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PROCYCLICAL FISCAL POLICY AND ASSET MARKET INCOMPLETENESS

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ABSTRACT

To explain the fact that government spending and tax policy are procyclical in emerging and developing countries, we develop a model for the joint behavior of optimal tax rates and government spending over the business cycle. Our set-up relies on financial frictions, which have been shown to be critical features of emerging markets, captured by various degrees of asset market incompleteness as well as varying levels of debt-elastic interest rate spreads. We first uncover a novel theoretical result within a simple static framework: incomplete markets can account for procyclical government spending but not necessarily procyclical tax policy. Explaining procyclical tax policy also requires that the ratio of private to public consumption comoves positively with the business cycle, which leads to larger fluctuations in the tax base. We then show that the procyclicality of tax policy holds in a more realistic DSGE model calibrated to emerging markets. Finally, we illustrate how larger financial frictions, which amplify the business cycle through more procyclical fiscal policies, have sizeable Lucas-type welfare costs.

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1 Introduction

There is, by now, overwhelming evidence that fiscal policy in emerging and developing countries is procyclical (i.e., expansionary in good times and contractionary in bad), compared to developed countries where fiscal policy is either acyclical or countercyclical.² Figure 1, updated from Reinhart *et al.* (2004), illustrates this stylized fact by plotting the correlation between the cyclical components of real government spending and real GDP for 121 countries: 99 developing (i.e., non-OECD) and 22 developed (i.e., OECD).³ Yellow (light) bars denote developing countries while black bars stand for developed countries. The visual message is striking: most of the yellow mass corresponds to positive values (indicating procyclical government spending) while most of the black mass corresponds to negative values (indicating countercyclical government spending). In fact, the average correlation for developing countries is 0.29 compared to -0.12 for developed countries (significant at the 1 and 5 percent levels, respectively).⁴ While the evidence on the tax side has been more difficult to come by due to the need of collecting data on tax *rates* (the policy instrument), Figure 2, updated from Vegh and Vuletin (2015), establishes the procyclicality of tax policy. Indeed, the average correlation between changes in the VAT rate and real GDP for developing countries is -0.22 (and significantly different from zero at the 1 percent level) and -0.06 for industrial countries (and not significant at the 5 percent level).⁵ ⁶

The heavily countercyclical policy on the spending side in industrial countries is to be expected. Textbook Keynesian models, of course, tell us that, in recessionary times, countercyclical fiscal policy is called for. More recent theoretical work has only confirmed this decades-old prescription. Christiano *et al.* (2011) and Nakata (2016), for example, show that the optimal fiscal policy in a stochastic model with sticky prices is countercyclical.

²For an early study, see Reinhart *et al.* (2004) and, more recently, Frankel *et al.* (2013) and the references therein.

³In this paper, OECD always refers to the original OECD countries.

⁴Of course, the question of causality is critical: is real GDP causing fiscal policy or viceversa? Ilzetzki and Vegh (2008) formally show that the causality from real GDP to government spending is indeed statistically significant.

⁵Section 4 shows that we can reject the null hypothesis that these two averages are the same.

⁶Notice that, in our terminology, a positive correlation between real GDP and the VAT rate indicates countercyclical tax policy (i.e., higher tax rates in good times and lower in bad times) and a negative correlation indicates procyclical tax policy (i.e., lower tax rates in good times or higher tax rates in bad times).

In fact, both papers conclude that countercyclical fiscal policy is even more effective when monetary policy has become powerless because of the zero lower bound. This is true whether monetary policy is taken as given (Christiano *et al.*, 2011) or is chosen optimally as well (Nakata, 2016).

The much more intriguing question, and the focus of this paper, is: why would policy-makers in developing countries conduct fiscal policy in a procyclical way? This is obviously puzzling since it amounts to making an already volatile business cycle even more pronounced. A popular explanation for this procyclical puzzle relies on political-economy considerations.⁷ In Tornell and Lane (1999), a positive shock may lead to a more-than-proportional increase in spending due to various units (e.g., ministries or provinces) staking competing claims on available resources (akin to socially excessive fishing from a common pond). In Alesina *et al.* (2008), voters demand more public goods and/or fewer taxes in good times to prevent the government from appropriating rents (the classic starving the Leviathan argument).

The other key explanation that has been put on the table is imperfections in international capital markets. In an early paper, Riascos and Vegh (2003) show that, in a calibrated model, incomplete markets can explain the procyclicality of government spending. Cuadra *et al.* (2010) show how the combination of incomplete markets with default risk leads to optimal procyclical fiscal policy (both on the spending and revenue sides). In turn, Bauducco and Caprioli (2014) introduce limited commitment in a small open economy model with exogenous government spending and show how this friction can lead to procyclical fiscal policy on the taxation side. More recently, Bianchi *et al.* (2019) show that, in the presence of high risk premia, nominal rigidities, and default risk, the optimal fiscal policy in bad times may be procyclical as the cost of high sovereign spreads dominates Keynesian gains of countercyclicality.

Our starting point is that while existing arguments based on imperfections in capital markets (like default risk and limited commitment) offer plausible and relevant insights into the procyclical fiscal policy puzzle, they miss a much more fundamental question: is there a role for standard incomplete markets (i.e., access to just a risk-free bond in an uncertain

⁷See, for example, Tornell and Lane (1999), Talvi and Vegh (2005), Alesina *et al.* (2008), and Ilzetzki (2011).

world) in explaining procyclical fiscal policy on both the spending and taxation sides? In other words, how far can we go with a canonical small open economy model with incomplete markets in explaining spending and tax procyclicality? In fact, Cuadra *et al.* (2010) argue that incomplete markets *per se* cannot explain procyclical tax policy.⁸ In other words, the literature has left the mistaken impression that incomplete markets may not be enough to generate procyclicality in both government consumption and tax rates.

In this paper, we go back to basics and examine what the workhorse model of a small open economy, operating under incomplete asset markets, has to say about procyclical fiscal policy. We quickly (even in a static setup) uncover a key and novel theoretical result: while incomplete markets are enough to generate procyclical government spending, the same is *not* true of tax rates. In fact, under incomplete markets, tax rates may be acyclical, countercyclical, or procyclical depending on how the ratio of private to public consumption comoves with real GDP over the business cycle. Intuitively, if this comovement is positive, then in good times the tax base (consumption) is increasing more than government spending, which will induce the government to reduce tax rates (procyclical tax policy). Conversely, if this comovement is negative, the tax base is increasing less than government spending in good times, which will prompt the fiscal authority to increase tax rates (countercyclical tax policy). We trace back this endogenous comovement to preference parameters over private and public consumption, which allows us to perform various experiments that shed further light on the mechanism involved. For expositional purposes, we will refer to this channel as the “consumption preference channel.” Importantly, we argue that, in practice, the ratio of private to public consumption comoves positively with the business cycle so that the empirically-relevant case is the one in which incomplete markets lead to procyclical tax rates. In other words, no other ingredients – sovereign risk, limited commitment, high risk premia – other than standard incomplete markets are needed to explain the procyclicality of both government spending and tax rates. It may well be, of course, that other ingredients are needed for quantitative purposes or related stylized facts but, from an applied theory perspective, all we need to explain the puzzle of procyclical fiscal policy in developing countries

⁸Specifically, they argue (p.455) that “[i]n Riascos and Vegh the government can commit to pay its debt, so it faces the same interest rate across states. Since the government always borrows at the international risk free rate, the model is not able to generate a negative correlation between output and tax rates.”

are incomplete markets.

Having used a static model to illustrate this novel result on the cyclicalities of tax rates in the simplest possible setup, we then build a standard DSGE model with incomplete markets to examine whether the intuition built with the static model holds for an infinite-horizon model. Further, to understand the precise role of the degree of market incompleteness, we also examine the benchmark cases of financial autarky and complete markets. To get rid of the unit root that characterizes the basic DSGE model, we incorporate an upward-sloping supply of funds that renders the model stationary and thus amenable to computer solutions (Schmitt-Grohe and Uribe, 2003). Interestingly, the intertemporal nature of the DSGE model brings a second channel into the picture: since households can, for example, borrow to smooth consumption in response to a negative shock, then consumption (and thus the tax base) will fall by less than otherwise, which will reduce the fiscal authority's need to increase taxes (i.e., tax policy will be, all else equal, less procyclical/more countercyclical). For expositional purposes, we will refer to this second channel as the "consumption smoothing channel."

How do these two channels interact in the DSGE model? The steeper the upward-sloping supply of funds, the less consumption smoothing will take place, and, hence, the more similar will be the results to the static case (i.e., the cyclicalities of tax policy will essentially depend on the consumption preference channel). Conversely, the flatter the upward-sloping supply of funds, the more important the consumption smoothing channel becomes and, hence, the less relevant the consumption preference channel. For the more empirically-relevant specification (based on estimations of the debt elasticity), the consumption preference channel clearly dominates and hence our insights from the static model go through. We thus conclude that the cyclicalities of tax policy will be essentially determined by the positive comovement of the ratio of private to public consumption over the business cycle.

We should note that the dominance of the consumption preference channel over the consumption smoothing channel is consistent with an extensive literature that has identified a steep upward-sloping supply of funds as a distinctive amplifying mechanism when accounting for EMEs' business cycles and linked this feature to the presence of stronger financial frictions

in DSGE models (Neumeyer and Perri, 2005; Uribe and Yue, 2006; García-Cicco *et al.*, 2010; Chang and Fernández, 2013; Fernández and Gulan, 2015).

We then examine the role of the degree of persistence of TFP shocks. The main motivation is the work of Aguiar and Gopinath (2007) who suggest that cycles in emerging markets are more volatile because shocks are more persistent. The question is then: how does more persistence affect optimal tax policy? Our analysis clearly indicates that more persistence is associated with more procyclical tax policy. The intuition is simple enough: the more persistent are TFP shocks, the more consumption will react. Hence, in response to a negative TFP shock, for example, consumption will fall by more than if the shock were not as persistent, which reduces the tax base for a prolonged period of time and forces the fiscal authority to increase taxes more. This is thus another channel that would make developing countries more procyclical.

Up to this point, the DSGE analysis will have illustrated the qualitative effects of various kinds of financial frictions on fiscal cyclicalities, without looking into how these dynamics may explain the actual data. To this effect, we conduct a matching moments exercise by fitting the model for non-OECD countries. We conclude that the model does quite a good job in matching our four targeted moments: the standard deviations of output and private consumption, and the correlations between government spending and output and tax rates and output. In addition, the model matches very well the (positive) correlation between GDP and the ratio of private to public consumption, even though this was not a targeted moment.

We then offer a formal quantification of the welfare costs of fiscal procyclicality. As stressed by Reinhart *et al.* (2004), fiscal procyclicality makes an already volatile business cycle in emerging markets even more pronounced (the “when it rains, it pours” phenomenon). To our knowledge, however, no paper has yet provided an estimate of this cost. By calibrating the model for non-OECD countries and varying the debt-elasticity (from low values typical of OECD countries to high values typical of non-OECD countries), we conclude that Lucas-type welfare costs of business cycles become at least twice as large as fiscal policy procyclicality increases accordingly. While higher debt-elasticity will also introduce other costs that may not be directly attributable to more procyclical fiscal policy (like making it

harder for households to smooth consumption), our analysis makes clear that more fiscal procyclicality will indeed be costly to the economy. This offers a compelling policy rationale to a vast literature that has emerged on the potential benefits of fiscal rules in reducing fiscal procyclicality.⁹

Finally, we offer empirical evidence in favor of (a) the results of the model and (b) the financial frictions emphasized in the theoretical and calibration analysis. In terms of the results of the model, we show that non-OECD countries exhibit statistically significant procyclical fiscal policy both on the spending and taxation sides. In terms of the financial frictions, we proxy market incompleteness with capital controls measures and show that such controls are significantly more prevalent in non-OECD than OECD countries (our point of reference). Further, we estimate the debt elasticities used in the model with two different proxies for debt and conclude that the debt elasticities are significantly higher in non-OECD than OECD countries. Finally, we show that GDP volatility is significantly higher in non-OECD than OECD countries. In sum, the empirical evidence is consistent with the idea that countries that exhibit procyclical fiscal policy are characterized by deeper financial frictions (i.e., more market incompleteness and higher debt elasticities) and display more output volatility.

The paper proceeds as follows. Section 2 develops the static model, which isolates the consumption preference channel. Section 3 turns to the DSGE model and focuses on how the consumption smoothing channel interacts with the consumption preference channel. Section 4 presents empirical evidence that supports our main findings. Section 5 offers concluding remarks.

2 A static model

To illustrate our main point, consider a simple, static, small open economy model of optimal fiscal policy.¹⁰ We will analyze two different asset market structures: (i) financial autarky and (ii) complete markets. The purpose of this model is to examine, in the simplest possible

⁹For an early and leading example, see Frankel (2011).

¹⁰The reader is referred to Appendix A for detailed derivations.

framework, how market incompleteness (in the form of financial autarky) affects the cyclical behavior of fiscal instruments (i.e., government spending and tax rates) in response to output fluctuations.¹¹ The key punchlines will be: (i) under complete markets, spending and tax policy are acyclical; and (ii) under financial autarky, while government spending is *always* procyclical, the same is *not* true of tax rates. In fact, the cyclical behavior of tax rates will depend on how the ratio of private to public consumption comoves with the business cycle.

2.1 Setup

Consider a small open economy perfectly integrated with the rest of world in good markets. Output is exogenous and stochastic, and follows the binomial distribution:

$$y = \begin{cases} y_H = \bar{y} + \gamma, \text{ with probability } p, \\ y_L = \bar{y} - \gamma, \text{ with probability } 1 - p, \end{cases} \quad (1)$$

where \bar{y} and γ are positive parameters, and H and L denote the high and low output states of nature, respectively. For simplicity, p is assumed to be equal to $1/2$. Since $E(y) = \bar{y}$ and $V(y) = \gamma^2$, an increase in γ represents a mean-preserving spread.

Following Baxter and King (1993), we assume that households' preferences are separable in private and public consumption:

$$U(c_i, g_i) = \begin{cases} \mathbb{E}_{i=H,L} \left[\alpha^{\frac{1-\frac{1}{\sigma_c}-1}{1-\frac{1}{\sigma_c}}} + (1-\alpha) \frac{\frac{1-\frac{1}{\sigma_g}-1}{1-\frac{1}{\sigma_g}}}{\frac{1-\frac{1}{\sigma_g}-1}{1-\frac{1}{\sigma_g}}} \right], & \sigma_c \neq 1 \text{ and } \sigma_g \neq 1, \\ \mathbb{E}_{i=H,L} \left[\alpha \ln(c_i) + (1-\alpha) \frac{\frac{1-\frac{1}{\sigma_g}-1}{1-\frac{1}{\sigma_g}}}{\frac{1-\frac{1}{\sigma_g}-1}{1-\frac{1}{\sigma_g}}} \right], & \sigma_c = 1 \text{ and } \sigma_g \neq 1, \\ \mathbb{E}_{i=H,L} \left[\alpha^{\frac{1-\frac{1}{\sigma_c}-1}{1-\frac{1}{\sigma_c}}} + (1-\alpha) \ln(g_i) \right], & \sigma_c \neq 1 \text{ and } \sigma_g = 1, \end{cases} \quad (2)$$

where $1/\sigma_c$ and $1/\sigma_g$ denote the coefficients of relative risk aversion for private and public consumption, respectively. The parameters σ_c and σ_g will determine how the ratio of private

¹¹It is worth stressing that, as emphasized by Reinhart *et al.* (2004), it would be misleading to use the ratio g/y to characterize fiscal policy cyclicity because of the endogeneity of y . For example, if g/y goes down in good times because output increases by more than g , we would wrongly conclude that government spending is countercyclical when in reality government spending is procyclical (since g is being increased in good times).

to public consumption correlates with output.¹² As will become clear below, the relative size of σ_c and σ_g will be crucial for our results.

The household's budget constraint in each state of nature is given by

$$y_i = (1 + \tau_i)c_i, \quad i = L, H, \quad (3)$$

where τ_i is a consumption tax.¹³

The government's budget constraints are thus

$$g_i = \tau_i c_i, \quad i = L, H. \quad (4)$$

Combining the household's constraints, given by (3), with the government's, given by (4), yields the economy's resource constraints:

$$y_i = c_i + g_i, \quad i = L, H. \quad (5)$$

For the sake of tractability, we will consider two polar cases in terms of asset market completeness: financial autarky and complete markets. We begin with the extreme case of financial autarky (i.e. full absence of financial instruments). Before proceeding, notice that we can solve this problem as a social planner because the consumption tax does not distort intertemporally (i.e., the model is static) or intratemporally (i.e., there is no labor/leisure choice). Once we have solved for the social planner's optimal allocation, we can use the government's constraints, given by (4), to recover the optimal consumption tax rates.

In the financial autarky case, the planner's problem consists in choosing $\{c_H, c_L, g_H, g_L\}$ to maximize households' utility, given by (2), subject to the economy's resource constraint. As shown in Appendix A, at an optimum, the marginal utilities of private and public consumption will be the same in each state of nature:

$$U_{c_i}(c_i, g_i) = U_{g_i}(c_i, g_i), \quad i = H, L. \quad (6)$$

¹² As illustrated in Appendix A for the case of $\alpha = 1/2$, $\sigma_c = \sigma_g$ implies $c = g$; $\sigma_c > \sigma_g$ implies $c > g$, and $\sigma_c < \sigma_g$ implies $c < g$. In other words, the relative size of σ_c and σ_g dictates the preferences for c and g .

¹³ As shown in Appendix A, in this simple world the results would be identical if we assumed an output (endowment) tax. Also, for simplicity, we assume that private and public sector initial assets are zero.

In the complete markets case, the economy may buy/sell state contingent claims that promise to deliver one unit of output if states H and L occur for a price q_H and q_L , respectively.¹⁴ Prices are assumed to be actuarially fair, which implies that

$$\frac{q_H}{q_L} = \frac{p}{1-p}.$$

The economy's resource constraint is thus

$$q_H y_H + q_L y_L = q_H(c_H + g_H) + q_L(c_L + g_L). \quad (7)$$

The social planner chooses $\{c_H, c_L, g_H, g_L\}$ to maximize households' utility, given by (2), subject to constraint (7). In addition to condition (6), it is also the case that

$$\begin{aligned} U_{c_H}(c_H, g_H) &= U_{c_L}(c_L, g_L), \\ U_{g_H}(c_H, g_H) &= U_{g_L}(c_L, g_L). \end{aligned}$$

In other words, the marginal utilities of private and public consumption are equalized across states of nature (which, by definition, implies full risk sharing under complete markets). Since the utility function is separable, these two optimality conditions imply that $c_H = c_L$ and $g_H = g_L$. The latter implies that government spending is the same across states of nature regardless of the relation between σ_c and σ_g . We now fully characterize the properties of fiscal policy across states of nature.

2.2 Cyclical properties of fiscal policy

Let θ_g and θ_τ capture, respectively, the cyclicity of government spending and tax rates:

$$\begin{aligned} \theta_g^X &\equiv \ln \left(\frac{g_H}{g_L} \right), \\ \theta_\tau^X &\equiv \ln \left(\frac{\tau_H}{\tau_L} \right). \end{aligned}$$

¹⁴The state contingent bonds are intra-period; that is, they are purchased at the beginning of the period, before the shock materializes, and the households receive the pay-off at the end of the period, once the shock materializes.

where $X \equiv CM$ in the case of complete markets and $X \equiv FA$ in the case of financial autarky. A positive (negative) value of θ_g ; that is, $g_H > g_L$ ($g_H < g_L$), would indicate that government spending is procyclical (countercyclical). If $g_H = g_L$, then $\theta_g = 0$, which implies acyclical government spending. A positive (negative) value of θ_τ ; that is, $\tau_H > \tau_L$ ($\tau_H < \tau_L$), would indicate that tax rates are countercyclical (procyclical). If $\tau_H = \tau_L$, then $\theta_\tau = 0$, implying acyclical tax policy.

We are now ready to fully characterize the cyclical properties of optimal fiscal policy in the static model under complete and incomplete markets (i.e., financial autarky). We will do so by establishing the following two propositions.

Proposition 1 *Government spending is acyclical under complete markets (i.e., $\theta_g^{CM} = 0$) and is procyclical under financial autarky ($\theta_g^{FA} > 0$) regardless of the values of σ_c and σ_g .*

Proof. See Appendix A. ■

Proposition 2 *Tax rates are acyclical under complete markets (i.e., $\theta_\tau^{CM} = 0$). Under financial autarky, the cyclicity of tax rates depends on the relative values of σ_c and σ_g . Tax rates are acyclical ($\theta_\tau^{FA} = 0$) if $\sigma_c = \sigma_g$, countercyclical ($\theta_\tau^{FA} > 0$) if $\sigma_c < \sigma_g$, and procyclical ($\theta_\tau^{FA} < 0$) if $\sigma_c > \sigma_g$.*

Proof. See Appendix A. ■

The key and novel results of this section are thus captured by Propositions 1 and 2, which establish that market incompleteness (as captured in this instance by financial autarky) is a necessary and sufficient condition for government spending to be procyclical. In contrast, market incompleteness is a necessary but not sufficient condition for tax policy to be procyclical. In fact, acyclicity, procyclicality, and countercyclicality are all possible depending on the relative values of $\sigma_c > \sigma_g$. In sum, financial autarky does *not* tell us anything, in principle, about the cyclicity of optimal tax policy.

