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# WHAT DRIVES DEMAND FOR STATE-RUN LOTTERIES? EVIDENCE AND WELFARE IMPLICATIONS 

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#### Abstract

We use natural experiments embedded in state-run lotteries and a new nationally representative survey to provide reduced-form and structural estimates of risk preferences and behavioral biases in lottery demand. We find that sales respond more to the expected value of the jackpot than to price, but are unresponsive to variation in the second prize-a pattern that implies probability weighting but is inconsistent with standard parameterizations. In the survey, we find that lottery spending decreases modestly with income and is strongly associated with measures of innumeracy, poor statistical reasoning, and other proxies for behavioral bias. These bias proxies decline with income and statistically account for 43 percent of lottery purchases, suggesting that at least some of lottery demand is due to behavioral bias, not just anticipatory utility or entertainment value. We use these empirical moments to estimate a model of socially optimal lottery design. In the model, current multi-state lottery designs increase welfare but may harm heavy spenders.


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"[A lottery] preys upon the hard earnings of the poor; it plunders the ignorant and simple."

- U.S. Supreme Court, in Phalen v. Virginia (1850)
"In our stressful world, the ability to dream is well worth the price of a lottery ticket ... The lottery is simply a form of entertainment that happens to benefit your state."
- National Association of State and Provincial Lotteries (2021b)
"Since this regressive, addictive, partially hidden tax is here to stay, might a little improvement still be conceivable? ... Here's a modest suggestion: States should consider reducing their skim of the wagers."
- Purdue University president and former Indiana governor Mitch Daniels (2019)

People have long debated whether states should run lotteries. Opponents argue that lotteries are a regressive tax on people who are bad at math. Proponents argue that lotteries are a win-win, generating both consumer surplus and government revenues. If states do run lotteries, there are further debates, such as the optimal "implicit tax" - the share of revenues that is allocated to the government instead of returned to prize winners. Economists are divided: a recent survey of the University of Chicago IGM experts panel found that 23 percent of leading economists believe that state-run lotteries increase social welfare, 28 percent disagree, and 45 percent were uncertain or had no opinion. ${ }^{1}$

These debates matter. Americans spent a remarkable $\$ 679$ per household (or $\$ 87$ billion in total) on lottery tickets in 2019, generating $\$ 25$ billion in state government revenues. This was more than the revenue raised by federal estate or tobacco taxes, and just less than the revenue raised by the federal gas tax (Internal Revenue Service 2021). Americans spend more on lottery tickets than they do on cigarettes, and more than they do on music, sports tickets, movie tickets, books, and video games combined (Isidore 2015).

Embedded within these debates is a series of core (behavioral) economics questions. How does people's demand for lotteries vary with their prize structure, and what does this imply about underlying risk preferences? How much of lottery consumption is driven by innumeracy or other behavioral biases, as opposed to entertainment and other normatively respectable preferences? Do lower-income people spend more on lotteries, and is this good (because it reflects consumer surplus for people with higher marginal utility) or bad (because it reflects exploitation of behavioral biases)? How should states design lotteries to maximize welfare, accounting for consumer surplus, possible behavioral biases, concerns about regressivity, and the value of public funds?

We address these questions with new data on how people respond to variation in prizes and sales, and a new nationally-representative survey linking lottery expenditures to household income and proxies for behavioral biases. Building on a growing literature in structural behavioral economics applied to field and natural experiments (e.g., DellaVigna, List, and Malmendier 2012; DellaVigna,

[^0]List, Malmendier, and Rao 2017, 2022), we complement the reduced-form results from these data by calibrating a structural model of lottery demand, including the probability weighting function (Kahneman and Tversky 1979; Prelec 1998). Finally, we illustrate the policy relevance of our empirical estimates by considering state-run lotteries in an optimal taxation model, and deriving implications for welfare and optimal design.

As we show in Section 2, demand elasticities with respect to prizes and prices of lottery tickets identify the probability weighting function. We thus begin by estimating the aggregate semielasticities of demand with respect to prizes and price, exploiting natural experiments built into the two large multi-state lotteries, Mega Millions and Powerball. When nobody wins the jackpot, the jackpot prize money is "rolled over" to the next drawing. Since the winning numbers are randomly selected, rollovers are random conditional on ticket sales, generating conditionally random variation in the jackpot over time. In California, lower prizes also roll over if they are not won, and winning the second prize is unlikely enough that it rolls over regularly. We identify the effect of ticket prices on demand from event studies of when Mega Millions and Powerball separately increased their prices from $\$ 1$ to $\$ 2$.

The semi-elasticities of demand with respect to the jackpot expected value, second prize expected value, and price are about 1.7 , statistically zero, and -0.5 , respectively. Strikingly, this means that ticket sales increase over three times as much if the jackpot expected value increases by $\$ 1$ than if the price decreases by $\$ 1$. This is the opposite of what would be expected for risk-averse consumers, and it is not explained by substitution across games or time. Of course, this is qualitatively consistent with the long literature on probability weighting. ${ }^{2}$ However, the jackpot elasticity is so large relative to the second prize elasticity that it cannot be fit by three standard functional forms from the literature (Goldstein and Einhorn 1987; Tversky and Kahneman 1992; Prelec 1998), although it can be fit by the neo-additive form formalized by Chateauneuf, Eichberger, and Grant (2007).

We complement the quasi-experimental analysis with a new nationally-representative survey to inform two complementary questions. First, how much of the apparent probability weighting is due to behavioral biases versus normative preferences like wishful thinking or the joy of playing? Second, does demand for lotteries vary by income? We carried out the survey on the AmeriSpeak panel, a high-quality probability-based sample that includes households that might not participate in cheaper opt-in surveys.

We find that the spending distribution is highly skewed, generating some imprecision in the estimated means, but point estimates suggest that lottery spending declines moderately with income: people with household income under $\$ 50,000$ spend 29 percent more on the lottery than people with household income above $\$ 100,000$. Measures of perceived self-control problems, financial illiteracy, statistical mistakes (such as the Gambler's Fallacy, non-belief in the Law of Large Numbers, and difficulty calculating expected values), and incorrect beliefs about expected returns from lottery

[^1]play are highly statistically significantly associated with more lottery spending. This holds both unconditionally and after controlling for demographics, risk aversion, and questions measuring how much people enjoy playing the lottery. Interestingly, not all of these relationships suggest that bias increases consumption: the average person actually underestimates the share of lottery revenues that is returned to winners. We measure and correct for imperfect test-retest reliability by resampling the same survey respondents one year later, building on other resampling designs such as Beauchamp, Cesarini, and Johannesson (2017), Gillen, Snowberg, and Yariv (2019), Chapman et al. (2020), and Stango and Zinman (Forthcoming).

Regression predictions suggest that Americans would spend 43 percent less on lotteries if they had perfect self-control, had the financial literacy and statistical ability of the highest-scoring people in our sample, and had correct beliefs about expected returns. Lower-income people score lower on financial literacy and statistical ability, so the point estimates suggest that a larger share of their spending is attributable to bias. Although we control for a rich array of demographics and preference measures, a key caveat is that these regression predictions are not the causal effects of behavioral biases. With that important caveat, these results are consistent with concerns that behavioral biases play a role in lottery spending.

These reduced-form results allow us to calibrate a structural model of lottery demand. The structural estimates imply that the weight people attach to the jackpot is 230 times as large as the objective probability of winning the jackpot. We estimate that on average, 30 percent of the difference between the decision weight and the probability is driven by bias, although there is significant heterogeneity in bias. The bias share ranges from 33 percent for heavy-spending high-income consumers to 17 percent for middle- and high-income consumers who only buy lottery tickets occasionally.

In the last part of the paper, we present a theoretical model for studying the welfare effects of different lottery designs. This model can be applied to goods other than lotteries, and it generalizes the "optimal sin tax" model of Allcott, Lockwood, and Taubinsky (2019) to settings where the government can also regulate or directly control a good's attributes-for example, cigarette nicotine content or lightbulb energy efficiency. In our setting, the policymaker has multiple instruments that determine the implicit tax on lotteries, including raising the price, reducing the jackpot or other prizes, and reducing the prize probabilities. Optimal policy takes into account how these instruments can both counteract bias (the usual corrective taxation logic) and affect bias-for example, if a change in the jackpot win probability affects the misperception of that probability.

We derive formulas that characterize optimal lottery prices and attributes as functions of several sufficient statistics. Stronger behavioral biases imply a higher optimal implicit tax, i.e., higher prices and/or lower payouts to winners. Relatively more behavioral bias among lower-income people implies a higher optimal implicit tax, because the corrective benefits of the implicit tax accrue to the poor. However, relatively more lottery consumption among lower-income people implies a lower optimal implicit tax, because the mechanical utility losses from taxation accrue to the poor. The relative importance of the corrective benefits versus the mechanical losses depends on the demand
slope: more responsive demand means that the corrective benefits of demand changes outweigh the mechanical losses, whereas less responsive demand means that the corrective benefits will be relatively small. Importantly, different means of adjusting the implicit tax affect different types of individuals. Optimal policy thus involves striking a balance between instruments most targeted to counteract bias and instruments with the best distributional properties.

Combining our empirical estimates with this model, we find that the optimal implicit tax rate on lotteries is close to the current Mega Millions and Powerball designs. In our model, actual consumer surplus is much smaller than consumers' perceived surplus at our baseline behavioral bias estimates, but the current Mega Millions and Powerball designs increase welfare and deliver close to maximal welfare levels. A key channel for these gains comes through the effect on revenues: lotteries raise public funds while also generating (much smaller) consumer surplus for lottery purchasers. These results hinge on the magnitude of behavioral bias. For example, if we alternatively assume zero bias, then low-income heavy spenders derive the most surplus. If bias is more than about twice as large as our baseline estimate, lotteries would reduce overall welfare. This highlights the importance of our survey evidence but also underscores the need for future work.

Our main contribution to the literature is the new empirical results on decisionmaking under risk and key parameters relevant for lottery design, including elasticities and the relationship between lottery consumption, income, and proxies for behavioral bias. Our paper is also the first to formalize and empirically implement a framework for studying optimal lottery policy with behavioral bias and redistributive concerns.

We build on six distinguished literatures. The first is a reduced-form empirical literature on state-run lotteries; see Clotfelter and Cook (1989, 1990), Kearney (2005a), and Grote and Matheson (2011) for overviews. This includes papers studying how lottery spending varies by income and other demographics. ${ }^{3}$ While lotteries have changed substantially in the past two decades, to our knowledge our AmeriSpeak survey is the first to measure individual-level lottery spending using a nationwide probability sample since Clotfelter et al. (1999). ${ }^{4}$ This literature also includes papers studying aggregate demand patterns and substitution across games. ${ }^{5}$ We extend that work by (i) using instrumental variables to address simultaneity bias in the relationship between jackpots and sales, (ii) using new data to estimate the elasticity with respect to the second prize, which we find to be very different than the jackpot elasticity, and (iii) exploiting recent Mega Millions and Powerball price changes to estimate the price elasticity of demand for those games. Both (ii) and (iii) are necessary for structural estimates of the probability weighting function.

The second literature we build on is the work in structural behavioral economics, particularly in its application to field and natural experiments. ${ }^{6}$ The third literature we build on studies

[^2]behavioral biases that might affect gambling and lottery demand in the field. ${ }^{7}$ While much of this literature studies one or two biases in isolation, our survey extends this literature by offering a more comprehensive measure of the many different biases that could affect lottery demand. The fourth literature studies the use of lotteries to encourage beneficial behaviors such as saving money (Kearney et al. 2011), charitable giving (Landry et al., 2006; Lange, List, and Price, 2007a,b), and healthy behaviors (Haisley et al. 2012). These behavior change-focused lotteries are distinct from the state-run lotteries that we study because the objective of these lotteries is behavior change, rather than striking an optimal balance between raising revenue and increasing consumer surplus through entertainment. The fifth literature is the work in behavioral public economics, studying other settings where behavioral biases affect optimal policy design. ${ }^{8}$ The sixth literature studies how survey measures of bias predict behavior. ${ }^{9}$

Sections 1-6 present the background, theoretical framework, quasi-experimental evidence on lottery demand, survey evidence on lottery expenditures and behavioral biases, model calibration, and welfare analysis, respectively.

## 1 Background on Lotteries in the United States

From 1995 to 2019, real lottery spending grew from $\$ 540$ per household ( $\$ 53$ billion in total) to $\$ 679$ per household ( $\$ 87$ billion in total); see Appendix Figure A1. There are two major types of lottery games: instant (or "scratch-off") games and draw games, where players choose numbers and win if their numbers match those selected in the next drawing. The largest draw games are two multi-state lotteries, Mega Millions and Powerball. From 1995 to 2019, instant games grew from 38 to 65 percent of sales, Mega Millions and Powerball grew from 3 to 9 percent, and all other games dropped from 59 to 26 percent.

Of the $\$ 87$ billion in 2019 sales, 60 percent was returned in prizes, 10 percent was overhead (6.5 percent commissions and 3.9 percent administrative costs), and the remaining 29 percent represented state government proceeds; see Appendix Figure A2. This 29 percent implicit tax has decreased from 34 percent since 1995, a trend that is associated with the growing market share of instant games, which return a larger share to winners. Lottery prizes are taxed as income, so an additional share of prize money returns to governments through the income tax.
(2017), Giaccherini et al. (2019), DellaVigna et al. (2022), Laibson et al. (2021). See DellaVigna (2018) for a review.
${ }^{7}$ Studies outside of laboratory settings include Clotfelter and Cook (1993), Haisley, Mostafa, and Loewenstein (2008), Guryan and Kearney (2008), Post et al. (2008), Snowberg and Wolfers (2010), and Suetens, Galbo-Jørgensen, and Tyran (2016). A separate large literature, e.g. Kahneman and Tversky (1979) and subsequent work, studies related biases in laboratory settings.
${ }^{8}$ See, e.g., Bernheim and Rangel (2004, 2009), Mullainathan, Schwartzstein, and Congdon (2012), Allcott, Mullainathan, and Taubinsky (2014), Allcott and Taubinsky (2015), Baicker, Mullainathan, and Schwartzstein (2015), Bernheim, Fradkin, and Popov (2015), Handel and Kolstad (2015), Ambuehl, Bernheim, and Lusardi (2022), Spinnewijn (2017), Taubinsky and Rees-Jones (2018), Allcott, Lockwood, and Taubinsky (2019), Handel, Kolstad, and Spinnewijn (2019), Beshears et al. (2020), Farhi and Gabaix (2020), Goldin and Reck (2020), and Rees-Jones and Taubinsky (2020); see Bernheim and Taubinsky (2018) for a review.
${ }^{9}$ See, e.g., Chabris et al. (2008), Meier and Sprenger (2010), Beauchamp, Cesarini, and Johannesson (2017), Gillen, Snowberg, and Yariv (2019), Chapman et al. (2020), and Stango and Zinman (Forthcoming).

Our aggregate demand estimation focuses on Mega Millions and Powerball from June 2010 through February 2020. In Mega Millions, players select five numbers from 1 to 70 and one Mega Ball number from 1 to 25 ; this is the " $5 / 70+1 / 25$ format." Powerball uses a $5 / 69+1 / 26$ format. Players win the jackpot if their numbers match all six balls selected in the next semi-weekly drawing; players can also win lower prizes from $\$ 2$ to $\$ 1,000,000$ by choosing one or more numbers correctly. Table 1 presents ticket price and prize information for the formats in place during our sample period. For both games, tickets now cost $\$ 2$, and the jackpot odds are about $1 / 300,000,000$. The ratios of ticket expected value to price are 0.30 and 0.33 , so accounting for 10 percent overhead gives implicit tax rates of 61 and 57 percent, respectively.

Jackpot amounts are determined on a parimutuel basis: the jackpot prize pool depends on ticket sales, and the jackpot is split equally among all winners. Jackpot winners can choose to receive 30 annual installments that increase by five percent per year or the discounted present value of that annuity at current interest rates; most choose the latter. Before each drawing, Mega Millions and Powerball advertise an "estimated jackpot," which is the undiscounted value of the annuity based on projected ticket sales. If no one wins the jackpot, the jackpot prize pool is rolled over to the next drawing. If someone wins the jackpot, the next drawing's jackpot returns to a reset value, which was $\$ 40$ million for both Mega Millions and Powerball at the end of our sample in February 2020. ${ }^{10}$

In most states, the lower prize amounts are fixed. However, the California Supreme Court ruled in 1996 that the California State Lottery Act allows parimutuel games where players play against other players, but not games where players play against the house. As a result, both Mega Millions and Powerball have California-specific parimutuel prize pools for each lower prize that roll over to the next draw if they are not won. The second prize odds are about $1 / 12,000,000$ for both games, so there are many drawings when no one wins the second prize and it thus rolls over. The third and lower prizes have high enough odds that they generally don't roll over.

Powerball began in 1992 with 15 states, and Mega Millions began in 2002 with 9 other states. Both games replaced earlier multi-state games with different names, and both have gradually added states over time. After a cross-selling agreement was reached, individual states began to offer both games in 2010. By June 2010, 42 states had joined Mega Millions and 41 had joined Powerball. From September 2014 through the end of our sample, both games were available through all 45 state lotteries and the D.C. and U.S. Virgin Islands lotteries. ${ }^{11}$

Mega Millions and Powerball have both adjusted their pricing and formats several times since 2010, as shown in Table 1. There are five key trends. First, both games increased prices from $\$ 1$ to $\$ 2$. Second, the jackpot odds have been reduced, increasing the frequency of rollovers: the jackpot was won every 7.4 drawings in 2010-2011 but only every 15.4 drawings in 2018-2019. Third, the

[^3]prize structure is being hollowed out: there is less expected value in the middle prizes and more in the jackpot and lowest prizes. Fourth, as a result of the previous two trends, average jackpots have grown: the average jackpot was $\$ 37$ million in 2010-2011 and $\$ 103$ million in 2018-2019. Fifth, the ratio of expected value to price is decreasing, i.e., winners receive a smaller share of the revenues. This pushes against the overall trend of lower implicit taxes described above.

## 2 Template for Analysis

In this section we present a simple model of preferences over lotteries and formalize the key concepts that guide our empirical analysis. We first formalize what we mean by probability weighting, and we show how it is identified from prize and price elasticities. This helps organize our analysis in Section 3. We then formalize a general framework for considering the role of behavioral biases, which helps organize our analysis in Section 4. We calibrate the structural model in Section 5, and in Section 6 we continue our analysis of the theoretical model presented here by deriving a sufficient statistics formula for optimal lottery design.

### 2.1 Model

Individuals choose labor supply (and thus earnings $z$ ), numeraire consumption $c$, and whether to buy a lottery ticket $x_{t} \in\{0,1\}$ in each of many choice occasions indexed by $t$. We disaggregate demand into a series of binary decisions on different choice occasions because probability weighting (Kahneman and Tversky 1979; Tversky and Kahneman 1992) most plausibly applies at the level of each individual ticket, rather than at the level of an aggregate portfolio. This is consistent with the evidence on narrow bracketing (Tversky and Kahneman 1981; Rabin and Weizsacker 2009), particularly as it applies to lottery tickets (Haisley, Mostafa, and Loewenstein 2008). This is also consistent with our empirical results that there is no substitution between different types of lotteries. To simplify exposition in the body of the paper, we assume that there is a single choice occasion; the more general case with multiple choice occasions is characterized in Appendix C.

Earnings are subject to a nonlinear income tax $T(z)$. The price of the numeraire is normalized to one, and the price of a lottery ticket is $p$. We define $\boldsymbol{a}$ as a vector of lottery attributes, including the $K$ prizes $\boldsymbol{w}=\left(w_{1}, \ldots, w_{K}\right)$ and corresponding probabilities $\boldsymbol{\pi}=\left(\pi_{1}, \ldots, \pi_{K}\right)$, as well as other possible attributes such as advertising.

Individuals have heterogeneous types $\theta$ capturing income-earning ability, preferences, and behavioral biases. On each choice occasion, individuals receive a lottery taste shock $\varepsilon$ drawn from a continuous distribution. We think of $\varepsilon$ as representing the transaction cost of buying a lottery ticket. We let $F(\theta, \varepsilon)$ denote the joint distribution over types, and we let $F_{\theta}$ denote the marginal distribution over $\theta$, and $F_{\varepsilon \mid \theta}$ the conditional distribution of $\varepsilon$. We define $u(\boldsymbol{a} ; \theta, \varepsilon)$ as the perceived subutility from purchasing a lottery ticket, normalized by the marginal utility of income at $z$. We define $\psi(z ; \theta)$ as the labor disutility of generating earnings $z$. Individuals maximize "perceived
utility"

$$
\begin{equation*}
U=G(c+x \cdot u(\boldsymbol{a} ; \theta, \varepsilon)-\psi(z ; \theta)), \tag{1}
\end{equation*}
$$

subject to budget constraint $z-T(z)=c+p x$, where $G$ is twice-differentiable and concave. For a given set of probabilities and prizes, the subutility is given by

$$
\begin{equation*}
u(\boldsymbol{a} ; \theta, \varepsilon)=\sum_{k} \Phi\left(\pi_{k} ; \theta\right) m\left(w_{k} ; \theta\right)-\varepsilon \tag{2}
\end{equation*}
$$

where $m\left(w_{k} ; \theta\right)$ is the incremental utility of $w_{k}$ more income, normalized by the marginal utility of income at $z .{ }^{12}$ The decision weight corresponding to outcome $k$ is $\Phi\left(\pi_{k}\right)$, so that $\Phi(\pi) \equiv \pi$ corresponds to expected utility preferences. Because we normalize by the local marginal utility of consumption, $\sum_{k} \Phi_{k}\left(\pi_{k} ; \theta\right) m\left(w_{k} ; \theta\right)$ can be interpreted as the certainty equivalent of the lottery, absent transaction costs. Beyond expected utility preferences, this specification nests many modifications proposed in the literature on prospect theory and cumulative prospect theory. ${ }^{13}$

Type $\theta$ 's demand function is the average across many binary purchase decisions of type- $\theta$ individuals with different taste shocks $\varepsilon$ :

$$
\begin{align*}
s(p, \boldsymbol{a} ; \theta) & =\operatorname{Pr}(u(\boldsymbol{a} ; \theta, \varepsilon)>p)  \tag{3}\\
& =F_{\varepsilon \mid \theta}\left[\sum_{k} \Phi\left(\pi_{k} ; \theta\right) m\left(w_{k} ; \theta\right)-p\right] . \tag{4}
\end{align*}
$$

We define $\bar{s}(p, \boldsymbol{a}):=\int s(p, \boldsymbol{a} ; \theta) d F_{\theta}(\theta)$ as population-level aggregate demand. When no ambiguity arises, we sometimes suppress arguments in the demand functions.

Two features of the utility specification in equation (1), separability and quasilinearity in numeraire consumption, imply that financial windfalls do not affect either lottery expenditures or labor supply. The negligible labor supply income effects assumption is supported by Gruber and Saez (2002) and Saez, Slemrod, and Giertz (2012). ${ }^{14}$ The more general model in Appendix C allows for income effects on lottery consumption, although empirically we find them to be small, and in our welfare simulations we find these effects have a negligible impact on our results.

We define $C(\boldsymbol{a}, \bar{s})$ as the cost to the government of selling $\bar{s}$ tickets with attributes $\boldsymbol{a}$; this includes the expected prize payout plus administration, marketing, and any other costs. We refer to the percent markup above average cost, $\frac{p \bar{s}-C(\boldsymbol{a}, \bar{s})}{p \bar{s}}$, as the "implicit tax rate."

[^4]
### 2.2 Using Elasticities to Identify Decision Weights

With minor abuse of notation, we sometimes write $s(z)$ and other statistics as functions of $z$, to denote consumption of $s$ among all $z$-earners, and so forth. We define $\zeta_{p}(z):=\frac{d \ln s(z)}{d p}$ as the semielasticity of demand with respect to ticket price, and we define $\zeta_{a}(z):=\frac{d \ln s(z)}{d a}$ as the semi-elasticity of demand with respect to attribute $a$. We define $\bar{\zeta}_{p}$ and $\bar{\zeta}_{a}$ as population-wide semi-elasticities. When there is no ambiguity, we use $\zeta_{k}$ to denote the elasticity with respect to the expected value of the $k$ th prize, $\pi_{k} w_{k}$. Consistent with our empirical analysis, this elasticity is defined with respect to variation in the prize $w_{k}$, not in the probability $\pi_{k} \cdot{ }^{15}$

Our demand model implies the following relationship between the decision weights $\Phi\left(\pi_{k}\right)$ and the measured semi-elasticities:

$$
\begin{equation*}
\frac{\zeta_{k}}{\left|\zeta_{p}\right|}=\frac{\Phi\left(\pi_{k}\right)}{\pi_{k}} m^{\prime}\left(w_{k}\right) . \tag{5}
\end{equation*}
$$

To see this, note that changing the ticket price by $-d p<0$ induces an increase in lottery purchases equal to $d s=\left|\zeta_{p}\right| s \cdot d p$. The increase in the expected value of prize $k$ that would generate the same change in demand, denoted $d x_{k}=\pi_{k} d w_{k}$, satisfies $d p=\frac{\Phi_{k}}{\pi_{k}} m^{\prime}\left(w_{k}\right) d x_{k}$. The change in demand generated by this jackpot increase is $d s=\zeta_{k} s \cdot d x_{k}$. Therefore, $\left|\zeta_{p}\right| s \frac{\Phi_{k}}{\pi_{k}} m^{\prime}\left(w_{k}\right) \cdot d x_{k}=$ $\zeta_{k} s \cdot d x_{k}$, which gives the equation in the text. This logic generalizes the simple principle that if $m$ is concave and decision weights equal objective probabilities, then people must be less responsive to a $\$ 1$ change in the expected value of a prize than they are to a $\$ 1$ change in the price. That is, a prize semi-elasticity $\zeta_{k}$ that is higher than $\left|\zeta_{p}\right|$ is inconsistent with the standard assumptions of $\Phi\left(\pi_{k}\right)=\pi_{k}$ and concave $m$.

Comparing prize semi-elasticities to each other also provides insight into the decision weights. A corollary of (5) is that

$$
\begin{equation*}
\frac{\zeta_{k}}{\zeta_{k^{\prime}}}=\frac{\Phi\left(\pi_{k}\right)}{\pi_{k}} \cdot \frac{\pi_{k^{\prime}}}{\Phi\left(\pi_{k^{\prime}}\right)} \cdot \frac{m^{\prime}\left(w_{k}\right)}{m^{\prime}\left(w_{k^{\prime}}\right)} \tag{6}
\end{equation*}
$$

Letting $w_{k}>w_{k}^{\prime}$, note that if utility from wealth is (weakly) concave, so that $\frac{m^{\prime}\left(w_{k}\right)}{m^{\prime}\left(w_{k^{\prime}}\right)} \leq 1$, then $\zeta_{k} \leq \zeta_{k^{\prime}}$ when decision weights equal objective probabilities. Thus, $\zeta_{k}>\zeta_{k^{\prime}}$ is evidence that decision weights do not equal objective probabilities, and in particular that people attach a relatively much larger decision weight to the larger prize than to the smaller prize.

If $m\left(w_{k}\right)$ is specified, the semi-elasticity estimates from Section 3 provide direct estimates of the decision weights on the jackpot and the second-highest prize. This is the key link between our reduced-form estimates and our structural model.

### 2.3 Bias

The function $u$ can reflect a variety of possible motives for playing the lottery. For example, it can include any entertainment derived from playing the lottery (Conlisk 1993; Kearney 2005b), or

[^5]anticipatory utility from thinking about a chance of winning (e.g., Loewenstein 1987; Caplin and Leahy 2001; Brunnermeier and Parker 2005; Gottlieb 2014). We also allow for perceptual distortions, such as over- or under-estimating the likelihood of winning or imperfect processing of small probabilities (Woodford 2012; Steiner and Stewart 2016). Biases induced by salience or focusing effects (Bordalo, Gennaioli, and Shleifer 2013; Kőszegi and Szeidl 2013; Bushong, Schwartzstein, and Rabin 2021) could affect demand as well. Probability weighting could be generated either by normatively relevant preferences such as anticipatory utility or the entertainment value of playing, or by perceptual distortions. For example, if the entertainment utility individuals receive from playing is bounded away from zero, as long as the likelihood of winning is positive, individuals will behave as if they overweight small probabilities.

To formalize the possibility of mistakes, we draw a distinction between perceived utility, which individuals maximize, and "normative utility," which enters the planner's objective function:

$$
\begin{equation*}
V=G(c+x \cdot v(\boldsymbol{a} ; \theta, \varepsilon)-\psi(z ; \theta)) . \tag{7}
\end{equation*}
$$

We define $s^{V}(p, \boldsymbol{a} ; \theta):=\operatorname{Pr}(v(\boldsymbol{a} ; \theta, \varepsilon)>p)$ as the type- $\theta$ demand function that would obtain if individuals maximized normative utility. We assume that $v$ is also additively separable in $\varepsilon$.

Bias is the difference between the perceived utility and normative utility from consuming a lottery ticket: $\gamma(\boldsymbol{a} ; \theta):=u(\boldsymbol{a} ; \theta, \varepsilon)-v(\boldsymbol{a} ; \theta, \varepsilon){ }^{16}$ This representation of bias mirrors Allcott, Lockwood, and Taubinsky (2019, henceforth "ALT"), where $\gamma$ is equivalent to the price decrease that would lead consumers maximizing $V$ to purchase as many lottery tickets as consumers maximizing $U: s^{V}(p-\gamma, \boldsymbol{a} ; \theta)=s(p, \boldsymbol{a}, \theta)$.

### 2.3.1 Examples

Suppose that the lottery offers a single large prize of amount $w$ with a probability $\pi$, so that $\boldsymbol{a}=\{w, \pi\}$, and consider demand from a given type of agent (with index $\theta$ suppressed for simplicity). Further suppose that utility is linear in the numeraire and the perceived subutility from a lottery ticket is $u(\boldsymbol{a} ; \varepsilon)=(1+\phi) \pi m(w)-\varepsilon$, where $m(w)$ is the utility gain from winning a prize of size $w$, and $(1+\phi) \pi$ is the decision weight that the individual applies to that utility. If $\phi=0$, the individual is an expected utility maximizer. We now provide examples of the types of biases that can be accommodated by this framework.

Misperceived probability of winning. Suppose the individual misperceives the probability of winning as $\tilde{\pi} \neq \pi$, and the normative decision weight (with no misperception) is $\left(1+\phi^{V}\right) \pi$, with $\phi^{V}=0$ for expected utility decision makers and $\phi^{V}>0$ for individuals who experience anticipatory utility. Then normative utility from lottery consumption is $v(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V}\right) \pi m(w)-\varepsilon$, perceived utility is $u(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V}\right) \tilde{\pi} m(w)-\varepsilon$, and bias is $\gamma(\boldsymbol{a})=u(\boldsymbol{a} ; \varepsilon)-v(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V}\right)(\tilde{\pi}-\pi) m(w)$.

Present focus (and addiction). Suppose that $\phi^{V} \pi m(w)$ is the immediate hedonic gain from anticipatory utility and joy of playing. Suppose further that individuals are quasi-hyperbolic

[^6]discounters who discount all consumption other than the immediate utility $\phi^{V} \pi m(w)$ by a factor $\beta<1$. Finally, suppose that normative utility corresponds to long-run preferences that set $\beta=1$. Then normative and perceived utility can be written, respectively, as $v(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V}\right) \pi m(w)-\varepsilon$ and $u(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V} / \beta\right) \pi m(w)-\varepsilon$. Bias is then $\gamma(\boldsymbol{a})=(1 / \beta-1) \phi^{V} \pi m(w)$.

Our framework can also accommodate the interaction between present focus and addiction, as studied by, e.g., Gruber and Kőszegi (2001) in the context of smoking. Suppose that buying a lottery ticket today imposes expected costs of $d$ on one's future self because prior experience with gambling makes it more painful not to gamble, and possibly also makes future gambling less enjoyable. If these expected costs are down-weighted by present focus $\beta$ (but additional anticipatory utility $\phi^{V} \pi m$ is not), then $u(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V}\right) \pi m(w)-\beta d-\varepsilon, v(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V}\right) \pi m(w)-d-\varepsilon$, and $\gamma(\boldsymbol{a})=(1-\beta) d$.

