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ABSTRACT

This paper studies voting in shareholders meetings. We focus on the informational efficiency of different voting mechanisms, taking into account that they affect both management's incentives before the meeting and shareholders' decisions at the meeting. We first focus on the case in which the management does not affect the proposal being voted on. We prove that, for any distribution of shareholdings, the one-share-one-vote mechanism (1S1V) dominates the one-person-one-vote mechanism (1P1V), independently of whether or how shareholdings correlate with information accuracy. We also show that 1S1V becomes efficient only if votes are fully divisible. Second, we consider the case in which the management decides whether to put the proposal to a vote. The properties of a voting mechanism then depend both on its voting efficiency and on how it affects managers' incentives to select good proposals. We uncover a trade-off between selection and voting efficiency underlying the comparison of 1S1V and 1P1V: the higher voting efficiency of 1S1V implies worse selection incentives. In some cases, the negative effect of worse selection incentives on shareholders' welfare can be large enough to wash out the higher voting efficiency of 1S1V.

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1 Introduction

This paper studies voting in shareholders meetings. Given that shareholders typically hold different numbers of shares,[1] corporations face a non-trivial choice of how to allocate voting rights. Should each shareholder be allocated a unique vote, following the *one-person-one-vote principle*? Or should shareholders instead hold multiple votes depending on the number of shares they own, following the *one-share-one-vote principle*? Nowadays, most corporations follow the latter approach and attach one vote to each share.[2] Proponents argue that this fosters good governance by aligning voting power and economic incentives, among other reasons (see Burkart and Lee 2008 for a thorough discussion of the literature). We consider a complementary perspective focusing on which allocations of voting rights (i.e., voting mechanisms) foster information aggregation, and hence produce the best outcome for shareholders.[3] Crucial to this question is how different voting mechanisms affect both shareholders' decisions at the meeting and the management incentives before the meeting.

In the first part of the paper, we focus on the meeting, taking the management proposal as given.[4] Since we allow for heterogeneity in shareholdings, shareholders differ in how much they care about the performance of the firm, and hence the decision at the meeting. Yet, in line with the literature on shareholders meetings, we focus on cases in which (most) shareholders have state-contingent preferences: if the management proposal increases the value of the firm, they are all better off if approved. Otherwise, they prefer to stick to the status quo. The issue is that, at the time of the vote, shareholders are only imperfectly informed about the desirability of the proposal. Moreover, some shareholders may be more precisely informed than others.

We establish two main results for this case with an exogenous proposal. First, we compare mechanisms with a finite ballot space in terms of their information aggregation efficiency (i.e., the likelihood of selection of the best alternative). We show that, for any

[1] For instance, 96% of the firms in a representative sample of US firms have at least one *blockholder*, i.e., a shareholder who own more than 5% of the outstanding shares (Holderness 2009). Similarly, as Edmans and Holderness (2017) discuss, many institutional investors hold less than 5% of the outstanding shares of a given firm but still hold a very substantial number of shares. By contrast, retail shareholders hold much fewer shares of a given firm. Moreover, the dispersion of shareholdings among retail shareholders is substantial (Brav et al. 2019).

[2] The hegemony of the one-share-one-vote principle is a 20th century phenomenon. In the 19th century most corporations used to restrict the voting power of any given shareholder; with many of them employing one-person-one-vote mechanisms (see, e.g., Hansmann and Pargendler 2013). The more recent trends goes in the opposite direction: deviations from the one-share-one-vote principle are now mostly progressive, e.g., dual-class stock, instead of regressive (see, e.g., Adams and Ferreira 2008, and Hayden and Bodie 2008).

[3] The heterogeneity in shareholdings, and hence the question of which voting mechanism to use, has been overlooked by most of the literature on voting in shareholders meetings. Bar-Isaac and Shapiro (2020) is a notable exception.

[4] As discussed in Christoffersen et al. (2007) and Yermack (2010), management's proposals include, e.g., possible mergers and acquisitions, the issuance of new shares, the sale of the firm, amendments to governance procedures, changes in voting rights of directors, and new compensation package for directors.

distribution of shareholdings and any informational asymmetry among shareholders, a voting mechanism with a richer ballot space dominates a voting mechanism with a poorer one. This is because shareholders can use the richer ballot space to adjust their impact on the voting outcome to the accuracy of their information. It directly follows from that result that the one-share-one-vote mechanism (*1S1V*) dominates the one-person-one-vote mechanism (*1P1V*), simply because the former has a richer ballot space.[5] If there is a positive correlation between shareholdings and information accuracy, this result is quite intuitive: under *1S1V* more informed shareholders have a larger influence over the outcome than under *1P1V*, and hence outcomes are better. What we prove, though, is that *1S1V* outperforms *1P1V* even when there is zero or negative correlation between information accuracy and shareholdings. Key to this result is that, under *1S1V*, shareholders have the possibility to vote some of their shares and abstain on others.[6]

However, because there is not necessarily a perfect correlation between the number of shares held by a shareholder and how well-informed she is, *1S1V* is not efficient (i.e., it does not guarantee full-information equivalence). This leads to our second main result: the one-share-one-vote mechanism with divisible votes (*1S1V-D*) is efficient.[7] This is so because, under this mechanism, all shareholders have the ability to reveal their information fully, even if they hold only a small number of shares. The efficiency of *1S1V-D* is robust to the presence of (i) partisan shareholders, (ii) super-majority thresholds, (iii) ambiguity about the information technology of other shareholders, and (iv) endogenous acquisition of information by shareholders.

In the second part of the paper, we explore how voting mechanisms affect the management's incentives to shape the proposal before the meeting. We focus on the extensive margin: the management decides whether to put a proposal to the vote after observing a signal about its quality. If the management blocks the proposal, then the status quo remains. The properties of a voting mechanism then depend both on (i) *selection* (i.e., the incentives it provides the management to select good proposals and veto bad ones), and (ii) *voting efficiency* (i.e., the quality of information aggregation at the meeting).

Following the corporate governance and finance literature, we allow for conflict between shareholders and the management (see Tirole 2005 and references therein). We consider two specific dimensions of conflict. First, the manager may be *misaligned*, in

[5] Under *1S1V*, shareholders are endowed with one point per share they own, and they can allocate these points in favor or against the proposal, or abstain. Under *1P1V*, shareholders are endowed with only one point. Under both rules, the proposal is approved if the number of points in favor is larger than a predetermined (super-)majority threshold (50% in the case of simple majority).

[6] The standard version of *1S1V* that we consider, which allows for partial abstention, also dominates its rigid counterpart (*1S1V-R*) under which shareholders have to vote either all their shares in favor of the proposal or all their shares against. As discussed in Bar-Isaac and Shapiro (2020), both partial and full abstention are costly (and maybe even illegal) for some types of shareholders.

[7] Under *1S1V-D* shareholders can, for every share they own, allocate any fraction of point between 0 and 1 in favor or against the management's proposal. The proposal is approved if and only if the total number of points in favor is larger than the ones against.

the sense that she wants the proposal to be adopted even when it is undesirable for the shareholders.[8] Second, the manager may incur a (reputation) cost if a proposal is rejected at the shareholder meeting.[9]

We uncover a trade-off between selection and voting efficiency underlying the comparison of *1S1V* and *1P1V* mechanisms. In particular, we find that the higher voting efficiency under *1S1V* implies worse selection incentives for managers, and conversely for *1P1V*. Higher voting efficiency reduces the incentives of an aligned manager to veto any reform because she trusts that the shareholders will choose correctly with high probability. Therefore, the proposals she puts forward under *1S1V* are, on average, inferior to the proposals she puts forward under *1P1V*. As we show, this tradeoff is preserved even when the manager is misaligned with some probability, and when rejection is costly. We find that, in some cases, the negative effect of worse selection incentives on shareholders' welfare can be large enough to wash out the higher voting efficiency of *1S1V*. This can only happen when the manager is misaligned and suffers a reputational cost when her proposal is rejected.

While our focus is on shareholders meetings, some of our results apply beyond that setting. For instance, mechanisms with rich ballot spaces are often used to aggregate the opinions of judges that have to select a winner in sport competitions (e.g., gymnastic and ice skating). Our results about exogenous proposal are relevant to that case. The main difference is that, in most of those competitions, judges have to evaluate more than two contestants. Yet, we can show that, under some conditions, voting mechanisms with rich ballot spaces feature similarly desirable information aggregation properties in multi-winner elections. Our results also suggest that the adoption of such mechanisms could be desirable in various situations in which some voters may be better informed than others. These include juries, expert panels, and debt restructuring votes. In some of these cases, when one of the players has agenda power, our results about the effect of the richness of the ballot space on the incentives of the proposers may also be relevant.

[8] For instance, as discussed in Becht et al. (2016), there is empirical evidence showing that a substantial share of corporate acquisisions are associated with negative returns for acquirer shareholders. One explanation is that, in the case of M&A, managerial wealth and shareholder wealth are decoupled (Grinstein and Hribar 2004; Harford and Li 2007; Fu, Lin, and Officer 2013). Another area of conflict is Say-on-Pay proposals (Cunat, Gine, and Guadalupe 2016). Finally, Bach and Metzger (2016) and Babenko, Choi, and Sen (2019) find evidence that managers manipulate the voting process to increase the success rate of their proposals. Those manipulations appear to be value-destroying. For a more thorough discussion of conflict between managers and shareholders see, e.g., Tirole (2005, Chapter 1).

[9] Becht et al. (2016) and Gantchev and Giannetti (2020) mention the existence of such reputational costs but we are not aware of direct evidence. However, there is suggestive evidence. For instance, Cai, Garner, and Walking (2009) and Aggarwal, Dahiya, and Prabhala (2017) find that dissent votes in uncontested director elections have negative consequences for both directors and executives. The rejection of a management proposal at the meeting could have similar implications for the management. Also, Li et al. (2018) find that managers trying to acquire another corporation implement various strategies to avoid shareholder voting on the acquisition, and hence the risk of an embarassing rebuke at the shareholder meeting.

1.1 Related Literature

There is a large empirical literature studying the effect of shareholder voting on firms performance and management's behavior. Overall, this literature finds that control by shareholders (i) affects positively firms' performance, and (ii) is key to provide proper incentives to the management.[10] The specifics of corporate governance rules and procedures appear to play an important role. As summarized by Yermack (2010, p. 106): research that studies the "[...] general effects of voting restrictions on firm value and performance, often [finds] that firms perform worse when the shareholder franchise is curtailed [...]. Notable recent papers in this large literature include Gompers et al. (2003), which examines a range of takeover defenses and voting restrictions; Bebchuk and Cohen (2005) and Faleye (2007), both of which focus on staggered boards; and Gompers et al. (2009), which studies dual-class voting structures."[11] More directly connected to us, Burkart and Lee (2008) review the theoretical literature on the one-share-one-vote principle and highlight three classes of effects of the security-voting structure (aka voting rules): effects on takeovers, effects on incentives of blockholders, and effects on the firms' choice of ownership and financing.[12] We complement that literature by highlighting the effect of voting mechanisms on the quality of decisions at shareholders meetings, and how that shapes managers' incentives.

The exogenous proposal part of our paper contributes to the literature on information aggregation in committees and elections with exogenous alternatives (see, e.g., Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1996, 1997, 1998, Fey 1997, Myerson 1998, Mandler 2012; Bouton and Castanheira 2012, Bhattacharya 2013, McMurray 2013, Bouton et al. 2018, Barelli et al. 2019, Herrera et al. 2019, and Bar-Isaac and Shapiro 2020). The closest paper to ours is Bar-Isaac and Shapiro (2020). They consider a setup with one blockholder who owns many shares, and many retail shareholders who own one share each. We generalize their model, and their result of the desirability of partial abstention, by considering any distribution of shares among shareholders, including but not limited to multiple blockholders. Two other key differences are that, first, we compare

[10] For instance, Appel, Gormley, and Keim (2016) find that even passive investors affect positively firms' longer-term performance through voting; Fos, Li, and Tsoutsoura (2018) find that the proximity to an election affects the behavior of directors; Li et al. (2018) finds that, in the US, shareholder voting help mitigate agency problems that plague corporate acquisitions; Becht et al. (2016) find that shareholder voting has a substantial positive effect on the quality of acquisitions by firms in the UK; Richardson (2000) finds a positive relationship between information asymmetry between managers and shareholders, which impedes the ability of the latter to control efficiently the former, and earnings management (i.e., funky accounting by managers); Cai et al. (2006) and Aggarwal et al. (2019) focus on directors elections in US firms and find that even uncontested elections affect the firms and directors in various dimensions; Conyon and Sadler (2010), Ferri and Maber (2012), and Alissa (2015) study the effects of say-on-pay votes in the UK and find that they constraint the size and the structure of top managers' pay.

[11] Note however that Frankenreiter et al. (2021) have recently cast some serious doubts on the reliability of the datasets used in this literature.

[12] See Adams and Ferreira (2008) for a survey of the empirical literature on the topic.

the properties of various voting mechanisms in our generalized setup. This includes the identification of the optimal voting mechanisms for any shareholdings. Second, we study the information aggregation properties of voting mechanisms taking into account of their effects on the management's incentives. That allows us to identify situations in which partial abstention, through its effects on the management's incentives, is detrimental to information aggregation.

The endogenous proposal part of our paper contributes to the literature on information aggregation with endogenous alternatives. We are only aware of two papers exploring that question: Henry (2008), and Bond and Eraslan (2010). The latter is the closest to our paper. It stresses the importance of endogenizing the proposal when one studies the information properties of a voting system. But, given our focus on shareholders meetings, our model and insights are quite different than those in Bond and Eraslan (2010). For instance, we permit the distribution of votes to depend on the shareholding structure. This allows us to compare *1P1V* to *1S1V* mechanisms instead of focusing on different types of *1P1V* mechanisms such as majority and unanimity rules, as Bond and Eraslan (2010) do. Moreover, by comparing two mechanisms which admit an unambiguous ranking in terms of voting efficiency (*1S1V* and *1P1V*), we uncover a general tradeoff between voting efficiency and selection incentives that is not possible to detect when limiting attention to classes of *1P1V* mechanisms.[13] Also, we focus on the extensive margin of action for the manager, instead of the intensive margin in Bond and Eraslan (2010). That allows us to show that selection incentives can upset the welfare ordering of two mechanisms even if the manager only controls the agenda and has limited power to adjust the details of the proposal.

2 Model

In this section we present our baseline model. Section 4.3 proves the robustness of our results to various extensions of the model. Section 6 provides a discussion of some of the key assumptions.

Consider a firm with a set $N = \{1, 2, ..., n\}$ of shareholders, with $n > 2$, and $m \geq n$ shares. A shareholder $i \in N$ holds $d_i \in \mathbb{N}$ shares, with $\sum_{i \in N} d_i = m$. Let $d = (d_1, d_2, ..., d_n)$ be the vector of the shares among the n shareholders, $h_i = \frac{d_i}{m}$ be the fraction of shares held by shareholder i, and $h = (h_1, h_2, ..., h_n)$ the vector of the fraction of shares held by the n shareholders.

At the shareholders meeting, they have to choose, through voting, whether to approve a proposal by the management, A, or keep the status quo, B. We denote the set of

[13] Super-majority rules–a prominent class of 1P1V mechanisms–do not admit a clear ranking in terms of voting efficiency: each super-majority rule is optimal under certain informational assumptions (see, e.g., Maug and Rydqvist 2009).

alternatives by $O = \{A, B\}$. For now, we assume that the proposal is exogenously given. In Section 5 we allow the management to shape the proposal.

Shareholders are uncertain about the quality of the proposal. There are two states of the world, $\omega \in \Omega = \{\alpha, \beta\}$ that are unobserved at the time of the vote. For the sake of simplicity, we assume that the states are equiprobable.

While shareholders do not necessarily have the same stakes, they all agree that the proposal is good in state α but bad in state β:

$$\begin{aligned} u_i(A|\alpha) &= h_i,\ u_i(A|\beta) = -h_i, \\ u_i(B|\alpha) &= u_i(B|\beta) = 0. \end{aligned}$$

We relax that assumption in Section 4.3.1.

Before the meeting, each shareholder i receives a signal $s_i \in S := [0,1]$ distributed according to a shareholder-specific distribution function, $F_i(\cdot|\omega)$, with density $f_i(\cdot|\omega)$.[14] Conditional on the state, signals are drawn independently. The type of shareholder i after the draw of the signal is $t_i = \frac{f_i(s_i|\alpha)}{f_i(s_i|\beta)} \in T_i = [\delta_i, 1/\delta_i]$. We make the following assumption about the signal technology:

Assumption 1 (Strong MLRP and Bounded Support) *For every shareholder $i \in N$, t_i is strictly increasing in s_i and there exists $\delta_i \in (0,1)$ such that $\frac{f_i(0|\alpha)}{f_i(0|\beta)} = \delta_i$ and $\frac{f_i(1|\alpha)}{f_i(1|\beta)} = 1/\delta_i$.*

The first part of Assumption 1 means that shareholders who receive higher signals attach a larger probability to the state of the world being state α, while the second part means that there is no shareholder with arbitrarily precise information about the state of the world. This assumption allows for various structures of shareholders' information technology. For instance, it allows for arbitrarily large differences in the (expected) information quality of two shareholders, and for any type of correlation between (expected) information quality of a shareholder and the number of shares she owns.

We consider a broad class of voting mechanisms. A voting mechanism V associates to any share distribution d a voting rule $V(d) = \{X, w\}$, where $X = (X_1, X_2, ..., X_n)$ with $X_i \subseteq \mathbb{R}$ and $w \in \mathbb{R}$, and is such that each shareholder $i \in N$ chooses $x_i \in X_i$ and the outcome is:

$$G^w(x) = \begin{cases} A & \text{if } \sum_{i \in N} x_i > w \\ AB & \text{if } \sum_{i \in N} x_i = w \\ B & \text{if } \sum_{i \in N} x_i < w, \end{cases}$$

where AB denotes the fair lottery between A and B.[15]

[14] In Section 4.3, we endogenize the acquisition of information by shareholders.

[15] All results can be easily extended to a more general formulation of the voting rule $V(d) = \{X, w_A, w_B\}$

This class of voting mechanisms nests most popular mechanisms, including some that are closer to one-person-one-vote procedures, like (super-)majority rules, and some that distribute voting power depending on the exact number of shares that each shareholder holds. The fact that we allow the voting rules to vary with the distribution of shares enriches our model.

While we formally consider simple capital structures consisting only of common shares, our model also accommodates dual class structures with both common shares (which carry voting rights) and preferential shares (which do not carry voting rights). Indeed, if each shareholder holds some common shares, and some preferential ones, then, again, the voting rules should depend only on the distribution of the common shares, leaving the model intact.[16]

It will be useful to distinguish between cases in which there is a shareholder who can always affect the outcome independently of the choices of the other shareholders, and the more interesting cases in which such a player is not present. In particular, we say that there is no decisive shareholder if, for a given voting mechanism V and a share distribution d, there exists a strategy profile such that A (B) wins with certainty and the outcome cannot be affected by any individual deviation. In all other cases, we say that a decisive shareholder exists.

Due to their empirical relevance in shareholders meetings (as discussed in the Introduction), two mechanisms are of particular interest to us.[17] First, the one-share-one-vote with partial abstention mechanism, V^{1S1V}, is such that $V^{1S1V}(d) = \{\times_{i \in N}\{-d_i, -d_i + 1, -d_i + 2, ..., d_i\}, 0\}$. Second, the one-person-one-vote mechanism, V^{1P1V}, is such that $V^{1P1V}(d) = \{\times_{i \in N}\{-1, 1\}, 0\}$. Under *1S1V*, each shareholder i can choose from a richer set of ballots: either fully supporting A (action d_i), fully supporting B (action $-d_i$), or intermediate intensities of support (integer numbers between $-d_i$ and d_i). Under *1P1V*, each shareholder has to choose among two ballots, voting for A (action 1) or voting for B (action -1).[18] Under both mechanisms, the proposal is approved if the total support for A is strictly larger than the total support for B, and ties are broken randomly.

