change in the savings-to-GDP ratio from the compositional effect between 2016 and 2100 across countries, reported in pp. Appendix F details the calculation.

9 shows the resulting compositional effects on savings rates through 2100. These are indeed negative in all countries except for Germany.[59]

In equilibrium, net saving in any period must equal the increase in the stock of assets. On a balanced growth path where K, B, and Y all grow at rate g, we therefore have

$$\frac{S}{Y} = g\frac{K}{Y} + g\frac{B}{Y}, \tag{25}$$

where the left is net savings, and the right represents net investment gK/Y and net public borrowing gB/Y (see appendix F for a formal derivation). Panel A of figure 10 depicts this relationship. The compositional effect from figure 9 implies a leftward shift in the savings curve. At first glance, this might seem to imply an increase in r, as represented by the hollow circle. However, since demographic change lowers the population growth rate and therefore g, the other curve also shifts left, and the overall effect is a decline in r. This effect is missed by Summers, Lane (2020), and Mian et al. (2021).

To understand this result, it is helpful to consider panel B, which depicts the same situation in the space of stocks. Here, only the asset demand curve shifts—to the right—and the unambiguous implication is a decline in r.[60] But the curves in panel A are identical to

cation of measurement error that stems from taking the difference between two large quantities, disposable income and consumption, that are themselves observed with error.

[59]Figure A.16 shows, however, that in many countries, the effect was positive prior to 2016. This gives some support to the common view that aging of baby boomers has pushed up savings in recent decades.

[60]For given r, standard neoclassical theory implies that K/Y is not affected by demographic change. In our model, B/Y is also constant. This is subject to debate, but we note that the effect of demographic change on B/Y could take either sign: if lawmakers hold deficits gB/Y constant, B/Y will rise, but if they hold net

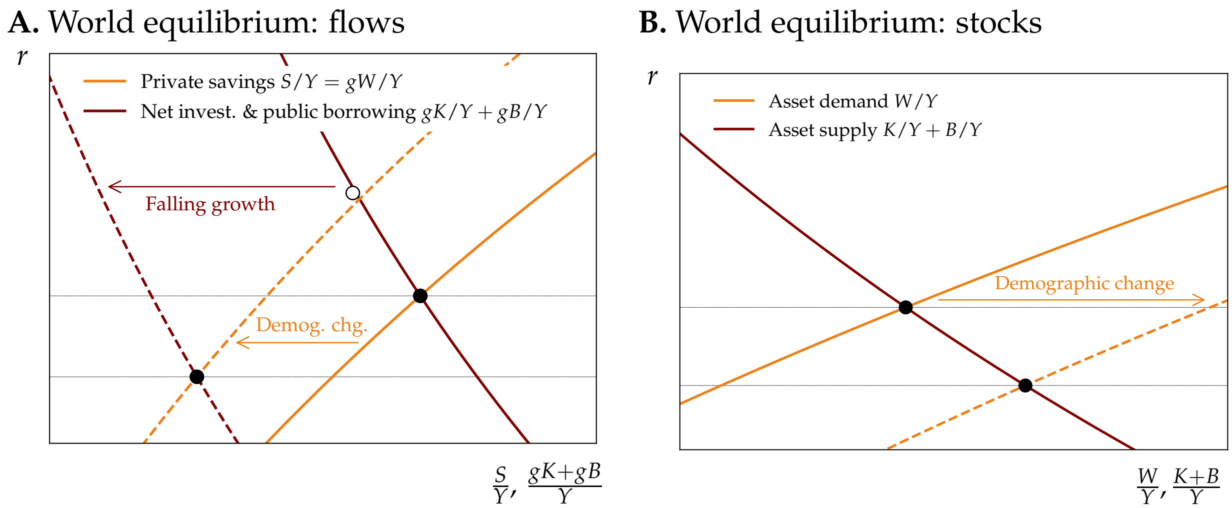

Figure 10: World asset market equilibrium

Notes: This figure represents asset market equilibrium in flow space (panel A) and in stock space (panel B). The growth rate g converts Panel B into Panel A. At given r, demographics increases W/Y and lowers g.

panel B, just both multiplied by g.[61] Hence, although both curves in panel A shift left, the net investment curve shifts left by more, producing the same decline in r as in panel B.[62]

We conclude that the "flow" view of equilibrium in panel A is in principle just as valid as the "stock" view of equilibrium in panel B, but *only* if we remember the effect of g on net investment. Ignoring this effect in the context of demographic change, which can significantly push down long-run g, may give the wrong sign for the change in r. This example illustrates the value of using a fully specified GE model to guide sufficient statistic analysis.

6 Conclusion

We project out the compositional effect of aging on the wealth-to-GDP ratio of 25 countries until the end of the twenty-first century. This effect is positive, large and heterogeneous across countries. According to our model, this will lead to capital deepening everywhere, falling real interest rates, and rising net foreign asset positions in India and China financed by declining asset positions in the United States.

payments $(r-g)B/Y$ constant, it will fall.

[61] Net savings-to-GDP is $S_t/Y_t = (W_{t+1} - W_t)/Y_t$. In steady state, this is $S/Y = g(W/Y)$, since $W_{t+1} = (1+g)W_t$. Similarly, net investment-to-GDP is $g(K/Y)$ and net public borrowing-to-GDP is $g(B/Y)$.

[62] Goodhart and Pradhan (2020) acknowledge that investment can also fall in response to demographics, but argue that savings will fall faster than investment. An important part of their argument is that the labor scarcity caused by aging will trigger a rise in labor-saving investment demand. In our model, the capital-labor ratio does increase in equilibrium, but only because the real interest rate falls.

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Appendix to "Demographics, Wealth and Global Imbalances in the Twenty-First Century"

A Appendix to Section 1

The total return on wealth r_t for the US from 1950–2016 in panel C of figure 1 is constructed as follows. We take:

- Capital K_t as total private fixed assets at current cost from line 1 of Table 2.1 in the BEA's Fixed Assets Accounts (FA).
- Output Y_t as gross domestic product from line 1 of Table 1.1.5 in the BEA's National Income and Product Accounts (NIPA).
- Wealth W_t as "net private wealth" from the World Inequality Database (WID).
- Net foreign assets NFA_t as the net worth of the "rest of the world" sector from line 147 of Table S.9.a in the Integrated Macroeconomic Accounts (IMA).[63]
- Government bonds B_t as gross federal debt held by the public, from the Economic Report of the President (accessed via FRED at FYGFDPUB).
- The safe real interest rate r_t^{safe} as the 10-year constant maturity interest rate—from Federal Reserve release H.15 (accessed via FRED at GS10), extended backward from 1953 to 1950 by splicing with the NBER macrohistory database's yield on long-term US bonds (accessed via FRED at M1333BUSM156NNBR)—minus a slow-moving inflation trend, calculated as the trend component of annual HP-filtered inflation in the PCE deflator, with smoothing parameter $\lambda = 100$.
- Net capital income $(s_K Y - \delta K)_t$ as corporate profits plus net interest and miscellaneous payments of the corporate sector (sum of lines 7 and 8 in NIPA Table 1.13), plus rental income and net interest by households and nonprofit organizations (sum of lines 46, 47, 53, 54), plus imputed net capital income from the noncorporate business sector, under the assumption that the ratio of net capital income to net factor income (line 10 minus line 17) in the noncorporate business sector is the same as the ratio of net capital income (defined above) to net factor income (line 3 minus line 9) in the corporate sector.[64]

We then calculate our baseline total return on wealth series as

$$r_t \equiv \frac{(s_K Y - \delta K)_t + r_t^{safe} B_t}{W_t - NFA_t} \tag{A.1}$$

i.e. as the ratio of net capital income plus real interest income on government debt to domestic assets. This calculation gives the total return on private wealth, excluding changes in asset valuation, under the assumption that the average return on net foreign assets is the same as the average return on private wealth.[65]

[63] This is very similar to the standard net international investment position computed by the BEA, but is chosen because it offers a longer time series.

[64] This imputation is a common way of splitting mixed income within the noncorporate sector between labor and capital, used e.g. by Piketty and Zucman (2014).

[65] This can be seen by rearranging (A.1) as $r_t = \frac{s_K Y - \delta K + r^{safe} B + rNFA}{W}$, which gives the total return r_t on private wealth if r_t equals the return on NFA_t. We take this route because data on capital income from foreign assets is not comparable to domestic data; for instance, the national accounts only measure dividend payments, not the total net capital income, on foreign equities (other than FDI) held in the US, and also only measure nominal rather than real interest payments on bonds. The trend in r_t, however, is not very sensitive to alternative assumptions on the average rate for NFA_t.

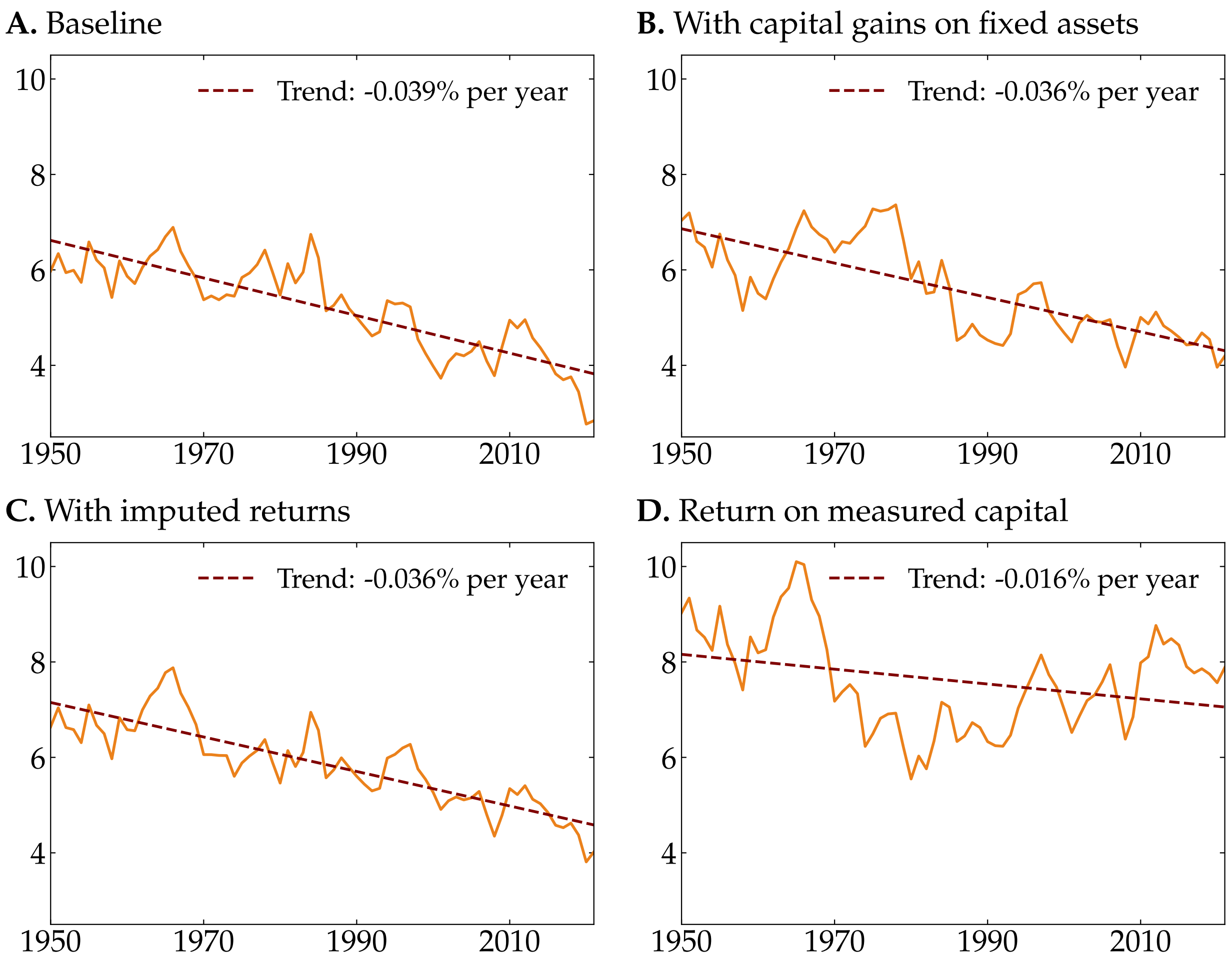


Figure A.1: Alternative ways of constructing the total return on wealth in the US

Notes: Panel A gives our baseline series for the total return on wealth in the US, as described in the text. Panel B adds capital gains on fixed assets, as measured in the fixed assets accounts. Panel C imputes an additional return on unmeasured wealth $W_t - K_t - B_t - NFA_t$ equal to trend growth. Panel D takes our baseline capital income series and divides it by capital measured in the fixed assets accounts.

This baseline r_t and its trend are displayed in panel A of figure A.1. The other three panels provide alternative ways to calculate r_t.

Panel B adds a slow-moving trend of capital good inflation minus PCE inflation, which we denote by π_{Kt}:

$$r_t \equiv \frac{(s_K Y - \delta K)_t + r_t^{safe} B_t + \pi_{Kt} K_t}{W_t - NFA_t}$$

Average inflation of goods in the capital stock is inferred by taking the ratio of changes in the nominal stock (FA Table 2.1, line 1) and changes in the quantity index (FA Table 2.2, line 1), and as with PCE inflation above, we take the slow-moving trend component using the HP filter with $\lambda = 100$. This accounts for expected capital gains on fixed capital (assuming that the expectation follows the trend).

Panel C assumes that there is some unmeasured return on the portion of wealth $W_t - K_t - B_t - NFA_t$ that cannot be accounted for by capital, bonds, or net foreign assets, which it sets equal to the trend real GDP growth rate g_t:

$$r_t \equiv \frac{(s_K Y - \delta K)_t + r_t^{safe} B_t + g_t (W_t - K_t - B_t - NFA_t)}{W_t - NFA_t}$$

where g_t is again calculated using the HP filter with $\lambda = 100$. If $W_t - K_t - B_t - NFA_t$ is the capitalized value of pure rents in the economy, for instance, its value might be expected to grow in line with output.

Finally, panel D simply divides net capital income by the measured capital stock:

$$r_t \equiv \frac{(s_K Y - \delta K)_t}{K_t}$$

Note that despite these alternative constructions, the 1950–2016 trends in panels A, B, and C of figure A.1 are almost identical: -.039, -.036, and -.036 percentage points, respectively. All show a steady decline.

The return on capital in panel D, on the other hand, is quite different: it has a smaller long-term trend decline, of -.016 percentage points per year, and since roughly 1980 it actually displays a mild increase. This post-1980 pattern of a constant or increasing return on capital has been widely remarked upon in the literature—for instance, Gomme, Ravikumar and Rupert (2011), Farhi and Gourio (2018), Eggertsson et al. (2021), and Marx, Mojon and Velde (2021).

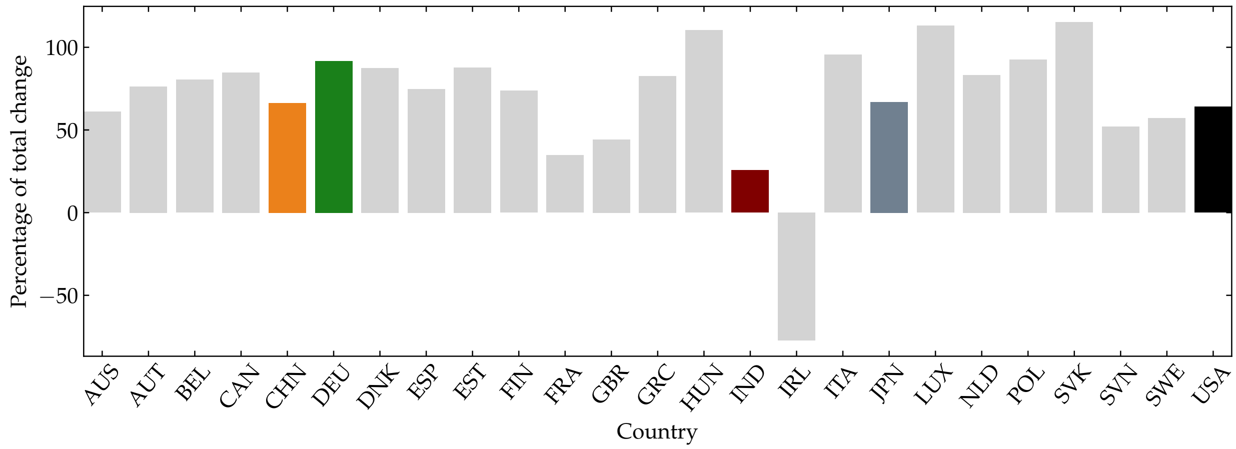
The main source of the disparity between panels A–C and panel D is that the former divides by wealth, while the latter divides only by measured capital. Our choice to use wealth in the denominator stems from our approach to estimating returns, which involves dividing payments to asset holders by the total value of assets. Since it is challenging to precisely separate payments to capital from other non-labor earnings, we include all non-labor net factor payments in the numerator. For consistency, we therefore include all private wealth in the denominator, as this represents the total value of assets generating these returns. Another advantage of using wealth in the denominator is that capital may be imperfectly measured in the fixed assets accounts.

B Appendix to Section 2

B.1 Contribution of changing fertility to aging, 1950-2100

Figure A.2 uses our model of the age distribution of the population in each country to decompose population aging into contributions from fertility, mortality, migration and the so-called momentum effect. Our measure of population aging is the changes in the share of the population aged 50 or above. Denote by $\Delta\pi$ the change in this share between two periods t_0 and t_1. To isolate the role of primitive forces for $\Delta\pi$, we start with an initial age distribution in year t_0. We obtain the contribution of fertility plus "momentum" by simulating the population distribution holding mortality and migration constant until t_1, and then computing the counterfactual change $\Delta^f\pi$ in the share of the 50+ year-old in this scenario. The ratio $\Delta^f\pi/\Delta\pi$ gives us the contribution of fertility and momentum to population aging, which our baseline model of section 2 includes, with the remainder accounted for by mortality and migration, which the baseline model abstracts from. We conduct this exercise over two separate time periods t_0-t_1: 1950-2016 and 2016-2100.

A. 1952-2016 change in the share of 50+ : percentage due to fertility and momentum



B. 2016-2100 change in the share of 50+ : percentage due to fertility and momentum

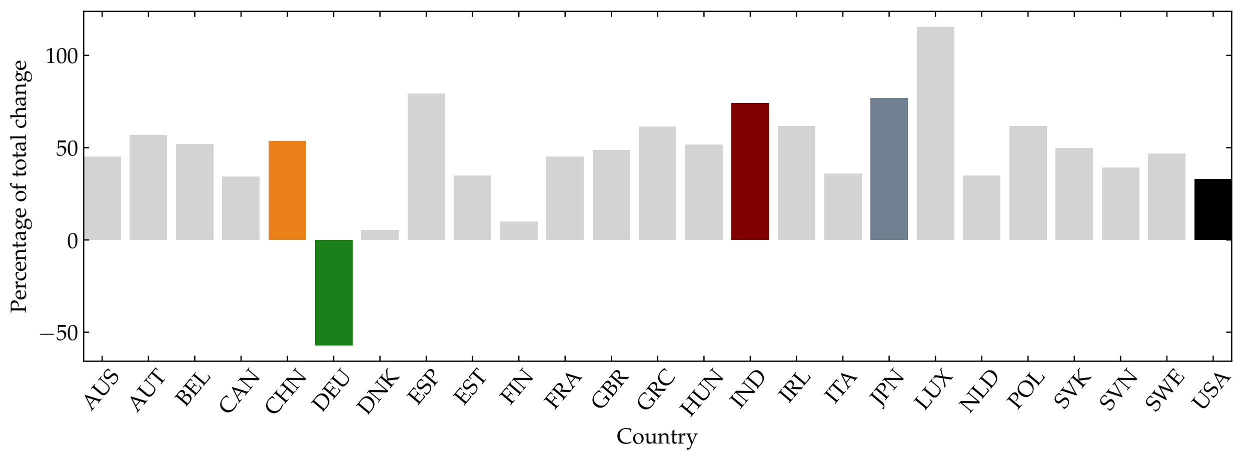


Figure A.2: Contribution of fertility and momentum to population aging

Notes: This figure presents the percentage of the change in the share of 50+ that is due to fertility changes and momentum. It is computed as the ratio between the change in this share under the assumptions of constant mortality rates and migration flows, and under the baseline assumptions for 1952-2016 (panel A) and 2016-2100 (panel B).

Figure A.2 presents the results, showing $\Delta^f \pi/\Delta\pi$ over these two time periods for the 25 countries in our sample. The top panel shows that, between 1950-2016, fertility and momentum contributed an average of 70% of population aging. The bottom panel shows that, between 2016 and 2100, their contributions are projected to shrink a little to an average of 46%, but still constitute the majority of the contribution. Hence, our baseline assumption of fixed mortality and migration is a useful first pass at the data, although decreasing mortality becomes increasingly important to population aging as we look towards the 21st century. Our model of section 4 allows for time variation in mortality and models the savings response to it.

B.2 Proofs of lemma 1 and proposition 1

The ratio K_t/Z_tL_t of capital to effective labor is constant over time, pinned down by constant r and the condition $r_t + \delta = F_K(K_t/(Z_tL_t), 1)$. From the condition $w_t = Z_tF_L(K_t/(Z_tL_t), 1)$, w_t is then proportional to Z_t and grows at the constant rate γ. It follows immediately that average pre-tax labor income $h_{jt} \equiv w_t\bar{\ell}_j = (1+\gamma)^t w_0\bar{\ell}_j$ grows at the constant rate γ.

Letting hats denote normalization of time-subscripted variables by $(1+\gamma)^t$, and defining $\hat{\beta}_j \equiv (1+\gamma)^{j(1-\frac{1}{\sigma})}\beta_j$, the household utility maximization problem (1) becomes

$$\begin{aligned} \max_{\hat{\mathfrak{c}}_{jt}, \hat{\mathfrak{a}}_{j+1,t+1}} \; & \mathbb{E}_k \left[\sum_{j=0}^{J} \hat{\beta}_j \Phi_j \frac{\hat{\mathfrak{c}}_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right] \\ \text{s.t.} \quad \hat{\mathfrak{c}}_{jt} + (1+\gamma)\phi_j \hat{\mathfrak{a}}_{j+1,t+1} & \le w_0 \left((1-\tau)\ell(z_j) + tr(z^j) \right) + (1+r)\hat{\mathfrak{a}}_{jt} \\ \hat{\mathfrak{a}}_{j+1,t+1} & \ge -Z_0\bar{a} \end{aligned} \tag{A.2}$$

This problem is no longer time-dependent: given the same asset holdings $\hat{\mathfrak{a}}_j$, state z^j and age j, households optimally choose the same $(\hat{\mathfrak{c}}_j, \hat{\mathfrak{a}}_{j+1})$ regardless of t. Regardless of their date of birth, every cohort born in this environment will have the same distribution of normalized assets $\hat{\mathfrak{a}}_j$ at each age j. Hence, once t is high enough that all living agents were born in this environment, there exists a balanced-growth distribution of assets at each age that grows at rate γ. Average assets normalized by productivity satisfy $a_{jt}/Z_t = (\mathbb{E}\mathfrak{a}_{jt})/Z_t = (\mathbb{E}\hat{\mathfrak{a}}_j)/Z_0 \equiv a_j(r)$ for some function $a_j(r)$. If, at date 0, already-living agents start with the joint balanced-growth distribution of assets and states, then this holds immediately.

The ratio of aggregate wealth to aggregate labor at time t is

$$\frac{W_t}{L_t} = \frac{\sum_j N_{jt}a_{jt}}{\sum_j N_{jt}\bar{\ell}_j} = \frac{\sum_j N_{jt}(1+\gamma)^t a_{j0}}{\sum_j N_{jt}h_{j0}/w_0} = (1+\gamma)^t w_0 \frac{\sum_j \pi_{jt}a_{j0}}{\sum_j \pi_{jt}h_{j0}} \tag{A.3}$$

The ratio of output to aggregate labor is

$$\frac{Y_t}{L_t} = \frac{F(K_t, Z_tL_t)}{L_t} = Z_tF\left(\frac{K_t}{Z_tL_t}, 1\right) = Z_tF\left(\frac{K_0}{Z_0L_0}, 1\right) \tag{A.4}$$

where we use the fact that the capital-to-effective-labor ratio is constant. Dividing (A.3) and (A.4), the wealth-to-output ratio is

$$\frac{W_t}{Y_t} = \frac{w_0}{Z_0F(K_0/Z_0L_0, 1)} \frac{\sum_j \pi_{jt}a_{j0}}{\sum_j \pi_{jt}h_{j0}} \tag{A.5}$$

where the first factor is constant with time. We conclude that $\frac{W_t}{Y_t}$ grows in proportion to $\frac{\sum_j \pi_{jt}a_{j0}}{\sum_j \pi_{jt}h_{j0}}$.

B.3 Proofs of propositions 2 and 3

Proof of proposition 2. Within each country c, for a constant rate of return r, lemma 1 shows that there exists a balanced-growth distribution of assets normalized by productivity. Assuming we start with this balanced-growth distribution, then at each t, (A.5) implies

$$\begin{aligned}\frac{W_t^c}{Y_t^c} &= \frac{w_0^c}{Z_0^c F^c(K_0^c/Z_0^c L_0^c,1)} \frac{\sum_j \pi_{jt}^c a_{j0}^c}{\sum_j \pi_{jt}^c h_{j0}^c} \\ &= \frac{F_L^c(K_0^c/Z_0^c L_0^c,1)}{F^c(K_0^c/Z_0^c L_0^c,1)} \frac{\sum_j \pi_{jt}^c a_{j0}^c}{\sum_j \pi_{jt}^c h_{j0}^c} \\ &= \frac{F_L^c(k^c(r),1)}{F^c(k^c(r),1)} \frac{\sum_j \pi_{jt}^c a_{j0}^c}{\sum_j \pi_{jt}^c h_{j0}^c} \equiv \frac{W^c}{Y^c}(r,\pi_t^c)\end{aligned}$$

where $\pi_t^c \equiv \{\pi_{jt}^c\}_j$, and $k(r)$ is the capital-to-effective-labor ratio associated with r, defined implicitly by $F_K^c(k(r),1) = r+\delta$.

Each country's share of world GDP is then given by

$$\frac{Y_t^c}{Y_t} = \frac{Z_t^c L_t^c y^c(r)}{\sum Z_t^c L_t^c y^c(r)} = \frac{Z_0^c \nu_t^c y^c(r) \sum \pi_{jt}^c \ell_j^c}{\sum Z_0^c \nu_t^c y^c(r) \sum \pi_{jt}^c \ell_j^c} \equiv \frac{Y^c}{Y}(r,\pi_t,\nu_t),$$

where $\nu_t^c \equiv N_t^c/N_t$ and π_t and ν_t denote vectors across all countries, and $y^c(r) \equiv F^c(k^c(r),1)$.

The capital-to-output ratio in every country can also be written as a function of r, $\frac{K^c}{Y^c}(r) \equiv k^c(r)/F^c(k^c(r),1)$, and we assume that government policy maintains a constant $\frac{B^c}{Y^c}$ in each country.

We assume that the economy is in balanced growth corresponding to long-run r_0 at date 0, which means that the initial wealth-to-output ratio is $\frac{W^c}{Y^c}(r_0,\pi_0^c)$ and that the initial capital-output ratio is $\frac{K^c}{Y^c}(r_0)$. We also assume that net foreign asset positions in each country are 0 at time 0, i.e. that

$$\frac{W^c}{Y^c}(r_0,\pi_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c} = 0.$$

In the long run, π_t^c and ν_t^c converge to constants π_{LR}^c and ν_{LR}^c in each country. Suppose that the real return r_t converges to a long-run value r_{LR}. Then the world asset market clearing condition is

$$0 = \sum_c \frac{Y^c}{Y}(r,\pi,\nu)\left[\frac{W^c}{Y^c}(r,\pi^c) - \frac{K^c}{Y^c}(r) - \frac{B^c}{Y^c}\right] \tag{A.6}$$

which holds for both $(r,\pi,\nu) = (r_0,\pi_0,\nu_0)$ and $(r,\pi,\nu) = (r_{LR},\pi_{LR},\nu_{LR})$. Subtracting the former from the latter, we have

$$\begin{aligned}0 = &\sum_c \frac{Y^c}{Y}(r_{LR},\pi_{LR},\nu_{LR})\left[\frac{W^c}{Y^c}(r_{LR},\pi_{LR}^c)\right.\\ &\left.-\frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c}\right] - \sum_c \frac{Y^c}{Y}(r_0,\pi_0,\nu_0)\left[\frac{W^c}{Y^c}(r_0,\pi_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c}\right] \\ = &\sum_c \left[\frac{Y^c}{Y}(r_{LR},\pi_{LR},\nu_{LR}) - \frac{Y^c}{Y}(r_0,\pi_0,\nu_0)\right]\left[\frac{W^c}{Y^c}(r_{LR},\pi_{LR}^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c}\right] \\ &+ \sum_c \frac{Y^c}{Y}(r_0,\pi_0,\nu_0)\left[\frac{W^c}{Y^c}(r_{LR},\pi_{LR}^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c} - \left(\frac{W^c}{Y^c}(r_0,\pi_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c}\right)\right]\end{aligned}$$

Note that $\frac{W^c}{Y^c}(r_0,\pi_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c}$ is 0 by the assumption of zero initial NFA. To first-order, therefore, the product of $\left[\frac{Y^c}{Y}(r_{LR},\pi_{LR},\nu_{LR}) - \frac{Y^c}{Y}(r_0,\pi_0,\nu_0)\right]$ and $\left[\frac{W^c}{Y^c}(r_{LR},\pi_{LR}^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c}\right]$ is zero as well. To first-order, the above then simplifies to the equivalent

$$0 = \sum_c \frac{Y_0^c}{Y_0} \left[\frac{W^c}{Y^c}(r_{LR}, \pi_{LR}^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c} - \left(\frac{W^c}{Y^c}(r_0, \pi_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c} \right) \right] \tag{A.7}$$

$$= \sum_c \frac{Y_0^c}{Y_0} \left[\frac{W^c}{Y^c}(r_{LR}, \pi_{LR}^c) - \frac{W^c}{Y^c}(r_0, \pi_{LR}^c) + \frac{W^c}{Y^c}(r_0, \pi_{LR}^c) - \frac{W^c}{Y^c}(r_0, \pi_0^c) - \left(\frac{K^c}{Y^c}(r_{LR}) - \frac{K^c}{Y^c}(r_0) \right) \right]$$

$$\simeq \sum_c \frac{Y_0^c}{Y_0} \left[\frac{\partial \frac{W^c}{Y^c}(r_0, \pi_{LR}^c)}{\partial r}(r_{LR} - r_0) + \frac{W^c}{Y^c}(r_0, \pi_0^c) \left(\exp(\Delta_{LR}^{comp,c}) - 1 \right) - \frac{\partial \frac{K^c}{Y^c}(r_0)}{\partial r}(r_{LR} - r_0) \right]$$

$$\simeq \sum_c \frac{W_0^c}{Y_0} \left[\frac{\partial \log \frac{W^c}{Y^c}(r_0, \pi_{LR}^c)}{\partial r}(r_{LR} - r_0) + \Delta_{LR}^{comp,c} - \frac{1}{\frac{W^c}{Y^c}(r_0, \pi_0)} \frac{\partial \frac{K^c}{Y^c}(r_0)}{\partial r}(r_{LR} - r_0) \right], \tag{A.8}$$

where we write $\frac{Y_0^c}{Y_0}$ and $\frac{W_0^c}{Y_0}$ to denote $\frac{Y^c}{Y}(r_0, \pi_0, \nu_0)$ and $\frac{W^c}{Y}(r_0, \pi_0^c, \nu_0^c)$.