The role of the relative values of σ_c and σ_g becomes clear when we realize that these parameters will determine how the ratio g/c moves over the business cycle. In order to understand precisely the role of these two parameters, we can use (4) and rewrite θ_τ as follows:

$$\theta_\tau \equiv \ln \left(\frac{\tau_H}{\tau_L} \right) = \ln \left(\frac{g_H/c_H}{g_L/c_L} \right). \quad (8)$$

Therefore, the tax rate cyclical is tightly linked to the optimal ratio g/c across states of nature which, in turn, is determined by σ_c and σ_g :

- When $\sigma_c = \sigma_g$, then g/c is constant across states of nature (i.e., $g_H/c_H = g_L/c_L$), and hence $\tau_H = \tau_L$. Since private and public consumption are equally valued, the two types of consumption increase by the same proportion in the good state of nature. The higher tax base (i.e., the higher private consumption) enables the fiscal authority to leave the tax rate unchanged and still finance the higher government spending. Tax policy is thus acyclical.
- When $\sigma_c < \sigma_g$, then $g_H/c_H > g_L/c_L$, and hence $\tau_H > \tau_L$. Since households have stronger preferences for public than private consumption, then private consumption increases proportionally less than public consumption in the good state of nature. Hence, if the fiscal authority kept the same tax rate, tax revenues would fall short. The fiscal authority needs to increase the tax rate to finance the higher public consumption ($\tau_H > \tau_L$), thus engaging in countercyclical tax policy.
- When $\sigma_c > \sigma_g$, then $g_H/c_H < g_L/c_L$, and hence $\tau_H < \tau_L$. Since households' preferences are stronger for private than public consumption, then c increases proportionately more than g in the good state of nature. The relatively higher tax base allows the fiscal authority to reduce the tax rate and still finance the higher public consumption (i.e., $\tau_H < \tau_L$; procyclical tax rates), thus engaging in procyclical fiscal policy. We will refer to this channel as the “consumption preference channel.” In other words, the households' stronger preference for private than public consumption leads to procyclical tax policy.

It will thus be of interest to relate the degree of cyclical of tax rates in the data to the dynamics of the ratio of private to public consumption over the business cycle. We will do so in the next section in the context of a fully-calibrated dynamic model.

Finally, the degree of asset market completeness will also matter for the extent to

which higher income volatility affects the cyclicalities of fiscal policy. The next proposition formalizes this.

Proposition 3 *Under financial autarky, the procyclicality of government spending increases with output volatility (i.e., $(d\theta_g^{FA}/d\gamma) > 0$). When tax rates are procyclical (i.e., $\sigma_c > \sigma_g$), then tax procyclicality increases with output volatility ((i.e., $(d\theta_\tau^{FA}/d\gamma) < 0$). When tax rates are countercyclical (i.e., $\sigma_c < \sigma_g$), then tax countercyclicality also increases with output volatility (i.e., $(d\theta_\tau^{FA}/d\gamma) > 0$). When tax rates are acyclical (i.e., $\sigma_c = \sigma_g$), then output volatility has no effect (i.e., $d\theta_\tau^{FA}/d\gamma = 0$). Under complete markets, output volatility does not affect the cyclicity of either government spending or tax rates.*

Proof. See Appendix A. ■

In sum, this simple model has achieved several goals. First, we have shown (for the first time, as far as we know) that incomplete markets do not necessarily imply that both government spending and tax rates are procyclical. It would therefore be incorrect, as a matter of applied macro-theory, to assert that incomplete asset markets explain fiscal procyclicality. Second, we have shown that, depending on the relative size of σ_c and σ_g , tax rates could result in acyclical, procyclical, or countercyclical tax policy. Hence, the theory, by itself, cannot tell us how tax rates are expected to move over the business cycle. In other words, we will eventually need empirical guidance to establish which case is the most plausible in the real world. In fact, we will see that, in practice, the ratio c/g comoves positively with the business cycle, which implies that the model's prediction would be that tax policy is procyclical.

Before we get there, however, we will first explore the natural question of whether the key theoretical insights that we have derived in the context of this simple and static model remain valid in a much richer and more realistic modeling environment.

3 A DSGE model

This section further explores the cyclical implications of optimal (Ramsey) fiscal policy in both tax rates and public spending and their relationship with the degree of asset market

completeness in open economies, but in a richer environment (i.e., the standard DSGE model used for the analysis of business cycles in small open economies). Agents will now operate within an infinite horizon setting, production will be endogenous, and households will choose labor supply optimally.

An important new feature (referred to henceforth as the “consumption smoothing channel”) is that households will now be able to smooth consumption by issuing risk-free debt in international markets, which will have a direct effect on the cyclicity of fiscal policy. While we will continue to explore the effects of different degrees of asset market completeness, the presence of debt opens up the possibility of studying the presence of a different, but related, type of financial friction: the existence of an upward sloping supply of external savings. We will also explore the extent to which the slope of this supply of funds – a proxy for other kinds of financial frictions – affects the cyclicity of fiscal policy in response to productivity shocks. Lastly, we will focus on how higher output volatility, driven by more persistent TFP shocks, affects the cyclicity of fiscal policy. Despite the richness of the new environment, results will show that the main takeaways from the model continue to be those from the much simpler static one in the previous section.

3.1 Setup

3.1.1 Households

Households maximize the expected present discounted value of utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t), \quad (9)$$

where c_t , g_t , l_t , and β denote, respectively, private consumption, public consumption, labor supply, and the discount factor.

In what will be our benchmark case henceforth, households will have access to international asset markets by issuing one period non-state contingent bonds. This assumption will be later relaxed when we consider, as in the previous static framework, the polar cases of complete markets and financial autarky.

The representative household's budget constraint is given by:

$$d_t = R_{t-1}d_{t-1} + \theta_t c_t - y_t, \quad (10)$$

where d_t is the stock of private debt at the end of period t ; R_t is the (gross) interest rate of debt contracted in period $t-1$ and repaid in period t ; and θ_t is the (gross) tax rate on consumption (and equal to $1 + \tau_t$).¹⁵ The standard no-Ponzi games condition applies. Lastly, output y_t , is given by

$$y_t = A_t l_t, \quad (11)$$

where A_t is a stochastic productivity factor.

3.1.2 Real interest rates

The gross international real interest rate faced by households, R_t , is assumed to be equal to the stochastic gross world real interest rate (R_t^*) and an endogenous risk premium, S_t , that depends on the stock of debt:

$$\begin{aligned} R_t &= R_t^* S_t, \\ S_t &= 1 + p(\tilde{d}_t). \end{aligned} \quad (12)$$

We follow Schmitt-Grohé and Uribe (2003) and assume that $p(\cdot)$ is a country-specific interest rate premium, and \tilde{d}_t is the aggregate level of foreign debt which, in equilibrium, is equal to household's debt. The functional form that we use for $p(\cdot)$ also follows their work:

$$p(d) = \psi^c \left(e^{d - \bar{d}} - 1 \right), \quad (13)$$

where ψ^c and \bar{d} are parameters.

Note that ψ^c governs the elasticity of the spread to changes in private debt (i.e., the slope of the supply of external funds). In our benchmark case, we will follow Schmitt-Grohé and Uribe (2003) and consider small values of ψ^c so as to render the model stationary. As

¹⁵Labor income taxation would deliver similar, though not identical, results because the distortions introduced are not exactly the same as in the previous section.

explained in more detail below, we will nonetheless consider also deviations from this level as an additional source of financial frictions (García-Cicco *et al.*, 2010; Chang and Fernández, 2013; and Fernández and Gulan, 2015).

3.1.3 Government

The government's flow budget constraint is given by

$$d_t^g = R_{t-1}^g d_{t-1}^g + g_t - (\theta_t - 1)c_t, \quad (14)$$

where d_t^g is the stock of public debt.

We will further assume that a similar debt-elastic interest rate premium applies to government debt; that is, $R_t^g = R_t^* S_t^g$, where $S_t^g = 1 + p(d_t^g) = \psi^g (e^{d_t^g - \bar{d}^g} - 1)$ and ψ^g and \bar{d}^g are parameters.

3.1.4 Driving processes

There are two independent sources of uncertainty modeled as stochastic driving forces. The first one is for TFP:

$$\ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2), \quad (15)$$

where \bar{A} is TFP in the steady state. The second one is for the world real interest rate:

$$\ln(R_t^*/\bar{R}^*) = \rho_R \ln(R_{t-1}^*/\bar{R}^*) + \varepsilon_t^{R^*}, \quad \varepsilon_t^{R^*} \sim NIID(0, \sigma_{R^*}^2), \quad (16)$$

where \bar{R}^* is the non-stochastic real interest rate.

3.1.5 Ramsey problem

The Ramsey planner maximizes the welfare of the representative agent (9), subject to the private and public budget constraints, (10) and (14), and the implementability constraints

from the household's problem:

$$\begin{aligned}\Gamma_t &= A_t, \\ \lambda_t &= \beta R_t E_t \lambda_{t+1},\end{aligned}$$

where $\Gamma_t(c_t, g_t, l_t, \theta_t)$ is the marginal rate of substitution between consumption and leisure and $\lambda_t = h(c_t, g_t, l_t, \theta_t)$ is the shadow price of wealth (further details can be found in Appendix B).

3.2 Financial frictions

As in the static case, we will study the effects of varying the level of asset market completeness. This is captured schematically in Figure 3. In addition to the benchmark case (point B in Figure 3), we will explore the two polar cases of financial autarky (point A) and complete markets (point C).

Following Schmitt-Grohé and Uribe (2003), in the case of complete asset markets, agents have access to a complete array of state-contingent claims so that the sequential budget constraint becomes

$$E_t r_{t+1} b_{t+1} = b_t + y_t - \theta_t c_t, \quad (17)$$

where b_{t+1} are the assets purchased in t to be delivered in each state of period $t+1$ and r_{t+1} denotes the period- t price of an asset that pays one unit of the good in a particular state of period $t+1$ divided by the probability of occurrence of that state given information available in period t . A no-Ponzi-game constraint exists, given by $\lim E_t q_{t+j} b_{t+j} \geq 0$ as $j \rightarrow \infty$, for all dates and for all contingencies, where $q_t = r_1 r_2 \dots r_t$, with $q_0 \equiv 1$.

On the other hand, the case of financial autarky assumes that neither households nor the government can buy or sell financial securities from/to the rest of the world. In other words, assets can only be exchanged within the country.¹⁶ Formally, then, the following

¹⁶In this case, and as detailed in Appendix B, we assume that both households and the government are subject to quadratic portfolio adjustment costs.

general equilibrium condition must be added for this case:

$$d_t + d_t^g = 0 \quad \forall t. \quad (18)$$

Furthermore, in this case the real interest rate R is no longer given by (12), (13), and (16), and instead adjusts endogenously to ensure that condition (18) is satisfied at all times.

The second type of financial friction, illustrated schematically by the vertical axis in Figure 3 and captured in reduced form by (13), is a debt-elastic risk premium, with ψ^c and ψ^g denoting the two elasticities considered. Larger values of this elasticity, which imply a stronger response of the real interest rate paid by the country to international creditors, appear to be empirically necessary to bring DSGE models closer to emerging markets data (Garcia-Cicco *et al.*, 2010; Chang and Fernandez, 2013).

While, conceptually, the model with debt-elastic interest rates converges to the financial autarky case as ψ^c and ψ^g become arbitrarily large (i.e., the vertical axis in Figure 3 rotates to the left, eventually converging to the horizontal arrow), the two types of financial frictions are not completely isomorphic. The existence of debt-elastic real interest rates requires at least *some* level of asset market incompleteness, but higher levels of this elasticity do not necessarily imply more incompleteness. Fernández and Gulan (2015), for instance, provide microfoundations for the debt elasticity parameter in an environment where private debt is defaultable due to asymmetric information between domestic entrepreneurs and external lenders, holding the level of asset market incompleteness constant. This justifies a separate analysis of these two types of frictions.

A third and final dimension that we will study within this richer setup is the presence of varying degrees of persistence in the TFP process, A_t , as illustrated by the diagonal axis in Figure 3. Based on the observation that small emerging economies have more volatile GDP, Aguiar and Gopinath (2007) hypothesized that such phenomenon could be related to higher persistence in TFP which, in turn, captures a myriad of frictions (whether of financial nature or not), such as frequent changes in fiscal, monetary, and trade policies. Our goal will be to study the extent to which a more persistent TFP process delivers fiscal procyclicality when the two kinds of financial frictions that we consider coexist.

3.3 Calibration

Following Baxter and King (1993), we continue to use separable preferences:

$$U(c, g, l) = \frac{c^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \frac{g^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \log(1 - l),$$

where σ_c and σ_g can now be also interpreted as the intertemporal elasticity of substitution of private and public consumption, respectively.

The calibration of the various parameters in the DSGE model is summarized in Table 1. We rely mostly on previous studies of small open economies. Without loss of generality, the steady-state level of TFP (\bar{A}) is normalized to one. Following Schmitt-Grohé and Uribe (2003), the steady-state level of the international real interest rate (\bar{R}^*) is set to 4 percent on an annual basis. The steady-state ratio of total debt (public and private) to (quarterly) income is set to 1.34, using Lane and Milesi-Ferretti's (2007) data on net foreign assets for the non-OECD countries in Figures 1 and 2.

The persistence of the TFP process, governed by the AR(1) coefficient ρ_A is set to 0.95 in our benchmark case, taken from Neumeyer and Perri (2005). The volatility of the shock to this process will vary throughout our various experiments, including one where we calibrate it so as to match certain moments in the data, but in our benchmark case it will be set to 0.0129, also taken from Schmitt-Grohé and Uribe (2003). The values chosen for ρ_{R^*} and σ_{R^*} , which govern the persistence and volatility of the real interest rate process, come from Uribe and Yue (2006) and are set to 0.83 and 0.007, respectively.

As usual, the discount factor (β) is calibrated as the inverse of the gross international real interest rate. In our baseline case of incomplete markets, the parameters capturing the debt elasticity of the real interest rate in both private and public debt, ψ^c and ψ^g , will be set to 0.125, using the estimated value to be discussed later in the empirical section. However, in the various experiments that we will conduct, the values considered for these parameters will change so as to capture varying degrees of financial frictions, and also consider alternative values that aim at better matching some of the moments in the data.

Last, but not least, we need to calibrate the parameters governing the intertemporal elasticities of substitution for private (σ_c) and public (σ_g) consumption. From the static

model, we know that the relative values chosen for these parameters are crucial for determining the cyclical dynamics of fiscal policy. We will therefore set σ_c to 1 and consider a range of values for σ_g from lower to higher than σ_c .¹⁷

3.4 Results

This section presents the quantitative results from the DSGE model. We study, separately, the effects of market incompleteness and varying debt elastic spreads on the optimal path of fiscal policy as it reacts to exogenous TFP disturbances. We do this through the analysis of second moments, simulations, and impulse response functions. In addition, we explore the fiscal consequences of higher income volatility resulting from more persistent TFP shocks. Lastly, we present an exercise that measures the performance of the DSGE model in matching some of the key moments in the data. All results come from a first-order Taylor approximation of the model around its non-stochastic steady state.

3.4.1 Second moments

We begin by exploring the two main second moments in our analysis: the contemporaneous correlations of government consumption (g) and tax rates (τ) with income (y). The key role played by the elasticities σ_g and σ_c in determining these moments and, thus, on the degree of fiscal procyclicality are illustrated in Figures 4 and 5 for, respectively, the cases when the debt elasticity (ψ^c and ψ^g) varies and for different degrees of market incompleteness.¹⁸

When varying the debt elasticity in Figure 4, in addition to our benchmark calibration for $\psi^c = \psi^g = 0.125$, we consider four alternative values: two are lower than our calibrated benchmark ($\psi^c = \psi^g = 0.01$, $\psi^c = \psi^g = 0.001$), while the remaining two are higher ($\psi^c = \psi^g = 1$, $\psi^c = \psi^g = 2.8$). The lowest value considered (0.001) can be thought as the minimum level needed to render the model stationary and hence amenable to computing second moments, while the highest value comes from the estimated value for Argentina in

¹⁷It is worth pointing out that the calibration of σ_c and σ_g not only has relevance for the cyclical dynamics of fiscal policy, but also for the steady state of the model. A calibration where $\sigma_c > \sigma_g$ ($\sigma_c < \sigma_g$) delivers a ratio of private-to-public consumption, c/g , that is lower (higher) than one, and hence tax rates that are relatively higher (lower).

¹⁸Since, in these exercises, we assume that $\psi^c = \psi^g$, we often use the notation $\psi^c = \psi^g = \psi$.

Garcia-Cicco *et al.* (2010).

The main message from Figure 4 is essentially the same derived in the static model of Section 2 (recall Propositions 1 and 2): fiscal procyclicality in government spending does not depend on the relative values of σ_g and σ_c , but the same is *not* true of tax rates, in which case the relative values of these parameters are crucial in determining the sign of the correlation. Indeed, as shown in the top panel of Figure 4, for the entire range considered for σ_g from 0.5 to 1.5 (recall that σ_c is fixed at 1), the correlation between g and y lies above 0.95 for all values of ψ considered. In contrast, as shown in the bottom panel, the sign of the correlation between τ and y changes drastically from -1 to 1 as σ_g increases. For example, in the extreme case where $\psi^c = \psi^g = \psi = 2.8$, and the economy is basically in financial autarky because it is too costly to issue debt (a case that will be explored in greater detail next), the results are identical to the static model: tax rates behave in a procyclical manner (i.e., the correlation is negative) for σ_g less than σ_c (which is fixed at 1) while they behave countercyclically (i.e., the correlation is positive) when the opposite occurs. Put differently, the consumption preference channel identified in the static model dominates the consumption smoothing channel.

An additional, and novel, feature of the correlation between τ and y that follows from Figure 4 is that the change in the sign of such correlation depends also on ψ^c and ψ^g . Indeed, as we move away from the polar case of $\psi^c = \psi^g = \psi = 2.8$ and ψ falls, so does the cut-off value of σ_g for which taxes change from being procyclical to countercyclical. Intuitively, agents in this dynamic model can issue debt in order to smooth out the effects of shocks on their consumption path, a mechanism that, by construction, was absent in the static case previously considered. For a given σ_g , as agents are confronted with a negative TFP shock, the cheaper it is for them to issue debt as ψ^c and ψ^g fall, and hence the smaller the effect of the TFP shock on consumption (i.e., the tax base). This reduces the need for taxes to increase in a procyclical manner, which will occur only when σ_g is low.

Figure 5 reports the correlations between public consumption and output (top panel) and tax rates and output (bottom panel) for the three cases of asset market completeness considered (financial autarky, incomplete markets, and complete markets). In the top panel, we see that the less complete markets are, the higher the correlation between output

and public consumption, regardless of the value of σ_g . Indeed, public consumption is uncorrelated with output under complete markets (i.e., covariance is zero) and perfectly correlated under financial autarky.¹⁹ As in the static framework, the correlation between tax rates and output (bottom panel) depends on the value of σ_g , more so as markets are less complete. In the extreme case of financial autarky, such correlation turns around completely from -1 to 1 as σ_g crosses 1 (note how similar this case is to the one in Figure 4 for $\psi^c = \psi^g = 2.8$). In the case of incomplete markets, the cross-over occurs at slightly lower levels of ψ^g because of the presence of the consumption smoothing channel analyzed above.

We report other second moments of the model in Table 2. In addition to the two moments already analyzed (correlation of tax rates and government spending with output), the table shows the standard deviation of these three variables and consumption and the correlation of the public spending ratio with output. The table reports the moments for the three cases of market completeness considered and the various possible calibrations of σ_g .

The case of market completeness is trivial as all moments (except income variability) are zero, regardless of the calibration of σ_g as consumption, public spending and, hence, tax rates, are perfectly smoothed thanks to complete markets.²⁰ For the case of financial autarky, as expected from the discussion above, the moments vary considerably depending on the calibration of σ_g . Low (high) values of this elasticity relative to σ_c render private consumption more (less) volatile, while public consumption becomes less (more) volatile. This, in turn, implies that the ratio c/g commoves positively (negatively) with income only if $\sigma_g \leq 1$ ($\sigma_g > 1$). The case of incomplete asset markets is an intermediate case, though closer to the case of financial autarky given the relatively high spread elasticity considered ($\psi^c = \psi^g = 0.125$).

A final look at the dynamics of fiscal variables is explored in the simulation presented in Figure 6 which illustrates, for the incomplete markets case, the deviations from the steady state for the key variables in the model from randomly drawing TFP shocks during 40 quarters. The particular pattern of the draws is such that roughly in the first half of the period a sequence of positive TFP shocks takes place while the opposite is true towards the

¹⁹For the complete markets case, the plot shows the covariance since the correlation cannot be computed.

²⁰Note that, for the case of complete markets, the table presents covariances instead of correlations, as the latter cannot be computed.

end of the sample (Panel A). Naturally, private consumption comoves positively with output (Panel B). The path of government spending (Panel C) increases in the period of positive TFP shocks and falls towards the end of the sample, indicating a procyclical behavior. As expected, the path of tax rates (Panel D) critically depends on whether σ_g is above or below $\sigma_c = 1$: (i) when $\sigma_g > \sigma_c$, the path of tax rates is countercyclical (i.e., tax rates comove positively with GDP); (ii) when $\sigma_g < \sigma_c$, the path of tax rates is procyclical (i.e., tax rates comove negatively with GDP); and (iii) when $\sigma_g = \sigma_c = 1$, the path of tax rates is acyclical. Such simulations were conducted for the extreme case where $\psi^c = \psi^g = 2.8$.