Misforecasted happiness. Suppose individuals overestimate the happiness they would gain from winning the prize $w$ by a factor of $b$. Then $u(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V}\right) \pi(1+b) m(w)-\varepsilon$, while $v(\boldsymbol{a} ; \varepsilon)=\left(1+\phi^{V}\right) \pi m(w)-\varepsilon$, and thus $\gamma(\boldsymbol{a})=\left(1+\phi^{V}\right) \pi b m(w)$.

## 3 Aggregate Lottery Demand

This section presents estimates of the aggregate semi-elasticities of demand $\bar{\zeta}_{a}$ and $\bar{\zeta}_{p}$ for the two major multi-state lotteries, Mega Millions and Powerball. We assume a static demand function with constant semi-elasticity with respect to price and the expected value of each prize; we see below that our data are consistent with these assumptions. $k, j$, and $t$ index prizes, games, and drawing dates, respectively, $\pi_{k j t}$ is the win probability from Table 1 , and $w_{k j t}$ is the prize amount. Aggregate demand is

$$
\begin{equation*}
\ln \bar{s}_{j t}=\sum_{k} \bar{\zeta}_{k} \pi_{k j t} w_{k j t}+\bar{\zeta}_{p} p_{j t}+\varepsilon_{j t} \tag{8}
\end{equation*}
$$

For reasons described below, we do not directly estimate equation (8). Instead, we separately estimate the prize elasticities $\bar{\zeta}_{k}$ in Section 3.2 and the price elasticity $\bar{\zeta}_{p}$ in Section 3.3.

### 3.1 Data: Lottery Prizes and Aggregate Sales

Our primary analyses use draw-level sales and prize data for Mega Millions and Powerball from June 2010 through February 2020. We scraped jackpot and sales data from the website LottoReport.com, and we scraped California second prize amounts from the California Lottery website. Jackpots are advertised jackpot amounts, and ticket sales exclude the Just the Jackpot, Power Play, and Megaplier add-ons.

Table 2 presents descriptive statistics. There are 2,035 observations: two draws per week for almost ten years, for both Mega Millions and Powerball. The California sample is slightly smaller, because California did not join Powerball until April 2013. The average jackpot was $\$ 68.8$ million, and the average California second prize was $\$ 948,200$. The average draw sold 23 million tickets
nationwide and 3.3 million in California.
We run 36 -month event study analyses around the four format changes described in Table 1. To maintain a consistent set of states in the national sales data as states join Mega Millions or Powerball, we construct separate national sales series for each event window that include total sales from only the states that participated for the full 36 -month period. For example, California, Florida, Louisiana, and the U.S. Virgin Islands joined Mega Millions and/or Powerball during the 36 -month window around the Powerball price increase on January 15, 2012, so we exclude them from the national sales totals used for that event study.

We also use data on lottery sales by game, state, and week from January 1995 through February 2020 purchased from La Fleur's, a standard data provider. To study substitution across games, we limit to a balanced panel including instant games plus the 85 other games that were offered continuously and for which we have complete data from June 2010 through February 2020 in the 41 states that had Mega Millions and Powerball over that period, representing 61 percent of total lottery sales reported in the Census of Governments over the sample period.

Because prize amounts are typically round numbers in nominal dollars, we use nominal dollars throughout the paper except for Appendix Figures A1 and A2. When there are multiple winners of the jackpot or any prize in California, the prize is split equally among all winners. To account for prize-sharing, we use our LottoReport sales data to approximate the average factor by which each prize is reduced in expectation by prize-sharing and apply that factor to prize amounts when computing expected values. The factors applied to jackpots and California second prizes are 0.91 and 0.71 , respectively. We also discount all jackpots to their present value using a conversion factor of 0.59 . We compute the conversion factor as the ratio between (i) the stock of cash required to purchase bonds that yield enough interest in each period to pay out the jackpot annuity as scheduled and (ii) the total annuity value of the jackpot. ${ }^{17}$

### 3.2 Prize Elasticities

### 3.2.1 Estimation Strategy

We identify the jackpot elasticity $\bar{\zeta}_{1}$ off of jackpot variation generated by randomness in whether someone won the jackpot in the previous draw, and we identify the second prize elasticity $\bar{\zeta}_{2}$ off of analogous variation in California second prizes. As an example, Figure 1 illustrates the identifying variation for Powerball in 2014. Over that year, the jackpot varied between $\$ 40$ million and $\$ 237$ million. In most drawings, nobody wins the jackpot, so the prize pool rolls over to the next drawing. Each of the 11 times that someone won the jackpot in 2014, it returned to its $\$ 40$ million reset value. The odds of winning the jackpot were about $1 / 200,000,000$ in 2014 , so the expected value of the jackpot varied from roughly $\$ 0.15$ to $\$ 1.25$. The California second prize varied from $\$ 183,000$ to $\$ 7.7$ million, and the second prize odds were about $1 / 5,000,000$, so the second prize expected

[^7]value varied from roughly $\$ 0.03$ to $\$ 1.00$. This illustrates that the California second prize expected value can be material (and even larger than the jackpot), although the jackpot expected value is about 3.3 times larger on average during our sample.

The figure also plots California ticket sales against the right axis. Sales and jackpots move together, and sales rise especially sharply when jackpots are unusually high. However, sales do not seem to respond to second prize variation, suggesting that $\bar{\zeta}_{2}$ is close to zero.

While there is some randomness in prize amounts, there are two reasons why we do not directly estimate equation (8). First, the draw $t$ advertised jackpot is directly determined by forecasted ticket sales for draw $t$, potentially generating simultaneity bias. Second, due to rollovers, the draw $t$ prize pool is also directly affected by previous draws' sales, which may be correlated with draw $t$ demand if demand shocks are serially correlated.

To isolate the random variation in prize amounts, we instrument for prize $k$ 's expected value $\pi_{k j t} w_{k j t}$ with a forecast $\pi_{k j t} Z_{k j t}$ based on whether or not the prize rolled over from the previous period. Define $r_{k j t}$ as an indicator for whether prize $k$ rolled over from $t-1$ into $t$. A rollover from $t-1$ is less likely when $t-1$ ticket sales $\bar{s}_{j, t-1}$ are higher, but the realization of $r_{k j t}$ is random conditional on $\bar{s}_{j, t-1}$. Let $\iota_{k j f(t)}$ denote the observed average percent increase in prize $k$ in game $j$ conditional on a rollover when format $f$ is in effect, and let $\underline{w}_{k j f(t)}$ denote prize $k$ 's reset value in game $j .{ }^{18}$ The prize forecast is

$$
\begin{equation*}
Z_{k j t}=r_{k j t} \cdot\left(1+\iota_{k j f(t)}\right) \cdot w_{k j, t-1}+\left(1-r_{k j t}\right) \cdot \underline{w}_{k j f(t)} \tag{9}
\end{equation*}
$$

Rollovers $r_{k j t}$ and past prizes $w_{k j, t-1}$ both depend on previous demand, and demand shocks may be serially correlated. We address this following Borusyak and Hull (2021) by including a control $\bar{Z}_{k j t}$ for the forecast of the prize prior to the realization of the rollover outcome. Borusyak and Hull (2021) formally show that including this control yields consistent estimates. This pre-rollover prize forecast is ${ }^{19}$

$$
\begin{equation*}
\bar{Z}_{k j t}=\left(1-\pi_{k j t-1}\right)^{\bar{s}_{j, t-1}} \cdot\left(1+\iota_{k j f(t)}\right) \cdot w_{k j, t-1}+\left(1-\left(1-\pi_{k j t-1}\right)^{\bar{s}_{j, t-1}}\right) \cdot \underline{w}_{k j f(t)} \tag{10}
\end{equation*}
$$

Thus, after controlling for $\bar{Z}_{k j t}$, we identify only off of conditionally random variation in the prize forecast $Z_{k j t}$ delivered by randomness in whether the prize rolled over. ${ }^{20}$

We also add a vector of fixed effects collectively denoted $\xi_{j t}$ : game-format fixed effects to soak up any changes in demand caused by changes in lower prize amounts and probabilities, gameregional coverage fixed effects to soak up changes in demand when new states join, game-quarter

[^8]of sample indicators to soak up changes in demand over time, and game-weekend fixed effects to soak up higher demand for Friday and Saturday draws. We do not include time fixed effects that are common across both games, because this could introduce bias if consumers substitute across games.

Incorporating these modifications into equation (8), our estimating equation is

$$
\begin{equation*}
\ln \bar{s}_{j t}=\bar{\zeta}_{1} \pi_{1 j t} w_{1 j t}+\bar{\beta}_{1} \pi_{1 j t} \bar{Z}_{1 j t}\left(+\bar{\zeta}_{2} \pi_{2 j t} w_{2 j t}+\bar{\beta}_{2} \pi_{2 j t} \bar{Z}_{2 j t}\right)+\xi_{j t}+\epsilon_{j t} . \tag{11}
\end{equation*}
$$

The second prize elasticity term in parentheses is included only in our California-specific estimates. We instrument for $\bar{\zeta}_{1} \pi_{1 j t} w_{1 j t}$ (and $\bar{\zeta}_{2} \pi_{2 j t} w_{2 j t}$ ) with the jackpot forecast $\pi_{1 j t} Z_{1 j t}$ (and the second prize forecast $\pi_{2 j t} Z_{2 j t}$ ).

### 3.2.2 Estimation Results

Figure 2 presents binned scatter plots of the variation that identifies the prize semi-elasticities $\bar{\zeta}_{1}$ and $\bar{\zeta}_{2}$, conditional on the controls in equation (11). Panel (a) shows that national ticket sales are highly responsive to the jackpot expected value fitted values $\widehat{\pi_{1 j t} w_{1 j t}}$ from the first stage of equation (11). The relationship is very close to linear, and the slope is slightly less than 2.0 , meaning that a $\$ 0.10$ increase in the jackpot expected value increases sales by nearly $20 \log$ points. However, Panel (b) shows that California ticket sales are not responsive to the California second prize expected value first-stage fitted values.

Table 3 presents estimates of equation (11). The first stages are very strong; see Appendix Table A1. In all estimates in this section, we use Newey-West standard errors with up to ten lags. Our standard errors do not change much if we allow more or fewer Newey-West lags or cluster standard errors at the intersection of game and month, quarter, or half year; see Appendix Table A2.

Panel (a) presents estimates of the jackpot elasticity $\bar{\zeta}_{1}$ using the nationwide sales data, while Panel (b) presents estimates of the jackpot and second prize elasticities $\bar{\zeta}_{1}$ and $\bar{\zeta}_{2}$ using California sales only. In both panels, column 1 presents the OLS estimates without a $\pi \bar{Z}$ pre-rollover prize forecast control, and column 2 presents our IV estimates with that control.

The OLS estimates are slightly larger, consistent with slight simultaneity bias. The estimates match the binned scatter plots in Figure 2. In both panels of Table 3, the jackpot semi-elasticity is around 1.7-1.8. In Panel (b), the second prize semi-elasticity is statistically indistinguishable from zero, and the 95 percent confidence intervals in column 2 exclude values larger than about 0.19.

As shown in equation (6) in Section 2.2, the large difference between these two elasticities implies that decision weights do not equal objective probabilities, and that the decision weight on the jackpot is relatively much larger than the decision weight on the smaller prize. A probability weighting function with this form could arise for a variety of reasons. First, individuals might underappreciate differences between the (small) second prize probability and the (very small) jackpot probability. Second, individuals could derive larger anticipatory utility from larger prizes, and
anticipatory utility might be insensitive to probabilities. Third, because the second prize expected value doesn't vary much (its standard deviation is only one-third of the jackpot's), individuals might be inattentive to the second prize, as in Gabaix (2014) and related theories of focusing and salience (Bordalo, Gennaioli, and Shleifer 2013; Kőszegi and Szeidl 2013; Bushong, Schwartzstein, and Rabin 2021); these theories have more recently been suggested as attentional microfoundations for the probability weighting function (Bordalo, Gennaioli, and Shleifer, 2012; Dertwinkel-Kalt and Koster, 2020; Bordalo, Gennaioli, and Shleifer, Forthcoming). Fourth, high jackpots generate extra media attention that may temporarily increase sales. Fifth, while jackpot amounts are covered in the media and heavily promoted on billboards and in online ads, the estimated second prize for the next draw must be inferred from results of the previous drawing. ${ }^{21}$ Of course, the second prize might not be promoted precisely because individuals would be unresponsive.

Substitution. As described in Clotfelter and Cook (1990), Grote and Matheson (2011), and the literature cited therein, it is ambiguous whether draw games such as Mega Millions and Powerball are substitutes or complements for other lottery games. While their similarity suggests that they might be substitutes, the attention to high jackpots could also spill over to other games. Furthermore, high jackpots bring additional consumers into lottery outlets, where they can immediately buy other games. It could also be that high jackpots primarily bring in new consumers who wouldn't otherwise buy lottery tickets.

Appendix D. 1 shows that Mega Millions or Powerball jackpots have tightly estimated zero effects on sales of other games. Our confidence intervals from Appendix Table A3 rule out diversion ratios of more than about four percent to the other multi-state game, major state-level draw games, instant games, and other state-level games. This limited substitution is consistent with our "narrow bracketing" model assumption under which people consider each lottery in isolation rather than forming a utility-maximizing portfolio from a combination of different lotteries.

Short-run versus long-run responses. As in many other studies, we have a well-identified short-run elasticity, but our policy analysis requires a long-run elasticity. To address this concern, Appendix D. 2 shows that the jackpot elasticity is very similar when we aggregate over time and that lagged jackpot amounts have positive but relatively small effects on current sales. While not dispositive, this is consistent with the idea that the long-run elasticity is not much different from the short-run elasticity.

### 3.3 Price Elasticity

### 3.3.1 Estimation Strategy

We identify the price semi-elasticity $\bar{\zeta}_{p}$ from the change in ticket sales when Mega Millions and Powerball raised their prices from $\$ 1$ to $\$ 2$. To use these event studies, we must also consider the simultaneous format changes described in Table 1. Both games substantially increased their

[^9]jackpot amounts and expected values. However, as shown in Table 1, the expected return (i.e., expected value per dollar spent) from all lower prizes did not change very much. Because of this limited change, and because we saw insignificant elasticity with respect to second prize variation in Table 3, we assume that the lower prize changes did not affect demand.

Define $W_{j t}$ as an indicator for the 12 -month period around game $j$ 's price change, six months before and six months after. Our estimating equation modifies equation (8) into an event study estimator that identifies the change in sales within a 12-month event window around the price change:

$$
\begin{equation*}
\ln \bar{s}_{j t}=\bar{\zeta}_{p} p_{j t} W_{j t}+\beta_{1} W_{j t}+\beta_{2} \pi_{1 j t} w_{1 j t}+\xi_{j}+\epsilon_{j t} \tag{12}
\end{equation*}
$$

where $\xi_{j}$ is now simply an event fixed effect. The jackpot expected value $\pi_{1 j t} w_{1 j t}$ control increases precision and controls for the simultaneous format change. Because lagged sales and jackpots are also affected by the price change, we do not control for $\pi_{1 j t} \bar{Z}_{1 j t}$, and we do not instrument for the jackpot. Since our OLS and IV results were quite similar in Table 3, this is unlikely to affect our results. To identify the coefficients on the controls with the most relevant data, we limit the regression sample to the 36 -month period around the price change.

### 3.3.2 Estimation Results

The red lines in Figure 3 present ticket sales residual of the jackpot expected value, i.e. $\ln \bar{s}_{j t}-$ $\hat{\beta}_{2} \pi_{1 j t} w_{1 j t}$, in the 36 -month event study window for the game whose price changed. Panel (a) presents the Powerball price change in January 2012, while Panel (b) presents the Mega Millions price change in November 2017. We recenter so that the average residual equals zero before the price change when the jackpot is within $\$ 10$ million of the reset value. In both event studies, sales drop by about $50 \log$ points immediately after the price increase.

The other lines on Figure 3 present ticket sales for the other multi-state game, major state draw games, and all other games, respectively. To reduce noise, the other multi-state game $-j$ 's sales are residual of that game's jackpot expected value control $\hat{\beta}_{2} \pi_{1-j t} w_{1-j t}$. Some long-run trends become visible over the full 36 month period: both Mega Millions and Powerball sales gradually decline after the other game's price increases, and other game sales (mostly instant games) gradually increase. However, the effect on own-game sales is much larger and more immediate than these gradual trends for other games, suggesting that the other games' trends are unrelated to the price change.

Table 4 presents estimates of equation (12). Column 1 presents our primary estimates pooling the two event studies, while columns 2 and 3 consider each event study separately. Consistent with Figure 3, the estimates suggest that the price changes decreased demand by 43 to $56 \log$ points.

While we model demand as a function of the jackpot expected value, it could be that people pay attention only to the jackpot amount and not the probability (Cook and Clotfelter 1993). From Table 1, we can calculate that the Mega Millions jackpot probability decreased by 14 percent when the price increased, while the Powerball jackpot probability increased by 12 percent. If demand
responds to the jackpot amount instead of the expected value, our price elasticity estimates would be biased, although in opposite directions for the two games. Column 4 shows that the pooled estimate changes little when we control for the jackpot amount.

Substitution. Consistent with the graphical evidence in Figure 3, Appendix Table A5 shows statistically zero substitution to other games after Mega Millions and Powerball increase prices. However, the estimates are not precise enough to rule out economically significant diversion ratios.

Placebo tests. As shown in Table 1, there were also two format changes during our sample period that did not involve price changes. We can use these as placebo tests: if our price elasticity estimates are unbiased, these format changes should not affect demand. Appendix Table A8 presents estimates of equation (12) for these two other format changes, where we substitute $p_{j t}$ for a post-format change indicator variable and again limit to the 36 months around the format change. These format changes have statistically zero effect on demand after controlling for the jackpot level, and the confidence intervals suggest that any confounding effects are small relative to the price change coefficients in Table $4 .{ }^{22}$

Summary. Lottery purchases are (i) highly elastic to jackpot variation, (ii) unresponsive to variation in smaller prizes, and (iii) less responsive to a $\$ 1$ price change than to a $\$ 1$ jackpot expected value change. The third result suggests that jointly raising price and jackpot expected value increases demand, consistent with the trends toward larger jackpots and higher ticket prices. Taken together, these three results are consistent with a probability weighting function that weights the jackpot higher than its objective probability and places little weight on smaller prizes. In the next section, we consider the extent to which these lottery purchasing patterns might be driven by innumeracy, confusion, or other types of mistakes.

## 4 Individual-Level Demand

### 4.1 AmeriSpeak Survey

This section provides evidence on how lottery expenditures vary with income and proxies for behavioral biases. We use a new survey that we designed to measure lottery spending and proxies for preferences and biases potentially related to lottery purchases. The survey was fielded on AmeriSpeak, a high-quality survey panel managed by the National Opinion Research Corporation (NORC). Unlike internet panels that allow anyone to opt in, AmeriSpeak is a probability sample that includes only U.S. households that have been randomly selected (and heavily incentivized) to participate. This helps to reduce sample selection biases that can make surveys unrepresentative on unobserved characteristics. The average spending estimates in Section 4.2 are weighted for national representativeness on age, sex, race, education, geography, and other key characteristics using sam-

[^10]ple weights provided by NORC. The bias proxy regressions in Section 4.3 are left unweighted to maximize precision, although the results with sample weights are similar.

The survey was fielded in April 2020, and a follow-up survey was fielded in April 2021. 3,013 people completed the 2020 survey, of whom 2,879 passed basic data quality checks. Table 5 presents descriptive statistics. Panel (a) presents panelist demographics, Panel (b) presents survey questions on spending and income effects, and Panel (c) presents questions that proxy for preferences and bias. Sample sizes differ on individual questions due to item non-response. Appendix E. 1 presents the text of the survey questions, which we summarize here.

Spending and income effects. Monthly lottery spending is the response to the question, "How many dollars did you spend in total on lottery tickets in an average month in 2019?" We asked panelists to "please include Mega Millions, Powerball, and other lotto/prize drawings, instant/scratch-off games, and any other lottery game offered by your state lottery." To ensure that large expenditures were correctly reported, any person who reported more than $\$ 500$ monthly lottery spending was asked to explicitly confirm or update their response.

To measure income effects, the survey asked people to report the percent change in their household income and lottery spending in 2019 compared to 2018 (income change and spending change), as well as how much they think their lottery spending would change "if you got a raise and your income doubled" (self-reported income effect).

Preference proxies. We construct three proxies of preferences for playing the lottery. First, we proxy for risk aversion using two questions: "In general, how willing or unwilling are you to take risks?" and a second question measuring aversion to financial risks when saving or investing money. Our risk aversion preference proxy is the average of these two measures after standardizing each to have a standard deviation equal to one. Second, our lottery seems fun preference proxy is the extent to which people agree or disagree that "For me, playing the lottery seems fun." Third, our enjoy thinking about winning preference proxy, which is intended to measure anticipatory utility, is the extent to which people agree or disagree that "I enjoy thinking about how life would be if I won the lottery."

We designed the survey questions to allow us to construct proxies for six biases that might be related to lottery purchases.

Self-control problems. Self-control problems might affect lottery purchases if the enjoyment of playing is in the present, while the cost of buying the ticket (reduced consumption of other goods) is incurred later. To measure perceived self-control problems, the survey said, "It can be hard to exercise self-control, and some people feel that there are things they do too much or too little - for example, exercise, save money, or eat junk food. Do you feel like you play the lottery too little, too much, or the right amount?" Our self-control problems bias proxy is the response to that question, on a seven-point scale from "far too little" (coded as -3 ) to "the right amount" (coded as 0) to "far too much" (coded as +3 ).

Financial illiteracy. Financial illiteracy and innumeracy might affect lottery purchases by making it harder to evaluate risky prospects and correctly perceive small probabilities. Financial
literacy is the share of correct answers on five standard questions from Lusardi and Mitchell (2014), and financial numeracy is the share of correct answers on three numeracy questions from Banks and Oldfield (2007). Our financial illiteracy bias proxy is the share of incorrect answers across all eight questions.

Statistical mistakes. Poor statistical reasoning might similarly make it harder to evaluate risky prospects and correctly perceive small probabilities. The survey included three measures of statistical reasoning. First, we measured the Gambler's Fallacy (Jarvik 1951; Tversky and Kahneman 1971; Rabin 2002) by eliciting beliefs about the probability that an unbiased coin lands heads after three different sequences of heads and tails. The true probability is of course 50 percent. Gambler's Fallacy is the share of answers that differ from 50 percent; Table 5 reports that people gave some other answer 29 percent of the time. Second, we measured non-belief in the Law of Large Numbers (Benjamin, Rabin, and Raymond 2016; Benjamin, Moore, and Rabin 2018) by asking the probability that out of 1000 coin flips, the number of heads would be between 481 and 519 (correct answer $=0.78$ ), 450 and 550 (correct answer $=0.9986$ ), and 400 and 600 (correct answer $=0.9999$ ). Non-belief in the Law of Large Numbers is the average absolute deviation from the correct answer. Third, we asked people to calculate the expected value of four simple example lotteries. Expected value miscalculation is the share of answers that are incorrect. To construct our statistical mistakes bias proxy, we standardize each of these three measures to have a standard deviation equal to one in the 2020 sample, take the average, standardize the average to have a standard deviation equal to one in 2020, and recenter so that zero is the best score in 2020.

Overconfidence. Overconfidence could increase lottery purchases by increasing people's perception of the chance of winning. The survey said, "Imagine you could keep buying whatever lottery tickets you want, over and over for a very long time. For every $\$ 1000$ you spend, how much do you think you would win back in prizes, on average?" The survey also asked people to report how much "the average lottery player" would win back. Overconfidence is the difference between own and average person expected winnings per $\$ 1$ spent.

Expected returns. Misunderstanding the expected returns for the average person might also affect lottery purchases. Expected returns is the response to the question, "Think about the total amount of money spent on lottery tickets nationwide. What percent do you think is given out in prizes?"

Predicted life satisfaction. As argued by Kahneman et al. (2006) and others, people may overestimate the effect of wealth on happiness, and such a bias would cause people to overestimate the utility gains from winning a lottery. Using random variation in lottery winnings conditional on winning some amount, Lindqvist, Ostling, and Cesarini (2020) estimate that the effect of an additional $\$ 100,000$ on life satisfaction (measured on a $0-10$ scale) is 0.071 points. The survey told panelists about the Lindqvist, Ostling, and Cesarini (2020) study, informed them that the sample average life satisfaction was 7.21 out of 10 , and asked them to predict the effect of an additional $\$ 100,000$ on life satisfaction. Predicted life satisfaction is the response to that question.

2021 follow-up survey. We fielded the April 2021 follow-up survey to help address measure-
ment error. NORC invited everyone who had completed the 2020 survey to participate. 2,186 people responded, representing a normal follow-up response rate for AmeriSpeak, of whom 2,124 passed our data quality checks. The 2021 survey asked the same preference and bias proxy questions as in 2020. It also re-elicited monthly lottery spending in 2019 for 104 people who reported outlying values in 2020; we use this to confirm or winsorize the outlying responses. ${ }^{23}$

### 4.2 Lottery Spending by Income

The spending distribution is highly skewed: in our survey data, the top 10 percent of spenders account for 56 percent of spending, while over 40 percent of people report no spending at all; see Appendix Figure A6. This skewness reduces the precision of our estimates and underscores the importance of our efforts to validate outlying self-reports of monthly spending. The average spending of $\$ 15$ per month multiplied by 255 million American adults gives $\$ 47$ billion, which is smaller than the $\$ 87$ billion total nationwide sales reported in Figure A1. Clotfelter et al. (1999, Table 6) also found that survey responses understate total nationwide sales. This understatement and other forms of measurement error would bias our conclusions only if correlated with income or bias proxies. As a suggestive test, we find no evidence that income or bias proxies are correlated with the change in 2021 vs. 2020 reports of monthly lottery spending; see Appendix Table A13.

Figure 4 presents average monthly lottery spending by income. People with household income above $\$ 100,000$ spend an average of $\$ 13$ per month on the lottery, while people with household income under $\$ 50,000$ spend an average of $\$ 17$, or 29 percent more. ${ }^{24}$ The cross-sectional income elasticity of lottery spending (from a regression of $\ln (1+$ spending) on $\ln ($ income $)$ ) is -0.111 . The share of people with non-zero spending also declines slightly with income; see Appendix Figure A7. Clotfelter et al. (1999, Table 10) also found that lower-income households spend more on lotteries, although their point estimates suggest a steeper decline in spending by income as of 1998. Golosov et al. (2021) find that U.S. lottery winners have wage earnings, employment status, and age similar to the average tax filer, although lottery winners are more likely to be single.

Proposition C. 1 in Appendix C distinguishes two reasons why lottery spending might vary with income: the causal effect of income and preference heterogeneity that is correlated with income. The survey offers two ways to measure causal income effects. First, regressing spending change on income change suggests a causal income elasticity of 0.194; see Appendix Table A11 for formal regression results. This should be interpreted cautiously because changes in life circumstances correlated with income changes might also change lottery consumption preferences. Second, the average of self-reported income effect (the amount by which people thought their lottery spending would change if their income doubled) is -1.4 percent, suggesting a causal income elasticity of

[^11]$d \ln s / d \ln z \approx-0.014 / \ln (2 / 1) \approx-0.02$. This should be interpreted cautiously because the question was hypothetical.

While the exact point estimates differ, both of these strategies are consistent in suggesting limited income effects, and they are both statistically less negative than the cross-sectional income elasticity of -0.111 illustrated in Figure 4.

### 4.3 Association Between Bias Proxies and Lottery Spending

This section presents the relationships between bias proxies and lottery spending. Although we control for confounders such as measures of preferences, caution is warranted in interpreting these relationships as causal.

Define $s_{i}$ as person $i$ 's monthly lottery spending, define $b_{i k}$ as person $i$ 's value of bias proxy $k$, and define $b_{k}^{V}$ as the benchmark value of $b_{k}$ for an unbiased consumer who does not have self-control problems, has high financial literacy and statistical reasoning ability, is not overconfident, and has correct beliefs about expected returns and the effect of lottery winnings on life satisfaction. Define the standardized bias proxy $\tilde{b}_{i k}:=\frac{b_{i k}-b_{k}^{V}}{S D\left(b_{i k}\right)}$ as the difference between person $i$ 's proxy $b_{i k}$ and the unbiased value $b_{k}^{V}$ in 2020 standard deviation units, and define $\tilde{\boldsymbol{b}}_{i}$ as a vector of the six standardized bias proxies. All bias proxies are signed so that a more positive value should cause more lottery consumption. Finally, define $\boldsymbol{x}_{i}$ as a vector of controls, including the three preference proxies (risk aversion, lottery seems fun, and enjoy thinking about winning), the demographic characteristics presented in Panel (a) of Table 5, and state fixed effects.

Test-retest reliability. To summarize test-retest reliability, Figure 5 presents binned scatter plots of (unstandardized) bias proxies $b_{i k}$ elicited in 2021 vs. 2020. The area of each circle is proportional to the share of observations in each bin. For overconfidence, the large mass at zero reflects the fact that 65 percent of people expect to win the same amount as the average lottery player, and there is close to zero correlation in relative optimism or pessimism across years. For the other five bias proxies, however, the correlations range from 0.34 to 0.75 . For comparison, Stango and Zinman (2021, Table 3) find within-person rank correlations of 0.04 to 0.59 for bias proxies similar to ours in surveys separated by three years, and Chapman et al. (2020, Table 2) find correlations of 0.30 to 0.96 between "econographic" preference measures elicited twice on the same survey.

Descriptive correlations. Figure 6 presents binned scatter plots of (unstandardized) bias proxies $b_{i k}$ against the natural log of $1+$ monthly lottery spending, using the 2020 survey data. We use natural logs because spending is skewed and it is natural to think of bias entering multiplicatively, as in the examples from Section 2.3.1, and we add 1 to spending to be able to include zero-spending observations. Our results in Table 6 below are similar when using spending levels or the inverse hyperbolic sine of spending. In each of the six panels, a vertical line indicates the unbiased benchmark $b_{k}^{V}$.

For self-control problems, the unbiased benchmark $b_{k}^{V}$ is playing the lottery "the right amount" instead of "too little" or "too much." The relationship between lottery spending and self-control
problems is not as close to linear as with the other five bias proxies. To the left of zero, a stronger feeling that one plays the lottery "too little" is not clearly associated with spending. We thus recode negative values as 0 for the regressions and predictions described below. To the right of zero, a stronger feeling that one plays "too much" is positively associated with spending, suggesting that self-control problems might contribute to overconsumption. This contribution may be limited, however, because the circle sizes indicate that 71 percent of people report playing "the right amount," and more people report playing "too little" than "too much."

For financial illiteracy and statistical mistakes, we define the unbiased benchmarks $b_{k}^{V}$ as the best scores in the sample. For financial illiteracy, this is answering all eight questions correctly, and statistical mistakes is constructed so that 0 is the best score in the sample. The best fit lines' upward slopes mean that people with higher financial illiteracy and more statistical mistakes spend more on lotteries. The figure suggests that this might contribute to overconsumption: many people score relatively poorly on these two scales, and people with the worst scores spend $100 \log$ points more on the lottery than people with the best scores.

For overconfidence, we define the unbiased benchmark $b_{k}^{V}$ as predicting no difference between one's own lottery winnings and the average person's lottery winnings. The figure suggests that overconfidence may not contribute much to overconsumption: there is little relationship between overconfidence and spending, and 65 percent of people reported the same expected earnings for themselves and the average player. The lack of relationship between overconfidence and spending is unsurprising given the low test-retest reliability.

For expected returns, the unbiased benchmark $b_{k}^{V}$ is 60 percent: the true share of state lottery ticket sales that are given out as prizes, using data reported in Appendix Figure A2. The best fit line's upward slope means that people who think the expected returns are higher spend more on lotteries. Most of the mass is to the left of $b_{k}^{V}$, and the average person believes that only 29 percent of lottery spending is returned to winners. This suggests that people might play more if they didn't underestimate the expected returns.

