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SIZING UP THE REAL INCOME CHANNEL

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Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel

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ABSTRACT

Introducing heterogeneous households into a New Keynesian small open economy model amplifies the real income channel of exchange rates: the rise in import prices from a depreciation lowers households' real income, and leads them to cut back on spending. When the sum of import and export elasticities is one, this channel is offset by a larger Keynesian multiplier, heterogeneity is irrelevant, and expenditure switching drives the output response. With plausibly lower short-term elasticities, however, the real income channel dominates, and the depreciation can be contractionary for output. This weakens monetary transmission and creates a dilemma for policymakers facing capital outflows. Endogenous portfolios and delayed import price pass-through weaken the real income channel, while heterogeneous consumption baskets can strengthen it.

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1 Introduction

How do open economies respond to exchange rate shocks, such as those caused by capital flows? What is the role of exchange rates in monetary transmission? The canonical answers to these questions are derived from models with a representative agent.¹ In these models, marginal propensities to consume are small, muting the income effects of exchange rates for shocks at business cycle frequencies.

In this paper, we revisit these questions in a Heterogeneous-Agent New Keynesian model that features higher marginal propensities to consume, in line with the empirical evidence.² We first provide novel neutrality results under which heterogeneity is irrelevant. We then argue that, in the empirically relevant case, heterogeneity generates a powerful *real income channel* that limits or even undoes the expansionary effects of depreciations and weakens monetary transmission. This provides an explanation for the common policy view that depreciations can cause declines in consumption or output, even when foreign currency borrowing is not an issue.³

To isolate the forces that make heterogeneity relevant, we take as our benchmark the canonical representative-agent, complete markets (RA-CM) model of [Galí and Monacelli \(2005\)](#). This is a model in which markets are complete both within and across countries. Departing from complete markets, we consider incomplete markets with respect to idiosyncratic risk, and potentially with respect to aggregate risk as well. A large mass of domestic households faces idiosyncratic income uncertainty and borrowing constraints; they cannot insure the idiosyncratic risk, but may be able to form asset portfolios to hedge aggregate risk. We consider two main types of aggregate shocks: exchange rate shocks (shocks to the foreign interest rate that do not affect foreign demand) and domestic monetary policy shocks.

For exchange rate shocks, we show, using a sequence-space representation of the model ([Auclert, Bardóczy, Rognlie and Straub 2021](#), [Auclert, Rognlie and Straub 2024a](#)) that the output response combines three effects: an expenditure switching channel, a real

¹See [Obstfeld and Rogoff \(1995\)](#), [Corsetti and Pesenti \(2001\)](#), [Clarida, Galí and Gertler \(2002\)](#) and [Galí and Monacelli \(2005\)](#), as well as the textbook treatments in [Galí \(2008\)](#) and [Uribe and Schmitt-Grohé \(2017\)](#).

²High MPCs have been documented in advanced economies and emerging markets alike, see for instance [Johnson, Parker and Souleles \(2006\)](#) for the United States, [Jappelli and Pistaferri \(2014\)](#) for Italy, [Fagereng, Holm and Natvik \(2021\)](#) for Norway, and [Hong \(2023\)](#) for Peru.

³On contractionary devaluations, [Frankel \(2005\)](#) says: “Why are devaluations so costly? Many of the currency crises of the last 10 years have been associated with output loss. Is this, as alleged, because of excessive reliance on raising the interest rate as a policy response? More likely, it is because of contractionary effects of devaluation.” Although widespread, this policy view is difficult to back up empirically because it is challenging to identify exogenous exchange rate shocks in the data. This makes it important to study the conditions under which contractionary depreciations can emerge in microfounded general equilibrium models.

income channel, and a Keynesian multiplier channel. The RA-CM model only has expenditure switching, whose magnitude is governed by the composite parameter χ , equal to the sum of the price elasticities of imports and exports (the *trade elasticity*).⁴ This channel is unchanged in the heterogeneous agent (HA) model. Instead, there are two new forces, both of which work through households' real income: the "real income channel" through which rising import prices reduce aggregate consumption, and the multiplier on aggregate output. Since the multiplier depends on the overall output response, its importance grows with χ . Our first neutrality result states that, when $\chi = 1$, the two new forces exactly cancel, and the RA-CM and HA models have identical responses to any exchange rate shock—in fact, the response is independent of the market structure both across and within countries. Intuitively, when the trade elasticity is equal to 1, the rise in output from expenditure switching is exactly enough to offset rising import prices, leaving each household's real income and therefore consumption unchanged. The trade balance also remains constant, as reallocation from foreign to domestic goods offsets higher prices on the foreign goods.

When the trade elasticity χ is below 1 instead, the real income channel dominates. This makes the output response in the HA model lower than in the RA-CM model. For χ sufficiently below one, this response turns negative: a contractionary depreciation emerges. Qualitatively, this effect is at play even in a representative agent model with incomplete market across countries (RA-IM), and in a heterogeneous agent model in which households can form portfolios to hedge aggregate risk (HA-CM). But we show that it is quantitatively much larger with heterogeneous agents who cannot insure idiosyncratic risk and who cannot form portfolios to hedge against aggregate risk (HA-IM). In other words, the *combination* of heterogeneity and incomplete markets "sizes up" the real income channel that [Díaz-Alejandro \(1963\)](#), [Cooper \(1968\)](#) and [Krugman and Taylor \(1978\)](#) had emphasized as a potential source of contractionary devaluations.⁵ By contrast, when $\chi > 1$, the multiplier effect dominates, and depreciations are even more expansionary. Hence, our theoretical result is one of *complementarity* between heterogeneity/incomplete markets and trade elasticities. Later, we argue that the relevant empirical counterpart of χ is the short-run trade elasticity, which tends to be less than 1.

Turning to monetary policy, we show that there also exists a threshold level of the

⁴This is the elasticity that enters the well-known Marshall-Lerner condition, which states that, in partial equilibrium, depreciations improve the trade balance when $\chi > 1$. We show that in our model, this condition also applies in general equilibrium.

⁵We show that a two agent (TA) model calibrated to the same MPCs as our HA-IM model generates much smaller contractionary depreciations than our HA-IM model due to the smaller "intertemporal marginal propensities to consume" ([Auclert, Rognlie and Straub 2024a](#)).

trade elasticity for which heterogeneity is irrelevant. This result requires an elasticity of intertemporal substitution of 1, and a trade elasticity of $\chi = 2 - \alpha$, with $\alpha \in (0, 1)$ denoting the openness of the country. As in the exchange rate case, this involves a constant trade balance; here we need a higher trade elasticity χ to offset the increase in import demand from rising consumption in a monetary expansion. The $\chi = 2 - \alpha$ level covers the Cole-Obstfeld parametrization, in which both domestic and foreign agents have unitary elasticities of substitution. In fact, our neutrality result is reminiscent of the original [Cole and Obstfeld \(1991\)](#) result, which established that with Cobb-Douglas elasticities, market structure was irrelevant for the effect of productivity shocks. Our result shows that the same is true for monetary policy shocks, and also for a much broader set of market structures, including within-country incomplete markets with respect to idiosyncratic shocks. In that sense, our result generalizes [Werning \(2015\)](#)'s seminal neutrality result for closed economies to an open economy setting.⁶

Away from this benchmark, when $\chi < 2 - \alpha$, the output response is lower in the HA-IM model than in the RA-CM model—driven in part by the real income effect. One way to understand this result is that, with elasticities below Cobb-Douglas, a temporary monetary expansion induces a current account deficit, as in [Tille \(2001\)](#). The resulting negative net foreign asset position must be repaid later. However, absent further monetary stimulus, repayment must occur without a depreciated exchange rate, and hence without increased exports. Instead, the trade balance improves via depressed imports—which are achieved through a domestic contraction. Thus, in an HA model with $\chi < 2 - \alpha$, monetary easing raises current demand at the expense of a future contraction: it “steals demand from the future”. This mechanism is reminiscent of the effects of durable goods or indebted demand in closed economies ([McKay and Wieland, 2021](#), [Mian, Straub and Sufi, 2021](#)), but it operates through the current account.

Our benchmark model allows for clean analytical results, but it says nothing of the empirically relevant level of the trade elasticity χ . A simple quantification is difficult because trade elasticities are well documented to be dynamic: smaller in the short run than in the medium to long run.⁷ We address this shortcoming of our baseline model by building a quantitative extension. In it, we incorporate a tractable model of *delayed substitution*, in which consumers can only substitute between goods with a given Calvo probability.⁸

⁶In appendix C.5, we show that our neutrality result can also be extended to productivity shocks, as in the original [Cole and Obstfeld \(1991\)](#) paper.

⁷See, e.g., [Hooper, Johnson and Marquez \(2000\)](#), [Feenstra, Luck, Obstfeld and Russ \(2017\)](#), [Fitzgerald and Haller \(2018\)](#), [Auer, Burstein, Erhardt and Lein \(2019\)](#), [Auer, Burstein and Lein \(2021\)](#), [Amiti, Itsikhoki and Konings \(2022\)](#), and [Boehm, Levchenko and Pandalai-Nayar \(2023\)](#).

⁸This approach complements a structural literature on models of delayed adjustment in the exporting and importing decisions of firms, as in e.g., [Baldwin \(1988\)](#), [Baldwin and Krugman \(1989\)](#), [Ruhl \(2008\)](#),

Calibrating to the evidence in [Boehm, Levchenko and Pandalai-Nayar \(2023\)](#), we find that our model generates a “J curve”, with a trade elasticity that is smaller than 1 in the short-run, but larger than 1 in the long-run.⁹ As a consequence, our quantitative model finds that transitory depreciation shocks are contractionary in the short run.

Aside from accounting for dynamic trade elasticities, the quantitative model also allows us to speak to several other issues: we show that when consumption baskets of the poor are skewed towards imported goods (as in e.g. [Cravino and Levchenko, 2017](#)) or when recessions disproportionately affect the poor (as in [Blanco, Drenik and Zaratiegui, 2024](#)), the real income channel is amplified and a depreciation is more likely to be contractionary; we find that the real income channel is larger than a balance sheet channel calibrated to the net currency exposure of a typical country (which has shrunk in recent decades, see e.g. [Lane and Shambaugh 2010](#)); and we find that the real income channel is stronger the faster exchange rates pass through to retail prices of imported goods—and hence, likely stronger in emerging markets.

Our model can speak to the common perception of a dilemma for policymakers facing capital outflows—captured in our model as exchange rate depreciation shocks. On the one hand, outflows are contractionary, and fighting them with accommodative monetary policy exacerbates the depreciation. On the other hand, stabilizing the exchange rate by tightening monetary policy comes with the negative side effects of higher interest rates domestically, as in [Gertler, Gilchrist and Natalucci \(2007\)](#) and [Gourinchas \(2018\)](#). We use our model to derive the unique output-stabilizing monetary policy. In our baseline parameterization, we find that it actually involves an interest rate cut and an even larger depreciation, but in a lower trade elasticity parameterization, it involves an interest rate hike that mitigates the depreciation instead.

Our paper is connected to an empirical literature that estimates the aggregate effects of devaluations on output (e.g. [Kamin and Rogers 2000](#), [An, Kim and Ren 2014](#), [Fukui, Nakamura and Steinsson 2023](#)). This literature has reached mixed conclusions about the sign of the effect of devaluations on output. Beyond the empirical difficulty of identifying exchange rate shocks—devaluations are often caused by shocks that also contract the economy—this is likely due to the fact that the aggregate effect of exchange rate shocks on output depends on a large number of factors that are difficult to control for. By performing comparative statics in a quantitatively realistic heterogeneous-agent New Keynesian model on a variety of parameters—including the monetary policy response, im-

[Drozd and Nosal \(2012\)](#), [Alessandria and Choi \(2021\)](#). See [Alessandria, Arkolakis and Ruhl \(2021\)](#) for a review of this literature.

⁹See [Backus, Kehoe and Kydland \(1994\)](#) for an alternative, equilibrium model of the J curve.

port price pass-through, trade elasticity, and many others—our paper can help rationalize these mixed findings and provide useful controls for this literature.

Our paper relates to a literature on the importance of the real income channel, which was studied by [Díaz-Alejandro \(1963\)](#), [Cooper \(1968\)](#) and [Krugman and Taylor \(1978\)](#) in the context of IS-LM models. It is well understood from this literature that when the trade elasticity is low, a depreciation can lower consumption through changes in the terms of trade. In fact, a textbook result from the Old Keynesian open-economy theory ([Polak 1947](#), [Harberger 1950](#), [Laursen and Metzler 1950](#)), as summarized in [Dornbusch \(1980\)](#), is that depreciations are contractionary for output whenever the Marshall-Lerner condition is not satisfied ($\chi < 1$ in our notation). We show that in a micro-founded model, contractionary depreciations are in fact *less* likely than implied by this framework, and that their magnitude depends crucially on heterogeneity and incomplete markets.

Modern literature has also studied the real income channel. Working with a first-generation new open economy model with incomplete markets and prices set one period in advance, [Corsetti and Pesenti \(2001\)](#) analytically showed that monetary accommodations have a “beggar-thyself effect” through this channel. Their model featured unitary elasticities, so while this effect reduced country welfare, it did not lower aggregate consumption or output. Later, [Tille \(2001\)](#) and [Corsetti, Pesenti, Roubini and Tille \(2000\)](#) extended this model to feature a non-unitary elasticity substitution between goods and noted that, when this elasticity was low enough, this model allowed for the possibility of a devaluation to reduce consumption and output. Our paper confirms this qualitative result for RA-IM, but shows the effect is quantitatively small unless one increases the MPC, for instance with HA-IM. Finally, [Corsetti, Dedola and Leduc \(2008\)](#) show in a model without nominal rigidities that a negative productivity shock generates a depreciation together with a fall in consumption and output when the trade elasticity is low; this is due to the real income channel generating a fall in demand for home goods in spite of the depreciation. Relative to these papers, ours clarifies the importance of heterogeneity and market incompleteness in determining the magnitude of the real income channel.

Our paper also relates to a large international macro literature that, building on the original [Cole and Obstfeld \(1991\)](#) result, studies how the structure of asset markets matters for the aggregate effects of international shocks ([Baxter and Crucini 1995](#), [Heathcote and Perri 2002](#)). In the context of a representative-agent model, [Itskhoki \(2021\)](#) generalizes the Cole-Obstfeld equivalence between complete markets and financial autarky to a broader range of shocks, including monetary policy shocks. We provide similar neutrality results for monetary policy and exchange rate shocks, showing that this requires different trade elasticities, and consider a broader set of market structures, including complete and

incomplete markets both within and across countries.

Finally, our paper relates to an emerging literature that analyzes the effects of international shocks in the context of heterogeneous-agent, New Keynesian open economy models.¹⁰ Most of this literature has focused on wealth effects through households' balance sheets. [de Ferra, Mitman and Romei \(2020\)](#) study the effects of sudden stops in capital inflows when households hold foreign currency debt. They show that sudden stops lead to a contraction of consumption when monetary policy lets the exchange rate depreciate. [Guo, Ottonello and Perez \(2023\)](#) study the distributional effects of international shocks when agents differ by their sector of work and their financial integration, and show that unequal access to international financial markets is the main driver of inequality in response to domestic and international shocks. [Otten \(2021\)](#) study the redistributive effects of shocks under different exchange rate regimes when households hold foreign-currency debt. [Zhou \(2022\)](#) studies the effects of foreign monetary policy shocks in a model where the foreign-currency debt exposure of households is disciplined with microdata. Other papers with heterogeneous agents include [Giagheddu \(2020\)](#), who compares the distributional effects of fiscal devaluations to that of nominal devaluations, [Druedahl, Ravn, Sunder-Plassmann, Sundram and Waldstrøm \(2022\)](#), who study the transmission of foreign demand shocks, and [Oskolkov \(2023\)](#), who compares the dynamics of inequality under different exchange rate regimes. Relative to these papers, ours focuses on the real income channel and provides analytical results on when heterogeneity and market incompleteness matter and when they do not. We show that depreciations can be contractionary even if households do not hold foreign currency debt.

Layout. Section 2 sets up our baseline heterogeneous-agent model, with and without endogenous portfolios. Section 3 considers the effect of exchange rate shocks, while section 4 considers the transmission of monetary policy. Section 5 introduces our quantitative model, which we use to study the role of delayed substitution, delayed import price pass-through, heterogeneous consumption baskets, unequal incidence of aggregate shocks, and the response of monetary policy to contractionary capital outflows. Appendices A–D contain additional model details and proofs by section. Appendix E presents four alternative models in which the real income channel has a similar effect as in our baseline: one with dollar currency pricing, one with nontraded goods, one with imported intermediates, and one in which the country is a commodity exporter.

¹⁰See [Farhi and Werning \(2016\)](#), [Farhi and Werning \(2017\)](#), and [Cugat \(2022\)](#) for New Keynesian open economy models with two agents.

2 Baseline model

Our model merges two New Keynesian traditions: the Heterogeneous-Agent (“HANK”) framework for closed economies and the New Open Economy macro framework for open economies. Specifically, our model builds on the open-economy model of [Galí and Monacelli \(2005\)](#). To this model we add heterogeneity on the household side and sticky wages, as in [Auclert, Rognlie and Straub \(2024a\)](#). We additionally allow for both incomplete and complete markets with respect to aggregate risk: with incomplete markets, household portfolios are chosen independently of aggregate risk; with complete markets, households use their portfolios to hedge aggregate risk. This nests the market structures considered in the international macro literature.¹¹

2.1 Model setup

Time is discrete and the horizon is infinite. We focus directly on the problem of a small open economy understood, as in [Galí and Monacelli \(2005\)](#), as part of a world economy consisting of a continuum of countries.¹² We denote variables with a star superscript when they correspond to the world economy as a whole. We consider perfect-foresight impulse responses to aggregate shocks starting from a steady state at date 0, where for the moment these shocks are unanticipated (the case of incomplete markets with respect to aggregate risk, or “MIT shocks”). We use the solution method from [Boppart, Krusell and Mitman \(2018\)](#) and [Auclert et al. \(2021\)](#), which linearizes with respect to these shocks.

There are two goods in the economy: domestically produced “home” goods H , which can be exported, and “foreign” goods F , which are produced abroad and imported.

Domestic households. The economy is populated by heterogeneous households that may be subject to idiosyncratic income risk in the form of productivity shocks e_{it} , which follow a first-order Markov chain with mean $\mathbb{E}e_{it} = 1$. Households can invest their wealth in three assets: a domestic stock, a domestic nominal bond, and a foreign nominal bond. The returns on these assets cannot be indexed to idiosyncratic productivity.

A household with incoming stock position s , domestic bond position B^H , foreign bond position B^F , and productivity level e at any time t optimally chooses her consumption of

¹¹[Galí and Monacelli \(2005\)](#) is a complete markets representative-agent model. For incomplete markets models with a representative agent, see e.g. [Obstfeld and Rogoff \(1995\)](#) and [Corsetti and Pesenti \(2001\)](#); with two agents see e.g. [Cugat \(2022\)](#); and with heterogeneous agents see e.g. [de Ferra, Mitman and Romei \(2020\)](#).

¹²See [Aggarwal, Auclert, Rognlie and Straub \(2023\)](#) for a version of this model with N large countries.

the two goods, c_H, c_F , and position in each asset $s', B^{H'}, B^{F'}$ for next period by solving the dynamic program

$$\begin{aligned}
\tilde{V}_t(s, B^H, B^F, e) &= \max_{c_F, c_H, s', B^{H'}, B^{F'}} u(c_F, c_H) - v(N_t) + \beta \mathbb{E}_t \left[\tilde{V}_{t+1}(s', B^{H'}, B^{F'}, e') | e \right] \\
\text{s.t.} \quad &P_{Ft}c_F + P_{Ht}c_H + \mathcal{P}_ts' + B^{H'} + \mathcal{E}_tB^{F'} \\
&= (\mathcal{P}_t + D_t)s + (1 + \iota_{t-1})B^H + (1 + \iota_{t-1}^*)\mathcal{E}_tB^F + eW_tN_t \\
&\mathcal{P}_ts + B^{H'} + \mathcal{E}_tB^{F'} \geq 0
\end{aligned} \tag{1}$$

Here, P_{Ft} is the price of foreign goods in domestic currency units, P_{Ht} is the price of domestic goods, \mathcal{P}_t is the nominal stock price, \mathcal{E}_t is the nominal exchange rate, D_t the nominal dividend, ι_t is the nominal interest rate on home bonds, ι_t^* the nominal interest rate on foreign bonds, W_t is the nominal wage, and N_t denotes labor supplied by households, determined by union demand as specified below. The exchange rate \mathcal{E}_t is defined as the price of the foreign currency in units of domestic currency, so that an increase in \mathcal{E}_t indicates a nominal depreciation at home. The household is subject to a borrowing constraint on its net asset position.

We assume that households share the common per period utility function

$$u(c_F, c_H) = \frac{c^{1-\sigma}}{1-\sigma}, \quad v(N) = \psi \frac{N^{1+\varphi}}{1+\varphi} \tag{2}$$

where c is the CES consumption basket¹³

$$c = \left[\alpha^{1/\eta} c_F^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_H^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \tag{3}$$

The parameter $\sigma > 0$ is the inverse elasticity of intertemporal substitution, $\varphi > 0$ the inverse Frisch elasticity of labor supply, and $\eta > 0$ is the elasticity of substitution between home and foreign goods. $\alpha \in (0, 1)$ measures the openness of the economy ($1 - \alpha$ is the degree of home bias in preferences). $\psi > 0$ is a normalization constant. Given these preferences, the consumer price index is

$$P_t \equiv \left[\alpha P_{Ft}^{1-\eta} + (1-\alpha)P_{Ht}^{1-\eta} \right]^{1/(1-\eta)} \tag{4}$$

¹³Since CES preferences are homothetic, households have the same consumption basket and share the same price index. Section 5 considers non-homothetic preferences, under which poor households can consume foreign goods in different proportions than rich households.

Since there is no aggregate uncertainty, by no arbitrage,¹⁴ for any $t \geq 0$, all assets must have equal nominal returns from t to $t + 1$:

$$1 + \iota_t = (1 + \iota_t^*) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{\mathcal{P}_{t+1} + D_{t+1}}{\mathcal{P}_t} \quad (5)$$

It is useful to express these equations in real terms. Denote by P_t^* the price index abroad, and by Q_t the real exchange rate, defined as the price of a foreign consumption basket relative to a domestic consumption basket, that is:

$$Q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t} \quad (6)$$

With this convention, an increase in Q_t indicates a real depreciation at home. Define the real domestic bond return from t to $t + 1$ as $1 + r_t \equiv (1 + \iota_t) \frac{P_t}{P_{t+1}}$, the real foreign bond return as $1 + r_t^* \equiv (1 + \iota_t^*) \frac{P_t^*}{P_{t+1}^*}$, the real stock price as $p_t \equiv \frac{P_t}{P_t}$, and the real dividend as $d_t \equiv \frac{D_t}{P_t}$. Then we can rewrite (5) as

$$1 + r_t = (1 + r_t^*) \frac{Q_{t+1}}{Q_t} = \frac{p_{t+1} + d_{t+1}}{p_t} \quad (7)$$

The first equations in (5) and (7) are the nominal and real uncovered interest parity (UIP) conditions, respectively. The second equations are the asset pricing equations for the stock.

Using the fact that all assets have common real return r_t , we can simplify the household problem (1). Denote by a'_t the real value of end-of-period assets, and by a_t^p the real value of beginning-of-period assets including returns. These are defined respectively as

$$a'_t \equiv \frac{\mathcal{P}_t s' + B^H' + \mathcal{E}_t B^F'}{P_t}, \quad a_t^p \equiv \frac{(\mathcal{P}_t + D_t) s + (1 + \iota_{t-1}) B^H + (1 + \iota_{t-1}^*) \mathcal{E}_t B^F}{P_t} \quad (8)$$

Using these definitions, we can rewrite (1) in the simple canonical form

$$\begin{aligned} V_t(a^p, e) &= \max_{c, a'} u(c) - v(N_t) + \beta \mathbb{E}_t [V_{t+1}((1 + r_t) a', e') | e] \\ \text{s.t.} \quad c + a' &= a^p + e \frac{W_t}{P_t} N_t \\ a' &\geq 0 \end{aligned} \quad (9)$$

¹⁴Since only the total value of assets enters the constraints in (1), agents could achieve unboundedly high returns if (5) did not hold by taking a levered position in the higher-return asset.

