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THE REAL EXPLANATION OF NOMINAL BOND-STOCK PUZZLES

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ABSTRACT

We present evidence that the mix of transitory and permanent shocks to consumption is changing over time. We study the implications of this finding for asset prices. The uncovered dynamics of consumption implies modestly upward sloping real bond and equity curves, upward sloping nominal yield curve, and sign-switching correlation between equities and bonds consistent with the stylized facts. This is achieved without relying on the nominal channel too much. That is, as in the data, the variation of inflation in the model is under 40% as a fraction of variation in nominal yields.

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1 Introduction

In this paper we propose a model of macroeconomic fluctuations. The novelty of the model is that it allows for changes in the relative contribution of permanent and transitory shocks to aggregate consumption. We find empirical support for such a specification and study implications of this evidence for asset prices. We show that the model goes a long way towards capturing the changing stock-bond correlation, real and nominal bond yield curves, as well as the equity yield curve.

The nature of macroeconomic fluctuations has preoccupied macroeconomists since at least Hayek (1933) and, more recently, Lucas (1977). While the starting point was that departures from the trend are of transitory nature, the literature has gradually shifted towards recognition that there are shocks to the trend itself, aka permanent shocks (Campbell and Mankew, 1987, Cochrane, 1988, among many others). Alvarez and Jermann (2005) demonstrate that asset prices are particularly sensitive to the differences in these perspectives on the macroeconomic dynamics.

The asset-pricing literature has primarily focused on the other end of the spectrum, that is, models where permanent shocks to consumption are the dominant driving force (e.g., Bansal and Yaron, 2004, Campbell and Cochrane, 1999). This permanent shock paradigm is experiencing difficulty matching some key asset-pricing facts, such as small positive slopes of the real yield curve and the equity yield curve, as well as the changing sign of the real equity-bond covariance.

We model the consumption level as the sum of two components that we label permanent and transitory, respectively. Regime-changing volatility of the shocks to these components leads to changes in the dominance of one of them. If the permanent component dominates, the economy behaves similar to workhorse asset-pricing models and features positive covariance between realized and expected consumption growth. In contrast, if the transitory component dominates, that covariance becomes negative.

When the model is confronted with data on consumption only, the preferred specification features three regimes. The changes between two of these regimes occur at a very low frequency. The "transitory" regime has prevailed throughout the early part of our sample, up until about 1995. The "permanent" regime has appeared sporadically in the early sample, primarily at the peak of expansions, and prevails in the post-1995 period. The third regime captures persistent disaster state, but its persistence and magnitude are modest relative to what is usually entertained in the disaster literature (Barro, 2006, Wachter, 2013).

We explore asset pricing implications of these dynamics in two steps. First, we assume a representative agent with Epstein and Zin (1989) preferences and explore assets whose price can be computed from consumption alone: real bond yields and the consumption claim. In the later "permanent" regime, negative shocks to the macroeconomy tends to be followed by lower than average growth (positive conditional covariance between realized and expected consumption growth), making real bonds hedges. The real yield curve is therefore downward-sloping in this regime. In the earlier "transitory" regime, however, negative shocks tend to be followed by higher than average growth (negative conditional covariance between realized and expected consumption growth), which makes real bonds risky. The disaster regime is even more strongly affected by such transitory shocks as crisis periods are followed by a recovery. The term structures in these states are flat and upward-sloping, respectively. The real curve averaged across the regimes is nearly flat with slightly positive slope. Because real bonds transition from being risky to becoming hedges, the model implied consumption claim - real bond correlation switches sign as well. In the sample this correlation switched from positive to negative around 1995.

Second, we link the model's implications to observable asset prices, namely, aggregate equity and nominal bonds. Doing so requires specification and calibration of cash flows, dividends and inflation, respectively. Because our focus is on the role of consumption dynamics for asset pricing, we posit cash flows dynamics that are simple and in line with the literature. We show that the model is capable of generating realistic baseline asset-pricing moments, including risk premiums, return volatility, and predictability of both equity and bond excess returns. Also, the estimated consumption combined with calibrated cash flows are consistent with the nominal bond (e.g., Bansal and Shaliastovich, 2013, Piazzesi and Schneider, 2006) and equity yield curves (e.g., van Binsbergen, Brandt, and Koijen, 2012, Giglio, Kelly, and Kozak, 2020).

Furthermore, the results obtained using consumption dynamics alone help us capture evidence emphasized in the recent literature. Because the model generates sign-switching correlations between the consumption claim and real bond returns, it continues doing so for the equity claim and nominal bonds. That is, the model generates this pattern via the real channel, as emphasized by Campbell, Shiller, and Viceira (2009), Duffee (2018a), and Liu (2020). Further, because the real bond curve is flat, the model does not require a lot of variation in inflation relative to overall variation in nominal yields to match the nominal bond curve. Thereby, our model is capable of addressing the Duffee (2018b) critique of the current mainstream asset-pricing models.

Campbell (1986) studies theoretically joint modeling of bonds and stocks allowing for both transitory and permanent effects in consumption. Existing empirical work typically addresses a subset of these asset-pricing moments. Campbell, Pflüger, and Viceira (2020), David and Veronesi (2013), and Fang, Liu, and Roussanov (2021) focus on stock-bond correlations. Song (2017) additionally focuses on the nominal yield curve vs consumption-inflation correlation. These papers rely on the nominal component as the key channel, which Duffee (2018a,b) argues is counter-factual. Gomez-Cram and Yaron (2021) specifically target that critique. They achieve success in matching the nominal and real yield curves via preferences that combine Epstein-Zin utility with shocks to the time preference parameter. They do not discuss joint comovement between bonds and stocks.

Bansal, Kiku, and Yaron (2010) and Hassler and Marfe (2016) advocate permanent, transitory, and disaster components albeit in a functional form that is different from ours. In particular, the composition of permanent and transitory shocks is time-invariant. Research questions are different as well. The former paper focuses on the single-horizon equity premium and interest rate. The latter focuses exclusively on the term structure of dividend strips.

In contemporaneous work, Jones and Pyun (2021) extend the Bansal and Yaron (2004) model with exogenously specified time-varying covariance between realized and expected consumption growth. The time-varying mix of the permanent and transitory components, which we advocate and explicitly model, is implicit in their specification and is referred to as time-varying consumption growth persistence. The authors primarily focus on the stock-bond covariance and on the leverage effect for equities. Implementation relies on calibration rather than on estimation of the consumption dynamics.

Nakamura, Steinsson, Barro, and Ursua (2013) and Nakamura, Sergeyev, and Steinsson (2017) estimate consumption dynamics using a cross-section of countries and a long sample. They focus on estimating consumption disasters, typically associated with wars, and time-variation in the conditional mean and its volatility. They do not allow for time-variation in the relative magnitude of permanent and transitory shocks as is our focus.

No-arbitrage models of stock-bond comovement are represented by Backus, Boyarchenko, and Chernov (2018), Campbell, Sunderam, and Viceira (2017), Koijen, Lustig, and Nieuwerburgh (2017), and Lettau and Wachter (2011).

2 Model

We start by presenting initial regression-based evidence about changing properties of consumption. Next, we use the evidence as a motivation for the posited consumption dynamics. We describe how we estimate the model. We conclude by presenting and discussing the implications of the estimated dynamics.

2.1 Preliminary evidence

The correlation between stock market and nominal Treasury bond returns changed sign around 1998, from positive before 1998 to negative from 1998 and onwards (e.g., Baele, Bekaert, and Inghelbrecht, 2010, Campbell, Shiller, and Viceira, 2009). A real explanation of this phenomenon suggests consumption dynamics are different in the two periods. We show that the autocorrelation of consumption growth indeed is significantly and economically different across these periods. According to our interpretation, this evidence arises from the changing mix of permanent versus transitory shocks to the economy.

Specifically, consider the regression:

$$\Delta c_{t+1} = \quad \alpha_0 + \alpha_1 \mathbb{1}_{t<1998} + \beta_0 \Delta c_t + \beta_1 \times (\mathbb{1}_{t<1998} \times \Delta c_t) + \epsilon_{t+1}, \tag{1}$$

where Δc_t is the change in log consumption growth, and $\mathbb{1}_{t<1998}$ equals one if observation t is in the pre-1998 sample and zero otherwise. If β_1 is significantly different from zero, there is different autocorrelation in the period before 1998 versus that after.

We use the real-time data series on real personal consumption expenditures (PCE) from 1947:Q2 to 2020:Q4, which are available from the Federal Reserve Bank of

Philadelphia. We use real-time consumption data to best align them with investors' information set at the time they set asset prices. PCE offers the longest span of such data.[1]

In our first set of regressions, we use data up until 2019:Q4 so the Covid period is excluded from the sample. A simple regression is not well-suited to handle the extreme movements in consumption growth occurring during that period. We also consider regressions where we exclude all NBER recession periods to check that the results are not due to the lower frequency of recessions in the later sample. When we do that, we ensure that the right hand side in the regression (observation t) is always the previous quarter relative to $t+1$ whether a NBER recession or not. Using NBER dates introduces a look-ahead bias, so the nature of these regressions is illustrative.

Table 1 shows the estimated coefficients. The results with and without NBER recessions are qualitatively similar. Our discussion focuses on the latter (right side of the Table). The column labeled (1) shows that $\beta_0 = 0.25$ and $\beta_1 = -0.39$, significant at the 5% and 1% level, respectively. Thus, the autocorrelation coefficient in the pre-1998 period is -0.14 ($\beta_0 + \beta_1 = 0.25 - 0.39$), while in the later period it is 0.25 (β_0). The significance of β_1 means this difference is statistically significant, and the economic magnitude is substantial.

The intercept is also statistically different across the two periods, as one would expect given that the slope is significantly different. In order to ensure that this intercept change does not reflect significantly different means of consumption growth, we also run a regression where we impose that the unconditional mean is the same across the two periods. In particular, we demean the consumption growth data we use in the regression across the whole sample and run the regression (1) setting $\alpha_1 = 0$. The

[1] As a robustness check, we develop the evidence using quarterly revised real per capita nondurable and services consumption. The results are similar. See Appendix A.

results are reported in Table 1 in the column labeled (2), which shows that the effect is slightly stronger in this case.

In this preliminary analysis, we made two short-cuts. First, we imposed the break-point as 1998, motivated by the evidence on the stock-bond correlation. Second, we excluded the Covid crisis as well as business cycle downturns in general. In the sequel, we present our model of consumption dynamics and estimate it, including the timing of different regimes and accounting for crisis periods. The conclusion remains the same.

2.2 Consumption dynamics

We model aggregate log consumption as the sum of a deterministic trend and two persistent components, $c_{p,t}$ and $c_{\tau,t}$, which we label permanent and transitory, respectively:

$$c_t = \mu_c t + c_{p,t} + c_{\tau,t}, \tag{2}$$

implying the following growth rate

$$\Delta c_{t+1} = \mu_c + \Delta c_{p,t+1} + \Delta c_{\tau,t+1}. \tag{3}$$

We model the permanent and transitory components as follows:

$$\begin{aligned} \Delta c_{p,t+1} &= \rho_p \Delta c_{p,t} + \epsilon_{p,t+1}, \\ c_{\tau,t+1} &= \rho_\tau c_{\tau,t} + \epsilon_{\tau,t+1}, \end{aligned} \tag{4}$$

where $\epsilon_{j,t+1}$ are mean-zero shocks and $j \in \{p, \tau\}$. Thus, the permanent component contains a unit root, while the transitory component does not. This specification is motivated by the preliminary evidence. Also, in general equilibrium models with production, (e.g., Blanchard, L'Huillier, and Lorenzoni, 2013, and Kaltenbrunner and Lochstoer, 2010), a combination of permanent and transitory shocks to productivity leads to endogenous consumption dynamics similar to those specified above.

The permanent component $c_{p,t}$ contains both permanent and transitory elements. To see this, consider the Beveridge-Nelson decomposition of $c_{p,t}$:

$$c_{p,t} = \frac{1}{1-\rho_p}\frac{1}{1-L} \cdot \epsilon_{p,t} + \frac{1}{1-L}\left[\frac{1}{1-\rho_p L} - \frac{1}{1-\rho_p}\right] \cdot \epsilon_{p,t}, \tag{5}$$

where L denotes the lag operator. The first element represents the permanent effect on consumption, while the second is transitory, which disappears if $\rho_p = 0$.

For simplicity, we set $\rho = \rho_p = \rho_\tau$ and note that

$$\rho \Delta c_{p,t+1} + (\rho - 1)\Delta c_{\tau,t+1} \equiv x_{t+1} = \rho x_t + \rho \epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1}. \tag{6}$$

We can then conveniently express consumption growth as

$$\Delta c_{t+1} = \mu_c + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}. \tag{7}$$

We allow the shocks to have time-varying volatility to capture any time-variation in the relative magnitude of the two consumption components. In particular, we assume $\epsilon_{j,t+1} \sim N(0, \sigma_j^2(S_{t+1}))$, where S_{t+1} is a discrete Markov state variable that takes on N values $S_{t+1} \in \{1, ..., N\}$. Agents observe the current regime S_{t+1} and make forecast of future regime based on the transition matrix below

$$\mathbb{P} = \begin{bmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & \ddots & \vdots \\ p_{N1} & \cdots & p_{NN} \end{bmatrix} \tag{8}$$

where $\sum_{i=1}^{N} p_{ji} = 1$.

2.3 Estimation

Our preferred model features three different Markov states. That is the most parsimonious specification where the first two states capture economic phenomena that are more slowly moving than business cycles. The remaining third state captures business cycle downturns (combined, for parsimony, with potential disasters in the economy).

We estimate the model via maximum likelihood. See Appendix B for the regime-switching state-space representation as well as the evaluation of the likelihood function. Regimes are labeled by imposing that

$$\sigma_\tau(1) < \sigma_\tau(2) < \sigma_\tau(3). \tag{9}$$

Also, to ensure that the third regime is associated with the worst states, we assume

$$\sigma_p(3) > \max\{\sigma_p(1), \sigma_p(2)\}. \tag{10}$$

In order to reduce the number of parameters to estimate, we impose that

$$var(x_{t+1}|x_t, S_{t+1} = 1) = var(x_{t+1}|x_t, S_{t+1} = 2).$$

Restricting expected consumption growth to be homoscedastic across regimes 1 and 2 means that these regimes are identified by the relative fraction of permanent versus transitory shocks, which is the economic effect we are focusing on in this paper, as opposed to the Great Moderation or business cycle variation in overall volatility. The restriction implies that

$$\sigma_p(1)^2 = \sigma_p(2)^2 + \left[\frac{1-\rho}{\rho}\right]^2 \left(\sigma_\tau(2)^2 - \sigma_\tau(1)^2\right). \tag{11}$$

Restrictions (9)-(11) together imply that the 1st (3rd) regime has the smallest (largest) conditional variance for realized consumption growth. Given this, we follow Sims, Waggoner, and Zha (2008) and reduce the number of parameters further by assuming that the economy cannot move between regimes 1 and 3 directly, but has to go through the intermediate volatility regime 2. This assumption implies that:

$$p_{1,3} = p_{3,1} = 0. \tag{12}$$

2.4 Evidence

Table 2 reports estimated parameters of consumption dynamics and Figure 1 displays the probabilities of the regimes.[2] The first regime, when compared to the second regime, features slightly higher volatility of the permanent shock (0.09% vs 0.08% per quarter) and lower volatility of the transitory shock (0.30% vs 0.73%). The third regime is a high volatility regime with the values of 0.12% for the permanent shock and 7.35% for the transitory one. Overall, our estimation implies that shocks to the transitory consumption component are the biggest drivers of short-run consumption volatility, but the first regime has the relatively highest proportion of permanent shocks. This is very different from the long-run risk literature, which relies on shocks to the permanent consumption component. The second regime prevails up to the mid-1990s with a few exceptions occurring usually at expansion peaks. After the

[2]We also estimate the model using quarterly revised real per capita nondurable and services consumption. The regimes and their intuition are very similar, though volatility is somewhat lower. See Appendix A.

mid-1990s, the first regime dominates with the exception of the 2008 crisis.

The third regime occurs at various points throughout the sample and is capable of generating much more adverse events than the other two regimes. This regime captures bad states of the economy that are associated with particularly volatile transitory shocks, and we therefore label it as a disaster regime. In our model this regime is distinct from regular recessions, including the Great Recession, which are driven relatively more by the permanent component of c_t. That is reflected in x_t becoming low and remaining so for an extended period. See the last panel of Figure 1. In contrast, disasters are associated with a high conditional mean of consumption growth. That is because the disaster shocks are transitory and the level of consumption therefore reverts to the trend relatively quickly. These episodes occur during the Korean war, oil shocks, the 1981 monetary recession, and the COVID crisis.