Finally, the unbiased benchmark $b_{k}^{V}$ for predicted life satisfaction is the actual effect from Lindqvist, Ostling, and Cesarini (2020): an additional $\$ 100,000$ of lottery winnings increased life satisfaction by 0.071 points on the $0-10$ scale. The slope of the best fit line suggests that predicting that additional winnings increase life satisfaction by 1 additional point on the $0-10$ scale is associated with spending about 4.3 log points more on lottery tickets.

Regression estimates. To test whether these relationships survive controls for demographics and preferences, we estimate the following regression:

$$
\begin{equation*}
\ln \left(s_{i}+1\right)=\boldsymbol{\tau} \tilde{\boldsymbol{b}}_{i}+\boldsymbol{\beta} \boldsymbol{x}_{i}+\epsilon_{i} . \tag{13}
\end{equation*}
$$

Table 6 presents results. Column 1 presents the full model, column 2 presents the regression without any controls, and columns 3 and 4 progressively add controls. Column 1 shows that four of the unconditional relationships from Figure 6 survive controls: self-control problems, financial illiteracy, statistical mistakes, and expected returns are all strongly conditionally associated with
lottery spending, while overconfidence and predicted life satisfaction are not. All the estimated $\hat{\tau}$ coefficients are positive.

Comparing columns 2 and 3 shows that the preference proxies explain a large share of the variation in lottery spending, increasing the $R^{2}$ from 0.16 to 0.36 . These controls also materially attenuate the $\boldsymbol{\tau}$ coefficients, which underscores the importance of having collected the preference control variables. Adding the demographic controls in column 4 increases the $R^{2}$ slightly and has limited effects on the $\boldsymbol{\tau}$ coefficients.

Measurement error can attenuate the relationships in columns 1-4. To address this, we use the Obviously Related Instrumental Variables (ORIV) approach of Gillen, Snowberg, and Yariv (2019): we estimate equation (13) in a stacked dataset with the $2021 \tilde{\boldsymbol{b}}_{i}$ and $\boldsymbol{x}_{i}$ below the $2020 \tilde{\boldsymbol{b}}_{i}$ and $\boldsymbol{x}_{i}$, instrumenting for the 2020 variables with their 2021 values and vice versa, while clustering standard errors by $i$. The ORIV approach is more efficient than an unstacked IV approach that instruments the 2020 variables with their 2021 values or vice versa, and it avoids the ambiguity that arises if those two unstacked estimates are different.

We drop overconfidence because the low test-retest reliability causes a weak instruments problem. After that drop, the first stage regressions involve highly statistically significant relationships between the resampled values of the same variable and limited correlations with other variables; see Appendix Table A12. The one exception is that financial illiteracy and statistical mistakes are moderately correlated.

Column 5 of Table 6 presents OLS estimates in the subsample that also responded in 2021; the coefficients change little relative to column 1. Column 6 presents the ORIV estimates using the same sample as column 5. The coefficient on self-control problems grows substantially, consistent with its lower test-retest reliability. The financial literacy coefficient also grows, helping make statistical mistakes insignificant. The coefficients on expected returns and predicted life satisfaction also become statistically insignificant. Interestingly, the coefficient on education shrinks substantially and becomes statistically insignificant, meaning that after correcting for measurement error, our bias proxies explain why higher-education people spend less on the lottery. Meanwhile, the Black and Hispanic indicators remain strongly positively associated with lottery spending, even after all other covariates are included.

Share of consumption attributable to bias. We can use the regression results to predict what lottery spending would be without systematic bias, i.e., if all individuals' bias proxies $b_{i k}$ equaled the unbiased benchmarks $b_{k}^{V}$. Define $\hat{s}^{V}$ as predicted consumption with $\tilde{b}_{i k}=0$. Equation (13) implies that $\ln \left(s_{i}+1\right)-\ln \left(\hat{s}_{i}^{V}+1\right)=\hat{\boldsymbol{\tau}} \tilde{\boldsymbol{b}}_{i}$, and thus $\hat{s}_{i}^{V}=\frac{s_{i}+1}{\exp \left(\hat{\boldsymbol{\tau}} \tilde{\boldsymbol{b}}_{i}\right)}-1$. We winsorize at $\hat{s}_{i}^{V} \geq 0$, and we fix $\hat{s}_{i}^{V}=0$ for people with zero spending $\left(s_{i}=0\right)$. Using the OLS $\hat{\boldsymbol{\tau}}$ in column 1 of Table 6 , the share of consumption statistically attributable to bias is $\frac{\sum_{i}\left(s_{i}-\hat{s}_{i}^{V}\right)}{\sum_{i} s_{i}} \approx 43$ percent.

We can also compute the share of consumption attributable to each individual bias $k$ by constructing $\hat{s}_{i}^{V}$ with $\exp \left(\hat{\tau}_{k} \tilde{b}_{i k}\right)$ instead of $\exp \left(\hat{\boldsymbol{\tau}} \tilde{\boldsymbol{b}}_{i}\right)$. Figure 7 presents estimates using the OLS $\hat{\tau}_{k}$. For self-control problems, financial literacy, and statistical mistakes, the average $\tilde{b}_{i k}>0$ and $\hat{\tau}_{k}$ is positive, so Figure 7 correspondingly attributes increased lottery spending to these biases. The
$\hat{\tau}_{k}$ coefficients for overconfidence and predicted life satisfaction are both very close to zero, so the figure attributes little spending to those biases. Since the average person underestimates expected lottery returns, the average $\tilde{b}_{i k}<0$ for expected returns, and the figure attributes about a 10 percent spending reduction to this.

As we will show in Section 6.1, optimal policy depends on whether lower-income people are more or less biased. Figure 8 presents binned scatter plots of each bias proxy by income. Self-control problems is the only bias proxy that becomes more positive with income: higher-income people are more likely to report that they play the lottery "too much." Financial illiteracy declines strongly with income: people with household incomes under $\$ 20,000$ incorrectly answered 44 percent of our eight financial literacy and financial numeracy questions, while people with household incomes over $\$ 100,000$ incorrectly answered only 17 percent. Similarly, people with household incomes under $\$ 20,000$ scored about 0.7 standard deviations worse on our statistical mistakes questions than people with household incomes over $\$ 100,000$. Overconfidence, expected returns, and predicted life satisfaction differ little across income groups.

Using these differences in bias proxies by income, we construct the share of consumption attributable to bias separately for each income group. Figure 9 shows that this share declines moderately by income, from 46 percent for people with household incomes under $\$ 50,000$ to 40 percent for people with household incomes over $\$ 100,000$.

We can also adjust these two figures for measurement error by using the instrumental variables estimates from column 6 of Table 6; see Appendix Figures A9 and A10. Consistent with the results in Table 6, the shares of consumption attributable to self-control problems and financial illiteracy grow, while the shares attributable to the other variables attenuate. The estimates are less precise, but they continue to suggest that a larger share of consumption is attributable to bias for lower-income people. On average across all incomes, the share of consumption attributable to bias increases slightly to 47 percent.

## 5 Structural Model Calibration

In this section, we use the reduced-form moments from Sections 3 and 4 to calibrate a model of lottery demand and bias. In Section 6, we use this model for policy evaluation.

### 5.1 Setup and Assumptions

We now calibrate the model from Section 2.1, as specified in equations (1)-(7).
In Section 2.3.1, we showed how misperceived probabilities, present focus, and some other biases entered utility by multiplying $m(w)$. In line with those examples, we assume that a share $\chi(\theta)$ of the difference $\Phi\left(\pi_{k} ; \theta\right)-\pi_{k}$ between the decision weight and the objective probability is due to bias, and the remaining share is due to normative factors such as anticipatory utility. Thus, normative
utility is

$$
\begin{equation*}
v\left(\boldsymbol{a} ; \theta, \varepsilon_{t}\right)=u\left(\boldsymbol{a} ; \theta, \varepsilon_{t}\right)-\underbrace{\chi(\theta) \sum_{k}\left(\Phi\left(\pi_{k} ; \theta\right)-\pi_{k}\right) m\left(w_{k} ; \theta\right)}_{\gamma(\boldsymbol{a} ; \theta)} . \tag{14}
\end{equation*}
$$

We now describe our functional form choices for the components of equations (3) and (14). For legibility, we sometimes suppress dependence on $\theta$.

Utility of prizes. We assume that $m\left(w_{k}\right)$ arises from a concave constant relative risk aversion (CRRA) utility function over continuation wealth. This rules out the possibility that lottery demand is driven by convexity in the value function, as proposed by Friedman and Savage (1948). The theory that utility might be convex at high amounts of money is largely inconsistent with the bulk of the evidence, as reviewed in Appendix F.3.

Appendix F. 2 presents the formal derivation of $m$ arising from continuation utility with a constant coefficient of relative risk aversion. Our baseline specification employs a CRRA parameter of 1 , the central estimate from Chetty (2006), corresponding to logarithmic utility over continuation wealth. In Appendix F. 3 we summarize evidence that CRRA parameters non-negligibly different from 1 generate unrealistic implications for consumers' willingness to pay for lottery tickets.

Decision weights. We utilize the "neo-additive" weighting function, axiomatized in Chateauneuf, Eichberger, and Grant (2007). This weighting function is discontinuous at the endpoints $\pi=0$ and $\pi=1$, and linear on the interval $(0,1)$, with intercept $b_{0}$ and slope $b_{1}$. As we show in Appendix F.1, other standard parameterizations of probability weighting functions (Tversky and Kahneman 1992; Goldstein and Einhorn 1987; Prelec 1998) cannot simultaneously fit the estimates of $\Phi\left(\pi_{1}\right)$ and $\Phi\left(\pi_{2}\right)$ implied by equation (5). These other parameterizations imply that the ratio of secondprize to jackpot expected value semi-elasticities must be significantly larger than what we estimate, and in fact larger than 1.

Distribution of taste shocks. We assume that $\theta$ and $\varepsilon$ are independently distributed. The distribution of $\varepsilon$ is then inferred from our estimates of the semi-elasticities at various levels of lottery demand. Note that in the utility specification from equation (4), any two lotteries with the same net-of-price certainty equivalent $\sum_{k} \Phi\left(\pi_{k}\right) m\left(w_{k}\right)-p$ generate identical demand. We assume that the semi-elasticity of demand with respect to this certainty equivalent is constant over the range of net certainty equivalents generated by the variation in our data. Appendix F. 2 provides further details on our procedure for calibrating the distribution of $\varepsilon$.

### 5.2 Model Calibration

We summarize the intuition behind the calibration here; Appendix F. 2 presents details.
We calibrate the model assuming a single, representative lottery game with price, prizes, and probabilities set to match the current values for Powerball from Table 1. We assume that all prizes are reduced by 30 percent to account for income taxes.

The positive model of demand is fully characterized by the weighting function parameters $b_{0}$
and $b_{1}$. We first calibrate a representative consumer model using our estimates of aggregate demand semi-elasticities from Section 3.

To estimate the bias share parameter $\chi$, we augment our observational data with an estimate of the average counterfactual level of lottery spending if consumers were unbiased, based on the survey results in Section 4.3. We assume that the share of consumption attributable to bias from Section 4.3 is the causal effect of bias, allowing us to compute the counterfactual normative level of consumption for each individual in our survey. This is a strong assumption, both because of the causal interpretation and because there may be other behavioral biases not captured by our bias proxies. Using equation (14), we then compute the bias share $\chi$ that would generate this normative level of consumption.

We allow for heterogeneity by exploiting the survey microdata on household income and lottery spending. We partition income into the three bins displayed in Figure 4: less than $\$ 50,000$, $\$ 50,000$ to $\$ 100,000$, and more than $\$ 100,000$. Within each income bin, we specify three types of individuals: those who consume zero lotteries, and among positive consumers, those with belowvs. above-median consumption. We assume non-consumers never purchase a lottery ticket in any counterfactual scenario. Among the remaining types, we draw the average level of lottery spending and counterfactual unbiased spending from our survey, and we assume our semi-elasticity estimates are homogeneous across types. This allows us to compute type-specific parameters.

### 5.3 Parameter Estimates

Table 7 presents the estimated model parameters $b_{0}, b_{1}$, and $\chi$ in both the representative agent and heterogeneous specifications. The probability weighting function intercept term $b_{0}$, though small in absolute terms, is large relative to the jackpot probability $\pi_{1}$, implying substantial overweighting of small probabilities. Approximately 30 percent of the wedge between decision weights and objective probabilities is attributable to bias, with above-median lottery consumers being the most biased in the heterogeneous specification.

## 6 Implications for Welfare and Optimal Lottery Design

We focus our analysis on the optimal choice of two attributes to which consumer responses are best identified in our data: ticket price and jackpot size. We begin by deriving and implementing sufficient statistics formulas for optimal attributes. As is standard in applications of sufficient statistics approaches, we employ the simplifying approximation that the elasticities and other key statistics are unaffected by the policy changes we consider; Kleven (Forthcoming) would call this a "structural" approach. Second, we jointly compute the optimal price and jackpot using the structural model from the previous section. The structural model allows the key statistics to adjust in response to policy changes and also allows us to estimate welfare. When separately computing the optimal ticket price or jackpot (while holding the other fixed), the structural model delivers similar answers to our sufficient statistics formulas, which suggests that the estimates are
not sensitive to specific parametric assumptions.

### 6.1 Optimal Lottery Design: A Sufficient Statistics Approach

### 6.1.1 Government Objective Function

The policymaker selects $T, p$, and $\boldsymbol{a}$ to maximize normative utility, aggregated across individuals using type-specific Pareto weights $\mu(\theta, \varepsilon)$ :

$$
\begin{equation*}
\max _{T, p, \boldsymbol{a}}\left[\int_{\theta, \varepsilon} \mu(\theta, \varepsilon) V(c, s, \boldsymbol{a}, z ; \theta, \varepsilon) d F(\theta, \varepsilon)\right], \tag{15}
\end{equation*}
$$

subject to a government budget constraint,

$$
\begin{equation*}
\int_{\theta}(p s(\theta)+T(z(\theta))) d F_{\theta}(\theta)-C(\boldsymbol{a}, \bar{s}) \geq R \tag{16}
\end{equation*}
$$

and to individuals' maximization of $U$.
We let $\lambda$ denote the marginal value of public funds (i.e., the multiplier on the government budget constraint in equation (16) at the optimum), and we define $g(\theta, \varepsilon):=\mu(\theta, \varepsilon) U_{c}^{\prime} / \lambda$ to denote the weighted marginal utility from consumption for type $(\theta, \varepsilon)$. Following Saez (2002a) and others, we assume that this "social marginal welfare weight" $g(\theta, \varepsilon)$ is equal for all individuals with a given level of earnings (and thus, all who have a common type $\theta$ ) under the optimal tax system. We thus use $g(z)$ to denote social marginal welfare weights as a function of income. We impute these welfare weights based on our survey measure of pre-tax income using the pre- to post-tax income mapping from Piketty, Saez, and Zucman (2018). We follow Saez (2002b) in setting these weights proportional to $c(\theta)^{-\nu}$, with $\nu=1$ in our baseline specifications. Under the assumption of negligible income effects on labor supply, $\mathbb{E}[g(z)]=1$.

### 6.1.2 Sufficient Statistics

We proceed by characterizing the first-order conditions for the optimal choice of price and each (continuously variable) attribute $a \in \mathbb{R}^{+}$from the attribute vector $\boldsymbol{a}$; for the remainder of this section we therefore write all expressions as functions of $a$, implicitly holding fixed attributes other than $a$. The socially optimal choices of attributes and price depend on three types of sufficient statistics: elasticities, a money-metric measure of bias, and the "progressivity of bias correction." These statistics are endogenous to the policy choices, though we suppress those arguments for notational simplicity. Because we assume that social marginal welfare weights depend only on income, the relevant statistics are functions of income $z$, rather than of types $\theta$. We define $s(z)$ as the lottery demand aggregated across all $z$-earners.

Our results here are not specific to lotteries; they apply to other settings where the government can also regulate or directly control a good's attributes - for example, cigarette nicotine content or lightbulb energy efficiency. In fact, our results apply under more general specifications of the utility function than assumed in Section 2. The more general result in Proposition C. 1 in Appendix C
applies to an arbitrarily dynamic model with any number of choice occasions, it does not assume additive separability in $\varepsilon$, and it allows for income effects.

We define $\kappa(\theta)$ and $\kappa(z)$, respectively, to be the average willingness-to-pay (WTP) for a marginal increase in $a$ of $\theta$-types and $z$-earners. These statistics can be obtained from empirically-measurable semi-elasticities, as $\kappa(\theta)=-\zeta_{a}(\theta) / \zeta_{p}(\theta) \cdot s(\theta)$ and $\kappa(z)=\mathbb{E}[\kappa(\theta) \mid z(\theta)=z]$. We define $\bar{\kappa}:=\mathbb{E}[\kappa(\theta)]$ as the average WTP of all buyers of the lottery ticket.

Following ALT, and with some abuse of notation, we define $\gamma_{p}(z):=\frac{\mathbb{E}\left[\left.\gamma(\theta) \frac{d s(\theta)}{d p} \right\rvert\, z(\theta)=z\right]}{\mathbb{E}\left[\left.\frac{d s(\theta)}{d p} \right\rvert\, z(\theta)=z\right]}$ and $\bar{\gamma}_{p}:=\frac{\mathbb{E}\left[\gamma(\theta) \frac{d s(\theta)}{d p}\right]}{\mathbb{E}\left[\frac{d(\theta)}{d p}\right]}$ to be the average bias of $z$-earners and all individuals, respectively, who are marginal to a price change. We define $\gamma_{a}(z)$ and $\bar{\gamma}_{a}$ analogously over individuals marginal to an attribute change.

Because bias can depend on the attribute $a$, individuals' perceived utility from a change in $a$ may be biased. The bias in $z$-earners' valuation of a marginal increase in $a$ is $\rho(z):=$ $\mathbb{E}\left[\left.\frac{d}{d a} \gamma(a ; \theta) \cdot s(\theta) \right\rvert\, z(\theta)=z\right]$. We define $\bar{\rho}:=\mathbb{E}[\rho(z)]$ as the population average. By definition, $\bar{\kappa}-\bar{\rho}$ is the average impact on normative utility of increasing $a$, measured in dollars.

Because our framework includes redistributive motives, the welfare effects of a change in consumption depend on whose consumption is changed. All else equal, the welfare change is more positive when more of the benefits of correcting bias accrue to low-income individuals. To capture these redistributive concerns, we define a statistic $\sigma_{p}:=\operatorname{Cov}\left[g(z), \frac{\gamma_{p}(z)}{\bar{\gamma}_{p}} \frac{\zeta_{p}(z) s(z)}{\zeta_{p} \bar{s}}\right]$, which we call the "progressivity of bias correction" from a price change. We define an analogous statistic for an attribute change: $\sigma_{a}:=\operatorname{Cov}\left[g(z), \frac{\gamma_{a}(z)}{\bar{\gamma}_{a}(z) s(z)} \frac{\zeta_{\bar{s}} \bar{s}}{\bar{\zeta}_{a}}\right]$. These statistics quantify the extent to which the benefits of bias correction accrue to low-income individuals, per unit change in $\bar{s}$.

### 6.1.3 Optimal Policy

Our main result can be stated in terms of intuitive mark-up formulas. This result is a special case of Proposition C. 1 in Appendix C.

Proposition 1. If $p$ and $a$ are interior optima, then

For intuition, notice first that equation (17) is a special case of equation (18) if we think of price $p$ as one of the attributes. Because we assume that people do not misperceive prices, $\rho(z) \equiv 0$.

Moreover, $\kappa(z)=s(z)$, as the mechanical effect on consumer surplus of lowering the price by $d p$ is simply $d p s(z)$. Finally, $\frac{\partial C}{\partial p}=\bar{s}$, as the mechanical revenue effect of lowering price by $d p$ is simply a decrease $d p \bar{s}$ in revenue. Thus, $\bar{\kappa}-\bar{\rho}-\frac{\partial C}{\partial a}=0$ when $a=p$, and equation (18) reduces to equation (17).

Equation (18) is a generalization of Ramsey-style optimal commodity tax formulas. The wedge $p-\frac{\partial C}{\partial \bar{s}}$ is analogous to a per-unit tax on a lottery ticket. This "tax" must equal the difference between two terms. The first term, $\bar{\gamma}_{a}\left(1+\sigma_{a}\right)$, is a Pigouvian correction that corresponds to the social marginal benefits of decreasing lottery consumption. This term is increasing in $\sigma_{a}$, the progressivity of bias correction. The second term is the mechanical effect on consumer welfare net of revenues; it consists of two parts. The first part, $\bar{\kappa}-\bar{\rho}-\frac{\partial C}{\partial a}$, is the direct effect on consumer surplus net of revenues. The second part is the extent to which the effects on consumer surplus are distributed in a regressive or progressive way. In the case of a price change, this is simply the extent to which lower-income individuals buy more lottery tickets, and are thus more impacted by the price change. In the case of an attribute change more generally, this is the extent to which lower-income individuals have a higher (normative) WTP for this attribute change.

To further flesh out intuition, Appendix B presents special cases of Proposition 1, including (i) no bias and homogeneous preferences, (ii) no bias and heterogeneous preferences, (iii) homogeneous bias and preferences, and (iv) the revenue-maximizing lottery structure.

### 6.1.4 Generalizations and Connection to the Atkinson-Stiglitz Theorem

Atkinson-Stiglitz. The generalization in Proposition C. 1 in Appendix C allows income effects on lottery consumption. Of particular note is the connection to the theorem of Atkinson and Stiglitz (1976) and related work, which states that under homogeneous preferences and weak separability between consumption and labor, it is efficient to address redistributive goals through income taxation, not through differential commodity taxes. The logic of the Atkinson-Stiglitz Theorem does not apply to the Proposition 1, because our utility specification implies that all variation in $s(z)$ across the income distribution is driven by how tastes for gambling vary with earnings ability.

At the same time, a version of the Atkinson-Stiglitz Theorem does hold in our setting under assumptions that parallel theirs. A corollary of Proposition C. 1 is that when all variation in $s(z)$ across $z$ is driven by causal income effects, rather than heterogeneous preferences, the optimal values of $p$ and $a$ are determined only by $\bar{\gamma}_{p}\left(1+\sigma_{p}\right)$ and $\bar{\gamma}_{a}\left(1+\sigma_{a}\right)$, respectively. Under more general assumptions that allow both preference heterogeneity and causal income effects on lottery consumption, Proposition C. 1 shows that Proposition 1 remains mostly in force, except with $s(z)$ and $\kappa(z)$ replaced by $s_{\text {pref }}(z)$ and $\kappa_{\text {pref }}(z)$, which we define as the amount of cross-sectional variation in $s$ and $\kappa$ that cannot be explained by causal income effects.

Substitution to other gambling. Our baseline formulas assume that when individuals substitute away from the lottery in question, they do not substitute to other goods over which they might misoptimize, such as other lotteries or casino gambling. This simplifying assumption is motivated by our empirical result in Section 3 that there is limited substitution between state-run
lottery games, and by Kearney's (2005b) result that the introduction of state-run lotteries has no detectable effect on non-state-sponsored gambling. ${ }^{25}$ That said, introducing substitution to other lotteries in Proposition 1 is easy. Substitution to other state-run lotteries subject to the same bias would not affect Proposition 1, provided the elasticities reflect total state-run lottery demand. Building on ALT's formulas, a diversion ratio $r$ to other (non-state-run) gambling subject to the same bias implies that $\bar{\gamma}$ should be replaced by $(1-r) \bar{\gamma}$ in Proposition 1.

### 6.2 Optimal Policy from Sufficient Statistics Formulas

From Proposition 1, the sufficient statistics formula for the lottery ticket's markup over the marginal cost is $\bar{\gamma}(1+\sigma)-\operatorname{Cov}[g(\theta), s(\theta)] /\left|\bar{\zeta}_{p}\right| \overline{\mid}$. We can rearrange this to give an expression for the optimal lottery ticket price, which we explain further below:

$$
\begin{align*}
p & =\bar{\gamma}(1+\sigma)-\frac{\operatorname{Cov}[g(\theta), s(\theta)]}{\left|\bar{\zeta}_{p}\right| \bar{s}}+\sum_{k} \pi_{k} w_{k}+o  \tag{19}\\
& =1.17(1+0.11)-\frac{0.34}{|-0.4953| \times 7.64}+0.46+0.2  \tag{20}\\
& \approx 2.04 \tag{21}
\end{align*}
$$

Equation (20) substitutes in our estimate of each statistic. The first term is the bias correction term. We approximate money-metric bias $\gamma(\theta)$ using our survey data from Section 4.3: we divide each individual's amount of consumption attributable to bias by our estimated price semi-elasticity of demand, $\gamma_{i}=\hat{\boldsymbol{\tau}} \tilde{\boldsymbol{b}}_{i} /\left|\bar{\zeta}_{p}\right|$. For example, if bias increases consumption by 60 percent and a $\$ 1$ price increase reduces consumption by 30 percent, then bias would be $\gamma_{i}=60 \% /(30 \% / \$ 1)=\$ 2$. ALT formalizes this approach. This implies an average marginal bias of $\bar{\gamma} \approx \$ 1.17$ per ticket. ${ }^{26}$ To estimate the progressivity of bias correction $\sigma$, we combine the bias estimates with each individual's welfare weight $g(z)$. This gives $\sigma=0.11$, reflecting the higher bias among lower-income individuals.

The second term, $-\frac{\operatorname{Cov}[g(\theta), s(\theta)]}{\left|\zeta_{p}\right| \bar{s}}$, quantifies the optimal price reduction due to redistributive concerns. Since lottery spending doesn't decline much with income, this covariance is small. We can use this term to quantify the importance of causal income effects, as discussed in Section 6.1.4. Our two estimates of causal income elasticities from Appendix Table A11 suggest that either 82 percent or 275 percent of the downward slope in lottery spending across incomes is attributable to income-correlated preference heterogeneity, rather than causal income effects. ${ }^{27}$ Assuming that

[^12]these percentages are constant across the income distribution, Proposition C. 1 in Appendix C shows that to account for causal income effects, we should rescale $\operatorname{Cov}[g(\theta), s(\theta)]$ by either 0.82 or 2.75 in the optimal price formula. But because $\operatorname{Cov}[g(z), s(z)]$ is small, these adjustments would change the optimal price by less than $\$ 0.16$.

The final two terms in equation (19) reflect the lottery ticket's marginal cost. $\sum_{k} \pi_{k} w_{k}$ is the net-of-income-tax expected payout of prizes, which we set to reflect the current Powerball format in Table 1, reduced by the 30 percent income tax rate. $o$ is the overhead cost, which we assume is 10 percent of the current $\$ 2$ price, following the discussion in Section 1.

This calculation gives an optimal price of $\$ 2.04$, which is close to the current Powerball ticket price of $\$ 2$. The estimate of money-metric bias matters a lot for the optimal price: the corrective term $\bar{\gamma}(1+\sigma)$ is about $\$ 1.30$, and so the optimal ticket price would be significantly lower in the absence of bias.

Similarly, we can use Proposition 1 to estimate the optimal jackpot amount. We rearrange equation (18) to isolate the jackpot expected value from the summation on the left-hand side, and we use the fact that the average cost of a ticket rises one-for-one with a change in the jackpot expected value, implying $\frac{\partial C}{\partial a}=\bar{s}$, where $a=\pi_{1} w_{1}$ is the jackpot expected value. This gives the following result:

$$
\begin{align*}
\pi_{1} w_{1} & =-\bar{\gamma}\left(1+\sigma_{1}\right)+\frac{\bar{\kappa}-\bar{\rho}+\operatorname{Cov}[g(\theta), \kappa(\theta)-\rho(\theta)]-\bar{s}}{\bar{\zeta}_{1} \bar{s}}+p-\sum_{k=2}^{K} \pi_{k} w_{k}-o  \tag{22}\\
& =-1.17(1+0.11)+\frac{38.06-22.24+(-0.73)-7.64}{2.47 \times 7.64}+2-0.22-0.2  \tag{23}\\
& \approx 0.68 . \tag{24}
\end{align*}
$$

The key new terms for this calculation are $\bar{\kappa}$, the average monthly willingness-to-pay to increase the jackpot expected value by $\$ 1$ (through $w_{1}$ rather than $\pi_{1}$ ), and $\bar{\rho}$, the average portion of that WTP that is driven by bias. From Section 6.1.2, $\bar{\kappa}=-\bar{\zeta}_{1} / \bar{\zeta}_{p} \cdot \bar{s}=\$ 38.06$ per month. To estimate $\bar{\rho}$, we assume that the normative share of WTP for a higher jackpot is equal to $(p-\gamma) / p$, i.e., the normative share of WTP for a lottery ticket for the marginal consumer, which is equal to the ticket price. This average normative share among lottery purchasers is 0.74 , and $\bar{\rho} \approx \$ 22.24$ per month. $\bar{\gamma}$ and $\sigma$ are computed as before, but weighting individuals by demand responses to jackpot expected value changes, rather than price changes. ${ }^{28} \bar{\zeta}_{1}$ is the net-of-tax jackpot semi-elasticity, computed as our estimate of the jackpot semi-elasticity from column 2 of Table 3a divided by one minus the income tax rate.
profile of spending in Figure 4. Thus our causal income elasticity of -0.02 accounts for 18 percent of the cross-sectional income elasticity ( -0.111 ) reported in Appendix Table A11, implying that the remaining 82 percent is attributable to preference heterogeneity. Our alternative causal income elasticity of 0.194 suggests that causal income effects have the opposite sign of the cross-sectional profile-i.e., lotteries are a normal good-implying that the preference for lotteries is declining steeply enough with income as to more than offset the positive causal income effects.
${ }^{28}$ Specifically, we assume a homogeneous prize semi-elasticity, so that $\bar{\gamma}=\frac{\mathbb{E}\left[\gamma_{i} \bar{\zeta}_{1} s_{i}\right]}{\zeta_{1} \bar{s}}$.

Using these values to compute equation (23), the optimal net-of-tax jackpot expected value is $\$ 0.68$, somewhat higher than the current average Powerball net-of-tax jackpot expected value of $\$ 0.24$.

### 6.3 Optimal Policy from Structural Model

We now study optimal policy using the heterogeneous structural model calibrated in Section 5.3. Figure 10 plots the estimated surplus per capita that each consumer type derives from each $\$ 100$ of per-capita spending on the status quo representative lottery. Total perceived surplus is decomposed into lottery revenues, which we assume are distributed evenly across individuals, and perceived consumer surplus, reflecting consumers' WTP for lotteries. Behavioral biases reduce actual (normative) consumer surplus below perceived surplus, to the point that actual surplus may be negative. We plot actual surplus both for our baseline bias estimates and under the alternative assumption that bias share $\chi(\theta)$ is 2 times as large. We find substantial heterogeneity in surplus conditional on income, with heavier consumers deriving far more perceived utility from lotteries, while also incurring larger bias costs. Total surplus is positive in our baseline specification, but declines when bias is scaled up from our estimates and is negative for the heaviest consumers in the " 2 x bias" specification. The threshold at which total surplus from the status quo becomes negative is 2.16 times our baseline bias estimates.

Figure 11 presents our baseline optimal policy estimates. Both panels display plots of social welfare (relative to a counterfactual with no lottery) across a range of prices (Panel a) and jackpots (Panel b). These figures report results for three different assumptions about bias. The dashed green lines report results under the assumption that all observed demand is fully normatively justified $(\chi(\theta) \equiv 0)$. Unsurprisingly, lotteries generate substantial surplus in this case. The optimal "effective tax rate" - by which we mean the implicit tax rate (share of ticket price that is a markup over marginal cost) after reducing marginal cost to account for revenues recovered by a 30 percent income tax-is lower than in the status quo because there is no corrective benefit from reducing consumption. In fact, redistributive considerations favor a small subsidy, because poorer consumers buy more lottery tickets. Relative to the current Powerball design, decreasing the price (Panel a) or increasing the jackpot (Panel b) would increase welfare in this case.

