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MUSSA PUZZLE REDUX

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ABSTRACT

The Mussa (1986) puzzle is the observation of a sharp and simultaneous increase in the volatility of both nominal and real exchange rates following the end of the Bretton Woods System of pegged exchange rates in 1973. It is commonly viewed as a central piece of evidence in favor of monetary non-neutrality because it is an instance in which a change in the monetary regime caused a dramatic change in the equilibrium behavior of a real variable (the real exchange rate) and is often further interpreted as direct evidence in favor of models with nominal rigidities in price setting. This paper shows that the data do not support this latter conclusion because there was no simultaneous change in the properties of the other macro variables, nominal or real. We show that an extended set of Mussa facts equally falsifies both conventional flexible-price RBC models and sticky-price New Keynesian models as explanations for the Mussa puzzle. We present a resolution to the broader Mussa puzzle based on a model of segmented financial market — a particular type of financial friction by which the bulk of the nominal exchange rate risk is held by financial intermediaries and is not shared smoothly throughout the economy. We argue that rather than discriminating between models with sticky versus flexible prices, or monetary versus productivity shocks, the Mussa puzzle provides sharp evidence in favor of models with monetary non-neutrality arising in the financial market, suggesting the importance of monetary transmission via the risk premium channel.

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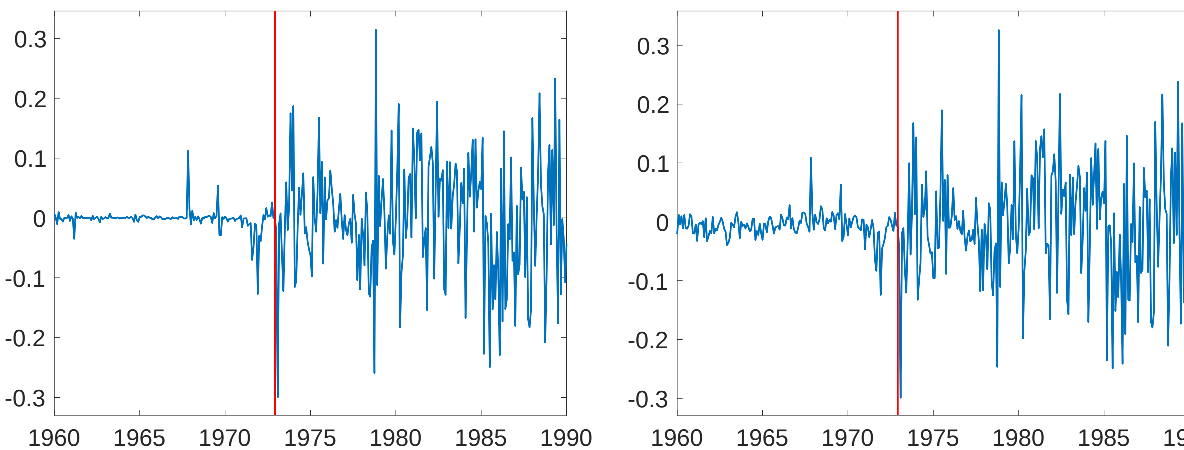
1 Introduction

Mussa (1986) famously observed that the end of the Bretton Woods System in the early 1970s and the change in the monetary policy regime away from pegged towards floating exchange rates naturally led to an increase in the volatility of nominal exchange rates (by an order of magnitude), but also instantaneously increased the volatility of *real* exchange rates nearly by the same factor (see Figure 1). This fact is commonly viewed by economists as a central piece of evidence in favor of monetary non-neutrality, since a change in the monetary regime has caused a dramatic change in the equilibrium behavior of a real variable — the real exchange rate.[1] Indeed, under the neutrality of money, properties of the real exchange rate should not be affected by changes in the monetary rule absent other contemporaneous changes. However, the Mussa fact is often further interpreted as direct evidence in favor of models with *nominal rigidities* in price setting (sticky prices). We show that this latter conclusion is not supported by the data, and propose an alternative explanation of the puzzle.

Figure 1: Nominal and real exchange rates (in log changes)

(a) Nominal exchange rate, Δe_t (b) Real exchange rate, Δq_t



Note: US vs the rest of the world (defined as G7 countries except Canada plus Spain), monthly data from IFM IFS database.

We first document empirically that while there was a change in the properties of the real exchange rate, there was no change in the properties of other macro variables — neither nominal like inflation, nor real like consumption and output (see Figure 2, which exhibits no evident structural break). One could interpret this as an extreme form of *neutrality*, where a major shift in the monetary regime, which increased the volatility of the nominal exchange rate by an order of magnitude, does not affect the equilibrium properties of any macro variables, apart from the real exchange rate. In fact, this is a considerably more puzzling part of the larger set of "Mussa facts". While the lack of change in

[1]When Nakamura and Steinsson (2018, pp.69–70) surveyed "prominent macroeconomists [on what is the most convincing evidence for monetary nonneutrality], the three most common answers were: the evidence presented in Friedman and Schwartz (1963) regarding the role of monetary policy in the severity of the Great Depression; the Volcker disinflation of the early 1980s and accompanying twin recession; and the sharp break in the volatility of the US real exchange rate accompanying the breakdown of the Bretton Woods System of fixed exchange rates in 1973." Note that the growing direct evidence on the duration of nominal prices does not immediately imply allocative effects of sticky prices and the ensuing monetary non-neutrality. See also a textbook treatment of the Mussa puzzle in Uribe and Schmitt-Grohé (2017, Chapter 9.12) from the perspective of discriminating between flexible-price and sticky-price models.

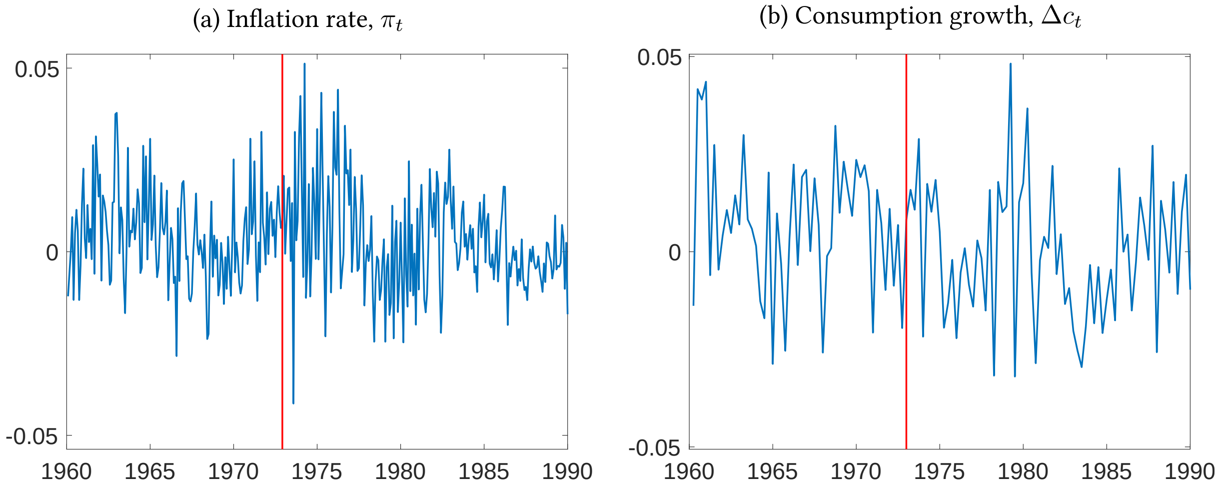


Figure 2: Inflation and consumption growth

Note: average inflation rates (monthly) and consumption growth rates (quarterly) for G7 countries except Canada plus Spain.

the volatility of inflation is inconsistent with models of monetary neutrality, the lack of change in the volatility of real variables, like consumption and output, is inconsistent with sticky-price models. Therefore, the combined evidence does not favor one type of model over the other, but rather rejects both types.

To provide intuition for this logic, consider two equilibrium conditions. The first is simply the definition of the real exchange rate (in logs):

$$q_t = e_t + p_t^* - p_t, \tag{1}$$

where p_t and p_t^* are consumer price levels at home and abroad, and e_t and q_t are the nominal and real exchange rates respectively. In models with monetary neutrality (e.g., international RBC), a change to the monetary policy rule should *not* affect the process for the real exchange rate q_t, and therefore (1) implies that the volatility of relative inflation, $\pi_t - \pi_t^* \equiv \Delta p_t - \Delta p_t^*$, must change along with the volatility of the nominal exchange rate, Δe_t. In the data, the volatility of Δq_t and Δe_t increased simultaneously, while the volatility of $\pi_t - \pi_t^*$ remained stable and low (see Figure 3 below). This pattern may, however, be consistent with conventional sticky-price models, which are at the core of the standard interpretation of the Mussa puzzle. This suggests that sticky price models dominate RBC models and that monetary policy must have real effects due to nominal rigidities.[2]

This interpretation, however, misses the second half of the picture. Equilibrium dynamics in a general class of models satisfy the following equilibrium relationship between relative consumption (with the rest of the world) and the real exchange rate:

$$z_t = \sigma(c_t - c_t^*) - q_t, \tag{2}$$

where $\sigma > 0$ and z_t can be interpreted as the equilibrium departure from efficient international risk

[2] Note that the classical indeterminacy result of Kareken and Wallace (1981) applies only to the nominal exchange rate, but not to real variables, and therefore cannot explain the increase in the volatility of the real exchange rate.

sharing.[3] Indeed, equation (2) with $z_t \equiv 0$ corresponds to the seminal Backus-Smith condition under separable utility with constant relative risk aversion σ. We show that equation (2) with a structural term z_t emerges as an equilibrium relationship in a general class of models independent from the completeness of asset markets and other features of the model. Furthermore, in a large class of *conventional* models — including both international RBC (IRBC) and New Keynesian Open Economy (NKOE) models — the structural residual z_t is independent of the monetary policy regime. Therefore, a shift in the monetary policy regime that changes dramatically the volatility of Δq_t, should necessarily change the volatility of $\Delta c_t - \Delta c_t^*$. In the data, however, the volatility of relative consumption growth, just like that of inflation, remained both stable and small (see Figure 3). Proposition 1 formalizes this logic and suggests an empirical test which we carry out in the data.[4]

Thus, models with monetary neutrality are consistent with the observed lack of change in the volatility of consumption, but for the wrong reason as they fail to predict the change in the volatility of the real exchange rate. In contrast, models with nominal rigidities can explain the changing behavior of the real exchange rate, but have the counterfactual implication for the missing change in the volatility of the real variables. Therefore, the extended Mussa facts are inconsistent with standard RBC and New Keynesian models alike, and we show that a single sufficient statistic z_t defined by (2) and directly measurable in the data allows us to falsify all conventional business cycle models at once.

We propose an alternative framework where monetary non-neutrality arises due to financial market segmentation with limits to arbitrage. This allows the model to be consistent with the umbrella of Mussa facts. The model features financial shocks in international asset markets, which our earlier work shows to be essential to explain the exchange rate disconnect under a floating regime and resolve a variety of exchange rate puzzles, including the Meese-Rogoff, PPP, Backus-Smith and UIP puzzles (Itskhoki and Mukhin 2021). Exogenous financial shocks are not suitable, however, to resolve the Mussa puzzle which requires that the equilibrium UIP deviations are endogenous to the exchange rate regime. Indeed, we show that under exogenous financial shocks, equilibrium exchange rate volatility must be offset by monetary policy, resulting in a counterfactually volatile inflation and/or output under the peg.

In contrast, in the limits-to-arbitrage model, a change in the exchange rate regime, and the associated change in the nominal exchange rate volatility, affects the quantity of risk faced by intermediaries in the international financial market. Greater nominal exchange rate volatility discourages intermediation and results in larger equilibrium UIP deviations under the floating regime, consistent with the empirical evidence. Vice versa, a lower nominal exchange rate volatility under the peg encourages intermediation, shielding the real exchange rate from financial shocks. As a result, a change in the

[3] Note the parallel between z_t and q_t, which can be viewed as defined by identities (2) and (1) respectively, where q_t is the departure from a (purchasing power) *parity* in the goods market and z_t is the departure from a (risk-sharing) "parity" in the financial market (in fact, z_t is related to the wedge in the uncovered interest rate parity, as we show below).

[4] Equilibrium relationship (2) emerges independently of trade openness of the economy, and thus the argument here does not rely on muted exchange rate pass-through at the border (i.e. the transmission of exchange rate volatility as emphasized by Baxter and Stockman 1989). It is instead the result of a general equilibrium relationship between aggregate macro variables (i.e. the source of exchange rate volatility as emphasized by Flood and Rose 1995). Put differently, a change in equilibrium exchange rate volatility requires a change in monetary policy, which in conventional models must be accompanied by changing properties of either inflation or output, or both, even in the closed-economy limit with zero aggregate exchange rate pass-through.

monetary regime has real consequences via the financial market, *even* when prices are fully flexible, thus affecting the volatility of both nominal and real exchange rates simultaneously.

Importantly, endogenous intermediation also implies that a credible commitment to a peg leads to an endogenous decline in equilibrium financial volatility, confronting the monetary authority with little need to compromise inflation stabilization under the peg. This is the reason why a segmented market model with endogenous financial shocks is consistent with a dramatic change in the exchange rate volatility unaccompanied by any comparable change in macroeconomic volatility, whether nominal or real. Equilibrium macroeconomic outcomes are primarily shaped by the fundamental macroeconomic forces, e.g. productivity shocks, and in turn are largely insensitive to the volatility in the international financial market and the resulting exchange rate volatility. This is true under *either* exchange rate regime, explaining the largely absent change in macroeconomic volatility after the breakdown of the Bretton Woods system.[5] The direct pass-through of exchange rate volatility into price levels, consumption and output is further dampened by the fact that economies are sufficiently closed to international trade. In our quantitative analysis, we calibrate the model to the average openness of the U.S. in 1960-90 and show the robustness of our results for the U.K., a smaller more open economy.

To summarize, the Mussa puzzle does not help discriminate between models with sticky versus flexible prices, or monetary versus productivity shocks, although realistic price and wage stickiness improve somewhat the quantitative fit of the model. Instead, the Mussa facts provide strong evidence in favor of models with monetary non-neutrality arising in the financial market, due to financial market segmentation — a particular type of financial friction by which the bulk of the *nominal* exchange rate risk is held by a group of financial intermediaries and is not shared smoothly throughout the economy.

The rest of the paper is organized as follows. After a brief literature review, Section 2 documents a set of empirical patterns of macroeconomic dynamics around the breakup of the Bretton Woods system which form the set of Mussa facts that allow us to discriminate between different classes of models of monetary non-neutrality. Section 3 sets up a general modeling framework used in our theoretical and quantitative analysis. Section 4 defines a class of conventional international business cycle models and a simple sufficient statistic z_t which allows us to falsify all such models at once using data from the end of the Bretton Woods. Section 5 introduces an alternative model of monetary non-neutrality with segmented financial market and limits to arbitrage and shows that this model can simultaneously match a full set of Mussa facts, including the overidentifying moments on macroeconomic comovement. Section 6 contains our quantitative analysis showing the robustness of the theoretical results of the earlier sections in a context of a general calibrated modeling framework. Finally, Section 7 offers a detailed discussion of the assumptions, alternative interpretations, as well as broader policy implications.

Contribution to the literature This paper aims to contribute to three strands of literature. First, we combine empirical evidence about the change in the dynamics of prices, quantities, and asset prices associated with the end of the Bretton Woods period and more broadly, with the switch between a peg

[5] Noticeable changes can be detected, however, in macroeconomic comovement including sign reversals of the Fama regression coefficient and the Backus-Smith correlation and a muted Balassa-Samuleson effect under the floating regime. All of these empirical facts are in line with the predictions of the segmented market model.

and a float, which together provide a set of moments that can sharply discriminate between alternative models. The empirical part of the paper builds on studies about the behavior of exchange rates by Mussa (1986), Stockman (1983, 1988), Berka, Devereux, and Engel (2012, 2018) and Bergin, Glick, and Wu (2014), the evidence about macroeconomic variables from Baxter and Stockman (1989), Flood and Rose (1995) and Devereux and Hnatkovska (2020), and additional facts about interest rates and financial variables from Frenkel and Levich (1975), Colacito and Croce (2013) and Kalemli-Özcan (2019).

Second, we derive a simple sufficient statistic that allows us to falsify a large class of conventional models that are often used to study the effects of floating and pegged exchange rates. This negative result echoes previous findings that sticky-price models, although successful at explaining a higher real exchange rate volatility under the float, yield counterfactual predictions about the behavior of other macroeconomic variables. The important advantage of our approach relative to the previous literature that relies on calibrated models (see Dedola and Leduc 2001, Chari, Kehoe, and McGrattan 2002, Duarte 2003, Monacelli 2004, Corsetti, Dedola, and Leduc 2008) or wedge accounting (see Kollmann 2005, which is an important precursor to our work) is that the sufficient statistic is essentially independent of the structural parameters and can be directly measured in the data. This allows us to test a large class of models under weaker assumptions.[6]

Finally, we provide an alternative model of monetary non-neutrality that relies on financial rather than nominal frictions and is consistent with the full set of Mussa facts. Similarly to De Long, Shleifer, Summers, and Waldmann (1990), Devereux and Engel (2002) and in particular Gabaix and Maggiori (2015), our setup features segmented financial markets with noise traders and limits to arbitrage, but in contrast to these models, generates asset prices and risk premia that are endogenous to monetary policy, which we show is necessary to account for the Mussa puzzle. This source of monetary non-neutrality is closely related to that of Jeanne and Rose (2002), but theirs is a partial equilibrium model and cannot be directly applied to explain the equilibrium properties of macroeconomic variables.[7] The transmission of monetary shocks via risk premia also relates our paper to the model of endogenously segmented markets in Alvarez, Atkeson, and Kehoe (2009), although our focus is on the effects of the exchange rate policy rather than of monetary (inflation) shocks.

2 Empirical Facts

We focus our empirical analysis on the end of the Bretton Woods system in 1973, comparing the dynamics of macroeconomic aggregates before and after the break. The break-up of the Bretton Woods system is indeed a unique natural experiment with a number of essential characteristics typically absent in other episodes of switching between a peg and a float. First, it constituted a large discontinuous break in the monetary regime from a near-perfect system of fixed exchange rates to a pure float between the U.S. dollar and other major currencies, in contrast to a more common alternation of exchange rate arrangements between partial and dirty pegs (see Ilzetzki, Reinhart, and Rogoff 2019). Second, the Bretton Woods system was more credible and persistent than most alternative pegs, again making the

[6]See also recent work by Ayres, Hevia, and Nicolini (2021), Ohanian, Restrepo-Echavarria, Van Patten, and Wright (2021).

[7]See also the related literature on target zones by Krugman (1991) and Krugman and Miller (1993).

experiment of a switch cleaner. Lastly, the breakup of the Bretton Woods system featured two large regions and multiple countries, as opposed to isolated small open economies typically entering and exiting pegs as part of a broader domestic policy shift.

The above features are at the core of our identification strategy. As is standard in regression discontinuity design (RDD; see e.g. Lee and Lemieux 2010), the identification does not rely on a randomness of the switch in the monetary policy and only requires that potential confounders evolve continuously around the end of the Bretton Woods system, and in particular there are no discontinuous changes in the volatility of non-monetary shocks that coincide with the adoption of the floating regime. This implies that a common narrative that the break-up of Bretton Woods was due to a gradual accumulation of monetary and fiscal imbalances (see e.g. Eichengreen 2007) is not a threat to our identification. Furthermore, structural VAR estimates from Bayoumi and Eichengreen (1992) and the subsequent literature indicate little difference in the incidence of fundamental macroeconomic shocks before and after the break-up of Bretton Woods.[8] For further discussion of identification see Section 7.

Data We briefly describe the construction of our dataset and provide further details in Appendix A.2. All monthly data (for nominal exchange rates, consumer prices, interest rates and stock prices) come from the IFM's IFS database, while all quarterly data (for GDP, consumption, imports and exports) are from the OECD database.[9] Net exports nx_t are defined as the ratio of exports minus imports to the sum of exports and imports in order to counter a mechanical increase in the volatility of the ratio of net exports to GDP due to increased openness of economies in later periods. All data are annualized to make standard deviations comparable across series. The rest of the world (RoW) for the U.S. is constructed as an average of log changes in series across France, Germany, Italy, Japan, Spain and the U.K., using the countries' average GDP over the sample period as weights.

Our sample starts from 1960 and does not include the "preconvertible phase" of the Bretton Woods which featured limited capital mobility and a high volatility of exchange rates (see e.g. Bordo 1993). There is some ambiguity over the exact end of the Bretton Woods System. While all countries officially allowed their exchange rates to float after January 1973, most of them were already adjusting their exchange rates since the "Nixon shock" in August 1971, which limited direct convertibility of the dollar to gold.[10] Therefore, we label the period 1960:01-1971:07 as the peg and the period 1973:01-1989:12 as the float, as used in the tables and figures below, excluding the intermediate period 1971:08–1972:12.[11] The regression discontinuity graphs are done for three alternative break points: 1973:01 in the main figures, and 1971:08 and 1980:01 as robustness in the Appendix Figure A1 (see Section 7).

Macroeconomic volatility Figure 3 displays the main empirical results of the paper: standard deviations of the variables using a rolling window that starts at 1973:01 and goes either forward or backward.

[8]Our theoretical results in Section 4 further relax the identifying assumptions by allowing for a discontinuous change in the dispersion of productivity and other domestic shocks between the regimes.

[9]All quantity variables (GDP, consumption, imports and exports) are real and seasonally-adjusted, while prices are not.

[10]There were also isolated devaluations in the U.K. and Spain in November 1967, a devaluation in France and an appreciation in Germany in August and October 1969, respectively.

[11]In Canada, the two exchange rate regimes occurred over different periods with free floating before 1962:06 and after 1970:05, and a peg in between. This is why we exclude Canada from the construction of the "rest of the world" in figures.

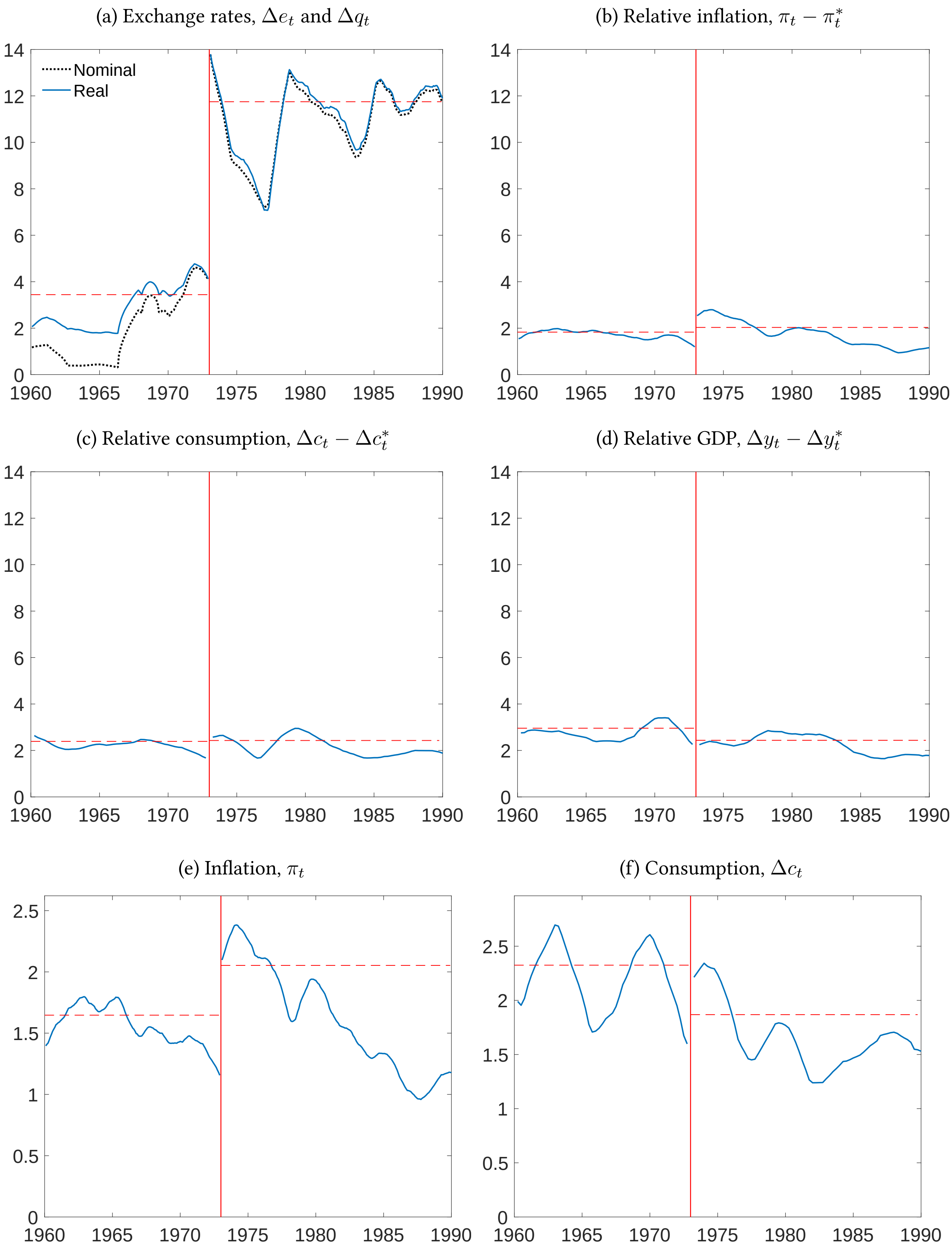


Figure 3: Macroeconomic volatility over time

Note: annualized standard deviations (in log points) for the RoW relative to the U.S. in panels a-d and for country-level variables in the RoW in panels e-f, estimated as triangular moving averages with a window over 18 months (panels a, b, e) or 10 quarters (panels c, d, f) before and after, treating 1973:01 as the end point for the two regimes; the dashed lines correspond to the average standard deviations under the two regimes. See Appendix Figure A2 for GDP and net exports.