The persistence of expected consumption growth has a coefficient of $\rho = 0.941$ with a wide 90% confidence interval. This value is similar to the classic Bansal and Yaron (2004) x's persistence of $0.979^3 = 0.938$. Unlike in the Bansal and Yaron (2004) model, the correlation between shocks to expected and realized consumption is non-zero and time-varying. That reflects the relative magnitude of the permanent and transitory components.

The expected duration of regime j is roughly $1/(1-p_{jj})$ (e.g., Kim and Nelson, 1999). Thus, the expected duration of the first two regimes is 10 and 5 years, respectively, compared to less than a year for the third regime. These values highlight that the low-frequency shifts in the economy are captured in the first two regimes.

2.5 Discussion

Our model contributes to the existing literature by documenting that a mix of transitory and permanent shocks to aggregate consumption is changing over time. That potentially explains disagreement in the earlier literature that reached different conclusions using classical time-series methods: a specific sample could have tilted analyses towards one of the configurations. Next, this finding raises questions of how that matters and why that happens.

As regards the first question, Cochrane (1988) points out that "... the size or existence of a random walk component in GNP cannot directly distinguish broad classes of economic theories of the business cycle ...". Yet, Alvarez and Jermann (2004, 2005) forcefully demonstrate that asset prices are informative about the marginal rate of substitution and macroeconomic fluctuations. Therefore, it is natural to take exploration of the consumption dynamics that we have uncovered to asset-pricing data. Specifically, we evaluate if one can make progress on capturing the most recent puzzles that pertain to the interaction between stocks and bonds.

As regards the second question, Kaltenbrunner and Lochstoer (2010) show that the permanent or transitory components arise in general equilibrium with production depending on the nature of productivity shocks. Specifically, a positive permanent shock to the level of productivity leads to dynamics that are similar to the permanent consumption component we specify exogenously here with $\rho_p > 0$. Indeed, in response to this shock investors initially increase investment to bring the capital stock to its new optimal long-run level, which in turn temporarily depresses consumption before it converges to its new optimal level. In contrast a transitory shock to productivity leads to a transitory shock in consumption, similar to the transitory consumption component we specify exogenously here. Both specifications of productivity can account for the standard macroeconomic moments, but have markedly different effects on asset prices.

3 Basic asset-pricing implications

The next natural step is to investigate implications of the estimated consumption dynamics with time-varying volatility of the transitory and permanent components. In this section we consider asset-pricing implications. We start by developing intuition, and then discuss applications to the bond and stock markets.

3.1 Intuition

It is useful to consider

$$Cov(\Delta c_{t+1}, x_{t+1}|S_{t+1}) = \rho\sigma_p^2(S_{t+1}) - (1-\rho)\sigma_\tau^2(S_{t+1}). \tag{13}$$

The covariance is clearly time-varying and could even switch signs because $\rho < 1$. This property of consumption growth has dramatic implications for the dynamics of asset prices within the model, such as the real term structure of interest rates. To see that we follow Piazzesi (2014) and consider (log) risk premium on a two-period real bond in the case of log preferences and constant shock volatilities:

$$E_t([q_{t+1}^{(1)} - q_t^{(2)}] + q_t^{(1)}) + \frac{1}{2}Var_t(q_{t+1}^{(1)}) = Cov_t(\Delta c_{t+1}, q_{t+1}^{(1)}) = -Cov_t(\Delta c_{t+1}, x_{t+1}),$$

where $q_t^{(n)}$ denotes a time t log price of a real bond maturing in n periods. See Appendix C for all the derivations pertaining to log preferences.

If $\Delta c_{t+1} = \Delta c_{p,t+1}$, consumption growth is positively serially correlated. Thus, states with low growth have low growth expectations resulting in a high bond price. Bonds act as hedges to equity, and the short rate is procyclical.

If $\Delta c_{t+1} = \Delta c_{\tau,t+1}$ consumption growth is negatively serially correlated. Thus, states with low growth have high growth expectations resulting in a low bond price. Bonds are no longer hedges to equity, and the short rate is countercyclical.

In our model, covariance in (13) can be negative for sufficiently large values of $\sigma_\tau(S_{t+1})$. That would result in consumption dynamics that are closer to the transitory component. Thus, regimes control which effects dominates.

Let's introduce the relative contribution of permanent and transitory components to shocks to expected consumption growth explicitly via

$$\eta = \frac{(1-\rho)\sigma_\tau^2}{\rho\sigma_p^2}.$$

Further, let's use a consumption claim with a log price q_t^c as a metaphor for a stock. Then the discussion above can be extended and summarized in terms of the sign of the covariance between returns on a real bond and a stock:

$$sign\left[Cov_t(q_{t+1}^{(n-1)} - q_t^{(n)}, q_{t+1}^c - q_t^c)\right] = sign[\eta - 1]. \tag{14}$$

Three special cases are of particular interest. First, consider $\rho_p \neq \rho_\tau$ in Equation (4). Non-zero ρ_p, which reflects the transitory element in $c_{p,t}$ per Equation (5), is crucial for the mechanism advocated in this paper. If $\rho_p = 0$ then $\eta = \infty$, and the bond-stock covariance is positive. Second, if $\sigma_\tau = 0$, we have the permanent component only, like in Bansal and Yaron (2004), $\eta = 0$, and the bond-stock covariance is negative. Third, Blanchard, L'Huillier, and Lorenzoni (2013) explicitly assume this channel for switching signs away in a single-regime setting by requiring ρ, σ_p and σ_τ to be such that the covariance in Equation (13) is equal to zero, or, equivalently, $\eta = 1$. As a result, the univariate process for c_t is a random walk. That property, again, shuts down the mechanism that we are exploring in this paper.

3.2 Real bonds and consumption claim

Full analysis of stocks and bonds requires additional assumptions about cash flows (dividends and inflation). Before we proceed with this task, we investigate the properties of consumption dynamics itself through the lens of asset pricing. To that end we assume workhorse preferences of a representative agent and, building on the discussion in the previous section, apply them to valuation of real bonds and the consumption claim.

Specifically, we consider an Epstein and Zin (1989) representative agent. The indirect utility, U_t, takes the form

$$U_t = \max_{C_t} \left[(1-\delta) C_t^{(1-\gamma)/\theta} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{1/\theta} \right]^{\theta/(1-\gamma)},$$

where C_t denotes consumption, γ is the coefficient of relative risk aversion, $\theta = (1-\gamma)/(1-\psi^{-1})$ with ψ denoting the elasticity of intertemporal substitution, and δ is the time discount factor. The associated log stochastic discount factor (marginal rate of substitution) is

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1},$$

where $z_{c,t}$ is the log price to consumption ratio and $r_{c,t+1}$ is the log return on the consumption claim. Appendix D derives bond and consumption claim prices implied by the consumption dynamics and these preferences.

We set $\delta = 0.999$ to match the low level of real bonds in the data (close to zero for the short-rate). We analyze the asset pricing moments implied from the consumption-only estimation, which in practice means less risk than in models that use financial asset data for estimation. As a result, we set risk aversion to $\gamma = 20$ to generate quantitatively meaningful asset pricing moments. Lastly, we assume that $\psi = 1.5$ implying that the agent prefers an early resolution of uncertainty.

Figure 2 displays theoretical model implications by regime. The "permanent" regime 1 features a correlation between the consumption claim and a 5-year real bond of -0.66, while the "transitory" regime 2 implies a correlation of 0.51. The real yield curve in regime 1 is downward-sloping, consistent with the intuition that bonds are hedges in this state.

In regime 2, the yield curve is nearly flat. That shape reflects two countervailing forces. On the one hand, the positive bond-consumption claim correlation indicates bonds are risky. On the other hand, bonds hedge transitions to regime 3. This regime features a strong precautionary savings motive due to high consumption volatility and thus lower yields.

The disaster regime is dominated by the transitory component (the consumption claim-bond correlation is 0.96) and, therefore, features an upward-sloping yield curve, consistent with the correlations between expected and realized consumption given in Table 3.

The lower panel of Figure 2 displays time series of the model implied consumption claim-bond correlation. The correlation averages 0.5 until 1992. Then it starts declining and by 1996 it averages -0.6. The occasional spikes up match up with regime 3. Spikes down correspond to switches from regime 2 to regime 1 throughout the sample. Thus, consistent with the analysis in Campbell, Shiller, and Viceira (2009), Duffee (2018a), and Liu (2020), the correlation's behavior is primarily driven by the real side of the economy. The first two regimes in our model that emphasize transitory vs permanent component of consumption, respectively, are responsible for this result.

4 Extended asset-pricing implications

The key objective of this section is to evaluate implications of consumption dynamics for observed asset prices. However observable prices depend, besides consumption and preferences, on cash flows (dividends and inflation). While there are many plausible specifications of inflation and dividends, the debate about the "best" specification continues unabated. Thus, instead of an extensive discussion, we assume something relatively simple in line with the literature (traditional conditional mean complemented with exposures to the consumption shocks).

Further, because of our main focus, we do not want to estimate cash flow growth and consumption growth jointly so that the inferred consumption dynamics are not affected by the choice or properties of the other series. Thus, we calibrate cash flow dynamics to illustrate the pricing implications. The model generates realistic slopes of the real and nominal yield curves, as well as the equity forward curve, with reasonable dividend growth and inflation dynamics and without sacrificing the Duffee (2018b) moment, which is the contribution of inflation to the overall variation in nominal yields, and the observed stock-bond correlation.

4.1 Cash flows

In a pure-exchange endowment economy such as ours cash flows have to be specified exogenously. For our purposes, we have to specify two types: aggregate stock dividends and inflation. Inflation can be viewed as a cash flow to a nominal bond.

Following Bansal, Kiku, and Yaron (2012) and Schorfheide, Song, and Yaron (2018), dividend growth has levered exposures to both x_t and shocks as follows

$$\Delta d_{t+1} = \mu_d + \alpha x_t + \varphi_{d,p}\epsilon_{p,t+1} + \varphi_{d,\tau}\epsilon_{\tau,t+1} + \epsilon_{d,t+1}. \tag{15}$$

Idiosyncratic shocks to dividends are captured by $\epsilon_{d,t+1} \sim N(0, \sigma_d^2)$.

Inflation dynamics are assumed to follow

$$\pi_{t+1} = \mu_\pi + \rho_\pi(\pi_t - \mu_\pi) + \varphi_{\pi,p}\epsilon_{p,t+1} + \varphi_{\pi,\tau}\epsilon_{\tau,t+1} + \epsilon_{\pi,t+1}. \tag{16}$$

Idiosyncratic shocks to inflation are captured by $\epsilon_{\pi,t+1} \sim N(0, \sigma_\pi^2)$.

In order to appreciate the effect of inflation on the bond-stock covariance, we can extend the simple log-preference example in Equation (14) to the case of a nominal bond with price $q_t^{\$,(n)}$:

$$sign\left[Cov_t(q_{t+1}^{\$,(n-1)} - q_t^{\$,(n)}, q_{t+1}^c - q_t^c)\right] = sign[\Gamma\eta - 1], \tag{17}$$

where explicit expression for Γ is provided in Appendix C.2. Γ reflects the sign of consumption-inflation covariance. If $\varphi_{\pi,p} < 0$ and $\varphi_{\pi,\tau} < 0$ that covariance is negative and $\Gamma > 1$, and vice versa. $\Gamma = 1$ corresponds to no inflation risk.

The previous literature emphasizes the importance of switching covariance between inflation and consumption growth for explaining the pattern in stock-bond covariance, that is, it entertains a fixed η and time-varying Γ (e.g., Campbell, Pfluger, and Viceira, 2020, David and Veronesi, 2013, Song, 2017). Equation (17) demonstrates that as long as η varies around the value of $1/\Gamma$, the sign of covariance between the nominal bond returns and consumption claim returns would be switching signs even if Γ itself varies around the value of 1. Thus, if we add time-varying Γ to our setup, our model has the potential of generating richer dynamics and implications.

4.2 Data

We choose the following set of data to evaluate the pricing implications of our model. First, we rely on the value-weighted with- and without-dividend annual returns of the Center for Research in Security Prices (CRSP) stock market indexes (NYSE/AMEX/NASDAQ/ARCA) to construct nominal dividend growth series following Hodrick (1992). We take the CPI inflation rates from the Federal Reserve Bank of St. Louis to convert nominal dividend growth into real terms. We compute sample averages, standard deviations, and first-order autocorrelations of real dividend growth and inflation series and compare with the model-implied counterparts.

Second, we collect the zero-coupon Treasury (1971:Q1 to 2020:Q4) from Gurkaynak, Sack, and Wright (2007) and TIPS (1999:Q1 to 2020:Q4) from Gurkaynak, Sack, and Wright (2010). Only maturities that are higher than two years are available for TIPS. We discard the initial four years and rely on the post-2003 TIPS data to alleviate any concern regarding the credibility of the TIPS data. We concatenate the TIPS data with the estimated real rates provided by Chernov and Mueller (2012) (1971:Q3 to 2002:Q4) for selective maturities to obtain long-sample averages of the real bond term structure. Also, we compute rolling correlation estimates between daily stock market returns and nominal bond returns of maturity five years over the rolling windows of five years (1963:Q1 to 2020:Q4).

Third, we consider two sets of equity strip yield data, one is the synthetic equity strip yield data provided by Giglio, Kelly, and Kozak (2020) available from 1975:Q4 to 2020:Q3 and the other is the traded equity strip yield data from Bansal, Miller, Song, and Yaron (2020) available from 2005:Q1 to 2016:Q4.[3] We take the sample average of the equity strip yield data.

Lastly, we take the inflation risk variation ratio estimates provided by Duffee (2018b), which are available for the maturities of 1, 5, and 10 years, respectively. We take the average of various different values of estimates in his Tables 1-3.

4.3 Calibration

We set the sensitivity of expected dividends to expected consumption growth to 4, as is common in the literature. The short-term dividend exposures to permanent and transitory shocks are both set to 8 to match the equity premium, while the mean dividend growth rate is set to be consistent with its empirical counterpart as well as a roughly flat dividend claim term structure, as in the data.

In the case of inflation, a positive inflation risk premium obtains if the loadings on inflation shocks, $\varphi_{\pi,p}$ and $\varphi_{\pi,\tau}$, are negative, as in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2006). We let the exposure of inflation to transitory shocks be more negative than the exposure to permanent shocks so that the slope of the nominal yield curve is sufficiently upward-sloping, while at the same time requiring the 10-year Duffee (2018b) moment, that is contribution of inflation to the nominal yield variation, not to exceed 40%. The mean and autocorrelation parameters are set as close as possible to their sample counterparts, while again respecting the restriction implied by the Duffee (2018b) moment as well as the sample mean of the nominal short-term interest rate.

Table 4 summarizes the calibrated parameters. Table 5 provides how the calibrated cash flow processes (inflation and dividends) relate to the data. Generally speaking,

[3] We thank Serhiy Kozak for providing data.

we find that the sample averages, standard deviations, and first-order autocorrelations computed from the calibrated series are close to their data counterparts. That is not entirely surprising given that we also targeted these moments in the calibration. Still, it is reassuring that the model is capable of meeting the targets.

4.4 Evidence

Figure 3 reports two versions of model implied stock-bond correlation (real and nominal) against the sample values of the stock-nominal bond correlation. The sample estimate is the last five years' realized correlation between stock and nominal bond returns. The evidence is in line with the consumption claim - real bond correlation reported earlier, and the model matches the broad magnitudes quite well.

In particular, as in the data, the stock-nominal bond correlation is mainly positive until the late 1990s, when it switches sign and becomes negative. This is driven by the real side of the economy and the consumption dynamics transitioning from regime 2, which is dominated by the transitory consumption component, to regime 1, which features a relatively higher magnitude of the permanent component. The latter makes bonds hedges, which is why the correlation turns negative.[4] Campbell, Pfluger, and Viceira (2020), David and Veronesi (2013), Song (2017) emphasizes the importance of switching covariance between inflation and consumption growth for explaining the pattern in stock-bond covariance. The effect is actually present in our model as well due to the changing volatility of transitory and permanent components that inflation loads on, but unlike in these papers the effect is minor relative to that coming from the real side of the economy.

[4]Consistent with the model, the empirical stock-bond correlation increases in regime 3 as well. That said, the realized correlation does not respond nearly as much as the conditional correlation in the model as it is constructed as a five-year backward-looking moving average, whereas the model numbers are the conditional forward-looking quarterly correlations.

Figure 4 compares baseline summary statistics about bonds and equity in our model and in the data. Also, we use extant models as a reference. There are many asset-pricing models that focus on matching different dimensions of asset price data. Thus, a benchmark can be selected in a number of ways. We decided to focus on models that feature (i) long-run risk because their functional form is in the same family; and (ii) an Epstein and Zin (1989) representative agent as is the case for our computations. In this context, Bansal and Shaliastovich (2013) is a classic reference for bonds. However, the equity pricing implications of that model are not explored. Thus, we select Bansal, Kiku, and Yaron (2012) as a reference model for equity pricing. Neither of these models is capable of generating the changing relation between stocks and bonds.