Two extreme variations of the *1S1V* voting mechanism will also prove of particular interest. First, the one-share-one-vote without partial abstention mechanism, V^{1S1V-R}, is

with $w_A \leq w_B$, where the outcome is A if $\sum_{i \in N} x_i > w_A$, B if $\sum_{i \in N} x_i < w_B$, and AB if $\sum_{i \in N} x_i \in [w_A, w_B]$.

[16] In this more general case one should allow h_i to depend on both kinds of shareholdings. As it will be made clear in our analysis, the exact value of h_i does not alter shareholder i's incentives, and hence the equilibria of the game.

[17] It is noteworthy that while the definitions of some mechanisms consider a simple majority threshold for the adoption of the proposal (i.e. $w = 0$), we could easily adapt the definitions and consider super majority versions of these mechanisms. That is, keep the same ballot structure X but introduce a larger (or smaller) quota, $w \neq 0$, for the passing of proposal.

[18] We do not allow for abstention for simplicity of exposure. However, as it will become clear in section 4, all our results are robust to allowing for abstention.

such that $V^{1S1V-R}(d) = \{\times_{i\in N}\{-d_i, d_i\}, 0\}$. Second, the one-share-one-vote with divisible votes mechanism, V^{1S1V-D}, is such that $V^{1S1V-D}(d) = \{\times_{i\in N}[-d_i, d_i], 0\}$. Under both mechanisms, the maximum number of votes that a shareholder can cast is equal to the total number of shares she owns, and hence both mechanisms conform to the one-share-one-vote principle. They are polar variations of the standard *1S1V* in the sense that *1S1V-R* is maximally rigid in terms of vote divisibility –each shareholder has to decide between fully supporting A (action d_i) or fully supporting B (action $-d_i$)– and *1S1V-D* allows for maximal vote divisibility –shareholders can choose any intermediate intensity of support, not only integers. The ballot space is thus richer under *1S1V-D* than *1S1V*, and richer under *1S1V* than *1S1V-R*. Again, all our results are robust to allowing for full abstention under *1S1V-R*.

The *1S1V-D* mechanism belongs to the class of continuous voting mechanisms:

Definition 1 *A voting mechanism V with associated rules $V(d) = \{X, w\}$ is a* ***continuous voting mechanism*** *if for every d there exists $(\psi_i)_{i\in N} \in \times_{i\in N} int(X_i)$, such that $\sum_{i\in N} \psi_i = w$.*

The class of continuous voting mechanisms is such that (i) all shareholders have access to a continuous ballot space, and (ii) the ballot space allows for a tie in which each voter can increase and decrease the net vote total by any arbitrarily small degree.

For each voting rule $\{X, w\}$ and each shareholder $i \in N$, a strategy is a function $\sigma_i : T_i \to \Delta(X_i)$. As it is standard in the literature, $\Delta(X_i)$ is the set of all probability distributions which support is a subset of X_i. When σ_i is a pure strategy, we sometimes abuse notation and denote by $\sigma_i(t_i)$ the action x that the shareholder picks with probability 1. Since σ_i can be a mixed strategy, it is useful to distinguish the random variable, σ_i, from a potential realization, $\widehat{\sigma}_i$: we say that $\widehat{\sigma}_i$ is a potential realization of σ_i if and only if $\widehat{\sigma}_i$ belongs in the support of σ_i. Consequently, when σ_i is a pure strategy we have $\sigma_i(t_i) = \widehat{\sigma}_i(t_i)$ for every $t_i \in T_i$.

We focus on (interim) *Bayesian Nash Equilibria* (BNE) such that equilibrium strategies are best responses at the interim stage (when each shareholder knows her own type).

3 Welfare Benchmarks

We consider two different welfare benchmarks, which serve different purposes: one is to assess the efficiency of a mechanism, and the other to compare mechanisms which are not efficient.

Our efficiency benchmark corresponds to the preferred outcome of shareholders when they have access to all the information dispersed in the electorate (i.e., if they were able to observe the signal profile). In that case, shareholders would prefer the alternative that is most likely to match the state of the world, conditional on the available information:

Definition 2 *Given a vector of signals $s = (s_1, s_2, ..., s_n)$, the efficient outcome, E, is equal to A if $\Pr(\alpha|s) > 1/2$, B if $\Pr(\beta|s) > 1/2$, and AB otherwise.*

This leads to a natural implementation notion:

Definition 3 *Given a voting rule $\{X, w\}$, a BNE $\sigma = (\sigma_1, ..., \sigma_n)$ is efficient if $G^w(\sum_{i \in N} \widehat{\sigma}_i(t_i)) = E$ for every $t \in \Pi_{i \in N} T_i$ and every potential realization $\widehat{\sigma}(t)$ of the random variables $\{\sigma_1(t_1), \sigma_2(t_2), ...\}$. A voting mechanism V implements the efficient outcome in equilibrium, if for every d, $V(d)$ admits an efficient BNE.*

Our second welfare benchmark corresponds to the preferred outcome of shareholders when they know the state of the world:

Definition 4 *Given the state of the world ω, the correct outcome is A in state α, and B in state β.*

The two welfare benchmarks are fully compatible: when the state of the world is unobservable, then the alternative that is most likely the correct outcome given the available information coincides with the efficient outcome. Moreover, in our common value environment, both these welfare benchmarks are aligned with utilitarian principles. The correct outcome is the utilitarian outcome (i.e. the alternative that maximizes the sum of ex-post utilities), and the efficient outcome is the outcome most likely to be the utilitarian one given all the shareholders' information.

We then compare the performance of voting mechanisms focusing on the ex-ante (i.e. before the state of the world and types are drawn) probability with which they implement the correct decision, considering both the best equilibria (in terms of selecting the correct outcome), and the worst ones:

Definition 5 *Voting mechanism V dominates voting mechanism V' given a share distribution d if (i) for every BNE of $V'(d)$, there is a BNE of $V(d)$ such that the ex-ante probability of implementing the correct outcome is higher under V than V'; and (ii) for every BNE of $V(d)$, there is a BNE of $V'(d)$ such that the ex-ante probability of implementing the correct outcome is lower under V' than V. If, moreover, either (i) or (ii) (or both) hold strictly, we say that V strictly dominates V'.*

Voting mechanisms typically admit multiple equilibria. Therefore, assessing the potential performance of a mechanism considering only the best (worst) equilibrium, in terms of the ex-ante probability of selecting the correct alternative, might be overly optimistic (pessimistic). For this reason, we opt for a comparative criterion that combines both the best and the worst possible equilibrium outcomes. We say that a mechanism dominates another if and only if the former is superior to the latter in both dimensions. This is particularly important in our case because, as we will illustrate below, mechanisms with richer ballot spaces allow for both better and worse outcomes compared to mechanisms with poorer ballot spaces. Hence, focusing only on the best (worst) equilibrium of each mechanism might not be very informative with respect to the range of equilibrium performances.

4 Equilibrium Analysis: Exogenous Proposal

In this section, we analyze the equilibrium performances of various voting mechanisms when the manager is passive, i.e., the proposal is exogenously given. We split the section in two parts. First, we compare the information aggregation properties of finite voting mechanisms, i.e., mechanisms with finite ballot spaces such as *1P1V*, *1S1V*, and *1S1V-R*. Second, we focus on the informational efficiency of all types of voting mechanisms.

We first introduce the following Lemma, which is not only useful to prove some of our key results, but also to understand the benefits and pitfalls of a mechanism with a richer ballot space.

Lemma 1 $\Pr(\alpha|s) > \frac{1}{2} \Leftrightarrow \sum_{i\in N} \ln(t_i) > 0$.

Proof. See Appendix A. ■

This is reminiscent of the results in Nitzan and Paroush (1982), which characterizes the weights that a rule should attach to votes in situations where agents have heterogeneous information precision in a common-value game.

This result suggests that if a voting mechanism allows shareholders with different information precision to cast votes in proportion to the logarithm of their type, then efficiency can be reached. For instance, let us consider a group of voters with the same type space $T = \{e^{-10}, e^{-1}, e, e^{10}\}$. The voting mechanism $V = \{\times_{i\in N}\{-10, -1, 1, 10\}, 0\}$ allows them to secure the efficient outcome if they each cast $\ln(t_i)$ votes in favor of A. Of course, such a mechanism also makes the "inefficient" outcome (i.e., A when $\Pr(\alpha|s) < \frac{1}{2}$ and B when $\Pr(\alpha|s) > \frac{1}{2}$) attainable, since a voter of type t_i could cast $-\ln(t_i)$ votes in favor of A. By contrast, the mechanism $V' = \{\times_{i\in N}\{-10, 10\}, 0\}$ does not allow the voters to reach the efficient outcome, nor the inefficient one. Hence, it is not obvious how to rank mechanisms V and V' in terms of potential outcomes. As we prove in the rest of this section, when we focus on equilibrium outcomes, this indeterminacy is resolved.

4.1 Comparison of Finite Mechanisms

We start this section by comparing two arbitrary finite mechanisms, with one having a ballot space that is a subset of the other's ballot space. To state our results in a compact manner it is useful to pin down this possible relationship between two voting mechanisms.

Definition 6 *Consider two voting mechanisms, V and V', with associated rules $V(d) = \{X, w\}$ and $V'(d) = \{X', w\}$. If $X'_i \subseteq X_i$ for every shareholder i and every share distribution d, then V is said to have a richer ballot space than V'.*

We are now ready to state our first main result:[19]

Proposition 1 *Under Assumption 1, if a finite mechanism V has a richer ballot space than mechanism V', and there is no decisive shareholder, then V dominates V'.*

Proof. See Appendix A. ∎

Why does the mechanism with the richer ballot space dominate the other? First, let us consider the comparison of the best equilibria under those two mechanisms. Recall from McLennan (1998) that in a pure common value environment, a strategy profile producing the maximal ex-ante utility must be an equilibrium.[20] Given that mechanism V has a richer ballot profile, any outcome under mechanism V' can be reproduced under V by simply replicating the strategy. Hence, there always exists an equilibrium under V that produces an ex-ante utility at least as high as in the best equilibrium under V'.[21] Notice that for this argument to hold it is not necessary that the best equilibrium of V is efficient, nor that a decisive player is absent. As long as V provides more ballot options to the shareholders than V', then the best equilibrium of V leads to the correct outcome with at least as high a probability as the best equilibrium of V'.

Second, let us consider the comparison of the worst equilibria under those two mechanisms. The proof has two main steps. In the first step, we prove the intuitive result that under any voting mechanism, a profile of monotone strategies is at least as good as a profile where all voters vote in favor of the same outcome independently of their signal. This last profile is in fact an equilibrium under any voting mechanism, provided that no

[19] We prove the result under Assumption 1 for clarity of exposition. The result would still hold under the milder assumption of Weak Monotone Likelihood Property. The proof can indeed be directly adapted to that case, by defining shareholders' strategies as functions defined on the (compact) signal space rather than on the type space.

[20] McLennan (1998) assumes that types are finite to guarantee the existence of such a utility-maximizing profile. We prove that profiles that maximize ex-ante utility under any finite mechanism also exist in settings with infinite types, like ours.

[21] Ahn and Oliveros (2016) use a similar argument to show the superiority of approval voting over plurality.

shareholder is a decisive shareholder (as defined above). The second step consists in proving that any equilibrium is welfare-equivalent to a profile of strategies increasing in the voters' types. We show that if an equilibrium exhibits strategies that are not monotonic, with a shareholder i casting x_i votes when of type t_i but a larger amount $x_i' > x_i$ when of lower type $t_i' < t_i$, then it must be that the difference between x_i' and x_i is small enough that it does not affect the outcome. By following this line of reasoning, the equilibrium can be shown to be equivalent to a profile of monotone strategies.

Proposition 1 has two corollaries that are relevant for the three discrete rules introduced in Section 2, *1P1V*, *1S1V*, and *1S1V-R*. First, given its richer ballot space, *1S1V* dominates both *1P1V* and *1S1V-R* when there is no decisive shareholder:[22]

Corollary 1 *If the share distribution d is such that there is no decisive shareholder under V^{1P1V}, V^{1S1V}, and V^{1S1V-R}, then V^{1S1V} dominates both V^{1P1V} and V^{1S1V-R}.*

Given Lemma 1, the dominance of *1S1V* over *1P1V* seems intuitive when shareholdings are positively correlated with information accuracy (especially, when d_is are proportional to $\ln(t_i)$s). But, if one relies on this intuition to try to assess which mechanism is better when there is no correlation between information accuracy and shareholdings, one might end up with the wrong conclusion that in such cases *1P1V* should perform better than *1S1V*. This would be true only if shareholders behaved in an unsophisticated manner (i.e., if they always cast the maximum allowed number of votes for the alternative they consider more likely to be correct). As we prove, and explain after Proposition 1, when the shareholders are strategic, then *1S1V* dominates *1P1V* independently of whether there is a correlation between information precision and shareholdings.

We can actually show, using numerical examples, that for some values of the parameters, the dominance is strict. Consider a firm with six shareholders. Shareholders 1, 2, and 3 hold $k \in \{1, 2, 3\}$ shares each, while shareholders 4, 5, and 6 hold $4 - k$ shares each. Signals are binary, $s_i \in \{a, b\}$, and the likelihood of receiving the right signal is $p(a|\alpha) = p(b|\beta) = p_H \in [.55, 1)$ for shareholders $1 - 3$, and $p(a|\alpha) = p(b|\beta) = .55$ for shareholders $4 - 6$.[23] That is, shareholders $1 - 3$ have more accurate information –they estimate correctly the true state of the world with a probability $p_H \geq 0.55$– while the other three shareholders are less accurately informed –they correctly infer the true state of the world with a probability 0.55. Figure 1 plots the probability of taking the right

[22] When a decisive shareholder exists under *1S1V* (i.e., a shareholder that holds more than 50% of the shares), then the dominance of *1S1V* over *1P1V* is reinforced as the worst equilibrium under *1S1V* is strictly better than the worst equilibrium under *1P1V*. Indeed, under *1P1V* the worst equilibrium is the uninformative one in which a sufficiently large majority of shareholders votes for the same alternative independently of their signals, while under *1S1V* the worst equilibrium should at least reflect the information of the decisive player.

[23] These parameters would be equivalent to having $f_i(s_i|\alpha) = f_i(1 - s_i|\beta) = 2p$ for every $s_i < \frac{1}{2}$ and $f_i(s_i|\alpha) = f_i(1 - s_i|\beta) = 2(1 - p)$ for every $s_i \geq \frac{1}{2}$, where $p = p_H$ for shareholders $1 - 3$ and $p = .55$ for shareholders $4 - 6$.

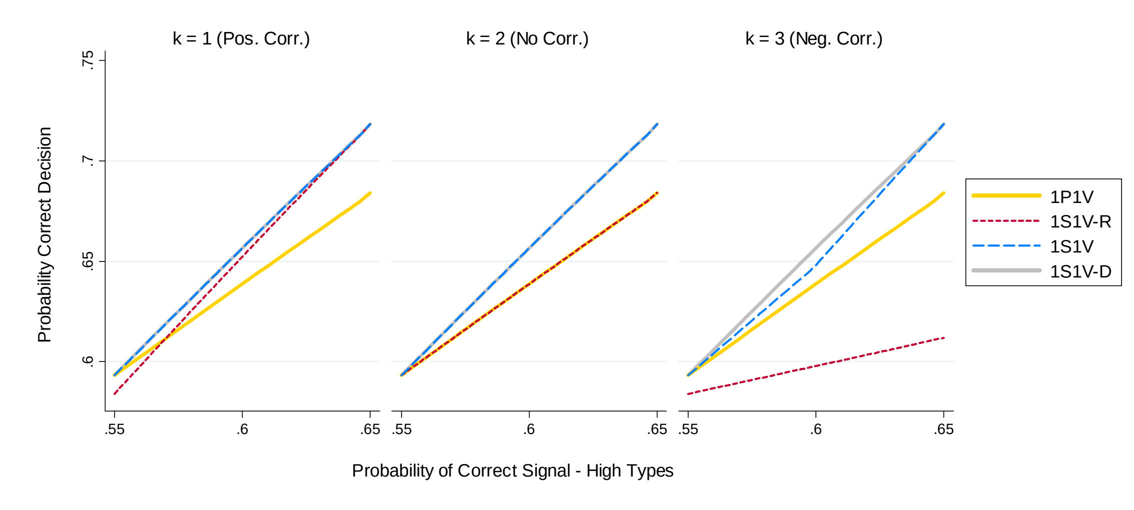

Figure 1: Probability of a correct decision under 1P1V, 1S1V-R, 1S1V and 1S1V-D.

decision under *1S1V*, *1S1V-R* and *1P1V* (and *1S1V-D* as the efficient benchmark, that will be discussed later).

As we know from Lemma 1, a key to aggregate information efficiently is to attach more weight to more informative signals. In our simple example, shareholders $1-3$ are always the ones that have more informative signals, and therefore their signals should have more weight in the group decision. Under *1P1V*, however, all votes have the same weight and this undermines the ability of shareholders to aggregate information well, especially compared to *1S1V*. Under *1P1V* there are two types of equilibria: (i) an equilibrium where all shareholders vote their signal and (ii) an equilibrium in which two of the lowly informed shareholders mute themselves by voting for opposite options. Each type of equilibria has a different type of inefficiencies: in the former, the cost is that lowly informed voters overwrite decisions by the highly informed while in the second some of the information by the lowly informed shareholders is lost.

Under *1S1V-R*, shareholders votes have weights proportional to their shares. The ability of this mechanism to aggregate information critically depends on the correlation between the number of shares and the quality of shareholders' information. When $k=2$, all shareholders hold the same number of shares, and *1S1V-R* mimics the properties of *1P1V*. When $k=1$, there is a positive correlation between shares and quality of information, and in this case *1S1V-R* outperforms *1P1V* only if shareholders with more shares are sufficiently better informed. When $k=3$, there is a negative correlation between shares and quality of information, and in this case it is *1P1V* that outperforms *1S1V-R*.

Finally, as one can see from Figure 1, *1S1V* dominates both *1P1V* and *1S1V-R*. The key for that result is that voters can *endogenously* determine the weight of their vote.

This allows the group's decision to depend more heavily on more informative signals. The difference is particularly significant in cases with negative correlation between shares and information ($k = 1$).[24]

Despite the superiority of *1S1V*, it may not be fully efficient (as one can see in the case of $k = 3$ in Figure 1). The sources of inefficiency of *1S1V* are the discreteness of the action space and, perhaps more importantly, the potential mismatch between the number of shares own by a shareholder and the quality of her information. When $k = 3$ in the previous example, *1S1V* is not efficient because better informed shareholders have too few shares in order to reveal their information through the voting process. This leads to our second corollary of Proposition 1: *1S1V* becomes more efficient when we multiply the shares that all shareholders hold (i.e., when we split stocks).[25]

Corollary 2 *Under V^{1S1V}, splitting votes weakly increases welfare. That is, $V^{1S1V}(k \cdot d)$ weakly dominates $V^{1S1V}(d)$ for any $k \in \mathbb{N}$.*

As discussed in the introduction, this result shows that, through their effect on the number of shares held by shareholders, stock splits indeed increase shareholders' ability to reveal their information about the quality of management proposals through voting.

Finally, Proposition 1 also has implications for dual class capital structures in which some shares do not carry voting rights. Allowing for shares with no voting rights reduces the effective ballot space of some shareholders, limiting their ability to convey information through voting. This affects negatively information aggregation. Therefore, *1S1V* where all shares have voting rights dominate dual class systems.

4.2 Efficiency of Continuous Mechanisms

In this section, we focus on the efficiency of the various voting mechanisms. We can achieve a full characterization, both of the voting mechanisms that lead to efficiency in equilibrium, and of the complete set of efficient equilibria corresponding to each such mechanism. The overall message is that, to be efficient, a voting mechanism must have a ballot space at least as rich as the signal space.