Denote the consumption and asset policy functions that solve (9) by $c_t(a^p, e)$ and $a'_t(a^p, e)$. Let $c_F(a^p, e)$ and $c_H(a^p, e)$ be equal to:

$$c_{Ft}(a^p, e) = \alpha \left(\frac{P_{Ft}}{P_t} \right)^{-\eta} c_t(a^p, e) \quad (10)$$

$$c_{Ht}(a^p, e) = (1 - \alpha) \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} c_t(a^p, e) \quad (11)$$

Then, the policy functions $\tilde{c}_{Ht}(s, B^H, B^F, e)$, $\tilde{c}_{Ft}(s, B^H, B^F, e)$ that solve the original problem in (1) are respectively given by $c_{Ht}(a_t^p, e)$ and $c_{Ft}(a_t^p, e)$, where a_t^p is given in (8); and the policy functions $\tilde{s}'_t(s, B^H, B^F, e)$, $\tilde{B}_t^{F'}(s, B^H, B^F, e)$ and $\tilde{B}_t^{H'}(s, B^H, B^F, e)$ can take any value that respects the constraint $P_t \tilde{s}'_t + \tilde{B}_t^{H'} + \tilde{B}_t^{F'} = P_t a'_t(a_t^p, e)$. Because of perfect foresight, the consumption-savings choice is well defined, but the portfolio choice is indeterminate.

Let $\mathcal{D}_t(a^p, e)$ denote the measure giving the joint distribution over beginning-of-period assets and productivity. We define aggregate consumption as $C_t \equiv \int c_t(a^p, e) d\mathcal{D}_t(a^p, e)$ and aggregate assets as $A_t \equiv \int a_t(a^p, e) d\mathcal{D}_t(a^p, e)$.

At date 0, there is an exogenous initial distribution $d\tilde{\mathcal{D}}_0(s, B^H, B^F, e)$ over stocks, domestic bonds, foreign bonds, and productivity. Given initial prices $\mathcal{P}_0, P_0, \mathcal{E}_0$, dividends D_0 , and the expression for a_0^p in (8), there is an induced distribution $d\mathcal{D}_0(a^p, e)$ over beginning-of-period assets at date 0. Then, for $t \geq 1$, the optimal policy from (9) induces a law of motion from \mathcal{D}_{t-1} to \mathcal{D}_t ¹⁵ which recursively determines the distribution \mathcal{D}_t at all dates. This fully characterizes the solution to the household problem for all periods $t \geq 0$.

Rest-of-the-world households. Households in the rest of the world face the same problem as domestic households, but for simplicity we assume that there is a single type of agent in the rest of the world facing no idiosyncratic risk or constraints on assets. That is, each country in the rest of the world has a representative agent with discount factor $(\beta^*)^t \mathcal{B}_t$, where \mathcal{B}_t is an exogenous shifter of intertemporal preferences. As in [Galí and Monacelli \(2005\)](#), the basket of foreign goods in each country is itself a CES aggregator of goods from all countries, with an elasticity of substitution of $\gamma > 0$ (see appendix A.1 for details). Let P_{Ht}^* denote the price in the rest of the world for goods produced in the home country, and C_t^* denote aggregate consumption in the rest of the world. Then, export

¹⁵Namely, $\mathcal{D}_t(a^{p'}, e') = \sum_e \mathcal{D}_{t-1} \left(a_{t-1}^{-1} \left(\frac{A^{p'}}{1+r_{t-1}}, e \right), e' \right) \Pi(e, e')$, where Π is the Markov chain for productivity. Note that the finer distribution $\tilde{\mathcal{D}}_t$ over stocks and bonds is indeterminate since agents are indifferent between assets giving the same return.

demand for home goods is given by

$$C_{Ht}^* = \alpha \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-\gamma} C_t^* \quad (12)$$

Production and price-setting for home goods. Home goods are produced from domestic labor with constant returns and a constant productivity level of 1,

$$Y_t = N_t \quad (13)$$

where N_t is aggregate labor supplied. There is a continuum of monopolistically competitive firms producing home goods with technology (13). Let ϵ denote the elasticity of substitution between varieties produced within a country. For now, we assume that prices are fully flexible, so that the price of home goods is set at a constant markup μ over nominal marginal costs,

$$P_{Ht} = \mu W_t \quad (14)$$

where $\mu = \epsilon / (\epsilon - 1)$. Firms pay real dividends equal to to:

$$d_t = \frac{P_{Ht} Y_t - W_t N_t}{P_t} + \frac{\mathcal{E}_t P_{Ht}^* - P_{Ht}}{P_t} C_{Ht}^* \quad (15)$$

The first term in (15) is domestic profits; the second term captures profits from exporters' unhedged currency exposure, if any. Firms have a unit mass of shares outstanding, with end-of-period real price p_t . As is usual, their objective is to maximize real firm value $d_t + p_t$.

Rest-of-the-world monetary policy and price-setting for imports and exports. Monetary policy in the rest of the world keeps the price of foreign goods in foreign currency constant, $P_{Ft}^* = P_t^* = 1$. Since technology is constant, this implies a constant level of consumption in the rest of the world $C_t = C^*$, and requires interest rates to satisfy

$$1 + \iota_t^* = 1 + r_t^* = \frac{1}{\beta^*} \frac{\mathcal{B}_t}{\mathcal{B}_{t+1}} \quad (16)$$

Hence, foreign nominal and real interest rates are equal, and move to offset shocks to intertemporal preferences. An exogenous increase in \mathcal{B}_t relative to \mathcal{B}_{t+1} raises ι_t^* and r_t^* , and transmits to the domestic economy via the UIP conditions (5)–(7). Because, given a sequence of ι_t^* converging to $\frac{1}{\beta^*} - 1$ sufficiently fast, we can form $\mathcal{B}_t = \prod_{s \geq t} (\beta^* (1 + \iota_s^*))$ to satisfy (16), we can alternatively take the ι_t^* sequence as an exogenous primitive instead.

For convenience, we refer to either the ι^* or the equivalent \mathcal{B} sequence as “ ι^* -shocks”, and use both interchangeably in what follows.

For now, we assume that imports are denominated in foreign currency and that there is perfect pass-through of exchange rates into domestic goods prices: the law of one price holds at the good level, so that $P_{Ft} = \mathcal{E}_t$, where \mathcal{E}_t is the nominal exchange rate. We assume that the law of one price holds for foreign goods as well, so that P_{Ht}^* is equal to the cost P_{Ht}/\mathcal{E}_t of a domestic good in foreign currency units:

$$P_{Ht}^* = \frac{P_{Ht}}{\mathcal{E}_t} \quad (17)$$

Our economy therefore features nominal rigidity only in domestic currency, with the law of one price holding at level of each good. This formulation makes our model similar to the *producer currency pricing* (PCP) in the celebrated Mundell-Fleming model, adopted by [Galí and Monacelli \(2005\)](#) and many others, in which exchange rates fully pass through to foreign-currency prices of exported goods. Later, we will also consider *dollar currency pricing* (DCP), where the foreign currency price of home goods P_{Ht}^* is sticky in foreign currency.

Sticky wages and unions. We assume a standard formulation for sticky wages with heterogeneous households, similar to [Auclert, Rognlie and Straub \(2024a\)](#).¹⁶ A union employs all households for an equal number of hours N_t , and sets the nominal wage rate W_t à la Calvo. We choose the union’s objective function so that it leads to the wage Phillips curve¹⁷

$$\pi_{wt} = \kappa_w \left(\frac{v'(N_t)}{\frac{1}{\mu_w} \frac{W_t}{P_t} u'(C_t)} - 1 \right) + \beta \pi_{wt+1} \quad (18)$$

where π_{wt} denotes nominal wage inflation, $\pi_{wt} \equiv W_t/W_{t-1} - 1$. Wage inflation rises when marginal disutility of average work $v'(N_t)$ is higher than the product of the marked-down real wage by the marginal utility of average consumption $\frac{1}{\mu_w} \frac{W_t}{P_t} u'(C_t)$, now or in the future. The Phillips curve parameter is $\kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$, where θ_w is the Calvo probability of keeping the wage fixed every period.

¹⁶As explained in [Auclert, Bardóczy and Rognlie \(2023\)](#) and [Broer, Hansen, Krusell and Öberg \(2020\)](#), the assumption of sticky wages and flexible prices is better suited to heterogeneous-agent models than the opposite assumption of sticky prices and flexible wages.

¹⁷In [Auclert, Rognlie and Straub \(2024a\)](#)’s formulation of the union problem, the consumption level that enters the Phillips curve in (18) is the aggregator $\bar{C}_t \equiv (u')^{-1}(\mathbb{E}[e_{it}u'(c_{it})])$ that takes into account inequality in labor earnings. Here we opt for the formulation in (18) for simplicity. All our analytical results hold if we use \bar{C}_t instead of C_t , with the exception of the neutrality result with a Taylor rule from appendix C.5, and the quantitative results from section 5 are nearly identical.

Domestic fiscal and monetary policy. The government does not spend, tax or use transfers, and domestic bonds are in zero net supply. The monetary authority sets the nominal interest rate according to a monetary rule. It is standard in the open-economy literature to consider a few of these rules. For the analytical results that we develop in the next two sections, we consider a specification in which monetary policy holds the real interest rate constant:

$$\iota_t = r_{ss} + \pi_{t+1} + \epsilon_t \quad (19)$$

This is a CPI-based Taylor rule with a coefficient of 1 on expected inflation. This monetary rule achieves a middle ground between standard CPI-based Taylor rules with responsiveness larger than 1, and zero-lower-bound specifications with a fixed nominal interest rate, and is widely used in the literature as a device to partial out the effects of monetary policy in the study of the effects of shocks to aggregate demand (e.g. [Woodford 2011](#), [McKay, Nakamura and Steinsson 2016](#), [Auclert, Rognlie and Straub 2024a](#)). In section 5, we consider, as an alternative, a standard Taylor rule with inertia, as in [Clarida, Galí and Gertler \(2000\)](#) and [Itskhoki and Mukhin \(2021\)](#),

$$\iota_t = \rho_m \iota_{t-1} + (1 - \rho_m)(r_{ss} + \phi \pi_{t+1}) + \epsilon_t \quad (20)$$

with $\phi > 1$, which, as we show below, yields similar results to (19). We also show in appendix C.5 that the results are robust to a Taylor rule based on producer prices, $\iota_t = r_{ss} + \phi \pi_{Ht} + \epsilon_t$ with $\pi_{Ht} = \frac{P_{Ht}}{P_{Ht-1}} - 1$ and $\phi > 1$, as in [Galí and Monacelli \(2005\)](#).

Equilibrium. Given sequences of rest-of-the-world discount shocks $\{\mathcal{B}_t\}$ (or alternatively foreign interest rates $\{\iota_t^*\}$) and monetary shocks $\{\epsilon_t\}$, and an initial wealth distribution $d\tilde{\mathcal{D}}_{i0}(s, B^H, B^F, e)$, an equilibrium is a path of policies $\{c_{Hit}(a^p, e), c_{Fit}(a^p, e), c_{it}(a^p, e), a_{it+1}(a^p, e)\}$ and distributions $\mathcal{D}_{it}(a^p, e)$ for households, prices $\{\mathcal{E}_t, Q_t, P_t, P_{Ht}, P_{Ft}, W_t, r_t, \iota_t\}$ and aggregate quantities $\{C_t, C_{Ht}, C_{Ft}, Y_t, A_t, p_t, d_t, \text{nfat}\}$, such that households optimize, distributions evolve consistently with $d\tilde{\mathcal{D}}_{i0}$ and household policies, firms optimize, and the domestic goods market clears:

$$C_{Ht} + C_{Ht}^* = Y_t \quad (21)$$

We focus on equilibria in which the long-run exchange rate returns to its initial steady state level, $Q_\infty = Q_{ss}$.¹⁸

¹⁸In appendix A.3, we show that there is always a unique such steady state, and that this is also the unique no-inflation steady state in stationary models (i.e. in the HA-IM and all the CM models.) For instance, in these models, the Taylor rule $\iota_t = r_{ss} + \phi \pi_{t+1} + \epsilon_t$ as $\phi \rightarrow 1$ selects this equilibrium while also enforcing

We define the net foreign asset position as the difference between the real value of assets accumulated domestically, A_t , and the total value of assets in net supply domestically, p_t , i.e.

$$\text{nfa}_t \equiv A_t - p_t \quad (22)$$

Appendix A.2 shows that, in equilibrium, the current account identity holds:

$$\text{nfa}_t - \text{nfa}_{t-1} = \frac{P_{Ht}}{P_t} Y_t - C_t + r_{t-1} \text{nfa}_{t-1} \quad (23)$$

for $t \geq 1$, where $\frac{P_{Ht}}{P_t} Y_t - C_t \equiv NX_t$ is the value of net exports (or, equivalently, the trade balance) in units of the consumption basket.

Steady state. We consider first-order shocks to the economy around a steady state with no inflation and a zero net foreign asset position (see appendix A.3 for a characterization of the steady states of this economy.) Without loss of generality, we normalize prices to 1 in this steady state, implying that $P_{Hss}, P_{Fss}, P_{ss}, P_{Hss}^*, \mathcal{E}_{ss}, Q_{ss}$ are all equal to 1. Moreover, we normalize domestic steady-state output Y_{ss} to 1. Hence, C_{ss} and C^* also equal 1.

Initial portfolios. We resolve the indeterminacy of steady-state portfolios by assuming, for now, that all agents hold all of their assets in domestic stocks, ie $a' = \mathcal{P}s'$.¹⁹ Note that these portfolios do not provide cross-country insurance, which is why we label this case an *incomplete-market equilibrium*. In the representative-agent case, this coincides with the standard notion of incomplete market equilibrium as defined in, e.g., [Mendoza \(1991\)](#), [Baxter and Crucini \(1995\)](#), and [Corsetti et al. \(2008\)](#).

Complete markets. So far, we have described the incomplete markets model, where shocks are fully unanticipated, portfolio choice is undetermined in the steady-state, and initial portfolios are exogenously given. We also consider a case where households can choose their portfolios optimally to hedge against aggregate shocks. In an environment, like ours, with a single type of shock, e.g. an exchange rate shock, this allows them to implement an allocation that has complete markets with respect to aggregate risk, but still incomplete markets with respect to idiosyncratic shocks. We do this, following [Auclert, Rognlie, Straub and Tapák \(2024b\)](#), by assuming that first-order shocks are anticipated

$r_t = r_{ss}$ at all t . More broadly, we can think of the $Q_\infty = Q_{ss}$ restriction as one enforced by a long-run monetary policy rule.

¹⁹We consider alternative initial exogenous portfolios in section 5.6.

to occur with a mean-0 distribution.²⁰ As appendix A.4 shows, this assumption pins down initial portfolios but otherwise maintains the same equations as the incomplete-markets case. The endogenous portfolios that hedge agents against aggregate shocks are determined by an agent-specific version of the classic Backus-Smith condition,

$$\frac{Q_0}{B_0} \mathbb{E} \left[c_0 (a_0^p, e')^{-\sigma} | e \right] = \mathbb{E} \left[c_{ss} (a_{ss}^p, e')^{-\sigma} | e \right] \quad (24)$$

which has to hold to first order at date 0.

2.2 Nested models

We consider three kinds of assumptions on the types of households inhabiting the economy and the parameters of the problem that they solve (“RA, TA, HA”). For each, we allow for either incomplete markets vis-à-vis aggregate risk (“IM”) or complete markets (“CM”).

Representative agent (RA). A special case of (9) drops the borrowing constraint and has no idiosyncratic income risk, $e_{it} = 1$. This model admits a single representative agent whose consumption is equal to its permanent income. As appendix A.6 shows, the problem from date-0 onward must satisfy the Euler equation

$$C_t^{-\sigma} = \beta (1 + r_t) C_{t+1}^{-\sigma} \quad (25)$$

for $t \geq 0$ as well as the budget constraint

$$C_t + A_t = (1 + r_{t-1}) A_{t-1} + \frac{W_t}{P_t} N_t \quad (26)$$

for $t \geq 1$, and a no-Ponzi condition $\lim_{t \rightarrow \infty} \prod_{s \leq t} (1 + r_s)^{-1} A_t = 0$. With incomplete markets (“RA-IM”), initial consumption C_0 is determined by the requirement that the initial value of assets is $p_0 + d_0$, consistent the country holding a 100% stock portfolio. With complete markets (“RA-CM”) instead, C_0 is determined by the Backus-Smith condition,

$$\frac{Q_t}{B_t} C_t^{-\sigma} = C_{ss}^{-\sigma} \quad (27)$$

which holds at all $t \geq 0$. There is an implied initial value of assets inclusive of insurance payments to or from the rest of the world, and an implied set of initial portfolios

²⁰See Bhandari, Bourany, Evans and Golosov (2023) for a related, state-space approach.

sustaining these payments.²¹

Two agents (TA). We also introduce two-agent models with incomplete markets (“TA-IM”) and complete markets (“TA-CM”). In these models, the respective representative-agent model is modified by introducing a fixed fraction $\lambda \cdot \mu$ of “hand-to-mouth” agents who are constrained to have zero total assets, $\mathcal{P}_t s_t^c + B_t^{H,c} + B_t^{F,c} = 0$. As appendix A.7 shows, hand-to-mouth agents derive their income from wages so they earn, at the margin, a fraction λ of aggregate income; in the CM case they can nevertheless form their portfolios to hedge aggregate shocks at date 0.²² We calibrate TA-IM and TA-CM by adjusting λ to match an income-weighted MPC of 0.10, as in our HA model.

Heterogeneous agents (HA). Our heterogeneous-agent model with incomplete markets (“HA-IM”) is as introduced above, with equal initial 100% stock portfolios across households, implying in particular no cross-country gross positions. With complete markets (“HA-CM”), agents are allowed to hedge against the aggregate risk realized at date 0, leading to the agent-specific Backus-Smith condition (24) and endogenous initial portfolios. We calibrate this model to match data on income dynamics and MPCs in Mexico; in particular, we target a cross-sectional average MPC of 0.20, implying an income-weighted MPC of 0.10. We defer a more detailed discussion of the calibration to section 5.2.

2.3 Intertemporal MPCs

In the spirit of [Ayclert et al. \(2024a\)](#), we now establish the existence of an intertemporal consumption function and discuss properties of its derivatives, the intertemporal marginal propensities to consume, in the six nested models we introduced. These iMPCs will turn out to be critical objects for the equilibrium analysis of the next sections.

We begin by observing that, combining the price-setting condition (14) with the production function (13), real wage income is always equal to

$$\frac{W_t}{P_t} N_t = \frac{1}{\mu} \frac{P_{Ht}}{P_t} Y_t \quad (28)$$

²¹Appendix A.6 shows that the RA-CM model admits the exact same log-linear equations as the [Galí and Monacelli \(2005\)](#) model, extended to allow for foreign discount factor shocks. In particular, since we are not considering productivity shocks, our assumption that wages rather than prices are sticky is innocuous.

²²An alternative formulation of the TA-CM model would restrict hand-to-mouth agents to hold zero gross position in all assets. We impose the constraint only on net worth to make it parallel to our assumption in the HA model.

i.e. a fixed fraction $\frac{1}{\mu}$ of aggregate real income $\frac{P_{Ht}}{P_t} Y_t$. Moreover, combining (28) with (15), and using the law of one price (17), we see that real dividends are equal to

$$d_t = \left(1 - \frac{1}{\mu}\right) \frac{P_{Ht}}{P_t} Y_t \quad (29)$$

i.e. also a fixed fraction of aggregate real income.

Next, we observe that, combining the relation between the preference shock \mathcal{B}_t and foreign real interest rates in (16) with the real UIP condition (7), we have $1 + r_t^* = \frac{1}{\beta^*} \frac{\mathcal{B}_t}{\mathcal{B}_{t+1}} = (1 + r_t) \frac{Q_t}{Q_{t+1}}$. Iterating on this equation with $Q_\infty = Q_{ss} = 1$, we therefore find that

$$Q_t = \mathcal{B}_t \cdot \prod_{s \geq t} \left(\frac{1 + r_{ss}}{1 + r_s} \right) \quad (30)$$

where $1 + r_{ss} = 1/\beta^*$ is the steady state real interest rate.

We can now state the following result:

Proposition 1. *For any calibration of RA, TA or HA, under either incomplete or complete markets, there exists an intertemporal consumption function \mathcal{C} such that the time path of aggregate consumption for any $t \geq 0$ is given by:*

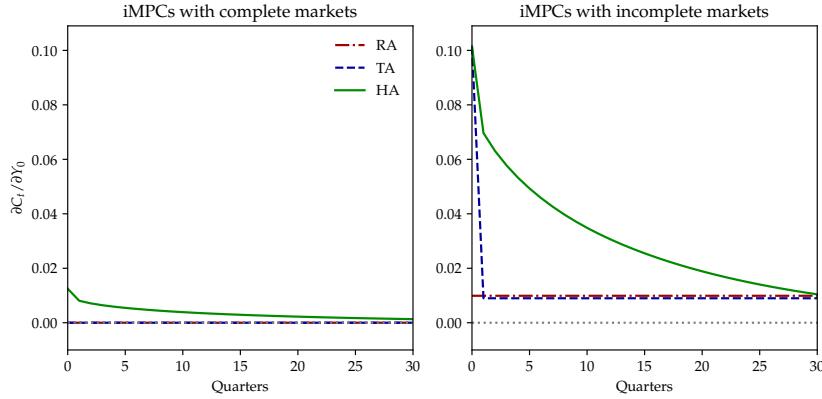
$$C_t = \mathcal{C}_t \left(\left\{ \frac{P_{Hs}}{P_s} Y_s \right\}_{s=0}^\infty, \{r_s\}_{s=0}^\infty \right) \quad (31)$$

We write \mathbf{M} for the Fréchet derivative of \mathcal{C} with respect to $\frac{P_H}{P} Y$ around the steady state, and similarly \mathbf{M}^r for $1 + r$ times the Fréchet derivative of \mathcal{C} with respect to r .

The proof is in appendix A.5. The argument is as follows: note first that the only way in which time-varying aggregate variables enter the household Bellman equation in (9) is through aggregate real wage income $\frac{W_t}{P_t} N_t$ and the ex-ante real interest rate r_t . But by (28) real wage income is a fixed fraction of aggregate real income. Hence, the household consumption and asset policy functions only depend on the paths of $\frac{P_{Ht}}{P_t} Y_t$ and r_t .

The realized consumption path of a given household, however, also depends on that household's initial asset position a_0^p . With incomplete markets, the household's portfolio coming into date 0 consists entirely of domestic equities. The date-0 value of these equities is the present discounted value of real dividends, which by (29) is also a fixed fraction of real income. With complete markets, the initial asset position a_0^p is pinned down by equation (24) with Q_0/\mathcal{B}_0 determined by equation (30). Hence, a_0^p depends on the path of real income via the policy function $c_0(\cdot, \cdot)$. In either case, we see that the entire distribution of a_0^p is, like the policy functions, only a function of the path of aggregate real

Figure 1: Intertemporal MPCs (first column of \mathbf{M}) in our six calibrated models



Note: This figure shows the first column of the \mathbf{M} matrix in our six calibrated models. The left panel shows RA-CM, TA-CM and HA-CM, the right panel RA-IM, TA-IM and HA-IM. TA and HA are calibrated so that the income-weighted partial equilibrium impact MPC is 0.10; under CM the impact response M_{00} is smaller due to endogenous insurance transfers. Appendix figure A.1 displays the other columns of \mathbf{M} .

income $\frac{P_{Ht}}{P_t} Y_t$ and real interest rates r_t . Hence, the path of aggregate consumption also only depends on these two sequences. We denote the implied ‘‘consumption function’’ by \mathcal{C}_t .²³

Let \mathbf{M} be the Frechet derivative of \mathcal{C} with respect to $\frac{P_H}{P} Y$ around the steady state. As in [Auclert, Rognlie and Straub \(2024a\)](#), this sequence-space Jacobian can be represented as a matrix with elements $M_{t,s} \equiv \frac{\partial \mathcal{C}_t}{\partial \left(\frac{P_{Hs}}{P_s} Y_s \right)}$, which characterize the date- t aggregate consumption response to a date- s change in aggregate real income. Following that paper, we refer to the elements of \mathbf{M} as ‘‘intertemporal MPCs’’. Appendices A.6–A.8 provide expressions for \mathbf{M} in our six calibrated models.

The first column of \mathbf{M} captures the response of aggregate consumption to a one-time marginal increase in real income at date 0. Figure 1 visualizes this response in our six models. In the RA-CM model this response is zero, since any real income gain is shared with the rest of the world (and in fact $\mathbf{M} = 0$).²⁴ In the TA-CM model this response is also zero, since even hand-to-mouth agents can insure this income shock. (They cannot, however, insure future income shocks, so $\mathbf{M} \neq 0$.) In the HA-CM model the insurance against the aggregate income shock is imperfect, but the iMPCs are still extremely small.

Under incomplete markets, agents can no longer insure aggregate income shocks with the rest of the world. Nevertheless, the RA-IM model still has low MPCs, as households

²³A similar logic underlies the consumption functions used in [Kaplan, Moll and Violante \(2018\)](#), [Farhi and Werning \(2019\)](#), and [Auclert, Rognlie and Straub \(2024a\)](#).

²⁴Note that the iMPCs are objects that incorporate features of general equilibrium, such as international consumption insurance. In IM models, $M_{0,0}$ is an income-weighted MPC. See also [Auclert et al. \(2024a\)](#).

can perfectly smooth out their consumption response. The TA-IM model generates a large date-0 response, but has no persistence. HA-IM is unique in generating both a large date-0 response as well as persistence, as households that are subject to idiosyncratic risk slowly spend down any transfer to get back to their buffer stocks. This is the crucial property of the HA-IM model that will be relevant in our analysis below.