The top left plot in Figure 4 shows the unconditional real yield curve, which in the data is slightly upward-sloping. In the model, the unconditional yield curve is approximately flat. That is consistent with the yield curves in each regime shown in Figure 2 in combination with the relative rarity of regime 3, which has a strongly upward-sloping curve. While the model cannot quite match the unconditional slope in the data, it is progress relative to the strongly downward-sloping curve in the benchmark model.

Importantly, even a flat real curve makes it much easier to generate a realistic nominal yield curve while satisfying the Duffee (2018b) moment, that is, the contribution of inflation to the overall variation in nominal yields. Indeed, that is what we observe in the panels on the right of Figure 4. The top right plot shows that the model generates a nominal bond yield curve that is upward-sloping and consistent with the data, while the bottom right shows that the model's implication for nominal bond's inflation risk is in line with the data. In contrast, the reference model generates a steeper nominal curve than the one observed in the data and the contribution of inflation to yield variation is close to 100%, which is strongly counter-factual.

The bottom left plot of Figure 4 shows the model's implications for forward equity yields. The evidence about the shape of this curve has been the subject of some controversy in the literature, but the most recent evidence, both data and model-based, is pointing towards mildly upward-sloping curve (Bansal, Miller, Song, and Yaron, 2020 and Giglio, Kelly, and Kozak, 2020). In our model the curve is upward-sloping as well, but more flat. The relatively large magnitude of transitory shocks make longer-horizon equity relatively less risky than in the Bansal and Yaron (2004) model (see also Belo, Collin-Dufresne, and Goldstein, 2015 for a similar effect coming from leverage). In fact, the reference model features a too steeply upward-sloping equity yield curve.

Figure 5 parses the real bond and forward equity yield curves by regimes and compares to the data. We see that qualitatively the model captures forward equity yields well also across regimes, with a strongly downward-sloping curve in regime 3. The difference between the curves across regimes 1 and 2 is smaller in the model than in the data. For real yields, the data features slightly upward-sloping curves in both regime 1 and 2, whereas the model curves are either slightly downward-sloping or flat. Quantitatively, however, the model is relatively close to the data. Regime 3 has an upward-sloping curve both in the model and the data. Overall, the model does a substantially better job than the reference models of matching these term structures.

Table 6 shows that the model also does a good job accounting for the standard unconditional moments of equity returns, such as the equity risk premium and volatility, which are computed by averaging over a simulation of length 100,000. In particular, equity returns are substantially more volatile than dividend growth. In fact, the price-dividend ratio is volatile and persistent, as in the data. The final rows of the table shows that the price-dividend ratio in the model indeed is negatively related to future excess market returns, as is widely documented in prior studies (e.g., Cochrane, 1994). Thus, the risk premium in the model is counter-cyclical as in the data.

The model also accounts for the failure of the expectations hypothesis for nominal bonds (Campbell and Shiller, 1991). In particular, Figure 6 gives the slope coefficient and R^2 of regressions of annual excess bond returns on the lagged spread between the log yield of 10- and 1-yr nominal bonds. The coefficients are positive and increasing with bond maturity and quite similar to the data. The same is the case for the regression R^2. The time-variation in conditional risk premiums is quantitatively mainly driven by the disaster regime 3, which features high conditional risk premiums.

5 Conclusion

Financial assets exhibit high single-horizon risk premiums, and modest term structures of these premiums (e.g., Backus, Chernov, and Zin, 2014). The term structure evidence suggests that persistence of the conditional moments of consumption and dividend growth cannot be as high as usually calibrated in the literature, at least without offsetting effects that render the pricing kernel less autocorrelated. We use consumption data to estimate a model of consumption dynamics that allows for time-varying conditional volatility of permanent and transitory shocks to the economy. We find that disasters are driven mainly by transitory shocks, while there are low frequency regimes that determine whether permanent or transitory shocks drive expected consumption growth. The permanent component of consumption dominates in the sample from the late 1990s and on, with the exception of the global financial and Covid crises.

This model can account for flat or modestly upward-sloping term structures of real and nominal bonds, as well as zero-coupon dividend claims, as observed in the data, along with the change in the stock-bond correlation that occurred in the late 1990s. Risk premiums in this model are driven by a bad state that we associate with disasters.

The model differs from existing models in the literature in that there are no highly persistent components that generate substantial risk. Risks come mainly from the relatively short-lived regime 3. In this regime, real bonds are risky as transitory shocks dominate the economy, which allows us to account for the Duffee (2018b) moment. Existing models struggle with capturing this behavior of equity and bonds in one unifying framework.

While our model does surprisingly well at matching a wide set of moments existing models cannot jointly match, our calibration also exposes shortcomings. For instance, the unconditional real-term structure is not sufficiently upward-sloping relative to the data, and the difference between the forward equity yield curves in regimes 1 and 2 are not as wide as in the data. That suggests that further extensions of our framework to other forms of preferences or more sophisticated cash flow dynamics may be a fruitful avenue of research.

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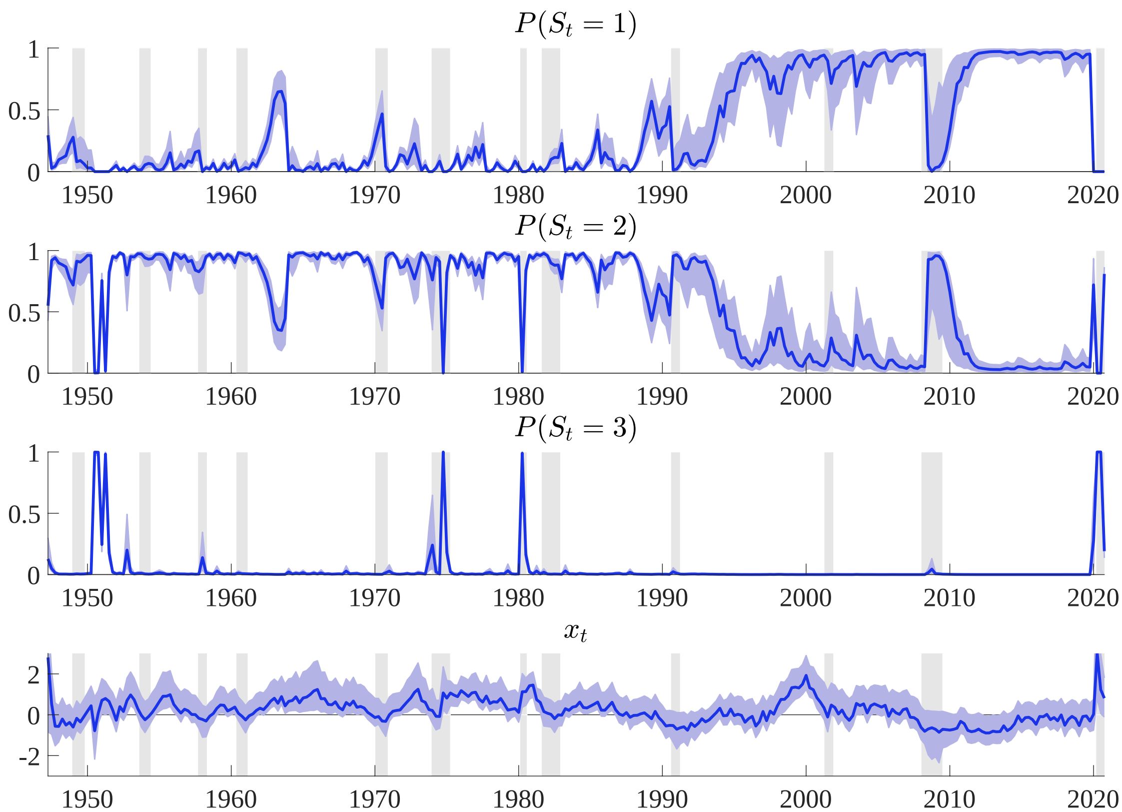
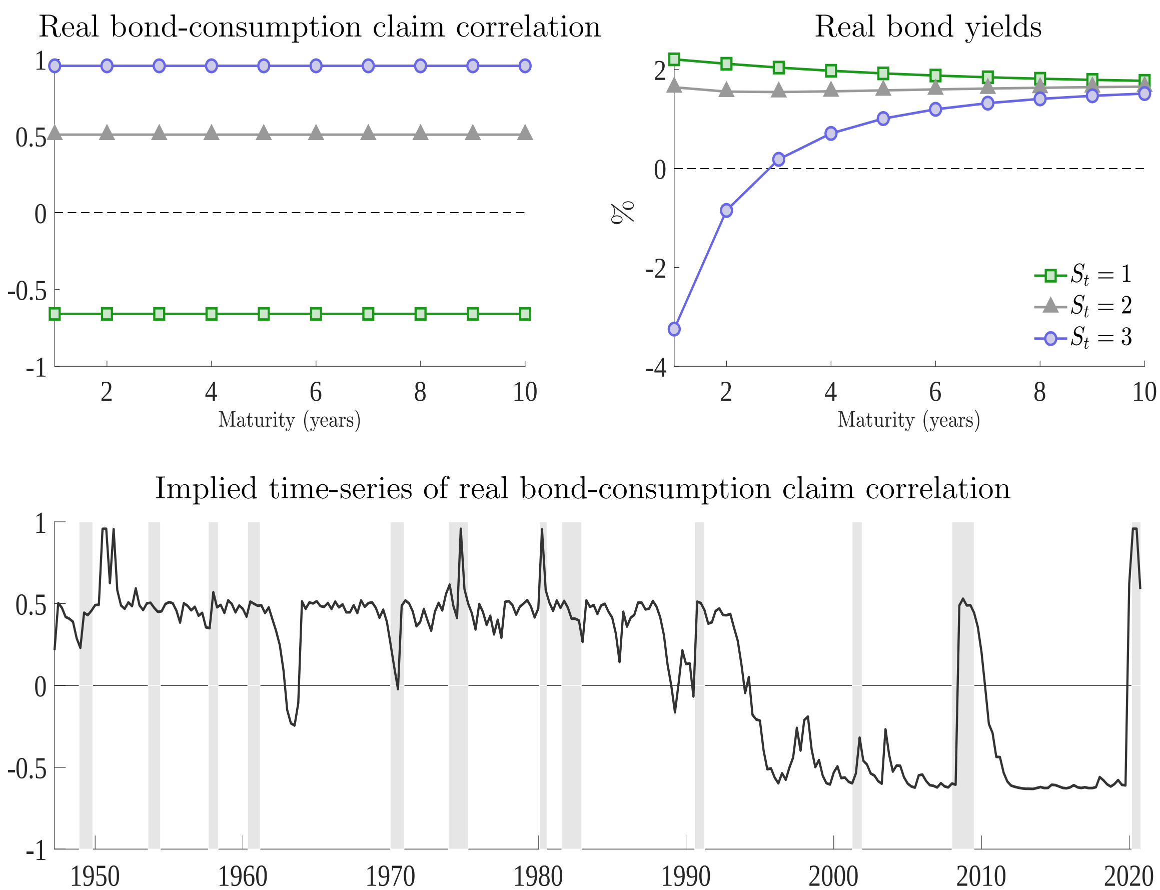
Figure 1: Estimated states



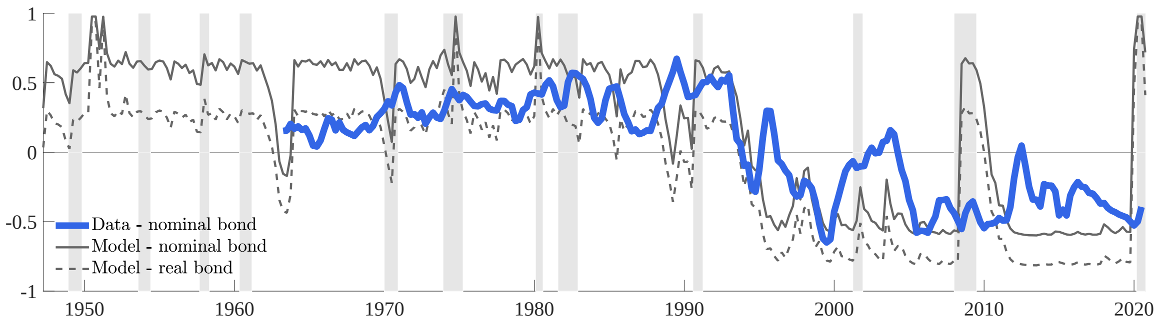
Notes: The plots show the filtering probabilities of each regime and expected consumption growth through the sample, along with 95% confidence bands. The grey bars represent NBER recessions. The first regime is the "permanent" regime, the second regime is the "transitory" regime, and the third regime is the "disaster" regime. The data frequency is quarterly and the sample is 1947:Q2-2020:Q4.

Figure 2: Model-implied asset pricing conditional moments



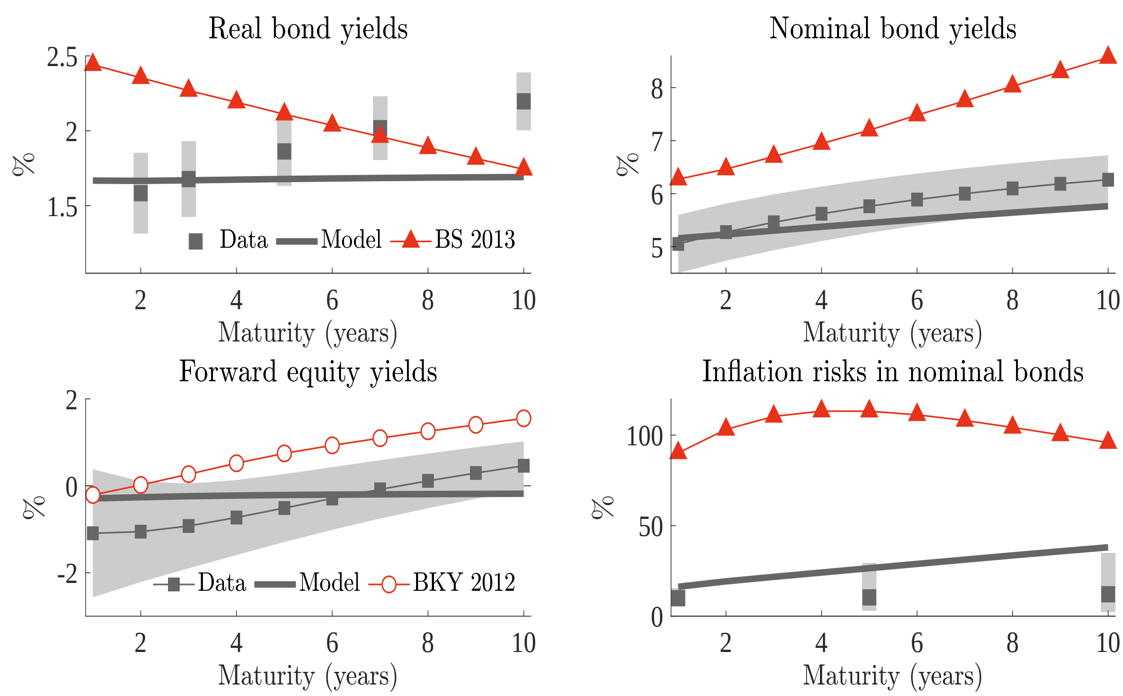
*N*otes: The top left plot shows the conditional correlation by regime between returns to real bonds and the consumption claim. The top left shows the real term structure of yields in each regime. In the top plots, all values are reported setting $x_t = 0$. The bottom plot shows the time-series of the conditional correlation between returns to the consumption claim and the 5-year real zero-coupon bond. Grey bars represent NBER recessions.

