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THE DECLINE OF DRUDGERY AND THE PARADOX OF HARD WORK

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### **ABSTRACT**

We develop a theory focusing on general equilibrium and long-run macroeconomic consequences of trends in job utility—the process benefits and costs of work. Given secular increases in job utility, work hours can remain approximately constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect. Secular improvements in job utility can be substantial relative to welfare gains from ordinary technological progress. These two implications are connected by an equation flowing from optimal hours choices: improvements in job utility that have a significant effect on labor supply tend to have large welfare effects.

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# 1 Introduction

In his 1930 essay, “Economic Possibilities for Our Grandchildren,” Keynes predicted that a large increase in leisure would take place over the following century, but at the aggregate level robust signs of such a leisure boom have failed to materialize.<sup>1</sup> As shown in Figure 1, for a large set of OECD countries, from 1960 through 2019 aggregate (real) private consumption per capita more than tripled, while (total) work hours per population have been relatively flat. Indeed, compared to 1960, in 2019 work hours in the US were a bit higher (thirteen percent) while work hours in the other OECD countries in our sample were a bit lower (fifteen percent). This means that in absolute value terms changes in work hours were only about five percent of the changes in consumption.

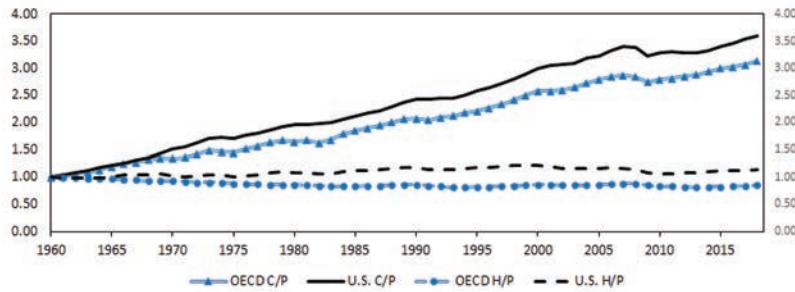


Figure 1: (Data are normalized at 1960.) Hours worked per population (H/P) and private real consumption per capita (C/P)—data are from the Conference Board’s Total Economy Database and the OECD (VPVOBARSA consumption)<sup>2</sup>—for the US and (a large number of non-U.S.) OECD countries.<sup>3</sup>

It is not unreasonable to think, as Keynes did, that the extent to which real consumption has increased, along with long-run growth in real wages, should have led to a prominent trend of declining work hours driven by the income

<sup>1</sup>As related to the US, see for example Neville and Ramey, 2009.

<sup>2</sup>Conference Board: <https://www.conference-board.org/data/economydatabase/> OECD: <https://stats.oecd.org/>

<sup>3</sup>Simple average of country-specific ratios over Australia, Austria, Belgium, Finland, France, Germany, Greece, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. Other standard weighting techniques, such as GDP weighting, yield similar results.

effect overtaking the substitution effect. Why are people still working so hard? And, what are the welfare implications of this *paradox of hard work*?<sup>4</sup>

Five alternative, but not mutually exclusive, explanations could explain the paradox of hard work theoretically. To frame these alternative explanations, consider a representative agent model with the following simple utility function:  $u(C, H) = \frac{C^{1-\frac{1}{s}}}{1-\frac{1}{s}} - v(H)$ , where  $C$  is real consumption,  $H$  is labor hours,  $s$  is the elasticity of intertemporal substitution (EIS),  $v' > 0$  and  $v'' > 0$ . Household optimization implies  $W = -u_H/u_C = C^{\frac{1}{s}}v'(H)$ , where  $W$  is the real wage. Rearranging and manipulating, this implies in turn that  $\frac{(WH/Y)}{(C/Y)}C^{1-\frac{1}{s}} = Hv'(H) = \phi(H)$ , where  $\frac{WH}{Y}$  is the labor share  $\alpha_H$ , and  $\frac{C}{Y}$  is the consumption-to-output ratio  $\zeta_C$ . It follows that  $H = \phi^{-1}\left(\frac{\alpha_H}{\zeta_C}C^{1-\frac{1}{s}}\right)$ . Because  $\phi$  is monotonically increasing, so is  $\phi^{-1}$ . Therefore, with an approximately constant labor share and an approximately constant consumption-to-output ratio (which are empirical regularities),  $s < 1$ , and increasing real consumption,  $H$  should be decreasing. That is what Keynes predicted. Given the other assumptions, with  $s < 1$  and  $C$  increasing, the only way to avoid decreasing labor hours  $H$  is to have constant increases in  $WH/C$ , which is not easy in general equilibrium.

What are possible ways out of this box?

1. *Assuming the elasticity of intertemporal substitution (EIS) is large*—close to 1 or greater than 1. This is a possibility—and the explanation most often resorted to (at least implicitly). However, as argued in Appendix A, the balance of the empirical evidence puts the EIS at less than 1, which if true would rule out this explanation.

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<sup>4</sup>A well known parallel strand of literature is motivated by *cross*-country differences in hours worked per population (in contrast, we focus on *within*-country trends in hours relative to domestic gains in wealth). This literature includes, among others, Prescott (2004), Alesina et al. (2005), Rogerson (2006, 2007, and 2009), Faggio and Nickell (2007), Ljungqvist and Sargent (2006), Ohanian et al. (2008), Shimer (2009), McDaniel (2011), and Epstein et al. (2021). Prescott (2004) and Ohanian et al. (2008) argue that in Europe a large fraction of the trend decline in work hours can be attributed to secular increases in labor taxes, value-added taxes and transfers.

2. *An increasing ratio of effective marginal wages to consumption.* This could be the result, for instance, of a reduction in the progressivity of the tax system, an intensification of competition for promotions within firms, and increasing educational debts.<sup>5</sup>
3. *Anything that keeps the marginal utility of consumption high.* This could be, for example, because of habit formation, “keeping up with the Joneses,” and the introduction of new goods.<sup>6</sup>
4. *The income and substitution effects are both basically equal to zero.* This, of course, is consistent with very curved disutility of labor and, ultimately, with flat work hours. That said, Appendix B argues that the marginal propensity to earn, which is a good measure of the income effect, is substantial, which if true rules out this possibility.
5. *Anything that serves to keep the marginal disutility of work low.* This includes some kinds of technological progress in household production, non-separability between consumption and leisure,<sup>7</sup> and jobs getting more pleasant.

Of these possible explanations for the paradox of hard work, in this paper we focus on jobs becoming more pleasant—one of the factors that can keep the disutility of work hours down. Economists have long understood that cross-sectional differences in job utility give rise to compensating differentials. We focus on a less-studied topic: the long-run macroeconomic consequences of trends in job utility.

We propose a growth framework for thinking about the causes and effects of secular increases in job utility, that is, of jobs getting more pleasant. Our

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<sup>5</sup>For additional discussion, see, for instance, Kimball and Shapiro (2008).

<sup>6</sup>See, for instance, Abel (1990), Fuhrer (2000), Luttmer (2005), Rayo and Becker (2007), Struck (2014) and Zhou (2021). We view external habit formation as a version of “keeping up with the Joneses.”

<sup>7</sup>See, among others, King, Plosser, and Rebelo (1988) and Basu and Kimball (2002).

framework is a natural, but novel, extension of the static theory of compensating differentials, which was spelled out originally in the first ten chapters of Book I of “The Wealth of Nations” (Smith, 1776) and for which a standard modern reference is Rosen (1986).

In our model, as is standard in macroeconomics, individuals obtain utility from consumption and non-work time. Our additional assumption is that work also has process benefits and process costs—what we call “job utility.” Job utility depends on work effort and amenities in the workplace, as well as on the nature of the job itself. Our definition of these variables is very broad, going beyond how they are usually understood.<sup>8</sup> Moreover, firm-side “job-enjoyment technology” affects the mapping of effort and amenities into overall job utility.

Some of the questions that our framework provides answers to, with clear growth implications, are the following.

