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Abstract

We develop a simple (incumbent versus entrant) strategic deterrence model to study the economic and geopolitical interactions underlying international trade-related infrastructure projects such as the Panama Canal. We study the incentives for global geopolitical players to support allied satellite countries where these projects are or could potentially be built. We show that even if no effective competitor emerges, the appearance of a geopolitical challenger capable of credibly supporting the entrant has a pro-competition economic effect which benefits consumers all over the world.

JEL classification codes: L1, F5.

Keywords: market entry, deterrence, international trade infrastructure, geopolitics.

1 Introduction

There is a growing and fascinating body of empirical literature on the effects of large-scale infrastructure projects (e.g., the expansion of the Panama Canal) on the volume and pattern of international and regional trade (Maurer and Yu, 2008; Hugot et al., 2016; Feyrer, 2021). These studies use disruptions in the operation of trade-related infrastructures or new infrastructure projects as invaluable exogenous shocks that affect trade costs across locations and products. This is definitely a well founded empirical approach to estimate the causal effect of trade costs on the volume and pattern of international and regional trade. In this paper, we adopt a completely different but complementary approach. Our goal is to explore the strategic economic and political forces that underlie some of these infrastructure projects. Strategic considerations are relevant for at least two reasons. First, the construction of large-scale trade-related infrastructure, such as ports and canals, tends to be undertaken on a non-competitive basis, as such projects are often carried out under monopolistic or oligopolistic conditions or are conducted by government-owned firms. Thus, the scope for strategic economic decisions is simply larger than it is, say, for standard shipping and transportation services. Second, since major infrastructure projects have the potential to redirect trade flows and foreign direct investment and, in the event of open conflict, to influence military operations, they are often considered to be of key importance for geopolitical reasons.

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As the Panama Canal provides such a strategic link between the Atlantic and Pacific Oceans, it is an excellent example of a trade-related infrastructure project that is subject to substantial economic and geopolitical strategic considerations. Ever since its construction, the Panama Canal has been an almost uncontested monopoly. Initially, it was owned by the United States and, although in 1999 it was transferred to the Republic of Panama, it is still considered to be within the orbit of influence of the United States (Sabonge and Sánchez, 2014). During the twentieth century, several projects to build alternative routes between the Atlantic and Pacific Oceans were envisioned, but it was not until the economic and geopolitical rise of China that a more serious challenge emerged. That challenge took the form of China's inclusion of a proposal for an alternative transoceanic canal running through Nicaragua as part of its Belt and Road Initiative. However, the project has since been postponed and the initial construction works have been suspended. We argue that the expansion of the Panama Canal played an important role in China's decision to suspend the project, but we also contend that the threat of a Chinese-financed rival canal through Nicaragua was a factor in Panama's decision to expand its canal and in the United States' decision to support that plan.

To formally capture the strategic interactions illustrated by the Panama Canal, we develop a formal game-theoretic model of strategic entry deterrence that includes a geopolitical component. In this model, there is one incumbent (e.g., Panama) and a potential entrant (e.g., Nicaragua) that play an entry game and two global powers (e.g., United States and China) that try to influence the outcome of this entry game for economic and geopolitical reasons. To do so, each global power subsidizes its geopolitical ally. When the global power allied with the incumbent wins the subsidy race, in equilibrium, there is deterrence (e.g., no canal is built in Nicaragua). This does not imply that geopolitics does not matter. Under deterrence, the incumbent, supported by its global ally, overinvests in capacity to deter the entrant that has received a credible promise of support from the other global power. In equilibrium, this credible promise is not acted upon, but it plays an important role in prompting the incumbent and its global ally to further expand capacity/ support the expansion of that capacity. Thus, even when no effective competitor emerges, the rise of a geopolitical challenger has a pro-competition economic effect which benefits consumers all over the world.

When the global power that is allied with the entrant wins the subsidy race, in equilibrium, there is accommodated entry. In other words, there is entry (e.g., a canal is built in Nicaragua with Chinese support) when the global ally of the entrant is willing to provide significant support and the global ally of the incumbent (e.g., the United States) is not willing to provide the substantial funds required to deter entry. In this case, the rise of a geopolitical challenger has economic as well as geopolitical effects. From an economic perspective, the market structure changes from a monopoly to a duopoly, which in turn leads to a reduction in the equilibrium price. Once again, consumers of all regions benefit from this change. From a geopolitical perspective, in equilibrium, there is effective entry by a new global power, which breaks up the geopolitical monopoly of the incumbent's global ally.

In the baseline model, we implicitly assume that the subsidies promised by both global powers are contingent but binding decisions. In an extension, we explore what happens when the global ally of the entrant cannot fully commit to support the entrant. That is, we consider the possibility that the rising global power (e.g., China) has a limited ability to convince the incumbent and its global ally (e.g., the United States and Panama) that the funds needed to support the entrant's efforts will actually be forthcoming. Limited credibility gives rise to three novel results. First, when the rising global power enjoys relatively high levels of commitment, entry is less likely to occur and deterrence becomes easier to sustain, as the established global power cannot be bullied with non-credible promises of large subsidies. Second, when the rising global power has intermediate levels of commitment, it does not present a geopolitical threat for the established global power because the incumbent is willing to deter entry even with no support. Then, at the margin, both global powers are better off if the rising global power gains some credibility, which prompts the incumbent to expand its capacity in order to deter entry. Finally, when the rising global power has a low level of commitment, in equilibrium, entry is blocked and, once again, both global powers will be better off if the rising global power gains enough credibility to induce deterrence.

1.1 Related Literature

There are two areas of the literature that are specifically related to this paper. First, there is an extensive body of literature on industrial organization and strategic entry deterrence. Second, in the area of international relationships, there is also an extensive body of literature on geopolitics and, in particular, on the interactions between an established global or regional power and a rising challenger.

The literature on industrial organization as it relates to strategic entry deterrence has highlighted two main mechanisms that an incumbent can use to deter entry. A first group of models focuses on pricing decisions, which can be used to build up the reputation of an incumbent (Kreps et al., 1982) or to signal the existence of a low cost to the potential entrant (Milgrom and Roberts, 1982). A second group of models considers that an incumbent can use strategic investments to deter entry. Spence (1977) formalizes the idea that investments in capacity are a credible commitment capable of deterring entry, while Dixit (1979) expands this model to allow the incumbent to choose between deterring and accommodating entry. Dixit (1980) goes on to explore different post-entry scenarios, including those involving a quantity leadership role for the entrant and price competition. Our model builds on Tirole (1988), who drew on the results of Kreps and Scheinkman (1983) and Fudenberg and Tirole (1984) to study a two-stage entry game where firms select their capacities in the first stage and then compete on prices in the second stage. We augment this model by introducing two new players (the global powers) with the ability and willingness to influence the incumbent and entrant, respectively.

The seminal models of entry deterrence have been extended in several directions. For example, Maggi (1996) introduced uncertainty regarding conditions in the contested market, while Bagwell and Ramey (1996) explored the role of avoidable costs, and Eaton and Ware (1987) looked at how the market structure might vary with technology. Additionally, several theoretical implications of these models have been tested in a variety of markets. For example, Thomas (1999) focused on cereals, Lieberman (1987) on chemical industries, Conlin and Kadiyali (2006) on lodging properties, and Ellison and Ellison (2011) on pharmaceuticals. However, to the best of our knowledge, models of strategic deterrence have not been employed to study trade infrastructure and/or extended to study how geopolitical considerations affect the equilibrium. At a pure theoretical level, our model also suggests that once we introduce a player with the ability and willingness to expand the equilibrium quantity (e.g., the rising global power in our model), blocked entry will never be an equilibrium of the deterrence model. The reason being that such a player can always induce deterrence without actually incurring any cost. The only remaining question is whether this player is interested in escalating its support to induce entry.

There is a vast body of literature within the field of international relations on the interactions between an established power and a rising challenger (e.g., Nye, 1991; Ikenberry, 2011). Our paper emphasizes the dilemma between economic gains and geopolitical threats. Overall, a rising economic power opens up excellent new economic opportunities for the established power via specialization, international trade and foreign direct investment. The cost for the established power is the sharing of political influence with the rising power. We make three contributions to this literature. First, we formally model one possible way in which an established power and a rising challenger can interact and explore under what conditions and why a dilemma between economic gains and geopolitical threats emerges. Second, our model also allows us to explore what the consequences are for the countries being influenced by the global powers as well as third countries. Finally, we identify a mechanism through which geopolitical competition and considerations shape strategic international trade infrastructure.

The rest of the paper is organized as follows. Section 2 develops a simple model of strategic economic deterrence taking the subsidies provided by the global powers as a given. Section 3 introduces the geopolitical dimension by looking at the equilibrium interactions between the two global powers. Section 4 explores an extension of the model in which the rising global power does not enjoy full credibility. Section 5 applies the model to the case of the Panama Canal. Section 6 presents the conclusions.

2 A Simple Model of Economic Deterrence

Consider two countries that, due to by virtue of their locations, could provide an strategic transportation service such as a connection between the Atlantic and Pacific Oceans (e.g., Panama and Nicaragua). The demand for this service comes from three countries and/or regions that we interpret as two global powers (e.g., the United States and China) and the rest of the world, respectively. To simplify things, suppose that the strategic transportation service is an homogenous product for which the demand in country j is a linear function of the price:

$$Q^j = A^j (a - P) \ \ for\ j \in J = \{G_1, G_2, RW\},$$

where $P \geq 0$ is the price of the service, $a > 0$, and $A^j > 0$ for all j. Therefore, the inverse demand of the service is $P = a - bQ$, where $b = \left(\sum_{j\in J} A^j\right)^{-1}$ and $Q = \sum_{j\in J} Q^j$.

The countries that are strategically located to provide this service are not symmetric. One country, denoted by I, is the market incumbent (e.g., Panama) and the other country, denoted by E, is a potential entrant (e.g., Nicaragua). I and E play a deterrence game. Specifically, countries first make a capacity decision (e.g., build or expand the canal) and later compete on prices. Let $k_i \geq 0$ denote the capacity decision of country $i \in \{I, E\}$. The cost of building capacity k_I for the incumbent is given by:

$$C_I(k_I) = ck_I - S_1(k_E),$$

where $c \in (0, a)$ and $S_1(k_E) = S_1 \geq 0$ if $k_E = 0$ and $S_1(k_E) = 0$ if $k_E > 0$. That is, S_1 is the subsidy provided by global power 1 if E does not enter. The cost of building capacity k_E for the potential entrant is given by:

$$C_E(k_E) = ck_E + F - S_2(k_E),$$

where $S_2(k_E) = S_2$ with $S_2 \geq 0$ if $k_E > 0$ and $S_2(k_E) = 0$ if $k_E = 0$. That is, $F > 0$ is the entry cost and S_2 is the subsidy provided by global power 2.

The timing of events is as follows: (i) I selects $k_I \geq 0$; (ii) E observes k_I and selects $k_E \geq 0$; (iii) Given (k_I, k_E), there is price competition. Specifically, I and E simultaneously and independently select prices (p_I, p_E) and the demand of each player is determined according to the efficient-rationing rule.[1]

To solve this game, we employ subgame Nash perfect equilibrium. Moreover, we impose restrictions on capacity choices and the set of parameters which ensure that, in equilibrium, under a duopoly, both countries set the same price and use all their installed capacity. (See Appendix A.1 for details.) That is, the equilibrium market price as a function of (k_I, k_E) is $P = a - b(k_I + k_E)$ and, hence, profit functions are given by:

$$\Pi_I(k_I, k_E) = [a - b(k_I + k_E) - c]\, k_I + S_1(k_E)$$
$$\Pi_E(k_I, k_E) = [a - b(k_I + k_E) - c]\, k_E - S_2(k_E)$$

To characterize the equilibrium it is useful to define the following thresholds[2,3]:

$$\bar{S}^b = F - \frac{(a-c)^2}{16b} \tag{1}$$
$$\bar{S}^d(S_2) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F - S_2}{b}} + 4(F - S_2) \tag{2}$$
$$\bar{S}^d_0 \in \left(\bar{S}^b, \bar{S}\right) \text{ such that } \bar{S}^d\left(\bar{S}^d_0\right) = 0 \tag{3}$$
$$\bar{S} = F - \frac{(2a-3c)^2}{36b} \tag{4}$$

The following proposition characterizes the economic equilibrium for any pair of subsidies (S_1, S_2).

Proposition 1 ***Economic equilibrium***. *Suppose that* $9c/7 \leq a \leq \left(6\sqrt{2}+3\right)c/7$.

1. *Suppose that* $0 \leq S_2 \leq \bar{S}^b$. *Then, the entry of E is* ***blocked***. *Specifically, in equilibrium,* $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ *and* $P = \frac{a+c}{2}$.

2. *Suppose that* $\bar{S}^b < S_2 \leq \bar{S}$.[4]

   (a) If $S_1 > \bar{S}^d(S_2)$, *then the entry of E is* ***deterred***. *Specifically, in equilibrium,* $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ *and* $P = c + 2\sqrt{b(F - S_2)}$.

[1]The efficient-rationing rule indicates that consumers with the highest willingness to pay will be served first. This rule has the advantage of maximizing the consumer surplus. For more details see Tirole (1988).

[2]In Appendix A.1 we prove that, under proper conditions, $\bar{S}^d(S_2)$ is strictly increasing and strictly convex in S_2 for all $S_2 \in \left[\bar{S}^b, \bar{S}\right]$. Moreover, there exists a unique $\bar{S}^d_0 \in \left(\bar{S}^b, \bar{S}\right)$ such that $\bar{S}^d(S_2) < 0$ for all $S_2 \in \left[\bar{S}^b, \bar{S}^d_0\right)$, $\bar{S}^d\left(\bar{S}^d_0\right) = 0$, and $\bar{S}^d(S_2) > 0$ for all $S_2 \in \left(\bar{S}^d_0, \bar{S}\right]$.

[3]$\bar{S}^b$ is the minimum subsidy that G_2 must offer to E before E considers entering when I behaves as an unchallenged monopoly. $\bar{S}^d(S_2)$ is the minimum subsidy that G_1 must offer to I in order to deter entry when G_2 is offering a subsidy of S_2 to E. $\bar{S}^d_0$ is the minimum subsidy that G_2 must offer to E in order for E to consider entering when I is willing to expand its capacity, but it does not receive any support from G_1. Finally, $\bar{S}$ is the maximum feasible subsidy for G_2.

[4]In Appendix A.1 we further characterize the equilibrium for $\bar{S}^b < S_2 \leq \bar{S}$. In particular, we prove that there exists $\bar{S}^d_0 \in \left(\bar{S}^b, \bar{S}\right)$ such that for all $\bar{S}^b < S_2 \leq \bar{S}^d_0$, I deters the entry of E.

(b) *If* $S_1 = \bar{S}^d(S_2)$*, then there are two equilibria: in one equilibrium the entry of* E *is* ***deterred****, while in the other* I ***accommodates*** *the entry of* E*. Under deterrence (accommodation),* (k_I, k_E, P) *is as in part a (c).*

(c) *If* $S_1 < \bar{S}^d(S_2)$*, then* I ***accommodates*** *the entry of* E*. Specifically, in equilibrium,* $(k_I, k_E) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$ *and* $P = \frac{a+3c}{4}$*.*

Proof*: See Appendix A.1.* ■

Figure 1 illustrates Proposition 1. When the subsidy provided to E by global power G_2 is below a certain threshold (formally, $0 \leq S_2 \leq \bar{S}^b$), then E will not enter even if I keeps capacity at the monopoly level. Under such circumstances, I does not need to invest in extra capacity to deter E. Then, the equilibrium outcome coincides with the standard equilibrium under a monopoly. For the case of the Panama Canal, this can be interpreted as a situation in which China is not seriously committed to subsidizing Nicaragua and, lacking China's backing, Nicaragua finds it too costly to build a new canal even when Panama does not expand its capacity.

When the subsidy provided to E by global power G_2 is above a certain threshold (formally, $S_2 > \bar{S}^b$) and if I keeps capacity at the monopoly level, then E will have incentives to enter. Under such circumstances, I's only choice is between accommodating and overinvesting in capacity to deter the entry of E. Indeed, when the subsidy provided by global power G_1 is generous enough (formally, $S_1 > \bar{S}^d(S_2)$), it is profitable for I to install extra capacity to deter E's entry. The market then becomes a monopoly. For the case of the Panama Canal, this can be interpreted as a situation in which the United States helps Panama to build extra capacity in order to deter Nicaragua from building a new canal with the support of China. It is worth mentioning that, although the market becomes a monopoly under both deterred and blocked entry, equilibrium quantities and prices are not the same. The reason for this is that when $S_2 > \bar{S}_2^b$, I must overinvest in capacity to deter E. When the subsidy provided by G_1 is not generous enough (formally, $S_1 < \bar{S}^d(S_2)$), I prefers to accommodate entry and the equilibrium outcome coincides with the equilibrium of the Stackelberg's model. For the case of the Panama Canal, this can be interpreted as a situation in which the United States does not provide enough support to Panama to deter Nicaragua from building a new canal with the support of China. Finally, $S_1 = \bar{S}^d(S_2)$ is a knife edge situation in which the subsidies are such that I is indifferent to the choice between deterrence and accommodation. This knife edge situation will prove to be important in Section 3, where we endogenize the subsidies provided by the global powers.

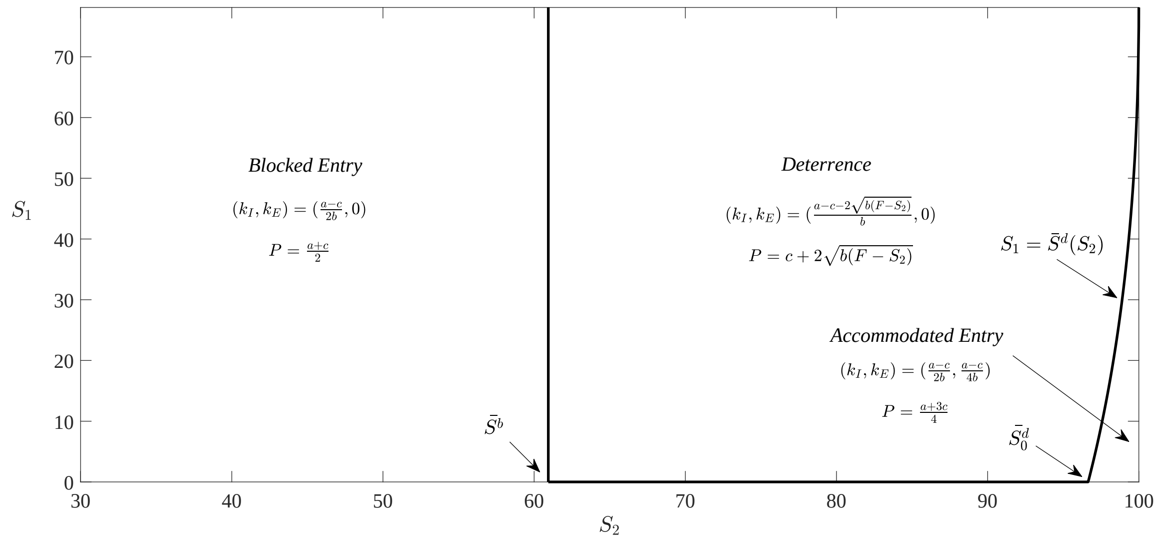

Figure 1. Economic equilibrium given (S_1, S_2). Note: The figure has been plotted assuming $a = 3.75$, $b = 1/400$, $c = 2.5$, and $F = 100$.

3 Geopolitics and Political Deterrence

This section introduces geopolitical conflict between the global powers that use the strategic transportation service. In particular, we introduce a payoff function for each global power that considers economic as well as geopolitical factors and we characterize the equilibrium subsidies and corresponding capacity choices.

Suppose that before I and E play the economic deterrence game, global powers play an international influence game in which they simultaneously and independently select (S_1, S_2). Let the payoff function of global power G_j be given by:

$$W_j(k_I, k_E) = CS_j(k_I, k_E) + B_j(k_I, k_E),$$

where CS_j is the consumer surplus enjoyed by country G_j, and B_j is the net geopolitical net benefits for G_j (i.e., geopolitical benefits minus subsidies). In particular, assume that:

$$B_1(k_I, k_E) = \begin{cases} B_1^M - S_1 & \textit{if } k_E = 0 \\ B_1^D & \textit{if } k_E > 0 \end{cases} \quad \textit{and } B_2(k_I, k_E) = \begin{cases} 0 & \textit{if } k_E = 0 \\ B_2^D - S_2 & \textit{if } k_E > 0 \end{cases}$$

where $B_1^M > B_1^D \geq 0$, $B_2^D > 0$. B_1^M is the geopolitical benefits enjoyed by G_1 when there is no entry, i.e., under a monopoly, while B_1^D is the geopolitical benefits enjoyed by G_1 when there is entry, i.e., under a duopoly. Thus, $B_1^M - B_1^D > 0$ is the geopolitical benefits for G_1 of avoiding entry. Similarly, $B_2^D > 0$ is the geopolitical benefits enjoyed by G_2 when there is entry, while, under no entry, G_2 has no geopolitical benefits.[5]

The payoff functions of the global powers encompass in a stylized fashion the perspectives on the goal of states supported by the two most influential schools of thought in International Relations: liberalism and realism. While liberals often emphasize the importance of economic gains through international cooperation, realists focus on security dilemmas (e.g., Shiraev and Zubok 2020). Since we assume that each global power values economic as well as geopolitical payoffs, our specification can handle both schools. Indeed, as geopolitical benefits rise (decrease), our payoff functions become more realists (liberal).