Figure 3: Model-implied stock-bond return correlation



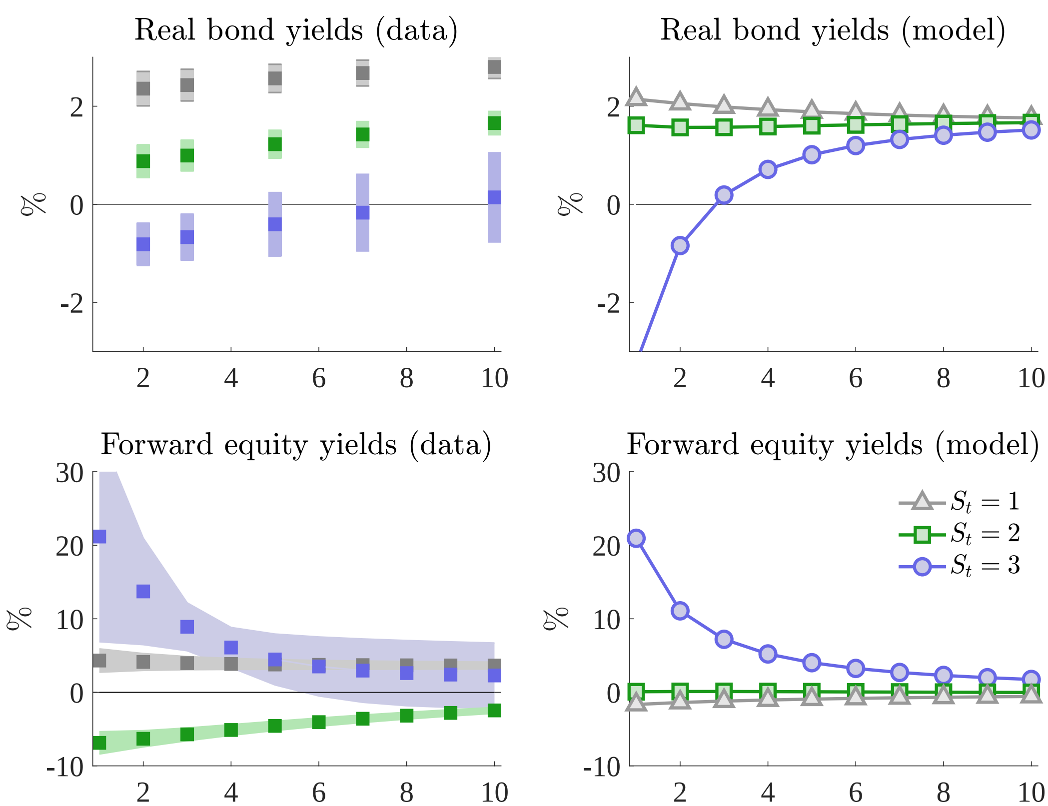
Notes: We compute rolling correlation estimates between market returns and nominal bond returns of 5-year maturity over the rolling windows of five years (Data - nominal bond). We compare with our model-implied correlation between market returns and returns of 5-year maturity for both nominal bond (Model - nominal bond) and real bond (Model - real bond), respectively.

Figure 4: Model-implied asset pricing sample moments



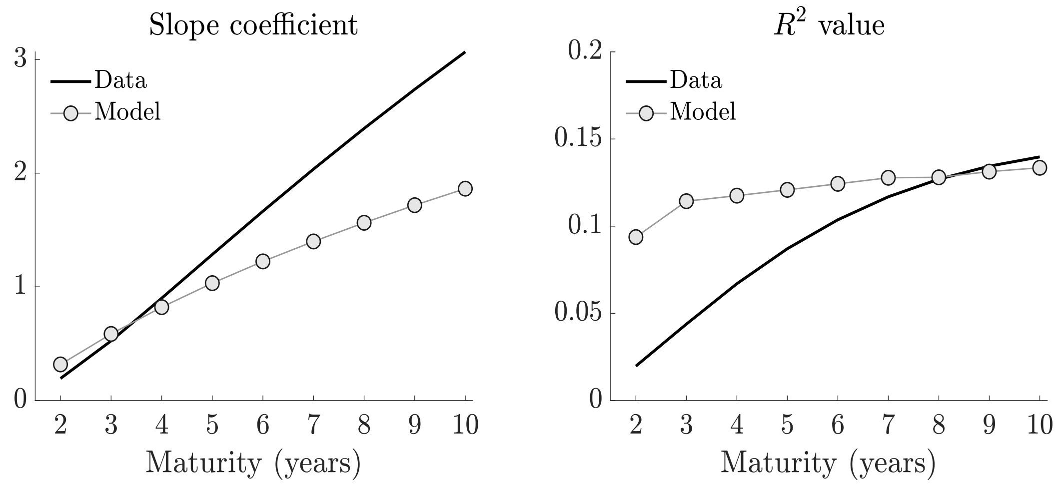
*N*otes: The top plots show average real and nominal yield curves. We compute the sample averages of the zero-coupon Treasury (1971:Q1 to 2020:Q4) from Gurkaynak, Sack, and Wright (2007) and TIPS (1999:Q1 to 2020:Q4) from Gurkaynak, Sack, and Wright (2010), respectively. For the TIPS, maturities higher than two years are available. We discard the initial four years and rely on the post-2003 TIPS data to alleviate any concern regarding the credibility of the TIPS data. We concatenate the TIPS data with the estimated real rates provided by Chernov and Mueller (2012) (1971:Q3 to 2002:Q4) for selective maturities to obtain long-sample averages of the real bond term structure. Red triangles represent the average yield curves in the Bansal and Shaliastovich (2013) model. In the bottom left plot, we show sample average the synthetic equity strip yield data provided by Giglio, Kelly, and Kozak (2020) from 1975:Q4 to 2020:Q3. The pattern is similar if we use the traded equity strip yield data from Bansal, Miller, Song, and Yaron (2020) which are available from 2005:Q1 to 2016:Q4. Finally, in the bottom right plot we show the inflation risk variation ratio estimates provided by Duffee (2018b), which are available for the maturities of 1, 5, and 10 years, respectively. We take the average of various different values of estimates in his Tables 1-3. Inflation variation ratio is computed in (A-81). Red circles represent implications of the Bansal, Kiku, and Yaron (2012) model.

Figure 5: Model-implied asset pricing conditional moments with data comparison



Notes: The data for the left plots in this figure are described in the legend for Figure 4. The averages are calculated for each regime by taking the average of the product of the yield and the respective regime's filtering probabilities and plotted with 95% confidence bands. The model-based yields displayed in the right plots are given for each regime assuming $x_t = 0$.

Figure 6: Excess bond return predictability



*N*otes: Excess nominal bond return is defined as the one-year holding period nominal bond return in excess of nominal one-year interest rate. We regress excess nominal bond returns of maturities 2 to 9 years on the nominal bond yield spread between maturities 10-year and 1-year, respectively. The left plot shows the slope coefficients from these regressions, while the right plot shows the R^2's. Model moments are averaged over a simulation of length 100,000.

Table 1: Consumption autocorrelation before and after 1998

	Full sample		Sample excluding recessions	
	(1)	(2)	(1)	(2)
α_0	0.0039***	0.0001	0.0058***	0.0002
	(0.0011)	(0.0005)	(0.0008)	(0.0005)
α_1	0.0040***		0.0053***	
	(0.0014)		(0.0014)	
β_0	0.4424***	0.4560***	0.2463**	0.3460***
	(0.1288)	(0.1317)	(0.1035)	(0.1014)
β_1	−0.4411***	−0.4541***	−0.3898***	−0.4848***
	(0.1605)	(0.1608)	(0.1585)	(0.1481)
R^2	0.0203	0.0191	0.0397	0.0286

*N*otes: The table shows the regression estimates from the regression in Equation (1). On the right side, business cycle downturns are excluded from the sample as explained in the main text. COVID period is excluded from all regressions. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient in the post-1998 sample, while the β_1 estimate gives the difference between the autocorrelation in the pre- vs. post-1998 samples. The specification in the column labeled (1) estimates the full regression, while the specification in the column labeled (2) is run using demeaned consumption growth and setting $\alpha_1 = 0$, as explained in the main text. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level. The data is real-time quarterly real PCE data, from 1947:Q2 to 2019:Q4.

Table 2: Estimated parameters of consumption

	MLE	[5%,	95%]
Consumption growth, Δc			
μ_c	0.0076	[0.0056,	0.0094]
ρ	0.9414	[0.8486,	0.9879]
$\sigma_p(1)$	0.0009	[0.0008,	0.0019]
$\sigma_p(2)$	0.0008	[0.0004,	0.0019]
$\sigma_p(3)$	0.0012	[0.0009,	0.0065]
$\sigma_\tau(1)$	0.0030	[0.0025,	0.0036]
$\sigma_\tau(2)$	0.0073	[0.0050,	0.0108]
$\sigma_\tau(3)$	0.0735	[0.0509,	0.1289]

Transition probabilities, $\mathbb{P}$

$$\begin{bmatrix} \underset{[0.9213,0.9922]}{0.9747} & \underset{[0.0078,0.0787]}{0.0253} & \underset{[-,-]}{0} \\ \underset{[0.0024,0.0558]}{0.0156} & \underset{[0.8983,0.9778]}{0.9515} & \underset{[0.0128,0.0642]}{0.0300} \\ \underset{[-,-]}{0} & \underset{[0.2244,0.3861]}{0.3079} & \underset{[0.6139,0.7756]}{0.6921} \end{bmatrix}$$

*N*otes: The table gives the maximum likelihood estimates (MLE) for the parameters governing the consumption process, along with the 5% and 95% confidence bands. The data frequency is quarterly. The estimated transition probability matrix gives the 5% and 95% confidence bands in brackets underneath the MLE. We order the regimes according to the transitory shock volatilities, and restrict the off-diagonal corner elements in the transition probability matrix to equal zero. The sample is 1947:Q2-2020:Q4.

Table 3: Correlation between realized and expected consumption growth

	$S_t = 1$	$S_t = 2$	$S_t = 3$
$Corr(\Delta c_{t+1}, x_{t+1} \mid S_{t+1}, x_t)$	0.10	-0.42	-0.98

*N*otes: The table reports the conditional correlation between shocks to realized and expected consumption growth within the model for each regime. The sample is 1947:Q2-2020:Q4.

Table 4: Calibrated parameters

Dividend growth		Inflation	
μ_d	0.003	μ_π	0.0085
α	4.0	ρ_π	0.9985
$\varphi_{d,p}$	8.0	$\varphi_{\pi,p}$	-0.01
$\varphi_{d,\tau}$	8.0	$\varphi_{\pi,\tau}$	-0.04
σ_d	0.0	σ_π	0.0

*N*otes: We report the calibrated quarterly frequency parameter values for dividend growth and inflation.

Table 5: Dividend growth and inflation moments

Dividend growth	Data	Model
Mean (%)	2.61	1.43
Standard deviation (%)	13.02	11.98
Autocorrelation	-0.26	0.03
Inflation	**Data**	**Model**
Mean (%)	2.52	3.03
Standard deviation (%)	2.13	1.54
Autocorrelation	0.69	0.89

*N*otes: Both data and model-implied dividend growth and inflation are aggregated to annual frequency. Then, we compute the sample averages, standard deviations, and first-order autocorrelations of annual dividend growth and inflation, respectively. The data sample ranges from 1947 to 2020.

Table 6: Standard equity moments

		Data	Model
Excess returns	Equity premium	7.48	7.32
	Volatility	15.85	21.11
	Sharpe ratio	0.47	0.35
Log pd ratio	Mean	3.49	3.07
	Volatility	0.43	0.19
	AR(1) coefficient	0.97	0.92
Predictability	1-year excess return on log pd ratio	-0.09	-0.13
	R^2 value	0.06	0.02

*N*otes: Excess stock return is defined as the one-quarter holding period stock return in excess of one-quarter risk-free rate. All excess return moments are annualized. The equity premium is the average sample return in the data and the unconditional average excess equity return in the model. Volatility refers to the standard deviation of excess returns, while the Sharpe ratio is the ratio of the mean excess return to its standard deviation. The log price-dividend (pd) ratio is calculated by summing the dividends over the last year. In the predictability regression, we regress one-year excess stock return on the lagged log pd-ratio and report the slope coefficient and the R^2. The sample is 1947:Q2-2020:Q4. Model moments are averaged over a simulation of length 100,000.

A Robustness of estimation to alternative consumption data

Since our goal is to estimate the dynamics of consumption growth as perceived by investors, we use real-time consumption data. However, historically, a number of papers use the ex post revised real per capita nondurables + services consumption data from NIPA. In Table A-1 we show that there is a significant change in the autocorrelation of consumption growth in 1998 also using these data when estimating the regression in Equation (1). The results are similar to those obtained using the real-time PCE dataset.

In Table A-2 we show the maximum likelihood estimates that obtain when using these data. This alternative data yields similar overall dynamics. In particular, the first regime features relatively more of the permanent component, regime 2 reflects a greater share of the transitory component, and regime 3 is a disaster regime. Again, the shift between regime 1 and regime 2 happens in the later 1990s, while regime 3 picks up disasters like the Covid-crisis (the filtering probabilities are not shown). That said, the volatility coefficients are somewhat smaller relative to our main estimation and the parameter and regime uncertainty is larger.

B Estimation of the regime-switching model

Consider the state-space model

$$\begin{aligned} Y_t &= D(S_t) + Z(S_t)\alpha_t + v_t, \quad v_t \sim N(0, U) \\ \alpha_t &= T(S_t)\alpha_{t-1} + R(S_t)\epsilon_t, \quad \epsilon_t \sim N(0, \Sigma(S_t)), \end{aligned} \tag{A-1}$$

where α_t is the latent state. See Chapter 5 of Kim and Nelson (1999) for detailed descriptions. Given $\alpha^i_{t-1|t-1}, P^i_{t-1|t-1}$, for $i, j, \in \{1, ..., N\}$,

Forecasting

$$\begin{aligned} \alpha^{(i,j)}_{t|t-1} &= T(S_t = j)\alpha^i_{t-1|t-1} \\ P^{(i,j)}_{t|t-1} &= T(S_t = j)P^i_{t-1|t-1}T(S_t = j)' + R(S_t = j)\Sigma(S_t = j)R(S_t = j)' \\ e^{(i,j)}_{t|t-1} &= Y_t - D(S_t = j) - Z(S_t = j)\alpha^{(i,j)}_{t|t-1} \\ F^{(i,j)}_{t|t-1} &= Z(S_t = j)P^{(i,j)}_{t|t-1}Z(S_t = j)' + U. \end{aligned} \tag{A-2}$$

Updating

$$\begin{aligned} \alpha^{(i,j)}_{t|t} &= \alpha^{(i,j)}_{t|t-1} + \left(P^{(i,j)}_{t|t-1}Z(S_t = j)'\right)\left(F^{(i,j)}_{t|t-1}\right)^{-1} e^{(i,j)}_{t|t-1} \\ P^{(i,j)}_{t|t} &= P^{(i,j)}_{t|t-1} - \left(P^{(i,j)}_{t|t-1}Z(S_t = j)'\right)\left(F^{(i,j)}_{t|t-1}\right)^{-1}\left(Z(S_t = j)P^{(i,j)}_{t|t-1}\right). \end{aligned} \tag{A-3}$$

Each iteration of the Kalman filter produces an N-fold increase in the number of cases to consider. It is necessary to introduce some approximations to make the above Kalman filter operable. The key is to collapse terms in the right way at the right time. Therefore, it remains to reduce the $N \times N$ posteriors $\alpha_{t|t}^{(i,j)}, P_{t|t}^{(i,j)}$ into N posteriors $\alpha_{t|t}^{j}, P_{t|t}^{j}$. Note that

$$\begin{aligned}
E\big(\alpha_t|S_t = j, Y_t\big) &= \frac{\sum_{i=1}^{N} Pr(S_{t-1} = i, S_t = j|Y_t)E\big(\alpha_t|S_t = j, S_{t-1} = i, Y_t\big)}{Pr(S_t = j|Y_t)} \qquad \text{(A-4)}\\
&= \sum_{i=1}^{N} \Delta_t^{(i,j)} E\big(\alpha_t|S_t = j, S_{t-1} = i, Y_t\big), \quad \Delta_t^{(i,j)} = \frac{Pr(S_{t-1} = i, S_t = j|Y_t)}{Pr(S_t = j|Y_t)}\\
\alpha_{t|t}^{j} &= \sum_{i=1}^{N} \Delta_t^{(i,j)} \alpha_{t|t}^{(i,j)}.
\end{aligned}$$

The variance of α_t conditional on $S_t = j, Y_t$ could be derived in the following way:

$$E\Big((\alpha_t - \alpha_{t|t}^{j})(\alpha_t - \alpha_{t|t}^{j})'|S_t = j, Y_t\Big) = \sum_{i=1}^{N} \Delta_t^{(i,j)} E\Big((\alpha_t - \alpha_{t|t}^{j})(\alpha_t - \alpha_{t|t}^{j})'|S_t = j, S_{t-1} = i, Y_t\Big) \qquad \text{(A-5)}$$

Note that

$$\begin{aligned}
&E\Big((\alpha_t - \alpha_{t|t}^{j})(\alpha_t - \alpha_{t|t}^{j})'|S_t = j, S_{t-1} = i, Y_t\Big) \qquad \text{(A-6)}\\
&= E\Big((\alpha_t - \alpha_{t|t}^{(i,j)} + \alpha_{t|t}^{(i,j)} - \alpha_{t|t}^{j})(\alpha_t - \alpha_{t|t}^{(i,j)} + \alpha_{t|t}^{(i,j)} - \alpha_{t|t}^{j})'|S_t = j, S_{t-1} = i, Y_t\Big)\\
&= E\Big((\alpha_t - \alpha_{t|t}^{(i,j)})(\alpha_t - \alpha_{t|t}^{(i,j)})'|S_t = j, S_{t-1} = i, Y_t\Big) + \Big((\alpha_{t|t}^{j} - \alpha_{t|t}^{(i,j)})(\alpha_{t|t}^{j} - \alpha_{t|t}^{(i,j)})'\Big)\\
&\quad + 2E\Big((\alpha_t - \alpha_{t|t}^{(i,j)})|S_t = j, S_{t-1} = i, Y_t\Big)(\alpha_{t|t}^{(i,j)} - \alpha_{t|t}^{j})'.
\end{aligned}$$