Let us also define

$$\epsilon^{d,c} \equiv \frac{\partial \log \frac{W^c}{Y^c}(r_0, \pi_{LR}^c)}{\partial r}$$

$$\epsilon^{s,c} \equiv -\frac{1}{\frac{W^c}{Y^c}(r_0, \pi_0^c)} \frac{\partial \frac{K^c}{Y^c}(r_0)}{\partial r}$$

$$\omega^c \equiv \frac{W^c}{W}(r_0, \pi_0, \nu_0)$$

and divide both sides of (A.8) by $\frac{W}{Y}(r_0, \pi_0, \nu_0)$ to obtain the first-order result

$$0 \simeq \sum_c \omega^c \left[\Delta_{LR}^{comp,c} + (\epsilon^{d,c} + \epsilon^{s,c})(r_{LR} - r_0) \right]$$

$$= \bar{\Delta}_{LR}^{comp} + (\bar{\epsilon}^d + \bar{\epsilon}^s)(r_{LR} - r_0) \tag{A.9}$$

where we let bars denote averages across countries with initial wealth weights ω^c. The equations (12) and (13) are rearrangements of (A.9).

Now, the change in W^c/Y^c in each country can be written to first-order as

$$\Delta_{LR} \log \left(\frac{W^c}{Y^c} \right) = \Delta_{LR}^{comp,c} + \epsilon^{d,c}(r_{LR} - r_0)$$

Summing up both sides with weights ω^c, this becomes

$$\overline{\Delta_{LR} \log \left(\frac{W^c}{Y^c} \right)} = \bar{\Delta}_{LR}^{comp} + \bar{\epsilon}^d (r_{LR} - r_0) \tag{A.10}$$

and using (A.9) to substitute out for $r_{LR} - r_0$, we obtain (14),

$$\overline{\Delta_{LR} \log \left(\frac{W^c}{Y^c} \right)} = \frac{\bar{\epsilon}^s}{\bar{\epsilon}^s + \bar{\epsilon}^d} \bar{\Delta}_{LR}^{comp} \tag{A.11}$$

Proof of proposition 3. The change in $NFA^c/Y^c = W^c/Y^c - K^c/Y^c - B^c/Y^c$ is given by

$$\begin{aligned}
\Delta_{LR}\frac{NFA^c}{Y^c} &= \frac{W_0^c}{Y_0^c}\left[\exp\left(\Delta_{LR}^{comp,c} + (\epsilon^{d,c}+\epsilon^{s,c})(r_{LR}-r_0)\right) - 1\right] \\
&= \frac{W_0^c}{Y_0^c}\left[\exp\left(\Delta_{LR}^{comp,c} - (\epsilon^{d,c}+\epsilon^{s,c})\frac{\bar{\Delta}_{LR}^{comp}}{\bar{\epsilon}^d+\bar{\epsilon}^s}\right) - 1\right] \\
&= \frac{W_0^c}{Y_0^c}\left[\exp\left(\Delta_{LR}^{comp,c} - \bar{\Delta}_{LR}^{comp} - (\epsilon^{d,c}+\epsilon^{s,c} - (\bar{\epsilon}^d+\bar{\epsilon}^s))\frac{\bar{\Delta}_{LR}^{comp}}{\bar{\epsilon}^d+\bar{\epsilon}^s}\right) - 1\right] \\
&= \frac{W_0^c}{Y_0^c}\left[\exp\left(\Delta_{LR}^{comp,c} - \bar{\Delta}_{LR}^{comp} + (\epsilon^{d,c}+\epsilon^{s,c} - (\bar{\epsilon}^d+\bar{\epsilon}^s))(r_{LR}-r_0)\right) - 1\right]
\end{aligned}$$

Rearranged, this gives the desired result, which is

$$\log\left(1 + \left(\Delta_{LR}\frac{NFA^c}{Y^c}\right)\Big/\frac{W_0^c}{Y_0^c}\right) = \Delta_{LR}^{comp,c} - \bar{\Delta}_{LR}^{comp} + (\epsilon^{d,c}+\epsilon^{s,c} - (\bar{\epsilon}^d+\bar{\epsilon}^s))(r_{LR}-r_0)$$

B.4 Alternative fiscal policy responses to demographic change

Our baseline model assumes that governments keep debt-to-output ratios fixed as demographic change unfolds. This section considers the implications of alternative hypotheses.

Change in debt-to-output ratios. Suppose that each country operates a fiscal rule that targets an exogenous sequence $\frac{B_t^c}{Y_t^c}$ which converges to some long-run value $\frac{B_{LR}^c}{Y_{LR}^c}$ in every country. The average change in bonds is a shifter of asset supply, and the new version of (12) is

$$\bar{\Delta}_{LR}^{comp} - \overline{\frac{\Delta_{LR}B^c/Y^c}{W_0^c/Y_0^c}} + \bar{\epsilon}^d(r_{LR}-r_0) \simeq -\bar{\epsilon}^s(r_{LR}-r_0), \tag{A.12}$$

where $\overline{\frac{\Delta_{LR}B^c/Y^c}{W_0^c/Y_0^c}} \equiv \sum_c \omega^c\left(\frac{B_{LR}^c}{Y_{LR}^c} - \frac{B_0^c}{Y_0^c}\right)$ is the average log change in debt-to-output ratios.

We can solve (A.12) to obtain $r_{LR} - r_0$, which is simply the original formula with this shifter in supply subtracted from the compositional effect:

$$r_{LR} - r_0 = \frac{\bar{\Delta}_{LR}^{comp} - \overline{\frac{\Delta_{LR}B^c/Y^c}{W_0^c/Y_0^c}}}{\bar{\epsilon}^d+\bar{\epsilon}^s} \tag{A.13}$$

The average change in wealth-to-GDP now becomes

$$\overline{\Delta_{LR}\log\frac{W^c}{Y^c}} \simeq \frac{\bar{\epsilon}^s}{\bar{\epsilon}^d+\bar{\epsilon}^s}\bar{\Delta}_{LR}^{comp} + \frac{\bar{\epsilon}^d}{\bar{\epsilon}^d+\bar{\epsilon}^s}\overline{\frac{\Delta_{LR}B^c/Y^c}{W_0^c/Y_0^c}} \tag{A.14}$$

which adds the direct impact of increasing debt to (14), and the change in NFA in each country is

$$\begin{aligned}
\log\left(1 + \frac{\Delta_{LR}NFA_{LR}^c}{W_0^c/Y_0^c}\right) \simeq \left(\Delta_{LR}^{comp,c} - \frac{\Delta_{LR}B^c/Y^c}{W_0^c/Y_0^c}\right) - \left(\bar{\Delta}_{LR}^{comp} - \overline{\frac{\Delta_{LR}B^c/Y^c}{W_0^c/Y_0^c}}\right) \\
+ \left(\epsilon^{d,c}+\epsilon^{s,c} - \left(\bar{\epsilon}^d+\bar{\epsilon}^s\right)\right)(r_{LR}-r_0)
\end{aligned} \tag{A.15}$$

which now subtracts the change in asset supply from bonds in each country from the compositional effect on asset demand, but is otherwise the same formula as (15).

Neutralizing debt-to-output policy. Equations (A.13) and (A.15) show that effects of demographics on interest rates and NFAs can be neutralized if governments conduct a debt policy that absorbs the shift in aggregate asset demand. More precisely, if all governments expand debt in line with their compositional effect

$$\Delta_{LR}^{comp,c} = \frac{\Delta_{LR} B^c / Y^c}{W_0^c / Y_0^c}$$

we obtain $r_{LR} - r_0 \simeq 0$ and $\log\left(1 + \frac{\Delta_{LR} NFA_{LR}^c}{W_0^c}\right) \simeq 0$ for every country c. Intuitively, if governments in every country expand debt to perfectly meet the new demand for assets, there is no change in net asset demand, so interest rates stay constant and NFAs do not change. In this case, the change in wealth equals the compositional effect in every country, since there is no general equilibrium feedback reducing the impact of increased asset demand on wealth.

An alternative specification is if each government increases the *level* of its debt-to-output ratio in line with the *average* compositional effect, so that for all c

$$\frac{W}{Y} \bar{\Delta}_{LR}^{comp} = \Delta_{LR} \frac{B^c}{Y^c}$$

In this case, we still have $r_{LR} - r_0 = 0$, but now (A.15) implies that NFAs change in line with the demeaned compositional effect across countries $\Delta^{comp,c} - \bar{\Delta}^{comp,c}$.

Strikingly, these findings are also true in the transition, not just in the long run. That is, if the sequence of debt holdings satisfies $\frac{\Delta_t B^c / Y^c}{W_0^c / Y_0^c} = \Delta_t^{comp,c}$ for every t, then interest rates and NFAs are constant over time, and the path of wealth-to-output ratios equals the path of the compositional effect. Moreover, if $\frac{\Delta_t B^c / Y^c}{W_0^c / Y_0^c} = \bar{\Delta}^{comp}$, then the interest rate change is zero at every point in time, and NFAs at every time period for each country is the demeaned compositional effect.

B.5 Proof of proposition 4

We start by stating the complete version of the proposition including explict expressions for the ϵ terms in cases with $r \neq g$, and then proceed to the proof. Writing $\hat{r} \equiv \frac{1+r}{1+g} - 1$, we define the present value versions of aggregates: $W^{PV} \equiv \sum_j \frac{\pi_j a_j}{(1+\hat{r})^j}$ and $C^{PV} \equiv \sum_j \frac{\pi_j c_j}{(1+\hat{r})^j}$, and Age_a^{PV} and Age_c^{PV} as random variables having probability masses at j proportional to $\frac{\pi_j a_j}{(1+\hat{r})^j}$ and $\frac{\pi_j c_j}{(1+\hat{r})^j}$ respectively. We then have the following complete version of proposition 4.

Proposition 6. *Consider a small open economy with a steady-state population distribution π. If individuals face no income risk or borrowing constraints, the long-run semielasticity of the steady-state W/Y to the rate of return is given by*

$$\epsilon^d \equiv \frac{\partial \log W/Y}{\partial r} = \sigma \epsilon^d_{substitution} + \epsilon^d_{income} + (\eta - 1)\epsilon^d_{laborshare}. \tag{A.16}$$

When $\hat{r} = 0$, $\epsilon^d_{substitution}$ and ϵ^d_{income} are given by (16). When $\hat{r} \neq 0$,

$$\epsilon^d_{substitution} = \frac{1}{1+r} \frac{C}{(1+g)W} \frac{\mathbb{E}Age_c - \mathbb{E}Age_c^{PV}}{\hat{r}} \tag{A.17}$$

$$\epsilon^d_{income} = \frac{1}{1+g} \frac{\frac{C/C^{PV}}{W/W^{PV}} - 1}{\hat{r}} \tag{A.18}$$

In both cases, $\epsilon^d_{laborshare}$ is given by

$$\epsilon^d_{laborshare} \equiv \frac{(1-s_L)/s_L}{r+\delta}, \qquad s_L \equiv \frac{wL}{Y}. \tag{A.19}$$

The proof proceeds in five steps.

B.5.1 Framework

Dropping idiosyncratic risk and the borrowing constraint, and writing assets and consumption (which are now common to all individuals of the same age at a given time) as a_{jt} and c_{jt} for convenience, the individual problem is

$$\max_{\{c_{jt},a_{j+1,t+1}\}} \sum_{j=0}^{J} \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$
$$\text{s.t. } c_{j,t} + \phi_j a_{j+1,t+1} \leq w_t \left((1-\tau)\ell_j + tr_j \right) + (1+r_t) a_{jt} \tag{A.20}$$

where $t \equiv k + j$ is time. Note that we assume agents start and end the lifecycle with zero assets: $a_{0,t} = 0$ and $\Phi_{J+1} a_{J+1,t} = 0$.

The only way in which time-varying macroeconomic aggregates enter this problem is through the real wage w_t and real interest rate r_t. Suppose that we have a balanced growth path by age with technology growth γ, so that $r_t = r$, $w_t = w(1+\gamma)^t$ for some w, and we can also write $a_{jt} = a_j(1+\gamma)^t$ and $c_{jt} = c_j(1+\gamma)^t$. Then (A.20) becomes

$$\max_{\{c_j,a_{j+1}\}} \sum_{j=0}^{J} \tilde{\beta}_j \Phi_j \frac{c_j^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$
$$\text{s.t. } c_j + \phi_j (1+\gamma) a_{j+1} \leq w y_j + (1+r) a_j \tag{A.21}$$

where we define $\tilde{\beta}_j \equiv \beta_j (1+\gamma)^{j(1-\frac{1}{\sigma})}$ and $y_j \equiv (1-\tau)\ell_j + tr_j$. Again, we have the initial and terminal conditions $a_0 = 0$ and $a_{J+1} = 0$.

B.5.2 Effects on wealth-to-GDP

We are interested in characterizing the semielasticity of steady-state W/Y with respect to steady-state r. Using balanced growth by age and a demographic steady state, we have both $W_t = W(1+g)^t$ and $Y_t = Y(1+g)^t$, where $W = \sum_{j=0}^{J} \pi_j a_j$ and $Y = F(k(r), 1) L_0$.

Thanks to linearity of the budget constraint and homotheticity of intertemporal preferences, the entire problem (A.21) scales in w. Hence, if we use W to denote aggregate wealth given the normalization $w = 1$, then for a different w, steady-state wealth will be wW.

We can now write the semielasticity of wealth-to-GDP with respect to r as

$$\frac{\partial \log(w(r)W(r)/Y(r))}{\partial r} = \frac{\partial \log W(r)}{\partial r} + \frac{\partial \log(w(r)/F(k(r),1))}{\partial r} \tag{A.22}$$

where the first term $\frac{\partial \log W(r)}{\partial r}$ is the semielasticity of wealth with respect to r, holding fixed wages at $w = 1$. Note that the second term, the semielasticity of the wage-output ratio with respect to r, will be zero in the Cobb-Douglas case. We will return to this term for the non-Cobb-Douglas case later, and focus on evaluating the first term $\frac{\partial \log W(r)}{\partial r}$ for now.

B.5.3 Budget constraint, Euler equation, and wealth

At the optimum, the budget constraint in (A.21) will hold with equality, and (recalling that we are now using the normalization $w = 1$) can be rewritten as

$$a_{j+1} = \frac{1}{\phi_j} \frac{1}{1+\gamma} \left(y_j - c_j + (1+r) a_j \right)$$

Multiply both sides by the survival probability $\Phi_{j+1} = \phi_j \cdot \Phi_j$ to obtain

$$\Phi_{j+1} a_{j+1} = \frac{1}{1+\gamma} \Phi_j \left(y_j - c_j + (1+r) a_j \right) \tag{A.23}$$

Now, a demographic steady-state implies that $\frac{\pi_{j+1}}{\Phi_{j+1}} = \frac{1}{1+n} \frac{\pi_j}{\Phi_j}$. Multiplying (A.23) by this gives

$$\begin{aligned} \pi_{j+1} a_{j+1} &= \frac{1}{1+g} \pi_j \left(y_j - c_j + (1+r) a_j \right) \\ &= \frac{1}{1+g} \pi_j (y_j - c_j) + \pi_j (1+\hat{r}) a_j \end{aligned} \tag{A.24}$$

where we use the steady-state relationship $1+g = (1+n)(1+\gamma)$ and the definition $1+\hat{r} = \frac{1+r}{1+g}$.

Also using $\Phi_{j+1} = \phi_j \cdot \Phi_j$, the optimization problem (A.21) has the Euler equation $\tilde{\beta}_j c_j^{-1/\sigma} = \tilde{\beta}_{j+1} \frac{1+r}{1+\gamma} c_{j+1}^{-1/\sigma}$. (Note that survival probablities drop out, since they appear symmetrically in the price of an annuity and in preferences.) This can be iterated forward to obtain

$$c_j = \left(\frac{\tilde{\beta}_j}{\tilde{\beta}_0} \left(\frac{1+r}{1+\gamma} \right)^j \right)^\sigma c_0 \tag{A.25}$$

We can solve for the $2J+1$ unknowns $c_0, \ldots, c_J$ and $a_1, \ldots, a_J$ (recalling $a_0 = a_{J+1} = 0$) using $2J+1$ equations, specifically (A.25) for $j = 1, \ldots, J$ and (A.24) for $j = 0, \ldots, J$.

Note that r enters these equations in two places: on the right in (A.24) (inside $1+\hat{r} = \frac{1+r}{1+g}$), and on the right in (A.25). To find the derivative of $\log W$ with respect to r, we will separately perturb r in each of these two places, find the effect on $\log W$, and then sum to find the overall derivative. The part of the derivative from perturbing r inside the Euler equation (A.25) can be thought of as the substitution effect, since it takes into account the effect of intertemporal substitution but ignores the effect of r in the budget constraint, and part from perturbing r inside (A.24) can be thought of as the income effect.

We will consider two cases of increasing complexity: first the special case where steady-state $\hat{r} = 0$ and production is Cobb-Douglas, and then the general case, for which we will also need to evaluate the second term in (A.22).

B.5.4 Special case with $\hat{r} = 0$ and Cobb-Douglas

Substitution effect. Given steady-state $\hat{r} = 0$, we can sum (A.24) from 0 to j to obtain

$$\pi_j a_j = \sum_{k=0}^{j-1} \pi_k \frac{1}{1+g} (y_k - c_k) \tag{A.26}$$

which for $j = J+1$ becomes the lifetime budget constraint

$$0 = \sum_{j=0}^{J} \pi_j \frac{1}{1+g} (y_j - c_j) \tag{A.27}$$

Summing up (A.26), we obtain

$$\begin{aligned} W = \sum_{j=0}^{J} \pi_j a_j &= \sum_{j=0}^{J} \sum_{k=0}^{j-1} \pi_k \frac{1}{1+g} (y_k - c_k) \\ &= \sum_{j=0}^{J} (J-j) \pi_j \frac{1}{1+g} (y_j - c_j) = \sum_{j=0}^{J} \pi_j j \frac{1}{1+g} (c_j - y_j) \end{aligned} \tag{A.28}$$

This simple result states that total wealth is the gap between the ages at which consumption occurs and the ages at which (after-tax-and-transfer) income is earned.[66] The intuition is simple: every year that income is deferred for later consumption requires holding an asset.[67]

Now suppose that we perturb r in (A.25). Log-differentiating gives

$$\frac{dc_j}{c_j} = \sigma j \frac{dr}{1+r} + \frac{dc_0}{c_0} \tag{A.29}$$

and substituting into (A.27) we get

$$0 = \frac{dc_0}{c_0} \sum_{j=0}^{J} \pi_j c_j + \sigma \sum_{j=0}^{J} j \pi_j c_j \frac{dr}{1+r}$$

which we can solve out to obtain

$$\frac{dc_0}{c_0} = -\sigma \frac{\sum_{j=0}^{J} \pi_j j c_j}{\sum_{j=0}^{J} \pi_j c_j} \frac{dr}{1+r}$$

and hence, plugging back into (A.29),

$$\frac{dc_j}{c_j} = \sigma \left(j - \frac{\sum_{k=0}^{J} \pi_k k c_k}{\sum_{k=0}^{J} \pi_k c_k} \right) \frac{dr}{1+r} \tag{A.30}$$

i.e. the proportional change in consumption at a given age j due to the substitution response to an interest rate shock $\frac{dr}{1+r}$ equals the elasticity of intertemporal substitution σ times the difference between age j and the average age of consumption. Here, the schedule of consumption by age rotates counterclockwise around the average age of consumption: in response to a rising r, individuals substitute so that their consumption increases at high ages and decreases at low ages, increasing by more as we get further from the average age.

Plugging (A.30) into (A.28) gives

$$\begin{aligned} dW &= \sigma \frac{1}{1+g} \sum_{j=0}^{J} \pi_j j c_j \left(j - \frac{\sum_k \pi_k k c_k}{\sum_k \pi_k c_k} \right) \frac{dr}{1+r} \\ &= \sigma \frac{1}{1+g} \sum_{j=0}^{J} \pi_j c_j \left(j - \frac{\sum_k \pi_k k c_k}{\sum_k \pi_k c_k} \right)^2 \frac{dr}{1+r} \end{aligned} \tag{A.31}$$

where in the second step we use the fact that $\sum_{j=0}^{J} \pi_j c_j \left(j - \frac{\sum_k \pi_k k c_k}{\sum_k \pi_k c_k} \right) = 0$. Finally, dividing both sides of (A.31) by W and multiplying and dividing the right by $C = \sum_{j=0}^{J} \pi_j c_j$, we get

$$d \log W = \sigma \frac{C}{(1+g)W} \sum_{j=0}^{J} \pi_j c_j \left(j - \frac{\sum_k \pi_k k c_k}{\sum_k \pi_k c_k} \right)^2 \frac{dr}{1+r}$$

Now, if we let Age_c be a random variable distributed across ages j with mass proportional to $\pi_j c_j$, then this

[66] This is multiplied by $1/(1+g)$, since W is incoming wealth, which when normalized by GDP growth is $1/(1+g)$ times smaller than the outgoing wealth from income exceeding consumption in prior periods.

[67] See, for instance, Willis (1988) and Lee (1994).

becomes simply

$$d \log W = \sigma \frac{C}{(1+g)W} \frac{\text{Var}Age_c}{1+r} dr \tag{A.32}$$

which gives us the substitution effect of dr.

Note that $\text{Var}Age_c$, which grows *quadratically* with the dispersion of consumption across ages, appears in (A.32). This reflects two forces. First, from (A.30) we see that when consumption is further from the average age, it changes by proportionally more in response to a change in r. Second, financing higher consumption later in life (and correspondingly lower consumption earlier in life) requires holding assets for longer, leading to a larger effect on aggregate assets. Together, these produce the quadratic effect in (A.32).[68]

Income effect. Write $1+r = (1+r_{ss})(1+\tilde{r})$, so that $1+\hat{r} = (1+\hat{r}_{ss})(1+\tilde{r})$. Substituting this into (A.24) and assuming that $\hat{r}_{ss} = 0$, we get

$$\pi_{j+1}a_{j+1} = \frac{1}{1+g}\pi_j(y_j - c_j) + \pi_j a_j \tilde{r} + \pi_j a_j \tag{A.33}$$

Noting that $a_j\tilde{r}$ enters (A.33) in the same way as $\frac{1}{1+g}(y_j - c_j)$ (i.e. this extra asset income acts as another form of net income), we can redo the same steps to obtain modified versions of (A.27) and (A.28):

$$0 = \sum_{j=0}^{J} \pi_j \left(a_j \tilde{r} + \frac{1}{1+g}(y_j - c_j) \right) \tag{A.34}$$

$$W = \sum_{j=0}^{J} \pi_j j \left(\frac{1}{1+g}(c_j - y_j) - a_j \tilde{r} \right) \tag{A.35}$$

Since the interest rate in the Euler equation (A.25) is unchanged, we must have $dc_j/c_j \equiv \hat{c}$ for some common $\hat{c}$ across all j. Totally differentiating, (A.34) thus becomes

$$\frac{1}{1+g}\hat{c} \sum_{j=0}^{J} \pi_j c_j = d\tilde{r} \sum_{j=0}^{J} \pi_j a_j \tag{A.36}$$

and (A.35) becomes

$$dW = \frac{1}{1+g}\hat{c} \sum_{j=0}^{J} \pi_j j c_j - d\tilde{r} \sum_{j=0}^{J} \pi_j j a_j \tag{A.37}$$

Dividing both sides of (A.37) by (A.36) (and recalling that $W = \sum_{j=0}^{J} \pi_j a_j$), we get

$$\begin{aligned} d \log W &= \left(\frac{\sum_{j=0}^{J} \pi_j j c_j}{\sum_{j=0}^{J} \pi_j c_j} - \frac{\sum_{j=0}^{J} \pi_j j a_j}{\sum_{j=0}^{J} \pi_j a_j} \right) d\tilde{r} \\ &= (\mathbb{E}Age_c - \mathbb{E}Age_a) \frac{dr}{1+r} \end{aligned} \tag{A.38}$$

[68] Although the $1+g$ and $1+r$ factors in the denominator of (A.32) are equal in this $r = g$ special case, we retain them to highlight their separate origin. The $1+r$ originates with (A.29), since $d\log(1+r) = dr/1+r$. Meanwhile, the $1+g$ originates with (A.24), since wealth is measured at the beginning of the period, and yesterday's net saving by a $1/(1+n)$ smaller generation when productivity was $1/(1+\gamma)$ as high translates into normalized beginning-of-period wealth today that is $1/(1+g)$ smaller relative to the normalized savings yesterday. (Of course, both factors will tend to be fairly small.)

where we define the random variable Age_c as before, and analogously Age_a as a variable with mass at each age j proportional to $\pi_j a_j$.

The basic intuition behind (A.38) is the same as in (A.28): total wealth is the gap between the ages at which consumption occurs and the ages at which income is earned. For the income effect, we can think of a rise in r as an increase in income proportional to assets in each period. Consumption will increase proportionally in every period in response to this extra income; this increased consumption will occur, on average, at the same age as existing consumption. The marginal change in wealth is proportional to the gap between the average age of the marginal consumption ($\mathbb{E}Age_c$) and the average age of the marginal income ($\mathbb{E}Age_a$).

Overall special-case result. Evaluating the semielasticity of wealth-to-GDP with respect to r in (A.22), noting that the second term is zero because of the Cobb-Douglas assumption, we combine (A.32) and (A.38) to obtain

$$\underbrace{\sigma \frac{C}{(1+g)W} \frac{\text{Var}Age_c}{1+r}}_{\equiv \epsilon^d_{substitution}} + \underbrace{\frac{\mathbb{E}Age_c - \mathbb{E}Age_a}{1+r}}_{\equiv \epsilon^d_{income}} \tag{A.39}$$

which is $\partial \log W(r)/\partial r$.

B.5.5 General case

Substitution effect. For the case $\hat{r} \neq 0$, sum both sides of (A.24) from $j = 0$ to $j = J$, making use of the boundary conditions $a_{J+1} = 0$ and $a_0 = 0$ and the definition $W = \sum_{j=0}^{J} \pi_j a_j$, to obtain

$$W = \frac{1}{1+g} \pi_j (y_j - c_j) + (1+\hat{r}) W$$

which can be rearranged as

$$W = \frac{1}{\hat{r}} \sum_{j=0}^{J} \pi_j \frac{1}{1+g} (c_j - y_j) \tag{A.40}$$

Applying (A.24), we can obtain the general version of the lifetime budget constraint (A.27)

$$0 = \sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} \frac{1}{1+g} (y_j - c_j) \tag{A.41}$$

The consumption response to a r shock in the Euler equation is still given by (A.29). Substituting into (A.41), we obtain

$$0 = \frac{dc_0}{c_0} \sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} c_j + \frac{dr}{1+r} \sigma \sum_{j=0}^{J} j \pi_j (1+\hat{r})^{-j} c_j$$

which we can solve out to obtain

$$\frac{dc_0}{c_0} = -\sigma \frac{\sum_{j=0}^{J} j \pi_j (1+\hat{r})^{-j} c_j}{\sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} c_j} \frac{dr}{1+r}$$

and hence, plugging back into (A.29),

$$\frac{dc_j}{c_j} = \sigma \left(j - \frac{\sum_{k=0}^{J} k \pi_k (1+\hat{r})^{-j} c_k}{\sum_{k=0}^{J} \pi_k (1+\hat{r})^{-j} c_k} \right) \frac{dr}{1+r} \tag{A.42}$$

which is a slight generalization of (A.30), replacing the average age of consumption $\frac{\sum_{k=0}^{J} k\pi_k c_k}{\sum_{k=0}^{J} \pi_k c_k}$ with the average age in present value terms discounted by $\hat{r}$, $\frac{\sum_{k=0}^{J} k\pi_k (1+\hat{r})^{-j} c_k}{\sum_{k=0}^{J} \pi_k (1+\hat{r})^{-j} c_k}$.

Plugging (A.42) into (A.40), we have

$$\begin{aligned} dW &= \frac{1}{\hat{r}} \sum_{j=0}^{J} \pi_j \frac{1}{1+g} \sigma \left(j - \frac{\sum_{k=0}^{J} k\pi_k (1+\hat{r})^{-j} c_k}{\sum_{k=0}^{J} \pi_k (1+\hat{r})^{-j} c_k} \right) c_j \frac{dr}{1+r} \\ &= \sigma \frac{dr}{1+r} \frac{1}{1+g} \frac{1}{\hat{r}} \left(\sum_{j=0}^{J} j\pi_j c_j - \sum_{j=0}^{J} \pi_j c_j \cdot \frac{\sum_{j=0}^{J} j\pi_j (1+\hat{r})^{-j} c_j}{\sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} c_j} \right) \end{aligned}$$

Dividing both sides by W and multiplying and dividing the right by $C = \sum_{j=0}^{J} \pi_j c_j$, we obtain

$$\begin{aligned} d \log W &= \frac{dr}{1+r} \frac{C}{(1+g)W} \frac{1}{\hat{r}} \left(\frac{\sum_{j=0}^{J} j\pi_j c_j}{\sum_{j=0}^{J} \pi_j c_j} - \frac{\sum_{j=0}^{J} j\pi_j (1+\hat{r})^{-j} c_j}{\sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} c_j} \right) \\ &= \frac{dr}{1+r} \frac{C}{(1+g)W} \frac{\mathbb{E}Age_c - \mathbb{E}Age_c^{PV}}{\hat{r}} \end{aligned} \tag{A.43}$$

where we define Age_c^{PV} as the random variable with probability mass on each age j proportional to $\pi_j (1+\hat{r})^{-j} c_j$.