- How do effort, amenities, job-enjoyment technology, and labor-augmenting technology interact?
- What are the key determinants of long-run labor supply in the light of job utility?
- How does job utility matter for firms’ optimization problems and firms’ ongoing ability to operate, attract workers, and establish job parameters given long-run changes in labor-augmenting technology and job-enjoyment technology?
- What are the long-run welfare effects of changes in job utility?

In turn, the answers to these questions lead to two contributions to the macro and labor economics literatures. First, we show that secular improvements in job utility—*the decline of drudgery*—can cause work hours to remain approximately constant over time even if the income effect of higher wages

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<sup>8</sup>In a narrower sense than our interpretation of amenities, a strand of literature shows that amenities are important. See, for instance, Coulibaly (2006), Becker (2011), Sullivan and To (2014), Taber and Vejlín (2016), Hall and Mueller (2018), and Sorkin (2018).

on labor supply exceeds the substitution effect of higher wages. Therefore, the paradox of hard work is not necessarily evidence that the elasticity of intertemporal substitution is large, that preferences are strongly non-separable, or that preferences have some other feature such as habit formation (though each of these *might* be part of the explanation of the paradox). Second, we argue that secular improvements in job utility can be substantial even in comparison to the welfare gains from ordinary (say, labor-augmenting) technological progress. These two implications are connected by an equation: improvements in job utility that have a significant effect on labor supply tend to have large welfare effects. It is worth noting that our theoretical analysis also yields many insights if, for instance, the income effect is less than the substitution effect, so we make a substantial theoretical contribution regardless of one’s specific views on parameter values.

This paper proceeds as follows. Section 2 relates our work to the static theory of compensating differentials. Section 3 provides a general overview of our framework. Section 3.3 discusses the variables we focus on and how our formulation maps into the real world. Sections 5 and 6 focus, respectively, on the optimization problems of individuals and firms. Section 8 highlights certain implications from our theory. Section 7 deals with the economy’s general equilibrium. Section 9 addresses the welfare consequence of changes in job utility. Section 10 concludes.

## 2 Static Theory of Compensating Differences

The natural point of reference for our analysis is the theory of compensating differentials, spelled out originally in the first ten chapters of Book I of “The Wealth of Nations” (Smith, 1776). A standard modern reference on compensating differentials is Rosen (1986).

## 2.1 Worker and Firm Choices

The solid line in the left panel of Figure 2 is a wage/job-utility frontier: jobs offering lower job utility will, in principle, compensate by offering higher real wages (in the figure  $W$  is the real wage and  $J$  is job utility). Thus, all else equal, individuals face a trade-off between these two variables. Conditional on individual preferences, a particular worker optimizes by choosing a feasible point on the (solid) frontier in the  $(W, J)$  plane.

The solid line in the right panel of Figure 2 is a job-utility/output frontier: in order to improve job utility firms must divert part of their resources away from the production of output ( $Y$ ). Given a firm's idiosyncratic costs of job utility in terms of output, a particular firm optimizes by choosing a feasible point on the (solid) frontier in the  $(Y, J)$  plane.

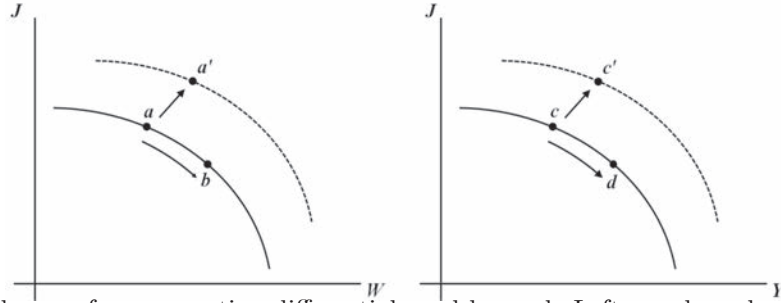


Figure 2: Theory of compensating differentials and beyond. Left panel: real wage ( $W$ )–job utility ( $J$ ) frontier faced by workers. Right panel: job utility–output ( $Y$ ) frontier faced by firms.

## 2.2 Movements Along the Frontiers

Suppose higher output and higher real wages came from movements *along* the solid frontiers ( $a$  to  $b$  in the left panel and  $c$  to  $d$  in the right panel). As argued in Kimball and Shapiro (2008), income effects on labor supply are substantial. So, the higher real wage implied by moving from point  $a$  to point  $b$  would tend to reduce work hours. In addition, if work hours are increasing in job utility,



then the lower job utility implied by moving from point  $a$  to point  $b$  also puts downward pressure on work hours.

## 2.3 Movements of the Frontiers

However, the frontiers themselves can shift (the dashed lines in Figure 2). As the economy's choice set expands, optimal choices can entail moving to points such as  $a'$  and  $c'$ , in which case job utility, output, and real wages can all rise, and increases in job utility emerge as potentially *offsetting* to income effects.

The theory we develop in this paper focuses on shedding light on the dynamic general equilibrium implications and endogenous foundations of such intertemporal changes in the economy's choice set. Understanding this is complementary to the long-standing static, partial equilibrium microeconomic framework of compensating differentials.

## 2.4 Implications of the Theory for the Cross-Section of Hours

Although we focus in this paper on the decline of drudgery over time, it is worth noting that cross-sectional differences in job utility can be helpful in explaining why differences in working hours are as small as they are for workers at different wage levels, even if income effects exceed substitution effects. Firms, in their efforts to provide workers a reservation utility level at minimum cost, are likely to offer both higher wages and pleasant job attributes to higher skill workers and both lower wages and unpleasant job attributes to lower skill workers. When the pleasantness of a job tends to covary positively with a job's wage, the pleasantness of a job can help keep a worker willing to work longer hours despite an income effect that exceeds the substitution effect of the higher wage.

### 3 The Model

There are no distortions in our model; therefore, the social planner's problem has the same behavior as a decentralized economy with perfect competition. Both perspectives are valuable. We begin with the social planning perspective.

The social planner's problem involves choosing consumption, capital, work hours devoted to particular jobs, effort demands by a particular job (per hour of work), and amenities provided by a particular job, in order to maximize a household's lifetime utility given firms' production structures and other standard constraints.

#### 3.1 Baseline Assumptions

We consider a small open economy in which agents can freely borrow and lend at an exogenously determined real interest rate  $r$  (equal to  $\rho$ , the rate at which all economic agents discount future utility). Capital is freely mobile across firms and borders. We assume that all benefits and costs to firms and workers other than the utility from leisure and consumption are proportional to work hours. Given fully mobile capital and the exogenous world interest rate, we can focus on steady state analysis since the absence of state variables implies that changes between steady states occur instantaneously. The model is cast in continuous time (we omit time indices in order to avoid notational clutter).

#### 3.2 Individuals and Firms

The economy is inhabited by  $i = 1, \dots, I$  firms, all of which are producers of the same final good, and a continuum of individuals whose mass is normalized to one. Households each have only one individual, so we will use the terms household and worker interchangeably.

Utility depends on consumption, the division of time between work time and non-work time, and job utility per hour of work. Job utility depends on effort, amenities, and *job-enjoyment technology* (we elaborate on all of these further below).

Firms produce output using capital and *effective labor input* (the product of hours, effort and labor-augmenting technology), and can vary in their real wage and job utility offerings.

### 3.3 Planning Problem

Table 1 below lays out some of our notation. Using this notation, the planning problem is:

$$\begin{aligned} & \max_{C, H_i, E_i, A_i, K_i} \int e^{-\rho t} \mathcal{U}(C, T - H, \sum_i H_i J_i(E_i, A_i, \Psi_i)) dt \\ \text{such that } & \sum_i Y_i(K_i, Z_i E_i H_i) + \Pi = C + \dot{K} + \delta K + \sum_i A_i H_i, \\ & \text{and } \sum_i H_i = H. \end{aligned}$$

For any variable  $X$ ,  $\dot{X}$  refers to its change over time.