Since consumption is a normal good in our models, it is natural to expect the iMPG matrix \mathbf{M} to be non-negative. We prove this in the tractable RA and TA models, and show this numerically for all reasonable parameterizations of the HA models (though we currently do not have a proof for a general HA model). We will use this property for some of our results below, so we state it as an assumption here.

Assumption 1. $\mathbf{M} \geq 0$.²⁵

Proposition 1 also defines \mathbf{M}^r as $1 + r$ times the Fréchet derivative of \mathcal{C} with respect to r . We defer a discussion of this object to section 4.

3 Exchange rate shocks

We start by considering preference shocks \mathcal{B}_t to foreign households—or equivalently, ι_t^* -shocks. Given the constant real rate rule (19), we have $r_t = r_{ss}$ for all t , so equation (30) implies that the real exchange rate is given by

$$Q_t = \mathcal{B}_t = \prod_{s \geq t} \left(\frac{1 + \iota_s^*}{1 + r_{ss}} \right) \quad (32)$$

Intuitively, when foreign households become more impatient (rising \mathcal{B}_t), they push up foreign interest rates ι_t^* , leading to capital outflows that depreciate the exchange rate (rising Q_t). Given (32), the real exchange rate is effectively exogenous in this section.

Our analysis is centered around the home goods market clearing condition (21). Aggregating the domestic demand equation for home goods (11) across agents, and combining with the foreign demand equation (12), our assumption on foreign monetary policy, which ensures $P_t^* = 1$ and $C_t^* = C^*$, and the law of one price (17), we can write this condition as

$$Y_t = (1 - \alpha) \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{P_{Ht}}{\mathcal{E}_t} \right)^{-\gamma} C^* \quad (33)$$

²⁵For vectors and matrices, we use the notation \geq to denote greater than equal element-by-element, the notation $>$ to add that at least one element is strictly positive, and \gg to denote strictly greater than, element-by-element.

The relative prices in equation (33) are tied to the real exchange rate Q_t . A depreciation lowers the price of home goods relative to the domestic CPI, P_{Ht}/P_t , and relative to the foreign CPI, P_{Ht}/\mathcal{E}_t . This leads domestic and foreign consumers to substitute towards home goods. In addition to these traditional expenditure switching effects, the volume of domestic spending C_t may change. In this section, we characterize how intertemporal MPCs affects this response.

We illustrate our results using our six calibrated models, subjected to an AR(1) shock to the foreign discount factor with a quarterly persistence of $\rho = 0.85$ (or equivalently, an AR(1) shock to ι_t^* with the same persistence), normalized to have an impact effect of 1 on the real exchange rate dQ_0 . The shock is shown in the left panel of figure 2. We follow [Galí and Monacelli \(2005\)](#) in setting the openness parameter to $\alpha = 0.40$, and leave the elasticities η, γ unspecified for now.

3.1 RA-CM benchmark

We first consider the representative-agent complete-market model ("RA-CM model"). Combining the exchange rate determination equation (32) with the Backus Smith condition (27), we immediately find that consumption does not respond to the shock, $C_t = C_{ss}$.²⁶ Equation (33) then implies that domestic production is only affected by expenditure switching.

Proposition 2. *In the representative-agent complete-market model with real interest rate rule (19), the linearized deviations of consumption over output $dC_t = (C_t - C_{ss}) / Y_{ss}$ and output $dY_t = (Y_t - Y_{ss}) / Y_{ss}$ in response to shocks to the real exchange rate $dQ_t = (Q_t - Q_{ss}) / Q_{ss}$ are given by*

$$dY_t^{RA-CM} = \frac{\alpha}{1-\alpha} \chi dQ_t \quad \forall t \quad (34)$$

$$dC_t^{RA-CM} = 0 \quad \forall t \quad (35)$$

where χ is the trade elasticity, defined as

$$\chi \equiv \eta (1 - \alpha) + \gamma \quad (36)$$

Proposition 2, proved in appendix B.1, captures a common view in the literature: depreciations are expansionary due to expenditure switching, with greater trade elasticity

²⁶Consumption comoves negatively with real exchange rates in response to other shocks that do not change foreign preferences ($\mathcal{B}_t = 1$), since the Backus-Smith condition implies that $Q_t C_t^{-\sigma}$ is constant in response to these shocks.

leading to more expenditure switching and therefore a greater expansion dY_t .

The *trade elasticity* χ , defined in (36), captures the combined effect of reduced imports and increased exports following a depreciation. Holding aggregate C_t and C_t^* fixed, the elasticity of imports with respect to the relative price of foreign goods P_{Ft}/P_{Ht} is $-\eta(1-\alpha)$, and the elasticity of exports is γ .²⁷ The trade elasticity is the export minus the import elasticity, and gives the response of net exports in quantity terms. Because a 1% real depreciation raises the relative price of foreign goods by $\frac{1}{1-\alpha}\%$, it therefore raises the volume of net exports by $\frac{\chi}{1-\alpha}\%$ of initial imports, so it has a $\frac{\alpha}{1-\alpha}\chi\%$ effect on real GDP. This explains equation (34). The middle panel of figure 2 plots the response of output in the RA-CM model to the exchange rate shock in the left panel, for $\chi = 1$ (black, solid) and $\chi = 0.1$ (red, dot-dashed). The scaling of this response in χ is evident in the figure.

While the *volume* of net exports increases by $\frac{\alpha}{1-\alpha}\chi dQ_t$, the *value* of net exports also reflects the rising cost of imports, so it only rises by $dNX_t = \frac{\alpha}{1-\alpha}(\chi - 1)dQ_t$. At $\chi = 1$, the volume and the price effect offset each other: this is the threshold in the well-known *Marshall-Lerner condition*. This will turn out to play a key role in the analysis that follows.

From now on, it will be convenient to express impulse responses as infinite vectors, e.g. $d\mathbf{Y} = (dY_0, dY_1, \dots)$. With this notation, (34)-(35) become $d\mathbf{C} = 0$ and $d\mathbf{Y} = \frac{\alpha}{1-\alpha}\chi d\mathbf{Q}$.

3.2 General case

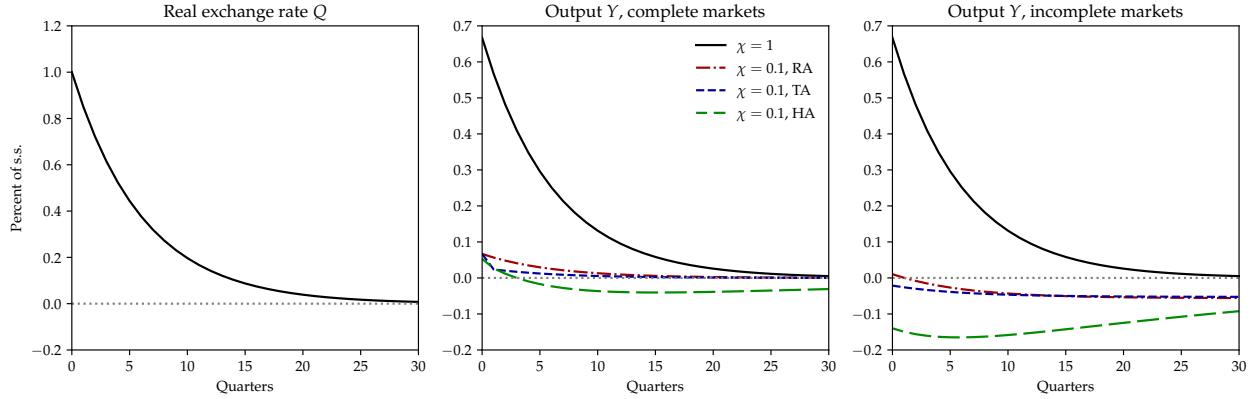
Away from the RA-CM benchmark, domestic consumption C_t does respond to depreciation, via the effect of depreciation on real income. Indeed, applying proposition 1 with the real interest rate rule $r_t = r_{ss}$, we see that C_t only depends on the time path of real income $\frac{P_{Ht}}{P_t}Y_t$. Moreover, we have the first-order relation $d\mathbf{C} = \mathbf{M}d\left(\frac{\mathbf{P}_H}{\mathbf{P}}\mathbf{Y}\right)$, with the matrix of intertemporal MPCs \mathbf{M} mapping real income changes into consumption. In turn, consumption affects income through (33). Hence, for general \mathbf{M} , the equilibrium response dY_t to an exchange rate shock dQ_t is the solution to the following fixed-point problem.

Proposition 3. *In response to a shock to the real exchange rate $d\mathbf{Q}$, the impulse response of consumption is given by*

$$d\mathbf{C} = - \underbrace{\frac{\alpha}{1-\alpha}\mathbf{M}d\mathbf{Q}}_{\text{Real income channel}} + \underbrace{\mathbf{M}d\mathbf{Y}}_{\text{Multiplier}} \quad (37)$$

²⁷Aggregating the home demand equation for foreign goods (10) across consumers, we find that aggregate imports are $C_{Ft} = \alpha \left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t$, and then $\frac{\partial \log C_{Ft}}{\partial \log P_{Ft}/P_{Ht}} = \eta \frac{\partial \log P_t/P_{Ft}}{\partial \log P_{Ft}/P_{Ht}} = -\eta(1-\alpha)$. Given (12), the elasticity of aggregate exports is $\frac{\partial \log C_{Ht}^*}{\partial \log P_{Ft}^*/P_{Ht}^*} = \gamma \frac{\partial \log P_t^*/P_{Ht}^*}{\partial \log P_{Ft}^*/P_{Ht}^*} = \gamma$, where the last equality follows from the fact that the home country is too small to affect the foreign CPI.

Figure 2: Effect of exchange rate shocks on output for various χ 's



Note: AR(1) shock to i_t^* with persistence 0.85, and corresponding impulse response of the real exchange rate Q_t in the left panel. The shock is normalized so that the real exchange rate depreciates by 1% on impact. χ is the trade elasticity (the sum of the import and export elasticity to the exchange rate). The HA model generates a contraction on impact for $\chi < \chi^* = 0.26$.

and the impulse response of output $d\mathbf{Y}$ is determined by an “international Keynesian cross”

$$d\mathbf{Y} = \underbrace{\frac{\alpha}{1-\alpha} \chi d\mathbf{Q}}_{\text{Exp. switching channel}} - \underbrace{\alpha \mathbf{M} d\mathbf{Q}}_{\text{Real income channel}} + \underbrace{(1-\alpha) \mathbf{M} d\mathbf{Y}}_{\text{Multiplier}} \quad (38)$$

Proposition 3 shows that the impulse responses of consumption and output only depend on the openness parameter α , the trade elasticity χ , and the matrix of intertemporal MPCs \mathbf{M} . Equation (37) finds that there are two ways in which real income $\frac{P_{Ht}}{P_t} Y_t$, and hence consumption $d\mathbf{C}$, are affected by an exchange rate depreciation $d\mathbf{Q}$. First, a depreciation lowers $\frac{P_{Ht}}{P_t}$ by $\frac{\alpha}{1-\alpha} dQ_t$, that is, it lowers the price of the goods that the country produces relative to the price of those that it buys. This reduces real income, leading agents to cut consumption by $\mathbf{M} \times \frac{\alpha}{1-\alpha} d\mathbf{Q}$. We refer to this as the *real income channel*. Second, a depreciation affects the path of output $d\mathbf{Y}$, which also enters real income, and changes consumption by $\mathbf{M} \times d\mathbf{Y}$. This is a standard (Keynesian) *multiplier* effect.

Linearizing goods market clearing (33) and substituting in (37), we obtain equation (38), whose form is like that of a standard Keynesian cross, where the relevant multiplier is the product of \mathbf{M} by home bias $(1-\alpha)$. Including expenditure switching, there are altogether three distinct channels that jointly determine the output response to any given shock. The next proposition, proved in appendix B.3, derives the general solution to (38).

Proposition 4. *The equilibrium consumption and output responses are unique and given by*

$$d\mathbf{C} = \frac{\alpha}{1-\alpha} (\chi - 1) \sum_{k \geq 0} (1-\alpha)^k \mathbf{M}^{k+1} d\mathbf{Q} \quad (39)$$

$$d\mathbf{Y} = \frac{\alpha}{1-\alpha} d\mathbf{Q} + \frac{\alpha}{1-\alpha} (\chi - 1) \sum_{k \geq 0} (1-\alpha)^k \mathbf{M}^k d\mathbf{Q} \quad (40)$$

Just like the ordinary Keynesian cross, the solution to our international Keynesian cross (38) involves infinitely many rounds, captured here by the powers $(1-\alpha)^k \mathbf{M}^k$. One way to understand (39)–(40) is that, holding consumption constant, the effect of the depreciation on the value of net exports is $d\mathbf{NX} = \frac{\alpha}{1-\alpha} (\chi - 1) d\mathbf{Q}$ (see section 3.1), where $\chi - 1$ is the distance to the Marshall-Lerner condition.²⁸ Through the national accounting identity $\frac{P_{Ht}}{P_t} Y_t = C_t + NX_t$, this change in net exports feeds into real income, which in turn affects consumption via \mathbf{M} , spilling back into real income, and so on, with the infinite series in (39) giving the outcome of this multiplier process.

This also explains why, when $\chi = 1$, \mathbf{M} drops out of the solutions: the consumption and output responses to the depreciation are independent of household behavior.

Proposition 5. *If $\chi = 1$, $d\mathbf{Y} = \frac{\alpha}{1-\alpha} d\mathbf{Q}$, $d\mathbf{C} = 0$, and $d\mathbf{NX} = 0$, all independent of \mathbf{M} .*

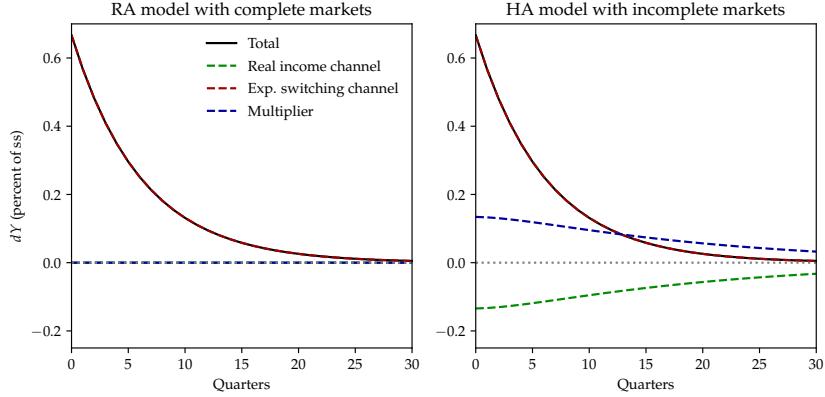
When we substitute $d\mathbf{Y} = \frac{\alpha}{1-\alpha} d\mathbf{Q}$ into (37), we get $d\mathbf{C} = 0$: for $\chi = 1$, the rise in output from expenditure switching is just large enough to offset the loss of real income from higher prices, leaving total real income and therefore consumption unchanged in every period. Since consumption is unchanged, the only effect on output is from expenditure switching, just as in the RA-CM model.

Figure 3 shows the responses to the exchange rate shock in the $\chi = 1$ case for both the RA-CM model ($\mathbf{M} = 0$) and the HA-IM model ($\mathbf{M} > 0$), displaying separately the three components of output in equation (38). The expenditure-switching channel is the same in both cases, but the HA-IM model has a negative real income effect that is exactly offset by a positive multiplier effect, while in the RA-CM model these two effects are zero.

This result suggests that the real income channel will play an economically more meaningful role when $\chi \neq 1$, a case we turn to next.

²⁸Observe that $\chi = 1$ does not correspond to the well-known Cole and Obstfeld (1991) parametrization, which, using (36), is given by $\chi = 2 - \alpha$. The Cole and Obstfeld (1991) parametrization turns out to be more relevant for the analysis of monetary policy (see section 4).

Figure 3: Exchange rate shock when $\chi = 1$ and its transmission channels



Note: impulse response in RA-CM and HA-IM to the shock to i_t^* from figure 2, with decomposition from proposition 3.

3.3 Complementarity between iMPCs and the trade elasticity

When $\chi \neq 1$, proposition 4 suggests that the aggregate response to depreciation is affected by \mathbf{M} . Specifically, there is a complementarity between having trade elasticities $\chi \neq 1$ and larger intertemporal MPCs \mathbf{M} . We formalize this result in the following proposition.

Proposition 6. *Consider two models with $\mathbf{M}, \widetilde{\mathbf{M}}$ such that $\mathbf{M} > \widetilde{\mathbf{M}}$, and a shock $d\mathbf{Q} \gg 0$. Then,*

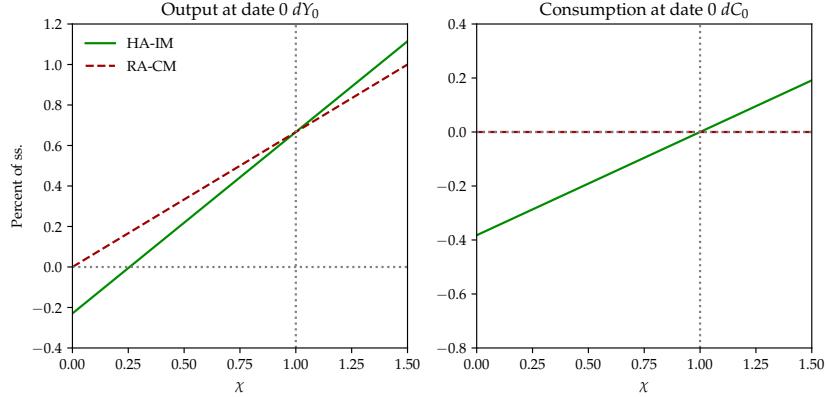
$$\chi < 1 \quad \Leftrightarrow \quad d\mathbf{Y} < d\widetilde{\mathbf{Y}} \quad \text{and} \quad d\mathbf{C} < d\widetilde{\mathbf{C}}$$

Higher intertemporal MPCs \mathbf{M} reduce the output response $d\mathbf{Y}$ if and only if $\chi < 1$, since they amplify the negative effect of the real income channel on consumption (symmetrically, when $\chi > 1$, higher iMPCs increase the output response.) Since both high \mathbf{M} and—as we argue in section 5—low short-run trade elasticity $\chi < 1$ are realistic assumptions for exchange rate shocks, this suggests that the RA-CM model is overstating the output and consumption response to a depreciation.

Figure 4 visualizes the complementarity described in proposition 6. It shows, for the RA-CM model ($\mathbf{M} = 0$) and the HA-IM model ($\mathbf{M} > 0$), the impact responses of output dY_0 and consumption dC_0 to the shock considered in figure 2 at various values of χ . A larger \mathbf{M} pivots the lines in these figures counter-clockwise around $\chi = 1$. As a result, in a model with $\mathbf{M} > 0$, we get an impact decline in consumption when $\chi < 1$, and a decline in output when χ is sufficiently below 1.

Which models have an \mathbf{M} large enough to meaningfully affect the output and consumption response to depreciation? Among the models that we consider, HA-IM stands out. Indeed, figure 2 shows that, at $\chi = 0.1$, it is the only model that generates a sizable decline in output on impact. Table 1 shows that this stems from its unique ability to gen-

Figure 4: Complementarity between expenditure switching elasticity χ and high MPCs



Note: changes on impact in output and consumption following the shock to i_t^* from figure 2. The HA-IM model generates a contraction for output on impact for $\chi < \chi^* = 0.26$ and a contraction for consumption for $\chi < 1$. The RA-CM model never generates a contraction.

	dC_0	RA	TA	HA
Complete markets	0.00	0.00	- 0.02	
Incomplete markets	-0.09	- 0.15	- 0.34	

Note: changes on impact of consumption to the shock to i_t^* displayed in Figure 2 with $\chi = 0.1$.

Table 1: Complementarity between market incompleteness and heterogeneity

erate a large decline in consumption on impact. In CM models, the effect of depreciation on aggregate consumption are insured.²⁹ In RA-IM, the impact MPC is small. In TA-IM, the impact MPC is large, but the iMPCs are small—there is very limited persistence in spending after an income shock (figure 1). We conclude that it is the *combination* of aggregate market incompleteness and heterogeneity that "sizes up" the real income channel, generating a meaningful contractionary effect of depreciation at low trade elasticities.

Extensions: dollar currency pricing, nontradable goods, imported intermediates. Appendix E.1 describes how our results change if we assume dollar currency pricing instead of producer currency pricing. Appendices E.2 and E.3 show that our results also apply in a model with nontradable goods or with imported intermediate goods. These models are isomorphic to our baseline model, under a reinterpretation of parameters.

²⁹In particular, a CM model always has a smaller consumption decline than an IM model at $\chi < 1$. This is because $\mathbf{M}^{CM} \leq \mathbf{M}^{IM}$, as we prove in appendix B.6.

3.4 Contractionary depreciations

We now summarize our results on contractionary depreciations due to the real income channel and contrast them to existing literature. First, when χ is below 1, depreciations lower real income and consumption.

Proposition 7. *If $\chi < 1$, the consumption and real income response to a depreciation shock $d\mathbf{Q} \geq 0$ is negative: $d\mathbf{C} \leq 0$ and $d\frac{\mathbf{P}_H \mathbf{Y}}{\mathbf{P}} \leq 0$*

This result shows that low trade elasticities and high intertemporal MPCs can help explain the empirical Backus-Smith correlation, complementing the recent findings of [It-skhoki and Mukhin \(2021\)](#).

Second, while expenditure switching always boosts output relative to consumption, there is a threshold elasticity χ^* such that output also contracts.

Proposition 8. *Given a depreciation shock $d\mathbf{Q} \geq 0$, there is a threshold χ^* between $(1 - \alpha)M_{0,0}$ and 1 such that for any $\chi < \chi^*$, the output response is negative on impact, $dY_0 \leq 0$. Moreover, if $\chi < 1 - \alpha$, in any incomplete market model, the present value of the output response is negative, $\sum_{t=0}^{\infty} (1 + r)^{-t} dY_t \leq 0$.*

For low enough χ , the real income channel overwhelms the expenditure switching channel in (38) on impact of the shock. Consider, for instance, a one-time shock to dQ_0 : here, the real income effect at $t = 0$ is $-\alpha M_{0,0} dQ_0$, compared to expenditure switching of $\frac{\alpha}{1-\alpha} \chi dQ_0$. When $\chi < (1 - \alpha)M_{0,0}$, the former dominates. This dominant real income channel is only reinforced by the multiplier in (38), since with high iMPCs the real income effect on consumption persists after the shock has passed, and this persistence feeds back to date 0 via the multiplier. Overall, for any depreciation, there is a threshold χ^* at which it becomes contractionary on impact: χ^* is at least $(1 - \alpha)M_{0,0}$, and usually greater due to multiplier effects.³⁰ Further, in an incomplete markets model, $\chi < 1 - \alpha$ is enough to generate an output response with negative present value.

Since this result is driven by the real income channel, it is different from, and complementary to, the commonly studied balance sheet channel with currency mismatch (e.g. [Aghion, Bacchetta and Banerjee 2004](#), [Céspedes, Chang and Velasco 2004](#)). It can potentially explain the continued relevance of fear of floating ([Ilzetzki, Reinhart and Rogoff 2019](#)) and reserve hoarding ([Bianchi and Lorenzoni 2022](#)) among countries for which currency mismatch is no longer an issue (see appendix D.6).

³⁰For instance, in our calibration of HA-IM and for the shock we consider, we have $\chi^* = 0.26$.

Relation to the old Keynesian literature. So far, we have stressed the contrast between a model with realistic iMPCs and the RA-CM model, in which depreciations never lower output. However, our result also contrasts with the old Keynesian analysis of open-economy IS-LM (Polak 1947, Harberger 1950, Laursen and Metzler 1950), as summarized, for instance, in Dornbusch (1980). According to this theory, depreciations always lower output when $\chi < 1$, irrespective of MPCs.

The reason for the discrepancy is as follows. Dornbusch (1980) wrote the equation determining the equilibrium level of output as $Y = E(Y) + T(p, Y)$, where E is domestic spending, T is the trade surplus, and p is the relative price of imports in terms of domestic goods. This equation implies that $dY = \frac{T_p}{1-E_Y-T_Y} dp$, and therefore, the condition under which output falls with depreciation ($dY/dp < 0$) is the same as that under which the trade balance deteriorates ($T_p < 0$), which corresponds to being on the wrong side of the Marshall-Lerner condition ($\chi < 1$). In this theory, MPCs only play a role in amplifying any given response of the trade balance on output.