Income effect. We define $\tilde{r}$ as before, so that $1+r = (1+r_{ss})(1+\tilde{r})$ and $1+\hat{r} = (1+\hat{r}_{ss})(1+\tilde{r})$, and the budget constraint (A.33) becomes

$$\pi_{j+1} a_{j+1} = \frac{1}{1+g} \pi_j (y_j - c_j) + \pi_j (1+\hat{r}) a_j \tilde{r} + (1+\hat{r}) \pi_j a_j \tag{A.44}$$

Since $(1+\hat{r}) a_j \tilde{r}$ enters into the budget constraint the same way as income net of consumption, $\frac{1}{1+g}(y_j - c_j)$, we can write modified versions of (A.41) and (A.40) that incorporate this term:

$$0 = \sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} \left((1+\hat{r}) a_j \tilde{r} + \frac{1}{1+g} (y_j - c_j) \right) \tag{A.45}$$

$$W = \frac{1}{\hat{r}} \sum_{j=0}^{J} \pi_j \left(\frac{1}{1+g} (c_j - y_j) - (1+\hat{r}) a_j \tilde{r} \right) \tag{A.46}$$

Now totally differentiate with respect to $\tilde{r}$. Since we are not perturbing the r in the Euler equation, (A.29) implies that $dc_j / c_j \equiv \hat{c}$ for some common $\hat{c}$ across all j. (A.45) can be solved out to obtain

$$\hat{c} = d\tilde{r} \frac{(1+\hat{r}) \sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} a_j}{\frac{1}{1+g} \sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} c_j}$$

Plugging this into the totally differentiated (A.46), we obtain

$$dW = \frac{1}{\hat{r}}\left(\sum_{j=0}^{J}\frac{1}{1+g}\pi_j c_j \hat{c} - d\tilde{r}(1+\hat{r})\sum_{j=0}^{J}\pi_j a_j\right)$$
$$= d\tilde{r}\left(\left(\sum_{j=0}^{J}\pi_j c_j\right)\frac{(1+\hat{r})\sum_{j=0}^{J}\pi_j(1+\hat{r})^{-j}a_j}{\sum_{j=0}^{J}\pi_j(1+\hat{r})^{-j}c_j} - (1+\hat{r})\sum_{j=0}^{J}\pi_j a_j\right)$$

Dividing both sides by W, this becomes

$$d\log W = \frac{dr}{1+r}(1+\hat{r})\frac{\frac{C/C^{PV}}{W/W^{PV}}-1}{\hat{r}} = \frac{dr}{1+g}\frac{\frac{C/C^{PV}}{W/W^{PV}}-1}{\hat{r}} \tag{A.47}$$

where we identify $C^{PV} \equiv \sum_{j=0}^{J}\pi_j(1+\hat{r})^{-j}c_j$ and $A^{PV} \equiv \sum_{j=0}^{J}\pi_j(1+\hat{r})^{-j}a_j$, and also write $d\tilde{r} = \frac{dr}{1+r}$.

Labor share effect. In the general, non-Cobb-Douglas case, the $\frac{\partial \log(w(r)/F(k(r),1))}{\partial r}$ term in (A.46), which is the semielasticity of the labor share with respect to r, is nonzero.

Normalizing $L = 1$ and letting $s_L \equiv w/F(k,1)$ be the labor share and $1 - s_L \equiv (r+\delta)k/F(k,1)$ be the capital share, we log-differentiate and use the definition of the local elasticity of substitution η to write

$$d\log s_L - d\log(1-s_L) = (1-\eta)\left(d\log w - d\log(r+\delta)\right) \tag{A.48}$$

Since F has constant returns to scale, the log change in output price (zero here, since output is the numeraire) must be the share-weighted log change in input prices, so that

$$s_L d\log w + (1-s_L)d\log(r+\delta) = 0 \tag{A.49}$$

implying that $d\log w = -\frac{1-s_L}{s_L}d\log(r+\delta)$. Using this and other simplifications, we can rewrite (A.48) as

$$\frac{1}{1-s_L}d\log s_L = -(1-\eta)\frac{1}{s_L}\frac{dr}{r+\delta}$$
$$d\log s_L = (\eta-1)\frac{1-s_L}{s_L}\frac{dr}{r+\delta} \tag{A.50}$$

giving us the semielasticity of the labor share.

Overall result. Combining (A.43), (A.47), and (A.50), the semielasticity (A.22) of wealth-to-GDP with respect to r is

$$\underbrace{\sigma\frac{1}{1+r}\frac{C}{(1+g)W}\frac{\mathbb{E}Age_c - \mathbb{E}Age_c^{PV}}{\hat{r}}}_{\equiv\epsilon^d_{substitution}} + \underbrace{\frac{1}{1+g}\frac{\frac{C/C^{PV}}{W/W^{PV}}-1}{\hat{r}}}_{\equiv\epsilon^d_{income}} + (\eta-1)\underbrace{\frac{(1-s_L)/s_L}{r+\delta}}_{\equiv\epsilon^d_{laborshare}} \tag{A.51}$$

which is our main result.

Continuity in the $\hat{r} \to 0$ limit. Taking the limit of $\frac{\mathbb{E}Age_c - \mathbb{E}Age_c^{PV}}{\hat{r}}$ as $\hat{r} \to 0$ using L'Hospital's rule, we get:

$$\lim_{\hat{r}\to 0} \frac{1}{\hat{r}} \left(\frac{\sum_{j=0}^{J} j\pi_j c_j}{\sum_{j=0}^{J} \pi_j c_j} - \frac{\sum_{j=0}^{J} j\pi_j (1+\hat{r})^{-j} c_j}{\sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} c_j} \right) = \mathbb{E}Age_c \left(\frac{\sum_{j=0}^{J} j^2 \pi_j c_j}{\sum_{j=0}^{J} j\pi_j c_j} - \frac{\sum_{j=0}^{J} j\pi_j c_j}{\sum_{j=0}^{J} \pi_j c_j} \right)$$

$$= \mathbb{E}Age_c \left(\frac{\mathbb{E}Age_c^2}{\mathbb{E}Age_c} - \mathbb{E}Age_c \right) = EAge_c^2 - (EAge_c)^2 = \text{Var}Age_c$$

which makes the $\epsilon^d_{substitution}$ term in (A.51) identical to (A.39).

Similarly, taking the limit of $\frac{\frac{C/C^{PV}}{W/W^{PV}} - 1}{\hat{r}}$ as $\hat{r} \to 0$ using L'Hospital's rule, we get:

$$\lim_{\hat{r}\to 0} \frac{1}{\hat{r}} \left(\frac{\sum_{j=0}^{J} \pi_j c_j \Big/ \sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} c_j}{\sum_{j=0}^{J} \pi_j a_j \Big/ \sum_{j=0}^{J} \pi_j (1+\hat{r})^{-j} a_j} - 1 \right) = \frac{\sum_{j=0}^{J} j\pi_j c_j}{\sum_{j=0}^{J} \pi_j c_j} - \frac{\sum_{j=0}^{J} j\pi_j a_j}{\sum_{j=0}^{J} \pi_j a_j} = \mathbb{E}Age_c - \mathbb{E}Age_a$$

which, when also using the fact that $\hat{r} = 0$ implies $1 + g = 1 + r$, makes the ϵ^d_{income} term in (A.51) identical to (A.39).

C Appendix to Section 3

C.1 Data sources

Demographics. Our population data and projections comes from the 2019 UN World Population Prospects.[69] We gather data between 1950 and 2100 on total number of births, number of births by age-group of the mother, population by 5-year age groups, and mortality rates by 5-year age groups. We interpolate to construct population distributions N_{jt} and mortality rates ϕ_{jt} in every country, every year, and for every age. We compute total population as $N_t = \sum_j N_{jt}$, population distributions as $\pi_{jt} = N_{jt}/N_t$, and population growth rates as $1 + n_t = N_t/N_{t-1}$. Finally, we compute the number of migrants by age M_{jt} as the residual of the population law of motion

$$N_{jt} = (N_{j-1,t-1} + M_{j-1,t-1})\phi_{j-1,t-1}.$$

Age-income profiles. We use the LIS to construct the base-year age-income profiles for all the countries we consider. For Australia, the LIS is based on the Survey of Income and Housing (SIH) and the Household Expenditure Survey (HES), for Austria on the Survey on Income and Living Conditions (SILC), for Canada on the Canadian Income Survey (CIS), for China on the Chinese Household Income Survey (CHIP), for Denmark on the Law Model (based on administrative records), for Estonia on the Estonian Social Survey (ESS) and the Survey on Income and Living Conditions (SILC), for Finland on the Income Distribution Survey (IDS) and the Survey on Income and Living Conditions (SILC), for France on the Household Budget Survey (BdF), for Germany on the German Socio-Economic Panel (GSOEP), for Greece on the Survey of Income and Living Conditions (SILC), for Hungary on the Tárki Household Monitor Survey, for India on the India Human Development Survey (IHDS), for Ireland on the Survey on Income and Living Conditions (SILC), for Italy on the Survey of Household Income and Wealth (SHIW), for Japan on the Japan Household Panel Survey (JHPS), for Luxembourg on the Socio-economic Panel "Living in Luxembourg" (PSELL III) and the Survey on Income and Living Conditions (SILC), for Netherlands on the Survey on Income and Living Conditions (SILC), for Norway on the Household Income Statistics, for Poland on the Household Budget Survey, for Slovakia on the Survey of Income and Living Conditions (SILC), for Slovenia on the Household Budget Survey (HBS), for Estonia on the Survey on Income and Living Conditions

[69] https://population.un.org/wpp/

(SILC), for Sweden on the Household Income Survey (HINK/HEK), and for the United Kingdom on the Family Resources Survey (FRS).

Age-wealth profiles. Our wealth data for the United States comes from the 2016 Survey of Consumer Finances. We gather data from other countries as follows. First, we take data from the Luxembourg Wealth Study (LWS)[70] for Australia in 2016, Canada in 2016, Germany in 2017, United Kingdom in 2017, Italy in 2016, and Sweden in 2005. For Australia the LWS is based on the Survey of Income and Housing (SIH) and the Household Expenditure Survey (HES), for Canada on the Survey of Financial Securities (SFS), for Germany on the German Socio-Economic Panel (GSOEP), for Italy on the Survey of Household Income and Wealth (SHIW), for Sweden on the Household Income Survey (HINK/HEK), and for United Kingdom on the Wealth and Assets Survey (WAS). We rescale the survey weights such that they sum up to the correct number of households according to, respectively, the Australian Bureau of Statistics, Statistics Canada, Statistisches Bundesamt, the Office for National Statistics, the Instituto Nazionale di Statistica, and the United Nations Economic Commission for Europe (UNECE). Next, we use 2014 data from the Household Finance and Consumption Survey (HFCS)[71] for Austria, Belgium, Estonia, Spain, Finland, France, Greece, Hungary, Ireland, Luxembourg, Netherlands, Poland, Slovenia, and Slovakia.. For China, we rely on the 2013 China Household Finance Survey (CHFS).[72] For India, we use the National Sample Survey (NSS).[73] For Japan, we construct a measure of total wealth by age of household head from Table 69 of the 2014 National Survey of Family Income and Expenditure (NFSIE) available on the online portal of Japanese Government Statistics[74]. For Denmark, we use the 2016 data from table "FORMUE11 Wealth by type of property, unit, age, sex and population (2014-2022)" produced by Statistics Denmark.

Treatment of defined benefit pensions in the United States. For the present value of all DB wealth by age, we use estimates provided by Sabelhaus and Volz (2019), and we set the funded share to 37.5% to ensure consistency with the aggregate amount of non-federal funded defined benefit assets in the US economy. We exclude unfunded DB liabilities since they do not affect the level of wealth a_{j0} that goes into asset demand; conceptually, we instead think of unfunded DBs as a future transfer tr_j in the household budget constraint (1). For the same reason, we do not include "social security wealth" in a_{j0} (Sabelhaus and Volz 2022, Catherine, Miller and Sarin 2024).

Aggregation. We cross-check the wealth data aggregated from the household survey with the aggregate wealth-to-GDP ratio provided by the WID or the OECD. Table A.1 provides details on the source of both survey and aggregate data, as well as the wealth-to-GDP ratio computed from the survey, compared to the official statistic.

C.2 Compositional effect in all countries

Figure A.3 redoes figure 2 for all 25 countries in our sample, rather than only the top 5 countries.

C.3 Robustness

In this section, we show that our results are robust to some of our main assumptions behind the calculation of compositional effects. In the interest of space, we focus here on the United States, where we have the most detailed data going back many years.

[70] https://www.lisdatacenter.org/our-data/lws-database/
[71] https://www.ecb.europa.eu/stats/ecb_surveys/hfcs/
[72] http://www.chfsdata.org/
[73] http://microdata.gov.in
[74] https://www.e-stat.go.jp

Table A.1: Wealth-to-GDP ratios from survey data and aggregate data

	Wealth survey data			Aggregate data		
Country	Year	Source	$\frac{W^c}{Y^c}$	Year	Source	$\frac{W^c}{Y^c}$
AUS	2016	LWS	3.87	2016	WID	5.24
AUT	2014	HFCS	2.76	2016	WID	3.87
BEL	2014	HFCS	3.82	2016	WID	5.27
CAN	2016	LWS	6.77	2016	WID	4.61
CHN	2013	CHFS	3.28	2016	WID	4.25
DEU	2017	LWS	2.12	2016	WID	4.36
DNK	2016	SD	2.57	2016	WID	3.67
ESP	2014	HFCS	4.91	2016	WID	5.7
EST	2014	HFCS	2.69	2016	WID	3.14
FIN	2014	HFCS	2.3	2016	WID	2.82
FRA	2014	HFCS	3.16	2016	WID	5.01
GBR	2017	LWS	6.14	2016	WID	5.41
GRC	2014	HFCS	2.5	2016	WID	3.52
HUN	2014	HFCS	1.79	2016	WID	3.22
IND	2013	NSS	4.05	2016	WID	4.84
IRL	2014	HFCS	3.32	2016	WID	2.61
ITA	2016	LWS	3.38	2016	WID	6.02
JPN	2014	NSFIE	5.57	2016	WID	5.05
LUX	2014	HFCS	3.78	2016	WID	2.92
NLD	2014	HFCS	1.77	2016	WID	4.39
POL	2014	HFCS	3.24	2016	WID	1.7
SVK	2014	HFCS	1.74	2016	WID	2.88
SVN	2014	HFCS	3.07	2016	WID	3.29
SWE	2005	LWS	1.97	2016	WID	2.69
USA	2016	SCF	5.04	2016	WID	4.4

Notes: This table summarizes our sources of wealth survey data and aggregate data. Abbreviations are described in the text. The survey-based wealth to GDP ratio W^c/Y^c is computed by aggregating household wealth using survey weights and dividing by GDP per household from the national accounts.

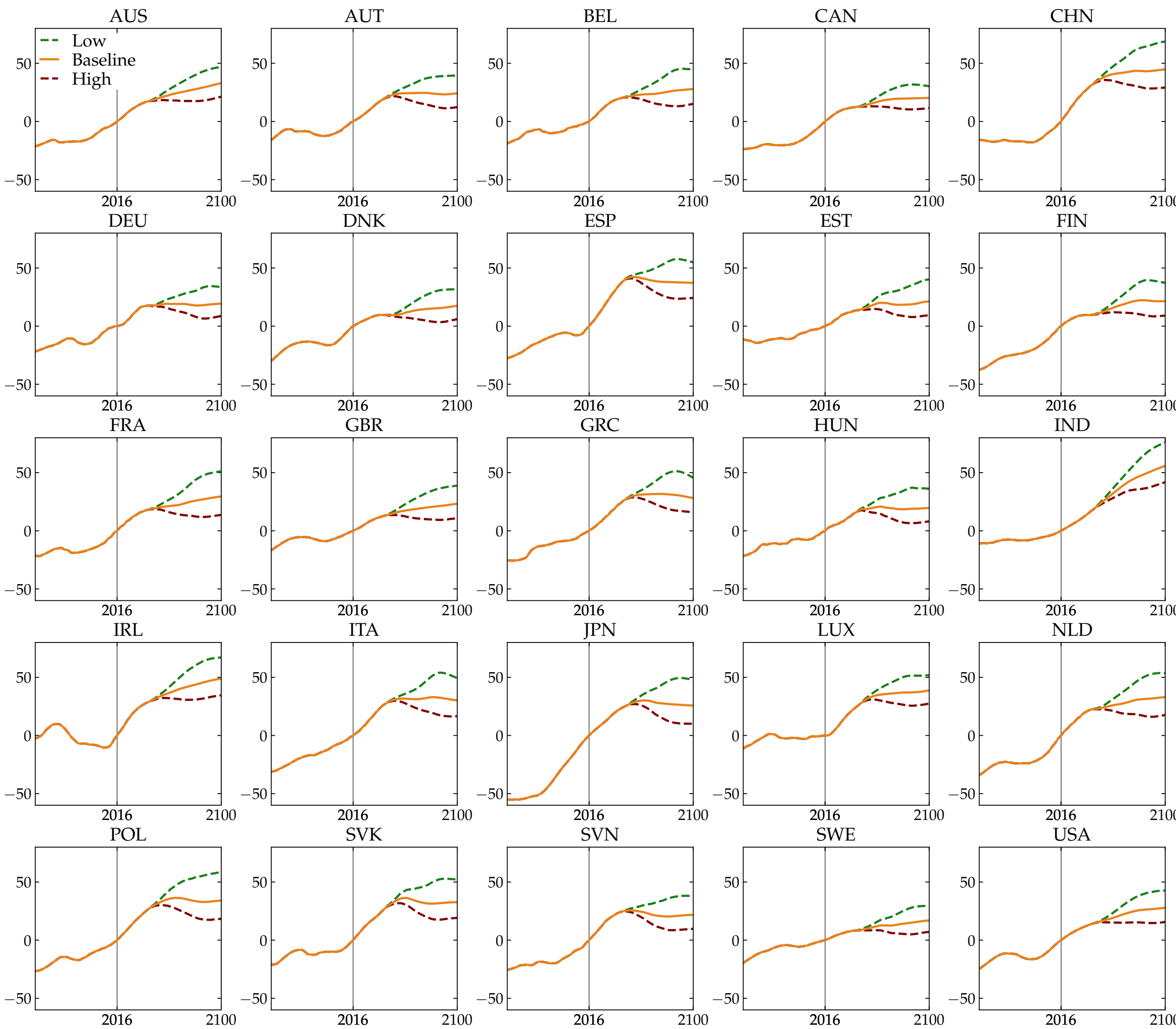


Figure A.3: Compositional effect of demographics, 1950 to 2100 (all countries)

Notes: This figure depicts the evolution of the compositional effect of demographic change on wealth-to-GDP, calculated using equation (10) for t =1950 to 2100, reported in log points (100 log). The base year is 2016 (vertical line). The solid orange line corresponds to the medium fertility scenario from the UN, the dashed green line to the low fertility scenario, and the dashed red line to the high fertility scenario.

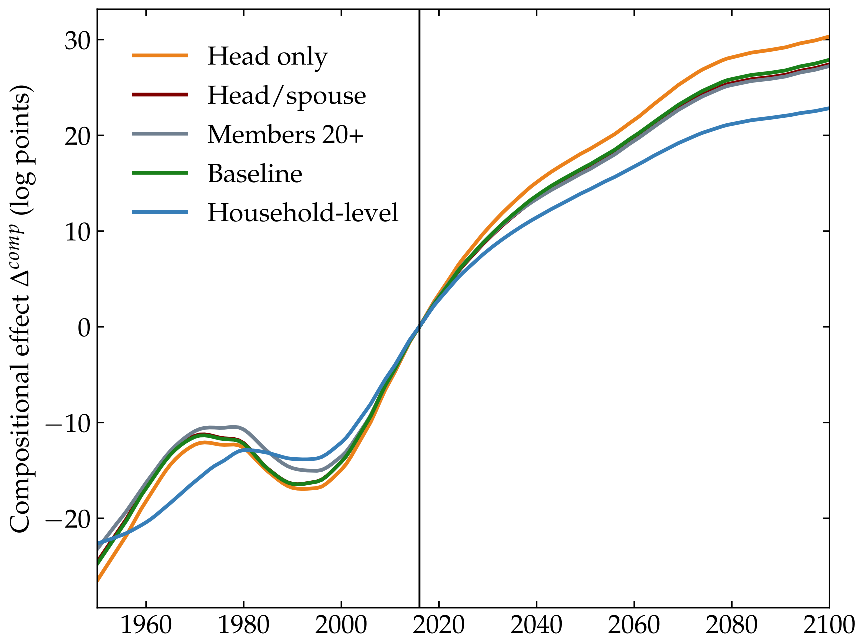


Figure A.4: US compositional effect under alternative assumptions on asset allocation

Notes: This figure depicts the evolution of the compositional effect, calculated using equation (10) from t =1950 to 2100. The orange line corresponds to our baseline case, where the wealth of households is allocated equally to all members at least as old as the head or the spouse. The red line shows the outcome when wealth is allocated to the head of household only, the gray line to the head and the spouse equally, and the green line to all members aged 20 or more. The blue line presents the outcome when the analysis is conducted at the household-level rather than at the individual level.

Alternative allocation of household wealth across individual members. All our surveys measure wealth at the household level. In the main text, we obtain individual wealth by splitting up all assets equally between all members of the household that are at least as old as the head or spouse. The orange line in figure A.4, labeled "baseline", reproduces the projection from the United States under the main fertility scenario (cf figures 2 and 5). The red line shows that allocating all household wealth to the head increases the compositional effect a little, since heads tend to be older on average; the grey line shows that allocating all wealth equally to head as spouse, as in Poterba (2001), or equally to all household members aged 20 or older. This delivers results extremely close to our baseline.

Constructing compositional effects at the household level. All our exercises in the main text of section 3.1, as well as the alternative considered in the previous paragraph, are conducted at the individual level. To gauge the importance of the household vs individual distinction, here we calculate compositional effects at the household level instead.

We first obtain the age-wealth and labor income profiles at the household level, summing the pre-tax labor income of each household member. To convert the age distribution of the population over individuals to an age distribution over households, we use the PSID to estimate a mapping that gives, for each age j, the age of the household head than an average individual of age j lives with.

With these data in hand, we recompute the compositional effect Δ^{comp}. Figure A.4 reports the projected change in W/Y from this exercise under the baseline fertility scenarios. The dashed line reproduces the central individual-level compositional effect from the main text. Overall, the timing of the projected changes in W/Y change slightly, but the overall magnitude remains close.

Table A.2: Sensitivity of US compositional effect Δ^{comp} to choice of base year

Panel A. Compositional effect between 2016 and 2100 (in log points)

	h_j year											
a_j year	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	DH
1989	26.4	26.0	25.7	25.5	25.4	25.1	25.1	24.6	24.2	24.0	23.5	23.4	27.7
1992	22.3	22.0	21.7	21.4	21.4	21.1	21.1	20.6	20.2	19.9	19.5	19.4	23.6
1995	25.4	25.0	24.7	24.5	24.4	24.1	24.1	23.6	23.2	23.0	22.5	22.4	26.7
1998	22.6	22.2	21.9	21.7	21.6	21.3	21.3	20.8	20.4	20.2	19.7	19.6	23.9
2001	22.3	22.0	21.7	21.4	21.4	21.1	21.1	20.6	20.2	19.9	19.5	19.4	23.6
2004	25.5	25.2	24.9	24.6	24.6	24.3	24.3	23.8	23.4	23.1	22.7	22.6	26.8
2007	24.3	24.0	23.7	23.4	23.4	23.1	23.0	22.6	22.2	21.9	21.5	21.4	25.6
2010	28.6	28.2	28.0	27.7	27.6	27.4	27.3	26.9	26.5	26.2	25.7	25.6	29.9
2013	28.1	27.7	27.4	27.2	27.1	26.8	26.8	26.3	25.9	25.7	25.2	25.1	29.4
2016	30.9	30.5	30.2	30.0	29.9	29.6	29.6	29.1	28.7	28.5	28.0	27.9	32.2
DH	28.1	27.8	27.5	27.2	27.2	26.9	26.9	26.4	26.0	25.7	25.3	25.2	29.4

Panel B. Compositional effect between 1950 and 2016 (in log points)

	h_j year											
a_j year	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	DH
1989	22.6	22.4	22.2	21.6	21.1	20.6	21.0	20.1	19.8	19.1	18.9	19.2	29.4
1992	22.6	22.3	22.1	21.6	21.0	20.5	20.9	20.0	19.7	19.0	18.9	19.1	29.3
1995	24.2	23.9	23.7	23.2	22.6	22.1	22.5	21.6	21.3	20.6	20.5	20.7	30.9
1998	23.5	23.2	23.1	22.5	21.9	21.5	21.9	20.9	20.6	19.9	19.8	20.1	30.2
2001	23.7	23.4	23.3	22.7	22.1	21.7	22.1	21.1	20.9	20.1	20.0	20.3	30.5
2004	25.3	25.0	24.8	24.3	23.7	23.2	23.6	22.7	22.4	21.7	21.6	21.8	32.0
2007	24.7	24.4	24.2	23.7	23.1	22.6	23.0	22.1	21.8	21.1	21.0	21.2	31.4
2010	27.8	27.5	27.3	26.8	26.2	25.7	26.1	25.2	24.9	24.2	24.1	24.4	34.5
2013	26.5	26.2	26.1	25.5	25.0	24.5	24.9	24.0	23.7	23.0	22.8	23.1	33.3
2016	28.2	27.9	27.7	27.2	26.6	26.1	26.5	25.6	25.3	24.6	24.5	24.8	34.9
DH	29.2	28.9	28.7	28.2	27.6	27.1	27.5	26.6	26.3	25.6	25.5	25.7	35.9

Notes: This table reports the US compositional effect Δ^{comp} on log W/Y, as defined in equation (10), for alternative base years of the age-wealth and the age-labor income profiles. Panel A considers our main period of interest 2016 to 2100, and panel B considers 1950 to 2016. Every column corresponds to an alternative base year for the age-labor income profile, and every row to an alternative base year for the age-wealth profile. The last row and column correspond to the cases where we use the average age effect from a time-age-cohort decomposition on the 1989–2016 SCF data for a and 1980-2018 annual data from the CPS for h, with all growth loading on time effects (DH, for "Deaton-Hall").

Table A.3: Sensitivity of US compositional effect Δ^{comp} to choice of earlier base year

Panel A. Compositional effect between 2016 and 2100 (in log points)

a_j year	h_j year 1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	DH
1958	21.7	21.3	21.0	20.8	20.7	20.4	20.4	19.9	19.5	19.3	18.8	18.7	23.0
1959	17.0	16.7	16.4	16.1	16.1	15.8	15.8	15.3	14.9	14.6	14.2	14.1	18.3
1960	17.5	17.2	16.9	16.6	16.6	16.3	16.3	15.8	15.4	15.1	14.7	14.6	18.8
1962	17.2	16.9	16.6	16.3	16.3	16.0	15.9	15.5	15.1	14.8	14.4	14.3	18.5
1965	20.1	19.7	19.5	19.2	19.1	18.9	18.8	18.4	18.0	17.7	17.2	17.1	21.4
1967	21.3	21.0	20.7	20.4	20.4	20.1	20.1	19.6	19.2	18.9	18.5	18.4	22.6
1968	19.4	19.1	18.8	18.5	18.5	18.2	18.1	17.7	17.3	17.0	16.6	16.5	20.7
1969	19.7	19.3	19.0	18.8	18.7	18.4	18.4	17.9	17.5	17.3	16.8	16.7	21.0
1970	24.0	23.6	23.3	23.1	23.0	22.7	22.7	22.2	21.8	21.6	21.1	21.0	25.3
1977	11.5	11.2	10.9	10.6	10.6	10.3	10.3	9.8	9.4	9.1	8.7	8.6	12.8
1983	23.9	23.6	23.3	23.0	23.0	22.7	22.7	22.2	21.8	21.5	21.1	21.0	25.2
2016	30.9	30.5	30.2	30.0	29.9	29.6	29.6	29.1	28.7	28.5	28.0	27.9	32.2

Panel B. Compositional effect between 1950 and 2016 (in log points)

a_j year	h_j year 1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	DH
1958	13.5	13.2	13.0	12.5	11.9	11.4	11.8	10.9	10.6	9.9	9.8	10.1	20.2
1959	16.0	15.7	15.5	15.0	14.4	13.9	14.3	13.4	13.1	12.4	12.3	12.5	22.7
1960	17.6	17.3	17.1	16.6	16.0	15.5	15.9	15.0	14.7	14.0	13.9	14.1	24.3
1962	17.8	17.6	17.4	16.8	16.3	15.8	16.2	15.3	15.0	14.3	14.1	14.4	24.6
1965	15.9	15.6	15.4	14.9	14.3	13.8	14.3	13.3	13.0	12.3	12.2	12.5	22.6
1967	17.9	17.6	17.4	16.9	16.3	15.8	16.2	15.3	15.0	14.3	14.2	14.4	24.6
1968	17.1	16.8	16.7	16.1	15.6	15.1	15.5	14.6	14.3	13.6	13.4	13.7	23.9
1969	17.9	17.6	17.5	16.9	16.3	15.9	16.3	15.3	15.1	14.3	14.2	14.5	24.7
1970	21.9	21.6	21.4	20.9	20.3	19.8	20.3	19.3	19.0	18.3	18.2	18.5	28.6
1977	10.6	10.4	10.2	9.6	9.1	8.6	9.0	8.1	7.8	7.1	6.9	7.2	17.4
1983	21.6	21.3	21.1	20.6	20.0	19.5	19.9	19.0	18.7	18.0	17.9	18.1	28.3
2016	28.2	27.9	27.7	27.2	26.6	26.1	26.5	25.6	25.3	24.6	24.5	24.8	34.9

Notes: This table reports the US compositional effect Δ^{comp} on $\log W/Y$, as defined in equation (10), for alternative base years of the age-wealth and the age-labor income profiles. Compared to table A.2, this table considers earlier SCF waves for the age-wealth profile, as constructed by Kuhn, Schularick and Steins (2020). Panel A considers our main period of interest 2016 to 2100, and panel B considers 1950 to 2016. Every column corresponds to an alternative base year for the age-labor income profile, and every row to an alternative base year for the age-wealth profile. The last column corresponds to the case where we use the average age effect from a time-age-cohort decomposition on the 1980-2018 annual data from the CPS, with all growth loading on time effects (DH, for "Deaton-Hall").

Alternative choice of base year profiles. Tabled A.2 and A.3 explores how the magnitude of the compositional effects Δ^{comp} changes when we change the base year 0 we use to construct the age profiles a_{j0} and h_{j0} in equation (10).