To prove the results in this section, we need one additional assumption:

Assumption 2 *For every shareholder $i \in N$, $T_i = T$ (i.e. $\delta_i = \delta \in (0, 1)$) and t_i is continuous in s_i.*

[24] It is worth pointing out that in the more general setup, it is not ex-ante clear which subjects will receive the most informative signal, and this makes 1S1V-R relatively worse.

[25] Note that increasing the number of shares proportionally has no effect under *1P1V* and *1P1V-R*.

This assumptions requires that all types that are possible for one shareholder are possible –but not necessarily equally likely– for any other. Even with this assumption, our model allows for arbitrarily large differences in the expected information quality of two shareholders.[26]

We first characterize the unique efficient equilibrium under *1S1V-D*:

Proposition 2 *Under Assumptions 1 and 2, for any share distribution d, the one-share-one-vote with divisible votes mechanism, $V^{1S1V-D}(d)$, admits a unique (up to admissible multiplicative and additive constants) efficient BNE such that $\sigma_i^{1S1V-D}(t_i) = c\ln t_i + \kappa_i$ with $\sum_{i\in N}\kappa_i = 0$ and $c \in (0, \min\{\frac{-d_i-\kappa_i}{\ln\delta}, \frac{-d_i+\kappa_i}{\ln\delta}\}]$ for every $i \in N$.*

Proof. See Appendix A. ■

The equilibrium strategy implies that A wins if and only if $\sum_{i\in N} c\ln(t_i) > 0$, which guarantees that the outcome is efficient. As mentioned above, we know from McLennan (1998) that a strategy that maximizes ex ante welfare must be an equilibrium.[27] Uniqueness (up to a multiplicative and an additive constant) follows from the fact that any efficient BNE σ must guarantee that $sgn\left(\sum_{i\in N}\sigma_i(t_i)\right) = sgn\left(\sum_{i\in N}\ln(t_i)\right)$. Only strategies such that $\sigma_i(t_i) = c\ln t_i + \kappa_i$ with $\sum_{i\in N}\kappa_i = 0$ and $c \in (0, \min\{\frac{-d_i-\kappa_i}{\ln\delta}, \frac{-d_i+\kappa_i}{\ln\delta}\}]$ for every $i \in N$ satisfy that condition.[28] The maximum value of c simply guarantees that the ballot of any given type of voter fits in the ballot space.

This equilibrium characterization provides interesting insights regarding the optimal majority threshold for the passing of a proposal (see, e.g., Maug and Rydqvist 2009). Indeed, under one-person-one-vote mechanisms or one-share-one-vote mechanisms that do not allow for vote divisibility, we know that the majority requirement maximizing the probability of implementing the correct alternative varies with the information structures (i.e. for different F_is). By contrast, under *1S1V-D* we have an efficient equilibrium for any majority requirement, w. Hence, *1S1V-D* remains efficient even if a super majority threshold is necessary for alternative reasons (e.g. to prevent aggressive acquisition attempts) or is mandated by law (e.g., for changes in the Bylaws of the firm).

[26] The continuity of the mapping from signals to types amounts to having a connected type space. This helps us to fully focus on the essential nature of the strategies in an efficient equilibrium, without being distracted by the possibility of multiple best responses. Indeed, when gaps in the type space are allowed all the equilibria that we identify still exist, but additional ones emerge due to indifferences of types close to points of discontinuity.

[27] We can even show that σ^{1S1V-D} is an ex-post equilibrium: no shareholder has incentives to deviate ex post, when all types are known.

[28] This result highlights a challenge for shareholders: they need to coordinate on the correct multiplicative and additive constants. It seems that a focal equilibrium is the one in which shareholders use the highest possible vote weight when they receive the most informative signal and zero votes when completely indifferent. This prediction could be tested in the laboratory.

Next, we show that continuous voting mechanisms, like *1S1V-D*, are the only efficient mechanisms:

Proposition 3 *Under Assumptions 1 and 2, a voting mechanism implements the efficient outcome in equilibrium if and only if it is a continuous voting mechanism.*

Proof. See Appendix A. ∎

The "if" part of the proposition follows from the fact that the efficient equilibrium of the *1S1V-D* mechanism can be properly rescaled to fit the ballot space of any continuous voting mechanism (i.e. to fit within any open set around any vector $(\psi_i)_{i\in N}$ such that $\sum_{i\in N}\psi_i = w$). The "only if" part of the proposition follows from the fact that any efficient equilibrium must be strictly increasing in the shareholder's type, and only continuous voting mechanisms satisfy that requirement. To understand why it must be strictly increasing, just consider two type profiles t and t' such that (i) $t_{-i} = t'_{-i}$, (ii) $\Pr(\alpha|t) > \Pr(\beta|t)$, and (iii) $\Pr(\alpha|t') < \Pr(\beta|t')$. It must then be that $t_i > t'_i$. But, if $\sigma_i(t_i) \leq \sigma_i(t'_i)$, then either the outcome for t or for t' is not efficient (or both).

Proposition 3 has the following implication for the comparison of *1S1V-D*, or any continuous voting mechanism, with *1S1V*, *1S1V-R* and *1P1V*:[29]

Corollary 3 *Any continuous voting mechanism admits a BNE that implements the correct outcome with a higher probability than any BNE of 1S1V, 1S1V-R, and 1P1V. For some values of the parameters, the inequality is strict.*

The intuition of this result is similar to the one of Proposition 2. First, note that the ballot space is richer under continuous voting mechanisms than under *1S1V*, *1S1V-R,* and *1P1V* (or any other mechanism under consideration). Hence, any strategy profile under *1S1V*, *1S1V-R,* or *1P1V* can be reproduced under a continuous voting mechanism. In particular, this is true for any efficient equilibrium under *1S1V*, *1S1V-R,* or *1P1V*. Thus continuous voting mechanisms are at least as good as *1S1V*, *1S1V-R,* and *1P1V*. On the other hand, the unique efficient equilibrium under any continuous voting mechanism cannot be reproduced under any other mechanism. This is simply because *1S1V*, *1S1V-R*, and *1P1V* do not have ballot spaces sufficiently rich to accommodate for every possible small change in the type profile.

There are various specific weaknesses of the discrete mechanisms we analyzed that *1S1V-D* solves. While we know that *1P1V* aggregates information asymptotically in various sets of situations (see, e.g., Feddersen and Pesendorfer 1997, Myerson 1998, Bhattacharya 2013, and Barelli et al. 2019), we also know that it does not typically aggregate

[29] Under the same assumptions used in Proposition 1, and with a general –but finite– type space, we can further establish that the worst equilibrium of a continuous voting mechanism is no worse than the worst equilibrium of any finite mechanism.

information well in relatively small groups (see, e.g., Feddersen and Pesendorfer 1996). It is easy to prove that the same is true for *1S1V-R*. In contrast, *1S1V-D* performs well both in large and small groups.

Another weakness of *1P1V*, *1S1V-R* and *1S1V* is that shareholders need to know the information technology of all other shareholders, i.e., $F_i(\cdot|\omega)$, in order to determine their optimal strategy. If they are mistaken or have ambiguous beliefs about the information technology of others, then *1P1V* may even fail to aggregate information asymptotically. By contrast, it is clear from Proposition 2 that the equilibrium strategy for any shareholder under *1S1V-D* is independent of the information technology of other shareholders. This directly implies that *1S1V-D* remains efficient even in presence of such mistakes or ambiguities.

4.3 Robustness

In this section, we explore the robustness of the above results when we allow for partisan shareholders, and when we endogenize the acquisition of information by shareholders.

4.3.1 Partisans

There is evidence that disagreement among shareholders may not only stem from information asymmetries, but also from differences in preferences (Bolton et al., 2020). As mentioned in Cvijnovic et al. (2019, p.3), preferences may vary due to differences in, e.g., portfolio allocation (Cohen and Schmidt 2009), business ties (Davis and Kim 2007, and Cvijnovic et al. 2016), reputational concerns (Chevalier and Ellison 2009), and political and social goals (Woidtke 2002).

We consider the same model as in Section 2 except that we allow for three types of shareholders: A, B, and C. The utility of the different shareholders are given by:

$$\forall O \in \{A, B\}, \forall \omega \in \{\alpha, \beta\}, \quad \begin{cases} u_i^A(O|\omega) & = h_i \times \mathbf{1}_{\{O=A\}} \\ u_i^B(O|\omega) & = h_i \times \mathbf{1}_{\{O=B\}} \\ u_i^C(O|\omega) & = h_i \times (\mathbf{1}_{\{O=A,\omega=\alpha\}} - \mathbf{1}_{\{O=A,\omega=\beta\}}). \end{cases} \tag{1}$$

That is, A (resp. B)-shareholders want A (resp. B) to be selected irrespective of the state ω, whereas C-shareholders are common-value shareholders who want the decision to match the state. We denote the fraction of shares held by A(resp. B)-shareholders by h_A (resp. h_B). Common-value shareholders own the remaining fraction of the stock, $h_C = 1-h_A-h_B$. Only C-shareholders receive a signal about the quality of the proposal.[30] In what follows, we consider two different cases: (i) h_A and h_B are common knowledge,

[30] Allowing partisan shareholders to be informed does not alter the results, but requires additional notation.

and (ii) h_A and h_B are **not** common knowledge.

Partisan shareholders (A and B types) are always (weakly) better off by giving the largest possible number of votes to their favorite alternative, independently of the behavior of the other shareholders (as in Nunez and Laslier 2014). To ensure that our results do not hinge on possible beliefs that make partisan voters indifferent between several actions, we consider that such voters always play that unique dominant strategy. This directly implies that when there are only partisan shareholders ($h_C = 0$), then *1S1V*, *1S1V-R*, and *1S1V-D* lead to the same outcome.

Another implication of partisan voters' dominant strategy is that if different mechanisms allow for a different maximum number of votes for different shareholders, partisans will have an asymmetric effect across mechanisms. For this reason, we limit attention to comparing voting mechanisms which result in the same distribution of power:

Definition 7 *If two mechanisms, V and V', with associated rules $V(d) = \{X, w\}$ and $V'(d) = \{X', w\}$, are such that $\max X_i = \max X'_i$ and $\min X_i = \min X'_i$ for every shareholder i and any distribution of shareholdings d, then V assigns power similarly to V'.*

That is, while different shareholders might have a different maximum number of votes, each shareholder must have the same maximum number of votes under any mechanism that we compare.

Finally, while the game now includes non-common value shareholders, we still use the notion of dominance and efficiency defined in Section 3. The main reason to do so is that our focus is on the informational efficiency of voting mechanisms. Another reason is that these welfare benchmarks, even if they are better suited for the analysis of common-value elections, also admit a utilitarian interpretation in certain contexts with heterogeneous preferences.[31]

Partisan Shareholders are Common Knowledge. When there is no uncertainty about the fraction of shares held by partisans, all our main results mostly hold. In particular, we can prove that (i) mechanisms with richer ballot spaces dominate mechanisms with poorer ballot spaces if these mechanisms do not differ in how they distribute power accross shareholders, and (ii) the *1S1V-D* mechanism is efficient and its superiority over other mechanisms might be reinforced.

[31] When there are partisan shareholders, the correct outcome (i.e., A in state α, and B in state β) does not necessarily coincide with the utilitarian one. Under *1S1V*, *1S1V-R*, and *1S1V-D*, when partisans have so many votes that common-value shareholders cannot affect the final outcome, then in one state the correct outcome coincides with the one that maximizes the sum of utilities; and in the other state they differ. Importantly, those situations are the same for *1S1V*, *1S1V-R*, and *1S1V-D*. In the remaining situations, when the behavior of the common-value shareholders matters for the final outcome, the correct outcome coincides with the utilitarian one. Hence, if one of the three voting mechanisms is found to admit a BNE that implements the correct outcome with higher probability than any BNE of the other mechanisms, then it follows that it also implements the utilitarian outcome with higher probability.

Proposition 4 *Consider that h_A and h_B are common knowledge. Under Assumption 1: (i) if a finite mechanism V has a richer ballot space than mechanism V' –but the two mechanisms distribute power similarly– and d is such that there is no decisive voter, then V dominates V'; and (ii) if d is such that $h_C > |h_A - h_B|$, then V^{1S1V-D}, admits an efficient BNE.*

Proof. See Appendix A. ■

To understand this result, let us first focus on the efficiency of *1S1V-D.* Shareholder i will choose $+d_i$ if she is a A-shareholder, and $-d_i$ if she is a B-shareholder. When h_A is larger than h_B, it is as if the voting mechanism were biased against the status quo, and conversely for $h_A < h_B$. To implement the efficient outcome, C-shareholders must then find a way to compensate for that bias. An easy way to do so is for common-value shareholders to make two modifications to the efficient equilibrium strategy under *1S1V-D* as characterized in Proposition 2. First, common-value shareholders have to rescale their strategy by a factor of $\left(1 - \frac{|h_A - h_B|}{h_C}\right)$ to leave room for compensation. Second, if say $h_B > h_A$, each common-value shareholder i includes $d_i \times \frac{h_B - h_A}{h_C}$ points in favor of A on her ballot.

For instance, consider a case with ten common-value shareholders holding 10 shares each, and partisan shareholders, all of them of type B, holding cumulatively 20 shares. Assume that, without the partisans, each common-value shareholder would give 10 points to A when receiving the most informative signal in favor of A, 2 points to A when receiving another, less informative signal in favor of A, and -5 points to A (i.e. 5 points to B) when receiving a signal in favor of B. With partisan shareholders, each common-value shareholder would rescale her strategy by a factor of 0.8 (i.e., giving 0.8 times the number of points she would have given without partisan shareholders), and add 2 points for A. Thus, she would still give 10 ($= 10 \times 0.8 + 2$) points to A when receiving the most informative signal in favor of A, but she would now give 3.6 ($= 2 \times 0.8 + 2$) points to A when receiving the less informative signal in favor of A, and -2 ($= -5 \times 0.8 + 2$) points to A (i.e. 2 points to B) when receiving a signal in favor of B.

We can also show that *1S1V* not only continues to dominate *1S1V-R* in the presence of a known number of partisans, but that this dominance may be reinforced. This is so because the presence of partisans makes information aggregation under *1S1V-R* less efficient. The problem comes from the fact that shareholders cannot at the same time reveal their information and compensate for the bias. This is particularly evident in the asymmetric equilibrium. In that equilibrium, under *1S1V-R*, some shareholders specialize in compensating for the bias and others in sharing information (i.e., they vote sincerely). Thus, the information of shareholders who compensate for the bias is lost (the same is true in symmetric BNE in which all shareholders mix between the compensating strategy and the sharing information one). We illustrate this with a numerical example in which,

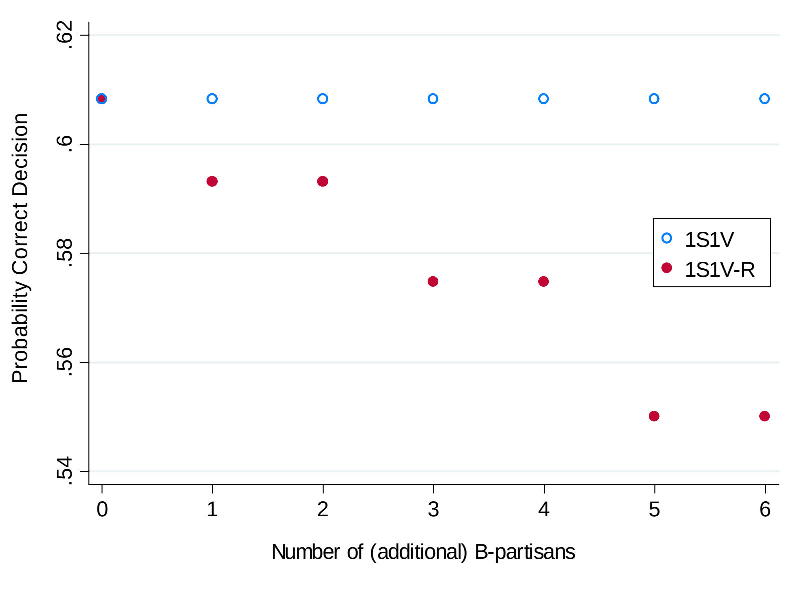

Figure 2: Probability of correct decision under 1S1V and under 1S1V-R. There are 7 symmetric non-partisan shareholders: they hold the same (large) amount of shares and have the same information technology. Signals are binary, and the likelihood of receiving the right signal is .55 in both states.

without partisan shareholders, *1S1V-R* would perform as well as *1S1V* (see Figure 2).

Partisan Shareholders are Not Common Knowledge. To explore this case, we assume that the distribution of shares $d = (d_i)_{i \in N}$ is common knowledge among shareholders, but the partisan type of each shareholder is unknown. Specifically, the partisan type of each shareholder $i \in N$ is drawn from a distribution $p_i \in \Delta(\{A, B, C\})$, and her utility is given by (1). We let $p = (p_i)_{i \in N}$ denote the shareholder-specific partisan-type distribution.

In this extended model, we show that mechanisms with richer ballot spaces still dominate mechanisms with poorer ballot spaces (provided that they distribute power similarly) when we restrict our attention on their best equilibria.

Proposition 5 *Consider two finite mechanisms V and V', a distribution of shareholdings d, and a partisan-type distribution p. Under Assumptions 1, if V has a richer ballot space than mechanism V' –but the two mechanisms distribute power similarly–, then for any BNE under $V'(d)$ there exists a BNE under $V(d)$ such that the probability of making the correct decision is higher.*

Proof. See Appendix A. ■

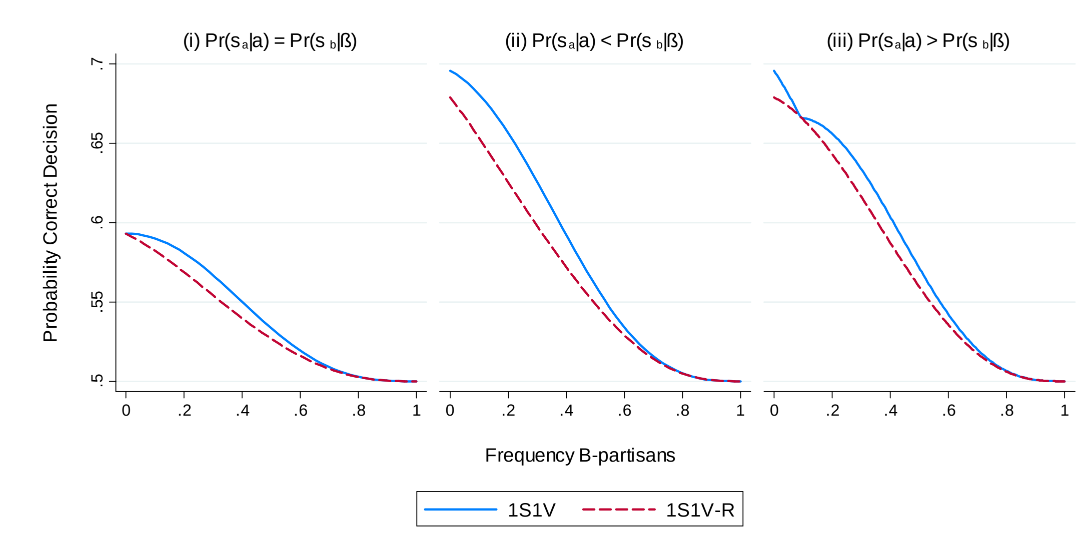

Figure 3: Probability of correct decision under 1S1V (with a large number of votes) and 1S1V-R with binary signals and a random number of B partisans. $n = 6$ and priors are even. The conditional probabilities of receiving the correct signals in each state are $Pr(s_a|\alpha) = Pr(s_b|\beta) = 55\%$ in (i), $Pr(s_a|\alpha) = 55\%$ and $Pr(s_b|\beta) = 65\%$ in (ii), and $Pr(s_a|\alpha) = 65\%$ and $Pr(s_b|\beta) = 55\%$ in (iii).