The solid blue lines in Figure 11 show the welfare gain from the lottery given our empirical estimates of bias. The lottery generates lower welfare gains, although welfare remains positive under the status quo price and jackpot. Holding prizes fixed, the status quo price is close to optimal, although the welfare effect of changing the price is small. Holding price fixed, the optimal jackpot is substantially higher than the status quo, reflecting the normative utility that consumers derive from larger jackpots. Both of these estimates are reassuringly consistent with our sufficient statistics formulas, suggesting that the welfare conclusions are not sensitive to the assumptions in either approach.

Finally, the dot-dashed red lines plot the welfare gains assuming that the bias share $\chi(\theta)$ is 2 times our empirical estimates. In this case, the welfare gains from the status quo lottery are barely
positive.
We also jointly solve for the optimal combination of ticket price and jackpot size. Table 8 presents results. The third column presents the effective tax rate. Row 1 presents estimates under our baseline assumptions. The optimal price is $\$ 2.88$, the optimal jackpot expected value is $\$ 0.78$, and the optimal effective tax rate is 58 percent. This optimal effective tax rate is close to the current Mega Millions and Powerball effective rates of about 60 percent. ${ }^{29}$ Our model's predicted optimal price and jackpot are both somewhat higher than in the status quo, however. This is because our estimates imply substantial normative utility from higher jackpots, so a revenueneutral perturbation that jointly raises jackpots and prices above status quo levels would increase consumer surplus.

The remaining rows present results under alternative assumptions. Rows 2 and 3 report alternative bias assumptions. When consumers are completely unbiased (row 2), the optimal effective tax rate is close to zero, and in fact is slightly negative due to regressivity concerns. When consumers are 2 times as biased as in the baseline (row 3), the optimal effective tax rate increases. For sufficiently high levels of bias, welfare would be maximized by eliminating the lottery altogether.

Rows 4 and 5 consider alternative assumptions about the curvature of utility over wealth. Our baseline assumption was a CRRA parameter of 1 (Chetty 2006), and we consider alternative values of 0.9 and 1.5. These values cover the CRRA range across which the model predicts reasonable willingness to pay for lottery tickets. ${ }^{30}$

Rows 6 and 7 consider weaker and stronger redistributive preferences, with welfare weights proportional to $c(\theta)^{-0.25}$ and $c(\theta)^{-4}$, rather than $c(\theta)^{-1}$ as in our baseline, where $c(\theta)$ denotes type $\theta$ 's net-of-tax income. Rows 8 and 9 consider alternative possibilities for the second prize semi-elasticity. Motivated by the fact that our baseline estimate for $\hat{\bar{\zeta}}_{2}$ in Table 3 is statistically indistinguishable from zero, row 8 assumes that $\bar{\zeta}_{2}=0$. To account for the possibility that our statistically insignificant estimate for $\hat{\bar{\zeta}}_{2}$ reflects inattention to variation in the California second prize, rather than a low weight on those prizes' expected value, row 9 assumes a "structural" second-prize semi-elasticity of $\bar{\zeta}_{2}=\bar{\zeta}_{1}$ as a conservative upper bound. Row 10 assumes $b_{1}=1$ for all consumers, implying that decision weights depart from expected utility maximization only for the jackpot and, correspondingly, only the jackpot decision weight is subject to bias. Row 11 examines the case where the jackpot varies over time, cycling over 10 values corresponding to the average value of each decile of Powerball jackpots in our data. In this case, jackpot expected value is adjusted by rescaling all values of the jackpot proportionally. Row 12 reports results when we estimate bias with the ORIV measurement error correction. The results in rows 6-12 are all broadly similar to row 1 .

[^13]The remaining rows explore the role of differences in bias and consumption across the income distribution. Rows 13 and 14 report results when the bias share $\chi(\theta)$ is assumed to be the same at all incomes, while still varying across the three consumption groups (row 13), or homogeneous across incomes and consumption groups (row 14). Row 13 reports results when we assume a steeper decline in expenditures across income groups, assuming consumption in the bottom (top) income bin is the highest (lowest) level in the 95 percent confidence intervals of consumption in each bin.

## 7 Conclusion

People have long debated whether state-run lotteries are a regressive "tax on people who are bad at math" or a win-win that generates both enjoyment and government revenues. In this paper, we provide a novel set of empirical results that are relevant for optimal policy. We find that aggregate demand responds more to a $\$ 1$ change in jackpot expected value than it does to a $\$ 1$ price change or to a $\$ 1$ change in second prize expected value, a result consistent with a particular form of probability weighting. In our new nationally-representative survey, lottery spending is correlated with proxies of behavioral bias such as innumeracy and poor statistical reasoning; our bias proxies account for 43 percent of lottery spending. Using these empirical moments, we calibrate a structural model of lottery demand and implement sufficient statistics formulas to study optimal policy.

We illustrate the relevance of our results in a theoretical model of optimal policy. In the model, current multi-state lotteries increase overall welfare, although the gains are unequally distributed: below-median spenders benefit from redistributed revenues, while above-median spenders receive the least - and not necessarily positive - surplus. This suggests that quantity restrictions such as monthly spending limits could increase welfare. The model's socially optimal implicit tax rate is slightly higher than the current Mega Millions and Powerball designs, which suggests that recent trends toward higher implicit taxes in those games may have increased welfare.

Our optimal policy and welfare results hinge on estimates of bias, which highlights the importance of our survey work but also motivates additional research in that area. There are many other caveats to our results, and we think of this paper as just a first step toward studying state-run lotteries through the lens of behavioral optimal taxation. Our theoretical and empirical techniques may also be more broadly useful for studying regulation of non-price attributes in the presence of behavioral bias.

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Table 1: Mega Millions and Powerball Prices and Prize Structures

| Start date | Mega Millions |  |  | Powerball |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | June 22, 2005 | October 19, 2013 | $\begin{gathered} \text { October } 28, \\ 2017 \end{gathered}$ | $\begin{gathered} \text { January } 7, \\ 2009 \end{gathered}$ | January 15, 2012 | $\begin{gathered} \text { October 7, } \\ 2015 \end{gathered}$ |
| Ticket price | \$1 | \$1 | \$2 | \$1 | \$2 | \$2 |
| Format | $5 / 56+1 / 46$ | $5 / 75+1 / 15$ | $5 / 70+1 / 25$ | $5 / 59+1 / 39$ | $5 / 59+1 / 35$ | $5 / 69+1 / 26$ |
| Jackpot (average) | \$34 million | $\$ 57$ million | \$102 million | \$39 million | \$66 million | \$101 million |
| Reset value | $\$ 7$ million | $\$ 9$ million | $\$ 24$ million | \$12 million | $\$ 24$ million | \$24 million |
| Probability | 1/175,711,536 | 1/258,890,850 | 1/302,575,350 | 1/195,249,054 | 1/175,223,510 | 1/292,201,338 |
| Expected value | \$0.19 | \$0.22 | \$0.34 | \$0.20 | \$0.38 | \$0.34 |
| Second prize | \$250,000 | \$1 million | \$1 million | \$200,000 | \$1 million | \$1 million |
| Probability | 1/3,904,701 | 1/18,492,204 | 1/12,607,306 | 1/5,138,133 | 1/5,153,633 | 1/11,688,054 |
| Expected value | \$0.064 | \$0.054 | \$0.079 | \$0.039 | \$0.19 | \$0.086 |
| Third prize | \$10,000 | \$5,000 | \$10,000 | \$10,000 | \$10,000 | \$50,000 |
| Probability | 1/689,065 | 1/739,688 | 1/931,001 | 1/723,145 | 1/648,976 | 1/913,129 |
| Expected value | \$0.015 | \$0.0068 | \$0.011 | \$0.014 | \$0.015 | \$0.055 |
| Fourth prize | \$150 | \$500 | \$500 | \$100 | \$100 | \$100 |
| Probability | 1/15,313 | 1/52,835 | 1/38,792 | 1/19,030 | 1/19,088 | 1/36,525 |
| Expected value | \$0.0098 | \$0.0095 | \$0.013 | \$0.0053 | \$0.0052 | \$0.0027 |
| Fifth prize | \$150 | \$50 | \$200 | \$100 | \$100 | \$100 |
| Probability | 1/13,781 | 1/10,720 | 1/14,547 | 1/13,644 | 1/12,245 | 1/14,494 |
| Expected value | \$0.011 | \$0.0047 | \$0.014 | \$0.0073 | \$0.0082 | \$0.0069 |
| Sixth prize | \$7 | \$5 | \$10 | \$7 | \$7 | \$7 |
| Probability | 1/306 | 1/766 | 1/606 | 1/359 | 1/360 | 1/580 |
| Expected value | \$0.023 | \$0.0065 | \$0.016 | \$0.019 | \$0.019 | \$0.012 |
| Seventh prize | \$10 | \$5 | \$10 | \$7 | \$7 | \$7 |
| Probability | 1/844 | 1/473 | 1/693 | 1/787 | 1/706 | 1/701 |
| Expected value | \$0.012 | \$0.011 | \$0.014 | \$0.0089 | \$0.0099 | \$0.01 |
| Eighth prize | \$3 | \$2 | \$4 | \$4 | \$4 | \$4 |
| Probability | 1/141 | 1/56 | 1/89 | 1/123 | 1/111 | 1/92 |
| Expected value | \$0.021 | \$0.035 | \$0.045 | \$0.032 | \$0.036 | \$0.043 |
| Ninth prize | \$2 | \$1 | \$2 | \$3 | \$4 | \$4 |
| Probability | 1/75 | 1/21 | 1/37 | 1/62 | 1/55 | 1/38 |
| Expected value | \$0.027 | \$0.047 | \$0.055 | \$0.049 | \$0.072 | \$0.10 |
| Probability, any prize | 1/40 | 1/15 | 1/24 | 1/35 | 1/32 | 1/25 |
| Lower prize expected value | \$0.18 | \$0.17 | \$0.25 | \$0.17 | \$0.36 | \$0.32 |
| Total expected value | \$0.38 | \$0.40 | \$0.59 | \$0.37 | \$0.74 | \$0.66 |

Notes: This table reports the prizes, win probabilities, and expected values corresponding to each prize level and the overall ticket for all Mega Millions and Powerball formats used since 2010. The non-jackpot prize amounts are the fixed prizes offered in states other than California. The expected value is computed simply as the advertised prize (or average prize) multiplied by the win probability.

Table 2: Descriptive Statistics: Mega Millions and Powerball Sales and Prize Data

|  | Obs. | Mean | Std. dev. | Min | Max |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Jackpot (\$millions) | 2,035 | 68.8 | 68.1 | 7.1 | 946.3 |
| California 2nd prize pool (\$000s) | 1,705 | 948.2 | $1,034.6$ | 82.7 | $7,660.3$ |
| Nationwide ticket sales (millions) | 2,035 | 23.0 | 30.6 | 8.8 | 651.9 |
| California ticket sales (millions) | 1,705 | 3.3 | 5.1 | 0.9 | 120.2 |

Notes: This table presents descriptive statistics for draw-level Mega Millions and Powerball sales and prize data from June 2010 through February 2020. Jackpot amounts and sales data are from LottoReport.com; California second prize amounts are from https://www.calottery.com/draw-games/. Jackpots are advertised jackpot amounts, and ticket sales exclude the Just the Jackpot, Power Play, and Megaplier add-ons.

Table 3: Prize Semi-Elasticity Estimates

|  | $\begin{gathered} \hline(1) \\ \text { OLS } \end{gathered}$ | $\begin{gathered} (2) \\ \text { IV } \end{gathered}$ |
| :---: | :---: | :---: |
| Jackpot expected value (\$) | $\begin{gathered} \hline 1.8648^{* * *} \\ (0.0576) \end{gathered}$ | $\begin{gathered} 1.7277^{* * *} \\ (0.0619) \end{gathered}$ |
| Observations | 2,035 | 2,035 |
| (b) Jackpot and Second Prize Semi-Elasticities: Califor nia Sales Only |  |  |
|  | (1) | $\stackrel{(2)}{\text { IV }}$ |
| Jackpot expected value (\$) | $\begin{gathered} \hline 2.0154^{* * *} \\ (0.0776) \end{gathered}$ | $\begin{gathered} \hline 1.8466^{* * *} \\ (0.0852) \end{gathered}$ |
| 2nd prize expected value (\$) | $\begin{aligned} & -0.0202 \\ & (0.0590) \end{aligned}$ | $\begin{gathered} 0.0837 \\ (0.0554) \end{gathered}$ |
| Observations | 1,705 | 1,701 |

Notes: This table presents estimates of equation (11), a regression of the natural $\log$ of sales on prize expected values, controlling for game-format, game-regional coverage, game-quarter of sample, and gameweekend fixed effects. The IV regressions instrument for prize expected values with a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period and additionally control for the expectation of the forecast prior to the realization of the rollover outcome. Panel (a) uses nationwide sales, while Panel (b) uses California sales only. The samples include all Mega Millions and Powerball drawings from June 2010 to February 2020; the sample in Panel (b) is smaller because California did not join Powerball until April 2013. Newey-West standard errors allowing up to ten lags are in parentheses. ${ }^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

Table 4: Price Semi-Elasticity

|  | $(1)$ <br> Pooled | $(2)$ <br> Powerball | $(3)$ <br> Mega Millions | $(4)$ <br> Pooled |
| :--- | :---: | :---: | :---: | :---: |
| Price $\times$ 12-month window | $-0.4953^{* * *}$ | $-0.5583^{* * *}$ | $-0.4300^{* * *}$ | $-0.4973^{* * *}$ |
|  | $(0.0437)$ | $(0.0578)$ | $(0.0569)$ | $(0.0475)$ |
| 12-month window | $0.7400^{* * *}$ | $0.8552^{* * *}$ | $0.6214^{* * *}$ | $0.7406^{* * *}$ |
|  | $(0.0748)$ | $(0.0976)$ | $(0.0957)$ | $(0.0771)$ |
| Jackpot expected value (\$) | $1.6387^{* * *}$ | $1.5739^{* * *}$ | $1.6843^{* * *}$ |  |
|  | $(0.0848)$ | $(0.0765)$ | $(0.1434)$ |  |
| Jackpot (\$millions) |  |  |  | $0.0057^{* * *}$ |
|  |  |  |  | $(0.0006)$ |
| Observations | 625 | 312 | 313 | 625 |

Notes: This table presents estimates of equation (12), a regression of the natural log of sales on ticket price interacted with an indicator for the 12-month window-six months before and six months after-around a price change event and the 12 -month window indicator, controlling for weekend fixed effects and either the jackpot expected value or the jackpot amount. Columns 1 and 4 pool the data from both the Powerball and Mega Millions price changes and also include a price-change event fixed effect, while columns 2 and 3 consider each price change in isolation. We use Newey-West standard errors with up to ten lags. *, ${ }^{* *}$, ***: statistically significant with 90,95 , and 99 percent confidence, respectively.

Table 5: Descriptive Statistics: 2020 Survey Data
(a) Demographics

|  | Obs. | Mean | Std. dev. | Min | Max |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Household income (\$000s) | 2,879 | 72.12 | 53.08 | 5 | 250 |
| Years of education | 2,879 | 14.32 | 2.26 | 4 | 20 |
| Age | 2,879 | 48.82 | 16.79 | 18 | 91 |
| 1(Male) | 2,879 | 0.50 | 0.50 | 0 | 1 |
| 1(White) | 2,879 | 0.66 | 0.47 | 0 | 1 |
| 1(Black) | 2,879 | 0.11 | 0.31 | 0 | 1 |
| 1(Hispanic) | 2,879 | 0.16 | 0.36 | 0 | 1 |
| Household size | 2,879 | 3.04 | 1.62 | 1 | 6 |
| 1(Married) | 2,879 | 0.53 | 0.50 | 0 | 1 |
| 1(Employed) | 2,879 | 0.63 | 0.48 | 0 | 1 |
| 1(Urban) | 2,879 | 0.83 | 0.37 | 0 | 1 |
| 1(Attend church) | 2,879 | 0.36 | 0.48 | 0 | 1 |
| Political ideology | 2,878 | 3.83 | 1.59 | 1 | 7 |

(b) Spending and Income Effects

|  | Obs. | Mean | Std. dev. | Min | Max |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Monthly lottery spending (\$) | 2,877 | 15.16 | 38.04 | 0 | 1,000 |
| Income change (\%) | 2,871 | 0.17 | 18.52 | -50 | 50 |
| Spending change (\%) | 2,870 | -5.46 | 19.17 | -50 | 50 |
| Self-reported income effect (\%) | 2,855 | -1.41 | 16.28 | -50 | 50 |

(c) Proxies for Preferences and Biases

|  | Obs. | Mean | Std. dev. | Min | Max |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Unwillingness to take risks | 2,879 | -3.93 | 1.38 | -7 | -1 |
| Financial risk aversion | 2,879 | 3.05 | 0.82 | 1 | 4 |
| Lottery seems fun | 2,875 | 0.16 | 1.83 | -3 | 3 |
| Enjoy thinking about winning | 2,871 | 0.81 | 1.93 | -3 | 3 |
| Self-control problems | 2,875 | -0.34 | 1.10 | -3 | 3 |
| Financial literacy | 2,879 | 0.77 | 0.25 | 0 | 1 |
| Financial numeracy | 2,879 | 0.63 | 0.32 | 0 | 1 |
| Gambler's Fallacy | 2,879 | 0.29 | 0.39 | 0 | 1 |
| Non-belief in Law of Large Numbers | 2,879 | 0.42 | 0.18 | 0.00 | 0.93 |
| Expected value miscalculation | 2,879 | 0.69 | 0.37 | 0 | 1 |
| Overconfidence | 2,865 | -0.01 | 0.51 | -4.95 | 4.95 |
| Expected returns | 2,871 | 0.28 | 0.20 | 0.05 | 0.95 |
| Predicted life satisfaction | 2,850 | 2.34 | 4.76 | -10 | 10 |

Notes: This table presents descriptive statistics for our 2020 AmeriSpeak survey. Panel (a) presents demographics, Panel (b) presents spending and income effects, and Panel (c) presents proxies for preferences and biases. Section 4.1 summarizes the coding of these variables.

Table 6: Regressions of Monthly Lottery Spending on Bias Proxies

|  | $\begin{gathered} \hline(1) \\ \text { OLS } \end{gathered}$ | $\begin{gathered} \hline(2) \\ \text { OLS } \end{gathered}$ | $\begin{gathered} \hline(3) \\ \mathrm{OLS} \end{gathered}$ | $\begin{gathered} \hline(4) \\ \text { OLS } \end{gathered}$ | $\begin{gathered} \hline(5) \\ \text { OLS } \end{gathered}$ | (6) ORIV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Self-control problems | $\begin{gathered} \hline 0.281^{* * *} \\ (0.029) \end{gathered}$ | $\begin{gathered} \hline 0.414^{* * *} \\ (0.033) \end{gathered}$ | $\begin{gathered} \hline 0.293^{* * *} \\ (0.031) \end{gathered}$ | $\begin{gathered} 0.280^{* * *} \\ (0.030) \end{gathered}$ | $\begin{gathered} \hline 0.279^{* * *} \\ (0.035) \end{gathered}$ | $\begin{gathered} \hline 0.708^{* * *} \\ (0.143) \end{gathered}$ |
| Financial illiteracy | $\begin{gathered} 0.123^{* * *} \\ (0.032) \end{gathered}$ | $\begin{gathered} 0.190^{* * *} \\ (0.034) \end{gathered}$ | $\begin{gathered} 0.142^{* * *} \\ (0.030) \end{gathered}$ | $\begin{gathered} 0.127^{* * *} \\ (0.032) \end{gathered}$ | $\begin{gathered} 0.137^{* * *} \\ (0.040) \end{gathered}$ | $\begin{aligned} & 0.182^{*} \\ & (0.099) \end{aligned}$ |
| Statistical mistakes | $\begin{gathered} 0.101^{* * *} \\ (0.027) \end{gathered}$ | $\begin{gathered} 0.157^{* * *} \\ (0.031) \end{gathered}$ | $\begin{gathered} 0.133^{* * *} \\ (0.027) \end{gathered}$ | $\begin{gathered} 0.097^{* * *} \\ (0.027) \end{gathered}$ | $\begin{gathered} 0.096^{* * *} \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.019 \\ (0.097) \end{gathered}$ |
| Overconfidence | $\begin{gathered} 0.029 \\ (0.023) \end{gathered}$ | $\begin{gathered} 0.034 \\ (0.029) \end{gathered}$ | $\begin{gathered} 0.032 \\ (0.024) \end{gathered}$ | $\begin{gathered} 0.027 \\ (0.024) \end{gathered}$ |  |  |
| Expected returns | $\begin{gathered} 0.068^{* * *} \\ (0.024) \end{gathered}$ | $\begin{gathered} 0.199^{* * *} \\ (0.028) \end{gathered}$ | $\begin{gathered} 0.098^{* * *} \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.068^{* * *} \\ (0.024) \end{gathered}$ | $\begin{aligned} & 0.051^{*} \\ & (0.029) \end{aligned}$ | $\begin{gathered} -0.025 \\ (0.079) \end{gathered}$ |
| Predicted life satisfaction | $\begin{gathered} 0.006 \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.153^{* * *} \\ (0.027) \end{gathered}$ | $\begin{gathered} 0.024 \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.005 \\ (0.024) \end{gathered}$ | $\begin{gathered} -0.009 \\ (0.029) \end{gathered}$ | $\begin{gathered} -0.082 \\ (0.097) \end{gathered}$ |
| Risk aversion | $\begin{gathered} -0.012 \\ (0.026) \end{gathered}$ |  | $\begin{gathered} -0.013 \\ (0.025) \end{gathered}$ | $\begin{gathered} -0.009 \\ (0.027) \end{gathered}$ | $\begin{aligned} & -0.012 \\ & (0.031) \end{aligned}$ | $\begin{gathered} 0.028 \\ (0.046) \end{gathered}$ |
| Lottery seems fun | $\begin{gathered} 0.614^{* * *} \\ (0.029) \end{gathered}$ |  | $\begin{gathered} 0.604^{* * *} \\ (0.029) \end{gathered}$ | $\begin{gathered} 0.608^{* * *} \\ (0.029) \end{gathered}$ | $\begin{gathered} 0.606^{* * *} \\ (0.033) \end{gathered}$ | $\begin{gathered} 1.082^{* * *} \\ (0.085) \end{gathered}$ |
| Enjoy thinking about winning | $\begin{gathered} 0.167^{* * *} \\ (0.029) \end{gathered}$ |  | $\begin{gathered} 0.171^{* * *} \\ (0.029) \end{gathered}$ | $\begin{gathered} 0.176^{* * *} \\ (0.029) \end{gathered}$ | $\begin{gathered} 0.175^{* * *} \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.064 \\ (0.084) \end{gathered}$ |
| $\ln$ (household income) | $\begin{gathered} 0.104^{* * *} \\ (0.036) \end{gathered}$ |  |  | $\begin{gathered} 0.104^{* * *} \\ (0.036) \end{gathered}$ | $\begin{aligned} & 0.099^{* *} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.096^{* *} \\ & (0.046) \end{aligned}$ |
| $\ln$ (years of education) | $\begin{gathered} -0.652^{* * *} \\ (0.170) \end{gathered}$ |  |  | $\begin{gathered} -0.635^{* * *} \\ (0.173) \end{gathered}$ | $\begin{gathered} -0.552^{* * *} \\ (0.192) \end{gathered}$ | $\begin{aligned} & -0.160 \\ & (0.211) \end{aligned}$ |
| 1(Black) | $\begin{gathered} 0.560^{* * *} \\ (0.095) \end{gathered}$ |  |  | $\begin{gathered} 0.572^{* * *} \\ (0.093) \end{gathered}$ | $\begin{gathered} 0.533^{* * *} \\ (0.115) \end{gathered}$ | $\begin{aligned} & 0.271^{* *} \\ & (0.121) \end{aligned}$ |
| 1(Hispanic) | $\begin{gathered} 0.387^{* * *} \\ (0.080) \end{gathered}$ |  |  | $\begin{gathered} 0.348^{* * *} \\ (0.075) \end{gathered}$ | $\begin{gathered} 0.441^{* * *} \\ (0.097) \end{gathered}$ | $\begin{gathered} 0.329^{* * *} \\ (0.103) \end{gathered}$ |
| Other demographics | Yes | No | No | Yes | Yes | Yes |
| State fixed effects | Yes | No | No | No | Yes | Yes |
| $R^{2}$ | 0.41 | 0.16 | 0.36 | 0.39 | 0.40 | 0.57 |
| Observations | 2,810 | 2,810 | 2,810 | 2,810 | 2,072 | 4,144 |
| Clusters | 2,810 | 2,810 | 2,810 | 2,810 | 2,072 | 2,072 |

Notes: This table presents estimates of equation (13), a regression of $\ln (1+$ monthly lottery spending) on bias proxies, preference proxies, demographic controls, and state fixed effects using data from our AmeriSpeak surveys. "Other demographics" includes age, household size, political ideology, and indicators for male, Black, Hispanic, other (non-white) race, married, employed, urban area, and attends religious services at least once a month. Columns 1-5 present OLS estimates. Column 6 presents Obviously Related Instrumental Variables estimates: we estimate equation (13) in a stacked dataset with the 2021 bias and preference proxies below the 2020 bias and preference proxies, instrumenting for the 2020 variables with their 2021 values and vice versa, while clustering standard errors by respondent. Robust standard errors are in parentheses. *, **, ${ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

Table 7: Estimates of Parameters in Structural Model

| Representative agent model |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $b_{0}$ | $b_{1}$ | $\chi$ |  |  |  |
|  | $7.85 \times 10^{-7}$ | 0.36 | 0.30 |  |  |  |
| Model with heterogeneity |  |  |  |  |  |  |
|  | Below-median $s$ |  |  | Above-median $s$ |  |  |
|  | $b_{0}$ | $b_{1}$ | $\chi$ | $b_{0}$ | $b_{1}$ | $\chi$ |
| Low incomes | $1.51 \times 10^{-6}$ | 0.47 | 0.20 | $1.51 \times 10^{-6}$ | 0.47 | 0.30 |
| Middle incomes | $7.58 \times 10^{-7}$ | 0.36 | 0.17 | $7.58 \times 10^{-7}$ | 0.36 | 0.29 |
| High incomes | $4.12 \times 10^{-7}$ | 0.30 | 0.17 | $4.12 \times 10^{-7}$ | 0.30 | 0.33 |

Notes: This table reports estimates of parameters for the structural models, both in the representative agent case, and in the heterogeneous agent case with three income levels and two levels of consumption. Parameters $b_{0}$ and $b_{1}$ are the intercept and slope parameters of the neo-additive probability weighting function. The parameter $\chi$ represents the share of the departure from expected utility weighting that is attributed to bias, as opposed to normative preferences. (Lottery non-consumers are omitted from the lower panel.) In the heterogeneous agent specification, semi-elasticities are assumed to be constant, resulting in homogeneous values of $b_{0}$ and $b_{1}$ conditional on income. See Section 5.1 for details.

Table 8: Optimal Lottery Tax and Attributes Under Alternative Assumptions

|  | Ticket <br> price $(\$)$ | Average jackpot <br> expected value (\$) | Effective <br> tax rate |
| :--- | :---: | :---: | :---: |
| 1. Baseline | 2.88 | 0.78 | 0.58 |
| 2. Completely unbiased | 1.45 | 1.10 | -0.05 |
| 3. 100 percent more biased | 3.96 | 0.45 | 0.78 |
| 4. CRRA $=0.9$ | 3.05 | 0.80 | 0.60 |
| 5. CRRA $=1.5$ | 2.37 | 0.61 | 0.56 |
| 6. Weaker redistribution | 2.86 | 0.79 | 0.58 |
| 7. Stronger redistribution | 2.93 | 0.77 | 0.59 |
| 8. Lower value of $\bar{\zeta}_{2}=0$ | 2.88 | 0.77 | 0.58 |
| 9. Higher value of $\bar{\zeta}_{2}=\bar{\zeta}_{1}$ | 2.89 | 0.87 | 0.55 |
| 10. All bias is on jackpot | 2.88 | 0.79 | 0.58 |
| 11. Variable jackpot | 2.80 | 0.60 | 0.57 |
| 12. Measurement error correction | 3.07 | 0.73 | 0.62 |
| 13. Same bias share across incomes | 2.91 | 0.77 | 0.59 |
| 14. Same bias share for everyone | 2.29 | 0.92 | 0.41 |
| 15. Steeper decline across incomes | 2.88 | 0.76 | 0.59 |

Notes: This table reports key features of the optimal representative lottery according to our structural model. The first two columns report the jointly optimal price and jackpot expected value of a lottery ticket resembling a current Powerball ticket. The third column reports the optimal effective tax rate, calculated as the share of price that is a mark-up over marginal cost (net-of-tax total ticket expected value plus overhead). See Section 6.3 for details about the specifications considered in each row.

Figure 1: Powerball Prizes and Ticket Sales in 2014


Notes: This figure presents the expected values of the jackpot and California second prize as well as California ticket sales for each Powerball drawing in 2014.

Figure 2: Responsiveness of Ticket Sales to Jackpot and California Second Prize
(a) National Sales versus Jackpot Expected Value First-Stage Fitted Values

(b) California Sales versus Second Prize Expected Value First-Stage Fitted Values


Notes: Panel (a) presents a binned scatter plot of the natural $\log$ of national sales against the jackpot expected value fitted values from the first stage of equation (11), residual of the controls in that equation. Panel (b) presents a binned scatter plot of the natural log of California sales against the California second prize expected value fitted values from the first stage of equation (11), residual of the jackpot expected value first-stage fitted values and the other controls in that equation. The sample includes all Mega Millions and Powerball drawings from June 2010 through February 2020.

Figure 3: Price Change Event Studies


Notes: These figures present the natural log of Mega Millions and Powerball ticket sales (residual of jackpot expected values and weekend fixed effects) and the natural log of other games' sales in dollars (residual of week fixed effects) before and after price increases, which are indicated by the vertical red lines. The levels of Mega Millions and Powerball sales are adjusted so that the average natural log of sales is zero before the price change when jackpots are within $\$ 10$ million of the reset value, while the levels of other games' sales are adjusted so that the average natural $\log$ of sales is zero before the price change. In Panel (a), the Powerball ticket price increased from $\$ 1$ to $\$ 2$ on January 15, 2012. In Panel (b), the Mega Millions ticket price increased from $\$ 1$ to $\$ 2$ on October 28, 2017. One value in Panel (b) from a Mega Millions drawing with a record jackpot is winsorized at -1 .

Figure 4: Lottery Spending by Income


Notes: This figure presents average monthly lottery spending within household income groups, with 95 percent confidence intervals, using data from our AmeriSpeak survey. Observations are weighted for national representativeness.

Figure 5: Test-Retest Reliability of Bias Proxies


Notes: This figure presents binned scatter plots of the 2020 vs. 2021 elicitations of our six bias proxies, using data from our AmeriSpeak surveys.

Figure 6: Relationship Between Lottery Spending and Bias Proxies


Notes: This figure presents binned scatter plots of $\ln (1+$ monthly lottery spending $)$ versus our six bias proxies, using data from our 2020 AmeriSpeak survey. The vertical line on each panel corresponds to the correct or "unbiased" value of the bias proxy.

Figure 7: Share of Lottery Spending Attributable to Biases


Notes: This figure plots the share of lottery spending attributable to each of our six bias proxies, with 95 percent confidence intervals. Predicted unbiased consumption is $\hat{s}_{i k}^{V}=\frac{s_{i}+1}{\exp \left(\tilde{\tau}_{k} \tilde{b}_{i k}\right)}-1$, where $s_{i}$ is monthly lottery spending, $\hat{\tau}_{k}$ is the OLS estimate from column 1 of Table 6 , and $\tilde{b}_{i k}=\frac{b_{i k}-b_{k}^{V}}{S D\left(b_{i k}\right)}$ is the difference between person $i$ 's proxy $b_{i k}$ and the unbiased value $b_{k}^{V}$ in standard deviation units. We winsorize at $\hat{s}_{i}^{V} \geq 0$, and we fix $\hat{s}_{i k}^{V}=0$ if $s_{i}=0$. The share of consumption attributable to each bias proxy is $\frac{\sum_{i}\left(s_{i}-\hat{s}_{i k}^{V}\right)}{\sum_{i} s_{i}}$.