In the last row and column, labeled "DH", we use the age effects extracted from a time-age-cohort decomposition in the style of Hall (1968) and Deaton (1997), imposing that all growth loads on time effects. It is important to load growth on time effects to recover the age profiles that are the correct input into Proposition 1.

Using earlier data for age-wealth profiles tends to imply smaller effects, since the age-wealth profile has steepened over time. (The 1977 data stands out as an outlier implying especially small effects; the age-wealth profile in that year declined much more rapidly at higher ages.) Using earlier data for age-labor income profiles tends to imply slightly larger effects, since the hump-shape in the age-labor income profile has moved to the right over time as generations retire later. Overall, using earlier data for both profiles implies mildly smaller effects. In contrast, using the age effects from our time-age-cohort decomposition ("DH-t") implies a slightly larger compositional effect.

C.4 Additional results for section 3.1

Historical compositional effects vs actual change in W/Y. Table A.4 contrasts, for a range of countries for which the World Inequality Database contains a sufficiently long time series of measured wealth-to-GDP ratios, the measured change in the log of W/Y (labeled "Data") relative to the compositional effect Δ_t^{comp} (labeled "Comp"). The latter is constructed from equation 10 using age profiles from the base year interacted with the actual change in population distributions over the period reported. The results are reported in log points, and the corresponding change in the level of W/Y is also reported for ease of interpretation. The compositional effect predicts an increase in W/Y in every country, consistent with what occurred. For countries like the United States, Austria, Greece, Spain and Sweden, the magnitudes also line up closely. For most countries the historical increase in W/Y is greater than the compositional effect alone would predict. If demographics was the only force driving wealth-to-GDP ratios then our theory suggests that the rise in W/Y should be less than what is predicted by the compositional effect due to the endogenous response of asset returns; the fact that many countries experienced larger increases suggests that other forces, such as declining productivity growth, have also been at play.

Role of heterogeneity in demographic change vs age profiles. Figure A.5 presents the implied change in log W/Y between 2016 and 2100 from the compositional effect and isolates the contributions from demographic forces and from the age-profiles. Panel A repeats the results from section 3.1, ranking countries from lowest to highest compositional effect. It also presents the results under the two UN fertility scenarios. To isolate the contribution from demographic forces, panel B computes the compositional effect where age-profiles in all countries are identical to the US profile. To isolate the contribution from the profiles, panel C computes the compositional effect where population distributions of the US are used in every country. Panels B and C show that both the shapes of the profiles and the changes in population distributions matter to the compositional effect, but that the demographic forces play a much more important role in generating shift-shares that are high and heterogeneous across countries.

Randomly reallocating age-wealth and age-income profiles. This section considers how stable the wealth-weighted compositional effect $\bar{\Delta}^{comp}$ is to random reshuffling of age-labor and age-wealth profiles. To do this, we randomly draw with replacement the age-wealth profiles in each country from the full set of age-wealth profiles considered in the paper (across both countries and time), and do the same for the income profiles; and calculate the value of $\bar{\Delta}^{comp}$ for each draw. Figure A.6 reports the distribution of $\bar{\Delta}^{comp}$'s across 100,000 draws. We find that the distribution of $\bar{\Delta}^{comp}$ tends to be very close to its baseline value of 32 log points, with a mean at 30 log points and most of the mass between 24 and 36 log points. This reflects the fact that the shape of age-wealth and age-income profiles is remarkably stable across countries and time, and that demographic change is the main driver of our compositional effects.

Table A.4: Historical change in W/Y vs compositional effect Δ^{comp}

Country	Period	Data (log)	Comp (log)	Data (level)	Comp (level)
AUS	1960-2016	63.7	17.4	246.9	83.7
AUT	1995-2016	11.8	12.4	43.2	45.0
BEL	1995-2016	25.5	8.5	118.7	42.7
CAN	1971-2016	78.9	19.7	251.7	82.3
CHN	1995-2016	85.7	17.6	244.8	68.6
DEU	1950-2016	85.6	21.9	250.9	85.9
DNK	1973-2016	87.2	14.0	213.4	47.8
ESP	1950-2016	34.9	27.6	167.8	137.7
EST	1995-2016	1.8	6.9	5.6	20.9
FIN	1995-2016	28.0	19.5	69.0	49.9
FRA	1950-2016	111.2	21.4	336.2	96.7
GBR	1950-2016	40.5	16.9	180.2	84.0
GRC	1995-2016	7.7	8.8	26.2	29.6
HUN	1995-2016	26.1	7.1	74.0	22.0
IND	1995-2016	25.7	7.0	109.8	32.8
IRL	1995-2016	2.0	7.9	5.1	19.8
ITA	1966-2016	114.0	23.5	409.5	126.0
JPN	1970-2016	68.9	52.4	251.6	206.1
LUX	1995-2016	13.0	2.5	35.7	7.3
NLD	1950-2016	95.9	34.1	270.8	126.8
POL	1995-2016	48.2	11.6	64.8	18.6
SVK	1995-2016	-1.8	9.9	-5.2	27.1
SVN	1995-2016	-35.5	15.9	-140.4	48.4
SWE	1950-2016	15.4	19.6	38.3	47.7
USA	1950-2016	31.7	24.8	119.5	96.5

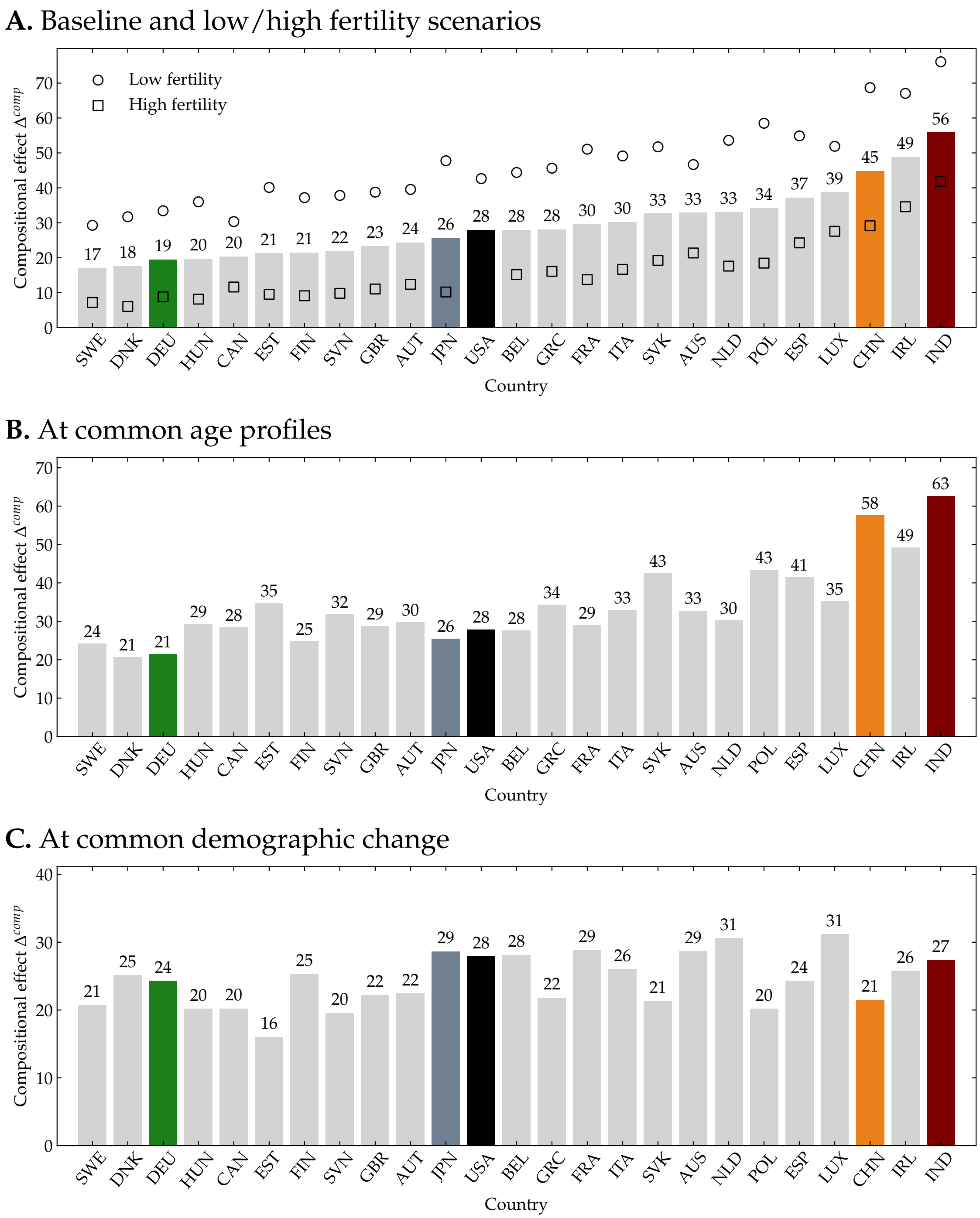


Figure A.5: World compositional effect between 2016 and 2100: alternative assumptions

Notes: Panel A presents the compositional effect on W/Y between 2016 and 2100 from equation (10) as well as its value using the low fertility (circles) and high fertility (squares) scenarios. Panel B does this calculation again, assuming that all countries have US age profiles of assets and income. Panel C does this calculation again, assuming all countries have the US age distribution in every year.

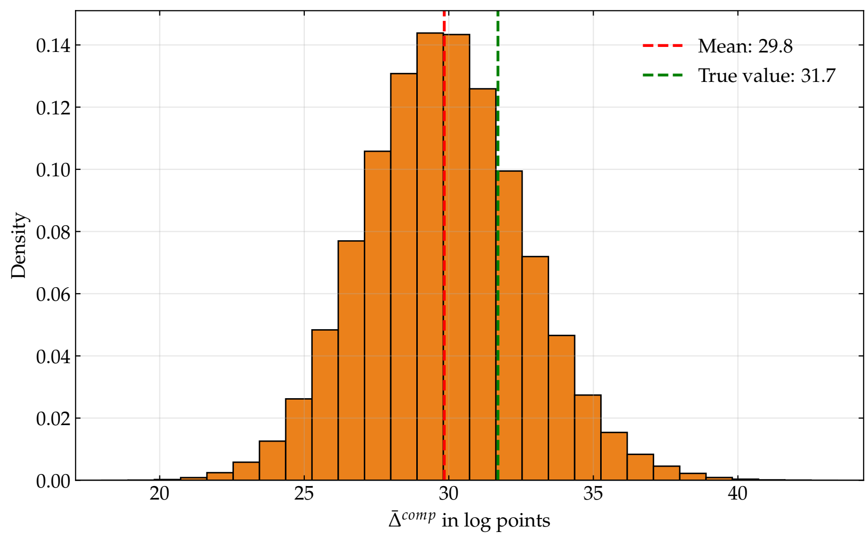


Figure A.6: Distribution of $\bar{\Delta}^{comp}$ with randomly sampled wealth and labor profiles

Notes: This figure presents the distribution of $\bar{\Delta}^{comp}$ as we randomly redraw 100,000 age-wealth and age-income profiles from the set of profiles in our dataset. "True value" corresponds to our baseline calculation for $\bar{\Delta}^{comp}$ using the actual profiles in each country, and "mean" reports the mean across the 100,000 draws.

C.5 Additional results for sections 3.2 and 3.3

Age profiles of consumption and assets. Figure A.7 presents the age distributions of consumption (orange lines) and asset holdings (red lines), constructed using the procedure described in section 3.2. The consumption profile is backed out of the asset profile and the profile of net income. Net income includes all taxes and transfers; since this measure is not available in most surveys, we back it out of aggregate information on taxes and transfers. In practice, we use net income from our quantitative model of section 4, which is constructed using that information for each country.

Applying equation (15) at each point in time to predict NFAs. Figure A.8 reproduces Figure 7, but we apply equation (15) at each point in time to predict NFAs. Specifically, we apply equation

$$\log\left(1+\frac{NFA_t^c/Y_t - NFA_0^c/Y_0}{W_0^c/Y_0^c}\right) \simeq \Delta_t^{comp,c} - \overline{\Delta}_t^{comp} + \left(\epsilon^{d,c}+\epsilon^{s,c} - \left(\bar{\epsilon}^d+\bar{\epsilon}^s\right)\right)(r_t - r_0) \qquad \text{(A.52)}$$

where $r_t - r_0$ is, in turn, calculated by applying equation (13) at each point in time,

$$r_t - r_0 \simeq -\frac{1}{\bar{\epsilon}^d+\bar{\epsilon}^s}\bar{\Delta}_t^{comp} \qquad \text{(A.53)}$$

and, in equations (A.52)–(A.53), we recalculate $\bar{\epsilon}^d$ using the age distribution at each t as well.[75]

The main findings from figure A.8 are unchanged relative to those from figure 7, indicating that the interest rate adjustment term does not play a major role when it comes to forecasting NFAs. This is because this interest adjustment only matters to the extent that elasticities of supply and demand differ across countries, and the heterogeneity we calculate from our sufficient statistics is relatively limited.

[75] The exact first-order approximation involves a sequence-space Jacobian matrix (Auclert, Bardóczy, Rognlie and Straub 2021). In practice, however, we are unaware of a sufficient statistic expression for the Jacobian that underlies $\bar{\epsilon}^d$. Figure 8 shows that the approximation in (A.52)–(A.53) works fairly well in

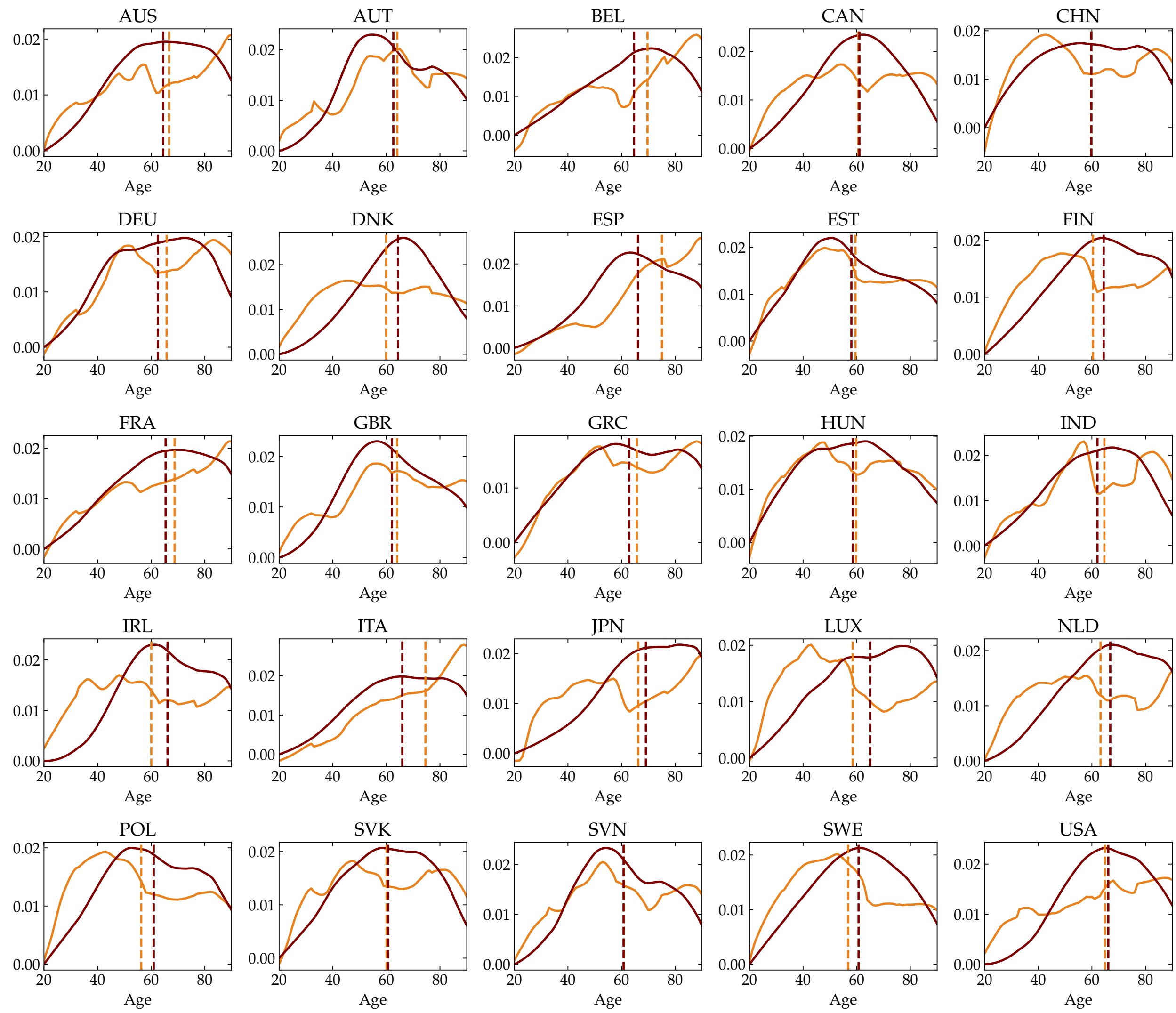


Figure A.7: Distribution of the ages of consumption and wealth in each country.

Notes: This figure presents the age distributions of consumption (orange lines) and asset holdings (red lines). The dashed vertical lines depict the average ages of consumption and asset holdings.

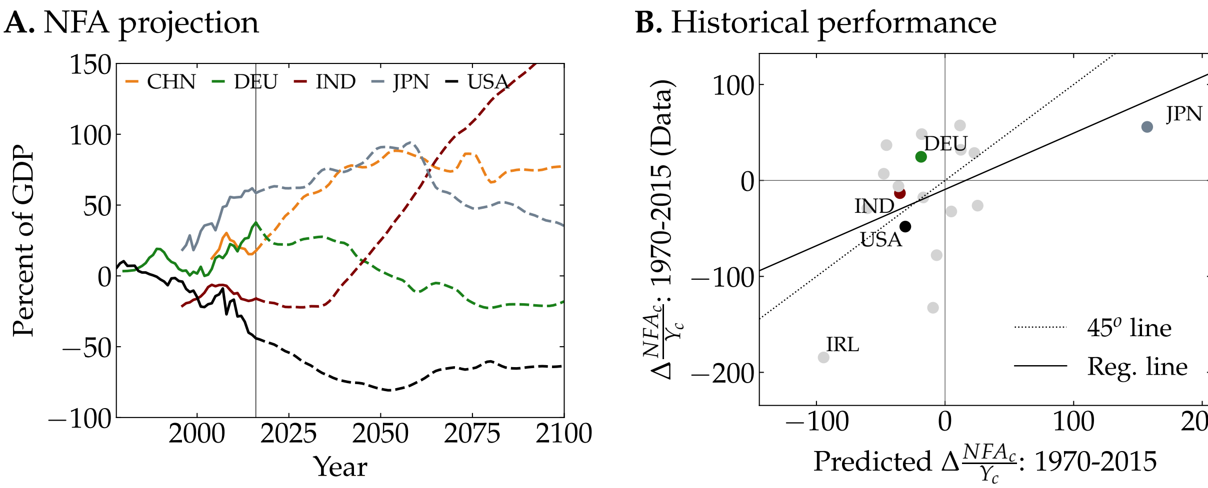


Figure A.8: Using a dynamic version of equation (15) to project NFAs

Notes: This reproduces figure 7, but uses (A.52)–(A.53), rather than $\Delta_t^{comp,c} - \overline{\Delta}_t^{comp}$, to project $\Delta_t \frac{NFA^c}{Y^c}$.

C.6 Historical validation

This section considers how well demographics explain the historical variation in interest rates, wealth, and net foreign asset positions through the lens of our model. We find that demographics can explain a sizable fraction of interest rates and NFAs, and a smaller fraction of wealth changes. Extending our supply-and-demand framework, we show that this finding is natural, given other forces that have also been at play over the period.

Interest rates and wealth. We begin with interest rates and wealth. We conduct the same exercise as in section 3.3, applying Proposition 2 using historical rather than forward-looking data. We begin calculating $\bar{\Delta}_t^{comp}$ for all years between $t = 1950$ and $t = 2016$. The green dash-dot line in Figure A.9, Panel B displays this path; we obtain in particular $\bar{\Delta}_{1950}^{comp} = -23.6$ log points in 1950. For a given choice of σ and η, we then calculate $\bar{\epsilon}_t^s$ and $\bar{\epsilon}_t^d$ for all years, applying (19) and (20) using the age distribution at date t. Finally, we apply the formulas from Proposition 2 to obtain a prediction for $r_t - r_{2016}$ and $\overline{\Delta_t \log W/Y}$ for all past dates t, and compare this to the historical change in r that we estimated in Figure 1, as well as the historical change in W/Y for the countries for which we have data going back to 1950 (i.e. Germany, Spain, France, the UK, the Netherlands, Sweden and the US; see Table A.4)

Figure A.9 shows the result of this exercise, using our central value of $\sigma = 0.5$ and $\eta = 1$ to calculate $\bar{\epsilon}_t^s$ and $\bar{\epsilon}_t^d$. For the rate of return, the solid yellow line in panel A shows that the demographic pressure summarized by $\bar{\Delta}_t^{comp}$ explains a sizable 1.12pp decline in the real interest rate since 1950, compared with a 2.15pp estimated decline in the dashed red line. Of course, we cannot predict the high frequency movements, but it is notable that there is a marked decline in the 1990s in both the data and the model.

For wealth, the solid yellow line in Figure A.9, panel B shows that demographics explains, through the lens of our model, about 7 log points out of the historical 47 log point increase. Mechanically, this is because $\bar{\Delta}_t^{comp}$ is 23.6 log points over the period (green dash-dot line), but the decline in interest rates attenuates the effect of this pressure on asset demand by a factor of $\frac{\bar{\epsilon}^s}{\bar{\epsilon}^s + \bar{\epsilon}^d}$ at each date. Since our calculated elasticities are $\bar{\epsilon}^s = 8.3$ and $\bar{\epsilon}^d = 14.3$ for 1950, this implies that our model predicts an increase in wealth of about a third of $\bar{\Delta}_t^{comp}$. While both the compositional effect and the elasticities take on somewhat different values, the magnitudes of these predicted changes for the 1950-2016 period are generally similar to those for the

the context of our structural model.

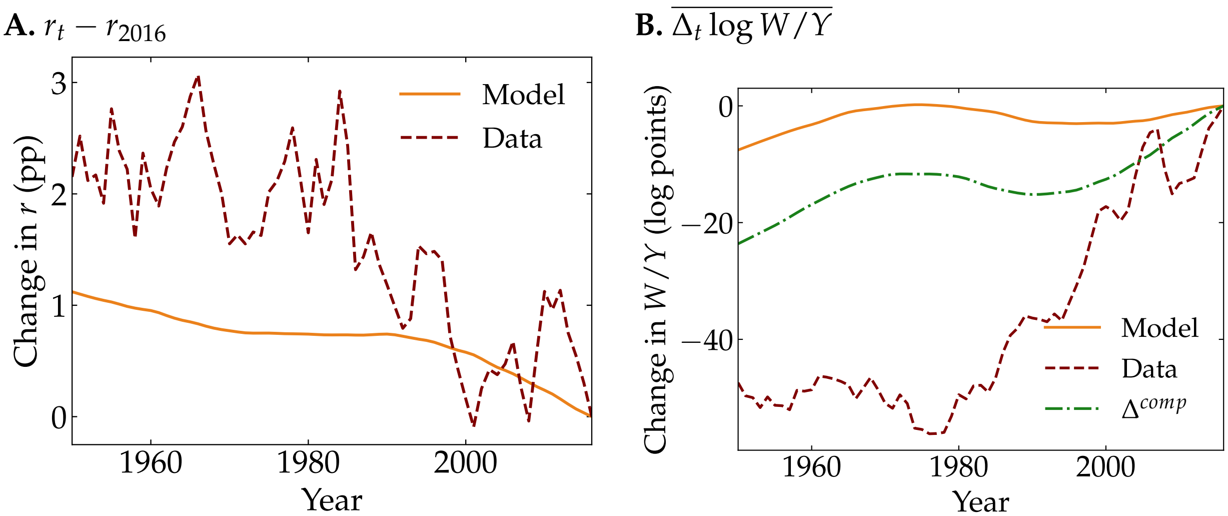


Figure A.9: Historical returns and wealth-to-GDP changes: model vs data

Notes: This figure considers historical changes in returns and average wealth-to-GDP ratios in the data (dashed lines) and our sufficient statistics model (solid line). Model predictions are obtained by applying Proposition 2 using the calculated average compositional effect $\bar{\Delta}_t^{comp}$ together with a calculation of $\bar{\epsilon}_t^d$ and $\bar{\epsilon}_t^s$ that uses the date-t age distribution with $\sigma = 0.5$ and $\eta = 1$. Panel B displays the data and model averages for the subset of countries (Germany, Spain, France, the UK, the Netherlands, Sweden and the US) where we have W/Y data going back to 1950. The model effect on $\log W/Y$ in 1950 is slightly larger (-9.3 log points vs -7.6 log points) when averaging across all countries.

2016-2100 period given in Table 1. (Table A.5 provides a full historical counterpart to table 1 for alternative σ and η.)

It is not surprising that our model can explain a much larger fraction of the historical decline in interest rates than that of the increase in wealth, because the 1950 to 2016 period has been affected by a number of major other trends beyond demographic change. These trends are not the direct focus of our paper, but it is helpful to categorize them into movements in asset supply and movements in asset demand. On the asset supply side, we have seen large increases in government debt, rising market power (which leads to larger capitalized values of rents), increased mechanization and automation, a rising importance of housing and intangible capital. On the asset demand side, beyond demographics, we have seen increases in inequality and falling productivity growth.[76] A simple way to estimate how important these forces have been historically is to postulate a supply and demand system of the following form:

$$\Delta \log\left(\frac{W}{Y}\right) = -\bar{\epsilon}^s \times \Delta r + \Delta^{supply} \tag{A.54}$$

$$\Delta \log\left(\frac{W}{Y}\right) = \bar{\epsilon}^d \times \Delta r + \Delta^{demand} \tag{A.55}$$

We can then use data on $\Delta \log W/Y$ and Δr between any two dates, as well as our estimates for $\bar{\epsilon}^s$ and $\bar{\epsilon}^d$, to back out Δ^{supply} and Δ^{demand} between these two dates. This gives us an estimate of the extent to which supply and demand forces have been at play.

[76]In representative-agent frameworks where steady-state asset demand is infinitely elastic, falling productivity growth is often thought of as directly decreasing r^*, because it decreases expected consumption growth in the Euler equation. In frameworks like ours where asset demand is imperfectly elastic, this force shows up as a rightward shift in the schedule.

Table A.5: Implied change in world interest rate and wealth-to-GDP: historical 1950–2016

	A. $r_{LR} - r_0$			**B.** $\overline{\Delta_{LR} \log\left(\frac{W}{Y}\right)}$		
		σ			σ	
η	0.25	0.50	1.00	0.25	0.50	1.00
0.60	-2.86	-1.50	-0.77	14.2	7.0	3.5
1.00	-1.74	**-1.12**	-0.66	14.0	**8.7**	4.9
1.25	-1.39	-0.97	-0.60	13.9	9.4	5.7

Notes: This table presents the implied change in the total return on wealth (r) and the wealth-weighted log wealth-to-GDP (W/Y) from the compositional effect between 1950 and 2016 using our sufficient statistic methodology. It is calculated in exactly the same way as table 1, but with the year 2100 replaced by 1950 (and the sign flipped such that the numbers describe the change from 1950 to 2016). Columns vary the assumption on the elasticity of intertemporal substitution σ, rows vary the assumption on the elasticity of capital-labor substitution η. Central estimates are in bold. r is expressed in percentage points, and wealth in percent ($100 \cdot \log$).

Table A.6 performs this exercise for different values of σ and η between 1950 and 2016, restricting to the subset of countries where W/Y is available. Irrespective of the particular choice of elasticities, large and positive shifts in both supply and demand are necessary to rationalize the data, with a larger shift in asset demand than supply. This is natural given the patterns that we are trying to explain: solving (A.54)–(A.55) for Δr and $\Delta \log(W/Y)$ given Δ^{supply} and Δ^{demand}, we must have:

$$\Delta r = -\frac{\Delta^{demand} - \Delta^{supply}}{\bar{\epsilon}^s + \bar{\epsilon}^d} \tag{A.56}$$

so $\Delta^{demand} > \Delta^{supply}$ is needed to explain the decline in r. Moreover, we have

$$\Delta \log\left(\frac{W}{Y}\right) = \frac{\bar{\epsilon}^s}{\bar{\epsilon}^s + \bar{\epsilon}^d}\Delta^{demand} + \frac{\bar{\epsilon}^d}{\bar{\epsilon}^s + \bar{\epsilon}^d}\Delta^{supply} \tag{A.57}$$

so the shift in Δ^{demand} must be large to explain the large historical 45 log point increase in the wealth-to-GDP ratio as a weighted average of Δ^{demand} and Δ^{supply} with a weight of $\bar{\epsilon}^s / \left(\bar{\epsilon}^s + \bar{\epsilon}^d\right)$ on Δ^{demand}, which is about $1/3$ in our central case of $\sigma = 0.5$ and $\eta = 1$.

In conclusion, rationalizing the historical data on wealth and interest rates requires a large shift in asset demand. Demographic change is one such shift, and our model implies that it accounts for $\bar{\Delta}^{comp}/\Delta^{demand}$ of the total. Since $\bar{\Delta}^{comp} = 23.6$ between 1950 and today, Table A.6, Panel A shows that, depending on our choice of elasticities, demographic change accounts for between 20% (=23.6/118.5) and 43% (=23.6/54.6) of the shift in asset demand, with 30% (=23.6/78.3) in our central case. This shows that, while other forces on the demand and supply sides are needed to fully account for the historical data, demographics does represent a sizable fraction of the shift in asset demand observed over the period.