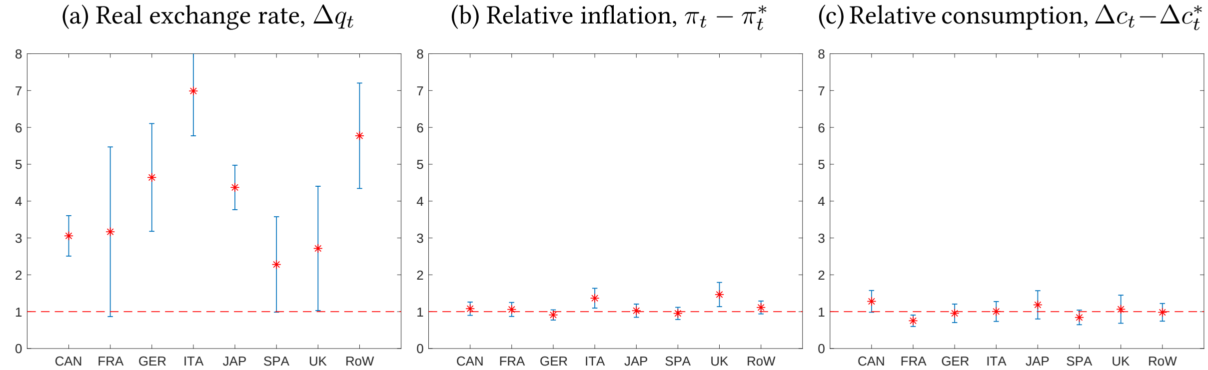


Figure 4: Volatility ratio float/peg across countries

Note: the ratios of standard deviations under the float and the peg across individual countries with 90% confidence intervals estimated using Newey-West (HAC) standard errors. See Appendix Figures A2 and A3 for GDP, NX and financial variables.

In line with the seminal Mussa (1986) evidence, the end of the Bretton Woods system is associated with a dramatic change in the volatility of both nominal and real exchange rates, from around 2% to 10%. What makes this fact much more puzzling, however, is the absence of any comparable change in the volatility of other macroeconomic variables — either nominal like inflation or real like consumption and GDP. We emphasize the relative magnitudes of volatility across different variables and regimes by keeping the same scale for standard deviations of all variables in Figures 3(a-d). While under the peg the volatility of the real exchange rate is of the same order of magnitude as for other macro variables, there is a clear disconnect between the real exchange rate and macroeconomic fundamentals under the float. Furthermore, even after zooming in on the volatility of inflation and consumption in Figures 3(e-f), we find little evidence of a discontinuity in the behavior of macroeconomic variables around 1973.[12]

The next pictures and tables expand on this finding and provide some additional details. In particular, although our primary focus is on the volatility of relative macroeconomic variables between the U.S. and the rest of the world, Figures 3(e-f) show that the same pattern holds for the country-level variables. Unpacking further the rest of the world into separate countries, Figure 4 shows the volatility ratios under the two regimes for each variable and country, on a common scale for comparability and with Newey-West (HAC) robust 90% confidence intervals. We find that the change in volatility of the real exchange rate was large (roughly five-fold) and highly significant in every country, while changes in volatility of other variables were small (typically within 10%) and generally insignificant.

Rather than emphasizing the lack of any change in macro variables, we emphasize the difference in the order of magnitudes. Table 1 shows that while nominal and real exchange rate volatility increased on average by about 8 and 6 times respectively, the volatility of other variables changed in different directions across countries and by an order of magnitude of about 10% — a stark difference. A notable exception is the volatility of relative interest rates, which roughly doubled after the end of the Bretton Woods, consistent with the decoupling of monetary policy from that of the U.S. under the float, but this change is still considerably smaller than that for the volatility of exchange rates.

[12]Note a slight increase in the volatility of inflation in the brief period after the break-up of Bretton Woods (due to the two large oil price shocks), which quickly comes back down so that the average inflation volatility before and after 1973 is about the same. There is also a slight increase in the volatility of consumption briefly after 1973 due to the 1974 recession in Japan.

Table 1: Macroeconomic volatility across policy regimes

	Δe_t			Δq_t			$\pi_t - \pi_t^*$			$\Delta c_t - \Delta c_t^*$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	0.8	4.4	5.7*	1.5	4.7	3.1*	1.3	1.4	1.1	1.7	2.1	1.3
France	3.4	11.8	3.4*	3.7	11.8	3.2*	1.2	1.3	1.1	2.4	1.8	0.8*
Germany	2.4	12.3	5.0*	2.7	12.4	4.6*	1.4	1.3	0.9	2.6	2.5	1.0
Italy	0.5	10.4	18.8*	1.5	10.3	7.0*	1.4	1.9	1.4*	2.1	2.1	1.0
Japan	0.8	11.6	14.2*	2.7	11.8	4.4*	2.7	2.8	1.0	2.3	2.7	1.2
Spain	4.4	10.8	2.4*	4.7	10.8	2.3*	2.7	2.6	1.0	2.4	2.0	0.8
U.K.	4.1	11.5	2.8*	4.4	12.0	2.7*	1.7	2.5	1.5*	2.7	2.9	1.1
RoW	1.2	9.9	8.4*	1.7	10.0	5.8*	1.3	1.4	1.1	1.7	1.7	1.0

	π_t			Δc_t			Δy_t			$i_t - i_t^*$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	1.3	1.4	1.1	1.7	1.8	1.1	1.7	1.8	1.0	0.8	1.6	2.1*
France	1.0	1.3	1.3*	1.7	1.5	0.9	1.8	1.1	0.6	0.9	1.8	1.9*
Germany	1.2	1.1	0.9	2.0	2.1	1.0	3.0	2.0	0.7	1.3	1.9	1.5*
Italy	1.0	2.1	2.0*	1.3	1.6	1.2	2.5	1.9	0.8	0.9	3.2	3.6*
Japan	2.6	2.9	1.1	2.0	2.6	1.3	2.2	2.1	0.9	1.4	2.5	1.8*
Spain	2.5	2.5	1.0	1.9	1.4	0.7	2.8	1.4	0.5*	0.7	5.4	7.4*
U.K.	1.6	2.6	1.6*	2.3	2.8	1.2	2.0	2.5	1.2	0.8	2.2	2.9*
U.S.	0.8	1.3	1.5*	1.4	1.6	1.1	1.8	2.0	1.1		—	
RoW	1.7	2.0	1.2	1.9	2.1	1.1	2.4	1.9	0.8	0.7	1.3	2.0*

Note: annualized standard deviations in log points; the peg corresponds to the period from 1960:01 to 1971:07 (except for Canada where it is from 1962:04 to 1970:01); the float is from 1973:08 to 1989:12; nx_t is the ratio of export minus imports over the sum of imports and exports; * indicates significance of the difference (ratio) of standard deviations under the float and the peg at the 5% level (`robvar` test in Stata). RoW for differences aggregates all non-U.S. countries into RoW and subtracts the U.S. before calculating moments; RoW for levels is a weighted average of the respective moment across non-U.S. countries.

These robust patterns of the differential change in the volatility of exchange rates and other macro variables constitute the main focus of our analysis below. In contrast, most correlations of macroeconomic variables across countries are typically not very strong or stable over time, and suggest only a weak pattern of change across the two monetary regimes (see Appendix Table A1). We use a few notable exceptions, including the Backus-Smith correlation and the Fama regression coefficient (characterizing the extent of UIP deviations), together with additional evidence on the behavior of trade balance and financial variables as overidentification tests of our model in Sections 5–7.

3 Theoretical Framework

We describe here the general theoretical framework which we use in Sections 4–6, where we consider its various special cases. We build on a standard New Keynesian open-economy model (NKOE) featuring capital, intermediate inputs, pricing to market, productivity and monetary shocks, wage and price stickiness with border prices sticky in producer, destination or dominant currency. The model features home bias in consumption with exogenous taste shocks for foreign goods and shocks to international risk sharing. We allow for various degrees of financial market (in)completeness including segmented financial markets.

There are two mostly symmetric countries — home (Europe) and foreign (US, denoted with a $*$). Each country has its nominal unit of account in which the local prices are quoted: for example, the home wage rate is W_t euros and the foreign wage rate is W_t^* dollars. The nominal exchange rate $\mathcal{E}_t$ is the price of dollars in terms of euros, hence an increase in $\mathcal{E}_t$ signifies a nominal devaluation of the home currency (euro). The monetary policy is conducted according to a conventional Taylor rule targeting inflation or the nominal exchange rate, depending on the monetary regime. In particular, the foreign country (US) always targets inflation, while the home country (Europe) switches from an exchange rate peg to inflation targeting ('float').

3.1 Model setup

Households A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} L_t^{1+\varphi} \right), \tag{3}$$

where σ is the relative risk aversion parameter and φ is the inverse Frisch elasticity of labor supply. The flow budget constraint is given by:

$$P_t C_t + \sum_{j \in J_t} \Theta_t^j B_{t+1}^j \leq W_t L_t + e^{-\zeta_t} \sum_{j \in J_{t-1}} \mathcal{D}_t^j B_t^j + \Pi_t + T_t, \tag{4}$$

where P_t is the consumer price index, W_t is the nominal wage rate, Π_t are profits of home firms, T_t are lump-sum transfers from the government. B_{t+1}^j is the quantity of asset $j \in J_t$ purchased at time t at price Θ_t^j with a state-contingent pay-out $\mathcal{D}_{t+1}^j$ at $t+1$, taxed at a state-contingent rate ζ_{t+1} which we interpret in the spirit of Chari, Kehoe, and McGrattan (2007) wedges.

The foreign households are symmetric, having access to a set J_t^* of state contingent assets with dividends taxed at a country-specific tax rate ζ_{t+1}^*. The assets $j \in J_t \cap J_t^*$ can be purchased by households of both countries at a common price Θ_t^j in units of home currency, or equivalently $\Theta_t^j/\mathcal{E}_t$ in units of foreign currency.[13] When there are no such assets ($J_t \cap J_t^*$ is empty), the households cannot trade assets *directly* across countries and the financial market is segmented.

Expenditure and demand Domestic households allocate their within-period consumption expenditure between home and foreign varieties of the goods, $P_t C_t = \int_0^1 \Big[P_{Ht}(i) C_{Ht}(i) + P_{Ft}(i) C_{Ft}(i) \Big] \mathrm{d}i$,

[13]For example, when a foreign-currency risk-free bond B_{t+1}^{f*} is available to both home and foreign households, its foreign-currency price is $\Theta_t^{f*} = 1/R_t^*$, where R_t^* is the foreign gross nominal interest rate, and its pay-out is $\mathcal{D}_{t+1}^{f*} \equiv 1$ state-by-state in foreign currency. Correspondingly, the home households can buy it at price $\mathcal{E}_t/R_t^*$ and receive a pay-out of $\mathcal{E}_{t+1}$ in home currency, resulting in a nominal rate of return equal to $\mathcal{E}_{t+1} R_t^*/\mathcal{E}_t$.

to maximize the CES consumption aggregator:

$$C_t = \left[\int_0^1 \left((1-\gamma)^{\frac{1}{\theta}} e^{-\frac{\gamma}{\theta}\xi_t} C_{Ht}(i)^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} e^{\frac{1-\gamma}{\theta}\xi_t} C_{Ft}(i)^{\frac{\theta-1}{\theta}}\right) \mathrm{d}i\right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where $\gamma \in [0, 1/2)$ captures the level of the *home bias*, which can be due to a combination of home bias in preferences, trade costs and non-tradable goods (see Obstfeld and Rogoff 2001), and ξ_t denotes the relative demand shock for the foreign good or other sources of time-varying home bias (see Pavlova and Rigobon 2007). In the quantitative model of Section 6, we extend the analysis from CES to Kimball (1995) demand system to allow for variable markups and *pricing to market*. The solution to the optimal expenditure allocation results in the conventional constant-elasticity demand schedules:

$$C_{Ht}(i) = (1-\gamma)e^{-\gamma\xi_t}\left(\frac{P_{Ht}(i)}{P_t}\right)^{-\theta} C_t \qquad \text{and} \qquad C_{Ft}(j) = \gamma e^{(1-\gamma)\xi_t}\left(\frac{P_{Ft}(j)}{P_t}\right)^{-\theta} C_t, \tag{6}$$

where the price index is given by $P_t = \left[\int_0^1 \left((1-\gamma)e^{-\gamma\xi_t} P_{Ht}(i)^{1-\theta} + \gamma e^{(1-\gamma)\xi_t} P_{Ft}(i)^{1-\theta}\right) \mathrm{d}i\right]^{1/(1-\theta)}$.

The expenditure allocation of foreign households is symmetrically given by:

$$C^*_{Ht}(i) = \gamma e^{(1-\gamma)\xi^*_t}\left(\frac{P^*_{Ht}(i)}{P^*_t}\right)^{-\theta} C^*_t \qquad \text{and} \qquad C^*_{Ft}(j) = (1-\gamma)e^{-\gamma\xi^*_t}\left(\frac{P^*_{Ft}(j)}{P^*_t}\right)^{-\theta} C^*_t, \tag{7}$$

where ξ^*_t is the foreign demand shock for home goods, $P^*_{Ht}(i)$ and $P^*_{Ft}(j)$ are the foreign-currency prices of the home and foreign goods in the foreign market, and P^*_t is the foreign price level. The *real exchange rate* is the relative consumer price level in the two countries:

$$\mathcal{Q}_t \equiv \frac{P^*_t \mathcal{E}_t}{P_t}, \tag{8}$$

with an increase in $\mathcal{Q}_t$ corresponding to a real depreciation, that is a decrease in the relative price of the home consumption basket.

Production and profits Home output is produced by a given pool of identical firms (hence we omit indicator i) according to a Cobb-Douglas technology in labor L_t, capital K_t and intermediate inputs X_t:

$$Y_t = \left(e^{a_t} K_t^{\vartheta} L_t^{1-\vartheta}\right)^{1-\phi} X_t^{\phi}, \tag{9}$$

where a_t is the aggregate productivity shock, and ϑ and ϕ determine the input expenditure shares. For simplicity, our exposition below focuses on the case of $\phi = \vartheta = 0$, where labor is the only production input, so that the marginal cost is given by $MC_t = e^{-a_t} W_t$. Appendix A.3 describes the general case, which we use in our quantitative analysis in Section 6.

Firm i profits (in home currency) from serving both home and foreign markets are given by:

$$\Pi_t(i) = (P_{Ht}(i) - MC_t)C_{Ht}(i) + (\mathcal{E}_t P^*_{Ht}(i) - MC_t)C^*_{Ht}(i), \tag{10}$$

where $P_{Ht}(i)$ and $P^*_{Ht}(i)$ are the home and foreign market prices charged by the firm, by convention expressed in respective local currencies, and $C_{Ht}(i)$ and $C^*_{Ht}(i)$ are the domestic and foreign absorption of the home good i, as characterized by (6) and (7). Goods market clearing requires that firms produce $Y_t(i) = C_{Ht}(i) + C^*_{Ht}(i)$. Aggregate profits of domestic firms, $\Pi_t = \int_0^1 \Pi_t(i)\mathrm{d}i$, are distributed to domestic households. We assume no entry or exit of firms, focusing on the medium-run dynamics.

Wage and price setting In the neoclassical (RBC) version of the model, wages and prices are flexible. In particular, the equilibrium wage rate clears the labor market by equalizing the labor demand of profit-maximizing firms with the household labor supply. The prices are set by monopolistically competitive firms as a markup over the marginal cost MC_t. In the New Keynesian version of the model, wages and prices are adjusted infrequently *à la* Calvo with a constant per-period non-adjustment hazard rate λ_w and λ_p, respectively. We adopt the conventional sticky wages and prices formulation, as described in e.g. Galí (2008). We allow border prices to be sticky in any currency, including producer (PCP), destination (local, LCP) and dominant (dollar, DCP) currency pricing, and thus the law of one price may or may not be satisfied across specifications. Under wage and price stickiness, the quantities are demand-determined: specifically, labor supply must satisfy labor demand given the preset wage rates, as well as the supply of goods must satisfy the demand given prices. We describe the respective equilibrium conditions in Appendix A.3.

Financial sector The financial sector features financial intermediaries and noise traders who participate in *currency carry trades* by taking zero-capital positions in home and foreign-currency bonds. For concreteness, we assume they return earned profits and losses to foreign (US) households along with foreign firm profits, Π^*_t. Whenever home and foreign households can trade some assets directly ($J_t \cap J^*_t$ is non-empty), the presence of financial intermediaries and noise traders does not materially affect macroeconomic allocations and leaves risk-sharing conditions between home and foreign households unchanged. All assets j are in zero net supply, and therefore for $j \in J_t \cap J^*_t$ market clearing requires:

$$B^j_{t+1} + B^{j*}_{t+1} + D^j_{t+1} + N^j_{t+1} = 0,$$

where D^j_{t+1} and N^j_{t+1} are the positions taken by intermediaries and noise traders respectively. When the financial market is *segmented* and the home households cannot trade assets directly with the foreign households ($J_t \cap J^*_t$ is empty), the presence of noise traders and financial intermediaries has an important effect on allocation and international risk sharing. We study this case in detail in Section 5, where we describe the behavior of both intermediaries and noise traders.

Government The fiscal authority is passive, collecting exogenous taxes ζ_t on financial positions of domestic households and returning the collected revenues to the households as a lump sum:

$$T_t = \sum\nolimits_{j \in J_{t-1}} \big(1 - e^{-\zeta_t}\big)\mathcal{D}^j_t B^j_t. \tag{11}$$

Monetary policy is implemented by means of a *generalized* Taylor rule:

$$i_t = \rho_m i_{t-1} + (1-\rho_m)\big[\phi_\pi \pi_t + \phi_e(e_t - \bar{e})\big] + \sigma_m \varepsilon_t^m, \tag{12}$$

where $i_t = \log R_t$ is the log nominal interest rate, $\pi_t = \Delta \log P_t$ is the inflation rate, $\varepsilon_t^m \sim iid(0,1)$ is the monetary policy shock with volatility parameter $\sigma_m \geq 0$, and the parameter ρ_m characterizes the persistence of the monetary policy rule. Coefficients $\phi_\pi > 1$ and $\phi_e \geq 0$ are the Taylor rule parameters which weigh the two nominal objectives of monetary policy — inflation and exchange rate stabilization. We assume that the foreign country (the US) only has the inflation objective, so that $\phi_e^* = 0$. The home country changes ϕ_e depending on the monetary policy regime, with a pure float corresponding to $\phi_e = 0$ and a partial peg featuring $\phi_e > 0$, which approaches a perfect peg as ϕ_e increases. We study the differential behavior of macro variables across monetary regimes of the home country, leaving unchanged the stochastic processes for all exogenous shocks.

3.2 International equilibrium conditions

We emphasize two equilibrium relationships that are specific to the open economy framework — the home country budget constraint and the international risk sharing conditions. These conditions link together the real exchange rate and relative consumption across countries. Changes to other parts of the equilibrium system, including to monetary policy, do not affect these equilibrium relationships.

The home country budget constraint derives from substituting firm profits (10) and government transfers (11) into the household budget constraint (4):

$$\mathcal{B}_{t+1} - \mathcal{R}_t \mathcal{B}_t = NX_t = \mathcal{E}_t P_{Ht}^* C_{Ht}^* - P_{Ft} C_{Ft}, \tag{13}$$

where P_{Ht}^* and P_{Ft} are the export and import price indexes and C_{Ht}^* and C_{Ft} are the aggregate export and import quantities.[14] The left hand-side of (13) is the evolution of home net foreign assets $\mathcal{B}_{t+1} \equiv \sum_{j \in J_t} \Theta_t^j B_{t+1}^j$ with the cumulative pre-tax realized return defined by $\mathcal{R}_t \mathcal{B}_t \equiv \sum_{j \in J_{t-1}} \mathcal{D}_t^j B_t^j$. Using the expressions for import demand (6) and (7), we can rewrite the expression for net exports as:

$$nx_t \equiv \frac{NX_t}{GDP} = \Lambda_t \cdot \left[e^{-(1-\gamma)\tilde{\xi}_t} \mathcal{Q}_t^\theta \mathcal{S}_t^{\theta-1} \frac{C_t^*}{C_t} - 1 \right], \tag{14}$$

where $\Lambda_t \equiv P_{Ft} C_{Ft}/GDP$ is the import-to-GDP ratio, $\mathcal{S}_t \equiv P_{Ft}/(\mathcal{E}_t P_{Ht}^*)$ is the terms of trade, and $\tilde{\xi}_t \equiv \xi_t - \xi_t^*$ is the relative taste shock for the foreign good (home imports). Equation (14) shows the link between net exports, relative consumption levels C_t/C_t^* shaping relative import demand and international relative prices $\mathcal{Q}_t$ and $\mathcal{S}_t$ governing expenditure switching between home and foreign goods. In particular, under financial autarky, the country budget constraint requires $NX_t \equiv 0$, and therefore C_t/C_t^* is directly related to $\mathcal{Q}_t^\theta \mathcal{S}_t^{\theta-1}$, conditional on the taste (home bias) shock $\tilde{\xi}_t$.

International risk sharing conditions, for each asset traded between home and foreign households,

[14] Using (6), aggregate imports are given by $P_{Ft} C_{Ft} = \int_0^1 P_{Ft}(i) C_{Ft}(i) \mathrm{d}i = \gamma e^{(1-\gamma)\xi_t} P_t^\theta P_{Ft}^{1-\theta} C_t$, where $P_{Ft} = \big[\int_0^1 P_{Ft}(i)^{1-\theta} \mathrm{d}i\big]^{1/(1-\theta)}$ is the import price index, and similarly aggregate exports $P_{Ht}^* C_{Ht}^* = \gamma e^{(1-\gamma)\xi_t^*} P_t^{*\theta} P_{Ht}^{*1-\theta} C_t^*$.

are given by:

$$\mathbb{E}_t \left\{ \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} - \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{\mathcal{Q}_t}{\mathcal{Q}_{t+1}} e^{\tilde{\zeta}_{t+1}} \right] e^{-\zeta_{t+1}} \mathcal{R}^j_{t+1} \right\} = 0 \qquad \forall j \in J_t \cap J^*_t, \tag{15}$$

where $\tilde{\zeta}_{t+1} \equiv \zeta_{t+1} - \zeta^*_{t+1}$ is the relative financial tax (wedge) across countries and $\mathcal{R}^j_{t+1} \equiv \frac{\mathcal{D}^j_{t+1}/\Theta^j_t}{P_{t+1}/P_t}$ is the pre-tax real return on asset j at home. Condition (15) derives from combining the home and foreign household Euler equations, reflecting that households trade available assets $j \in J_t \cap J^*_t$ until the home and foreign stochastic discount factors (SDFs) are aligned as much as possible.[15] Under complete international asset markets, the set of tradable assets $J_t \cap J^*_t$ allows agents to replicate a full set of Arrow securities for each state of the world, and (15) simply becomes

$$\left(\frac{C_{t+1}/C_t}{C^*_{t+1}/C^*_t} \right)^{\sigma} = \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \cdot e^{-\tilde{\zeta}_{t+1}}. \tag{16}$$

This is effectively a static relationship between the relative consumption growth and real depreciation, which generalizes the Backus and Smith (1993) condition by introducing state-dependent risk sharing wedges $\tilde{\zeta}_{t+1}$. More generally, conditions (13) and (15) together characterize the joint equilibrium dynamics of relative consumption and the real exchange rate under a variety of asset market structures that range from financial autarky to complete international asset markets, depending on the richness of the $J_t \cap J^*_t$ set.

4 Conventional Models: A Falsification

We now consider a class of conventional international DSGE models, including both standard international real business cycle (IRBC) and New Keynesian open economy (NKOE) models, in which a particular equilibrium relationship between relative consumption and the real exchange rate holds independently of monetary policy and the exchange rate regime, and thus is falsified by the data on the end of the Bretton Woods. Importantly, this *sufficient statistic* does not depend on the supply side of the economy — in particular, on the presence and nature of price and wage stickiness, the set and statistical properties of exogenous shocks, and the degree of openness of the economies — and holds for a general structure of the international asset market. To prove the exact result, we specialize the model to the case of the Cole and Obstfeld (1991) parameter restriction, which is ubiquitously used in international economics (see e.g. Galí and Monacelli 2005, Heathcote and Perri 2013). While the Cole-Obstfeld case is clearly special, the sharp analytical result that we establish in this section holds approximately true in a much richer quantitative environment and guides our quantitative analysis in Section 6.