The difference with our framework can be best understood by recognizing that the national accounting identity is given by $\frac{P_{Ht}}{P_t} Y_t = C_t + NX_t$, where C_t and NX_t depend on real income $\frac{P_{Ht}}{P_t} Y_t$ rather than output directly. Hence, the result derived by Dornbusch (1980) should be interpreted as the effect of depreciation on real income, not output, consistent with our proposition 7. In fact, in the special case of the TA-CM model, we show in appendix A.7 that, in periods $t \geq 1$, real income is given by the analogue of the Dornbusch (1980) condition

$$d \left(\frac{P_{Ht}}{P_t} Y_t \right) = \frac{\alpha}{1-\alpha} \frac{\chi - 1}{1 - (1-\alpha) \lambda} dQ_t \quad (41)$$

where λ is the effective MPC. However, because the relative price P_{Ht}/P_t falls with depreciation, the equation determining the volume of output for $t \geq 1$ is in fact

$$dY_t = \frac{\alpha}{1-\alpha} \frac{\chi - (1-\alpha) \lambda}{1 - (1-\alpha) \lambda} dQ_t \quad (42)$$

Hence, the threshold determining whether output contracts is $\chi^* = (1-\alpha) \lambda < 1$, which increases with the effective MPC λ . Relative to the open-economy IS-LM logic, our micro-founded model therefore requires *lower* trade elasticities to generate contractionary output depreciations.

4 Monetary policy

We next ask how monetary transmission is affected by the real income channel under heterogeneity and incomplete markets. For this section, we assume log preferences, $\sigma = 1$, and a real interest rate rule, which allow for a clean analytical characterization.³¹ We also focus on the contrast between the RA-CM and HA-IM models, which we saw provided two polar cases for the importance of the real income channel. Appendix C.4 covers the intermediate RA-IM case.

4.1 Transmission of real interest rates

Given our monetary policy rule (19), monetary policy affects aggregate activity by directly changing the path of domestic real interest rates $\{r_t\}$. This has two distinct effects on household behavior. First, it affects the path of the real exchange rate $\{Q_t\}$ through the real UIP condition. Given no preference shock $\mathcal{B}_t = 1$, (30) now implies

$$Q_t = \prod_{s \geq t} \left(\frac{1 + r_{ss}}{1 + r_s} \right) \quad (43)$$

These changes in the real exchange rate operate through the expenditure switching and real income channels analyzed in section 3.

Second, changes in domestic interest rates also affect the economy directly, since r_t moves asset prices and returns at all dates, including, under incomplete markets, by revaluing wealth at date 0. The resulting income and substitution effects are well-studied in the closed economy literature (e.g. [Auclet 2019](#)). We refer to this set of effects as the *interest rate channel*. The interest rate response matrix \mathbf{M}^r , defined in section 2.3 as $M_{t,s}^r \equiv (1 + r) \cdot \partial \mathcal{C}_t / \partial r_s$, captures these closed-economy effects.

To characterize the effect of monetary policy on output, we proceed again by linearizing the goods market clearing condition. Consider a change $\{dr_t\}$ to real interest rates, and let $d\mathbf{r} \equiv \left(\frac{dr_0}{1+r}, \frac{dr_1}{1+r}, \dots \right)'$. Given (43), the real exchange rate responds by $dQ_t = -\sum_{s \geq t} \frac{dr_s}{1+r}$, or in matrix notation, $d\mathbf{Q} = -\mathbf{U}d\mathbf{r}$, where \mathbf{U} is the matrix with 1's on and above the diagonal. Linearizing (21), we now obtain a generalized version of the international Keynesian cross (38):

³¹These assumptions can be relaxed slightly; for instance, appendix C.5 extends our main equivalence result to Taylor rules and productivity shocks.

$$d\mathbf{Y} = \underbrace{(1-\alpha) \mathbf{M}^r d\mathbf{r}}_{\text{Interest rate channel}} + \underbrace{\frac{\alpha}{1-\alpha} \chi d\mathbf{Q}}_{\text{Exp. switching channel}} - \underbrace{\alpha \mathbf{M} d\mathbf{Q}}_{\text{Real income channel}} + \underbrace{(1-\alpha) \mathbf{M} d\mathbf{Y}}_{\text{Multiplier}} \quad (44)$$

The representative-agent model with complete markets is still covered as a special case of equation (44), for which $\mathbf{M} = 0$ and $\mathbf{M}^r = -\mathbf{U}$.³² In that case, equation (44) delivers the simple expression $d\mathbf{Y}^{RA-CM} = -((1-\alpha) + \frac{\alpha}{1-\alpha}\chi) \mathbf{U} d\mathbf{r}$.

Contrast this with the HA-IM model. In that model, it is well-understood from the closed-economy literature that the interest rate channel is less powerful, since agents have less ability to substitute intertemporally. In a closed economy, Werning (2015) has shown that this weaker interest rate channel tends to be offset by a stronger multiplier. In the open economy, however, the multiplier is weaker, since only a share $1-\alpha$ of domestic demand is spent on home goods. Hence, with $\chi = 1$, the HA model has a weaker output response to monetary policy. However, as we prove next, equivalence is restored at a greater value of χ , namely $\chi = 2-\alpha$.

Proposition 9. *Assume $\sigma = 1$, and consider an arbitrary first-order monetary policy shock $d\mathbf{r}$. If $\chi = 2-\alpha$, all aggregate quantities and prices are identical in the RA-CM and HA-IM models. Moreover, provided that $\mathbf{M} > 0$, for an accommodative shock $d\mathbf{r} < 0$, the output response in the HA-IM model satisfies*

$$\chi < 2-\alpha \Leftrightarrow d\mathbf{Y}^{HA-IM} < d\mathbf{Y}^{RA-CM}$$

Proposition 9 is the analogue of Proposition 5 for monetary policy. The neutral case, $\chi = 2-\alpha$, includes the commonly-studied Cole and Obstfeld (1991) parameterization with $\eta = \gamma = \sigma = 1$.³³ This result generalizes the representative-agent result in Itskhoki (2021) to heterogeneous-agent models, and the closed economy result of Werning (2015) to the open economy. Appendix C.4 shows that this proposition applies equally to the comparison between RA-CM and RA-IM.³⁴

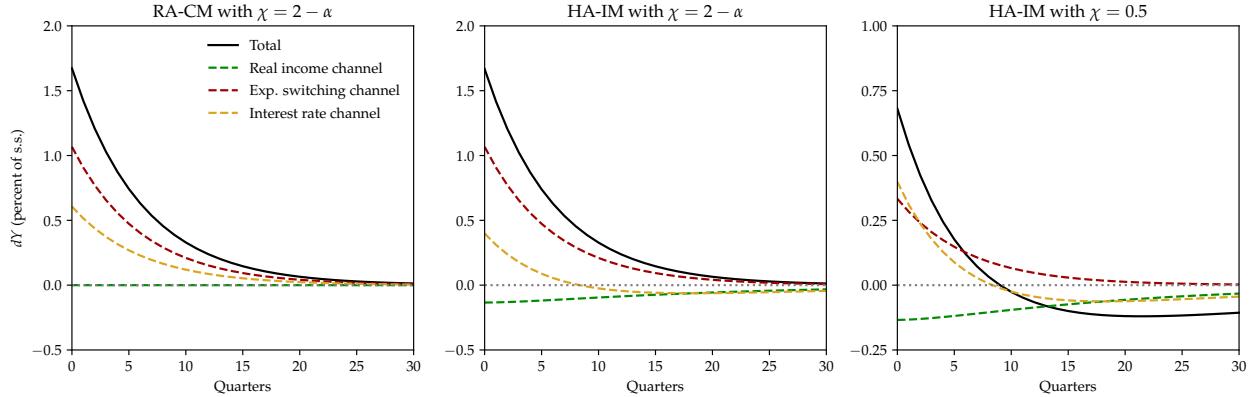
To understand this result, it is helpful to consider the effects of monetary policy on the trade balance. Suppose that consumption changes as in the RA-CM model: the Backus-

³²Recall that $\sigma = 1$ in this section. In a RA-CM model with general σ , we have $\mathbf{M}^r = -\frac{1}{\sigma}\mathbf{U}$.

³³In fact, in this Cole-Obstfeld case, it is even possible to prove that proposition 9 holds for unanticipated non-linear shocks. We can generalize our results further when $\chi = 2-\alpha$, e.g. by allowing for arbitrary monetary policy rules. See appendix C.5.

³⁴Bianchi and Coulibaly (2023) independently arrived at a decomposition similar to (44) for the consumption response after a monetary policy shock in a tradable-nontradable RA-IM model. See appendix E.4 for an analogous model.

Figure 5: The effects of monetary policy



Note: impulse response to a r_t shock that is identical to the ι_t^* shock from figure 2, but with opposite sign. This leads to the same Q_t path as in the left panel of that figure. The decomposition follows equation (44) but the multiplier effect is omitted from the figure.

Smith condition then implies that $dC_t = dQ_t$. Then, applying the relationship between the trade balance, the real exchange rate and consumption (see appendix B.1), we have

$$dNX_t = \frac{\alpha}{1-\alpha} (\chi - 1) dQ_t - \alpha dC_t = \frac{\alpha}{1-\alpha} (\chi - (2-\alpha)) dQ_t \quad (45)$$

Hence, at $\chi = 2 - \alpha$, expenditure switching offsets *both* the increase in import prices and the higher import demand, and the economy behaves as if it were a closed economy. Given this, we can apply Werning (2015)'s result for closed economies to validate our guess that consumption behaves as if there was a representative agent. By contrast, when $\chi < 2 - \alpha$, the net export decline causes a real income decline, which in general equilibrium results in weaker consumption and output responses than the RA-CM model.

Figure 5 illustrates proposition 9 by showing the output response as well as its decomposition using equation (44). For simplicity, we consider an accommodative interest rate shock that generates the same path for the real exchange rate as that considered in section 3. The left and middle panels illustrate the case with $\chi = 2 - \alpha$. Relative to the RA-CM model (left panel), there is a clearly negative real income effect and a weaker interest rate effect in the HA-IM model (middle panel). However, both are exactly offset by a positive multiplier effect from the increased production (not shown), so that the output response is identical. The right panel shows what happens in the HA-IM model when $\chi = 0.5 < 2 - \alpha$ instead. The interest rate and real income channels are unchanged relative to $\chi = 2 - \alpha$ but the expenditure switching channel is muted. As a result, the positive multiplier effect no longer offsets the negative influence of the interest rate and real income channels in HA-IM relative to RA-CM. This is the reason why the output response in HA-IM is below that of the RA model everywhere, consistent with proposition 9.

4.2 Stealing demand from the future through current account deficits

An intriguing aspect of the bottom right panel of figure 5 is that the output response in the HA model turns negative after 9 quarters, until it returns to steady state much later. In other words, monetary stimulus successfully raises aggregate demand for a few quarters, but at the cost of lowering it afterward. It “steals” demand from the future.

What explains this pattern? As discussed above, when $\chi < 2 - \alpha$, monetary stimulus generates a current account deficit: agents borrow from abroad, both to finance higher spending today, spurred by the low rates, and to smooth the real income losses from higher import prices. These current account deficits accumulate into a negative net foreign asset position over time, which remains even after the interest rate and exchange rate have converged most of the way back to steady state. To rebalance the current account, agents cut back on spending, causing a downturn in aggregate demand, and the economy eventually converges back to its initial steady state.

The following proposition derives a simple expression for the present value of the consumption adjustment needed to close a given net foreign asset position $dnfa_t$.

Proposition 10. *If the real exchange rate is at steady state from date $t + 1$ onward ($dQ_{t+s} = 0$ for $s \geq 1$), the date- t present value of consumption and output is given by*

$$\sum_{s=1}^{\infty} (1 + r_{ss})^{-s} dC_{t+s} = \frac{1}{\alpha} dnfa_t \quad \sum_{s=1}^{\infty} (1 + r_{ss})^{-s} dY_{t+s} = \frac{1 - \alpha}{\alpha} dnfa_t \quad (46)$$

Intuitively, in present value terms, any negative net foreign asset position will eventually be repaid. If there is no depreciation, this must involve a recession. Proposition 10 shows that the more closed the economy (the smaller α), the larger the reduction of spending and output required for repayment, since most of the reduced spending falls on home goods, which does not contribute to the international adjustment.³⁵

Our “stealing demand from the future” effect is a close cousin to the “limited ammunition” effect in closed-economy models that has been recently described by [McKay and Wieland \(2021\)](#), [Caballero and Simsek \(2021\)](#) and [Mian, Straub and Sufi \(2021\)](#). There is one crucial difference, however. In our open economy setting, the effect of monetary policy can be so weak that the present value of the output response to monetary stimulus, $PV(dY)$, is *negative*. Appendix C.3 shows that this happens in our model when $\chi < 1 - \alpha$. To conclude, with high MPCs and low trade elasticities, the real income channel substantially alters the transmission of monetary policy in open economies.

³⁵See [Krugman \(1987\)](#) for an earlier articulation of this point. Of course, a less open economy is less likely to accumulate a large negative NFA in the first place.

5 Quantitative model

We have shown that the importance of heterogeneity for the effects of exchange rates or monetary policy depends on the level of the trade elasticity χ . We derived these results under the standard assumption of static CES demand, for which χ is a constant structural parameter. Yet, a host of empirical evidence suggests that the response of the trade balance to exchange rate shocks takes time to play out and depends on the nature of the shock, notably on agents' expectations of its persistence (e.g. [Ruhl 2008](#), [Fitzgerald and Haller 2018](#)). For transitory shocks to exchange rates, the elasticity can be close to 0 in the short run (e.g. [Hooper, Johnson and Marquez 2000](#)); for more permanent shocks, such as tariff changes, it can be 4 or more in the long run (e.g. [Caliendo and Parro 2015](#)). Any plausible quantification exercise needs to confront this evidence.

In this section, we develop a quantitative version of the benchmark model studied so far. To this benchmark, we add a stylized model of "delayed substitution," which exhibits shock-dependent and time-varying elasticities of imports and exports to movements in relative prices. The model's aggregate dynamics are similar to those of the richer models in [Ruhl \(2008\)](#), [Drozd and Nosal \(2012\)](#) and [Alessandria and Choi \(2021\)](#), but it abstracts away from the behavior of firms and focuses directly on that of households. In doing so, it captures the essence of these theories in reduced form, and is straightforward to integrate into broader general equilibrium environments, such as that of our heterogeneous-agent model.³⁶

In addition to delayed substitution, our quantitative model allows for price rigidities on top of wage rigidity (and hence intermediate degrees of exchange rate pass-through), non-homotheticities in consumption, heterogeneous incidence of aggregate shocks on households' labor income, and a standard Taylor rule for monetary policy.

5.1 Additional model elements

We next introduce our new model elements.

Non-homothetic preferences. [Cravino and Levchenko \(2017\)](#) document that, in Mexico, households at the bottom of the income distribution consume a larger share of imported goods than households at the top, implying that they experience larger declines in real income during a depreciation.³⁷ Since poor households typically have higher MPCs, ac-

³⁶See [Arkolakis, Eaton and Kortum \(2012\)](#) and [Drozd, Kolbin and Nosal \(2021\)](#) for alternative reduced-form models that share the same objective.

³⁷The importance of this phenomenon in other countries is subject to an empirical debate. [Borusyak and Jaravel \(2021\)](#) argue that the share of imports in consumption baskets is flat across the income distribution in the United States. [Bems and di Giovanni \(2016\)](#) argue that the fall in aggregate income during the 2008

counting for this fact could magnify the importance of the real income channel. To allow for this possibility, we follow [Carroll and Hur \(2020\)](#) and [Fanelli and Straub \(2021\)](#) and assume agents consume a Stone-Geary CES bundle, with a positive subsistence need \underline{c} for imported goods,³⁸

$$c = \left[\alpha^{1/\eta} (c_F - \underline{c})^{(\eta-1)/\eta} + (1 - \alpha)^{1/\eta} c_H^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \quad (47)$$

Unequal incidence of aggregate shocks on workers. [Blanco et al. \(2024\)](#) show that the devaluation of 2002 in Argentina had heterogeneous incidence on the real income of workers across the income distribution. To allow for this, we follow [Auclert and Rognlie \(2018\)](#) and assume that the total labor income of a worker with productivity e is³⁹

$$\text{labor income for } e, t = \frac{W_t}{P_t} N_t \frac{e^{1+\zeta \log\left(\frac{W_t}{P_t} N_t / \frac{W_{ss}}{P_{ss}} N_{ss}\right)}}{\int \tilde{e}^{1+\zeta \log\left(\frac{W_t}{P_t} N_t / \frac{W_{ss}}{P_{ss}} N_{ss}\right)} d\tilde{e}} \quad (48)$$

In the data, the elasticity of individual household income to aggregate income $\frac{W_t}{P_t} N_t$ is decreasing in the productivity e of workers, so ζ is negative. This implies that the labor income of low-productivity workers falls more in recessions and rises more in booms relative to high-productivity workers. Given that low-productivity workers also have higher marginal propensities to consume, this additional source of heterogeneity will amplify the real income and the multiplier channels.

Monetary policy. [Clarida et al. \(2000\)](#) show that US monetary policy is well approximated by a policy rule targeting expected CPI-inflation with inertia. In this spirit, we replace the constant- r monetary rule in the previous sections by the Taylor rule (20).

Sticky prices and imperfect exchange rate pass-through. We allow for price stickiness in domestic prices, modeled a la Calvo with a price stickiness coefficient of θ_H . This leads to a Phillips curve for inflation in domestic prices P_{Ht} of

$$\pi_{Ht} = \kappa_H \left(\mu \frac{W_t}{Z_t P_{Ht}} - 1 \right) + \frac{1}{1+r} \mathbb{E}_t [\pi_{H,t+1}] \quad (49)$$

with $\kappa_H = (1 - \theta_H) \left(1 - \frac{1}{1+r} \theta_H \right) / \theta_H$.

We also allow for imperfect pass-through of the exchange rate into import prices as

crisis in Latvia caused consumers to shift towards lower-quality, domestically produced goods.

³⁸Appendix D.1 describes how to modify our solution method to incorporate non-homothetic demand.

³⁹Cyclical movements in aggregate labor income could affect workers differently because of differences in fundamental productivity, or differences in hours worked.

in [Monacelli \(2005\)](#). To model imperfect pass-through to import prices, we assume that foreign exporters produce differentiated goods at a flat cost of \mathcal{E}_t per unit and sell them domestically at a sticky price P_{Ft} . The elasticity of substitution between these varieties is $\mu_F / (\mu_F - 1)$. This formulation leads to a Phillips curve for imported goods P_{Ft} of

$$\pi_{Ft} = \kappa_F \left(\mu_F \frac{\mathcal{E}_t}{P_{Ft}} - 1 \right) + \frac{1}{1+r} \mathbb{E}_t [\pi_{F,t+1}] \quad (50)$$

with $\kappa_F = (1 - \theta_F) \left(1 - \frac{1}{1+r} \theta_F \right) / \theta_F$.

We make the same assumption for home exporters to model imperfect pass-through into export prices. Then, inflation in the price $P_{H,t}^*$ of home goods that foreigners see, expressed in their currency, is

$$\pi_{Ht}^* = \kappa_{H^*} \left(\mu_{H^*} \frac{P_{Ht}}{\mathcal{E}_t P_{Ht}^*} - 1 \right) + \frac{1}{1+r} \mathbb{E}_t [\pi_{H,t+1}^*] \quad (51)$$

with $\kappa_{H^*} = (1 - \theta_{H^*}) \left(1 - \frac{1}{1+r} \theta_{H^*} \right) / \theta_{H^*}$. Domestic equity earns the dividends of both home producers and home exporters.

Delayed substitution. We introduce delayed substitution by modifying the household problem. Instead of being able to flexibly adjust their relative consumption of different countries' goods in each period, we now assume that households can only do so with a certain probability $1 - \theta$. With probability θ , they are forced to keep the ratio of each country's good to total consumption constant, although they can still adjust overall expenditure.

We obtain general results for this delayed substitution model in [appendix D.3](#). In our problem, this model generates first-order dynamics for the *target ratios* \hat{x}_{Ht} and \hat{x}_{Ht}^* , for households who can adjust, of domestic and foreign home good consumption to overall consumption.⁴⁰

$$d \log \hat{x}_{Ht} = -(1 - \beta\theta) \eta d \log \frac{P_{Ht}}{P_t} + \beta\theta d \log \hat{x}_{Ht+1} \quad (52)$$

$$d \log \hat{x}_{Ht}^* = -(1 - \beta^*\theta) \gamma d \log P_{Ht}^* + \beta^*\theta d \log \hat{x}_{Ht+1}^* \quad (53)$$

This is similar to a Calvo model of price-setting, but here, consumers reset their consumption *bundles* based on their perceptions of current and future relative prices. The aggregate ratios C_{Ht}/C_t and C_{Ht}^*/C^* , in turn, evolve sluggishly as only a fraction $1 - \theta$ adjusts each

⁴⁰For the heterogeneous domestic households, the simple form of (52) and (54) requires our assumption that $\sigma = 1$. Otherwise, as shown in [appendix D.3](#), there is a more complex expression for the target ratio, which can vary between heterogeneous households.

Parameter	Benchmark model	Quantitative model	Parameter	Benchmark model	Quantitative model
σ	1	1	μ	1.043	1.041
φ	2	2	σ_e	0.883	0.883
$\eta = \gamma$	$\frac{\{0.1, 0.5, 1, 2 - \alpha\}}{2 - \alpha}$	4	ρ_e	0.912	0.912
θ	n.a.	0.976	θ_w	0.938	0.938
β^*	0.990	0.990	θ_H	0	0.66
β	0.965	0.962	θ_{H^*}	0	0.66
ζ	n.a.	-0.196	θ_F	0	0
α	0.4	0.344	ρ_m	n.a.	0.8
ε	0	0.085	ϕ	n.a.	1.5
Moment		Data	Benchmark model	Quantitative Model	
Average quarterly MPC		0.20	0.20	0.20	
Std of log annual post-tax income		0.84	0.84	0.84	
Autocorrelation of order 1 of log annual post-tax income		0.78	0.78	0.78	
Average import share		0.40	0.40	0.40	
Std of import share		0.042	n.a.	0.042	
Std of elasticity to aggregate income		0.15	n.a.	0.15	

Notes: all parameters are for the quarterly calibration. Average and standard deviations are computed across deciles of income. The income process is discretized with 7 points.

Table 2: Calibration

period, with dynamics described by

$$d \log \frac{C_{Ht}}{C_t} = (1 - \theta) d \log \hat{x}_{Ht} + \theta d \log \frac{C_{Ht-1}}{C_{t-1}} \quad (54)$$

$$d \log C_{Ht}^* = (1 - \theta) d \log \hat{x}_{Ht}^* + \theta d \log C_{Ht-1}^* \quad (55)$$

This delivers a model in which the trade elasticity is both shock- and time-dependent. For instance, for foreign consumption C_{Ht}^* of the home good, the long-run elasticity to a permanent shock is simply γ . By contrast, the short-run elasticity to the same permanent shock is lower, at $\gamma(1 - \theta)$, since it takes time for consumers to adjust. Finally, the short-run elasticity to a one-time shock is even lower, at $\gamma(1 - \theta)(1 - \beta\theta)$, since even those who change their bundles choose to adjust little, as they anticipate wanting to adjust back in the other direction after the shock has passed.

5.2 Calibration

Aggregate calibration. We calibrate the model at a quarterly frequency. Our aggregate calibration is standard. Our goal is to capture the essential features of a typical Latin

American economy such as Mexico, since emerging economies have been the focal point of policy discussions on the consequences of exchange rate depreciations. Furthermore, as we will show in section 5.5, many of the features that make contractionary depreciations more likely (e.g. incomplete financial markets, openness to trade) tend to prevail in emerging economies.

Table 2 summarizes this calibration. We set β^* to achieve an annualized real interest rate of $r = 4\%$ in steady state and β to clear the asset market at home. We set the initial steady state net foreign asset position to 0, with all assets invested in domestic stocks, to avoid interactions between exchange rates and pre-existing trade deficits. We consider standard values of $\sigma = 1$ for the elasticity of intertemporal substitution, and $\varphi^{-1} = 0.5$ for the Frisch elasticity of labor supply. For the elasticity of substitution across goods, we proceed as follows. Since there is limited evidence that this elasticity is different for imported vs domestic goods relative to between imported goods, we set $\gamma = \eta$. This implies that $\chi = (2 - \alpha) \gamma$. In our benchmark model, we considered a range of values for χ . By contrast, our quantitative model relies on delayed substitution, which we calibrate below.

MPCs. To calibrate the aggregate consumption behavior of the model, and in the absence of MPC evidence from Mexico, we target a quarterly MPC of 0.20 measured by Hong (2023) for Peru⁴¹. We assume an AR(1) process for log income, with a persistence ρ_e and a cross-sectional standard deviation of logs σ_e set to match the persistence and standard deviation of residual log annual income for Mexico from the GRID project (Govenen, Pistaferri and Violante 2022), which we adjust for progressive income taxation using estimates from De Magalhaes, Martorell-Toledano and Santaeulalia-Llopis (2022).⁴² We adjust the markup μ , which mostly affects the level of steady-state liquidity in the model, so as to target the average quarterly MPC, and set the borrowing constraint to $\underline{a} = 0$.⁴³ This delivers $\mu = 1.041$, so an average liquid wealth to GDP ratio of 98%.⁴⁴ We set the markups of importers and exporters to $\mu_F = \mu_{H^*} = 1$ to keep the steady states of the baseline and quantitative models similar.