To characterize equilibrium subsidies, it is useful to employ a selection criterion to deal with multiple economic equilibria for the knife edge situation. Recall that when $S_1 = \bar{S}^d(S_2)$, deterrence and accommodation are both subgame perfect Nash equilibria (see Proposition 2.2.b). A convenient criterion is to assume that if $S_1 = \bar{S}^d(S_2)$, then the economic equilibrium will be accommodation when accommodation strictly dominates deterrence for G_2. Otherwise, the economic equilibrium will be deterrence. One advantage of this criterion is that G_2 always has a best response for any S_1.

The following proposition characterizes the equilibrium subsidies chosen by the global powers. To do so, we define:

$$\Delta(S) = \frac{9(a-c)^2}{32} - \frac{\left[a - c - 2\sqrt{b(F-S)}\right]^2}{2} \tag{5}$$

where $A^j\Delta(S)$ is the change in the consumer surplus experienced by consumers of country j when the economic equilibrium moves from deterrence to accommodation.

Proposition 2 ***Geopolitical equilibrium***.[6] *Suppose that* $9c/7 \le a \le \left(6\sqrt{2}+3\right)c/7$, $A^1 b < 2\left(\sqrt{2}-1\right)$, *and* $B_2^D \in \left(\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right), \bar{S} - A^2\Delta\left(\bar{S}\right)\right]$. *Then, the equilibrium subsidy profiles are those that satisfy:*

$$S_1 = \bar{S}^d(S_2)\,,\; S_2 \in \left[\bar{S}_0^d, \bar{S}\right] \tag{6}$$

and

$$[B_2^D > S_2 - A^2\Delta(S_2) \;\; \textit{and} \;\; B_1^M - B_1^D \le A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1] \tag{7}$$

or

$$[B_2^D \le S_2 - A^2\Delta(S_2) \;\; \textit{and} \;\; B_1^M - B_1^D \ge A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1] \tag{8}$$

Moreover, if (7) holds there is accommodated entry, while if (8) holds, entry is deterred. ***Proof:*** *See Appendix A.2.* ■

To see the intuition behind Proposition 2, we must understand the logic behind equations (6), (7) and (8). In equilibrium, it is always the case that $S_1 = \bar{S}^d(S_2)$ and $S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$. This is because

[5] One possible way to motivate the geopolitical payoff function is to assume that geopolitical benefits are determined by a contest as follows: $B_1(k_E, k_I) = \frac{(k_I)^m}{(k_I)^m + (k_E)^m}B - S_1(k_E)$ and $B_2(k_E, k_I) = \frac{(k_E)^m}{(k_I)^m + (k_E)^m}B - S_2(k_E)$ with $m \in (0,1]$. Then, $B_1^M = B$, $B_1^D = \frac{(2)^m}{(2)^m+1}B$, and $B_2^D = \frac{1}{(2)^m+1}B$.

[6] In Appendix A.2 we prove a more general version of this proposition that fully characterizes equilibrium subsidies for any set of parameters.

$S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ is the best response function of G_2. Figure 2 illustrates why this is the case. Panel a shows the payoff of G_2 as a function of S_2 for any $S_2 \in \left[0, \bar{S}\right]$, while Panel b zooms in to the key range $S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$. The intuition is as follows. Given S_1, from Proposition 1, we know that if G_2 offers $S_2 \in \left[0, \left(\bar{S}^d\right)^{-1}(S_1)\right)$, then entry will be deterred, while if G_2 offers $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), \bar{S}\right]$, then there will be accommodation. Offering $S_2 \in \left[0, \left(\bar{S}^d\right)^{-1}(S_1)\right)$ is not a best response to S_1. With this offer E will not enter and, hence, G_2 will not need to pay any subsidy. However, the higher the subsidy offered by G_2, the greater the amount of capacity that I will need to install to deter E and, hence, the lower the equilibrium price. Formally, $W_2(S_1, S_2)$ is strictly increasing in S_2 for all $S_2 \in \left[0, \left(\bar{S}^d\right)^{-1}(S_1)\right)$. Offering $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), \bar{S}\right]$ is not a best response to S_1 either. With this offer, E will enter and, hence, G_2 will need to pay the subsidy. However, the equilibrium price under accommodation does not depend on S_2. Formally, $W_2(S_1, S_2)$ is strictly decreasing in S_2 for all $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), \bar{S}\right]$. Thus, the only remaining possibility is $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$. But are we sure that $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ is the best response function of G_2? In particular, note that $W_2(S_1, S_2)$ is not continuous at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ (see Figure 2.b). Our selection criterion, however, implies that $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to the economic equilibrium that maximizes $W_2(S_1, S_2)$, which ensures that $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ is indeed the best response function of G_2. (For further details, refer to Lemma 2 in Appendix A.2).

Does $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ lead to deterrence or accommodation? There are two possible situations to consider. Suppose that G_1 offers a relatively low subsidy (formally, S_1 such that $B_2^D > \left(\bar{S}^d\right)^{-1}(S_1) - A^2\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$). Then, $W_2(S_1, S_2)$ adopts its maximum at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ when there is accommodation (see Figure 2.b). On the other hand, suppose that G_1 offers a relatively high subsidy (formally, S_1 such that $B_2^D \leq \left(\bar{S}^d\right)^{-1}(S_1) - A^2\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$). Then, $W_2(S_1, S_2)$ adopts its maximum at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ when there is deterrence (see Figure 2.b). Intuitively, when G_1 offers a relatively low (high) subsidy, it is (not) worth it for G_2 to pay $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ in order to enjoy the economic as well as geopolitical gains associated with E's entry. Summing up, in order for accommodation (deterrence) to be an equilibrium it must be the case that $B_2^D > S_2 - A^2\Delta(S_2)$ ($B_2^D \leq S_2 - A^2\Delta(S_2)$).

What about G_1's incentives? Considering the best response function of G_2, there are two types of candidates for equilibrium subsidy profiles. For any profile in which $S_1 = \bar{S}^d(S_2)$, $S_2 \in \left[\bar{S}_0^d, \bar{S}\right)$ and $B_2^D > S_2 - A^2\Delta(S_2)$ leads to accommodation. For those profiles, $B_1^M - B_1^D \leq A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ ensures that G_1 does not have an incentive to unilaterally deviate to $S_1 < \bar{S}^d(S_2)$, which would lead to deterrence. For any profile in which $S_1 = \bar{S}^d(S_2)$, $S_2 \in \left[\bar{S}_0^d, \bar{S}\right)$ and $B_2^D \leq S_2 - A^2\Delta(S_2)$ leads to deterrence. For those profiles, $B_1^M - B_1^D \geq A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ ensures that G_1 does not have an incentive to unilaterally deviate to $S_1 > \bar{S}^d(S_2)$, which would lead to accommodation. The intuition behind these inequalities is as follows. $B_1^M - B_1^D > 0$ is the geopolitical gain for G_1 associated with maintaining its geopolitical monopoly. To enjoy those benefits, G_1 must incur two costs: a reduction in the consumer surplus of $A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)$ (the equilibrium price under accommodation is lower than under deterrence) and the payment of a subsidy of S_1 to the incumbent.

Several remarks regarding Proposition 2 are called for here. First, consider the equilibria that induce deterrence. In those equilibria, G_2 does not actually pay any subsidy to E. G_2 just offers a subsidy, which triggers a response from G_1 and I, which move to overinvest in capacity to deter E's entry. Of course, this raises the question as to how credible is G_2's offer to subsidize the entrance of E actually is. The model implicitly assumes that S_2 is fully credible, but it is not difficult to envision situations in which G_2 must at least incur some cost in order to signal its commitment. Similarly, in the equilibria that induce accommodated entry, G_1 does not actually pay any subsidy to I, but the subsidy promised by G_1 is not completely irrelevant either. Indeed, the higher S_1, the more generous S_2 needs to be in order to induce E's entrance.

Second, Proposition 2 suggests a simple but coherent explanation for the expansion of the Panama Canal. (See Section 5 for further details.) China threatened to support Nicaragua's effort to build a new canal, and Panama reacted by expanding its canal to deter entry. Does the United States need to subsidize the expansion of the Panama Canal in order for this to be an equilibrium? According to Proposition 2, not necessarily. Depending on the parameters of the model, $(S_1, S_2) = \left(0, \bar{S}_0^d\right)$ could be a Nash equilibrium that leads to deterrence.

Finally, G_2's geopolitical challenge (i.e., its willingness and commitment to support E's entry) has a pro-competition economic effect (i.e., lower equilibrium price), which benefits consumers all over the world (including consumers who are not associated with any global power). This is an example of good economic outcomes resulting from political competition.

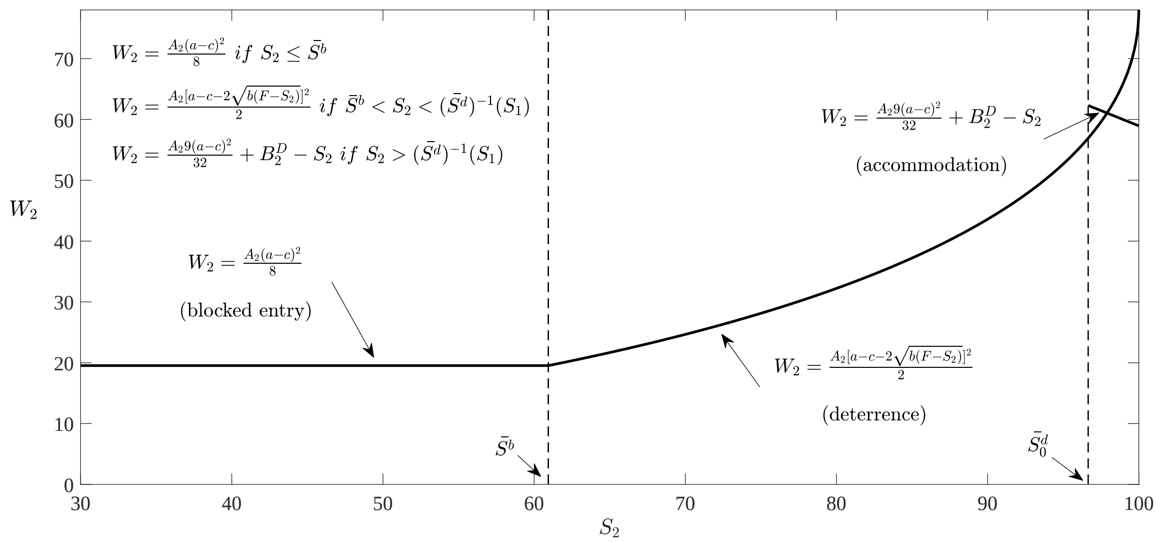

Figure 2.a. Geopolitical equilibrium. Notes: The figure has been plotted assuming $a = 3.75$, $b = 1/400$, $c = 2.5$, $F = 100$, $A^2 = 100$, and $B_2^D = 115$.

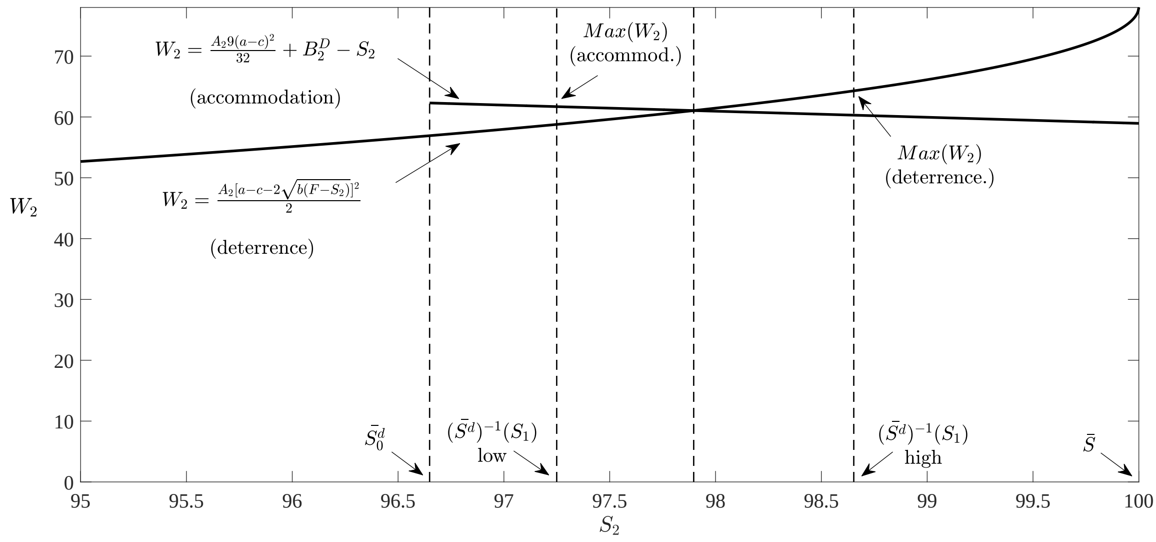

Figure 2.b. Geopolitical equilibrium. Notes: The figure has been plotted assuming $a = 3.75$, $b = 1/400$, $c = 2.5$, $F = 100$, $A^2 = 100$, and $B_2^D = 115$.

3.1 Comparative Statics Analysis

Proposition 2 states necessary and sufficient conditions for a profile of equilibrium subsidies to induce accommodation and deterrence. Next, we further characterize these conditions and explore how geopolitical factors affect the equilibrium. The following proposition summarizes the results.

Proposition 3 ***Comparative statics.*** *Suppose that* $9c/7 \leq a \leq \left(6\sqrt{2}+3\right)c/7$*,* $A^1 b < 2\left(\sqrt{2}-1\right)$*,* $B_2^D \in \left(\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right), \bar{S} - A^2\Delta\left(\bar{S}\right)\right]$ *and* $B_1^M - B_1^D \in \left[A^1\Delta\left(\bar{S}_0^d\right), A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)\right]$*. Let* $\tilde{S}_1 \in \left(0, \bar{S}^d\left(\bar{S}\right)\right]$ *and* $\tilde{S}_2 \in \left(\bar{S}_0^d, \bar{S}\right]$ *be the unique solution to:*

$$B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1 \tag{9}$$

$$B_2^D = -A^2\Delta\left(\tilde{S}_2\right) + \tilde{S}_2 \tag{10}$$

1. *If* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$*, then the equilibrium subsidy profiles are those that satisfy:* $S_1 = \bar{S}^d\left(S_2\right)$ *and* $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$*. Moreover, in all these equilibria entry is deterred.*

2. *If* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$*, then the equilibrium subsidy profiles are those that satisfy:* $S_1 = \bar{S}^d\left(S_2\right)$ *and* $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$*. Moreover, in all these equilibria there is accommodated entry.*

3. $\tilde{S}_1$ *(*$\tilde{S}_2$*) is strictly increasing in* $B_1^M - B_1^D$ *(*B_2^D*);* $\tilde{S}_1$ *and* $\tilde{S}_2$ *are both strictly increasing in* F*; and the effect of* c *(*a*) on* $\tilde{S}_1$ *and* $\tilde{S}_2$ *is ambiguous.*

Proof: *See Appendix A.2.* ■

The intuition behind Proposition 3 is as follows. $\tilde{S}_1$ is the maximum subsidy that G_1 is willing to pay in order to deter entry. Indeed, (9) simply equates the geopolitical benefits derived from deterrence (i.e., $B_1^M - B_1^D$) with its economic costs (i.e., $A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$). Analogously, $\tilde{S}_2$ is the maximum subsidy that G_2 is willing to pay in order to induce entry, while (10) equates the geopolitical and economic benefits of entry (i.e., $B_2^D + A^2\Delta\left(\tilde{S}_2\right)$) with its cost (i.e., $\tilde{S}_2$). There are two possible situations. When $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, G_1 is willing to offer a subsidy higher than or equal to $\bar{S}^d\left(\tilde{S}_2\right)$ in order to deter entry, while G_2 is not willing to pay more than $\tilde{S}_2$ to induce entry. Then, G_1 outbids G_2 in the subsidy race and, in equilibrium, entry is always deterred. On the other hand, when $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, G_1 is not willing to offer more than $\bar{S}^d\left(\tilde{S}_2\right)$ in order to deter entry, while G_2 is willing to pay up to $\tilde{S}_2$ to induce entry. Then, G_2 outbids G_1 in the subsidy race and, in equilibrium, there is accommodated entry.

How do geopolitical benefits affect equilibrium subsidies and, ultimately, the entry decision? An increase in $B_1^M - B_1^D$ makes G_1 more willing to pay a higher subsidy in order to deter entry. Formally, $\tilde{S}_1$ is strictly increasing in $B_1^M - B_1^D$ (Proposition 3.3). If it was initially the case that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, then a rise in $B_1^M - B_1^D$ does not affect the nature of the equilibrium, i.e., before as well as after the increase in $B_1^M - B_1^D$ entry is deterred. However, the rise in $B_1^M - B_1^D$, increases the maximum equilibrium subsidy offered by G_2, which reduces the lowest possible equilibrium price. Thus, the rise in $B_1^M - B_1^D$ opens the way for improving the situation for consumers all over the world.[7] On the other hand, if it was initially the case that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, then a marginal rise in $B_1^M - B_1^D$ does not affect the nature of the equilibrium. Before, as well as after, the increase in $B_1^M - B_1^D$, there is accommodated entry. Moreover, since, under accommodated entry, neither capacity choices nor the equilibrium price depend on the subsidies, a marginal rise in $B_1^M - B_1^D$ has no effect on the well-being of consumers. Starting from $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, a sufficiently large rise in $B_1^M - B_1^D$ reverses this inequality and, hence, the equilibrium changes from accommodated entry to deterrence. Since the equilibrium price under deterrence is always higher than under accommodated entry, this large rise in $B_1^M - B_1^D$ makes consumers all over the world worse off. Summing up, a rise in the geopolitical benefits of G_1 has an ambiguous effect on the well-being of consumers.

An increase in B_2^D makes G_2 more willing to pay a higher subsidy in order to induce entry. Formally, $\tilde{S}_2$ is strictly increasing in B_2^D (Proposition 3.3). If it was initially the case that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, a rise in B_2^D does not affect the nature of the equilibrium. Before, as well as after, the increase in B_2^D, there is accommodated entry. Moreover, since, under accommodated entry, neither capacity choices nor the equilibrium price depend on the subsidies, a rise in B_2^D has no effect on the well-being of consumers. On the other hand, if it was initially the case that $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$, then a marginal a rise in B_2^D does not affect the nature of the equilibrium. Before, as well as after, the increase in B_2^D, there is deterrence. However,

[7] Since there are multiple equilibrium subsidy profiles, we cannot state that consumers will be better off after the increase in $B_1^M - B_1^D$. More formally, every equilibrium subsidy profile before the rise in $B_1^M - B_1^D$ will also be an equilibrium subsidy profile after the rise in $B_1^M - B_1^D$. In addition, after the rise in $B_1^M - B_1^D$, there will be a new range of equilibrium subsidy profiles with higher S_2 than in the equilibrium subsidy profiles before the rise in $B_1^M - B_1^D$.

this marginal rise in B_2^D increases the minimum equilibrium subsidy offered by G_2, which reduces the highest possible equilibrium price. Thus, the rise in B_2^D opens way for improving the situation for consumers.[8] Starting from $\tilde{S}_1 > \bar{S}^d\left(\tilde{S}_2\right)$, a sufficiently large rise in B_2^D makes $\tilde{S}_2$ greater than or equal to $\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_2\right)$ and, hence, the equilibrium changes from deterrence to accommodated entry. Since the equilibrium price under accommodated entry is always lower than under deterrence, this change unambiguously makes consumers better off. Summing up, a rise in the geopolitical benefits of G_2 has a benign pro-competition economic effect that tends to improve the well-being of consumers all over the world.