[15] Home and foreign Euler equations are $\mathbb{E}_t\{\mathcal{M}_{t+1} e^{-\zeta_{t+1}} \mathcal{R}^j_{t+1}\} = 1$ for all assets $j \in J_t$ and $\mathbb{E}_t\{\mathcal{M}^*_{t+1} e^{-\zeta^*_{t+1}} \mathcal{R}^{j*}_{t+1}\} = 1$ for all assets $j \in J^*_t$, where $\mathcal{M}_t \equiv \beta(C_{t+1}/C_t)^{-\sigma}$ and $\mathcal{M}^*_t \equiv \beta(C^*_{t+1}/C^*_t)^{-\sigma}$ are the home and foreign real SDFs respectively. Subtracting one from the other, and noting that $\mathcal{R}^{j*}_{t+1} = \mathcal{R}^j_{t+1} \frac{\mathcal{Q}_t}{\mathcal{Q}_{t+1}}$, results in (15).

Dynamic system We now combine the two equilibrium conditions derived in Section 3.2 to establish an equilibrium relationship between consumption and the real exchange rate. We rewrite (13) and (15) in log-deviation terms from a non-stochastic symmetric equilibrium (with $\overline{NX} = \bar{\mathcal{B}} = 0$ and $\bar{\mathcal{R}} = 1/\beta$):

$$\mathbb{E}_t\big\{\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1}\big\} = \hat{\psi}_t, \tag{17}$$

$$\beta b_{t+1} - b_t = nx_t = \gamma\big[\hat{\theta} q_t - (c_t - c^*_t) - (1-\gamma)\hat{\xi}_t\big], \tag{18}$$

where $nx_t \equiv NX_t/(\bar{P}\bar{Y})$ and $b_t \equiv B_t/(\beta\bar{P}\bar{Y})$, and $\hat{\theta}$ is the projection coefficient of $\log(\mathcal{Q}_t^{\theta}\mathcal{S}_t^{\theta-1})$ on $q_t = \log \mathcal{Q}_t$.[16] We view these conditions as exact equations with $\hat{\psi}_t$ and $\hat{\xi}_t$ defining the residual terms, which include both exogenous shocks or wedges ($-\mathbb{E}_t\tilde{\zeta}_{t+1}$ and $\tilde{\xi}_t$), as well as higher order terms such as risk premia in (15). Since we do not impose any statistical properties on the co-evolution of $\{\hat{\psi}_t, \hat{\xi}_t\}_t$, this interpretation is without loss of generality.

Definition 1 (Conventional Models) *We call conventional the models in which changes in monetary policy or exchange rate regime do not change the stochastic path of the residual terms $\{\hat{\psi}_t, \hat{\xi}_t\}_t$ in (17)–(18).*

This definition is useful because it nests a large class of popular international business cycle models. In particular, all linearized DSGE models, whether IRBC or NKOE, fall into this class. Similarly, all complete market models with separable expected utility (e.g., CRRA) fall into this class as well. Furthermore, this definition accurately approximates most standard international macro models, which typically have quantitatively negligible size and volatility of risk premia (the equity premium puzzle).

With this definition, the system (17)–(18) characterizes equilibrium dynamics of $\{b_{t+1}, q_t, c_t - c^*_t\}$ conditional on initial NFA position b_0, a no-bubble condition $\lim_{t\to\infty}\beta^t b_t = 0$, and an exogenous path of wedges $\{\hat{\psi}_t, \hat{\xi}_t\}_{t\ge 0}$. This system is, in general, incomplete as it does not include any of the domestic equilibrium conditions, focusing on the two international equilibrium conditions only. Nonetheless, under certain circumstances, this system is already sufficient to characterize the equilibrium dynamics of a particular linear combination of relative consumption $c_t - c^*_t$ and the real exchange rate q_t. Intuitively, while these international conditions do not characterizes the equilibrium properties of consumption levels, c_t and c^*_t, they provide a link between the international variables, namely q_t and net foreign assets b_{t+1}, and relative consumption across countries, $c_t - c^*_t$.

We now focus on the case in which international conditions (17)–(18) are sufficient to characterize the equilibrium dynamics of a particular statistic (for formal proof, see Appendix A.5):

Proposition 1 *In conventional models, under the Cole-Obstfeld parameter restriction $\sigma = \theta = 1$, the statistical properties of $z_t \equiv \sigma(c_t - c_t) - q_t$ do not change with a change in the monetary policy rule and exchange rate regime, in particular, the volatility of Δz_t remains unchanged.*

[16] Note that $\hat{\theta} = 1$ when $\theta = 1$, as in this case we simply have $\log(\mathcal{Q}_t^{\theta}\mathcal{S}_t^{\theta-1}) = q_t$. Furthermore, when the law of one price holds (e.g., under PCP), we have $\mathcal{S}_t = \mathcal{Q}_t^{1/(1-2\gamma)}$ (at least up to second order), and thus $\hat{\theta} = \theta + \frac{\theta-1}{1-2\gamma}$; note that the residual $\hat{\xi}_t$ in (18) includes the residual term from the projection, which is in general non-zero. Models with variable markups and law of one price violations exhibit the same qualitative property, just with $\hat{\theta}$ having a more complex structure.

The significance of this result is that it emphasizes an existence of a simple sufficient statistic, $z_t = \sigma(c_t - c_t^*) - q_t$, that is readily measurable in the data. The proposition predicts that its statistical properties, and in particular the simple unconditional standard deviation of Δz_t, are independent of the monetary regime, and should not change with a shift between a peg and a float. As we show in Figure 5, this implication is strongly rejected by the data — which suggest a dramatic increase in the volatility of z_t after the end of the Bretton Woods — and thus falsifies a class of conventional business cycle models, including both flexible-price IRBC and sticky-price NKOE models.

Importantly, this insight does not depend on most modeling details, including the presence and the nature of nominal rigidities, openness of the economy and the set and dynamic properties of shocks.[17] We discuss the role of the Cole-Obstfeld parameter restriction below. Note that the stable statistical properties of the sufficient statistic z_t do not imply the same for the behavior of c_t, c_t^* and q_t separately which could all change considerably equilibrium properties with a shift in monetary policy. We illustrate this in Appendix Figure A9, where a change in the exchange rate regime affects the dynamics of both $c_t - c_t^*$ and q_t (but not z_t), and does so differentially depending on the type of border price stickiness. This emphasizes the sufficient statistic's role, as individual data series are insufficient to evaluate the class of conventional models while a particular cointegration vector z_t is.

What is the logic behind Proposition 1? It is easiest to see it starting from the limiting case of complete markets.[18] In this case, international risk sharing leads to (16) resulting in $\Delta z_{t+1} = -\tilde{\zeta}_{t+1}$, which reduces to the Backus-Smith condition $z_t = \sigma(c_t - c_t^*) - q_t = 0$ in the absence of risk-sharing wedges. We argue that this logic extends in a particular way to a much larger class of models that allow for incomplete markets and risk sharing shocks. Specifically, instead of a perfect correlation between relative consumption and the real exchange rate, we obtain the result in Proposition 1, namely that a particular linear combination of relative consumption and the real exchange rate does not depend on the monetary regime and a variety of other features and parameters of the model.

Formally, this can be seen by iterating the general risk-sharing condition (17) forward:

$$z_t = \sum\nolimits_{j=0}^{\infty} \mathbb{E}_t \hat{\psi}_{t+j} + z_t^{\infty}, \qquad \text{where} \qquad z_t^{\infty} \equiv \lim_{j\to\infty} \mathbb{E}_t z_{t+j}. \tag{19}$$

Therefore, $\{\hat{\psi}_{t+j}\}$ determines the properties of z_t up to its long-run expectation z_t^{∞}, which unlike in the complete market case, depend on the rest of the equilibrium system. Under the Cole-Obstfeld restriction, the country budget constraint is sufficient to pin down the long-run expectation z_t^{∞}. The reason this is possible and that no other equilibrium condition is needed is that net exports (14) in this case are also shaped by z_t, namely $nx_t = -\gamma[z_t - (1-\gamma)\hat{\xi}_t]$, as $\hat{\theta} = \theta = 1/\sigma = 1$. Therefore, the intertemporal budget constraint offers an integral condition on the equilibrium path of $\{z_{t+j}\}$. As a result, this combination of the two open-economy equilibrium conditions under a general asset market structure replaces the Backus-Smith condition in shaping equilibrium properties of $z_t = \sigma(c_t - c_t^*) - q_t$.

[17] In particular, even discontinuous shifts in productivity shocks and other macroeconomic shocks (apart from $\hat{\psi}_t$ and $\hat{\xi}_t$) do not result in a change in the statistical properties of z_t.

[18] While the dynamic system (17)–(18) does not formally nest the limiting cases of complete markets and financial autarky, we verify directly that Proposition 1 also holds for these two special cases. Note that financial autarky with $nx_t = 0$, which implies $\hat{\theta} q_t - (c_t - c_t^*) = (1-\gamma)\hat{\xi}_t$, is qualitatively similar to the complete market case with $\sigma(c_t - c_t^*) - q_t = \hat{\psi}_t$.

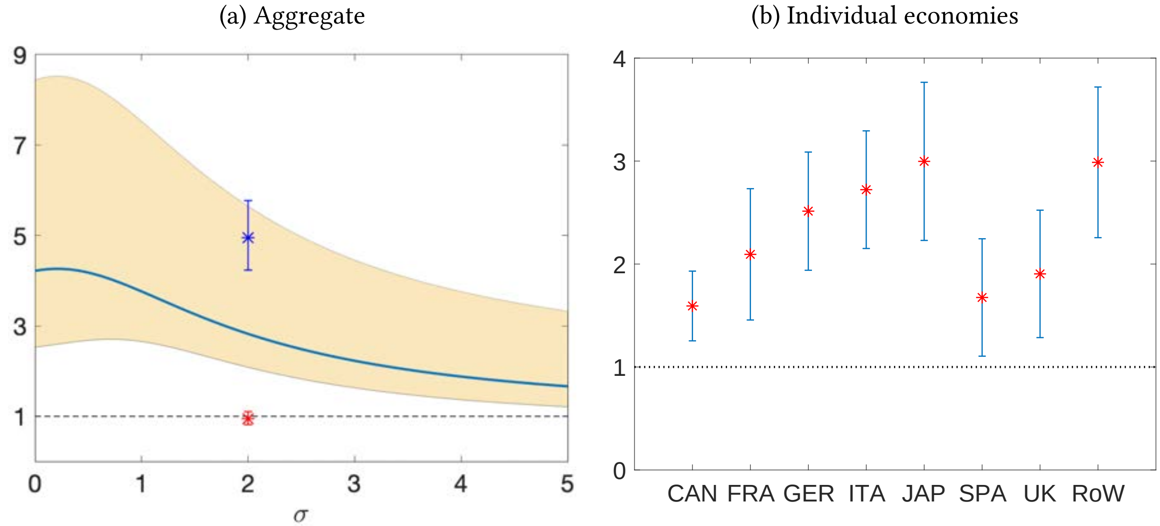


Figure 5: Ratio of $\text{std}(\Delta z_t)$ after/during the Bretton Woods System

Note: $\text{std}(\Delta z_t)$, where $z_t \equiv \sigma(c_t - c_t^*) - q_t$, is computed for 1960–72 and 1973–89 for the RoW vs the U.S. for different values of σ with a 90% confidence interval. The dashed line at 1 illustrates the prediction of Proposition 1. The red asterisk (and the simulated 90% confidence interval around it) correspond to the calibrated conventional model from Section 6, which relaxes the Cole-Obstfeld parameter restriction (specifically, $\sigma = 2$, $\theta = 1.5$; see Table 2). The blue asterisk and confidence interval correspond to the calibrated segmented markets model (IRBC$^+$ in Table 2).

How important is the Cole-Obstfeld parameter restriction? First, note that the equilibrium in the Cole-Obstfeld case is *not* equivalent to the complete market allocation in the presence of taste shocks $\hat{\xi}_t$ and/or risk-sharing wedges $\hat{\psi}_t$ (see e.g. Pavlova and Rigobon 2007). Second, while the Cole-Obstfeld restriction is necessary for the exact result in Proposition 1, quantitatively the prediction that Δz_{t+1} does not change its statistical properties holds approximately in calibrated conventional models outside the Cole-Obstfeld case, as we illustrate in Figure 5 and explore further in Section 6. Furthermore, the result of Proposition 1 still holds exactly in important limiting cases away from the Cole-Obstfeld parameter restriction. This is trivially true in the case of complete markets and under financial autarky, as well as in the limits of flexible or fully sticky prices (wages). More generally, one needs to complement the risk-sharing condition and the budget constraint with a dynamic price-setting equation to pin down the long-run expectation z_t^∞ in (19). However, the role of the latter condition is vanishingly small as $\beta \to 1$, and in this limit z_t^∞ is again independent of this additional dynamic equation.

Lastly, we discuss briefly which models are ruled out by Proposition 1 in light of its quantitative robustness outside the Cole-Obstfeld case. A necessary property for a model to account for the Mussa facts is to feature an equilibrium process for $\hat{\psi}_t$ (or $\hat{\xi}_t$) that changes with the monetary regime, which makes the model *unconventional* according to Definition 1. Standard models may feature such effects, as changes in monetary policy can, in general, affect equilibrium risk premia under *incomplete* financial markets.[19] However, in standard business cycle models — whether with flexible or sticky prices and monetary or productivity shocks — such effects are quantitatively small, and thus Proposition 1 provides an accurate approximation for such models as well (see Section 6). As a result, the data on the end

[19]Similarly, failure in the law of one price in the goods market may render $\hat{\xi}_t$ endogenous to the monetary regime, yet such effects are typically small for macroeconomic quantities.

of Bretton Woods falsifies all conventional business cycle frameworks, requiring an unconventional approach to modeling the international financial market.

5 An Alternative Model of Non-neutrality

We now present an alternative explanation to the broad set of Mussa facts documented in Section 2. The negative result of Proposition 1 has a constructive nature, as it emphasizes the need to depart from conventional business cycle models, and in particular introduce equilibrium variation in risk premium $\hat{\psi}_t$, endogenous to the exchange rate regime. Towards this goal, we develop a model where monetary non-neutrality emerges due to financial market segmentation, rather than as a result of goods-market nominal rigidities.[20] We maintain the general modeling environment of Section 3, but to emphasize our point assume away nominal rigidities altogether. The only new feature is the modeling of the international financial market, as we describe next.

5.1 Segmented financial market

Our model of the financial sector builds on Jeanne and Rose (2002) and Gabaix and Maggiori (2015), and features three types of agents: households, noise traders and professional intermediaries.[21] Specifically, we assume that the home and foreign households can only trade their respective local-currency bonds, and thus cannot directly trade assets with each other, resulting in a *segmented* financial market. Formally, this corresponds to the case with $J_t \cap J_t^* = \emptyset$ in the general notation of Section 3, with the home households holding B_{t+1} units of the home-currency bond and the foreign households holding B_{t+1}^* units of the foreign-currency bond at time t. Both B_{t+1} and B_{t+1}^* can take positive or negative values, depending on whether the households save or borrow. The bonds pay out $\mathcal{D}_{t+1} = 1$ euro and $\mathcal{D}_{t+1}^* = 1$ dollar at period $t+1$, and hence their period t prices are $\Theta_t = 1/R_t$ euros and $\Theta_t^* = 1/R_t^*$ dollars, where R_t and R_t^* are the respective nominal interest rates. We assume away exogenous risk-sharing wedges in (4), $\zeta_{t+1} = \zeta_{t+1}^* \equiv 0$, as they emerge endogenously in a segmented market equilibrium.

In addition to the household *fundamental* demand for currency (bonds), the financial market features a *liquidity* currency demand — independent of the expected currency return and the other macroeconomic fundamentals — from a measure n of symmetric *noise* traders. In particular, noise traders follow a zero-capital strategy by taking a long position in the foreign currency and shorting equal value in the home currency, or vice versa if they have excess demand for the home currency. The overall position of the noise traders is $\frac{N_{t+1}^*}{R_t^*}$ dollars invested in the foreign-currency bond, matched by $\frac{N_{t+1}}{R_t} = -\frac{\mathcal{E}_t N_{t+1}^*}{R_t^*}$

[20] While Proposition 1 does not discriminate between $\hat{\psi}_t$ and $\hat{\xi}_t$ wedges, our model builds on endogenous risk premium, which is consistent with larger UIP deviations and little change in volatility of trade balance under the floating regime. The data also favor the segmented markets approach as the covariance of the exchange rate with aggregate macro variables is generally negligible and did not feature any noticeable change after the end of Bretton Woods (see Appendix Figure A10), suggesting that representative-agent models of risk premia are unlikely to be successful at this task (see Section 7).

[21] We follow Jeanne and Rose (2002) in modeling the financial intermediaries, who take limited asset positions due to exposure to the exchange rate risk rather than due to financial constraints as in Gabaix and Maggiori (2015). In contrast, we follow Gabaix and Maggiori (2015) in modeling the segmented participation of the households. Lastly, the exogenous liquidity needs of the noise trader are akin to the exogenous 'portfolio flows' in Gabaix and Maggiori (2015) but can equally emerge from biased expectations about the exchange rate, $\mathbb{E}_t^n \mathcal{E}_{t+1} \neq \mathbb{E}_t \mathcal{E}_{t+1}$, as in Jeanne and Rose (2002).

euros invested in the home-currency bond, and we model it as an exogenous process:

$$\frac{N^*_{t+1}}{P^*_t} = n\left(e^{\psi_t} - 1\right) \qquad \text{with} \qquad \psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \varepsilon^\psi_t. \tag{20}$$

We refer to the noise-trader demand shock ψ_t as the *financial shock*, with $\rho_\psi \in [0,1]$ and $\sigma_\psi \geq 0$ parametrizing its persistence and volatility, respectively.

The trades of the households and the noise traders are intermediated by a measure m of symmetric risk-averse arbitrageurs, or market makers. These intermediaries adopt a zero-capital *carry trade* strategy by taking a long position in the foreign-currency bond and a short position of equal value in the home-currency bond, or vice versa. The return on the carry trade is given by:

$$\tilde{R}^*_{t+1} = R^*_t - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \tag{21}$$

per dollar invested in the foreign-currency bond and $\mathcal{E}_t$ euros sold of the home-currency bond at time t. We denote the size of an individual position by d^*_{t+1}, which may take positive or negative values, and assume that intermediaries maximize the CARA utility of the real return in units of the foreign good:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} \right) \right\}, \tag{22}$$

where $\omega \geq 0$ is the risk aversion parameter.[22] In aggregate, all m intermediaries invest $\frac{D^*_{t+1}}{R^*_t} = m\frac{d^*_{t+1}}{R^*_t}$ dollars in foreign-currency bond, and take an offsetting position of $\frac{D_{t+1}}{R_t} = -\frac{\mathcal{E}_t D^*_{t+1}}{R^*_t}$ euros in home-currency bond, resulting in a zero-capital portfolio at time t.

Both currency bonds are in zero net supply, and therefore financial market clearing requires that the positions of the households, noise traders and intermediaries balance out:

$$B_{t+1} + N_{t+1} + D_{t+1} = 0 \qquad \text{and} \qquad B^*_{t+1} + N^*_{t+1} + D^*_{t+1} = 0. \tag{23}$$

In equilibrium, the intermediaries absorb the demand for home and foreign currency of both households and noise traders. If intermediaries were risk neutral, $\omega = 0$, they would do so without a risk premium, resulting in the *uncovered interest parity* (UIP), or equivalently a zero expected real return, $\mathbb{E}_t\{\tilde{R}^*_{t+1}/P^*_{t+1}\} = 0$. Risk-averse intermediaries, however, require an appropriate compensation for taking a risky carry trade, which results in equilibrium risk premia and deviations from UIP:

[22] CARA utility provides tractability as it results in a portfolio choice that does not depend on the level of wealth of the intermediaries, thus avoiding the need to carry it as an additional state variable. The tradeoff of working with CARA-utility, however, is that intermediaries need to be short-lived, maximizing a one-period return on their investment.

Lemma 1 *The optimal portfolio choice of intermediaries and the resulting equilibrium condition in the financial market, log-linearized around a symmetric steady state with* $\bar{B} = \bar{B}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{\mathcal{Q}} = 1$ *and a finite nonzero* $\omega\sigma_e^2$*, are given respectively by:*

$$\frac{d_{t+1}^*}{P_t^*} = -\frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}}{\omega\sigma_e^2}, \tag{24}$$

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \tag{25}$$

where $i_t - i_t^* \equiv \log(R_t/R_t^*)$, $b_t \equiv B_t/(\beta\bar{P}\bar{Y})$, *and the coefficients* $\chi_1 \equiv n\frac{\omega\sigma_e^2}{m}$ *and* $\chi_2 \equiv \beta\bar{Y}\frac{\omega\sigma_e^2}{m}$, *with* $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ *denoting the volatility of the log nominal exchange rate,* $e_t \equiv \log \mathcal{E}_t$.

We provide a formal proof of this lemma in Appendix A.4. The solution to the portfolio problem (22) of an individual arbitrageur results in (24), namely each arbitrageur invests in a zero-capital portfolio d_{t+1}^* long in dollar (bonds) and $R_t\frac{\mathcal{E}_t d_{t+1}^*}{R_t^*}$ short in euro, or vice versa ($d_{t+1}^* < 0$) if the dollar is the low interest rate currency. Intuitively, the optimal size of an arbitrageur's carry trade position is proportional to the expected log return on the carry trade, $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$, adjusted by the volatility of carry trade returns σ_e^2 (the exchange rate risk) and arbitrageur's risk aversion ω, as is standard in portfolio choice theory (see Campbell and Viceira 2002). The overall position of all intermediaries is given by $D_{t+1}^* = m d_{t+1}^*$, and thus their combined *risk-absorption capacity* of the UIP deviations is proportional to $m/(\omega\sigma_e^2)$, decreasing in the volatility of the nominal exchange rate.

In equilibrium, to clear the financial market (23), the combined position of intermediaries must offset the combined positions of households and noise traders, namely $D_{t+1}^* = -(B_{t+1}^* + N_{t+1}^*)$. This results in the equilibrium relationship (25), which is a *modified UIP* condition in our model with imperfect financial intermediation, characterizing the equilibrium size of UIP deviations. Note that as the risk absorption capacity of the intermediary sector $m/(\omega\sigma_e^2)$ increases, the UIP deviations disappear in the limit, as $\chi_1, \chi_2 \to 0$. With $\omega\sigma_e^2/m > 0$, the UIP deviations remain first order and hence affect the first-order equilibrium dynamics. Note that both $\psi_t > 0$ and $b_{t+1} < 0$ correspond to the excess demand for the foreign-currency bond — by noise traders and households, respectively — resulting in a negative expected return on the foreign currency bond.

International risk sharing Note that as both noise traders and intermediaries hold zero-capital positions, financial market clearing (23) implies a balanced position for the home and foreign households combined, $\frac{B_{t+1}}{R_t} + \mathcal{E}_t\frac{B_{t+1}^*}{R_t^*} = 0$. In other words, even though the home and foreign households do not trade any assets directly, the financial market acts to intermediate the intertemporal borrowing between them. However, this intermediation is *frictional*, as there is a wedge between interest rates faced by the home and foreign households, R_t and R_t^*, namely the (expected) departures from the UIP in (25). If interest rate parity held, the equilibrium would correspond to a conventional IRBC model with incomplete markets, whereas the UIP wedge further limits the extent of international risk sharing.

Condition (25) characterizes equilibrium in the financial market. It can be combined with the household Euler equations, to obtain a version of the international risk-sharing condition (17) with an endogenous risk-sharing wedge $\hat{\psi}_t$:[23]

$$\mathbb{E}_t\big\{\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1}\big\} = \hat{\psi}_t \equiv \chi_1\psi_t - \chi_2 b_{t+1}, \tag{26}$$

When monetary policy affects the equilibrium volatility of the nominal exchange rate, σ_e^2, it changes the volatility of the risk-sharing wedge, $\hat{\psi}_t$, via its effects on $\chi_1 = \chi_1(\sigma_e^2)$ and $\chi_2 = \chi_2(\sigma_e^2)$, as defined in Lemma 1. Despite fully flexible prices, monetary policy is *not* neutral in our model because of financial frictions, as it affects the extent of international risk-sharing. Indeed, a shift to an exchange rate peg stabilizes the nominal exchange rate and encourages financial intermediation, as arbitrageurs are willing to take larger positions (24), reducing the extent of equilibrium UIP deviations (25) and risk-sharing wedges (26).[24]

The *unconventional* feature of this model is that monetary policy affects equilibrium risk premia and allocations in the financial market, and financial market segmentation magnifies these effects. Indeed, for financial intermediaries who earn carry trade returns, the relevant measure of risk is the volatility of the *nominal* exchange rate, rather than its covariance with aggregate consumption as would be the case in a conventional model. While intermediaries maximize the *real* return on their carry trade in (22), it is nonetheless the volatility of the *nominal* exchange rate that constitutes the source of risk in their investment, and therefore a change in the nominal exchange rate regime has real consequences. This is the source of monetary non-neutrality in this model without nominal rigidities.[25]

5.2 The Mussa puzzle resolution

We now study the general equilibrium properties of the model with segmented financial markets under alternative exchange rate regimes. The goal is to offer a simple *qualitative* resolution to the set of Mussa facts documented in Section 2 in a tractable analytical environment, with a comprehensive *quantitative* analysis deferred to Section 6. Specifically, we consider a monetary policy rule which fully stabilize either consumer prices or the nominal exchange rate, depending on the policy regime. That is, the foreign country always chooses $\pi^*_t \equiv 0$, while the home country adopts either a peg with $\Delta e_t \equiv 0$ or a float with $\pi_t \equiv 0$. As a result, under the peg $\sigma_e^2 = 0$, and thus $\chi_1 = \chi_2 = 0$ in (25) and (26), while under the float $\sigma_e^2 > 0$ and $\chi_1, \chi_2 > 0$.