Non-homothetic preferences. To calibrate the spending behavior of households across

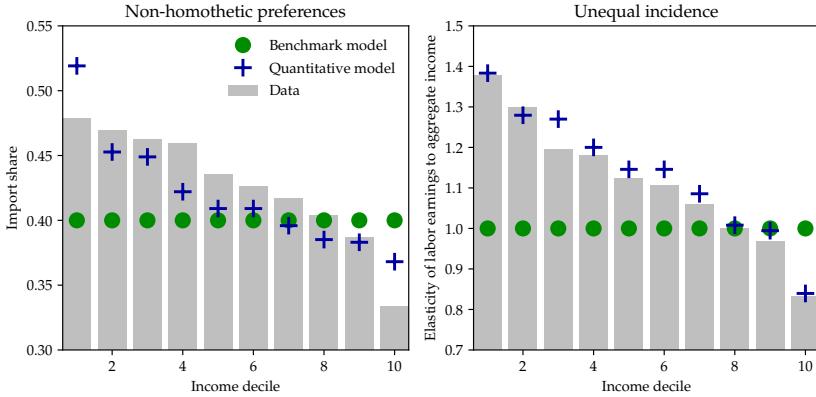
⁴¹A quarterly MPC of 0.20 is similar to estimates from other countries (e.g. see Sokolova 2023).

⁴²Since our quantitative model has subsistence needs, we make sure that our discretization procedure respects the constraint that the agent at the lowest level of income can always afford the subsistence level of consumption.

⁴³Setting a different borrowing constraint \underline{a} would leave our quantitative results almost unaffected, because we would recalibrate markups to match the same average quarterly MPC.

⁴⁴This compares to a Mexican wealth-GDP ratio of 350% in 2018. Our estimate is smaller and best understood as capturing liquid wealth. We decided not to target aggregate wealth to GDP in order to hit realistic MPCs, whose importance is emphasized by our theoretical results.

Figure 6: Calibration targets across the income distribution



Note: the quantitative model targets the average and standard deviation across income deciles of consumption share of foreign goods and the standard deviation across income deciles of the elasticity of labor income to aggregate income.

goods, we target moments of the Mexican spending survey reported in [Cravino and Levchenko \(2017\)](#). From their data, we obtain the average tradable share at each income decile, as well as that decile's share of aggregate consumption. We then assume that the share of imports within tradables is the same across the income distribution, and compute income-specific import shares so that the economy-wide share lines up with the Mexican import/GDP ratio of 40%, as reported in appendix table A.1.⁴⁵ In the benchmark model, we set $\alpha = 0.4$, while in the quantitative model we adjust α , the asymptotic import share, and c , the subsistence level on the imported good, to target an average import share of 0.4 together with the standard deviation of import shares across income deciles.

Unequal incidence of aggregate shocks. We use information from [Blanco et al. \(2024\)](#) to calibrate ζ , which governs in the model the elasticity of labor income with respect to aggregate income for workers with different productivity e . The authors generously provided us with estimates for the elasticity of real labor income with respect to aggregate income for different income deciles following the depreciation of 2002 in Argentina. They find that the elasticity is larger for low-income workers than high-income workers, which implies that $\zeta < 0$. Our strategy is to find $\zeta < 0$ such that the standard deviation of the elasticity across income deciles is identical in the model and in the data. This yields $\zeta = -0.196$. Appendix D.2 provides details.

Figure 6 shows the import share and elasticity of labor earnings to aggregate income across deciles of the income distribution. Overall our model does a good job at capturing the joint variation in the import share and the elasticity of labor earnings. In particular, both are declining in income, as emphasized by [Cravino and Levchenko \(2017\)](#) and

⁴⁵Appendix E.2 spells out a formal model with nontradables, domestically produced tradables and imported tradables, and shows that it is equivalent to our model if α is calibrated to the import/GDP ratio.

Blanco et al. (2024).

Phillips curve parameters. Appendix D.5 provides details on our calibration of Phillips curve parameters. Among the price rigidity parameters, the Calvo coefficient for import prices θ_F is the most important as it directly affects the magnitude of the real income effect. We calibrate θ_F using evidence from the 1994 Mexican devaluation as reported by Burstein and Gopinath (2015). For this particular devaluation, we find perfect pass-through to import prices, so $\theta_F = 0$.⁴⁶ By contrast, given the widespread evidence in Boz et al. (2022) for dollar pricing of exports in Latin American countries, we set $\theta_{H^*} > 0$. We assume that the degree of price rigidity in dollar prices, like the price rigidity of domestic goods prices, corresponds to an average price reset frequency of 9 months, as is standard in the literature. This leads us to set $\theta_H = \theta_{H^*} = 0.66$. We then find the wage stickiness parameter that is able to replicate the path of home good prices after the Mexican devaluation (see figure A.3).

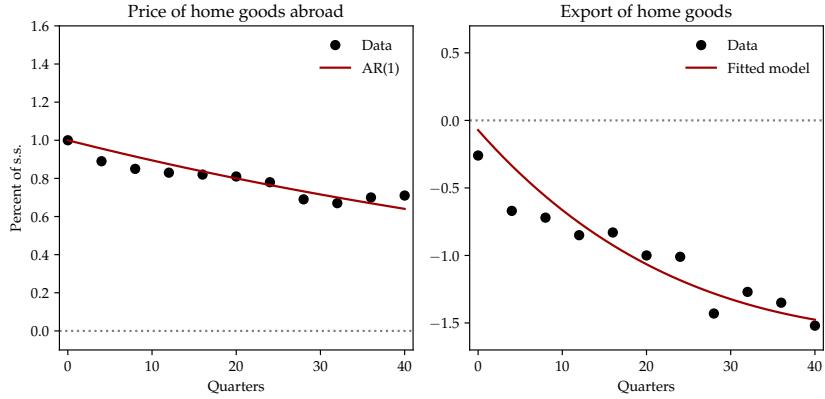
Monetary policy. We set the Taylor rule coefficient on expected inflation to a standard value of $\phi = 1.5$. We calibrate the persistence in the monetary policy rule to $\rho_m = 0.8$, following estimates from Clarida et al. (2000).

Delayed substitution model. We assume that our delayed substitution model applies equally to domestic and foreign households, with the same parameter θ . We calibrate the model to the evidence in Boehm, Levchenko and Pandalai-Nayar (2023) (henceforth, BLP). BLP identify plausibly exogenous changes in tariffs and trace out the entire dynamic response of trade flows. To be precise, BLP observe how a country A's exports within an industry to a specific importing country B respond to a persistent increase in tariffs levied by B on imports from A. This elasticity captures γ , the elasticity of export demand by the rest of the world. Figure 7 plots the evidence from their estimates. The left panel shows the changes in tariffs. The right panel shows the response of trade flows.

We replicate this experiment in our model as follows. We begin by setting γ to 4, delivering a long-run trade elasticity of 6, well within the consensus range for the long-term trade elasticity (e.g. Caliendo and Parro 2015). We then interpret the tariff change in the BLP data as a change in the relative price of home goods abroad $d \log P_{Ht}^*$, which we assume follows an AR(1) with persistence ρ . We choose ρ to minimize the sum of squared distances to the tariff response in the left panel in figure 7, finding $\rho = 0.989$ quarterly. We then feed this process into (53) and (55) and calibrate θ to minimize the sum of squared distances to the estimates displayed in the right panel in figure 7. This delivers $\theta = 0.976$ quarterly, which implies a trade elasticity of 0.15 on impact, 0.3 after one quarter, and 0.7 after one year.

⁴⁶We recalibrate to evidence from other countries in appendix D.5.

Figure 7: Calibrating delayed substitution in the quantitative model



Note: calibration of delayed substitution in the quantitative model. We fit an AR(1) to the change in tariffs from [Boehm, Levchenko and Pandalai-Nayar \(2023\)](#) and estimate θ to minimize the sum of squared residuals between the model response and the data response of the foreign demand for home goods.

5.3 Revisiting contractionary depreciations

We use our quantitative model to revisit the effect of exogenous depreciation shocks. Proposition 8 showed that, in the benchmark model, these shocks generate output contractions when the trade elasticity χ is small enough. Figure 8 shows that in our quantitative model with a Taylor rule (green line) depreciations are contractionary for one year, and then expansionary. This is because, in the short run, the quantitative model behaves similarly to a model with a low static trade elasticity. Over time, households respond to the depreciation by substituting away from foreign goods, and towards home goods. This stimulates net exports, and eventually output after 4 quarters. Thus in the medium to long run, the quantitative model behaves similarly to a model with a high static trade elasticity.⁴⁷

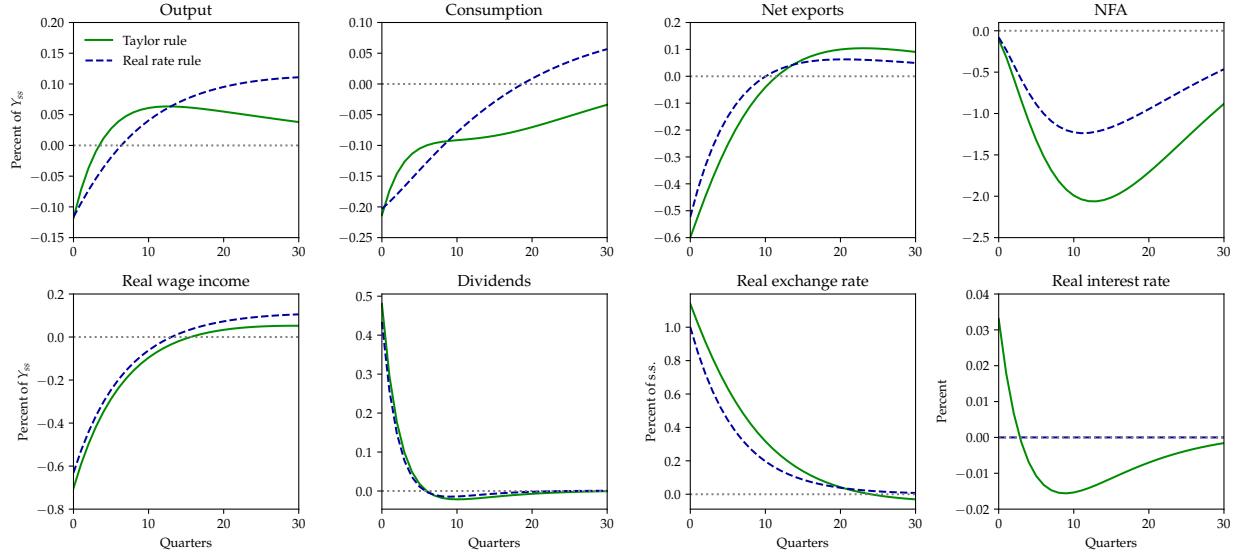
5.4 Managing contractionary depreciations

Our analysis shows that depreciations can be contractionary in the short run. We now discuss how monetary policy should respond if its goal is to stabilize output. The question is non-trivial, due to the following dilemma: should monetary policy hike interest rates to fight the depreciation, which is the root cause of the recession? Or should it stimulate by cutting interest rates, as is traditional to fight a recession? To illustrate this trade-off, we consider two simple policies.

The red line in figure 9 shows what happens when the central bank stabilizes the ex-

⁴⁷Appendix D.8 shows that depreciations generate a boom in the quantitative model with a representative agent.

Figure 8: Contractionary depreciations



Note: impulse response in the quantitative model to the shock to ι_t^* from figure 2. The model with Taylor rule is our quantitative model; the one with real rate rule is our quantitative model without the Taylor rule.

change rate. This policy leads to an even worse recession. The intuition for this finding is that hiking rates replaces one evil (contractionary depreciation) with another (contractionary monetary policy), as highlighted by [Gourinchas \(2018\)](#) and [Kalemlı-Özcan \(2019\)](#).

The blue line in figure 9 shows the policy that fully stabilizes output.⁴⁸ In our quantitative model, cutting the real interest rate allows to stabilize output despite making the depreciation worse. This is because the real income channel is not strong enough to offset the positive effects of accommodative monetary policy at our calibrated trade elasticity. Under a lower trade elasticity, however, the output-stabilizing policy involves an interest rate hike that mitigates the depreciation instead (e.g. with $\theta = 0.99$).

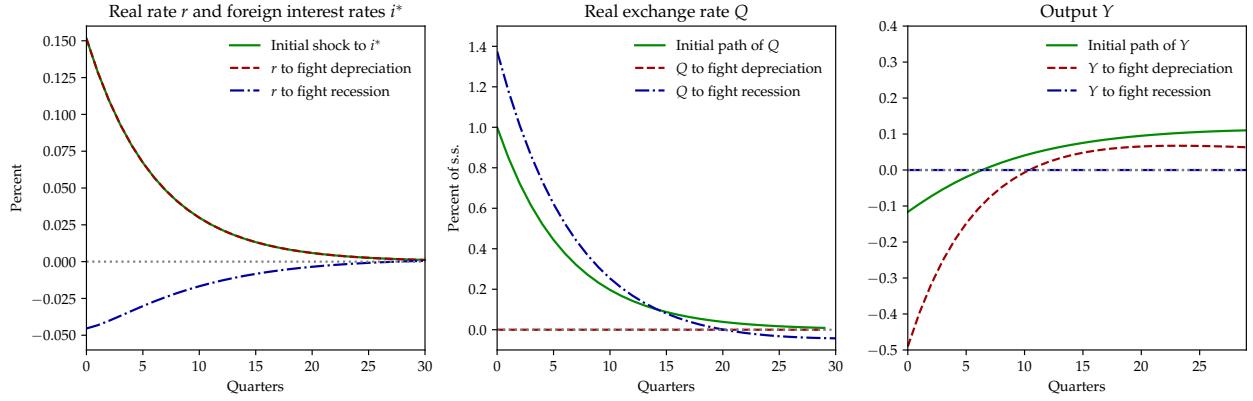
5.5 When does the real income channel matter?

Table 3 explores the role of economy-wide characteristics more systematically. For each column, we vary one characteristic and re-calibrate the model to match the other moments from table 2. We report both the impact response of output and its one-year cumulative response. The first column corresponds to our quantitative model, as displayed in the green line of Figure 8.

The second column shows that lower openness makes the effect of an exchange rate

⁴⁸This is the unique output-stabilizing policy, which is simple to obtain with our methods by setting the path of the real interest rate as an additional unknown, and the stable output path as an additional target. Optimal policy would be very interesting to study, but is still out of reach at present.

Figure 9: Policies to deal with contractionary depreciations



Note: impulse responses to the shock to i_t^* from figure 2 with and without monetary policy response. The green line shows the impulse without monetary policy response. The red dashed line shows the responses when monetary policy stabilizes the real exchange rate. The blue dash-dotted line shows the responses when monetary policy stabilizes output.

shock on output less contractionary. This is natural, as both the real income channel and the expenditure switching channel of exchange rates scale with α . Next, we consider an economy with higher MPC. This amplifies the real income and multiplier channels, leading depreciations to be significantly more contractionary. The fourth column removes Dollar Currency Pricing that, as appendix E.1 shows, reduces expenditure switching and stimulates the profits of exporters. Overall, Dollar Currency Pricing attenuates the contraction in our quantitative model. This occurs because households have a relatively high MPC out of capital income in our calibration, and as a result the increased profits from exports, visible in the dividend panel of figure 8, stimulate aggregate consumption significantly. Homothetic preferences and equal incidence of aggregate shocks on workers both reduce the size of the contraction. A lower short-term substitution elasticity makes the contraction significantly larger by dampening expenditure switching even more. Finally, less exchange rate pass-through into import prices reduces the strength of the real income channel. Since it also dampens domestic expenditure switching, the output response is scaled down, rather than flipping sign.

These patterns suggest that different countries are likely to respond differently to exchange rate depreciations. In appendix D.5, we calibrate the model to seven countries that have experienced depreciation episodes. We find that the degree of inferred import price pass-through is the most important cross-country determinant of the magnitude of the contraction after a depreciation.

Quantitative model	Low trade openness	High MPC	No DCP	Homothetic	Equal incidence	Low ST χ	Low \mathcal{E} pass-through
dY_0	- 0.12	- 0.06	- 0.29	- 0.19	- 0.07	- 0.10	- 0.36
$\sum_{t=0}^3 dY_t$	- 0.24	- 0.13	- 0.83	- 0.45	- 0.04	- 0.18	- 1.16

Note: change on impact and 1-year cumulative impulse response of output to the shock to t_t^* from figure 2 for various parametric assumptions. For low trade openness we target a share of import of 20% instead of 40%; for high MPC we target an average quarterly MPC of 40% instead of 20%; for no DCP we assume full pass-through of domestic prices into export prices ($\theta_{H^*} = 0$) instead of an intermediate pass-through ($\theta_{H^*} = 0.66$); for homothetic we target a constant import share across the income distribution ($\underline{\zeta} = 0$); for equal incidence we assume a constant elasticity of labor income to aggregate income across the income distribution ($\zeta = 1$); for low short-term elasticity we set $\theta = 0.99$ to target a trade elasticity of 0.3 after 1 year, instead of 0.7; for low \mathcal{E} pass-through, we assume limited pass-through of exchange rates into import prices ($\theta_F = 0.8$) instead of full pass-through ($\theta_F = 0$).

Table 3: Effects of exchange rate shock under various assumptions

5.6 Comparison with balance sheet effects

A well-documented feature of international investment positions is that the net foreign asset position consists of the difference between gross assets and gross liabilities that are both very large, and often differ in terms of their risk profile and currency composition (e.g. [Gourinchas and Rey 2007](#), [Lane and Shambaugh 2010](#)). While we cannot easily capture the risk dimension, we can accommodate currency mismatch in the net foreign asset position.

We relax the assumption that domestic households hold 100% of their assets in domestic stocks, and that the government has no gross assets or liabilities. Instead, we assume that one of these has initially borrowed in foreign currency to invest in nominal domestic bonds, while keeping their position in domestic equities unchanged. Throughout, we assume that gross foreign currency liabilities are 50% of GDP and that all bonds have an average duration of 18 quarters. Appendix D.6 provides details and shows that this calibration is an upper bound on the magnitude of valuation effects: data from [Bénétrix, Gautam, Juvenal and Schmitz \(2020\)](#) show that few countries have historically had such large gross currency mismatches in their external balance sheets, and that most countries have dramatically reduced these gross mismatches in the past two decades⁴⁹.

In Table 4 we report how our results for the output effect of the devaluation (repeated in the first column) are altered in this scenario. We consider four cases. In the first, called “Gross positions”, the gross foreign currency debt is held by households in proportion to their net asset holdings. This brings down the output response by a further 0.09% on impact, and by 0.31% over 1 year. In other words, foreign currency debt causes amplification

⁴⁹In appendix D.6, we show that currency mismatch does not generate a contraction in the model with a static trade elasticity equal to $\chi = 1$. Thus, it is the addition of the real income channel and the currency mismatch that generates large contractions.

	Baseline	Gross positions	Gov, lump-sum	Gov, proportional tax	Gov, deficit-finance
dY_0	- 0.12	- 0.21	- 0.36	- 0.26	- 0.24
$\sum_{t=0}^3 dY_t$	- 0.24	- 0.55	- 0.79	- 0.65	- 0.63

Note: change on impact and 1-year cumulative impulse response of output to the shock to i_t^* from figure 2 for different balance sheet specifications. The baseline corresponds to our quantitative model. In the second column we assume that households hold the equivalent of 50% of annual GDP in debt denominated in foreign currency; for government with lump-sum transfers we assume that the government owes foreign currency debt and owns local currency assets, and adjusts following the depreciation using lump sum taxes to balance budget period by period; for government with proportional taxes we assume that taxes are proportional to labor income; for government deficit-financed we assume that the government does not balance budget period by period but can run a deficit. In all our specifications we assume that debt takes the form of long-term bonds with average duration of 18 quarters.

Table 4: Balance sheet effects under various distribution assumptions

of the contractionary effect of the depreciation.

In the next three columns, we consider what happens if instead the foreign currency exposure is held on the government balance sheet, and then financed by households according to various tax schemes. The first two columns report the effect of immediately taxing households lump sum or proportionally, while the third reports the effect of deficit-financing and taxing later with a proportional tax. The amplification is largest with an immediate lump-sum tax, which is most regressive. This echoes the findings in [de Ferra, Mitman and Romei \(2020\)](#) and [Zhou \(2022\)](#), who show that valuation effects are especially powerful at reducing output when they are concentrated on high-MPC households.

6 Conclusion

We introduce heterogeneous households in a New Keynesian model of a small open economy and we show that this new feature is critical to understand the effects of capital flows and monetary policy. When depreciations pass through quickly to consumer prices but it takes time for consumers to substitute toward domestic varieties, depreciations become contractionary. Monetary easing comes with the negative side effects of a depreciated exchange rate; it leads to current account deficits, and depresses demand in the future. Stabilization policy balances the costs of high interest rates against the costs of depreciation created by the real income channel.

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Online Appendix for “Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel”

A Appendix to section 2

A.1 Model setup

Here we provide additional details on the setup of the model in section 2.

Preferences across goods. In our baseline model, consumption c_{it} of any agent i living in any country aggregates their home good H and a composite foreign good F with elasticity η ,

$$c_{it} = \left[(1 - \alpha)^{\frac{1}{\eta}} (c_{iHt})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (c_{iFt})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

home consumption aggregates goods j produced at home, while foreign consumption aggregates goods produced in a continuum of countries k :

$$c_{iHt} = \left(\int_0^1 c_{iHt}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \quad c_{iFt} = \left(\int_0^1 c_{ikt}(j)^{\frac{\gamma-1}{\gamma}} dk \right)^{\frac{\gamma}{\gamma-1}}$$

with $\epsilon > 1$, $\gamma > 0$ and $\eta > 0$. In turn, consumption from country k aggregates goods produced there with the same elasticity ϵ as that used to aggregate goods produced at home,

$$c_{ikt} = \left(\int_0^1 c_{ikt}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

Writing \mathcal{A}_{it} for the nominal value of end-of-period assets and \mathcal{A}_{it}^p for the nominal value of beginning-of-period assets including returns, the agent's budget constraint is

$$\int_0^1 P_{Ht}(j) c_{iHt}(j) dj + \int_0^1 \int_0^1 P_{kt}(j) c_{ikt}(j) dj dk + \mathcal{A}_{it+1} \leq \mathcal{A}_{it}^p + e_{it} W_t N_t$$

hence, consumer i 's demand for good j in country k is

$$c_{ikt}(j) = \alpha \left(\frac{P_{kt}(j)}{P_{kt}} \right)^{-\epsilon} \left(\frac{P_{kt}}{P_{Ft}} \right)^{-\gamma} \left(\frac{P_{Ft}}{P_t} \right)^{-\eta} c_{it}$$

while their demand for good j in the home country is

$$c_{iHt}(j) = (1 - \alpha) \left(\frac{P_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} c_{it}$$

Applying this demand system to the heterogeneous agents at home, indexed by their state (a, e) , delivers equations (10) and (11). Applying this demand system to the representative foreign

agents, noting that all foreign countries are symmetric and prices are flexible abroad so that $P_{Ft}^* = P_t^*$, delivers

$$C_{Ht}^* = \alpha \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-\gamma} C_t^*$$

which is equation (12).

Foreign agents. All foreign countries are symmetric. As discussed in section 2.1, in each foreign country lives a representative foreign agent with utility

$$\sum_{t=0}^{\infty} (\beta^*)^t \mathcal{B}_t \{u(C_t^*) - v(N_t^*)\} \quad (\text{A.1})$$

where β^* is the foreign discount factor, and \mathcal{B}_t is a utility modifier capturing time-varying patience for the foreign household. We assume that \mathcal{B}_t has initial value $\mathcal{B}_{-1} = 1$, is nonnegative and bounded, $\mathcal{B}_t \in (0, \bar{B})$ for $\bar{B} > 0$, and reverts to 1 in the long run: $\mathcal{B}_\infty = 1$.

Foreign countries produce their own good under constant returns to scale with production function

$$Y_t^* = Z^* N_t^*$$

Prices and wages are flexible abroad, so that

$$P_t^* = \mu \frac{W_t^*}{Z^*}$$

The home country is infinitesimal, so that market clearing for the composite foreign good is

$$C_t^* = Y_t^*$$

The first order conditions for a representative foreign agent are

$$\frac{v'(N_t^*)}{u'(C_t^*)} = \frac{W_t^*}{P_t^*}$$

and

$$\mathcal{B}_t (C_t^*)^{-\sigma} = \beta^* (1 + r_t^*) \mathcal{B}_{t+1} (C_{t+1}^*)^{-\sigma} \quad (\text{A.2})$$

where r_t^* denotes the foreign interest rate. It follows that the world equilibrium features a constant level of consumption C^* (and output $Y^* = C^*$) given by

$$\frac{v'(C^*/Z^*)}{u'(C^*)} = \frac{Z^*}{\mu}$$

and that the real interest rate r_t^* is given by (A.2) when $C_t^* = C_{t+1}^* = C^*$. The central bank targets a constant price index P^* , which it achieves by setting the foreign interest rate according to price-level targeting rule with the natural rate r_t^* as an intercept, $\iota_t^* = r_t^* + \phi \log(P_t^*/P^*)$. In equilibrium, the foreign nominal and real interest rates are equal, and relate to the discount factor shocks \mathcal{B}_t according to (16). The primitive shocks in our economy are the sequence of \mathcal{B}_t 's. Alternatively, given (16), we can construct this sequence for a given exogenous sequence of foreign interest rates

ι_t^* 's and the fact that $\lim \mathcal{B}_t = 1$, from

$$\mathcal{B}_t = \prod_{s \geq t} \left(\frac{1 + \iota_s^*}{1/\beta^*} \right)$$

so that high \mathcal{B}_t corresponds to high current or future foreign interest rate ι_t^* relative to the steady state foreign interest rate $1/\beta^*$.