What are the effects of a change in entry costs F? A rise in F makes both global powers more willing to pay a higher subsidy. Formally, $\tilde{S}_1$ and $\tilde{S}_2$ are both strictly increasing in F (Proposition 3.3). The intuition behind this result is as follows. Consider the economic and geopolitical calculus of G_1. The geopolitical benefits derived from deterrence (i.e., $B_1^M - B_1^D$) are not affected by a change in F, while its economic costs (i.e., $A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$) decrease with a rise in F. This might seem counterintuitive given that $A^1\Delta$ is decreasing in F (as the consumer surplus obtained by G_1 under entry is not affected by F while the consumer surplus obtained by G_1 under deterrence decreases with F). However, F also influences $\left(\bar{S}^d\right)^{-1}$. Indeed, an increase in F leads to a decrease in $\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and this 'indirect' change dominates the direct effect of F on Δ. Thus, a higher F leads to a higher $\tilde{S}_1$. For G_2, neither the geopolitical benefits of entry (i.e., B_2^D) nor its cost (i.e., $\tilde{S}_2$) are affected by F, while the economic benefits of entry (i.e., $A^2\Delta\left(\tilde{S}_2\right)$) increase with F as the consumer surplus obtained by G_2 under entry is not affected by F while the consumer surplus obtained by G_2 under deterrence decreases with F. Thus, a higher F leads to a higher $\tilde{S}_2$.

What about the nature of the equilibrium? It is easy to verify that if it was initially the case that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, then this inequality will also hold after a rise in F. Thus, if before the rise in F there was deterrence, there will also be deterrence after the rise in F. Moreover, it is also possible to prove that a change in F does not affect $F - \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$ and, hence, has no impact on well-being of consumers. (See Appendix A.2 for details). In other words, the rise in F is fully neutralized by G_1 with no effect on consumers or geopolitical outcomes. If, on the contrary, it was initially the case that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, we must distinguish two possible situations. First, if after the rise in F it is still the case that $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, then there is accommodated entry before as well as after the change in F. Since under accommodated entry, neither capacity choices nor the equilibrium price depend on the subsidies or the entry cost, a rise in F has no effect on the well-being of consumers. Second, if after the rise in F we have $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, then the equilibrium changes from accommodated entry to deterrence, making consumer worse off.

[8]Since there are multiple equilibrium subsidy profiles, we cannot state that consumers will be better off after the increase in B_2^D. More formally, the rise in B_2^D eliminates a range of equilibrium subsidy profiles with the lowest S_2 and, hence, the highest equilibrium prices.

4 Extension: Non-binding Subsidies

One concern with the set-up in Section 3 is that subsidies are considered to be fully credible promises. However, in equilibrium, one of the global powers does not actually pay any subsidy. For example, global power G_2 can push the incumbent to increase its capacity just by threatening to subsidize the entrant, without actually paying any subsidy. The reason for this is that we have assumed that the threat is credible because S_2 is a contingent but binding decision. That is, in the event that E decides to enter, G_2 must fulfill its commitment to pay S_2, but if there is no entry, then no subsidy is paid. Similarly, S_1 is a contingent but binding decision made by global power G_1. If there is no entry, G_1 must still fulfill its commitment to pay S_2, but if E enters, then no subsidy is paid.

To illustrate the importance of credibility for the global powers, we consider a situation in which G_2 has only limited credibility.[9] In particular, we assume that the actual subsidy paid by G_2 cannot be larger than a fraction $\rho \in (0,1)$ of the maximum possible subsidy $\bar{S}$. Formally, any promise above $\rho\bar{S}$ will not be credible.

It is not difficult to see how G_2's limited commitment affects Proposition 1. (See Appendix A.3 for details). When $\rho \in \left[\bar{\rho}_0^d, 1\right)$, where $\bar{\rho}_0^d = \bar{S}_0^d/\bar{S}$, it is possible for entry to be blocked, deterred or accommodated depending on S_1 and S_2, as it is the case in Proposition 1. Thus, for $\rho \in \left[\bar{\rho}_0^d, 1\right)$, limited commitment has no major impact on Proposition 1. When $\rho \in \left(\bar{\rho}^b, \bar{\rho}_0^d\right)$, where $\bar{\rho}^b = \bar{S}^b/\bar{S}$, G_2 can only credibly commit to pay an amount lower than $\bar{S}_0^d$, which could be enough to induce I to increase its capacity to deter entry, but it will never be enough to induce accommodation. In other words, for intermediate values of ρ, entry will be either blocked or deterred. Finally, when $\rho \in (0, \bar{\rho}]$, entry is always blocked for all values of S_2. Intuitively, with a low enough ρ, G_2 can only credibly commit to pay an amount lower than $\bar{S}^b$, which is never enough to induce the incumbent to deter entry or to induce an accommodated entry.

The following proposition characterizes the Nash equilibrium subsidies chosen by the global powers for different values of ρ.

Proposition 4 *Suppose that $9c/7 \leq a \leq \left(6\sqrt{2}+3\right)c/7$, and that the maximum credible subsidy that G_2 can promise is $\rho\bar{S}$. Let*

$$\bar{\rho}^b = \frac{\bar{S}^b}{\bar{S}} \text{ and } \bar{\rho}_0^d = \frac{\bar{S}_0^d}{\bar{S}}$$

1. *Suppose that $\bar{\rho}_0^d \leq \rho < 1$, $B_2^D \in \left(\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right), \bar{S} - A^2\Delta\left(\bar{S}\right)\right]$ and $B_1^M - B_1^D \in \left[A^1\Delta\left(\bar{S}_0^d\right), A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)\right]$.*

   (a) *If $\tilde{S}_1 \geq \bar{S}^d\left(\rho\bar{S}\right)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d\left(\rho\bar{S}\right)$ and $S_2 \in \left[\rho\bar{S}, \bar{S}\right]$. Moreover, in all these equilibria entry is deterred.*

   (b) *If $\tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right)$ and $\tilde{S}_2 \leq \rho\bar{S}$, then Proposition 3 holds.*

   (c) *If $\tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right)$ and $\tilde{S}_2 > \rho\bar{S}$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d\left(S_2\right)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho\bar{S}\right)$. Moreover, in all these equilibria there is accommodated entry.*

[9] To some extent, it is arbitrary to restrict the credibility of G_2, but not the credibility of G_1. One possible justification is that G_2 represents a rising global power that is still building up its reputation in the international arena.

2. *Suppose that* $\bar{\rho}^b < \rho < \bar{\rho}_0^d$. *Then, the set of equilibrium subsidies is given by* $S_1 = 0$ *and* $S_2 \in \left[\rho\bar{S}, \bar{S}\right]$. *Moreover, in equilibrium, entry is deterred.*

3. *Suppose that* $0 < \rho \leq \bar{\rho}^b$. *Then, the set of equilibrium subsidies is given by* $S_1 = 0$ *and* $S_2 \in \left[0, \bar{S}\right]$. *Moreover, in equilibrium, entry is blocked.*

Proof: *See Appendix A.3.* ■

Proposition 4.1 is similar to Proposition 3. In equilibrium, entry is deterred when global power G_1 wins the subsidy race, and there is accommodated entry when global power G_2 wins the subsidy race. The difference is that while in Proposition 3 the winner is the global power that is willing to go farther in the subsidy race, now G_2 faces a credibility problem that restricts how much it can credibly promise to offer to E. As a consequence, if G_1 is willing to offer $S_1 \geq \bar{S}^d\left(\rho\bar{S}\right)$ (formally, if $\tilde{S}_1 \geq \bar{S}^d\left(\rho\bar{S}\right)$), then there is nothing that G_2 can do to induce entry. In equilibrium, entry is deterred, even when G_2 would be willing to outbid G_1, (formally, when $\tilde{S}_2 > \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)$). The problem is that G_2 cannot credibly promise to offer its willingness to pay to induce entry. When G_1 is not willing to offer $S_1 \geq \bar{S}^d\left(\rho\bar{S}\right)$ (formally, when $\tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right)$), then there are two possible situations. If G_2's credibility constraint is not binding (formally, if $\tilde{S}_2 \leq \rho\bar{S}$), then Proposition 3 holds. All that matters is the global players' willingness to pay to deter or to induce entry. If G_2's credibility constraint is binding (formally, if $\tilde{S}_2 > \rho\bar{S}$), then it must be the case that G_2 is willing to and capable of outbidding G_1. Then, in equilibrium, there is accommodated entry. The only difference with Proposition 3 is that now $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho\bar{S}\right)$ instead of $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$.

Table 1 summarizes the differences between Proposition 3 and Proposition 4.1. First, suppose that $\bar{S}^d\left(\rho\bar{S}\right) \leq \tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$. Then, when the maximum credible subsidy that G_2 can offer is $\bar{S}$, there is accommodated entry, while when the maximum credible subsidy that G_2 can offer is $\rho\bar{S}$, there is deterrence. In other words, G_2's limited commitment changes the nature of the equilibrium outcome (from accommodation to deterrence). This induces a rise in the equilibrium price, which negatively affects consumers. Second, suppose that $\bar{S}^d\left(\rho\bar{S}\right) \leq \bar{S}^d\left(\tilde{S}_2\right) \leq \tilde{S}_1$. Then, G_2's limited commitment does not change the nature of the equilibrium outcome (i.e., with or without limited commitment there is deterrence). However, under limited commitment, the equilibrium price is higher than under full commitment because G_2 has to bid a subsidy lower than its willingness to pay to induce entry. Once again, limited commitment negatively affects consumers. Finally, suppose that $\tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right) < \bar{S}^d\left(\tilde{S}_2\right)$. Then, limited commitment does not change the nature of the equilibrium outcome (i.e., with or without limited commitment, there is accommodated entry). However, under limited commitment, there is a lower maximum subsidy that G_2 pays to support entry. Since, under accommodation, subsidies do not change the equilibrium price, consumers are not affected.

	Maximum credible S_2		
Situation	$\bar{S}$ (Proposition 2) $S_1 = \bar{S}^d(S_2)$ and $S_2 \in$	$\rho\bar{S}$ (Proposition 4) $S_1 = \bar{S}^d(\min\{S_2, \rho\bar{S}\})$ and $S_2 \in$	Main effects of limited commitment
$\bar{S}^d(\rho\bar{S}) \leq \tilde{S}_1 < \bar{S}^d(\tilde{S}_2)$	$\left[(\bar{S}^d)^{-1}(\tilde{S}_1), \tilde{S}_2\right)$ (accommodation)	$[\rho\bar{S}, \bar{S}]$ (deterrence)	- From accommodation to deterrence - Higher price
$\bar{S}^d(\rho\bar{S}) \leq \bar{S}^d(\tilde{S}_2) \leq \tilde{S}_1$	$\left[\tilde{S}_2, (\bar{S}^d)^{-1}(\tilde{S}_1)\right)$ (deterrence)	$[\rho\bar{S}, \bar{S}]$ (deterrence)	- Lower S_1 - Higher price
$\bar{S}^d(\tilde{S}_2) \leq \tilde{S}_1 < \bar{S}^d(\rho\bar{S})$	$\left[\tilde{S}_2, (\bar{S}^d)^{-1}(\tilde{S}_1)\right)$ (accommodation)	$\left[\tilde{S}_2, (\bar{S}^d)^{-1}(\tilde{S}_1)\right)$ (accommodation)	- No effect
$\tilde{S}_1 < \bar{S}^d(\tilde{S}_2) \leq \bar{S}^d(\rho\bar{S})$	$\left[(\bar{S}^d)^{-1}(\tilde{S}_1), \tilde{S}_2\right)$ (deterrence)	$\left[(\bar{S}^d)^{-1}(\tilde{S}_1), \tilde{S}_2\right)$ (deterrence)	- No effect
$\tilde{S}_1 < \bar{S}^d(\rho\bar{S}) < \bar{S}^d(\tilde{S}_2)$	$\left[(\bar{S}^d)^{-1}(\tilde{S}_1), \tilde{S}_2\right)$ (accommodation)	$\left[(\bar{S}^d)^{-1}(\tilde{S}_1), \rho\bar{S}\right)$ (accommodation)	- Lower S_2

Table 1: Binding versus non-binding subsidies from G_2 when $\bar{\rho}_0^d \leq \rho < 1$.

Proposition 4.2 brings about new results. For $\bar{\rho}^b < \rho < \bar{\rho}_0^d$, G_2 can only credibly commit to pay an amount lower than $\bar{S}_0^d$, which implies that E will not enter, even when $S_1 = 0$. This does not imply that there is no room for strategic subsidies, however. In particular, to induce the incumbent to expand its capacity, G_2 has an incentive to offer the highest possible subsidy to E (i.e., $S_2 \in [\rho\bar{S}, \bar{S}]$). On the other hand, G_1 does not need to offer any subsidy to induce deterrence. Thus, in equilibrium, $S_1 = 0$, $S_2 \in [\rho\bar{S}, \bar{S}]$ and entry is deterred. In the context of the Panama Canal, this would be an scenario where China, by promising to support Nicaragua, forces deterrence by Panama without the need for any subsidy from the United States. Compared with Proposition 4.1, now limited commitment has a more radical impact on the equilibrium outcome. For $\bar{\rho}^b < \rho < \bar{\rho}_0^d$, G_2 does not pose any geopolitical threat for G_1. This is because there is no promise that G_2 can make that will induce E to enter. Moreover, in economic terms, G_1 benefits from G_2's support to E because it forces I to increase its capacity, which reduces the equilibrium price of the transportation service. Indeed, it is easy to verify that when $\bar{\rho}^b < \rho < \bar{\rho}_0^d$, the payoffs for both global powers are increasing in ρ. Thus, this is a situation in which the United States would prefer that China gains credibility up to $\rho < \bar{\rho}_0^d$.

Proposition 4.3 also brings about novel results. For $0 < \rho \leq \bar{\rho}^b$, regardless of the subsidy offered by G_2, entry will be blocked. Then, G_1 does not have any incentives to offer a positive subsidy. G_2, on the other hand, is indifferent to any promised subsidy because, given its low level of credibility level, G_2's promises will not affect capacity decisions. In the context of the Panama Canal, this would be an scenario in which China lacks credibility and entry remains blocked. Once again, this is not a good outcome for the global powers. Both would be better off if China were to gain credibility and I were forced to increase its capacity in order to deter entry.

5 The Case of the Panama Canal

The Panama Canal's monopoly on passage between the Atlantic and Pacific Oceans has periodically been threatened by the possibility of a project to build a new canal through Nicaragua. In the last decade, this threat became more credible because such a project was part of China's Belt and Road worldwide infrastructure initiative aimed at developing logistical infrastructure to facilitate Chinese engagement in foreign markets and military actions (Cai, 2017).

Panama' existence as a state and an economy that are backed by American interests in transoceanic travel has been defined by the Panama Canal ever since its construction. Sigler (2014) shows just how much the Panama Canal has shaped Panama's national economy and its internal politics and goes on to show how disruptive a rival, such as a canal in Nicaragua, could be for that country.

The geopolitical implications of the possibility of constructing canals to span Central America are closely linked to the inception of the state of Panama itself. The Panamanian isthmus was part of the sovereign territory of Colombia and became a place of interest to the French government in the late nineteenth century when France started dredging a trans-American canal through the swamps and jungles of that territory to create a sea lane to connect the Atlantic and Pacific Oceans. The French eventually failed when malaria and yellow fever decimated their workers. This opened up an opportunity for the United States, under President Theodore Roosevelt, to take over the project. As Panama was part of Colombia at the time, the negotiations concerning the building of the canal took place between the United States and Colombia. Those talks led to the signing of the Hay-Herrán Treaty, which, however, ended up being rejected by the Colombian Senate. This set the stage for the separation of Panama from Colombia and resulted in the Hay-Bunau-Varilla Treaty, which was signed by the French plenipotentiary ambassador of Panama to Washington. The United States then bought the French interest in Panama for US$40 million (Sabonge and Sánchez, 2014). When the United States purchased the rights to the canal project, the population of the isthmus rebelled against Colombia and declared independence in 1903. Colombia tried to retake the isthmus, but the new state of Panama was shielded by a fleet of US Navy ships (Sánchez, 2019).

The Hay-Bunau-Varilla Treaty gave the United States the rights, in perpetuity, to a strip of land (the Canal Zone) where the laws of the United States would apply. The arrangement for operating the canal did not allow for Panama to share in the revenue or other financial benefits derived from it. All that Panama received was a modest lease payment (Sabonge and Sánchez, 2014). All this changed, however, with the signing of the Torrijos-Carter Treaty in 1977, which provided for the Canal Zone to be abolished and for the Panama Canal to be handed over to the Republic of Panama at the end of 1999 (Sabonge and Sánchez, 2014).

Since the construction of the Panama Canal in 1914, the value of that route has changed over time. In the beginning, the Canal was primarily of strategic value from a military standpoint. In the years following the Second World War, it gained increasing economic and commercial value. And since its handover to the Republic of Panama, it has become a significant generator of wealth for Panama, whose monopoly position has essentially been uncontested until fairly recently, when a robust push for a canal through Nicaragua began to emerge.

The Nicaragua Interoceanic Grand Canal Master Plan was aimed at creating a faster route through the Americas while also industrializing the adjacent corridor. As it would be located to the north of the Panama Canal, the Nicaraguan canal would provide a faster route for ships bound for the Northern Hemisphere and would be able to accommodate ships that are too large to fit through the Panama Canal. The project was to be organized by the Hong Kong Nicaragua Canal Development Investment Company (HKND). In 2013, a 100-year concession contract for the management of the Nicaraguan Canal Authority was signed between HKND and the Government of Nicaragua. The first stages of the canal's construction began the following year. It has often been speculated that the HKND receives funds directly from the Chinese government (Sabonge and Sánchez, 2014). Arturo Cruz, the former Ambassador of Nicaragua to the United States, has said that "if the canal goes ahead… it will be because the Chinese government wants it to, and the financing will come from China's various state firms" (Sánchez, 2019).

However, although the Nicaraguan canal project nominally still forms part of China's Belt and Road Initiative, China has distanced itself from the project, and construction has been suspended. At the same time, Panama has effectively doubled the capacity of the Panama Canal by adding a new lane of traffic so that a larger number of ships can transit the canal at the same time and increasing the width and depth of the lanes and locks in order to accommodate larger container ships. The new ships, called New Panamax, are about one and a half times the previous Panamax size and can carry over twice as much cargo. The expansion was approved by a national referendum in 2006, but because of the 2008 financial crisis, construction did not actually begin until later, and the expanded facilities were finally completed in 2016.

In 2006, the Panama Canal Authority (PCA) estimated the cost of the third set of locks at US$5.25 billion. The PCA also estimated that the investment could be recouped thanks to the increased revenues that the project would yield. Opponents of the project contend that these estimates are based on uncertain projections of maritime trade and world economic trends. Indeed, Former President Jorge Illueca, former Assistant Administrator of the Panama Canal Commission Fernando Manfredo, shipping consultant Julio Manduley, and industrial entrepreneur George Richa M. have said that the expansion was not necessary and claimed that the construction of a mega-port on the Pacific side would be sufficient to meet probable future demand. At the moment, the projections presented to support the financial viability of the project appear to be grounds for optimism; the delay in the construction works has also substantially altered the initial financial estimates. External finance for the project was provided by several international financial institutions in which the United States Government has a great deal of influence, such as the Inter-American Development Bank (IDB) and the International Finance Corporation (IFC), as well as by the Japan Bank of International Cooperation (JBIC) and the European Investment Bank (EIB).

Although it is often argued that China has stepped back from the Nicaragua canal project in response to Panama's decision to cut diplomatic ties with Taiwan and to recognize the People's Republic of China as the only sovereign Chinese republic (Cheng and Lohman, 2017), Propositions 2-4 offer a more plausible explanation. Indeed, these propositions suggest several mechanisms that explain the observed behavior of the parties involved. First, most likely the Panama Canal has very high geopolitical value for the United States and much more limited geopolitical value for China. For example, some works in International Relations indicate that powerful countries put special interest in keeping other powerful countries out of their areas of influence (e.g., Mearsheimer 2003). In terms of our model, this translates into $B_1^M - B_1^D$

relatively higher than B_2^D, which makes deterrence more likely. Second, the entry cost for Nicaragua-China was probably very high. Some initial estimates for the Nicaragua Canal were US$ 50 billion (almost 10 times the cost of the Panama Canal expansion). As we discussed after Proposition 3, a rise in F makes deterrence more likely. This, however, does not imply that China should have not considered doing the project. As Proposition 4 shows, even when China knew that, in equilibrium, entry will be deterred, it was rational to include the Nicaragua Canal in the Belt and Road Initiative, start serious conversations with the Nicaraguan government about the project, and sign a contract for the concession of the Nicaraguan Canal Authority to HKND. We interpret these decisions as strategic moves to establish the credibility of China's intentions. Ultimately, China did not finance the Nicaragua Canal, but creating a credible threat was probably useful to influence the expansion of the Panama Canal, a non-negligible improvement as China is the second most important user of the canal.[10]

6 Concluding Remarks

We have developed a simple model of strategic deterrence between an incumbent country in which strategic trade-related infrastructure is located and a potential entrant. An established global power allied with the incumbent and a rising global power aligned with the entrant strategically influence the game by making funding available in order to advance their economic and geopolitical interests. Our main finding is that, even if the entrant is deterred, a geopolitical challenger that credibly commits to supporting the entrant has a pro-competition economic effect on the market for this type of strategic transportation service. This effect makes consumers of the transportation service in all regions better off, reduces the profits of the incumbent, and has no effect on the entrant. The established global power might be forced to pay out more generous subsidies in order to support the incumbent's deterrence effort, but it will not suffer a geopolitical loss. The rising global power will enjoy a larger consumer surplus at no cost, but it will not secure any geopolitical advantage.