We allow for two types of shocks — the noise trader shock ψ_t introduced in (20) and country-specific productivity shocks (a_t, a^*_t), which are possibly correlated and also follow AR(1) processes with persistence ρ_a and standard deviation of innovations σ_a. For simplicity, we assume a common persistence parameter for all shocks, $\rho_a = \rho_\psi = \rho \in [0, 1]$. The equilibrium system consists of two dynamics equations, still given by the risk-sharing condition (17) and the intertemporal budget

[23] The log-linearized home household Euler equation is $i_t = \mathbb{E}_t\{\sigma\Delta c_{t+1} + \Delta p_{t+1}\}$, and similarly for the foreign household, so that that UIP deviation equals the risk-sharing wedge, $i_t - i^*_t - \mathbb{E}_t\Delta e_{t+1} = \mathbb{E}_t\big\{\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1}\big\}$.

[24] Note that, despite the changing properties of the UIP violations with the monetary regime, the *covered interest parity* (CIP) holds in the model independently of the monetary regime. Indeed, any CIP violations would result in arbitrage profit opportunities, and intermediaries would be willing to take unbounded asset positions in order to exploit them.

[25] Despite flexible prices, monetary policy determines real returns on nominal bonds and, thus, alters the spanning of states by nominal assets, which in turn changes equilibrium risk sharing and real allocations in the two economies.

constraint (18). In the former, the overall risk-sharing wedge $\hat{\psi}_t$ satisfies (26), while in the latter we set $\hat{\xi}_t \equiv 0$ for simplicity. We do not restrict the parameter values; the derived elasticity of net exports with respect to the real exchange rate in (18) is given by $\hat{\theta} \equiv \frac{2\theta(1-\gamma)-1}{1-2\gamma}$.[26]

The remaining equilibrium condition linking relative consumption and the real exchange rate derives from market clearing in labor and product markets, and can be written as (see Appendix A.6):

$$c_t - c_t^* = \kappa_a(a_t - a_t^*) - \gamma\kappa_q q_t, \tag{27}$$

where $\kappa_a \equiv \frac{1+\varphi}{\sigma+(1-2\gamma)\varphi}$ and $\kappa_q \equiv \frac{2}{1-2\gamma}\frac{2\theta(1-\gamma)\varphi+1}{\sigma+(1-2\gamma)\varphi}$. Relative home consumption increases with relative home productivity, shaping the relative supply of the home good, and decreases with a real depreciation (increase in q_t), which shifts expenditure towards home goods globally.

The equilibrium system defined in (18), (26) and (27) determines the dynamics of the real exchange rate q_t, in each monetary regime respectively. The equilibrium nominal exchange rate is given by $e_t = q_t$ under the float (as $\pi_t = \pi_t^* = 0$), and is fully stabilized as $e_t \equiv 0$ under the peg (and thus $\pi_t = -\Delta q_t$). As a result, $\sigma_e^2 = 0$ under the peg and $\sigma_e^2 = \sigma_q^2 \equiv \mathrm{var}_t(\Delta q_{t+1})$ under the float, and the equilibrium process for q_t takes full account of this fixed point between the real exchange rate process and the nominal exchange rate volatility. Lastly, given the path of q_t, equilibrium relative consumption $c_t - c_t^*$ is characterized by market clearing in (27), independently of the monetary regime. With this, we prove the main qualitative result of this section:

Proposition 2 *With a segmented financial market, the equilibrium exists under both a peg and a float policy regime. A change in the monetary policy rule from a peg to a float leads to* (a) *an arbitrary large increase in the volatility of both nominal and real exchange rates; and* (b) *a vanishingly small change in the behavior of all other macro variables.*

This 'order-of-magnitude' result shows that a model with a segmented financial market can be consistent with the broad set of Mussa facts documented in Section 2. We discuss below the circumstances when this possibility result applies, while Section 6 shows that this proposition also offers a relevant point of approximation for a quantitative model that matches business cycle properties of exchange rates and macroeconomic variables. A formal proof of Proposition 2, in particular, describes a closed-form solution for the exchange rate process and macroeconomic allocations under both policy regimes, and is contained in Appendix A.6.[27]

Part (a) of Proposition 2 focuses on the original Mussa (1986) fact about the discontinuity in the volatility of exchange rates — both nominal and real — across a float and a peg regime, and shows that a segmented financial market model can be consistent with this pattern even in the absence of nominal rigidities. The intuition is that the equilibrium real exchange rate can be decomposed as $q_t = q_t^a + q_t^\psi$,

[26] Recall from (14) that nx_t increases in $[\theta q_t + (\theta-1)s_t]$, and under flexible prices, which ensure the law of one price, we have $s_t = q_t/(1-2\gamma)$ (see Appendix A.6). Therefore, nx_t increases in $\hat{\theta}q_t$, with the expression for $\hat{\theta}$ given in the text and $\hat{\xi}_t \equiv 0$ in the absence of preference shocks (ξ_t, ξ_t^*).

[27] For example, in the limit of persistent random-walk shocks ($\rho \to 1$), we have $q_t = q_t^a + q_t^\psi$ with $q_t^a = \frac{\kappa_a}{\hat{\theta}+\gamma\kappa_q}(a_t - a_t^*)$ and $(1-\delta L)q_t^\psi = \frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta\delta}{1-\beta\delta}\big(1-\frac{1}{\beta}L\big)\chi_1\psi_t$, where L is a lag operator, $\delta = \delta(\sigma_e^2) \in (0,1]$ and $\chi_1 = \chi_1(\sigma_e^2) \geq 0$ with $\delta \to 1$ and $\chi_1 \to 0$ as $\sigma_e^2 \to 0$, while q_t^a does not change its properties with the policy regime.

with the two components driven by productivity and financial shocks respectively, where the second component $q_t^\psi \equiv 0$ under the peg. The latter property arises from the fact that financial shocks ψ_t are featured only in the international risk-sharing condition (26) with a coefficient that is endogenous to the monetary regime, $\chi_1 = \chi_1(\sigma_e^2)$ such that $\chi_1 > 0$ for $\sigma_e^2 > 0$ and $\chi_1 \to 0$ as $\sigma_e^2 \to 0$.[28]

To the extent that financial shocks account for the bulk of exchange rate volatility under the float — a property that Itskhoki and Mukhin (2021) argue is essential for a successful model of exchange rates — a switch to the peg that mutes such shocks would result in an arbitrarily less volatile real exchange rate.[29] A switch to a nominal peg endogenously alters the behavior of financial intermediaries, who are willing to take much larger gross currency positions when $\sigma_e^2 = \text{var}_t(\Delta e_{t+1})$ is small, resulting in a smaller (and zero in the limit) equilibrium risk-sharing wedge $\hat{\psi}_t \propto \chi_1(\sigma_e^2) \cdot \psi_t$. This endogenous change in the relative contribution of shocks that drive the real exchange rate across policy regimes constitutes the source of the monetary non-neutrality that accounts for the Mussa facts.

Part (b) of Proposition 2 focuses on the complementary set of Mussa facts that we emphasized in Section 2, namely the lack of a noticeable change in the business cycle properties of macro variables associated with a change in the policy regime. With flexible prices, the only channel through which monetary policy affects real variables is through its effect on international risk sharing and the properties of the real exchange rate. This effect is, however, proportional to the openness of the economy γ, and is vanishingly small in the autarky limit ($\gamma \to 0$), where movements in the real exchange rate are irrelevant for macroeconomic allocations. This logic can be seen directly from the goods market clearing condition (27), which displays the sources of equilibrium volatility in relative consumption. Small γ ensures that macro variables are not responsive to the exchange rate volatility, and instead are driven by macro-fundamental shocks under both the float and the peg, consistent with exchange rate disconnect. As a result, a discontinuous drop in the real exchange rate volatility associated with a switch to the peg has only minor consequences for real allocations.

This interpretation, however, describes only the partial equilibrium channel of transmission based on low exchange rate pass-through into macroeconomic quantities when countries are sufficiently closed to international trade.[30] A deeper question, however, is how a change in monetary policy — from stabilizing inflation to stabilizing highly volatile nominal exchange rate — can have no consequences for macroeconomic variables, nominal or real. Indeed, even in the closed economy limit ($\gamma \approx 0$), an attempt to use monetary policy to stabilize a volatile nominal variable should translate into volatile inflation under flexible prices and additionally volatile real variables in the presence of nominal rigidities. The lack of this general equilibrium spillover from a change in monetary policy is perhaps the most surprising part of Proposition 2.

Since our model here features no nominal rigidities, we focus on domestic inflation, which equals

[28]Note that this also implies that the result is not driven by a discontinuity at a fully fixed nominal exchange rate with $\sigma_e^2 = 0$, but also applies continuously as σ_e^2 gradually declines towards zero, as is the case in our calibration in Section 6.

[29]A sufficient requirement for part (a) of Proposition 2 is that $\text{var}(\Delta q_{t+1}^a)/\text{var}(\Delta q_{t+1})$ is sufficiently small in the variance decomposition of q_t under the float, which is also a necessary requirement for the model to be consistent with e.g. the Meese and Rogoff (1983) disconnect and a negative Backus and Smith (1993) correlation.

[30]Low trade openness is complemented by incomplete pass-through due to variable markups and local-currency price stickiness, which effectively reduce κ_q in (27) and are both featured in the quantitative model of Section 6.

$\pi_t = 0$ under the float (inflation targeting) and $\pi_t = -\Delta q_t$ under the peg as $\Delta e_t = \pi_t^* = 0$. Under the peg, $q_t^\psi = 0$, and therefore $q_t = q_t^a$, which accounts for a vanishing small portion of the overall real exchange rate volatility under the float (according to part (a) of Proposition 2). As a result, the change in the volatility of inflation can be arbitrary small, at least relative to the floating exchange rate volatility. Note that this would not be possible if $\hat{\psi}_t$ did not change its properties with the monetary regime. In this case, the volatility of q_t would remain unchanged across policy regimes (violating part (a)), and it would translate into a dramatic increase in the volatility of inflation $\pi_t = -\Delta q_t$ under the peg (violating part (b)). This is a pure general equilibrium effect of monetary policy, which operates independently of the value of γ and other parameters, and with nominal rigidities additionally translates into an increased volatility of consumption and output under the peg. Therefore, it is the endogenous response of the risk-sharing wedge $\hat{\psi}_t$ which results in both changing properties of the real exchange rate and largely unchanged properties of the macro variables. In other words, an endogenous decline in the volatility of $\hat{\psi}_t$ under the peg permits the monetary authority to stabilize the nominal exchange rate without much compromising its ability to stabilize inflation, consumption and output.

5.3 Additional evidence

Our analysis has so far focused on the volatility of exchange rates and macro variables across the two monetary regimes. This choice of moments is driven mostly by robust discontinuities, or the lack thereof, around the end of Bretton Woods in the data. As Table A1 makes clear, the patterns are less obvious for other moments, namely the correlations. Nevertheless, changes in empirical correlations are important *overidentifying* tests of the theoretical mechanism, and as we argue next are consistent with the predictions of the segmented market model.

In particular, a key property of our model is that financial shocks are central to exchange rate dynamics under the floating regime, and become significantly less important under the peg. It follows that the main drivers of the real exchange rate under the peg are 'fundamental' macroeconomic shocks, such as productivity. Given conventional transmission of these shocks, the model predicts that most exchange rate puzzles that emerge under a floating regime should disappear under a peg. This is true in particular for the forward premium puzzle (Fama 1984), the Backus-Smith puzzle (Backus and Smith 1993, Kollmann 1995), and the Balassa-Samuelson effect (Balassa 1964, Samuelson 1964):[31]

Proposition 3 *A change in the monetary policy rule from a peg to a float results in the emergence of* (a) *the forward premium puzzle,* (b) *the Backus-Smith puzzle, and* (c) *a weaker Balassa-Samuelson effect.*

Consider first the forward premium puzzle. Clearly, this anomaly cannot emerge when risk premium is zero, and therefore one would expect smaller deviations from the UIP under the peg, as $\chi_1 \to 0$ in the generalized UIP condition (25). The empirical evidence is consistent with this prediction of the model. Using historical data for the U.K and the U.S., Colacito and Croce (2013) show that the estimated

[31]Note that the Meese and Rogoff (1983) disconnect puzzle and the PPP puzzle (Rogoff 1996) trivially disappear under the peg, as nominal exchange rate becomes stable, and thus nearly perfectly predictable, while the real exchange rate satisfies $\Delta q_t = \pi_t^* - \pi_t$, thus sharing the volatility and persistence properties of the relative inflation rates.

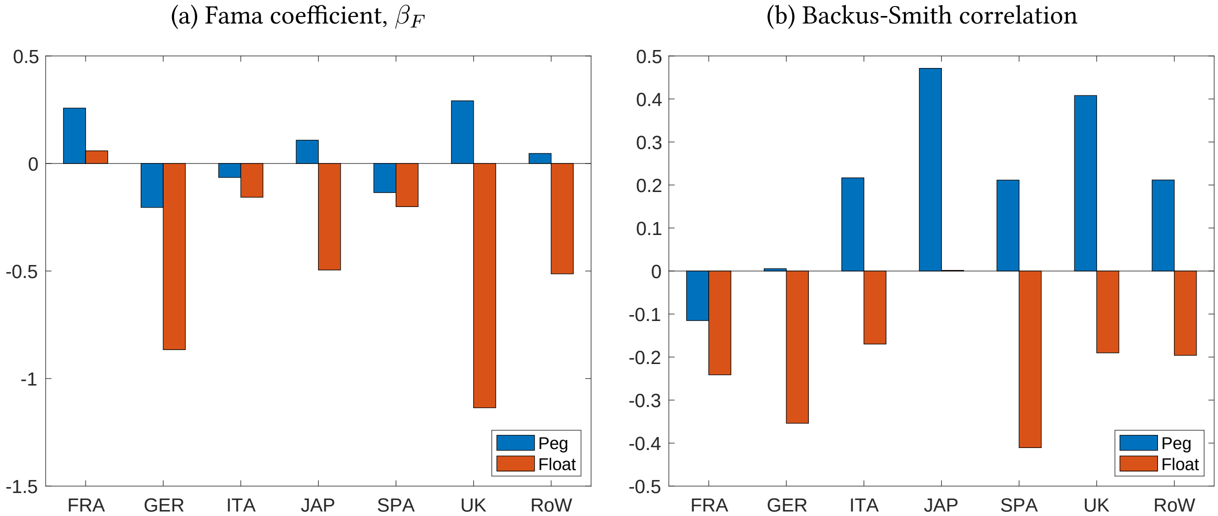


Figure 6: Fama coefficient and Backus-Smith correlation before and after the end of the Bretton Woods

Note: The left panel displays Fama regression coefficient β_F, obtained from an OLS regression of Δe_{t+1} on $(i_t - i_t^*)$, using monthly data for 1960:01–1971:07 for Peg and 1973:01–1989:12 for Float. The right panel displays the Backus-Smith correlation, $\text{corr}(\Delta c_t - \Delta c_{t+1}, \Delta q_t)$, using annual data for 1960–71 for Peg and 1973–1989 for Float.

UIP coefficient was close to one during most of the Bretton-Woods period and became negative afterwards. Kollmann (2005) documents a discontinuous increase in the UIP wedge following the break-up of Bretton Woods, consistent with the endogenous mechanism in our model. We present the estimated Fama regression coefficients during and after Bretton Woods in the left panel of Figure 6, showing that it turned from mildly positive to pronouncedly negative for most countries in our sample. In contrast to the changing pattern of UIP violations, Marston (2007) documents that CIP held equally well across the two monetary regimes, in line with the predictions of the model.

Similarly, the model predicts that the Backus-Smith condition should hold, at least conditionally in expected terms, in the absence of risk-premium shocks in the financial market, which is the case under the peg as $\hat{\psi}_t \to 0$ in (26). The right panel of Figure 6 shows that the annual Backus-Smith correlation between the real exchange rate and relative consumption is indeed higher under the peg than under the float for every country in our sample, and flips sign from positive to negative under the float in all cases but one. This is one of the central moments in our calibration in Section 6, which in particular ensures that the model reproduces exchange rate disconnect properties under the float. This pattern of a changing Backus-Smith correlation and the emergence of the Backus-Smith puzzle is also consistent with the findings of Colacito and Croce (2013) based on longer historical series for the U.S. and the U.K. In addition, Devereux and Hnatkovska (2020) provide empirical evidence using an alternative quasi-experiment, namely the formation of the Eurozone. In particular, they show that the Backus-Smith risk-sharing condition holds much better for the members of the currency union than for the same countries before the formation of the Eurozone or for countries with different currencies.

Finally, in a straightforward extension of the baseline model with tradable and non-tradable goods, the real exchange rate appreciates according to Balassa-Samuelson forces when a country's productivity in the tradable sector increases relative to the productivity in the non-tradable sector.[32] While true under both monetary regimes, this correlation is harder to identify under the float because of the relatively small overall contribution of productivity shocks to the exchange rate dynamics. The empirical evidence is again in line with this prediction: while the Balassa-Samuelson effect has almost no explanatory power under the float (Rogoff 1996, Engel 1999), recent literature has shown that this effect is notable in the Eurozone countries with a fixed exchange rate (Berka, Devereux, and Engel 2012, 2018).

6 Quantitative Exploration

This section shows that both the positive results in Propositions 2 and 3 and the negative result in Propositions 1 are robust in a quantitative version of the model. We compare three classes of models — without financial shocks, with exogenous financial shocks, and with financial shocks endogenous to the monetary policy regime (as in the segmented market model of Section 5). We show that *only* the latter class of models is consistent with the umbrella of Mussa facts documented in Section 2. At the same time, whether models feature nominal rigidities or have flexible prices, and whether the fundamental macro shocks are due to productivity or monetary policy does not qualitatively change the ability of the model within each class to match the empirical patterns. The quantitative modeling framework augments the baseline model from Section 3 with intermediate inputs, capital and investment with adjustment costs, variable markups and pricing-to-market due to Kimball demand, and Calvo sticky wages and local-currency sticky prices, as we describe in Appendix A.3. Monetary policy is conducted according to a Taylor rule (12), where the shift in a policy regime corresponds to a change in the weight ϕ_e that the monetary authority puts on the nominal exchange rate.

6.1 Calibration

For most parameters we use conventional values in the literature, as summarized in Appendix Tables A2 and A3. In particular, we set the relative risk aversion $\sigma = 2$, the Frisch elasticity of labor supply $1/\varphi = 1$, the *quarterly* discount factor $\beta = 0.99$, the intermediate input share $\phi = 0.5$, the capital share $\vartheta = 0.3$, and the quarterly capital depreciation rate $\delta = 0.02$. For each specification of the model, we calibrate the capital adjustment cost parameter κ to have the volatility of investment equal 2.5 times that of GDP.

Given the intermediate share ϕ, we set the openness of the economy to $\gamma = 0.035$ to match the average import-to-GDP ratio of 7% for the U.S. for the period from 1960 to 1990, and we also consider an alternative calibration for the U.K. with a much higher import-to-GDP ratio of 20%. We set the elasticity of variable markups at 0.67 resulting in a 60% pass-through rate following the estimates of Amiti, Itskhoki, and Konings (2018). The elasticity of substitution $\theta = 1.5$ is set based on the evidence

[32] In particular, one can show that $q_t = \big(1 - 2\gamma(1-\omega)\big)\big[e_t + w_t^* - w_t - (a_t^* - a_t)\big] + \omega\nu_t^N$, where ω is the expenditure share on non-tradables and ν_t^N is the Balassa-Samuelson term, i.e. the relative non-tradable productivity across countries (see Itskhoki 2021). Under the float, the nominal exchange rate e_t dominates the volatility of q_t, while ν_t^N becomes relatively more important under the peg.

from Feenstra, Luck, Obstfeld, and Russ (2014) and the original calibrations in Backus, Kehoe, and Kydland (1994) and Chari, Kehoe, and McGrattan (2002).

We consider three versions in each model class: a flexible-price version with productivity shocks (IRBC), and a sticky price and wage versions with productivity shocks (IRBC$^+$) and with monetary shocks (NKOE). In the versions of the model with nominal rigidities, we assume that prices adjust on average once a year, and thus set $\lambda_p = 0.75$, while wages adjust on average every six quarters, $\lambda_w = 0.85$, following standard calibrations in the literature (see e.g. Galí 2008). We set the Taylor-rule parameter $\phi_\pi = 2.15$ and the interest-rate smoothness parameter $\rho_m = 0.95$ following the estimates in Clarida, Galí, and Gertler (2000). The weight of the nominal exchange rate in the Taylor rule of foreign country is always zero, while for home country it is zero under the float and is calibrated to match an eight-fold reduction in annualized $\mathrm{std}(\Delta e_t)$, from 10% to 1.25%, under the peg. We keep all other parameters constant across policy regimes. In the class of models with endogenous financial shocks, we scale coefficients χ_1 and χ_2 in the UIP and risk-sharing conditions (25) and (26) in proportion with the change in σ_e^2 across the two monetary regimes, as required by Lemma 1.[33]

The model features three types of shocks — country-specific productivity or monetary shocks, (a_t, a_t^*) or $(\varepsilon_t^m, \varepsilon_t^{m*})$, relative taste shocks for home versus foreign goods, $\tilde{\xi}_t = \xi_t - \xi_t^*$, and financial shocks, ψ_t. We assume that all types of shocks are orthogonal to each other and follow AR(1) processes with the same autoregressive coefficient $\rho = 0.97$, which is consistent with the observed persistence of both macroeconomic variables, such as GDP and interest rates, as well as risk premia in international financial markets (namely, $\hat{\psi}_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$). The other elements of the covariance matrix of shocks are identified using the following empirical moments under the floating regime: (i) the variance of shocks is calibrated to match the annualized volatility of the nominal exchange rate $\mathrm{std}(\Delta e_t) = 10\%$ under the float; (ii) the relative volatility of productivity (monetary) shocks is set to match the Backus-Smith correlation, $\mathrm{corr}\,(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.2$; and (iii) the cross-country correlation of productivity (monetary) shocks is set to match $\mathrm{corr}\,(\Delta gdp_t, \Delta gdp_t^*) = 0.3$. For simplicity, we keep the volatility of the taste shock constant across simulations, $\mathrm{std}(\tilde{\xi}_t) = 0.12$, which makes the model consistent with the observed volatility of net exports under the peg.

6.2 Results

Our quantitative results are summarized in Table 2, which displays the volatilities of exchange rates and macro variables under the two monetary regimes for alternative versions of the model, contrasting them with empirical counterparts.

No financial shocks We consider first the class of models without the financial shock, namely with $\hat{\psi}_t \equiv 0$ in (17). Calibrated to match the volatility of the nominal exchange rate, all three specifications reproduce a high volatility in the real exchange rate under the floating regime. The flexible-price IRBC model, however, fails to generate differential behavior of the real exchange rate across the two monetary

[33] We set $\chi_2 = 0.001$ under the float, making the model consistent with high persistence (0.95) in the growth rate of net foreign assets, Δb_{t+1}; a small positive χ_2 renders the model long-run stationary without changing its quantitative properties in the short and medium run.

regimes, and thus is expectedly inconsistent with the original Mussa (1986) observation. In contrast, the two specifications with nominal rigidities capture well the large drop in the volatility of the real exchange rate under the peg, which is why such models were often viewed as promising for explaining the Mussa puzzle (see Dedola and Leduc 2001, Duarte 2003, Monacelli 2004). Furthermore, and perhaps less expectedly, these models are also consistent with only minor changes in the extent of volatility of the other macro variables.

Nonetheless, all three specifications in this class — irrespectively of the presence or absence of nominal rigidities — are inconsistent with the basic disconnect properties of exchange rates under the floating regime and imply counterfactually large volatility of macro variables under both monetary regimes.[34] Indeed, in the absence of financial shocks, the model requires very large productivity (monetary) shocks to explain the volatile exchange rate under the float, and as a result the implied volatility of consumption and GDP is roughly 5 times larger than in the data. Furthermore, such models imply that the correlation between relative consumption $c_t - c_t^*$ and the real exchange rate q_t is close to one, even when asset markets are incomplete, which is at odds with the mildly negative correlation measured in the data (the Backus-Smith puzzle; see Appendix Table A4).

To summarize, neither model in this class can explain a broader set of exchange rate facts, and in particular this class of models is uniformly falsified by the properties of the sufficient statistic $z_t = \sigma(c_t - c_t^*) - q_t$, as we emphasized in Proposition 1. Despite a substantial departure of our calibration from the Cole-Obstfeld parameter restriction, the falsification result of Proposition 1 applies quantitatively, and thus proves to be useful in realistically-calibrated quantitative models. As displayed in the last column of Table 2, the volatility of the sufficient statistic z_t barely changes with the monetary regime in each of the models in this class, in sharp contrast with the observed empirical discontinuity.