Table 1: Notation for Functions, Variables, and Parameters

Notation	Description	Notation	Description
$C$	Total consumption of final output	$J_i$	Job utility function of $i$ th firm
$H_i$	Work hours devoted to $i$ th firm	$\Psi_i$	Job-enjoyment technology of $i$ th firm
$E_i$	Effort demands of $i$ th firm	$Y_i$	Final output of $i$ th firm
$A_i$	Amenities provision of $i$ th firm	$Z_i$	Labor-augmenting technology of $i$ th firm
$K_i$	Capital use of $i$ th firm	$\Pi$	Non-labor, non-interest income (including from foreign sources)
$\rho$	Discount rate	$\delta$	Depreciation rate
$t$	Time		
$\mathcal{U}$	Instantaneous utility		
$T$	Time endowment		
$H$	Total work hours		

## 4 The Model’s Abstractions and the Real World

Our objective is to deal with many real world features of jobs without adding too much complexity to the model. We accomplish this by having a broad interpretation of consumption, work hours, effort, amenities, and job utility that allows each to address multiple dimensions of the real world. For example, job-enjoyment technology is meant to capture both innovations in the nature of work proper and innovations in the nature of the work environment.

### 4.1 Consumption

Consumption,  $C$ , is meant to capture all the richness of how resources other than time affect life outside of working hours. For instance, our broad notion of consumption takes into account fringe benefits.

### 4.2 Work Hours

Work hours,  $H$ , is meant to capture every way in which a person’s job interferes with the quantity and enjoyment of non-work time and home production. For example, if an individual is unable to stop thinking about work issues while at home and this interferes with other activities at home, then this can be considered an effective reduction in leisure—and hence an increase in  $H$ . Also, consider time spent away from home due to work-related travel. Travel may boost the utility of non-work time if it provides pleasant and interesting experiences. However, work-related travel can also hamper the enjoyment of non-work time because of being away from friends and family. In either case, an adjustment to  $H$  may be warranted.

Commuting, in particular, is a way in which a person’s job can interfere with the enjoyment of time outside of formal work time. The Covid-19 pandemic has

accelerated experimentation by firms with options for remote work. Freedom to do remote work some or all of the time reduces commuting time, enables more interactions with family interspersed during the work day and also changes other dimensions of an employee’s work experience that can be considered part of effort (*e.g.*, Zoom fatigue) and amenities (the comforts of home instead of the comforts of the office).

### **4.3 Effort**

Effort,  $E$ , is meant to capture all aspects of a job that generate proportionate changes in effective productive input from labor. Effort has many dimensions. Hence, for example, all of the following count as reductions in effort. Lack of concentration on a task while at one’s work station, time spent at the water cooler or in other forms of on-the-job leisure, own time spent cleaning and beautifying the work place in ways that don’t contribute to production, time spent in office parties, and time spent pursuing worker interests that have some productivity to the firm but would not be the boss’s first priority.

### **4.4 Amenities**

Amenities,  $A$ , are job characteristics whose cost to the firm is in terms of goods. The real-world characterization of amenities is just as rich as the characterization of effort. For instance, amenities include the number of parking spots, the quality of air conditioning, and the quality—and capacity relative to number of employees—of the office gym.

## **4.5 Job-Enjoyment Technology**

Job-enjoyment technology affects the mapping of effort and amenities into overall job utility. Therefore, changes in job-enjoyment technology can be interpreted as capturing both innovations in the nature of work proper and innovations in the work environment.

### **4.5.1 Innovations in the Nature of Work Proper**

Innovations in the nature of work proper come in many forms. For example,

- working in groups,
- establishing clear guidelines about what is expected from the worker,
- allowing workers to have greater discretion in the way projects are carried out,
- developing creative ways to give workers feedback on their performance (including constructive criticism techniques rather than, say, yelling at the worker about what he or she is doing wrong),
- improving the organizational structure of the firm in terms of who does what, how they do it and when they do it,
- allowing individuals greater flexibility in determining the time during which work is carried out,

all count historically as innovations in the nature of work proper.

### **4.5.2 Innovations in the Nature of the External Work Environment**

Innovations related to the external work environment come in many forms as well. In particular, think of

- the advent of air conditioning,
- the provision of on-site childcare, exercise, and laundry facilities,
- the institution of measures to reduce the incidence of sexual harassment.

There is also one set of innovations that have had an important effect on both the nature of work proper and on the work environment:

- the distribution, design, and allocation of physical work space (such as cubicalization or open office environments).

## 4.6 Aggregation Over Techniques

The job utility function  $J_i$  itself is the optimum over many possible ways of doing things. For example, consider two production techniques, as shown in Figure 3 in  $(E, J)$  space. Production technique 1, yielding  $\mathcal{J}_i^1$ , results in relatively higher job utility at lower levels of effort, while production technique 2, yielding  $\mathcal{J}_i^2$ , results in relatively higher job utility at higher levels of effort. Then,  $J_i$  itself is the upper envelope (bold) of these two techniques. The analytical framework that we develop is robust to such non-concavities in job-utility functions.

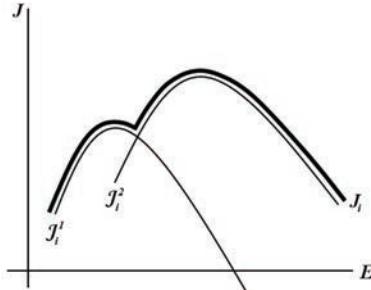


Figure 3: The job-utility function,  $J_i$ , as the upper envelope of the two different production techniques  $\mathcal{J}_i^1$  and  $\mathcal{J}_i^2$ .

## 4.7 Aggregation Over Many Dimensions of Job Utility

The function  $J_i = \mathcal{J}_i(\mathcal{E}_i, \mathcal{A}_i, \Psi_i)$  maps  $\mathcal{E}_i$ ,  $\mathcal{A}_i$ , and  $\Psi_i$  into the hourly utility associated with being at work.  $\mathcal{E}_i$  is a vector describing aspects of effort—all dimensions of what the average hour of work is like that affect productivity (including the fraction of time spent in each different activity at work).  $\mathcal{A}_i$  is

a vector of aspects of amenities.  $\Psi_i$  is job-utility technology as above.  $\mathcal{E}_i$  and  $\mathcal{A}_i$  are determined optimally by firms.

The reduced form job utility function comes from maximizing over these vectors, subject to keeping effort-related productivity and the cost of amenities the same, that is,

$$J_i(E_i, A_i, \Psi_i) = \max_{\mathcal{E}_i, \mathcal{A}_i} \{ \mathcal{J}_i(\mathcal{E}_i, \mathcal{A}_i, \Psi_i) \}$$

such that  $E_i = E_i(\mathcal{E}_i)$  and  $A_i = p_{\mathcal{A}_i} \cdot \mathcal{A}_i$ ,

where  $p_{\mathcal{A}_i}$  is a vector of real amenity prices. So, the number  $E_i$ —hourly effort per worker—gives effective productive input from an hour of labor before multiplication by labor-augmenting technology, while the number  $A_i$  summarizes the expenditure on amenities per hour of work.<sup>9</sup>

We allow for the possibility of  $J_i$  being either positive or negative and for the possibility that job utility is increasing in effort at relatively small levels of effort.<sup>10</sup> However, we assume it must be decreasing in effort at relatively high levels of effort if only because physical and mental exhaustion eventually push  $J_i$  toward  $-\infty$  (otherwise there would be no upper limit to feasible  $E_i$ ). We also assume that  $\partial J_i / \partial A_i > 0$  and  $\partial J_i / \partial \Psi_i > 0$ .

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<sup>9</sup>Thus, using a vertically integrated perspective, one can treat the relative price of amenities as part of the job-enjoyment technology  $\Psi_i$ . Think of the production function for firm  $i$ 's  $k$ th amenity as

$$\mathcal{A}_i^k = \theta_i^k Y_i^k,$$

where  $\theta_i^k$  is multiplicative technology and  $Y_i^k$  is the part of the firm's total output,  $Y_i$ , devoted to producing the amenity. Then, the firm's total expenditure on amenity  $k$  is  $(1/\theta_i^k)\mathcal{A}_i^k = Y_i^k$  and we define  $p_{\mathcal{A}_i}^k \equiv (1/\theta_i^k)$ . Thus, for instance, an improvement in the amenity technology  $\theta_i^k$  decreases the relative price of the  $k$ th amenity. Except when the real prices of amenities are visible in markets it might be impossible to distinguish between an improvement in job-enjoyment technology proper and a fall in the price of an amenity.