To understand this result, first note that partisan shareholders behave in the same way under V and V': they give as many points as possible to their favorite alternative. Thus, they create the same noise under the two voting mechanisms. To implement their desired outcomes, common-value shareholders have to correct for that noise while still finding a way to reveal their information. The relatively richer ballot space under V than V' gives more leeway to common-value shareholders to achieve this.

Focusing on the comparison between *1S1V-R* and *1S1V*, we can show, using numerical examples, that the latter sometimes strictly dominates the former (Figure 3). Figure 3 also illustrates that the presence of partisan shareholders creates inefficiencies, even under *1S1V*: the probability of reaching the correct decision given the collective information decreases with the frequency of partisan shareholder B under both *1S1V-R* and *1S1V*. This suggests an explanation for the empirical finding that firms with more heterogeneous shareholder base under-perform (see, e.g., Kandel et al. 2011, and Schwartz-Ziv and Volkova 2020).

4.3.2 Endogenous Information

Consider now that the signal is not free. Specifically, we introduce a pre-stage to the game in which shareholders independently and simultaneously decide whether to acquire an informative signal at cost $l > 0$, or to stay only with the prior. In the next stage of the game, the shareholders observe who acquired a signal, and then vote. For tractability, we focus on equilibria in which shareholders use pure strategies in the first stage, and the BNE that maximizes the probability of making the correct decision in each subgame.

This is the model considered in Persico (2004), with two differences: (i) we allow for more general signal structures and a more general class of voting mechanisms, and (ii) he allows for imperfect information (i.e. shareholders do not necessarily observe who acquired a signal). This simplifying assumption helps us deal more easily with the large variety of voting rules and signals that we consider here, but we argue after the statement of the result why it is robust to having unobservable information acquisition at least as far as the comparison of best equilibria of different mechanisms is concerned.

In this setup, we can show that continuous voting mechanisms (like *1S1V-D*) still dominate all the other voting mechanisms but with a different welfare criterion in mind: the probability of making the correct decision net of total information acquisition costs. It seems reasonable that shareholders should care both about the accuracy of their choice and the cost of information acquisition. Now, this does not exclude the possibility that another mechanism is superior in terms of maximizing the probability of making the correct decision independent of the information acquisition costs.

Proposition 6 *Under Assumptions 1 and 2, the probability of making the correct decision net of total information costs is higher under a continuous voting mechanism compared to any other voting mechanism.*

Proof. See Appendix A. ■

The intuition behind this result relies on the following fact. Consider a given profile of information acquisition decisions of the other shareholders. Then, an increase in the expected utility of a shareholder if she acquires information corresponds to the increase in the probability of making the correct decision net of the increase in total information acquisition costs. Hence, the information acquisition game is a potential game. Its potential is the probability of making the correct decision net of the total information acquisition costs.

Now, let us consider shareholders using the same profile of information acquisition decisions under a continuous voting mechanism as in the equilibrium of another voting mechanism. The value of the potential corresponding to the continuous voting mechanism must be at least as high as that of the other mechanism. This follows from the superior information aggregation properties of continuous voting mechanisms. For that profile of information acquisition decisions, the information costs are the same under the two mechanisms, but the probability of making the correct decision is higher under continuous voting mechanisms. Hence, in every equilibrium under continuous voting mechanisms the value of the potential must be strictly larger compared to the value of the potential in any equilibrium of any other mechanism.

The above result is robust to information acquisition decisions being unobservable. To see this, notice that the best equilibrium of the game with observable information acquisition decisions remains an equilibrium of the game with unobservable information decisions under any voting mechanism. Indeed, under observable information acquisition, in the best equilibrium of a voting mechanism when a shareholder who is expected to acquire information does not acquire information, she decreases her expected utility (otherwise, we would not be in equilibrium), but less so compared to the case of unobservable information acquisition decisions: in the first case, all shareholders adjust and use the welfare maximizing BNE of the voting subgame, but in the latter they cannot do so and the probability of making the correct decision decreases even more. Hence, continuous voting mechanisms deliver a higher probability of making the correct choice net of total information acquisition costs, even when information acquisition decisions are unobservable.

5 Endogenous Proposal

As we discussed in the Introduction, a key role of shareholder meetings is to provide proper incentives to managers.[32] The first part of our analysis abstracted from the issue of managers' incentives with respect to the design and selection of proposals, to instead focus on the information aggregation properties of voting mechanisms. In this section, we explore the effect of the voting mechanism on the managers' incentives. In particular, we allow the management to decide whether to put a proposal (of exogenously given quality) to the vote.[33] The management thus has veto power on the proposal. We allow for management's preferences to differ from shareholders' preferences over two dimensions. First, the manager may prefer the proposal to be adopted in both states. We then say that the manager is *misaligned.* Second, the manager may incur a cost if a proposal is rejected at the shareholder meeting. In order to disentangle the impact of those two dimensions, we first consider the case with costless rejection. We then consider the case with costly rejection.

[32] The proper use of information by shareholders appears to be crucial. As mentioned in Harford et al. (2018, p. 425): "Monitoring by imperfectly informed market participants can lead managers to make myopic investment decisions (Stein, 1988). Indeed, most managers admit that they are willing to sacrifice long-term shareholder value for short-term profits (Graham et al., 2005)." Similarly, Richardson (2000) finds a positive relationship between information asymmetry, between shareholders and managers, and earnings management (i.e., funky accounting practices by managers).

[33] In a previous version of the paper, we explored another margin of action for the management: the intensive margin. To do so, we allowed the management to exert costly effort in order to increase the quality of the proposal.

5.1 A General Model

We introduce a general model that encompasses all the cases that will be discussed in this section. The shareholders are modeled as in our baseline model. For the sake of conciseness, we mostly restrict our attention to two voting mechanisms: *1P1V* and *1S1V-D*, which are the two polar cases in terms of vote divisibility, and hence informational efficiency.

We introduce a new player: the manager, denoted by M. She does not belong to the set of shareholders.[34] After receiving a signal (more details below), the manager decides to either put the proposal to a vote ($x_M = P$) or veto it ($x_M = V$). If the manager vetoes, the proposal is not considered by the shareholders, and the outcome is B. If the manager calls for a vote, shareholders decide whether to accept it (outcome A) or reject it (outcome B).

Before making her decision, the manager receives a signal $s_M \in [0,1]$. In any state ω, the signal s_M is drawn from a distribution $F_M(\cdot|\omega)$, with density $f_M(\cdot|\omega)$, independently from the signals of the shareholders. We assume that the manager's type t_M is weakly increasing in s_M.

The utility of the manager u_M can be decomposed into two parts: an outcome-utility u_M^o and a potential (reputation) cost c. The outcome-utility depends on whether the manager is *aligned* ($a = 1$) with the shareholders or *misaligned* ($a = 0$). When aligned, the manager has the same outcome-utility as a shareholder holding one share. In particular, for any decision $O \in \{A, B\}$ and state $\omega \in \{\alpha, \beta\}$, we have:

$$u_M^o(O|\omega, a = 1) = \mathbf{1}_{\{O=A,\omega=\alpha\}} - \mathbf{1}_{\{O=A,\omega=\beta\}}.$$

When misaligned, the manager only wants the reform to pass:

$$u_M^o(O|\omega, a = 0) = \mathbf{1}_{\{O=A\}}.$$

We assume that the manager has an ex-ante probability $\mu \in [0,1]$ to be misaligned, and that the draw of a is independent of both the state and the signals. Only the manager knows whether she is aligned.

The manager also incurs a cost $c \geq 0$ if the proposal is turned down by shareholders at the meeting. Shareholders know the value of c. The utility of the manager can be written as:

$$u_M(O, x_M|\omega, a) = u_M^o(O|\omega, a) - c \times \mathbf{1}_{\{x_M=P,O=B\}}.$$

While the game has now an additional player, we still use the notion of dominance defined for the exogenous proposal case: a voting mechanism dominates another if the

[34] This assumption is not crucial for our results except for the case of a fully aligned manager under *1S1V*. Without that assumption, the manager never vetoes in that case, making it less interesting.

best and worst equilibrium of the former mechanism (in terms of informational efficiency, taking both shareholders' and the manager's signals into account), outperform the best and worst equilibria of the latter.

In order to analyze the endogenous proposal case, we need to consider asymmetric priors. In particular, shareholders may attach different probabilities to each state because of the strategic behavior of the manager. Crucial to the analysis that follows, our result that *1S1V-D* dominates *1P1V* is robust to asymmetric priors. In fact, *1S1V-D* remains efficient in that case.[35]

5.2 Costless Rejection

We start by investigating the case for which the manager does not suffer from a cost if her proposal is turned down at the shareholders meeting ($c = 0$). The only potential source of conflict with shareholders is then whether the proposal should pass only in state $\omega = \alpha$, or in both states.

The analysis of this case allows us (i) to show that the informational efficiency of a voting mechanism affects the incentives of the manager, whether or not there is conflict with the shareholders; (ii) to identify a new rationale for the selection of good proposals by the manager and hence the high approval rate of management's proposal in practice; and (iii) to highlight a new testable prediction about the relationship between the approval rate of proposals at the meeting and the alignment of manager and shareholders.

5.2.1 Comparison of Voting Mechanisms

The following proposition shows that, when rejection of the proposal at the meeting is costless for the manager, *1S1V* continues to perform better than *1P1V*:

Proposition 7 *Under Assumptions 1 and 2, for every probability $\mu \in [0,1]$ that the manager is misaligned, the one-share-one-vote with partial abstention mechanism (V^{1S1V}) dominates the one-person-one-vote mechanism (V^{1P1V}). For some parameter values, the dominance is strict.*

Proof. See Appendix B. ■

To understand this result, let us first suppose that the manager is aligned with shareholders with probability one (i.e., $\mu = 0$). When an aligned manager takes the behavior of

[35] A simple modification of Lemma 1 to allow for asymmetric priors shows that the posterior $\Pr(\alpha|s) > \frac{1}{2}$ if and only if $\sum_{i \in N} \ln(t_i) > \ln\left(\frac{1-\Pr(\alpha)}{\Pr(\alpha)}\right)$. Thus, in order to implement the efficient decision at the voting stage, shareholders need to compensate for the different likelihood across states. Under *1S1V-D*, shareholders can still implement the efficient decision with the equilibrium $\sigma_i^{1S1V-D}(t_i) = c \ln t_i + \kappa_i$, with $\sum \kappa_i = c \ln\left(\frac{\Pr(\alpha)}{1-\Pr(\alpha)}\right)$.

shareholders as given, she puts the proposal to a vote only if she believes that it is sufficiently likely to be a good proposal (i.e., that the state is α). The definition of "sufficiently likely" depends on the magnitude of type-I (outcome A in state β) and type-II (outcome B in state α) errors at the voting stage. The higher the overall probability of error, the higher the manager's incentives to veto. As we have seen in the previous section, for any given information structure, errors at the voting stage are higher under *1P1V* than *1S1V*. Hence, the aligned manager has stronger incentives to veto the proposal under *1P1V* than *1S1V*.

Given that the manager is (partially) informed about the state of the world, her decision of whether to veto influences the beliefs of shareholders about the quality of the proposal. The higher propensity of the manager to veto under *1P1V* implies that the decision to call for a vote is a stronger signal that the proposal is good under *1P1V* than *1S1V*. Hence, shareholders start their meeting with more precise information (and more favorable to A) under *1P1V* than *1S1V*, which increases the probability that they make a correct decision. There is thus a trade-off between the informational efficiency of *1S1V* in the voting phase and the poorer selection incentives it gives to the manager before the meeting. In Proposition 7, we prove that, despite this trade-off, *1S1V* continues to dominate *1P1V*. For the case of a perfectly aligned manager, the intuition is similar to that of previous results: conditional on the state, all players prefer the same outcome. Since *1S1V* gives more flexibility to transmit information than *1P1V*, it dominates.

The case with a potentially misaligned manager is less straightforward. The trade-off between selection and voting efficiency remains when the manager is misaligned with positive probability, i.e., $\mu > 0$. The only difference is that under both *1S1V* and *1P1V*, when the manager is misaligned ($a = 0$), she always puts the proposal to a vote. This is true even if she receives a very precise signal that the proposal is undesirable. Hence, the ratio of bad proposals over good ones proposed by a misaligned manager is higher than the same ratio for an aligned managers. This affects the shareholders' beliefs at the meeting. But, since the behavior of the misaligned manager is the same under *1S1V* and *1P1V*, this effect does not change differently the beliefs of shareholders under the two voting mechanisms.

5.2.2 Empirical Implications

This analysis sheds a new light on the very high approval rate of management's proposals by shareholders in practice (see, e.g., Maug and Rydqvist 2009, Babenko et al. 2018, and Bach and Metzger 2019). It has been argued (informally) that such a high rate can be explained by the selection of proposals by managers (see, e.g., Becht et al. 2016). The idea is that the fear of having a proposal turned down, which has negative consequences for the managers, gives managers incentives to withhold low quality proposals. This means that only high quality proposals are put to a vote, and are approved at a very high rate.

In our model, shareholders approve the proposal more frequently when the manager has the power to veto it than when she does not. Yet, the mechanism at play is different. As we explained above, it does not necessarily rely on the management incurring a cost when its proposal is rejected by shareholders, as we assumed $c = 0$ in this subsection. It instead relies on the manager's willingness to make decision that are beneficial for the firm and shareholders. This is another source of selection for proposals at shareholders meetings that has implications for the empirical literature studying the effects of shareholders voting on firms' performance.

Our analysis also produces a testable prediction: the approval rate of proposals at the shareholder meeting is decreasing in μ, the probability that the manager is misaligned. Different measures of alignment between shareholders and management could be used to test this prediction, such as the extent of the CEO equity-based compensation as in Datta et al. (2001), bonus-compensation as in Grinstein and Hribar (2004), and the sensitivity of the CEO compensation to stock performance post acquisition as in Harford and Li (2007).

5.3 Costly Rejection

We now analyze the case in which the manager incurs a cost when her proposal is rejected at the meeting ($c > 0$). There are thus two sources of conflicts with shareholders: the manager may want the proposal to pass in both states of the world (when she is misaligned), and she fears a no vote by shareholders (independently of whether she is misaligned).

Costly rejection moderates the incentives of the manager to call for a vote. This moderating effect is stronger (i) the higher the cost of rejection c, and (ii) the higher the probability of rejection by shareholders. Given that the probability of rejection is generically different under *1S1V* and *1P1V*, the moderating effect affects manager's behavior differently under the two mechanisms. Our main finding is that, in some cases, the cost of rejection alters the trade-off between selection and voting efficiency so that the selection effect under *1P1V* becomes sufficiently strong to overturn the dominance of *1S1V*.

To simplify the analysis, we work under the assumption that the manager is perfectly informed about the quality of the proposal (i.e., she knows the state of the world). Thus, the aligned manager (i) always vetoes a bad proposal, and (ii) has the same incentives as the misaligned manager when the proposal is good.

5.3.1 Equilibrium Behavior

Given that the probability of rejection is higher in the bad state (under both *1S1V* and *1P1V*), a misaligned manager has stronger incentives to veto when the proposal is bad. Hence, she must veto with a higher probability in the bad state. This narrows down the potential equilibrium strategy to three forms: (i) the manager never vetoes, (ii) the manager never vetoes in the good state but vetoes with positive probability in the bad state, and (iii) the manager vetoes in both states. That latter case, which occurs only with specific out-of-equilibrium beliefs for shareholders or very large c, is uninteresting. We thus focus on the two other types of equilibria in what follows.

In these equilbria, the manager never vetoes in the good state (regardless of her type), she always veto a bad reform when aligned, and vetoes in the bad state with probability $\gamma_V \in [0,1)$ when misaligned.[36] Together, μ and γ_V determine the shareholders' prior that the reform is good conditional on a vote, which in turn determines the shareholders' optimal behavior at the meeting. In equilibrium, it must be that the probability of vetoing in the bad state is optimal given the best response of shareholders.

A key driver of the manager's behavior is the probability that the proposal is rejected by shareholders. For the misaligned manager considering whether to veto the proposal in the bad state, the decision relies on the probability that the proposal is rejected in that state, i.e., 1 minus the probability of type-I error, p_I. The expected payoff of the manager when she calls for a vote is, therefore, $p_I - c\,(1 - p_I)$. This has to be compared to a payoff of 0 if she vetoes the proposal. As a result, the manager strictly prefers to veto if $p_I > \frac{c}{1+c}$, and is indifferent if $p_I = \frac{c}{1+c}$.

How does p_I vary across voting mechanisms? Depending on the situation, it may be higher or lower under *1S1V* than *1P1V*. This is illustrated in Figure 4, which shows the probability of type-I errors as a function of the prior for the case of an exogenous proposal. The parameters used for this example are $n = 5$, and binary signals with $\Pr(s_\alpha|\alpha) = \Pr(s_\beta|\beta) = 0.6$. The figure shows that, depending on shareholders' prior (α), type-I error can be more or less likely under *1P1V* than *1S1V*. Thus, incentives to veto in the bad state are stronger under *1P1V* only in some cases.[37]

5.3.2 Comparison of Mechanisms

When the probability of type-I error is higher under *1S1V*, the better selection of proposals by the manager reinforces the voting efficiency advantage of *1S1V* (which is defined in terms of both types of errors). *1S1V* then dominates *1P1V* even more than when the proposal is exogenous. By contrast, when the probability of type-I error is higher under *1P1V*, the better selection of proposals by the manager under *1P1V* compensates for its lower voting efficiency. As we show below with a numerical example, in equilibrium, this

[36] Note that if $\gamma_V = 1$, the shareholders can make the inference that conditional on voting, the state must be good. In that case, the incentives of shareholders is to always approve the reform. But this would give the manager incentives to deviate from $\gamma_V = 1$. Hence, this cannot be an equilibrium.

[37] In the case of binary signals, the optimal decision is simply implemented with a threshold: the reform is only accepted given a number of signals in favor of the reform. All priors that have the same optimal threshold generate the same probability of type-I error. This, together with the fact that the optimal threshold decreases with the prior explains the step function under *1S1V* in Figure 4.

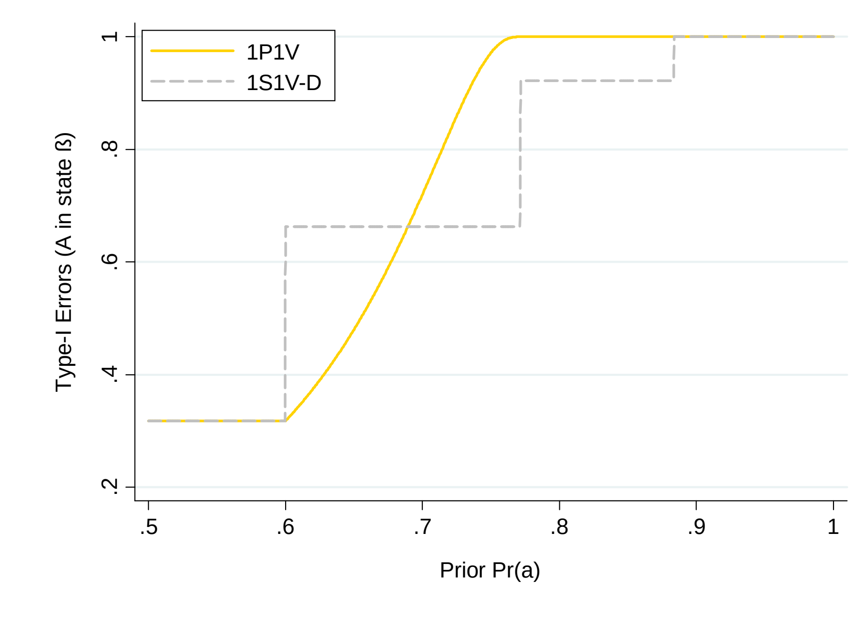

Figure 4: Probability of a type-I error under 1P1V and 1S1V as a function of the prior on state α (with an exogenous agenda). Parameters assumed: $n = 5$, and binary signals with $\Pr(s_\alpha|\alpha) = \Pr(s_\beta|\beta) = 0.6$.

selection effect can be strong enough to overturn the higher voting efficiency of *1S1V*.