The model used in this paper and the resulting findings are just the tip of the iceberg for a more ambitious research agenda focusing on the international political economy of strategic trade-related infrastructure, in particular, and geopolitics and international trade, more generally. That research should address questions such as the following: When does rivalry between global powers lead to market restrictions that distort international trade flows (e.g., colonial powers and mercantilist policies) and when does it generate pro-competition economic effects by breaking up monopoly positions or forcing agents to engage in more competitive behaviors?

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Online Appendix to "Geopolitics and International Trade Infrastructure"

This appendix presents the proofs of all lemmas and propositions.

A.1 Proof of Proposition 1

Proposition 1 ***Economic equilibrium***. *Suppose that* $9c/7 \leq a \leq \left(6\sqrt{2}+3\right)c/7$.

1. *Suppose that* $0 \leq S_2 \leq \bar{S}^b$. *Then the entry of* E *is* **blocked**. *Specifically, in equilibrium* $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ *and* $P = \frac{a+c}{2}$.
2. *Suppose that* $\bar{S}^b < S_2 \leq \bar{S}$.
   (a) *If* $S_1 > \bar{S}^d(S_2)$, *then the entry of* E *is* ***deterred***. *Specifically, in equilibrium,* $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ *and* $P = c + 2\sqrt{b(F-S_2)}$.
   (b) *If* $S_1 = \bar{S}^d(S_2)$, *then there are two equilibria: in one equilibrium the entry of* E *is deterred, while in the other* I *accommodates the entry of* E. *Under deterrence (accommodation),* (k_I, k_E, P) *is as in part a (c).*
   (c) *If* $S_1 < \bar{S}^d(S_2)$, *then* I ***accommodates*** *the entry of* E. *Specifically, in equilibrium,* $(k_I, k_E) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$ *and* $P = \frac{a+3c}{4}$.

Proof. We proceed through backward induction.

Efficient-rationing rule and price competition: According to the efficient-rationing rule, demands are given by:

$$Q_I(p_E, p_I) = \begin{cases} \min\left\{\max\left\{\frac{a-p_I}{b} - k_E, 0\right\}, k_I\right\} & \text{if } p_I > p_E \\ \min\left\{\max\left\{\frac{a-p}{2b}, \frac{a-p}{b} - k_E\right\}, k_I\right\} & \text{if } p_E = p_I = p \\ \min\left\{\frac{a-p_I}{b}, k_I\right\} & \text{if } p_I < p_E \end{cases}$$

$$Q_E(p_E, p_I) = \begin{cases} \min\left\{\max\left\{\frac{a-p_E}{b} - k_I, 0\right\}, k_E\right\} & \text{if } p_E > p_I \\ \min\left\{\max\left\{\frac{a-p}{2b}, \frac{a-p}{b} - k_I\right\}, k_E\right\} & \text{if } p_E = p_I = p \\ \min\left\{\frac{a-p_E}{b}, k_E\right\} & \text{if } p_E < p_I \end{cases}$$

To see the logic behind the efficient-rationing rule, assume that $p_E = p_I$ and focus on I (analogous logic applies to E). Then, demand will be split evenly between both countries at $(a-p)/2b$, unless E is capacity-constrained. If so, I will be the only service provider over the excess of demand $(a-p)/b - k_E$. Since I also needs to consider its own capacity constraint, we have $Q_I(p_E, p_I) = \min\left\{\max\left\{\frac{a-p}{2b}, \frac{a-p}{b} - k_E\right\}, k_I\right\}$. Next, assume that $p_E > p_I$. Then, as consumers try to buy from the low-priced firm first, I's demand is $(a-p_I)/b$, provided that its capacity constraint (k_I) does not bind. Therefore, $Q_I(p_E, p_I) = \min\left\{\frac{a-p_I}{b}, k_I\right\}$. E obtains the residual demand $\max\left\{\frac{a-p_E}{b} - k_I, 0\right\}$ (if any) after taking into account its own capacity constraint (k_E). Then, $Q_E(p_E, p_I) = \min\left\{\max\left\{\frac{a-p_E}{b} - k_I, 0\right\}, k_E\right\}$. A similar reasoning follows for $p_E < p_I$.

Suppose that I and E have selected capacity levels $k_I \geq 0$ and $k_E \geq 0$, respectively. We will prove that, under proper conditions, it is a Nash equilibrium for I and E to set $p_I = p_E = p^* = a - b(k_I + k_E)$. To do so, suppose that I sets $p_I = p^* = a - b(k_I + k_E)$ and recall that E's demand is given by

$$\begin{aligned}
&\text{If } p_E > p_I, \text{ then } x_E = \min\left\{\max\left\{\frac{a-p_E}{b} - k_I, 0\right\}, k_E\right\} \\
&\text{If } p_E < p_I\ , \text{ then } x_E = \min\left\{\frac{a-p_E}{b}, k_E\right\} \\
&\text{If } p_E = p_I = p, \text{ then } x_E = \min\left\{\max\left\{\frac{a-p}{2b}, \frac{a-p}{b} - k_I\right\}, k_E\right\}
\end{aligned}$$

Then, E has three possible choices to consider:

1. If E also sets $p_E = p^*$, then E's demand is given by $x_E = \min\{\max\{(k_I + k_E)/2, k_E\}, k_E\} = k_E$ and, therefore, E's revenue is $R_E(p^*) = p^* k_E = [a - b(k_I + k_E)] k_E$.

2. If E sets $p_E < p^*$, then E's demand is given by $x_E = \min\{(a - p_E)/b, k_E\}$ and, therefore, E's revenue is $R_E = p_E \min\{(a - p_E)/b, k_E\}$. Since $p_E < p^*$, it must be the case that $(a - p_E)/b > (k_I + k_E)$ and, hence, $R_E = p_E k_E < p^* k_E$. Thus, E obtains higher revenues of it sets $p_E = p^*$.

3. If E sets $p_E > p^*$, then E's demand is given by $x_E = \min\{\max\{(a - p_E)/b - k_I, 0\}, k_E\}$ and, therefore, E's revenue is $R_E = p_E \min\{\max\{[(a - p_E)/b] - k_I, 0\}, k_E\}$. Since $p_E > p^*$, it must be the case that $[(a - p_E)/b - k_I] < k_E$ and, hence, $R_E = p_E\{[(a - p_E)/b] - k_I\}$. This implies that E's maximum revenue is attained at $p_E = \hat{p} = (a - bk_I)/2$. In order for $p_E = \hat{p}$ not to be a possible deviation, we need that $\hat{p} \leq p^*$, which holds if and only if $k_E \leq (a - bk_I)/2b$.

Summing up, E's best response to $p_I = p^* = a - b(k_I + k_E)$ is to set $p_E = p^*$ if and only if $k_E \leq (a - bk_I)/2b$. Following the same steps it is easy to prove that I's best response to $p_E = p^*$ is to set $p_I = p^*$ if and only if $k_I \leq (a - bk_E)/2b$. For these conditions to hold for every profile of capacity choices, we impose that

$$k_I \in \left[0, \frac{a}{3b}\right] \text{ and } k_E \in \left[0, \frac{a}{3b}\right]$$

Capacity. Next we study the capacity choices. Assume that I has selected $k_I \in \left[0, \frac{a}{3b}\right]$. Then, the problem of E is given by:

$$\max_{k_E \in \left[0, \frac{a}{3b}\right]} \left\{ \pi_E = [a - b(k_I + k_E) - c] k_E - \left\{ \begin{array}{ll} F - S_2 & if\ k_E > 0 \\ 0 & if\ k_E = 0 \end{array} \right\} \right.$$

If E selects $k_E > 0$, its best response is $k_E = (a - bk_I - c)/2b$ (see parameter restrictions at the end of the proof to ensure that $(a - bk_I - c)/2b \leq a/3b$). Thus, E's profits are $\pi_E = \left[(a - bk_I - c)^2/4b\right] - (F - S_2)$. On the contrary, if E selects $k_E = 0$, E's profits are $\pi_E = 0$. Thus, E's best response is given by (see parameter restrictions at the end of the proof to ensure that $\bar{k}^d \leq a/3b$):

$$k_E(k_I) = \left\{ \begin{array}{ll} 0 & if\ \bar{k}^d \leq k_I \leq \frac{a}{3b} \\ \frac{a - bk_I - c}{2b} & if\ 0 \leq k_I < \bar{k}^d \end{array} \right. , \text{ where } \bar{k}^d = \frac{a - c - 2\sqrt{b(F - S_2)}}{b}$$

Given the reaction function of E, the problem of I is:

$$\max_{k_I \in \left[0, \frac{a}{3b}\right]} \left\{ \pi_I = \left\{ \begin{array}{ll} \pi_I^m = (a - bk_I - c) k_I + S_1 & if\ k_I \geq \bar{k}^d \\ \pi_I^s = \left(\frac{a - bk_I - c}{2}\right) k_I & if\ k_I < \bar{k}^d \end{array} \right. \right\}$$

Let $\bar{k}^m = (a-c)/2b$ be the monopoly capacity level (see parameter restrictions at the end of the proof to ensure that $\bar{k}^m \leq a/3b$). It is easy to verify that π_I^s is increasing in k_I for all $k_I \in [0, \bar{k}^m)$, decreasing in k_I for all $k_I \in (\bar{k}^m, a/3b]$ and it has a maximum at $k_I = \bar{k}^m$. Similarly, π_I^m is increasing in k_I for all $k_I \in [0, \bar{k}^m)$, decreasing in k_I for all $k_I \in (\bar{k}^m, a/3b]$ and it has a maximum at $k_I = \bar{k}^m$. Thus, to solve this problem we must consider two possible cases.

Case 1 (blocked entry): Suppose that $\bar{k}^d \leq \bar{k}^m$, which holds if and only if

$$S_2 \leq \bar{S}^b = F - \frac{(a-c)^2}{16b}$$

Then π_I^d is increasing in k_I for all $k_I < \bar{k}^d$ and π_I^m has a global maximum at $k_I = \bar{k}^m$. Since $\pi_I^m(\bar{k}^m) \geq \pi_I^m(\bar{k}^d) > \pi_I^s(\bar{k}^d)$, π_I has a global maximum at $k_I = \bar{k}^m$. Summing up, when $\bar{k}^d \leq \bar{k}^m$, the unique subgame perfect Nash equilibrium outcome is $k_I = \bar{k}^m$, $k_E = 0$, the equilibrium price is $P = a - b\bar{k}^m$, and the equilibrium profits of I and E are $\pi_I = \left[(a-c)^2/4b\right] + S_1$ and $\pi_E = 0$, respectively.

Case 2 (deterred or accommodated entry): Suppose that $\bar{k}^m < \bar{k}^d$, which holds if and only if

$$S_2 > \bar{S}^b = F - \frac{(a-c)^2}{16b}$$

Then, π_I^s has a global maximum at $k_I = \bar{k}^m$ and π_I^m is decreasing in k_I for all $k_I \geq \bar{k}^d$, which means that π_I^m has a global maximum at $k_I = \bar{k}^d$. If I selects $k_I = \bar{k}^m$, then it gets $\pi_I^s(\bar{k}^m) = (a - b\bar{k}^m - c)\bar{k}^m/2$. If I selects $k_I = \bar{k}^d$, then it gets $\pi_I^m(\bar{k}^d) = (a - b\bar{k}^d - c)\bar{k}^d + S_1$. $\pi_I^m(\bar{k}^d) > \pi_I^s(\bar{k}^m)$ if and only if $S_1 > \left[(a-c)^2/8b\right] - 2(a-c)\sqrt{(F-S_2)/b} + 4(F-S_2)$, $\pi_I^m(\bar{k}^d) = \pi_I^s(\bar{k}^m)$ when $S_1 = \left[(a-c)^2/8b\right] - 2(a-c)\sqrt{(F-S_2)/b} + 4(F-S_2)$, and $\pi_I^m(\bar{k}^d) < \pi_I^s(\bar{k}^m)$ if and only if $S_1 < \left[(a-c)^2/8b\right] - 2(a-c)\sqrt{(F-S_2)/b} + 4(F-S_2)$. Therefore, we have the following cases:

Case 2.a (deterred entry). Suppose that

$$S_1 > \bar{S}^d(S_2) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F-S_2}{b}} + 4(F-S_2)$$

Then, the unique subgame perfect Nash equilibrium outcome is $k_I = \bar{k}^d$, $k_E = 0$, the equilibrium price is $P = a - b\bar{k}^d$, and the equilibrium profits of I and E are $\pi_I = \left\{\left[2\sqrt{b(F-S_2)}\right]\left[a - c - 2\sqrt{b(F-S_2)}\right]/b\right\} + S_1$ and $\pi_E = 0$, respectively.

Case 2.b (deterred or accommodated entry). Suppose that

$$S_1 = \bar{S}^d(S_2) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F-S_2}{b}} + 4(F-S_2)$$

Then, there are two subgame perfect Nash equilibrium outcomes: the equilibrium described in case 2.a and the equilibrium described in case 2.c.

Case 2.c (accommodated entry): Suppose that

$$S_1 < \bar{S}^d(S_2) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F-S_2}{b}} + 4(F-S_2)$$

Then, the unique subgame perfect Nash equilibrium outcome $k_I = \bar{k}^m$, $k_E = (a-c)/4b$, the equilibrium price is $P = (a+3c)/4$, and the equilibrium profits of I and E are $\pi_I = (a-c)^2/8b$ and $\pi_E = \left[(a-c)^2/16b\right] - (F - S_2)$, respectively.

Parameter restrictions and characterization of $\bar{S}^d(S_2)$: To ensure that, if E selects $k_E > 0$, then its best response is $k_E = (a - bk_I - c)/2b \leq a/3b$, we need to impose that $a \leq 3c$. To ensure that $\bar{k}^d \leq a/3b$ we need to impose that $S_2 \leq \bar{S} = F - \left[(2a-3c)^2/36b\right]$. Moreover, note that $\bar{S}^b \leq \bar{S}$ if and only if $9c/7 \leq a \leq 3c$. Finally, $\bar{k}^m \leq a/3b$ if and only if $a \leq 3c$.

It is possible to further characterize the equilibrium for $\bar{S}^b < S_2 \leq \bar{S}$. In particular, note that:

- $\bar{S}^d$ is a continuous function of S_2 for all $\bar{S}^b \leq S_2 \leq \bar{S}$.
- $\bar{S}^d\left(\bar{S}^b\right) = -(a-c)^2/8b < 0$.
- $d\bar{S}^d(S_2)/dS_2 = \left[(a-c)/\sqrt{(F-S_2)\,b}\right] - 4 > 0$ if and only if $S_2 > \bar{S}^b$. Thus, $\bar{S}^d(S_2)$ is strictly increasing in S_2 for all $\bar{S}^b \leq S_2 \leq \bar{S}$.
- $d^2\bar{S}^d(S_2)/(dS_2)^2 = (a-c)/2b^{1/2}(F-S_2)^{3/2} > 0$. Thus, $\bar{S}^d(S_2)$ is strictly convex in S_2 for all $\bar{S}^b \leq S_2 \leq \bar{S}$.
- $\bar{S}^d\left(\bar{S}\right) = \left(-7a^2 + 6ac + 9c^2\right)/72b > 0$, which holds provided that $a < \left(6\sqrt{2}+3\right)c/7 \approx 1.64c$.

Therefore, there exists $\bar{S}_0^d \in \left(\bar{S}^b, \bar{S}\right)$ such that $\bar{S}^d(S_2) < 0$ for all $S_2 \in \left[\bar{S}^b, \bar{S}_0^d\right)$, $\bar{S}^d\left(\bar{S}_0^d\right) = 0$, and $\bar{S}^d(S_2) > 0$ for all $S_2 \in \left(\bar{S}_0^d, \bar{S}\right]$. Moreover, $\bar{S}^d(S_2)$ has a continuous inverse and, hence, $S_1 = \bar{S}^d(S_2)$ if and only if $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$.

Summary of equilibrium outcomes:
If $0 \leq S_2 \leq \bar{S}^b$, then entry is blocked.
If $\bar{S}^b < S_2 < \left(\bar{S}^d\right)^{-1}(S_1)$, then entry is deterred.
If $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$, then entry is either deterred or accommodated.
If $\left(\bar{S}^d\right)^{-1}(S_1) < S_2 \leq \bar{S}$ entry is accommodated. ■

A.2 Proof of Propositions 2 and 3

We begin proving two lemmas that help us characterize the geopolitical trade-off faced by each global power. Then, we prove a general version of Proposition 2. Finally, Propositions 2 and 3 in the text are derived as corollaries of Proposition 2 (general version).

Lemma 1 ***Geopolitical trade-off for*** G_1**.** *Suppose that* $A^1 b < 2\left(\sqrt{2}-1\right)$.

1. *If* $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$, *then* $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ *for all* $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$.
2. *If* $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$, *then there exists a unique* $\tilde{S}_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ *such that* $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ *for all* $S_1 \in \left[0, \tilde{S}_1\right)$, $B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$, *and* $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ *for all* $S_1 \in \left(\tilde{S}_1, \bar{S}^d\left(\bar{S}\right)\right]$.

3. *If* $B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$*, then* $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ *for all* $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$*.*

Proof: Define

$$\Delta W_1(S_1) = B_1^M - B_1^D - A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) - S_1,$$

where $\Delta(S) = \frac{9(a-c)^2}{32} - \frac{\left[a-c-2\sqrt{b(F-S)}\right]^2}{2}$ and $\left(\bar{S}^d\right)^{-1}$ is the inverse of $\bar{S}^d(S) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F-S}{b}} + 4(F-S)$. $\Delta W_1(S_1)$ is continuously differentiable for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. Take the derivative of $\Delta W_1(S_1)$ with respect to S_1:

$$\frac{\partial \Delta W_1(S_1)}{\partial S_1} = -A^1\left[\frac{\partial\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)}{\partial\left(\bar{S}^d\right)^{-1}(S_1)}\right]\left[\frac{\partial\left(\bar{S}^d\right)^{-1}(S_1)}{\partial S_1}\right] - 1$$

where

$$\frac{\partial\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)}{\partial\left(\bar{S}^d\right)^{-1}(S_1)} = -\frac{\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]\sqrt{b}}{\sqrt{F-\left(\bar{S}^d\right)^{-1}(S_1)}}$$

Due to the implicit function theorem,

$$\frac{d\left(\bar{S}^d\right)^{-1}(S_1)}{dS_1} = \left[\frac{d\bar{S}^d\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)}{dS_1}\right]^{-1} = \left[\frac{a-c}{\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}} - 4\right]^{-1}$$

It is easy to verify that $\partial\Delta W_1(S_1)/\partial S_1 < 0$ if and only if $\left(\bar{S}^d\right)^{-1}(S_1) > S_2' = F - \left[\left(1-A^1b\right)^2(a-c)^2/4\left(2-A^1b\right)^2 b\right]$. Thus, $\partial\Delta W_1(S_1)/\partial S_1 < 0$ if and only if $S_1 > \bar{S}^d(S_2')$. Note that $\bar{S}^d(S_2') < 0$ if and only if $A^1b < 2\left(\sqrt{2}-1\right) \approx 0.828$, which we assume holds. Therefore, $\partial\Delta W_1(S_1)/\partial S_1 < 0$ for all $S_1 \geq 0$, which implies that $\Delta W_1(S_1)$ is an strictly decreasing function of S_1 for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. Since $\Delta W_1(S_1)$ is a continuous and strictly decreasing function of S_1 for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$, there are three possible cases to consider:

Case 1: Suppose that $\Delta W_1(0) < 0$ or, which is equivalent, $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$.