<sup>10</sup>We consider this to be the more intuitive case for many jobs, although our results are unaltered by assuming that job utility is always decreasing in effort.



## 5 Household Optimization

We now focus on the decentralized version of the representative worker's optimization problem. We show that this problem can be broken into three optimization subproblems that jointly answer the following question: what are the key determinants of labor supply when job utility is taken into account?

### 5.1 Main Problem

We assume household utility is additively separable between consumption  $C$ , leisure  $T - H$  and all the dimensions of labor (in the Appendix we show that our main messages are robust to relaxing this assumption). Given financial wealth  $M$  and job opportunities, the worker chooses consumption  $C$ , total work hours  $H$ , and work hours devoted to each job  $H_i$ , to maximize utility:

$$\begin{aligned} \max_{C, H, H_i} \quad & \int e^{-\rho t} [U(C) + \Phi(T - H) + \sum_i H_i J_i] dt, \\ \text{such that} \quad & \dot{M} = rM + \Pi + \sum_i W_i H_i - C, \\ & \sum_i H_i = H, \text{ and } H_i \geq 0. \end{aligned}$$

Overall flow utility comes from consumption utility  $U$ , utility from off-the-job leisure  $\Phi$ , and the job utility from each job multiplied by hours on that job  $\sum_i H_i J_i$ .  $W_i$  is the real wage offered by the  $i$ th job, which the worker takes as given. We assume that  $U' > 0$ ,  $U'' < 0$ ,  $\Phi' > 0$ , and  $\Phi'' < 0$ . The choice of job is represented simply as the choice of whether to devote strictly positive work hours to any one job in particular.

## 5.2 Optimization Subproblems

The current-value Hamiltonian associated with the worker's problem is

$$\begin{aligned}\mathcal{H} = & U(C) + \Phi(T - H) + \sum_i H_i J_i \\ & + b(H - \sum_i H_i) + \sum_i \mu_i H_i + \lambda(rM + \Pi + \sum_i W_i H_i - C).\end{aligned}$$

This maximization problem can be broken down into three optimization subproblems:

$$\begin{aligned}\max \mathcal{H} = & \max_C \{U(C) - \lambda C\} + \lambda(rM + \Pi) \\ & + \max_H \{\Phi(T - H) + bH\} \\ & + \max_{H_i} \{\sum_i \mu_i H_i + \sum_i H_i \underbrace{(J_i + \lambda W_i)}_{=B_i} - b \sum_i H_i\}.\end{aligned}$$

Above,  $\lambda$  is the costate variable giving the marginal value of real wealth; the Euler equation is  $\dot{\lambda} = \rho - r = 0$ .  $b$  is the multiplier on the overall time constraint.  $\mu_i$  is the multiplier on the nonnegativity constraint for hours at each possible job.<sup>11</sup> Finally,  $B_i$  denotes the *marginal hourly net job benefits* associated with a job of type  $i$ . In the order we will discuss them, the three optimization subproblems nested within the overall maximization of the current-value Hamiltonian are: (1) the consumption decision; (2) job choice and the decision about work hours for each job; and (3) the overall hours decision.

In the additively separable case here we can normalize the accounting between  $J_i$  and  $\Phi$  so that  $\Phi'(T) = 0$ .<sup>12</sup> Given this normalization,  $J_i > 0$  means

<sup>11</sup>The worker's problem would be dramatically different if it were possible to devote negative work hours to unpleasant, badly paid jobs.

<sup>12</sup>Consider  $U + \tilde{\Phi} + H\tilde{J}_i$  with  $\tilde{\Phi}'(T) = \kappa$ , where  $\kappa$  is a constant. Define  $\Phi(X) = \tilde{\Phi}(X) - \kappa X$  and  $J_i = \tilde{J}_i + \kappa$ . Then,  $\Phi'(T) = 0$ , and  $\mathcal{U} = U + \tilde{\Phi}(T - H) + \sum_i H_i \tilde{J}_i - \kappa T$ , which in turn equals  $U + \Phi(T - H) + \underbrace{\sum_i H_i J_i + \kappa(H - \sum_i H_i)}_{=0}$ .

that a worker would be willing to spend at least some time on a job even if unpaid, should that be the only job available. On the other hand,  $J_i < 0$  means that a worker would never spend time on such a job unless paid.

### 5.2.1 Choice of Consumption

As shown in the left panel of Figure 4, the solution to the first optimization sub-problem,  $\max_C \{U(C) - \lambda C\}$ , is to choose consumption to satisfy the first order condition  $U' = \lambda$ .

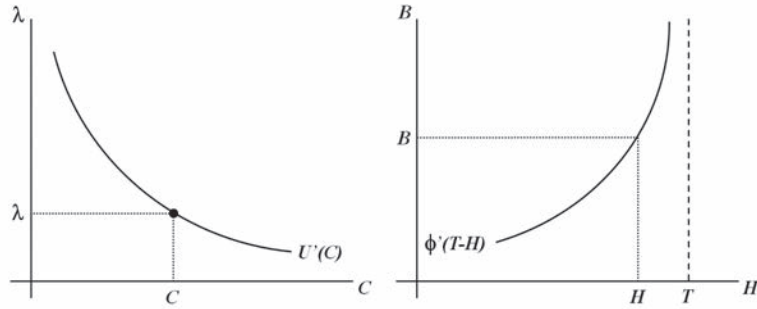


Figure 4: Household solution to choice of consumption,  $C$ , and total work hours,  $H$ .

### 5.2.2 Choice of Jobs and Hours at Each Job

Job choice involves surveying all possible job types and choosing the job or jobs with the highest  $B_i$ . Define  $B = \max_i B_i$ . It follows that if total work hours are spread across more than one job type, each job with positive hours for the individual must be offering the same level of (hourly marginal net) job benefits—although they need not be offering the same combination of real wage and job utility. We elaborate on the fraction of time devoted to each job later.

### 5.2.3 Choice of Overall Work Hours

Combining job choice with the choice of work hours at each job, optimization requires  $H_i = 0$  if  $J_i + \lambda W_i < b$  and  $J_i + \lambda W_i = b$  when  $H_i > 0$ . This implies that  $b = B$ : the marginal benefit of overall work hours is equal to the marginal

benefit of hours at the job with the highest job benefits. Therefore, total work hours should be chosen to satisfy  $\Phi' = B$ . In words, at the optimal level of work hours, the marginal utility from off-the-job leisure is equal to the job benefits  $B$  of the most attractive job. The right panel of Figure 4 shows the determination of the optimal choice of  $H$ . Note that the *labor-hours supply function* is  $\Phi'$ , and the equivalent in this model to a market clearing price for work hours is job benefits  $B$ . (Section 7 discusses the determination of the general equilibrium value of  $B$ .)

## 6 Firm Optimization

In the decentralized version of the optimization problem for firms, the firms are price takers in the product market. Each firm's production function takes as inputs capital and effective labor input (the product of hours, effort, and labor-augmenting technology). The firm rents capital at an exogenous rental rate (determined by the depreciation rate and the international real interest rate). The hourly cost of labor is captured by the inclusive wage: the sum of the real wage and the hourly cost of amenities. That is, writing  $\mathcal{W}_i$  for the inclusive wage,  $\mathcal{W}_i = W_i + A_i$ ; in payment for their labor, workers receive the real wage  $W_i$  (which includes fringe benefits), and as indirect payment—through job utility—amenities  $A_i$ . The solution to the firm's cost minimization problem implies that its cost function can be stated as a function of the rental rate of capital and the effective wage: the ratio of the inclusive wage to effective labor productivity (which in turn is the product of effort and labor-augmenting technology). Minimization of the effective wage is the focus of the firm's optimization subproblems.

## 6.1 Cost Minimization

Consider a representative firm providing a job with job-enjoyment technology  $\Psi_i$ . The firm's production function is  $Y_i = K_i^\alpha (Z_i E_i H_i)^{1-\alpha}$ , where capital's share is  $\alpha \in (0, 1)$  and other variables are as defined earlier. Let  $R$  denote the rental rate of capital, which is exogenous to the firm. (There are no adjustment costs, so  $R = r + \delta$ ).