Exogenous financial shocks The next class of models allows for exogenous financial shocks $\hat{\psi}_t$ in (17) with the same volatility under the two monetary regimes. Table 2 shows that all three specifications in this class are successful in addressing the disconnect puzzle under the floating regime, as the volatility of exchange rates is an order of magnitude higher than the volatility of consumption, GDP and inflation. Nonetheless, these models struggle to match the broad set of Mussa facts. Expectedly, the flexible-price IRBC model produces no change in the behavior of the real exchange rate, which remains equally volatile under the peg. Furthermore, the shift in the monetary policy rule to stabilize the nominal exchange rate results in a counterfactually volatile inflation rate under the peg — an equally important observation that falsifies this version of the model.

Specifications with sticky prices in this class, on the other hand, perform much better in matching the drop in the real exchange rate volatility and stable inflation rates under the peg. However, these specifications have counterfactual predictions for real macro variables — consumption and GDP — which feature roughly a five-fold increase in volatility under the peg. This is again the general equilibrium implication of a shift in the monetary policy rule which stabilizes the nominal exchange rate, and thus shifts volatility to real variables in the presence of nominal rigidities (recall the discussion in

[34]Conversely, an alternative calibration that targets the level of volatility of macro variables, understates the extent of the exchange rate volatility by an order of magnitude.

Table 2: Quantitative results: standard deviations

	Δq_t			π_t			Δc_t			Δgdp_t			Δnx_t			$i_t - i_t^*$			Δz_t		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Data	1.7	10.0	5.8	1.7	2.0	1.2	1.9	2.1	1.1	2.4	1.9	0.8	3.4	4.2	1.2	0.7	1.3	2.0	3.8	11.4	3.0
No financial shocks, $\hat{\psi}_t \equiv 0$ in (17)																					
IRBC	13.3	13.3	1.0	10.9	3.0	0.3	8.6	8.6	1.0	14.1	14.1	1.0	4.2	4.2	1.0	0.7	2.6	3.5	7.3	7.3	1.0
IRBC$^+$	4.0	11.2	2.8	3.0	2.0	0.7	6.3	7.5	1.2	12.3	12.6	1.0	6.8	5.0	0.7	0.5	2.7	5.6	6.6	7.1	1.1
NKOE	1.4	9.7	7.0	1.1	1.0	0.9	4.2	4.3	1.0	6.9	6.9	1.0	3.8	3.8	1.0	0.6	1.3	2.3	2.9	2.8	0.9
Exogenous financial shocks, $\hat{\psi}_t$ in (17)																					
IRBC	9.4	9.4	1.0	8.6	0.8	0.1	1.8	1.8	1.0	2.7	2.7	1.0	8.2	8.2	1.0	1.8	0.7	0.4	11.1	11.1	1.0
IRBC$^+$	2.0	9.9	5.0	1.3	0.4	0.3	4.9	1.5	0.3	12.8	2.6	0.2	8.8	6.7	0.8	1.7	0.6	0.4	11.7	11.0	0.9
NKOE	1.8	9.9	5.4	1.2	0.4	0.3	4.8	1.3	0.3	7.5	2.2	0.3	6.4	6.6	1.0	1.6	0.5	0.3	11.4	10.8	1.0
Segmented financial markets, ψ_t in (26)																					
IRBC	3.7	9.4	2.5	2.2	0.8	0.4	1.7	1.8	1.1	2.7	2.7	1.0	3.7	8.2	2.2	0.4	0.7	1.8	2.2	11.1	5.1
IRBC$^+$	1.6	9.9	6.0	0.6	0.4	0.7	1.4	1.5	1.1	3.3	2.6	0.8	3.7	6.7	1.8	0.3	0.6	2.0	2.2	11.0	4.9
NKOE	1.4	9.9	7.3	0.5	0.4	0.8	1.3	1.3	1.0	2.6	2.2	0.9	4.1	6.6	1.6	0.3	0.5	1.7	2.1	10.8	5.1
Robustness																					
Alt. $\chi_1(\sigma_e^2)$	1.7	9.9	5.9	0.7	0.4	0.6	1.7	1.5	0.9	4.2	2.6	0.6	3.9	6.7	1.7	0.3	0.6	1.8	3.6	11.0	3.0
DCP	1.7	9.7	5.6	0.7	0.6	0.9	1.7	1.8	1.1	3.4	2.9	0.9	3.8	7.9	2.1	0.3	0.7	2.7	2.3	11.2	4.8
UK openness	1.8	9.9	5.5	0.7	0.7	0.9	1.7	1.9	1.1	4.5	3.9	0.9	2.6	6.3	2.4	0.3	0.9	3.2	2.1	11.6	5.6

Note: see text and notes to Appendix Table A3. Cross-country empirical moments are computed for the U.S. against the RoW, while moments for country-level inflation, consumption and GDP are weighted averages across economies from the RoW (see Tables 1 and A1). See Appendix Table A4 for correlations).

the end of Section 5.2). As for the previous class of models, the insight from Proposition 1 holds quantitatively in this class as well, and the sufficient statistic z_t remains stable across monetary regimes in all three specifications, at odds with the discontinuity in the data. Note the variety of ways in which different model specifications in these two classes fail, and the robustness of our simple sufficient statistic z_t to identify all such failures.

Segmented financial market We finally turn to the three specifications that feature an endogenous financial shock in (25)–(26) due to the segmented financial market introduced in Section 5. Under the float, this class of models is isomorphic to the one with exogenous financial shocks discussed above. Therefore, these models are consistent with the empirical patterns of exchange rate disconnect, including a large gap in volatility between exchange rates and macro variables, a weak negative Backus-Smith correlation, and a negative Fama regression coefficient, as summarized in Tables 2 and A4.

However, in contrast with previous specifications with exogenous financial shocks, this class of models also matches the data under the peg — the volatility of the real exchange rate drops discontinuously under the peg along with that of the nominal exchange rate, while the volatility of other macro variables changes only modestly by about 10%. In addition, these models are also consistent with a two-fold increase in the volatility of the interest rate differential, $i_t - i_t^*$, under the float relative to the peg, reflecting a noticeable, yet mild, change in the monetary policy rule associated with a shift to the float when financial shocks are endogenous to the monetary regime.[35]

Furthermore, in this class of models, the sufficient statistic z_t from Proposition 1 exhibits a sharp increase in its volatility from a shift to the float, in line with the empirical patterns. In fact, the model implied increase in the volatility of z_t is statistically indistinguishable from that observed in the data, as we illustrate in Figure 5. This confirms the potency of our sufficient statistic to distinguish between models in their ability to match a broad set of Mussa facts. Notably, the results are similar across model specifications in this class, and do not qualitatively change with the type of the macro shock (productivity vs monetary) or the presence of nominal rigidities. This is the sense in which nominal rigidities are neither necessary, nor sufficient to explain the Mussa puzzle, and it is the segmented financial market that gives rise to the associated monetary non-neutrality. Sticky prices do improve the quantitative fit of the model, and overall our preferred specification is IRBC$^+$ featuring sticky prices and wages and productivity shocks.[36]

To aid with the intuition behind these results, Table 3 displays an equilibrium variance decomposition for consumption and the real exchange rate in the class of models with endogenous financial

[35] Contrast this with the case of exogenous financial shocks, where the volatility of $i_t - i_t^*$ counterfactually increases under the peg as monetary policy stabilizes the nominal exchange rate in face of volatile UIP deviations: $i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1} + \hat{\psi}_t = \hat{\psi}_t$ under the full peg. With a segmented financial market, the UIP deviations $\hat{\psi}_t$ endogenously shrink towards zero with the announcement of the peg, relieving the monetary authority from the need to increase volatility of i_t under the peg.

[36] Table 2 also reveals two main limitations of the models. First, the price level is counterfactually stable, possibly due to the lower efficiency of inflation targeting and larger Phillips-curve shocks in the 1960–80s (relative to the period of the Great Moderation post 1995). Second, the floating regime features counterfactually volatile net exports, likely reflecting a lacking mechanism of slow adjustment in trade quantities (the J-curve). Our model, however, is consistent with the positive low-frequency comovement between the real exchange rate and trade balance emphasized by Alessandria and Choi (2019), as we illustrate in Figure A6. While the sample period is too short to test for differences across the regimes, both the data and the model suggest that this relationship is less strong and perhaps even flips the sign under the peg.

Table 3: Variance decomposition

	peg			float		
	ψ	$\tilde{\xi}$	a or m	ψ	$\tilde{\xi}$	a or m
Real exchange rate, $\mathrm{var}(\Delta q_t)$						
IRBC	0	49	51	82	11	7
IRBC$^+$	0	40	60	90	7	3
NKOE	0	39	61	88	7	5
Consumption, $\mathrm{var}(\Delta c_t)$						
IRBC	0	1	99	10	1	89
IRBC$^+$	0	16	84	4	0	96
NKOE	0	30	70	6	0	94

Note: This table shows a variance decomposition of the real exchange rate and consumption into contribution shares (%) of various shocks in three model specifications with endogenous financial shocks. IRBC and IRBC$^+$ specifications feature productivity shocks (a_t, a_t^*) and NKOE specification features monetary shocks $(\varepsilon_t^m, \varepsilon_t^{m*})$.

shocks. Under the float, over 80% of the real exchange rate volatility is driven by financial shocks. A switch to the peg removes most of the carry trade risk and almost fully eliminates financial shocks, resulting in a drastic fall in the real exchange rate volatility. At the same time, the dynamics of other variables do not change much for two reasons. Due to low openness of the economies and limited exchange rate pass-through, international financial shocks account for only a modest share of macro volatility even under the float (e.g., no more than 10% of aggregate consumption volatility). As a result, the decreasing importance of financial shocks under the peg has only minor implications for macro aggregates. Nonetheless, a change in monetary policy *per se* could significantly change the behavior of inflation (under flexible prices) and real variables (under sticky prices), as we saw was the case with exogenous financial shocks. This does not happen here, however, thanks to small changes in the *equilibrium* monetary policy. Indeed, with UIP deviations largely eliminated by arbitrageurs under the peg, the policy does not need to change much to secure a stable nominal exchange rate. In other words, the government's commitment to a peg, if credible, goes a long way towards stabilizing the exchange rate even without large monetary interventions along the equilibrium path, thus confronting the monetary authority with little tradeoff between exchange rate and inflation stabilization.

6.3 Robustness

The lower panel of Table 2 complements the analysis with three alternative versions of our preferred IRBC$^+$ model with a segmented financial market. First, we relax the assumption that the perceived carry trade risk under the peg, which shapes the policy function of the intermediaries (24), is proportional to the *ex post* observed volatility of the nominal exchange rate. Instead, we assume that intermediaries expect a break up in the Bretton Woods system of fixed exchange rates with a positive probability, and hence consider the carry trade risky even in the absence of any observed exchange rate volatility. Specifically, we calibrate the proportion reduction in χ_1 and χ_2 under the peg to match exactly the ratio of $\text{std}(\Delta z_t)$ for the sufficient statistic $z_t = \sigma(c_t - c_t^*) - q_t$ across the two policy regimes. A back-of-the-envelope calculation suggests that the implied probability of a switch from the peg to a float must equal 5.7% at the quarterly horizon to rationalize this calibration. Table 2 shows that the simulated moments remain largely unchanged, except for a somewhat higher volatility of GDP under the peg.

Second, motivated by recent evidence that most international prices are set in dollars (see Gopinath et al. 2020), we replace the conventional LCP assumption with an alternative assumption that all international trade prices are sticky in the foreign currency, namely the dollar (DCP). The quantitative results barely change, with just a slightly increasing implied volatility of inflation, net exports and GDP. Low openness of countries in the period around the break up of the Bretton Woods limits the importance of border price stickiness for aggregate macroeconomic outcomes.

Lastly, while our baseline calibration targets the openness of the U.S. against the rest of the world, the empirical evidence of Section 2 demonstrates the robustness of the Mussa puzzle for economies of different sizes and trade openness. In order to address this, we relax the assumption of symmetric home and foreign, and calibrate the model to the U.K. with the global share of GDP of 5% and the import-to-GDP ratio of 20%, a 5-to-6-fold difference relative to the U.S. For transparency of the comparison, all other parameters are kept unchanged and the covariance matrix of shocks is calibrated to match the same moments as before. The last row of Table 2 shows that a lower home bias of the economy results in a higher pass-through of exchange rate volatility into domestic macro variables. These effects are quantitatively small and consistent with empirical evidence (see Itskhoki and Mukhin 2021), while the relative volatilities across the monetary regimes remain almost the same as in the main specification, confirming the model's ability to reproduce the broad set of Mussa facts for small open economies.

7 Discussion

We propose a model with a segmented financial market, in which monetary non-neutrality arises due to the effects of monetary policy on risk premia and financial intermediation. This allows the model to account for a broad set of Mussa facts on the end of the Bretton Woods system of fixed exchange rates. We now discuss possible alternative empirical and theoretical interpretations of these stylized facts.

Policy implementation Our modeling focuses on a change in monetary policy implemented by means of an interest rate rule, where a shift in the monetary regime corresponds to a changing weight on the nominal exchange rate in the generalized Taylor rule (12). We discuss here alternative policy instruments and implementation, in particular the gold standard and capital controls. Crucially, our negative result in Proposition 1 applies independently of the way monetary policy is carried out, and thus it does not hinge on the assumption of interest rate implementation.[37] Take the gold standard as an alternative monetary policy regime, at least preceding 1973. This can be captured with an exogenous stochastic process for the U.S. inflation rate π_t^*, which reflects fluctuations in the market price of gold,

[37] In Section 6, we discuss evidence on the change in the volatility of relative interest rates, $i_t - i_t^*$, across monetary regimes in excess of the associated change in relative inflation (see the left panel of Figure A3), which arguably favors the view of a change in the interest rate rule between the two regimes.

while the other countries peg to the dollar, $\Delta e_t = 0$. The results in Propositions 1, 2 and 3 hold independently of this change in assumptions, and thus our conclusions are robust to whether countries follow a gold standard or inflation targeting by means of an interest rate rule.

Consider next an implementation of the peg by means of capital controls. In particular, one can view the shift around 1973 as also involving a transition from financial autarky to a more complete set of international financial markets, as modeled by Colacito and Croce (2013). While there were indeed significant changes in the intensity of capital controls and international mobility of capital in 1970s, a closer look shows that the timing of these changes varies across countries and does not coincide with the switch to the floating regime: restrictions on capital flows were maintained from 1961 to 1979 in the U.K., from 1970 to 1974 in Germany, and from 1966 to 1974 in the U.S. (Marston 2007). The evidence in Gourinchas and Rey (2014) suggests, if anything, a discontinuity in 1980, after which there was an intense acceleration in the pace of the build up of gross international asset positions. To address this possibility, in Appendix Figure A1, we consider an alternative break point in 1980 and show that there was almost no change in the behavior of the nominal or real exchange rates and equally no change in the behavior of the other macro variables. This is consistent with our view that changes in capital controls could not have produced the Mussa discontinuity observed after the break up of the Bretton Woods in 1973.[38]

Macroeconomic implications of capital controls depend on the set of agents that are subject to regulation. A likely scenario is one in which capital controls, whether quantity or tax-based, are imposed on cross-border trades of domestic agents (households), whereas the financial sector can effectively escape them by e.g. trading in offshore financial markets, propagating further the segmented nature of financial markets that we emphasize. Alternatively, capital controls can be imposed on the financial sector reducing profitability of the carry trade. In both cases, however, the removal of capital controls in 1973 would predict a counterfactual reduction in the UIP deviations and, if anything, aggravate the Mussa puzzle.

Structure of the financial market We model a segmented financial market that features three types of actors — households representing macroeconomic demand for currency, noise traders representing non-fundamental (liquidity) demand for currency, and arbitrageurs intermediating trades of the other two types of agents — with the change in monetary regime affecting directly the optimal behavior of arbitrageurs. We now additionally introduce government foreign exchange interventions F^*_{t+1}, generalizing financial market clearing condition (23) as $B^*_{t+1}+N^*_{t+1}+D^*_{t+1}+F^*_{t+1} = 0$. The essential feature that differentiates the models in their ability to account for the Mussa evidence is whether the household sector (with positions B^*_{t+1}) is effectively segmented from international risk sharing (whether due to missing markets, participation costs and/or capital controls). In particular, the trades of arbitrageurs, noise traders and the government are of limited consequence for exchange rates and international risk

[38] A similar argument, which involves a typically continuous build up of macroeconomic trends, rules out many other changes in the equilibrium environment that occurred in the 1970 and 1980s. One such change is the increase in the volatility of commodity prices (see e.g. Ayres, Hevia, and Nicolini 2021). In order for such explanations to resolve the Mussa facts, it is essential not just that the macro trends had a discontinuity, but that it perfectly coincides with the abrupt shift in the monetary regime, as the timing of the change in the volatility of the real exchange rate is perfectly aligned with it.

sharing in conventional models with no segmentation of households, generalizing the negative result of Proposition 1 (*cf.* Wallace 1981).

Beyond this segmentation requirement, the macroeconomic implications are not sensitive to the type of agent(s) affected by the shift in the policy regime, emphasizing robustness of our main positive result about the transmission of monetary policy via the financial market. The three possibilities are that: (i) governments actively intervene under the peg to counterbalance noise-trader currency demand; (ii) noise traders reduce their activity in the absence of exchange rate volatility under the peg, and (iii) arbitrageurs are more active in their intermediation under the peg. While our main insights rely little on the choice between these three possibilities, or a mix thereof, available evidence favors the third arbitrageur-centric possibility, which we adopt in our analysis. Consider first governments: pegs may require more active foreign exchange interventions, which in turn suggest larger and more volatile official foreign reserves. The data reveals no discontinuity — either in levels or in volatility — of foreign reserves around 1973, as we show in Appendix Figure A7 (see also Flood and Rose 1995). This is consistent with the mechanism in our model, which requires little official interventions in the currency market in equilibrium, provided commitment to the peg is credible.

Could it be, instead, that noise traders' liquidity demand for international bonds is discontinuously lower under the peg? A salient implication of this hypothesis is a discontinuously lower turnover in international asset markets. Note that alternative versions of this hypothesis include informational frictions and expectational errors (as in e.g. Gourinchas and Tornell 2004, Bacchetta and van Wincoop 2006). A nominal peg anchors expectations and eliminates the source of disagreement in carry trades, which should reduce UIP deviations, consistent with the evidence, yet also reduce equilibrium turnover in international asset markets as expectations are more closely aligned across agents. While we have little data from the 1970s on transactions in international asset markets, including offshore markets, the more recent experience from the Swiss peg in 2012–15 suggests little change in financial market turnover relative to the floating periods before and after, as we show in Appendix Figure A8. This is the reason we opt in favor of modeling the change in the behavior of arbitrageurs rather than noise traders, which in equilibrium results in no change in the the volatility of N^*_{t+1} or D^*_{t+1}, yet a pronounced change in the associated prices (UIP deviations).

Alternative theoretical mechanisms Our theoretical analysis establishes that, on the one hand, the Mussa puzzle rejects conventional IRBC and NKOE models, and on the other hand, is consistent with a particular model of a segmented financial market. This naturally raises the question whether the same facts can be explained with alternative models that do not belong to either of the two classes. Our results provide guidance on the possible alternative mechanisms, as we briefly discuss next.

Beyond conventional DSGE models, Proposition 1 implies that the Mussa facts are inconsistent with models of currency risk premia which are exogenous to the monetary policy regime. This includes flexible-price models of bonds in the utility, convenience yield and liquidity premium (e.g. Valchev 2020, Jiang, Krishnamurthy, and Lustig 2021, Bianchi, Bigio, and Engel 2020), as well as models with complete asset markets where risk premia are amplified by means of high risk aversion ($\sigma \gg 1$; e.g. Lustig and Verdelhan 2011), habits (e.g. Verdelhan 2010), long-run risk (e.g. Colacito and Croce 2011) or rare disasters (e.g. Farhi and Gabaix 2016). While not suitable when taken as they are, these models can potentially generate risk premia that are endogenous to monetary policy if augmented with nominal rigidities (e.g. Caballero, Farhi, and Gourinchas 2015). Given a strong aggregate demand channel in the goods market, it is however unlikely that a switch from a peg to a float in such environments would change only the forward premium without affecting other macroeconomic and financial variables.[39] Perhaps a more promising avenue for future research is to extend these models of risk premia to environments with incomplete and segmented financial markets.

Frameworks with financial frictions as in Gabaix and Maggiori (2015) and Bruno and Shin (2015) are closer to our model of non-neutrality. Instead of relying on risk-averse arbitrageurs, these models emphasize balance sheet constraints as the source of limits to arbitrage, but have similar predictions for currency risk premia under a floating regime. When augmented with financial constraints that are endogenous to monetary policy, e.g. due to a higher value-at-risk of the carry trade under a volatile exchange rate, such models can potentially explain the Mussa puzzle as well. Even more promising are models with endogenously segmented markets (see e.g. Jeanne and Rose 2002, Alvarez, Atkeson, and Kehoe 2009), where a switch in the monetary policy regime affects the identity of the marginal trader and via this channel can generate large fluctuations in risk premia without a substantial change in macroeconomic volatility.

Policy implications In this paper, we consider a major policy shift from a fixed to a floating exchange rate regime, emphasizing the transmission via the financial market. It is intriguing to study, both theoretically and empirically, such transmission mechanism for more ubiquitous types of monetary shocks (see e.g. Alvarez, Atkeson, and Kehoe 2007, Gourinchas, Ray, and Vayanos 2019, Greenwood, Hanson, Stein, and Sunderam 2020, Drechsler, Savov, and Schnabl 2018). We conclude with a brief discussion of the possible normative implications of this mechanism to address in future research.

Our model emphasizes an important tradeoff for monetary policy associated with the two transmission channels — one conventional via demand in the product market and the other unconventional via risk premia in the financial market. In particular, a floating exchange rate regime improves allocations in an open economy in response to conventional productivity shocks (Friedman 1953), yet it possibly results in excessive exchange rate volatility in response to financial shocks, which mutes the extent of international risk sharing. This tradeoff raises several policy questions: Should the monetary authorities partially stabilize exchange rates in such circumstances? Can a combination of conventional monetary policy with FX interventions achieve an efficient allocation? Does optimal policy depend on the specific nature of noise trader demand and the country of origin of arbitrageurs? Furthermore, the policy implications are not limited to an open economy environment. The ability of a peg to stabilize the risk premium on the carry trade raises the question of whether monetary policy can also stabilize the volatility in the *equity* risk premium by targeting the stock market index. How such policy affects the economy and whether it is desirable are important questions for future research.

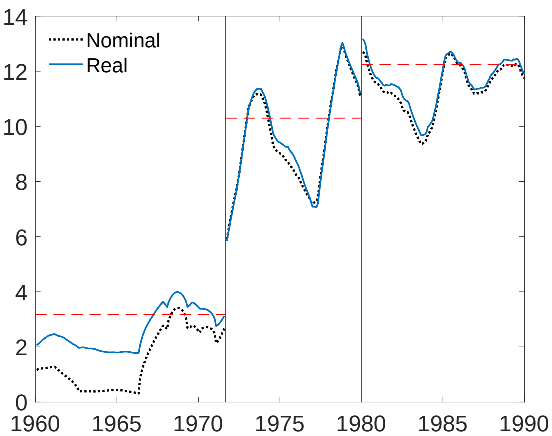
[39] Note that statistical properties of financial variables, just like those of macro variables, do not exhibit discontinuity around 1973, as we illustrate in the right panel of Figure A3 for the relative stock market returns across countries.