3.4.2 Impulse responses

A complementary analysis of fiscal policy is conducted by means of impulse response functions (IRFs) following a fall of TFP by one percentage point relative to the steady state. Figures 7A, 7B, and 8 show the IRFs under different degrees of asset market completeness (Figures 7A and 7B) and varying debt elasticities of spreads (Figure 8). The figures present the dynamics of the key variables in the model: productivity, private consumption, public consumption, output, private debt, tax rates, real interest rates, and private and public debt.²¹ ²²

The columns of Figures 7A and 7B illustrate the three cases considered in terms of asset market completeness: the left-most column is the case of financial autarky, the middle column is the benchmark case (i.e., a non-state contingent bond) and the right-most column is the case of complete markets. Focusing first on the top row of the figures that deals with the case of $\sigma_g = 0.5$, the key result is that, again, moving from complete markets to incomplete markets and financial autarky increases the procyclicality of both public expenditures as well as tax rates. Under financial autarky and incomplete markets, and in response to a fall of TFP by 1 percent below steady state, tax rates increase and public consumption falls (Figure 7A). On the other hand, under complete markets (right column), neither public nor private consumption and, thus, tax rates react as debt increases automatically given that

²¹Notice that in the financial autarky case, there is a single (domestic) real interest rate, whereas in the incomplete markets case, the real interest rates faced by the private and public sector are not the same.

²²Units are in percentage deviations from steady state levels, except for real interest rates and debt (linear deviations).

debt is no longer a state variable. In the case of incomplete markets (middle column), debt is now a state variable and hence does not react following the shock. However, next period debt increases in order to smooth the effect on consumption of the shock (Figure 7B). Note that under the benchmark calibration, issuing debt is relatively expensive (high ψ^c and ψ^g) and the shock is quite persistent (high ρ_A) so that the amount of debt issued following the shock does not help much in smoothing its effect, delivering a result that is quantitatively similar to that of financial autarky (Figure 7A). Note also that, under financial autarky, the public borrowing needed to pay for public consumption, in addition to higher tax rates, pushes the domestic real interest rate up (Figure 7B).²³

The second and third rows of Figures 7A and 7B illustrate the effects of different values of σ_g , the only parameter that varies as we move across rows, with the second row showing results for $\sigma_g = 1$, and the third for $\sigma_g = 2$. In the financial autarky case, left column (when $\sigma_g = \sigma_c = 1$), the fall of public and private consumption is identical and, in turn, the same as output. This implies that tax rates do not need to change and become acyclical. The same is essentially true in the incomplete markets case due to debt being relatively expensive. When σ_g is twice as large as σ_c , the Ramsey planner can actually use tax rates in a countercyclical way because the fall in public consumption is larger than that in private consumption. Quantitatively similar results are observed under incomplete markets. Lastly, as expected, increases in σ_g do not have any relevance in the complete markets case as neither public nor private consumption react.

The effects of increasing the spread elasticity are illustrated in the first row of Figure 8, comparing the benchmark case (middle column) with the case where ψ^c and ψ^g increase to 2.8 (left column) and the case where ψ^c and ψ^g fall to 0.001 (right column). As it becomes more costly to issue debt, a negative TFP shock has larger negative effects on private and public consumption. For relatively low values of σ_g , as is the case of the first row in Figure 8, tax rates become procyclical because the effects on public consumption are smaller than on private consumption. The opposite occurs, however, when σ_g increases, as in the second and third rows, to the point that taxes become countercyclical (i.e., they fall as the recession

²³The considerable increase of the (now endogenous) interest rate in the financial autarky model following a negative TFP shock is explained by the outward shift in the demand for debt by the government. This effect is stronger the lower is the elasticity of substitution of public consumption.

takes place). Public consumption always responds procyclically by falling relative to its steady state.

3.4.3 Output volatility

As mentioned above, higher persistence in the TFP residual is viewed as a potential source of the higher output volatility observed in emerging economies (Aguiar and Gopinath, 2007). This higher persistence is thus a simple way of capturing various frictions (financial and non-financial), such as frequent changes in fiscal, monetary, and trade policies. Motivated by this hypothesis, we now study the extent to which changes in the persistence of this shock affect the impact of financial frictions on fiscal procyclicality.

Figure 9 illustrates the IRFs following, as before, a fall in TFP of 1 percentage point below the steady state under the three alternative cases of market completeness. We focus on the case in which the persistence of the TFP process, captured by the AR(1) coefficient, falls from 0.95 to 0.42 as in Mendoza (1991). The results are shown in the plots on the lower row of the figure and, for comparison, the benchmark cases are reported on top. The fall in persistence implies that the half life of a TFP shock falls from close to seven quarters to less than a quarter, the volatility of the Solow residual decreases from 1.7 to 1.3 percent, and the standard deviation of income falls from 2.9 to 2.5 percent in our benchmark case.

Overall, the lower persistence reduces the tendency for financial imperfections to increase the procyclicality of fiscal policy. The mechanism is simple and intuitive: as the shock becomes less persistent, agents react less vigorously in terms of private consumption, calling for a less pronounced fiscal response (i.e., a reduction in public consumption and an increase in tax rates). The exception is, of course, the case of complete markets (right column) where the shock does not have any effect except a smaller increase in debt. This contrasts with the case of financial autarky (left column) where the increase in taxes is considerably less persistent following the shock.²⁴ For the case of market incompleteness, results on public spending and tax rates change drastically relative to the benchmark case as the persistence of the shock diminishes (centered subplot in middle column). The strong procyclical tax rates

²⁴In fact, in the financial autarky case, tax rates fall on impact. This is related to the increase in the real interest rate that provides income to the government, which is a net creditor in the steady state. (Not shown, but available upon request.)

largely vanishes. However, when we interact the fall in persistence with a higher spread elasticity (bottom plot, middle column), the procyclicality in tax rates is restored and tax rates increase in the wake of the shock.

The role of persistence in TFP is further illustrated in Figure 10, which focuses on the two key correlations between public consumption and tax rates with output, and reports their values as the AR(1) coefficient in the TFP process increases from 0.4 to near 1 (from below). Results are qualitatively robust for the correlation of government expenditures and income (top panel) and the correlation is always positive regardless of the persistence in the shock.

Regarding the correlation between tax rates and income (bottom panel), for the intermediate case of incomplete markets, we see that, as persistence decreases, the degree of procyclicality falls. In fact, for relatively low levels of the debt elasticity ($\psi = 0.125$), the lack of persistence of the shock is enough for consumption not to drop as much, which leads to countercyclical tax rates (i.e., the correlation turns positive). In the extreme case of financial autarky, it is also the case that procyclicality falls as output persistence falls.

3.4.4 Moment matching

The analysis presented thus far has illustrated the effect of various kinds of financial frictions on fiscal cyclicalities, without asking how these dynamics account for those observed in the data. Matching the data, however, is important to discipline the calibration of the model. In the static model, for example, we showed how the behavior of the ratio c/g was relevant for pinning down the relative values of σ_g and σ_c which, in turn, determines the degree of procyclicality of tax rates.

This subsection addresses the issue of data matching by calibrating some of the key parameters in the DSGE model to match the most salient moments in the data. The four parameters to be calibrated are: (i) the volatility and persistence of TFP shocks (σ_A and ρ_A , respectively), (ii) the intertemporal elasticity of substitution for government consumption (σ_g), and (iii) the debt elasticity of the spread ($\psi^c = \psi^g = \psi$). The targeted moments in the data are the standard deviations of income and private consumption (σ_y and σ_c , respectively) and the correlations of taxes and public spending with real GDP ($\rho_{\tau,y}$ and $\rho_{g,y}$, respectively).

The calibration procedure consists of the following steps. First, a grid is defined over all possible combinations of values that the four parameters to be calibrated can take.²⁵ Second, the model is simulated for each point of the grid (based on random TFP and world real interest rate shocks) and the four targeted moments are computed using the HP-filtered series. Third, a quadratic loss function – defined as the sum of the (percentage) squared deviations of simulated (HP-filtered) moments from their empirical counterparts – is minimized over a four-dimensional grid defined for each of the parameters. Finally, we select the combination of values of $\{\sigma_A, \rho_A, \sigma_g, \psi\}$ for which the loss function takes its minimum value. We use the data for non-OECD countries (see Figures 1 and 2) and restrict the sample to balanced panel observations (within country) of more than ten annual consecutive observations.

The results of this exercise are reported in Table 3. The first column shows the second moments in the data. The second column reports the results from the simulated moments. The model performs rather well, considering how parsimonious it is. It captures the higher volatility of private consumption relative to that of output, as well as the positive correlation of government spending with output (procyclical government spending) and the negative correlation of tax rates with output (procyclical tax policy). Remarkably, the model matches the comovement between the ratio c/g and output, a moment that the exercise was not targeted to match. This implies that, as anticipated in Section 2, the empirically relevant case is the one in which c/g comoves positively with the cycle, which yields procyclical tax policy. As shown in Table 3, the good performance of the model is based on a relative inelastic government consumption intertemporal elasticity ($\sigma_g = 0.25$), relatively high debt elasticity of the spread ($\psi = 1$), and persistent TFP shocks ($\rho_A = 0.95$).

The model, however, is too parsimonious to account for the high volatility of government spending, which is about one tenth of that in the data. It also accounts for only a fifth of the observed volatility in taxes. Clearly, the true data generating process captures other sources of volatility in the fiscal instruments, in addition to the Ramsey dynamics implied by our model. Likewise, the model accounts for only a fraction of the correlation between c and

²⁵We use a 1,925-point grid that provides at least five different values for each calibrated parameter. The rest of the parameters are taken from Table 1.

y , signalling perhaps that additional shocks are needed that can increase the comovement between these two variables.

3.4.5 Welfare costs

Finally, we assess to what extent fiscal procyclicality is associated with more volatility and welfare costs in emerging economies. As discussed in Reinhart *et al.* (2004), procyclical fiscal policy could be a source of amplification of business cycles in emerging economies (the “when it rains, it pours phenomenon”). In other words, expansionary (contractionary) fiscal policy in good (bad) times would be expected to magnify the underlying business cycle. We now evaluate how procyclicality might influence welfare costs for different levels of financial frictions (ψ). We follow Lucas (1987) and compute welfare costs as the share of (steady state) consumption that households living in an economy without shocks would have to forgo to equate lifetime utility in an economy with TFP shocks and varying degrees of fiscal procyclicality induced by different levels of the debt-elasticity (ψ). Formally, welfare costs (ω) are pinned down by the following expression:²⁶

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\bar{c}(1 - \omega), \bar{g}, \bar{l}) = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t), \quad (19)$$

where a bar over a variable denotes its steady-state value.

Figure 11 presents the results. The left-most and middle panels illustrate how as ψ increases, the procyclicality of both government spending and tax rates becomes larger (i.e., the correlation of cyclical government spending and cyclical output increases and that of cyclical tax rates and cyclical output falls). This reflects the “when it rains, it pours” syndrome as fiscal policy amplifies shocks in a small open economy with incomplete markets. The right-most panel reports the resulting welfare costs (ω). In the benchmark case of incomplete markets but no financial frictions (i.e., an arbitrarily small value of ψ), welfare

²⁶To compute welfare costs, we simulate the model for 100,000 quarters randomly drawing TFP shocks and using the following parameters: $\sigma_g = 0.5$, $\sigma_c = 1$, $\sigma_A = 0.0129$, and $\rho_A = 0.95$. We use HP-filtered series to calculate the moments relevant to determining the second order approximation of ω , $Std(C_{HP})$, $Std(G_{HP})$, and $Std(L_{HP})$, according to the following equation: $\omega = \left[1 - \exp \left(\frac{-\sigma_c c^{1-\sigma_c} * Std(C_{HP})^2 - (\frac{l}{1-l})^2 Std(L_{HP})^2 - \sigma_g g^{1-\sigma_g} * Std(G_{HP})^2}{2} \right) \right]$.

costs are close to 0.015 percent of lifetime consumption. As we increase the value of ψ to empirically plausible levels found in emerging economies (e.g. $\psi = 0.125$, as shown in the empirical section below), welfare costs nearly double to 0.03 percent.

4 Empirical evidence

This section complements our previous theoretical analysis by providing empirical evidence supporting the links between financial frictions and fiscal procyclicality in spending and tax rates. Specifically, we now test whether several plausible empirical proxies for the two types of financial frictions (asset market incompleteness and debt elasticity) considered in our theoretical framework differ between OECD and non-OECD countries.

4.1 Data and estimation

As a proxy for market incompleteness, we use the dataset on restrictions on capital inflows and outflows from Fernández *et al.* (2016), which quantifies *de jure* restrictions on cross border flows across 32 types of transactions in 10 different assets (equity, bonds, FDI, and so forth), over the period 1995-2015. We use the following four specific indices: (i) overall assets inflow restrictions index (kai); (ii) overall assets outflow restrictions index (kao); (iii) bond inflow restrictions (boi); and (iv) bond outflow restrictions (boi). The value of these four measures is an average of several restrictions indices within the corresponding category (e.g., assets outflows). Each index varies between 0 (no restrictions) and 1 (restrictions on all assets).

Our proxy for the spread elasticity is derived from estimating the functional form used in the model (see equation (13)):

$$S_{i,t} = \phi_i + \psi \left[\exp \left(\frac{D_{i,t}}{Y_{i,t}} - \left(\frac{\overline{D_i}}{\overline{Y_i}} \right) \right) - 1 \right] + \varepsilon_{i,t},$$

where i and t are, respectively, country and time indices, $S_{i,t}$ is the country spread, ϕ_i is a constant, ψ is the estimated elasticity, $D_{i,t}$ is debt, $Y_{i,t}$ is output, and $\varepsilon_{i,t}$ is a mean zero *i.i.d* disturbance. The above equation is estimated using panel fixed effects for two different

samples of countries: OECD and non-OECD.

Two measures of debt are used: (i) total public debt from the World Bank's WDI; and (minus) net foreign assets from Lane and Milesi-Ferretti (2015). $S_{i,t}$ is proxied with the EMBIG for non-OECD countries; the 10-year T-bill spread with respect to German T-bills for OECD countries in the EU; and a UIP condition between the domestic 10-year T-bills and the U.S. 10-year T-bills for the remaining OECD countries in the sample.

Lastly, we use the two measures of fiscal procyclicality plotted in Figures 1 and 2. The first, $\rho(y, g)$, is the correlation between the cyclical components of real GDP and real government consumption expenditure (using the HP filter). The second, $\rho(y, \tau)$, is the correlation between the cyclical component of real GDP and the VAT rate (also using the HP filter).

4.2 Results

The key takeaway from the results summarized in Table 4 is that, as discussed in the Introduction, non-OECD countries, characterized by procyclical fiscal policies (unlike OECD countries where government spending and tax policy are countercyclical or acyclical) also exhibit larger financial frictions as captured by the two proxies that we quantify and display more macroeconomic volatility.

Specifically, the top panel in Table 4 reports the average cyclical measures across the two groups of countries. The average correlations of government spending and the business cycle in non-OECD and OECD countries are 0.29 and -0.12, respectively. Further, we can reject the null hypothesis that the correlations are the same in the two groups of countries. A similar result holds for the VAT rate in the sense that non-OECD countries behave in a more procyclical manner.

The second panel in Table 4 reports the results on asset market incompleteness. As shown, all four non-OECD indices that proxy countries' inability to participate in capital markets are higher than those of OECD countries by an order of magnitude, and the null hypothesis of equal degrees of completeness is statistically rejected.

The third panel presents our estimated debt spread elasticities for the two groups of countries using two debt proxies. In both cases, the elasticities are considerably higher for

non-OECD countries and the null hypothesis of equality is easily rejected. For the case of total public debt, the estimate for the non-OECD countries is $\psi = 0.125$ and significant at the one percent level, whereas the point estimate for OECD countries is 0.002 and not significant at the ten percent level. Using (negative) NFA as proxies of debt delivers similar qualitative results.

Lastly, average GDP volatility in non-OECD countries, measured by the standard deviation of the filtered real GDP process, 3.28, is more than twice as large as that of OECD countries, 1.47. This is consistent with existing work that documents the relatively higher business cycle volatility in developing countries. This basic empirical evidence complements our previous theoretical analysis by providing evidence in favor of the links between financial frictions and fiscal procyclicality in spending and tax rates.

5 Concluding remarks

The evidence shows that fiscal policy in developing countries is procyclical both on the spending and taxation sides. Since procyclical fiscal policy only exacerbates an already volatile business cycle, why would policymakers pursue such a policy? This is a puzzle in search of an explanation. Several explanations in the literature have focused on capital market imperfections, in the form, for example, of sovereign risk or limited commitment. While there may certainly be merit in such explanations, we believe that, from an applied theory point of view, an obvious but critical question is: can a canonical small open economy model with incomplete markets explain fiscal procyclicality? In other words, do we need to go beyond such an off-the-shelf model to explain the basic stylized facts of fiscal procyclicality?

To answer this question, we have examined the effects of asset market incompleteness on optimal fiscal policy à la Ramsey. We first established theoretically in a static model that (i) incomplete markets can explain procyclical government spending policy and (ii) whether tax policy is procyclical depends on the relative preference for private versus public consumption. Specifically, if the ratio of private to public consumption comoves positively with the cycle, then optimal tax policy is procyclical. If the comovement is zero, then tax policy is acyclical. If the comovement is negative, then tax policy is countercyclical. We

isolate this channel in a static model and refer to it as the “consumption preference channel.” Since, in practice, the ratio of private to public consumption comoves positively with the cycle, the theoretical presumption is that incomplete markets can also explain procyclical tax policy.

When we turn to a DSGE model with market incompleteness, a second channel comes into the picture: the “consumption smoothing channel.” The ability to smooth consumption implies that in response to, for example, a positive shock, consumption increases less than otherwise, which implies that the tax rate is reduced by less (thus making tax procyclicality less likely, all else equal). Once we add an upward-sloping supply of funds to the model and calibrate it to a typical developing country, we find that the consumption preference channel prevails. Hence, for realistic parameterizations of the model, optimal tax policy will be procyclical when the ratio of private to public consumption comoves positively with the cycle, which is the case for developing countries. We thus conclude that the simplest model of incomplete markets is capable of explaining procyclical fiscal policy in emerging markets (both on the spending and taxation sides). In other words, more complicated set-ups with sovereign risk and limited commitment are not needed for this purpose.

Further, we offer empirical support for the main frictions introduced into the model: market incompleteness and an upward-sloping supply of funds. We proxy market incompleteness by using different measures of controls on capital flows and show that such controls are much more prevalent in developing countries than in industrial countries. In the same vein, we estimate an upward-sloping supply of funds and show that it is significantly steeper in developing than in industrial countries.

Finally, we show that fiscal procyclicality is costly in terms of welfare. Indeed, in our model, Lucas-type welfare costs of fiscal procyclicality may be twice as much in the typical non-OECD country compared to the typical OECD country. This result offers a clear theoretical rationale for developing countries to “complete” markets as much as possible and reduce frictions (such as an upward supply of funds) that render more costly a given degree of market incompleteness.

In this respect, having access to sovereign wealth funds or contingent credit lines with international financial institutions could allow developing countries to access funds in bad

times, resulting in more “complete” markets and reducing the extent of fiscal procyclicality. In addition, countries may benefit from implementing fiscal rules that take into account market incompleteness. For example, structural rules that force the fiscal authority to save in good times (and repay debt) would “complete” markets by increasing self-insurance and incentivizing creditors to lend in bad times. The role of fiscal rules in reducing procyclical fiscal policy by completing markets is thus an important avenue of future research.

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Appendix A: Static model (for online publication)

5.1 Financial autarky case

Consider a static model with a stochastic endowment and consumption tax. Output (endowment) can take two different values with probability p and $1 - p$:

$$y = \begin{cases} y_H = \bar{y} + \gamma, \text{ with probability } p, \\ y_L = \bar{y} - \gamma, \text{ with probability } 1 - p. \end{cases} \quad (20)$$

For simplicity, p is assumed to be equal to $1/2$. It can be easily checked that

$$\begin{aligned} E(y) &= \bar{y}, \\ V(y) &= \gamma^2. \end{aligned}$$

Preferences are given by

$$U(c_i, g_i) = \begin{cases} \mathbb{E}_{i=H,L} \left[\alpha^{\frac{1-\frac{1}{\sigma_c}-1}{1-\frac{1}{\sigma_c}}} + (1-\alpha) \frac{g_i^{1-\frac{1}{\sigma_g}-1}}{1-\frac{1}{\sigma_g}} \right], & \sigma_c \neq 1 \text{ and } \sigma_g \neq 1, \\ \mathbb{E}_{i=H,L} \left[\alpha \ln(c_i) + (1-\alpha) \frac{g_i^{1-\frac{1}{\sigma_g}-1}}{1-\frac{1}{\sigma_g}} \right], & \sigma_c = 1 \text{ and } \sigma_g \neq 1, \\ \mathbb{E}_{i=H,L} \left[\alpha^{\frac{1-\frac{1}{\sigma_c}-1}{1-\frac{1}{\sigma_c}}} + (1-\alpha) \ln(g_i) \right], & \sigma_c \neq 1 \text{ and } \sigma_g = 1. \end{cases} \quad (21)$$

We can solve this problem as a social planner because the consumption tax does not distort intertemporally (i.e., the model is static) or intratemporally (i.e., there is no labor/leisure choice). The economy's resource constraints take the form

$$y_i = c_i + g_i, \quad i = L, H. \quad (22)$$

The planner's choice variables are $\{c_H, g_H, c_L, g_L\}$. The Lagrangian is given by

$$\begin{aligned}\mathcal{L} = & p \left[\alpha \frac{c_H^{1-\frac{1}{\sigma_c}} - 1}{1 - \frac{1}{\sigma_c}} + (1 - \alpha) \frac{g_H^{1-\frac{1}{\sigma_g}} - 1}{1 - \frac{1}{\sigma_g}} \right] + (1 - p) \left[\alpha \frac{c_L^{1-\frac{1}{\sigma_c}} - 1}{1 - \frac{1}{\sigma_c}} + (1 - \alpha) \frac{g_L^{1-\frac{1}{\sigma_g}} - 1}{1 - \frac{1}{\sigma_g}} \right] \\ & + \lambda_H(y_H - c_H - g_H) + \lambda_L(y_H - c_H - g_H).\end{aligned}$$

Once we have solved the planner's problem, the government constraint in each state of nature can be used to find out the corresponding tax rates:

$$g_i = \tau_i c_i, \quad i = L, H. \quad (23)$$

The first-order conditions for $\{c_H, g_H, c_L, g_L\}$ are given by, respectively,

$$\alpha c_H^{-\frac{1}{\sigma_c}} = \lambda_H, \quad (24)$$

$$(1 - \alpha) g_H^{-\frac{1}{\sigma_g}} = \lambda_H, \quad (25)$$

$$\alpha c_L^{-\frac{1}{\sigma_c}} = \lambda_L, \quad (26)$$

$$(1 - \alpha) g_L^{-\frac{1}{\sigma_g}} = \lambda_L. \quad (27)$$

These first-order conditions imply that the marginal utilities of private and public consumption are equalized in each state of nature (i.e., $U_{c^H} = U_{g^H}$ and $U_{c^L} = U_{g^L}$) but not across states of nature (because there is no full insurance).