Historical NFAs. Next, we explore how well our framework can explain the historical variation in NFAs. Due to data limitations, and because capital flow movements were highly limited until the end of the 1960s, we focus on the period 1970–2015. Our exercise in the main text, reported in panel B of figure 7, shows the effect of regressing the historical change in NFA/Y across countries on the prediction from our model that this should be approximately equal to the demeaned compositional effect scaled by $\frac{W}{Y}$ (equation (21)). We see that this exercise performs quite well, with a regression coefficient of $\hat{\beta} = 0.605$ that is statistically significant (standard error = 0.294). In particular, we cannot reject the $\beta = 1$ coefficient

Table A.6: Estimated shifters in asset demand and asset supply, 1950-2016

	A. Δ^{demand}			**B.** Δ^{supply}		
		σ			σ	
η	0.25	0.50	1.00	0.25	0.50	1.00
0.60	54.6	73.0	110.0	36.7	36.7	36.7
1.00	59.8	**78.3**	115.2	29.5	**29.5**	29.5
1.25	63.1	81.5	118.5	25.0	25.0	25.0

Notes: This table presents estimates of the shifters of aggregate asset supply Δ^{supply} and aggregate asset demand Δ^{demand} as defined in equations (A.54)–(A.55), following the procedure described in the text. Columns vary the assumption on the elasticity of intertemporal substitution σ, rows vary the assumption on the elasticity of capital-labor substitution η. Both supply and demand shifts are expressed in percent ($100 \cdot \log$).

predicted by equation (21). We note in particular that demographics predicts the evolution of NFAs with the correct sign. By contrast, the allocation puzzle literature shows that productivity growth tends to predict the evolution of NFAs with the wrong sign relative to neoclassical theory (see e.g. Gourinchas and Jeanne 2013).

Even then, our basic regression raises a few concerns. First, our exercise above documented other supply and demand forces affecting interest rates and wealth-to-GDP; we may also expect these to affect NFAs. Second, the literature has discussed valuation effects from fluctuations in nominal exchange rates and relative stock market performance as important drivers of NFAs historically, and those are not allowed by our model. Finally, Ireland and Japan appear as outliers in figure 7B, and while Japan may appear as a poster child for the effects of demographics, it is well-known that Ireland experienced large inflows due to its growing status as a tax haven. We address these concerns with a more elaborate regression exercise in table A.7.

Results. We begin in panel A, column (1), with the simple regression of the historical change in NFA/Y on the demeaned compositional effect from figure 7B.

The first concern is that additional supply and demand forces, which also affect NFA, may be correlated with demographic change in the data. To see the effect of this omitted variable bias, we control for the change in debt-to-GDP as an important asset supply force, and the change in the top 10% income share as an asset demand force stressed in the literature. Column (2) shows that the debt-GDP control pushes the main coefficient even closer to 1, with debt-GDP showing up with the expected negative sign. Controlling for the change in inequality in addition requires dropping more countries and makes all estimates noisy, but there is no good evidence that inequality increases NFAs.

Next, to address the concern that Japan and Ireland are driving the relation in figure 7B, columns (4)–(6) repeat the exercise while dropping these two data points. Indeed, the simplest regression loses statistical significance when the influence of Japan and Ireland is removed, though the coefficients remain positive and we can still not reject that they are equal to 1.

Panel B redoes the exercise on a shorter sample, 1993 to 2015, where we have 4 more countries. The results are generally much stronger in this sample, with a regression coefficient close to 1 in each of the specifications, even as we drop Japan and Ireland. Especially in the latter case, Debt-to-GDP shows up with a coefficient close to -1, as predicted by the theory, while inequality continues not to have any clear effect.

Another concern is that NFA movements may be driven by valuation effects as opposed to cumulated changes in current accounts, with our model only accounting for the latter. Panel C redoes our regressions when the historical change in NFA/Y is measured by cumulating the current account changes between the initial and the final period. The results in this case are similar to those already discussed, though not as statistically significant.

Our conclusion from this broad exercise is that the compositional effect is a relatively effective predictor of the variation in NFAs across countries and over time, even after other determinants of NFAs are accounted for. While we sometimes lose statistical significance, almost all point estimates are positive, and '1' always remains inside the confidence interval. We find this especially notable in light of the allocation puzzle literature, which has highlighted the difficulty of explaining NFAs with neoclassical mechanisms.

Table A.7: Historical changes in NFA-to-GDP vs demeaned compositional effect

	Full Sample			Excluding JPN & IRL		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Baseline						
Demeaned compositional effect	0.605*	1.082**	0.862	0.156	0.381	0.641
	(0.294)	(0.426)	(0.512)	(0.496)	(0.450)	(0.698)
Change in Debt/GDP		-0.547	-0.485		-0.508	-0.559
		(0.426)	(0.474)		(0.423)	(0.526)
Top 10% Income Share			-0.549			0.366
			(2.795)			(3.484)
N	18	15	10	16	13	9
R^2	0.209	0.384	0.413	0.007	0.159	0.227
Panel B. Shorter sample with additional countries						
Demeaned compositional effect	0.966**	1.220**	1.344*	0.943**	0.706**	0.625
	(0.416)	(0.539)	(0.699)	(0.424)	(0.264)	(0.371)
Change in Debt/GDP		-0.348	-0.355		-1.002***	-1.025***
		(0.354)	(0.369)		(0.233)	(0.254)
Top 10% Income Share			-2.509			1.226
			(8.443)			(3.707)
N	22	15	15	20	13	13
R^2	0.213	0.299	0.304	0.216	0.765	0.768
Panel C. Alternative measure of NFA/Y						
Demeaned compositional effect	0.290	0.858	0.174	-0.130	0.904	0.058
	(0.407)	(0.558)	(0.845)	(0.823)	(0.869)	(1.177)
Change in Debt/GDP		-0.451	0.106		-0.654	0.068
		(0.541)	(0.781)		(0.733)	(0.887)
Top 10% Income Share			-3.027			-2.546
			(4.610)			(5.878)
N	18	14	10	16	12	9
R^2	0.031	0.200	0.252	0.002	0.145	0.038

Notes: This table presents estimates of the relationship between the change in net foreign assets over GDP ratio ($\Delta\frac{NFA}{Y}$) between 1970 and 2015 and the demeaned compositional effect. The shorter sample is 1993 to 2015. The NFA data comes from Lane and Milesi-Ferretti (2017). The debt-to-GDP ratio is obtained from the historical macrodatabase. The top 10% of income share is from the World Inequality Database. We use the simple difference between 1970 and 2015, or from the first available year to 2015 if data starts later. Standard errors are reported in parentheses. *, **, and *** correspond to significance at the p=0.1, 0.05, and 0.01 thresholds, respectively.

D Model extensions

This section shows how the framework of section 2 is altered when some key assumptions are relaxed. It then shows how this affects the results from the quantitative implementation of the framework in section 3.

D.1 Allowing for nonzero initial NFAs

With non-zero initial NFAs, there is a first-order compositional effect of aging on net asset demand insofar as the change in relative GDP across countries is correlated with initial NFAs.

Theory. If NFA_0^c is not zero in every country c, we would retain an additional term in (A.7), equal to first-order to

$$\sum_c \left[\frac{Y^c}{Y}(r_{LR}, \pi_{LR}, \nu_{LR}) - \frac{Y_0^c}{Y_0}\right]\frac{NFA_0^c}{Y_0^c} = \sum_c \frac{Y_0^c}{Y_0}\Delta_{LR}\log\frac{Y^c}{Y}(r_{LR}, \pi_{LR}, \nu_{LR})\frac{NFA_0^c}{Y_0^c}$$

When we divide by $\frac{W_0}{Y_0}$ as in our derivation of (A.9), this becomes

$$\sum_c \omega^c \frac{NFA_0^c}{W_0^c}\Delta_{LR}\log\frac{Y^c}{Y}(r_{LR}, \pi_{LR}, \nu_{LR}) \tag{A.58}$$

which will show up as an additional term in (A.9). Since the wealth-weighted average of $\frac{NFA_0^c}{W_0^c}$ is zero by global market clearing, this can be written as a wealth-weighted covariance

$$\text{Cov}_{\omega^c}\left(\frac{NFA_0^c}{W_0^c}, \Delta_{LR}\log\frac{Y^c}{Y}(r_{LR}, \pi_{LR}, \nu_{LR})\right) \tag{A.59}$$

If we define

$$\Delta_L^{demog} R\frac{Y^c}{Y} \equiv \frac{\partial(\log\frac{Y^c}{Y})}{\partial\pi}\Delta_{LR}\pi + \frac{\partial(\log\frac{Y^c}{Y})}{\partial\nu}\Delta_{LR}\nu$$

to be the change in GDP shares caused by demographic change alone, holding r constant, and

$$\bar{\epsilon}^{weight} \equiv \text{Cov}_{\omega^c}\left(\frac{NFA_0^c}{W_0^c}, \frac{\partial(\log\frac{Y^c}{Y})}{\partial r}\right) \tag{A.60}$$

then the modified (A.9) becomes

$$\bar{\Delta}_{LR}^{comp} + \text{Cov}_{\omega^c}\left(\frac{NFA_0^c}{W_0^c}, \Delta_{LR}^{demog}\frac{Y^c}{Y}\right) + (\bar{\epsilon}^d + \bar{\epsilon}^s + \bar{\epsilon}^{weight})(r_{LR} - r_0) = 0 \tag{A.61}$$

and we can solve to obtain

$$r_{LR} - r_0 = \frac{\bar{\Delta}_{LR}^{comp} + \text{Cov}_{\omega^c}\left(\frac{NFA_0^c}{W_0^c}, \Delta_{LR}^{demog}\frac{Y^c}{Y}\right)}{\bar{\epsilon}^d + \bar{\epsilon}^s + \bar{\epsilon}^{weight}} \tag{A.62}$$

Note that the two departures from our previous result, the covariance in (A.61) and the covariance in the definition (A.60) of $\bar{\epsilon}^{weight}$, both involve wealth-weighted covariances between initial net foreign asset positions as shares of wealth, $\frac{NFA_0^c}{W_0^c}$, and some change in each country's GDP weight (either in response to demographics or endogenously in response to r). A priori, there is no particular reason to have a covariance in either direction here, and indeed we will show that these terms are fairly small in practice.

Our previous simplification for the average change in wealth-to-GDP no longer holds, but we can still

Table A.8: Change in world r and $\log W/Y$ under non-zero initial NFAs

	A. $r_{LR} - r_0$			**B.** $\overline{\Delta_{LR} \log\left(\frac{W}{Y}\right)}$		
		σ			σ	
η	0.25	0.50	1.00	0.25	0.50	1.00
0.60	-2.30	-1.24	-0.65	13.3	8.1	5.2
1.00	-1.60	**-1.01**	-0.58	15.2	**10.3**	6.8
1.25	-1.35	-0.90	-0.54	15.9	11.3	7.6

Notes: This table recalculates Table 1 but allowing for non-zero initial NFAs as in the data. The table presents predictions for the change in the total return on wealth (r) and the wealth-weighted log wealth-to-GDP (W/Y) between 2016 ($t = 0$) and 2100 ($t = LR$) using our sufficient statistic methodology modified to account for the effect of non-zero NFAs. Columns vary the assumption on the elasticity of intertemporal substitution σ, rows vary the assumption on the elasticity of capital-labor substitution η. Central estimates are in bold. r is expressed in percentage points, and wealth in percent ($100 \cdot \log$).

write

$$\overline{\Delta_{LR} \log \frac{W^c}{Y^c}} \simeq \bar{\Delta}^{comp} + \bar{\epsilon}^d (r_{LR} - r_0). \tag{A.63}$$

The change in NFA in each country is

$$\Delta \log\left(1 + \frac{\Delta_{LR} NFA^c / Y^c}{W_0^c / Y_0^c}\right) = \Delta_{LR}^{comp,c} + (\epsilon^{d,c} + \epsilon^{s,c})(r_{LR} - r_0)$$

Application. We use the same 2016 NFA data as in our extended model to calculate the two new terms $\text{Cov}_{\omega^c}\left(\frac{NFA_0^c}{W_0^c}, \Delta_{LR}^{demog} \frac{Y^c}{Y}\right)$ and $\bar{\epsilon}^{weight}$. The second term, $\bar{\epsilon}^{weight}$, turns out to be so small (-0.08) in comparison to $\bar{\epsilon}^d + \bar{\epsilon}^s \approx 30$ that it is effectively irrelevant. The first term, $\text{Cov}_{\omega^c}\left(\frac{NFA_0^c}{W_0^c}, \Delta_{LR}^{demog} \frac{Y^c}{Y}\right)$, is approximately -2 log points, which when added to $\bar{\Delta}_{LR}^{comp} \approx 32$ log points leads to a slightly smaller numerator in (A.62). This implies a slightly smaller decline in r, which goes from -1.07pp for our central case in table 1 to -1.01pp in table A.8 here.

This value for $\text{Cov}_{\omega^c}\left(\frac{NFA_0^c}{W_0^c}, \Delta_{LR}^{demog} \frac{Y^c}{Y}\right)$ reflects a mild negative correlation in the data between NFAs in 2016 and projected population growth from 2016 to 2100. This negative correlation, in turn, is entirely due to the US, Japan, and China, and flips sign if these three countries are removed: the US is a large economy with a negative NFA that will have relatively healthy population growth through 2100, while Japan and China have positive NFAs and will shrink in relative terms by 2100. The relative demographic decline of Japan and China means that there will be less of a global savings glut, while the relative growth of the US means that are more willing borrowers to absorb it. Intuitively, this pushes up the real interest rate slightly, although the effect is not large enough to offset the standard compositional effect by much.

The effect on the average change in wealth-to-GDP is somewhat more noticeable: in our central case, it increases from 8.9 log points in table 1 to 10.3 log points in table A.8 here. To understand this, note that the average increase in wealth-to-GDP in (A.63) reflects two largely offsetting forces: the positive compositional effect $\bar{\Delta}^{comp}$, and the negative asset demand effect $\bar{\epsilon}^d(r_{LR} - r_0)$ from r falling in equilibrium. When r falls by a bit less (-1.01pp instead of -1.07pp), the latter negative effect shrinks—and this leads to a larger effect in relative terms for the net increase in wealth-to-GDP.

Overall, the difference between table A.8 and table 1 is relatively minor, which is why we simplify by assuming zero initial NFAs in our main exercise. The additional effect here is also conceptually different, stemming from changes in relative country sizes rather than population aging per se.

D.2 Model with markups and land

We now generalize the production side of the economy to allow for rents. To do so, we allow both for an additional fixed factor, "land", in the production function, and also for a constant markup μ. For simplicity, here we will restrict the production function to be Cobb-Douglas (our central case in the main analysis), and focus directly on balanced growth path comparisons.

The production function is now $Y_t = F(K_t, Z_t L_t, X) \equiv K_t^{\alpha_K}(Z_t L_t)^{\alpha_L} X_t^{\alpha_X}$, where X_t is land and $\alpha_K + \alpha_L + \alpha_X = 1$. On a balanced-growth path, pure profits and land rents are given by $(1-\mu^{-1})Y_t$ and $\mu^{-1}\alpha_X Y_t$, respectively. Given constant r and a constant growth rate $g = (1+\gamma)(1+n) - 1$ of Y_t, the capitalized value of these rents at the beginning of period t satisfies

$$\frac{\Pi_t}{Y_t} = (1-\mu^{-1}+\mu^{-1}\alpha_X)\sum_{s=0}^{\infty}\left(\frac{1+g}{1+r}\right)^s = (1-\mu^{-1}+\mu^{-1}\alpha_X)\frac{1+r}{r-g} \tag{A.64}$$

Similarly to before, $\frac{K_t}{Y_t} = \mu^{-1}\frac{\alpha_K}{r+\delta}$, with an added constant factor μ^{-1}. Also, as before, along a balanced-growth path, $W_t/w_t L_t$ is a function only of r and demographics. Since the Cobb-Douglas assumption together with constant μ imply that $w_t L_t$ is a constant share of GDP, we can write W_t/Y_t as a function only of r and demographics and apply the same analysis as before.

For each country, following the analysis in proposition 2, we can write the balanced-growth NFA-to-GDP ratio in (A.6)

$$\frac{W^c}{Y^c}(r,\pi) - \frac{K^c}{Y^c}(r) - \frac{\Pi^c}{Y^c}(r,g) - \frac{B^c}{Y^c}$$

and the same logic leads us to an expression similar to (A.8), except that we have the following additional term inside the brackets:

$$\Delta_{LR}^{g,c} - \frac{1}{\frac{W^c}{Y^c}(r_0,\pi_0)}\frac{\partial\frac{\Pi^c}{Y^c}(r_0,g_{LR})}{\partial r}(r_{LR}-r_0) \tag{A.65}$$

where $\Delta_{LR}^{g,c} \equiv -\frac{1}{\frac{W^c}{Y^c}(r_0,\pi_0)}\left(\frac{\Pi^c}{Y^c}(r_0,g_{LR}) - \frac{\Pi^c}{Y^c}(r_0,g_0)\right)$ is the magnitude of the reduction in asset supply due to the change in the growth rate, working through the capitalized value of rents. Like the compositional effect, this term measures the exact nonlinear effect of demographics on asset market clearing, holding interest rates fixed at r_0. The second term in (A.65) parallels the already-used term for capital, $-\frac{1}{\frac{W^c}{Y^c}(r_0,\pi_0)}\frac{\partial\frac{K^c}{Y^c}(r_0)}{\partial r}(r_{LR}-r_0)$.

The semielasticity of asset supply is now

$$\epsilon^{s,c} \equiv \frac{1}{\frac{W^c}{Y^c}(r_0,\pi_0)}\left(\frac{\partial\frac{K^c}{Y^c}(r_0)}{\partial r} + \frac{\partial\frac{\Pi^c}{Y^c}(r_0,g_{LR})}{\partial r}\right) = \frac{K^c}{W^c}\frac{1}{r_0+\delta} + \frac{\Pi^c}{W^c}\frac{1+g_{LR}}{1+r_0}\frac{1}{r_0-g_{LR}} \tag{A.66}$$

where all ratios of wealth are taken in the base year. We can also simplify to obtain $\Delta_{LR}^{g,c} = \frac{\Pi^c}{W^c}\frac{g_0-g_{LR}}{r_0-g_{LR}}$.

With these ingredients, the first-order result for $r_{LR} - r_0$ in (A.9) is unchanged except that $\bar{\Delta}_{LR}^{comp}$ is replaced by $\bar{\Delta}_{LR}^{comp} + \bar{\Delta}_{LR}^{g}$, and we use the new $\bar{\epsilon}^s$ from (A.66):

$$r_{LR} - r_0 = -\frac{\bar{\Delta}_{LR}^{comp} + \bar{\Delta}_{LR}^{g}}{\bar{\epsilon}^d + \bar{\epsilon}^s} \tag{A.67}$$

To get the average change in wealth-to-GDP, we can then use (A.10), which is unchanged: $\overline{\Delta_{LR}\log\left(\frac{W^c}{Y^c}\right)} = \bar{\Delta}_{LR}^{comp} + \bar{\epsilon}^d(r_{LR}-r_0)$.

Calibration. To implement this extension, we need some estimate for Π^c in each country. To ensure that asset market clearing holds given measured W^c, B^c, and NFA^c, this also requires recalibrating K^c.

Previously, we assumed that K^c equaled $W^c - B^c - NFA^c$, the value of privately supplied assets in each

Table A.9: Change in world r and $\log W/Y$ in the presence of rents

	A. $r_{LR} - r_0$			**B.** $\overline{\Delta_{LR} \log\left(\frac{W}{Y}\right)}$		
		σ			σ	
κ	0.25	0.50	1.00	0.25	0.50	1.00
0.625	-1.47	-1.04	-0.66	16.7	9.8	3.8
0.812	-1.51	-1.03	-0.63	16.2	10.0	4.8
1.000	-1.71	-1.07	-0.62	14.1	8.9	5.1

Notes: This table recalculates Table 1 but allowing for a calibration where fraction $1-\kappa$ is rents.

country. In the extension, some share of these assets now reflects the capitalized value of rents. Assuming the share κ is common across all countries, this implies $K^c = \kappa(W^c - B^c - NFA^c)$ and $\Pi^c = (1-\kappa)(W^c - B^c - NFA^c)$. We calibrate κ by dividing K^{USA}, measured from the BEA private fixed asset accounts, by $W^{USA} - B^{USA} - NFA^{USA}$.[77] This gives $\kappa = 0.625$.

We also need to modify our calibration of r to take into account the capital gains earned by households on Π^c, and we therefore use the 2016 estimate from Panel C of Figure A.1, which includes imputed capital gains, rather than our baseline in Panel A.

Results. Using $\kappa = 0.625$ rather than our original $\kappa = 1$, we calculate the asset supply reduction from declining population growth from 2016 to 2100 to be $\bar{\Delta}^g_{LR} = 0.065$. This reflects the fact that the capitalized value of future rents, holding r constant, shrinks when there is less growth g. We also calculate that $\bar{\epsilon}^s$ rises from 8.3 to 15.7, since the $1/(r_0 - g_{LR})$ in A.66 is much larger than $1/(r_0+\delta)$. Intuitively, the capitalized value of rents is much more sensitive to r than is the capital-output ratio (at least under Cobb-Douglas), because the user cost of capital includes an additional depreciation term δ, whereas we assume that rents actually grow with the economy (a form of negative depreciation).

For r, these two effects push in opposite directions: the fall in asset supply adds to the compositional effect in the numerator of (A.67), but the rise in the asset supply semielasticity $\bar{\epsilon}^s$ adds to the denominator. Overall, for our central case of $\sigma = 0.5$, the net effect on r is small: $r_{LR} - r_0$ shrinks from -1.07 to -1.04. Since the asset demand side of the economy is unchanged, this slightly smaller decline in r implies a slightly larger increase in average wealth-to-GDP (9.8 log points rather than 8.9).

Table A.9 fully recalculates table 1 with Cobb-Douglas production $\eta = 1$, now letting rows indicate varying κ. We consider our new $\kappa = 0.625$, our original case of $\kappa = 1$ (which corresponds to the $\eta = 1$ case in the original table), and a halfway case of $\kappa = 0.812$. We note that for low κ, the higher $\bar{\epsilon}^s$ makes the choice of σ (which affects $\bar{\epsilon}^d$) less important, but otherwise the results are mostly unchanged.

D.3 Portfolio choice model

This section extends our setup to have multiple assets and an endogenous risk premium. In this case, the effect of aging can be analyzed using a multidimensional version of our shift-share method, with aging increasing the risk premium since older households hold a lower share of risky assets. In addition, we show that this phenomenon is largely orthogonal to predictions about average returns, which are similar to those in the one-asset model.

We assume that households hold a portfolio of two annuity assets, b ("safe") and k ("risky"), with returns r^f and r^r respectively.[78] To capture risk aversion, we assume that there is a reduced-form utility

[77] In principle, we could calculate κ^c separately for each country, but we could not find a source that provided private (as opposed to private + public) capital for each country in our base year.

[78] For simplicity, here we disregard the transition and think about moving between balanced growth

cost of holding risky assets. Writing $a_j \equiv b_j + k_j$ for total asset holdings and $s_j \equiv k_j/a_j$ for the portfolio share of risky assets, the household solves the following problem:

$$\max_{a_{j+1}, s_{j+1}, c_j} \sum_{j=0}^{T} \Phi_j \beta_j \frac{\tilde{c}_j^{1-1/\sigma}}{1-1/\sigma}, \qquad \tilde{c}_j \equiv c_j - v_j(s_j) \tag{A.68}$$

subject to

$$c_j + \phi_j a_{j+1} \leq h_j w + a_j[1 + (1-s_j)r^f + s_j r^r], \qquad a_0 = 0.$$

The utility cost of holding risk, v_j, is given by $v_j(s_j) = \overline{rp}(s_j - \bar{s}) + \frac{1}{2\phi}(s_j - \bar{s}_j)^2$, implying a risky asset share $s_j = \bar{s}_j + \psi[(r^r - r^f) - \overline{rp}]$. Here, Ψ captures the elasticity of risky asset demand, and $\overline{rp}$ and $\bar{s}_j$ encode a baseline risk premium and risky share by age, which we set to agree with 2016 data. We set $\bar{s}$ equal to the 2016 global risky share. Substituting in the optimal portfolio choice, the household chooses $\tilde{c}_j$ and a_{j+1} to maximize (A.68) subject to

$$\tilde{c}_j + \phi_j a_{j+1} \leq y_j + a_j[1 + r + (\bar{s}_j - \bar{s}) \times \Delta rp] + \frac{\psi}{2}\left[\Delta rp\right]^2, \tag{A.69}$$

where $r \equiv (1-\bar{s})r^f + \bar{s}r^r$ is the average return to wealth and $\Delta rp = (r^r - r^f) - \overline{rp}$ is the deviation of the risk premium from its baseline value.

Holding other parameters constant, the household problem maps $\{r, rp\}$ to wealth and portfolio shares $\{a_j, s_j\}$. Consider a small open economy with constant r and rp, facing demographic change as population shares move from π_j to π'_j. As in section 2, changes in aggregate wealth-to-GDP and the aggregate risky portfolio share will be given by compositional effects:

$$\Delta \log\left(\frac{W}{Y}\right) = \Delta^{comp} \equiv \log\left(\frac{\sum_j \pi'_j a_j}{\sum_j \pi'_j h_j}\right) - \log\left(\frac{\sum_j \pi_j a_j}{\sum_j \pi_j h_j}\right)$$
$$\Delta \log\left(\frac{K}{W}\right) = \Delta^{risk} \equiv \log\left(\frac{\sum_j \pi'_j s_j a_j}{\sum_j \pi'_j a_j}\right) - \log\left(\frac{\sum_j \pi_j s_j a_j}{\sum_j \pi_j a_j}\right)$$

where the first line is our standard compositional effect and the second is a new one for the risk share, using initial risk shares s_j.

In an integrated world economy, r and rp will adjust to clear both asset markets. For the overall level of assets, as in section 2 we have to first order $\bar{\Delta}^{comp} + (\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r})\Delta r + (\bar{\epsilon}^{d,rp} + \bar{\epsilon}^{s,rp})\Delta rp = 0$, where as before bars denote averages across countries weighted by initial wealth, and we now have semielasticities of asset supply and demand with respect to the risk premium rp in addition to the overall rate r. By similar logic, we can show that clearing in the risky asset market implies to first order $\bar{\Delta}^{risk} + (\bar{\epsilon}^{d,risk,r} + \bar{\epsilon}^{s,risk,r})\Delta r + (\bar{\epsilon}^{d,risk,rp} + \bar{\epsilon}^{s,risk,rp})\Delta rp = 0$, where here the semielasticities are for demand and supply of K/W and bars denote averages weighted by initial capital. Stacking the $\bar{\epsilon}^d$ in $\bar{\Sigma}^d \equiv \begin{pmatrix} \bar{\epsilon}^{d,r} & \bar{\epsilon}^{d,rp} \\ \bar{\epsilon}^{d,risk,r} & \bar{\epsilon}^{d,risk,rp} \end{pmatrix}$ and similarly the $\bar{\epsilon}^s$ in $\bar{\Sigma}^s$, we obtain a multi-dimensional version of proposition 4:

$$\begin{pmatrix} \Delta r \\ \Delta rf \end{pmatrix} = -\left(\bar{\Sigma}^d - \bar{\Sigma}^s\right)^{-1} \begin{pmatrix} \bar{\Delta}^{comp} \\ \bar{\Delta}^{risk} \end{pmatrix}. \tag{A.70}$$

To calculate $\bar{\Delta}^{risk}$, we calibrate risky asset shares by age using the 2016 SCF. Our definition of risky assets consists of real estate, other physical assets, and equity, including directly held equity, business interests, and equity held indirectly through life insurance, funded defined benefit pensions, and retirement assets.

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paths, which each have constant r^f and r^r. We also drop income risk and the borrowing constraint, as previously needed for proposition 4.

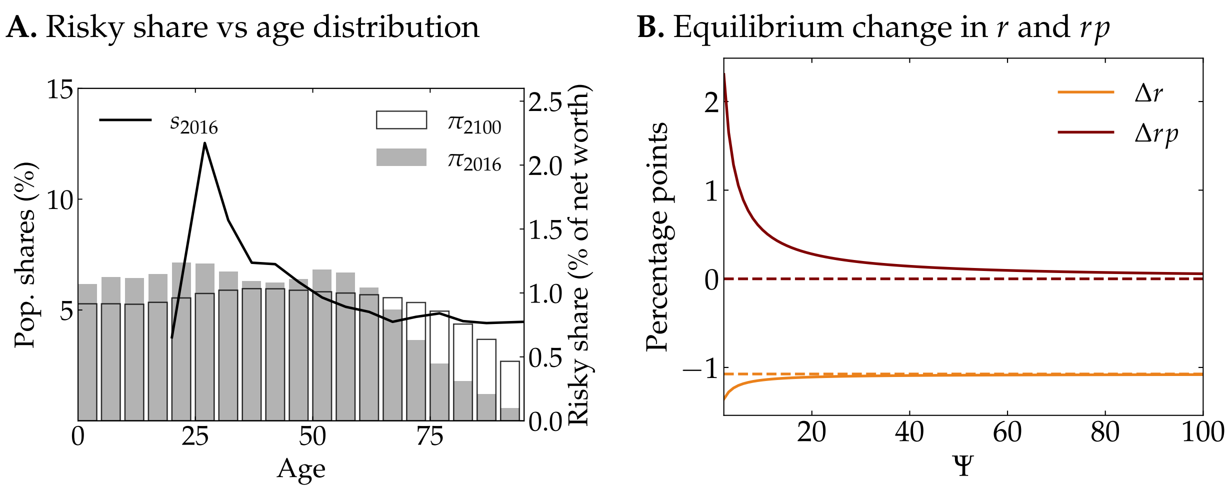

Figure A.10: Shift-share on risky share $\bar{\Delta}^{risk}$ and equilibrium Δr, Δrp as a function of Ψ

Notes: This reproduces figure 4, but plotting the risky share in 2016 s_{2016} against the evolving age distribution. The risky share is defined as the ratio, for each age group, of risky assets to net worth. See the main text for our definition of risky assets.

For indirect holdings, we assume an age-dependent equity share, decreasing from 75% for the youngest households to 55% for the oldest based on estimates in Parker, Schoar, Cole and Simester (2022) (see their figure 5). All other assets are classified as safe, with debt considered a negative safe asset. For each country in our analysis, we base s_j^c on the SCF-estimated shape of the risky share profile by age, rescaled to align with the country's risky share of total asset supply $K^c/(K^c + B^c)$. Figure A.10A illustrates the SCF risky share profile alongside the evolving age distribution in the US between 2016 and 2100. Early in life, households have a high risky share, sometimes exceeding 200% due to leverage, which gradually decreases as they pay off loans and shift towards fixed income securities. This profile means that risky demand falls as the population ages, and we find $\bar{\Delta}^{risk} = -6.5$ log points.

To evaluate equation (A.70) for Δr and Δrf, we need $\bar{\Sigma}^d$ and $\bar{\Sigma}^s$. The main challenge lies in calculating Σ_d, which requires knowing how asset profiles a_j and risk shares s_j respond to changes in returns.[79] The response of risk shares is straightforward: $ds_j = \Psi \times drp$, which only affects $\epsilon^{d,risk,rp}$. For asset profiles, we express the household problem using age-specific returns $r_j = r + (\bar{s}_j - \bar{s})(rp - \bar{rp})$.