A.2 Key equilibrium relations

Current account identity. Start by aggregating up household budgets in equation (9), using $\mathbb{E} e_{it} = 1$,

$$C_t + A_t = A_t^p + \frac{W_t}{P_t} N_t$$

Next, note that, for all $t \geq 1$, (8) together with the asset pricing equation (7) implies $A_t^p = (1 + r_{t-1}) A_{t-1}$. Using the definition of the NFA in (22), we obtain, for $t \geq 1$,

$$C_t + p_t + \text{nfa}_t = (1 + r_{t-1}) p_{t-1} + (1 + r_{t-1}) \text{nfa}_{t-1} + \frac{W_t}{P_t} N_t$$

Using the asset pricing equation (7) again, $(1 + r_{t-1}) p_{t-1} = p_t + d_t$, so we can rearrange the equation above as

$$\text{nfa}_t - \text{nfa}_{t-1} = d_t + \frac{W_t}{P_t} N_t - C_t + r_{t-1} \text{nfa}_{t-1}$$

Using the expression for dividends (15) together with the law of one price (17), which implies that domestic producers do not have any unhedged currency exposure, we finally have $d_t + \frac{W_t}{P_t} N_t = \frac{P_{Ht}}{P_t} Y_t$, so

$$\text{nfa}_t - \text{nfa}_{t-1} = \underbrace{\frac{P_{Ht}}{P_t} Y_t - C_t}_{NX_t} + r_{t-1} \text{nfa}_{t-1} \tag{A.3}$$

which is the current account identity.⁵⁰ Note that we have defined net exports NX_t (the trade balance) in units of the consumption basket. Alternatively, since consumption is defined as $C_t \equiv \frac{P_{Ft}}{P_t} C_{Ft} + \frac{P_{Ht}}{P_t} C_{Ht}$, and using the goods market clearing condition (21), we find:

$$NX_t \equiv \frac{P_{Ht}}{P_t} Y_t - C_t = \frac{P_{Ht}}{P_t} C_{Ht}^* - \frac{P_{Ft}}{P_t} C_{Ft} \tag{A.4}$$

Other key relations. Combining the goods market clearing condition (21) with the equations for demand for domestic goods (11) and (12), the law of one price (17), and the foreign monetary policy condition, implying $P_t^* = 1$ and $C_t^* = C^*$, we see that domestic output is always given by

$$Y_t = (1 - \alpha) \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{P_{Ht}}{\mathcal{E}_t} \right)^{-\gamma} C^* \tag{A.5}$$

⁵⁰At date 0, the right-hand side of (A.3) also includes a term $A_0^p - (1 + r_{-1}) A_{-1}$, reflecting valuation effects.

which is equation (33) in the main text. Combining instead the net export equation (A.4) with these same equations, together with the law of one price for foreign goods $P_{Ft} = \mathcal{E}_t$, we see that the trade balance is always given by

$$NX_t = \alpha \frac{P_{Ht}}{P_t} \left(\frac{P_{Ht}}{\mathcal{E}_t} \right)^{-\gamma} C^* - \alpha \frac{\mathcal{E}_t}{P_t} \left(\frac{\mathcal{E}_t}{P_t} \right)^{-\eta} C_t \quad (\text{A.6})$$

We next relate all relative prices in equations (A.5) and (A.6) to the real exchange rate Q_t . First, we note that definition of the real exchange rate in (6) combined with $P_t^* = 1$ implies $Q_t = \frac{\mathcal{E}_t}{P_t}$. Manipulating the price index equation (4), we then see that Q_t is connected to the relative price of home goods P_{Ht}/P_t through

$$1 = \left[(1 - \alpha) \left(\frac{P_{Ht}}{P_t} \right)^{1-\eta} + \alpha Q_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (\text{A.7})$$

We denote by $p_H(Q)$ the mapping between $\frac{P_{Ht}}{P_t}$ and Q implicit in equation (A.7). We can also rewrite (A.7) to relate the real exchange rate Q_t to the relative price of home and foreign goods (the inverse of the terms of trade) P_{Ht}/\mathcal{E}_t via

$$Q_t^{-1} = \left[(1 - \alpha) \left(\frac{P_{Ht}}{\mathcal{E}_t} \right)^{1-\eta} + \alpha \right]^{\frac{1}{1-\eta}} \quad (\text{A.8})$$

We let $p_H^*(Q)$ denote the mapping between the inverse terms of trade P_H/\mathcal{E} and the real exchange rate Q implicit in this equation.

Taken together, equations (A.5), (A.6), (A.7) and (A.8) imply that the level of the real exchange rate Q_t and aggregate domestic spending C_t uniquely determine the level of domestic output on the one hand,

$$Y_t = (1 - \alpha) (p_H(Q_t))^{-\eta} C_t + \alpha (p_H^*(Q_t))^{-\gamma} C^* \quad (\text{A.9})$$

and the trade balance on the other,

$$NX_t = \alpha p_H(Q_t) (p_H^*(Q_t))^{-\gamma} C^* - \alpha (Q_t)^{1-\eta} C_t \quad (\text{A.10})$$

Combining the price-setting condition (14) with the production function (13), we see that real wage income is

$$\frac{W_t}{P_t} N_t = \frac{1}{\mu} \frac{P_{Ht}}{P_t} Y_t \quad (\text{A.11})$$

i.e. a fixed fraction $\frac{1}{\mu}$ of aggregate real income $\frac{P_{Ht}}{P_t} Y_t$. Then, combining (A.11) with (15), and using the law of one price (17), we see that real dividends are equal to

$$d_t = \left(1 - \frac{1}{\mu} \right) \frac{P_{Ht}}{P_t} Y_t \quad (\text{A.12})$$

i.e. also a fixed fraction of aggregate real income.

Finally, combining the relation between \mathcal{B}_t and foreign real interest rates in (16) with the real UIP condition (7), we find that $1 + r_t^* = \frac{1}{\beta^*} \frac{\mathcal{B}_t}{\mathcal{B}_{t+1}} = (1 + r_t) \frac{Q_t}{Q_{t+1}}$, and therefore for all $t \geq 0$,

$$\frac{Q_t}{\mathcal{B}_t} = \frac{1}{\beta^*(1 + r_t)} \frac{Q_{t+1}}{\mathcal{B}_{t+1}} \quad (\text{A.13})$$

In a steady state, equation (A.13) implies that the domestic real interest rate is $r_{ss} = (\beta^*)^{-1} - 1$. Given a sequence $\{r_t, \mathcal{B}_t\}$, with $\mathcal{B}_\infty = 1$, and our focus on a steady state with $Q_\infty = Q_{ss}$, we can therefore always solve for Q_t as

$$Q_t = \mathcal{B}_t \cdot \prod_{s \geq t} \left(\frac{1 + r_{ss}}{1 + r_s} \right) \cdot Q_{ss} \quad (\text{A.14})$$

A.3 Characterizing steady states

Consider a steady state of this model, with a constant level of all aggregates $\{C, C_H, C_F, Y, A, p, d, \text{nfa}\}$ and relative real prices $\{Q, P_H/P, W/P, r, i^*\}$, for given constant foreign discount factor shocks \mathcal{B} and productivity Z . The steady-state version of equation (A.13) implies that the domestic real interest rate is $r = i^* = (\beta^*)^{-1} - 1$. Equations (A.11) and (A.12) imply that the long-run real wage and dividends are given, respectively, by

$$\begin{aligned} \frac{W}{P} &= \frac{1}{\mu} \frac{P_H}{P} \frac{Y}{N} = \frac{1}{\mu} p_H(Q) \frac{Y}{N} \\ d &= \left(1 - \frac{1}{\mu}\right) p_H(Q) Y \end{aligned}$$

The asset pricing equation (7) then implies that the domestic stock price is

$$p = \frac{1}{r} \left(1 - \frac{1}{\mu}\right) p_H(Q) Y$$

The current account identity (A.3) implies that, in steady state,

$$r \cdot \text{nfa} = -NX = C - p_H(Q) Y \quad (\text{A.15})$$

Goods market clearing (A.9) implies

$$Y = (1 - \alpha) (p_H(Q))^{-\eta} C + \alpha (p_H^*(Q))^{-\gamma} C^* \quad (\text{A.16})$$

Finally, the wage Phillips curve (18), together with the production function in (13), $Y = ZN$, implies that long-run wage (and price) inflation rate is equal to

$$\pi_w = \frac{1}{1 - \beta} \kappa_w \left(\frac{v' (Y/Z)}{\frac{1}{\mu_w} \frac{1}{\mu} p_H(Q) Z u'(C)} - 1 \right) \quad (\text{A.17})$$

A *steady state* in our model is characterized by a 4-tuple (Y, Q, C, nfa) for output, the real exchange rate, consumption and the net foreign position, that simultaneously satisfies equations (A.15) and (A.16). A *zero-inflation steady state* additionally restricts this tuple to satisfy equation (A.17) with $\pi_w = 0$. In principle, there is a one-dimensional family of zero-inflation steady states, but stationary models add one equation.

Stationary models. In a stationary model, there exists an aggregate consumption function $C^{ss}(r, p_H(Q) Y)$ such that aggregate consumption in the steady state is only a function of the real

interest rate r , the real exchange rate Q , and the level of output Y , ie:

$$C = \mathcal{C}^{ss}(r, Q, Y) \quad (\text{A.18})$$

Hence, in a stationary model, equation (A.18) gives an additional relation between (Y, Q, C, nfa) that a steady state must satisfy. Given that a zero-inflation steady state must generally satisfy all four of (A.15)–(A.18), there is generally a unique such steady state. Given $r \neq 0$, in a stationary model (A.15) pins down a unique long-run level of nfa for any level of real income $p_H(Q)Y$. By contrast, in a non-stationary model, any long-run level of nfa can be consistent with a given $(r, p_H(Q)Y)$ pair provided that (A.15) is satisfied. RA-IM and TA-IM are nonstationary; all other models in the paper are stationary.

Initial steady state. We pick the initial steady-state of the economy as follows. We choose β^* to deliver our target for the real interest rate r , and normalize $C^* = Q = 1$ so that all relative prices are 1. Then, equation (A.15) implies that $C = Y + r\text{nfa}$, while equation (A.16) implies that $Y = (1 - \alpha)C + \alpha$, so that

$$Y = 1 + \frac{1 - \alpha}{\alpha} \cdot r \cdot \text{nfa} \quad \text{and} \quad C = 1 + \frac{1}{\alpha} \cdot r \cdot \text{nfa} \quad (\text{A.19})$$

We finally set $Z = 1$ and solve for the scaling parameter in labor disutility v' such that equation (A.17) holds for these values of Y and C , given our choice for μ_w, μ , and the initial steady state nfa. In our baseline calibration we set $\text{nfa} = 0$, so that these normalizations imply $Y = C = 1$. Finally, we pick the discount rate at home β so that (A.18) holds with $C = Q = Y = 1$ and our target r .

Unique steady state with $Q = 1$. After transitory foreign impatience or monetary policy shocks, the model always returns to a steady state, though not necessarily a zero inflation steady state. In stationary models, there is a one-dimensional set of such steady states, characterized by the 4-tuples (Y, Q, C, nfa) such that (A.18), (A.15) and (A.16) simultaneously hold. The unique steady state with $Q = 1$ is therefore the initial steady state, which in our baseline features $Y = C = 1$, $\text{nfa} = 0$ and $\pi^w = 0$, i.e. it is also the zero-inflation steady state. In our analysis, we focus on transitions that converge to a steady state with $Q_\infty = 1$. This is equivalent to studying a model in which monetary policy selects a long-run steady state with zero inflation, or one in which monetary policy follows the Taylor rule $\iota_t = r_{ss} + \phi\pi_{t+1} + \epsilon_t$ with $\phi \rightarrow 1$.

In non-stationary models, there is a unique C and nfa consistent with any given path $\{r_t, p_H(Q_t)Y_t\}$ (see for instance the RA-IM model in section A.6). Given $\{r_t^*, r_t\}$, any given long-run Q pins down the path of Q_t and therefore also C , nfa, and Y via (A.16). Hence, in a non-stationary model, there is again a unique steady-state with $Q = 1$, but it generally features non-zero nfa and inflation.

A.4 Complete markets

We follow Auclert et al. (2024b) and set up the model with complete markets with respect to aggregate risk by considering a situation where aggregate shocks occur only at date 0, and are anticipated to do so with some mean-0 distribution at date $t = -1$. Taking the limit as the standard deviation of shocks goes to 0, we recover the same equations as under linearized perfect-foresight, together with an additional condition pinning down portfolios in the steady state.

General setting. We begin by slightly generalizing the setting of section 2 so that it can nest RA, TA and HA models. We do this by allowing for different fixed types of households i , with their own discount factor β_i and asset constraints $\underline{a}_i, \bar{a}_i$. At any time $t = -\infty \dots \infty$, the Bellman equation for household type i is

$$\begin{aligned} \tilde{V}_{it} (s, B^H, B^F, e) &= \max_{c_F, c_H, s', B^{H'}, B^{F'}} u(c_F, c_H) - v(N_t) + \beta_i \mathbb{E}_t \left[\tilde{V}_{it+1} (s', B^{H'}, B^{F'}, e') \mid e \right] \\ \text{s.t.} \quad &P_{Ft} c_F + P_{Ht} c_H + \mathcal{P}_t s' + B^{H'} + \mathcal{E}_t B^{F'} \\ &= (\mathcal{P}_t + D_t) s + (1 + \iota_{t-1}) B^H + (1 + \iota_{t-1}^*) \mathcal{E}_t B^F + e W_t N_t \\ &\mathcal{P}_t s + B^{H'} + \mathcal{E}_t B^{F'} \in [P_t \underline{a}_i, P_t \bar{a}_i] \end{aligned} \quad (\text{A.20})$$

Here, the $\mathbb{E}_t [\cdot]$ operator takes expectations with respect to idiosyncratic risk and, at date $t = -1$, also aggregate risk.

At date 0, an aggregate shock realizes, and there is no uncertainty going forward. Hence, as in the main text, for $t \geq 0$, households solve the perfect foresight problem:

$$\begin{aligned} V_{it} (a^p, e) &= \max_{c, a'} u(c) - v(N_t) + \beta_i \mathbb{E}_t \left[V_{it+1} ((1 + r_t) a', e') \mid e \right] \\ \text{s.t.} \quad &c + a' = a^p + e \frac{W_t}{P_t} N_t \\ &a' \in [\underline{a}_i, \bar{a}_i] \end{aligned} \quad (\text{A.21})$$

We write ϵ for the random variable representing the aggregate shock realizing at date 0, and assume ϵ to have a mean-0 distribution. The problem of a household coming into period -1 with their consolidated position a^p and skill level e is (making the dependence on the realization of ϵ explicit):

$$\begin{aligned} V_{i-1} (a^p, e) &= \max_{c, s', B^{H'}, B^{F'}} u(c) + \beta_i \mathbb{E}_\epsilon \left[V_{0i} \left(\frac{(\mathcal{P}_0(\epsilon) + D_0(\epsilon)) s' + (1 + \iota_0(\epsilon)) B^{H'} + (1 + \iota_0^*(\epsilon)) \mathcal{E}_0(\epsilon) B^{F'}}{P_0(\epsilon)}, e'; \epsilon \right) \mid e \right] \\ \text{s.t.} \quad &P_{-1} c + \mathcal{P}_{-1} s' + B^{H'} + \mathcal{E}_{-1} B^{F'} = P_{-1} a^p + e W_{-1} N_{-1} \\ &\mathcal{P}_{-1} s' + B^{H'} + \mathcal{E}_{-1} B^{F'} \in [P_{-1} \underline{a}_i, P_{-1} \bar{a}_i] \end{aligned} \quad (\text{A.22})$$

We split the problem in (A.22) into two subproblems. First, we consider the portfolio allocation problem, for given asset choice a' :

$$\begin{aligned} W_i (a', e) &\equiv \max \mathbb{E}_\epsilon \left[V_{0i} \left(\frac{(\mathcal{P}_0(\epsilon) + D_0(\epsilon)) s' + (1 + \iota_0(\epsilon)) B^{H'} + (1 + \iota_0^*(\epsilon)) \mathcal{E}_0(\epsilon) B^{F'}}{P_0(\epsilon)}, e'; \epsilon \right) \mid e \right] \\ \text{s.t.} \quad &\frac{\mathcal{P}_{-1} s' + B^{H'} + \mathcal{E}_{-1} B^{F'}}{P_{-1}} = a' \end{aligned} \quad (\text{A.23})$$

We note that the problem in (A.22) rewrites as:

$$\begin{aligned}
V_{i-1}(a^p, e) &= \max_{c, a'} u(c) + \beta_i W_i(a', e) \\
\text{s.t. } c + a' &= a^p + e \frac{W_{-1}}{P_{-1}} N_{-1} \\
a' &\in [\underline{a}_i, \bar{a}_i]
\end{aligned} \tag{A.24}$$

which is similar to the steady state version of (9), with the value function $W_i(a', e)$ instead of $\mathbb{E}[V_{i,ss}((1+r_{ss})a', e')|e]$ and aggregate wage income $\frac{W_{-1}}{P_{-1}}N_{-1}$ instead of $\frac{W_{ss}}{P_{ss}}N_{ss}$.

We write the date-0 distribution of shocks as $\epsilon = \bar{\epsilon}\sigma$, and consider a first-order expansion in σ of the solution to the model. Since all derivatives of objects in the model after date 0 are multiples of $\bar{\epsilon}$, and since $\mathbb{E}_{\bar{\epsilon}}[\bar{\epsilon}] = 0$, derivatives of objects with respect to σ prior to date 0 are zero. Hence, we have that

$$\begin{aligned}
W_i(a', e) &= \mathbb{E}_\epsilon [V_{i,ss}((1+r_{ss})a', e')|e] + O(\sigma^2) \\
\frac{W_{-1}}{P_{-1}}N_{-1} &= \frac{W_{ss}}{P_{ss}}N_{ss} + O(\sigma^2)
\end{aligned}$$

so to first order in σ , the problem in (A.24) is the steady state problem. In other words, to first order, the consumption-savings problem at date -1 is the same as the consumption-savings problem in the steady state with no aggregate risk. This is an instance of first-order certainty equivalence.

Rewriting the portfolio choice problem. Define $\omega_s \equiv \frac{\mathcal{P}_{-1}s'}{P_{-1}}$, $\omega_{B^H} \equiv \frac{B^H}{P_{-1}}$, and $\omega_{B^F} \equiv \frac{\mathcal{E}_{-1}B^F}{P_{-1}}$, to be total exposure to stocks, domestic and foreign bonds in the portfolio. Note that

$$\omega_s + \omega_{B^H} + \omega_{B^F} = a'$$

and that, defining the real stock return as

$$1 + r^s(\epsilon) \equiv \frac{(\mathcal{P}_0(\epsilon) + D_0(\epsilon))}{\mathcal{P}_{-1}} \frac{P_{-1}}{P_0(\epsilon)}$$

and denoting *excess* real returns of the domestic and foreign bond over the stock as

$$\begin{aligned}
r^{x,B^H}(\epsilon) &\equiv (1 + \iota_0(\epsilon)) \frac{P_{-1}}{P_0(\epsilon)} - (1 + r^s(\epsilon)) \\
r^{x,B^F}(\epsilon) &\equiv (1 + \iota_0^*(\epsilon)) \frac{\mathcal{E}_0(\epsilon)}{\mathcal{E}_{-1}} \frac{P_{-1}}{P_0(\epsilon)} - (1 + r^s(\epsilon))
\end{aligned}$$

we can rewrite real cash-on-hand coming into period 0 as:

$$\begin{aligned}
&\frac{(\mathcal{P}_0(\epsilon) + D_0(\epsilon))s' + (1 + \iota_0(\epsilon))B^{H'} + (1 + \iota_0^*(\epsilon))\mathcal{E}_0(\epsilon)B^{F'}}{P_0(\epsilon)} \\
&= r^{x,B^H}(\epsilon)\omega_{B^H} + r^{x,B^F}(\epsilon)\omega_{B^F} + (1 + r^s(\epsilon))a'
\end{aligned} \tag{A.25}$$

Next, define the value function

$$J(x, a', e; \epsilon) \equiv \mathbb{E}[V_0(x + (1 + r^s(\epsilon))a', e'; \epsilon)|e] \tag{A.26}$$

as a function of excess returns x over the domestic stock.⁵¹ The portfolio problem in (A.23) can then be rewritten as

$$W(a', e) = \max_{\omega_s, \omega_{B^H}, \omega_{B^F}} \mathbb{E}_\epsilon \left[J \left(r^{x, B^H}(\epsilon) \omega_{B^H} + r^{x, B^F}(\epsilon) \omega_{B^F}, a', e; \epsilon \right) \right] \quad (\text{A.27})$$

The first-order condition for ω_k in this problem is:

$$\mathbb{E}_\epsilon \left[r^{x, k}(\epsilon) J_x \left(r^{x, B^H}(\epsilon) \omega_{B^H} + r^{x, B^F}(\epsilon) \omega_{B^F}, a', e; \epsilon \right) \right] = 0 \quad k \in \{B^H, B^F\} \quad (\text{A.28})$$

Consider two agents i, j , with different (a', e) , choosing portfolios ω_k^i, ω_k^j . Dividing by the value of J_x at $\epsilon = 0$, and subtracting the two conditions (A.28) gives:

$$\mathbb{E}_\epsilon \left[r^{x, k}(\epsilon) g^{ij}(\epsilon) \right] = 0 \quad (\text{A.29})$$

where we have defined

$$\begin{aligned} g^{ij}(\epsilon) &\equiv \frac{J_x^i \left(r^{x, B^H}(\epsilon) \omega_{B^H}^i + r^{x, B^F}(\epsilon) \omega_{B^F}^i; \epsilon \right)}{J_x^i(0; 0)} \\ &\quad - \frac{J_x^j \left(r^{x, B^H}(\epsilon) \omega_{B^H}^j + r^{x, B^F}(\epsilon) \omega_{B^F}^j; \epsilon \right)}{J_x^j(0; 0)} \end{aligned}$$

Perturbation to get the portfolio. We want to perform a second-order expansion of (A.29) around $\sigma = 0$, where $\epsilon = \bar{\epsilon}\sigma$. Given any k, i, j , define

$$f(\sigma) \equiv \mathbb{E}_{\bar{\epsilon}} \left[r^{x, k}(\sigma \bar{\epsilon}) g^{ij}(\sigma \bar{\epsilon}) \right]$$

Note that we have:

$$f'(\sigma) = \mathbb{E}_\epsilon \left[\bar{\epsilon} r_{\epsilon}^{x, k}(\sigma \bar{\epsilon}) g^{ij}(\sigma \bar{\epsilon}) + \bar{\epsilon} r^{x, k}(\sigma \bar{\epsilon}) g_{\epsilon}^{ij}(\sigma \bar{\epsilon}) \right]$$

so, since $\mathbb{E}_\epsilon [\bar{\epsilon}] = 0$, we have $f'(0) = 0$. Moreover, we have:

$$f''(\sigma) = \mathbb{E}_\epsilon \left[\bar{\epsilon}^2 r_{\epsilon \epsilon}^{x, k}(\sigma \bar{\epsilon}) f^{ij}(\sigma \bar{\epsilon}) + \bar{\epsilon} r_{\epsilon}^{x, k}(\sigma \bar{\epsilon}) g_{\epsilon}^{ij}(\sigma \bar{\epsilon}) + \bar{\epsilon}^2 r^{x, k}(\sigma \bar{\epsilon}) g_{\epsilon \epsilon}^{ij}(\sigma \bar{\epsilon}) \right]$$

so, since $g^{ij}(0) = r^{x, k}(0) = 0$, we have

$$f''(0) = \mathbb{E}_\epsilon \left[\bar{\epsilon}^2 r_{\epsilon \epsilon}^{x, k}(0) f^{ij}(0) + \bar{\epsilon}^2 r_{\epsilon}^{x, k}(0) g_{\epsilon}^{ij}(0) + \bar{\epsilon}^2 r^{x, k}(0) g_{\epsilon \epsilon}^{ij}(0) \right] = \mathbb{E}_\epsilon [\bar{\epsilon}^2] r_{\epsilon}^{x, k}(0) g_{\epsilon}^{ij}(0)$$

Hence, (A.29) implies that, to second order in σ , we have

$$\mathbb{E}_\epsilon [\bar{\epsilon}^2] r_{\epsilon}^{x, k}(0) g_{\epsilon}^{ij}(0) = 0$$

⁵¹Note that we could have picked any benchmark asset against which to measure excess returns. Here we choose the domestic stock, which is natural here because 100% domestic stock portfolios is our incomplete markets assumption.