Case 2: Suppose that $\Delta W_1\left(\bar{S}^d\left(\bar{S}\right)\right) \leq 0 \leq \Delta W_1(0)$ or, which is equivalent, $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, there exists a unique $\tilde{S}_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ such that $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \tilde{S}_1\right)$, $B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$, and $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left(\tilde{S}_1, \bar{S}^d\left(\bar{S}\right)\right]$.

Case 3: Suppose that $\Delta W_1\left(\bar{S}^d\left(\bar{S}\right)\right) > 0$ or, which is equivalent, $B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. ■

Lemma 2 ***Geopolitical trade-off for*** G_2***.***

1. *If* $B_2^D \leq \bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right)$*, then* $B_2^D < S_2 - A^2\Delta\left(S_2\right)$ *for all* $S_2 \in \left(\bar{S}_0^d, \bar{S}\right]$.

2. *If* $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$*, then, there exists a unique* $\tilde{S}_2 \in \left(\bar{S}_0^d, \bar{S}\right]$ *such that* $B_2^D > S_2 - A^2\Delta\left(S_2\right)$ *for all* $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$, $B_2^D = \tilde{S}_2 - A^2\Delta\left(\tilde{S}_2\right)$*, and* $B_2^D < S_2 - A^2\Delta\left(S_2\right)$ *for all* $S_2 \in \left(\tilde{S}_2, \bar{S}\right]$.

3. *If* $B_2^D > \bar{S} - A^2\Delta\left(\bar{S}\right)$*, then* $B_2^D > S_2 - A^2\Delta\left(S_2\right)$ *for all* $S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$.

Proof: Define

$$\Delta W_2\left(S_2\right) = B_2^D + A^2\Delta\left(S_2\right) - S_2,$$

where $\Delta\left(S_2\right) = \frac{9(a-c)^2}{32} - \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}$. Note that $\Delta W_2\left(S_2\right)$ is a continuous and strictly decreasing function of S_2 for all $S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$. Thus, that there are three possible cases to consider.

Case 1: Suppose that $\Delta W_2\left(\bar{S}_0^d\right) \leq 0$ or, which is equivalent, $B_2^D \leq \bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right)$. Then, $B_2^D < S_2 - A^2\Delta\left(S_2\right)$ for all $S_2 \in \left(\bar{S}_0^d, \bar{S}\right]$.

Case 2: Suppose that $\Delta W_2\left(\bar{S}\right) \leq 0 < \Delta W_2\left(\bar{S}_0^d\right)$ or, which is equivalent, $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$. Then, there exists a unique $\tilde{S}_2 \in \left(\bar{S}_0^d, \bar{S}\right]$ such that $B_2^D > S_2 - A^2\Delta\left(S_2\right)$ for all $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$, $B_2^D = \tilde{S}_2 - A^2\Delta\left(\tilde{S}_2\right)$, and $B_2^D < S_2 - A^2\Delta\left(S_2\right)$ for all $S_2 \in \left(\tilde{S}_2, \bar{S}\right]$.

Case 3: Suppose that $\Delta W_2\left(\bar{S}\right) > 0$ or, which is equivalent, $B_2^D > \bar{S} - A^2\Delta\left(\bar{S}\right)$. Then, $B_2^D > S_2 - A^2\Delta\left(S_2\right)$ for all $S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$. ■

Proposition 2 (***General version***)**.** *Suppose that* $9c/7 \leq a \leq \left(6\sqrt{2}+3\right)c/7$ *and* $A^1 b < 2\left(\sqrt{2}-1\right)$. *Let*

$$\Delta\left(S\right) = \frac{9\left(a-c\right)^2}{32} - \frac{\left[a-c-2\sqrt{b\left(F-S\right)}\right]^2}{2}$$

1. *Suppose that* $B_2^D \leq \bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right)$. *Then, the set of equilibrium subsidies is given by* $S_1 = \bar{S}^d\left(S_2\right)$ *with:*

$$\begin{array}{ll} S_2 = \bar{S}_0^d & \text{if } B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right) \\ S_2 \in \left[\bar{S}_0^d, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right] & \text{if } A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right) \\ S_2 \in \left[\bar{S}_0^d, \bar{S}\right] & \text{if } B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right) \end{array}$$

Moreover, in all these equilibria entry is deterred.

2. *Suppose that* $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$. *Then, the set of equilibrium subsidies is given by* $S_1 = \bar{S}^d\left(S_2\right)$ *with:*

$$\begin{array}{ll} S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right) & \text{if } B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right) \\ S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right) & \text{if } A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right) \text{ and } \tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right) \\ S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right] & \text{if } A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right) \text{ and } \tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right) \\ S_2 \in \left[\tilde{S}_2, \bar{S}\right] & \text{if } B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right) \end{array}$$

where $\tilde{S}_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ *is the unique solution to* $B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$ *and* $\tilde{S}_2 \in \left(\bar{S}_0^d, \bar{S}\right]$ *is the unique solution to* $B_2^D = \tilde{S}_2 - A^2\Delta\left(\tilde{S}_2\right)$. *Moreover, in all the equilibria in which* $S_2 \in \left[\tilde{S}_2, \bar{S}\right]$ *entry is deterred, while in all the equilibria in which* $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$ *there is accommodated entry.*

3. *Suppose that* $B_2^D > \bar{S} - A^2\Delta\left(\bar{S}\right)$. *Then, the set of equilibrium subsidies is given by* $S_1 = \bar{S}^d\left(S_2\right)$ *with:*

$$\begin{array}{ll} S_2 \in \left[\bar{S}_0^d, \bar{S}\right] & \text{if } B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right) \\ S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \bar{S}\right] & \text{if } A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right) \\ S_2 = \bar{S} & \text{if } B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right) \end{array}$$

Moreover, in all these equilibria entry is accommodated.

Proof:

Payoff functions: The consumer surplus of country j as a function of the price is $CS_j(P) = A^j(a-P)^2/2$. Thus, employing Proposition 1, the consumer surplus of each country as a function of (S_1, S_2) is given by:

$$CS_j\left(S_1, S_2\right) = A^j \begin{cases} \frac{(a-c)^2}{8} & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}_0^d \leq S_2 < \left(\bar{S}^d\right)^{-1}\left(S_1\right) \\ \frac{\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} \text{ or } \frac{9(a-c)^2}{32} & \text{if } \bar{S}_0^d \leq S_2 = \left(\bar{S}^d\right)^{-1}\left(S_1\right) \\ \frac{9(a-c)^2}{32} & \text{if } \bar{S}_0^d \leq \left(\bar{S}^d\right)^{-1}\left(S_1\right) < S_2 \leq \bar{S} \end{cases}$$

Once again, employing Proposition 1, the geopolitical payoff of each global power as a function of (S_1, S_2) is given by:

$$B_1\left(S_1, S_2\right) = \begin{cases} B_1^M - S_1 & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ B_1^M - S_1 & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ B_1^M - S_1 & \text{if } \bar{S}_0^d \leq S_2 < \left(\bar{S}^d\right)^{-1}\left(S_1\right) \\ B_1^M - S_1 \text{ or } B_1^D & \text{if } \bar{S}_0^d \leq S_2 = \left(\bar{S}^d\right)^{-1}\left(S_1\right) \\ B_1^D & \text{if } \bar{S}_0^d \leq \left(\bar{S}^d\right)^{-1}\left(S_1\right) < S_2 \leq \bar{S} \end{cases}$$

$$B_2\left(S_1, S_2\right) = \begin{cases} 0 & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ 0 & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ 0 & \text{if } \bar{S}_0^d \leq S_2 < \left(\bar{S}^d\right)^{-1}\left(S_1\right) \\ B_2^D - S_2 \text{ or } 0 & \text{if } \bar{S}_0^d \leq S_2 = \left(\bar{S}^d\right)^{-1}\left(S_1\right) \\ B_2^D - S_2 & \text{if } \bar{S}_0^d \leq \left(\bar{S}^d\right)^{-1}\left(S_1\right) < S_2 \leq \bar{S} \end{cases}$$

Finally, the payoff function of each global power as a function of (S_1, S_2) is given by:

$$W_j\left(S_1, S_2\right) = CS_j\left(S_1, S_2\right) + B_j\left(S_1, S_2\right)$$

Selection criterion: From Proposition 1.2.b, if $S_1 = \bar{S}^d(S_2)$, deterrence and accommodation are both subgame perfect Nash equilibria. In such a case, the equilibrium with accommodation is selected when it strictly dominates the equilibrium with deterrence for G_2. Otherwise, the economic equilibrium with deterrence is selected. Thus,

$$W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = \max\left\{\begin{array}{c} \frac{A^2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2}, \\ \frac{A^2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) \end{array}\right\}$$

Best response correspondence of G_2: Employing the above selection criterion, the payoff function of G_2 as a function of (S_1, S_2) is given by:

$$W_2(S_1, S_2) = \begin{cases} \frac{A^2(a-c)^2}{8} & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{A^2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \frac{A^2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}_0^d \leq S_2 < \left(\bar{S}^d\right)^{-1}(S_1) \\ \max\left\{\begin{array}{c} \frac{A^2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2}, \\ \frac{A^2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) \end{array}\right\} & \text{if } S_2 = \left(\bar{S}^d\right)^{-1}(S_1) \\ \frac{A^2 9(a-c)^2}{32} + B_2^D - S_2 & \text{if } \left(\bar{S}^d\right)^{-1}(S_1) < S_2 \leq \bar{S} \end{cases}$$

$W_2(S_1, S_2)$ is a constant for all $S_2 \in \left[0, \bar{S}^b\right]$, it is strictly increasing in S_2 for all $S_2 \in \left[\bar{S}^b, \left(\bar{S}^d\right)^{-1}(S_1)\right)$, and it is strictly decreasing in S_2 for all $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), \bar{S}\right]$. This does not immediately imply that $W_2(S_1, S_2)$ has its unique global maximum at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$. The reason is that $W_2(S_1, S_2)$ might not be continuous at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$.[11] However, note that $W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right)$ adopts the maximum between the left and right limits of the function at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ and both of these limits exist. Therefore, it is always the case that $W_2(S_1, S_2)$ adopts its unique global maximum at $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$. Thus, the best response correspondence of G_2 is given by:

$$S_2 = \left(\bar{S}^d\right)^{-1}(S_1) \text{ for all } S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$$

Economic equilibrium selection under $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$: To determine if $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to deterrence or accommodated entry, we must study $W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right)$. Note that

$$W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = \begin{cases} \frac{A^2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2} & \text{if } \Delta W_2\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) \leq 0 \\ \frac{A^2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) & \text{if } \Delta W_2\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) > 0 \end{cases}$$

[11] $W_2(S_1, S_2)$ is always a continuous function of S_2 for all $S_2 \in \left[0, \left(\bar{S}^d\right)^{-1}(S_1)\right)$ and $S_2 \in \left(\left(\bar{S}^d\right)^{-1}(S_1), \bar{S}\right]$. In particular, it is continuous at $S_2 = \bar{S}^b$.

where $\Delta W_2(S_2) = B_2^D + A^2\Delta(S_2) - S_2$. Employing Lemma 2, there are three possible cases to consider.

Case 1: Suppose that $B_2^D \leq \bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right)$. Then, $B_2^D < S_2 - A^2\Delta(S_2)$ for all $S_2 \in \left(\bar{S}_0^d, \bar{S}\right]$. Therefore, $W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = A^2\left[a - c - 2\sqrt{b\left(F - \left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2/2$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. That is, $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to deterrence.

Case 2: Suppose that $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$. Then, there exists a unique $\tilde{S}_2 \in \left(\bar{S}_0^d, \bar{S}\right]$ such that $B_2^D > S_2 - A^2\Delta(S_2)$ for all $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$, $B_2^D = \tilde{S}_2 - A^2\Delta\left(\tilde{S}_2\right)$, and $B_2^D < S_2 - A^2\Delta(S_2)$ for all $S_2 \in \left(\tilde{S}_2, \bar{S}\right]$. Therefore,

$$W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = \begin{cases} \frac{A^2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1) & \textit{if } 0 \leq S_1 < \bar{S}^d\left(\tilde{S}_2\right) \\ \frac{A^2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}(S_1)\right)}\right]^2}{2} & \textit{if } \bar{S}^d\left(\tilde{S}_2\right) \leq S_1 \leq \bar{S}^d\left(\bar{S}\right) \end{cases}$$

That is, $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to accommodated entry when $S_1 < \bar{S}^d\left(\tilde{S}_2\right)$ and to deterrence when $S_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$.

Case 3: Suppose that $B_2^D > \bar{S} - A^2\Delta\left(\bar{S}\right)$. Then, $B_2^D > S_2 - A^2\Delta(S_2)$ for all $S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$. Therefore, $W_2\left(S_1, \left(\bar{S}^d\right)^{-1}(S_1)\right) = \frac{A^2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}(S_1)$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. That is, $S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$ leads to accommodated entry.

Best response correspondence of G_1. The payoff function of G_1 as a function of (S_1, S_2) is given by:

$$W_1(S_1, S_2) = \begin{cases} \frac{A^1(a-c)^2}{8} + B_1^M - S_1 & \textit{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & \textit{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & \textit{if } \bar{S}_0^d \leq S_2 \leq \bar{S} \textit{ and } S_1 > \bar{S}^d(S_2) \\ \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 \textit{ or } \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } \bar{S}_0^d \leq S_2 \leq \bar{S} \textit{ and } S_1 = \bar{S}^d(S_2) \\ \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } \bar{S}_0^d \leq S_2 \leq \bar{S} \textit{ and } S_1 < \bar{S}^d(S_2) \end{cases}$$

If $0 \leq S_2 \leq \bar{S}^b$, then, $W_1(S_1, S_2) = \left[A^1(a-c)^2/8\right] + B_1^M - S_1$, which is strictly decreasing in S_1. Thus, the best response to $0 \leq S_2 \leq \bar{S}^b$ is always $S_1 = 0$. Similarly, if $\bar{S}^b < S_2 < \bar{S}_0^d$, then $W_1(S_1, S_2) = A^1\left[a - c - 2\sqrt{b(F - S_2)}\right]^2/2 + B_1^M - S_1$, which is strictly decreasing in S_1. Thus, the best response to $\bar{S}^b < S_2 < \bar{S}_0^d$ is always $S_1 = 0$.

If $\bar{S}_0^d \leq S_2 \leq \bar{S}$, there are three possible cases to consider and for each case, we have three possible subcases.

Case 1: Suppose that $B_2^D \leq \bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right)$. Then:

$$W_1(S_1,S_2) = \begin{cases} \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & \textit{if } S_1 > \bar{S}^d(S_2) \\ \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - \bar{S}^d(S_2) & \textit{if } S_1 = \bar{S}^d(S_2) \\ \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } S_1 < \bar{S}^d(S_2) \end{cases}$$

$W_1(S_1,S_2)$ adopts its maximum at $S_1 = \bar{S}^d(S_2)$ if and only if $B_1^M - B_1^D \geq A^1\Delta(S_2) + \bar{S}^d(S_2)$, while it adopts its maximum at $S_1 \in \left[0, \bar{S}^d(S_2)\right)$ if and only if $B_1^M - B_1^D \leq A^1\Delta(S_2) + \bar{S}^d(S_2)$. Thus, employing Lemma 1, we must consider three possible subcases:

Case 1.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, from Lemma 1 $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 \leq \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \bar{S}_0^d < S_2 \leq \bar{S} \end{cases}$$

Case 1.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, from Lemma 1, there exists a unique $\tilde{S}_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ such that $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \tilde{S}_1\right)$, $B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$, and $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left(\tilde{S}_1, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 \leq \bar{S}_0^d \\ \bar{S}^d(S_2) & \textit{if } \bar{S}_0^d < S_2 \leq \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 \leq \bar{S} \end{cases}$$

Case 1.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, from Lemma 1, $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 \leq \bar{S}_0^d \\ \bar{S}^d(S_2) & \textit{if } \bar{S}_0^d < S_2 \leq \bar{S} \end{cases}$$

Case 2: Suppose that $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$. Then:

$$W_1(S_1,S_2) = \begin{cases} \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & \textit{if } S_1 > \bar{S}^d(S_2) \\ \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } S_1 = \bar{S}^d(S_2) \textit{ and } S_2 < \tilde{S}_2 \\ \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - \bar{S}^d(S_2) & \textit{if } S_1 = \bar{S}^d(S_2) \textit{ and } S_2 \geq \tilde{S}_2 \\ \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } S_1 < \bar{S}^d(S_2) \end{cases}$$

If $S_2 < \tilde{S}_2$, $W_1(S_1,S_2)$ adopts its maximum at $S_1 \in \left[0, \bar{S}^d(S_2)\right]$ if and only if $B_1^M - B_1^D \leq A^1\Delta(S_2) + \bar{S}^d(S_2)$. Otherwise, there is no $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ that maximizes $W_1(S_1,S_2)$. If $S_2 \geq \tilde{S}_2$, $W_1(S_1,S_2)$

adopts its maximum at $S_1 = \bar{S}^d(S_2)$ if and only if $B_1^M - B_1^D \geq A^1\Delta(S_2) + \bar{S}^d(S_2)$, while it adopts its maximum at $S_1 \in \left[0, \bar{S}^d(S_2)\right)$ if and only if $B_1^M - B_1^D \leq A^1\Delta(S_2) + \bar{S}^d(S_2)$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \bar{S}_0^d \leq S_2 < \tilde{S}_2 \textit{ and } B_1^M - B_1^D \leq A^1\Delta(S_2) + \bar{S}^d(S_2) \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \leq S_2 \leq \bar{S} \textit{ and } B_1^M - B_1^D \geq A^1\Delta(S_2) + \bar{S}^d(S_2) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq \bar{S} \textit{ and } B_1^M - B_1^D \leq A^1\Delta(S_2) + \bar{S}^d(S_2) \end{cases}$$

Thus, employing Lemma 1, we must consider three possible subcases:

Case 2.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, from Lemma 1 $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \bar{S}_0^d \leq S_2 < \tilde{S}_2 \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq \bar{S} \end{cases}$$

Case 2.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, from Lemma 1, there exists a unique $\tilde{S}_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ such that $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \tilde{S}_1\right)$, $B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$, and $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left(\tilde{S}_1, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \tilde{S}_2 \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \leq S_2 \leq \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \textit{if } \max\left\{\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right\} \leq S_2 \leq \bar{S} \end{cases}$$

Case 2.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, from Lemma 1, $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d(S_2) & \textit{if } \tilde{S}_2 \leq S_2 \leq \bar{S} \end{cases}$$

Case 3: Suppose that $B_2^D > \bar{S} - A^2\Delta\left(\bar{S}\right)$. Then:

$$W_1(S_1, S_2) = \begin{cases} \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} + B_1^M - S_1 & \textit{if } S_1 > \bar{S}^d(S_2) \\ \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } S_1 = \bar{S}^d(S_2) \\ \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } S_1 < \bar{S}^d(S_2) \end{cases}$$

If $\bar{S}_0^d \leq S_2 < \bar{S}$, $W_1(S_1, S_2)$ adopts its maximum at $S_1 \in \left[0, \bar{S}^d(S_2)\right]$ if and only if $B_1^M - B_1^D \leq A^1\Delta(S_2) + \bar{S}^d(S_2)$. Otherwise, there is no $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ that maximizes $W_1(S_1, S_2)$. If $S_2 = \bar{S}$, then

$W_1(S_1,S_2)$ adopts its maximum at $S_1 \in \left[0, \bar{S}^d(S_2)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \le S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \text{if } \bar{S}_0^d \le S_2 < \bar{S} \text{ and } B_1^M - B_1^D \le A^1\Delta(S_2) + \bar{S}^d(S_2) \\ \left[0, \bar{S}^d\left(\bar{S}\right)\right] & \text{if } S_2 = \bar{S} \end{cases}$$

Thus, employing Lemma 1, we must consider three possible subcases:

Case 3.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, from Lemma 1 $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \le S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \text{if } \bar{S}_0^d \le S_2 \le \bar{S} \end{cases}$$

Case 3.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \le B_1^M - B_1^D \le A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, from Lemma 1, there exists a unique $\tilde{S}_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ such that $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \tilde{S}_1\right)$, $B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$, and $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left(\tilde{S}_1, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \le S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \text{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \le S_2 \le \bar{S} \end{cases}$$

Case 3.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, from Lemma 1, $B_1^M - B_1^D > A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$. Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \le S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(\bar{S}\right)\right] & \text{if } S_2 = \bar{S} \end{cases}$$

Nash equilibrium: We must consider three possible cases and for each case, we have three possible subcases.

Case 1: Suppose that $B_2^D \le \bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right)$.

Case 1.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \le S_2 \le \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right) & \text{if } \bar{S}_0^d < S_2 \le \bar{S} \end{cases} \text{ and } S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = 0 \text{ and } S_2 = \bar{S}_0^d$$

Moreover, in this equilibrium entry is deterred.