Specifically, combining first-order conditions (24) and (25), we obtain:

$$c_H = g_H^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1 - \alpha} \right)^{\sigma_c}. \quad (28)$$

By the same token, from (26) and (27):

$$c_L = g_L^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1 - \alpha} \right)^{\sigma_c}. \quad (29)$$

²⁷To show that the relative size of σ_c and σ_g captures preferences for c/g , divide equation (28) by equation (29) and rewrite this equation as $\log(c_H/c_L) = (\sigma_c/\sigma_g) \log(g_H/g_L)$. Then, $\sigma_c = \sigma_g$ implies $(c_H/g_H) = (c_L/g_L)$; $\sigma_c > \sigma_g$ implies $(c_H/g_H) > (c_L/g_L)$, and $\sigma_c < \sigma_g$ implies $(c_H/g_H) < (c_L/g_L)$.

Combining (22), (28), and (29), it follows that

$$\begin{aligned} y_H &= g_H^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c} + g_H, \\ y_L &= g_L^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c} + g_L. \end{aligned}$$

Define

$$\begin{aligned} \phi(g_i) &\equiv g_i^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c} + g_i, \quad i = L, H, \\ \phi'(g_i) &= 1 + \left(\frac{\alpha}{1-\alpha} \right)^{\sigma_c} \frac{\sigma_c}{\sigma_g} g_i^{\frac{\sigma_c}{\sigma_g}-1} > 0. \quad i = L, H. \end{aligned}$$

Then,

$$\begin{aligned} y_i &= \phi(g_i), \quad i = L, H, \\ g_i &= \Gamma(y_i) > 0, \quad i = L, H, \\ \Gamma'(y_i) &> 0, \quad i = L, H, \end{aligned} \tag{30}$$

where $\Gamma \equiv \phi^{-1}$ and

$$\frac{dg_i}{dy_i} = \frac{1}{\phi'} > 0, \quad i = L, H.$$

Define the cyclicality of government spending as

$$\theta_g \equiv \log \left(\frac{g_H}{g_L} \right) = \log(g_H) - \log(g_L).$$

Using (30), this expression can be rewritten as

$$\theta_g = \log(\Gamma(y_H)) - \log(\Gamma(y_L)) > 0,$$

because $\Gamma'(y_i) > 0$ and $y_H > y_L$, which shows that government spending is procyclical regardless of the values of σ_c and σ_g (Proposition 1 in the text).

Taking into account the binomial distribution, given by (1), we now show that θ_g is

increasing in γ (Proposition 3 in the text):

$$\frac{d\theta_g}{d\gamma} = \frac{1}{\Gamma(y_H)} \Gamma'(y_H) + \frac{1}{\Gamma(y_L)} \Gamma'(y_L) > 0.$$

To obtain a reduced form for τ_i , combine (23) with (28) and (29) to obtain

$$\tau_i = \left(\frac{1-\alpha}{\alpha} \right)^{\sigma_c} g_i^{1-\frac{\sigma_c}{\sigma_g}}, \quad i = L, H. \quad (31)$$

Now define the cyclicality of the tax rate as

$$\theta_\tau \equiv \log \left(\frac{\tau_H}{\tau_L} \right)$$

which, using (31), can be rewritten as

$$\theta_\tau = \log \left(\frac{\frac{1-\frac{\sigma_c}{\sigma_g}}{g_H}}{\frac{1-\frac{\sigma_c}{\sigma_g}}{g_L}} \right) = \left(1 - \frac{\sigma_c}{\sigma_g} \right) \log \left(\frac{g_H}{g_L} \right) = \left(1 - \frac{\sigma_c}{\sigma_g} \right) \theta_g \gtrless 0. \quad (32)$$

As stated in Proposition 2, it follows that

$$\theta_\tau = \begin{cases} + & (\text{countercyclical}), \quad \sigma_c < \sigma_g, \\ 0 & (\text{acyclical}), \quad \sigma_c = \sigma_g, \\ - & (\text{procyclical}), \quad \sigma_c > \sigma_g. \end{cases}$$

What happens in response to a mean-preserving spread in output? Differentiating (32) with respect to τ ,

$$\frac{d\theta_\tau}{d\gamma} = \left(1 - \frac{\sigma_c}{\sigma_g} \right) \frac{d\theta_g}{d\gamma},$$

where $(d\theta_g/d\gamma) > 0$. Hence,

$$\frac{d\theta_\tau}{d\gamma} = \begin{cases} + & \sigma_c < \sigma_g, \\ 0 & \sigma_c = \sigma_g, \\ - & \sigma_c > \sigma_g. \end{cases}$$

As stated in Proposition 3 in the text, if θ_τ is countercyclical (i.e., positive), it will become more countercyclical. If it is zero, it remains zero of course. If it is negative (procyclical), it becomes more negative (i.e., more procyclical). In other words, a mean-preserving spread always amplifies the cyclical of tax rates.

5.2 Complete markets

While the model is static, we assume that households have access to contingent claims that can insure them against the outcomes in each state of nature (i.e., high and low output). The state-contingent bonds are intra-period; that is, they are purchased at the beginning of the period (i.e., before the shock materializes), and the households receive the pay-off at the end of the period (i.e., after the shock takes place).

As in the case of financial autarky, we can solve the planner's problem (with the planner having access to complete markets abroad) since, in the absence of any distortions, the government will be able to implement the first-best policy.

The planner chooses $\{c_H, g_H, c_L, g_L\}$ to maximize

$$pU(c_H, g_H) + (1 - p)U(c_L, g_L),$$

subject to

$$q_H y_H + q_L y_L = q_H(c_H + g_H) + q_L(c_L + g_L),$$

where $U(c_i, g_i)$ is given by (21).

The Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & pU(c_H, g_H) + (1 - p)U(c_L, g_L) \\ & + \lambda [q_H y_H + q_L y_L - q_H(c_H + g_H) - q_L(c_L + g_L)]. \end{aligned}$$

The first-order conditions are given by

$$pU_{c_H}(c_H, g_H) = \lambda q_H, \quad (33)$$

$$pU_{g_H}(c_H, g_H) = \lambda q_H, \quad (34)$$

$$(1-p)U_{c_L}(c_L, g_L) = \lambda q_L, \quad (35)$$

$$(1-p)U_{g_L}(c_L, g_L) = \lambda q_L. \quad (36)$$

Notice that, as in the financial autarky case, the marginal utilities of private and public consumption are equalized in each state of the world; that is, $U_{c_H} = U_{g_H}$ and $U_{c_L} = U_{g_L}$.

Combining first-order conditions across states of the world (i.e., (33) and (34) on the one hand, and (35) and (36) on the other), we obtain

$$\begin{aligned} \frac{pU_{c_H}(c_H, g_H)}{q_H} &= \frac{(1-p)U_{c_L}(c_L, g_L)}{q_L}, \\ \frac{pU_{g_H}(c_H, g_H)}{q_H} &= \frac{(1-p)U_{g_L}(c_L, g_L)}{q_L}. \end{aligned}$$

Assuming actuarially fair insurance (i.e., $q_H/q_L = p/(1-p)$), we can rewrite these optimality conditions as

$$U_{c_H}(c_H, g_H) = U_{c_L}(c_L, g_L), \quad (37)$$

$$U_{g_H}(c_H, g_H) = U_{g_L}(c_L, g_L). \quad (38)$$

Marginal utilities of private and public consumption are equalized across states of nature (implying full risk sharing). Since the utility function, given by (21), is separable, conditions (37) and (38) imply, respectively, that $c_H = c_L$ and $g_H = g_L$. The latter implies that government spending is acyclical (Proposition 1). In other words, under complete markets, government spending is acyclical regardless of the relation between σ_c and σ_g . Further, given (23), $c_H = c_L$ and $g_H = g_L$ imply that $\tau_H = \tau_L$. Hence, tax policy is also acyclical for any values of σ_c and σ_g (Proposition 2). In either case (spending or tax rates), fiscal policy is acyclical regardless of the variance of the distribution (Proposition 3).

5.3 Income tax

Suppose that we have an income tax in the form of an endowment tax (to keep it symmetrical with the consumption tax case). In other words, the consumer's budget constraints are given by:

$$c_i = (1 - \xi_i)y_i, \quad i = L, H,$$

where ξ_i is the endowment tax.

Other than this, the model is exactly the same as in the case analyzed above. As in the consumption tax case, this endowment tax is non-distortionary. We can thus solve the planner's problem as we did for the consumption tax case. All results for both financial autarky and complete markets go through since they do not depend on what is the government's source of tax income as long as it is non-distortionary.

Given the planner's optimal choices (which will be the same as in the consumption tax case), the income tax will follow from the government budget constraint:

$$g_i = (1 - \xi_i)y_i, \quad i = L, H.$$

We conclude that Propositions 1-3 would also hold for the case of an income (i.e., endowment) tax.

6 Appendix B: DSGE model (for online publication)

This appendix formally sets up and solves the three variations of the main DSGE model used in the text. First, we consider a small open economy operating under financial autarky (i.e., no borrowing from/lending to the rest of the world). Second, we deal with a standard small open economy that has access to a world risk-free bond (i.e., incomplete asset markets). Third, we consider a small open economy that is operating under complete markets.

6.1 Financial autarky model

6.1.1 Households' problem

The household's problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

subject to

$$\begin{aligned} d_t &= (1 + r_{t-1}) d_{t-1} - y_t + (1 + \tau_t) c_t + \Phi(d_t) - \Pi_t, \\ y_t &= A_t l_t, \\ \ln(A_t/\bar{A}) &= \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2), \end{aligned} \tag{39}$$

and a no-Ponzi condition.

We assume households face convex portfolio transactions costs, $\Phi(d_t)$. These administrative services (for either assets or liabilities) are provided by a government agency at zero cost. Profits (Π) are transferred to households in a lump-sum way so as to get rid of any wealth effects associated with these portfolio adjustment/transaction costs.

Notice also that here the portfolio costs are internalized by both the household and the Ramsey planner. In the SOE case (see below), only the Ramsey planner internalizes the upward sloping supply of funds.

6.1.2 Lagrangian

The Lagrangian is given by

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t [d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t - (1 + \tau_t) c_t - \Phi(d_t) + \Pi_t] \right\}.$$

The first-order conditions are given by

$$\begin{aligned}
 [c_t] &: \beta^t U_c(c_t, g_t, l_t) - \beta^t \lambda_t (1 + \tau_t) = 0, \\
 [l_t] &: \beta^t U_l(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0, \\
 [d_t] &: \beta^t \lambda_t [1 - \Phi'(d_t)] - (1 + r_t) \beta^{t+1} E_t \lambda_{t+1} = 0.
 \end{aligned}$$

Simplifying the first-order conditions, we obtain:

$$\begin{aligned}
 U_c(c_t, g_t, l_t) &= \lambda_t (1 + \tau_t), \\
 U_l(c_t, g_t, l_t) &= -\lambda_t A_t, \\
 \lambda_t [1 - \Phi'(d_t)] &= (1 + r_t) \beta E_t \lambda_{t+1}.
 \end{aligned}$$

6.1.3 Government's flow budget constraint

The government's flow budget constraint is given by

$$d_t^g = (1 + r_{t-1}) d_{t-1}^g - \tau_t c_t + g_t - \Phi(d_t) + \Pi_t. \quad (40)$$

6.1.4 Aggregate constraints

Financial autarky implies:

$$d_t + d_t^g = 0, \quad \forall t.$$

Combining the household's and government's flow budget constraints, given by (39) and (40), respectively, with the restriction above (in t and $t - 1$) yields:

$$c_t + g_t = y_t.$$

6.1.5 Implementability conditions

The implementability conditions follow from the first-order conditions and are given by:

$$\begin{aligned} A_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)} (1 + \tau_t) = \Gamma_t \equiv \Gamma(c_t, g_t, l_t, \tau_t), \\ \frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} &= \lambda_t \equiv \lambda(c_t, g_t, l_t, \tau_t) = \frac{1 + r_t}{1 - \Phi'(d_t)} \beta E_t \lambda(c_{t+1}, g_{t+1}, l_{t+1}, \tau_{t+1}). \end{aligned}$$

6.1.6 Ramsey problem

The Lagrangian for the Ramsey problem is given by

$$\mathcal{L} = E_0 \left\{ \begin{array}{l} \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) \\ + \beta^t \mu_{1,t} (\Gamma_t - A_t) \\ + \beta^t \mu_{2,t} \left[\lambda_t - \frac{1+r_t}{1-\Phi'(d_t)} \beta E_t \lambda_{t+1} \right] \\ + \beta^t \mu_{3,t} [d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t - (1 + \tau_t) c_t] \\ + \beta^t \mu_{4,t} (A_t l_t - c_t - g_t) \end{array} \right\}.$$

The first-order conditions are given by

$$\begin{aligned} [c_t] &: U_{ct} + \mu_{1,t} \Gamma_{ct} + \mu_{2,t} \lambda_{ct} - \mu_{3,t} (1 + \tau_t) - \mu_{4,t} = 0, \\ [l_t] &: U_{lt} + \mu_{1,t} \Gamma_{lt} + \mu_{2,t} \lambda_{lt} + \mu_{3,t} A_t + \mu_{4,t} A_t = 0, \\ [g_t] &: U_{gt} + \mu_{1,t} \Gamma_{gt} + \mu_{2,t} \lambda_{gt} - \mu_{4,t} = 0, \\ [\tau_t] &: \mu_{1,t} \Gamma_{\tau_t} + \mu_{2,t} \lambda_{\tau_t} - \mu_{3,t} c_t = 0, \\ [d_t] &: \mu_{2,t} \left[-\frac{1+r_t}{[1-\Phi'(d_t)]^2} \Phi''(d_t) \beta E_t \lambda_{t+1} \right] + \mu_{3,t} - \beta (1 + r_t) E_t \mu_{3,t+1} = 0, \\ [r_t] &: -\frac{1}{1-\Phi'(d_t)} \mu_{2,t} \beta E_t \lambda_{t+1} - \beta d_t E_t \mu_{3,t+1} = 0, \\ [\mu_{1,t}] &: \Gamma_t - A_t = 0, \\ [\mu_{2,t}] &: \lambda_t - \frac{1+r_t}{1-\Phi'(d_t)} \beta E_t \lambda_{t+1} = 0, \\ [\mu_{3,t}] &: d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t - (1 + \tau_t) c_t = 0, \\ [\mu_{4,t}] &: A_t l_t - c_t - g_t = 0. \end{aligned}$$

6.1.7 Parameterization

The portfolio transactions costs take the following quadratic form (see, for example, Schmitt-Grohe and M. Uribe, 2003):

$$\begin{aligned}\Phi(d_t) &= \frac{\phi}{2} (d_t - \bar{d})^2, \\ \Phi'(d_t) &= \phi (d_t - \bar{d}), \quad \Phi''(d_t) = \phi.\end{aligned}$$

where ϕ and \bar{d} are parameters.

$$\begin{aligned}U(c_t, g_t, l_t) &= \frac{c_t^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \frac{g_t^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \ln(1 - l_t), \\ U_{c_t} &= c_t^{-1/\sigma_c}; \quad U_{g_t} = g_t^{-1/\sigma_g}; \quad U_{l_t} = -\frac{1}{1 - l_t}.\end{aligned}$$

The derivatives of Γ_t and λ_t are given by, respectively,

$$\begin{aligned}\Gamma_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)} (1 + \tau_t) = \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma_c}}, \\ \Gamma_{c_t} &= \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}}; \quad \Gamma_{l_t} = \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}}; \quad \Gamma_{g_t} = 0; \quad \Gamma_{\tau_t} = \frac{1}{(1 - l_t)c_t^{-1/\sigma_c}}. \\ \lambda_t &= \frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t}, \\ \lambda_{c_t} &= -\frac{c_t^{-1/\sigma_c-1}}{\sigma_c(1 + \tau_t)}; \quad \lambda_{l_t} = 0; \quad \lambda_{g_t} = 0; \quad \lambda_{\tau_t} = -\frac{c_t^{-1/\sigma_c}}{(1 + \tau_t)^2}.\end{aligned}$$

6.1.8 Dynamic system

We have a system of 12 endogenous and 1 exogenous variables:

$\{c_t, \tau_t, l_t, g_t, d_t, r_t, y_t, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t\}$ and $\{A_t\}$, respectively,.

that are determined by 13 equations:

$$\begin{aligned}
 U_{c_t} + \mu_{1,t} \Gamma_{c_t} + \mu_{2,t} \lambda_{c_t} - \mu_{3,t} (1 + \tau_t) - \mu_{4,t} &= 0, \\
 [1] \quad c_t^{-1/\sigma_c} + \mu_{1,t} \frac{1 + \tau_t}{\sigma_c (1 - l_t) c_t^{1-1/\sigma_c}} - \mu_{2,t} \frac{c_t^{-1/\sigma_c-1}}{\sigma_c (1 + \tau_t)} - \mu_{3,t} (1 + \tau_t) - \mu_{4,t} &= 0,
 \end{aligned}$$

$$\begin{aligned}
 U_{l_t} + \mu_{1,t} \Gamma_{l_t} + \mu_{2,t} \lambda_{l_t} + \mu_{3,t} A_t + \mu_{4,t} A_t &= 0, \\
 [2] \quad - \frac{1}{1 - l_t} + \mu_{1,t} \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}} + \mu_{3,t} A_t + \mu_{4,t} A_t &= 0,
 \end{aligned}$$

$$\begin{aligned}
 U_{g_t} + \mu_{1,t} \Gamma_{g_t} + \mu_{2,t} \lambda_{g_t} - \mu_{4,t} &= 0, \\
 [3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} &= 0,
 \end{aligned}$$

$$\begin{aligned}
 \mu_{1,t} \Gamma_{\tau_t} + \mu_{2,t} \lambda_{\tau_t} - \mu_{3,t} c_t &= 0, \\
 [4] \quad \mu_{1,t} \frac{1}{(1 - l_t) c_t^{-1/\sigma_c}} - \mu_{2,t} \frac{c_t^{-1/\sigma_c}}{(1 + \tau_t)^2} - \mu_{3,t} c_t &= 0,
 \end{aligned}$$

$$\begin{aligned}
 \mu_{2,t} \left[-\frac{1 + r_t}{[1 - \Phi'(d_t)]^2} \Phi''(d_t) \beta E_t \lambda_{t+1} \right] + \mu_{3,t} - \beta (1 + r_t) E_t \mu_{3,t+1} &= 0, \\
 [5] \quad \mu_{2,t} \left[-\frac{1 + r_t}{[1 - \phi(d_t - \bar{d})]^2} \phi \beta E_t \lambda \right] + \mu_{3,t} - \beta (1 + r_t) E_t \mu_{3,t+1} &= 0,
 \end{aligned}$$

$$\begin{aligned}
 -\frac{1}{1 - \Phi'(d_t)} \mu_{2,t} \beta E_t \lambda_{t+1} - \beta d_t E_t \mu_{3,t+1} &= 0, \\
 [6] \quad -\frac{1}{1 - \phi(d_t - \bar{d})} \mu_{2,t} \beta E_t \lambda_{t+1} - \beta d_t E_t \mu_{3,t+1} &= 0,
 \end{aligned}$$

$$\begin{aligned}
\Gamma_t - A_t &= 0, \\
[7] \quad \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma_c}} - A_t &= 0,
\end{aligned}$$

$$\begin{aligned}
\lambda_t - \frac{1 + r_t}{1 - \Phi'(d_t)} \beta E_t \lambda_{t+1} &= 0, \\
[8] \quad \lambda_t - \frac{1 + r_t}{1 - \phi(d_t - \bar{d})} \beta E_t \lambda_{t+1} &= 0,
\end{aligned}$$

$$[9] \quad d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t - (1 + \tau_t) c_t = 0,$$