Hence,

$$\begin{aligned}
P_{t|t}^{j} &= \sum_{i=1}^{N} \Delta_t^{(i,j)} E\Big((\alpha_t - \alpha_{t|t}^{j})(\alpha_t - \alpha_{t|t}^{j})'|S_t = j, S_{t-1} = i, Y_t\Big) \qquad \text{(A-7)}\\
&= \sum_{i=1}^{N} \Delta_t^{(i,j)} \Big[E\Big((\alpha_t - \alpha_{t|t}^{(i,j)})(\alpha_t - \alpha_{t|t}^{(i,j)})'|S_t = j, S_{t-1} = i, Y_t\Big) + \Big((\alpha_{t|t}^{j} - \alpha_{t|t}^{(i,j)})(\alpha_{t|t}^{j} - \alpha_{t|t}^{(i,j)})'\Big)\Big]\\
&\quad +2\sum_{i=1}^{N} \Delta_t^{(i,j)} \underbrace{E\Big((\alpha_t - \alpha_{t|t}^{(i,j)})|S_t = j, S_{t-1} = i, Y_t\Big)}_{=0}(\alpha_{t|t}^{(i,j)} - \alpha_{t|t}^{j})'\\
&= \sum_{i=1}^{N} \Delta_t^{(i,j)} \Big[\underbrace{E\Big((\alpha_t - \alpha_{t|t}^{(i,j)})(\alpha_t - \alpha_{t|t}^{(i,j)})'|S_t = j, S_{t-1} = i, Y_t\Big)}_{=P_{t|t}^{(i,j)}} + \Big((\alpha_{t|t}^{j} - \alpha_{t|t}^{(i,j)})(\alpha_{t|t}^{j} - \alpha_{t|t}^{(i,j)})'\Big)\Big]\\
&= \sum_{i=1}^{N} \Delta_t^{(i,j)} \Big[P_{t|t}^{(i,j)} + (\alpha_{t|t}^{j} - \alpha_{t|t}^{(i,j)})(\alpha_{t|t}^{j} - \alpha_{t|t}^{(i,j)})'\Big].
\end{aligned}$$

Merging

$$\alpha^j_{t|t} = \frac{\sum_{i=1}^N Pr(S_{t-1}=i, S_t=j|Y_t)}{Pr(S_t=j|Y_t)}\Big(\alpha^{(i,j)}_{t|t}\Big) \tag{A-8}$$

$$P^j_{t|t} = \frac{\sum_{i=1}^N Pr(S_{t-1}=i, S_t=j|Y_t)}{Pr(S_t=j|Y_t)}\Big(P^{(i,j)}_{t|t} + \big(\alpha^j_{t|t} - \alpha^{(i,j)}_{t|t}\big)\big(\alpha^j_{t|t} - \alpha^{(i,j)}_{t|t}\big)'\Big).$$

Finally, the likelihood density of observation Y_t is given by,

Likelihood

$$l(Y_t|Y_{1:t-1}) = \sum_{j=1}^N \sum_{i=1}^N f(Y_t|S_{t-1}=i, S_t=j, Y_{1:t-1}) Pr(S_{t-1}=i, S_t=j|Y_{t-1}) \tag{A-9}$$

$$f(Y_t|S_{t-1}=i, S_t=j, Y_{1:t-1}) = (2\pi)^{-\frac{n}{2}} \det\Big(F^{(i,j)}_{t|t-1}\Big)^{-\frac{1}{2}} \exp\Big[-\frac{1}{2}\big(e^{(i,j)}_{t|t-1}\big)'\Big(F^{(i,j)}_{t|t-1}\Big)^{-1}\big(e^{(i,j)}_{t|t-1}\big)\Big].$$

C An illustration with log utility preferences

For ease of illustration, we assume homoscedasticity

$$\Delta c_{t+1} = \mu_c + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}, \quad \epsilon_p \sim N(0, \sigma_p^2), \tag{A-10}$$

$$x_{t+1} = \rho x_t + \rho \epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1}, \quad \epsilon_\tau \sim N(0, \sigma_\tau^2),$$

$$\pi_{t+1} = \mu_\pi + \rho_\pi(\pi_t - \mu_\pi) + \varphi_{\pi,p}\epsilon_{p,t+1} + \varphi_{\pi,\tau}\epsilon_{\tau,t+1},$$

and consider log utility preference implying the following log stochastic discount factor

$$m_{t+1} = \ln\delta - \Delta c_{t+1}. \tag{A-11}$$

In this environment, the unexpected component of the return on consumption claim is

$$r_{c,t+1} - E_t r_{c,t+1} = \Delta c_{t+1} - E_t \Delta c_{t+1} \tag{A-12}$$

because the log price to consumption ratio is constant.

C.1 Covariance between returns on a real bond and a stock

The n-maturity log real bond price is

$$q_t^{(n)} = q_{n,0} + q_{n,1} x_t \tag{A-13}$$

and the return on the n-maturity real bond is

$$r_{n,t+1} \equiv q_{t+1}^{(n-1)} - q_t^{(n)}. \tag{A-14}$$

We can deduce that its unexpected component is

$$r_{n,t+1} - E_t r_{n,t+1} = q_{n-1,1}\big(x_{t+1} - E_t x_{t+1}\big). \tag{A-15}$$

From (A-12) and (A-15), we can express the sign of the conditional covariance as

$$\begin{aligned} sign\big(cov_t(r_{n,t+1}, r_{c,t+1})\big) &= sign\big(q_{n-1,1}\big) \cdot sign\big(cov_t(\Delta c_{t+1}, x_{t+1})\big), \\ &= -sign\big(cov_t(\Delta c_{t+1}, x_{t+1})\big), \\ &= sign(\eta - 1), \end{aligned} \tag{A-16}$$

where the relative contribution of permanent and transitory components is introduced via

$$\eta = \frac{1-\rho}{\rho} \cdot \frac{\sigma_\tau^2}{\sigma_p^2}. \tag{A-17}$$

C.2 Covariance between returns on a nominal bond and a stock

The n-maturity log nominal bond price is

$$q_t^{\$,(n)} = q_{n,0}^{\$} + q_{n,1}^{\$} x_t + q_{n,2}^{\$} \pi_t. \tag{A-18}$$

From the Euler equation

$$q_t^{\$,(n)} = \ln E_t\big[\exp(m_{t+1} - \pi_{t+1} + q_{t+1}^{\$,(n-1)})\big], \tag{A-19}$$

we can solve for

$$q_{n-1,1}^{\$} = -\left[\frac{1-\rho^{n-1}}{1-\rho}\right], \qquad q_{n-1,2}^{\$} = -\rho_\pi \left[\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\right]. \tag{A-20}$$

The unexpected component of the return on the n-maturity nominal bond is

$$r_{n,t+1}^{\$} - E_t r_{n,t+1}^{\$} = q_{n-1,1}^{\$}\big(x_{t+1} - E_t x_{t+1}\big) + q_{n-1,2}^{\$}\big(\pi_{t+1} - E_t \pi_{t+1}\big). \tag{A-21}$$

From (A-12), (A-20), and (A-21), we can express the conditional covariance as

$$\begin{aligned} cov_t(r_{n,t+1}^{\$}, r_{c,t+1}) &= q_{n-1,1}^{\$} cov_t(\Delta c_{t+1}, x_{t+1}) + q_{n-1,2}^{\$} cov_t(\Delta c_{t+1}, \pi_{t+1}) \\ &\propto \left[\frac{1-\rho}{\rho} \cdot \frac{\sigma_\tau^2}{\sigma_p^2} - 1\right] - \left[\varphi_{\pi,p} + \varphi_{\pi,\tau} \cdot \frac{\sigma_\tau^2}{\sigma_p^2}\right] \cdot \left[\frac{\rho_\pi\big(\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\big)}{\rho\big(\frac{1-\rho^{n-1}}{1-\rho}\big)}\right] \\ &\propto \left[\frac{1 - \varphi_{\pi,\tau} \cdot [\frac{\rho}{1-\rho}] \cdot \left[\frac{\rho_\pi\big(\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\big)}{\rho\big(\frac{1-\rho^{n-1}}{1-\rho}\big)}\right]}{1 + \varphi_{\pi,p} \cdot \left[\frac{\rho_\pi\big(\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\big)}{\rho\big(\frac{1-\rho^{n-1}}{1-\rho}\big)}\right]}\right] \cdot \eta - 1. \end{aligned} \tag{A-22}$$

For ease of expression, we define

$$\Gamma(\varphi_{\pi,\tau}, \varphi_{\pi,p}; \rho, \rho_\pi) = \frac{1 - \varphi_{\pi,\tau} \cdot [\frac{\rho}{1-\rho}] \cdot \Psi(\rho, \rho_\pi)}{1 + \varphi_{\pi,p} \cdot \Psi(\rho, \rho_\pi)}, \qquad \Psi(\rho, \rho_\pi) = \frac{\rho_\pi \left(\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\right)}{\rho\left(\frac{1-\rho^{n-1}}{1-\rho}\right)} \tag{A-23}$$

to show that the sign of the conditional covariance is

$$sign\big(cov_t(r^{\$}_{n,t+1}, r_{c,t+1})\big) = sign\big(\Gamma(\varphi_{\pi,\tau}, \varphi_{\pi,p}; \rho, \rho_\pi) \cdot \eta - 1\big). \tag{A-24}$$

Two remarks can be made. First, when inflation has no exposures to consumption shocks, i.e., $\varphi_{\pi,p} = \varphi_{\pi,\tau} = 0$, then (A-24) is identical to (A-16) as $\Gamma(\varphi_{\pi,\tau} = 0, \varphi_{\pi,p} = 0; \rho, \rho_\pi) = 1$. Second, we find that $\forall j \in \{p, \tau\}$,

$$\frac{\partial \Gamma(\varphi_{\pi,\tau}, \varphi_{\pi,p}; \rho, \rho_\pi)}{\partial \varphi_{\pi,j}} < 0. \tag{A-25}$$

Thus, holding η constant, (A-24) is a decreasing function of $\varphi_{\pi,j}$.

D Model solution with recursive preferences

This section provides approximate analytical solutions for the equilibrium asset prices.

D.1 Exogenous dynamics

The joint dynamics of consumption and dividend growth, and inflation are

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}, \\
\Delta d_{t+1} &= \mu_d + \alpha x_t + \varphi_{d,p}(S_{t+1})\epsilon_{p,t+1} + \varphi_{d,\tau}(S_{t+1})\epsilon_{\tau,t+1} + \epsilon_{d,t+1}, \quad \epsilon_{d,t+1} \sim N(0, \sigma_d^2) \\
\pi_{t+1} &= \mu_\pi + \rho_\pi(\pi_t - \mu_\pi) + \varphi_{\pi,p}(S_{t+1})\epsilon_{p,t+1} + \varphi_{\pi,\tau}(S_{t+1})\epsilon_{\tau,t+1} + \epsilon_{\pi,t+1}, \quad \epsilon_{\pi,t+1} \sim N(0, \sigma_\pi^2) \\
x_t &= \rho x_{t-1} + \rho\epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1}, \quad \epsilon_{\tau,t+1} \sim N(0, \sigma_\tau(S_{t+1})^2), \quad \epsilon_{p,t+1} \sim N(0, \sigma_p(S_{t+1})^2)
\end{aligned} \tag{A-26}$$

and the transition matrix is given by

$$\mathbb{P} = \begin{bmatrix} p_{11} & p_{12} & 1 - p_{11} - p_{12} \\ p_{21} & p_{22} & 1 - p_{21} - p_{22} \\ p_{31} & p_{32} & 1 - p_{31} - p_{32} \end{bmatrix}. \tag{A-27}$$

D.2 Derivation of the approximate analytical solutions

The Euler equation for the economy is

$$1 = E_t\left[\exp\left(m_{t+1} + r_{c,t+1}\right)\right] \tag{A-28}$$

where the log stochastic discount factor is

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{A-29}$$

$z_{c,t}$ is the log price to consumption ratio and $r_{c,t+1}$ is the log return on the consumption claim

$$r_{c,t+1} \quad = \quad \kappa_{0,c} + \kappa_{1,c} z_{c,t+1} - z_{c,t} + \Delta c_{t+1}. \tag{A-30}$$

Derivation of (A-28) follows Bansal and Zhou (2002), which make repeated use of the law of iterated expectations and log-linearization.

$$\begin{aligned} 1 &= E\Big(E\left[\exp\left(m_{t+1} + r_{c,t+1}\right) \mid S_{t+1}\right] \mid S_t\Big) \qquad \text{(A-31)} \\ &= \sum_{j=1}^{2} \mathbb{P}_{ij} E\Big(\exp\left(m_{t+1} + r_{c,t+1}\right) \mid S_{t+1} = j, S_t = i\Big) \\ 0 &= \sum_{j=1}^{2} \mathbb{P}_{ij} \Big(E\big[m_{t+1} + r_{c,t+1} \mid S_{t+1} = j, S_t = i\big] + \frac{1}{2} Var\big[m_{t+1} + r_{c,t+1} \mid S_{t+1} = j, S_t = i\big]\Big). \end{aligned}$$

The first line uses the law of iterated expectations; the second line uses the definition of Markov chain; and the third line relies on the log-normality assumption and applies log-linearization (i.e., $\exp(B) - 1 \approx B$).

D.3 Linearization parameters

Let $\bar{p}_j = \frac{1-p_l}{2-p_l-p_j}$. The linearization parameters are determined endogenously by the following system of equations

$$\bar{z}_c = \sum_{j=1}^{2} \bar{p}_j A_{0,c}(j) \tag{A-32}$$

$$\kappa_{1,c} = \frac{\exp(\bar{z}_c)}{1 + \exp(\bar{z}_c)} \tag{A-33}$$

$$\kappa_{0,c} = \log(1 + \exp(\bar{z}_c)) - \kappa_{1,c} \bar{z}_c. \tag{A-34}$$

The solution is determined numerically by iteration until reaching a fixed point of $\bar{z}_c$.