Case 1.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \le B_1^M - B_1^D \le A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \le S_2 \le \bar{S}_0^d \\ \bar{S}^d(S_2) & \text{if } \bar{S}_0^d < S_2 \le \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \text{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \le S_2 \le \bar{S} \end{cases} \text{ and } S_2 = \left(\bar{S}^d\right)^{-1}(S_1)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d (S_2) \;\; and \;\; S_2 \in \left[\bar{S}_0^d, \left(\bar{S}^d\right)^{-1} \left(\tilde{S}_1\right)\right]$$

Moreover, in all these equilibria entry is deterred.

Case 1.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & if \;\; 0 \leq S_2 \leq \bar{S}_0^d \\ \bar{S}^d (S_2) & if \;\; \bar{S}_0^d < S_2 \leq \bar{S} \end{cases} \;\; and \;\; S_2 = \left(\bar{S}^d\right)^{-1} (S_1)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d (S_2) \;\; and \;\; S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$$

Moreover, in all these equilibria entry is deterred.

Case 2: Suppose $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$.

Case 2.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & if \;\; 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d (S_2)\right] & if \;\; \bar{S}_0^d \leq S_2 < \tilde{S}_2 \\ \left[0, \bar{S}^d (S_2)\right) & if \;\; \tilde{S}_2 \leq S_2 \leq \bar{S} \end{cases} \;\; and \;\; S_2 = \left(\bar{S}^d\right)^{-1} (S_1)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d (S_2) \;\; and \;\; S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$$

Moreover, in all these equilibria entry is accommodated.

Case 2.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & if \;\; 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d (S_2)\right] & if \;\; \left(\bar{S}^d\right)^{-1} \left(\tilde{S}_1\right) \leq S_2 < \tilde{S}_2 \\ \bar{S}^d (S_2) & if \;\; \tilde{S}_2 \leq S_2 \leq \left(\bar{S}^d\right)^{-1} \left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d (S_2)\right) & if \;\; \max\left\{\tilde{S}_2, \left(\bar{S}^d\right)^{-1} \left(\tilde{S}_1\right)\right\} \leq S_2 \leq \bar{S} \end{cases} \;\; and \;\; S_2 = \left(\bar{S}^d\right)^{-1} (S_1)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$[S_1 = \bar{S}^d (S_2) \;\; and \;\; S_2 \in \left[\left(\bar{S}^d\right)^{-1} \left(\tilde{S}_1\right), \tilde{S}_2\right)] \;\; when \;\; \tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$$

$$[S_1 = \bar{S}^d (S_2) \;\; and \;\; S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1} \left(\tilde{S}_1\right)\right]] \;\; when \;\; \tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$$

Moreover, in all the equilibria in which $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$, entry is accommodated, while in the equilibria in which $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$ entry is deterred.

Case 2.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d\left(S_2\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq \bar{S} \end{cases} \quad \textit{and } S_2 = \left(\bar{S}^d\right)^{-1}\left(S_1\right)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d\left(S_2\right) \ \textit{and } S_2 \in \left[\tilde{S}_2, \bar{S}\right]$$

Moreover, in all these equilibria entry is deterred.

Case 3: Suppose that $B_2^D > \bar{S} - A^2\Delta\left(\bar{S}\right)$.

Case 3.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(S_2\right)\right] & \textit{if } \bar{S}_0^d \leq S_2 \leq \bar{S} \end{cases} \quad \textit{and } S_2 = \left(\bar{S}^d\right)^{-1}\left(S_1\right)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d\left(S_2\right) \ \textit{and } S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$$

Moreover, in all these equilibria entry is accommodated.

Case 3.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(S_2\right)\right] & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 \leq \bar{S} \end{cases} \quad \textit{and } S_2 = \left(\bar{S}^d\right)^{-1}\left(S_1\right)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$[S_1 = \bar{S}^d\left(S_2\right) \ \textit{and } S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \bar{S}\right]$$

Moreover, in all these equilibria entry is accommodated.

Case 3.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(\bar{S}\right)\right] & \textit{if } S_2 = \bar{S} \end{cases} \quad \textit{and } S_2 = \left(\bar{S}^d\right)^{-1}\left(S_1\right)$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d\left(\bar{S}\right) \ \textit{and } S_2 = \bar{S}$$

Moreover, in this equilibrium entry is accommodated. This completes the proof of Proposition 2 (general version). ■

Proposition 2 in the text is an immediate corollary of Proposition 2.2 (general version), Lemma 1, and Lemma 2. In particular, Lemma 1 implies that $S_1 \in \left[0, \tilde{S}_1\right]$ if and only $B_1^M - B_1^D \geq A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(S_1\right)\right) +$

S_1, while $S_1 \in \left[\tilde{S}_1, \bar{S}^d\left(\bar{S}\right)\right]$ if and only if $B_1^M - B_1^D < A^1\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right) + S_1$; and Lemma 2 implies that $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$ if and only if $B_2^D < S_2 - A^2\Delta(S_2)$, while $S_2 \in \left[\tilde{S}_2, \bar{S}\right]$ if and only if $B_2^D \geq S_2 - A^2\Delta(S_2)$).

Proposition 3 *Suppose that* $9c/7 \leq a \leq \left(6\sqrt{2}+3\right)c/7$, $A^1 b < 2\left(\sqrt{2}-1\right)$, $B_2^D \in \left(\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right), \bar{S} - A^2\Delta\left(\bar{S}\right)\right]$ *and* $B_1^M - B_1^D \in \left[A^1\Delta\left(\bar{S}_0^d\right), A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)\right]$. *Let* $\tilde{S}_1 \in \left(0, \bar{S}^d\left(\bar{S}\right)\right]$ *and* $\tilde{S}_2 \in \left(\bar{S}_0^d, \bar{S}\right]$ *be the unique solution to:*

$$B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$$
$$B_2^D = -A^2\Delta\left(\tilde{S}_2\right) + \tilde{S}_2$$

1. *If* $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$, *then the equilibrium subsidy profiles are those that satisfy* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. *Moreover, in all these equilibria entry is deterred.*
2. *If* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, *then the equilibrium subsidy profiles are those that satisfy* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. *Moreover, in all these equilibria there is accommodated entry.*
3. $\tilde{S}_1$ *(*$\tilde{S}_2$*) is strictly increasing in* $B_1^M - B_1^D$ *(*B_2^D*);* $\tilde{S}_1$ *and* $\tilde{S}_2$ *are both strictly increasing in* F*; and the effect of* c *(*a*) on* $\tilde{S}_1$ *and* $\tilde{S}_2$ *is ambiguous.*

Proof:

Nash equilibrium: Parts 1 and 2 are immediate from Proposition 2.2 (general version), Lemma 1 and Lemma 2.

Comparative statics with respect to $B_1^M - B_1^D$ ***and*** B_2^D: $\tilde{S}_1$ and $\tilde{S}_2$ are given by:

$$B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$$
$$B_2^D = \tilde{S}_2 - A^2\Delta\left(\tilde{S}_2\right)$$

Employing the implicit function theorem we have:

$$\frac{\partial \tilde{S}_1}{\partial\left(B_1^M - B_1^D\right)} = \left\{\left[A^1\frac{\partial\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\partial\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)}\right]\left[\frac{\partial\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right)}{\partial\tilde{S}_1}\right] + 1\right\}^{-1}$$
$$\frac{\partial\tilde{S}_2}{\partial B_2^D} = \frac{1}{1 - A^2\left[\frac{\partial\Delta\left(\tilde{S}_2\right)}{\partial\tilde{S}_2}\right]} > 0$$

We have already proved that $\frac{\partial\Delta W_1(S_1)}{\partial S_1} = -\left\{A^1\left[\frac{\partial\Delta\left(\left(\bar{S}^d\right)^{-1}(S_1)\right)}{\partial\left(\bar{S}^d\right)^{-1}(S_1)}\right]\left[\frac{\partial\left(\bar{S}^d\right)^{-1}(S_1)}{\partial S_1}\right] + 1\right\} < 0$ for all $S_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ (see the proof of Lemma 1). Therefore, $\frac{\partial\tilde{S}_1}{\partial\left(B_1^M - B_1^D\right)} > 0$. We have already proved that $\frac{\partial\Delta W_1(S_2)}{\partial S_2} = 1 - A^2\left[\frac{\partial\Delta\left(\tilde{S}_2\right)}{\partial\tilde{S}_2}\right] > 0$ for all $S_2 \in \left[\bar{S}_0^d, \bar{S}\right]$ (see the proof of Lemma 2). Therefore, $\frac{\partial\tilde{S}_2}{\partial B_2^D} > 0$.

Comparative statics with respect to F: To make calculations easier, define $\tilde{S}_1 = \bar{S}^d\left(\check{S}_2, F\right)$. Then:

$$B_1^M - B_1^D = A^1\Delta\left(\check{S}_2, F\right) + \bar{S}^d\left(\check{S}_2, F\right)$$
$$B_2^D = \tilde{S}_2 - A^2\Delta\left(\tilde{S}_2, F\right)$$

where

$$\Delta\left(\check{S}_2, F\right) = \frac{9\left(a-c\right)^2}{32} - \frac{\left[a-c-2\sqrt{b\left(F-\check{S}_2\right)}\right]^2}{2}$$
$$\bar{S}^d\left(\check{S}_2, F\right) = \frac{\left(a-c\right)^2}{8b} - 2\sqrt{\frac{F-\check{S}_2}{b}}\left[a-c-2\sqrt{b\left(F-\check{S}_2\right)}\right]$$

Employing the implicit function theorem we have:

$$\left[A^1\frac{\partial\Delta\left(\check{S}_2, F\right)}{\partial\check{S}_2} + \frac{\partial\bar{S}^d\left(\check{S}_2, F\right)}{\partial\check{S}_2}\right]d\check{S}_2 + \left[A^1\frac{\partial\Delta\left(\check{S}_2, F\right)}{\partial F} + \frac{\partial\bar{S}^d\left(\check{S}_2, F\right)}{\partial F}\right]dF = 0$$
$$\left[1 - \frac{A^2\partial\Delta\left(\tilde{S}_2, F\right)}{\partial\tilde{S}_2}\right]d\tilde{S}_2 - \frac{A^2\partial\Delta\left(\tilde{S}_2, F\right)}{\partial F}dF = 0$$

Using $d\tilde{S}_1 = \frac{\partial\bar{S}^d\left(\check{S}_2,F\right)}{\partial\check{S}_2}d\check{S}_2$ and solving we obtain:

$$\frac{\partial\tilde{S}_1}{\partial F} = -\left[\frac{A^1\frac{\partial\Delta\left(\check{S}_2,F\right)}{\partial F} + \frac{\partial\bar{S}^d\left(\check{S}_2,F\right)}{\partial F}}{A^1\frac{\partial\Delta\left(\check{S}_2,F\right)}{\partial\check{S}_2} + \frac{\partial\bar{S}^d\left(\check{S}_2,F\right)}{\partial\check{S}_2}}\right]\frac{\partial\bar{S}^d\left(\check{S}_2, F\right)}{\partial\check{S}_2}$$
$$\frac{\partial\tilde{S}_2}{\partial F} = \frac{A^2\frac{\partial\Delta\left(\tilde{S}_2,F\right)}{\partial F}}{1 - A^2\left[\frac{\partial\Delta\left(\tilde{S}_2,F\right)}{\partial\check{S}_2}\right]}$$

where

$$\frac{\partial\Delta\left(\check{S}_2, F\right)}{\partial F} = \frac{\sqrt{b}\left[a-c-2\sqrt{b\left(F-\check{S}_2\right)}\right]}{\sqrt{\left(F-\check{S}_2\right)}} > 0,\ \frac{\partial\Delta\left(\check{S}_2, F\right)}{\partial\check{S}_2} = \frac{-\partial\Delta\left(\check{S}_2, F\right)}{\partial F} < 0$$
$$\frac{\partial\bar{S}^d\left(\check{S}_2, F\right)}{\partial F} = \frac{-\left(a-c\right)}{\sqrt{b\left(F-\check{S}_2\right)}} + 4 < 0,\ \frac{\partial\bar{S}^d\left(\check{S}_2, F\right)}{\partial\check{S}_2} = \frac{-\partial\bar{S}^d\left(\check{S}_2, F\right)}{\partial F} > 0$$

Then:

$$\frac{\partial\tilde{S}_1}{\partial F} = \frac{-\partial\bar{S}^d\left(\check{S}_2, F\right)}{\partial\check{S}_2} > 0,\ \frac{\partial\tilde{S}_2}{\partial F} = \frac{A^2\frac{\partial\Delta\left(\tilde{S}_2,F\right)}{\partial F}}{1 - A^2\left[\frac{\partial\Delta\left(\tilde{S}_2,F\right)}{\partial\tilde{S}_2}\right]} > 0$$

Finally, we study the effect of F on $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2, F\right)$. Note that $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2, F\right)$ if and only if $\check{S}_2 = \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1, F\right) \geq \tilde{S}_2$. define

$$H(F) = \check{S}_2(F) - \tilde{S}_2(F)$$

and take the derivative of H with respect to F:

$$\frac{\partial H(F)}{\partial F} = \frac{\partial \check{S}_2(F)}{\partial F} - \frac{\partial \tilde{S}_2(F)}{\partial F} = 1 - \frac{A^2 \frac{\partial \Delta\left(\tilde{S}_2, F\right)}{\partial F}}{1 - A^2\left[\frac{\partial \Delta\left(\tilde{S}_2, F\right)}{\partial \check{S}_2}\right]} = \frac{1}{1 + A^2\left[\frac{\partial \Delta\left(\tilde{S}_2, F\right)}{\partial F}\right]} > 0$$

where we have used that $\frac{\partial \Delta\left(\tilde{S}_2, F\right)}{\partial \check{S}_2} = \frac{-\partial \Delta\left(\tilde{S}_2, F\right)}{\partial F}$ and $\frac{\partial \Delta\left(\tilde{S}_2, F\right)}{\partial F} > 0$. Thus, if $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2, F\right)$ holds, then $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2, F'\right)$ for $F' > F$.

Comparative statics with respect to c: Using the same procedure we employed for a change in F we obtain:

$$\frac{\partial \tilde{S}_1}{\partial c} = -\left[\frac{A^1 \frac{\partial \Delta\left(\check{S}_2, c\right)}{\partial c} + \frac{\partial \bar{S}^d\left(\check{S}_2, c\right)}{\partial c}}{A^1 \frac{\partial \Delta\left(\check{S}_2, c\right)}{\partial \check{S}_2} + \frac{\partial \bar{S}^d\left(\check{S}_2, c\right)}{\partial \check{S}_2}}\right] \frac{\partial \bar{S}^d\left(\check{S}_2, c\right)}{\partial \check{S}_2}$$

$$\frac{\partial \tilde{S}_2}{\partial c} = \frac{A^2 \frac{\partial \Delta\left(\tilde{S}_2, c\right)}{\partial F}}{1 - A^2\left[\frac{\partial \Delta\left(\tilde{S}_2, c\right)}{\partial \check{S}_2}\right]}$$

where

$$\Delta\left(\check{S}_2, c\right) = \frac{9(a-c)^2}{32} - \frac{\left[a - c - 2\sqrt{b\left(F - \check{S}_2\right)}\right]^2}{2}$$

$$\frac{\partial \Delta\left(\check{S}_2, c\right)}{\partial c} = \frac{7(a-c) - 32\sqrt{b\left(F - \check{S}_2\right)}}{16}$$

$$\frac{\partial \Delta\left(\check{S}_2, c\right)}{\partial \check{S}_2} = \frac{-\sqrt{b}\left[a - c - 2\sqrt{b\left(F - \check{S}_2\right)}\right]}{\sqrt{F - \check{S}_2}} < 0$$

and

$$\bar{S}^d\left(\check{S}_2, c\right) = \frac{(a-c)^2}{8b} - 2(a-c)\sqrt{\frac{F - \check{S}_2}{b}} + 4\left(F - \check{S}_2\right)$$

$$\frac{\partial \bar{S}^d(S, c)}{\partial c} = \frac{-(a-c) + 8\sqrt{b\left(F - \check{S}_2\right)}}{4b}$$

$$\frac{\partial \bar{S}^d\left(\check{S}_2, c\right)}{\partial \check{S}_2} = \frac{a - c - 4\sqrt{b\left(F - \check{S}_2\right)}}{\sqrt{b\left(F - \check{S}_2\right)}} > 0$$

Note that $\frac{\partial \tilde{S}_1}{\partial c} > 0$ if and only if $A^1 \frac{\partial \Delta(\check{S}_2,c)}{\partial c} + \frac{\partial \bar{S}^d(\check{S}_2,c)}{\partial c} < 0$ or, which is equivalent, $A^1 b < \frac{4}{7}$ *and* $\check{S}_2 > F - \frac{1}{b}\left[\frac{7(a-c)\left(\frac{4}{7}-A^1 b\right)}{32(1-A^1 b)}\right]^2$. Since $\tilde{S}_1 = \bar{S}^d\left(\check{S}_2, c\right)$, $\check{S}_2 > F - \frac{1}{b}\left[\frac{7(a-c)\left(\frac{4}{7}-A^1 b\right)}{32(1-A^1 b)}\right]^2$ if and only if $\tilde{S}_1 > \bar{S}^d\left(F - \frac{1}{b}\left[\frac{7(a-c)\left(\frac{4}{7}-A^1 b\right)}{32(1-A^1 b)}\right]^2, c\right)$. Thus, $\frac{\partial \tilde{S}_1}{\partial c} > 0$ if and only

$$A^1 b < \frac{4}{7} \textit{and } \tilde{S}_1 > \frac{\left[16\left(5A^1 b - 2\right)\left(1 - A^1 b\right) + 7\left(4 - 7A^1 b\right)^2\right]^2}{(16)^2 b\left(1 - A^1 b\right)^2}$$

$\frac{\partial \tilde{S}_2}{\partial c} > 0$ if and only if $\frac{\partial \Delta\left(\tilde{S}_2,F\right)}{\partial F} > 0$ or, which is equivalent,

$$\tilde{S}_2 > F - \frac{1}{b}\left[\frac{7(a-c)}{32}\right]^2$$

***Comparative statics with respect to** a*: It is easy to verify that $\frac{\partial \tilde{S}_1}{\partial a} = -\frac{\partial \tilde{S}_1}{\partial c}$ and $\frac{\partial \tilde{S}_2}{\partial a} = -\frac{\partial \tilde{S}_2}{\partial c}$. This completes the proof of Proposition 3. ■

A.3 Proof of Proposition 4

We begin reconsidering Proposition 1 when the maximum credible subsidy that G_2 can promise is $\rho \bar{S}$. Then, we prove a general version of Proposition 4. Finally, Proposition 4 in the text is deduced as a corollary of Proposition 4 (general version).

Let $S_2 \to \left[0, \bar{S}\right]$ denote the subsidy promised by G_2. Then, the subsidy that G_2 will actually pay if E enters and, hence, the credible component of S_2, is given by:

$$S_2^c = \min\left\{\rho \bar{S}, S_2\right\}$$

Proposition 1bis *Suppose that* $9c/7 \leq a \leq \left(6\sqrt{2}+3\right)c/7$ *and the maximum credible subsidy that* G_2 *can promise is* $\rho \bar{S}$*. Let*

$$\bar{\rho}_0^d = \frac{\bar{S}_0^d}{\bar{S}} \textit{ and } \bar{\rho}^b = \frac{\bar{S}^b}{\bar{S}}$$

1. *Suppose that* $\bar{\rho}_0^d \leq \rho < 1$*. Then:*
   (a) *If* $0 \leq S_2 \leq \bar{S}^b$ *entry is blocked,* $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ *and* $P = \frac{a+c}{2}$.
   (b) *If* $\bar{S}^b < S_2 < \bar{S}_0^d$ *entry is deterred,* $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b(F-S_2)}}{b}, 0\right)$ *and* $P = c + 2\sqrt{b\left(F - S_2\right)}$.
   (c) *If* $\bar{S}_0^d \leq S_2 \leq \bar{S}$ *and* $S_1 > \bar{S}^d\left(S_2^c\right)$ *entry is deterred,* $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b\left(F-S_2^c\right)}}{b}, 0\right)$ *and* $P = c + 2\sqrt{b\left(F - S_2^c\right)}$.
   (d) *If* $\bar{S}_0^d \leq S_2 \leq \bar{S}$ *and* $S_1 = \bar{S}^d\left(S_2^c\right)$*, then there are two equilibria: in one equilibrium entry is deterred, while in the other entry is accommodated. Under deterrence (accommodation),* (k_I, k_E, P) *is as in part c (e).*

(e) *If* $\bar{S}_0^d \leq S_2 \leq \bar{S}$ *and* $S_1 < \bar{S}^d(S_2^c)$ *entry is accommodated,* $(k_I, k_E) = \left(\frac{a-c}{2b}, \frac{a-c}{4b}\right)$ *and* $P = \frac{a+3c}{4}$.