Figure 8: Relationship Between Income and Bias Proxies


Notes: This figure presents binned scatter plots of our six bias proxies by household income, using data from our 2020 AmeriSpeak survey. The horizontal line on each panel corresponds to the correct or "unbiased" value of the bias proxy.

Figure 9: Share of Lottery Spending Attributable to Bias within Income Groups


Notes: This figure plots the share of lottery spending attributable to bias within household income groups, with 95 percent confidence intervals. Predicted unbiased consumption is $\hat{s}_{i}^{V}=\frac{s_{i}+1}{\exp \left(\hat{\boldsymbol{\tau}} \tilde{\boldsymbol{b}}_{i}\right)}-1$, where $s_{i}$ is monthly lottery spending, $\hat{\boldsymbol{\tau}}$ is the OLS estimate from column 1 of Table 6, and $\tilde{b}_{i k}=\frac{b_{i k}-b_{k}^{V}}{S D\left(b_{i k}\right)}$ is the difference between person $i$ 's proxy $b_{i k}$ and the unbiased value $b_{k}^{V}$ in standard deviation units. We winsorize at $\hat{s}_{i}^{V} \geq 0$, and we fix $\hat{s}_{i k}^{V}=0$ if $s_{i}=0$. The share of consumption attributable to bias is $\frac{\sum_{i}\left(s_{i}-\hat{s}_{i}^{V}\right)}{\sum_{i} s_{i}}$.

Figure 10: Estimated Surplus from Lotteries in the Status Quo


Notes: This figure plots the estimated surplus per capita that each consumer type derives from each $\$ 100$ of per-capita spending on the status quo representative lottery in our structural model, relative to a setting with no lottery. Income bins are partitioned into those who purchase no lottery tickets, those who purchase less than the median amount (conditional on purchasing), and those who purchase more than the median.

Figure 11: Effect of Lottery Attributes on Social Welfare
(a) Variation in Ticket Price

(b) Variation in Jackpot Size


Notes: These figures plot the simulated social welfare gain from a representative lottery relative to no lottery, when varying ticket price (Panel a) or jackpot size (Panel b). The baseline representative lottery is based on a standard $\$ 2$ Powerball ticket with a jackpot pool of $\$ 101$ million. Prizes are reduced by 30 percent to account for income taxes.

## Online Appendix

## The Optimal Design of State-Run Lotteries

Benjamin B. Lockwood © Hunt Allcott (r) Dmitry Taubinsky © Afras Sial

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## A Background Appendix

Figure A1: Lottery Sales by Game Type over Time


Notes: This figure presents total U.S. lottery sales by type of game, using data from La Fleur's. Census data are from the Census of Governments, inflated to account for the assumption that retailers receive 6.5 percent of sales as commissions, the midpoint of the typical range (North American Association of State and Provincial Lotteries 2021a). Monetary amounts are in real 2019 dollars.

Figure A2: Lottery Sales Allocation and Implicit Tax Rate over Time


Notes: This figure presents the allocation of the proceeds of U.S. lottery sales and the implicit lottery tax rate using data from the Census of Governments, assuming that commissions equal 6.5 percent of sales, the midpoint of the typical range (North American Association of State and Provincial Lotteries 2021a). The implicit tax rate equals state government proceeds divided by total sales. Monetary amounts are in real 2019 dollars.

## B Special Cases of Proposition 1

To provide intuition for the implications of our formulas, we consider a number of special cases.

No bias, homogeneous preferences. When $\gamma(\theta) \equiv 0$ and $s(z), \kappa(z)$ and $\rho(z)$ are constant across the population, Proposition 1 implies $p=C_{s}^{\prime}(\boldsymbol{a}, \bar{s})$ and $\bar{\kappa}(a)=C_{a}^{\prime}(a, \bar{s})$. In other words, the price is equal to the marginal cost of an additional lottery ticket, while $a$ is set such that lottery buyers' surplus from an increase in $a$ is equal to the marginal cost of increasing $a$.

In the simple example where there is a single prize of size $w=a$ and expected payout $\pi a$, the conditions reduce to $p=\pi a$ and $\bar{\kappa}(a)=\pi \bar{s}$. With utility function $u(a)=(1+\phi) \pi m(a)$, this implies that $p$ and $a$ are determined by the conditions $p=\pi a+o$ and $(1+\phi) \pi m^{\prime}(a)=\pi$, where $o$ is the additional administrative cost from each lottery ticket. The second condition uniquely determines
$a$, which then determines the price $p$. For example, if $m(x)=\ln (1+x)$, then the optimal lottery structure is $a=\max \{\phi, 0\}$ and $p=\pi \max \{\phi, 0\}+o$. This implies that $a>0$ at the optimum if and only if $\phi>0$; i.e., if additional entertainment utility leads individuals to value lotteries above their monetary value.

No bias, heterogeneous preferences. When $\gamma(\theta) \equiv 0$, Proposition 1 implies the conditions $p-C_{s}^{\prime}(a, \bar{s})=-\frac{\operatorname{Cov}[s(z), g(z)]}{\left|\bar{\zeta}_{p}\right| \bar{s}}$ and $\bar{\kappa}=C_{a}^{\prime}(a, \bar{s})+\operatorname{Cov}[s(z), g(z)] \frac{\bar{\zeta}_{a}}{\left|\bar{\zeta}_{p}\right|}-\operatorname{Cov}[\kappa(z), g(z)]$. The first condition is analogous to Diamond's (1975) "many-person Ramsey tax rule," which states that the tax is proportional to its degree of progressivity and is inversely proportional to the elasticity. The condition for $\bar{\kappa}$, which results from substituting (17) into (18), is new, and states that lottery buyers' surplus from a marginal increase in $a$ must equal the marginal cost of increasing $a$ plus the degree to which increasing $a$ is more progressive than decreasing $p$.

Homogeneous bias and preferences. When $s(z), \gamma(z), \kappa(z)$ and $\rho(z)$ are homogeneous across the income distribution, Proposition 1 implies $p-C_{s}^{\prime}(a, \bar{s})=\bar{\gamma}$ and $\bar{\kappa}(a)=C_{a}^{\prime}(a, \bar{s})+\bar{\rho}(a)$. In this case, the price is set above a lottery ticket's expected value when $\bar{\gamma}>0$, so as to discourage lottery consumption. Moreover, the optimal level of attribute $a$ is set such that $\bar{\kappa}(a)>C_{a}^{\prime}$ when individuals overvalue not just the absolute utility of the lottery ticket but also changes in $a$ (i.e., $\bar{\rho}(a)>0)$. This implies that the optimal choice of $a$ is lower than what it would be when individuals correctly evaluate lottery tickets (under the reasonable assumption that $\bar{\kappa}(a)$ is decreasing in $a$ due to the concavity of $m$ ). Thus, individuals are effectively taxed in two ways relative to the no-bias benchmark. First, the price is set to be higher than the expected monetary value of a lottery ticket. Second, utility-increasing attributes of the lottery ticket (e.g., its prize levels) are set to be lower than what would be optimal in the absence of bias.

Consider the simple example where there is a single prize of size $w=a$, and $u=(1+\phi) \pi m(a)$ and $v=\left(1+\phi^{V}\right) \pi m(a)$. We have $\bar{\kappa}(a)=(1+\phi) \pi m^{\prime}(a) \bar{s}, \bar{\rho}(a)=\left(\phi-\phi^{V}\right) \pi m^{\prime}(a) \bar{s}$, and $\gamma=(\phi-$ $\left.\phi^{V}\right) \pi m(a)$. At an interior optimum, we thus have the first-order conditions $p-\pi a=\left(\phi-\phi^{V}\right) \pi m(a)$ and $(1+\phi) m^{\prime}(a)=1+\left(\phi-\phi^{V}\right) m^{\prime}(a)$. Note that if $\phi^{V}$ is sufficiently low, it is optimal to choose $a=0$.

Revenue-maximizing lottery structure. The revenue-maximizing lottery structure can be obtained from our calculations by ignoring the effects on consumer surplus. The revenue effects of changing $a$ and $p$ are $p \frac{d \bar{s}}{d a}-\frac{d}{d a} C(a, \bar{s}(a))$ and $p \frac{d \bar{s}}{d p}-\frac{d}{d p} C(a, \bar{s}(a))$, respectively. This leads to the conditions $p-C_{s}^{\prime}(a, \bar{s})=\frac{1}{\left|\bar{\zeta}_{p}\right|}$ and $\frac{C_{a}^{\prime}(a, \bar{s})}{\bar{s}}=\frac{\bar{\zeta}_{a}}{\left|\bar{\zeta}_{p}\right|}$. The first condition is just the standard inverse elasticity rule for product pricing. The second condition states that the per-ticket marginal cost of increasing $a$ has to equal the ratio of the semi-elasticities. The intuition for the second condition is that increasing $a$ by $d a$ and increasing $p$ by $d p=d a \frac{C_{a}^{\prime}(a, \bar{s})}{\bar{s}}$ has a zero direct effect on government revenue; thus, this perturbation cannot affect consumer demand if $a$ and $p$ are set optimally. For example, when $a$ corresponds to expected payout of a lottery ticket, so that $C_{s}^{\prime}=a$ and $C_{a}^{\prime} / \bar{s}=1$,
the optimal choice of $p$ and $a$ must satisfy $p-a=1 /\left|\bar{\zeta}_{p}\right|$ and $\left|\bar{\zeta}_{p}\right|=\bar{\zeta}_{a}$.

## C Theory Appendix: A More General Model and Generalizations of Proposition 1

## C. 1 A More General Model

We consider a more general model in which individuals first choose income $z$ and then choose whether or not to buy lottery tickets on various occasions. Specifically, we assume that individuals choose their income in period $t=0$, and then choose whether or not to buy a lottery ticket on choice occasions $t=1, \ldots, t^{*}$. Individuals realize a taste shock $\varepsilon_{t}$ at the beginning of each period that determines their utility from the lottery ticket. Individuals' utility given a vector of shocks $\boldsymbol{\varepsilon}$ and a vector $\boldsymbol{x}=\left(x_{1}, \ldots, x_{t^{*}}\right) \in\{0,1\}^{t^{*}}$ of lottery ticket purchase decisions is given by $U(x, c, z ; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon})=G\left(n(c ; \theta)+\sum_{t} u\left(x_{t} ; \boldsymbol{a}, \theta, \varepsilon_{t}\right)-\psi(z ; \theta)\right)$, where $\boldsymbol{a}$ is a vector of attributes, with $j$ th component $a_{j}$. Without loss of generality, we can consider individuals' optimization problem as a static problem where the vector of shocks $\boldsymbol{\varepsilon}$ is realized in period 1 and individuals choose a consumption plan $\boldsymbol{x}$ for all $t^{*}$ periods.

Lottery demand is a random variable $s_{\theta}(\varepsilon)$ that maps shocks $\varepsilon$ to a total number of lottery tickets purchased. We let $\bar{s}_{\theta}$ denote expected lottery purchases. We assume that $\boldsymbol{\varepsilon}$ is smoothly distributed, so that $\bar{s}_{\theta}$ is smooth in $p$ and $a_{j}$. For shorthand, we will sometimes write $\boldsymbol{u}(s ; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon})=$ $\operatorname{argmax}_{\boldsymbol{x}}\left\{\sum_{t} u\left(x_{t} ; \boldsymbol{a}, \theta, \varepsilon_{t}\right) \mid \sum_{t} x_{t}=s\right\}$; that is, $\boldsymbol{u}$ is utility obtained after a vector of shocks $\boldsymbol{\varepsilon}$ is realized and the person executes an optimal consumption plan given the constraint that $s$ lottery tickets are purchased.

In contrast to the body of the paper, this functional form allows for exogenous shocks to income to change lottery consumption, but we still maintain the assumption of weak separability. The weak separability assumption could also be relaxed by following the approach of ALT, and replacing the (causal) income elasticity of lottery demand with the elasticity of lottery demand with respect to changes in earnings $z$, where appropriate.

We let $\kappa_{j}(p, \boldsymbol{a}, z ; \theta)$ denote the valuation of a marginal increase in the $j$ th attribute of $\boldsymbol{a}$. We let $V(x, c, z ; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon})=G\left(n(c ; \theta)+\sum_{t} v\left(x_{t} ; \boldsymbol{a}, \theta, \varepsilon_{t}\right)-\psi(z ; \theta)\right)$ denote normative utility. We define $\boldsymbol{v}(s ; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon})$ analogous to $\boldsymbol{u}(s ; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon})$.

We let $C(\boldsymbol{a}, \bar{s})$ be the cost of supplying lottery tickets, with $C_{j}^{\prime}$ denoting the derivative with respect to the $j$ th component $a_{j}$ of $\boldsymbol{a}$, and $C_{s}^{\prime}$ denoting the derivative with respect to $\bar{s}$.

Note that we assume that the utility from gambling does not depend on the history of prior gambling decisions (i.e., no habit formation) for expositional simplicity. Our results, which apply the Envelope Theorem to the expected perceived utility function, do not require this form of stationarity.

## C. 2 Assumptions

We make the following assumptions:
Assumption 1. Utility from numeraire consumption, $n(c)$, has bounded relative risk aversion: there is $r>0$ such that $\left|c n^{\prime \prime}(c) / n^{\prime}(c)\right|<r$ for all $c$.

Assumption 2. Lottery expenditures are a small share of the total budget, so that terms of order $\frac{\bar{s}_{\theta}}{z(\theta)-T(z(\theta))-p \bar{s}_{\theta}}$ and $\frac{p V a r\left[s_{\theta}(\varepsilon)\right] / \bar{s}_{\theta}}{z(\theta)-T(z(\theta))-p \bar{s}_{\theta}}$ are negligible.
Assumption 3. Constant social marginal welfare weights conditional on income: $g(\theta, \boldsymbol{\varepsilon})=g\left(\theta^{\prime}, \boldsymbol{\varepsilon}\right)$ if $z(\theta)=z\left(\theta^{\prime}\right)$.

Assumption 4. $U$ and $V$ are smooth functions that are strictly concave in $c$, $s$, and $z$, and $\mu$ is differentiable with full support.

Assumption 5. The optimal income tax function $T(\cdot)$ is twice differentiable, and each consumer's choice of income $z$ admits a unique global optimum, with the second-order condition holding strictly at the optimum.

Assumption 6. $\bar{s}_{\theta}$ and $\kappa_{j}(\theta)$ are orthogonal to the income elasticity $\zeta_{z}$ conditional on income.
Assumption 7. Income effects on labor supply are negligible.
Assumptions 1 and 2 ensure unpredicted variation in an individual's lottery expenditures does not have consequential effects on her marginal utility from numeraire consumption, as is clarified in Lemma C. 1 and its proof. Assumption 2 also implies that the difference between compensated and uncompensated demand for lottery tickets is negligible (see ALT). The term $\frac{\bar{s}_{\theta}}{z(\theta)-T(z(\theta))-p \bar{s}_{\theta}}$ is negligible simply when lotteries are a small share of total expenditures. By the Central Limit Theorem, the term $\frac{p \operatorname{Var}\left[s_{\theta}(\varepsilon)\right] / \bar{s}_{\theta}}{z(\theta)-T(z(\theta))-p \bar{s}_{\theta}}$ approaches zero when the number of choice occasions grows large while $\bar{s}_{\theta}$ stays constant. Thus, the second part of Assumption 2 mechanically holds when there are many choice occasions and $\bar{s}_{\theta}$ is assumed to not exceed a certain fixed share of expenditures.

Assumption 3 is analogous to Assumption 1 in Saez (2002a). It holds immediately if types are homogeneous conditional on income. More generally, Saez (2002a) argues this is a reasonable normative requirement even under heterogeneity "if we want to model a government that does not want to discriminate between different consumption patterns..." Therefore we sometimes write $g(z)$ to denote the welfare weight directly as a function of earnings.

Assumptions 4 and 5 ensure that the income distribution does not exhibit any atoms and consumers' labor supply and consumption decisions respond smoothly to perturbations of the tax system (Jacquet and Lehmann 2021).

## C. 3 Elasticity Concepts and Sufficient Statistics

All statistics are understood to be endogenous to the tax regime $(t, T)$, though we suppress those arguments for notational simplicity. We begin by defining the elasticities related to sin good consumption.

## Price and attribute elasticities

- $\zeta_{p}(\theta)$ : the price semi-elasticity of demand for $s$ from type $\theta$, formally equal to $\left(\frac{d \bar{s}_{\theta}}{d p}\right) \frac{1}{\bar{s}_{\theta}}$
- $\zeta_{a_{j}}(\theta)$ : the price semi-elasticity of demand for $s$ from type $\theta$, formally equal to $\left(\frac{d \bar{s}_{\theta}}{d a_{j}}\right) \frac{1}{\overline{s_{\theta}}}$
- $\xi(\theta)$ : the causal income elasticity of demand for $s$, equal to $\frac{d}{d z} \bar{s}_{\theta}(p, \boldsymbol{a}, z ; \theta) \cdot \frac{z}{s}$.


## Income Elasticities

We define labor supply responses to include any "circularities" due to the curvature of the income tax function, which is assumed to be differentiable. Thus, following Jacquet and Lehmann (2021), we define a tax function $\hat{T}$ which has been locally perturbed around the income level $z_{0}$ by raising the marginal tax rate by $\tau$ and reducing the tax level by $\nu$ :

$$
\begin{equation*}
\hat{T}\left(z ; z_{0}, \tau, \nu\right):=T(z)+\tau\left(z-z_{0}\right)-\nu . \tag{25}
\end{equation*}
$$

Let $z^{*}(\theta)$ denote a type $\theta$ 's choice of earnings under the status quo income tax $T$, and let $\hat{z}(\theta ; \tau, \nu)$ denote $\theta$ 's choice of earnings under the perturbed income $\operatorname{tax} \hat{T}\left(z ; z^{*}(\theta), \tau, \nu\right)$. Then the compensated elasticity of taxable income is defined in terms of the response of $\hat{z}$ to $\tau$, evaluated at $\tau=\nu=0$ :

$$
\begin{equation*}
\zeta_{z}^{c}(\theta):=\left(-\left.\frac{\partial \hat{z}(\theta ; \tau, 0)}{\partial \tau}\right|_{\tau=0}\right) \frac{1-T^{\prime}\left(z^{*}(\theta)\right)}{z^{*}(\theta)} \tag{26}
\end{equation*}
$$

The income tax is similarly defined in terms of the response of $\hat{z}$ to a tax credit $\nu$ (this statistic will be nonpositive if leisure is a non-inferior good):

$$
\begin{equation*}
\eta_{z}(\theta):=\left(\left.\frac{\partial \hat{z}(\theta ; 0, \nu)}{\partial \nu}\right|_{\nu=0}\right)\left(1-T^{\prime}\left(z^{*}(\theta)\right)\right) . \tag{27}
\end{equation*}
$$

These definitions are comparable to those in Saez (2001), except that they include circularities and thus permit a representation of the optimal income tax in terms of the actual earnings density, rather than the "virtual density" employed in that paper.

## Bias

We continue defining bias $\gamma$ analogous to the definition in the body of the paper: it is the value that $\gamma(p, a, y ; \theta, \varepsilon)$ that satisfies

$$
u(1 ; \boldsymbol{a}, \theta, \varepsilon)-v(1 ; \boldsymbol{a}, \theta, \varepsilon)=n(y-p+\gamma ; \theta)-n(y-p ; \theta)
$$

where $y$ is disposable income. In other words, $\gamma$ is the degree, in units of dollars, by which the individual overestimates the value of the lottery ticket.

We define

$$
\begin{aligned}
\gamma(z ; \boldsymbol{a}, \theta) & =\mathbb{E}\left[\gamma\left(p, \boldsymbol{a}, z-T(z)-s_{\theta}(\boldsymbol{\varepsilon}) ; \theta, \varepsilon\right)\right. \\
& \mid \boldsymbol{u}\left(s_{\theta}(\boldsymbol{\varepsilon})+1 ; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon}\right)-\boldsymbol{u}\left(s_{\theta}(\boldsymbol{\varepsilon}) ; \boldsymbol{a}, \theta, \boldsymbol{\varepsilon}\right)=n\left(z-T(z)-p\left(s_{\theta}(\boldsymbol{\varepsilon})+1\right) ; \theta\right) \\
& \left.-n\left(z-T(z)-s_{\theta}(\boldsymbol{\varepsilon}) ; \theta\right), z(\theta)=z\right]
\end{aligned}
$$

In other words, $\gamma(z)$ is the average bias of $z$-earners who are on the margin of purchasing an additional lottery ticket. The statistics $\bar{\gamma}, \sigma_{p}$ and $\sigma_{a_{j}}$ are constructed as in the body of the paper. By definition, the social welfare impact of inducing a marginal $z$-earner to purchase one fewer lottery ticket is $\gamma(z) g(z)$.

We define $\rho_{j}$ more generally as the difference $\rho_{j}=\kappa_{j}-\kappa_{j}^{V}$, where $\kappa_{j}^{V}$ is the willingness to pay for a marginal change in $a_{j}$ that would result if consumers chose according to normative preferences V.

## Aggregation

With some abuse of notation, we write $s(z), \gamma(z), \kappa_{j}(z)$ and so forth to denote the averages among $z$-earners. We denote population averages of these statistics using "bar" notation. For example, average consumption of $s$ is denoted $\bar{s}$. The cumulative density function of the income distribution is denoted $H(z)$, which we assume possesses a density function $h(z)$.

## Causal Income Effects and Preference Heterogeneity

Following ALT, we distinguish between two sources of cross-sectional variation in $s(z)$ : income effects and (decision) preference heterogeneity. Let $\bar{s}^{\prime}(z)$ denote the cross-sectional change in $s$ with respect to income $z$ at a particular point in the income distribution. This total derivative can be decomposed into two partial derivatives: the (causal) income effect, $s_{i n c}^{\prime}(z)$, and between-income preference heterogeneity $s_{p r e f}^{\prime}(z)$. The causal income effect depends on the empirically estimable income elasticity of $s: s_{i n c}^{\prime}(z)=\mathbb{E}[\xi(\theta) / z \mid z(\theta)=z]$. Between-income preference heterogeneity is the residual: $s_{p r e f}^{\prime}(z)=\bar{s}^{\prime}(z)-s_{i n c}^{\prime}(z)$. The key sufficient statistic for preference heterogeneity, "cumulative between-income preference heterogeneity" is defined as:

$$
\begin{aligned}
s_{\text {pref }}(z) & :=\int_{x=z_{\min }}^{z} s_{\text {pref }}^{\prime}(x) d x \\
s_{\text {inc }}(z) & :=\int_{x=z_{\min }}^{z} s_{\text {inc }}^{\prime}(x) d x
\end{aligned}
$$

The $s_{\text {pref }}(z)$ term quantifies the amount of lottery consumption at income $z$, relative to the lowest income level $z_{\min }$, that can be attributed to preference heterogeneity rather than income effects.

Analogously, we define $\kappa_{j}^{\prime}(z)$ to be the cross-sectional heterogeneity in the valuation of a marginal increase in $a_{j}$. We define $\kappa_{j, i n c}^{\prime}(z):=\mathbb{E}\left[\left.\frac{\partial}{\partial z} \kappa_{j}(p, \boldsymbol{a}, z ; \theta) \right\rvert\, z(\theta)=z\right]$, and let $\kappa_{j, p r e f}^{\prime}(z)=$
$\kappa_{j}^{\prime}(z)-\kappa_{j, \text { inc }}^{\prime}(z)$ be the residual. We define

$$
\begin{aligned}
\kappa_{j, p r e f}(z) & :=\int_{x=z_{\min }}^{z} \kappa_{j, p r e f}^{\prime}(x) d x \\
\kappa_{j, \text { inc }}(z) & :=\int_{x=z_{\text {min }}}^{z} \kappa_{j, \text { inc }}^{\prime}(x) d x .
\end{aligned}
$$

## C. 4 Results and Derivations

## Preliminary Lemmas

Although in contrast to standard optimal tax models, ours features discrete choice of a commodity, we show that we can still establish an approximate Roy Identity in our model under assumptions 1 and 2 , and thus derive a simple expression for how changes in lottery attributes affect labor supply. This is the content of Lemma C. 1 below.

Lemma C.1. The change in earnings of type $\theta$ induced by a small change da $a_{j}$ is equal to the change in earnings that would be induced by imposing a type-specific $d T^{\theta}(z)=-d a_{j} \cdot \kappa_{j}(p, \boldsymbol{a}, z ; \theta)$. Under assumptions 1 and 2, the change in earnings of type $\theta$ induced by a small change $d p$ in the price is equal to $d T^{\theta}(z)=d p \cdot \bar{s}(p, \boldsymbol{a}, z ; \theta)$ up to negligible terms.

Proof. The first statement follows from Lemma 1 of Saez (2002a). To prove the second statement, assume, without loss, that $G$ is linear. The Envelope Theorem implies that a change $d p$ in the price of the lottery has an expected utility impact of $\mathbb{E}_{\varepsilon}\left[n^{\prime}\left(z-T(z)-p s_{\theta}(\varepsilon)\right) s_{\theta}(\varepsilon)\right] d p$. Similarly, a change $d y=d p \bar{s}_{\theta}$ in after-tax income has an expected utility impact of $\mathbb{E}_{\varepsilon}\left[n^{\prime}\left(z-T(z)-p s_{\theta}(\varepsilon)\right)\right] \bar{s}_{\theta} d p$. Up to second order, the difference between these two terms is

$$
\begin{aligned}
& \mid \mathbb{E}\left[n^{\prime \prime}\left(z-T(z)-\bar{s}_{\theta}\right)\left(\left(s_{\theta}(\varepsilon)\right)^{2}-s_{\theta}(\varepsilon) \bar{s}_{\theta}\right)\right] d p \mid \\
&=\left|n^{\prime \prime}\left(z-T(z)-\bar{s}_{\theta}\right) d p\right| \operatorname{Var}\left[s_{\theta}\right] \\
& \leq r n^{\prime}\left(z-T(z)-\bar{s}_{\theta}\right) \frac{p \operatorname{Var}\left[s_{\theta}(\varepsilon)\right] / \bar{s}_{\theta}}{z(\theta)-T(z(\theta))-p \bar{s}_{\theta}}|d p|
\end{aligned}
$$

Thus, for $d T^{\theta}(z)=d p \cdot \bar{s}(p, \boldsymbol{a}, z ; \theta)$,

$$
\frac{d z(\theta)}{d p} / \frac{d z(\theta)}{d T^{\theta}}=1+O\left(\frac{p \operatorname{Var}\left[s_{\theta}(\varepsilon)\right] / \bar{s}_{\theta}}{z(\theta)-T(z(\theta))-p \bar{s}_{\theta}}\right)
$$

The next lemma will also prove useful in the derivations below.
Lemma C.2. Let $q(x)$ be any continuously differentiable function with $q\left(z_{\text {min }}\right)=0$. Then

$$
\int_{z=z_{\min }}^{\infty} \int_{x=z}^{\infty}(1-g(x)) h(x) d x q^{\prime}(z) d z=\int_{z=z_{\min }}^{\infty} q(z)(1-g(z)) h(z) d z .
$$

Proof. This follows from integration by parts and the fact that $\int_{z=z_{\text {min }}}^{\infty}(1-g(z)) h(z)=0$ at the optimum.

## The Main Result

Proposition C.1. If $p, \boldsymbol{a}$, and $T$ are set optimally, then
for all $a_{j}$, with equality when $a_{j}>0$.
If the income tax $T$ is not necessarily optimal, but $p$ and $\boldsymbol{a}$ are set optimally, then

$$
\begin{aligned}
p-C_{s}^{\prime} & =\bar{\gamma}\left(1+\sigma_{p}\right)-\frac{\mathbb{E}[s(z)(g(z)-1)]}{\left|\bar{\zeta}_{p}\right| \bar{s}}-\frac{1}{\left|\bar{\zeta}_{p}\right| \bar{s}} \mathbb{E}\left[\frac{T^{\prime}(z)}{1-T^{\prime}(z)} \zeta_{z}(z) z s_{\text {inc }}^{\prime}(z)\right] \\
p-\frac{\partial C}{\partial a_{j}} & =\bar{\gamma}\left(1+\sigma_{a_{j}}\right)-\frac{\mathbb{E}\left[\left(\kappa_{j}(z)-\rho_{j}(z)\right) g(z)\right]-\frac{\partial C}{\partial a_{j}}}{\bar{\zeta}_{a_{j}} \bar{s}}+\frac{1}{\bar{\zeta}_{a_{j}} \bar{s}} \mathbb{E}\left[\frac{T^{\prime}(z)}{1-T^{\prime}(z)} \zeta_{z}(z) z \kappa_{j, i n c}^{\prime}(z)\right]
\end{aligned}
$$

The intuition behind the "regressivity" and "consumer surplus" terms comes from considering a joint reform where a change in $p$ or $a_{j}$ is accompanied by a corresponding change in the income tax $T$ that leaves labor supply preserved. When there are no causal income effects, a change in $p$ or $a_{j}$ has no effect on labor supply, and thus no accompanying change in the income $\operatorname{tax} T$ is necessary; in this case, $s_{\text {pref }}(z)=s(z)$ and $\kappa_{j, p r e f}(z)=\kappa_{j}(z)$. When lotteries are a normal good, an increase in the price, for example, generates a higher tax burden on those choosing to earn more, and thus creates disincentives for higher labor supply equivalent to the change produced by an increase in the marginal income tax rate - this is formalized in Lemma 1. Thus, an increase in $p$ must be accompanied by a decrease in the income tax, which leads the net tax burden of the reform to be proportional to $s_{p r e f}(z)$ rather than $s(z)$.

Proposition C. 1 generalizes the classic Atkinson and Stiglitz (1976) result in three ways. First, note that in the case of no correlated preference heterogeneity, $s_{\text {pref }} \equiv 0$, and thus the optimal price equals the marginal cost. This is analogous to the classic Atkinson and Stiglitz (1976) result that when consumption preferences are homogeneous, commodity taxes are not useful for redistribution in the presence of nonlinear income taxation. Second, our results allow us to establish an AtkinsonStiglitz type result for optimal attribute regulation. In the absence of biases, and when $\kappa_{j, p r e f} \equiv$ 0 , meaning that preferences for the attribute are uncorrelated with earnings ability, the optimal attribute choice must satisfy $\bar{\kappa}_{j}=C_{j}^{\prime}+\left(p-C_{s}^{\prime}\right)\left|\bar{\zeta}_{a_{j}}\right| \bar{s}$. In other words, consumers' average marginal
valuation of each attribute component must equal the marginal cost of increasing that attribute component. This again parallels the classic Atkinson and Stiglitz (1976) results, but extends to the case of attribute regulation. Third, while the case of $s_{\text {pref }}=0$, and $\kappa_{j, p r e f} \equiv 0$ is a special case corresponding to the assumptions of Atkinson and Stiglitz (1976), our general result in Proposition C. 1 provides a characterization of optimal regulation under a much broader set of assumptions.