$$[10] \quad A_t l_t - c_t - g_t = 0,$$

$$[11] \quad y_t = A_t l_t,$$

$$[12] \quad \lambda_t = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t},$$

$$[13] \quad \ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0; \sigma_A^2),$$

with associated 8 parameters

$$\{\sigma_c, \sigma_g, \beta, \rho_A, \sigma_A^2, \bar{A}, \phi, \bar{d}\}.$$

6.1.9 Steady state

The steady state is a system of 13 equations with 21 unknowns, comprised of 13 steady-state variables given by

$$\{c, \tau, l, g, d, r, y, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, A\},$$

and 8 parameters $\{\sigma_c, \sigma_g, \beta, \rho_A, \sigma_A^2, \bar{A}, \phi, \bar{d}\}$:

$$\begin{aligned}
c_t^{-1/\sigma_c} + \mu_{1,t} \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}} - \mu_{2,t} \frac{c_t^{-1/\sigma_c-1}}{\sigma_c(1 + \tau_t)} - \mu_{3,t}(1 + \tau_t) - \mu_{4,t} &= 0, \\
[1] \quad c^{-1/\sigma_c} + \mu_1 \frac{1 + \tau}{\sigma_c(1 - l)c^{1-1/\sigma_c}} - \mu_2 \frac{c^{-1/\sigma_c-1}}{\sigma_c(1 + \tau)} - \mu_3(1 + \tau) - \mu_4 &= 0,
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{1-l_t} + \mu_{1,t} \frac{1+\tau_t}{(1-l_t)^2 c_t^{-1/\sigma_c}} + \mu_{3,t} A_t + \mu_{4,t} A_t = 0, \\
[2] \quad & -\frac{1}{1-l} + \mu_1 \frac{1+\tau}{(1-l)^2 c^{-1/\sigma_c}} + \mu_3 A + \mu_4 A = 0,
\end{aligned}$$

$$\begin{aligned}
& g_t^{-1/\sigma_g} - \mu_{4,t} = 0, \\
[3] \quad & g^{-1/\sigma_g} - \mu_4 = 0,
\end{aligned}$$

$$\begin{aligned}
& \mu_{1,t} \frac{1}{(1-l_t) c_t^{-1/\sigma_c}} - \mu_{2,t} \frac{c_t^{-1/\sigma_c}}{(1+\tau_t)^2} - \mu_{3,t} c_t = 0, \\
[4] \quad & \mu_1 \frac{1}{(1-l) c^{-1/\sigma_c}} - \mu_2 \frac{c^{-1/\sigma_c}}{(1+\tau)^2} - \mu_3 c = 0,
\end{aligned}$$

$$\begin{aligned}
& \mu_{2,t} \left[-\frac{1+r_t}{[1-\phi(d_t-\bar{d})]^2} \phi \beta E_t \lambda_{t+1} \right] + \mu_{3,t} - \beta (1+r_t) E_t \mu_{3,t+1} = 0, \\
[5] \quad & \mu_2 \left[-\frac{1+r}{[1-\phi(d-\bar{d})]^2} \phi \beta \lambda \right] + \mu_3 - \beta (1+r) \mu_3 = 0,
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{1-\phi(d_t-\bar{d})} \mu_{2,t} \beta E_t \lambda_{t+1} - \beta d_t E_t \mu_{3,t+1} = 0, \\
[6] \quad & -\frac{1}{1-\phi(d-\bar{d})} \mu_2 \lambda - d \mu_3 = 0,
\end{aligned}$$

$$\begin{aligned}
& \frac{1+\tau_t}{(1-l_t) c_t^{-1/\sigma_c}} - A_t = 0, \\
[7] \quad & \frac{1+\tau}{(1-l) c^{-1/\sigma_c}} - A = 0,
\end{aligned}$$

$$\begin{aligned}
& \lambda_t - \frac{1+r_t}{1-\phi(d_t-\bar{d})} \beta E_t \lambda_{t+1} = 0, \\
[8] \quad & 1 - \frac{1+r}{1-\phi(d-\bar{d})} \beta = 0,
\end{aligned}$$

$$[9] \quad d - (1 + r) d + Al - (1 + \tau) c = 0,$$

$$[10] \quad Al - c - g = 0,$$

$$[11] \quad y = Al,$$

$$[12] \quad \frac{c^{-1/\sigma_c}}{1 + \tau} = \lambda,$$

$$[13] \quad A = \bar{A}.$$

After calibrating some of these parameters (see text for details), we solve this system numerically.

6.2 Small open economy with incomplete asset markets

6.2.1 Households' problem

The household's problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

subject to

$$\begin{aligned} d_t &= (1 + r_{t-1}) d_{t-1} - y_t + (1 + \tau_t) c_t, \\ y_t &= A_t l_t, \\ r_t &= r_t^* + p(\tilde{d}_t), \\ \ln(A_t/\bar{A}) &= \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2), \\ \ln\left(R_t^*/\bar{R}^*\right) &= \rho_{R^*} \ln\left(R_{t-1}^*/\bar{R}^*\right) + \varepsilon_t^{R^*}, \quad \varepsilon_t^{R^*} \sim NIID(0, \sigma_{R^*}^2). \end{aligned}$$

and a no-Ponzi condition.

Note that, in equilibrium, $\tilde{d}_t = d_t$. This is not internalized by the household, but is internalized by the Ramsey planner (below).

6.2.2 Lagrangian

The Lagrangian is given by

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t [d_t - (1 + r_{t-1}) d_{t-1} + A_t l_t - (1 + \tau_t) c_t] \right\}.$$

The first-order conditions are given by

$$\begin{aligned} [c_t] &: \beta^t U_c(c_t, g_t, l_t) - \beta^t \lambda_t (1 + \tau_t) = 0, \\ [l_t] &: \beta^t U_l(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0, \\ [d_t] &: \beta^t \lambda_t - (1 + r_t) \beta^{t+1} E_t \lambda_{t+1} = 0. \end{aligned}$$

The first-order conditions reduce to

$$\begin{aligned} U_c(c_t, g_t, l_t) &= \lambda_t (1 + \tau_t), \\ U_l(c_t, g_t, l_t) &= -\lambda_t A_t, \\ \lambda_t &= (1 + r_t) \beta E_t \lambda_{t+1}. \end{aligned}$$

6.2.3 Government's flow budget constraint

The government's flow budget constraint is given by

$$\begin{aligned} d_t^g &= (1 + r_{t-1}^g) d_{t-1}^g - \tau_t c_t + g_t, \\ r_t^g &= r_t^* + p(d_t^g). \end{aligned}$$

6.2.4 Implementability conditions

The implementability conditions are given by

$$\begin{aligned} A_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)} (1 + \tau_t) = \Gamma_t \equiv \Gamma(c_t, g_t, l_t, \tau_t), \\ \frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} &= \lambda_t \equiv \lambda(c_t, g_t, l_t, \tau_t) = (1 + r_t) \beta E_t \lambda(c_{t+1}, g_{t+1}, l_{t+1}, \tau_{t+1}). \end{aligned}$$

6.2.5 Ramsey problem

The Ramsey problem takes the form:

$$\mathcal{L} = E_0 \left\{ \begin{array}{l} \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) \\ \quad + \beta^t \mu_{1,t} (\Gamma_t - A_t) \\ \quad + \beta^t \mu_{2,t} [\lambda_t - (1 + r_t^* + p(d_t)) \beta E_t \lambda_{t+1}] \\ \quad + \beta^t \mu_{3,t} [d_t - (1 + r_{t-1}^* + p(d_{t-1})) d_{t-1} + A_t l_t - (1 + \tau_t) c_t] \\ \quad + \beta^t \mu_{4,t} [d_t^g - (1 + r_{t-1}^* + p(d_{t-1}^g)) d_{t-1}^g + \tau_t c_t - g_t] \end{array} \right\}.$$

The first-order conditions are given by

$$\begin{aligned}
[c_t] &: U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\lambda_{c_t} - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t = 0, \\
[l_t] &: U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\lambda_{l_t} + \mu_{3,t}A_t = 0, \\
[g_t] &: U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\lambda_{g_t} - \mu_{4,t} = 0, \\
[\tau_t] &: \mu_{1,t}\Gamma_{\tau_t} + \mu_{2,t}\lambda_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t = 0, \\
[d_t] &: -\mu_{2,t}p'(d_t)\beta E_t\lambda_{t+1} + \mu_{3,t} - \beta(p'(d_t)d_t + 1 + r_t^* + p(d_t))E_t\mu_{3,t+1} = 0, \\
[d_t^g] &: \mu_{4,t} - \beta(p'(d_t^g)d_t^g + 1 + r_t^* + p(d_t^g))E_t\mu_{4,t+1} = 0, \\
[\mu_{1,t}] &: \Gamma_t - A_t = 0, \\
[\mu_{2,t}] &: \lambda_t - (1 + r_t^* + p(d_t))\beta E_t\lambda_{t+1} = 0, \\
[\mu_{3,t}] &: d_t - (1 + r_{t-1}^* + p(d_{t-1}))d_{t-1} + A_t l_t - (1 + \tau_t)c_t = 0, \\
[\mu_{4,t}] &: d_t^g - (1 + r_{t-1}^* + p(d_{t-1}^g))d_{t-1}^g + \tau_t c_t - g_t = 0.
\end{aligned}$$

6.2.6 Parameterization

Preferences are given by:

$$\begin{aligned}
U(c_t, g_t, l_t) &= \frac{c_t^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \frac{g_t^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \ln(1 - l_t), \\
U_{c_t} &= c_t^{-1/\sigma_c}; \quad U_{g_t} = g_t^{-1/\sigma_g}; \quad U_{l_t} = -\frac{1}{1 - l_t}.
\end{aligned}$$

The derivatives of Γ_t and λ_t are given by, respectively,

$$\begin{aligned}
\Gamma_t &= -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)}(1 + \tau_t) = \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma_c}}, \\
\Gamma_{c_t} &= \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}}; \quad \Gamma_{l_t} = \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}}; \quad \Gamma_{g_t} = 0; \quad \Gamma_{\tau_t} = \frac{1}{(1 - l_t)c_t^{-1/\sigma_c}}; \\
\lambda_t &= \frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t}, \\
\lambda_{c_t} &= -\frac{c_t^{-1/\sigma_c-1}}{\sigma_c(1 + \tau_t)}; \quad \lambda_{l_t} = 0; \quad \lambda_{g_t} = 0; \quad \lambda_{\tau_t} = -\frac{c_t^{-1/\sigma_c}}{(1 + \tau_t)^2}.
\end{aligned}$$

Following Uribe and Schmitt-Grohé (2003), the debt-elastic specification for the household is given by:

$$\begin{aligned} p(d_t) &= \psi^c [\exp(d_t - \bar{d}) - 1], \\ p'(d_t) &= \psi^c \exp(d_t - \bar{d}), \end{aligned}$$

and for the government:

$$\begin{aligned} p(d_t^g) &= \psi^g [\exp(d_t^g - \bar{d}^g) - 1], \\ p'(d_t^g) &= \psi^g \exp(d_t^g - \bar{d}^g). \end{aligned}$$

6.2.7 Dynamic system

The dynamic system consists of 13 endogenous and 2 exogenous variables, respectively:

$$\{c_t, \tau_t, l_t, g_t, d_t, d_t^g, r_t, y_t, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t\} \text{ and } \{A_t, r_t^*\},$$

that are determined by 15 equations:

$$\begin{aligned} U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\lambda_{c_t} - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t &= 0, \\ [1] \quad c_t^{-1/\sigma_c} + \mu_{1,t} \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}} - \mu_{2,t} \frac{c_t^{-1/\sigma_c-1}}{\sigma_c(1 + \tau_t)} - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t &= 0, \end{aligned}$$

$$\begin{aligned} U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\lambda_{l_t} + \mu_{3,t}A_t &= 0, \\ [2] \quad \frac{-1}{1 - l_t} + \mu_{1,t} \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}} + \mu_{3,t}A_t &= 0, \end{aligned}$$

$$U_{gt} + \mu_{1,t}\Gamma_{gt} + \mu_{2,t}\lambda_{gt} - \mu_{4,t} = 0,$$

$$[3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} = 0,$$

$$\mu_{1,t}\Gamma_{\tau_t} + \mu_{2,t}\lambda_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t = 0,$$

$$[4] \quad \mu_{1,t} \frac{1}{(1-l_t)c_t^{-1/\sigma_c}} - \mu_{2,t} \frac{c_t^{-1/\sigma_c}}{(1+\tau_t)^2} - \mu_{3,t}c_t + \mu_{4,t}c_t = 0,$$

$$-\mu_{2,t}p'(d_t)\beta E_t\lambda_{t+1} + \mu_{3,t} - \beta(p'(d_t)d_t + 1 + r_t^* + p(d_t))E_t\mu_{3,t+1} = 0,$$

$$[5] \quad -\mu_{2,t}\psi \exp(d_t - \bar{d}) \beta E_t\lambda_{t+1} + \mu_{3,t} - \beta(\psi \exp(d_t - \bar{d})d_t + 1 + r_t^* + \psi [\exp(d_t - \bar{d}) - 1])E_t\mu_{3,t+1} = 0,$$

$$\mu_{4,t} - \beta(p'(d_t^g)d_t^g + 1 + r_t^* + p(d_t^g))E_t\mu_{4,t+1} = 0,$$

$$[6] \quad \mu_{4,t} - \beta(\psi_g \exp(d_t^g - \bar{d}^g)d_t^g + 1 + r_t^* + \psi_g [\exp(d_t^g - \bar{d}^g) - 1])E_t\mu_{4,t+1} = 0,$$

$$\Gamma_t - A_t = 0,$$

$$[7] \quad \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma_c}} - A_t = 0,$$

$$[8] \quad \lambda_t - (1 + r_t^* + \psi [\exp(d_t - \bar{d}) - 1])\beta E_t\lambda_{t+1} = 0,$$

$$[9] \quad d_t - (1 + r_{t-1}^* + \psi [\exp(d_{t-1} - \bar{d}) - 1])d_{t-1} + A_t l_t - (1 + \tau_t)c_t = 0,$$

$$[10] \quad d_t^g - (1 + r_{t-1}^* + \psi_g [\exp(d_{t-1}^g - \bar{d}^g) - 1])d_{t-1}^g + \tau_t c_t - g_t = 0,$$

$$[11] \quad y_t = A_t l_t,$$

$$[12] \quad \lambda_t = \frac{c_t^{-1/\sigma_c}}{1 + \tau_t},$$

$$[13] \quad r_t = r_t^* + \psi [\exp(d_t - \bar{d}) - 1],$$

$$[14] \quad \ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0; \sigma_A^2)$$

$$[15] \quad \ln(1 + r_t^*) = (1 - \rho_{R^*}) \ln(1 + \bar{r}^*) + \rho_{R^*} \ln(1 + r_{t-1}^*) + \varepsilon_t^{r^*}, \quad \varepsilon_t^{r^*} \sim NIID(0; \sigma_{r^*}^2)$$

with associated 13 parameters

$$\{\sigma_c, \sigma_g, \beta, \psi, \bar{d}, \psi_g, \bar{d}^g, \rho_A, \sigma_A^2, \rho_r^*, \sigma_{r^*}^2, \bar{A}, \bar{r}^*\}.$$

6.2.8 Steady state

The steady state is a system of 15 equations with 28 unknowns, comprised of 15 steady-state variables

$$\{c, \tau, l, g, d, d^g, r, y, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, A, r^*\},$$

and 13 parameters $\{\sigma_c, \sigma_g, \beta, \psi, \bar{d}, \psi_g, \bar{d}^g, \rho_A, \sigma_A^2, \rho_{r^*}, \sigma_{r^*}^2, \bar{A}, \bar{r}^*\}$:

$$[1] \quad c^{-1/\sigma_c} + \mu_1 \frac{1 + \tau}{\sigma_c (1 - l) c^{1-1/\sigma_c}} - \mu_2 \frac{c^{-1/\sigma_c - 1}}{\sigma_c (1 + \tau)} - \mu_3 (1 + \tau) + \mu_4 \tau = 0,$$

$$[2] \quad - \frac{1}{1 - l} + \mu_1 \frac{1 + \tau}{(1 - l)^2 c^{-1/\sigma_c}} + \mu_3 A = 0,$$

$$[3] \quad g^{-1/\sigma_g} - \mu_4 = 0,$$

$$[4] \quad \mu_1 \frac{1}{(1 - l) c^{-1/\sigma_c}} - \mu_2 \frac{c^{-1/\sigma_c}}{(1 + \tau)^2} - \mu_3 c + \mu_4 c = 0,$$

$$[5] \quad - \mu_2 \psi \exp(d - \bar{d}) \beta \lambda + \mu_3 - \beta (\psi \exp(d - \bar{d}) d + 1 + r^* + \psi [\exp(d - \bar{d}) - 1]) \mu_3 = 0,$$

$$[6] \quad \mu_4 - \beta (\psi_g \exp(d^g - \bar{d}^g) d^g + 1 + r^* + \psi_g [\exp(d^g - \bar{d}^g) - 1]) \mu_4 = 0,$$

$$[7] \quad \frac{1 + \tau}{(1 - l) c^{-1/\sigma_c}} - A = 0,$$

$$[8] \quad \lambda - (1 + r^* + \psi [\exp(d - \bar{d}) - 1]) \beta \lambda = 0,$$

$$[9] \quad d - (1 + r^* + \psi [\exp(d - \bar{d}) - 1]) d + A l - (1 + \tau) c = 0,$$

$$[10] \quad d^g - (1 + r^* + \psi_g [\exp(d^g - \bar{d}^g) - 1]) d^g + \tau c - g = 0,$$

$$[11] \quad y = A l,$$

$$[12] \quad r = r^* + \psi \left[\exp \left(d - \bar{d} \right) - 1 \right],$$

$$[13] \quad \frac{c^{-1/\sigma_c}}{1 + \tau} = \lambda,$$

$$[14] \quad \ln A = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A,$$

$$[15] \quad \ln(1 + r^*) = (1 - \rho_{R^*}) \ln(1 + \bar{r}^*) + \rho_{R^*} \ln(1 + r^*).$$

This system may be reduced to the following 8 equations:

$$[1] \quad c^{-1/\sigma_c} + \mu_1 \frac{1 + \tau}{\sigma_c (1 - l) c^{1-1/\sigma_c}} - \mu_2 \frac{c^{-1/\sigma_c-1}}{\sigma_c (1 + \tau)} - \mu_3 (1 + \tau) + \mu_4 \tau = 0,$$

$$[2] \quad - \frac{1}{1 - l} + \mu_1 \frac{1 + \tau}{(1 - l)^2 c^{-1/\sigma_c}} + \mu_3 \bar{A} = 0,$$

$$[3] \quad g^{-1/\sigma_g} - \mu_4 = 0,$$

$$[4] \quad \mu_1 \frac{1}{(1 - l) c^{-1/\sigma_c}} - \mu_2 \frac{c^{-1/\sigma_c}}{(1 + \tau)^2} - \mu_3 c + \mu_4 c = 0,$$

$$[5] \quad - \mu_2 \psi \beta \frac{c^{-1/\sigma_c}}{1 + \tau} + \mu_3 - \beta (\psi \bar{d} + 1 + r^*) \mu_3 = 0,$$

$$[6] \quad \frac{1 + \tau}{(1 - l) c^{-1/\sigma_c}} - \bar{A} = 0,$$

$$[7] \quad \bar{d} - (1 + r^*) \bar{d} + \bar{A} l - (1 + \tau) c = 0,$$

$$[8] \quad d^g - (1 + r^*) d^g + \tau c - g = 0,$$

with the following 9 unknowns

$$\{c, \tau, l, g, \bar{d}, \mu_1, \mu_2, \mu_3, \mu_4\}$$

and the following equation added to close the system

$$(\bar{d} + d^g) / \bar{A} l = DtoY,$$

where $DtoY$ is calibrated to 1.34 (see Table 1).

Note that this system can be further simplified by setting $d^g = 0$. To see this, note that equation (4) above, repeated here for convenience:

$$\mu_4 - \beta \left(\psi_g \exp \left(d^g - \bar{d}^g \right) d^g + 1 + r^* + \psi_g \left[\exp \left(d^g - \bar{d}^g \right) - 1 \right] \right) \mu_4 = 0,$$

can be reduced to

$$\exp \left(d^g - \bar{d}^g \right) (d^g + 1) = 1.$$

Let us assume that the parameter \bar{d}^g takes the value zero. Then:

$$\exp (d^g) (d^g + 1) = 1,$$

which implies that $d^g = 0$. In other words, one can assume that $\bar{d}^g = 0$ and then it follows endogenously that $d^g = 0$.

After introducing this further simplification and calibrating some of the parameters in this model (see text for details), we solve this system numerically.

6.3 Small open economy with complete asset markets

6.3.1 Households' problem

The household's problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

subject to:

$$\begin{aligned} E_t d_{t+1} r_{t+1} &= d_t + A_t l_t - (1 + \tau_t) c_t, \\ y_t &= A_t l_t, \\ \ln (A_t / \bar{A}) &= \rho_A \ln (A_{t-1} / \bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2), \end{aligned}$$

and a no-Ponzi condition.