To maximize profits the firm must minimize costs. The rental rate of capital is exogenous, but the effective wage depends on the firm's choices. Minimizing the effective wage requires optimization over effort and amenities. In what follows we go through each of these steps in order.

For any output level  $\bar{Y}_i$  a firm's cost minimization problem involves choosing capital  $K_i$ , and total work hours  $H_i$ , to minimize total cost  $RK_i + \mathcal{W}_i H_i$  subject to  $K_i^\alpha (Z_i E_i H_i)^{1-\alpha} = \bar{Y}_i$ . The solution to the firm's cost minimization problem is standard. The firm's total cost is a function of the desired level of output,  $Y_i$ , the rental rate of capital,  $R$ , and the *effective wage*,  $\omega_i$ . The effective wage  $\omega_i$  is equal to the ratio of the inclusive wage and labor effectiveness:  $\omega_i = \mathcal{W}_i / (Z_i E_i)$ . Thus, the firm's cost function is

$$\mathcal{C}(\omega_i, R, Y_i) = R^\alpha \omega_i^{1-\alpha} Y_i / (\alpha^\alpha (1-\alpha)^{1-\alpha}). \quad (1)$$

## 6.2 Minimizing the Effective Wage

Given equation (1), a firm should minimize its effective wage subject to the constraint on job benefits:

$$\begin{aligned} & \min_{\mathcal{W}_i, E_i} \omega_i = \mathcal{W}_i / (Z_i E_i) \\ \text{such that } & \underbrace{\lambda (\mathcal{W}_i - A_i) + J_i(E_i, A_i, \Psi_i)}_{=B_i} \geq B, \end{aligned}$$

where we have substituted in  $\mathcal{W}_i - A_i$  for  $W_i$ .

In solving this optimization subproblem, firms take the marginal value of wealth  $\lambda$ , the rental rate of capital  $R$ , and equilibrium job benefits  $B$ , as given. However, the inclusive real wage  $\mathcal{W}_i$ , amenities  $A_i$ , and effort  $E_i$  are choice variables. For simplicity, let us assume here additive separability in job utility between effort and amenities:

$$J_i(E_i, A_i, \Psi_i) = F_i(E_i, \psi_i^E) + G_i(A_i, \psi_i^A, p_{\mathcal{A}_i}), \quad (2)$$

where  $\psi_i^E$  captures innovations in the nature of work proper and  $\psi_i^A$  captures innovations in the nature of the work environment. We will write  $\Psi_i = (\psi_i^E, \psi_i^A, p_{\mathcal{A}_i})$  and  $\Psi_i^A = (\psi_i^A, p_{\mathcal{A}_i})$ .

### 6.3 Choice of Amenities

By the definitions of the inclusive and effective wages  $\lambda W_i = \lambda(\mathcal{W}_i - A_i) = \lambda(\omega_i Z_i E_i - A_i)$ . Substituting this last equation into the firm's problem of meeting the market level of  $B$  so it can attract workers (which must bind at the optimal solution) it follows that

$$\lambda \omega_i Z_i E_i + \underbrace{F_i(E_i, \psi_i^E) + G_i(A_i, \Psi_i^A)}_{=J_i(E_i, A_i, \Psi_i)} - \lambda A_i = B.$$

This implies the nested subproblem:  $\max_{A_i} G(A_i, \Psi_i^A) - \lambda A_i$ . Thus, the choice of amenities should satisfy the tangency condition  $\partial G_i / \partial A = \lambda$ . This optimality condition is shown graphically in  $(A, G)$  space in the left panel of Figure 5.

It is helpful to define  $S(\lambda, \Psi_i^A) \equiv \max_{A_i} \{G(A_i, \Psi_i^A) - \lambda A_i\}$ : the individual surplus received from the firm's optimal choice of amenities. Note that  $S_\lambda < 0$  by the envelope theorem. Thus, the lower the marginal value of wealth

(intuitively, the richer a worker is), the greater the surplus from amenities.

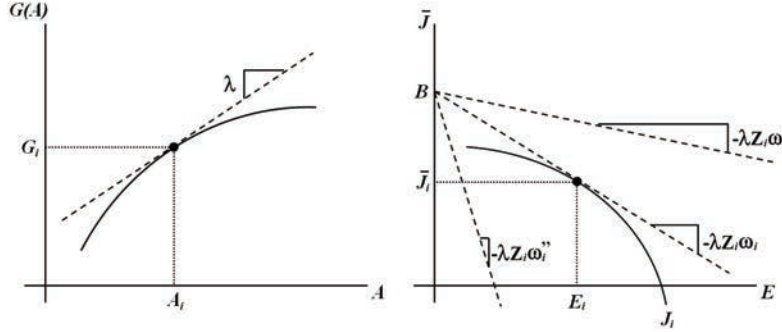


Figure 5: Solution to a representative firm's optimization subproblems.

## 6.4 Choice of Effort

Given the optimal choice of amenities, the firm's problem of minimizing the effective wage reduces to a second nested subproblem:

$$\begin{aligned} \min_{W_i, E_i} \quad & \omega_i = \mathcal{W}_i / (Z_i E_i) \\ \text{such that} \quad & \lambda \omega_i Z_i E_i + \underbrace{F(E_i, \psi_i^E) + S(\lambda, \Psi_i^A)}_{\bar{J}_i(E_i, \lambda, \Psi_i)} = B, \end{aligned} \quad (3)$$

where  $\bar{J}_i$  is the net job utility function (net of the costs of amenity provision measured in utils). Rearranging,

$$B - \lambda Z_i \omega_i E_i = \bar{J}_i(E_i, \lambda, \Psi_i) \quad (4)$$

and the objective is to find a feasible value of  $\bar{J}_i$  corresponding to the lowest  $\omega_i$ . In  $(E, \bar{J})$  space equation (4) traces out all effort and job-utility combinations that are consistent with any given effective wage: the firm's isocost lines are shown as downward sloping lines in the right panel of Figure 5. In that same panel the job utility function is shown as a concave curve, but none of the key qualitative results depend on whether it is concave or not. The firm's objective

is to find the tangency that yields an isocost line with the intercept at  $B$  and minimum negative slope that touches the job utility curve.

In other words, given  $B$ , the solution to the firm's optimization subproblem is implicitly captured by the isocost line that has the flattest (algebraically greatest) feasible slope. Feasibility is determined by the net job utility function, which captures all net job utility and effort combinations that a firm is able to offer. As seen in the right panel of Figure 5,  $\omega_i'' > \omega_i > \omega_i'$  and  $\omega_i$  is the firm's optimal effective wage: it can do better than  $\omega_i''$ , and although  $\omega_i'$  is preferred to  $\omega_i$ , the former is not feasible given the firm's net job utility function.

## 7 Equilibrium: Labor Earnings Demand and Labor Earnings Supply

Given the solution to the firm and worker problems, the economy's general equilibrium is summarized by the determination of equilibrium job benefits  $B$  and the marginal value of real wealth  $\lambda$ . These two variables can be pinned down by using two novel functions: labor earnings demand ( $LE^D$ ) and labor earnings supply ( $LE^S$ ). Both involve household behavior. Labor earnings supply also involves firm behavior.

### 7.1 Labor Earnings Demand

Intuitively, labor earnings demand can be interpreted as summarizing the worker's optimal choice of total labor earnings as a function of the marginal value of real wealth and for any given non-labor income. As such, labor earnings demand is decreasing in the marginal value of real wealth (rising wealth allows the worker to achieve any given level of utility with lower labor income).



Mathematically,

$$\mathbb{W}H = U'^{-1}(\lambda) - rM - \Pi \equiv LE^D, \quad (5)$$

where  $\mathbb{W} = \sum_i \chi_i W_i$  is the average wage. Here,  $\chi_i$  is the fraction of total work hours that the worker devotes to firm  $i$  ( $\sum_i \chi_i = 1$ ). (Note that our small open economy general equilibrium framework has  $r = \rho$ , so  $C$  is constant at  $C = rM + \Pi + H \sum_i \chi_i W_i$ .)