A Appendix

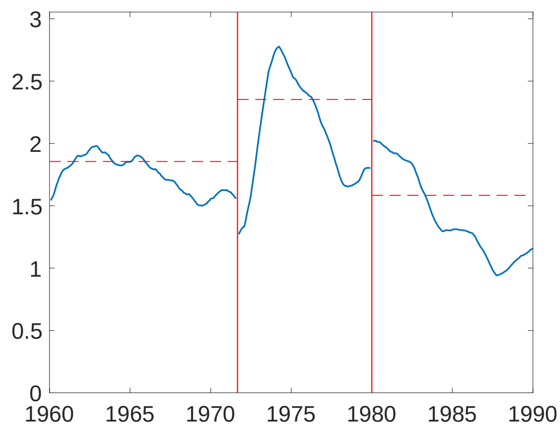
A.1 Additional Figures and Tables

Figure A1: Volatility of macroeconomic variables over time: alternative breakpoints at 1971:08 and 1980:01

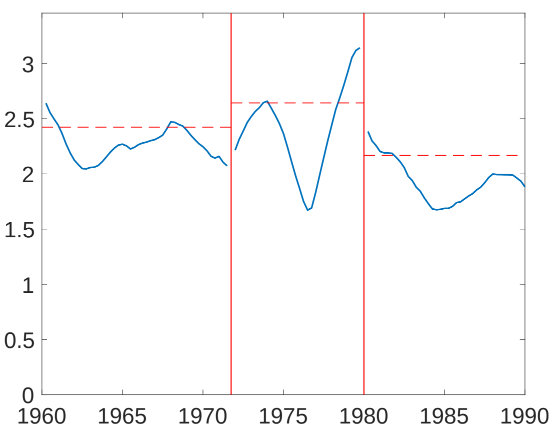
(a) Exchange rates, Δe_t and Δq_t



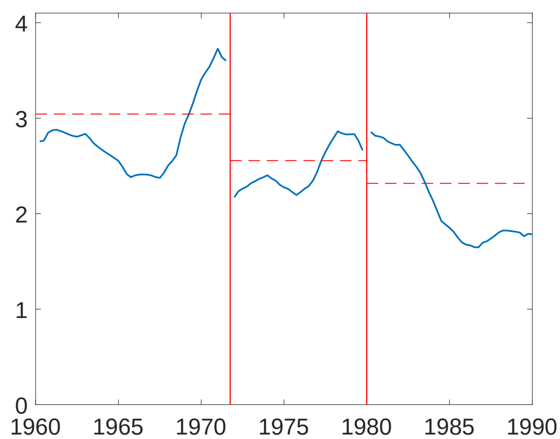
(b) Relative inflation, $\pi_t - \pi_t^*$



(c) Relative consumption, $\Delta c_t - \Delta c_t^*$

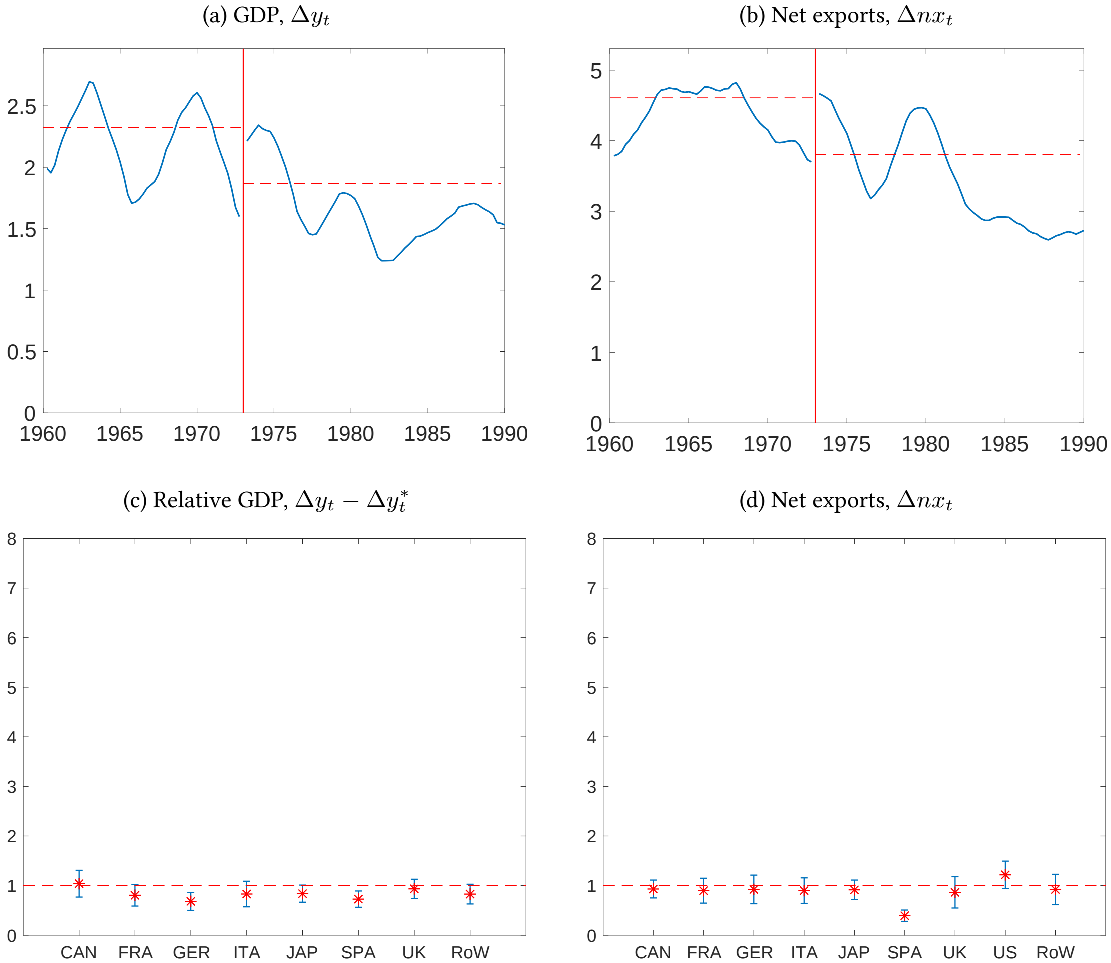


(d) Relative GDP, $\Delta gdp_t - \Delta gdp_t^*$



Note: as in Figure 3, annualized standard deviations for the U.S. against the RoW, estimated as triangular moving averages with a window over 18 months (panels a, b) or 10 quarters (panels c, d) before and after, treating 1971:08 and 1980:01 as the end points for the three regimes; the dashed lines correspond to average standard deviations within each interval.

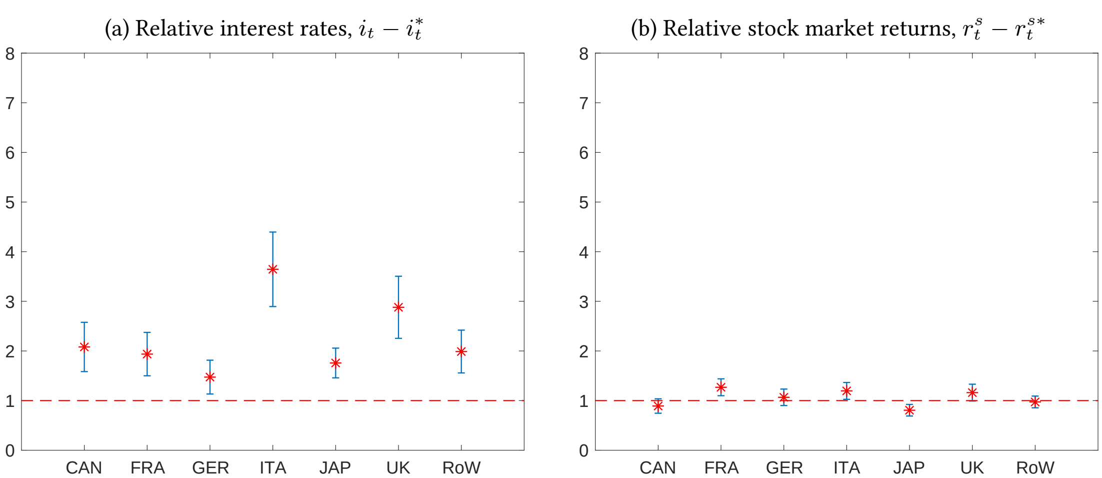
Figure A2: Macroeconomic volatility across policy regimes: GDP and net exports



(a) GDP, Δy_t (b) Net exports, Δnx_t (c) Relative GDP, $\Delta y_t - \Delta y_t^*$ (d) Net exports, Δnx_t

Note: see notes to Figure 3 for upper panels (moving average standard deviations, in log points) and notes to Figure 4 for lower panels (ratio of standard deviations float/peg).

Figure A3: Volatility ratio float/peg for financial variables

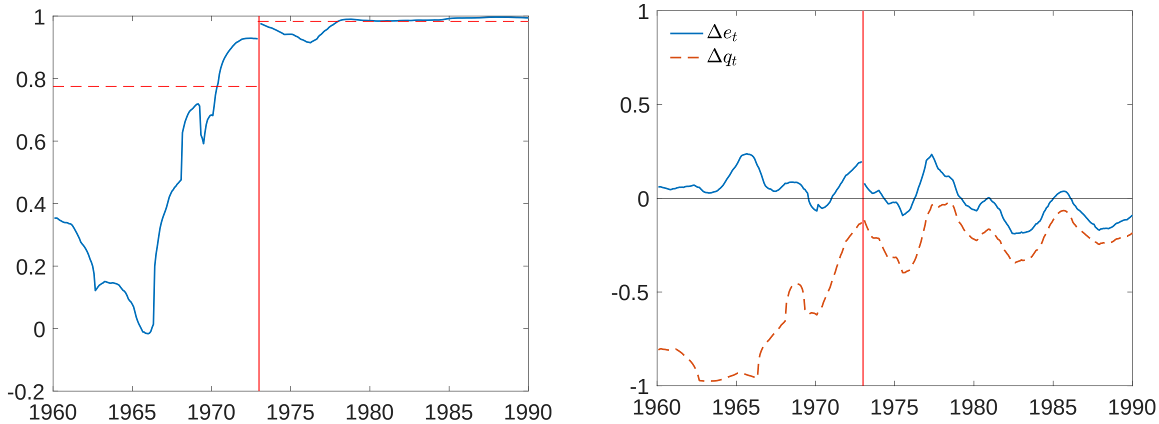


(a) Relative interest rates, $i_t - i_t^*$ (b) Relative stock market returns, $r_t^s - r_t^{s*}$

Note: see notes to Figure 4.

Figure A5: Correlations of exchange rates and prices over time

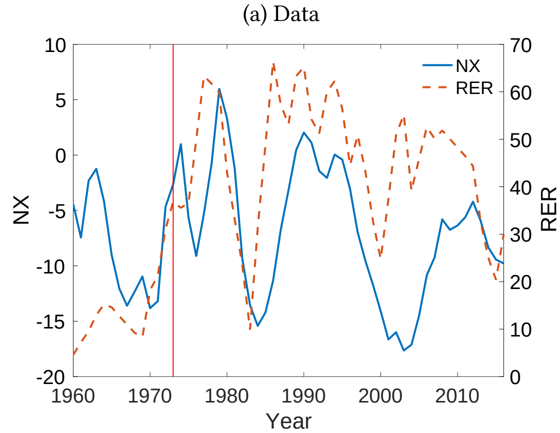
(a) $\text{corr}(\Delta q_t, \Delta e_t)$ (b) $\text{corr}(\bullet, \Delta\pi_t - \Delta\pi_t^*)$ for Δe_t and Δq_t



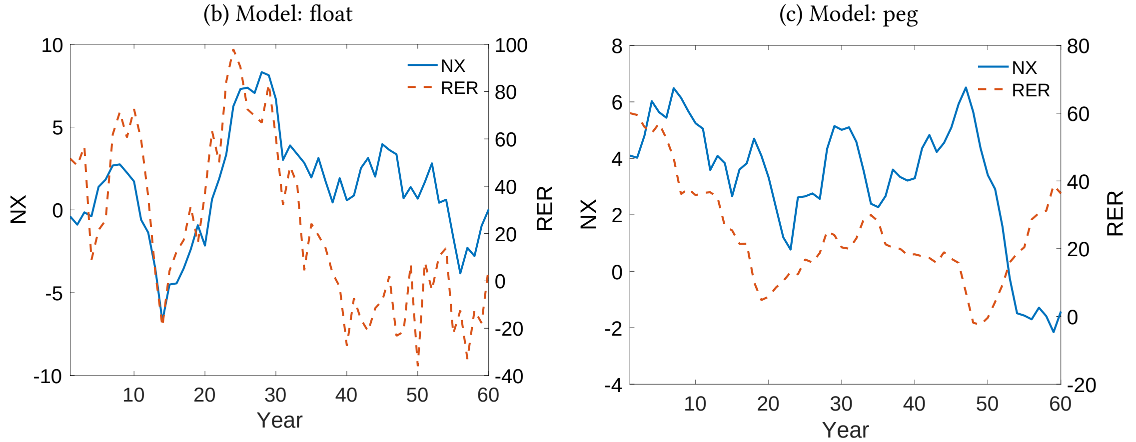
Note: triangular moving average correlations estimated with a window over 18 months before and after, treating 1973:01 as the end point for the two regimes; the dashed lines in the left panel correspond to average values under the two regimes.

Figure A6: The real exchange rate and the trade balance

(a) Data

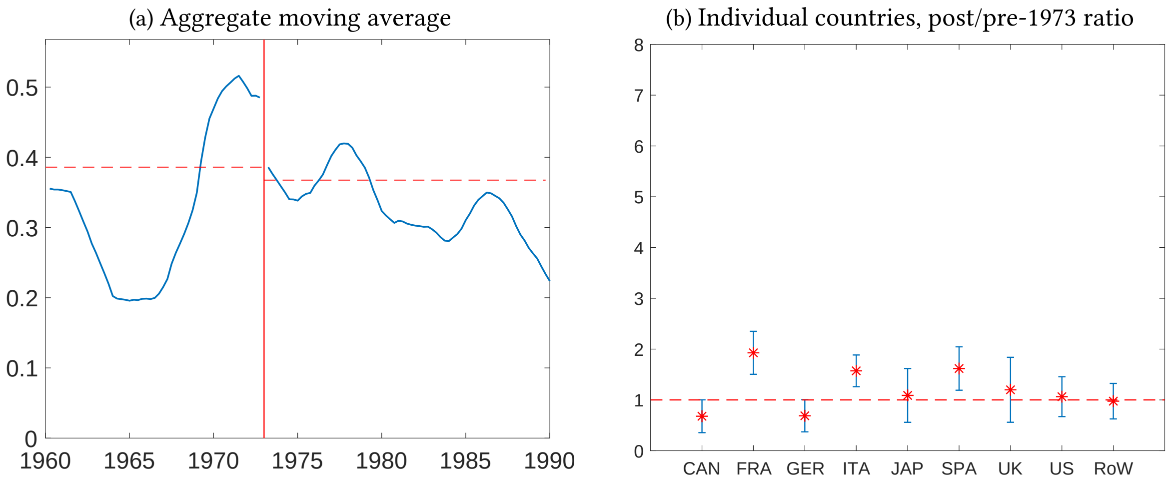


(b) Model: float (c) Model: peg



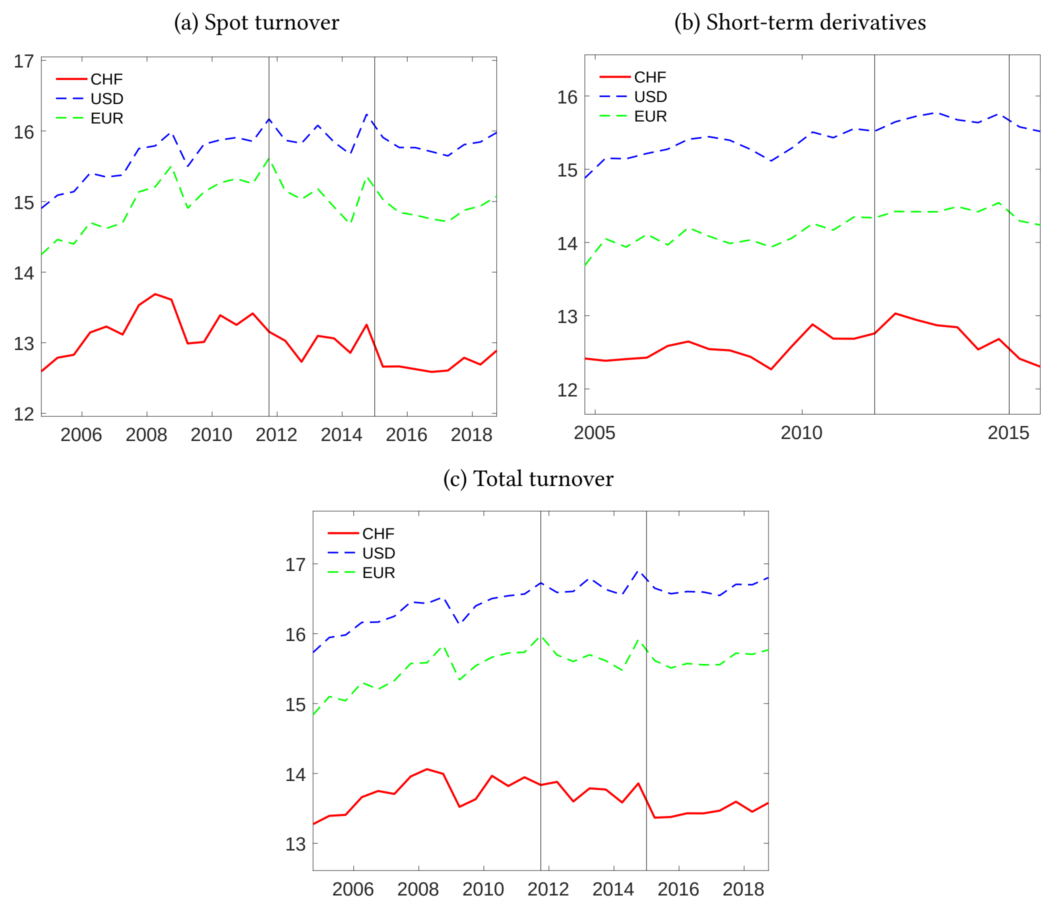
Note: panel (a) extends the figure from Alessandria and Choi (2019) using annual data for the U.S., while panels (b) and (c) show the series simulated from the IRBC$^+$ version of the model with endogenous financial shocks under the two exchange rate regimes.

Figure A7: Volatility of official foreign reserves-to-GDP ratio



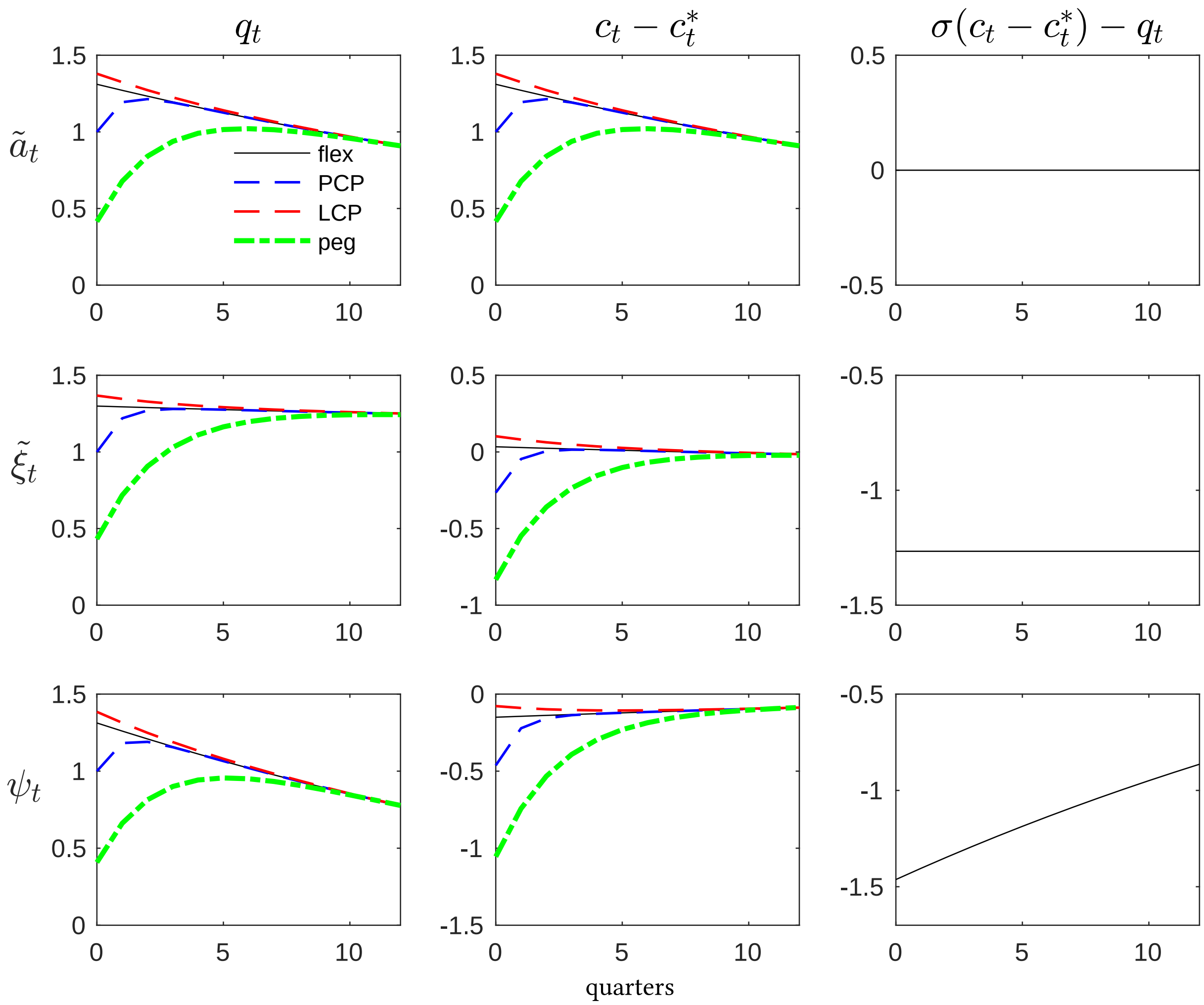
Note: std$\left(\Delta \frac{\text{FX}_t}{\text{GDP}_t}\right)$, using quarterly data on official foreign reserves from IMF IFS database, constructed as in Figures 3 and 4.

Figure A8: Turnover in the international asset markets (Swiss peg of 2012–15)



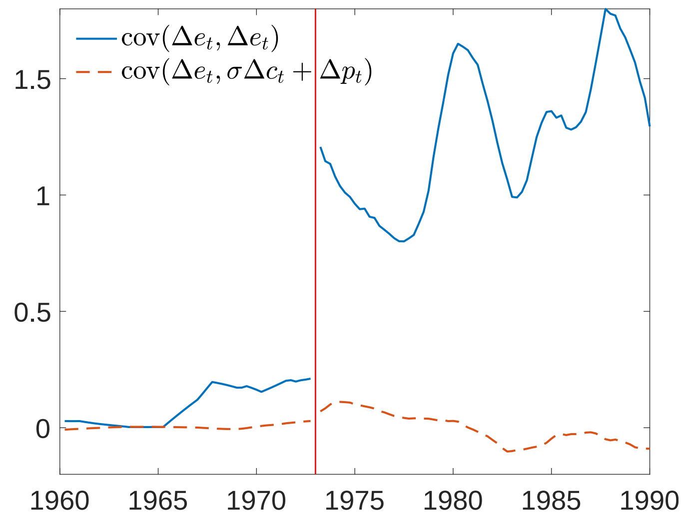
Note: turnover of the Swiss frank (CHF), dollar (USD) and euro (EUR) against other currencies in spot market (panel a), short-term ($\leq$ 1 month) derivative markets (panel b) and in all markets (panel c), expressed in logs of millions of dollars; the vertical lines show the start and the end of the Swiss peg to the euro.

Figure A9: Impulse response functions



Note: Impulse responses of q_t, $c_t - c_t^*$ and $z_t = \sigma(c_t - c_t^*) - q_t$ (columns) to shocks $\tilde{a}_t = a_t - a_t^*$, $\tilde{\xi}_t = \xi_t - \xi_t^*$ and exogenous $\hat{\psi}_t$ (rows) under (1) flexible prices (independent of monetary regime), (2) peg (independently of PCP or LCP), (3) PCP-float and (4) LPC-float, in 'conventional' models under the Cole-Obstfeld parameter restriction. Note that the impulse responses of q_t and $c_t - c_t^*$ change with both the supply side (flex prices vs PCP vs LCP) and the monetary policy regime (peg vs float), however, the IRF of z_t does not depend on these details of equilibrium environment, and hence the unconditional statistical properties of z_t also do not depend on the monetary regime, illustrating Proposition 1.

Figure A10: Covariance of the nominal exchange rate



Note: Triangular moving average covariances of the nominal exchange rate changes with itself (i.e., the variance) and with the representative-agent stochastic discount factor ($\sigma\Delta c_t + \Delta p_t$ for $\sigma = 2$), treating 1973:01 as the end point for the two regimes; quarterly data.

Table A1: Empirical moments: correlations

	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta nx_t$		$\Delta gdp_t, \Delta gdp_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta gdp_t$	
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float
Canada	0.88	0.96	0.03	−0.07	0.01	0.05	0.31	0.47	0.40	0.25	0.28	0.57
France	0.96	0.99	0.05	−0.08	0.23	0.12	0.09	0.30	−0.24	0.29	0.51	0.48
Germany	0.87	0.99	0.04	−0.19	−0.06	0.00	−0.01	0.28	−0.11	0.11	0.57	0.58
Italy	0.59	0.98	0.07	−0.13	0.02	−0.01	0.04	0.17	−0.18	0.13	0.64	0.45
Japan	0.58	0.97	0.21	−0.00	0.03	0.21	−0.08	0.24	0.11	0.23	0.70	0.71
Spain	0.84	0.97	−0.09	−0.18	−0.06	0.16	0.05	0.09	−0.06	0.05	0.56	0.63
U.K.	0.94	0.98	0.09	−0.10	−0.39	−0.16	−0.11	0.30	−0.02	0.22	0.59	0.71
RoW	0.78	0.98	0.05	−0.19	−0.20	0.20	−0.03	0.39	−0.12	0.31	0.63	0.71

Note: see notes to Table 1; cross-country correlation are with the U.S. as the foreign counterpart (indicated with a star). Moving average correlations between exchange rates and relative inflation rates are plotted in Appendix Figure A5.