6.3.2 Lagrangian

The Lagrangian is given by

$$\mathcal{L} = E_0 \sum_{t=1}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t [d_t + A_t l_t - (1 + \tau_t) c_t - d_{t+1} r_{t+1}].$$

The first-order conditions are given by

$$\begin{aligned} [c_t] &: \beta^t U_{c_t}(c_t, g_t, l_t) - \beta^t \lambda_t (1 + \tau_t) = 0, \\ [l_t] &: \beta^t U_{l_t}(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0, \\ [d_{t+1}] &: -\beta^t \lambda_t E_t r_{t+1} + \beta^{t+1} E_t \lambda_{t+1} = 0. \end{aligned} \quad (41)$$

These first-order conditions can be simplified to

$$\begin{aligned} U_{c_t}(c_t, g_t, l_t) &= \lambda_t (1 + \tau_t), \\ U_{l_t}(c_t, g_t, l_t) &= -\lambda_t A_t, \\ \lambda_t E_t r_{t+1} &= \beta E_t \lambda_{t+1}. \end{aligned}$$

6.3.3 Governments' flow budget constraint

The government's flow budget constraint is given by

$$E_t d_{t+1}^g r_{t+1} = d_t^g - g_t + \tau_t c_t.$$

6.3.4 Implementability conditions

The implementability conditions are given by

$$\begin{aligned} A_t &= -\frac{U_{l_t}(c_t, g_t, l_t)}{U_{c_t}(c_t, g_t, l_t)} (1 + \tau_t) = \Gamma_t \equiv \Gamma(c_t, g_t, l_t, \tau_t), \\ \frac{U_{c_t}(c_t, g_t, l_t)}{1 + \tau_t} &= \lambda_t \equiv \lambda(c_t, g_t, l_t, \tau_t). \end{aligned}$$

Following Schmitt-Grohé and Uribe (2003),

$$\frac{U_{c_t}}{1 + \tau_t} = \Psi_{CAM}.$$

From condition (41), it follows that

$$E_t r_{t+1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}.$$

Multiplying on both sides by debt:

$$\begin{aligned} E_t r_{t+1} d_{t+1} &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} d_{t+1}, \\ E_t r_{t+1} d_{t+1}^g &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} d_{t+1}^g. \end{aligned}$$

Define

$$\begin{aligned} s_t &\equiv E_t r_{t+1} d_{t+1}, \\ s_t^g &\equiv E_t r_{t+1} d_{t+1}^g. \end{aligned}$$

Hence:

$$\begin{aligned} s_t &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} d_{t+1}, \\ s_t^g &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} d_{t+1}^g. \end{aligned}$$

6.3.5 Ramsey problem

The Ramsey problem's Lagrangian takes the form:

$$\mathcal{L} = E_0 \left\{ \begin{array}{l} \sum_{t=1}^{\infty} \beta^t U(c_t, g_t, l_t) \\ \quad + \beta^t \mu_{1,t} [\Gamma_t - A_t] \\ \quad + \beta^t \mu_{3,t} [d_t + A_t l_t - (1 + \tau_t) c_t - s_t] \\ \quad + \beta^t \mu_{4,t} [d_t^g - g_t + \tau_t c_t - s_t^g] \end{array} \right\}$$

The first-order conditions are given by:

$$\begin{aligned}
[c_t] &: U_{c_t} + \mu_{1,t}\Gamma_{c_t} - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t = 0, \\
[l_t] &: U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{3,t}A_t = 0, \\
[g_t] &: U_{g_t} + \mu_{1,t}\Gamma_{g_t} - \mu_{4,t} = 0, \\
[\tau_t] &: \mu_{1,t}\Gamma_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t = 0, \\
[d_{t+1}] &: -\mu_{3,t}E_t(\lambda_{t+1}/\lambda_t) + E_t\mu_{3,t+1} = 0, \\
[d_{t+1}^g] &: -\mu_{4,t}E_t(\lambda_{t+1}/\lambda_t) + E_t\mu_{4,t+1} = 0, \\
[\mu_{1,t}] &: \Gamma_t - A_t = 0, \\
[\mu_{3,t}] &: d_t + A_t l_t - (1 + \tau_t)c_t - s_t = 0, \\
[\mu_{4,t}] &: d_t^g - g_t + \tau_t c_t - s_t^g = 0.
\end{aligned}$$

6.3.6 Parameterization

Preferences are given by

$$\begin{aligned}
U(c_t, g_t, l_t) &= \frac{c_t^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \frac{g_t^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \log(1 - l_t), \\
U_{c_t} &= c_t^{-1/\sigma_c}; \quad U_{g_t} = g_t^{-1/\sigma_g}; \quad U_{l_t} = -\frac{1}{1 - l_t}.
\end{aligned}$$

The derivatives of Γ_t and λ_t are given by

$$\begin{aligned}
\Gamma_t &= -\frac{U_{l_t}(c_t, g_t, l_t)}{U_{c_t}(c_t, g_t, l_t)}(1 + \tau_t) = \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma_c}}, \\
\Gamma_{c_t} &= \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}}, \quad \Gamma_{l_t} = \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}}, \\
\Gamma_{g_t} &= 0; \quad \Gamma_{\tau_t} = \frac{1}{(1 - l_t)c_t^{-1/\sigma_c}}.
\end{aligned}$$

6.3.7 Dynamic system

The dynamic system consists of 13 endogenous and 1 exogenous variable, given by

$\{c_t, l_t, g_t, \tau_t, d_t, d_t^g, s_t, s_t^g, \mu_{1,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t, y_t\}$ and A_t , respectively.

that are determined by 14 equations:

$$\begin{aligned} U_{c_t} + \mu_{1,t}\Gamma_{c_t} - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t &= 0, \\ [1] \quad c_t^{-1/\sigma_c} + \mu_{1,t} \frac{1 + \tau_t}{\sigma_c(1 - l_t)c_t^{1-1/\sigma_c}} - \mu_{3,t}(1 + \tau_t) + \mu_{4,t}\tau_t &= 0, \end{aligned}$$

$$\begin{aligned} U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{3,t}A_t &= 0, \\ [2] \quad -\frac{1}{1 - l_t} + \mu_{1,t} \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma_c}} + \mu_{3,t}A_t &= 0, \end{aligned}$$

$$\begin{aligned} U_{g_t} + \mu_{1,t}\Gamma_{g_t} - \mu_{4,t} &= 0, \\ [3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} &= 0, \end{aligned}$$

$$\begin{aligned} \mu_{1,t}\Gamma_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0, \\ [4] \quad \mu_{1,t} \frac{1}{(1 - l_t)c_t^{-1/\sigma_c}} - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0, \end{aligned}$$

$$[5] \quad -\mu_{3,t}E_t(\lambda_{t+1}/\lambda_t) + E_t\mu_{3,t+1} = 0,$$

$$[6] \quad -\mu_{4,t}E_t(\lambda_{t+1}/\lambda_t) + E_t\mu_{4,t+1} = 0,$$

$$\Gamma_t - A_t = 0,$$

$$[7] \quad \frac{1 + \tau_t}{(1 - l_t)c^{-1/\sigma_c}} - A_t = 0,$$

$$[8] \quad d_t + A_t l_t - (1 + \tau_t)c_t - s_t = 0,$$

$$[9] \quad d_t^g - g_t + \tau_t c_t - s_t^g = 0,$$

$$[10] \quad s_{t+1} = \beta E_t(\lambda_{t+1}/\lambda_t)d_{t+1},$$

$$[11] \quad s_{t+1}^g = \beta E_t(\lambda_{t+1}/\lambda_t)d_{t+1}^g,$$

$$[12] \quad \lambda_t - \Psi_{CAM} = 0,$$

$$[13] \quad y_t - A_t l_t = 0,$$

$$[14] \quad \ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2),$$

with associated 6 parameters

$$\{\sigma_c, \sigma_g, \beta, \rho_A, \sigma_A^2, \bar{A}\}.$$

6.3.8 Steady state

The steady state is a system of 14 equations with 19 unknowns, comprised of 14 steady-state variables

$$\{c, l, g, \tau, d, d^g, \mu_1, \mu_3, \mu_4, A, y, \lambda, s, s^g\}$$

and 5 parameters $\{\sigma_c, \sigma_g, \beta, \rho_A, \bar{A}\}$:

$$[1] \quad c^{-1/\sigma_c} + \mu_1 \frac{1 + \tau}{\sigma_c(1 - l)c^{1-1/\sigma_c}} - \mu_3(1 + \tau) + \mu_4\tau = 0,$$

$$[2] \quad -\frac{1}{1 - l} + \mu_1 \frac{1 + \tau}{(1 - l)^2 c^{-1/\sigma_c}} + \mu_3 A = 0,$$

$$[3] \quad g^{-1/\sigma_g} - \mu_4 = 0,$$

$$[4] \quad \mu_1 \frac{1}{(1-l)c^{-1/\sigma_c}} - \mu_3 c + \mu_4 c = 0,$$

$$[5] \quad \frac{1+\tau}{(1-l)c^{-1/\sigma_c}} - A = 0,$$

$$[6] \quad d + Al - (1+\tau)c - \beta d = 0,$$

$$[7] \quad d^g - g + \tau c - \beta d^g = 0,$$

$$[8] \quad \frac{d^g + d}{Al} - DtoY = 0,$$

$$[9] \quad \lambda_t - \Psi_{CAM} = 0,$$

$$[10] \quad s = \beta d,$$

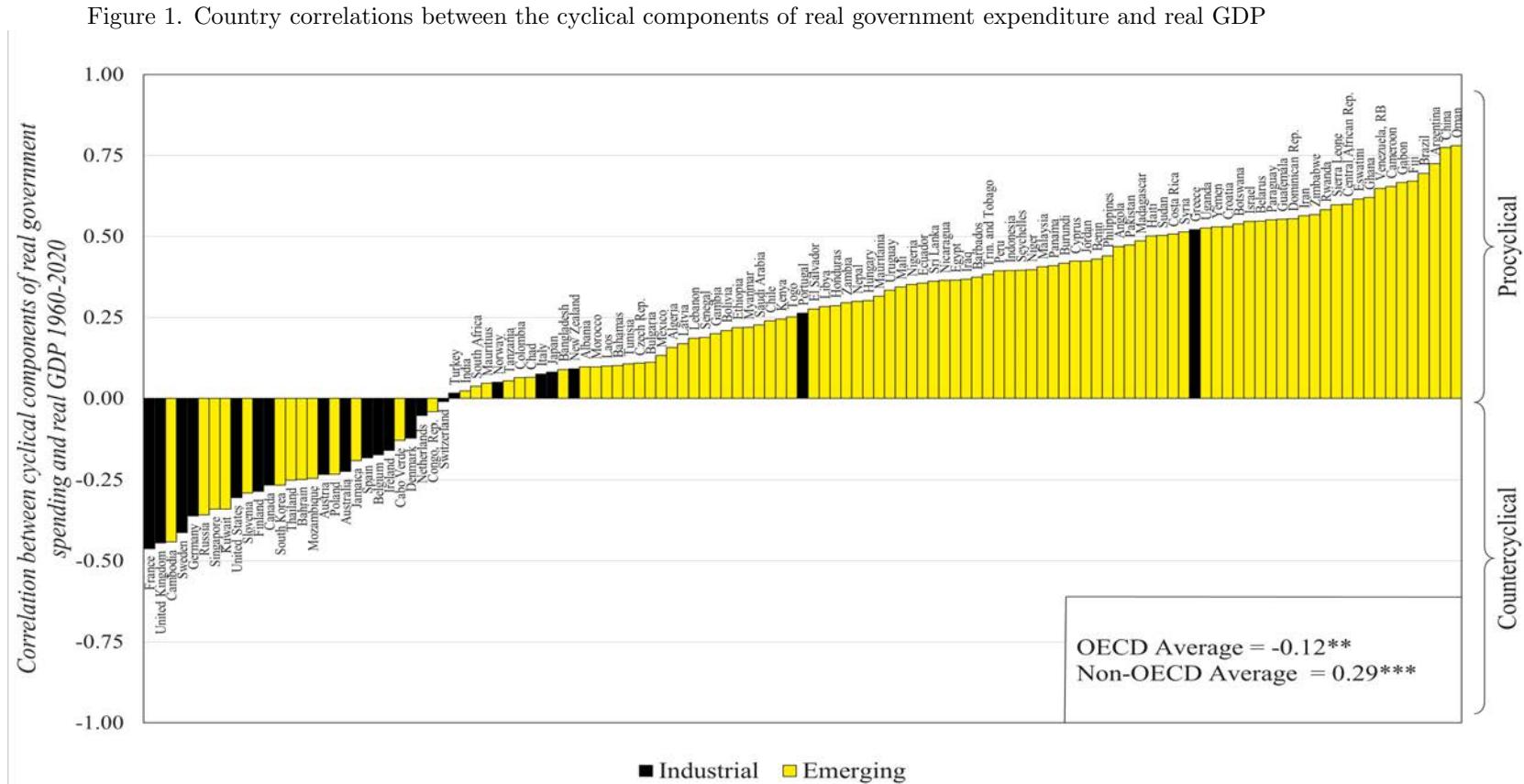
$$[11] \quad s^g = \beta d^g,$$

$$[12] \quad y = Al,$$

$$[13] \quad \ln A = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A,$$

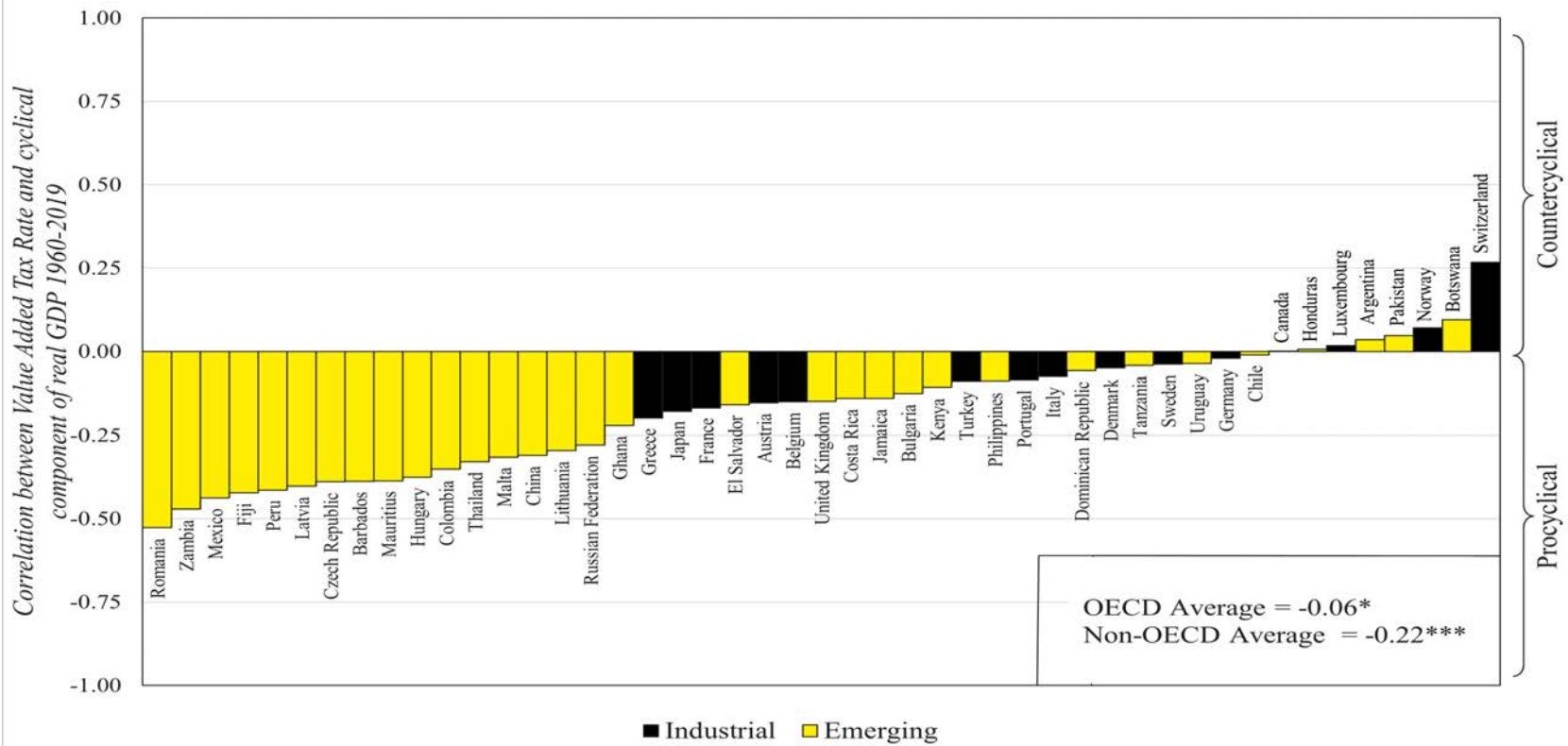
$$[14] \quad d^g = 0,$$

where we have added equations [8] and [14] so as to make the steady state consistent with that of the incomplete asset markets case: $d^g = 0$ and $DtoY = 1.34$. After calibrating the remaining parameters (see text for details), we solve this system numerically.



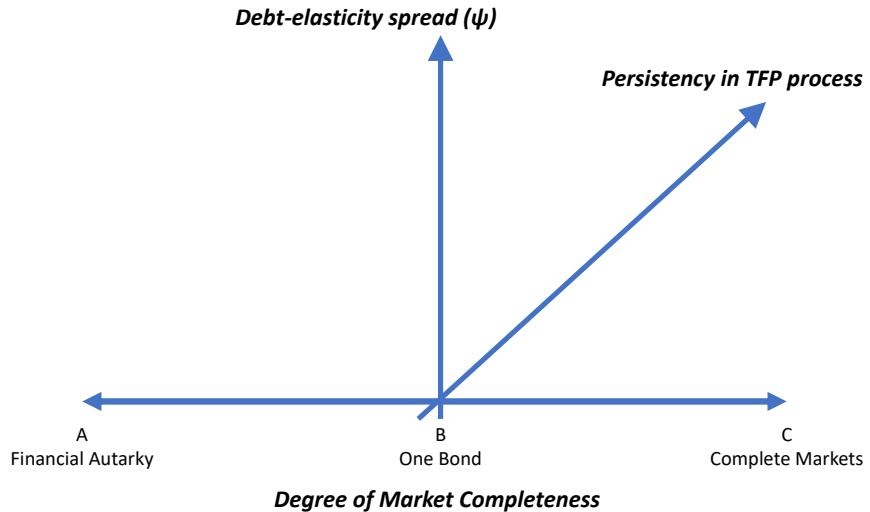
Notes: Sample is based on data availability and comprises 121 countries (22 OECD and 99 non-OECD) for the period 1960-2020. Black (dark) bars denote OECD countries and yellow (light) ones denote non-OECD (emerging) countries. OECD countries are those that were part of the OECD by 1973. The cyclical components were computed using the Hodrick-Prescott filter. Real government expenditure is defined as central government expenditure and net lending deflated by the GDP deflator. A positive (negative) correlation indicates procyclical (countercyclical) fiscal policy. The statistics shown in the figure correspond to the average of the country correlations for each group. The significance levels result from testing the hypothesis that the mean correlation is equal to zero within each group. Source: WEO (IMF).

Figure 2: Country correlations between the percentage changes in VAT and real GDP



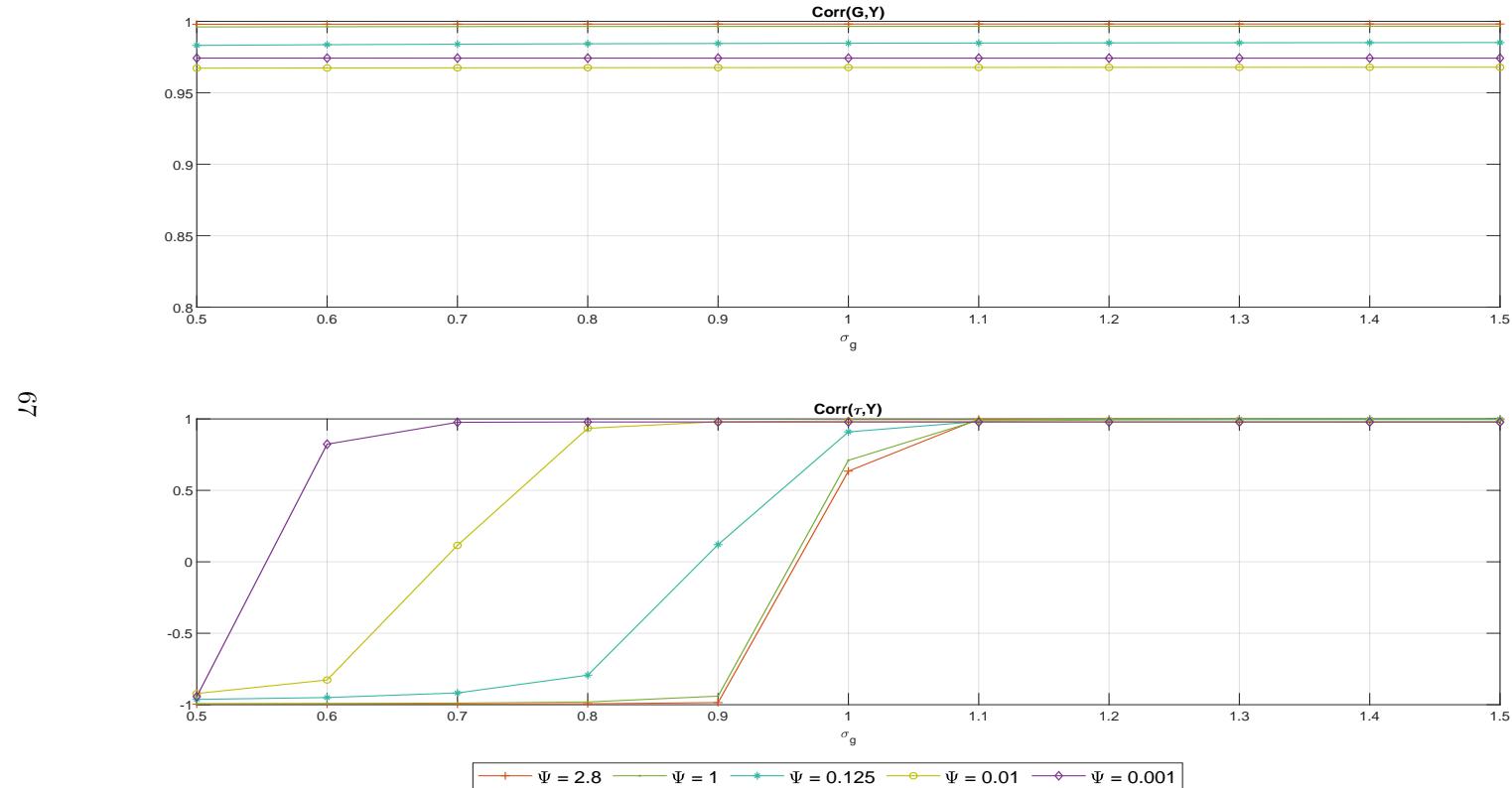
Notes: Sample is based on data availability and comprises 47 countries (15 OECD and 32 non-OECD) for the period 1960-2019. Black (dark) bars denote OECD countries and yellow (light) ones denote non-OECD (emerging) countries. OECD countries are those that were part of the OECD by 1973. A negative (positive) correlation indicates procyclical (countercyclical) fiscal policy. The cyclical components were computed using the Hodrick-Prescott filter. The statistics shown in the figure correspond to the average of the country correlations for each group. The significance levels result from testing the hypothesis that the mean correlation is equal to zero within each group. Sources of raw data are WEO (real GDP) and Végh and Vuletin's (2015) updated tax database (VAT), available at <http://www.guillermovuletin.com/data>.