The case of two jobs gives all the important insights. Consider a worker working for two firms: 1 and 2. Suppose  $B_1 = B_2$  with  $W_1 > W_2$  but  $J_1 < J_2$ . At an interior optimum for a worker  $B_1 = \lambda W_1 + J_1 = \lambda_2 W_2 + J_2 = B_2$ . Therefore, the worker's budget constraint can be written as

$$\underbrace{C}_{= U'^{-1}(\lambda)} = \underbrace{\left( T - \Phi'^{-1}(B) \right)}_{= \text{optimal } H} \cdot (\chi W_1 + (1 - \chi) W_2) + rM + \Pi,$$

which after rearrangement yields this case's optimal  $\chi$ :

$$\chi = \frac{1}{W_1 - W_2} \left( \frac{U'^{-1}(\lambda) - rM - \Pi}{T - \Phi'^{-1}(B)} - W_2 \right).$$

## 7.2 Labor Earnings Supply

Turning to labor earnings supply, this function can be interpreted as summarizing the optimal choice of firms' total labor costs as a function of the marginal value of real wealth. These costs depend on exogenous labor productivity and job-enjoyment technology, and these costs also depend on the effective wage and firms' optimal choices of effort and amenities. Intuitively, as the economy becomes wealthier, the worker demands higher wages to be willing to work a given amount. Therefore, in order to achieve any given level of production firms must expend a higher amount of total labor costs. This makes labor earnings supply increasing in the marginal value of real wealth. Mathematically, since

$W_i = Z_i \omega E_i - A_i$  it follows that

$$\mathbb{W}H = \sum_i [Z_i \omega \cdot E_i(\omega \lambda Z_i, \Psi_i) - A_i(\lambda, \Psi_i^A)] \cdot H_i(B(\omega \lambda Z_i, \Psi_i)) \equiv LE^S. \quad (6)$$

(further details regarding the mathematical derivation of labor-earnings supply are in the Appendix). Note that in labor earnings supply our notation indicates that in equilibrium there is a single effective wage  $\omega$  across firms. The reason behind this is the following. Perfect competition in both product and capital rental markets equalizes across firms not only prices but also the effective wage  $\omega_i$  for all firms with positive output. Given the cost function in equation (1), this means that for firms with positive output

$$\underbrace{1}_{= \text{output price}} = (R^\alpha / (\alpha^\alpha (1 - \alpha)^{1-\alpha})) \omega_i^{1-\alpha} \Rightarrow \underbrace{\frac{W_i + A_i}{Z_i E_i}}_{=\omega_i} = \underbrace{\left( \frac{\alpha^\alpha (1 - \alpha)^{1-\alpha}}{R^\alpha} \right)^{1/(1-\alpha)}}_{=\omega \text{ (market value of } \omega)},$$

where  $\omega$  is the market value of the effective wage. It is straightforward to extend the intuition from Figure 5 to this case in which, as far as a representative firm is concerned, the slope of an isocost line  $-\lambda Z_i \omega$  is exogenously determined. Now, the intersection of the firm's isocost line with the horizontal axis determines equilibrium job benefits  $B$ . Clearly, then, we can write  $E_i = E_i(\omega \lambda Z_i, \lambda, \Psi_i)$  and  $B_i = B_i(\omega \lambda Z_i, \lambda, \Psi_i)$  as we did in the definition of our  $LE^S$  function (the appearance of  $\lambda$  is only about the amenities choice, which is as earlier).<sup>13</sup>

### 7.3 Equilibrium

Figure 6 shows equilibrium as pinned down by labor earnings demand and labor earnings supply. For simplicity, here we assume that all operating firms

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<sup>13</sup>Note that the firms that are able to offer the highest job benefits are the firms that implicitly set the economy's equilibrium level of job benefits.

are clones of each other, meaning that  $LE^S$  is given by

$$\mathbb{W}H = [Z_i\omega \cdot E_i(\omega\lambda Z_i, \Psi_i) - A_i(\lambda, \Psi_i^A)] \cdot H_i(B(\omega\lambda Z_i, \Psi_i)).$$

In the Appendix we address general equilibrium in the case in which firms are heterogeneous, which is a straightforward extension of this case in which we assume clone firms.

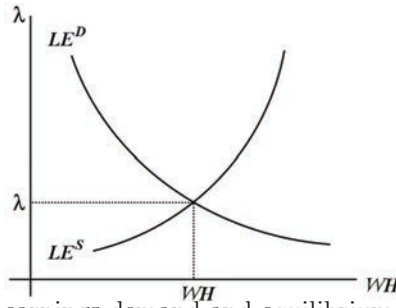


Figure 6: Equilibrium labor earnings demand and equilibrium labor earnings supply.

## 8 Insights From This Framework

### 8.1 Effort Being Unpleasant at the Margin is an Equilibrium Phenomenon

Despite the fact that job utility can be increasing in effort for some part of the range, the tangency condition shown in the right panel of Figure 5 implies that additional effort will be unpleasant at the optimum. Indeed, at the optimum:

$$\partial \bar{J}_i / \partial E_i = -\lambda Z_i \omega_i = \partial J_i / \partial E_i \Rightarrow \partial J_i / \partial E_i \cdot E_i = -\lambda (W_i + A_i).$$

Then,  $\lambda > 0$  and  $E_i > 0$  imply that for positive wages (and nonnegative amenities),  $\partial J_i / \partial E_i < 0$  at the optimal choice of effort. That is, the optimal choice of effort occurs where job utility is decreasing in effort. In other words,

since effort is productive it would make no sense to limit effort when additional effort is also pleasant. Effort should be increased until additional effort is painful enough that it counterbalances the extra productivity.

## 8.2 Improvements in Job Utility can Help Counteract the Income Effect From Economic Growth

Recall that  $B = \lambda W_i + J_i$ , and that work hours are increasing in  $B$  (as shown in the right panel of Figure 4). If the income effect dominates the substitution effect, then  $\lambda W_i$  is decreasing ( $W_i$  is growing, but  $\lambda$  is declining with increases in consumption at a faster rate than  $W_i$  increases). All else equal that makes  $B$ —and therefore work hours—decrease as well. But if job utility,  $J_i$ , is rising sufficiently, then the income effect seen in the fall in  $\lambda$  can be counterbalanced by the combination of the increase in  $W_i$  and the increase in  $J_i$ .

## 8.3 Even With a Strong Income Effect and Unchanging Job Utility, Hours Can Have a Positive Asymptote

There is another surprising implication. Even if  $\lambda W_i \rightarrow 0$  because the income effect overwhelms the substitution effect (that is,  $\lambda$  declines more quickly than  $W_i$  increases), work hours will tend to some constant  $\bar{H} > 0$  as long as job utility  $J_i$  tends to some positive constant  $\bar{J}_i > \Phi'(0) = 0$ . That is, even if people face quickly declining marginal utility for additional consumption, a positive asymptote for work hours can exist if there are jobs people enjoy as much as the marginal non-work activity they would otherwise fill out their days with.

## 8.4 Higher Compensating Differentials Raise the Effective Frisch Elasticity

At any given level of job benefits  $B$ , having a more pleasant, lower-paying job will result in a lower (Frisch) labor supply elasticity. To see this, rewrite  $B = \lambda W_i + J_i$  as  $\lambda(W_i(1 - \zeta_i))$ , where  $\zeta_i \equiv -J_i/\lambda W_i$  is the fraction of the wage that is a compensating differential for a job being unpleasant. Define the elasticity of work hours with respect to  $B$  by  $\bar{\eta} = \frac{\Phi'(T-H)}{H\Phi''(T-H)}$ . Then  $d \ln H = \bar{\eta} d \ln B$ . Holding everything constant except wages,  $d \ln B = d \ln W_i / (1 - \zeta_i)$ .<sup>14</sup> So, with  $J_i$  and  $\lambda$  held constant the Frisch elasticity of labor supply is for job  $i$  is

$$\eta_i = \frac{d \ln H}{d \ln W_i} = \frac{d \ln H}{d \ln B} \frac{d \ln B}{d \ln W_i} = \frac{\bar{\eta}}{(1 - \zeta_i)}. \quad (7)$$

Intuitively, if much of a wage is just making up for it being an unpleasant job, a 1% higher wage makes net job benefits more than 1% higher. On the other hand, if much of the attraction of a job is in its non-wage features, a 1% increase in the wage leads to less than a 1% increase in job benefits. Also, as the economy gets richer, if  $J$  increases while  $\bar{\eta}$  (as determined by the curvature of  $\Phi$ ) stays relatively constant, the volatility of work hours will fall relative to the volatility of temporary changes in the real wage.