Table A2: Calibrated parametrers

β	discount factor	0.99
σ	inverse of the IES	2
γ	openness of economy	0.035
φ	inverse Frisch elasticity	1
ϕ	intermediate share in production	0.5
ϑ	capital share	0.3
δ	capital depreciation rate	0.02
θ	elasticity of substitution between H and F goods	1.5
ϵ	elasticity of substitution between different types of labor	4
λ_w	Calvo parameter for wages	0.85
λ_p	Calvo parameter for prices	0.75
ρ	autocorrelation of shocks	0.97
ρ_m	Taylor rule: persistence of interest rates	0.95
ϕ_π	Taylor rule: reaction to inflation	2.15

Table A3: Estimated parameters

	σ_ψ	$\sigma_{\tilde{\xi}}$	σ_a	σ_m	ϱ_{a,a^*}	ϱ_{m,m^*}	κ	ϕ_e
NO FINANCIAL SHOCKS, $\hat{\psi}_t \equiv 0$ in (17)								
IRBC	—	12	7.7	—	0.27	—	11	13.5
IRBC$^+$	—	12	6.4	—	0.21	—	7	2.2
NKOE	—	12	—	0.63	—	0.30	22	5
EXOGENOUS FINANCIAL SHOCKS, $\hat{\psi}_t$ in (17)								
IRBC	0.49	12	1.46	—	0.29	—	13	14
IRBC$^+$	0.48	12	1.24	—	0.39	—	7	3.5
NKOE	0.47	12	—	0.18	—	0.48	20	3.5
SEGMENTED FINANCIAL MARKETS, ψ_t in (26)								
IRBC	0.49	12	1.46	—	0.29	—	13	0.85
IRBC$^+$	0.48	12	1.24	—	0.39	—	7	0.18
NKOE	0.47	12	—	0.18	—	0.48	20	0.35
ROBUSTNESS								
Alt. $\chi_1(\sigma_e^2)$	0.48	12	1.24	—	0.39	—	7	0.38
DCP	0.49	12	1.52	—	0.35	—	9	0.25
UK openness	0.56	12	1.56	—	0.26	—	6	0.23

Note: In all calibrations, shocks are normalized to obtain $\mathrm{std}(\Delta e_t) = 10\%$ under the float; parameter ϕ_e in the Taylor rule is calibrated to generate eightfold reduction in $\mathrm{std}(\Delta e_t)$, to 1.25% under the peg. Relative volatility of productivity (monetary) shocks is calibrated to match $\mathrm{corr}(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.2$ under the float; cross-country correlation ϱ_{a,a^*} (ϱ_{m,m^*}) matches $\mathrm{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.3$ under the float. Capital adjustment parameter κ ensures that $\frac{\mathrm{std}(\Delta inv_t)}{\mathrm{std}(\Delta gdp_t)} = 2.5$ under the float. The moments are calculated by simulating the model for $T = 100{,}000$ quarters.

Table A4: Quantitative results: correlations

	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta nx_t$		$\Delta gdp_t, \Delta gdp_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta gdp_t$		$\Delta gdp_t, \Delta nx_t$		β_F	
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float
Data	0.78	0.98	0.05	−0.19	−0.20	0.20	−0.03	0.39	−0.12	0.31	0.63	0.71	−0.31	−0.21	0.1	−0.5
No financial shocks, $\hat{\psi}_t \equiv 0$ in (17)																
IRBC	0.86	0.99	0.98	0.98	0.46	0.46	0.30	0.30	0.34	0.34	1.00	1.00	0.40	0.40	0.8	0.9
IRBC$^+$	0.65	0.98	0.86	0.97	0.53	0.52	0.84	0.30	0.77	0.33	0.89	0.99	−0.24	0.45	0.6	1.0
NKOE	0.91	0.99	0.22	0.96	−0.19	−0.14	0.87	0.30	0.94	0.33	1.00	1.00	0.38	0.16	1.0	1.0
Exogenous financial shocks, $\hat{\psi}_t$ in (17)																
IRBC	0.86	0.99	−0.20	−0.20	0.73	0.73	0.30	0.30	0.22	0.22	0.91	0.91	0.14	0.14	0.0	−0.9
IRBC$^+$	0.78	1.00	−0.80	−0.20	0.75	0.68	0.43	0.30	0.08	0.43	0.99	0.89	−0.77	0.32	−0.1	−0.8
NKOE	0.79	1.00	−0.84	−0.20	0.59	0.63	0.34	0.30	0.07	0.41	0.99	0.87	−0.50	0.33	−0.1	−1.5
Segmented financial markets, ψ_t in (26)																
IRBC	0.96	0.99	0.83	−0.20	−0.52	0.73	0.30	0.30	0.37	0.22	0.97	0.91	0.25	0.14	1.0	−0.9
IRBC$^+$	0.94	1.00	0.41	−0.20	−0.38	0.68	0.30	0.30	0.68	0.43	0.88	0.89	0.71	0.32	1.0	−0.8
NKOE	0.95	1.00	0.08	−0.20	−0.55	0.63	0.38	0.30	0.77	0.41	0.97	0.87	0.72	0.33	1.0	−1.5
Robustness																
Alt. $\chi_1(\sigma_e^2)$	0.91	1.00	−0.17	−0.20	0.00	0.68	0.28	0.30	0.58	0.43	0.92	0.89	0.39	0.32	0.1	−0.8
DCP	0.91	1.00	0.54	−0.20	−0.18	0.78	0.38	0.30	0.68	0.39	0.91	0.92	0.64	0.21	1.0	−0.5
UK openness	0.91	1.00	0.63	−0.20	−0.09	0.81	0.37	0.30	0.73	0.38	0.84	0.64	0.63	0.51	1.0	−0.7

Note: see text and Table 2.

A.2 Data

Quarterly data for FX reserves and monthly data for nominal exchange rates, consumer prices, production index, discount interest rates and stock market returns come from the IFM IFS database, while monthly data for stock market prices and quarterly data for GDP, consumption, imports and exports are from the OECD database. Finally, monthly data on currency turnover come from the New York Fed's Foreign Exchange Committee. Our analysis focuses on the "convertible phase" of the Bretton Woods period from 1960 to 1973 and the period of floating from 1973 to 1990, where the end date is chosen to keep the length of the two periods comparable and to exclude the Great Moderation of the 1990s. Before estimating empirical moments, we use extrapolation to replace missing data in the raw series and the following two outliers: (1) civil unrests in France in May-June 1968, which led to over a 20% fall in production and (2) missing values of GDP, imports and exports for Canada in 1960. The outliers in stock returns and changes in interest rates are eliminated using winsorization. We compute first differences of net exports normalized by total trade and log first differences of all other variables, and annualize the log changes by multiplying the quarterly series by $\sqrt{4}$ and the monthly series by $\sqrt{12}$. The series for France, Germany, Italy, Japan, Spain and the U.K. are aggregated into the RoW variables using the average PPP-adjusted GDP shares in 1960–1990 as weights.

A.3 Full Quantitative Model

This section provides a complete description of the general modeling framework. For simplicity, we focus on home households and firms with the understanding that the problems of foreign agents are symmetric.

Households A representative home household maximizes the expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+1/\nu} L_t^{1+1/\nu} \right), \tag{A1}$$

where $\nu \equiv 1/\varphi$ is the Frisch elasticity, subject to the flow budget constraint:

$$P_t C_t + P_t I_t + \frac{B_{t+1}}{R_t} \leq W_t L_t + R_t^K K_t + B_t + \Pi_t, \tag{A2}$$

where R_t^K is the nominal rental rate of capital and I_t is the gross investment into the domestic capital stock K_t, which accumulates according to a standard rule with depreciation δ and quadratic capital adjustment costs:

$$K_{t+1} = (1-\delta)K_t + \big[I_t - \frac{\kappa}{2} \frac{(\Delta K_{t+1})^2}{K_t}\big]. \tag{A3}$$

The domestic households allocate their within-period consumption expenditure $P_t C_t$ between home and foreign varieties of the goods

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft} = \int_0^1 \Big[P_{Ht}(i) C_{Ht}(i) + P_{Ft}(i) C_{Ft}(i) \Big] \mathrm{d}i \tag{A4}$$

to minimize expenditure on aggregate consumption, defined implicitly by a Kimball (1995) aggregator:[40]

$$\int_0^1 \left[(1-\gamma)e^{-\gamma\xi_t} g\left(\frac{C_{Ht}(i)}{(1-\gamma)e^{-\gamma\xi_t}C_t}\right) + \gamma e^{(1-\gamma)\xi_t} g\left(\frac{C_{Ft}(i)}{\gamma e^{(1-\gamma)\xi_t}C_t}\right)\right] \mathrm{d}i = 1, \tag{A5}$$

where the aggregator function $g(\cdot)$ in (A5) has the following properties: $g'(\cdot) > 0$, $g''(\cdot) < 0$ and $-g''(1) \in (0,1)$, and two normalizations: $g(1) = g'(1) = 1$. The solution to the optimal expenditure allocation results in the following homothetic demand schedules:

$$C_{Ht}(i) = (1-\gamma)e^{-\gamma\xi_t} h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right) C_t \qquad \text{and} \qquad C_{Ft}(j) = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}(j)}{\mathcal{P}_t}\right) C_t, \tag{A6}$$

where $h(\cdot) = g'^{-1}(\cdot) > 0$ and satisfies $h(1) = 1$ and $h'(\cdot) < 0$. The function $h(\cdot)$ controls the curvature of the demand schedule, and we denote its point elasticity with $\theta \equiv -\frac{\partial \log h(x)}{\partial \log x}\big|_{x=1} = -h'(1) > 1$. The consumer price level P_t and the auxiliary variable $\mathcal{P}_t$ in (A6) are two alternative measures of average prices in the home market (different by a second-order term in cross-sectional price dispersion), which are defined implicitly by (A4) and (A5) after substituting in the demand schedules (A6). The taste shock ξ_t in (A5) is defined such that it has no first-order effects on the consumer prices level P_t.

Production Home output is produced according to a Cobb-Douglas technology in labor L_t, capital K_t and intermediate inputs X_t:

$$Y_t = \left(e^{a_t} K_t^{\vartheta} L_t^{1-\vartheta}\right)^{1-\phi} X_t^{\phi}, \tag{A7}$$

where ϑ is the elasticity of the value added with respect to capital and ϕ is the elasticity of output with respect to intermediates. Intermediates (as well as investment goods) are the same bundle of home and foreign varieties as the final consumption bundle (A5). The marginal cost of production is thus:

$$MC_t = \tfrac{1}{\varpi}\left[e^{-a_t}(R_t^K)^{\vartheta} W_t^{1-\vartheta}\right]^{1-\phi} P_t^{\phi}, \quad \text{where } \varpi \equiv \phi^{\phi}\left[(1-\phi)\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}\right]^{1-\phi}. \tag{A8}$$

The aggregate *value-added productivity* follows an AR(1) process in logs:

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \qquad \varepsilon_t^a \sim iid(0,1), \tag{A9}$$

where $\rho_a \in [0,1]$ is the persistence parameter and $\sigma_a \geq 0$ is the volatility of the innovation.

Profits and price setting The firm maximizes profits from serving the home and foreign markets:

$$\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \mathbf{M}_t \Pi_t(i), \quad \text{where} \quad \Pi_t(i) = \left(P_{Ht}(i) - MC_t\right) Y_{Ht}(i) + \left(P_{Ht}^*(i)\mathcal{E}_t - MC_t\right) Y_{Ht}^*(i), \tag{A10}$$

where $\mathbf{M}_t \equiv \beta^t \frac{C_t^{-\sigma}}{P_t}$ is the nominal present-value stochastic discount factor. In the absence of nominal frictions, profit maximization results in the markup pricing rules, with a common price across all

[40] The CES demand is nested as a special case of the Kimball aggregator (A5) with $g(z) = 1 + \frac{\theta}{\theta-1}\left(z^{1-1/\theta} - 1\right)$, resulting in the demand schedule $h(x) = x^{-\theta}$ and price index $P_t = \mathcal{P}_t = \left(\int_0^1 \left[(1-\gamma)P_{Ht}(i)^{1-\theta} + \gamma P_{Ft}(i)^{1-\theta}\right]\mathrm{d}i\right)^{1/(1-\theta)}$.

domestic firms $i \in [0,1]$ in a given destination market and expressed in the destination currency:

$$P_{Ht}(i) = P_{Ht} = \mu\left(\frac{P_{Ht}}{\mathcal{P}_t}\right) \cdot MC_t \qquad \text{and} \qquad P^*_{Ht}(i) = P^*_{Ht} = \mu\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right) \cdot \frac{MC_t}{\mathcal{E}_t}, \tag{A11}$$

where $\mu(x) \equiv \frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}$ is the markup function (with $\mu'(\cdot) \le 0$) and $\tilde{\theta}(x) = -\frac{\partial \log h(x)}{\partial \log x}$ is the elasticity schedule for the demand curve in (A6).

Nominal rigidities We introduce Calvo sticky prices and wages in a conventional way (see e.g. Galí 2008). Denote with ϵ the elasticity of substitution between varieties of labor, and let λ_p and λ_w be the Calvo probability of price and wage non-adjustment. Then the resulting New Keynesian Phillips Curves (NKPC) for nominal-wage and domestic-prices inflation can be written respectively as:

$$\pi^w_t = k_w\left[\sigma c_t + \frac{1}{\nu}\ell_t + p_t - w_t\right] + \beta\mathbb{E}_t\pi^w_{t+1}, \quad \text{where} \quad k_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w(1+\epsilon/\nu)},$$

$$\pi_{Ht} = k_p\Big[(1-\alpha)mc_t + \alpha p_t - p_{Ht}\Big] + \beta\mathbb{E}_t\pi_{Ht+1}, \quad \text{where} \quad k_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p},$$

where $\alpha \in [0,1)$ is the strategic complementarity elasticity defined by $\alpha = \frac{-\mu'(x)}{1-\mu'(x)}\Big|_{x=1}$, and $(1-\alpha) = \frac{1}{1-\mu'(x)}\Big|_{x=1}$ is the cost pass-through elasticity (under flexible prices), and $\mu'(\cdot)$ is the elasticity of the markup function in (A11). The NKPC for export prices depends on the currency of invoicing and is given by:

$$\pi^*_{Ht} = k_p\Big[(1-\alpha)(mc_t - e_t) + \alpha p^*_t - p^*_{Ht}\Big] + \beta\mathbb{E}_t\pi^*_{Ht+1}, \qquad \text{under LCP,}$$

$$(\pi^*_{Ht} + \Delta e_t) = k_p\Big[(1-\alpha)mc_t + \alpha(p^*_t + e_t) - (p^*_{Ht} + e_t)\Big] + \beta\mathbb{E}_t\left(\pi^*_{Ht+1} + \Delta e_{t+1}\right), \quad \text{under PCP.}$$

Note that the DCP case with all international trade invoiced in foreign currency can be expressed as a mix of the two other regimes — home exporters use LCP and foreign exporters use PCP.

Good and factor market clearing The labor market clearing requires that L_t equals simultaneously the labor supply of the households and the labor demand of the firms, and equivalently for L^*_t in foreign. Similarly, equilibrium in the capital market requires that K_t (and K^*_t) equals simultaneously the capital supply of the households and the capital demand of the local firms. Goods market clearing requires that the total production by the home firms is split between supply to the home and foreign markets respectively, $Y_t = Y_{Ht} + Y^*_{Ht}$, and satisfies the local demand in each market for the final, intermediate and capital goods:

$$Y_{Ht} = C_{Ht} + X_{Ht} + I_{Ht} = (1-\gamma)h\left(\frac{P_{Ht}}{\mathcal{P}_t}\right)\big[C_t + X_t + I_t\big], \tag{A12}$$

$$Y^*_{Ht} = C^*_{Ht} + X^*_{Ht} + I^*_{Ht} = \gamma h\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right)\big[C^*_t + X^*_t + I^*_t\big]. \tag{A13}$$

Lastly, we combine the household budget constraint (A2) with profits (A10), aggregated across all home firms, as well as the market clearing conditions above to obtain the home country budget constraint:

$$\frac{B_{t+1}}{R_t} - B_t = NX_t \qquad \text{with} \qquad NX_t = \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - P_{Ft} Y_{Ft}, \tag{A14}$$

where NX_t denotes net exports expressed in units of the home currency.

A.4 Segmented financial market

The structure of the financial markets is as described in Section 5, and we provide here:

Proof of Lemma 1 The proof follows two steps. First, it characterizes the solution to the portfolio problem (22) of the arbitrageurs to derive their policy function (24). Second, it combines this solution with the financial market clearing (23) to derive the equilibrium condition (25).

(a) Portfolio choice: *The solution to the portfolio choice problem* (22) *when the time periods are short is given by:*

$$\frac{d^*_{t+1}}{P^*_t} = -\frac{i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}, \tag{A15}$$

where $i_t - i^*_t \equiv \log(R_t/R^*_t)$, $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ *and* $\sigma_{e\pi^*} = \mathrm{cov}_t(\Delta e_{t+1}, \Delta p^*_{t+1})$.

Proof: The proof follows Campbell and Viceira (2002, Chapter 3 and Appendix 2.1.1). Consider the objective of the arbitrageur's problem (22) and rewrite it as:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega\big(1 - e^{x^*_{t+1}}\big) e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t} \right) \right\}, \tag{A16}$$

where we used the definition of $\tilde{R}^*_{t+1}$ in (21) and the following algebraic manipulation:

$$\frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} = \frac{\tilde{R}^*_{t+1}/R^*_t}{P^*_{t+1}/P^*_t} \frac{d^*_{t+1}}{P^*_t} = \frac{1 - \frac{R_{t+1}}{R^*_t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}}{e^{\pi^*_{t+1}}} \frac{d^*_{t+1}}{P^*_t} = \left(1 - e^{x^*_{t+1}}\right) e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t}$$

and defined the log Carry trade return and foreign inflation rate as

$$x^*_{t+1} \equiv i_t - i^*_t - \Delta e_{t+1} = \log(R_t/R^*_t) - \Delta \log \mathcal{E}_{t+1} \qquad \text{and} \qquad \pi^*_{t+1} \equiv \Delta \log P^*_{t+1}.$$

When time periods are short, (x^*_{t+1}, π^*_{t+1}) correspond to the increments of a vector normal diffusion process $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ with time-varying drift $\boldsymbol{\mu}_t$ and time-invariant conditional variance matrix $\boldsymbol{\sigma}$:

$$\begin{pmatrix} \mathrm{d}\mathcal{X}^*_t \\ \mathrm{d}\mathcal{P}^*_t \end{pmatrix} = \boldsymbol{\mu}_t \mathrm{d}t + \boldsymbol{\sigma} \mathrm{d}\mathcal{W}_t, \tag{A17}$$

where $\mathcal{W}_t$ is a standard two-dimensional Brownian motion. Indeed, as we show below, in equilibrium x^*_{t+1} and π^*_{t+1} follow stationary linear stochastic processes (ARMAs) with correlated innovations, and therefore

$$(x^*_{t+1}, \pi^*_{t+1}) \,\big|\, \mathcal{I}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\sigma}^2),$$

where $\mathcal{I}_t$ is the information set at time t, and the drift and variance matrixes are given by:

$$\boldsymbol{\mu}_t = \mathbb{E}_t \begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} \\ \mathbb{E}_t \pi^*_{t+1} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\sigma}^2 = \mathrm{var}_t \begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} \sigma_e^2 & -\sigma_{e\pi^*} \\ -\sigma_{e\pi^*} & \sigma_{\pi^*}^2 \end{pmatrix},$$

where $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$, $\sigma^2_{\pi^*} \equiv \mathrm{var}_t(\Delta p^*_{t+1})$ and $\sigma_{e\pi^*} \equiv \mathrm{cov}_t(\Delta e_{t+1}, \Delta p^*_{t+1})$ are time-invariant (annualized) conditional second moments. Following Campbell and Viceira (2002), we treat (x^*_{t+1}, π^*_{t+1}) as discrete-interval differences of the continuous process, $(\mathcal{X}^*_{t+1} - \mathcal{X}^*_t, \mathcal{P}^*_{t+1} - \mathcal{P}^*_t)$. With short time periods, the solution to (A16) is equivalent to

$$\max_{d^*} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \big(1 - e^{\mathrm{d}\mathcal{X}^*_t}\big) e^{-\mathrm{d}\mathcal{P}^*_t} \frac{d^*}{P^*_t} \right) \right\}, \tag{A18}$$

where $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ follow (A17). Using Ito's Lemma, we rewrite the objective as:

$$\begin{aligned}
&\mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega\Big(-\mathrm{d}\mathcal{X}^*_t - \tfrac{1}{2}(\mathrm{d}\mathcal{X}^*_t)^2 \Big) \Big(1 - \mathrm{d}\mathcal{P}^*_t + \tfrac{1}{2}(\mathrm{d}\mathcal{P}^*_t)^2 \Big) \frac{d^*}{P^*_t} \right) \right\} \\
&= \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega\Big(-\mathrm{d}\mathcal{X}^*_t - \tfrac{1}{2}(\mathrm{d}\mathcal{X}^*_t)^2 + \mathrm{d}\mathcal{X}^*_t \mathrm{d}\mathcal{P}^*_t \Big) \frac{d^*}{P^*_t} \right) \right\} \\
&= -\frac{1}{\omega} \exp\left(\left[\omega\big(\mu_{1,t} + \tfrac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}\big) \frac{d^*}{P^*_t} + \frac{\omega^2 \sigma_e^2}{2} \left(\frac{d^*}{P^*_t} \right)^2 \right] \mathrm{d}t \right),
\end{aligned}$$

where the last line uses the facts that $(\mathrm{d}\mathcal{X}^*_t)^2 = \sigma_e^2 \mathrm{d}t$ and $\mathrm{d}\mathcal{X}^*_t \mathrm{d}\mathcal{P}^*_t = -\sigma_{e\pi^*} \mathrm{d}t$, as well as the property of the expectation of an exponent of a normally distributed random variable; $\mu_{1,t}$ denotes the first component of the drift vector $\boldsymbol{\mu}_t$. Therefore, maximization in (A18) is equivalent to:

$$\max_{d^*} \left\{ -\omega\big(\mu_{1,t} + \tfrac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}\big) \frac{d^*}{P^*_t} - \tfrac{1}{2}\omega^2\sigma_e^2 \left(\frac{d^*}{P^*_t} \right)^2 \right\} \quad \text{w/solution} \quad \frac{d^*}{P^*_t} = -\frac{\mu_{1,t} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}.$$

This is the portfolio choice equation (A15), which obtains under CARA utility in the limit of short time periods, but note it is also equivalent to the exact solution under mean-variance preferences. The extra terms in the numerator correspond to Jensen's Inequality corrections to the expected real log return on the carry trade. Assuming $\boldsymbol{\sigma} \to 0$, yet $\omega \to \infty$ such that $\omega\sigma_e^2$ stays bounded away from zero, this solution converges to the policy function in (24), as we discuss below ■

(b) Equilibrium condition: To derive the modified UIP condition (25), we combine the portfolio choice solution (A15) with the market clearing condition (23) and the noise-trader currency demand (20) to obtain:

$$B^*_{t+1} + P^*_t n\big(e^{\psi_t} - 1\big) - m P^*_t \frac{i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2} = 0. \tag{A19}$$

The market clearing conditions in (23) together with the fact that both intermediaries and noise traders take zero capital positions, that is $\frac{D_{t+1}+N_{t+1}}{R_t} = -\mathcal{E}_t \frac{D^*_{t+1}+N^*_{t+1}}{R^*_t}$. This results in the equilibrium balance between home and foreign household asset positions, $\frac{B_{t+1}}{R_t} = -\mathcal{E}_t \frac{B^*_{t+1}}{R^*_t}$. Therefore, we can rewrite (A19) as:

$$\frac{i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2/m} = n\big(e^{\psi_t} - 1\big) - \frac{R_t^*}{R_t}\frac{Y_t}{\mathcal{Q}_t}\frac{B_{t+1}}{P_tY_t},$$

where we normalized net foreign assets by nominal output P_tY_t and used the definition of the real exchange rate $\mathcal{Q}_t$ in (8). We next log-linearize this equilibrium condition around a symmetric equilibrium with $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{B} = \bar{B}^* = 0$, $\mathcal{Q} = 1$, and some $\bar{Y}$ and $\bar{P} = \bar{P}^*$. As shocks become small, the (co)variances σ_e^2 and $\sigma_{e\pi^*}$ become second order and drop out from the log-linearization. We adopt the asymptotics in which as σ_e^2 shrinks, ω/m increases proportionally leaving the risk premium term $\omega\sigma_e^2/m$ constant, finite and separated from zero in the limit.[41] As a result, the log-linearized equilibrium condition is:

$$\frac{1}{\omega\sigma_e^2/m}\Big(i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1}\Big) = n\psi_t - \beta\bar{Y}b_{t+1}, \tag{A20}$$

where $b_{t+1} = \frac{\bar{R}}{\bar{P}\bar{Y}}B_{t+1} = -\frac{\bar{R}^*}{\bar{P}\bar{Y}}B_{t+1}^*$. After rearranging, this yields the modified UIP condition (25), completing the proof of the lemma. ■

Income and losses in the financial market Consider the income and losses of the non-household participants in the financial market — the intermediaries and the noise traders:

$$\frac{D_{t+1}^* + N_{t+1}^*}{R_t^*}\tilde{R}_{t+1}^* = \Big(md_{t+1}^* + n(e^{\psi_t} - 1)\Big)\big(1 - e^{x_{t+1}}\big),$$

where we used the definition of $\tilde{R}_{t+1}^*$ in (21) and the log Carry trade return $x_{t+1} \equiv i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = \log(R_t/R_t^*) - \Delta\log\mathcal{E}_{t+1}$. Using the same steps as in the proof of Lemma 1, we can approximate this income as:

$$\left(-m\frac{\mathbb{E}_t x_{t+1}}{\omega\sigma_e^2} + n\psi_t\right)\big(-x_{t+1}\big) = -\beta\bar{Y}b_{t+1}x_{t+1},$$

where the equality uses (A20). Therefore, while the UIP deviations (realized x_{t+1} and expected $\mathbb{E}_t x_{t+1}$) are first order, the income and losses in the financial markets are only second order, as $b_{t+1} = B_{t+1}/(\beta\bar{P}\bar{Y})$ is first order around $\bar{B} = 0$. Intuitively, the income and losses in the financial market are equal to the realized UIP deviation times the gross portfolio position — while both are first order, their product is second order, and hence negligible from the point of view of the country budget constraint.