2. *Suppose that* $\bar{\rho}^b < \rho < \bar{\rho}_0^d$. *Then,*

   (a) *If* $0 \leq S_2 \leq \bar{S}^b$ *entry is blocked,* $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ *and* $P = \frac{a+c}{2}$.

   (b) *If* $\bar{S}^b < S_2 \leq \bar{S}$ *entry is deterred,* $(k_I, k_E) = \left(\frac{a-c-2\sqrt{b\left(F-S_2^c\right)}}{b}, 0\right)$ *and* $P = c + 2\sqrt{b\left(F - S_2^c\right)}$.

3. *Suppose that* $0 < \rho \leq \bar{\rho}^b$. *Then, entry is blocked,* $(k_I, k_E) = \left(\frac{a-c}{2b}, 0\right)$ *and* $P = \frac{a+c}{2}$ *for all* $0 \leq S_2 \leq \bar{S}$.

Proof. From Proposition 1 we have: If $0 \leq S_2^c \leq \bar{S}^b$, then entry is blocked; if $\bar{S}^b < S_2^c < \bar{S}_0^d$, then entry is deterred; if $\bar{S}_0^d \leq S_2^c < \left(\bar{S}^d\right)^{-1}(S_1)$ (or, which is equivalent, $\bar{S}_0^d \leq S_2^c \leq \bar{S}$ and $S_1 > \bar{S}^d(S_2^c)$] entry is deterred; if $\bar{S}_0^d < S_2^c = \left(\bar{S}^d\right)^{-1}(S_1)$ [or which is equivalent, $\bar{S}_0^d \leq S_2^c \leq \bar{S}$ and $S_1 = \bar{S}^d(S_2^c)$], then entry is either deterred or accommodated; and, finally, if $\left(\bar{S}^d\right)^{-1}(S_1) < S_2^c \leq \bar{S}$ [or which is equivalent $\bar{S}_0^d \leq S_2^c \leq \bar{S}$ and $S_1 < \bar{S}^d(S_2^c)$] entry is accommodated. Let $\bar{\rho}^b = \bar{S}^b/\bar{S}$, $\bar{\rho}^d = \bar{S}_0^d/\bar{S}$. Then, Proposition 1bis follows by the definition of $S_2^c = \min\left\{\rho\bar{S}, S_2\right\}$. ■

Proposition 4 (*General version*) *Suppose that* $9c/7 \leq a \leq \left(6\sqrt{2}+3\right)c/7$, $A^1 b < 2\left(\sqrt{2}-1\right)$, *and the maximum credible subsidy that* G_2 *can promise is* $\rho\bar{S}$. *Let*

$$\bar{\rho}^b = \frac{\bar{S}^b}{\bar{S}} \text{ and } \bar{\rho}_0^d = \frac{\bar{S}_0^d}{\bar{S}}$$

1. *Suppose that* $\bar{\rho}_0^d \leq \rho < 1$.

   (a) *Suppose that* $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \rho\bar{S} - A^2\Delta\left(\rho\bar{S}\right)$. *Then, the set of equilibrium subsidies is given by* $S_1 = \bar{S}^d\left(\min\left\{S_2, \rho\bar{S}\right\}\right)$ *with:*

$S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$	*if* $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$
$S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$	*if* $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$ *and* $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$
$S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$	*if* $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$ *and* $\bar{S}^d\left(\tilde{S}_2\right) \leq \tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right)$
$S_2 \in \left[\rho\bar{S}, \bar{S}\right]$	*if* $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$ *and* $\tilde{S}_1 = \bar{S}^d\left(\rho\bar{S}\right)$
$S_2 \in \left[\tilde{S}_2, \bar{S}\right]$	*if* $B_1^M - B_1^D > A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$

   where $\tilde{S}_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ *is the unique solution to* $B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$ *and* $\tilde{S}_2 \in \left(\bar{S}_0^d, \bar{S}\right]$ *is the unique solution to* $B_2^D = \tilde{S}_2 - A^2\Delta\left(\tilde{S}_2\right)$. *Moreover, in all the equilibria in which* $S_2 \in \left[\tilde{S}_2, \bar{S}\right]$ *entry is deterred, while in all the equilibria in which* $S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$ *there is accommodated entry.*

(b) *Suppose that* $\rho\bar{S} - A^2\Delta\left(\rho\bar{S}\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$. *Then, the set of equilibrium subsidies is given by* $S_1 = \bar{S}^d\left(\min\left\{S_2, \rho\bar{S}\right\}\right)$ *with:*

$$\begin{array}{ll} S_2 \in \left[\bar{S}_0^d, \rho\bar{S}\right) & \text{if } B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right) \\ S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho\bar{S}\right) & \text{if } A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right) \\ S_2 \in \left[\rho\bar{S}, \bar{S}\right] & \begin{array}{l}\text{if } A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right) \\ \text{and } \tilde{S}_1 = \bar{S}^d\left(\rho\bar{S}\right)\end{array} \\ S_2 \in \left[\rho\bar{S}, \bar{S}\right] & \text{if } B_1^M - B_1^D > A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right) \end{array}$$

where $\tilde{S}_1 \in \left[0, \bar{S}^d\left(\bar{S}\right)\right]$ *is the unique solution to* $B_1^M - B_1^D = A^1\Delta\left(\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right) + \tilde{S}_1$. *Moreover, in all the equilibria in which* $S_2 \in \left[\rho\bar{S}, \bar{S}\right]$ *entry is deterred, while in all the equilibria in which* $S_2 \in \left[\bar{S}_0^d, \rho\bar{S}\right)$ *there is accommodated entry.*

2. *Suppose that* $\bar{\rho}^b < \rho \leq \bar{\rho}_0^d$. *Then, the set of equilibrium subsidies is given by* $S_1 = 0$ *and* $S_2 \in \left[\rho\bar{S}, \bar{S}\right]$. *Moreover, in equilibrium, entry is deterred.*

3. *Suppose that* $0 < \rho \leq \bar{\rho}^b$. *Then, the set of equilibrium subsidies is given by* $S_1 = 0$ *and* $S_2 \in \left[0, \bar{S}\right]$. *Moreover, in equilibrium, entry is blocked.*

Proof of Part 1: Suppose that $\bar{\rho}_0^d \leq \rho < 1$.

Selection criterion: From Proposition 1bis, if $S_1 = \bar{S}^d\left(S_2\right)$, deterrence and accommodation are both subgame perfect Nash equilibria. In such a case, the equilibrium with accommodation is selected when it strictly dominates the equilibrium with deterrence for G_2, provided that $\bar{S}^d\left(S_2\right) < \rho\bar{S}$. Otherwise, the economic equilibrium with deterrence is selected. Thus,

$$W_2\left(S_1, \left(\bar{S}^d\right)^{-1}\left(S_1\right)\right) = \begin{cases} \max\left\{\begin{array}{c} \frac{A^2\left[a-c-2\sqrt{b\left(F-\left(\bar{S}^d\right)^{-1}\left(S_1\right)\right)}\right]^2}{2}, \\ \frac{A^2 9(a-c)^2}{32} + B_2^D - \left(\bar{S}^d\right)^{-1}\left(S_1\right) \end{array}\right\} & \text{if } \left(\bar{S}^d\right)^{-1}\left(S_1\right) < \rho\bar{S} \\ \frac{A^2\left[a-c-2\sqrt{b\left(F-\rho\bar{S}\right)}\right]^2}{2} & \text{if } \left(\bar{S}^d\right)^{-1}\left(S_1\right) \geq \rho\bar{S} \end{cases}$$

Best response correspondence of G_2. Suppose that $\left(\bar{S}^d\right)^{-1}\left(S_1\right) \geq \rho\bar{S}$ (equivalently, $S_1 \geq \bar{S}^d\left(\rho\bar{S}\right)$). Then, employing the above selection criteria, the payoff function of G_2 as a function of $\left(S_1, S_2\right)$ is given by:

$$W_2\left(S_1, S_2\right) = \begin{cases} \frac{A^2(a-c)^2}{8} & \text{if } 0 \leq S_2 \leq \bar{S}^b \\ \frac{A^2\left[a-c-2\sqrt{b\left(F-S_2\right)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 < \bar{S}_0^d \\ \frac{A^2\left[a-c-2\sqrt{b\left(F-\min\left\{S_2,\rho\bar{S}\right\}\right)}\right]^2}{2} & \text{if } \bar{S}_0^d \leq \min\left\{S_2, \rho\bar{S}\right\} \leq \left(\bar{S}^d\right)^{-1}\left(S_1\right) \end{cases}$$

which adopts a maximum at $S_2 \in \left[\rho\bar{S}, \bar{S}\right]$. Suppose that $\left(\bar{S}^d\right)^{-1}\left(S_1\right) < \rho\bar{S}$ (equivalently, $S_1 < \bar{S}^d\left(\rho\bar{S}\right)$). Then, employing the above selection criterion, the payoff function of G_2 as a function of $\left(S_1, S_2\right)$ is given by:

$$W_2(S_1,S_2)=\begin{cases} \frac{A^2(a-c)^2}{8} & \text{if } 0\le S_2\le \bar{S}^b \\ \frac{A^2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^b< S_2<\bar{S}^d_0 \\ \frac{A^2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2} & \text{if } \bar{S}^d_0\le S_2<\left(\bar{S}^d\right)^{-1}(S_1) \\ \max\left\{ \begin{array}{c} \frac{A^2\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}, \\ \frac{A^2 9(a-c)^2}{32}+B_2^D-S_2 \end{array}\right\} & \text{if } S_2=\left(\bar{S}^d\right)^{-1}(S_1) \\ \frac{A^2 9(a-c)^2}{32}+B_2^D-S_2 & \text{if } \left(\bar{S}^d\right)^{-1}(S_1)<S_2\le\rho\bar{S} \\ \frac{A^2 9(a-c)^2}{32}+B_2^D-\rho\bar{S} & \text{if } \rho\bar{S}<S_2\le\rho\bar{S} \end{cases}$$

which adopts a maximum at $S_2=\left(\bar{S}^d\right)^{-1}(S_1)$. Thus, the best response correspondence of G_2 is given by:

$$S_2=\begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & \text{if } 0\le S_1<\bar{S}^d\left(\rho\bar{S}\right) \\ \left[\rho\bar{S},\bar{S}\right] & \text{if } S_1\ge\bar{S}^d\left(\rho\bar{S}\right) \end{cases}$$

Economic equilibrium selection under the best response correspondence of G_2. We must consider three possible cases:

Case 1: Suppose that $S_1\ge\bar{S}^d\left(\rho\bar{S}\right)$. Then, using Proposition 1.bis and the economic selection criterion, $S_2\in\left[\rho\bar{S},\bar{S}\right]$ leads to deterrence.

Case 2: Suppose that $0\le S_1<\bar{S}^d\left(\rho\bar{S}\right)$. To determine if $S_2=\left(\bar{S}^d\right)^{-1}(S_1)$ leads to deterrence or accommodated entry, we use lemma 2. There are two possible cases to consider:

Case 2.a: Suppose that $\bar{S}^d_0<\tilde{S}_2\le\rho\bar{S}$ or, which is equivalent, $\bar{S}^d_0-A^2\Delta\left(\bar{S}^d_0\right)<B_2^D\le\rho\bar{S}-A^2\Delta\left(\rho\bar{S}\right)$. Then, $S_2=\left(\bar{S}^d\right)^{-1}(S_1)$ leads to accommodated entry when $S_1<\bar{S}^d\left(\tilde{S}_2\right)$ and to deterrence when $S_1\ge\bar{S}^d\left(\tilde{S}_2\right)$.

Case 2.b: Suppose that $\rho\bar{S}<\tilde{S}_2\le\bar{S}$ or, which is equivalent, $\rho\bar{S}-A^2\Delta\left(\rho\bar{S}\right)<B_2^D\le\bar{S}-A^2\Delta\left(\bar{S}\right)$. Then, $S_2=\left(\bar{S}^d\right)^{-1}(S_1)$ leads to accommodated entry.

Best response correspondence of G_1. If $0\le S_2\le\bar{S}^b$, then, $W_1(S_1,S_2)=\left[A^1(a-c)^2/8\right]+B_1^M-S_1$, which is strictly decreasing in S_1. Thus, the best response to $0\le S_2\le\bar{S}^b$ is always $S_1=0$. Similarly, if $\bar{S}^b<S_2<\bar{S}^d_0$, then $W_1(S_1,S_2)=A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2/2+B_1^M-S_1$, which is strictly decreasing in S_1. Thus, the best response to $\bar{S}^b<S_2<\bar{S}^d_0$ is always $S_1=0$.

If $\bar{S}^d_0\le S_2\le\bar{S}$, there are two possible cases to consider:

Case 1: Suppose that $\bar{S}^d_0-A^2\Delta\left(\bar{S}^d_0\right)<B_2^D\le\rho\bar{S}-A^2\Delta\left(\rho\bar{S}\right)$. Then, $\bar{S}^d_0<\tilde{S}_2\le\rho\bar{S}$ and, hence,

$$W_1(S_1,S_2)=\begin{cases}\frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-S_1 & \text{if } \bar{S}_0^d\le S_2<\tilde{S}_2 \text{ and } S_1>\bar{S}^d(S_2)\\ \frac{A^1 9(a-c)^2}{32}+B_1^D & \text{if } \bar{S}_0^d\le S_2<\tilde{S}_2 \text{ and } S_1\le\bar{S}^d(S_2)\\ \frac{A^1\left[a-c-2\sqrt{b(F-S_2)}\right]^2}{2}+B_1^M-S_1 & \text{if } \tilde{S}_2\le S_2\le\rho\bar{S} \text{ and } S_1\ge\bar{S}^d(S_2)\\ \frac{A^1 9(a-c)^2}{32}+B_1^D & \text{if } \tilde{S}_2\le S_2\le\rho\bar{S} \text{ and } S_1<\bar{S}^d(S_2)\\ \frac{A^1\left[a-c-2\sqrt{b\left(F-\rho\bar{S}\right)}\right]^2}{2}+B_1^M-S_1 & \text{if } \rho\bar{S}<S_2\le\bar{S} \text{ and } S_1\ge\bar{S}^d\left(\rho\bar{S}\right)\\ \frac{A^1 9(a-c)^2}{32}+B_1^D & \text{if } \rho\bar{S}<S_2\le\bar{S} \text{ and } S_1<\bar{S}^d\left(\rho\bar{S}\right)\end{cases}$$

For $\bar{S}_0^d \le S_2 < \tilde{S}_2$, $W_1(S_1,S_2)$ adopts its maximum at $S_1 \in \left[0,\bar{S}^d(S_2)\right]$ if and only if $B_1^M - B_1^D \le A^1\Delta(S_2)+\bar{S}^d(S_2)$. Otherwise, there is no $S_1 \in \left[0,\bar{S}^d\left(\bar{S}\right)\right]$ that maximizes $W_1(S_1,S_2)$. For $\tilde{S}_2 \le S_2 \le \rho\bar{S}$, adopts its maximum at $S_1 = \bar{S}^d(S_2)$ if and only if $B_1^M - B_1^D \ge A^1\Delta(S_2)+\bar{S}^d(S_2)$, while it adopts its maximum at $S_1 \in \left[0,\bar{S}^d(S_2)\right)$ if and only if $B_1^M - B_1^D \le A^1\Delta(S_2)+\bar{S}^d(S_2)$. For $\rho\bar{S} < S_2 \le \bar{S}$, adopts its maximum at $S_1 = \bar{S}^d\left(\rho\bar{S}\right)$ if and only if $B_1^M - B_1^D \ge A^1\Delta\left(\rho\bar{S}\right)+\bar{S}^d\left(\rho\bar{S}\right)$, while it adopts its maximum at $S_1 \in \left[0,\bar{S}^d\left(\rho\bar{S}\right)\right)$ if and only if $B_1^M - B_1^D \le A^1\Delta\left(\rho\bar{S}\right)+\bar{S}^d\left(\rho\bar{S}\right)$. Therefore, the best response correspondence of G_1 is given by:

$$S_1=\begin{cases}0 & \text{if } 0\le S_2<\bar{S}_0^d\\ \left[0,\bar{S}^d(S_2)\right] & \text{if } \bar{S}_0^d\le S_2<\tilde{S}_2 \text{ and } B_1^M-B_1^D\le A^1\Delta(S_2)+\bar{S}^d(S_2)\\ \bar{S}^d(S_2) & \text{if } \tilde{S}_2\le S_2\le\rho\bar{S} \text{ and } B_1^M-B_1^D\ge A^1\Delta(S_2)+\bar{S}^d(S_2)\\ \left[0,\bar{S}^d(S_2)\right) & \text{if } \tilde{S}_2\le S_2\le\rho\bar{S} \text{ and } B_1^M-B_1^D\le A^1\Delta(S_2)+\bar{S}^d(S_2)\\ \left[0,\bar{S}^d\left(\rho\bar{S}\right)\right) & \text{if } \rho\bar{S}<S_2\le\bar{S} \text{ and } B_1^M-B_1^D\le A^1\Delta\left(\rho\bar{S}\right)+\bar{S}^d\left(\rho\bar{S}\right)\\ \bar{S}^d\left(\rho\bar{S}\right) & \text{if } \rho\bar{S}<S_2\le\bar{S} \text{ and } B_1^M-B_1^D\ge A^1\Delta\left(\rho\bar{S}\right)+\bar{S}^d\left(\rho\bar{S}\right)\end{cases}$$

Thus, employing Lemma 1, we must consider three possible subcases:

Case 1.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, the best response correspondence of G_1 is given by:

$$S_1=\begin{cases}0 & \text{if } 0\le S_2<\bar{S}_0^d\\ \left[0,\bar{S}^d(S_2)\right] & \text{if } \bar{S}_0^d\le S_2<\tilde{S}_2\\ \left[0,\bar{S}^d(S_2)\right) & \text{if } \tilde{S}_2\le S_2\le\rho\bar{S}\\ \left[0,\bar{S}^d\left(\rho\bar{S}\right)\right) & \text{if } \rho\bar{S}<S_2\le\bar{S}\end{cases}$$

Case 1.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \le B_1^M - B_1^D \le A^1\Delta\left(\rho\bar{S}\right)+\bar{S}^d\left(\rho\bar{S}\right)$. Then, the best response correspondence of G_1 is given by:

$$S_1=\begin{cases}0 & \text{if } 0\le S_2<\bar{S}_0^d\\ \left[0,\bar{S}^d(S_2)\right] & \text{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\le S_2<\tilde{S}_2\\ \bar{S}^d(S_2) & \text{if } \tilde{S}_2\le S_2\le\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\\ \left[0,\bar{S}^d(S_2)\right) & \text{if } \max\left\{\tilde{S}_2,\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right\}\le S_2\le\rho\bar{S}\\ \left[0,\bar{S}^d\left(\rho\bar{S}\right)\right) & \text{if } \rho\bar{S}<S_2\le\bar{S}\\ \bar{S}^d\left(\rho\bar{S}\right) & \text{if } \rho\bar{S}<S_2\le\bar{S} \text{ and } \tilde{S}_1=\bar{S}^d\left(\rho\bar{S}\right)\end{cases}$$

Case 1.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d\left(S_2\right) & \textit{if } \tilde{S}_2 \leq S_2 \leq \rho\bar{S} \\ \bar{S}^d\left(\rho\bar{S}\right) & \textit{if } \rho\bar{S} < S_2 \leq \bar{S} \end{cases}$$

Case 2: Suppose that $\rho\bar{S} - A^2\Delta\left(\rho\bar{S}\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$. Then, that $\rho\bar{S} < \tilde{S}_2 \leq \bar{S}$ and, hence,

$$W_1\left(S_1, S_2\right) = \begin{cases} \frac{A^1\left[a-c-2\sqrt{b\left(F-S_2\right)}\right]^2}{2} + B_1^M - S_1 & \textit{if } \bar{S}_0^d \leq S_2 < \rho\bar{S} \textit{ and } S_1 > \bar{S}^d\left(S_2\right) \\ \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } \bar{S}_0^d \leq S_2 < \rho\bar{S} \textit{ and } S_1 \leq \bar{S}^d\left(S_2\right) \\ \frac{A^1\left[a-c-2\sqrt{b\left(F-\rho\bar{S}\right)}\right]^2}{2} + B_1^M - S_1 & \textit{if } \rho\bar{S} \leq S_2 \leq \bar{S} \textit{ and } S_1 \geq \bar{S}^d\left(\rho\bar{S}\right) \\ \frac{A^1 9(a-c)^2}{32} + B_1^D & \textit{if } \rho\bar{S} \leq S_2 \leq \bar{S} \textit{ and } S_1 < \bar{S}^d\left(\rho\bar{S}\right) \end{cases}$$

Therefore, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(S_2\right)\right] & \textit{if } \bar{S}_0^d \leq S_2 < \rho\bar{S} \textit{ and } B_1^M - B_1^D \leq A^1\Delta\left(S_2\right) + \bar{S}^d\left(S_2\right) \\ \left[0, \bar{S}^d\left(\rho\bar{S}\right)\right) & \textit{if } \rho\bar{S} \leq S_2 \leq \bar{S} \textit{ and } B_1^M - B_1^D \leq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right) \\ \bar{S}^d\left(\rho\bar{S}\right) & \textit{if } \rho\bar{S} \leq S_2 \leq \bar{S} \textit{ and } B_1^M - B_1^D \geq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right) \end{cases}$$

Thus, employing Lemma 1, we must consider three possible subcases:

Case 2.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(S_2\right)\right] & \textit{if } \bar{S}_0^d \leq S_2 < \rho\bar{S} \\ \left[0, \bar{S}^d\left(\rho\bar{S}\right)\right) & \textit{if } \rho\bar{S} \leq S_2 \leq \bar{S} \end{cases}$$

Case 2.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d\left(S_2\right)\right] & \textit{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \rho\bar{S} \\ \left[0, \bar{S}^d\left(\rho\bar{S}\right)\right) & \textit{if } \rho\bar{S} \leq S_2 \leq \bar{S} \\ \bar{S}^d\left(\rho\bar{S}\right) & \textit{if } \rho\bar{S} \leq S_2 \leq \bar{S} \textit{ and } \tilde{S}_1 = \bar{S}^d\left(\rho\bar{S}\right) \end{cases}$$

Case 2.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & \textit{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d\left(\rho\bar{S}\right) & \textit{if } \rho\bar{S} \leq S_2 \leq \bar{S} \end{cases}$$

Nash equilibrium: We must consider two possible cases:

Case 1: Suppose that $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \rho\bar{S} - A^2\Delta\left(\rho\bar{S}\right)$.