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<sup>14</sup>Note that

$$\begin{aligned} B = \lambda W_i + J_i &\Rightarrow dB|_{dJ_i=0} = \lambda dW_i \Rightarrow dB/B = (W_i \lambda / B) dW_i / W_i \\ &\Rightarrow d \ln B = W_i \lambda (\lambda W_i + J_i)^{-1} d \ln W_i \Rightarrow d \ln B = [(\lambda W_i + J_i) / (W_i \lambda)]^{-1} d \ln W_i \\ &\Rightarrow d \ln B = (1 + \underbrace{J_i / (W_i \lambda)}_{=-\zeta_i})^{-1} d \ln W_i. \end{aligned}$$

## 8.5 Growth Leads to Growing Compensating Differentials

Define  $\gamma \equiv -CU_{CC}/U_C$ . (That is,  $1/\gamma$  is the elasticity of intertemporal substitution). Then  $d\lambda/\lambda = -\gamma dC/C$ . Consider an individual working two jobs satisfying  $J_2 > J_1$ , and recall equation (7), which was derived for multiple jobs. Then,  $\lambda W_1 + J_1 = \lambda W_2 + J_2$ , implying that

$$\lambda = \frac{J_2 - J_1}{W_2 - W_1} \Rightarrow \frac{d\lambda}{\lambda} = \frac{dJ_1 - dJ_2}{J_1 - J_2} - \frac{dW_1 - dW_2}{W_1 - W_2}.$$

For any individual with  $dJ_1 - dJ_2 = 0$ , as, for example, when  $dJ_1 = dJ_2 = 0$ ,

$$\lambda^{-1}d\lambda = -(dW_1 - dW_2) / (W_1 - W_2),$$

and using  $d\lambda/\lambda = -\gamma dC/C$  it follows that

$$\gamma = [(dW_1 - dW_2) / (W_1 - W_2)] / (dC/C). \quad (8)$$

That is, the elasticity with respect to consumption of an individual's compensating differential is equal to  $\gamma$ . If  $\gamma > 1$ , Crucially, this means that for jobs that stay the same, economic growth will make compensating differentials a bigger fraction  $\zeta$  of the relevant wages.

Thus, if income effects exceed substitution effects, we can look forward to a future in which compensating differentials become a more and more important aspect of labor markets—except to the extent that unpleasant jobs simply fade away. Conversely, if, in general, compensating differentials between pairs of relatively unchanging jobs are growing faster than the wages—and therefore faster than the consumption that can be supported by those jobs—it provides important evidence that  $\gamma > 1$ .

Indeed, equation (8) points to a way to calibrate  $\gamma$  from growth in compensating differentials to cross-check the usual method of calibrating  $\gamma$  (or its reciprocal the EIS) from the effects of interest rates on the path of consumption.

## 8.6 Additional Insights

Our framework allows us to address several additional questions. For instance: How does a firm’s overall technology matter for its competitiveness? What are the effects of changes in labor-augmenting technology and job-enjoyment technology on labor earnings and the marginal value of wealth? Which changes in technology are consistent with higher real wages and trendless labor hours if the income effect outweighs the substitution effect?

In the Appendix, we show the following. First, within our framework, differences in job-enjoyment technology between firms can counterbalance differences in labor-augmenting technology, and vice versa. In particular, a firm that falls behind in labor-augmenting technological progress can keep up its ability to attract workers even with lower wages if its job enjoyment technology advances sufficiently. Second, within our framework, each of (a) a permanent increase in labor augmenting technology, (b) a permanent positive innovation in the nature of work proper, or (c) a permanent positive innovation in the nature of the work environment can lead simultaneously to higher (i) labor earnings, (ii) a lower marginal value of real wealth, and (iii) trendless or nearly trendless work hours. In essence, then, anything that “regular” technology can do, job enjoyment technology can do as well.<sup>15</sup>

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<sup>15</sup>To the extent that higher job utility matters for competitiveness, it is even plausible that firms might set what would otherwise be slightly above-optimal effort requirements in order to induce workers themselves to think of ways to increase job utility. This amounts to a low cost form of research and development in job enjoyment technology.

## 9 Welfare Gains

We argue above that an upward trend in job utility makes it possible for work hours to remain approximately constant over time even if the income effect of higher real wages on labor supply exceeds the substitution effect of higher real wages. The question that immediately follows is: “What are the welfare effects of such changes?” To answer that question, we need to algebraically isolate the part of welfare improvement due to improvements in on-the-job utility and compute that value.

In this section, we elaborate on the relationship between job utility and welfare, point to ways that theoretical relationships can be operationalized and give a numerical example for the potential welfare gains associated with secular changes in job utility.

### 9.1 Measuring Welfare

Because transitions between steady states are instantaneous, changes in welfare induced by changes in exogenous parameters are well assessed via comparative steady-state analysis. In steady state, given  $r = \rho$ , an individual’s problem is equivalent to the static optimization problem

$$\begin{aligned} & \max_{C, H, H_i \geq 0} U(C) + \Phi(T - H) + \sum_i H_i J_i \\ & \text{such that } C = rM + \Pi + \sum_i W_i H_i, \text{ and } H = \sum_i H_i. \end{aligned}$$

Given the multipliers  $\lambda$  and  $b$ , let

$$\mathcal{L}^* = \max_{C, H, H_i \geq 0} \left\{ \begin{aligned} & U(C) + \Phi(T - H) + \sum_i H_i J_i + b(H - \sum_i H_i) \\ & + \lambda(rM + \Pi + \sum_i W_i H_i - C) \end{aligned} \right\}.$$



Recall that the optimal choice of  $H_i$  yields two cases:  $H_i = 0$  and  $J_i + \lambda W_i < b$ , or  $H_i > 0$  and  $J_i + \lambda W_i = b$ . Therefore,  $b = B$ , where,  $B$  denotes the economy's level of equilibrium marginal net job benefits.

## 9.2 Using the Envelope Theorem

The envelope theorem implies that

$$\lambda^{-1} d\mathcal{L}^* = \lambda^{-1} \sum_i H_i dJ_i + \sum_i H_i dW_i + d(\Pi + rM). \quad (9)$$

Each of the three terms on the right-hand side of equation (9) highlights distinct ways in which the economy's opportunity set becomes larger. Changes in welfare from changes in job utility are captured by the first term; changes in welfare from changes in wages are reflected in the second term; and changes in welfare from changes in exogenous wealth appear in the last term. The first term  $\sum_i H_i dJ_i / \lambda$  can be interpreted as the portion of the change in the maximized value of utility that answers the question: "How much would the worker have to be paid annually in order to be willing to go back to working in yesterday's conditions?"

## 9.3 Paying Attention to Compositional Shifts

To better understand the implications of the envelope theorem as laid out in equation (9), note that the second term on the right-hand side of equation (9) is the change in wages for narrowly defined job categories and satisfies

$$\sum_i H_i dW_i = d(\sum_i H_i W_i) - \sum_i W_i dH_i.$$

Therefore, to gauge this component of welfare, we need to adjust the change in overall labor earnings by subtracting not only extra earnings from people work-

ing longer hours overall, but also extra earnings coming from people switching towards jobs that are more highly paid and have lower job utility. If  $\lambda W$  is moving down, then the overall trend should involve compositional shifts toward jobs with higher job utility and relatively lower pay than other available jobs. This means that the increase in labor earnings will tend to understate the true increase in welfare (leaving aside changes in overall hours, which obviously need to be accounted for).