Consider the following case: there are 5 shareholders ($n = 5$), the prior of the good state is $\Pr(\alpha) = 0.5$, and there are binary signals with $\Pr(s_\alpha|\alpha) = \Pr(s_\beta|\beta) = 0.6$. Suppose, moreover, that the manager is misaligned with probability $\mu = 0.6$, and that the cost of rejection $c \in [0, 1]$. Figure 5 shows how the utility of shareholders varies with c.[38] For sufficiently low values of c (below 0.63), when the manager calls for a vote with probability 1 in the bad state under both mechanisms, then $p_I^{1S1V}, p_I^{1P1V} > \frac{c}{1+c}$. Hence, it is a best response for the manager not to veto under both *1S1V* and *1P1V*. Since there is no differential selection, the only difference across mechanisms comes from their ability to aggregate the information dispersed among shareholders. Consistent with what we have seen in the case of an exogenous proposal, *1S1V* then dominates *1P1V*. By contrast, when c is higher than 0.63, the same strategy by the manager implies $p_I^{1S1V} > \frac{c}{1+c} > p_I^{1P1V}$. It is thus not a best response for the manager to always call for a vote under *1P1V* (but it is under *1S1V*). In equilibrium, she calls for a vote in the bad state with probability $1 - \gamma_V \in (0, 1)$ under *1P1V*, with γ_V increasing in c.[39] This selection of proposal by the manager is beneficial to shareholders. For c sufficiently large (above 0.85), *1P1V* dominates *1S1V*.

[38] In these numerical examples we focus on the optimal symmetric equilibria at the voting stage.

[39] For a given c, there is a unique probability of type-I error such that the manager is indifferent under *1P1V*: $p_I^{1P1V} = \frac{c}{1+c}$.

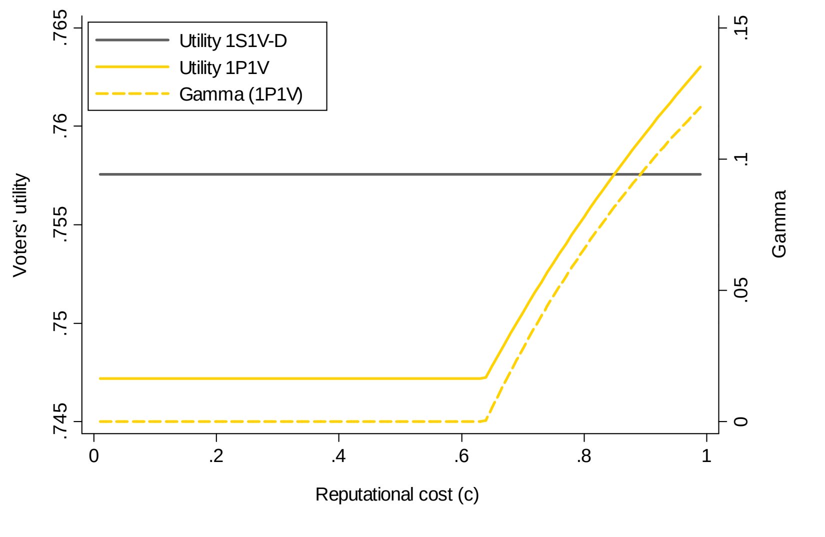

Figure 5: Welfare comparison between 1S1V and 1P1V. The parameters assumed for this comparison are $n = 5$, $\Pr(\alpha) = 0.5$, and binary signals with $\Pr(s_\alpha|\alpha) = \Pr(s_\beta|\beta) = 0.6$.

Expanding on the previous example, we can compare the utility of shareholders under *1S1V* and *1P1V* for various values of c and μ. Figure 6 shows that *1P1V* dominates *1S1V* when both c and μ are sufficiently high. The high value of c guarantees that the manager vetoes in some situations, and hence that the stronger selection advantage of *1P1V* is present. The high value of μ guarantees that the manager is often misaligned, and hence that the selection advantage of *1P1V* is large enough.[40]

5.3.3 Empirical Implications

In equilibrium, the quality of the proposal conditional on a vote being called is increasing in c under both *1S1V* and *1P1V*. Thus, the probability of approval of the proposal is also increasing in c. This confirms the informal argument in the literature (see discussion above) that the presence of a reputation cost for managers helps explain the high approval rate of the management's proposals. This result also suggests an explanation for the much higher approval rate of management's proposals than shareholders' proposals in practice (Bach and Metzger 2019): shareholders do not suffer (as high) reputation cost when their

[40] When aligned, the manager vetoes for sure when the state is bad. She does so under both *1S1V* and *1P1V*. When the state is good she has similar incentives as the misaligned manager (that is, she trades off the risk of rejection with the gain in case the proposal is adopted). Thus, the decision to put the proposal to a vote has a different effect on shareholders' beliefs than when the manager is always misaligned. In particular, due to the veto by the aligned manager in the bad state, the expected quality of the proposal conditional on a vote being called increases. This is beneficial for shareholders. Given that this positive selection effect when the manager is aligned occurs both under *1S1V* and *1P1V*, it results in a decrease of the overall advantage of *1P1V* in terms of selection (this advantage only materializes when the manager is misaligned).

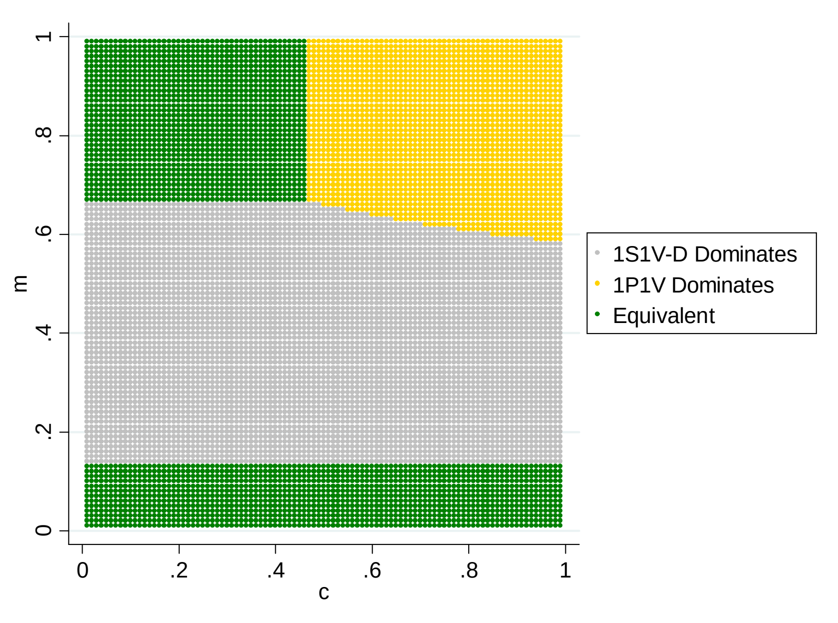

Figure 6: Welfare comparison across systems for different combinations of c (horizontal axes) and μ (vertical axes). The parameters assumed for this comparison are $n = 5$, $\Pr(\alpha) = 0.5$, and binary signals with $\Pr(s_\alpha|\alpha) = \Pr(s_\beta|\beta) = 0.6$.

proposals are turned down at the meeting. Differences in this cost among shareholders could potentially help explain why the approval rate of shareholder proposals is strongly associated with the identity of the sponsor (Gillan and Starks 2000).

Note also that the aforementioned testable prediction that the approval rate of managers' proposals at the meeting is decreasing in the probability that the manager is misaligned (μ), also holds when $c > 0$.

6 Discussion of Modeling Assumptions

6.1 State-Contingent Preferences

A central assumption of our baseline model is that shareholders have state-contingent preferences: conditional on the state of the world, they all agree whether the management's proposal should be approved or rejected. This is a standard assumption in the literature on shareholders voting (see, e.g., Maug and Yilmaz 2002, Marquez and Yilmaz 2008, Levit and Malenko 2011, Eso, Hansen, and White 2015, Malenko and Malenko 2019, Bar-Isaac and Shapiro 2019, Meirowitz and Pi 2020, Ma and Xiong 2020). It indeed seems natural to assume that (most) shareholders share the common goal of maximizing the value of the firm.

There are various pieces of empirical evidence that are coherent with the state-contingent preferences assumption. More precisely, the literature uncovers facts that are in line with models of strategic voting making that assumption, similar to the one developed above. For instance, Maug and Rydqvist (2009) structurally estimate such a model of strategic voting using data about U.S. shareholders meetings between 1994 and 2003. They find that the voting behavior of shareholders at those meetings is in line with their model. As predicted: (i) shareholders vote more in favor of proposal when the supermajority threshold increases, and (ii) there is essentially no effect of supermajority thresholds on acceptance rate.

Christoffersen et al. (2007) study vote trading in the US and the UK and find patterns that are in line with information aggregation theory of voting (see, e.g., Eso, Hansen, and White 2015). They indeed uncover an active market for votes, both in the US and the UK, where the average vote sells for a price of *zero*. Moreover, as predicted by the theory, vote trading increases (i) with asymmetric information among shareholders, (ii) the importance of the proposal at stake (proxied by poor performance of firm), and (iii) if the pivot probability is high. Finally, warnings of votes that violate corporate governance standards (which they interpret as a negative public signal about the proposal that reduces information asymmetry among shareholders) reduce vote trading.

Calluzzo and Dudley (2019) study the influence of proxy advisors on firm voting outcomes, policies and values. They find that, as predicted by Malenko and Malenko (2019) based on a model including shareholders with state-contingent preferences, proxy advisors have a large influence when shareholders have weak incentives to acquire information.

There is also evidence that, at first sight, appears to contradict the predictions of a model of strategic voting including shareholders with state-contingent preferences: Li et al. (2021) find that there is a lot of trading by mutual funds after shareholder meetings. Yet, Meirowitz and Pi (2020) show that this is actually consistent with such a model once one takes into account that shareholders who vote in the meetings are also traders after the meeting. In such a setting, shareholders do not fully reveal information through their vote, which prevents information aggregation. This creates opportunities to trade after shareholder meetings. In our model, under *1P1V*, there would be a different reason for trading after the vote. Depending on the precision of their signal, shareholders have different beliefs about the probability that the decision at the meeting was correct. Hence, shareholders with sufficiently precise signals against the decision made at the meeting would be willing to sell their shares, and those with sufficiently precise signals aligned with the decision made at the meeting would be willing to buy more shares. These trading patterns are in line with the findings of Li et al. (2021) about the behavior of mutual funds after the meetings.[41]

[41] Our model also predicts that the trading patterns would be systematically different under *1S1V-D* and *1P1V*. Under *1S1V-D*, when voting fully aggregates information, shareholders' posteriors are identical.

Last but not least, it is important to stress that we are not trying to argue that every single shareholder has state-contingent preferences. Indeed, as we mentioned in Section 4.3.1, there is evidence suggesting that disagreement among shareholders may not only stem from information asymmetries. This is exactly the reason why, in that section, we consider an extension of our model that allows for the presence of partisan shareholders. And we have shown that, in the presence of such shareholders, *1S1V-D* still outperforms other voting mechanisms in terms of information aggregation.

6.2 Information Asymmetry

Another key assumption of our model is that some shareholders are better informed than others. We view this assumption as uncontroversial. First, as explained in Knyazeva, Knyazeva, and Kostovetsky (2018, p. 681): "the precision of a trader's [...] private information may be a function of the trader's overall or company specific investment experience, local knowledge, or the extent of resources that the trader can allocate to information gathering." And, indeed, Kim and Verrecchia (1991) show that traders react differently to release of public information about a given firm, i.e., less informed traders, who revise the beliefs more, react more. Also, Iliev and Lowry (2015) and Iliev, Kalodimo, and Lowry (2018) find that mutual funds vary greatly in their reliance on proxy advisory recommendations, with the more informed voting less in line with the recommendations.

Second, the literature provides evidence that shareholders have different incentives to invest in acquisition of information (see, e.g., Chen, Harford, and Kai 2007, and Fich, Harford and Tran 2015).

Third, information asymmetries among shareholders help explain phenomena that are difficult to explain without such asymmetries (see, e.g., Glosten and Milgrom 1985 discussion of the bid-ask spread).

Finally, there is an empirical literature studying information asymmetry among shareholders, using different measures (see, e.g., Brown and Han 1992, Healy, Palep, and Sweeney 1995, Welker 1995, Iliev and Lowry 2014, and Knyazeva, Knyazeva, and Kostovetsky 2018). It points toward substantial information asymmetries among shareholders/investors. This is true both across types of shareholders (see, e.g., Sias, Starks, and Titman (2006) for evidence of the informational advantage of institutional investors over other types of investors), and within a given type (see, e.g., Knyazeva, Knyazeva, and Kostovetsky (2018) for evidence of heterogeneity among institutional investors).

There is then no room for trade after the meeting.

6.3 No Communication

In our baseline model, we assume that shareholders cannot communicate before the vote. This is not an innocuous assumption. If costless, communication can indeed improve information aggregation (see, e.g., Coughlan 2001), and mute differences between voting mechanisms (see, e.g., Gerardi and Yariv 2007). The idea is simple: when shareholders have state-contingent preferences, they have incentives to truthfully reveal their private information to one another, and then vote unanimously for the efficient outcome.

There are nonetheless several hurdles to communication among shareholders. First, in the presence of partisan shareholders, communication is impeded (Coughlan 2001). The problem is that those shareholders have incentives to pretend that they have state-contingent preferences but that they have received a signal in favor of their preferred alternatives. And, as we have shown above, *1S1V-D* still dominates other voting mechanisms in the presence of such shareholders (without communication).

Second, even if there are no partisan shareholders, communication among shareholders is far from costless. In the case of most public firms, shares are distributed among many, scattered, individuals and institutions. It is thus logistically challenging to organize communication. Moreover, as explained in Malenko and Malenko (2019, p.2470), "[...] investors fear that communication with others can be considered "forming a group", which would trigger costly administrative filling requirements and, in some cases, a poison pill. There could also be a cost of publicly disclosing your information: "[...] investors are often reluctant to publicly disclose their intention to vote against management, fearing that doing so would be viewed as an activist campaign and lead to managerial retaliation."

6.4 Vote Trading

Our baseline model does not allow shareholders to trade votes before the meeting. Yet, we know that there is an active market for votes (Christoffersen et al. 2007) and that vote trading can be beneficial for information aggregation. Eso, Hansen, and White (2015) study vote trading and, assuming one share per shareholder, prove the existence of an efficient equilibrium under *1P1V* in which vote trades at a price of zero. In that equilibrium, uninformed shareholders sell their votes to informed shareholders. As in the case of communication, allowing for vote trading could then mute differences between *1S1V-D* and other voting mechanisms. But, there are reasons to believe this is not the case.

First, note that vote trading is irrelevant under *1S1V-D*: there is no gain from trade because the equilibrium is efficient. Second, there are various hurdles to vote trading under other voting mechanisms. For instance, the efficient equilibrium in Eso, Hansen, and White (2015) is not robust to the presence of sufficiently many partisan shareholders. Moreover, we conjecture that differences in signal precision would also prevent efficient aggregation of information. The problem in that case is that shareholders need to know how precise their information is compared to that of other shareholders in order to decide optimally whether to "buy" or "sell" votes. There is no clear way for shareholders to do so. This issue becomes even worse if there is ambiguity about the information technology of other shareholders.

7 Conclusions

Shareholders typically hold different number of shares. This fact, which has been overlooked by most of the literature on voting at those meetings, raises questions about which voting mechanism should be used. In this paper, we have explored this question with a special focus on the informational efficiency of different voting mechanisms. We first considered the case in which the management is passive and does not select the proposal being voted on. We proved two main results. First, for any distribution of shareholdings, the one-share-one-vote mechanism (*1S1V*) dominates the one-shareholder-one-vote mechanism (*1P1V*) independently of whether information accuracies and shareholdings are correlated. Yet, *1S1V* is not always efficient. Second, the one-share-one-vote mechanism with divisible votes (*1S1V-D*) –and any other continuous voting mechanism– is efficient. We then considered the case in which the management decides whether to put the proposal to a vote. We uncovered a trade-off between selection and voting efficiency underlying the comparison of *1S1V* and *1P1V*: *1S1V*'s higher voting efficiency implies worse selection incentives for the management. We found that the negative effect of worse selection incentives on shareholders' welfare can be large enough to wash out the higher voting efficiency of *1S1V*.

Beyond possible calls for allowing perfect vote-divisibility at shareholders meetings, our results also have implications for the consequences, and hence desirability, of dual class capital structures and stock buybacks and splits. Through their effect on the number of voting shares held by shareholders, dual class shares, stock buybacks and splits indeed affect shareholders' ability to reveal their information about the quality of management proposals through voting. Hence, preferential shares and stock buybacks should affect negatively the efficiency of decisions at shareholder meetings, while stock splits should affect it positively. Through their influence on voting efficiency, they should also affect the incentives of managers to select proposals that benefit shareholders. These effects complement the common arguments in favor and against dual class capital structures, stock buybacks and splits.[42]

[42] Stock buybacks are often considered as a more flexible way than dividend to return money to shareholders. They are particularly appealing for firms sitting on unused cash and facing poor growth opportunities (see, e.g., Dittmar 2000, Grullon and Michaely 2004, and Brav et al. 2005). The typical argument against stock buybacks is that they are used by the management of the company to manipulate the short-term stock prices (see, e.g., Hribar, Jenkins, and Johnson 2006, and Almeida, Fos, and Kronlund 2016).

Our analysis also informs the empirical literature and the interpretation of existing empirical results. For instance, we found that the strategic selection of proposal by managers can explain the very high approval rate of management's proposals in practice. Our result that this selection becomes more stringent as the (reputation) cost of a proposal rejection at the meeting increases suggests that the much higher approval rate of management's proposals than shareholders' proposals in practice may be due to a lower reputation cost for shareholders. Our model also predicts that the approval rate of proposals at the shareholder meeting is decreasing in the misalignment of the manager with shareholders; a testable prediction. Finally, our result that heterogeneity in preferences among shareholders impedes the aggregation of information at the meeting proposes a mechanism for the empirical finding that firms with more heterogeneous shareholder base under-perform.

Our analysis inevitably abstracts from important real-life features of shareholder meetings. We view the trading of shares before (and after) the meeting as particularly interesting and important.[43] Under the one-share-one-vote mechanism, the trading of shares before the meeting determines the distribution of votes among shareholders. This could affect the outcome of the vote dramatically. The effect could be beneficial if better informed shareholders have stronger incentives to acquire shares, but detrimental if the opposite is true. With continuous voting mechanisms such an effect of share trading before the meeting is potentially less impactful (or even mute). We plan to explore this issue further in future research.

Stock splits are more of a puzzle in the literature (see discussion in Easley, O'Hara, and Saar 2001). It has been argued that they are used to increase the liquidity of the company's shares and to get the stock's price in an acceptable trading range (see, e.g., Copeland 1979, and Baker and Gallagher 1980). Another explanation is that the company's management prefers diffuse ownership in order to avoid control by large shareholders (see, e.g., Powell and Baker 1993). Yet another explanation is that stock splits aim at reducing information asymmetry, either by directly revealing some private information or by attracting more attention to the company (see, e.g., see Grinblatt, Masulis, and Titman 1984, Brennan and Copeland 1988, Brennan and Hughes 1991). Empirical evidence in support of those different theories are mixed.

[43] Recent theoretical papers on the topic include Meirowitz and Pi (2020), and Levit, Malenko, and Maug (2021a,b).