Case 1.a: Suppose that $B_1^M - B_1^D < A^1\Delta\left(\bar{S}_0^d\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \text{if } \bar{S}_0^d \leq S_2 < \tilde{S}_2 \\ \left[0, \bar{S}^d(S_2)\right) & \text{if } \tilde{S}_2 \leq S_2 \leq \rho\bar{S} \\ \left[0, \bar{S}^d\left(\rho\bar{S}\right)\right) & \text{if } \rho\bar{S} < S_2 \leq \bar{S} \end{cases} \quad \text{and } S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & \text{if } 0 \leq S_1 < \bar{S}^d\left(\rho\bar{S}\right) \\ \left[\rho\bar{S}, \bar{S}\right] & \text{if } S_1 \geq \bar{S}^d\left(\rho\bar{S}\right) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \text{ and } S_2 \in \left[\bar{S}_0^d, \tilde{S}_2\right)$$

Moreover, in all these equilibria entry is accommodated.

Case 1.b: Suppose that $A^1\Delta\left(\bar{S}_0^d\right) \leq B_1^M - B_1^D \leq A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & \text{if } \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \tilde{S}_2 \\ \bar{S}^d(S_2) & \text{if } \tilde{S}_2 \leq S_2 \leq \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \\ \left[0, \bar{S}^d(S_2)\right) & \text{if } \max\left\{\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right\} \leq S_2 \leq \rho\bar{S} \\ \left[0, \bar{S}^d\left(\rho\bar{S}\right)\right) & \text{if } \rho\bar{S} < S_2 \leq \bar{S} \\ \bar{S}^d\left(\rho\bar{S}\right) & \text{if } \rho\bar{S} < S_2 \leq \bar{S} \text{ and } \tilde{S}_1 = \bar{S}^d\left(\rho\bar{S}\right) \end{cases}$$

$$S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & \text{if } 0 \leq S_1 < \bar{S}^d\left(\rho\bar{S}\right) \\ \left[\rho\bar{S}, \bar{S}\right] & \text{if } S_1 \geq \bar{S}^d\left(\rho\bar{S}\right) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \text{ and } S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$$
$$S_1 = \bar{S}^d(S_2) \text{ and } S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$$
$$S_2 \in \left(\rho\bar{S}, \bar{S}\right] \text{ and } S_1 = \bar{S}^d\left(\rho\bar{S}\right) = \tilde{S}_1$$

Moreover, in all the equilibria in which $\tilde{S}_1 \geq \bar{S}^d\left(\tilde{S}_2\right)$ entry is deterred, while in all the equilibria in which $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, entry is accommodated. In the equilibrium in which $S_2 \in \left(\rho\bar{S}, \bar{S}\right]$ and $S_1 = \bar{S}^d\left(\rho\bar{S}\right) = \tilde{S}_1$, entry is deterred.

Case 1.c: Suppose that $B_1^M - B_1^D > A^1\Delta\left(\rho\bar{S}\right) + \bar{S}^d\left(\rho\bar{S}\right)$. Then, best response correspondences are given by:

$$S_1 = \begin{cases} 0 & \text{if } 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d(S_2) & \text{if } \tilde{S}_2 \leq S_2 \leq \rho\bar{S} \\ \bar{S}^d\left(\rho\bar{S}\right) & \text{if } \rho\bar{S} < S_2 \leq \bar{S} \end{cases} \quad \text{and } S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & \text{if } 0 \leq S_1 < \bar{S}^d\left(\rho\bar{S}\right) \\ \left[\rho\bar{S}, \bar{S}\right] & \text{if } S_1 \geq \bar{S}^d\left(\rho\bar{S}\right) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \;\; and \;\; S_2 \in \left[\tilde{S}_2, \rho\bar{S}\right)$$
$$S_1 = \bar{S}^d(\rho\bar{S}) \;\; and \;\; S_2 \in \left[\rho\bar{S}, \bar{S}\right]$$

Moreover, in all these equilibria entry is deterred.

Case 2: Suppose that $\rho\bar{S} - A^2\Delta(\rho\bar{S}) < B_2^D \leq \bar{S} - A^2\Delta(\bar{S})$. $\rho\bar{S} < \tilde{S}_2 \leq \bar{S}$

Case 2.a: Suppose that $B_1^M - B_1^D < A^1\Delta(\bar{S}_0^d)$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & if \; 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & if \; \bar{S}_0^d \leq S_2 < \rho\bar{S} \\ \left[0, \bar{S}^d(\rho\bar{S})\right) & if \; \rho\bar{S} \leq S_2 \leq \bar{S} \end{cases} \quad and \;\; S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & if \; 0 \leq S_1 < \bar{S}^d(\rho\bar{S}) \\ \left[\rho\bar{S}, \bar{S}\right] & if \; S_1 \geq \bar{S}^d(\rho\bar{S}) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \;\; and \;\; S_2 \in \left[\bar{S}_0^d, \rho\bar{S}\right)$$

Moreover, in all these equilibria entry is accommodated.

Case 2.b: Suppose that $A^1\Delta(\bar{S}_0^d) \leq B_1^M - B_1^D \leq A^1\Delta(\rho\bar{S}) + \bar{S}^d(\rho\bar{S})$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & if \; 0 \leq S_2 < \bar{S}_0^d \\ \left[0, \bar{S}^d(S_2)\right] & if \; \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right) \leq S_2 < \rho\bar{S} \\ \left[0, \bar{S}^d(\rho\bar{S})\right) & if \; \rho\bar{S} \leq S_2 \leq \bar{S} \\ \bar{S}^d(\rho\bar{S}) & if \; \rho\bar{S} \leq S_2 \leq \bar{S} \; and \; \tilde{S}_1 = \bar{S}^d(\rho\bar{S}) \end{cases}$$
$$S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & if \; 0 \leq S_1 < \bar{S}^d(\rho\bar{S}) \\ \left[\rho\bar{S}, \bar{S}\right] & if \; S_1 \geq \bar{S}^d(\rho\bar{S}) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(S_2) \;\; and \;\; S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho\bar{S}\right)$$
$$S_1 = \bar{S}^d(\rho\bar{S}) = \tilde{S}_1 \;\; and \;\; S_2 \in \left[\rho\bar{S}, \bar{S}\right]$$

Moreover, in all the equilibria in which $\tilde{S}_1 < \bar{S}^d(\rho\bar{S})$, entry is accommodated, while in the equilibrium in which $S_2 \in \left(\rho\bar{S}, \bar{S}\right]$ and $S_1 = \bar{S}^d(\rho\bar{S}) = \tilde{S}_1$, entry is deterred.

Case 2.c: Suppose that $B_1^M - B_1^D > A^1\Delta(\rho\bar{S}) + \bar{S}^d(\rho\bar{S})$. Then, the best response correspondence of G_1 is given by:

$$S_1 = \begin{cases} 0 & if \; 0 \leq S_2 < \bar{S}_0^d \\ \bar{S}^d(\rho\bar{S}) & if \; \rho\bar{S} \leq S_2 \leq \bar{S} \end{cases} \quad and \;\; S_2 = \begin{cases} \left(\bar{S}^d\right)^{-1}(S_1) & if \; 0 \leq S_1 < \bar{S}^d(\rho\bar{S}) \\ \left[\rho\bar{S}, \bar{S}\right] & if \; S_1 \geq \bar{S}^d(\rho\bar{S}) \end{cases}$$

Therefore, the set of Nash equilibrium subsidies is given by:

$$S_1 = \bar{S}^d(\rho\bar{S}) \;\; and \;\; S_2 \in \left[\rho\bar{S}, \bar{S}\right]$$

Moreover, in all these equilibria entry is deterred.

Proof of Part 2: Suppose that $\bar{\rho}^b < \rho < \bar{\rho}_0^d$. Then, employing Proposition 1bis (Part 2), the consumer surplus of each country as a function of (S_1, S_2) is given by:

$$CS_j(S_1,S_2) = A^j \begin{cases} \frac{(a-c)^2}{8} & \text{if } 0 \le S_2 \le \bar{S}^b \\ \frac{\left[a-c-2\sqrt{b\left(F-\min\{\rho\bar{S},S_2\}\right)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 \le \bar{S} \end{cases}$$

while the geopolitical payoff of each global power as a function of (S_1, S_2) is given by:

$$B_1(S_1,S_2) = \begin{cases} B_1^M - S_1 & \text{if } 0 \le S_2 \le \bar{S}^b \\ B_1^M - S_1 & \text{if } \bar{S}^b < S_2 \le \bar{S} \end{cases}, \quad B_2(S_1,S_2) = \begin{cases} 0 & \text{if } 0 \le S_2 \le \bar{S}^b \\ 0 & \text{if } \bar{S}^b < S_2 \le \bar{S} \end{cases}$$

Therefore, the payoff function of each global power is given by:

$$W_1(S_1,S_2) = \begin{cases} \frac{A^1(a-c)^2}{8} + B_1^M - S_1 & \text{if } 0 \le S_2 \le \bar{S}^b \\ \frac{A^1\left[a-c-2\sqrt{b\left(F-\min\{\rho\bar{S},S_2\}\right)}\right]^2}{2} + B_1^M - S_1 & \text{if } \bar{S}^b < S_2 \le \bar{S} \end{cases}$$

$$W_2(S_1,S_2) = \begin{cases} \frac{A^2(a-c)^2}{8} & \text{if } 0 \le S_2 \le \bar{S}^b \\ \frac{A^2\left[a-c-2\sqrt{b\left(F-\min\{\rho\bar{S},S_2\}\right)}\right]^2}{2} & \text{if } \bar{S}^b < S_2 \le \bar{S} \end{cases}$$

Best response correspondence of G_1. Fix $S_2 \ge 0$. Suppose that $0 \le S_2 \le \bar{S}^b$. Then, $W_1(S_1,S_2) = \left[A^1(a-c)^2/8\right] + B_1^M - S_1$, which is strictly decreasing in S_1. Thus, the best response to $0 \le S_2 \le \bar{S}^b$ is $S_1 = 0$. Suppose that $\bar{S}^b < S_2 \le \bar{S}$. Then, $W_1(S_1,S_2) = A^1\left[a-c-2\sqrt{b\left(F-\min\{\rho\bar{S},S_2\}\right)}\right]^2/2 + B_1^M - S_1$, which is strictly decreasing in S_1. Thus, the best response to $\bar{S}^b < S_2 \le \bar{S}$ is $S_1 = 0$.

Best response correspondence of G_2. Fix $S_1 \ge 0$. $W_2(S_1,S_2)$ is a continuous function of S_2 for all $S_2 \in \left[0,\bar{S}\right]$ (in particular, $W_2(S_1,S_2)$ is continuous for $S_2 = \bar{S}^b$); $W_2(S_1,S_2)$ is a constant for all $S_2 \in \left[0,\bar{S}^b\right]$; $W_2(S_1,S_2)$ is strictly increasing in S_2 for all $S_2 \in \left[\bar{S}^b,\rho\bar{S}\right]$; and $W_2(S_1,S_2)$ is constant for all $S_2 \in \left[\rho\bar{S},\bar{S}\right]$. Thus, the best response to $S_1 \ge 0$ is $S_2 = \left[\rho\bar{S},\bar{S}\right]$.

Nash equilibrium. The set of Nash equilibrium profiles is given by $S_1 = 0$ and $S_2 \in \left[\rho\bar{S},\bar{S}\right]$.

Most preferred equilibrium for each global power. In any Nash equilibrium it must be the case that $S_1 = 0$, which implies that the payoffs of the global powers as a function of the equilibrium profile of subsidies are given by:

$$W_1(0,S_2) = \frac{A^1\left[a-c-2\sqrt{b\left(F-\rho\bar{S}\right)}\right]^2}{2} + B_1^M$$

$$W_2(0,S_2) = \frac{A^2\left[a-c-2\sqrt{b\left(F-\rho\bar{S}\right)}\right]^2}{2}$$

Thus, G_1 and G_2 are indifferent among the Nash equilibrium profiles (S_1, S_2). This completes the proof of Proposition 4.2.

Proof of Part 3: Suppose that $0 < \rho \le \bar{\rho}^b$. Then, employing Proposition 1bis (Part 3), the consumer surplus of each country as a function of (S_1, S_2) is given by:

$$CS_j(S_1, S_2) = A^j \frac{(a-c)^2}{8}$$

while the geopolitical payoff of each global power as a function of (S_1, S_2) is given by:

$$B_1(S_1, S_2) = B_1^M - S_1,\ B_2(S_1, S_2) = 0$$

Therefore, the payoff function of each global power is given by:

$$W_1(S_1, S_2) = \frac{A^1(a-c)^2}{8} + B_1^M - S_1,\ W_2(S_1, S_2) = \frac{A^2(a-c)^2}{8}$$

Best response correspondence of G_1. Fix $S_2 \ge 0$. Then, $W_1(S_1, S_2) = \left[A^1(a-c)^2/8\right] + B_1^M - S_1$, which is strictly decreasing in S_1. Thus, the best response to $S_2 \ge 0$ is $S_1 = 0$.

Best response correspondence of G_2. Fix $S_1 \ge 0$. Then, $W_2(S_1, S_2) = A^2(a-c)^2/8$, which does not depend on S_2. Thus, the best response to $S_1 \ge 0$ is $S_2 \in \left[0, \bar{S}\right]$.

Nash equilibrium. The set of Nash equilibrium profiles is given by $S_1 = 0$ and $S_2 \in \left[0, \bar{S}\right]$.

Most preferred equilibrium for each global power: In any Nash equilibrium it must be the case that $S_1 = 0$, which implies that the payoffs of the global powers as a function of the equilibrium profile of subsidies are given by:

$$W_1(0, S_2) = \frac{A^1(a-c)^2}{8} + B_1^M,\ W_2(0, S_2) = \frac{A^2(a-c)^2}{8}$$

Thus, G_1 and G_2 are indifferent among the Nash equilibrium profiles (S_1, S_2). This completes the proof of Proposition 4.3. ■

Proposition 4 (***Simplified version in text***) *Suppose that* $9c/7 \le a \le \left(6\sqrt{2}+3\right)c/7$, $A^1 b < 2\left(\sqrt{2}-1\right)$, *and the maximum credible subsidy that* G_2 *can promise is* $\rho\bar{S}$. *Let*

$$\bar{\rho}^b = \frac{\bar{S}^b}{\bar{S}} \text{ and } \bar{\rho}_0^d = \frac{\bar{S}_0^d}{\bar{S}}$$

1. *Suppose that* $\bar{\rho}_0^d \le \rho < 1$, $B_2^D \in \left(\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right), \bar{S} - A^2\Delta\left(\bar{S}\right)\right]$ *and* $B_1^M - B_1^D \in \left[A^1\Delta\left(\bar{S}_0^d\right), A^1\Delta\left(\bar{S}\right) + \bar{S}^d\left(\bar{S}\right)\right]$.
   (a) *If* $\tilde{S}_1 \ge \bar{S}^d\left(\rho\bar{S}\right)$, *then the equilibrium subsidy profiles are those that satisfy* $S_1 = \bar{S}^d\left(\rho\bar{S}\right)$ *and* $S_2 \in \left[\rho\bar{S}, \bar{S}\right]$. *Moreover, in all these equilibria entry is deterred.*
   (b) *If* $\tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right)$ *and* $\tilde{S}_2 \le \rho\bar{S}$, *then Proposition 3 holds.*
   (c) *If* $\tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right)$ *and* $\tilde{S}_2 > \rho\bar{S}$, *then the equilibrium subsidy profiles are those that satisfy* $S_1 = \bar{S}^d(S_2)$ *and* $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho\bar{S}\right)$. *Moreover, in all these equilibria there is accommodated entry.*

2. *Suppose that* $\bar{\rho}^b < \rho \leq \bar{\rho}_0^d$. *Then, the set of equilibrium subsidies is given by* $S_1 = 0$ *and* $S_2 \in [\rho\bar{S}, \bar{S}]$. *Moreover, in equilibrium, entry is deterred.*

3. *Suppose that* $0 < \rho \leq \bar{\rho}^b$. *Then, the set of equilibrium subsidies is given by* $S_1 = 0$ *and* $S_2 \in [0, \bar{S}]$. *Moreover, in equilibrium, entry is blocked.*

Proof of Part 1: The proof of Part 1 is almost immediate from Proposition 4 (general version). We must consider several cases:

Case 1: Suppose that $\bar{S}_0^d - A^2\Delta\left(\bar{S}_0^d\right) < B_2^D \leq \rho\bar{S} - A^2\Delta\left(\rho\bar{S}\right)$ (i.e., $\tilde{S}_2 \leq \rho\bar{S}$). Then, from Proposition 4.1.a (general version) we have:

Case 1.a: If $\tilde{S}_1 \geq \bar{S}^d\left(\rho\bar{S}\right)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d\left(\rho\bar{S}\right)$ and $S_2 \in [\rho\bar{S}, \bar{S}]$. Moreover, in all these equilibria entry is deterred.

Case 1.b: If $\bar{S}^d\left(\tilde{S}_2\right) \leq \tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d\left(S_2\right)$ and $S_2 \in \left[\tilde{S}_2, \left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right)\right]$. Moreover, in all these equilibria entry is deterred. That is, Proposition 3.1 holds.

Case 1.c: If $\tilde{S}_1 < \bar{S}^d\left(\tilde{S}_2\right)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d\left(S_2\right)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \tilde{S}_2\right)$. Moreover, in all these equilibria there is accommodated entry. That is, Proposition 3.2 holds.

Case 2: Suppose that $\rho\bar{S} - A^2\Delta\left(\rho\bar{S}\right) < B_2^D \leq \bar{S} - A^2\Delta\left(\bar{S}\right)$ (i.e., $\tilde{S}_2 > \rho\bar{S}$). Then, from Proposition 4.1.b (general version) we have:

Case 2.a: If $\tilde{S}_1 \geq \bar{S}^d\left(\rho\bar{S}\right)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d\left(\rho\bar{S}\right)$ and $S_2 \in [\rho\bar{S}, \bar{S}]$. Moreover, in all these equilibria entry is deterred.

Case 2.b: If $\tilde{S}_1 < \bar{S}^d\left(\rho\bar{S}\right)$, then the equilibrium subsidy profiles are those that satisfy $S_1 = \bar{S}^d\left(S_2\right)$ and $S_2 \in \left[\left(\bar{S}^d\right)^{-1}\left(\tilde{S}_1\right), \rho\bar{S}\right)$. Moreover, in all these equilibria there is accommodated entry.

Proof of Parts 2 and 3: Parts 2 and 3 are identical to Proposition 4.2 and 4.3 (general version). ■