Provided that the excess return of at least one of the two assets $k \in \{B^H, B^F\}$ is sensitive to ϵ to first order, ie $r_\epsilon^{x,k}(0) \neq 0$, we therefore must have, at the optimal portfolio choice:

$$g_\epsilon^{ij}(0) = 0$$

This also says that, to first order in σ , we must have *for every* ϵ

$$d \log J_x^i(r^{x,B^H}(\epsilon) \omega_{B^H}^i + r^{x,B^F}(\epsilon) \omega_{B^F}^i, \epsilon) = d \log J_x^j(r^{x,B^H}(\epsilon) \omega_{B^H}^j + r^{x,B^F}(\epsilon) \omega_{B^F}^j, \epsilon) \quad (\text{A.30})$$

for all agents i, j . Note that, by the envelope theorem applied to (A.26), we also have:

$$J_x^i(r^{x,B^H}(\epsilon) \omega_{B^H}^i + r^{x,B^F}(\epsilon) \omega_{B^F}^i, \epsilon) \equiv \mathbb{E} \left[u' \left(c^i(\epsilon, e') \right) | e \right]$$

where $c^i(\epsilon, e')$ is the consumption of individual i if aggregate and individual states are realized as (ϵ, e') .

So equation (A.30) says that, at the optimal portfolio choice, for any realization of ϵ (which we index by time for convenience), there exists a constant λ_0 such that, for all (a', e) , we have:

$$\frac{\mathbb{E}[u'(c_{0,i}(a_0^p, e')) | e]}{\mathbb{E}[u'(c_{ss,i}(a_{ss}^p, e')) | e]} = \lambda_0 \quad (\text{A.31})$$

Foreigner's problem. The same logic applies to foreigners, but their marginal utility in terms of the domestic consumption bundle adds a factor of $\frac{\mathcal{B}_0}{Q_0}$, where \mathcal{B}_0 is the foreigner's intertemporal utility shifter and Q_0 is the real exchange rate, converting the domestic consumption basket into the foreigner's consumption basket. Hence, (A.31) for the foreigner reads

$$\frac{\frac{\mathcal{B}_0}{Q_0} u'(C_0^*)}{\frac{\mathcal{B}_{ss}}{Q_{ss}} u'(C_{ss}^*)} = \lambda_0$$

Since we normalize $\mathcal{B}_{ss} = Q_{ss} = 1$ and since our assumptions imply that $C_0^* = C_{ss}^*$, this simplifies to

$$\frac{\mathcal{B}_0}{Q_0} = \lambda_0 \quad (\text{A.32})$$

Equating λ_0 in (A.31) and (A.32), in the special case in which there is a single type of agent, we finally obtain (24). The case with many types of agents follows straightforwardly.

A.5 Proof of Proposition 1

We show that aggregate domestic consumption depends only on the path of the value of home production deflated by the domestic CPI, $\left\{ \frac{P_{Ht}}{P_t} Y_t \right\}$, and the path of ex-ante real interest rates $\{r_t\}$.

We first consider policies. Observe that the only time-varying aggregate inputs that affect consumption decisions in (9) are real labor earnings $\frac{W_t}{P_t} N_t$, which equals $\frac{1}{\mu} \frac{P_{Ht}}{P_t} Y_t$, and ex-ante real interest rates r_t . Hence, for all $t \geq 0$, consumption policies c_t over the states (a^p, e) depend only on the time path of future $\left\{ \frac{P_{Hs}}{P_s} Y_s \right\}$ and $\{r_s\}$, for $s \geq t$.

We next consider aggregation for dates $t \geq 1$. For these dates, the optimal policy from (9) induces a law of motion from $\mathcal{D}_{t-1}(a^p, e)$ to $\mathcal{D}_t(a^p, e)$. Given our result on policies, we therefore also have that the distribution $\mathcal{D}_t(a^p, e)$ depends only on $\left\{ \frac{P_{Hs}}{P_s} Y_s \right\}$ and $\{r_s\}$ for $s \geq 0$. Hence, for

all $t \geq 1$, aggregate consumption $C_t \equiv \int c_t(a^p, e) d\mathcal{D}_t(a^p, e)$ only depends on $\{\frac{P_{Hs}}{P_s} Y_s\}$ and $\{r_s\}$ for $s \geq 0$

We finally consider aggregation at $t = 0$. Here, we consider separately the incomplete and complete markets cases.

Incomplete markets. Since domestic agents are assumed to initially hold 100% of their portfolios in domestic stocks, the only other determinant of aggregate consumption is the date-0 real value of the stock, which is $\frac{P_0 + D_0}{P_0}$, determining a_0^p through (8). Iterating forward on (7), this equals the present value of the profit share of output:

$$\frac{P_0 + D_0}{P_0} = \sum_{t \geq 0} \prod_{s \leq t} \left(\frac{1}{1 + r_s} \right) \left(1 - \frac{1}{\mu} \right) \frac{P_{Ht}}{P_t} Y_t \quad (\text{A.33})$$

Hence, aggregate consumption at date 0 is also a function of $\{\frac{P_{Hs}}{P_s} Y_s\}$ and $\{r_s\}$, for $s \geq 0$. In conclusion, we can write $\mathcal{C}_t \left(\left\{ \frac{P_{Hs}}{P_s} Y_s \right\}, \{r_s\} \right)$ for all $t \geq 0$.

Complete markets. Here, equation (30) implies that we have $Q_0 = \mathcal{B}_0 \cdot \prod_{s \geq 0} \left(\frac{1 + r_{ss}}{1 + r_s} \right)$, so that (24) becomes

$$\mathbb{E} \left[c_0 (a_0^p, e')^{-\sigma} | e \right] = \left[\prod_{s \geq 0} \left(\frac{1 + r_{ss}}{1 + r_s} \right) \right]^{-1} \mathbb{E} \left[c_{ss} (a_{ss}^p, e')^{-\sigma} | e \right]$$

for every agent choosing a', e . This equation determines a_0^p in (9) as a function of the time path of $\{\frac{P_{Hs}}{P_s} Y_s\}$ and $\{r_s\}$. Hence, again, we can write aggregate consumption at date t as $\mathcal{C}_t \left(\left\{ \frac{P_{Hs}}{P_s} Y_s \right\}, \{r_s\} \right)$ for all $t \geq 0$. This completes the proof.

A.6 Representative-agent model

In the representative agent model, under both CM and IM, there is a single type of agent facing no idiosyncratic income risk ($e = 1$), and no borrowing or saving constraints ($\underline{a} = -\infty$ and $\bar{a} = \infty$). The problem in (A.21) therefore reduces to:

$$\begin{aligned} V_t(A^p) &= \max_{C, A'} u(C) - v(N_t) + \beta V_{t+1}((1 + r_t) A') \\ \text{s.t.} \quad C + A' &= A^p + \frac{W_t}{P_t} N_t \end{aligned} \quad (\text{A.34})$$

Given initial A_0^p , the solution implies the Euler equation (25) for $t \geq 0$, the budget constraint (26) for $t \geq 1$, and the initial budget constraint:

$$C_0 + A_0 = A_0^p + \frac{W_0}{P_0} N_0 \quad (\text{A.35})$$

Starting from (A.35), iterating on the flow budget constraints (26), and using the no-Ponzi condi-

tion $\lim_{t \rightarrow \infty} \prod_{s \leq t} (1 + r_s)^{-1} A_t = 0$, we obtain the country's present value budget constraint,

$$\sum_{s \geq 0} q_s C_s = A_0^p + \sum_{s \geq 0} q_s \frac{W_s}{P_s} N_s \quad (\text{A.36})$$

where $q_s \equiv \prod_{u=0}^{s-1} \frac{1}{1+r_u}$ is the price of date- s consumption in units of date-0 consumption. Moreover, the Euler equation (25) implies that for all $t \geq 0$,

$$C_t = (q_t)^{-\frac{1}{\sigma}} \beta^{\frac{t}{\sigma}} C_0 \quad (\text{A.37})$$

Equation (A.11) implies that $\frac{W_t}{P_t} N_t = \frac{1}{\mu} p_H(Q_t) Y_t$, where $p_H(\cdot)$ is the relationship defined in (A.7). Using this relation, we solve (A.36) and (A.37) to obtain C_0 as a function of A_0^p and the time paths of $\{q_t\}$ and $\{p_H(Q_t) Y_t\}$:

$$C_0(A_0^p) = \frac{1}{\sum_{s \geq 0} (q_s)^{\frac{\sigma-1}{\sigma}} \beta^{\frac{s}{\sigma}}} \left(A_0^p + \frac{1}{\mu} \sum_{s \geq 0} q_s p_H(Q_s) Y_s \right) \quad (\text{A.38})$$

Finally, we have from equation (8) that A_0^p relates to the initial portfolio (s, B^H, B^F) via

$$A_0^p = (p_0 + d_0) s + (1 + \iota_{-1}) \frac{1}{P_0} B^H + (1 + \iota_{-1}^*) \frac{\mathcal{E}_0}{P_0} B^F \quad (\text{A.39})$$

where $\iota_{-1} = \iota_{-1} = r_{ss}$, $\mathcal{E}_0/P_0 = Q_0$, and the real stock price $p_0 + d_0$ is given, iterating on the asset pricing condition (7) and using equation (A.12), by

$$p_0 + d_0 = \sum_{s \geq 0} q_s \left(1 - \frac{1}{\mu} \right) p_H(Q_s) Y_s \quad (\text{A.40})$$

Given $\{r_t, \mathcal{B}_t\}$, which determine $\{q_t\}$ and $\{Q_t\}$ via (A.14), and an initial portfolio (s, B^H, B^F) , a RA equilibrium is a sequence $\{C_t, Y_t\}$ that satisfies the Euler equation (25), the goods market clearing condition (A.9), as well as (A.38), (A.39) and (A.40).

Note that in a steady state, the RA Euler equation (25) implies $\beta(1 + r_{ss}) = 1$, and equation (A.13) implies that $\beta^*(1 + r_{ss}) = 1$, so in particular we have $\beta = \beta^*$. Combining equation (A.13) with the Euler equation (25) and using $\beta = \beta^*$, we find that for all $t \geq 0$,

$$\frac{Q_{t+1}}{\mathcal{B}_{t+1}} C_{t+1}^{-\sigma} = \frac{Q_t}{\mathcal{B}_t} C_t^{-\sigma} \quad (\text{A.41})$$

Hence, in both RA-IM and RA-CM, the dynamics of consumption are similar. The key difference between these two cases is the way in which the initial level of consumption C_0 is determined.

Complete markets (RA-CM). With complete markets, the level of initial consumption C_0 is pinned down by the Backus-Smith condition, and the level of A_0^p is endogenously determined to make sure that (A.38) holds given this C_0 .

More specifically, the Backus-Smith condition (24) implies that the initial level of consumption is

$$\frac{Q_0}{\mathcal{B}_0} C_0^{-\sigma} = C_{ss}^{-\sigma} \quad (\text{A.42})$$

Combining (A.42) with (A.41) implies that, for all $t \geq 0$,

$$\frac{Q_t}{\mathcal{B}_t} C_t^{-\sigma} = C_{ss}^{-\sigma} \quad (\text{A.43})$$

which is equation (27) in the main text. Given C_t and Q_t , the goods market clearing condition (A.9) implies the level of Y_t . All the elements of (A.38) are known except for A_0^p . This allows to solve for the endogenous level of A_0^p . Finally, we can solve for asset prices $p_0 + d_0$, P_0 , and \mathcal{E}_0 , and find a portfolio (s, B^H, B^F) that ensures that equation (A.39) is satisfied with this A_0^p .

Note in particular that (A.42) holds in the steady state, which ensures that the long-run level of consumption is the steady state level, $C_\infty = C_{ss}$. In the notation of section A.3, there is a simple \mathcal{C}^{ss} function giving the long-run level of consumption as a function of Q ,

$$C = \mathcal{C}^{ss}(r, Q, Y) = Q^{1/\sigma} C_{ss}$$

Hence, the model is stationary, so there is a unique steady state with $Q = 1$, and this is the initial steady state.

Incomplete markets (RA-IM). With incomplete markets, the exogenous initial portfolio pins down A_0^p , which implies the initial level of consumption C_0 via (A.38).

More specifically, since all wealth is invested in domestic equity, the initial portfolio is $s = 1$. Equations (A.36), (A.39) and (A.40) imply that

$$\sum_{s \geq 0} q_s C_s = \sum_{s \geq 0} q_s p_H(Q_s) Y_s \quad (\text{A.44})$$

which is the country's intertemporal budget constraint under incomplete markets. Equations (A.38), (A.39) and (A.40) with $s = 1$ imply that

$$C_0 = \frac{1}{\sum_{s \geq 0} (q_s)^{\frac{\sigma-1}{\sigma}} \beta^{\frac{s}{\sigma}}} \left(\sum_{s \geq 0} q_s p_H(Q_s) Y_s \right) \quad (\text{A.45})$$

and from (A.41) we obtain the level of consumption C_t at all t .

Equation (A.45) and (A.41) determine the entire path of C_t given $\{r_t, \mathcal{B}_t, Q_t, Y_t\}$. An RA-IM equilibrium is a sequence $\{Y_t\}$ such that market clearing (A.9) holds at all t .

Note that here, applying (A.41), the long-run level of consumption C_∞ is equal to C_0 in (A.45), and hence depends on the entire sequence of shocks. This reflects the non-stationarity of the model. There is still a unique equilibrium with $Q = 1$, but in contrast to the RA-CM model, in general the long level of (Y, C, nfa) differs from the initial steady state.

M matrices. We now derive the M matrices for RA-CM and RA-IM.

Note first that, since $\mathbf{M} = \frac{\partial C_t}{\partial Y_s}$ holds $r_t = r_{ss}$ constant, we can replace $q_s = \left(\frac{1}{1+r}\right)^s = \beta^s$ in the equations above. Equation (30) also implies that we have $Q_t = \mathcal{B}_t$.

In the RA-CM model, equation (A.43) therefore implies that we have $C_t = C_{ss}$ for all $t \geq 0$. In particular, this implies $\frac{\partial C_t}{\partial Y_s} = 0$ for all s and t . Hence, we have $\mathbf{M}^{RA-CM} = 0$.

In the RA-IM model, using $q_t = \beta^t$ and $Q_t = \mathcal{B}_t$ into equations (A.41) and (A.45), we have

$$C_t = (1 - \beta) \sum_{s \geq 0} \beta^s \frac{P_{Hs}}{P_s} Y_s$$

hence, the matrix $M_{ts} \equiv \frac{\partial C_t}{\partial \left(\frac{P_{Hs}}{P_s} Y_s \right)}$ is given by:

$$\mathbf{M}^{RA-IM} = \begin{pmatrix} 1 - \beta & (1 - \beta) \beta & (1 - \beta) \beta^2 & \dots \\ 1 - \beta & (1 - \beta) \beta & (1 - \beta) \beta^2 & \dots \\ 1 - \beta & (1 - \beta) \beta & (1 - \beta) \beta^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = (1 - \beta) \cdot \mathbf{1} \mathbf{q}' = \frac{\mathbf{1} \mathbf{q}'}{\mathbf{q}' \mathbf{1}} \quad (\text{A.46})$$

Equivalence of the RA-CM model with Galí and Monacelli. Here, we log-linearize the equilibrium conditions of the RA-CM model in the spirit of [Galí and Monacelli \(2005\)](#). Denote $b_t \equiv d \log \frac{B_t}{B_{ss}}$, $c_t \equiv d \log \frac{C_t}{C_{ss}}$, $q_t \equiv d \log \frac{Q_t}{Q_{ss}}$, $e_t \equiv d \log \frac{\varepsilon_t}{\varepsilon_{ss}}$ and so on, for log deviations of aggregate variables from their steady state. The equation for the price index (4) log-linearizes as

$$p_t = \alpha p_{Ft} + (1 - \alpha) p_{Ht}$$

so, using that $p_{Ft} = e_t$ and $q_t = e_t - p_t$, we obtain

$$p_{Ht} - p_t = -\frac{\alpha}{1 - \alpha} q_t \quad (\text{A.47})$$

in particular, the difference between PPI inflation $\pi_{Ht} = p_{Ht} - p_{Ht-1}$ and CPI inflation $\pi_t = p_t - p_{t-1}$ is given by

$$\pi_{Ht} - \pi_t = -\frac{\alpha}{1 - \alpha} \Delta q_t \quad (\text{A.48})$$

where $\Delta q_t = q_t - q_{t-1}$. Log-linearizing the demand equation (33), we have

$$y_t = (1 - \alpha) \left(\eta \frac{\alpha}{1 - \alpha} q_t + c_t \right) + \alpha \gamma \frac{1}{1 - \alpha} q_t$$

and using (A.47), together with the definition of χ in (36), we find that output is

$$y_t = \frac{\alpha}{1 - \alpha} \left(\underbrace{(1 - \alpha) \eta + \gamma}_{\chi} \right) q_t + (1 - \alpha) c_t \quad (\text{A.49})$$

Log-linearizing the Euler equation (25), we obtain

$$c_t = c_{t+1} - \frac{1}{\sigma} (\iota_t - \pi_{t+1} - \rho) \quad (\text{A.50})$$

where $\rho = -\log \beta$, while the log-linearized Backus-Smith condition (27) reads

$$c_t = \frac{1}{\sigma} (q_t - b_t) \quad (\text{A.51})$$

Finally, to derive the Phillips curve, combine equations (18) and (14) to obtain that $\pi_{Ht} = \pi_{wt}$, with

$$\pi_{Ht} = \kappa_w \left(\frac{v'(N_t)}{\frac{Z}{\mu \mu_w} \frac{P_{Ht}}{P_t} u'(C_t)} - 1 \right) + \beta \pi_{Ht+1}$$

Log-linearizing around the steady state with zero inflation so that $\frac{Z}{\mu\mu_w} \frac{P_H}{P} \frac{u'(C)}{v'(N)} = 1$, this results in

$$\pi_{Ht} = \kappa_w (\varphi n_t + \sigma c_t - (p_{Ht} - p_t)) + \beta \pi_{t+1}$$

Combining this equation with production (13),

$$y_t = n_t$$

and the expression for the relative price of home goods in (A.47), we obtain

$$\pi_{Ht} = \kappa_w \left(\varphi y_t + \sigma c_t + \frac{\alpha}{1-\alpha} q_t \right) + \beta \pi_{t+1} \quad (\text{A.52})$$

This is exactly equations (32) and (33) in Gali-Monacelli (since we do not consider productivity or foreign spending shocks, and the terms of trade in their notation is $s_t = e_t - p_{Ht} = q_t + p_t - p_{Ht} = q_t + \frac{\alpha}{1-\alpha} q_t = \frac{1}{1-\alpha} q_t$). In particular there was no loss of generality in considering sticky wages rather than sticky prices. The equivalence between the slopes of the Phillips curves obtains provided that we set

$$\kappa_w = \frac{(1-\beta\theta)(1-\theta)}{\theta} \quad (\text{A.53})$$

where θ is the Calvo probability of keeping a domestic price fixed in Gali-Monacelli.

Equations (A.48), (A.49), (A.50), (A.51), and (A.52), characterize the log-linear model, delivering $\pi_t, \pi_{Ht}, y_t, c_t, q_t$ as a function of the foreign preference shock b_t for a given monetary policy rule for ι_t .

Reduced-form two-equation system. To see the connection with the equations derived in [Galí and Monacelli \(2005\)](#), observe that we can use the Backus-Smith condition (A.51) and the market clearing condition (A.49) to solve for c_t and q_t as a function of y_t and the exogenous variable b_t . This gives us

$$\begin{pmatrix} 1 & -\frac{1}{\sigma} \\ 1-\alpha & \frac{\alpha}{1-\alpha}\chi \end{pmatrix} \begin{pmatrix} c_t \\ q_t \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sigma}b_t \\ y_t \end{pmatrix}$$

which can be inverted to read

$$\begin{pmatrix} c_t \\ q_t \end{pmatrix} = (1-\alpha) \sigma_\alpha \begin{pmatrix} -\frac{\alpha}{1-\alpha}\frac{\chi}{\sigma}b_t + \frac{1}{\sigma}y_t \\ \frac{1-\alpha}{\sigma}b_t + y_t \end{pmatrix}$$

where we have defined, as in in Gali-Monacelli,

$$\sigma_\alpha \equiv \frac{\sigma}{1-\alpha + \alpha\omega} \quad \text{with} \quad \omega \equiv \sigma\chi - (1-\alpha)$$

This implies that

$$\sigma c_t + \frac{\alpha}{1-\alpha} q_t = \sigma_\alpha \left(y_t - \frac{\alpha}{\sigma} \omega b_t \right) \quad (\text{A.54})$$

Since substituting (A.48) into the Euler equation (A.50) implies

$$\sigma \Delta c_{t+1} + \frac{\alpha}{1-\alpha} \Delta q_{t+1} = (\iota_t - \pi_{Ht+1} - \rho)$$

we have, using (A.54), that

$$\sigma_\alpha \Delta y_{t+1} = \iota_t - \pi_{Ht+1} - \rho + \alpha \frac{\sigma_\alpha}{\sigma} \omega \Delta b_{t+1} \quad (\text{A.55})$$

Next, substituting (A.54) into (A.52), the Phillips curve is

$$\pi_{Ht} = \kappa_w \left((\varphi + \sigma_\alpha) y_t - \alpha \frac{\sigma_\alpha}{\sigma} \omega b_t \right) + \beta \pi_{Ht+1}$$

Under flexible wages, output solves

$$(\varphi + \sigma_\alpha) y_t^n = \alpha \frac{\sigma_\alpha}{\sigma} \omega b_t \quad (\text{A.56})$$

hence, the Phillips curve is

$$\pi_{Ht} = \kappa_w (\varphi + \sigma_\alpha) (y_t - y_t^n) + \beta \pi_{Ht+1} \quad (\text{A.57})$$

which is exactly equation (36) in [Galí and Monacelli \(2005\)](#) augmented to include foreign discount factor shocks, which affect the natural output level y_t^n according to (A.56). Finally, equation (A.55) implies that, defining the natural real interest rate as

$$r_t^n \equiv \rho + \sigma_\alpha \Delta y_{t+1}^n - \alpha \frac{\sigma_\alpha}{\sigma} \omega \Delta b_{t+1} \quad (\text{A.58})$$

the Euler equation reads

$$\Delta y_{t+1} = \sigma_\alpha^{-1} (\iota_t - \pi_{Ht+1} - r_t^n) \quad (\text{A.59})$$

which is the counterpart of equation (37) in [Galí and Monacelli \(2005\)](#), where the natural PPI-based real rate of interest is affected by foreign discount factor shocks per equation (A.56).

A.7 Two-agent model

In the two-agent model, there is a fixed fraction $\lambda \mu$ of "hand-to-mouth" agents who are constrained to have zero total assets, $\mathcal{P}_t s_t^c + B_t^{H,c} + B_t^{F,c} = 0$ (as will be clear, we make this assumption so that a fraction λ of income goes to the hand-to-mouth agents in the steady state). The other agents are permanent income agents solving the Bellman equation (A.34). In the general notation of equation (A.20), constrained agents have $\underline{a}^c = 0$, $\bar{a}^c = 0$ and unconstrained agents have $\underline{a}^u = -\infty$, $\bar{a}^u = \infty$.

As in our baseline setup of section 2, union labor demand imposes that constrained and unconstrained work an equal amount of hours. Thus,

$$N_t^u = N_t^c = N_t$$

For unconstrained agents, consumption c_t^u is determined just like in section A.6. In particular, in the TA-CM model, unconstrained agent consumption is given by the equivalent of equation (A.43) at all $t \geq 0$,

$$(c_t^u)^{-\sigma} = (c_{ss}^u)^{-\sigma} \frac{\mathcal{B}_t}{Q_t} \quad (\text{A.60})$$

In the TA-IM model, we assume $B_t^{H,u} = B_t^{F,u} = 0$. c_0^u is then obtained by combining equations (A.38) and (A.39) with the initial stock position $s^u = \frac{1}{1-\lambda\mu}$ that ensures stock market clearing. This

implies that:

$$c_0^u = \frac{1}{\sum_{s \geq 0} (q_s)^{\frac{\sigma-1}{\sigma}} \beta^{\frac{s}{\sigma}}} \left(\frac{1 - \frac{1}{\mu}}{1 - \lambda\mu} + \frac{1}{\mu} \right) \left(\sum_{s \geq 0} q_s \frac{P_{Hs}}{P_s} Y_s \right) \quad (\text{A.61})$$

and then c_t^u at all t follows from the recursion $\frac{Q_{t+1}}{B_{t+1}} (c_{t+1}^u)^{-\sigma} = \frac{Q_t}{B_t} (c_t^u)^{-\sigma}$.