[41]Note that σ_e^2/m is the quantity of risk per intermediary and ω is their aversion to risk; alternatively, ω/m can be viewed as the effective risk aversion of the whole sector of intermediaries who jointly hold all exchange rate risk. Our approach follows Hansen and Sargent (2011) and Hansen and Miao (2018), who consider the continuous-time limit in the models with ambiguity aversion. The economic rationale of this asymptotics is not that second moments are zero and effective risk aversion ω/m is infinite, but rather that risk premia terms, which are proportional to $\omega\sigma_e^2/m$, are finite and nonzero. Indeed, the first-order dynamics of the equilibrium system result in well-defined second moments of the variables, including σ_e^2, as in Devereux and Sutherland (2011) and Tille and van Wincoop (2010). An important difference of our solution concept is that it allows for a non-zero first-order component of the return differential, namely a non-zero expected Carry trade return. We characterize the equilibrium σ_e^2 below in Appendix A.6.

A.5 Proof of Proposition 1

This proposition follows from the dynamic system (17)–(18), which transforms the risk sharing condition (15) and the flow budget constraint (13)–(14) by defining the residual terms $\hat{\psi}_t$ and $\hat{\xi}_t$. Define $z_t \equiv \sigma(c_t - c_t) - q_t$. The Cole-Obstfeld parameter restriction $\sigma = \theta = 1$ implies $\hat{\theta} = \theta = 1$.[42] In this case, (13)–(14) result in $\beta b_{t+1} - b_t = nx_t = \gamma\big[-z_t - (1-\gamma)\hat{\xi}_t\big]$ with $\hat{\xi}_t = \tilde{\xi}_t$ up to higher order terms, which is a special case of (18). Iterating this condition forward and using the no-bubble condition $\lim_{j\to\infty}\beta^j b_{t+j} = 0$, we obtain

$$\gamma\sum_{j=0}^{\infty}\beta^j z_{t+j} = b_t - \gamma(1-\gamma)\sum_{j=0}^{\infty}\beta^j\hat{\xi}_{t+j}.$$

Condition (17), in turn, results in a martingale property $\mathbb{E}_t\Delta z_{t+1} = \hat{\psi}_t$, or equivalently

$$\mathbb{E}_t z_{t+j} = z_t + \sum_{\ell=0}^{j-1}\mathbb{E}_t\hat{\psi}_{t+\ell} \qquad \text{for any } j > 0.$$

Combining the two expressions, we obtain:

$$z_t = \frac{1-\beta}{\gamma}b_t - \sum_{j=0}^{\infty}\beta^j\mathbb{E}_t\big\{\beta\hat{\psi}_{t+j} + (1-\beta)(1-\gamma)\hat{\xi}_{t+j}\big\}.$$

Substituting into (18), we solve for Δb_{t+1} and Δz_{t+1}, yielding:

$$\frac{\beta}{\gamma}\Delta b_{t+1} = (1-\gamma)\hat{\xi}_t + \sum_{j=0}^{\infty}\beta^j\mathbb{E}_t\big\{\beta\hat{\psi}_{t+j} + (1-\beta)(1-\gamma)\hat{\xi}_{t+j}\big\}, \tag{A21}$$

$$\begin{aligned}\Delta z_{t+1} = {}& \frac{1-\beta}{\beta}\Big[(1-\gamma)\hat{\xi}_t + \sum_{j=0}^{\infty}\beta^j\mathbb{E}_t\big\{\beta\hat{\psi}_{t+j} + (1-\beta)(1-\gamma)\hat{\xi}_{t+j}\big\}\Big] \\ & - \sum_{j=0}^{\infty}\beta^j(\mathbb{E}_{t+1}-\mathbb{E}_t)\big\{\beta\hat{\psi}_{t+1+j} + (1-\beta)(1-\gamma)\hat{\xi}_{t+1+j}\big\}\end{aligned} \tag{A22}$$

which only depends on the path of $\{\hat{\psi}_t, \hat{\xi}_t\}_t$. Therefore, in conventional models according to Definition 1, the properties of Δz_t do not depend on the monetary policy or exchange rate regime.

The cases of complete markets and financial autarky need to be considered separately. In the case of complete markets, we have from (16) that $\Delta z_t = -\tilde{\zeta}_{t+1}$, where $\tilde{\zeta}_{t+1}$ is a component of $\hat{\psi}_t$ corresponding to the relative exogenous risk-sharing wedges. In the case of financial autarky, we have $nx_t = 0$, which from derivation above implies $z_t = -(1-\gamma)\hat{\xi}_t$. Therefore, the result of Proposition 1 applies as well in these two limiting cases. ■

[42] Note that a weaker parameter restriction $\sigma\hat{\theta} = 1$ is a sufficient requirement for Proposition 1.

A.6 Derivations and Proofs for Section 5

In order to prove Propositions 2 and 3, we first derive the equilibrium system and solve for the equilibrium exchange rate process. A lot of the derivations build on Itskhoki and Mukhin (2021) and we refer the reader to that paper for a more detailed description of the equilibrium conditions and log-linearization of the equilibrium system around a symmetric steady state.

Market clearing First, we derive (27). We combine together the linearized goods market clearing, $y_t = (1-\gamma)c_{Ht} + \gamma c^*_{Ht}$, with home and foreign demand for the home good in (6)–(7), which in the absence of taste shocks (ξ_t, ξ^*_t) can be written as:

$$c_{Ht} = -\theta(p_{Ht} - p_t) + c_t \qquad \text{and} \qquad c^*_{Ht} = -\theta(p^*_{Ht} - p^*_t) + c^*_t.$$

From the definitions of the price index, we obtain $p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}$ and $p^*_t = (1-\gamma)p^*_{Ft} + \gamma p^*_{Ht}$, and therefore:

$$p_{Ht} - p_t = \gamma(p_{Ht} - p_{Ft}) = -\gamma s_t \qquad \text{and} \qquad p^*_{Ht} - p^*_t = (1-\gamma)(p^*_{Ht} - p^*_{Ft}) = -(1-\gamma)s_t,$$

where, due to the law of one price ($p_{Ht} = p^*_{Ht} + e_t$ and $p_{Ft} = p^*_{Ft} + e_t$), the terms of trade are:

$$s_t = p_{Ft} - p^*_{Ht} - e_t = (p^*_t + e_t - p_t)/(1-2\gamma) = q_t/(1-2\gamma).$$

Substituting these expressions into the market clearing results in:

$$y_t = \frac{2\theta\gamma(1-\gamma)}{1-2\gamma} q_t + (1-\gamma)c_t + \gamma c^*_t,$$

which equalizes aggregate supply and aggregate demand for the home good. Combining it together with the foreign counterpart, we have:

$$y_t - y^*_t = \frac{2\gamma}{1-2\gamma} 2\theta(1-\gamma) q_t + (1-2\gamma)(c_t - c^*_t), \tag{A23}$$

where the term with the real exchange rate is the expenditure switching term. Equations (A23) characterizes the locus of (relative) output and consumption combinations which clear the product market (for home and foreign goods).

The second step is to use the labor market clearing condition to solve out aggregate output. Labor market clears when ℓ_t satisfies simultaneously the household labor supply, $\sigma c_t + \frac{1}{\nu}\ell_t = w_t - p_t$, and the firm labor demand given by the production function, $y_t = a_t + \ell_t$, which together result in:

$$y_t + \sigma\nu c_t = \nu(w_t - p_t) + a_t.$$

Combining this with its foreign counterpart, we have:

$$(y_t - y^*_t) + \sigma\nu(c_t - c^*_t) = \nu(q_t - q^W_t) + (a_t - a^*_t) = -\frac{2\gamma\nu}{1-2\gamma} q_t + (1+\nu)(a_t - a^*_t), \tag{A24}$$

where $q_t^W = w_t^* + e_t - w_t$ is the wage-based real exchange rate and we used the relationship between $q_t = (1-2\gamma)[q_t^W + (a_t - a_t^*)]$.[43] Equation (A24) characterizes the locus of output and consumption combinations which clear the labor market. Combined together with (A23), the two conditions characterize the general labor and product market clearing, which we rewrite in the relative consumption and real exchange rate space as:

$$(1-2\gamma+\sigma\nu)(c_t - c_t^*) = -\frac{2\gamma}{1-2\gamma}\left[2\theta(1-\gamma)+\nu\right]q_t + (1+\nu)(a_t - a_t^*),$$

which is equivalent to (27) in the text after noting that $\varphi = 1/\nu$ is the inverse Frisch elasticity.

Equilibrium exchange rate process We next use (27) to solve out relative consumption, $c_t - c_t^*$, from the dynamic system (18) and (26), which results in two equations in (q_t, b_t):[44]

$$-(1+\gamma\sigma\kappa_q)\mathbb{E}_t\Delta q_{t+1} = -\sigma\kappa_a\mathbb{E}_t\Delta\tilde{a}_{t+1} + \chi_1\psi_t - \chi_2 b_{t+1},$$
$$\beta b_{t+1} - b_t = \gamma[(\hat{\theta}+\gamma\kappa_q)q_t - \kappa_a\tilde{a}_t],$$

where $\tilde{a}_t \equiv a_t - a_t^*$ and $\mathbb{E}_t\Delta\tilde{a}_{t+1} = -(1-\rho)\tilde{a}_t$ as (a_t, a_t^*) follow AR(1)s with persistence ρ.

We next rewrite this dynamic system in matrix form:

$$\begin{pmatrix} 1 & -\hat{\chi}_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathbb{E}_t q_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1/\beta \end{pmatrix}\begin{pmatrix} q_t \\ \hat{b}_t \end{pmatrix} - \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix},$$

where for brevity we make the following substitution of variables:

$$\hat{b}_t \equiv \frac{\beta}{\gamma(\hat{\theta}+\gamma\kappa_q)}b_t, \qquad \hat{a}_t \equiv \frac{\kappa_a}{\hat{\theta}+\gamma\kappa_q}\tilde{a}_t, \tag{A25}$$
$$\hat{\chi}_1 \equiv \frac{\chi_1}{1+\gamma\sigma\kappa_q}, \qquad \hat{\chi}_2 \equiv \frac{\gamma(\hat{\theta}+\gamma\kappa_q)}{\beta(1+\gamma\sigma\kappa_q)}\chi_2, \qquad k \equiv \frac{\sigma(\hat{\theta}+\gamma\kappa_q)}{1+\gamma\sigma\kappa_q}.$$

Diagonalizing the dynamic system, we have:

$$\mathbb{E}_t x_{t+1} = Bx_t - C\begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix}, \quad \text{where} \quad B \equiv \begin{pmatrix} 1+\hat{\chi}_2 & \hat{\chi}_2/\beta \\ 1 & 1/\beta \end{pmatrix}, \quad C \equiv \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k+\hat{\chi}_2 \\ 0 & 1 \end{pmatrix},$$

and we denoted $x_t \equiv (q_t, \hat{b}_t)'$. The eigenvalues of B are:

$$\mu_{1,2} = \frac{(1+\hat{\chi}_2+1/\beta) \mp \sqrt{(1+\hat{\chi}_2+1/\beta)^2 - 4/\beta}}{2/\beta} \qquad \text{such that} \qquad 0 < \mu_1 \le 1 < \frac{1}{\beta} \le \mu_2,$$

and $\mu_1 + \mu_2 = 1 + \hat{\chi}_2 + 1/\beta$ and $\mu_1 \cdot \mu_2 = 1/\beta$. Note that when $\chi_2 = 0$, and hence $\hat{\chi}_2 = 0$, the two

[43] Under constant-markup pricing, the linearized pricing equations are $p_{Ht} = w_t - a_t$ and $p_{Ft} = w_t^* + e_t - a_t^*$, so that $p_t = (1-\gamma)(w_t - a_t) + \gamma(w_t^* + e_t - a_t^*)$. Together with the foreign counterpart, it results in the relationship between q_t and q_t^W in the text. See Itskhoki and Mukhin (2021) and Itskhoki (2021) for derivations of these relationships in a more general model with variable markups, pricing to market and Balassa-Samuelson terms.

[44] Recall that $nx_t = \gamma[\theta q_t + (\theta-1)s_t - (c_t - c_t^*)]$ in the absence of taste shocks, $\xi_t = \xi_t^* = 0$, and since $s_t = q_t/(1-2\gamma)$ as derived above, we have $nx_t = \gamma[\hat{\theta}q_t - (c_t - c_t^*)]$ with $\hat{\theta} = \theta + \frac{\theta-1}{1-2\gamma} = \frac{2\theta(1-\gamma)-1}{1-2\gamma}$, as stated in the text.

roots are simply $\mu_1 = 1$ and $\mu_2 = 1/\beta$. In the text (footnote 27), we denote $\delta \equiv \mu_1$.

The left eigenvalue associated with $\mu_2 > 1$ is $v = (1, 1/\beta - \mu_1)$, such that $vB = \mu_2 v$. Therefore, we can pre-multiply the dynamic system by v and rearrange to obtain:

$$vx_t = \frac{1}{\mu_2}\mathbb{E}_t\{vx_{t+1}\} + \frac{1}{\mu_2}\hat{\chi}_1\psi_t + \left[\frac{(1-\rho)k + \hat{\chi}_2}{\mu_2} + \frac{1/\beta - \mu_1}{\mu_2}\right]\hat{a}_t.$$

Using the facts that $\hat{\chi}_2 + 1/\beta - \mu_1 = \mu_2 - 1$ and $1/\mu_2 = \beta\mu_1$, we solve this dynamic equation forward to obtain the equilibrium cointegration relationship:

$$vx_t = q_t + (1/\beta - \mu_1)\hat{b}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{1-\beta\mu_1+\beta(1-\rho)k\mu_1}{1-\beta\rho\mu_1}\hat{a}_t. \tag{A26}$$

Combining this with the second dynamic equation for $\hat{b}_{t+1}$, we solve for:

$$\hat{b}_{t+1} - \mu_1\hat{b}_t = \overbrace{q_t + \left(\tfrac{1}{\beta} - \mu_1\right)\hat{b}_t}^{=vx_t} - \hat{a}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t, \tag{A27}$$

Note that $\hat{b}_{t+1}$ in (A27) follows a stationary AR(2) with roots ρ and μ_1.

Finally, we apply the lag operator $(1-\mu_1 L)$ to (A26) and use (A27) to solve for:

$$\begin{aligned}(1-\mu_1 L)q_t &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t\right] + (1-\mu_1 L)\hat{a}_t \\ &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)\mu_1}{1-\beta\rho\mu_1}k\hat{a}_t\right] + \frac{1-\beta\mu_1}{1-\beta\rho\mu_1}(1-\rho\mu_1 L)\hat{a}_t, \end{aligned} \tag{A28}$$

where L is the lag operator such that $Lq_t = q_{t-1}$. Therefore, equilibrium RER q_t follows a stationary ARMA(2,1) with autoregressive roots $\delta = \mu_1$ and ρ. In the limit $\chi_2 \to 0$, which implies $\mu_1 \to 1$, this process for q_t becomes an ARIMA(1,1,1), which nonetheless has impulse responses that are arbitrarily close to a stationary ARMA(2,1) with a large $\mu_1 \lesssim 1$.

Furthermore, one can partition the components of q_t in (A28) driven by ψ_t and $\tilde{a}_t$ into two subprocesses q_t^ψ and q_t^a such that $q_t = q_t^\psi + q_t^a$:

$$(1-\mu_1 L)q_t^\psi = (1-\beta^{-1}L)\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t, \tag{A29}$$

$$(1-\mu_1 L)q_t^a = \left[(1-\beta^{-1}L)\frac{\beta(1-\rho)\mu_1}{1-\beta\rho\mu_1}k + \frac{1-\beta\mu_1}{1-\beta\rho\mu_1}(1-\rho\mu_1 L)\right]\hat{a}_t. \tag{A30}$$

Note that:

(i) as $\chi_1 \to 0$ (and hence $\hat{\chi}_1 \to 0$), $q_t^\psi \to 0$ and $q_t = q_t^a$;

(ii) the two components in q_t^a correspond to the effects of productivity shocks on the Euler equation and the budget constraint respectively, with the former component disappearing in the limit of persistent shocks $\rho \to 1$, such that the productivity component of the real exchange rate is simply $q_t^a = \hat{a}_t = \frac{\kappa_a}{\theta+\gamma\kappa_q}\tilde{a}_t$, a random walk that does not depend on χ_1 or χ_2. As a result, in this case, $\chi_1 \to 0$ implies $q_t = q_t^a = \hat{a}_t$.

Equilibrium variance of the exchange rate Solution (A28) characterizes the behavior of q_t for given values of χ_1 and χ_2 (and hence μ_1, μ_2), which from (25) themselves depend on $\sigma_e^2 = \text{var}_t(\Delta e_{t+1})$. Under the peg, $\sigma_e^2 = 0$ and hence $\chi_1 = \chi_2 = 0$. Under the float, monetary policy stabilizes inflation, ensuring $e_t = q_t$, and hence we have $\sigma_e^2 = \text{var}_t(\Delta q_{t+1})$. We now solve for the equilibrium value of σ_e^2, and thus of $(\chi_1, \chi_2, \mu_1, \mu_2)$.

Using (A28), we calculate $\sigma_e^2 = \text{var}_t(\Delta q_{t+1})$ for given χ_1 and χ_2:

$$\sigma_e^2 = \text{var}_t(\Delta q_{t+1}) = \left(\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\right)^2\sigma_\psi^2 + \left(\frac{\beta(1-\rho)\mu_1 k + (1-\beta\mu_1)}{1-\beta\rho\mu_1}\right)^2\sigma_a^2 = \frac{\hat{\chi}_1^2\sigma_\psi^2 + \big((1-\rho)k + (\mu_2-1)\big)^2\sigma_a^2}{(\mu_2-\rho)^2},$$

where the second line used the fact that $\beta\mu_1 = 1/\mu_2$. In addition, recall that:

$$\hat{\chi}_1 = \frac{n}{1+\gamma\sigma\kappa_q}\frac{\omega\sigma_e^2}{m}, \qquad \hat{\chi}_2 \equiv \frac{\hat{\theta}+\gamma\kappa_q}{1+\gamma\sigma\kappa_q}\gamma\bar{Y}\frac{\omega\sigma_e^2}{m} \quad \text{and} \quad \mu_2 = \frac{(1+\beta\hat{\chi}_2+\beta)+\sqrt{(1+\beta\hat{\chi}_2+\beta)^2-4\beta}}{2}.$$

We therefore can rewrite the fixed point equation for $\sigma_e^2 > 0$ as follows:

$$F(x,\tilde{\omega}) = \big(\mu_2(\tilde{\omega}x)-\rho\big)^2 x - b(\tilde{\omega}x)^2 - c = 0, \tag{A31}$$

where we used the following notation:

$$x \equiv \sigma_e^2 \geq 0, \qquad \tilde{\omega} = \frac{\omega}{m}, \qquad b \equiv \left(\frac{n}{1+\gamma\sigma\kappa_q}\right)^2\sigma_\psi^2, \qquad c \equiv \big((1-\rho)k + (\mu_2-1)\big)^2\sigma_a^2 \geq 0,$$

and $\mu_2(\cdot)$ is a function which gives the equilibrium values of μ_2 defined above as a function of $\tilde{\omega}\sigma_e^2$ for given values of the model parameters. Note that for any given $\tilde{\omega} > 0$:

$$\lim_{x\to 0} F(x,\tilde{\omega}) = -c \leq 0,$$
$$\lim_{x\to\infty}\frac{F(x,\tilde{\omega})}{x^3} = \lim_{x\to\infty}\left(\frac{\mu_2(\tilde{\omega}x)}{x}\right)^2 = \left(\frac{\beta\hat{\chi}_2^2}{\sigma_e^2}\right) = \left(\frac{\hat{\theta}+\gamma\kappa_q}{1+\gamma\sigma\kappa_q}\gamma\bar{Y}\tilde{\omega}\right)^2 > 0.$$

Therefore, by continuity at least one fixed-point $F(\sigma_e^2,\tilde{\omega}) = 0$ with $\sigma_e^2 \geq 0$ exists, and all such that $\sigma_e^2 > 0$ whenever $c > 0$ (that is, when $\sigma_a > 0$). One can further show that when σ_a/σ_ψ is not too small, this equilibrium is unique, which is in particular the case under our calibration.[45]

Finally, we consider the limit of log-linearization in Lemma 1, where $(\sigma_a,\sigma_\psi) = \sqrt{\epsilon}\cdot(\bar{\sigma}_a,\bar{\sigma}_\psi) = \mathcal{O}(\sqrt{\epsilon})$ as $\epsilon \to 0$, where $(\bar{\sigma}_a,\bar{\sigma}_\psi)$ are some fixed numbers. Then in (A31), $(b,c) = \mathcal{O}(\epsilon)$, as (b,c) are linear in $(\sigma_a^2,\sigma_\psi^2)$. This implies that for any given fixed point $(\bar{\sigma}_e^2,\bar{\omega})$, with $F(\bar{\sigma}_e^2,\bar{\omega};\bar{\sigma}_a^2,\bar{\sigma}_\psi^2) = 0$, there exists a sequence of fixed points $F(\epsilon\bar{\sigma}_e^2,\bar{\omega}/\epsilon;\epsilon\bar{\sigma}_a^2,\epsilon\bar{\sigma}_\psi^2) = 0$ as $\epsilon \to 0$, for which $\sigma_e^2 = \epsilon\bar{\sigma}_e^2 = \mathcal{O}(\epsilon)$, $\tilde{\omega} = \bar{\omega}/\epsilon = \mathcal{O}(1/\epsilon)$ and $\tilde{\omega}\sigma_e^2 = \bar{\omega}\bar{\sigma}_e^2 = const$. To verify this, one can simply divide (A31) by ϵ and note that, for a given $\tilde{\omega}x$, $F(x,\tilde{\omega})$ is linear in (x,b,c), which means that the fixed point x scales with (b,c) provided that $\tilde{\omega}x$ stays constant. This confirms the conjecture used in the proof of Lemma 1. ■

[45]For $\sigma_a/\sigma_\psi \approx 0$, there typically exist three equilibria. In particular, when $\sigma_a = 0$, there always exists an equilibrium with $\sigma_e^2 = \chi_1 = 0$, in addition to two other potential equilibria with $\sigma_e^2 > 0$, which exist when σ_ψ is not too small (see Itskhoki and Mukhin 2017).

Proof of Proposition 2 The proof follows directly from results above. First, the existence of equilibria under both the float and the peg follows from the equilibrium exchange rate process (A28) together with the fixed point argument for σ_e^2 established above. Part (a) of the proposition follows from the decomposition of $q_t = q_t^\psi + q_t^a$ in (A29)–(A30), which implies:

$$\mathrm{var}(\Delta q_t) = \mathrm{cov}(\Delta q_t^\psi, \Delta q_t) + \mathrm{cov}(\Delta q_t^a, \Delta q_t),$$

with $\mathrm{cov}(\Delta q_t^\psi, \Delta q_t) = 0$ under the peg as $q_t^\psi \equiv 0$. Thus, it is sufficient to require that $\mathrm{cov}(\Delta q_t^\psi, \Delta q_t) \gg \mathrm{cov}(\Delta q_t^a, \Delta q_t)$ under the float, which is the case as σ_ψ/σ_a increases, and thus can be always guaranteed.

Part (b) of the proposition follows from (27): as $\gamma \to 0$, $c_t - c_t^* \to \frac{1+\varphi}{\sigma+\varphi}(a_t - a_t^*)$, independently of the process for q_t and the exchange rate regime. The same applies for output, with $y_t - y_t^* \to \frac{1+\varphi}{\sigma+\varphi}(a_t - a_t^*)$. Finally, inflation $\pi_t - \pi_t^* \equiv 0$ under the float, and under the peg $\pi_t - \pi_t^* = -\Delta q_t = -\Delta q_t^a$, with volatility arbitrary close to zero relative to the volatility of Δq_t under the float, as follows from part (a). ■

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