D.4 Real consumption claim

Conjecture that the log price to consumption ratio follows

$$z_{c,t}(S_t) = A_{0,c}(S_t) + A_{1,c}(S_t) x_t. \tag{A-35}$$

From (A-26), (A-30), and (A-35), we can express the return on consumption claim by

$$\begin{aligned} r_{c,t+1} = &\kappa_{0,c} + \kappa_{1,c}A_{0,c}(S_{t+1}) - A_{0,c}(S_t) + \mu_c + (\kappa_{1,c}A_{1,c}(S_{t+1})\rho - A_{1,c}(S_t) + 1)x_t \\ &+ (1 + \kappa_{1,c}\rho A_{1,c}(S_{t+1}))\epsilon_{p,t+1} + (1 + \kappa_{1,c}(\rho - 1)A_{1,c}(S_{t+1}))\epsilon_{\tau,t+1}. \end{aligned} \tag{A-36}$$

Using (A-36), we can re-express the log SDF (A-29) by

$$\begin{aligned} m_{t+1} = &\theta \ln\delta + (\theta - 1)\big(\kappa_{0,c} - A_{0,c}(S_t) + \kappa_{1,c}A_{0,c}(S_{t+1})\big) - \gamma\mu_c \\ &+ \big((\theta - 1)(\kappa_{1,c}A_{1,c}(S_{t+1})\rho - A_{1,c}(S_t)) - \gamma\big)x_t \\ &+ \big((\theta - 1)\kappa_{1,c}\rho A_{1,c}(S_{t+1}) - \gamma\big)\epsilon_{p,t+1} + \big((\theta - 1)\kappa_{1,c}(\rho - 1)A_{1,c}(S_{t+1}) - \gamma\big)\epsilon_{\tau,t+1}. \end{aligned} \tag{A-37}$$

The solutions for $A_{0,c}$ and $A_{1,c}$ that describe the dynamics of the price-consumption ratio are determined from (A-31) which are

$$\begin{bmatrix} A_{1,c}(1) \\ A_{1,c}(2) \\ A_{1,c}(3) \end{bmatrix} = \begin{bmatrix} \mathbb{I}_3 - \rho\kappa_{1,c}\mathbb{P} \end{bmatrix}^{-1} \begin{bmatrix} (1 - \frac{1}{\psi}) \\ (1 - \frac{1}{\psi}) \\ (1 - \frac{1}{\psi}) \end{bmatrix} \tag{A-38}$$

$$\begin{bmatrix} A_{0,c}(1) \\ A_{0,c}(2) \\ A_{0,c}(3) \end{bmatrix} = \begin{bmatrix} \mathbb{I}_3 - \kappa_{1,c}\mathbb{P} \end{bmatrix}^{-1} \mathbb{P} \begin{bmatrix} \ln\delta + \kappa_{0,c} + (1 - \frac{1}{\psi})\mu_c + \frac{\theta}{2}\big(1 - \frac{1}{\psi} + \kappa_{1,c}\rho A_{1,c}(1)\big)^2\sigma_p(1)^2 \\ + \frac{\theta}{2}\big(1 - \frac{1}{\psi} + \kappa_{1,c}(\rho - 1)A_{1,c}(1)\big)^2\sigma_\tau(1)^2 \\ \ln\delta + \kappa_{0,c} + (1 - \frac{1}{\psi})\mu_c + \frac{\theta}{2}\big(1 - \frac{1}{\psi} + \kappa_{1,c}\rho A_{1,c}(2)\big)^2\sigma_p(2)^2 \\ + \frac{\theta}{2}\big(1 - \frac{1}{\psi} + \kappa_{1,c}(\rho - 1)A_{1,c}(2)\big)^2\sigma_\tau(2)^2 \\ \ln\delta + \kappa_{0,c} + (1 - \frac{1}{\psi})\mu_c + \frac{\theta}{2}\big(1 - \frac{1}{\psi} + \kappa_{1,c}\rho A_{1,c}(3)\big)^2\sigma_p(3)^2 \\ + \frac{\theta}{2}\big(1 - \frac{1}{\psi} + \kappa_{1,c}(\rho - 1)A_{1,c}(3)\big)^2\sigma_\tau(3)^2 \end{bmatrix}. \tag{A-39}$$

D.5 Real bond prices

Conjecture that $b_{n,t}$ depends on the regime S_t and x_t,

$$b_{n,t} = b_{n,0}(S_t) + b_{n,1}(S_t)x_t. \tag{A-40}$$

Exploit the law of iterated expectations

$$b_{n,t} = \ln E_t\Big(E[\exp(m_{t+1} + b_{n-1,t+1})|S_{t+1}]\Big)$$

and log-linearization to solve for $b_{n,t}$

$$b_{n,t} \approx \sum_{j=1}^{3}\mathbb{P}_{ij}\Big(E[m_{t+1} + b_{n-1,t+1}|S_{t+1}] + \frac{1}{2}Var[m_{t+1} + b_{n-1,t+1}|S_{t+1}]\Big). \tag{A-41}$$

The solution to (A-40) is

$$\begin{bmatrix} b_{n,1}(1) \\ b_{n,1}(2) \\ b_{n,1}(3) \end{bmatrix} = \mathbb{P} \begin{bmatrix} b_{n-1,1}(1)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(1)\rho \\ b_{n-1,1}(2)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(2)\rho \\ b_{n-1,1}(3)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(3)\rho \end{bmatrix} - \begin{bmatrix} (\theta-1)A_{1,c}(1)+\gamma \\ (\theta-1)A_{1,c}(2)+\gamma \\ (\theta-1)A_{1,c}(3)+\gamma \end{bmatrix} \tag{A-42}$$

$$\begin{aligned}
\begin{bmatrix} b_{n,0}(1) \\ b_{n,0}(2) \\ b_{n,0}(3) \end{bmatrix} &= \begin{bmatrix} \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(1) - \gamma\mu_c \\ \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(2) - \gamma\mu_c \\ \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(3) - \gamma\mu_c \end{bmatrix} \\
&+ \mathbb{P} \begin{bmatrix} b_{n-1,0}(1) + (\theta-1)\kappa_{1,c}A_{0,c}(1) \\ b_{n-1,0}(2) + (\theta-1)\kappa_{1,c}A_{0,c}(2) \\ b_{n-1,0}(3) + (\theta-1)\kappa_{1,c}A_{0,c}(3) \end{bmatrix} \\
&+ \mathbb{P} \begin{bmatrix} \frac{1}{2}\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(1) - \gamma + b_{n-1,1}(1)\rho\big)^2\sigma_p^2(1) \\ \frac{1}{2}\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(2) - \gamma + b_{n-1,1}(2)\rho\big)^2\sigma_p^2(2) \\ \frac{1}{2}\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(3) - \gamma + b_{n-1,1}(3)\rho\big)^2\sigma_p^2(3) \end{bmatrix} \\
&+ \mathbb{P} \begin{bmatrix} \frac{1}{2}\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(1) - \gamma + b_{n-1,1}(1)(\rho-1)\big)^2\sigma_\tau^2(1) \\ \frac{1}{2}\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(2) - \gamma + b_{n-1,1}(2)(\rho-1)\big)^2\sigma_\tau^2(2) \\ \frac{1}{2}\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(3) - \gamma + b_{n-1,1}(3)(\rho-1)\big)^2\sigma_\tau^2(3) \end{bmatrix}
\end{aligned}$$

with the initial condition $b_{0,0}(i) = 0$ and $b_{0,1}(i) = 0$ for $i \in \{1,2,3\}$. The real yield of the maturity n-period bond is $y^r_{n,t} = -\frac{1}{n}b_{n,t}$.

D.6 Real bond risk premium

Define the one-period HPR of an n-maturity real bond in excess of a one-period risk-free rate as

$$\begin{aligned}
rx^{rb}_{n,t+1} = {} & b_{n-1,0}(S_{t+1}) - b_{n,0}(S_t) + b_{1,0}(S_t) + \Big(b_{n-1,1}(S_{t+1})\rho - b_{n,1}(S_t) + b_{1,1}(S_t)\Big)x_t \\
& + b_{n-1,1}(S_{t+1})\rho\epsilon_{p,t+1} + b_{n-1,1}(S_{t+1})(\rho-1)\epsilon_{\tau,t+1}.
\end{aligned} \tag{A-43}$$

The real bond risk premium is

$$\begin{aligned}
& -cov_t(m_{t+1}, rx^{rb}_{n,t+1}) \\
& \approx -\mathbb{P} \begin{bmatrix} b_{n-1,1}(1)\rho((\theta-1)\kappa_{1,c}\rho A_{1,c}(1) - \gamma)\sigma_p^2(1) + b_{n-1,1}(1)(\rho-1)((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(1) - \gamma)\sigma_\tau^2(1) \\ b_{n-1,1}(2)\rho((\theta-1)\kappa_{1,c}\rho A_{1,c}(2) - \gamma)\sigma_p^2(2) + b_{n-1,1}(2)(\rho-1)((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(2) - \gamma)\sigma_\tau^2(2) \\ b_{n-1,1}(3)\rho((\theta-1)\kappa_{1,c}\rho A_{1,c}(3) - \gamma)\sigma_p^2(3) + b_{n-1,1}(3)(\rho-1)((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(3) - \gamma)\sigma_\tau^2(3) \end{bmatrix}.
\end{aligned} \tag{A-44}$$

D.7 Price to dividend ratio of zero coupon equity

Conjecture that the log price to dividend ratio of zero coupon equity $z_{n,t}$ depends on the regime S_t and persistent component x_t,

$$z_{n,t} = z_{n,0}(S_t) + z_{n,1}(S_t)x_t. \tag{A-45}$$

Exploit the law of iterated expectations

$$Z_{n,t} = E_t\Big(E[M_{t+1}Z_{n-1,t+1}\frac{D_{t+1}}{D_t}|S_{t+1}]\Big) \quad \text{(A-46)}$$

Take log

$$z_{n,t} = \ln E_t\Big(E[\exp(m_{t+1}+z_{n-1,t+1}+\Delta d_{t+1})|S_{t+1}]\Big) \quad \text{(A-47)}$$

and log-linearization to solve for $z_{n,t}$

$$z_{n,t} \approx \sum_{j=1}^{3}\mathbb{P}_{ij}\Big(E[m_{t+1}+z_{n-1,t+1}+\Delta d_{t+1}|S_{t+1}] + \frac{1}{2}Var[m_{t+1}+z_{n-1,t+1}+\Delta d_{t+1}|S_{t+1}]\Big). \quad \text{(A-48)}$$

The solution is

$$\begin{bmatrix} z_{n,1}(1) \\ z_{n,1}(2) \\ z_{n,1}(3) \end{bmatrix} = \mathbb{P}\begin{bmatrix} z_{n-1,1}(1)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(1)\rho \\ z_{n-1,1}(2)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(2)\rho \\ z_{n-1,1}(3)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(3)\rho \end{bmatrix} - \begin{bmatrix} (\theta-1)A_{1,c}(1)-(\alpha-\gamma) \\ (\theta-1)A_{1,c}(2)-(\alpha-\gamma) \\ (\theta-1)A_{1,c}(3)-(\alpha-\gamma) \end{bmatrix} \quad \text{(A-49)}$$

$$\begin{aligned}
\begin{bmatrix} z_{n,0}(1) \\ z_{n,0}(2) \\ z_{n,0}(3) \end{bmatrix} &= \begin{bmatrix} \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(1) - \gamma\mu_c + \mu_d \\ \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(2) - \gamma\mu_c + \mu_d \\ \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(3) - \gamma\mu_c + \mu_d \end{bmatrix} \\
&+ \mathbb{P}\begin{bmatrix} z_{n-1,0}(1) + (\theta-1)\kappa_{1,c}A_{0,c}(1) + \frac{\sigma_d^2}{2} \\ z_{n-1,0}(2) + (\theta-1)\kappa_{1,c}A_{0,c}(2) + \frac{\sigma_d^2}{2} \\ z_{n-1,0}(3) + (\theta-1)\kappa_{1,c}A_{0,c}(3) + \frac{\sigma_d^2}{2} \end{bmatrix} \\
&+ \mathbb{P}\begin{bmatrix} \frac{1}{2}\big(\{z_{n-1,1}(1) + (\theta-1)\kappa_{1,c}A_{1,c}(1)\}\rho + \varphi_{d,p}(1) - \gamma\big)^2\sigma_p^2(1) \\ \frac{1}{2}\big(\{z_{n-1,1}(2) + (\theta-1)\kappa_{1,c}A_{1,c}(2)\}\rho + \varphi_{d,p}(2) - \gamma\big)^2\sigma_p^2(2) \\ \frac{1}{2}\big(\{z_{n-1,1}(3) + (\theta-1)\kappa_{1,c}A_{1,c}(3)\}\rho + \varphi_{d,p}(3) - \gamma\big)^2\sigma_p^2(3) \end{bmatrix} \\
&+ \mathbb{P}\begin{bmatrix} \frac{1}{2}\big(\{z_{n-1,1}(1) + (\theta-1)\kappa_{1,c}A_{1,c}(1)\}(\rho-1) + \varphi_{d,\tau}(1) - \gamma\big)^2\sigma_\tau^2(1) \\ \frac{1}{2}\big(\{z_{n-1,1}(2) + (\theta-1)\kappa_{1,c}A_{1,c}(2)\}(\rho-1) + \varphi_{d,\tau}(2) - \gamma\big)^2\sigma_\tau^2(2) \\ \frac{1}{2}\big(\{z_{n-1,1}(3) + (\theta-1)\kappa_{1,c}A_{1,c}(2)\}(\rho-1) + \varphi_{d,\tau}(2) - \gamma\big)^2\sigma_\tau^2(3) \end{bmatrix}
\end{aligned}$$

with the initial condition $z_{0,0}(i) = 0$ and $z_{0,1}(i) = 0$ for $i \in \{1,2,3\}$.

D.7.1 m-holding-period and hold-to-maturity expected return

The price of zero coupon equity is $P_{n,t} = Z_{n,t}D_t$. Define the m-holding period return of the n-maturity equity is

$$R_{n,t+m} = \frac{Z_{n-m,t+m}}{Z_{n,t}}\frac{D_{t+m}}{D_t}. \quad \text{(A-50)}$$

The corresponding log expected return is defined by

$$E_t[r_{n,t+m}] = \frac{1}{m} E_t\big(z_{n-m,t+m} - z_{n,t} + \sum_{i=1}^{m} \Delta d_{t+i}\big) \tag{A-51}$$

To compute the excess return, we subtract the real rate of the same maturity

$$E_t[r_{n,t+m}] - y^r_{m,t}. \tag{A-52}$$

We consider two cases

- $m \neq n$: This is the m-holding-period expected excess return of the n-maturity equity.

$$\begin{aligned} E_t[g_{d,t+m}] &= \frac{1}{m} E_t\big(\sum_{i=1}^{m} \Delta d_{t+i}\big) \\ e_{n,m,t} &= \frac{1}{m} E_t\big(z_{n-m,t+m} - z_{n,t}\big) \\ E_t[r_{n,t+m}] &= e_{n,m,t} + E_t[g_{d,t+m}] \\ E_t[rx_{n,t+m}] &= E_t[r_{n,t+m}] - y^r_{m,t}. \end{aligned} \tag{A-53}$$

- $m = n$: This is the hold-to-maturity expected excess return of the n-maturity equity. Define

$$\begin{aligned} E_t[g_{d,t+n}] &= \frac{1}{n} E_t\big(\sum_{i=1}^{n} \Delta d_{t+i}\big) \\ e_{n,t} &= \frac{1}{n} E_t\big(-z_{n,t}\big) \\ E_t[r_{t+n}] &= e_{n,t} + E_t[g_{d,t+n}] \\ E_t[rx_{t+n}] &= E_t[r_{t+n}] - y^r_{n,t}. \end{aligned} \tag{A-54}$$

For illustrative purpose, we compute the one-period-HPR

$$\begin{aligned} r_{n,t+1} &= z_{n-1,0}(S_{t+1}) - z_{n,0}(S_t) + \mu_d + \Big(z_{n-1,1}(S_{t+1})\rho - z_{n,1}(S_t) + \alpha\Big)x_t \\ &\quad + \Big(z_{n-1,1}(S_{t+1})\rho + \varphi_{d,p}S_{t+1})\Big)\epsilon_{p,t+1} + \Big(z_{n-1,1}(S_{t+1})(\rho-1) + \varphi_{d,\tau}(S_{t+1})\Big)\epsilon_{\tau,t+1} + \epsilon_{d,t+1}, \\ rx_{n,t+1} &= r_{n,t+1} - y^r_{1,t} \\ &= z_{n-1,0}(S_{t+1}) - z_{n,0}(S_t) + \mu_d + b_{1,0}(S_t) + \Big(z_{n-1,1}(S_{t+1})\rho - z_{n,1}(S_t) + \alpha + b_{1,1}(S_t)\Big)x_t \\ &\quad + \Big(z_{n-1,1}(S_{t+1})\rho + \varphi_{d,p}(S_{t+1})\Big)\epsilon_{p,t+1} + \Big(z_{n-1,1}(S_{t+1})(\rho-1) + \varphi_{d,\tau}(S_{t+1})\Big)\epsilon_{\tau,t+1} + \epsilon_{d,t+1}. \end{aligned} \tag{A-55}$$

The risk premium is

$$-cov_t(m_{t+1}, rx_{n,t+1}) \approx -\mathbb{P}\begin{bmatrix} \big(z_{n-1,1}(1)\rho + \varphi_{d,p}(1)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(1) - \gamma\big)\sigma_p^2(1) \\ \big(z_{n-1,1}(2)\rho + \varphi_{d,p}(2)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(2) - \gamma\big)\sigma_p^2(2) \\ \big(z_{n-1,1}(3)\rho + \varphi_{d,p}(3)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(3) - \gamma\big)\sigma_p^2(3) \end{bmatrix} \tag{A-56}$$

$$- \mathbb{P}\begin{bmatrix} \big(z_{n-1,1}(1)(\rho-1) + \varphi_{d,\tau}(1)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(1) - \gamma\big)\sigma_\tau^2(1) \\ \big(z_{n-1,1}(2)(\rho-1) + \varphi_{d,\tau}(2)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(2) - \gamma\big)\sigma_\tau^2(2) \\ \big(z_{n-1,1}(3)(\rho-1) + \varphi_{d,\tau}(3)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(3) - \gamma\big)\sigma_\tau^2(3) \end{bmatrix}.$$