Figure 3: Frictions in DSGE model



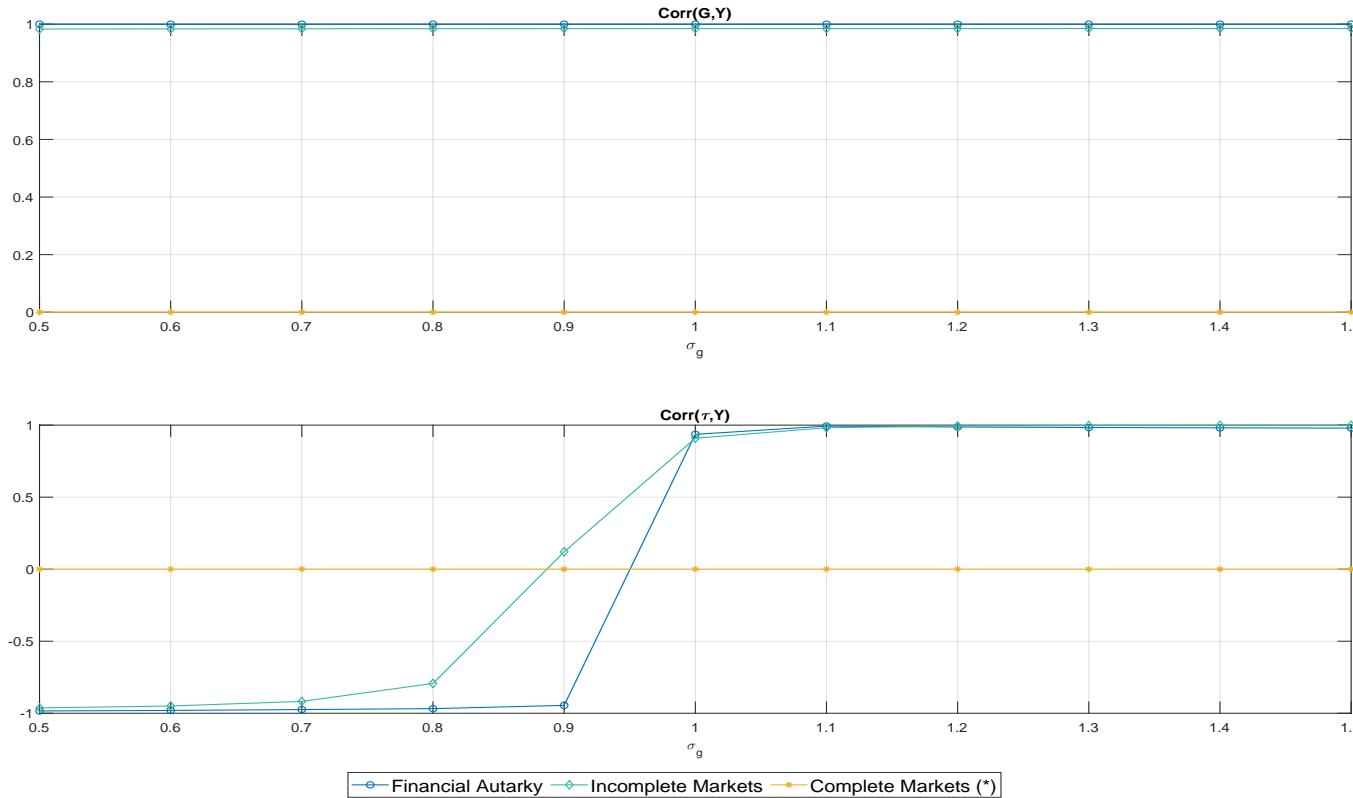
Notes: The figure illustrates, in a schematic format, the three frictions considered in the DSGE model and their interactions. The horizontal axis captures the various degrees of completeness in financial markets, with financial autarky and complete markets as two opposite extremes and a one-period, non-state contingent bond as an intermediate case. In the latter case, a varying debt elasticity of spreads interacts as the second friction considered, captured by the vertical axis. The third friction, illustrated by the diagonal axis, captures varying degrees of persistence (and, hence, volatility) of the TFP process.

Figure 4: Fiscal procyclicality and the intertemporal substitution of government spending in incomplete asset markets case



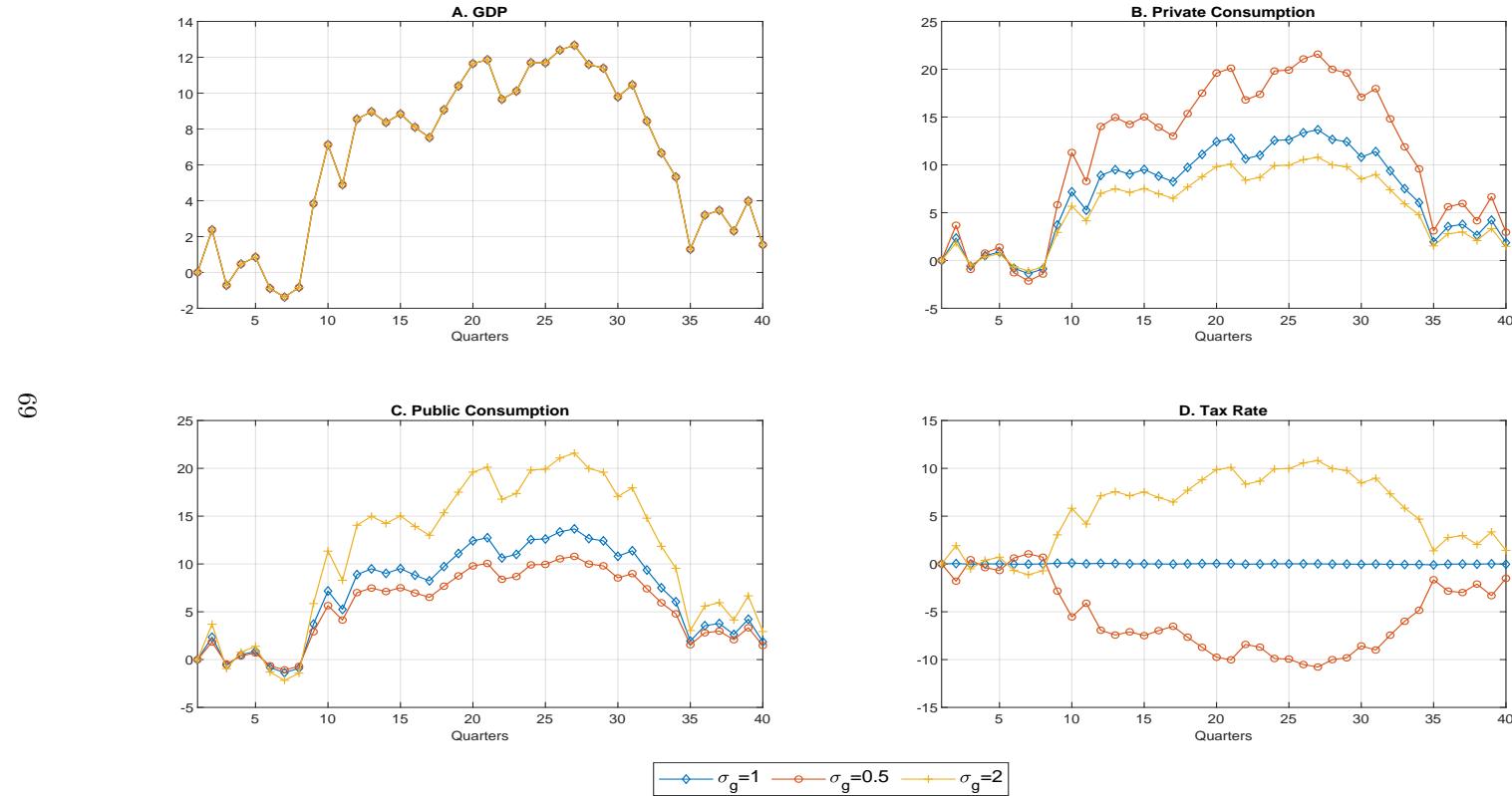
Notes: The figure shows the correlation between government consumption and output (top panel) and between tax rates and output (bottom panel) for various levels of the intertemporal elasticity of substitution of government spending in the incomplete asset markets model. Correlations are computed from (HP-filtered) simulated data for 100,000 quarters, randomly drawing TFP shocks only. Correlations are shown as a function of σ_g for different values of the debt elasticity of the interest rate (ψ). σ_c is fixed at one.

Figure 5: Fiscal procyclicality and the intertemporal substitution of government spending for different degrees of market completeness



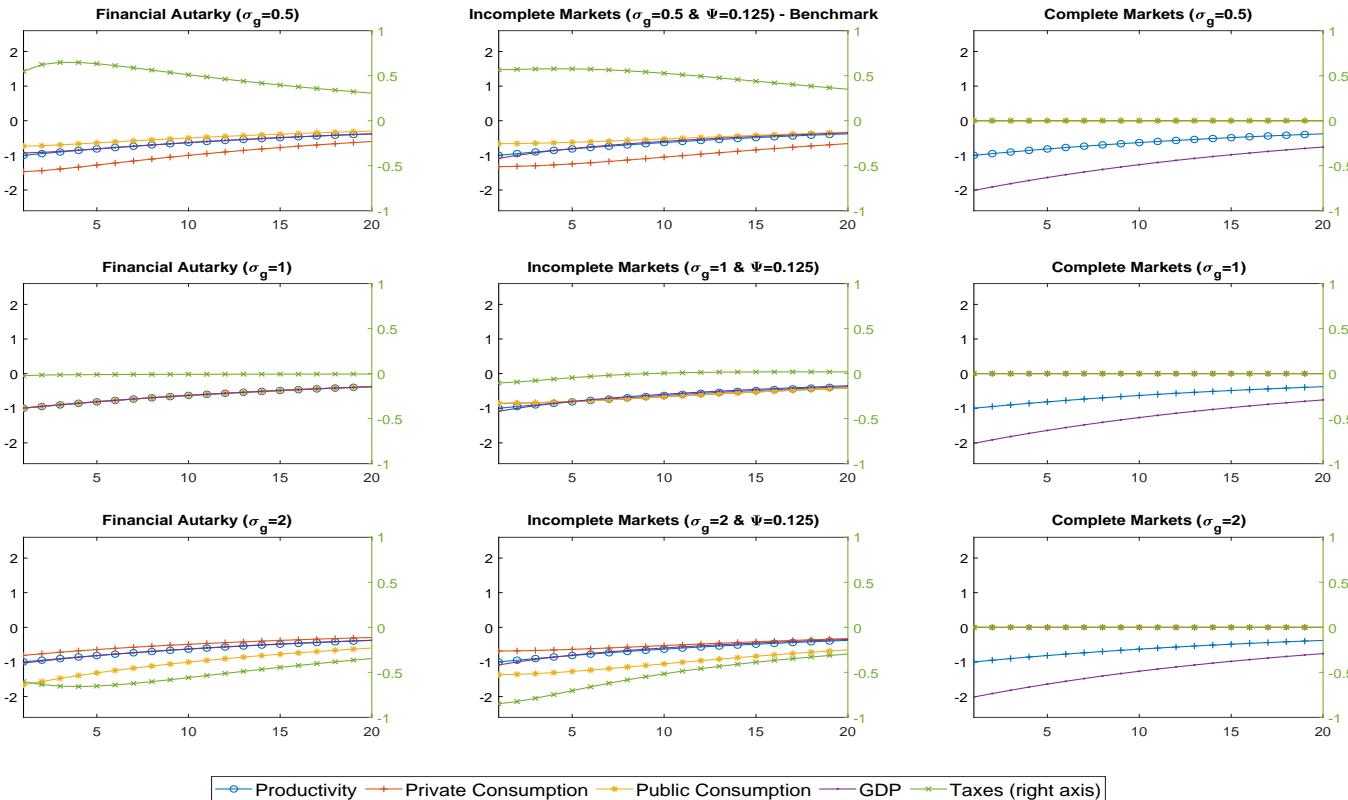
Notes: The figure shows the correlation between government consumption and output (top panel) and between tax rates and output (bottom panel) for various levels of the intertemporal elasticity of substitution of government spending (σ_g). σ_c is fixed at one. Correlations are computed from (HP-filtered) simulated data for 100,000 quarters, randomly drawing TFP shocks only. Results are shown for the three alternative cases of market completeness considered. Given that, under complete markets, government expenditure and consumption are fully smoothed, it is not possible to calculate the correlations between government expenditure and output and between tax rates and output. Therefore, in the complete markets case, the plot shows the covariance term.

Figure 6: Cyclical dynamics and the intertemporal substitution of government spending: A simulation



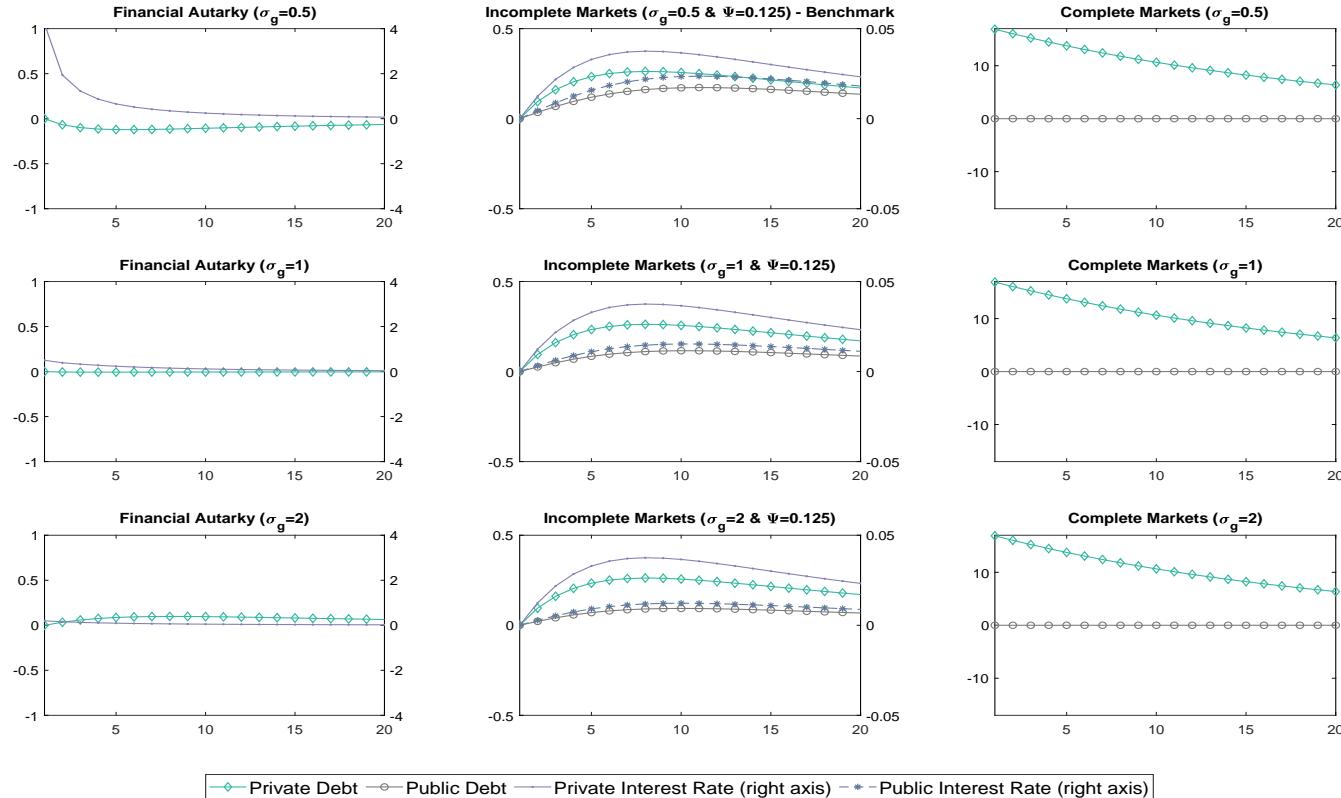
Notes: The figure shows, for the incomplete markets case, the deviations from the steady state for GDP, private consumption, public consumption, and tax rates from randomly drawing TFP shocks for 40 quarters. For visual purposes, in the simulation the parameter governing the debt elasticity of interest rates (ψ) was set at a higher value (2.8) than in our benchmark calibration. Each of the three time series illustrated per variable is associated with an alternative value of the intertemporal elasticity of substitution of government spending (σ_g).

Figure 7A: Varying degrees of market completeness: Impulse response functions



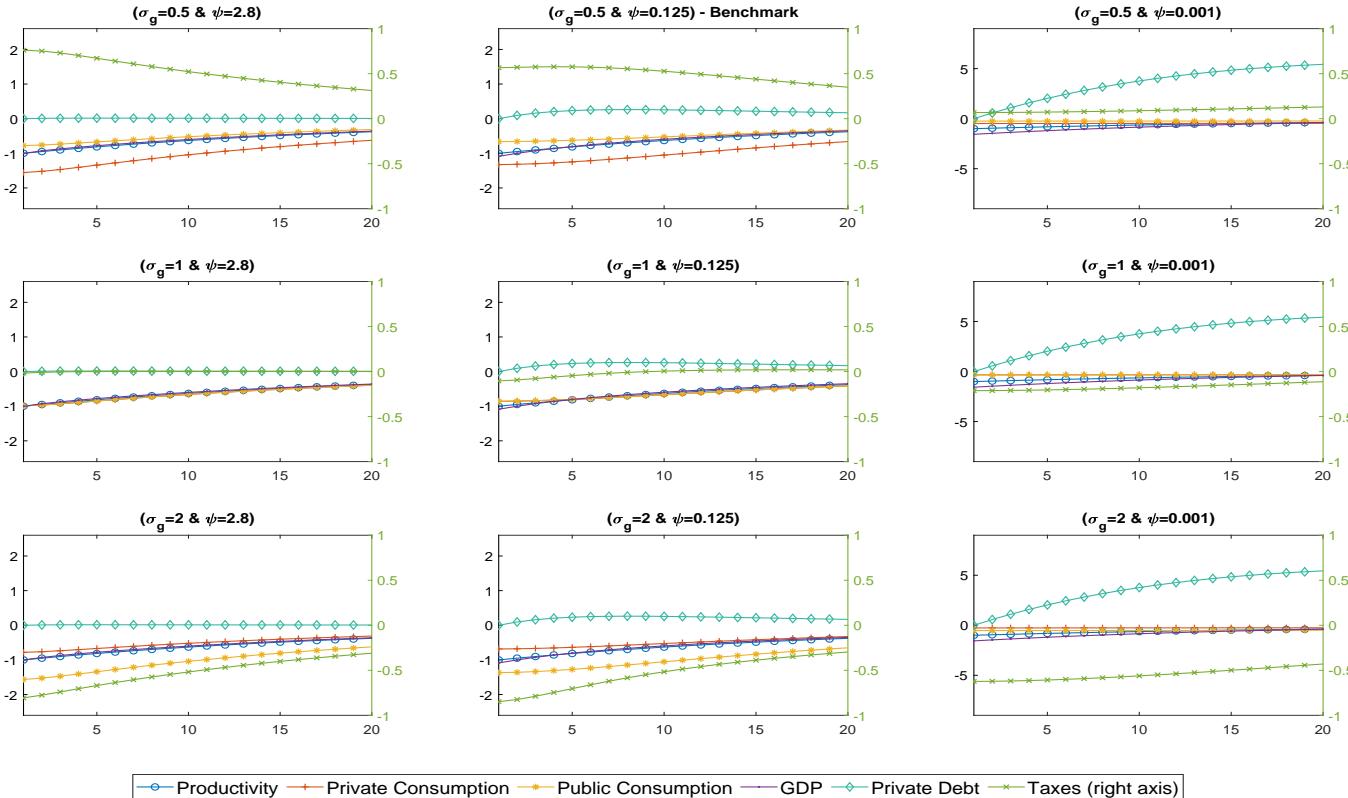
Notes: The figure shows the IRFs of private consumption, public consumption, GDP, and tax rates following a fall in TFP of one percentage point relative to the steady state. The responses are expressed in percentage deviation from steady state levels. The figure plots the IRFs for 20 quarters, including the initial one ($t = 1$) when the shock occurs. The three columns illustrate the three alternative cases of market completeness considered in the analysis. The difference across rows is the value of the intertemporal elasticity of substitution of government consumption (σ_g).