Appendix A: Proofs for Section 4

Proof of Lemma 1. First, note that

$$\mathbb{P}(\alpha|\mathbf{s}) = \frac{\prod\limits_{i\in N} f_i(s_i|\alpha)\,\mathbb{P}(\alpha)}{\prod\limits_{i\in N} f_i(s_i|\alpha)\,\mathbb{P}(\alpha) + \prod\limits_{i\in N} f_i(s_i|\beta)\,\mathbb{P}(\beta)}.$$

Thus, $\mathbb{P}(\alpha|\mathbf{s}) > 1/2$ requires $\prod\limits_{i\in N} f_i(s_i|\alpha) > \prod\limits_{i\in N} f_i(s_i|\beta)$, or equivalently $\sum_{i\in N} \ln\left(\frac{f_i(s_i|\alpha)}{f_i(s_i|\beta)}\right) > 0$. ■

Proof of Proposition 1 . In the sequel, we denote the conditional density of shareholder i's type t_i in state ω by $g_i(t_i \mid \omega)$, it is defined by $g_i(t_i \mid \omega) = f_i\left((s_i)^{-1}(t_i) \mid \omega\right)$. We denote by $G_i(\cdot \mid \omega)$ the corresponding cumulative distribution function. Moreover, we denote the (unconditional) density associated to shareholder i's type t_i by $g_i(t_i)$, i.e. defined by $g_i(t_i) = \mathbb{P}(\alpha)g_i(t_i \mid \alpha) + \mathbb{P}(\beta)g_i(t_i \mid \beta)$.

We know from Lemma 2 (in this Appendix) that the best BNE under $\{X, w\}$ is weakly better than the best BNE under $\{X', w\}$. Applying Lemma 3 (in this Appendix), we obtain that the worst BNE under $\{X, w\}$, σ, yields at least an expected utility of 0 to each shareholder. As there exists a BNE under $\{X', w\}$, σ', which exactly yields an expected utility of 0 to each shareholder (as there is no decisive shareholder by assumption, any profile yielding a sure outcome with no decisive shareholder is an equilibrium), we also obtain that the worst BNE under $\{X, w\}$ (σ) is no worse than the worst BNE under $\{X', w\}$ (σ'). ■

Lemma 2 *Let $\{X, w\}$ and $\{X', w\}$ be finite voting rules with $X'_i \subseteq X_i$ for all i. Then, for any BNE σ' under $\{X', w\}$, there exists a BNE σ under $\{X, w\}$ such that the ex-ante probability of implementing the correct outcome is (weakly) higher at σ than at σ'.*

Proof. As the utility of every shareholder is linearly increasing with the probability that the correct outcome is implemented, we employ in this proof as in the later proofs the terms "welfare" and "more efficient" to refer to this common utility. The main task of the proof is to show that a welfare-maximizing (and thus a BNE) exists for any finite voting rule $\{X, w\}$. This is shown in two steps.

Claim 1: for any profile σ, there is a profile σ' such that $\mathbb{E}[u_i(\sigma')] \geq \mathbb{E}[u_i(\sigma)]$ and where for all $i \in N$, $\sigma'_i : T_i \to X_i$ is a pure, weakly increasing strategy.

For any $i \in N$, we may write:

$$\begin{aligned}\mathbb{E}[u_i(x, \sigma_{-i}) \mid t_i] = h_i \times \Bigg(& \mathbb{P}(\alpha \mid t_i) \Bigg(\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j > -x \mid \alpha) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j = -x \mid \alpha) \Bigg) \\ & - (1 - \mathbb{P}(\alpha \mid t_i)) \Bigg(\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j > -x \mid \beta) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j = -x \mid \beta) \Bigg) \Bigg).\end{aligned}$$

Hence,

$$\frac{\partial \mathbb{E}[u_i(x,\sigma_{-i}) \mid t_i]}{\partial t_i} = h_i \times \frac{\partial \mathbb{P}(\alpha \mid t_i)}{\partial t_i} \Bigg(\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j > -x \mid \alpha) + \mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j > -x \mid \beta)$$
$$+\frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j = -x \mid \alpha) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j\neq i} x_j = -x \mid \beta) \Bigg).$$

We have $\mathbb{P}(\alpha \mid t_i) = \frac{g_i(t_i|\alpha)}{g_i(t_i|\alpha)+g_i(t_i|\beta)} = \frac{t_i}{1+t_i} = 1 - \frac{1}{1+t_i}$, and we get $\frac{\partial \mathbb{P}(\alpha|t_i)}{\partial t_i} = \frac{1}{(1+t_i)^2} > 0$. This implies that $\frac{\partial \mathbb{E}[u_i(x,\sigma_{-i})|t_i]}{\partial t_i}$ is weakly increasing in x (*increasing differences*). As X_i is finite, let us define the best reply σ'_i to σ_{-i} as the smallest value of $x \in X_i$ maximizing the expected utility (we abuse notation as σ'_i is a pure strategy) :

$$\forall t_i \in T_i, \quad \sigma'_i(t_i) = \min\{x \in X_i \mid \mathbb{E}[u_i(x,\sigma_{-i}) \mid t_i] \geq \mathbb{E}[u_i(y,\sigma_{-i}) \mid t_i], \quad \forall y \in X_i\}.$$

The strategy σ'_i must be weakly increasing. Assume by contradiction that $x = \sigma'_i(t_i) > y = \sigma'_i(t'_i)$ for $t_i < t'_i$. Then, by definition of $\sigma'_i(t_i)$ as a minimum, we have $\mathbb{E}[u_i(x,\sigma_{-i})|t_i] > \mathbb{E}[u_i(y,\sigma_{-i})|t_i]$ and thus, using the property of *increasing differences*, we obtain :

$$\mathbb{E}[u_i(x,\sigma_{-i})|t'_i) = \mathbb{E}[u_i(x,\sigma_{-i})|t_i] + \int_{t_i}^{t'_i} dt \frac{\partial \mathbb{E}[u_i(x,\sigma_{-i})|t]}{\partial t}$$
$$> \mathbb{E}[u_i(y,\sigma_{-i})|t_i] + \int_{t_i}^{t'_i} dt \frac{\partial \mathbb{E}[u_i(y,\sigma_{-i})|t]}{\partial t} = \mathbb{E}[u_i(y,\sigma_{-i})|t'_i],$$

a contradiction with y being a best reply at t'_i. Hence, the (pure) strategy σ'_i is weakly increasing in t'_i. By applying the same reasoning iteratively for $i = 1,\ldots,n$, we obtain the profile σ', as desired.

Claim 2: the rule $\{X,w\}$ admits a welfare-maximizing strategy profile, which is thus a BNE.

Let us consider the family of profiles consisting in pure, weakly increasing strategies. Let us write $X_i = \{x^1,\ldots,x^k\}$ with $x^1 < \ldots < x^k$. A pure, weakly increasing strategy σ_i is thus described by a series of cutoffs $(t_i^j)_{0\leq j\leq k} \in (T_i)^{k+1}$, with $t_i^0 = \delta_i$ and $t_i^k = \frac{1}{\delta_i}$, and such that $\forall j$, $t_i^j \leq t_i^{j+1}$ and $t_i \in (t_i^j, t_i^{j+1}) \Rightarrow \sigma_i(t_i) = x^{j+1}$. A profile of such strategies is thus described by a series of cutoffs for each shareholder $i \in N$. Now, as each distribution $G_i(\cdot \mid \omega)$ does not admit any atom, the expected utility attached to such profile is a continuous function of its cutoffs. As cutoffs are taken in a compact set, there is a profile σ^* maximizing the expected utility among all profiles in the family. By application of Claim 1, σ^* maximizes the expected utility among all profiles. As the game is of common interest, the profile σ^* must be a BNE, and hence a welfare-maximizing BNE (this is the original argument of Mc Lennan, 1998).

Finally, to conclude, whenever two rules $\{X,w\}$ and $\{X',w\}$ are such that $\forall i \in N$, $X'_i \subseteq X_i$, then each profile under rule $\{X',w\}$ can be reproduced under rule $\{X,w\}$. It follows that the welfare-maximizing profile (BNE) σ under $\{X,w\}$ achieves at least as much expected utility as the welfare-maximizing profile (BNE) σ' under $\{X',w\}$. This concludes the proof. ■

Lemma 3 *For any BNE σ under a finite rule $\{X, w\}$, we have $\forall i \in N$, $u_i(\sigma) \geq 0$.*

Proof. We introduce a couple of notations for the proof. For a strategy profile σ and a type vector $\mathbf{t} = (t_i)_{i \in N}$, we denote by $p^A(\sigma(\mathbf{t}))$ the probability that A is implemented given the votes $\sigma(t)$:

$$p^A(\sigma(\mathbf{t})) = \mathbb{P}\left(\sum_{i \in N} \widehat{\sigma}_i(t_i) > 0\right) + \frac{1}{2}\mathbb{P}\left(\sum_{i \in N} \widehat{\sigma}_i(t_i) = 0\right).$$

Note that we have $\int p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i = \mathbb{P}(O = A|\sigma)$.

Claim 1: For any profile of pure, weakly increasing strategies σ, we have $u_i(\sigma) \geq 0$ for all i.

Let σ be a profile of pure, weakly increasing strategies. Let us denote by $U(\sigma) = u_i(\sigma)/h_i$ the common utility. We may then write:

$$\begin{aligned} U(\sigma) &= \int \left(\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t})\right) p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i \\ &= \int \left(2\mathbb{P}(\alpha|\mathbf{t}) - 1\right) p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i \\ &= \underbrace{2\int \mathbb{P}(\alpha|\mathbf{t}) p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i}_{\widetilde{U}(\sigma)} - \mathbb{P}(O = A|\sigma). \end{aligned}$$

To prove the claim that $U(\sigma) \geq 0$, it thus suffices to show that $\widetilde{U}(\sigma) \geq \frac{1}{2}\mathbb{P}(O = A|\sigma)$. We first observe that, for any $k \in N$, the function $g^k : t_k \mapsto \int \mathbb{P}(\alpha|\mathbf{t}) \prod_{i=1}^{k-1} g_i(t_i) dt_i$ is non-negative and weakly increasing (for each t_{-k}). Moreover, for any $k \in N$, the function $h^k : t_k \mapsto \int p^A(\sigma(\mathbf{t})) \prod_{i=1}^{k-1} g_i(t_i) dt_i$ is non-negative and weakly increasing (for each t_{-k}) since σ_k is weakly increasing. By repeated application of Lemma 4, we thus obtain:

$$\begin{aligned} \widetilde{U}(\sigma) &= \int \mathbb{P}(\alpha|\mathbf{t}) p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i \\ &\geq \int \left(\int \mathbb{P}(\alpha|\mathbf{t}) g_1(t_1) dt_1\right) \times \left(\int p^A(\sigma(\mathbf{t})) g_1(t_1) dt_1\right) \prod_{i=2}^n g_i(t_i) dt_i \\ &\geq \dots \\ &\geq \int \left(\int \mathbb{P}(\alpha|\mathbf{t}) \prod_{i=1}^k g_i(t_i) dt_i\right) \times \left(\int p^A(\sigma(\mathbf{t})) \prod_{i=1}^k g_i(t_i) dt_i\right) \prod_{i=k+1}^n g_i(t_i) dt_i \\ &\geq \dots \\ &\geq \left(\int \mathbb{P}(\alpha|\mathbf{t}) \prod_{i=1}^n g_i(t_i) dt_i\right) \times \left(\int p^A(\sigma(\mathbf{t})) \prod_{i=1}^n g_i(t_i) dt_i\right) = \frac{1}{2}\mathbb{P}(O = A|\sigma). \end{aligned}$$

This concludes the proof of Claim 1.

Claim 2: For any BNE σ, there exists a profile σ^+ in pure, weakly increasing strategies such that $\forall i \in N,\ u_i(\sigma) = u_i(\sigma^+)$.

Let σ be a BNE. For any strategy σ_i of shareholder i, we consider a re-ordering σ_i^+, i.e. a strategy such that:

- σ_i^+ is pure and weakly increasing
- for any ballot $x_i \in X_i$, we have $\mathbb{P}(\widehat{\sigma}_i \leq x_i) = \mathbb{P}(\sigma_i^+ \leq x_i)$.

To construct such a re-ordering, we define σ_i^+ by (abusing notation as σ_i^+ is pure):

$$\forall t_i \in T_i, \qquad \sigma_i^+(t_i) = \min \left\{ x_i \in X_i \mid \sum_{x \in X_i, x \leq x_i} \int_\delta^{\frac{1}{\delta}} \sigma_i(t_i')(x) g_i(t_i') dt_i' > \int_\delta^{t_i} g_i(t_i') dt_i' \right\}.$$

The strategy σ_i^+ is pure, weakly increasing and continuous (and even flat) everywhere but on a finite number of points. We shall prove that for almost any type vector $\mathbf{t} = (t_i)_{i \in N}$, the sign of $\sum_{i \in N} \widehat{\sigma}_i(t_i)$ is the same as that of $\sum_{i \in N} \sigma_i^+(t_i)$. In the sequel, we refer to the sign of a number x as positive if $x > 0$, negative if $x < 0$, and null (neither positive nor negative) if $x = 0$.

Let $t_i \in T_i$ be a type such that σ_i^+ is continuous at t_i and assume that there exists $x_i \in X_i$ for which $\sigma_i(t_i)(x_i) > 0$ and $x_i \neq \sigma_i^+(t_i)$. We focus on the case for which $\sigma_i^+(t_i) > x_i$ (the other case can be treated analogously) and we further assume that $x_i = \min\{x \in X_i \mid \sigma_i(t_i)(x) > 0\} := \min \widehat{\sigma}_i(t_i)$. Observe that there must exist $t_i' < t_i$ such that $\sigma_i(t_i')(y_i) > 0$ with $y_i \geq \sigma_i^+(t_i) > x_i$. Indeed, if this type t_i' didn't exist, we would have $\forall t_i' < t_i, \widehat{\sigma}_i(t_i') < \sigma_i^+(t_i)$ for any realization of $\sigma_i(t_i')$, which would imply:

$$\sum_{x < \sigma_i^+(t_i)} \int_\delta^{\frac{1}{\delta}} \sigma_i(t"_i)(x) g_i(t"_i) dt"_i \geq \int_\delta^{t_i} \left(\sum_{x < \sigma_i^+(t_i)} \sigma_i(t"_i)(x) \right) g_i(t"_i) dt"_i = \int_\delta^{t_i} g_i(t"_i) dt"_i.$$

We would then have for any $t < t_i$ (since g_i is positive on T_i):

$$\sum_{x < \sigma_i^+(t_i)} \int_\delta^{\frac{1}{\delta}} \sigma_i(t"_i)(x) g_i(t"_i) dt"_i > \int_\delta^{t} g_i(t"_i) dt"_i.$$

By definition of σ_i^+, we would have $\forall t < t_i, \sigma_i^+(t) < \sigma_i^+(t_i)$. This contradicts the fact that σ_i^+ is continuous at t_i. We thus obtained the existence of $y_i \geq \sigma_i^+(t_i) > x_i$ such that $\sigma_i(t_i')(y_i) > 0$ for some $t_i' < t_i$.

As σ is a BNE, x_i must be optimal for i at t_i and y_i must be optimal for i at t_i':

$$\begin{aligned} \Delta_i &:= \frac{1}{h_i} \left(u_i(x_i, \sigma_{-i} | t_i) - u_i(y_i, \sigma_{-i} | t_i) \right) \geq 0 \\ \Delta_i' &:= \frac{1}{h_i} \left(u_i(y_i, \sigma_{-i} | t_i') - u_i(x_i, \sigma_{-i} | t_i') \right) \geq 0. \end{aligned}$$

By summation, we obtain that $\Delta_i + \Delta_i' \geq 0$. Now, we may write:

$$\Delta_i = \int \left(2\mathbb{P}(\alpha | t_i, t_{-i}) - 1 \right) \left(p^A \left(x_i, \sigma_{-i}(t_{-i}) \right) - p^A \left(y_i, \sigma_{-i}(t_{-i}) \right) \right) \prod_{j \neq i} g_j(t_j) dt_j.$$

Similarly,

$$\Delta_i' = \int (2\mathbb{P}(\alpha|t_i', t_{-i}) - 1) \left(p^A(y_i, \sigma_{-i}(t_{-i})) - p^A(x_i, \sigma_{-i}(t_{-i}))\right) \prod_{j\neq i} g_j(t_j)dt_j.$$

We thus have:

$$\begin{aligned}\Delta_i + \Delta_i' &= 2\int (\mathbb{P}(\alpha|t_i', t_{-i}) - \mathbb{P}(\alpha|t_i, t_{-i})) \left(p^A(y_i, \sigma_{-i}(t_{-i})) - p^A(x_i, \sigma_{-i}(t_{-i}))\right) \prod_{j\neq i} g_j(t_j)dt_j \\ &\geq 0.\end{aligned}$$

As $t_i' < t_i$, we have that for all t_{-i}, $\mathbb{P}(\alpha|t_i', t_{-i}) - \mathbb{P}(\alpha|t_i, t_{-i}) < 0$. Moreover, as $y_i > x_i$, we have by definition of p^A that for all t_{-i}, $p^A(y_i, \sigma_{-i}(t_{-i})) \geq p^A(x_i, \sigma_{-i}(t_{-i}))$. To reconcile the three inequalities, it must be that $\Delta_i + \Delta_i' = 0$ and that for almost all t_{-i}, $p^A(y_i, \sigma_{-i}(t_{-i})) = p^A(x_i, \sigma_{-i}(t_{-i}))$. This last equality implies, by definition of p^A, that $\mathbb{P}\left(x_i \leq -\sum_{j\neq i}\widehat{\sigma}_j \leq y_i\right) = 0$. As $x_i = \min\widehat{\sigma}_i(t_i)$ and $\sigma_i^+(t_i) \leq y_i$, we obtain

$$\mathbb{P}\left(\min\widehat{\sigma}_i(t_i) \leq -\sum_{j\neq i}\widehat{\sigma}_j \leq \sigma_i^+(t_i)\right) = 0.$$

Following a symmetrical argument, we also obtain

$$\mathbb{P}\left(\sigma_i^+(t_i) \leq -\sum_{j\neq i}\widehat{\sigma}_j \leq \max\widehat{\sigma}_i(t_i)\right) = 0.$$

It follows that $\mathbb{P}\left(sgn\left(\widehat{\sigma}_i(t_i) + \sum_{j\neq i}\widehat{\sigma}_j\right) \neq sgn\left(\sigma_i^+(t_i) + \sum_{j\neq i}\widehat{\sigma}_j\right)\right) = 0$. Moreover, by construction of the strategies $(\sigma_j^+)_{j\neq i}$, the probability of the previous event remains null if some strategy realizations $\widehat{\sigma}_j$ are transformed into σ_j^+ (the transformation from σ_j to σ_j^+ is measure-preserving by design). This can be written: for all $S \subseteq N\backslash\{i\}$,

$$\mathbb{P}\left(sgn\left(\widehat{\sigma}_i(t_i) + \sum_{j\in S}\widehat{\sigma}_j + \sum_{j\in(N\backslash S)\backslash\{i\}}\sigma_j^+\right) \neq sgn\left(\sigma_i^+(t_i) + \sum_{j\in S}\widehat{\sigma}_j + \sum_{j\in(N\backslash S)\backslash\{i\}}\sigma_j^+\right)\right) = 0. \tag{2}$$

We have just shown that (2) holds whenever σ_i^+ is continuous at t_i. As σ_i^+ is continuous almost everywhere, we have: for all $S \subseteq N\backslash\{i\}$,

$$\mathbb{P}\left(sgn\left(\widehat{\sigma}_i + \sum_{j\in S}\widehat{\sigma}_j + \sum_{j\in(N\backslash S)\backslash\{i\}}\sigma_j^+\right) \neq sgn\left(\sigma_i^+ + \sum_{j\in S}\widehat{\sigma}_j + \sum_{j\in(N\backslash S)\backslash\{i\}}\sigma_j^+\right)\right) = 0. \tag{3}$$

To conclude, we observe that $sgn(\sum_{j\in N}\widehat{\sigma}_j) \neq sgn(\sum_{j\in N}\sigma_j^+)$ can be satisfied only if there exists some index k for which $sgn(\sum_{j=1}^{k-1}\widehat{\sigma}_j + \sum_{j=k}^{n}\sigma_j^+) \neq sgn(\sum_{j=1}^{k}\widehat{\sigma}_j + \sum_{j=k+1}^{n}\sigma_j^+)$. Hence, we may write, applying (3):

$$\mathbb{P}\left(sgn(\sum_{j\in N}\widehat{\sigma}_j)\neq sgn(\sum_{j\in N}\sigma_j^+)\right)\leq\sum_{k=1}^{n}\mathbb{P}\left(sgn(\sum_{j=1}^{k-1}\widehat{\sigma}_j+\sum_{j=k}^{n}\sigma_j^+)\neq sgn(\sum_{j=1}^{k}\widehat{\sigma}_j+\sum_{j=k+1}^{n}\sigma_j^+)\right)=0.$$

Therefore, σ and σ^+ lead to the same outcome with probability one, i.e. for almost any type vector t. It follows that $\forall i\in N$, $u_i(\sigma)=u_i(\sigma^+)$. This concludes the proof of Claim 2.