Proof. Consider first increasing the marginal tax rate between $z^{*}$ and $z^{*}+d z$ by a small amount $d \tau$. Assumption 2 implies that the effects of small changes in $z$ induced by this perturbation have negligible effects on the consumption $s$ (this is a simple extension of the derivations in ALT of the proof of Proposition 1). Following the derivations of Saez (2001) or ALT, and utilizing Assumption 7, the optimal income tax $T$ must thus satisfy

$$
\begin{equation*}
\frac{T^{\prime}\left(z^{*}\right)}{1-T^{\prime}\left(z^{*}\right)}=\frac{\int_{x=z^{*}}^{\infty}(1-g(x)) d H(x)}{\bar{\zeta}_{z}\left(z^{*}\right) z^{*} h\left(z^{*}\right)} \tag{28}
\end{equation*}
$$

Consider now the effect of increasing the price $p$ by $d p$. The total welfare effect, written in terms of the marginal value of public funds, can be decomposed into the following components:

- Mechanical revenue effect: the reform mechanically raises revenue from each consumer by $d p \cdot s(\theta)$, for a total of $d p \bar{s}$.
- Mechanical welfare effect: As in the proof of Lemma 1, the reform mechanically reduces each consumer's net income by $d p \cdot s_{\theta}(\varepsilon)$. To isolate the mechanical effect, we compute the loss in welfare as if this reduction all comes from composite consumption $c$. Under Assumption 2 , and using the derivations in the proof of Lemma 1 , we can write this $d p \cdot \bar{s}_{\theta} g(\theta)$ up to negligible higher-order terms. Thus the total mechanical welfare effect is $-d p \mathbb{E}[s(z) g(z)]$
- Revenue effects of substitution. The reform changes costs by $C_{s}^{\prime} \frac{d \bar{s}}{d p} d p$, and changes earnings by $p \frac{d \bar{s}}{d p}$. The net effect is thus $\left(p-C_{s}^{\prime}\right) \bar{\zeta} \bar{s} d p$
- Bias-correcting effects of substitution: the reform causes each consumer to decrease their $s$ consumption by $d p \cdot \zeta(\theta) / p$. This generates a behavioral welfare effect equal to

$$
-d p \mathbb{E}\left[g(z) \gamma(z) \zeta_{p}(z) s(z)\right]=-d p \bar{\gamma}_{p}\left(1+\sigma_{p}\right) \bar{\zeta} \bar{s}
$$

- Effect on earnings: The reform causes a change in income tax revenue collected from type $\theta$ equal to $\frac{d z(\theta)}{d p} T^{\prime}(z(\theta))$. Lemma 1 implies that $\frac{d z(\theta)}{d p}=-\zeta_{z}(\theta)\left(\frac{z(\theta)}{1-T^{\prime}(z(\theta))}\right) \frac{\partial s(p, \boldsymbol{a}, z ; \theta)}{\partial z}$. By Assumption 6, this generates a total fiscal externality through the income tax equal to $-d p$. $\mathbb{E}\left[\frac{T^{\prime}(z)}{1-T^{\prime}(z)} \zeta_{z}(z) z s_{\text {inc }}^{\prime}(z)\right]$.
- Indirect effects on sin good consumption: The change in earnings affects consumption indirectly. However, relative to the other effects above, these effects are of order $\frac{\bar{s}_{\theta}}{z(\theta)-T(z(\theta))-p \bar{s}_{\theta}}$
and therefore negligible by Assumption 2 (formally, this is easily proven by extending the calculations of ALT in their proof of Proposition 1).

Combining these components, and taking into account that the income tax $T$ is set optimally, the total welfare effect of the price change is equal to

$$
\begin{align*}
\frac{d W}{d p} & =\mathbb{E}[s(z)(1-g(z))]+\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{p} \bar{s}-\bar{\gamma}\left(1+\sigma_{p}\right) \bar{\zeta}_{p} \bar{s}  \tag{29}\\
& -\mathbb{E}\left[\frac{T^{\prime}(z)}{1-T^{\prime}(z)} \zeta_{z}(z) z s_{\text {inc }}^{\prime}(z)\right]  \tag{30}\\
& =\mathbb{E}[s(z)(1-g(z))]+\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{p} \bar{s}-\bar{\gamma}_{p}\left(1+\sigma_{p}\right) \bar{\zeta}_{p} \bar{s} \\
& -\int_{z=z_{\text {min }}}^{\infty} \int_{x=z}^{\infty}(1-g(x)) h(x) d x s_{\text {inc }}^{\prime}(z) d z  \tag{31}\\
& =\mathbb{E}[s(z)(1-g(z))]+\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{p} \bar{s}-\bar{\gamma}_{p}\left(1+\sigma_{p}\right) \bar{\zeta}_{p} \bar{s} \\
& -\int_{z=z_{\text {min }}}^{\infty}(1-g(z)) s_{\text {inc }}(z) h(z) d z  \tag{32}\\
& =-\operatorname{Cov}\left[s_{\text {pref }}(z), g(z)\right]+\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{p} \bar{s}-\bar{\gamma}_{p}\left(1+\sigma_{p}\right) \bar{\zeta}_{p} \bar{s}
\end{align*}
$$

In the computations above, expression (31) follows from (28), while expression (32) follows from Lemma C.2. At the optimum, $\frac{d W}{d p}=0$, which implies the first-order condition

$$
\begin{equation*}
p-C_{s}^{\prime}=\bar{\gamma}_{p}\left(1+\sigma_{p}\right)-\frac{\operatorname{Cov}\left[s_{p r e f}(z), g(z)\right]}{\left|\bar{\zeta}_{p}\right| \bar{s}} \tag{33}
\end{equation*}
$$

Next consider the effects of increasing $a_{j}$. The total welfare effect, written in terms of the marginal value of public funds, can be decomposed into the following components:

- Mechanical welfare effect: The reform mechanically reduces changes consumers' perceived utility by $\kappa_{j}(\theta)$ dollars, and consumers' normative utility by $\kappa_{j}(\theta)-\rho_{j}(\theta)$ dollars. Thus the total mechanical welfare effect is $\mathbb{E}\left[\left(\bar{\kappa}_{j}(z)-\bar{\rho}_{j}(z)\right) g(z)\right] d a_{j}$
- Revenue effects. The reform changes costs by $C_{j}^{\prime} d a_{j}+C_{p}^{\prime} \frac{d \bar{s}}{d a_{j}} d a_{j}$, and changes earnings by $p \frac{d \bar{s}}{d a_{j}}$. The net effect is thus $\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{a_{j}} \bar{s} d a_{j}-C_{j}^{\prime} d a_{j}$
- Bias-correcting effects of substitution: The reform causes each consumer to decrease their $s$ consumption by $d p \cdot \zeta_{a_{j}}(\theta) / p$. This generates a behavioral welfare effect equal to

$$
-d a_{j} \mathbb{E}\left[g(z) \bar{\gamma}(z) \zeta_{a_{j}}(z) s(z)\right]=-d a_{j} \bar{\gamma}_{a_{j}}\left(1+\sigma_{a_{j}}\right) \bar{\zeta}_{a_{j}} \bar{s}
$$

- Effect on earnings: The reform causes a change in income tax revenue collected from type $\theta$ equal to $\frac{d z(\theta)}{d a_{j}} T^{\prime}(z(\theta))$. Lemma 1 implies that $\frac{d z(\theta)}{d a_{j}}=\zeta_{z}(\theta)\left(\frac{z(\theta)}{1-T^{\prime}(z(\theta))}\right) \frac{\partial k_{j}(p, \boldsymbol{a}, z ; \theta)}{\partial z}$. By Assumption 6, this generates a total fiscal externality through the income tax equal to $d a_{j}$. $\mathbb{E}\left[\frac{T^{\prime}(z)}{1-T^{\prime}(z)} \zeta_{z}(z) z \kappa_{j, i n c}^{\prime}(z)\right]$.
- Indirect effects on sin good consumption: The change in earnings affects consumption indirectly. However, relative to the other effects above, these effects are of order $\frac{\bar{s}_{\theta}}{z(\theta)-T(z(\theta))-p \bar{s}_{\theta}}$ and therefore negligible by Assumption 2 (formally, this is easily proven by extending the calculations of ALT in their proof of Proposition 1).

Combining these components, and taking into account that the income tax $T$ is set optimally, the total welfare effect of the price change is equal to

$$
\begin{align*}
\frac{d W}{d a_{j}} & =\mathbb{E}\left[\left(\kappa_{j}(z)-\rho_{j}(z)\right) g(z)\right]+\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{a_{j}} \bar{s}-C_{j}^{\prime}-\bar{\gamma}_{a_{j}}\left(1+\sigma_{a_{j}}\right) \bar{\zeta}_{a_{j}} \bar{s}  \tag{34}\\
& +\mathbb{E}\left[\frac{T^{\prime}(z)}{1-T^{\prime}(z)} \zeta_{z}(z) z \kappa_{j, i n c}^{\prime}(z)\right]  \tag{35}\\
& =\mathbb{E}\left[\left(\kappa_{j}(z)-\rho_{j}(z)\right) g(z)\right]+\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{a_{j}} \bar{s}-C_{j}^{\prime}-\bar{\gamma}_{a_{j}}\left(1+\sigma_{a_{j}}\right) \bar{\zeta}_{a_{j}} \bar{s} \\
& +\int_{z=z_{\min }}^{\infty} \int_{x=z}^{\infty}(1-g(x)) h(x) d x \kappa_{j, i n c}^{\prime}(z) d z \\
& =\mathbb{E}\left[\left(\kappa_{j}(z)-\rho_{j}(z)\right) g(z)\right]+\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{a_{j}} \bar{s}-C_{j}^{\prime}-\bar{\gamma}_{a_{j}}\left(1+\sigma_{a_{j}}\right) \bar{\zeta}_{a_{j}} \bar{s} \\
& +\int_{z=z_{\min }}^{\infty}(1-g(z)) \kappa_{j, i n c}(z) h(z) d z \\
& =\bar{\kappa}_{j}-\bar{\rho}_{j}+\operatorname{Cov}\left[\left(\kappa_{j, p r e f}(z)-\rho_{j}(z)\right), g(z)\right]+\left(p-C_{s}^{\prime}\right) \bar{\zeta}_{a_{j}} \bar{s}-C_{j}^{\prime}-\bar{\gamma}_{a_{j}}\left(1+\sigma_{a_{j}}\right) \bar{\zeta}_{a_{j}} \bar{s}
\end{align*}
$$

At the optimum $\frac{d W}{d a_{j}}=0$ if $a_{j}>0$ and $\frac{d W}{d a_{j}}<0$ if $a_{j}=0$, which implies the first-order condition

$$
\bar{\kappa}_{j}-\bar{\rho}_{j}+\operatorname{Cov}\left[\left(\kappa_{j, p r e f}(z)-\rho_{j}(z)\right), g(z)\right] \leq \bar{\gamma}_{a_{j}}\left(1+\sigma_{p}\right)\left|\bar{\zeta}_{a_{j}}\right| \bar{s}-\left(p-C_{s}^{\prime}\right)\left|\bar{\zeta}_{a_{j}}\right|+C_{j}^{\prime}
$$

with equality when $a_{j}>0$. Rearranging gives the second condition in the Proposition.
Finally, note that the first-order conditions implied by (29)-(30) and (34)-(35) also allow us to characterize the optimal $p$ and $\boldsymbol{a}$ even when the income tax is not optimal.

## D Aggregate Lottery Demand Appendix

## Table A1: First Stages for Prize Semi-Elasticity Estimates

(a) Jackpot Semi-Elasticity: Nationwide Data

|  | $(1)$ |
| :--- | :---: |
|  | Jackpot |
|  | EV $(\$)$ |
| Jackpot expected value forecast (\$) | $1.0884^{* * *}$ |
|  | $(0.0466)$ |
| F-statistic | 545 |
| $R^{2}$ | 0.96 |
| Observations | 2,035 |

(b) Jackpot and Second Prize Semi-Elasticities: California Data

|  | $(1)$ <br> Jackpot | 2nd prize <br> EV $(\$)$ |
| :--- | :---: | :---: |
| Jackpot expected value forecast (\$) | $1.0968^{* * *}$ | $0.1475^{* * *}$ |
|  | $(0.0533)$ | $(0.0339)$ |
| 2nd prize expected value forecast (\$) | -0.0162 | $1.0281^{* * *}$ |
|  | $(0.0175)$ | $(0.0300)$ |
| F-statistic | 6,875 | 1,183 |
| $R^{2}$ | 0.96 | 0.81 |
| Observations | 1,701 | 1,701 |

Notes: This table presents first stage estimates of equation (11). The first stages regress prize expected values on a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period, controlling for the expectation of the forecast prior to the realization of the rollover outcome as well as game-format, game-regional coverage, game-quarter of sample, and game-weekend fixed effects. Panel (a) uses nationwide data, while Panel (b) uses California data only. The samples include all Mega Millions and Powerball drawings from June 2010 to February 2020; the sample in Panel (b) is smaller because California did not join Powerball until April 2013. Newey-West standard errors allowing up to ten lags are in parentheses. ${ }^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

# Table A2: Prize Semi-Elasticity Estimates with Alternative Standard Errors 

| (a) Jackpot Semi-Elasticity: |  | National Sales |
| :--- | :---: | :---: |
|  | $(1)$ | $(2)$ |
|  | OLS | IV |
| Jackpot expected value (\$) | 1.8648 | 1.7277 |
|  | $(0.0577)$ | $(0.0615)$ |
|  | $(0.0576)$ | $(0.0619)$ |
|  | $(0.0588)$ | $(0.0626)$ |
|  | $(0.0546)$ | $(0.0603)$ |
|  | $(0.0574)$ | $(0.0611)$ |
|  | $(0.0563)$ | $(0.0591)$ |
| Observations | 2,035 | 2,035 |

(b) Jackpot and Second Prize Semi-Elasticities: California Sales Only

|  | OLS | (1) |
| :--- | :---: | :---: |
|  | IV |  |
| Jackpot expected value (\$) | 2.0154 | 1.8466 |
|  | $(0.0764)$ | $(0.0837)$ |
|  | $(0.0776)$ | $(0.0852)$ |
|  | $(0.0805)$ | $(0.0862)$ |
|  | $(0.0744)$ | $(0.0841)$ |
|  | $(0.0789)$ | $(0.0842)$ |
|  | $(0.0789)$ | $(0.0807)$ |
| 2nd prize expected value (\$) | -0.0202 | 0.0837 |
|  | $(0.0587)$ | $(0.0586)$ |
|  | $(0.0590)$ | $(0.0554)$ |
|  | $(0.0613)$ | $(0.0504)$ |
|  | $(0.0576)$ | $(0.0546)$ |
|  | $(0.0555)$ | $(0.0518)$ |
|  | $(0.0565)$ | $(0.0413)$ |
| Observations | 1,705 | 1,701 |

Notes: This table presents estimates of equation (11), a regression of the natural $\log$ of sales on prize expected values, controlling for game-format, game-regional coverage, game-quarter of sample, and gameweekend fixed effects. The IV regressions instrument for prize expected values with a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period and additionally control for the expectation of the forecast prior to the realization of the rollover outcome. Panel (a) uses nationwide sales, while Panel (b) uses California sales only. The samples include all Mega Millions and Powerball drawings from June 2010 to February 2020; the sample in Panel (b) is smaller because California did not join Powerball until April 2013. The standard errors, in order from top to bottom, are Newey-West with five lags, Newey-West with ten lags, Newey-West with twenty-five lags, robust standard errors clustered by game and month, robust standard errors clustered by game and quarter, and robust standard errors clustered by game and half year.

Figure A3: Prize Expected Values Accounting for Prize Sharing


Notes: This figure presents "simplified" and "actual" expected values of the Powerball jackpot and California second prize for each drawing in 2014. The "simplified" expected value is the product of the win probability and prize amount. The "actual" expected value approximates the expected value accounting for the possibility of prize sharing among multiple winners, assuming players select ticket numbers randomly.

## Figure A4: Actual versus Predicted Jackpot and California Second Prize Rollovers



Notes: This figure presents a binned scatter plot of the predicted probability of a rollover assuming that players select the numbers on their tickets randomly and an indicator for observed rollovers. A dashed 45degree line is included for reference. Panel (a) uses nationwide jackpot data, while Panel (b) uses California second prize data only. The sample includes all Mega Millions and Powerball drawings from June 2010 through February 2020.

## D. 1 Substitution Across Games

## D.1.1 Substitution in Response to Jackpot Variation

Our modeling considers only one good subject to behavioral bias. If changes in prices or attributes cause substitution to other goods subject to behavioral bias, optimal policy would have to account for this (Allcott, Lockwood, and Taubinsky 2019). To test for substitution between Mega Millions and Powerball, we re-estimate equation (11) with two changes. First, the dependent variable is in levels instead of logs, which allows us to easily construct a diversion ratio by dividing regression coefficients. Second, we add the jackpot for game $-j$. The regression is

$$
\begin{equation*}
\bar{s}_{j t}=\bar{\zeta}_{j} \pi_{1 j t} w_{1, j, t}+\bar{\zeta}_{c} \pi_{1,-j, t} w_{1,-j, t}+\beta_{j} \pi_{1 j t} \bar{Z}_{1, j, t}+\beta_{c} \pi_{1,-j, t} \bar{Z}_{1,-j, t}+\xi_{j t}+\epsilon_{j t} . \tag{36}
\end{equation*}
$$

As before, we instrument for $\pi_{1 j t} w_{1 j t}$ and $\pi_{1,-j, t} w_{1,-j, t}$ with $\pi_{1 j t} Z_{1 j t}$ and $\pi_{1,-j, t} Z_{1,-j, t}$, and we control for $\pi_{1 j t} \bar{Z}_{1 j t}$ and $\pi_{1,-j, t} \bar{Z}_{1,-j, t}$ in order to isolate random variation in jackpot amounts. Mega Millions draws are on Tuesday and Friday, while Powerball draws are on Wednesday and Saturday. We define $t$ by matching the Tuesday-Wednesday draws and Friday-Saturday draws for a given week.

Panel (a) of Appendix Table A3 presents OLS and IV estimates. Column 2 shows that when a game's jackpot expected value increases by $\$ 1$, that game's sales increase by 107.14 million tickets. However, the other game's ticket sales are statistically unaffected, and the 95 percent confidence intervals exclude effects larger than about 7.9 million tickets.

We can also estimate substitution to lottery games other than the multi-state games. To do that, we collapse the balanced panel of games in the La Fleur's data to the nationwide weekly level. Now let $\bar{s}_{j t}$ be sales in units of dollars, let $\pi_{1 t} w_{1 t}$ denote the average jackpot expected value across the four draws of Mega Millions and Powerball in week $t$. The regression is

$$
\begin{equation*}
\bar{s}_{j t}=\bar{\zeta}_{c} \pi_{1 t} w_{1 t}+\beta_{c} \pi_{1 j, t} \bar{Z}_{1 j, t}+\xi_{j t}+\epsilon_{j t} . \tag{37}
\end{equation*}
$$

We instrument for $\pi_{1 t} w_{1 t}$ with a weekly version of the rollover instrument and only include the weekly version of the pre-rollover expectation of the instrument $\pi_{1 j, t} \bar{Z}_{1 j, t}$ in our IV regressions. ${ }^{31}$ In these weekly data, $\xi_{j t}$ represents quarter-of-sample fixed effects and 52 week-of-year fixed effects, which we have found to improve precision by soaking up seasonality.

Panel (b) of Appendix Table A3 presents the OLS estimates. The IV estimates are very similar; see Appendix Table A4. Each column considers sales of different games. Column 1 shows that when the average Mega Millions and Powerball jackpot expected values increase by $\$ 1$, their combined weekly ticket sales increase by $\$ 500.36$ million. Column 2 shows that sales of 13 major state-level draw games increase by $\$ 1.79$ million, suggesting statistically significant but economically small

[^14]complementarity. ${ }^{32}$ Columns 3 and 4 show no statistically significant effects on instant games and on the combination of all games other than Mega Millions and Powerball. The 95 percent confidence interval in column 4 rules out that a $\$ 1$ increase in the Mega Millions and Powerball jackpots increases sales of other games by more than $\$ 9.6$ million or decreases sales by more than $\$ 11.9$ million, implying economically very limited substitution. Appendix Figure A5 presents visual examples of these null effects for the 2014 data, paralleling Figure 1.

Table A3: Cross-Game Substitution
(a) Mega Millions and Powerball

|  | $(1)$ | $(2)$ |
| :--- | :---: | :---: |
|  | OLS | IV |
| Own game jackpot | $108.32^{* * *}$ | $107.14^{* * *}$ |
| expected value (\$) | $(14.62)$ | $(16.76)$ |
| Other game jackpot | 0.06 | 1.94 |
| expected value (\$) | $(2.44)$ | $(3.01)$ |
| Observations | 2,035 | 2,035 |
| Dependent variable mean | 23.0 | 23.0 |

(b) Different Game Types

|  | $(1)$ <br> Mega Millions <br> \& Powerball | $(2)$ <br> Major state <br> draw games | $(3)$ <br> Instant <br> games | $(4)$ <br> Other state- <br> level games |
| :--- | :---: | :---: | :---: | :---: |
| Jackpot expected value (\$) | $500.36^{* * *}$ | $1.79^{* * *}$ | 0.01 | -1.16 |
|  | $(74.03)$ | $(0.32)$ | $(4.99)$ | $(5.47)$ |
| Observations | 508 | 508 | 508 | 508 |
| Dependent variable mean | 108.2 | 12.4 | 509.5 | 669.2 |

Notes: Panel (a) of this table presents estimates of equation (36), a regression of the level of sales of a multi-state game on the prize expected values of both multi-state games, controlling for game-format, gameregional coverage, game-week, weekend, and quarter-of-sample fixed effects, using nationwide game-by-draw data. The IV regression instruments for prize expected values with a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period and additionally controls for the expectations of the forecasts prior to the realization of rollover outcomes. Panel (b) presents estimates of equation (37), a regression of the aggregate level of sales of the lottery games indicated in each column on the week-average prize expected values of Mega Millions and Powerball, controlling for week and quarter-ofsample fixed effects, using a balanced panel of nationwide game-by-week data. Newey-West standard errors allowing up to ten lags are in parentheses. Sales are in millions of tickets in Panel (a) and millions of dollars in Panel (b). ${ }^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

[^15]Table A4: Cross-Game Substitution: IV

|  | $(1)$ <br> Mega Millions <br> \& Powerball | $(2)$ <br> Major state <br> draw games | $(3)$ <br> Instant <br> games | $(4)$ <br> Other state- <br> level games |
| :--- | :---: | :---: | :---: | :---: |
| Jackpot expected value (\$) | $470.69^{* * *}$ | $1.68^{* * *}$ | -9.26 | -13.96 |
|  | $(106.77)$ | $(0.49)$ | $(13.01)$ | $(14.51)$ |
| Observations | 508 | 508 | 508 | 508 |
| Dependent variable mean | 108.2 | 12.4 | 509.5 | 669.2 |

Notes: This table presents estimates of equation (37), a regression of the aggregate level of sales in millions of dollars of the lottery games indicated in each column on the week-average prize expected values of Mega Millions and Powerball, using a balanced panel of nationwide game-by-week data. Week-average prize expected values are instrumented with week averages of a forecast based on the previous period prize amount and an indicator for whether the prize was won in the previous period. Controls for the week averages of the expectation of the forecast prior to the realization of each rollover outcomes as well as game-week and quarter-of-sample fixed effects are included. Newey-West standard errors allowing up to ten lags are in parentheses. ${ }^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

## Figure A5: Powerball Prizes and Other Games Ticket Sales in 2014



Notes: This figure presents the expected values of the Powerball jackpot, the natural log of Powerball and Mega Millions California ticket sales for each drawing in 2014, and the natural log of aggregate sales from a balanced panel of all other California game-by-week data. The levels of sales are adjusted so that the average natural log of sales is zero.

## D.1.2 Substitution in Response to Price Variation

We can also test for substitution by estimating the effect of game $j$ 's price change on other games. To do this, we estimate an analogue to equation (12), except with the substitute game's sales level on the left-hand side, a control for the substitute game's jackpot expected value, and a vector of week-of-year and event fixed effects collectively denoted $\xi_{t}$ :

$$
\begin{equation*}
\bar{s}_{-j t}=\bar{\zeta}_{p} p_{j t} W_{j t}+\beta_{1} W_{j t}\left(+\beta_{2} \pi_{1-j t} w_{1-j t}\right)+\xi_{t}+\epsilon_{j t} \tag{38}
\end{equation*}
$$

The substitute game expected value control $\pi_{1-j t} w_{1-j t}$ is used only to study substitution to the other major multi-state game, not when we estimate substitution to other games in the La Fleur's data. Table A5 presents results.

Table A5: Cross-Price Demand Responses

|  | $(1)$ <br> Own-price <br> response | $(2)$ <br> Other multi- <br> state game | $(3)$ <br> Major state <br> draw games | $(4)$ <br> All other <br> games |
| :--- | :---: | :---: | :---: | :---: |
| Price $\times$ 12-month window | $-19.90^{* * *}$ | 5.84 | -0.03 | 7.85 |
|  | $(4.47)$ | $(6.70)$ | $(0.37)$ | $(11.52)$ |
| 12-month window | $27.76^{* * *}$ | -4.75 | 0.28 | -7.91 |
|  | $(7.46)$ | $(9.63)$ | $(0.61)$ | $(18.42)$ |
| Jackpot expected value (\$) | $119.42^{* * *}$ | $167.40^{* * *}$ |  |  |
|  | $(14.27)$ | $(40.56)$ |  |  |
| Observations | 312 | 312 | 312 | 312 |
| Dependent variable mean | 37.8 | 38.5 | 12.6 | 650.5 |

Notes: This table presents estimates of equation (38), a regression of the aggregate level of sales of the lottery games indicated in each column header on the ticket price of a multi-state game interacted with an indicator for the 12 -month window-six months before and six months after-around a price change event for that multi-state game and the 12-month window indicator, controlling for week fixed effects and a price-change event fixed effect. Columns 1 and 2 also include controls for the week-average jackpot expected value of the game indicated in the column header. Each column pools data from a 36 -month window around both the Powerball and Mega Millions price changes, using a balanced panel of nationwide game-by-week data. Newey-West standard errors allowing up to ten lags are in parentheses. Sales are in millions of tickets in columns 1-2 and millions of dollars in columns $3-4 .^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

## D. 2 Long-Run vs. Short-Run Elasticity

As in many other studies, we have a well-identified short-run elasticity, but our policy analysis requires a long-run elasticity. Consider two models in which these elasticities might differ. First, consumption might be substitutable or complementary over time, e.g. if previous purchases cause people to tire or get excited in the future. Second, consumers might have a desired average spending (e.g. $\$ \mathrm{X}$ per month) that they allocate across draws to maximize expected value. In the limiting case, demand might be fully inelastic to the average jackpot level but highly elastic to variation across draws.

To address these issues, we test for effects of lagged jackpot amounts and also aggregate over time. Define a "jackpot spell" as a group of draws beginning after a jackpot is won and continuing until the next win. The sawtooth pattern in Figures 1 and A5 illustrates that while the length of each jackpot spell varies, these spells are well-defined units of analysis that capture the variation we want to use. We collapse the data to the average $\ln \bar{s}_{j t}$ and average $\pi_{k j t} w_{k j t}$ over each of the 196 complete jackpot spells in our sample, which are 4.7 weeks long on average. Now using $t$ to index jackpot spells, we estimate an analogue of equation (11) including lags indexed by $l$ :

$$
\begin{equation*}
\ln \bar{s}_{j t}=\sum_{l=0}^{L} \bar{\zeta}_{1 l} \pi_{1 j, t-l} w_{1 j, t-l}+\sum_{l=0}^{L} \beta_{1 l} \pi_{1 j, t-l} \bar{Z}_{1 j, t-l}+\xi_{j t}+\epsilon_{j t}, \tag{39}
\end{equation*}
$$

where the fixed effects $\xi_{j t}$ are now game-format, game-regional coverage, and game-year of sample fixed effects based on the first draw in the spell. We instrument for $\pi_{1 j t} w_{1 j t}$ with a forecast of the spell midpoint jackpot expected value using the reset value $\underline{w}_{1 j f(t)}$, the average percent increase $\iota_{1 j f(t)}$, and the number of rollovers $R_{j t}$ in the spell: $\left(1+\iota_{1 j f(t)}\right)^{R_{j t} / 2} \cdot \pi_{1 j t} \underline{w}_{1 j f(t)}$. In our IV regressions, we control for the expectation of the spell midpoint jackpot expected value conditional on $\leq \bar{R}$ rollovers in the spell, where $\bar{R}$ is the observed number of rollovers in the spell, and assuming players randomly select the numbers on their tickets: $\left(1+\iota_{1 j f(t)}\right)^{\mathbb{E}\left[R_{j t} / 2 \mid R_{j t} \leq \bar{R}\right]} \cdot \pi_{1 j t} \underline{w}_{1 j f(t)}$. The first stages are strong, and the jackpot expected value forecast instrument for lag $t-l$ strongly predicts the actual jackpot expected value for lag $t-l$ but not for other lags; see Appendix Table A7.

Table A6 presents results. Columns 1 and 2 present the OLS and IV estimates with no lags, while columns 3 and 4 add three lags. In case aggregating to jackpot spells is still not enough to identify a long-run elasticity, columns 5 and 6 present estimates after aggregating to groups of three jackpot spells. There are 64 complete three-spell groups in our sample, which are 15.2 weeks long on average.

The contemporaneous effects in columns 1, 2, 5, and 6 are comparable to the draw-level estimates from Table 3. The lag coefficients in columns 3 and 4 imply intertemporal complementarity: higher jackpots in recent prior spells cause higher demand now, with an effect that decays toward zero by the third lag. The OLS and IV estimates are similar, suggesting limited simultaneity bias.