We then numerically differentiate the solution to the household problem to obtain $\frac{da_j}{dr} = \sum_{j'} \frac{\partial a_j}{\partial r_{j'}}$ and $\frac{da_j}{drf} = \sum_{j'} \frac{\partial a_j}{\partial r_j}(s_j - \bar{s})$, using that shifting r is a uniform shift to age-specific returns, while rf shifts r_j proportional to $s_j - \bar{s}$. With these shifts in a_j and s_j from dr and $d(rp)$, we can calculate the full Σ^d. For the asset supply elasticity, we consider that changes in capital depend only on changes in the risky rate r^r, with $\frac{dr^r}{dr} = 1$ and $\frac{dr^r}{drf} = (1 - \bar{s})$. This allows us to derive expressions for the semielasticities of both $(K^c + B^c)/Y^c$ and $K^c/(B^c + K^c)$.

Table A.10 presents results for different values of Ψ. Two scenarios are presented: $\Psi = \infty$ (an infinitely elastic benchmark) and $\Psi = 15$ (a lower bound defined by imposing a non-positive desired risky share at $rp = 0$, with the idea being that people would want 100% non-risky assets if there were no return premium).

[79] For the asset supply elasticity, we use that the change in capital only depends on changes in the risky rate r^r, and that $\frac{dr^r}{dr} = 1$ and $\frac{dr^r}{drf} = (1 - \bar{s})$. Given this, it is straightforward to obtain expressions for the semi-elasticities of both $(K^c + B^c)/Y^c$ and $K^c/(B^c + K^c)$. The semi-elasticity of $(K^c + B^c)/Y^c$ is aggregated using wealth shares and the semi-elasticity of $K^c/(B^c + K^c)$ using capital shares. Compared to the main paper, we also lower δ to ensure that user cost of capital is the same in the two exercises.

Table A.10: Dissecting changes in r and rp for $\Psi = \infty$ vs $\Psi = 15$

	$\begin{pmatrix}\bar{\Delta}^{comp} \\ \bar{\Delta}^{risk}\end{pmatrix}$	$(\bar{\Sigma}^d - \bar{\Sigma}^s)$	$-(\bar{\Sigma}^d - \bar{\Sigma}^s)^{-1}$	$\begin{pmatrix}\Delta r \\ \Delta rp\end{pmatrix}$
$\Psi = \infty$	$\begin{pmatrix}31.679 \\ -6.520\end{pmatrix}$	$\begin{pmatrix}29.5 & 3.6 \\ 1.2 & \infty\end{pmatrix}$	$\begin{pmatrix}-0.0339 & 0.0000 \\ 0.0000 & 0.0000\end{pmatrix}$	$\begin{pmatrix}-1.07 \\ 0.00\end{pmatrix}$
$\Psi = 15$		$\begin{pmatrix}29.5 & 3.6 \\ 1.2 & 21.1\end{pmatrix}$	$\begin{pmatrix}-0.0341 & 0.0059 \\ 0.0019 & -0.0478\end{pmatrix}$	$\begin{pmatrix}-1.12 \\ 0.37\end{pmatrix}$

The first column shows shift-shares, with the wealth shift-share matching the single-asset analysis and the risk shift-share being negative, reflecting de-risking as people age. The matrices are identical except for the lower right corner, which captures the change in risk share in response to the risk premium. The upper-left corner of both matrices equals $\bar{\epsilon}^d + \bar{\epsilon}^s$ from the one-asset model, reflecting an identical household problem when $\Delta rp = 0$. The lower-left ($\bar{\epsilon}^{d,risk,r}$) and upper-right ($\bar{\epsilon}^{d,risk,r}$) corners show small positive effects, primarily because a higher r depresses risky asset supply, and a higher rp depresses total asset supply.

The third column displays the negative inverse $-(\bar{\Sigma}^d - \bar{\Sigma}^s)^{-1}$ of the net elasticity matrix, which maps $\{\bar{\Delta}^{comp}, \bar{\Delta}^{risk}\}$ to $\{\Delta r, \Delta rp\}$, which is displayed in the fourth column. For $\Psi = \infty$, the infinitely elastic risk demand prevents changes to the risk premium and related feedback mechanisms, resulting in $\Delta rp = 0$ and Δr matching the one-asset model.

For $\Psi = 15$, by contrast, the risk premium rises by 0.37pp in response to composition-induced shift away from risky assets, $\bar{\Delta}^{risk} < 0$. Interestingly, however, the effect on the average return is little changed, now at -1.12pp rather than -1.07pp.

Why is the average return so little affected by the introduction of portfolio choice? Mechanically, the reason is that the top right element of $-(\bar{\Sigma}^d - \bar{\Sigma}^s)^{-1}$ is fairly small, and when multiplied by $\bar{\Delta}^{risk}$ (which is already much smaller than $\bar{\Delta}^{comp}$) it makes only a minor contribution.

Indeed, $-(\bar{\Sigma}^d - \bar{\Sigma}^s)^{-1}$ as a whole is fairly close to diagonal, which follows from the fact that $\bar{\Sigma}^d - \bar{\Sigma}^s$ is close to diagonal. Intuitively, this means that the average return r governs the overall level of world net asset demand at the margin, while the risk premium rp governs the overall riskiness of the net world portfolio (largely through Ψ). The fact that off-diagonal terms are small—r has little effect on desired riskiness, and rp has little effect on net asset demand—means that the effects of $\bar{\Delta}^{comp}$ on r and of $\bar{\Delta}^{risk}$ on rp can be analyzed more or less independently. This suggests that it is reasonable for our analysis in the main text to focus on the effect of $\bar{\Delta}^{comp}$ on r in isolation, without a portfolio choice dimension.

Finally, for the $\Psi = 15$ case, table A.11 calculates changes in r and rp for all combinations of η and σ from table 1. For our central $\sigma = 0.5$ and $\eta = 1$, figure A.10B shows Δr and Δrp as a function of Ψ. For Ψ far below 15 and close to 0, there can be a much larger effect on the risk premium, and also a moderately larger effect on r—but we view such low Ψs as implausible, since they imply that individuals would want to hold a very large risky share even with a negative risk premium.

Table A.11: Change in world r and risk premium rp in portfolio choice model ($\Psi = 15$)

	A. $r_{LR} - r_0$			**B.** $rp_{LR} - rp_0$		
		σ			σ	
η	0.25	0.50	1.00	0.25	0.50	1.00
0.60	-2.53	-1.36	-0.70	0.36	0.33	0.31
1.00	-1.80	**-1.12**	-0.64	0.42	**0.37**	0.34
1.25	-1.53	-1.01	-0.60	0.44	0.39	0.35

Notes: This table recalculates the world r (as in table 1), as well as the change in the risk premium Δrp, in the portfolio choice model with our baseline calibration of $\Psi = 15$.

E Appendix to Section 4

E.1 Full model setup

Here, we describe the model in section 4. We first describe the full model for one country, omitting the country superscript c, and define a small open economy equilibrium for a fixed sequence $\{r_t\}$. The world equilibrium is defined as a sequence $\{r_t\}$ that clears the global asset market.

Demographics. The demographics are given by a sequence of births $\{N_{0t}\}_{t\geq-1}$, a sequence of age- and time-specific survival rates $\{\phi_{jt}\}_{t\geq-1}$ for individuals between age j and $j+1$, a sequence of net migration levels $\{M_{jt}\}_{0\leq t, 0\leq j\leq T-1}$, as well as an initial number of agents by age $N_{j,-1}$. The assumption that demographic variables start at $t = -1$ is made for technical reasons; it allows us to account for bequests received at time $t = 0$. Given these parameters, the population variables evolve according to the exogenous N_{0t} and

$$N_{jt} = N_{j-1,t-1}\phi_{j-1,t-1} + M_{j,t}, \quad \forall t \geq 0, j > 0. \tag{A.71}$$

As in section 2, we write $N_t \equiv \sum_j N_{jt}$ for the total population at time t, and $\pi_{jt} \equiv \frac{N_{jt}}{N_t}$ for the age distribution of the population.

Agents' problem. The basic setup is the same as in section 2, with heterogeneous individuals facing idiosyncratic income risk. We restrict the income process so that effective labor supply ℓ_{jt} is the product of a deterministic term ℓ_j that varies across ages, a fixed effect θ, and a transitory component ϵ, where both the fixed effect and the transitory component have a mean of 1. The log transitory component follows a finite-state Markov process with a transition matrix across years $\Pi^\epsilon(\epsilon|\epsilon_-)$ from ϵ_- to ϵ, calibrated to have a persistence χ_ϵ and a standard deviation v_ϵ, while the log permanent component follows a discrete Markov process across generations with a transition matrix $\Pi(\theta|\theta_-)$ from θ_- to θ calibrated to have a persistence χ_θ and a standard deviation v_θ. The processes are independent, and we write $\pi^\epsilon(\epsilon)$ and $\pi^\theta(\theta)$ for the corresponding stationary probability mass functions.[80]

We assume that individuals become economically active at age J^w, so that labor income at age j at time t is $w_t(1-\rho_{jt})\theta\epsilon\ell_{jt}$, where w_t is the wage per efficiency unit as in section 2, and $\rho_{jt} \in [0,1]$ is a parameter of the retirement system indicating the fraction of labor that households of age j are allowed to supply at time t. After retirement, agents receive social security payments $w_t\rho_{jt}\theta d_t$ in proportion to their permanent type, where d_t encodes a time-varying social security replacement rate.

[80] Discrete processes are used to facilitate notation. The calibration to the persistence and standard deviation is done using Tauchen's method applied to a Gaussian AR(1) process with a given persistence, standard deviation, and mean.

The state for an individual at age j and time t is given by the fixed effect θ, the transitory effect ϵ, and asset holdings $\mathfrak{a}$, and their value function is given by

$$\begin{aligned} V_{jt}(\theta,\epsilon,\mathfrak{a}) = \max_{\mathfrak{c},\mathfrak{a}'} & \frac{\mathfrak{c}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \mathrm{Y} Z_t^{\nu-\frac{1}{\sigma}} \left(1-\phi_{jt}\right) \frac{(\mathfrak{a}')^{1-\nu}}{1-\nu} + \phi_{jt}\frac{\beta_{j+1}}{\beta_j}\mathbb{E}\left[V_{j+1,t+1}(\theta,\epsilon',\mathfrak{a}')|\epsilon\right] \\ & \mathfrak{c}+\mathfrak{a}' \leq w_t\theta\left[(1-\rho_{jt})(1-\tau_t)\ell_{jt}\epsilon+\rho_{jt}d_t\right] + (1+r_t)[\mathfrak{a}+b^r_{jt}(\theta)] \\ & -\bar{\mathfrak{a}}Z_t \leq \mathfrak{a}', \end{aligned} \tag{A.72}$$

which determines the decision function $\mathfrak{c} = \mathfrak{c}_{jt}(\theta,\epsilon,\mathfrak{a})$ and $\mathfrak{a}' = \mathfrak{a}_{j+1,t+1}(\theta,\epsilon,\mathfrak{a})$ for consumption and next-period assets.

The term $\frac{\mathfrak{c}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$ represents the flow utility of consumption, and $\mathrm{Y}Z_t^{\nu-\frac{1}{\sigma}}\left(1-\phi_{jt}\right)(\mathfrak{a}')^{1-\nu}/(1-\nu)$ represents the utility from giving bequests $\mathfrak{a}'$. The bequest utility is scaled by mortality risk $1-\phi_{jt}$, since agents only give bequests if they die, and $\nu \geq \frac{1}{\sigma}$ captures potential non-homotheticities in bequests, which has been shown to generate more realistic levels of wealth inequality (De Nardi, 2004). The scaling factor $Z_t^{\nu-\frac{1}{\sigma}}$ ensures balanced growth in spite of this non-homotheticity. The term $b^r_{jt}(\theta)$ represents bequests received, and is allowed to vary according to the agent's permanent type.

State distribution. To determine the evolution of states, we assume that the distribution of individuals across θ and ϵ is in the stationary distribution for all ages and times, as well as for arriving and leaving migrants. This implies that the joint distribution across $(\theta,\epsilon,\mathfrak{a})$ is fully characterized by

$$H_{jt}(\mathfrak{a}|\theta,\epsilon) = \mathbb{P}(\mathfrak{a}_{jt} \leq \mathfrak{a}|\theta,\epsilon),$$

where H_{jt} is the conditional probability distribution of assets given θ and ϵ.[81]

Over time, the distribution evolves according to

$$H_{j+1,t+1}(\mathfrak{a}|\theta,\epsilon) = \sum_{\epsilon_-}\frac{\Pi^\epsilon(\epsilon|\epsilon_-)\pi^\epsilon(\epsilon_-)}{\pi^\epsilon(\epsilon)}\int_{\mathfrak{a}'}\mathbb{I}(\mathfrak{a}_{j+1,t+1}(\mathfrak{a}',\theta,\epsilon)\leq\mathfrak{a})dH_{j,t}(\mathfrak{a}'|\theta,\epsilon) \quad \forall j > J^w, \tag{A.73}$$

where $\mathfrak{a}_{j+1,t+1}$ is the decision function for assets implied by the agents' problem (A.72). Note that (A.73) implicitly assumes that death is independent of asset holdings and that migrants have the same distribution of assets as residents. At time zero, there is an exogenous distribution of assets $H_{j0}(\cdot|\theta,\epsilon)$ for each age group. As a boundary condition, we assume that individuals do not have any assets before working life starts:

$$H_{j,t}(\mathfrak{a}|\theta,\epsilon) = \mathbb{I}(\mathfrak{a}\geq 0) \quad \forall\theta,\epsilon, j\leq J^w, 0\leq t, \tag{A.74}$$

where $\mathbb{I}$ is the indicator function.

Bequest distribution. We model partial intergenerational wealth persistence by assuming that all bequests from individuals of type θ are pooled and distributed across the types θ' of survivors in accordance with the intergenerational transmission of types. Formally, the total amount of bequests received by agents of type θ of age j at time t satisfies

$$\begin{aligned} N_{jt}b^r_{jt}(\theta) = & F_j\sum_{\theta_-}\left(\frac{\Pi^\theta(\theta|\theta_-)\pi^\theta(\theta_-)}{\pi^\theta(\theta)}\right)\times \\ & \sum_{k=0}^{T} N_{k,t-1}(1-\phi_{k,t-1})\times\sum_{\epsilon}\pi^\epsilon(\epsilon)\int_{\mathfrak{a}}\mathfrak{a}dH_{kt}(\mathfrak{a}|\theta_-,\epsilon) \end{aligned} \tag{A.75}$$

[81] Formally, given H_{jt}, the joint distribution function $\tilde{H}_{jt}$ of $(\theta,\epsilon,\mathfrak{a})$ can be written $\tilde{H}_{jt}(\theta,\epsilon,\mathfrak{a}) = \sum_{\theta'\leq\theta,\epsilon'\leq\epsilon}\pi^\theta(\theta')\pi^\epsilon(\epsilon')H_{jt}(\mathfrak{a}|\theta',\epsilon')$.

Here, $\sum_k N_{k,t-1}(1-\phi_{k,t-1})\sum_\epsilon \int_\mathfrak{a} \pi^\epsilon(\epsilon)\mathfrak{a}dH_{kt}(\mathfrak{a};\theta_-,\epsilon)$ captures the total amount of bequests given by individuals of type θ_-, which is distributed to different ages j according to F_j, with $\sum_j F_j = 1$. The timing is that interest income accrues after the death event. A share $\frac{\Pi^\theta(\theta|\theta_-)\pi^\theta(\theta_-)}{\pi^\theta(\theta)}$ of these bequests is given to agents of type θ, capturing partial intergenerational transmission by using the probability that an agent of type θ has a parent of type θ_-. We assume that migrants also share in the bequest pool of their permanent type.

Note that an aging population alters the relative number of agents that give relative to the number of agents that receive bequests, which ceteris paribus increases bequest sizes.

Aggregation. Given the decision functions $\mathfrak{c}_{jt}$ and $\mathfrak{a}_{j+1,t+1}$ and the distribution across states, aggregate consumption and assets satisfy

$$W_t = \sum_{j=0}^{J} N_{jt} \times \sum_{\epsilon,\theta} \pi^\epsilon(\epsilon)\pi^\theta(\theta) \int_\mathfrak{a} [\mathfrak{a} + b^r_{jt}(\theta)]dH_{jt}(\mathfrak{a};\theta,\epsilon)$$

$$C_t = \sum_{j=0}^{J} N_{jt} \times \sum_{\epsilon} \pi^\epsilon(\epsilon) \int_\mathfrak{a} \mathfrak{c}_{jt}(\theta_-,\epsilon,\mathfrak{a})dH_{jt}(\mathfrak{a}|\theta_-,\epsilon). \tag{A.76}$$

Note that bequests received are included in the definition of today's incoming assets.

Production. As in section 2, markets are competitive, there are no adjustment costs in capital, and there is labor-augmenting growth at a constant rate γ. Production is given by a CES aggregate production function. We obtain the following equations:

$$Y_t = F(K_t, Z_t L_t) \equiv \left(\alpha K_t^{\frac{\eta-1}{\eta}} + (1-\alpha)[Z_t L_t]^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{A.77}$$

$$Z_t = (1+\gamma)^t Z_0 \tag{A.78}$$

$$r_t = F_K(K_t, Z_t L_t) - \delta \tag{A.79}$$

$$w_t = Z_t F_L(K_t, Z_t L_t) \tag{A.80}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{A.81}$$

$$L_t = \sum_{j=0}^{J} N_{jt}\ell_j, \tag{A.82}$$

where the last line uses that $\mathbb{E}\theta\epsilon = 1$ to obtain that average effective labor supply is ℓ_j of individuals of age j.

Government. The government purchases G_t goods and sets the retirement policy ρ_{jt}, the tax rate τ_t, and the benefit generosity d_t. It faces the flow budget constraint

$$G_t + \sum_{j=0}^{J} N_{jt} w_t \rho_{jt} d_t + (1+r_t)B_t = w_t \sum_{j=0}^{J} N_{jt}[\rho_{jt} + \tau_t(1-\rho_{jt})]\ell_{jt} + B_{t+1}, \tag{A.83}$$

where a positive B_t denotes government borrowing. In the aggregation, we again use that $\mathbb{E}\theta\epsilon = \mathbb{E}\theta = 1$ for each j to obtain that average benefits and labor income per age-j person are $w_t\rho_{jt}d_t$ and $w_t(1-\rho_{jt})\bar{\ell}_{jt}$ respectively.

The government targets an eventually converging sequence $\left\{\frac{B_{t+1}}{Y_{t+1}}\right\}_{t\geq 0}$. To reach this target, we assume that the government uses a fixed sequence of retirement policies $\{\rho_{jt}\}_{t\geq 0}$, and adjusts the other instruments using a fiscal rule defined in term of the "fiscal shortfall" SF_t, defined as

$$\frac{SF_t}{Y_t} \equiv \frac{\bar{G}}{Y} + \frac{\sum_{j=0}^{T}[\rho_{j,t}\bar{d} - \bar{\tau}(1-\rho_{j,t})\bar{\ell}_{jt}]N_{jt}w_t}{Y_t} + (r_t - g_t)\frac{B_t}{Y_t} - (1+g_t)\left[\frac{B_{t+1}}{Y_{t+1}} - \frac{B_t}{Y_t}\right], \tag{A.84}$$

where $g_t = \frac{Y_{t+1}}{Y_t} - 1$. The fiscal shortfall is positive at time t if expenditures minus revenues is too high to reach the debt target when the instruments G, d, and τ are set at some reference levels $\bar{G}$, $\bar{d}$ and $\bar{\tau}$. Given a non-zero fiscal shortfall, the fiscal rule consists of three weights $\varphi^G, \varphi^\tau, \varphi^d$ and an updating rule for instruments

$$\varphi^G SF_t = -(G_t - \bar{G}) \quad \forall t \geq 0 \tag{A.85}$$

$$\varphi^\tau SF_t = (\tau_t - \bar{\tau}) \times w_t \sum_{j=0}^{J} N_{jt}\ell_{jt}(1-\rho_{jt}) \quad \forall t \geq 0 \tag{A.86}$$

$$\varphi^d SF_t = -(d_t - \bar{d}) \times \left(w_t \sum_{j=0}^{J} N_{jt}\rho_{jt}\right) \quad \forall t \geq 0 \tag{A.87}$$

$$1 = \varphi^G + \varphi^\tau + \varphi^d, \tag{A.88}$$

where the weights capture the share of the shortfall covered by each instrument.

Market clearing. The assets in the economy consist of capital K_t, government bonds B_t, and foreign assets NFA_t. The asset market clearing condition is

$$K_t + B_t + NFA_t = W_t. \tag{A.89}$$

Given the other equilibrium conditions, (A.89) can be used to derive the goods market clearing condition[82]

$$NFA_{t+1} - NFA_t = NX_t + r_t NFA_t + W_{t+1}^{mig}, \tag{A.90}$$

where $NX_t \doteq Y_t - I_t - C_t - G_t$ is net exports at time t and

$$W_t^{mig} \doteq \sum_{j=0}^{J} M_{j,t} \sum_{\theta,\epsilon} \pi^\theta(\theta)\pi^\epsilon(\epsilon) \int_{\mathfrak{a}} \mathfrak{a} dH_{j,t}(\mathfrak{a},\theta,\epsilon)$$

is the assets at time t that comes from migrants.

Small open economy equilibrium. A small open economy equilibrium is defined for:

- A sequence of interest rates $\{r_t\}_{t=0}^{\infty}$
- A government fiscal rule $\{B_{t+1}/Y_{t+1}, \rho_{jt}, \varphi^G, \varphi^\tau, \varphi^d, \bar{G}, \bar{\tau}, \bar{d}\}_{t=0}^{\infty}$
- A sequence of average effective labor supplies $\{\bar{\ell}_{jt}\}_{0 \leq t, J^w \leq j \leq J}$
- An initial distribution of assets $\{H_{j0}(a|\theta,\epsilon)\}_{j=0}^{J}$
- Technology parameters $\{Z_0, \gamma, \delta, \nu, \alpha\}$
- Demographics: initial $\{N_{j,-1}\}_{j=0}^{J}$ and forcing parameters $\{M_{jt}, \phi_{jt}, N_{0,t+1}\}_{-1 \leq t, 0 \leq j \leq J}$
- Initial aggregate variables K_0, B_0, A_0

[82] Combine the aggregated household budget constraint with the government budget constraint (A.83), the capital evolution equation (A.81), and the asset market clearing condition (A.89).

The equilibrium consists of:

- Individual decision functions: $c_{jt}(\theta, \epsilon, \mathfrak{a}), \mathfrak{a}'_{jt}(\theta, \epsilon, \mathfrak{a})$
- A sequence of asset distribution functions $\{H_{jt}(a; \theta, \epsilon)\}_{1 \le t, J^w \le j \le J}$
- Government policy variables $\{G_t, \tau_t, d_t\}_{t \ge 0}$
- A sequence of wages $\{w_t\}_{t \ge 0}$
- A sequence of bequests received $\{b_{jt}(\theta)\}_{t \ge 0}$
- A sequence of aggregate quantities $\{Y_t, L_t, I_t, K_{t+1}, W_t, C_t, NFA_t\}_{t \ge 0}$

It is characterized by requiring that:

- r_0 is consistent with $K_0 \implies$ (A.79) holds given K_0 and $L_0 = \sum_j N_{j0}(1 - \rho_{j0})\ell_{j0}$
- W_0 is consistent with H_{j0}, that is, (A.76) holds
- Individual decision functions solve (A.72).
- The set of H_{jt}'s satisfies the evolution equation (A.73) and the boundary condition (A.74)
- The government policy variables satisfy (A.84)-(A.87).
- Equations (A.77)-(A.82) hold.
- A_t satisfies (A.76) for $t \ge 0$
- $NFA_t = W_t - K_t - B_t$, with B_0 given by the initial condition, and B_{t+1}/Y_{t+1} by the government fiscal rule.
- Bequests received $b_{jt}(\theta)$ satisfy (A.75)

World-economy equilibrium. Given a set of countries $c \in \mathcal{C}$, a *world-economy equilibrium* is a sequence of returns $\{r_0, \{r_t\}_{t \ge 1}\}$ and a set of corresponding sequences of prices and allocations $\mathcal{S}^c$ for each economy c such that each $\mathcal{S}^c$ is a small open economy equilibrium, and that their NFAs satisfy

$$\sum_{c \in \mathcal{C}} NFA_t^c = 0 \quad \forall t \ge 0 \tag{A.91}$$

E.2 Proof of proposition 5

Let Φ_t^c capture all demographic variables in a country: population shares, fertility, mortality, migration. Given fixed r and B^c/Y^c, long-run government policy only depends on Φ^c. Wages per unit of effective labor only depend on r. Assuming that the steady state of the household problem is unique conditional on demographics, wages, and government policy, we can express it as a function of (r, Φ^c).[83] Let $\frac{W^c}{Y^c}(r, \Phi^c)$ denote the resulting steady-state wealth-to-output ratio.

Output, normalized by technology, only depends on aggregate effective labor supply, which is a function of Φ^c (both directly through the number of people at each age and indirectly through government retirement policy), and the capital-to-effective-labor ratio, which is a function of r. Hence we can write each country's share of global GDP as $\frac{Y^c}{Y}(r, \nu, \Phi)$.

From here on, the proofs of propositions 2 and 3 in appendix B.3 apply, provided that, in equations (A.6) and later, we replace π with Φ, $\frac{W^c}{Y^c}(r_0, \Phi_{LR}^c) - \frac{W^c}{Y^c}(r_0, \Phi_0^c)$ with $\Delta_{LR}^{soe,c}$, as well as $\sum_c \omega^c \left(\frac{W^c}{Y^c}(r_0, \Phi_{LR}^c) - \frac{W^c}{Y^c}(r_0, \Phi_0^c)\right)$ with $\bar{\Delta}_{LR}^{soe}$ everywhere.

[83] Aside from bequests, we have a standard incomplete markets household problem and this would be a standard result. Bequests introduce some complication, since bequests depend on the endogenous distribution of assets, but household asset policy also depends on realized and expected bequests. The solution to the household problem is a fixed point of this process. We assume that the fixed point is unique and a global attractor; in practice, we have found that this assumption is always satisfied.

E.3 Steady-state equations and calibration details

Steady-state equations. Our calibration targets a stationary equilibrium associated with a constant rate of return r. Most elements are standard: we assume constant technology parameters $\{\gamma, \delta, \nu, \alpha, \ell_j\}$, a constant bond-to-output ratio $\frac{B}{Y}$, retirement policy ρ_j, tax rate τ, social security generosity d, and government consumption-to-output ratio G/Y. We also assume that there is a fixed distribution of assets $H_j(\tilde{\mathfrak{a}}|\theta, \epsilon)$, where $\tilde{\mathfrak{a}}$ is assets normalized by technology (again, we drop the country superscripts in the description of each country, and reintroduce them when we define the world equilibrium).

The non-standard element is that we introduce a counterfactual flow of migrants to ensure a time-invariant population distribution and growth rate at their 2016 levels. In particular, demography consists of constant mortality rates, a fixed age distribution, a constant population growth rate, and a constant rate of migration by age m_j:

$$\phi_{jt} \equiv \phi_j, \quad \pi_{jt} \equiv \phi_j, \quad N_t = (1+n)^t N_0, \quad m_j \equiv \frac{M_j}{N},$$

and the net migration by age is given by

$$m_{j-1} \equiv \frac{M_{j-1}}{N} = \pi_j \frac{1+n}{\phi_{j-1}} - \pi_{j-1}, \tag{A.92}$$

which ensures that (A.71) holds given a fixed age distribution of population. The notation $\frac{M_{j-1}}{N}$ without a time index is used to indicate the constant ratio $\frac{M_{j-1,t}}{N_t}$. Throughout, we use an analogous notation whenever the ratio of two variables is constant over time.

In normalized form, the consumer problem is

$$\begin{aligned} \tilde{V}_j(\theta, \epsilon, \tilde{\mathfrak{a}}) = \max_{\tilde{\mathfrak{c}}, \tilde{\mathfrak{a}}'} \frac{\tilde{c}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \Upsilon(1+\gamma)^{1-\nu}\left(1-\phi_j\right)\frac{(\tilde{\mathfrak{a}}')^{1-\nu}}{1-\nu} + \frac{\beta_{j+1}}{\beta_j}(1+\gamma)^{1-\frac{1}{\sigma}}\phi_j \mathbb{E}\left[\tilde{V}_{j+1}(\theta, \epsilon', \tilde{\mathfrak{a}}')|\epsilon\right] \\ \tilde{\mathfrak{c}} + (1+\gamma)\tilde{\mathfrak{a}}' \le \tilde{w}_t \theta\left[(1-\rho_j)(1-\tau)\bar{\ell}_j \epsilon + \rho_j d\right] + (1+r)\tilde{\mathfrak{a}} + \tilde{b}_j^r(\theta) \\ -\bar{\mathfrak{a}} \le \mathfrak{a}'(1+\gamma), \end{aligned} \tag{A.93}$$

where a variable with a $\sim$ denotes normalization by Z_t, except for $\tilde{V}_j \equiv \frac{V_{jt}}{Z_t^{1-\frac{1}{\sigma}}}$. As elsewhere in the paper, we write g for the overall growth rate of the economy

$$1 + g \equiv (1+n)(1+\gamma).$$

The consumer problem implies decision functions $\tilde{\mathfrak{c}}_j(\cdot)$ and $\tilde{\mathfrak{a}}'_j(\cdot)$, where the latter denotes the choice of next period's normalized assets as a function of the state at age j. From the evolution and boundary conditions of assets (A.73) and (A.74), the stationary distribution of assets satisfies

$$H_j(\tilde{\mathfrak{a}}|\theta, \epsilon) = \begin{cases} \sum_{\epsilon_-} \frac{\Pi^\epsilon(\epsilon|\epsilon_-) \times \pi^\epsilon(\epsilon_-)}{\pi^\epsilon(\epsilon)} \int_{\tilde{\mathfrak{a}}'} \mathbb{I}\left[\tilde{\mathfrak{a}}'_{j-1}(\tilde{\mathfrak{a}}', \theta, \epsilon) \le \tilde{\mathfrak{a}}\right] dH_{j-1}(\tilde{\mathfrak{a}}'|\theta, \epsilon) & \text{if } j > J^w \\ \mathbb{I}(\tilde{\mathfrak{a}} \ge 0) & \text{if } j = J^w \end{cases},$$

Normalized bequests satisfy

$$\pi_j \tilde{b}_j^r(\theta) = F_j \sum_{\theta_-} \left(\frac{\Pi^\theta(\theta|\theta_-)\pi^\theta(\theta_-)}{\pi^\theta(\theta)} \right) \times \sum_{k=0}^{T} \frac{[\pi_k + m_k](1-\phi_k)}{1+n} \times \int_{\tilde{\mathfrak{a}}} \sum_{\epsilon} \pi^\epsilon(\epsilon) \tilde{\mathfrak{a}} dH_k(\tilde{\mathfrak{a}}; \theta_-, \epsilon) \tag{A.94}$$

Aggregate consumption and assets are

$$\frac{C}{NZ} = \sum_{j=0}^{T} \pi_j \sum_{\theta,\epsilon} \pi^\theta(\theta)\pi^\epsilon(\epsilon) \int_{\tilde{\mathfrak{a}}} \mathfrak{c}_j(\tilde{\mathfrak{a}}, \theta, \epsilon) dH_j(\tilde{\mathfrak{a}}, \theta, \epsilon)$$

$$\frac{W}{NZ} = \sum_{j=0}^{T} \pi_j \sum_{\theta,\epsilon} \pi^\theta(\theta)\pi^\epsilon(\epsilon) \left(\int_{\tilde{\mathfrak{a}}} \tilde{\mathfrak{a}} dH_j(\tilde{\mathfrak{a}}, \theta, \epsilon) + b_j^r(\theta) \right)$$

Finally, since we assume that steady state migrants have the same distribution of assets as regular households, we have

$$\frac{A^{mig}}{NZ} = \sum_{j=1}^{T} m_{j-1} \sum_{\theta,\epsilon} \pi^\theta(\theta)\pi^\epsilon(\epsilon) \left(\int_{\tilde{\mathfrak{a}}} \tilde{\mathfrak{a}} dH_j(\tilde{\mathfrak{a}}, \theta, \epsilon) + b_j^r(\theta) \right) \tag{A.95}$$

where we recall that m_j is the number of migrants as a share of age group j at time t, and W_j is the total amount of assets of age-j individuals.