To conclude the proof, note that any BNE σ yields the same utilities as a profile σ^+ of pure, weakly increasing strategies (Claim 2), under which all expected utilities are positive (Claim 1). Thus for all $i\in N$, $u_i(\sigma)\geq 0$. ■

Lemma 4 *Let f be a density function on a real interval T, let $g,h:T\to\mathbb{R}$ be two non-negative and weakly increasing functions. Then:*

$$\int g(t)h(t)f(t)dt\geq\left(\int g(t)f(t)dt\right)\times\left(\int h(t)f(t)dt\right).$$

Proof. If $\int h(t)f(t)dt=0$, we also have $\int g(t)h(t)f(t)dt=0$, so that the inequality is valid. Assume now that $\int h(t)f(t)dt>0$. Let $\hat{f}$ be the density on T defined by

$$\forall t\in T,\hat{f}(t)=\frac{h(t)f(t)}{\int h(z)f(z)dz}.$$

As h is weakly increasing, there exists a threshold $t^*\in T$ such that $\hat{f}(t)\leq f(t)$ for $t\leq t^*$ and $\hat{f}(t)\geq f(t)$ for $t>t^*$.

Our objective is to show that $\int g(\hat{f}-f)\geq 0$, as this will imply the desired inequality $\int hgf\geq(\int gf)(\int hf)$. We write: $\int g(\hat{f}-f)=\int_{-\infty}^{t^*}g(\hat{f}-f)+\int_{t^*}^{+\infty}g(\hat{f}-f)$. Now, as g is weakly increasing and $\hat{f}-f$ is non-positive on $(-\infty,t^*]$, we have $\int_{-\infty}^{t^*}g(\hat{f}-f)\geq g(t^*)\int_{-\infty}^{t^*}(\hat{f}-f)$. Similarly, as $\hat{f}-f$ is non-negative on $(t^*,+\infty)$, we have $\int_{t^*}^{+\infty}g(\hat{f}-f)\geq g(t^*)\int_{t^*}^{+\infty}(\hat{f}-f)$. As f and $\hat{f}$ are two densities, we have $\int(\hat{f}-f)=0$, so that $\int_{-\infty}^{t^*}(\hat{f}-f)=-\int_{t^*}^{+\infty}(\hat{f}-f)$. We thus obtain:

$$\int g(\hat{f}-f)\geq g(t^*)\left(\int_{-\infty}^{t^*}(\hat{f}-f)+\int_{t^*}^{+\infty}(\hat{f}-f)\right)=0.$$

■

Proof of Proposition 2. The fact that each $i\in N$ employing $\sigma_i^{1S1V\text{-}D}(t_i)=c\ln t_i$ with $c=\min_{i\in N}\{\frac{-d_i}{\ln(\delta)}\}$ is an efficient BNE of the game is straightforward, by application of Lemma 1. We can easily rule out the existence of a non-degenerate mixed efficient BNE. If σ is a non-degenerate mixed BNE of the game, there exists at least one $i\in N$ and at least one $y\in T$ such that the random variable $\sigma_i(y)$ admits at least two distinct potential realizations. Consider without loss of generality that this player is the first shareholder. If σ is efficient then for $\mathbf{t}=(t_1,t_2,t_3,t_4,...)=(y,\frac{1}{y},1,1,...)$ we have that the efficient alternative is AB. Hence, given any two vectors of potential realizations $\widehat{\sigma}=(\widehat{\sigma}_1(y),\widehat{\sigma}_2(\frac{1}{y}),\widehat{\sigma}_3(1),\widehat{\sigma}_4(1),...)$ and $\widetilde{\sigma}=(\widetilde{\sigma}_1(y),\widetilde{\sigma}_2(\frac{1}{y}),\widetilde{\sigma}_3(1),\widetilde{\sigma}_4(1),...)$ with $\widehat{\sigma}_i(t_i)=\widetilde{\sigma}_i(t_i)$ for every $i > 1$, we must have that $\sum_{i\in N}\widehat{\sigma}(t_i) = 0$ and $\sum_{i\in N}\widetilde{\sigma}(t_i) = 0$. But this means that $\widehat{\sigma}_1(y)$ must be identical to $\widetilde{\sigma}_1(y)$ and hence $\sigma_1(y)$ cannot admit at least two distinct potential realizations, which contradicts the assumption above. Hence, if σ is an efficient BNE, it must be pure.

We now turn attention to pure equilibria. First we argue that an efficient pure BNE σ must be symmetric across shareholders up to an additive constant (i.e. there exist $\phi_{i,j}$ such that $\sigma_i(y) = \sigma_j(y) + \phi_{i,j}$, for every $i, j \in N$ and every $y \in T$). If σ is efficient then for every $y \in T$, for $\mathbf{t} = (t_1, t_2, t_3, t_4, ...)$ with $t_i = y$, $t_j = \frac{1}{y}$ and $t_k = 1$ for all $k \notin \{i, j\}$, we need to have $\sum_{k\in N}\sigma_k(t_k) = 0$ and thus $\sigma_i(y) = -\sigma_j(\frac{1}{y}) - \sum_{k\in N-\{i.j\}}\sigma_k(1)$. By keeping j fixed and varying i we get that all players, except possibly j, employ the same strategy up to an additive constant. By varying j as well, we get that all players use the same strategy up to an additive constant. That is, in an efficient pure BNE σ a player i uses the strategy $\sigma_i(y) = \theta(y)+\kappa_i$, with $\sum_{i\in N}\kappa_i = 0$ and $\theta(1) = 0$.

Notice that if σ is an efficient equilibrium of the standard *1S1V-D* rule characterized by some θ and $\boldsymbol{\kappa} = (\kappa_1, \kappa_2, .., \kappa_n)$ such that $\kappa_i \neq 0$ for at least one $i \in N$, it follows that σ', characterized by the same θ and $\boldsymbol{\kappa} = (0, 0, ..., 0)$, is an efficient equilibrium of the *1S1V-D* rule $v' = \{(\mathbb{R}, \mathbb{R}, ..., \mathbb{R}), 0\}$. Hence, to characterize all efficient equilibria of the standard *1S1V-D* rule, it suffices to characterize all admissible θs that lead to full information equivalence when $\kappa = (0, 0, ..., 0)$ under rule v'. In the remaining part of the proof, we slightly abuse terminology, and instead of saying "a pure-strategy efficient equilibrium of v characterized by θ and $\kappa = (0, 0, ..., 0)$" we simply say "an efficient equilibrium θ."

In an efficient equilibrium θ, it must be the case that for every $\mathbf{t} \in T^n$ we have $sgn(\sum_{i\in N}\theta(t_i)) = sgn(\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}))$. But we know from Lemma 1 that $\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}) > 0 \Leftrightarrow \sum_{i\in N}\ln(t_i) > 0$, $\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}) < 0 \Leftrightarrow \sum_{i\in N}\ln(t_i) < 0$, and $\mathbb{P}(\alpha|\mathbf{t}) - \mathbb{P}(\beta|\mathbf{t}) = 0 \Leftrightarrow \sum_{i\in N}\ln(t_i) = 0$. In other words, for every $\mathbf{t} \in T^n$ it must hold that $sgn(\sum_{i\in N}\theta(t_i)) = sgn(\sum_{i\in N}\ln(t_i))$.

First, we prove that every efficient equilibrium θ is monotone (increasing, in particular) and symmetric (i.e., $\theta(y) = -\theta(\frac{1}{y})$ for every $y \in T$), then that it is differentiable on $int(T) = (\delta, \frac{1}{\delta})$ and continuous on $T = [\delta, \frac{1}{\delta}]$, and, finally, we provide a full characterization by showing that each efficient equilibrium θ is equal to the natural logarithm multiplied by some positive constant.

Monotonicity and symmetry of equilibria: For every $t_i < \frac{1}{\delta}$ there exists a $t_{-i} \in T^{n-1}$ such that $\mathbb{P}(\alpha|\mathbf{t}) = \mathbb{P}(\beta|\mathbf{t})$, so that $\sum_{i\in N}\ln(t_i) = 0$. Hence, for such a $\mathbf{t} = (t_i, t_{-i})$ and every $\varepsilon \in (0, \frac{1}{\delta} - t_i]$, it is true that, $\mathbb{P}(\alpha|(t_i + \varepsilon, t_{-i})) > \mathbb{P}(\beta|(t_i + \varepsilon, t_{-i}))$, so that $\sum_{j\in N-\{i\}}\ln(t_j) + \ln(t_i + \varepsilon) > 0$. Since every efficient equilibrium θ delivers the efficient outcome, it follows that for every $y < \frac{1}{\delta}$ and $\varepsilon \in (0, \frac{1}{\delta} - y]$ there exists a $t_{-i} \in T^{n-1}$ such that $\sum_{j\in N-\{i\}}\theta(t_j) + \theta(y) = 0$ and $\sum_{j\in N-\{i\}}\theta(t_j) + \theta(y + \varepsilon) > 0$. In other words, $\theta(y + \varepsilon) > \theta(y)$ for every $y < \frac{1}{\delta}$ and $\varepsilon \in (0, \frac{1}{\delta} - y]$; θ is strictly increasing in the player's type. To establish symmetry, consider that $\mathbf{t} \in T^n$ is such that $t_i = 1$ for every $i \in N$. In this case the efficient alternative is AB. Therefore, $\sum_{i\in N}\theta(t_i) = n\theta(1) = 0$, which implies $\theta(1) = 0$. Now consider a $\mathbf{t} \in T^n$ such that $t_1 = y \in T$, $t_2 = \frac{1}{y} \in T$ and $t_i = 1$ for every $i > 2$. We have $\sum_{i\in N-\{1,2\}}\ln(1) + \ln(y) + \ln(\frac{1}{y}) = 0$ and hence $\sum_{i\in N-\{1,2\}}\theta(1) + \theta(y) + \theta(\frac{1}{y}) = 0$, which implies $(n - 2) \times 0 + \theta(y) + \theta(\frac{1}{y}) = 0$, for every $y \in T$. In other words, $\theta(y) = -\theta(\frac{1}{y})$, for every $y \in T$.

Differentiability: By Lebesgue's theorem for the differentiability of monotone functions defined over open intervals we have that every equilibrium $\theta : T \to \mathbb{R}$ is differentiable at almost every $y \in int(T) = (\delta, \frac{1}{\delta})$. We will now establish that θ is actually differentiable at every $y \in (\delta, \frac{1}{\delta})$. Notice that in all profiles with $t_1 = y \in (\delta, 1]$, $t_2 = y' \in (1, \frac{1}{\delta})$, $t_3 = \frac{1}{yy'} \in (\delta, \frac{1}{\delta})$, and $t_i = 1$ for every $i > 3$, it must hold that $\theta(y) + \theta(y') + \theta(\frac{1}{yy'}) = 0 \Leftrightarrow \theta(y) = -\theta(y') - \theta(\frac{1}{yy'})$ by the fact $\theta(1) = 0$ and that $\sum_{i\in N} \ln(t_i) = 0$. Assume that θ is not differentiable at a particular $\tilde{y} \in (\delta, 1]$. Then it follows that $-\theta(y') - \theta(\frac{1}{yy'})$ is not differentiable with respect to y at $\tilde{y}$, for every $y' \in (1, \frac{1}{\delta})$. But due to the fact that θ is differentiable at almost every $y \in int(T)$, it follows that for every $y \in (\delta, 1]$, there exists $y' \in (1, \frac{1}{\delta})$ such that θ is differentiable at $\frac{1}{yy'}$. This contradicts the claim that there exists $\tilde{y} \in (\delta, 1]$ at which θ is not differentiable, and, by symmetry it follows that θ is differentiable at every $y \in int(T)$.

Continuity at the boundary: We know that θ is differentiable, and thus continuous, on $int(T) = (\delta, \frac{1}{\delta})$. Let us show that it is continuous at $y = \frac{1}{\delta}$. Suppose by contradiction, that there is a discontinuity. As θ is increasing, it must be of the form: $\theta(y) - \theta(y - \varepsilon) > \tilde{\varepsilon}$ for every $\varepsilon \in (0, \bar{\varepsilon}]$, where $\bar{\varepsilon}$ and $\tilde{\varepsilon}$ are positive constants. Then there exists $\lambda \in (0, 1 - \delta)$ such that $(n-2)\theta(1 - \lambda) + \theta(y) + \theta(\frac{1}{y-\varepsilon}) > 0$ for every $\varepsilon \in (0, \bar{\varepsilon}]$. But for every $\lambda \in (0, 1 - \delta)$ one can find $\varepsilon > 0$ small enough such that $(n-2)\ln(1 - \lambda) + \ln(y) + \ln(\frac{1}{y-\varepsilon}) < 0$. This contradicts the fact that θ leads to the efficient outcome for every possible realization of types. Thus θ is continuous at $\frac{1}{\delta}$, and for the same reason, it must be continuous at δ. We conclude that θ is continuous on $T = [\delta, \frac{1}{\delta}]$.

Characterization: We fix an efficient equilibrium θ and an arbitrary pair of values $(y', \tilde{y}) \in (\delta, \frac{1}{\delta})^2$, such that $y' < 1$ and $\tilde{y} > 1$. Consider now a $\mathbf{t} \in (\delta, \frac{1}{\delta})^n$ such that $t_1 = y'$, $t_2 = \tilde{y}$, $t_3 = \frac{1}{y'\tilde{y}} \in (\frac{1}{\tilde{y}}, \frac{1}{y'}) \subset (\delta, \frac{1}{\delta})$, and $t_i = 1$ for every $i > 3$. If we define $r = y' \times \tilde{y}$ we get $\sum_{i\in N-\{1,2,3\}} \theta(1) + \theta(y') + \theta(\frac{r}{y'}) + \theta(\frac{1}{r}) = 0$. Since, $\ln(y) + \ln(\frac{r}{y}) + \ln(\frac{1}{r}) = 0$ for every y in an open ball around y', and since θ is differentiable at y', it follows that we can take the derivative of $\sum_{i\in N-\{1,2,3\}} \theta(1) + \theta(y) + \theta(\frac{r}{y}) + \theta(\frac{1}{r}) = 0$ with respect to y and evaluate it at y'. By doing that, we get, $\theta'(y') + \theta'(\frac{r}{y'})(-\frac{r}{y'^2}) = 0$. This can be written as $y' \times \theta'(y') = \tilde{y} \times \theta'(\tilde{y})$. But since this holds for any pair of values $(y', \tilde{y}) \in int(T)^2$, such that $y' < 1$ and $\tilde{y} > 1$, it is true that, for any fixed $\tilde{y} \in (1, \frac{1}{\delta})$, we have $y \times \theta'(y) = \tilde{y} \times \theta'(\tilde{y})$ for every $y \in (\delta, 1)$. In other words, for every $y \in (\delta, 1)$ we have $y \times \theta'(y) = c \Longrightarrow \theta'(y) = \frac{c}{y} \Longrightarrow \theta(y) = c\ln y + \hat{c}$, for some $c > 0$ and $\hat{c} \in \mathbb{R}$. By the fact that $\theta(1) = 0$, it follows that $\hat{c} = 0$ and, hence, $\theta(y) = c\ln y$ for every $y \in (\delta, 1)$, with $c > 0$. By symmetry of θ it follows that for every $y \in (1, \frac{1}{\delta})$, we have $\theta(y) = -\theta(\frac{1}{y}) = -c\ln\frac{1}{y} = c\ln y$. That is, $\theta(y) = c\ln y$ for every $y \in (\delta, \frac{1}{\delta})$, with $c > 0$, and by continuity at the boundary, the formula must hold for every $y \in [\delta, \frac{1}{\delta}]$.

By the fact that in every equilibrium σ of the rule $V^{1S1V\text{-}D}(d)$, we must have $\sigma_i(\frac{1}{\delta}) \leq d_i$ and $\sigma_i(\delta) \geq -d_i$, and by the above analysis, it follows that in an efficient equilibrium it should hold that $\sigma_i(t_i) = c\ln t_i + \kappa_i$ with $\sum_{i\in N} \kappa_i = 0$ and $c \in (0, \min_{i\in N}\{\frac{\min(d_i - \kappa_i, d_i + \kappa_i)}{-\ln(\delta)}\}]$. ■

Proof of Proposition 3. Let $\{X, w\}$ be a continuous voting rule: there exists $(\psi_i)_{i\in N} \in \times_{i\in N} int(X_i)$ such that $\sum_{i\in N} \psi_i = w$. To see why $\{X, w\}$ admits an efficient equilibrium, notice that any efficient equilibrium σ of the voting rule $V^{1S1V-D}(1, \ldots, 1)$, as characterized in Proposition 2, can be properly re-scaled so that, for each $i \in N$, σ_i fits within any open set around ψ_i.

To understand why only continuous voting rules admit an efficient equilibrium, let $v = \{X, w\}$ be a voting rule with an efficient equilibrium σ. Since every strategy that is feasible according to this rule is also feasible under the rule $v' = \{(\mathbb{R}, \mathbb{R}, ..., \mathbb{R}), w\}$, it must be the case that σ is an efficient equilibrium of v' too. We replicate the reasoning in the proof of Proposition 2, and get the following result: as σ is an efficient equilibrium of v', it should hold that $\sum_{i \in N} \sigma_i(1) = w$ and, for every $i \in N$, $\sigma_i(y)$ should be continuous and strictly increasing in an open ball around $y = 1$. Hence, noting $\psi_i = \sigma_i(1)$, we have that $\sum_{i \in N} \psi_i = w$ and $\psi_i \in int(X_i)$ for every $i \in N$, that is $\{X, w\}$ is a continuous voting rule. ■

Proof of Proposition 4 . In this proof, we denote by N_A (resp. N_B) the set of A-partisans (resp. B-partisans) and by N_C the set of common value voters.

(i) for any distribution d, we may write $V(d) = \{X, w\}$ and $V'(d) = \{X', w\}$ with for all $i \in N$, $X_i \subseteq X'_i$. The proof's strategy consists in re-writing the voting games associated to these rules as voting games taking place among common value shareholders, and to apply Proposition 1.

For a C-shareholders' vote profile $\mathbf{x} = (x_i)_{i \in N_C}$, a proposal passes (for sure) if and only if:

$$\sum_{i \in N_A} (\max X_i) + \sum_{i \in N_B} (\min X_i) + \sum_{i \in N_C} x_i > w \quad \Leftrightarrow \quad \sum_{i \in N_C} x_i > w - \sum_{i \in N_A} (\max X_i) - \sum_{i \in N_B} (\min X_i).$$

Let $\tilde{w} = w - \sum_{i \in N_A} (\max X_i) - \sum_{i \in N_B} (\min X_i)$. We consider the voting rules $\{X_C = (X_i)_{i \in N_C}, \tilde{w}\}$ and $\{X'_C = (X'_i)_{i \in N_C}, \tilde{w}\}$. By application of Proposition 1, the first rule dominates the second one.