Table A6: Intertemporal Substitution

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | OLS | IV | OLS | IV | OLS | IV |
| Jackpot expected value, | $1.6699^{* * *}$ | $2.0158^{* * *}$ | $1.7178^{* * *}$ | $2.0001^{* * *}$ | $1.7928^{* * *}$ | $2.6474^{* * *}$ |
| $t(\$)$ | $(0.0797)$ | $(0.1133)$ | $(0.0943)$ | $(0.1075)$ | $(0.2589)$ | $(0.4104)$ |
| Jackpot expected value, |  |  | $0.2831^{* * *}$ | $0.7027^{* * *}$ |  |  |
| $t-1(\$)$ |  |  | $(0.0697)$ | $(0.1259)$ |  |  |
| Jackpot expected value, |  |  | $0.1624^{* *}$ | $0.3870^{* * *}$ |  |  |
| $t-2(\$)$ |  |  | $(0.0665)$ | $(0.1480)$ |  |  |
| Jackpot expected value, |  |  | $0.0890^{* *}$ | 0.0542 |  |  |
| $t-3(\$)$ |  |  | $(0.0366)$ | $(0.0745)$ |  |  |
| $R^{2}$ | 0.87 | 0.89 | 0.88 | 0.92 | 0.74 | 0.81 |
| Observations | 193 | 191 | 187 | 185 | 61 | 61 |

Notes: This table presents estimates of equation (39), a regression of the average natural $\log$ of sales on contemporaneous and lagged average jackpot expected values, controlling for game-format, game-regional coverage, and game-year of sample fixed effects based on the first draw in the period of interest. In columns $1-4$, we use nationwide game-by-jackpot spell data, where a jackpot spell is defined as a group of draws beginning after a jackpot is won and continuing until the next win. In columns 5-6, we use nationwide game-by-three-spell data, averaging across series of three jackpot spells. The sample includes all complete Mega Millions and Powerball jackpot spells from June 2010 to February 2020. Observation counts exclude singletons. Newey-West standard errors allowing up to ten lags are in parentheses. ${ }^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

Table A7: First Stages for Intertemporal Substitution Estimates

|  | (1) <br> Spell <br> jackpot <br> EV, $t$ | (2) Spell jackpot EV, $t$ | (3) Spell jackpot EV, $t-1$ | (4) Spell jackpot EV, $t-2$ | (5) Spell jackpot EV, $t-3$ | (6) 3 -spell jackpot EV, $t$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Jackpot expected value midpoint forecast, $t$ (\$) | $\begin{gathered} 0.9462^{* * *} \\ (0.0876) \end{gathered}$ | $\begin{gathered} 1.0234^{* * *} \\ (0.2586) \end{gathered}$ | $\begin{aligned} & 0.1431^{* *} \\ & (0.0616) \end{aligned}$ | $\begin{gathered} 0.0450 \\ (0.1087) \end{gathered}$ | $\begin{aligned} & -0.0627 \\ & (0.1298) \end{aligned}$ | $\begin{gathered} 1.6058^{* * *} \\ (0.4377) \end{gathered}$ |
| Jackpot expected value midpoint |  | 0.1900* | $1.0700^{* * *}$ | 0.1201 | 0.0499 |  |
| forecast, $t-1$ (\$) |  | (0.1059) | (0.2803) | (0.0839) | (0.0953) |  |
| Jackpot expected value midpoint |  | 0.0698 | 0.1052 | $1.1054^{* * *}$ | 0.1360 |  |
| forecast, $t-2$ (\$) |  | (0.0657) | (0.1218) | (0.2730) | (0.1030) |  |
| Jackpot expected value midpoint |  | 0.0501 | 0.0659 | 0.0693 | 1.1240*** |  |
| forecast, $t-3$ (\$) |  | (0.0814) | (0.0925) | (0.1229) | (0.2749) |  |
| F-statistic | 16.3 | 25.8 | 18.1 | 57.4 | 28.4 | 13.5 |
| $R^{2}$ | 0.69 | 0.79 | 0.80 | 0.79 | 0.71 | 0.79 |
| Observations | 193 | 185 | 185 | 185 | 185 | 61 |

Notes: This table presents first stage estimates of equation (39). The first stages regress average jackpot expected values on forecasts of the midpoint of the jackpot expected value within a jackpot spell or averaged across a three-spell period based on the number of rollovers during the period as well as the expected value of the jackpot when it resets and the average percent jackpot increase after a rollover corresponding to the game-format of the first draw in the period of interest. Controls for the approximate expectation of the expected value midpoint forecasts as well as game-format, game-regional coverage, and game-year of sample fixed effects based on the first draw in the period of interest are included. In columns 1-5, we use nationwide game-by-jackpot spell data, where a jackpot spell is defined as a group of draws beginning after a jackpot is won and continuing until the next win. In column 6, we use nationwide game-by-three-spell data, averaging across series of three jackpot spells. The sample includes all complete Mega Millions and Powerball jackpot spells from June 2010 to February 2020. Observation counts exclude singletons. Newey-West standard errors allowing up to ten lags are in parentheses. ${ }^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

## D. 3 Format Change Appendix

Table A8: Placebo Tests: Non-Price Format Change Event Studies

|  | $(1)$ <br> Mega <br> Millions | $(2)$ <br> Mega <br> Millions | $(3)$ <br> Miega <br> Millions | $(4)$ <br> Powerball | $(5)$ <br> Powerball | $(6)$ <br> Powerball |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Post format change $\times$ 12-month | $0.3273^{*}$ | $0.1557^{* * *}$ | -0.0081 | 0.3352 | $0.2947^{* * *}$ | 0.0551 |
| window | $(0.1811)$ | $(0.0498)$ | $(0.0496)$ | $(0.2466)$ | $(0.0658)$ | $(0.0760)$ |
| 12-month window | 0.0137 | $-0.1140^{* * *}$ | -0.0101 | $-0.1912^{*}$ | $-0.1544^{* * *}$ | -0.0471 |
|  | $(0.1024)$ | $(0.0341)$ | $(0.0375)$ | $(0.1054)$ | $(0.0268)$ | $(0.0377)$ |
| Jackpot expected value (\$) |  | $2.2831^{* * *}$ |  |  | $1.7424^{* * *}$ |  |
|  |  | $(0.1056)$ |  | $0.0084^{* * *}$ |  | $(0.0783)$ |
| Jackpot (\$millions) |  |  | $(0.0003)$ |  |  | $0.0060^{* * *}$ |
|  | 313 | 313 | 313 | 313 | 313 | 313 |
| Observations |  |  |  |  |  |  |

Notes: This table presents estimates of a regression of the natural $\log$ of sales on an indicator for a postformat change period interacted with an indicator for the 12 -month window-six months before and six months after-around a game-format change event and the 12 -month window indicator, controlling for weekend fixed effects in all columns, jackpot expected value in columns 2 and 5 , and jackpot amounts in columns 3 and 6 . In columns 1-3, the sample includes all Mega Millions draws in a 36 -month window around the October 19, 2013 format change event. In columns 4-6, the sample includes all Powerball draws in a 36 -month window around the October 7, 2015 format change event. We use Newey-West standard errors with up to ten lags. ${ }^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

## E Survey Appendix

## E. 1 AmeriSpeak Survey Question Text

| Variable | Question text |
| :---: | :---: |
| Spending and income effects |  |
| Monthly lottery spending | How many dollars did you spend in total on lottery tickets in an average month in 2019? |
| Income change | How much income did you earn in 2019 compared to 2018? In 2019, I earned ... [Less than half as much, $25 \%$ to $50 \%$ less, $10 \%$ to $25 \%$ less, $5 \%$ to $10 \%$ less, $1 \%$ to $5 \%$ less, The exact same amount, $1 \%$ to $5 \%$ more, $5 \%$ to $10 \%$ more, $10 \%$ to $25 \%$ more, $25 \%$ to $50 \%$ more, Over $50 \%$ more] |
| Spending change | How much money did you spend in total on lottery tickets in 2019 compared to 2018 ? In 2019, I spent . . [Less than half as much, $25 \%$ to $50 \%$ less, $10 \%$ to $25 \%$ less, $5 \%$ to $10 \%$ less, $1 \%$ to $5 \%$ less, The exact same amount, $1 \%$ to $5 \%$ more, $5 \%$ to $10 \%$ more, $10 \%$ to $25 \%$ more, $25 \%$ to $50 \%$ more, Over $50 \%$ more] |


| Self-reported | Imagine you got a raise and your income doubled. How do you think your |
| :--- | :--- |
| income effect | lottery spending would change? I would spend ... [Less than half as much, |
| $25 \%$ to $50 \%$ less, $10 \%$ to $25 \%$ less, $5 \%$ to $10 \%$ less, $1 \%$ to $5 \%$ less, The exact |  |
|  | same amount, $1 \%$ to $5 \%$ more, $5 \%$ to $10 \%$ more, $10 \%$ to $25 \%$ more, $25 \%$ to |
|  | $50 \%$ more, Over $50 \%$ more] |


|  | Preferences |
| :---: | :---: |
| Unwillingness to take risks | In general, how willing or unwilling are you to take risks? [1 Very unwilling, $2,3,4,5,6,7$ Very willing] |
| Financial risk aversion | Which of the following statements comes closest to the amount of financial risk that you are willing to take when you save or make investments? [Substantial financial risks expecting to earn substantial returns, Above-average financial risks expecting to earn above-average returns, Average financial risks expecting to earn average returns, No financial risks] |
| Lottery seems fun | To what extent do you agree or disagree with the following statement: For me, playing the lottery seems fun. [-3 Strongly disagree, $-2,-1,0$ Neutral, 1, 2, 3 Strongly agree] |
| Enjoy thinking about winning | To what extent do you agree or disagree with the following statement: I enjoy thinking about how life would be if I won the lottery. [-3 Strongly disagree, -2 , -1, 0 Neutral, 1, 2, 3 Strongly agree] |

## Bias proxies

Self-control problems

Financial illiteracy

It can be hard to exercise self-control, and some people feel that there are things they do too much or too little - for example, exercise, save money, or eat junk food. Do you feel like you play the lottery too little, too much, or the right amount? [-3 Far too little, $-2,-1,0$ The right amount, 1, 2, 3 Far too much]

Normally, which asset displays the highest fluctuations over time? [Savings accounts, Bonds, Stocks]

When an investor spreads her money among different assets, does the risk of losing money: [Increase, Decrease, Stay the same]

A second hand car dealer is selling a car for $\$ 6,000$. This is two-thirds of what it cost new. How much did the car cost new? [\$_]

If 5 people all have the winning numbers in the lottery and the prize is $\$ 2$ million, how much will each of them get? [\$_]

Let's say you have $\$ 200$ in a savings account. The account earns $10 \%$ interest per year. How much will you have in the account at the end of two years? [\$-]

Suppose you had $\$ 100$ in a savings account and the interest rate was $2 \%$ per year. After 5 years, how much do you think you would have in the account if you left the money to grow? [More than $\$ 102$, Exactly $\$ 102$, Less than \$102]

Imagine that the interest rate on your savings account was $1 \%$ per year and inflation was $2 \%$ per year. After 1 year, how much would you be able to buy with the money in this account? [More than today, Exactly the same as today, Less than today]

Do you think that the following statement is true or false? "Buying a single company stock usually provides a safer return than a stock mutual fund." [True, False]

For the next few questions, imagine flipping a coin that has a $50 \%$ chance of landing heads and a $50 \%$ chance of landing tails. Imagine that after eight flips, you observe the patterns described in the table below. What is the probability, in percent from $0-100$, that the next flip is tails? [tails-tails-tails-heads-tails-heads-heads-heads _\%, heads-heads-heads-heads-heads-heads-heads-heads _\%, heads-tails-heads-tails-tails-tails-tails-tails _\%]

Now imagine starting over and flipping a coin 1000 times. What are the chances, in percent from $0-100$, that the total number of heads will lie within the following ranges? [Between 481 and 519 heads _\%, Between 450 and 550 heads _\%, Between 400 and 600 heads _\%]

Now we are going to ask you how much people might win from different lotteries. For each lottery described in the table below, please give us your best estimate of what percent (from 0-100) of the lottery revenues are returned to the winners. [Tickets cost $\$ 1$, and 1 out of every 10 tickets wins $\$ 10$. $\%$, Tickets cost $\$ 1$, and 1 out of every 1,000 tickets wins $\$ 500$. $\%$, Tickets cost $\$ 1$, 1 out of every $400,000,000$ tickets wins $\$ 200,000,000$, and 1 out of every 1,000 tickets wins $\$ 100$. $\%$, Tickets cost $\$ 1$, and 1 out of every $300,000,000$ tickets wins $\$ 200,000,000$. $\%$ ]

Overconfidence

Expected returns

Predicted life satisfaction

Imagine you could keep buying whatever lottery tickets you want, over and over for a very long time. For every $\$ 1000$ you spend, how much do you think you would win back in prizes, on average? [ $\$ 0$ to $\$ 99, \$ 100$ to $\$ 199, \$ 200$ to $\$ 299, \$ 300$ to $\$ 399, \$ 400$ to $\$ 499, \$ 500$ to $\$ 599, \$ 600$ to $\$ 699, \$ 700$ to $\$ 799$, $\$ 800$ to $\$ 899, \$ 900$ to $\$ 999, \$ 1000$ to $\$ 1499, \$ 1500$ to $\$ 1999, \$ 2000$ to $\$ 5000$, More than \$5000]

Imagine that the average lottery player in the country could keep buying whatever lottery tickets they want, over and over for a very long time. For every $\$ 1000$ they spend, how much do you think they would win back in prizes, on average? [ $\$ 0$ to $\$ 99, \$ 100$ to $\$ 199, \$ 200$ to $\$ 299, \$ 300$ to $\$ 399, \$ 400$ to $\$ 499, \$ 500$ to $\$ 599, \$ 600$ to $\$ 699, \$ 700$ to $\$ 799, \$ 800$ to $\$ 899, \$ 900$ to $\$ 999$, $\$ 1000$ to $\$ 1499, \$ 1500$ to $\$ 1999, \$ 2000$ to $\$ 5000$, More than $\$ 5000$ ]

Think about the total amount of money spent on lottery tickets nationwide. What percent do you think is given out in prizes? [0-9\%, 10-19\%, $20-$ $29 \%, 30-39 \%, 40-49 \%, 50-59 \%, 60-69 \%, 70-79 \%, 80-89 \%, 90-100 \%$ ]

A recent study surveyed Swedish lottery winners. A typical person in the study had won between $\$ 100,000$ and $\$ 800,000$ in the lottery about 12 years before the survey. The study compared people who had won more vs. less money to determine the effect of additional lottery winnings.

The survey asked the following question about life satisfaction: "Taking all things together in your life, how satisfied would you say that you are with your life these days?" People responded on a scale from 0 ("Extremely dissatisfied") to 10 ("Extremely satisfied"). The average response was 7.21 out of 10 .

Do you think lottery winnings increased life satisfaction, decreased life satisfaction, or had exactly zero effect? [Increased life satisfaction, Decreased life satisfaction, Had exactly zero effect]

By how much do you think an additional $\$ 100,000$ in lottery winnings [increased/decreased] average life satisfaction on the 0-10 scale?

## E. 2 Additional Tables and Figures from AmeriSpeak Survey

Table A10: Descriptive Statistics: 2021 Survey Data

|  | Obs. | Mean | Std. dev. | Min | Max |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Unwillingness to take risks | 2,124 | -3.96 | 1.37 | -7 | -1 |
| Financial risk aversion | 2,124 | 3.04 | 0.80 | 1 | 4 |
| Lottery seems fun | 2,124 | -0.04 | 1.76 | -3 | 3 |
| Enjoy thinking about winning | 2,122 | 0.71 | 1.86 | -3 | 3 |
| Self-control problems | 2,124 | -0.66 | 1.27 | -3 | 3 |
| Financial literacy | 2,124 | 0.80 | 0.25 | 0 | 1 |
| Financial numeracy | 2,124 | 0.66 | 0.32 | 0 | 1 |
| Gambler's Fallacy | 2,124 | 0.27 | 0.39 | 0 | 1 |
| Non-belief in Law of Large Numbers | 2,124 | 0.39 | 0.17 | 0.00 | 0.93 |
| Expected value miscalculation | 2,124 | 0.66 | 0.38 | 0 | 1 |
| Overconfidence | 2,123 | 0.01 | 0.46 | -4.95 | 4.65 |
| Expected returns | 2,123 | 0.27 | 0.20 | 0.05 | 0.95 |
| Predicted life satisfaction | 2,112 | 2.28 | 4.82 | -10 | 10 |

Notes: This table presents descriptive statistics for our 2021 AmeriSpeak survey, which resampled proxies for preferences and biases. Section 4.1 summarizes the coding of these variables.

Figure A6: Distribution of Monthly Lottery Spending


Notes: This figure presents a histogram of monthly lottery spending, using data from our AmeriSpeak survey. For this figure, spending is winsorized at $\$ 100$ per month.

Figure A7: Non-Zero Lottery Spending by Income


Notes: This figure reports the share of people with non-zero monthly lottery spending in 2019 within household income groups, with 95 percent confidence intervals, using data from our AmeriSpeak survey. Observations are weighted for national representativeness.

Figure A8: Lottery Spending by Income with and without Winsorization


Notes: This figure presents average monthly lottery spending within household income groups, with 95 percent confidence intervals, using data from our 2020 AmeriSpeak survey. The winsorized values are the same as in Figure 4. The unwinsorized values are the original survey responses. Observations are weighted for national representativeness.

Table A11: Causal and Cross-Sectional Income Effects

|  | $(1)$ <br> Spending <br> change | $(2)$ <br> Self-reported <br> income effect | $(3)$ <br> $\ln (1+$ monthly <br> lottery spending $)$ |
| :--- | :---: | :---: | :---: |
| Income change | $0.194^{* * *}$ |  |  |
|  | $(0.025)$ |  | $-0.111^{* * *}$ |
| $\ln$ (household income) |  |  | $(0.035)$ |
|  |  |  | $1.911^{* * *}$ |
| Constant | $-5.483^{* * *}$ | $-0.014^{* * *}$ | $(0.144)$ |
| Observations | $(0.353)$ | $(0.003)$ | 2,877 |

Notes: Columns 1 and 2 report estimates of the causal elasticity of lottery spending with respect to income, using data from our AmeriSpeak survey. Income change and spending change refer to the self-reported percent change in household income and lottery spending in 2019 compared to 2018, respectively. Selfreported income effect is the answer to the question "Imagine you got a raise and your income doubled. How do you think your lottery spending would change?" in percent. Column 3 reports the cross-sectional elasticity of lottery spending with respect to income.

Table A12: First-Stage Estimates for Preference and Bias Proxies

|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Self-control problems | Financial illiteracy | Statistical mistakes | Expected returns | Predicted life satisfaction | Risk aversion | Lottery seems fun | Enjoy thinking about winning |
| Self-control problems | $0.353^{* * *}$ | 0.025** | 0.008 | 0.022 | -0.023 | -0.002 | 0.008 | 0.003 |
|  | (0.058) | (0.010) | (0.013) | (0.015) | (0.017) | (0.010) | (0.013) | (0.012) |
| Financial illiteracy | $0.064^{* * *}$ | $0.551^{* * *}$ | $0.248^{* * *}$ | $0.050 * * *$ | -0.003 | 0.017 | 0.004 | 0.007 |
|  | (0.020) | (0.018) | (0.015) | (0.017) | (0.019) | (0.011) | (0.015) | (0.015) |
| Statistical mistakes | 0.021 | $0.155^{* *}$ | 0.439*** | -0.017 | 0.026 | 0.008 | $0.036^{* * *}$ | 0.038*** |
|  | (0.018) | (0.011) | (0.019) | (0.016) | (0.017) | (0.010) | (0.013) | (0.013) |
| Expected returns | 0.023 | 0.020** | -0.018 | $0.356^{* * *}$ | 0.011 | 0.012 | $0.036^{* * *}$ | 0.009 |
|  | (0.015) | (0.009) | (0.011) | (0.023) | (0.014) | (0.010) | (0.012) | (0.012) |
| Predicted life satisfaction | -0.016 | 0.005 | 0.019* | 0.010 | $0.286{ }^{* * *}$ | -0.005 | 0.028** | $0.054^{* *}$ |
|  | (0.015) | (0.009) | (0.011) | (0.014) | (0.023) | (0.010) | (0.012) | (0.012) |
| Risk aversion | -0.027* | $0.028^{* * *}$ | 0.011 | 0.002 | -0.015 | $0.720^{* * *}$ | -0.050*** | 0.009 |
|  | (0.014) | (0.008) | (0.011) | (0.015) | (0.015) | (0.014) | (0.011) | (0.011) |
| Lottery seems fun | 0.028 | 0.004 | 0.021 | $0.071^{* * *}$ | 0.028 | -0.034*** | $0.472^{* * *}$ | $0.115^{* * *}$ |
|  | (0.019) | (0.011) | (0.014) | (0.018) | (0.018) | (0.012) | (0.021) | (0.017) |
| Enjoy thinking about winning | 0.017 | 0.005 | 0.025* | 0.005 | 0.103*** | 0.022* | 0.122*** | $0.491^{* * *}$ |
|  | (0.016) | (0.011) | (0.014) | (0.018) | (0.018) | (0.011) | (0.016) | (0.023) |
| $\ln$ (household income) | 0.029 | -0.059*** | $-0.031^{* *}$ | 0.024 | -0.011 | -0.055 *** | -0.016 | 0.003 |
|  | (0.019) | (0.011) | (0.014) | (0.018) | (0.019) | (0.009) | (0.013) | (0.013) |
| $\ln$ (years of education) | -0.174** | $-0.469^{* * *}$ | -0.281*** | -0.152* | $-0.269^{* * *}$ | -0.105** | $-0.178^{* * *}$ | $-0.196^{* * *}$ |
|  | (0.083) | (0.047) | (0.070) | (0.087) | (0.092) | (0.041) | (0.062) | (0.061) |
| 1(Black) | 0.088 | $0.227^{* * *}$ | $0.161^{* * *}$ | $0.153^{* *}$ | 0.102** | -0.050** | 0.081** | 0.043 |
|  | (0.054) | (0.028) | (0.033) | (0.046) | (0.050) | (0.021) | (0.033) | (0.029) |
| 1(Hispanic) | 0.055 | 0.190*** | 0.069** | 0.088** | 0.105** | -0.041** | 0.024 | 0.022 |
|  | (0.038) | (0.025) | (0.028) | (0.037) | (0.043) | (0.019) | (0.028) | (0.028) |
| Other demographics | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| State fixed effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| F-statistic | 50.4 | 506.9 | 299.0 | 225.0 | 132.7 | 2,570.9 | 363.9 | 312.2 |
| $R^{2}$ | 0.22 | 0.82 | 0.91 | 0.77 | 0.30 | 0.60 | 0.38 | 0.40 |
| Observations | 4,144 | 4,144 | 4,144 | 4,144 | 4,144 | 4,144 | 4,144 | 4,144 |
| Clusters | 2,072 | 2,072 | 2,072 | 2,072 | 2,072 | 2,072 | 2,072 | 2,072 |

Notes: This table presents first stage estimates of the Obviously Related Instrumental Variables version of equation (13): regressions of 2020 and 2021 bias and preference proxies on 2021 and 2020 bias and preference proxies plus demographic controls and state fixed effects, using data from our AmeriSpeak surveys. "Other demographics" includes age, household size, political ideology, and indicators for male, Black, Hispanic, other (non-white) race, married, employed, urban area, and attends religious services at least once a month. Robust standard errors, clustered by respondent, are in parentheses. ${ }^{*},{ }^{* *},{ }^{* * *}$ : statistically significant with 90, 95, and 99 percent confidence, respectively.

Table A13: Association of Income and Bias Proxies with Differences in Reported Lottery Spending

|  | $(1)$ <br> Household <br> income $(\$ 000 \mathrm{~s})$ | $(2)$ <br> Self-control <br> problems | Financial <br> illiteracy | $(4)$ <br> Statistical <br> mistakes | $(5)$ <br> Over- <br> confidence | $(6)$ <br> Expected <br> returns | Predicted life <br> satisfaction |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta$ monthly | -0.000080 | -0.000060 | -0.000046 | 0.000001 | 0.000225 | 0.000430 | 0.000061 |
| lottery spending | $(0.013641)$ | $(0.000865)$ | $(0.000303)$ | $(0.000328)$ | $(0.000290)$ | $(0.000393)$ | $(0.000268)$ |
| $R^{2}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 |
| Observations | 79 | 79 | 79 | 79 | 79 | 79 | 79 |

Notes: This table reports the associations of household income and bias proxies with the difference in 2019 monthly lottery spending reported on the 2020 vs. 2021 surveys. The sample includes all respondents from whom we re-elicited 2019 monthly lottery spending in the 2021 survey, which included only people who had reported spending more than $\$ 150$ per month or more than 10 percent of their income on lottery tickets in our 2020 survey. Robust standard errors are in parentheses. ${ }^{*}$, ${ }^{* *}$, ${ }^{* * *}$ : statistically significant with 90,95 , and 99 percent confidence, respectively.

Figure A9: Share of Lottery Spending Attributable to Biases (Instrumental Variables Estimates)


Notes: This figure plots the share of lottery spending attributable to each of our six bias proxies, with 95 percent confidence intervals. Predicted unbiased consumption is $\hat{s}_{i k}^{V}=\frac{s_{i}+1}{\exp \left(\tilde{\tau}_{k} \tilde{b}_{i k}\right)}-1$, where $s_{i}$ is monthly lottery spending, $\hat{\tau}_{k}$ is the IV estimate from column 6 of Table 6 , and $\tilde{b}_{i k}=\frac{b_{i k}-b_{k}}{S D\left(\left(_{i k}\right)\right.}$ is the difference between person $i$ 's proxy $b_{i k}$ and the unbiased value $b_{k}^{V}$ in standard deviation units. We winsorize at $\hat{s}_{i}^{V} \geq 0$, and we fix $\hat{s}_{i k}^{V}=0$ if $s_{i}=0$. The share of consumption attributable to each bias proxy is $\frac{\sum_{i}\left(s_{i} \hat{s}_{i k}^{V}\right)}{\sum_{i} s_{i}}$.

Figure A10: Share of Lottery Spending Attributable to Bias within Income Groups (Instrumental Variables Estimates)


Notes: This figure plots the share of lottery spending attributable to bias within household income groups, with 95 percent confidence intervals. Predicted unbiased consumption is $\hat{s}_{i}^{V}=\frac{s_{i}+1}{\exp \left(\tilde{\boldsymbol{\tau}} \tilde{\boldsymbol{b}}_{i}\right)}-1$, where $s_{i}$ is monthly lottery spending, $\hat{\boldsymbol{\tau}}$ is the IV estimate from column 6 of Table 6 , and $\tilde{b}_{i k}=\frac{b_{i k}-b_{k}^{v}}{S D\left(b_{i k}\right)}$ is the difference between person $i$ 's proxy $b_{i k}$ and the unbiased value $b_{k}^{V}$ in standard deviation units. We winsorize at $\hat{s}_{i}^{V} \geq 0$, and we fix $\hat{s}_{i k}^{V}=0$ if $s_{i}=0$. The share of consumption attributable to bias is $\frac{\sum_{i}\left(s_{i}-\hat{s}_{i}^{V}\right)}{\sum_{i} s_{i}}$.

## E. 3 Supplemental Survey

We administered a brief survey on the online platform Prolific in June 2021 with a final sample of 200 respondents. We restricted recruitment to respondents with (a) residency in the U.S., (b) at least 15 prior submissions on Prolific, and (c) a prior-submission approval rate of at least $95 \%$. We did not allow respondents to take the survey on a mobile device to maintain the legibility of graphics in the survey. The average respondent took 2.8 minutes to complete the survey and was paid $\$ 1.50$ for their participation.

The survey proceeded in the following steps. First, respondents consented to participate in the survey and entered their unique identifier used for anonymous compensation and communication on the platform. They then answered a set of four questions in which they expressed whether they would hypothetically prefer to receive a smaller certain dollar amount or a chance of receiving a larger dollar amount. Specifically, we asked whether they would prefer (i) $\$ 110$ for sure or a $50 \%$ chance of $\$ 200$, (ii) $\$ 210$ for sure or a $50 \%$ chance of $\$ 400$, (iii) $\$ 110$ million for sure or a $50 \%$
chance of $\$ 200$ million, and (iv) $\$ 210$ million or a $50 \%$ chance of $\$ 400$ million. The order in which the four questions were presented was randomized at the respondent level.

Next, we provided respondents with information about a Mega Millions drawing. We informed them that the cost of a Mega Millions ticket is $\$ 2$ and displayed a graphic stating that the next estimated jackpot was valued at $\$ 252$ million, with a cash option of $\$ 153.9$ million. We then displayed another graphic explaining how Mega Millions is played, the fixed lower prize amounts, and the odds of winning at each prize level as well as overall. We elicited respondents' hypothetical WTP for a ticket in the Mega Millions drawing we described.

Finally, we collected standard demographic information from respondents and asked whether they had purchased any lottery games with prize drawings (i.e., "any lottery game in which you pick numbers and win if you match the numbers from a drawing") in the past 12 months. The survey concluded with a request for feedback before redirecting respondents back to the Prolific platform. We excluded 20 respondents because of implausible WTP values of $\geq \$ 50$ that seem more consistent with inattention. These high-WTP respondents were just as risk-averse in all of the binary gamble decisions as the subjects in our main sample, which is internally inconsistent behavior that is highly suggestive of these high-WTP responses being "noise."

## F Details of Structural Estimation

## F. 1 Calibration of Decision Weights

This appendix explores the ability of common probability weighting function parameterizations to fit the decision weights implied by our empirical estimates. Since our elasticity estimates are measured at the population level, this calibration considers representative agent calibrations, and we therefore omit the dependence on type $\theta$. We consider the four common probability weighting function parameterizations discussed in Wakker's (2010) textbook treatment (see Section 7.2: "Parametric Forms of Weighting Functions"), and Fehr-Duda and Epper (2012). These are applied cumulatively as proposed in Tversky and Kahneman (1992), so that the decision weight on the jackpot is $\mathcal{W}\left(\pi_{1}\right)$, the weight on the second prize is $\mathcal{W}\left(\pi_{1}+\pi_{2}\right)-\mathcal{W}\left(\pi_{1}\right)$, and more generally,

$$
\begin{equation*}
\Phi\left(\pi_{k}\right)=\mathcal{W}\left(\sum_{j=1}^{k} \pi_{j}\right)-\mathcal{W}\left(\sum_{j=1}^{k-1} \pi_{j}\right) \tag{40}
\end{equation*}
$$

The four candidate parameterizations are:

- Tversky and Kahneman (1992):

$$
\begin{equation*}
\mathcal{W}(\pi)=\frac{\pi^{b}}{\left(\pi^{b}+(1-\pi)^{b}\right)^{1 / b}} \tag{41}
\end{equation*}
$$

- Goldstein and Einhorn (1987):

$$
\begin{equation*}
\mathcal{W}(\pi)=\frac{a \pi^{b}}{a \pi^{b}+(1-\pi)^{b}} \tag{42}
\end{equation*}
$$

- Prelec (1998):

$$
\begin{equation*}
\mathcal{W}(\pi)=\left(\exp \left(-(-\ln (\pi))^{a}\right)\right)^{b} \tag{43}
\end{equation*}
$$

- The "neo-additive" form studied in Chateauneuf, Eichberger, and Grant (2007):

$$
\mathcal{W}(\pi)= \begin{cases}0 & \text { if } \pi=0  \tag{44}\\ b_{0}+b_{1} \pi & \text { if } 0<\pi<1\end{cases}
$$

Note that in our application, the behavior at $\pi=1$ is irrelevant, since the residual worst-case outcome is a payoff of zero, with $m(0)=0$. When applied cumulatively as in equation (40), this neo-additive specification takes a particularly simple form with

$$
\Phi\left(\pi_{k}\right)= \begin{cases}b_{0}+b_{1} \pi_{1} & \text { if } k=1  \tag{45}\\ b_{1} \pi_{k} & \text { if } k>1\end{cases}
$$

Each of the first three specifications features a continuous "inverse S-shape" on the interval [0, 1]. Figure A11, Panel (a), illustrates this shape, plotting the Tversky and Kahneman (1992) and Prelec (1998) specifications for parameter values previously estimated in the literature, in solid blue and red. ${ }^{33}$ (The other displayed parameterizations will be described below.) Panel (b) is constructed by zooming in on the bottom left corner of Panel (a), in order to display the behavior of these parameterizations across the very low probabilities relevant for top lottery prizes; the solid red and blue lines rise so steeply from zero that they are indistinguishable from the vertical axis.

To compare the ability of these functional forms to fit our empirical estimates, our starting point is equation (5) from the text:

$$
\begin{equation*}
\frac{\zeta_{k}}{\left|\zeta_{p}\right|}=\frac{\Phi\left(\pi_{k}\right)}{\pi_{k}} m^{\prime}\left(w_{k}\right) \tag{46}
\end{equation*}
$$

We can use our semi-elasticity estimates and our specification of $m(\cdot)$ to compute the decision weights $\Phi\left(\pi_{1}\right)$ and $\Phi\left(\pi_{2}\right)$ non-parametrically; these correspond to points on the probability weighting function. For this calculation, we use values for $\pi_{k}$ and $w_{k}$ that correspond to a current Powerball ticket (net of income taxes), and we assume $m(\cdot)$ comes from a CRRA utility function, all as described in Section 5.1. Using our price semi-elasticity estimate from Section 3 of $\zeta_{p}=-0.4953$,

[^16]and our prize semi-elasticity estimates of $\zeta_{1}=1.7277$ and $\zeta_{2}=0.0837$, which we divide by 1 minus the assumed income tax rate of 0.3 as described in Section 5.1, this calculation implies
\[

$$
\begin{equation*}
\frac{\Phi\left(\pi_{1}\right)}{\pi_{1}}=323 . \tag{47}
\end{equation*}
$$

\]

and

$$
\begin{equation*}
\frac{\Phi\left(\pi_{2}\right)}{\pi_{2}}=0.41 . \tag{48}
\end{equation*}
$$

Figure A11b plots the points on the probability weighting function $\mathcal{W}$ implied by these weights, $\left(\pi_{1}, \mathcal{W}\left(\pi_{1}\right)\right)$ and $\left(\pi_{2}, \mathcal{W}\left(\pi_{2}\right)\right)$.