The stationary analogues of the production sector equations (A.77)-(A.82) are

$$\frac{Y}{ZN} = F\left[\frac{K}{ZN}, \frac{L}{N}\right] \tag{A.96}$$

$$r + \delta = F_K\left[\frac{K}{ZN}, \frac{L}{N}\right] = \alpha \left(\frac{K}{Y}\right)^{-1/\eta} \tag{A.97}$$

$$\frac{w}{Z} = F_L\left[\frac{K}{ZN}, \frac{L}{N}\right] \tag{A.98}$$

$$(g+\delta)\frac{K}{Y} = \frac{I}{Y} \tag{A.99}$$

$$\frac{L}{N} = \sum_{j=0}^{T} \pi_j (1-\rho_j)\ell_j, \tag{A.100}$$

The steady-state government budget constraint is derived from (A.83) given a fixed debt-to-output ratio

$$\frac{G}{Y} + \frac{w \times d \times \sum_j N_j \rho_j}{Y} + (r-g)\frac{B}{Y} = \tau \times \frac{wL}{Y}, \tag{A.101}$$

and the asset market and goods market clearing conditions are derived from (A.89) and (A.90):

$$\frac{W}{Y} = \frac{K}{Y} + \frac{B}{Y} + \frac{NFA}{Y} \tag{A.102}$$

$$0 = \frac{NX}{Y} + (r-g)\frac{NFA}{Y} + \frac{A^{mig}}{Y(1+g)}. \tag{A.103}$$

The world asset market clearing condition is

$$\sum_c \omega^c \frac{NFA^c}{Y^c} = 0, \qquad \omega^c \equiv \frac{Y^c}{\sum_c Y^c} \tag{A.104}$$

E.4 Calibration details

All demographic data is from the UN World Population Prospects, interpolated across years and ages to obtain data for each combination of year and age. For each country, we use the 2016 values for age-specific survival rates ϕ_j^c and population shares π_j^c. The population growth rate is defined as

$$1 + n^c = \frac{N_{2016}^c}{N_{2015}^c}$$

where N_{2016}^c and N_{2015}^c are the populations of country c in 2016 and 2015.

Debt-to-output is from the October 2019 IMF World Economic Outlook, and the net foreign asset position from the IMF Balance of Payments and International Investment Positions Statistics, deflated by nominal GDP from the Penn World Table 10.01.

For each country, the labor-augmenting productivity growth γ^c is defined as the average growth rate between 2000 and 2016 in real GDP divided by effective labor supply. For each country, we measure real GDP as expenditure-side real GDP from the Penn World Table 10.01, effective labor supply as $L_t^c = \sum_j N_{jt}^c h_j^c$, with N_{jt}^c taken from the UN World Population Prospects, and h_j^c given by the labor income profiles defined in section 3. We define the world γ as the average of γ^c, weighted by real GDP.

Given γ^c and the elasticity of substitution between capital and labor η, the growth rate of each economy is

$$g^c = (1 + n^c)(1 + \gamma^c) - 1,$$

and we calibrate the investment-to-output ratios, the share parameter in the production function, and the labor share

$$\begin{aligned} \frac{I^c}{Y^c} &= \frac{K^c}{Y^c}(\delta + g_c) \\ \alpha^c &= (r + \delta)\left(\frac{K^c}{Y^c}\right)^{\frac{1}{\eta}} \\ s^{L,c} &= 1 - (r + \delta)\frac{K^c}{Y^c}, \end{aligned}$$

where the expression for investment and α use (A.99) and (A.97). Note that this calibration ensures that the world asset market clearing condition (A.104) holds for r.

For government policy, we use the average labor wedge from the OECD Social Expenditure Database 2019 to target τ.[84] This measure includes both employer and employee social security contribution, which is consistent with treating w_t as the labor cost for employers. For d, we use data on the share of GDP spent on old age benefits, using data on benefits net of taxes from the OECD Social Expenditure Database. Our main source for the retirement age is OECD's data on "Effective Age of Labor Market Exit" from the OECD Pensions at a Glance guide.[85] For some countries, the age provided by the OECD implies that labor market exit happens after the age at which aggregate labor income falls below implied benefit income. In those cases, we define the latter age as the date of labor market exit. Formally, this is done by calibrating the implied benefit levels for each possible retirement age and choosing the highest age at which retirement benefits are weakly lower than net-of-tax income. Last, G/Y is calibrated residually to target (A.101) given B/Y, τ, d, and the retirement age.

[84] OECD SOCX Manual, 2019 edition. For China and India, which are not in the database, we use the labor wedge calculated in Gandullia, Iacobone and Thomas (2012).

[85] Pensions at a Glance 2019: OECD and G20 Indicators.

For individuals, we use Auclert and Rognlie (2018) and De Nardi (2004) to target the standard deviations v_ϵ, v_θ and the persistence parameters $\chi_\epsilon, \chi_\theta$. The processes are discretized using Tauchen's method, using three states for θ and 11 states for ϵ. Both processes are rescaled to ensure that they have a mean of 1.

Model outcomes and fit. Figure A.11 and A.12 show the model fit of age profiles of wage and labor income across all countries. For the labor income profile, the orange depicts labor income $(1-\rho_{j0})\ell_j$ in the initial steady state, and the white hollow dots depict ℓ_j which become relevant as the retirement age increases.

Table A.12 provides the main parameters for all countries, table A.13 provides additional parameters for all countries. Last, figure A.13 shows the outcomes for bequests and wealth inequality in the US. Panel A compares the distribution of bequests in the model to the empirical distribution in the data. We measure it as the value of bequests at certain percentiles divided by average bequests. We take the empirical distribution from Table 1 in Hurd and Smith (2002). The legend also reports the resulting model aggregate bequests-to-GDP ratio $\frac{Beq}{Y} = 8.8\%$. Panel B compares the model Lorenz curve to the one obtained in the SCF. We see that our model produces substantial wealth inequality, with the richest 20% holding roughly 70% of wealth. However, it does not go all the way to fit the wealth inequality in the US data.

E.5 Transitional dynamics in response to demographic change

Solution method. We solve for the perfect foresight transition path between 2016 ($t = 0$) and 2300 ($t = 284 \equiv T$) as follows.

In every country, we simulate demographics forward using the initial population distribution $\{N_{j,-1}\}_{j=0}^{J}$ and the forcing variables $\{M_{jt}, \phi_{jt}, N_{0,t+1}\}_{-1\le t\le T, 0\le j\le J}$ to obtain $\{\pi_{j,t}, N_{j,t}\}_{0\le t\le T, 0\le j\le J}$ and population growth rates $\{n_t\}_{j=0}^{T}$. The forcing variables are obtained from the UN World Population Prospects until 2100. From 2100 on, we assume that the survival rates ϕ_{jt} and migration rates $\frac{M_{jt}}{N_t}$ are kept constant at their 2100 levels. We further assume that the growth rate of the number of births, $N_{0,t+1}/N_{0t}$, adjusts linearly in every country from its 2100 level to a common long-run rate of -0.5% by 2200. Given the effective labor supply profile and the retirement policy, the demographic projections imply a path for aggregate labor $\{L_t\}_{t=0}^{T}$ from (A.82).

Next, given a path for the interest rate $\{r_t\}_{t=0}^{T}$, technological parameters, and aggregate labor, we can obtain the optimal capital-labor ratio from (A.79) and other production aggregates as well as the wage rate $\{\frac{K_t}{L_t}, K_t, Y_t, I_t, w_t\}_{t=0}^{T}$ follow from (A.77)-(A.81).

Given a government fiscal rule $\{B_{t+1}/Y_{t+1}, \rho_{jt}, \varphi^G, \varphi^\tau, \varphi^d, \bar{G}, \bar{\tau}, \bar{d}\}_{t=0}^{T}$, we obtain the path for the policies $\{G_t, \tau_t, d_t\}_{t=0}^{T}$ from (A.85)-(A.87) such that the government budget constraint (A.83) is satisfied for every t.

Then, we solve the household problem as follows. Given a guess for total bequests received by type θ across all ages $\{Beq_t^r(\theta)\}_{t\ge 0,\theta}$.[86], a path of prices $\{r_t, w_t\}_{t=0}^{T}$, government policy $\{\rho_{jt}, \tau_t, d_t\}_{t=0}^{T}$, demographic variables $\{n_t, \pi_{j,t}, \phi_{j,t}\}_{0\le t\le T, 0\le j\le J}$, we solve the household problem (A.72) in two steps. First, we use Carroll (2006)'s Endogenous Grid Point Method (EGM) to determine the decision functions $\{c_{jt}(\theta,\epsilon,\mathfrak{a})\}_{t\ge 0, 0\le j\le T}$ and $\{a_{j+1,t+1}(\theta,\epsilon,\mathfrak{a})\}_{t\ge 0, 0\le j\le T}$, assuming constant prices after 2300. Second, we obtain the distributions following Young (2010). We start from an initial distribution, which we take from the 2016 steady state, and iterate forward using the asset decision function and the law of motion of the state (θ, ϵ). We then compute aggregates following (A.76).

To solve for the world economy equilibrium, we use a Newton-based method, with Jacobians calculated with the algorithm from Auclert et al. (2021), to ensure that bequests received equals bequests given by type θ and that the asset market clearing condition (A.91) is satisfied. We iterate on a 285×1 path for the interest rate by year $\{r_t\}_t$, and a $285 \times 25 \times 3$ path for bequest by year, country and type $\{B^{r,c}(\theta)\}_{t,c,\theta}$ until convergence.

[86] $Beq_t^r(\theta) \equiv \sum_{\theta_-}\left(\frac{\Pi^\theta(\theta|\theta_-)\pi^\theta(\theta_-)}{\pi^\theta(\theta)}\right) \times \sum_{k=0}^{T}\left[N_{k,t-1} + M_{k,t-1}\right](1-\phi_{k,t-1}) \times \sum_\epsilon \pi^\epsilon(\epsilon)\int_{\mathfrak{a}} \mathfrak{a} dH_{kt}(\mathfrak{a}|\theta_-,\epsilon)$, so that bequests per age-j person of type θ is $b_{jt}^r(\theta) = \frac{F_j}{N_{jt}} Beq_t^r(\theta)$.

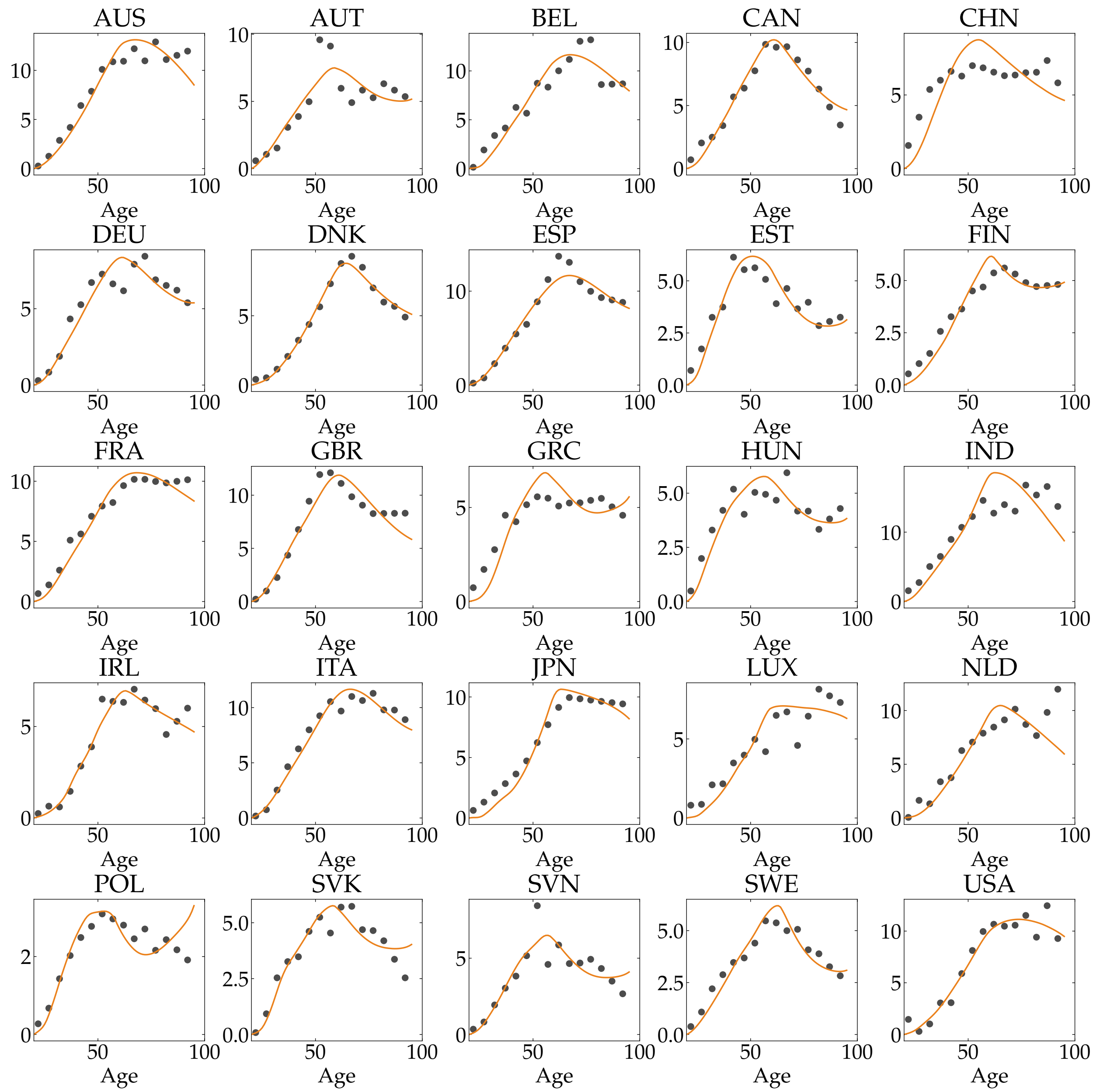

Figure A.11: Calibration outcomes: wealth

Notes: This figure presents the empirical age-wealth profiles (gray dots) and the calibrated model age-wealth profiles in the baseline calibration (orange line) for the 25 countries we consider.

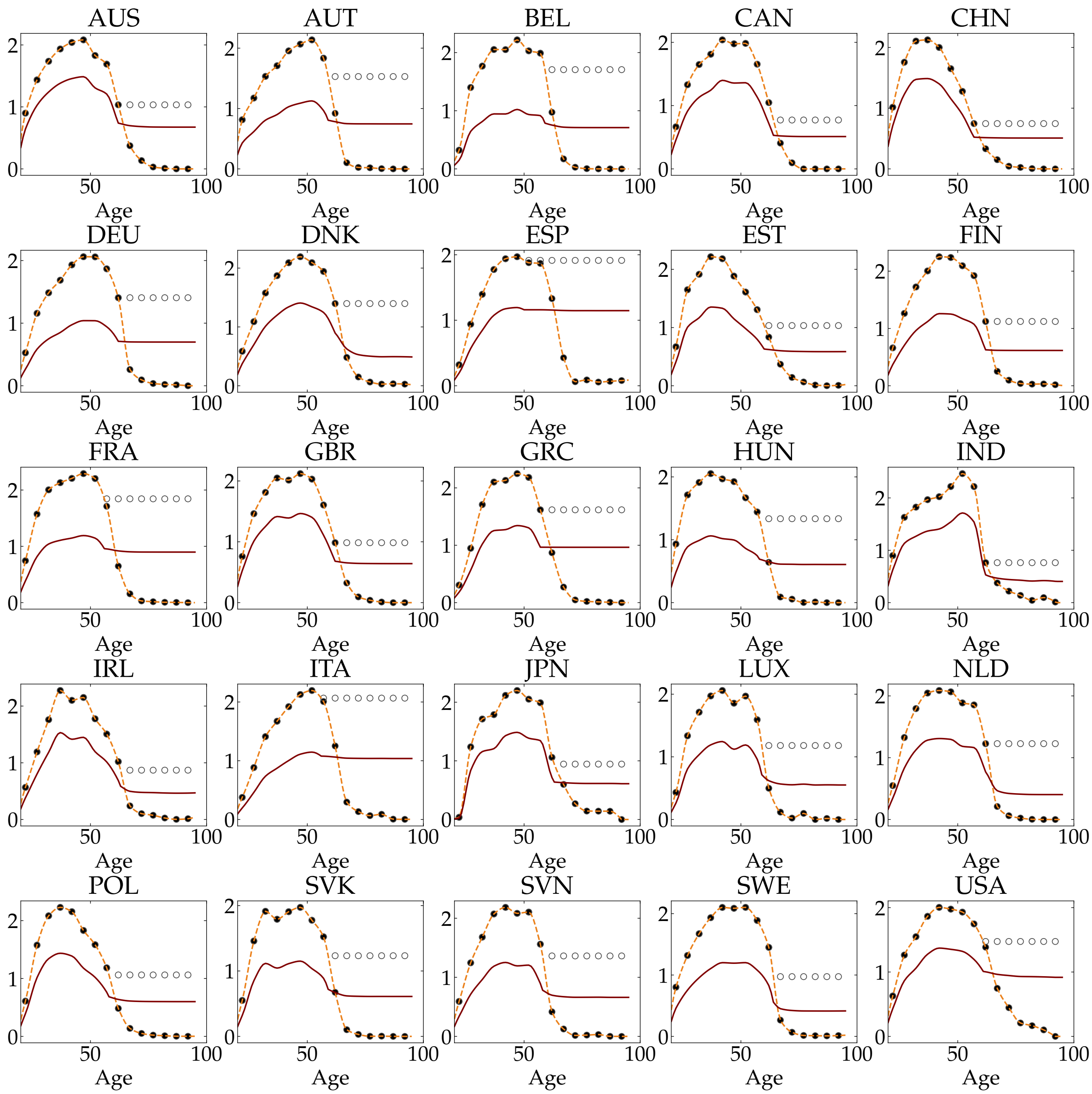

Figure A.12: Calibration outcomes: labor income

Notes: This figure presents the empirical age-labor supply profile from LIS used in section 2 (black dots), as well as the model gross age-labor supply profile (dashed orange line) and the net-of-taxes profile (red line).

Table A.12: World economy calibration (part 1)

	$\Delta_{2100}^{comp,c}$		Components of wealth			Government policy	
Country	Model	Data	$\frac{W^c}{Y^c}$	$\frac{B^c}{Y^c}$	$\frac{NFA^c}{Y^c}$	τ^c	$\frac{Ben^c}{Y^c}$
AUS	33	33	5.24	0.40	-0.49	0.29	0.04
AUT	25	24	3.87	0.83	0.12	0.47	0.11
BEL	28	28	5.27	1.06	0.60	0.54	0.09
CAN	21	20	4.61	0.92	0.28	0.31	0.04
CHN	45	45	4.25	0.44	0.26	0.30	0.04
DEU	19	19	4.36	0.69	0.46	0.50	0.10
DNK	18	18	3.67	0.37	0.58	0.36	0.06
ESP	38	37	5.70	0.99	-0.73	0.39	0.10
EST	21	21	3.14	0.09	-0.29	0.39	0.07
FIN	22	21	2.82	0.63	0.13	0.44	0.09
FRA	30	30	5.01	0.98	-0.04	0.48	0.13
GBR	23	23	5.41	0.88	0.16	0.31	0.06
GRC	30	28	3.52	1.81	-1.25	0.40	0.16
HUN	20	20	3.22	0.76	-0.49	0.48	0.09
IND	62	56	4.84	0.68	-0.08	0.30	0.01
IRL	49	49	2.61	0.74	-1.58	0.33	0.03
ITA	31	30	6.02	1.31	-0.04	0.48	0.13
JPN	26	26	5.05	2.36	0.66	0.32	0.09
LUX	40	39	2.92	0.21	0.70	0.40	0.07
NLD	30	33	4.39	0.62	0.68	0.37	0.05
POL	36	34	1.70	0.54	-0.48	0.36	0.10
SVK	33	33	2.88	0.52	-0.59	0.42	0.07
SVN	22	22	3.29	0.79	-0.19	0.43	0.11
SWE	17	17	2.69	0.42	0.06	0.43	0.06
USA	29	28	4.40	1.07	-0.36	0.32	0.06

Table A.13: World economy calibration (part 2)

Country	$\bar{\beta}^c$	ξ^c	Y^c	ν^c	$1 - s_L^c$	G^c/Y^c	n^c	$J^{r,c}$
AUS	1.00	0.00051	78.5	1.92	0.51	0.09	0.011	63
AUT	1.03	-0.00124	78.5	1.92	0.28	0.22	0.005	60
BEL	1.03	-0.00020	78.5	1.92	0.34	0.25	0.005	60
CAN	1.00	-0.00051	78.5	1.92	0.33	0.16	0.009	65
CHN	0.98	0.00004	78.5	1.92	0.34	0.15	0.005	58
DEU	1.04	-0.00101	78.5	1.92	0.31	0.23	-0.000	63
DNK	0.94	0.00040	78.5	1.92	0.26	0.20	0.003	63
ESP	1.10	-0.00120	78.5	1.92	0.52	0.07	0.003	51
EST	0.99	-0.00175	78.5	1.92	0.32	0.20	-0.002	61
FIN	0.92	0.00035	78.5	1.92	0.20	0.25	0.001	63
FRA	1.01	-0.00003	78.5	1.92	0.39	0.15	0.002	57
GBR	1.02	-0.00059	78.5	1.92	0.42	0.11	0.005	63
GRC	1.02	-0.00163	78.5	1.92	0.28	0.10	0.000	58
HUN	0.99	-0.00120	78.5	1.92	0.28	0.24	0.000	59
IND	1.03	0.00003	78.5	1.92	0.40	0.17	0.011	63
IRL	0.90	0.00127	78.5	1.92	0.33	0.18	0.008	64
ITA	1.09	-0.00122	78.5	1.92	0.45	0.10	-0.002	57
JPN	0.90	0.00148	78.5	1.92	0.19	0.13	-0.000	64
LUX	0.90	0.00147	78.5	1.92	0.19	0.25	0.023	60
NLD	0.94	0.00095	78.5	1.92	0.29	0.20	0.002	63
POL	0.86	-0.00092	78.5	1.92	0.16	0.19	-0.000	59
SVK	0.97	-0.00077	78.5	1.92	0.28	0.22	0.001	60
SVN	1.00	-0.00132	78.5	1.92	0.26	0.19	0.002	59
SWE	0.95	-0.00056	78.5	1.92	0.21	0.28	0.007	65
USA	0.97	0.00071	78.5	1.92	0.35	0.13	0.005	62

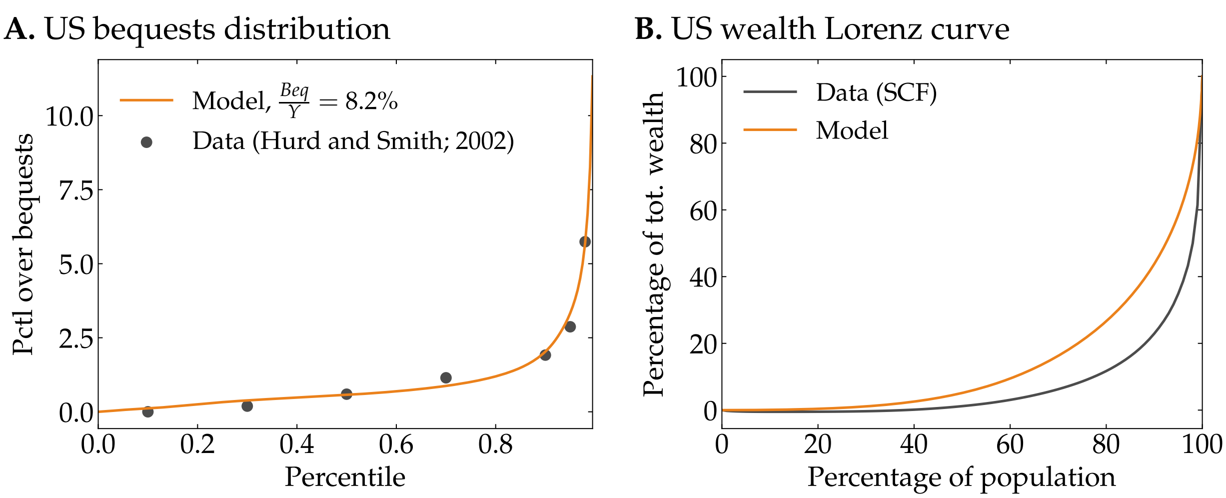

Figure A.13: Distribution of bequests and wealth Lorenz curve in the US

To solve for the small open economy, we hold fixed the path of the interest rate, i.e. $r_t = r_0, \forall t > 0$.

Details on table 3. Below, we provide details on the results in table 3, starting with the construction of each column, and then the details on the various experiments. The description of the columns applies to the full model analysis; for the sufficient statistic analysis, some columns have a slightly different interpretation, which is clarified when we discuss this experiment. For all columns, the changes refer to differences between 2016 and 2100. In the left panel, Δr is the change in the rate of return, $\overline{\Delta \log \frac{W}{Y}} \equiv \sum_c \omega^c \Delta_{2100} \log\left(\frac{W^c}{Y^c}\right)$ is the average change in the wealth-to-output ratio, weighted by initial shares of wealth.

In the right panel, $\bar{\Delta}^{comp} \equiv \sum_c \omega^c \Delta_{2100}^{c,comp}$ is the average compositional effect between 2016 and 2100, weighted by initial wealth levels. The term $\bar{\Delta}^{soe} \equiv \sum_c \omega^c \Delta_{2100}^{c,soe}$ is the equivalent average for the small open economy effect. For each country c, $\Delta^{c,soe}$ is defined as the change in $\frac{W^c}{Y^c}$ between 2016 and 2100 in a small open economy equilibrium with a fixed interest rate r_{2016}.

The asset supply and demand semielasticities $\bar{\epsilon}^d = \sum_c \omega^c \epsilon^{c,d}$ and $\bar{\epsilon}^s = \sum^c \omega_c \epsilon^{c,s}$ are the averages of the country semielasticities weighted by initial wealth levels. For each country c, the asset demand sensitivity $\epsilon^{d,c}$ is defined as the semielasticity of the steady-state $\frac{W^c}{Y^c}$ with respect to the steady state interest rate r.[87] The asset supply semielasticities are given by $\epsilon^{s,c} = \frac{1}{W^c/Y^c}\frac{\eta}{r+\delta}\frac{K^c}{Y^c}$.

The list below describes the sufficient statistic analysis and the various extended model experiments. All extended model experiments start from a steady-state equilibrium calibrated to 2016.

- **Extended model.** This is the full quantitative model discussed in this section, with a fiscal rule whose adjustment places equal weight on consumption, taxes, and retirement benefits.
- **Sufficient statistic analysis.** This row reproduces the baseline exercise in section 3 for our central parameter choices of $\sigma = 0.5$ and $\eta = 1$.
- **Drop annuities, add bequests.** This modifies the baseline model in section 2 by removing annuities and instead assuming that dying individuals leave bequests, which deliver joy-of-giving utility as in the extended model. It also replaces the calibration strategy in the sufficient statistic model (which implicitly varies β_j at each age to perfectly target the age-wealth profile in each country) with the strategy from our extended model (which calibrates a quadratic profile for $\log \beta_j$ and also a utility from bequests Y in each country to minimize squared deviation from the age-wealth profile), except that there is still no income heterogeneity within age (θ and ϵ are both constant at 1), and ν is exogenously chosen to be the same as in the full extended model because it is not well-identified in the absence of income heterogeneity. The bequest distribution rule is the same as in the full model, but here we assume that bequests received b^r_{jt}/w_t by age j, relative to wages, do not vary in the transition.[88] We also assume that when optimizing, individuals do not perceive any changes in the mortality rate $1-\phi_{jt}$ relative to its initial level (like in the baseline model); the time-varying ϕ_{jt} only shows up ex-post in the evolution of the population by age.
- **Adjust bequests received.** $\frac{b^r_{jt}}{w_t}$ now adjusts in the transition in response to demographic change, including time-varying mortality $1-\phi_{jt}$, like in the full model (although note that there is still not heterogeneity in types θ).
- **Add income risk.** This now adds income risk ϵ and permanent income types θ. We now calibrate ν as well to hit US bequest inequality from Hurd and Smith (2002) in the initial steady state, which is now the same as in the full extended model.
- **Change perceived mortality.** Individuals now perceive the true dynamic path of ϕ_{jt} when making their savings and consumption decisions.

[87] In practice, we calibrate a steady-state to 2100 demographics, and perturb r_{2016} and re-solve for a new stationary equilibrium, using the resulting perturbation to $\frac{W^c}{Y^c}$ to calculate the derivative.