(ii) for any d such that $h_C > |h_A - h_B|$, an efficient equilibrium can be constructed under $V^{1S1V-D}(d)$, following the same argument as the one described in the main text (re-scaling of the efficient equilibrium of Proposition 2). ■

Proof of Proposition 5. Let $\{X, w\}$ be a finite voting rule. The voting game can then be re-written as one of common values. In this game, each shareholder $i \in N$ has utility u_i^C (we then have common values in the sense that $u_i^C = \frac{h_j}{h_i} u_j^C$). When she chooses an action $x_i \in X_i$, the action is recorded with probability p_i^C, while it is transformed into $(\max X_i)$ with probability p_i^A and into $(\min X_i)$ with probability p_i^B.

The existence of a BNE for that game is obtained by the same argument as in the proof of Lemma 2. The only difference arises with the expressions (in the proof of Claim 1) $\mathbb{P}_{\sigma_{-i}}(\sum_{j \neq i} x_j > -x \mid \omega) + \frac{1}{2}\mathbb{P}_{\sigma_{-i}}(\sum_{j \neq i} x_j = -x \mid \omega)$ which should be replaced by $\mathbb{P}_{\tilde{\sigma}_{-i}}(\sum_{j \neq i} x_j > -x \mid \omega) + \frac{1}{2}\mathbb{P}_{\tilde{\sigma}_{-i}}(\sum_{j \neq i} x_j = -x \mid \omega)$, where the strategy $\tilde{\sigma}_j$ is defined for each shareholder j by:

$$\tilde{\sigma}_j = \begin{cases} \sigma_j & \text{with probability } 1 - p_j^A - p_j^B \\ (\max X_j) & \text{with probability } p_j^A \\ (\min X_j) & \text{with probability } p_j^B. \end{cases}$$

We thus obtain the existence of a BNE under the finite rule $\{X, w\}$. To conclude, if we have two finite mechanisms V and V' that distribute power similarly, then for each d, the common value games associated to $V(d)$ and $V'(d)$ are such that: the action spaces of the second game are included in the action spaces of the first game (for each shareholder); the payoffs associated to any profile is the same in the two games (since by assumption they distribute power similarly). As before, we thus obtain that for any BNE under $V'(d)$, there exists a BNE under V such that the probability of making the correct decision is higher. ■

Proof of Proposition 6. In this proof, we abuse notation and write simply *1S1V-D* for the voting rule $V^{1S1V-D}(d)$. A subgame is essentially defined by the set of shareholders that acquire a signal. Let us denote the set of informed shareholders by I. By Proposition 1 and by the argument of Mc Lennan (1998) we have that, from an ex-ante point of view, the probability of the firm making the correct decision in subgame I under voting rule v, denoted by $\mathbb{P}(I, v)$, must satisfy $\mathbb{P}(I, v) \geq \mathbb{P}(I', v)$ when $\#I > \#I'$ for any v, and $\mathbb{P}(I, \textit{1S1V-D}) \geq \mathbb{P}(I, v)$ for any I and any v. We also notice that since we have fixed a certain BNE in each subgame, the whole game may be viewed as a single stage game in which the shareholders only decide whether to draw an informative signal or not. This simplified version of the game is a potential game with potential function $\mathbb{P}(I(x), v) - \#I(x) \times l$, where x is the vector of information acquisition decisions with $x_i = 1$ when shareholder i acquires a signal and $x_i = 0$ otherwise; and $\#I(x)$ is the number of informed shareholders. By the fact that there are finitely many alternative vectors x, the potential function obtains a maximum value for (at least) one of these vectors, which is also a pure strategy equilibrium of this simplified game (see for instance Monderer and Shapley, 1996). Moreover, every pure strategy equilibrium of this simplified game must be a maximizer of this potential function. Assume now, that a pure strategy equilibrium, $x^*_{\textit{1S1V-D}}$, exists under *1S1V-D* such that $\mathbb{P}(I(x^*_{\textit{1S1V-D}}), \textit{1S1V-D}) - \#I(x^*_{\textit{1S1V}}) \times l < \mathbb{P}(I(x^*_v), V) - \#I(x^*_v) \times l$, where x^*_v is an equilibrium of some other rule v. Since $\mathbb{P}(I, \textit{1S1V-D}) \geq \mathbb{P}(I, v)$ for any I and any v, it follows that $\mathbb{P}(I(x^*_v), \textit{1S1V-D}) - \#I(x^*_v) \times l \geq \mathbb{P}(I(x^*_v), v) - \#I(x^*_v) \times l$, which contradicts the fact that $x^*_{\textit{1S1V-D}}$ is an equilibrium – and, thus, a maximizer of the corresponding potential function – under *1S1V-D*. Therefore, there is no rule v that admits a better equilibrium than *1S1V-D*. ■

Appendix B: Proofs of Section 5

Proof of Proposition 7. As for Proposition 1, we divide the statement in two lemmas (Lemma 5 and 6 in this Appendix), focusing respectively on the best and the worst equilibria under each voting mechanism. Throughout the proof, we treat the game between the manager and the shareholders as a simultaneous game and we continue to apply the equilibrium notion of a BNE, as in Section 4. We note however that the same results hold for the equilibrium concept of weak perfect Bayesian equilibrium in the sequential version of the game when there is no decisive shareholder. ■

Lemma 5 *For any $\mu \in [0, 1]$ and any share distribution d, for any BNE σ under $V^{1P1V}(d)$, there exists a BNE σ' under $V^{1S1V-D}(d)$ that makes all shareholders (weakly) better off in expectation.*

Proof. The strategy of the proof is to construct a two-player common value game between an aligned manager and an aggregate shareholder (holding all the shareholders' signals). When voting under *1S1V-D*, shareholders can implement (i.e. decentralize) the aggregate shareholder's strategy of the most efficient equilibrium of the two-player game. The corresponding profile is then the most efficient equilibrium of the original game under *1S1V-D*. Using the argument of Mc Lennan (1998), shareholders' welfare at equilibrium cannot improve with *1P1V*.

Two-player game. We consider a game with two players: a manager M and an aggregate shareholder AS. The manager receives the signal s_M, while the aggregate shareholder receives the signals $s_1, \ldots, s_n$. After receiving their (private) signals, players simultaneously choose to pass ($x_j = P$) or to veto ($x_j = V$) the proposal. The proposal is accepted with probability 1 if both players choose P, and it is accepted with probability μ if the manager vetoes while the aggregate shareholder passes.[44] Both players share the same utility: for all $j \in \{M, AS\}$,

$$\begin{aligned} &u_j(A|\alpha) = 1,\ u_j(A|\beta) = -1, \\ &u_j(B|\alpha) = u_j(B|\beta) = 0. \end{aligned}$$

Claim 1: the two-player game admits a most efficient strategy profile, which is also an equilibrium of that game. The expected utility of each player is at least 0 at this equilibrium.

We first show the existence of a most efficient equilibrium, and we start by computing players' best replies. Given a strategy σ_{AS} for the aggregate shareholder, the expected utility difference between actions P and V for the manager, $\Delta u_M := \mathbb{E}[u_M(x_M = P, \sigma_{AS}) \mid s_M] - \mathbb{E}[u_M(x_M = V, \sigma_{AS}) \mid s_M]$, can be written as:

$$\Delta u_M = (1-\mu)\mathbb{P}(\sigma_{AS} = P)\left(2\mathbb{P}(\omega = \alpha \mid \sigma_{AS} = P, s_M) - 1\right).$$

Indeed, the only difference between the actions $x_M = P$ and $x_M = V$ arises when the aggregate shareholder chooses to pass ($\sigma_{AS} = P$) and the manager has control over her own veto (with probability $1-\mu$). Hence, playing $x_M = P$ is a best reply for the manager whenever $\mathbb{P}(\omega = \alpha \mid \sigma_{AS} = P, s_M) \geq 1/2$, or equivalently $\frac{\mathbb{P}(\omega=\alpha|\sigma_{AS}=P,s_M)}{\mathbb{P}(\omega=\beta|\sigma_{AS}=P,s_M)} \geq 1$. Noting $t_M = \frac{f_M(s_M|\alpha)}{f_M(s_M|\beta)}$ and $t_{\sigma_{AS}} = \frac{\mathbb{P}(\sigma_{AS}=P|\alpha)}{\mathbb{P}(\sigma_{AS}=P|\beta)}$, we may write $\frac{\mathbb{P}(\omega=\alpha|\sigma_{AS}=P,s_M)}{\mathbb{P}(\omega=\beta|\sigma_{AS}=P,s_M)} = t_M t_{\sigma_{AS}}$. We obtain that there is a cutoff $\bar{t}_M := \frac{1}{t_{\sigma_{AS}}}$ such that $x_M = P$ is a best reply whenever $t_M \geq \bar{t}_M$, and that $x_M = V$ is a best reply otherwise.

Similarly, we denote the aggregate shareholder's type by $t_{AS} = \frac{\prod_{i=1}^n f_i(s_i|\alpha)}{\prod_{i=1}^n f_i(s_i|\beta)} = \prod_{i=1}^n t_i$. As for the manager, we obtain that $\Delta u_{AS} := \mathbb{E}[u_{AS}(\sigma_M, x_{AS} = P) \mid s_1, \ldots, s_n] - \mathbb{E}[u_{AS}(\sigma_M, x_{AS} = V) \mid s_1, \ldots, s_n]$, can be written as:

$$\begin{aligned} \Delta u_{AS} = {} & (\mathbb{P}(\sigma_M = P) + \mu\mathbb{P}(\sigma_M = V)) \times \\ & \left(2\mathbb{P}\left(\omega = \alpha | \tilde{\mathbb{P}}(\sigma_M = P) = \frac{\mathbb{P}(\sigma_M = P)}{\mathbb{P}(\sigma_M = P) + \mu\mathbb{P}(\sigma_M = V)}, s_1, \ldots, s_n\right) - 1\right), \end{aligned}$$

where $\tilde{\mathbb{P}}$ denotes the posterior probability once one knows that the manager has passed the reform (either because she chose to do so, $x_M = P$, or because she tried to veto, $x_M = V$, but the veto was not registered, which arises with probability μ). Hence, as for the manager, there is a cutoff $\bar{t}_{AS}$, function of the manager's strategy σ_M, such that $x_{AS} = P$ is a best reply to σ_M whenever

[44] In other words, the manager has incomplete control over her own veto: with probability μ, she is "transformed" into a misaligned manager, who automatically passes the proposal.

$t_{AS} \geq \bar{t}_{AS}$, and that $x_{AS} = V$ is a best reply otherwise.

We have shown that the best reply of each player $j \in \{M, AS\}$ is characterized by a cutoff $\bar{t}_j$ above which j plays P, and below which j plays V. Now, observe that the players' type spaces $T_M = [\delta_M, \frac{1}{\delta_M}]$ and $T_{AS} = [\delta^n, \frac{1}{\delta^n}]$ are compact. Moreover, if one denotes by $g_j(t_j \mid \omega)$ the density according to which player j is of type t_j in state ω, we obtain the expected utility of both players given the cutoffs $(\bar{t}_M, \bar{t}_{AS})$ as:

$$\mathbb{E}[u_j \mid \bar{t}_M, \bar{t}_{AS}] = \int_{\bar{t}_{AS}}^{\frac{1}{\delta^n}} dt_{AS} \int_{\delta_M}^{\frac{1}{\delta_M}} dt_M \left(\mu + (1-\mu)\mathbf{1}_{\{t_M \geq \bar{t}_M\}}\right) \times$$
$$(\mathbb{P}(\omega = \alpha) g_{AS}(t_{AS}|\alpha) g_M(t_M|\alpha) - \mathbb{P}(\omega = \beta) g_{AS}(t_{AS}|\beta) g_M(t_M|\beta)) \, .$$

As in each state, the conditional distributions of t_M and t_{AS} are continuous, the expected utility is continuous in both players' cutoffs. It follows that there exists an optimal couple of cutoffs $(\bar{t}^*_M, \bar{t}^*_{AS})$ which maximizes the common utility. The corresponding strategy profile must thus be an equilibrium, the most efficient equilibrium, and also the most efficient strategy profile. As the strategy profile $(x_M = V, x_{AS} = V)$ yields an expected utility of 0, the expected utility of the most efficient profile must be at least 0.

Claim 2: The most efficient profile of the two-player game is replicable in the original game under *1S1V-D*. In that game, the corresponding profile is both the most efficient profile such that the misaligned manager always proposes and the most efficient equilibrium.

In the original game under *1S1V-D*, consider the strategy profile σ^* where: the aligned manager behaves as the manager of the two-player game; the misaligned manager always proposes; the shareholders decentralize the aggregate shareholder's strategy of the two-player game by playing the log-strategy identified in Proposition 2.

First, observe that σ^* is an equilibrium. Indeed, we know from Claim 1 that the aligned manager and the shareholders are playing optimally (if one individual shareholder could improve her utility, then the aggregate shareholder could do it as well in the two-player game, a contradiction). Moreover, passing is always a best reply for the misaligned manager.

Second, σ^* is the most efficient profile among those for which the misaligned manager always proposes. Indeed, if there was a (strictly) more efficient such profile, then there would be a (strictly) more efficient profile than the one identified in the two-player game, a contradiction with Claim 1.

Third, assume by contradiction that there is a (strictly) more efficient equilibrium. By virtue of the previous assertion, it must be a strategy profile such that the misaligned manager proposes with probability strictly less than one. For such a strategy to be a best reply, it must be that shareholders always turn the reform down. Such a profile yields an expected utility of 0 for the shareholders (and the aligned manager), and thus cannot be a strict improvement over σ^*, since σ^* yields at least 0 (applying Claim 1).

Claim 3: The original game under *1P1V* admits equilibria, but none of them is more efficient than the most efficient profile under *1S1V-D*.

First, observe that the profile in which every manager's type vetoes and all shareholders vote against the proposal is an equilibrium, thus an equilibrium exists.

Second, any profile under *1P1V* for which the misaligned manager always proposes is replicable in the game under *1S1V-D*, and thus cannot be (strictly) more efficient than σ^* (applying Claim 2). Thus, no equilibrium under *1P1V* for which the misaligned manager always proposes can improve upon σ^*.

Finally, any equilibrium under *1P1V* such that the misaligned manager proposes with a probability strictly less than one must yield an expected utility of 0 for the shareholders and the aligned manager (same argument as under *1S1V-D*). Therefore, no equilibrium under *1P1V* can improve upon σ^*. ■

Lemma 6 *For any $\mu \in [0,1]$ and any share distribution d, for any BNE σ under $V^{1S1V-D}(d)$, there exists a BNE σ' under $V^{1P1V}(d)$ that makes all shareholders (weakly) worse off in expectation.*

Proof. First, observe that the profile for which any manager vetoes the proposal and all shareholder vote against it is an equilibrium under *1P1V*, with an expected utility of 0 for each shareholder.

Second, let $\sigma = (\sigma_M, \sigma_1, \ldots, \sigma_n)$ be a BNE under *1S1V-D*. We will show that this equilibrium yields an expected utility of at least 0 to all shareholders. If $\mathbb{P}(\sum_{i=1}^n \widehat{\sigma}_i(t_i) \geq 0) = 0$, the profile yields an expected utility of 0 (the proposal is never accepted), and the previous statement holds. We may thus focus on the case for which $\mathbb{P}(\sum_{i=1}^n \widehat{\sigma}_i(t_i) \geq 0) > 0$. As σ is an equilibrium, it must be that the misaligned manager always proposes. Then, by applying the same argument as in the proof of Lemma 5 (Claim 1), we obtain that the aligned manager's strategy must be weakly increasing.

Now, given the (aligned and misaligned) managers' strategies, the game among shareholders can be seen as a game with an exogenous proposal (as in Section 4), albeit with a possibly biased prior $\mathbb{P}(\omega = \alpha) \geq 1/2$. In that game, we can apply the same reasoning as in the proof of Lemma 2 (Claim 1), and we obtain that there exists a strategy profile σ' for the shareholders, such that each σ'_i is pure, weakly increasing and: $\forall i \in N, u_i(\sigma_M, \sigma_1, \ldots, \sigma_n) = u_i(\sigma_M, \sigma'_1, \ldots, \sigma'_n)$. As each player's strategy under the profile $(\sigma_M, \sigma'_1, \ldots, \sigma'_n)$ is weakly increasing, a similar argument as the one used in Lemma 3 (Claim 1) shows that this profile yields an expected utility of at least 0 to all shareholders. Therefore, $\forall i \in N, u_i(\sigma) \geq 0$. This concludes the proof.

The dominance statement in Proposition 7 is obtained by conjunction of Lemmas 5 and 6. To establish strict dominance, consider an instance for which, when the proposal is exogenous, the best BNE under *1S1V-D*, σ^{1S1V} , implements the correct outcome with a probability p^{1S1V} strictly higher than p^{1P1V}, attained at the best BNE under *1P1V*, σ^{1P1V}. We may further assume $p^{1S1V} > p^{1P1V} > \frac{1}{2}$. We will establish the strict dominance of *1S1V-D* over *1P1V* when the proposal is endogenous by taking δ_M sufficiently close to 1.

First, observe that for δ_M close enough to 1, the profile where the manager (either aligned or misaligned) always proposes and the shareholders play a BNE σ of the exogenous proposal game, such that $p^\sigma > 1/2$, must be an equilibrium. Indeed, on the shareholders' side, the game is the same as the one with an exogenous proposal since the manager always proposes. On the aligned manager's side, the utility of vetoing is 0, while the utility of proposing can be made arbitrarily close to $2p^\sigma - 1 > 0$ (for δ_M close to 1), in which case proposing is indeed a best reply. Finally, as the probability of accepting the reform is always positive (since $p^\sigma > 1/2$), the misaligned manager's best reply is also to propose.

Second, assume that there is a BNE σ under *1P1V* which implements the correct outcome with a probability $p^\sigma > p^{1P1V}$. As $p^\sigma > 1/2$, the misaligned manager always proposes (for δ_M close enough to 1) . As p^{1P1V} is the highest probability to implement the correct outcome under *1P1V* when the proposal is exogenous, it must be that the aligned manager sometimes vetoes the proposal, for some type $t_M \in T_M$. It must thus be that $\mathbb{P}(\alpha|t_M)\mathbb{P}(O = A|\alpha) + \mathbb{P}(\beta|t_M)\mathbb{P}(O = B|\beta) \leq \frac{1}{2}$, where $\mathbb{P}(O|\omega)$ denotes the probability of alternative O passing in state ω when the proposal is passed to the shareholders. By choosing δ_M sufficiently close to 1, we can make $\mathbb{P}(\alpha|t_M)$ and $\mathbb{P}(\beta|t_M)$ arbitrarily close to 1/2 for all $t_M \in [\delta_M, \frac{1}{\delta_M}]$, and, as $p^{1P1V} > \frac{1}{2}$, we can thus make sure that

$$\begin{aligned}&\left(\exists t_M \in T_M, \quad \mathbb{P}(\alpha|t_M)\mathbb{P}(O = A|\alpha) + \mathbb{P}(\beta|t_M)\mathbb{P}(O = B|\beta) \leq \frac{1}{2}\right) \\ &\qquad \Rightarrow \quad \left(\forall t_M \in T_M, \quad \mathbb{P}(\alpha|t_M)\mathbb{P}(O = A|\alpha) + \mathbb{P}(\beta|t_M)\mathbb{P}(O = B|\beta) < p^{1P1V}\right).\end{aligned}$$

If σ_M denotes the aligned manager's strategy, we obtain:

$$\begin{aligned}p^\sigma = \int &(\mu + (1-\mu)\sigma_M(t_M)(P)) \times \\ &(\mathbb{P}(\alpha|t_M)\mathbb{P}(O = A|\alpha) + \mathbb{P}(\beta|t_M)\mathbb{P}(O = B|\beta))\, g_M(t_M) dt_M < p^{1P1V}.\end{aligned}$$

Hence a contradiction. We have shown that for δ_M close enough to 1, the dominance of *1S1V-D* over *1P1V* can be strict. ■

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