D.8 Market return and equity premium

We derive the market return via Campbell-Shiller approximation

$$r_{d,t+1} = \kappa_{0,d} + \kappa_{1,d}A_{0,d}(S_{t+1}) - A_{0,d}(S_t) + \mu_d + \Big(\kappa_{1,d}A_{1,d}(S_{t+1})\rho - A_{1,d}(S_t) + \alpha\Big)x_t \tag{A-57}$$

$$+ \Big(\kappa_{1,d}A_{1,d}(S_{t+1})\rho + \varphi_{d,p}(S_{t+1})\Big)\epsilon_{p,t+1} + \Big(\kappa_{1,d}A_{1,d}(S_{t+1})(\rho-1) + \varphi_{d,\tau}(S_{t+1})\Big)\epsilon_{\tau,t+1} + \epsilon_{d,t+1}$$

where the log price-dividend ratio is given by

$$z_{d,t} = A_{0,d}(S_t) + A_{1,d}(S_t)x_t. \tag{A-58}$$

We solve for the coefficients

$$\begin{bmatrix} A_{1,d}(1) \\ A_{1,d}(2) \\ A_{1,d}(3) \end{bmatrix} = \begin{bmatrix} \mathbb{I}_3 - \rho\kappa_{1,d}\mathbb{P} \end{bmatrix}^{-1} \left(\begin{bmatrix} (\alpha-\gamma) - (\theta-1)A_{1,c}(1) \\ (\alpha-\gamma) - (\theta-1)A_{1,c}(2) \\ (\alpha-\gamma) - (\theta-1)A_{1,c}(3) \end{bmatrix} + \mathbb{P}\begin{bmatrix} (\theta-1)\kappa_{1,c}A_{1,c}(1)\rho \\ (\theta-1)\kappa_{1,c}A_{1,c}(2)\rho \\ (\theta-1)\kappa_{1,c}A_{1,c}(3)\rho \end{bmatrix} \right) \tag{A-59}$$

$$\begin{bmatrix} A_{0,d}(1) \\ A_{0,d}(2) \\ A_{0,d}(3) \end{bmatrix} = \begin{bmatrix} \mathbb{I}_3 - \kappa_{1,d}\mathbb{P} \end{bmatrix}^{-1}$$

$$\left(\begin{bmatrix} \kappa_{0,d} + \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(1) + \mu_d - \gamma\mu_c + \frac{1}{2}\sigma_d^2 \\ \kappa_{0,d} + \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(2) + \mu_d - \gamma\mu_c + \frac{1}{2}\sigma_d^2 \\ \kappa_{0,d} + \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(3) + \mu_d - \gamma\mu_c + \frac{1}{2}\sigma_d^2 \end{bmatrix} + \mathbb{P}\begin{bmatrix} (\theta-1)\kappa_{1,c}A_{0,c}(1) \\ (\theta-1)\kappa_{1,c}A_{0,c}(2) \\ (\theta-1)\kappa_{1,c}A_{0,c}(3) \end{bmatrix} \right.$$

$$+ \frac{1}{2}\mathbb{P}\begin{bmatrix} \big((\theta-1)\kappa_{1,c}\rho A_{1,c}(1) + \kappa_{1,d}\rho A_{1,d}(1) + \varphi_{d,p}(1) - \gamma\big)^2\sigma_p^2(1) \\ \big((\theta-1)\kappa_{1,c}\rho A_{1,c}(2) + \kappa_{1,d}\rho A_{1,d}(2) + \varphi_{d,p}(2) - \gamma\big)^2\sigma_p^2(2) \\ \big((\theta-1)\kappa_{1,c}\rho A_{1,c}(3) + \kappa_{1,d}\rho A_{1,d}(3) + \varphi_{d,p}(3) - \gamma\big)^2\sigma_p^2(3) \end{bmatrix}$$

$$\left. + \frac{1}{2}\mathbb{P}\begin{bmatrix} \big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(1) + \kappa_{1,d}(\rho-1)A_{1,d}(1) + \varphi_{d,\tau}(1) - \gamma\big)^2\sigma_\tau^2(1) \\ \big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(2) + \kappa_{1,d}(\rho-1)A_{1,d}(2) + \varphi_{d,\tau}(2) - \gamma\big)^2\sigma_\tau^2(2) \\ \big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(3) + \kappa_{1,d}(\rho-1)A_{1,d}(3) + \varphi_{d,\tau}(3) - \gamma\big)^2\sigma_\tau^2(3) \end{bmatrix} \right).$$

The market equity premium $E_t[r_{d,t+1}] - y^r_{1,t} + \frac{1}{2}V_t[r_{d,t+1}]$ is

$$
\begin{aligned}
-Cov_t(rx_{d,t+1}, m_{t+1}) &\approx -E[Cov(rx_{d,t+1}, m_{t+1}|S_{t+1})] \\
&= -\mathbb{P} \begin{bmatrix} \big(\kappa_{1,d}A_{1,d}(1)\rho + \varphi_{d,p}(1)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(1) - \gamma\big)\sigma_p^2(1) \\ \big(\kappa_{1,d}A_{1,d}(2)\rho + \varphi_{d,p}(2)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(2) - \gamma\big)\sigma_p^2(2) \\ \big(\kappa_{1,d}A_{1,d}(3)\rho + \varphi_{d,p}(3)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(3) - \gamma\big)\sigma_p^2(3) \end{bmatrix} \\
&\quad - \mathbb{P} \begin{bmatrix} \big(\kappa_{1,d}A_{1,d}(1)(\rho-1) + \varphi_{d,\tau}(1)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(1) - \gamma\big)\sigma_\tau^2(1) \\ \big(\kappa_{1,d}A_{1,d}(2)(\rho-1) + \varphi_{d,\tau}(2)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(2) - \gamma\big)\sigma_\tau^2(2) \\ \big(\kappa_{1,d}A_{1,d}(3)(\rho-1) + \varphi_{d,\tau}(3)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(3) - \gamma\big)\sigma_\tau^2(3) \end{bmatrix}.
\end{aligned} \tag{A-60}
$$

D.9 Real bond and stock return correlation

Define the one-period HPR of an n-maturity real bond as

$$
\begin{aligned}
r^{rb}_{n,t+1} = b_{n-1,0}(S_{t+1}) - b_{n,0}(S_t) + \Big(b_{n-1,1}(S_{t+1})\rho - b_{n,1}(S_t)\Big)x_t \\
+ b_{n-1,1}(S_{t+1})\rho\epsilon_{p,t+1} + b_{n-1,1}(S_{t+1})(\rho-1)\epsilon_{\tau,t+1}.
\end{aligned} \tag{A-61}
$$

The market return is

$$
\begin{aligned}
r_{d,t+1} = \kappa_{0,d} + \kappa_{1,d}A_{0,d}(S_{t+1}) - A_{0,d}(S_t) + \mu_d + \Big(\kappa_{1,d}A_{1,d}(S_{t+1})\rho - A_{1,d}(S_t) + \alpha\Big)x_t \\
+ \Big(\kappa_{1,d}A_{1,d}(S_{t+1})\rho + \varphi_{d,p}(S_{t+1})\Big)\epsilon_{p,t+1} + \Big(\kappa_{1,d}A_{1,d}(S_{t+1})(\rho-1) + \varphi_{d,\tau}(S_{t+1})\Big)\epsilon_{\tau,t+1} + \epsilon_{d,t+1}.
\end{aligned} \tag{A-62}
$$

The conditional covariance between two returns is

$$
\begin{aligned}
Cov_t(r_{d,t+1}, r^{rb}_{n,t+1}) &\approx E[Cov(r_{d,t+1}, r^{rb}_{n,t+1}|S_{t+1})] \\
&= \mathbb{P} \begin{bmatrix} \big(\kappa_{1,d}A_{1,d}(1)\rho + \varphi_{d,p}(1)\big)\big(b_{n-1,1}(1)\rho\big)\sigma_p^2(1) \\ \big(\kappa_{1,d}A_{1,d}(2)\rho + \varphi_{d,p}(2)\big)\big(b_{n-1,1}(2)\rho\big)\sigma_p^2(2) \\ \big(\kappa_{1,d}A_{1,d}(3)\rho + \varphi_{d,p}(3)\big)\big(b_{n-1,1}(3)\rho\big)\sigma_p^2(3) \end{bmatrix} \\
&\quad + \mathbb{P} \begin{bmatrix} \big(\kappa_{1,d}A_{1,d}(1)(\rho-1) + \varphi_{d,\tau}(1)\big)\big(b_{n-1,1}(1)(\rho-1)\big)\sigma_\tau^2(1) \\ \big(\kappa_{1,d}A_{1,d}(2)(\rho-1) + \varphi_{d,\tau}(2)\big)\big(b_{n-1,1}(2)(\rho-1)\big)\sigma_\tau^2(2) \\ \big(\kappa_{1,d}A_{1,d}(3)(\rho-1) + \varphi_{d,\tau}(3)\big)\big(b_{n-1,1}(3)(\rho-1)\big)\sigma_\tau^2(3) \end{bmatrix}.
\end{aligned} \tag{A-63}
$$

The conditional variance of bond return is

$$
Var_t(r^{rb}_{n,t+1}) \approx E[Var(r^{rb}_{n,t+1}|S_{t+1})] = \mathbb{P} \begin{bmatrix} \big(b_{n-1,1}(1)\rho\big)^2\sigma_p^2(1) + \big(b_{n-1,1}(1)(\rho-1)\big)^2\sigma_\tau^2(1) \\ \big(b_{n-1,1}(2)\rho\big)^2\sigma_p^2(2) + \big(b_{n-1,1}(2)(\rho-1)\big)^2\sigma_\tau^2(2) \\ \big(b_{n-1,1}(3)\rho\big)^2\sigma_p^2(3) + \big(b_{n-1,1}(3)(\rho-1)\big)^2\sigma_\tau^2(3) \end{bmatrix}. \tag{A-64}
$$

The conditional variance of market return is

$$\begin{aligned}Var_t(r_{d,t+1}) &\approx E[Var(r_{d,t+1}|S_{t+1})] \qquad \text{(A-65)}\\ &= \mathbb{P}\begin{bmatrix} \left(\kappa_{1,d}A_{1,d}(1)\rho+\varphi_{d,p}1)\right)^2\sigma_p^2(1)+\left(\kappa_{1,d}A_{1,d}(1)(\rho-1)+\varphi_{d,\tau}(1)\right)^2\sigma_\tau^2(1)+\sigma_d^2 \\ \left(\kappa_{1,d}A_{1,d}(2)\rho+\varphi_{d,p}(2)\right)^2\sigma_p^2(2)+\left(\kappa_{1,d}A_{1,d}(2)(\rho-1)+\varphi_{d,\tau}(2)\right)^2\sigma_\tau^2(2)+\sigma_d^2 \\ \left(\kappa_{1,d}A_{1,d}(3)\rho+\varphi_{d,p}(3)\right)^2\sigma_p^2(3)+\left(\kappa_{1,d}A_{1,d}(3)(\rho-1)+\varphi_{d,\tau}(3)\right)^2\sigma_\tau^2(3)+\sigma_d^2 \end{bmatrix}.\end{aligned}$$

D.10 Nominal bonds

Conjecture that $q_{n,t}$ depends on the regime S_t, x_t, and π_t

$$q_{n,t} = q_{n,0}(S_t) + q_{n,1}(S_t)x_t + q_{n,2}(S_t)\pi_t. \qquad \text{(A-66)}$$

Exploit the law of iterated expectations

$$q_{n,t} = \ln E_t\Big(E[\exp(m_{t+1} - \pi_{t+1} + q_{n-1,t+1})|S_{t+1}]\Big)$$

and log-linearization to solve for $q_{n,t}$

$$q_{n,t} \approx \sum_{j=1}^{2}\mathbb{P}_{ij}\Big(E[m_{t+1} - \pi_{t+1} + q_{n-1,t+1}|S_{t+1}] + \frac{1}{2}Var[m_{t+1} - \pi_{t+1} + q_{n-1,t+1}|S_{t+1}]\Big). \qquad \text{(A-67)}$$

The solution to (A-66) is

$$\begin{bmatrix} q_{n,1}(1)\\ q_{n,1}(2)\\ q_{n,1}(3)\end{bmatrix} = \mathbb{P}\begin{bmatrix} q_{n-1,1}(1)\rho+(\theta-1)\kappa_{1,c}A_{1,c}(1)\rho \\ q_{n-1,1}(2)\rho+(\theta-1)\kappa_{1,c}A_{1,c}(2)\rho \\ q_{n-1,1}(3)\rho+(\theta-1)\kappa_{1,c}A_{1,c}(3)\rho\end{bmatrix} - \begin{bmatrix}(\theta-1)A_{1,c}(1)+\gamma\\ (\theta-1)A_{1,c}(2)+\gamma\\ (\theta-1)A_{1,c}(3)+\gamma\end{bmatrix} \qquad \text{(A-68)}$$

$$\begin{bmatrix} q_{n,2}(1)\\ q_{n,2}(2)\\ q_{n,2}(3)\end{bmatrix} = \mathbb{P}\begin{bmatrix}(q_{n-1,2}(1)-1)\rho_\pi\\ (q_{n-1,2}(2)-1)\rho_\pi\\ (q_{n-1,2}(3)-1)\rho_\pi\end{bmatrix}.$$

$$\begin{aligned}\begin{bmatrix} q_{n,0}(1)\\ q_{n,0}(2)\\ q_{n,0}(3)\end{bmatrix} &= \begin{bmatrix}\theta\ln\delta+(\theta-1)\kappa_{0,c}-(\theta-1)A_{0,c}(1)\\ \theta\ln\delta+(\theta-1)\kappa_{0,c}-(\theta-1)A_{0,c}(2)\\ \theta\ln\delta+(\theta-1)\kappa_{0,c}-(\theta-1)A_{0,c}(3)\end{bmatrix} \qquad \text{(A-69)}\\ &+\mathbb{P}\begin{bmatrix} q_{n-1,0}(1)+(\theta-1)\kappa_{1,c}A_{0,c}(1)-\gamma\mu_c+(q_{n-1,2}(1)-1)\mu_\pi(1-\rho_\pi)+\frac{\sigma_\pi^2}{2}(q_{n-1,2}(1)-1)^2\\ q_{n-1,0}(2)+(\theta-1)\kappa_{1,c}A_{0,c}(2)-\gamma\mu_c+(q_{n-1,2}(2)-1)\mu_\pi(1-\rho_\pi)+\frac{\sigma_\pi^2}{2}(q_{n-1,2}(2)-1)^2\\ q_{n-1,0}(3)+(\theta-1)\kappa_{1,c}A_{0,c}(3)-\gamma\mu_c+(q_{n-1,2}(3)-1)\mu_\pi(1-\rho_\pi)+\frac{\sigma_\pi^2}{2}(q_{n-1,2}(3)-1)^2\end{bmatrix}\\ &+\mathbb{P}\begin{bmatrix}\frac{1}{2}\left((\theta-1)\kappa_{1,c}\rho A_{1,c}(1)+q_{n-1,1}(1)\rho-\gamma+(q_{n-1,2}(1)-1)\varphi_{\pi,p}(1)\right)^2\sigma_p^2(1)\\ \frac{1}{2}\left((\theta-1)\kappa_{1,c}\rho A_{1,c}(2)+q_{n-1,1}(2)\rho-\gamma+(q_{n-1,2}(2)-1)\varphi_{\pi,p}(2)\right)^2\sigma_p^2(2)\\ \frac{1}{2}\left((\theta-1)\kappa_{1,c}\rho A_{1,c}(3)+q_{n-1,1}(3)\rho-\gamma+(q_{n-1,2}(3)-1)\varphi_{\pi,p}(3)\right)^2\sigma_p^2(3)\end{bmatrix}\\ &+\mathbb{P}\begin{bmatrix}\frac{1}{2}\left((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(1)+q_{n-1,1}(1)(\rho-1)-\gamma+(q_{n-1,2}(1)-1)\varphi_{\pi,\tau}(1)\right)^2\sigma_\tau^2(1)\\ \frac{1}{2}\left((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(2)+q_{n-1,1}(2)(\rho-1)-\gamma+(q_{n-1,2}(2)-1)\varphi_{\pi,\tau}(2)\right)^2\sigma_\tau^2(2)\\ \frac{1}{2}\left((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(3)+q_{n-1,1}(3)(\rho-1)-\gamma+(q_{n-1,2}(3)-1)\varphi_{\pi,\tau}(2)\right)^2\sigma_\tau^2(3)\end{bmatrix}\end{aligned}$$

with the initial condition $q_{0,0}(i) = 0$, $q_{0,1}(i) = 0$, and $q_{0,2}(i) = 0$ for $i \in \{1, 2\}$. The real yield of the maturity n-period bond is $y_{n,t} = -\frac{1}{n} q_{n,t}$.