[88] Concretely, this is implemented with an age-specific lump sum tax offsets any change in bequests, so that individuals receive the same as they would have in the 2016 steady state. As in the baseline model, the government balances its budget in light of this tax by adjusting G.

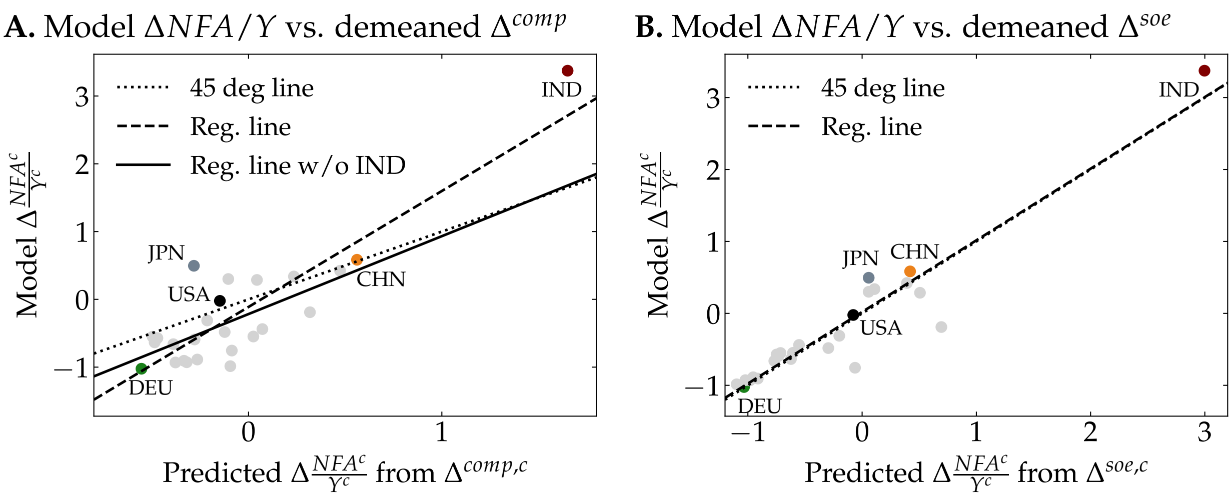

Figure A.14: Predicting change in net foreign asset position

Notes: Panel A presents the model-implied change in NFA/Y between 2016 and 2100 on the y-axis, and on the x-axis the change in NFA/Y predicted from the demeaned model compositional effect, $NFA/Y \approx \exp(\Delta^{comp,c} - \bar{\Delta}^{comp}) - 1$, over the same period. The dotted line is a 45 deg line. The dashed line is a regression line, and the solid line is this same regression line when India is excluded. Panel B also shows the model $\Delta NFA/Y$ on the y-axis, but the x-axis presents the change in NFA/Y predicted from the demeaned model small open economy effect, $NFA/Y \approx \exp(\Delta^{soe,c} - \bar{\Delta}^{soe}) - 1$.

- **Increase retirement age.** The retirement policy ρ_{jt} in each country now shifts to higher ages by 1 month per year for the first 60 years of the transition starting in 2016, for a cumulative increase of 5 years, as in the extended model.
- **Change taxes and transfers (= extended model).** To balance their budgets subject to maintaining a constant B/Y, governments now adjust a mix of G, taxes τ, and the level $\bar{d}$ of social security payments, closing one-third of the budget cap with each. This brings us to the full extended model (so that this line is the same as the first "extended model" line of the table).
- **Alternative fiscal rules.** Here, rather than fiscal adjustment happening through equal weight on G, τ, and $\bar{d}$ (as in the full model), it occurs entirely through either G, τ, or $\bar{d}$ alone. (Note that fiscal adjustment through G alone is the same as the "Increase retirement age" line above, prior to the final line implementing the equal adjustment rule.)

Changes to net foreign asset positions. Appendix Figure A.14 summarizes the model's predictions for the change in net foreign asset position in each country from 2016–2100. Panel A compares the full model findings to the method used in section 3 by plotting the full model results on the vertical axis, and the prediction based on demeaned compositional effects $\Delta^{comp,c} - \bar{\Delta}^{comp}$ on the horizontal axis. The compositional predictions are generally quite accurate, and the line of best fit excluding India is close to 45 degrees. In India, however, the model predicts even larger net foreign asset position growth than expected from the compositional effect.

Panel B shows that this discrepancy disappears, and the fit is even closer, when we use the demeaned small open economy effect $\Delta^{soe,c}$ for predictions on the horizontal axis instead. This shows that discrepancies in panel A, including for India, are mostly due to the non-compositional effects $\Delta^{soe,c} - \Delta^{comp,c}$ of aging in our model, rather than non-linearities or heterogeneity in elasticities.

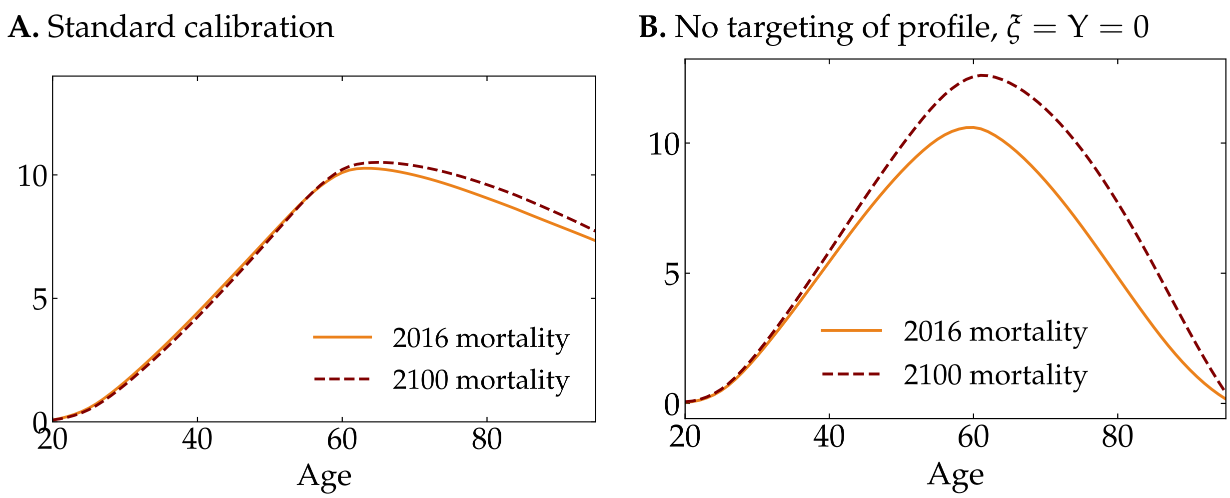

Figure A.15: Effect of changing perceived mortality on average steady-state asset profiles

E.6 Effect of mortality absent bequest preferences.

Figure A.15 plots the effect of changing perceived mortality on average steady-state asset profiles under two different calibrations: first, the main calibration of our extended model, and second, an alternative calibration that still targets wealth-to-GDP, but does not target age-wealth profiles, and instead sets the parameters we used to target those profiles (most importantly bequest utility Υ, and also a quadratic discount factor term ξ) to zero. For each calibration, we solve for household's asset profiles relative to GDP per capita in each country, and define the world age-wealth profile as the GDP-weighted average of these profiles. We do this for two levels of perceived mortality, the 2016 level and the projected 2100 level, holding all other inputs to the household problem fixed at 2016 levels in every case.

Panel A shows the results under our main calibration. In line with the results in table 3, there is a positive response of asset accumulation to lower mortality, but it is relatively muted. For the ages before 60, the effect is actually slightly *negative*, since bequests outcompete the traditional life-cycle savings motive, reflecting the fact that households can put off savings until later once there is a low risk of dying at an early age.

Panel B shows the result under the alternative calibration. Two things stand out. First, there is rapid decumulation of assets late in life, reflecting the well-known phenomenon that a standard life-cycle model struggles to rationalize limited decumulation without adding other features such as bequest motives or late-in-life medical expenses. Second, the effect of reducing mortality is much stronger, with assets being 50% higher at age 80 for 2100 mortality compared to for 2016 mortality.

F Appendix to Section 5

We first prove the results in the main text. Defining savings for an individual of age j in state (z^j, a_{jt}) at time t as

$$\mathfrak{s}_{jt} \equiv r\mathfrak{a}_{jt} + w_t \left((1-\tau)\ell(z_j) + tr(z^j) \right) - \mathfrak{c}_{jt}$$

and using the budget constraint (1), we see that aggregate savings for agents of age j is given by

$$s_{jt} = \mathbb{E}\mathfrak{s}_{jt} = \phi_j a_{j+1,t+1} - a_{jt} \tag{A.105}$$

Next, since lemma 1 implies $a_{jt} = a_j(r)Z_t$, we have

$$s_{jt} = \left(\phi_j(1+\gamma)a_{j+1} - a_j(r)\right) Z_t = s_j(r) Z_t$$

Hence, defining aggregate savings as

$$S_t \equiv \sum N_{jt} s_{jt} \tag{A.106}$$

we have that

$$\frac{S_t}{N_t} = \sum \pi_{jt} s_{jt} = \sum \pi_{jt} \underbrace{s_j(r)\, Z_0}_{s_{j0}} (1+\gamma)^t = \sum \pi_{jt} s_{j0} (1+\gamma)^t$$

Taking the ratio of this expression to equation (8), we obtain the equivalent of Proposition 1,

$$\frac{S_t}{Y_t} = \frac{F_L(k(r),1)}{F(k(r),1)} \cdot \frac{\sum \pi_{jt} s_{j0}}{\sum \pi_{jt} h_{j0}} \tag{A.107}$$

which delivers equation (24).

Next, combining (A.105), (A.106), and the population dynamics equation $N_{j+1t+1} = \phi_j N_{jt}$, we have

$$S_t \equiv \sum N_{jt} s_{jt} = \sum N_{jt}\phi_j a_{j+1,t+1} - \sum N_{jt} a_{jt} = \sum N_{j+1,t+1} a_{j+1,t+1} - \sum N_{jt} a_{jt} = W_{t+1} - W_t$$

where the last line uses the initial and terminal condition on wealth by age. Hence, the aggregate savings rate is:

$$\frac{S_t}{Y_t} = \frac{W_{t+1} - W_t}{Y_t} = \frac{Y_{t+1}}{Y_t}\frac{W_{t+1}}{Y_{t+1}} - \frac{W_t}{Y_t} = (1+g_{t+1})\frac{W_{t+1}}{Y_{t+1}} - \frac{W_t}{Y_t}$$

where g_t is the growth rate of aggregate GDP, the sum of productivity growth, population growth and changing composition,

$$1 + g_{t+1} \equiv \frac{Y_{t+1}}{Y_t} = (1+\gamma)\frac{N_{t+1}}{N_t}\frac{\sum_j \pi_{jt+1} h_{j0}}{\sum_j \pi_{jt} h_{j0}} = (1+\gamma)(1+n_{t+1})\frac{\sum_j \pi_{jt+1} h_{j0}}{\sum_j \pi_{jt} h_{j0}}$$

In steady state, therefore, we have

$$\frac{S}{Y} = g\frac{W}{Y} \tag{A.108}$$

where $1 + g = (1+\gamma)(1+n)$. This is the famous Solow (1956)–Piketty and Zucman (2014) formula for the relationship between the net savings rate W/Y, the growth rate of GDP g, and the wealth-to-GDP ratio W/Y. Furthermore, since $W/Y = K/Y + B/Y$ in equilibrium, we recover equation (25).

Finally, towards our implementation, we show that S_t/Y_t can be calculated from the cross-sectional profiles of assets a_{jt} and demographic projections alone. We first show that S_t/Y_t in equation (24) can be calculated from cross-sectional age profiles of assets $a_{j,0}$. Indeed, starting from $S_t = W_{t+1} - W_t$, we have

$$\begin{aligned}
\frac{S_t}{N_t (1+\gamma)^t} &= \frac{W_{t+1}}{N_t (1+\gamma)^t} - \sum \pi_{jt} a_{j0} \\
&= (1+n_{t+1})(1+\gamma) \sum \pi_{jt+1} a_{j0} - \sum \pi_{jt} a_{j0} \\
&= ((1+n_{t+1})(1+\gamma) - 1) \sum \pi_{jt} a_{j0} + (1+n_{t+1})(1+\gamma) \sum (\pi_{jt+1} - \pi_{jt}) a_{j0} \\
&= g_{t+1}^{ZN} \sum \pi_{jt} a_{j0} + \left(1 + g_{t+1}^{ZN}\right) \sum (\Delta \pi_{jt+1}) a_{j0}
\end{aligned}$$

where we have defined $1 + g_{t+1}^{ZN} \equiv (1+n_{t+1})(1+\gamma)$. Taking the ratio of this expression to equation (8), we have the following expression for the aggregate savings rate:

$$\frac{S_t}{Y_t} = \frac{F_L (k(r), 1)}{F(k(r), 1)} \left(\frac{g_{t+1}^{ZN} \sum \pi_{jt} a_{j0} + (1 + g_{t+1}^{ZN}) \sum (\Delta \pi_{jt+1}) a_{j0}}{\sum \pi_{jt} h_{j0}} \right) \tag{A.109}$$

which is an alternative to equation (A.107).

In principle, to project savings rates from demographic composition, we could equally well implement equation (A.107) or equation (A.109). Summers and Carroll (1987), Auerbach and Kotlikoff (1990), and Bosworth et al. (1991) follow the first route. We prefer to follow the second because it only requires only information that we have already used so far in the paper, and because the computation of age-specific savings rates is subject to a large amount of measurement error.

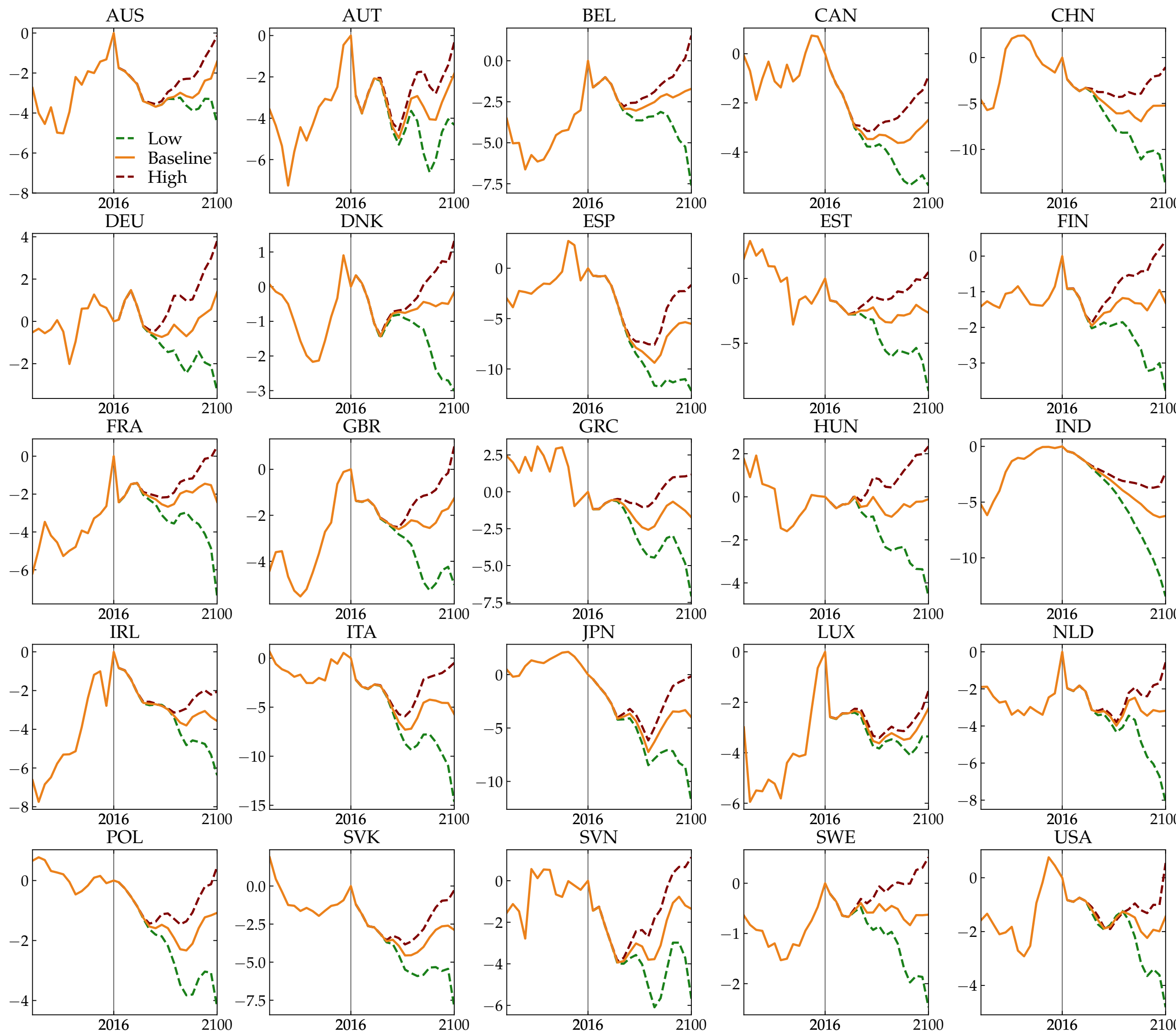

Figure A.16: Compositional effects on savings-to-GDP

Notes: This figure depicts the evolution of the implied change in the savings-to-GDP ratio from the compositional effect for t =1950 to 2100, reported in percentage points. The base year is 2016 (vertical line). The solid orange line corresponds to the medium fertility scenario from the UN, the dashed green line to the low fertility scenario, and the dashed red line to the high fertility scenario.

G Interpreting literature findings

In this appendix, we show that our results are useful to understand existing findings in the literature. First, across papers that conduct a similar exercise, we trace results back to their inputs, and show why different assumptions about the compositional effect are a critical driver of the differences in general equilibrium outcomes. Second, within papers that consider the role of parameter changes, we show that our results are useful in explaining the functional form relationship between these parameters and general equilibrium outcomes. In the interest of space, we focus on the effect of demographic change on the total return r (sometimes referred to as the natural interest rate, or r^*, in the literature).

G.1 Explaining different magnitudes across papers

Eggertsson et al. (2019) (EMR) and Gagnon et al. (2021) (GJLS) are two recent papers that find very different effects of demographics on real interest rates. Both study the US economy using closed-economy general equilibrium models, but EMR find that demography reduced real interest rates by 3.44 percentage points between 1970 and 2015, while GJLS only find an effect of 0.92 percentage points, a difference of 2.52 percentage points. We use publicly available replication files[89] to create table A.14, which applies the framework of proposition 5 to explain these results in terms of the underlying differences in compositional effects Δ^{comp}, non-compositional effects $\Delta^{soe} - \Delta^{comp}$, and semielasticities ϵ^d and ϵ^s.

The single most important difference is that the compositional effect in EMR is more than three times as large as that in GJLS. If EMR had the same compositional effect as GJLS, more than half of the gap between the two estimates would be closed. EMR also have a far lower asset supply semielasticity ϵ^s, one-fourth as large as GJLS. If EMR also had the same ϵ^s as GJLS, the first-order approximation would imply $\Delta r = -1.12\%$. very close to GJLS.

The results on compositional effects can be interpreted using figure A.17, which shows the asset profiles by age and the population distribution shifts in the two papers and in the data. Two forces explain the large compositional effect in EMR. First, the age-wealth profile is much steeper than in the data, staying below zero until age 46 and then rising sharply. This inflates the effect of shifting the age distribution toward older ages. Second, the shift in age composition itself is very large, because the exercise compares a steady state based on 2015 fertility and mortality with a steady state based on 1970 demographics (for which EMR take 1970 mortality and, since agents in the model come of age after 25 years, 1945 fertility). Due to the slow convergence rate of the empirical age distribution, these two steady states have larger differences in age distribution than the actual change that occurred between 1970 and 2015.[90]

For the asset supply semielasticity ϵ^s, the lower value in EMR partly reflects their assumption of a lower elasticity of substitution between capital and labor relative to GJLS ($\eta = 0.6$ versus $\eta = 1$). However, even with $\eta = 1$, EMR would only have $\epsilon^s = 4.6$, less than half that of GJLS. The remaining difference reflects a second, more subtle, reason for EMR having a low ϵ^s, namely that ϵ^s scales with the share of capital in total wealth K/W, which is 1 in GJLS and only 0.51 in EMR. Capital is a small part of wealth in EMR because high (uncapitalized) markups mean that capital owners only receives $\sim 10\%$ of total output, with a resulting low capital-output ratio of $K/Y = 124\%$. Combined with a high level of bonds $B/Y = 117\%$, capital becomes a small part of total wealth, lowering the responsiveness of asset supply to changes in r.

For comparison, we also include the results of the sufficient statistic analysis from section 3 applied to the same time period. For Δ^{comp}, the sufficient statistic result comes directly from the data and is closer to GJLS than to EMR. This reflects the fact that GJLS closely target the change in age distribution over time, and also do a good job fitting the age profile of wealth for all but the highest ages, which are of limited

[89]Replication repositories: `https://www.openicpsr.org/openicpsr/project/114159/version/V1/view` (EMR) and `https://sites.google.com/site/etigag/gjls-replication-materials` (GJLS).

[90]In addition to this comparison of steady states, EMR also perform an exercise with explicit transitional dynamics. This exercise features a smaller Δ^{comp} for 1970 to 2015—albeit one that is still somewhat overstated, due to the steep age-wealth profile and since the exercise starts with the 1970 steady state. Overall, however, the decline in r in this exercise from 1970 to 2015 is quite similar to the decline in r in the steady state exercise.

Table A.14: Decomposing change in r for US closed economy in existing papers

	Eggertsson et al. (2019)	Gagnon et al. (2021)	Sufficient statistic
Time-period	1970–2015	1970–2015	1970–2015
GE transition			
Δr^{GE}	-3.44%	-0.92%	
First-order approximation $\Delta r = \frac{-\Delta^{soe}}{\epsilon^d + \epsilon^s}$			
Δr	-2.96%	-0.97%	-0.45%
Δ^{comp}	45.4%	13.4%	10.8%
$\Delta^{soe} - \Delta^{comp}$	21.1%	25.3%	0%
ϵ^s	2.8	11.1	8.1
ϵ^d	19.7	28.5	15.6
σ	0.75	0.5	0.5
η	0.6	1.0	1.0

Notes: This table analyzes two key results from Eggertsson et al. (2019) (EMR) and Gagnon et al. (2021) (GJLS) using the framework of proposition 5. In GJLS, we analyze the 1970 to 2015 segment of the paper's main experiment, which is a simulation of the effects of demographic change between 1900 and 2030. In EMR, we analyze jointly the two demographic experiments from table 6 ("mortality rate" and "total fertility rate"). These are steady state experiments that consider the effect of changing fertility and mortality from their 2015 to their 1970 level. For both experiments, Δr^{GE} is the general equilibrium change in r from 1970 to 2015, Δ^{comp} is our compositional effect measure, implemented using the two papers' 2015 age profiles and the age distributions for 1970 and 2015, and ϵ^s is the semielasticity of asset supply $(B+K)/W$ in 2015 with respect to r. For EMR, Δ^{soe} is given by the change in W/Y between the 1970 and 2015 steady state when both have $r = r_{2015}$ and ϵ^d is the derivative of $\log W/Y$ to r in the 1970 steady state (similar to our exercise in the paper). For GJLS, Δ^{soe} is the counterfactual change in W/Y in a simulation where r is fixed after 1970, and ϵ^d is the derivative of $\log W/Y$ to r around a steady state defined to have the same population age distribution as the one observed in 2015. The sufficient statistic column applies the method in section 3 to 1970-2015, constructing Δ^{comp} from observed changes in the age distribution from 1970 to 2015 together with age profiles of assets and labor income from 2016, and asset semielasticities from (19) and proposition 4, for ϵ^s using the 2016 value of K/W, and for ϵ^d using the 2016 profiles of assets and labor income and 1970 demographics, together with $\sigma = 0.5$ and $\eta = 1$.

quantitative importance before 2015. For ϵ^s, the results in the sufficient statistic analysis lie above EMR and below GJLS. Apart from having a higher η than EMR, this mainly reflects the fact that our assumed share of capital in wealth $K/W = 0.76$ is between the values in GJLS and EMR.

While the non-compositional effects $\Delta^{soe} - \Delta^{comp}$ are zero in the sufficient statistic analysis, they are positive in EMR (21.1%) and GJLS (25.3%), and relatively large compared to what we find in the quantitative analysis in section 4. The non-compositional effect is especially pronounced in GJLS, where it is twice as large as the compositional effect. This reflects a very strong response of asset accumulation to falling mortality. This is largely due to the lack of bequest motive in GJLS, which implies that all saving is for personal consumption needs, which scale proportionally with survival probabilities. In our model in section 4, the bequest motive scales with mortality and counterbalances this effect; the role of saving for personal consumption in retirement is further diluted by the presence of a social security system.

G.2 Understanding the role of parameter changes

Our results in section 2 uncover a structural relationship between primitive parameters, calibration moments, and general equilibrium counterfactuals. For instance, combining the results in equations (13) and (17), the inverse effect on the interest rate of a change in demographics that creates a compositional effect

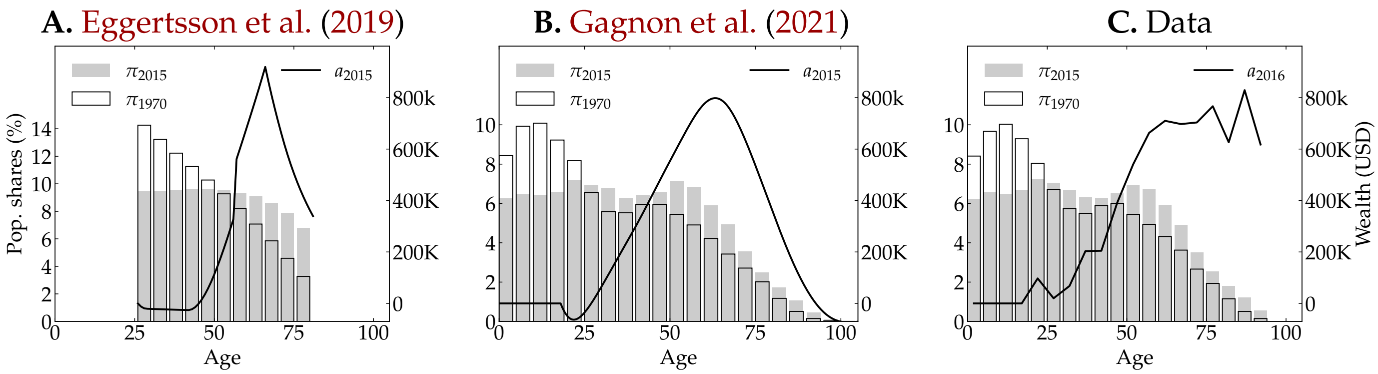

Figure A.17: Age-wealth profiles in papers vs the data

of $\bar{\Delta}^{comp}$ is given by a simple affine function,

$$\frac{1}{dr} = -\frac{\bar{\epsilon}^{income} - \bar{\epsilon}^{laborshare}}{\bar{\Delta}^{comp}} - \sigma\frac{\bar{\epsilon}^{substitution}}{\bar{\Delta}^{comp}} - \eta\frac{\bar{\epsilon}^{laborshare} + \frac{1}{r+\delta}\frac{\bar{K}}{\bar{W}}}{\bar{\Delta}^{comp}} \quad \text{(A.110)}$$

Plugging in the elasticity values from section 3.2, we obtain

$$\frac{1}{dr} = \frac{7.5}{\bar{\Delta}^{comp}} - \sigma\frac{39.5}{\bar{\Delta}^{comp}} - \eta\frac{13.5}{\bar{\Delta}^{comp}}$$

For the 2016-2100 period, we can take $\bar{\Delta}^{comp} = 32\%$ from section 3, and obtain (for r in %)

$$\frac{1}{dr} = 0.23 - 1.23 \cdot \sigma - 0.42 \cdot \eta$$

Equation (A.110) shows that, conditional on having recalibrated the model to hit the same data moments and therefore the same $\bar{\Delta}^{comp}$ and $\bar{\epsilon}$'s,[91] the effects of σ and η are additively separable for the inverse general equilibrium effect on interest rates, $1/dr$.

To illustrate the potential of this equation for interpreting findings in other papers, we study the results in Papetti (2021a), who provides a comprehensive structural OLG quantitative model of the Euro Area. In tables 2 and 3 of the working paper version (Papetti 2019), the author reports his model's predicted effect of demographics on the change in the real interest rate change over the period 1990 – 2030, which we call dr, first as a function of risk aversion $1/\sigma$, and then as a function of capital-labor substitution η. We reproduce his results in table A.15. Observe that all his estimates of the effect of demographics on interest rates over this period are all negative.

Note further that the inverse effect on the interest rate, $1/dr$, appears to be linear in both σ and η, just like equation (A.110) predicts. To confirm this, we run a linear regression of $1/dr$ on σ and η and obtain:

$$\frac{1}{dr} = 0.67 - 2.22 \cdot \sigma - 0.81 \cdot \eta$$

with an R^2 of 0.993. The quality of the fit of the functional form is remarkable. The coefficients are around two times larger than our coefficient for 2016-2100, so the interest rate effects are about half in our model what they are in his. One obvious distinction is that our results are for an 80 year period, while his are for a 40 year period. In addition, the fundamental inputs into $\epsilon^{substitution}$, ϵ^{income} are different, and the compositional effects $\bar{\Delta}^{comp}$ in his model appear to be lower than in ours, perhaps because Papetti (2019) does not directly target wealth profiles in his calibration.

[91] In practice, changing η does does not change the steady state so does not require a recalibration, while changing σ requires adjusting parameters to keep r and the age profiles of assets unchanged.

Table A.15: Understanding the functional form relationship between σ, η and dr

$1/\sigma$	σ	dr for $\eta = 1$	$1/dr$	η	dr for $1/\sigma = 2.5$	$1/dr$
1	1.00	-1.30	-0.77	0.50	-1.34	-0.75
2	0.50	-2.32	-0.43	1.00	-1.30	-0.77
2.5	0.40	-2.71	-0.37	1.25	-1.23	-0.81

Notes: This table presents Papetti (2019)'s findings for the equilibrium change in the real interest rate between 1990 and 2030 (dr) as a function of risk aversion $1/\sigma$ and capital-labor substitution η. The numbers are taken from his tables 2 and 3, and then transformed to make the additively linear relationship between $1/dr$ and σ and η, which is implied by our framework, appear.