As discussed in Section 3.2, our estimate of $\zeta_{2}$ may be affected by the low salience of variation in the second prize in California, with the implication that if the second prize were as heavily advertised as the jackpot, the resulting $\zeta_{2}$ would be higher. This concern rests on the observation that variation in the second prize (in California) lacks salience; there is no reason to suppose that the average level of the prize - which is stable across years - is similarly non-salient. Yet the low level of the second prize (relative to the jackpot) provides information about a natural upper bound for the elasticity $\zeta_{2}$. Put simply: if the "full salience" semi-elasticity $\zeta_{2}$ were in fact higher than the (observed) full salience jackpot semi-elasticity $\zeta_{1}$, then lottery administrators could raise demand at zero cost by reallocating the prize pool away from the jackpot and toward the second prize, while holding the total ticket expected value constant. Yet despite frequent format revisions, we generally do not see adjustments in this direction; on the contrary, there is usually a relative reallocation toward a higher jackpot expected value relative to the second prize. We interpret this trend as suggestive evidence that the second prize semi-elasticity is weakly lower than the jackpot semielasticity, and thus we plot an alternative value for $\mathcal{W}\left(\pi_{2}\right)$ in Panel (b) assuming, as a conservative upper bound, that $\zeta_{2}=\zeta_{1}$

We are interested in the features of probability weighting function parameterizations which can match the plotted points in Panel (b). The most straightforward case is the Tversky and Kahneman (1992) weighting function in equation (41): since this specification has only a single parameter $b$, there is a unique parameterization which passes through our estimated $\mathcal{W}\left(\pi_{1}\right)$; it is plotted as the dotted blue line. An immediate observation is that due to the steep slope of the dotted blue line across low probabilities, it predicts decision weights at $\pi_{2}$ that are far higher than even our upper bound estimate for $\mathcal{W}\left(\pi_{2}\right)$.

The red dot-dashed line shows how the failure of the Prelec (1998) class of functions is somewhat different. This two-parameter specification actually can be adjusted to pass through both our estimates of $\mathcal{W}\left(\pi_{1}\right)$ and $\mathcal{W}\left(\pi_{2}\right)$. (We use the upper bound in Panel (b); the specification passing through the lower point estimate is even more extreme.) Although this specification can technically fit both points, it results in an implausible weighting function beyond these two points. The extreme nature of this specification can be seen by returning to Panel (a), which also plots this specification, demonstrating that it places exceedingly low weights on almost the entire interval $[0,1]$, before rising sharply at high probabilities to reach $\mathcal{W}(1)=1$. Such a probability weighting
function would imply that an agent would be unwilling to pay more than $\$ 1$ for a lottery ticket that pays out $\$ 100$ with a probability of $90 \%$. This leads us to conclude that the Prelec (1998) specification cannot match our results under conventional parameterizations. The two-parameter Goldstein and Einhorn (1987) parameterization produces results very similar to the Prelec (1998) specification, and we thus omit it to minimize redundancy.

Other papers have found slightly different estimates of the three inverse S-shape functions than the ones reported plotted in Figure A11. See, e.g., also, Camerer and Ho (1994), Wu and Gonzalez (1996), Abdellaoui (2000), and Filiz-Ozbay et al. (2015). However, our arguments above show that none of these slightly different calibrations of the standard inverse S-shape functions can match our semi-elasticity estimates.

Finally, we consider the neo-additive parameterization. This weighting function simply corresponds to a straight line with a positive vertical intercept (at which the probability weighting function is discontinuous) passing through the estimated points for $\mathcal{W}\left(\pi_{1}\right)$ and $\mathcal{W}\left(\pi_{2}\right)$ in Panel (b). As such, it is readily apparent that this specification can easily match either set of points. Yet it is also apparent that although our estimate of $\mathcal{W}\left(\pi_{1}\right)$ provides a tight estimate of the vertical intercept parameter $b_{0}$, the uncertainty in our estimate of $\zeta_{2}$ admits a wide range of potential values for the slope parameter $b_{1}$. As we show in our structural simulations, the key implications for optimal lottery design are insensitive to this slope parameter within this wide range of values.

## Figure A11: Probability Weighting Function Parameterizations



Notes: These figures plot two common parameterizations of common probably weighting functions. The solid lines correspond to parameter values favored in the prospect theory literature. The dashed lines plot the parameterizations that match one or both of the probability weighting values consistent with our estimated demand elasticities. Panel (a) plots these functions across the full range of probabilities from 0 to 1 , while Panel (b) "zooms in" on the bottom left corner of Panel (a), while displaying points motivated by the elasticity estimates from Section 3 (see text for details).

## F. 2 Implementation of Structural Model

This appendix describes implementation details of the structural model described in Section 5. Equations are written as functions of individual types $\theta$, in order to span the heterogeneous agent model.

Equation (4) in the text illustrates that once we have specified decision weights $\Phi\left(\pi_{k} ; \theta\right)$ and the value function $m(w ; \theta)$, the net-of-price certainty equivalent (in brackets) is a sufficient statistic for type $\theta$ 's lottery demand. That is, any two lotteries with the same net-of-price certainty equivalent will generate the same demand. We will use this feature repeatedly, so it is useful to formally define this "net certainty equivalent":

$$
\begin{equation*}
N C E(L ; \theta)=\sum_{k} \Phi\left(\pi_{k} ; \theta\right) m\left(w_{k} ; \theta\right)-p \tag{49}
\end{equation*}
$$

In what follows, we calibrate the structural model above for a single representative lottery, which we then manipulate to find characteristics of the optimal representative lottery. To describe the model calibration, we proceed in two steps. In Step 1, we demonstrate how this model is exactly identified by the functional form assumptions in Section 5.1 and by estimates for the following typespecific parameters:

- $s(\theta)$ : consumption of a representative lottery,
- $s^{V}(\theta)$ : debiased consumption of that representative lottery,
- $\zeta_{1}(\theta), \zeta_{2}(\theta)$ : semi-elasticities of demand with respect to the expected value of the jackpot and the second prize, generated by (local) variation in the prizes $w_{1}$ and $w_{2}$ of the representative lottery,
- $\zeta_{p}(\theta)$ : semi-elasticity of demand with respect to a price change in the representative lottery,
- $y(\theta)$ : individual expected continuation wealth,
- $g(\theta)$ : welfare weights.

In Step 2, we describe how we translate our empirical estimates into the necessary estimates of $\left\{s(\theta), s^{V}(\theta), \zeta_{p}(\theta), \zeta_{1}(\theta), \zeta_{2}(\theta), y(\theta), g(\theta)\right\}$ above, for each representative agent $\theta$ in a discretized grid of types $\Theta$.

## Step 1: Model Identification

Value function. The robust stylized fact, first postulated as part of the "fourfold pattern of risk attitudes" by Tversky and Kahneman (1992), is that individuals are risk-seeking over lowprobability gains, but risk-averse over moderate to high probability gains. This pattern is consistent with typical probability weighting functions, but not with convex utility functions. To illustrate, we conducted a supplementary survey with 200 subjects, summarized in Appendix E.3. In this
survey, subjects could choose between a certain prize and an option with a 50 percent chance of a higher prize that yielded slightly lower expected value than the certain option; thus, choosing the risky option implies risk-seeking preferences. We find that for gambles on the order of hundreds of dollars, 12 percent of respondents chose the risky option (13 percent among those who purchased a lotto ticket in the past 12 months), while for gambles on the order of hundreds of millions of dollars, 2 percent of respondents chose the risky option ( 6 percent among those who purchased a lotto ticket in the past 12 months). These results are consistent with the fourfold pattern of risk attitudes, and inconsistent with the convexity conjecture of Friedman and Savage (1948).

We therefore assume agents have a concave utility function over continuation wealth $W$ with a constant coefficient of relative risk aversion (CRRA) $\eta$ :

$$
\mathcal{U}(W)= \begin{cases}\ln (W) & \text { if } \eta=1  \tag{50}\\ \frac{W^{1-\eta}-1}{1-\eta} & \text { if } \eta \neq 1\end{cases}
$$

Our value function $m(w ; \theta)$ corresponds to the utility gain from winning a prize $w$, normalized by one's local marginal utility of wealth, so that the decision-weighted gain can be expressed in dollars. For an agent whose non-prize continuation wealth is $y(\theta)$, this value function can be written

$$
\begin{align*}
m(w ; \theta) & =\frac{\mathcal{U}(y(\theta)+w)-\mathcal{U}(y(\theta))}{\mathcal{U}^{\prime}(y(\theta))}  \tag{51}\\
& = \begin{cases}\frac{\ln (y(\theta)+w)-\ln y(\theta)}{y(\theta)-1} & \text { if } \eta=1 \\
\frac{1}{1-\eta}\left((y(\theta)+w)^{1-\eta}-y(\theta)^{1-\eta}\right) \\
y(\theta)^{-\eta} & \text { if } \eta \neq 1\end{cases}  \tag{52}\\
& = \begin{cases}y(\theta) \ln \left(\frac{y(\theta)+w}{y(\theta)}\right) & \text { if } \eta=1 \\
\frac{y(\theta)}{1-\eta}\left(\left(\frac{y(\theta)+w}{y(\theta)}\right)^{1-\eta}-1\right) & \text { if } \eta \neq 1\end{cases} \tag{53}
\end{align*}
$$

We can also compute the derivative, which proves useful for the calibrations below:

$$
\begin{equation*}
m^{\prime}(w ; \theta)=\frac{\mathcal{U}^{\prime}(y(\theta)+w)}{\mathcal{U}^{\prime}(y(\theta))}=\left(\frac{y(\theta)}{y(\theta)+w}\right)^{\eta} . \tag{54}
\end{equation*}
$$

Decision weights. To identify decision weights, we can exploit the insight formalized in equation (5) from the text: decision weights imply a relationship between the relative responsiveness of demand to prices and prizes, quantified empirically by our estimates of $\zeta_{p}$ and $\zeta_{k}$. Intuitively, if demand reacts more strongly to variation in the size of the jackpot expected value than to a change in ticket price (as we find), that is evidence of a high decision weight $\Phi\left(\pi_{1}\right)$ on the jackpot. Formally, substituting equation (45), with $k=1$ and $k=2$, into equation (5) gives two equations which identify the parameters of the neo-additive decision weighting function $b_{0}(\theta)$ and $b_{1}(\theta)$ from
type-specific estimates of $\zeta_{1}(\theta), \zeta_{2}(\theta)$, and $\zeta_{p}(\theta)$ :

$$
\begin{align*}
& \frac{\zeta_{1}(\theta)}{\left|\zeta_{p}(\theta)\right|}=\frac{b_{0}(\theta)+b_{1}(\theta) \pi_{1}}{\pi_{1}} m^{\prime}\left(w_{1} ; \theta\right) \Rightarrow b_{0}(\theta)=\left(\frac{\zeta_{1}(\theta)}{\left|\zeta_{p}(\theta)\right|} \frac{1}{m^{\prime}\left(w_{1} ; \theta\right)}-b_{1}(\theta)\right) \pi_{1}  \tag{55}\\
& \frac{\zeta_{2}(\theta)}{\left|\zeta_{p}(\theta)\right|}=\frac{b_{1}(\theta) \pi_{2}}{\pi_{2}} m^{\prime}\left(w_{2} ; \theta\right) \Rightarrow b_{1}(\theta)=\frac{\zeta_{2}(\theta)}{\left|\zeta_{p}(\theta)\right|} \frac{1}{m^{\prime}\left(w_{2} ; \theta\right)} \tag{56}
\end{align*}
$$

By first computing $b_{1}$ and then $b_{0}$, we can compute these parameters. Cumulatively applying these decision weights as in Chateauneuf, Eichberger, and Grant (2007), these parameters fully identify the certainty equivalent,

$$
\begin{equation*}
\sum_{k} \Phi\left(\pi_{k} ; \theta\right) m\left(w_{k} ; \theta\right)=b_{0}(\theta) m\left(w_{1} ; \theta\right)+b_{1}(\theta) \sum_{k} \pi_{k} m\left(w_{k} ; \theta\right), \tag{57}
\end{equation*}
$$

and they thus fully specify the net certainty equivalent function in equation (49).

Preference shocks. Equations (3) and (4) imply that the preference shock distribution $F_{\varepsilon \mid \theta}$ is equivalent to specifying a $\mathbb{R} \rightarrow \mathbb{R}^{+}$function mapping $\operatorname{NCE}(L ; \theta)$ to demand $s$, which we denote

$$
S(N C E ; \theta)=F_{\varepsilon \mid \theta}[N C E] .
$$

Rather than characterizing $F_{\varepsilon \mid \theta}$ directly, we directly characterize the function $S(N C E ; \theta)$, which implicitly defines $F_{\varepsilon \mid \theta}$. Here we employ a simple structural assumption: that the semi-elasticity of demand is constant with respect to changes in NCE, and thus also in price. This assumption has two appealing features. First, it generates plausible patterns of demand across the range of variation in our data, in ways that other constant elasticity and semi-elasticity assumptions do not; see Figure A12 for plots of the structural predictions of both observed demand (thick lines) and latent debiased demand (thin lines) across variation in price and jackpot expected value for each type in our model. Second, this assumption implies that the hypothetical change in price required to induce a debiased consumer to purchase the observed (biased) quantity remains constant across changes in price and jackpot expected value. That hypothetical price change is equal to the moneymetric bias $\gamma$-which plays an important role in our optimal policy formulas-and thus by using this specification for demand, we ensure that our results are not driven by changes in bias which are generated by structural functional form assumptions.

To compute demand for an arbitrary lottery $L$, we first compute the difference in net certainty equivalent from the status quo lottery $L^{0}$,

$$
\begin{equation*}
\Delta N C E(\theta)=N C E(L ; \theta)-N C E\left(L^{0} ; \theta\right) . \tag{58}
\end{equation*}
$$

We then compute demand as

$$
\begin{equation*}
S(N C E ; \theta)=s(\theta) \cdot \exp \left(\left|\zeta_{p}(\theta)\right| \cdot \Delta N C E\right) . \tag{59}
\end{equation*}
$$

Bias. To calibrate bias, and thus normative (debiased) demand, we assume that a constant share $\chi$ of the difference $\Phi\left(\pi_{k}\right)-\pi_{k}$, i.e., the difference between decision weights and expected utility maximization, is driven by behavioral biases. Therefore we compute the $\chi(\theta)$ for each agent that would rationalize a given debiased level of demand $\ln s^{V}(\theta)$. To do this, note that we can write the "debiased certainty equivalent" for lottery $L$ as a function of the bias share $\chi(\theta)$ :

$$
N C E^{V}(L, \chi(\theta) ; \theta):=\sum_{k}\left(\pi_{k}+\left(\Phi\left(\pi_{k} ; \theta\right)-\pi_{k}\right)(1-\chi(\theta))\right) m\left(w_{k} ; \theta\right)
$$

Using the estimate of debiased demand for the baseline lottery, $s^{V}(\theta)$, we thus compute the bias share $\chi(\theta)$ for each type that satisfies

$$
s^{V}(\theta)=S\left(N C E^{V}\left(L^{0}, \chi(\theta)\right)\right) .
$$

Having thus identified $\chi(\theta)$, we can then compute normative demand for an arbitrary lottery $L$ as $S\left(N C E^{V}(L, \chi(\theta) ; \theta) ; \theta\right)$.

Welfare. To compute consumer surplus, and thus welfare, it suffices to show how welfare is computed for a given type; it is straightforward to aggregate across types by weighting these changes by the welfare weights $g(\theta)$. Here we can make use of the fact that surplus can be measured as the integral under the Hicksian demand curve, from the point where the quantity demanded is equal to zero. In this model, there are no income effects, and therefore the Hicksian demand curve is found simply by varying $p$ in the demand curve we have already derived: $S\left(\sum_{k} \Phi\left(\pi_{k} ; \theta\right) m\left(w_{k} ; \theta\right)-p ; \theta\right)$. Therefore we can compute surplus by integrating under this demand curve from the price at which demand is zero up to the actual price. And since $N C E$ varies one-for-one with price, this is equivalent to integrating under $S$ from the $N C E$ at which demand is zero. Here we impose the assumption that quantity demanded falls to zero when $N C E=0$, reflecting that we do not expect positive demand for a lottery that has a price of zero and prizes of zero. Then the integral under the Hicksian demand curve reflecting welfare from a given lottery $L$ is identical to the integral $\int_{x=0}^{N C E(L ; \theta)} S(x ; \theta) d x$. This provides the perceived utility surplus, based on consumers' willingness to pay for lottery tickets. To compute normative utility, we subtract the bias costs $\chi(\theta) \sum_{k}\left(\Phi\left(\pi_{k}\right)-\right.$ $\left.\pi_{k}\right) m\left(w_{k}\right)$ from perceived utility.

## Step 2: Estimation of Input Parameters

We now describe how we use our empirical results to arrive at estimates of the parameters $s(\theta)$, $s^{V}(\theta), \zeta_{p}(\theta), \zeta_{1}(\theta), \zeta_{2}(\theta), y(\theta)$, and $g(\theta)$ which identify the model.

We assume a baseline CRRA parameter of $\eta=1$, corresponding to log utility of continuation wealth.

To translate our empirical estimates into the context of this model with a single representative lottery game, we abstract from the diversity of games in the data and assume that our empirical
estimates (of demand, elasticities, and the other statistics described below) align with those that would arise from a single lottery game with average attributes. Specifically, we specify a "representative lottery" with the features of a current Powerball lottery ticket described in Table 1, with a price of $\$ 2$ per ticket and a jackpot pool equal to the empirical average of $\$ 101$ million. To account for administration and overhead costs, which are typically between $5 \%$ and $15 \%$ of total lottery revenues in the U.S., we assume each lottery ticket has an additional cost of $\$ 0.20$ (see Appendix Figure A2).

We then specify a discretized type space. Our heterogeneous agent specification employs nine types, corresponding to the three income bins displayed in Figure 4, with three partitions of agents within each income bin. A share of agents purchase no lottery tickets (we assume these agents are expected utility maximizers, for whom $\Phi\left(\pi_{k} ; \theta\right)=\pi_{k}$, implying that their demand is zero for any lottery with a price greater than expected value), while the remaining lottery purchasers are partitioned into two groups-above- and below-median consumers-at each income level. Population shares and type-specific averages are computed within each partition of the type space.

We calibrate type-specific estimates of $s(\theta), s^{V}(\theta), y(\theta)$, and $g(\theta)$ as follows. We draw our estimates of lottery consumption in the status quo, $s(\theta)$, directly from the reported expenditures in our survey, reported in Figure 4. To calibrate debiased consumption $s^{V}(\theta)$, we interpret the share of consumption attributable to bias (as estimated in Section 4.3) as the causal effect of bias. Thus for each individual, we reduce their reported level of consumption by their estimated quantity effect of bias to arrive at a debiased consumption, which we average within each partition of the type space to reach $s^{V}(\theta)$. To compute net continuation wealth $y(\theta)$, we compute the average reported income (according to our survey) within each partition of the type space. We then convert this into a measure of net (of tax) income $c(\theta)$ using the mapping between gross and net income from Piketty, Saez, and Zucman (2018). To arrive at continuation wealth, we multiple each type's net income by 20 , to coarsely represent expected wealth accounting for future earnings for a representative worker. Following Saez (2002b), we set welfare weights proportional to $c(\theta)^{-\nu}$, with $\nu=1$ in our baseline specifications, and with robustness checks for $\nu=0.25$ and $\nu=4$.

Our estimates of semi-elasticities with respect to price and the jackpot and second prizes, presented in Sections 3.2 and 3.3, rely on population data, and are not estimated separately across individuals. We assume these elasticities are homogeneous, with $\zeta_{p}(\theta)=\bar{\zeta}_{p}$ for each $\theta$, etc. (It is straightforward to extend this model to data with heterogeneous elasticity estimates, as they would simply affect the values of $b_{0}(\theta)$ and $b_{1}(\theta)$ identified by equations (55) and (56) above.) We divide prize elasticities by one minus the assumed income tax rate of $30 \%$, to convert these into elasticities with respect to the net-of-tax prize in the model.

This completes the description of the structural model calibration. Figure A12 illustrates the behavior of the structural demand model, plotting average simulated monthly demand for lottery tickets in each income bin in response to variation in the price and the jackpot of the representative lottery. Thin lines plot the latent "debiased" demand from each type. Several identifying features
of the model are apparent from these plots. The status quo price and jackpot expected value for the baseline representative lottery ticket are displayed as vertical lines, and the plotted demand curves intersect these status quo values at the quantities corresponding to the expenditure levels in each income bin from Figure 4. (Since the representative ticket price is $\$ 2$, these quantities are equal to half the level of dollar expenditures.)

The vertical axis of both plots is displayed with a log scale, so that a constant semi-elasticity of demand with respect to price appears as a constant slope in Panel (a). Our estimate of $\zeta_{p}$ therefore controls the slope of these demand curves, which is assumed to be constant across groups.

A ticket price of $\$ 1$ is shown with a dashed line - this depicts the discretely different price at which demand is measured prior to the Mega Millions and Powerball price changes described in Table 1. The model is calibrated so that the difference in demand at the price of $\$ 1$ vs. the status quo price of $\$ 2$ corresponds to our empirical estimate of $\zeta_{p}$. Perceived surplus in the status quo can be understood as the integral under the demand curves in Panel (a) from the status quo price rightward, to the point where the net certainty equivalent falls to zero.

Figure A12: Simulated Purchases and Expenditures by Income Bin
(a) Tickets Purchased varying Price

(b) Tickets Purchased varying Jackpot Size


Notes: This figure plots the simulated average number of tickets purchased in each of three income bins from our structural model, across different values of the representative lottery ticket price and jackpot. Ticket purchases are plotted on a log scale, so that Panel (b) illustrates the structural assumption of a constant semi-elasticity of demand with respect to jackpot expected value across the range of jackpot values in our data. Latent demand that would obtain if consumers were counterfactually debiased is plotted with thin lines. (See the discussion in Section 5.1 and Appendix F. 2 for additional details.)

## F. 3 Curvature of $m$

The robust stylized fact, first postulated as part of the "fourfold pattern of risk attitudes" by Tversky and Kahneman (1992), is that individuals are risk-seeking over low-probability gains, but risk-averse over moderate to high probability gains. This pattern is consistent with typical probability weighting functions, but not with convex utility functions. To illustrate, we conducted a supplementary survey with 200 subjects, summarized in Appendix E.3. In this survey, subjects could choose between a certain prize and an option with a 50 percent chance of a higher prize that yielded slightly lower expected value than the certain option; thus, choosing the risky option implies risk-seeking preferences. We find that for gambles on the order of hundreds of dollars, 12 percent of respondents chose the risky option (13 percent among those who purchased a lotto ticket in the past 12 months), while for gambles on the order of hundreds of millions of dollars, 2 percent of respondents chose the risky option ( 6 percent among those who purchased a lotto ticket in the past 12 months). These results are consistent with the fourfold pattern of risk attitudes, and inconsistent with the convexity conjecture of Friedman and Savage (1948).

In our model, the CRRA parameter shapes WTP for lottery tickets. With more curvature, the value function $m(w)$ is less sensitive to variation in the jackpot, requiring a higher decision weight $\Phi\left(\pi_{1}\right)$ to rationalize our empirical estimates of $\hat{\bar{\zeta}}_{1}$. This in turn implies more utility from the jackpot and a higher WTP. CRRA values below 0.8 imply low WTP for the representative lottery ticket, to the point that if the jackpot declines to the Powerball jackpot reset value, demand falls to zero. In our baseline specification with $\operatorname{CRRA}=1$, individuals' average WTP for the representative lottery ticket conditional on purchase - which must be higher than the $\$ 2$ ticket price - is $\$ 3.43$. On the other hand, a CRRA value of 1.5 implies a much higher WTP of $\$ 4$ for each purchased ticket. In the short 200-subject supplementary survey described in Appendix E.3, we find that for a Mega Millions lottery with a $\$ 250$ million jackpot, participants have a mean and median WTP of $\$ 6.11$ (with 95 percent confidence interval ( $\$ 5.25, \$ 6.97$ )) and $\$ 4.00$, respectively. The mean and median WTP among those who have purchased a lotto ticket in the last 12 months are similar: $\$ 6.22$ (with 95 percent confidence interval ( $\$ 4.52, \$ 7.92$ ) ) and $\$ 4.00$, respectively. These estimates are in the ballpark of our model's prediction at a CRRA parameter of 1 , and reinforce our claim that alternative assumptions would deliver implausible predictions about WTP for lottery tickets.


[^0]:    ${ }^{1}$ See www.igmchicago.org/surveys/state-run-lotteries.

[^1]:    ${ }^{2}$ See, for example, Kahneman and Tversky (1979), Tversky and Kahneman (1992), Prelec (1998), Gonzalez and Wu (1999), Wakker (2010), Filiz-Ozbay et al. (2015), and Bernheim and Sprenger (2020).

[^2]:    ${ }^{3}$ See, for example, Clotfelter and Cook (1987), Clotfelter et al. (1999), Farrell and Walker (1999), Price and Novak (1999, 2000), and Oster (2004).
    ${ }^{4}$ Lottery spending is "drastically underreported" in the Consumer Expenditure Survey (Kearney 2005b).
    ${ }^{5}$ See, for example, Clotfelter and Cook (1989), Cook and Clotfelter (1993), Farrell, Morgenroth, and Walker (1999), Farrell et al. (2000), Kearney (2005b), Grote and Matheson (2006), Guryan and Kearney (2010), and Knight and Schiff (2012).
    ${ }^{6}$ See, e.g., Conlin, O'Donoghue, and Vogelsang (2007), DellaVigna, List, and Malmendier (2012), DellaVigna et al.

[^3]:    ${ }^{10}$ In March 2020, lottery sales dropped substantially due to the coronavirus pandemic, and both games temporarily lowered their reset values to $\$ 20$ million at the beginning of April. We say "current design" to refer to the design in place just before the pandemic.
    ${ }^{11}$ Alabama, Alaska, Hawaii, Nevada, and Utah do not have state lotteries. The Puerto Rico lottery only offers Powerball.

[^4]:    ${ }^{12}$ Formally, if $\mathcal{U}(W)$ is the utility function over continuation wealth $W$ and $y$ denotes the agent's (net) continuation wealth absent any prize, then $m(w)=\frac{\mathcal{U}(y+w)-\mathcal{U}(y)}{\mathcal{U}^{\prime}(y)}$.
    ${ }^{13}$ See, e.g., Kahneman and Tversky (1979), Prelec (1998), Gonzalez and Wu (1999), Wakker (2010), Filiz-Ozbay et al. (2015), and Bernheim and Sprenger (2020).
    ${ }^{14}$ Saez, Slemrod, and Giertz (2012) review the empirical literature on labor supply elasticities and argue that "in the absence of compelling evidence about significant income effects in the case of overall reported income, it seems reasonable to consider the case with no income effects."

[^5]:    ${ }^{15}$ Thus, this semi-elasticity is $1 / \pi_{k}$ multiplied by the semi-elasticity with respect to the prize $w_{k}$. The $1 / \pi_{k}$ factor is a normalization that improves exposition by ensuring that all of our estimated elasticities are of similar orders of magnitude.

[^6]:    ${ }^{16}$ Note that $\gamma$ is not a function of $\varepsilon$ because both $u$ and $v$ are additively separable in $\varepsilon$.

[^7]:    ${ }^{17}$ We use the average 30-year Treasury yield curve rate as the interest rate in our calculations. We also set the number of annual payments to 30 and require payments to increase by 5 percent each year, reflecting the most common annuity structure.

[^8]:    ${ }^{18}$ The second prize pool in California does not have a fixed reset value: it is allocated a pre-determined share of revenues from each draw. We set $\underline{w}_{2 j f(t)}$ to the average second prize amount in the first draw after the second prize is won during format $f$.
    ${ }^{19}$ This prediction assumes that players randomly select the numbers on their tickets. Appendix Figure A4 shows that the frequency of rollovers predicted under this assumption is close to the observed frequency of rollovers; the mean predicted and observed likelihood of a jackpot rollover are 0.91 and 0.90 , respectively, while both the predicted and observed likelihood of a California second prize rollover are 0.72 .
    ${ }^{20} \bar{Z}_{k j t}$ is computed using a lag of the dependent variable. While serial correlation would bias our estimate of the regression coefficient on $\bar{Z}_{k j t}$, we do not use or interpret that coefficient.

[^9]:    ${ }^{21}$ People can estimate the California games' upcoming second prize amounts by using each game's webpage to look up the previous second prize pool and whether the prize was won, and then adding a guess about the increase in the prize pool from the upcoming draw's sales.

[^10]:    ${ }^{22}$ In both of these format changes, the jackpot probabilities decreased substantially - much more than the jackpot probabilities changed when the games increased prices. This means that controlling for jackpot level vs. jackpot expected value matters more. Appendix Table A8 shows that controlling for the jackpot expected value (instead of level) does not explain the demand increases after the format changes. This is consistent with a model in which individuals respond to the jackpot level but are not responsive to the jackpot probability.

[^11]:    ${ }^{23}$ Specifically, we resampled the 104 people who had reported spending more than $\$ 150$ per month or more than 10 percent of their income on lottery tickets. If the 2021 report was within 50 percent of the 2020 report, we use the average. Otherwise, we use the minimum. We then regress this modified monthly spending on the 2020 report and use the prediction for the 25 out of the 104 people who did not take the 2021 survey.
    ${ }^{24}$ If we do not winsorize outliers using the second elicitation in 2021, average monthly spending increases by about $\$ 9$. However, our policy analyses use only the differences in spending across income groups, and those differences are not significantly affected by winsorization; see Appendix Figure A8.

[^12]:    ${ }^{25}$ Interestingly, the introduction of new state-run lotteries did affect cross-border demand for existing state-run lotteries (Knight and Schiff, 2012).
    ${ }^{26}$ Specifically, we assume a homogeneous price semi-elasticity, so that $\bar{\gamma}=\frac{\mathbb{E}\left[\gamma(\theta) \bar{\zeta}_{p} s(\theta)\right]}{\bar{\zeta}_{p} \bar{s}}$. To align with the approach in the structural model and to limit the effect of extreme outliers, we compute this and the other statistics in this section by first collapsing our data into the three levels of income and three income-conditional levels of consumption (nonconsumers, below-median, and above-median) and then compute population-weighted averages and covariances. Our survey measures expenditures, which must be converted into a measure of quantity $s(\theta)$ to compute the covariances in equation (19); we use $p=\$ 2$. The covariance terms in which $s(\theta)$ appears are divided by the mean of $s(\theta)$, so the results are insensitive to the price used for this conversion.
    ${ }^{27}$ Causal income effects plus between-income preference heterogeneity must sum to the observed cross-sectional

[^13]:    ${ }^{29}$ The status quo effective taxes on instant games are somewhat lower.
    ${ }^{30}$ In our model, the CRRA parameter affects WTP for lottery tickets. With more curvature, the value function $m(w)$ is less sensitive to variation in the jackpot, requiring a higher decision weight $\Phi\left(\pi_{1}\right)$ to rationalize our empirical estimates of the jackpot EV semi-elasticity $\hat{\bar{\zeta}}_{1}$. In Appendix F.3, we describe how CRRA values from 1 to 1.5 appear consistent with lottery purchasers' willingness to pay for a ticket, as measured in the short 200-subject supplementary survey described in Appendix E.3. In contrast, CRRA values below 0.9 imply low WTP for the representative lottery ticket, to the point that if the jackpot declines to the Powerball jackpot reset value, demand falls to zero.

[^14]:    ${ }^{31}$ The weekly version of the instrument is the product of the jackpot win probability and the draw-level jackpot forecast, constructed as described in equation (9), averaged over weeks in the same way as the jackpot expected values.

[^15]:    ${ }^{32}$ We selected these 13 games because they were the most likely substitutes for Mega Millions and Powerball. We first selected the largest draw game in each state plus additional games where jackpot data were available, then limited to a balanced panel in states that were always Mega Millions and Powerball members. The 13 games are Lotto from Colorado, Lotto! from Connecticut, Lotto from Illinois, Megabucks Doubler from Massachusetts, Multi-Match from Maryland, Tri-State Megabucks Plus from Maine and New Hampshire, Gopher 5 from Minnesota, Lotto from New York, Classic Lotto and Rolling Cash 5 from Ohio, Megabucks from Oregon, Lotto Texas from Texas, and Lotto from Washington.

[^16]:    ${ }^{33}$ The Tversky and Kahneman (1992, p. 312) specification (solid blue) plots the parameterization with $b=0.61$, the preferred estimate in that paper. The Prelec (1998) specification (solid red) plots the parameterization at preferred values reported in Wakker (2010): "Ongoing empirical research suggests that the parameters $a=0.65$ and $b=1.0467$, giving intersection with the diagonal at 0.32 , are good parameter choices for gains."