Figure 7B: Varying degrees of market completeness: Impulse response functions (cont.)



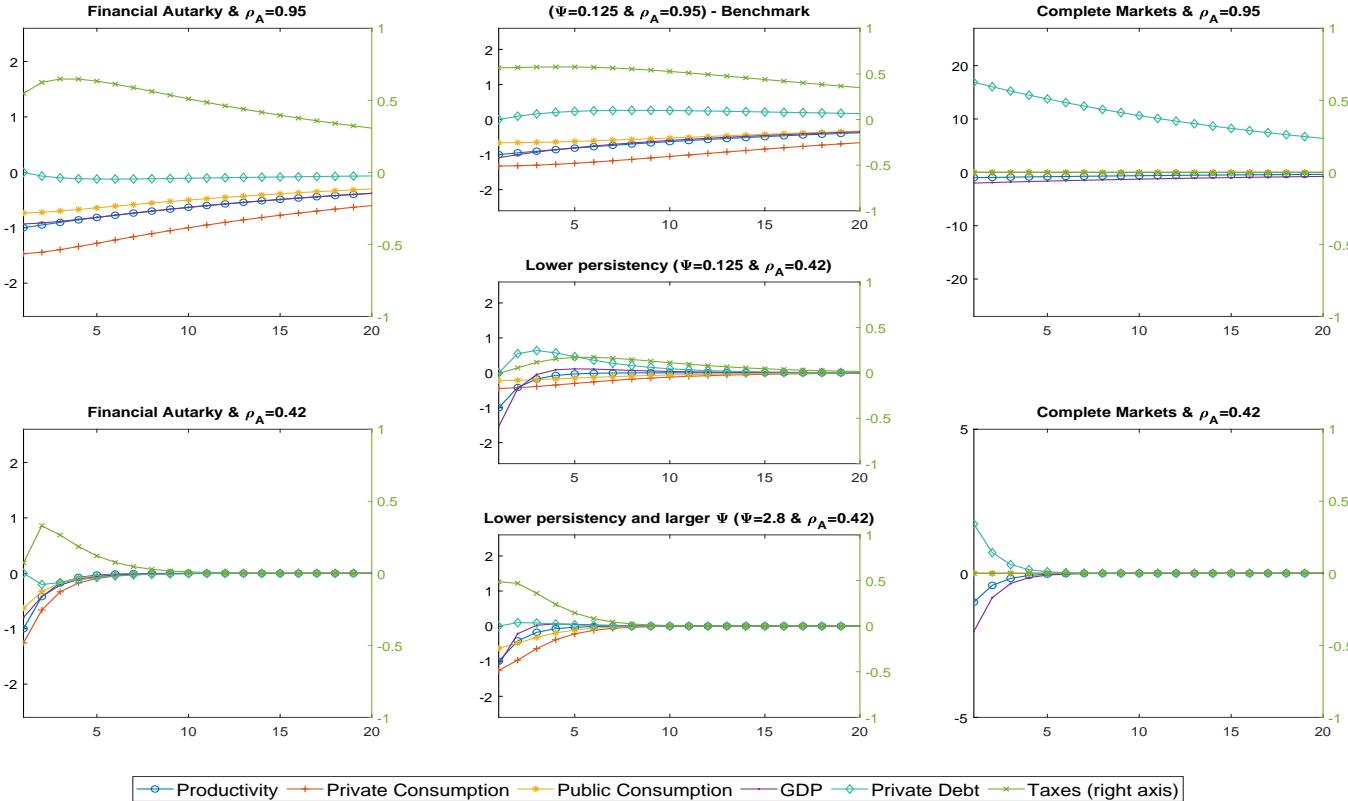
Notes: The figure shows the IRFs of private debt, public debt, private interest rate, and public interest rate following a fall in TFP of one percentage point relative to the steady state. Responses are expressed in percentage deviations from steady state levels, except for real interest rates and debt (linear deviations). The figure plots the IRFs for 20 quarters, including the initial one ($t = 1$) when the shock occurs. The three columns illustrate the three alternative cases of market completeness considered in the analysis. The difference across rows is the value of the intertemporal elasticity of substitution of government consumption (σ_g).

Figure 8: Varying degrees of market completeness: Impulse response functions (cont.)



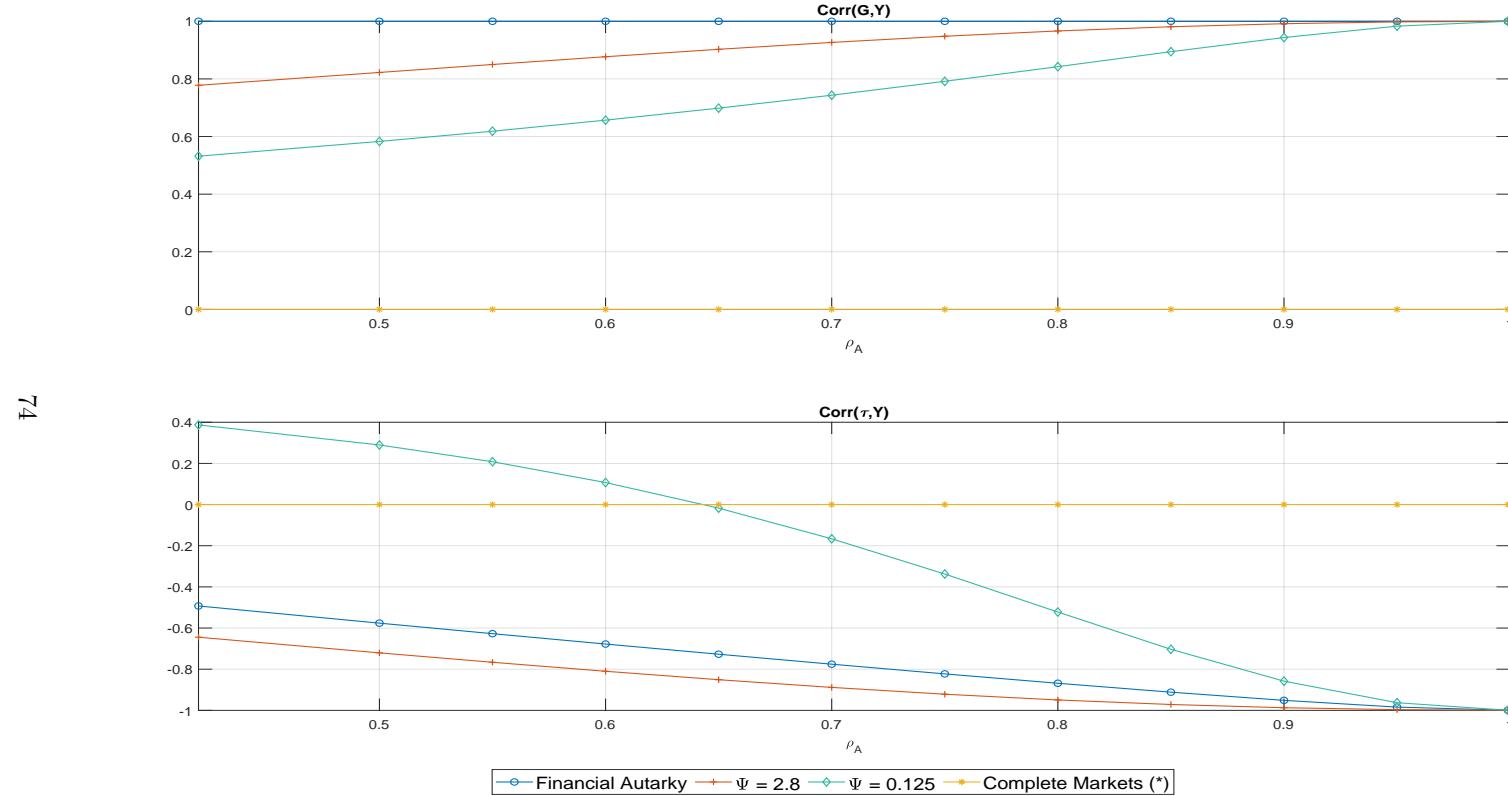
Notes: The figure shows, for the incomplete markets case, the IRFs of private consumption, public consumption, GDP, private debt, and tax rates following a fall in TFP of one percentage point relative to the steady state. Responses are expressed in percentage deviations from steady state levels, except for real interest rates and debt (linear deviations). The figure plots the IRFs for 20 quarters, including the initial one ($t = 1$) when the shock occurs. The three columns assume three different values of the debt elasticity of the interest rate (ψ). The difference across rows is the value of the intertemporal elasticity of substitution of government consumption (σ_g).

Figure 9: Varying TFP persistence: Impulse response functions



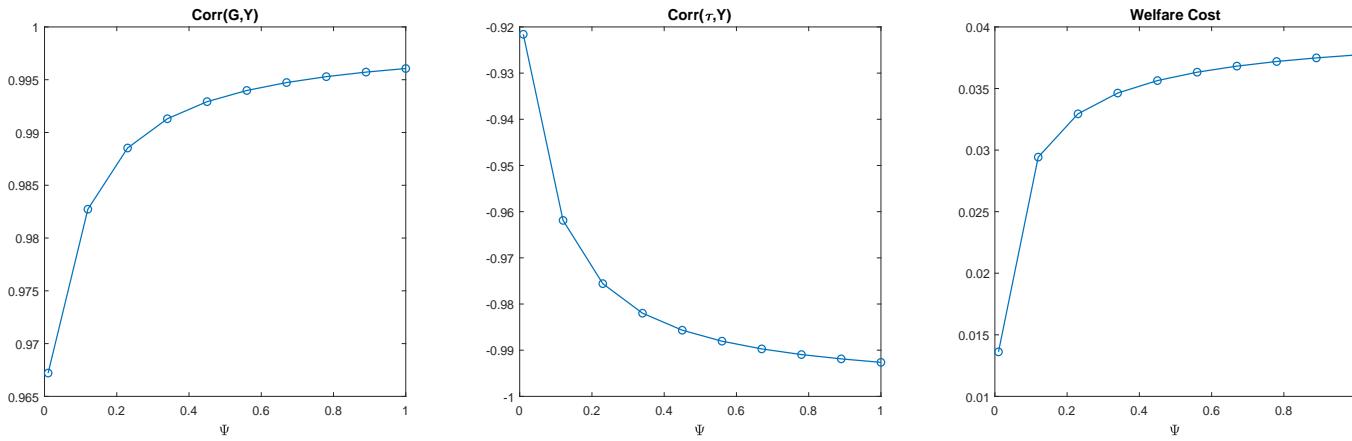
Notes: The figure shows, for the incomplete markets case, the IRFs of private consumption, public consumption, GDP, private debt, and tax rates following a fall in TFP of one percentage point relative to the steady state. Responses are expressed in percentage deviations from steady state levels, except for real interest rates and debt (linear deviations). The figure plots the IRFs for 20 quarters, including the initial one ($t = 1$) when the shock occurs. The three columns illustrate the three alternative cases of market completeness considered in the analysis. The difference across rows in the leftmost and rightmost columns is the value for the persistence of the TFP process (ρ_A). In the middle column, for the incomplete asset markets case, both ρ_A and the debt elasticity of the interest rate (ψ) vary.

Figure 10: Fiscal procyclicality and persistence of TFP



Notes: The figure shows the correlation between government consumption and output (top panel) and between tax rates and output (bottom panel) for different levels of persistence in the TFP process, as governed by the AR(1) coefficient of the corresponding law of motion, plotted on the horizontal axis. Correlations are computed from (HP-filtered) simulated data for 100,000 quarters, randomly drawing TFP shocks only. Results are reported for the three different cases of market incompleteness covered in the analysis, though, for the case of incomplete markets, two values of the debt elasticity of the interest rate (ψ) are considered. Given that, under complete markets, government expenditure and consumption are fully smoothed, it is not possible to compute the correlations between government expenditure and output and between tax rates and output. Therefore, the plot shows the covariance term. For this exercise, σ_g and σ_c are set at 0.5 and 1, respectively.

Figure 11: Welfare costs



Notes: The figure shows the correlation between government consumption and output (leftmost plot), between tax rates and output (middle plot), and welfare costs expressed in percentage (rightmost plot) for different levels of financial frictions (ψ). Correlations and welfare costs are computed from (HP-filtered) simulated data for 100,000 quarters, randomly drawing TFP shocks only. Parameter values are the following: $\sigma_g = 0.5$, $\sigma_c = 1$, $\sigma_A = 0.0129$, and $\rho_A = 0.95$.

Table 1: Calibration

Parameter	Description	Value	Source
\bar{A}	TFP in steady state	1	Assumed
\bar{R}^*	Real interest rate	$1.04^{\frac{1}{4}}$	Schmitt-Grohé and Uribe (2003)
$(\bar{d} + \bar{d}^g)/y$	Foreign debt share	1.34	NFA data (Lane and Milesi-Ferretti, 2015)
ρ_A	TFP AR(1)	varies	Neumeyer and Perri (2005)/MSM
σ_A	SD TFP Shock	varies	Schmitt-Grohé and Uribe (2003)/MSM
ρ_{R^*}	Interest rate AR(1)	0.83	Uribe and Yue (2006)
σ_{R^*}	SD Interest rate shock	0.007	Uribe and Yue (2006)
β	Discount factor	$1/1.04^{\frac{1}{4}}$	Endogenous ($\beta \bar{R}^* = 1$)
$\Psi^c = \Psi^g = \Psi$	Debt-elasticity	varies	Garcia-Cicco et al. (2010)/Own estimates
σ_c	IES Private consumption	1	Assumed
σ_g	IES Public consumption	varies	MSM
ϕ	Debt adjustment cost parameter in the financial autarky model	0.1	Assumed

Notes: This table summarizes the quarterly calibration of the parameters in the DSGE model. These values mostly rely on previous studies of small open economies noted in the column "Source." The persistence of the TFP process (ρ_A), the volatility of the TFP shocks (σ_A), the debt elasticity of the interest rate in both private and public debt (ψ^c and ψ^g), and the intertemporal elasticity of substitution of public consumption (σ_g) will vary across different experiments carried out in the analysis. In the benchmark case, however, these parameters are set at 0.95, 0.0129, 0.125, and 0.5, respectively. Results from matching second moments (MSM) are presented in Table 3.

Table 2: Varying degree of market completeness: Second moments

(1) $\sigma_c = 1; \sigma_g = 0.5$			
Moments	A. Fin. autarky	B. Incomplete markets	C. Complete markets (*)
Std(y)	0.016	0.018	0.034
Std(c)	0.026	0.023	0.000
Std(g)	0.013	0.012	0.000
Std(τ)	0.011	0.010	0.000
Corr(τ, y)	-0.984	-0.963	0.000
Corr(g,y)	1.000	0.983	0.000
Corr(c,y)	1.000	0.983	0.000
Corr((c/g),y)	1.000	0.983	0.000
(2) $\sigma_c = 1; \sigma_g = 1$			
Moments	A. Fin.autarky	B. Incomplete markets	C. Complete markets (*)
Std(y)	0.017	0.018	0.034
Std(c)	0.017	0.015	0.000
Std(g)	0.017	0.015	0.000
Std(τ)	0.000	0.002	0.000
Corr(τ, y)	0.936	0.909	0.000
Corr(g,y)	1.000	0.985	0.000
Corr(c,y)	1.000	0.985	0.000
Corr((c/g),y)	-0.031	0.993	0.000
(3) $\sigma_c = 1; \sigma_g = 2$			
Moments	A. Fin. autarky	B. Incomplete markets	C. Complete markets (*)
Std(y)	0.017	0.018	0.034
Std(c)	0.014	0.012	0.000
Std(g)	0.028	0.024	0.000
Std(τ)	0.012	0.015	0.000
Corr(τ, y)	0.974	0.997	0.000
Corr(g,y)	1.000	0.985	0.000
Corr(c,y)	1.000	0.985	0.000
Corr((c/g),y)	-1.000	-0.985	0.000

Notes: This table reports second moments of key variables in the model. These moments are computed from (HP-filtered) simulated data for 100,000 quarters, randomly drawing TFP shocks only. The three columns show the three alternative cases of market completeness considered in the analysis. The difference across panels is the value of the intertemporal elasticity of substitution of government consumption (σ_g). Given that, under complete markets, government expenditure and consumption are fully smoothed, it is not possible to compute the correlations between government expenditure and output and between tax rates and output. Therefore, in the complete markets case, the table reports the covariance term.

Table 3: Matching second moments

Targeted moments	$\sigma_y; \sigma_c; \rho_{\tau,y}; \rho_{g,y}$	
Calibration obtained via MSM	$\sigma_g = 0.25, \sigma_A = 0.005,$ $\psi = 1, \rho_A = 0.95$	
Moments	Data	Model
Std(y)	0.017	0.010
Std(c)	0.026	0.018
Std(g)	0.056	0.005
Std(τ)	0.054	0.014
Corr(τ, y)	-0.104	-0.058
Corr(g,y)	0.142	0.143
Corr(c,y)	0.629	0.143
Corr((c/g),y)	0.126	0.143

Notes: The calibration procedure for this table consists of the following steps: (1) a grid is defined with all the possible combinations of values that the parameters to be calibrated can take. These parameters are the intertemporal elasticity of substitution of government consumption (σ_g), the volatility of TFP shocks (σ_A), the debt elasticity of the interest rate (ψ), and the persistence of the TFP process (ρ_A); (2) for each point in the grid, the dynamics of the model are simulated and the targeted moments (the volatility of output (σ_y), the volatility of consumption (σ_c), the correlation between tax rates and output ($\rho_{\tau,y}$), and the correlation between government spending and output ($\rho_{g,y}$)) are computed from the Hodrick-Prescott filtered series. Both the simulation of the model and the computation of the moments were carried out for simultaneous random TFP shocks and world interest rate shocks; (3) a quadratic loss function based on the estimated theoretical moments and the moments in the data is computed; and (4) the combination of values of σ_g , σ_A , ψ , and ρ_A for which the loss function takes the lowest value is selected.

Table 4: Empirical Results

(1) Procyclicality Measure							
Procyclicality Measure	Non-OECD		OECD		T Stat	C.V	Test Result
	Average	SE	Average	SE			
	G	0.29***	0.029	-0.118*	0.052	6.916	1.306
(2) Market Incompleteness							
Capital Controls Measure	Non-OECD		OECD		T Stat	C.V	Test Result
	Average	SD	Average	SD			
	kai	0.412	0.301	0.099	0.093	7.421	1.292
(3) Debt Elasticity - Panel							
Reg	Non-OECD		OECD		Wald Stat	C.V	Test Result
	Coef	Coef	Coef	Coef			
	1 (Total Public Debt)	0.125***		0.002	61.18	3.84	Reject H_0
(4) GDP Volatility							
Business Cycle Measure	Non-OECD		OECD		T Stat	C.V	Test Result
	Average	SE	Average	SE			
	σ_Y	3.283***	0.306	1.472***	0.098	5.64	1.289

Note: In panels 1, 3, and 4, ***, **, and * indicate significance at, respectively, 1%, 5%, and 10% levels. Notes Panel 1: The statistics displayed in this panel are based on the full sample of countries for each procyclicality measure. For G and tax rates (VAT) the number of countries is, respectively, 121 and 47. The last column reports the result of the (one-tail) hypothesis test that the means for both groups of countries are equal for each procyclicality measure. Notes Panel 2: We use the capital controls indices from the updated version of Fernandez et al. (2016), which has a set of 87 countries that also have information on measures of procyclicality. The four specific indices considered are: overall assets inflow restrictions index (kai); bond inflow restrictions (boi); overall assets outflow restrictions index (kao); and bond outflow restrictions (boo). The indices for each country are normalized from zero to one, with one indicating the highest degree of capital controls. The last column shows the result of the (one-tail) hypothesis test that the means for both groups of countries, non-OECD and OECD, are equal for each capital controls index. Notes Panel 3: OECD countries are those that were part of the OECD by 1973. Non-OECD countries are the remaining countries for which we have data on their spread index (EMBIG or UIP spread in the case of countries considered developed by the BofA Merrill Lynch Bond Index Guide that are not in our OECD group). The number of countries for each regression is 16 OECD and 20 non-OECD in regression 1, and 17 OECD and 29 non-OECD in regression 2. The last column reports the result of the (one-tail) hypothesis test that the coefficients for both groups of countries are equal in each regression. Notes Panel 4: The statistics displayed in this panel are based on a sample of 123 countries. σ_Y corresponds to the volatility of the cyclical component of the HP-filtered data for the yearly real GDP of those countries. The last column reports the result of the (one-tail) hypothesis test that the means of σ_Y for both groups of countries are equal.