D.11 Nominal bond risk premium

Define the one-period HPR of an n-maturity nominal bond in excess of a one-period nominal risk-free rate as

$$\begin{aligned} rx^{nb}_{n,t+1} = {} & q_{n-1,0}(S_{t+1}) - q_{n,0}(S_t) + q_{n-1,2}(S_{t+1})\mu_\pi(1-\rho_\pi) + q_{1,0}(S_t) \\ & + \Big(q_{n-1,1}(S_{t+1})\rho - q_{n,1}(S_t) + q_{1,1}(S_t)\Big)x_t \\ & + \Big(q_{n-1,2}(S_{t+1})\rho_\pi - q_{n,2}(S_t) + q_{1,2}(S_t)\Big)\pi_t + q_{n-1,2}(S_{t+1})\epsilon_{\pi,t+1} \\ & + \Big(q_{n-1,1}(S_{t+1})\rho + q_{n-1,2}(S_{t+1})\varphi_{\pi,p}(S_{t+1})\Big)\epsilon_{p,t+1} \\ & + \Big(q_{n-1,1}(S_{t+1})(\rho-1) + q_{n-1,2}(S_{t+1})\varphi_{\pi,\tau}(S_{t+1})\Big)\epsilon_{\tau,t+1}. \end{aligned} \tag{A-70}$$

The nominal bond risk premium is

$$\begin{aligned} -cov_t(m_{t+1}, rx^{nb}_{n,t+1}) \approx {} & -\mathbb{P}\begin{bmatrix} \big(q_{n-1,1}(1)\rho + q_{n-1,2}(1)\varphi_{\pi,p}(1)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(1) - \gamma\big)\sigma_p^2(1) \\ \big(q_{n-1,1}(2)\rho + q_{n-1,2}(2)\varphi_{\pi,p}(2)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(2) - \gamma\big)\sigma_p^2(2) \\ \big(q_{n-1,1}(3)\rho + q_{n-1,2}(3)\varphi_{\pi,p}(3)\big)\big((\theta-1)\kappa_{1,c}\rho A_{1,c}(3) - \gamma\big)\sigma_p^2(3) \end{bmatrix} \\ & - \mathbb{P}\begin{bmatrix} \big(q_{n-1,1}(1)(\rho-1) + q_{n-1,2}(1)\varphi_{\pi,\tau}(1)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(1) - \gamma\big)\sigma_\tau^2(1) \\ \big(q_{n-1,1}(2)(\rho-1) + q_{n-1,2}(2)\varphi_{\pi,\tau}(2)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(2) - \gamma\big)\sigma_\tau^2(2) \\ \big(q_{n-1,1}(3)(\rho-1) + q_{n-1,2}(3)\varphi_{\pi,\tau}(3)\big)\big((\theta-1)\kappa_{1,c}(\rho-1)A_{1,c}(3) - \gamma\big)\sigma_\tau^2(3) \end{bmatrix}. \end{aligned} \tag{A-71}$$

D.12 Nominal bond and stock return correlation

Define the one-period HPR of an n-maturity nominal bond as

$$\begin{aligned} r^{nb}_{n,t+1} = {} & q_{n-1,0}(S_{t+1}) - q_{n,0}(S_t) + q_{n-1,2}(S_{t+1})\mu_\pi(1-\rho_\pi) + \Big(q_{n-1,1}(S_{t+1})\rho - q_{n,1}(S_t)\Big)x_t \\ & + \Big(q_{n-1,2}(S_{t+1})\rho_\pi - q_{n,2}(S_t)\Big)\pi_t + q_{n-1,2}(S_{t+1})\epsilon_{\pi,t+1} \\ & + \Big(q_{n-1,1}(S_{t+1})\rho + q_{n-1,2}(S_{t+1})\varphi_{\pi,p}(S_{t+1})\Big)\epsilon_{p,t+1} \\ & + \Big(q_{n-1,1}(S_{t+1})(\rho-1) + q_{n-1,2}(S_{t+1})\varphi_{\pi,\tau}(S_{t+1})\Big)\epsilon_{\tau,t+1}. \end{aligned} \tag{A-72}$$

The market return is

$$r_{d,t+1} = \kappa_{0,d} + \kappa_{1,d}A_{0,d}(S_{t+1}) - A_{0,d}(S_t) + \mu_d + \Big(\kappa_{1,d}A_{1,d}(S_{t+1})\rho - A_{1,d}(S_t) + \alpha\Big)x_t \quad \text{(A-73)}$$
$$+ \Big(\kappa_{1,d}A_{1,d}(S_{t+1})\rho + \varphi_{d,p}(S_{t+1})\Big)\epsilon_{p,t+1} + \Big(\kappa_{1,d}A_{1,d}(S_{t+1})(\rho-1) + \varphi_{d,\tau}(S_{t+1})\Big)\epsilon_{\tau,t+1} + \epsilon_{d,t+1}.$$

The conditional covariance between two returns is

$$Cov_t(r_{d,t+1}, r^{nb}_{n,t+1}) \approx E[Cov(r_{d,t+1}, r^{nb}_{n,t+1}|S_{t+1})] \quad \text{(A-74)}$$
$$= \mathbb{P}\begin{bmatrix} \big(\kappa_{1,d}A_{1,d}(1)\rho + \varphi_{d,p}(1)\big)\big(q_{n-1,1}(1)\rho + q_{n-1,2}(1)\varphi_{\pi,p}(1)\big)\sigma_p^2(1) \\ \big(\kappa_{1,d}A_{1,d}(2)\rho + \varphi_{d,p}(2)\big)\big(q_{n-1,1}(2)\rho + q_{n-1,2}(2)\varphi_{\pi,p}(2)\big)\sigma_p^2(2) \\ \big(\kappa_{1,d}A_{1,d}(3)\rho + \varphi_{d,p}(3)\big)\big(q_{n-1,1}(3)\rho + q_{n-1,2}(3)\varphi_{\pi,p}(3)\big)\sigma_p^2(3) \end{bmatrix}$$
$$+ \mathbb{P}\begin{bmatrix} \big(\kappa_{1,d}A_{1,d}(1)(\rho-1) + \varphi_{d,\tau}(1)\big)\big(q_{n-1,1}(1)(\rho-1) + q_{n-1,2}(1)\varphi_{\pi,\tau}(1)\big)\sigma_\tau^2(1) \\ \big(\kappa_{1,d}A_{1,d}(2)(\rho-1) + \varphi_{d,\tau}(2)\big)\big(q_{n-1,1}(2)(\rho-1) + q_{n-1,2}(2)\varphi_{\pi,\tau}(2)\big)\sigma_\tau^2(2) \\ \big(\kappa_{1,d}A_{1,d}(3)(\rho-1) + \varphi_{d,\tau}(3)\big)\big(q_{n-1,1}(3)(\rho-1) + q_{n-1,2}(3)\varphi_{\pi,\tau}(3)\big)\sigma_\tau^2(3) \end{bmatrix}.$$

The conditional variance of nominal bond return is

$$Var_t(r^{nb}_{n,t+1}) \approx E[Var(r^{nb}_{n,t+1}|S_{t+1})] \quad \text{(A-75)}$$
$$= \mathbb{P}\begin{bmatrix} \big(q_{n-1,1}(1)\rho + q_{n-1,2}(1)\varphi_{\pi,p}(1)\big)^2\sigma_p^2(1) + \big(q_{n-1,1}(1)(\rho-1) + q_{n-1,2}(1)\varphi_{\pi,\tau}(1)\big)^2\sigma_\tau^2(1) \\ \big(q_{n-1,1}(2)\rho + q_{n-1,2}(2)\varphi_{\pi,p}(2)\big)^2\sigma_p^2(2) + \big(q_{n-1,1}(2)(\rho-1) + q_{n-1,2}(2)\varphi_{\pi,\tau}(2)\big)^2\sigma_\tau^2(2) \\ \big(q_{n-1,1}(3)\rho + q_{n-1,2}(3)\varphi_{\pi,p}(3)\big)^2\sigma_p^2(3) + \big(q_{n-1,1}(3)(\rho-1) + q_{n-1,2}(3)\varphi_{\pi,\tau}(3)\big)^2\sigma_\tau^2(3) \end{bmatrix}.$$

The conditional variance of market return is

$$Var_t(r_{d,t+1}) \approx E[Var(r_{d,t+1}|S_{t+1})] \quad \text{(A-76)}$$
$$= \mathbb{P}\begin{bmatrix} \big(\kappa_{1,d}A_{1,d}(1)\rho + \varphi_{d,p}1)\big)^2\sigma_p^2(1) + \big(\kappa_{1,d}A_{1,d}(1)(\rho-1) + \varphi_{d,\tau}(1)\big)^2\sigma_\tau^2(1) + \sigma_d^2 \\ \big(\kappa_{1,d}A_{1,d}(2)\rho + \varphi_{d,p}(2)\big)^2\sigma_p^2(2) + \big(\kappa_{1,d}A_{1,d}(2)(\rho-1) + \varphi_{d,\tau}(2)\big)^2\sigma_\tau^2(2) + \sigma_d^2 \\ \big(\kappa_{1,d}A_{1,d}(3)\rho + \varphi_{d,p}(3)\big)^2\sigma_p^2(3) + \big(\kappa_{1,d}A_{1,d}(3)(\rho-1) + \varphi_{d,\tau}(3)\big)^2\sigma_\tau^2(3) + \sigma_d^2 \end{bmatrix}.$$

D.13 Inflation risks in nominal bond yields

We decompose shocks to nominal bond yields $\epsilon_{y,n,t}$ into news about expected inflation $\epsilon_{\pi,n,t}$, news about expected future real short rates $\epsilon_{y_1,n,t}$, and expected excess returns $\epsilon_{x,n,t}$. A yield shock is the sum of news

$$\epsilon_{y,n,t} = \epsilon_{\pi,n,t} + \epsilon_{y_1,n,t} + \epsilon_{x,n,t} \quad \text{(A-77)}$$

where

$$\epsilon_{y,n,t} = y_{n,t} - E_{t-1}(y_{n,t}) \tag{A-78}$$
$$\epsilon_{\pi,n,t} = (E_t - E_{t-1})\frac{1}{n}\sum_{i=1}^{n}\pi_{t+i}$$
$$\epsilon_{y_1,n,t} = (E_t - E_{t-1})\frac{1}{n}\sum_{i=1}^{n}y_{1,t+i-1}$$
$$\epsilon_{x,n,t} = (E_t - E_{t-1})\frac{1}{n}\sum_{i=1}^{n}rx_{n-i+1,t+i}$$

denotes the news. For ease of exposition, denote $y_{n,j}(S_t) = -\frac{1}{n}q_{n,j}(S_t)$ for $j \in \{0, 1, 2\}$. We can deduce that

$$\begin{aligned}\epsilon_{y,n,t} &= (E_t - E_{t-1})\Big(y_{n,0}(S_t) + y_{n,2}(S_t)\mu_\pi(1-\rho_\pi) + y_{n,1}(S_t)\rho x_{t-1} + y_{n,2}(S_t)\rho_\pi\pi_{t-1}\Big) \\ &\quad + \Big(y_{n,1}(S_t)\rho + y_{n,2}(S_t)\varphi_{\pi,p}(S_t)\Big)\epsilon_{p,t} + \Big(y_{n,1}(S_t)(\rho-1) + y_{n,2}(S_t)\varphi_{\pi,\tau}(S_t)\Big)\epsilon_{\tau,t} + y_{n,2}(S_t)\epsilon_{\pi,t} \\ \epsilon_{\pi,n,t} &= \frac{1}{n}\frac{\rho_\pi(1-\rho_\pi^n)}{1-\rho_\pi}\Big(\varphi_{\pi,p}(S_t)\epsilon_{p,t} + \varphi_{\pi,\tau}(S_t)\epsilon_{\tau,t} + \epsilon_{\pi,t}\Big).\end{aligned} \tag{A-79}$$

For simplicity, we condition on $x_{t-1} = 0$ and $\pi_{t-1} = 0$. Then,

$$\begin{aligned}&Var(\epsilon_{y,n,t}|S_{t-1}) \\ &\approx \left(\begin{bmatrix} y_{n,0}(1) \\ y_{n,0}(2) \\ y_{n,0}(3)\end{bmatrix} - \mathbb{P}\begin{bmatrix} y_{n,0}(1) \\ y_{n,0}(2) \\ y_{n,0}(3)\end{bmatrix}\right)^{.2} \\ &+ \mathbb{P}\begin{bmatrix} \{y_{n,1}(1)\rho + y_{n,2}(1)\varphi_{\pi,p}(1)\}^2\sigma_p^2(1) + \{y_{n,1}(1)(\rho-1) + y_{n,2}(1)\varphi_{\pi,\tau}(1)\}^2\sigma_\tau^2(1) + (y_{n,2}(1)\sigma_\pi)^2 \\ \{y_{n,1}(2)\rho + y_{n,2}(2)\varphi_{\pi,p}(2)\}^2\sigma_p^2(2) + \{y_{n,1}(2)(\rho-1) + y_{n,2}(2)\varphi_{\pi,\tau}(2)\}^2\sigma_\tau^2(2) + (y_{n,2}(2)\sigma_\pi)^2 \\ \{y_{n,1}(3)\rho + y_{n,2}(3)\varphi_{\pi,p}(3)\}^2\sigma_p^2(3) + \{y_{n,1}(3)(\rho-1) + y_{n,2}(3)\varphi_{\pi,\tau}(3)\}^2\sigma_\tau^2(3) + (y_{n,2}(3)\sigma_\pi)^2 \end{bmatrix} \\ &Var(\epsilon_{\pi,n,t}|S_{t-1}) \\ &= \mathbb{P}\begin{bmatrix} \varphi_{\pi,p}^2(1)\sigma_p^2(1) + \varphi_{\pi,\tau}^2(1)\sigma_\tau^2(1) + \sigma_\pi^2 \\ \varphi_{\pi,p}^2(2)\sigma_p^2(2) + \varphi_{\pi,\tau}^2(2)\sigma_\tau^2(2) + \sigma_\pi^2 \\ \varphi_{\pi,p}^2(3)\sigma_p^2(3) + \varphi_{\pi,\tau}^2(3)\sigma_\tau^2(3) + \sigma_\pi^2 \end{bmatrix} \cdot \left(\frac{1}{n}\frac{\rho_\pi(1-\rho_\pi^n)}{1-\rho_\pi}\right)^2\end{aligned} \tag{A-80}$$

A measure of inflation risk can be defined as

$$IR_{t-1} \equiv \frac{Var(\epsilon_{\pi,n,t}|S_{t-1})}{Var(\epsilon_{y,n,t}|S_{t-1})}. \tag{A-81}$$

Table A-1: Robustness: Consumption autocorrelations with revised data

	Full sample		Sample excluding recessions	
	(1)	(2)	(1)	(2)
α_0	0.0012***	0.0002	0.0022***	0.0003
	(0.0004)	(0.0003)	(0.0004)	(0.0003)
α_1	0.0030***		0.0039***	
	(0.0008)		(0.0010)	
β_0	0.6308***	0.6877***	0.4884***	0.6126***
	(0.0821)	(0.0868)	(0.0825)	(0.0918)
β_1	−0.4337***	−0.4855***	−0.4461***	−0.5573***
	(0.1212)	(0.1209)	(0.1415)	(0.1548)
R^2	0.1240	0.1171	0.1004	0.0809

Notes: The table shows the regression estimates from the regression in Equation (1). On the right side, business cycle downturns are excluded from the sample as explained in the main text. COVID period is excluded from all regressions. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient in the post-1998 sample, while the β_1 estimate gives the difference between the autocorrelation in the pre- vs. post-1998 samples. The specification in the column labeled (1) estimates the full regression, while the specification in the column labeled (2) is run using demeaned consumption growth and setting $\alpha_1 = 0$, as explained in the main text. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level. The data is the revised real, per capita, nondurables+services consumption data from 1947:Q2 to 2019:Q4.

Table A-2: Estimated parameters of NIPA revised nondurables+services consumption

	50%	[5%,	95%]
Consumption growth, Δc			
μ_c	0.0048	[0.0037,	0.0058]
ρ	0.8591	[0.7537,	0.9226]
$\sigma_p(1)$	0.0017	[0.0012,	0.0025]
$\sigma_p(2)$	0.0014	[0.0008,	0.0019]
$\sigma_p(3)$	0.0025	[0.0016,	0.0038]
$\sigma_\tau(1)$	0.0017	[0.0013,	0.0024]
$\sigma_\tau(2)$	0.0049	[0.0041,	0.0071]
$\sigma_\tau(3)$	0.0690	[0.0380,	0.0966]

Transition probabilities, $\mathbb{P}$

$$\begin{bmatrix} \underset{[0.9354,0.9954]}{0.9752} & \underset{[0.0046,0.0646]}{0.0248} & \underset{[-,-]}{0} \\ \underset{[0.0113,0.0179]}{0.0168} & \underset{[0.9312,0.9881]}{0.9630} & \underset{[0.0051,0.0486]}{0.0199} \\ \underset{[-,-]}{0} & \underset{[0.3507,0.5928]}{0.4717} & \underset{[0.4072,0.6493]}{0.5293} \end{bmatrix}$$

*N*otes: The table gives the maximum likelihood estimates (MLE) for the parameters governing the consumption process using the NIPA revised, per capita, real nondurables+services consumption data. The data frequency is quarterly. We order the regimes according to the transitory shock volatilities, and restrict the off-diagonal corner elements in the transition probability matrix to equal zero. The sample is 1947:Q2-2020:Q4.