## 9.4 Identifying How Much More Pleasant Jobs Are Becoming

To understand the remaining terms for the change in welfare, note that  $B = J_i + \lambda W_i$  implies

$$\begin{aligned} dB &= dJ_i + \lambda dW_i + W_i d\lambda \\ \Rightarrow \lambda^{-1} dJ_i &= \lambda^{-1} dB - \lambda^{-1} W_i d\lambda - dW_i. \end{aligned} \tag{10}$$

Thus, improvements in job utility are indicated by a wage moving lower than what would be predicted on the basis of labor market conditions and trends in the marginal value of wealth. Pinning down  $dJ_i$  calls for looking at labor hours (to gauge  $dB$ ), consumption (to gauge  $d\lambda/\lambda$ ), and at hourly wages. Substituting this last equation into equation (9) and rearranging yields

$$\frac{d\mathcal{L}^*}{\lambda \sum_i H_i W_i} = \frac{H}{\sum_i H_i W_i} \frac{dB}{\lambda} - \frac{d\lambda}{\lambda} + \frac{d(\Pi + rM)}{\sum_i H_i W_i}. \tag{11}$$

The last term on the right-hand side—the value of extra non-labor income—is easy to understand. Hence, we will focus on getting measures for the first two terms on the right-hand side of equation (11).

## 9.5 Accounting for Changes in Overall Work Hours

Recall that  $\gamma \equiv -CU_{CC}/U_C$ ,  $\eta_i = \bar{\eta}/(1 - \zeta_i)$ , where  $\eta_i$  is the Frisch elasticity of labor supply for job  $i$ ,  $\zeta_i$  is the fraction of the wage that is a compensating differential, and  $\bar{\eta} = \Phi'(T - H) / [H\phi''(T - H)]$ . Also, the definition of  $\zeta_i$  as the fraction of the wage that is a compensating differential can be rewritten as  $\frac{B}{\lambda W} = 1 - \zeta_i$ . It follows that

$$dB/B = (1/\bar{\eta})(dH/H) \Rightarrow dB = \frac{(1 - \zeta_i)\lambda W_i}{\bar{\eta}}(dH/H) \Rightarrow dB/\lambda = (W_i/\eta_i)(dH/H).$$

Substituting the appropriate expressions into equation (11) and simplifying yields

$$\frac{d\mathcal{L}^*}{\lambda \sum_i H_i W_i} = \frac{(W_i/\eta_i) dH}{\sum_i H_i W_i} + \frac{\gamma dC}{C} + \frac{d(\Pi + rM)}{\sum_i H_i W_i}. \quad (12)$$

The intuition for equation (12) is that (in the additively separable case)  $\gamma$  tells how many times bigger the income effect is than the substitution effect. If hours are relatively constant despite increasing wages, then there must be substantial increases in job utility to counteract the income effect associated with increases in consumption. On the other hand, if hours  $H$  decline as urged by the income effect, it gives less hint of improvements in job utility. (If  $\gamma = 1$ , income and substitution effects cancel, but increases in consumption still have the usual effect on welfare.)

## 9.6 Illustrating the Calculation of Welfare Gains: Putting It All Together

It is helpful to work through a numerical example. (The derivations below are also helpful in working through other numerical examples.) Suppose that the long-run elasticity of intertemporal substitution is 0.5, in which case  $\gamma = 2$ .

Using this value for  $\gamma$  along with equation (12) implies that for  $d\Pi = 0$ ,  $dM = 0$ , and  $dH = 0$ ,  $\frac{d\mathcal{L}^*}{\lambda \sum_i H_i W_i} = 2 \frac{dC}{C}$  and, therefore, a 1% increase in consumption would be associated with a welfare increase of 2%.

A natural question that follows is what fraction of welfare gains are attributable to higher job utility. To see this, continue to assume that  $d\Pi = dM = 0$ , but allow for the possibility that  $dH \neq 0$ . Dividing equation (9) by  $\sum_i H_i W_i$  and using equation (12) yields the following expression:

$$\frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} + \frac{\sum_i H_i dW_i}{\sum_i H_i W_i} = \frac{(W_i/\eta_i) dH}{\sum_i H_i W_i} + \frac{\gamma dC}{C}.$$

Using the identity

$$\sum_i d(H_i W_i) = \sum_i W_i dH_i + \sum_i H_i dW_i \quad (13)$$

to substitute out the second term in the left-hand side of the preceding equation yields, after rearranging,

$$\frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} - \frac{\sum_i W_i dH_i}{\sum_i H_i W_i} = \left[ \gamma \frac{dC}{C} - \frac{\sum_i d(H_i W_i)}{\sum_i H_i W_i} \right] + \frac{(W_i/\eta_i) dH}{\sum_i H_i W_i}. \quad (14)$$

The left hand side of this equation captures changes in job utility: the first term indicates changes in job utility from particular jobs becoming more pleasant, while the second term indicates the additions to job utility coming from switching between jobs (since willingness to take lower wages indicates higher job utility in the new jobs), both measured in relation to total labor income. For convenience, the right-hand side of this equation can be thought of as having two terms: the term in square brackets and the term with  $dH$ . Suppose that consumption and labor income are increasing at the same rate. Then, the term in square brackets is  $\gamma - 1$  times this rate of change. It follows that this term in brackets is the change in the marginal rate of substitution between

labor and consumption that one would expect if  $\gamma \neq 1$  and there were no improvement in job utility (be it from jobs becoming more pleasant or from switching jobs). Indeed, in this case equation (14) becomes

$$\frac{(W_i/\eta_i) dH}{\sum_i H_i W_i} = - \underbrace{\left[ \gamma \frac{dC}{C} - \frac{\sum_i d(H_i W_i)}{\sum_i H_i W_i} \right]}_{=(\gamma-1)dC/C}.$$

That said, in the case in which labor hours are approximately trendless, meaning that  $dH = 0$ , then the term in square brackets indicates how great a change in job utility is needed to explain the behavior of labor hours. In this case equation (14) becomes

$$\frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} - \frac{\sum_i W_i dH_i}{\sum_i H_i W_i} = \underbrace{\left[ \gamma \frac{dC}{C} - \frac{\sum_i d(H_i W_i)}{\sum_i H_i W_i} \right]}_{=(\gamma-1)dC/C} \quad (15)$$

In words: given approximately trendless labor hours,  $\gamma - 1$  times the rate of increase of consumption and labor income must be entirely accounted for by improvements in job utility. (The welfare effects of this change in job utility are measured in terms of consumption that would yield the same amount of extra utility.)

Coming full circle with our numerical example, we began this section of the paper noting that with  $\gamma = 2$  along with  $d\Pi = 0$ ,  $dM = 0$ , and  $dH = 0$ , then  $\frac{d\mathcal{L}^*}{\lambda \sum_i H_i W_i} = 2 \frac{dC}{C}$  and welfare gains were 2 percent. Continuing to assume that the rate of increase of consumption and labor income is the same, inspection of equation (15) implies that in this case improvements in job utility account for half of the increase in total welfare (1 percentage point of the 2%).

Here, we emphasize trends in how pleasant jobs are as an explanation for why labor hours might be trendless even if income effects are stronger than

substitution effects. Bringing other explanations into the picture would change the calculations. But what is likely to remain true in extensions of the model designed to allow for other possible explanations is the idea that it is difficult for a trend in the pleasantness of jobs to have a big effect on the trend in labor hours without also having a big effect on the trend in welfare.

## 10 Conclusions

The paradox of hard work is this: for decades, work hours per population among adults have remained roughly trendless, despite strong trends in consumption and real wages. In principle, the paradox of hard work can be rationalized in several different ways. Of these alternatives, we focus on the general equilibrium effects of secular changes that make work more pleasant. Economists have long understood that cross-sectional differences in job utility at a particular time give rise to compensating differentials. In this paper, we develop a theory that focuses on the less-studied long-run macroeconomic consequences of trends in job utility.

Two key implications emerge. First, secular improvements in job utility imply that work hours can remain approximately constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect of higher wages. Second, secular improvements in job utility can themselves be a substantial component of the welfare gains from technological progress. These two implications are connected by an equation flowing from optimal hours choices: improvements in job utility that have a significant effect on labor supply tend to have large welfare effects.

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