For constrained agents, the constraint that net wealth has to be zero imposes $A_t^{p,c} = A_t^c = 0$ for all $t \geq 1$, so that the budget constraint implies

$$c_t^c = \frac{W_t}{P_t} N_t = \frac{1}{\mu} \frac{P_{Ht}}{P_t} Y_t \quad (\text{A.62})$$

Moreover, at date 0, we have

$$c_0^c = A_0^{p,c} + \frac{W_0}{P_0} N_0$$

In the TA-IM model, we exogenously assume the initial portfolio $B^{H,c} = B^{F,c} = 0$, so the no-wealth constraint also imposes $s^c = 0$. Hence, $A_0^p = 0$, and equation (A.62) holds for all $t \geq 0$. In the TA-CM model, by contrast, we let constrained agents choose their portfolio in the steady state to hedge aggregate shocks. Hence, their consumption at date 0 is pinned down by the Backus-Smith condition,

$$(c_0^c)^{-\sigma} = (c_{ss}^c)^{-\sigma} \frac{B_0}{Q_0} \quad (\text{A.63})$$

while, for $t \geq 1$, it is given by equation (A.62). This makes period 0 special in the TA-CM model.

In all cases, aggregate consumption is given by

$$C_t = (1 - \lambda\mu) c_t^u + \lambda\mu \cdot c_t^c$$

In the steady state, we have $\beta = \beta^*$, as in the RA model, as well as

$$\begin{aligned} c_{ss}^c &= \frac{1}{\mu} \frac{P_{Hss}}{P_{ss}} Y_{ss} = \frac{1}{\mu} \\ c_{ss}^u &= \left(\frac{1 - \frac{1}{\mu}}{1 - \lambda\mu} + \frac{1}{\mu} \right) \frac{P_{Hss}}{P_{ss}} Y_{ss} = \frac{1 - \lambda}{1 - \lambda\mu} \end{aligned}$$

M matrices. Since \mathbf{M} holds $r_t = r_{ss}$ constant, as in the RA model, we have $q_t = \beta^t$ and $Q_t = B_t$.

In the TA-CM model, equations (A.60) and (A.63) therefore imply $c_t^u = c_{ss}^u$ and $c_0^c = c_{ss}^c$. Moreover, equation (A.62) determines c_t^c for $t \geq 1$. Aggregation then implies that $M_{t,s} = \frac{\partial C_t}{\partial Y_s} = \frac{\lambda\mu}{\mu} = \lambda$ for $t \geq 1$. Hence, we have the following iMPC matrix for the TA-CM model:

$$\mathbf{M}^{TA-CM} = \lambda \begin{pmatrix} 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (\text{A.64})$$

In the TA-IM model, using $q_t = \beta^t$ and $Q_t = \mathcal{B}_t$ into equation (A.61) gives

$$c_t^u = \frac{1-\lambda}{1-\lambda\mu}(1-\beta) \sum_{s \geq 0} \beta^s \frac{P_{Hs}}{P_s} Y_s$$

Meanwhile, equation (A.62) determines c_t^c at all dates. Aggregating, we therefore obtain

$$M_{t,s}^{TA-IM} = (1-\lambda)(1-\beta) \sum_{s \geq 0} \beta^s + \lambda 1_{t=s}$$

Note that this implies that the \mathbf{M} matrix of the TA-IM model can just be written as the convex combination of the RA-IM \mathbf{M} matrix and the identity matrix,

$$\mathbf{M}^{TA-CM} = (1-\lambda)\mathbf{M}^{RA-IM} + \lambda \mathbf{I} = (1-\lambda) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} + \lambda \mathbf{I} \quad (\text{A.65})$$

A.8 Heterogeneous-agent model

Here, we describe how we solve for the \mathbf{M} matrices of the HA-IM and the HA-CM model, ie the Jacobian of the consumption function with respect to real income. Recall that, the \mathbf{M} matrix holds $r_t = r_{ss}$ fixed, which implies in particular $Q_t = \mathcal{B}_t$ at all $t \geq 0$.

HA-IM model. In the IM model, given our baseline assumption that initial portfolios are 100% in domestic equity, equation (8) implies that

$$a_0^p = \frac{P_0 + D_0}{P_0} s \quad (\text{A.66})$$

Given (9), aggregate consumption at each date t is only a function of the sequence $\left\{ \frac{W_s}{P_s} N_s \right\}$ of aggregate labor income and the initial distribution of a_0^p . The latter, in turn, is solely influenced by $\frac{P_0 + D_0}{P_0}$ due to (A.66). Hence, we obtain a function \tilde{C}_t expressing aggregate consumption at t in terms of these two aggregate variables alone:

$$\tilde{C}_t \left(\left\{ \frac{W_s}{P_s} N_s \right\}, \frac{P_0 + D_0}{P_0} \right)$$

Let $M_{t,s}^{labor} \equiv \frac{\partial \tilde{C}_t}{\partial \left(\frac{W_s}{P_s} N_s \right)}$ denote the Jacobian of consumption to labor income, and $m_t^{cap} \equiv \frac{\partial \tilde{C}_t}{\partial \left(\frac{P_0 + D_0}{P_0} \right)}$ the vector of consumption responses to capital gains. These are the standard definitions of intertemporal marginal propensities to consume, as introduced in [Auclert et al. \(2024a\)](#), and can be calculated using the standard sequence-space algorithm described in [Auclert et al. \(2021\)](#). Now note that we have $\frac{W_s}{P_s} N_s = \frac{1}{\mu} \frac{P_{Hs}}{P_s} Y_s$ for all s , and that equation (A.33) gives $\frac{P_0 + D_0}{P_0}$ as $1 - \frac{1}{\mu}$ times the present discounted value $\mathbf{q}' \frac{\mathbf{P}_H}{\mathbf{P}} \mathbf{Y}$ of real income. Hence, applying the chain rule, the Jacobian of the consumption function $C_t = \tilde{C}_t \left(\left\{ \frac{P_{Hs}}{P_s} Y_s \right\} \right)$ with respect to income is given by:

$$\mathbf{M}^{HA-IM} = \frac{1}{\mu} \mathbf{M}^{labor} + \left(1 - \frac{1}{\mu} \right) \mathbf{m}^{cap} \mathbf{q}' \quad (\text{A.67})$$

HA-CM model. For the HA-CM, the procedure to obtain the Jacobian follows that described in [Aucle et al. \(2024b\)](#), in the special case with an exogenous stochastic discount factor due to our open economy assumption. We start from the Backus-Smith condition (24), substituting $\mathcal{B}_0 = Q_0$, and obtain:

$$\frac{\mathbb{E}[u'(c_0(a_0^p, e'))|e]}{\mathbb{E}[u'(c_{ss}(a_{ss}^p, e'))|e]} = 1 \quad (\text{A.68})$$

Consider a marginal shock to real income, which changes the policy by $dc_0(a_{ss}^p, e')$. Let $da_0^p(a^p, e)$ be the adjustment in the incoming asset position of an agent that, in the steady state, chooses asset position a^p in income state e (we suppress the ss subscript for ease of notation.) Differentiating (A.68), we have:

$$\mathbb{E}[u''(c(a^p, e'))(dc_0(a^p, e') + MPC(a^p, e')da_0^p(a^p, e))|e] = 0$$

Solving this equation for $da_0^p(a^p, e)$, we obtain:

$$da_0^p(a^p, e) = -\frac{\mathbb{E}[u''(c(a^p, e'))dc_0(a^p, e')|e]}{\mathbb{E}[u''(c(a^p, e'))MPC(a^p, e')|e]} \quad (\text{A.69})$$

Next, we calculate the aggregate effect of this change in incoming asset position on aggregate consumption at date t . To obtain this, let us define the expectation function for consumption t periods ahead as

$$\mathcal{E}_t(a^p, e) \equiv \mathbb{E}[c_t(a_t^p, e_t) | (a_0^p, e_0) = (a^p, e)] \quad (\text{A.70})$$

computed by assuming that agents follow their steady state policies. Note that (A.70) can be calculated recursively using the law of iterated expectations (see eg [Aucle et al. 2021](#) and [Aucle, Rigato, Rogne and Straub 2024c](#)). Next, define the date- t MPC for an agent in state (a^p, e) as

$$MPC_t(a^p, e) \equiv \frac{\partial \mathcal{E}_t(a^p, e)}{\partial a^p}$$

Then, the effect on aggregate consumption at t from the adjustment to asset positions induced by the shock is given by:

$$\int_{a^p, e} \sum_{e'} \Pi(e'|e) MPC_t(a^p, e') da_0^p(a^p, e) d\bar{\mathcal{D}}(a^p, e) \quad (\text{A.71})$$

where $d\bar{\mathcal{D}}(a^p, e)$ is the end-of-period distribution in the steady state.

The logic of equation (A.71) is as follows. Consider an agent in state e , choosing asset position a^p at the end of period -1 . The shock will change that agent's asset position by $da_0^p(a^p, e)$. The effect on aggregate consumption of this change is given by $\bar{MPC}_t(a^p, e) \equiv \sum_{e'} \Pi(e'|e) MPC_t(a^p, e')$, which is the date- t MPC for that agent. Integrating across all agents in the end-of-period distribution gives (A.71). Note that the end of period distribution is related to the beginning of period distribution via:

$$\bar{\mathcal{D}}(A^p, e) = \mathcal{D}\left(a^{-1}\left(\frac{A^p}{1+r}, e\right), e\right)$$

This is connected to, but different from, the beginning-of-period stationary distribution, which satisfies the relationship in footnote 15.

Calculating the Jacobian. We can now calculate the Jacobian \mathbf{M} of consumption with respect to real income. First, we calculate the Jacobian of consumption to labor income. As in the incomplete markets case, there are two effects: the direct effect \mathbf{M}^{labor} , and the indirect effect through the initial distribution of a_0^p , which we call $\widehat{\mathbf{M}}$. Given $\frac{W}{P} \mathbf{N} = \frac{1}{\mu} \frac{P_H}{P} \mathbf{Y}$, the complete-market Jacobian with respect to real income is then:

$$\mathbf{M} = \frac{1}{\mu} \mathbf{M}^{labor} + \frac{1}{\mu} \widehat{\mathbf{M}} \quad (\text{A.72})$$

To calculate $\widehat{\mathbf{M}}$, we proceed as follows. Consider a shock dx to labor income $\frac{W_s}{P_s} N_s$ at any date s . First, we can calculate the effect $dc_0^s(a^p, e')$ on the date-0 consumption policy function, and then the implied $da_0^{p,s}(a^p, e)$ using equation (A.69). Evaluating (A.71) then gives the effect on aggregate consumption at each date t .

In practice, we solve the model on a discretized grid of N states (a^p, e) . Let $\overline{\mathbf{mpc}}$ denote the $N \times T$ matrix where each column is the vector \overline{MPC}_t of date- t MPCs over all states. Let $\overline{\mathbf{D}}$ be a length- N vector representing the end-of-period distribution $\overline{\mathcal{D}}$, giving the mass of households in each discretized state. Finally, let α^p be the $N \times T$ matrix with column s given by $\alpha^s(a^p, e) = \frac{da_0^{p,s}(a^p, e)}{dx}$. Then, we can evaluate (A.71) on our discrete representation by taking the inner product⁵²

$$\widehat{\mathbf{M}} \equiv \overline{\mathbf{mpc}}' \text{diag}(\overline{\mathbf{D}}) \alpha^p \quad (\text{A.73})$$

Combining (A.73) and (A.74) we finally obtain

$$\mathbf{M}^{HA-CM} = \frac{1}{\mu} \mathbf{M}^{labor} + \frac{1}{\mu} \overline{\mathbf{mpc}}' \text{diag}(\overline{\mathbf{D}}) \alpha^p \quad (\text{A.74})$$

Recovering portfolios. Let $dr_0^s = d\left(\frac{P_0 + D_0}{P_{ss}}\right)$, $dr_0^H = d\left(\frac{1 + \iota_0}{P_0 / P_{ss}}\right)$, and $dr_0^F = d\left(\frac{(1 + \iota_0^*)\mathcal{E}_0}{P_0 \mathcal{E}_{ss} / P_{ss}}\right)$ denote the change in the returns to value of the stock and the return to home and foreign bond returns induced by the shock. Then, for each household with incoming savings $a' = \omega_s + \omega_{B^H} + \omega_{B^F}$, equation (8) implies that the choice of amounts invested in shocks, home and foreign bonds $\omega_s, \omega_{B^H}, \omega_{B^F}$ must satisfy:

$$da_0^p = dr_0^s \omega_s + dr_0^H \omega_{B^H} + dr_0^F \omega_{B^F} \quad (\text{A.75})$$

where da_0^p is pinned down by (A.69). In general, this will have a continuum of solutions: because there are three assets, households have two degrees of freedom in allocating their portfolio in order to hedge against the single shock.

To get one solution, we fix $\omega_s = a'$. Then, we have $\omega_{B^H} = -\omega_{B^F}$, and (A.75) implies that the amount invested in home bonds is

$$\omega^H = \frac{da_0^p - dr_0^s a'}{dr_0^H - dr_0^F}$$

This allows us to solve for the portfolios of all agents.

⁵²In practice, our code evaluates (A.73) by calculating the expectation function (A.70) at each horizon t , and then taking the inner product of this with the perturbation to the one-period-ahead discretized distribution, starting from the steady state, implied by each $\alpha^s(a^p, e)$. This is equivalent to (A.73), since the change in policy leads to a shift in mass between adjacent gridpoints of the discretized distribution, and taking the inner product of this with the expectation function is tantamount to evaluating the derivative MPC_t of the expectation function using numerical differentiation.

A.9 Additional results and summary

A.9.1 Present-value result for IM models

We have the following important result for incomplete market models. Recall that we have defined $\mathbf{q}' = \left(1, \frac{1}{1+r}, \frac{1}{(1+r)^2}, \dots\right)$ as the row vector that, when applied to a sequence, gives its net present value, i.e. $PV(\mathbf{Y}) = \mathbf{q}'\mathbf{Y}$

Proposition 11. *For all incomplete market models, we have $\mathbf{q}'\mathbf{M}^{IM} = \mathbf{q}'$.*

This is a consequence of budget constraints, which have to be respected under incomplete markets. In the RA model, it follows from equation (A.46), since $\frac{\mathbf{q}'\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} = \mathbf{q}'$. In the TA model, it follows similarly from equation (A.65), since

$$\mathbf{q}' \left((1 - \lambda) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} + \lambda I \right) = (1 - \lambda)\mathbf{q}' + (1 - \lambda)\mathbf{q}' = \mathbf{q}'$$

For the HA model, the proof is as follows. First, budget constraints imply that $\mathbf{q}'\mathbf{M}^{labor} = \mathbf{q}'$ and $\mathbf{q}'\mathbf{m}^{cap} = 1$ (see e.g. [Aucle et al. \(2024a\)](#), propositions 1 and 8, for a proof). Then equation (A.67) implies $\mathbf{q}'\mathbf{M} = \frac{1}{\mu}\mathbf{q}' + \left(1 - \frac{1}{\mu}\right)\mathbf{q}' = \mathbf{q}'$.

Note that the proposition does not apply for CM models, since under complete markets agents have insurance arrangements with the rest of the world with respect to aggregate income shocks. For instance, in the RA-CM model we have $\mathbf{M} = 0$ so also $\mathbf{q}'\mathbf{M} = 0$.

A.9.2 Summary: iMPCs in all models

Figure A.1 summarizes the results of the past three sections by displaying columns of the \mathbf{M} matrix in the six models we analyzed, complementing figure 1 which just display the first column. Recall that we calibrate TA and HA so the income-weighted partial equilibrium impact MPC is XXX. This corresponds to M_{00}^{labor} in (A.67) and (A.72). In TA-IM and HA-IM, this is the majority of M_{00} , which also depends on the response to capital gains. In TA-CM and HA-CM M_{00} is smaller due to endogenous insurance transfers captured by \tilde{M}_{00} . However, the ability of agents to insure future shocks in TA-CM and HA-CM is reduced: as equations (A.69) and (A.74) show, this is because they only have insurance against initial consumption movements, which get smaller and smaller for later shocks. This explains why, for these later shocks, the responses under CM and IM are closer.

B Appendix to section 3

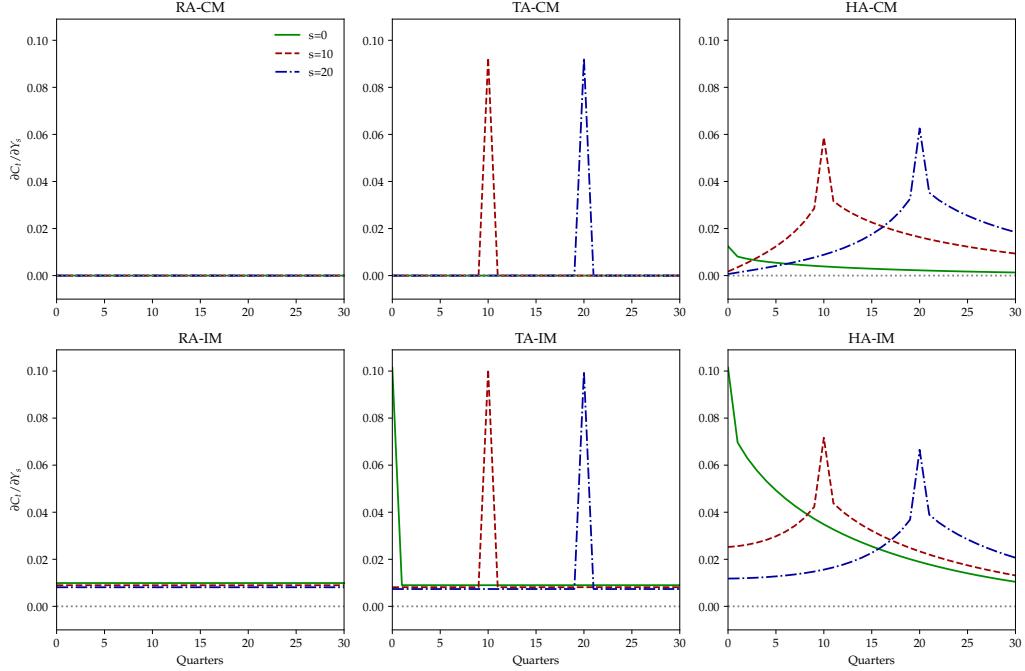
B.1 Proof of proposition 2

We first linearize the mappings (A.7) and (A.8) between the real exchange rate Q and relative prices P_H/P and P_H/\mathcal{E} around the steady state where $Q = P_H = P_F = \mathcal{E} = C = C^* = Y = 1$. This gives:

$$d \left(\frac{P_{Ht}}{P_t} \right) = -\frac{\alpha}{1-\alpha} dQ_t \quad d \left(\frac{P_{Ht}}{\mathcal{E}_t} \right) = -\frac{1}{1-\alpha} dQ_t \quad (\text{A.76})$$

in other words, we have $p'_H(1) = -\frac{\alpha}{1-\alpha}$ and $p_H^{*\prime}(1) = -\frac{1}{1-\alpha}$.

Figure A.1: Intertemporal MPCs (columns of \mathbf{M}) in our six calibrated models



Note: This figure shows several columns of the \mathbf{M} matrix in our six calibrated models.

We next linearize the goods market clearing condition (A.9) around the steady state. This gives

$$\begin{aligned}
 dY_t &= (1 - \alpha) (-\eta) \left(-\frac{\alpha}{1 - \alpha} \right) dQ_t + (1 - \alpha) dC_t + \alpha (-\gamma) \left(-\frac{1}{1 - \alpha} \right) dQ_t \\
 &= \frac{\alpha}{1 - \alpha} (\eta (1 - \alpha) + \gamma) dQ_t + (1 - \alpha) dC_t \\
 &= \frac{\alpha}{1 - \alpha} \chi dQ_t + (1 - \alpha) dC_t
 \end{aligned} \tag{A.77}$$

where $\chi \equiv \eta (1 - \alpha) + \gamma$, as defined in (36).

Similarly, we linearize the equation for the trade balance in (A.10). Using (A.76), we obtain

$$dNX_t = \alpha \left(-\frac{\alpha}{1 - \alpha} dQ_t + \frac{\gamma}{1 - \alpha} dQ_t \right) - \alpha (1 - \eta) dQ_t - \alpha dC_t$$

Simplifying, we find

$$dNX_t = \frac{\alpha}{1 - \alpha} (\chi - 1) dQ_t - \alpha dC_t \tag{A.78}$$

Finally, in the complete-market, representative-agent model, we know from the Backus-Smith condition (27) combined with the exchange rate equation (32) that $dC_t = 0$ at all t . Plugging this in (A.77) delivers equation (34). Plugging this into (A.78) delivers $dNX_t = \frac{\alpha}{1 - \alpha} (\chi - 1) dQ_t$, as claimed in the text. Finally, the definition of net exports in (A.4) implies $d(\frac{P_{Ht}}{P_t}) = dNX_t + dC_t$, which with $dC_t = 0$ implies $d(\frac{P_{Ht}}{P_t}) = dNX_t$ —that is, the change in real income is also the change in net exports.

B.2 Proof of proposition 3

In this section, we derive the “international Keynesian Cross” assuming a fixed real interest rate policy. From proposition 1, we know that aggregate domestic consumption C_t depends only on the path of aggregate real income (the value of home production deflated by the domestic CPI) $\left\{ \frac{P_{Ht}}{P_t} Y_t \right\}$. We can thus write it as $C_t = \mathcal{C}_t \left(\left\{ \frac{P_{Hs}}{P_s} Y_s \right\} \right)$.

Let \mathbf{M} denote the Jacobian of the aggregate consumption function $\mathcal{C}_t \left(\left\{ \frac{P_{Hs}}{P_s} Y_s \right\} \right)$ with respect to the path of real income $\frac{P_{Hs}}{P_s} Y_s$. Using equation (A.76), we can totally differentiate the equation $C_t = \mathcal{C}_t \left(\left\{ \frac{P_{Hs}}{P_s} Y_s \right\} \right)$ around the steady state with $C = Y = P_H = P = 1$ to obtain

$$d\mathbf{C} = -\frac{\alpha}{1-\alpha} \mathbf{M} d\mathbf{Q} + \mathbf{M} d\mathbf{Y} \quad (\text{A.79})$$

which is equation (37). The first term reflects the dependency of real income on the relative price of home goods $\frac{P_{Hs}}{P_s}$ (which we call the “real income channel”), and the second term reflects the dependency on the volume of output Y_s (which we call the “multiplier channel”).

Next, we write equation (A.77), which is the linearized version of equation (A.9), in vector form, and obtain:

$$d\mathbf{Y} = \frac{\alpha}{1-\alpha} \chi d\mathbf{Q} + (1-\alpha) d\mathbf{C} \quad (\text{A.80})$$

Substituting $d\mathbf{C}$ from (37), we obtain the full international Keynesian cross (38).

Alternative derivation via the trade balance. Here we propose an instructive alternative derivation of the international Keynesian cross via the trade balance. Writing equation (A.78) for the trade balance in vector form, we have:

$$d\mathbf{NX} = \frac{\alpha}{1-\alpha} (\chi - 1) d\mathbf{Q} - \alpha d\mathbf{C}$$

reflecting the two ways in which the value of net imports can increase: via a change in relative prices $d\mathbf{Q}$, leading to countervailing volume and price effects, or via a decline in domestic spending $d\mathbf{C}$, pushing down the volume of imports. On the other hand, linearizing the national accounting identity (A.4) we have:

$$-\frac{\alpha}{1-\alpha} d\mathbf{Q} + d\mathbf{Y} = d\mathbf{NX} + d\mathbf{C}$$

which says that the sum of the value of net exports and consumption adds up to real income. Combining these two equations, we again obtain (A.80).

B.3 Proof of propositions 4 and 5

The international Keynesian cross (38) can be rewritten as

$$(\mathbf{I} - (1-\alpha)\mathbf{M}) d\mathbf{Y} = \frac{\alpha}{1-\alpha} \chi d\mathbf{Q} - \alpha \mathbf{M} d\mathbf{Q}$$

Our first step in solving the equation is therefore the inversion of the (infinite) matrix on the left hand side of the equation. After that, we solve for $d\mathbf{Y}$ and characterize the solution as stated in

proposition 3.

Characterizing the inverse $(\mathbf{I} - (1 - \alpha)\mathbf{M})^{-1}$. We will work using the sequence norm $\|x\| \equiv \sum_{t=0}^{\infty} (1 + r)^{-t} |x_t|$, and let us consider the Banach space of sequences such that $\|x\|$ is finite. (This is like ℓ^1 , but with discounting in the norm, since $r > 0$.)

We first show this for incomplete market models, for which we have the identity $\sum(1 + r)^{s-t} M_{ts} = 1$, which states that the \mathbf{M} matrix preserves present value. We can then write

$$\begin{aligned} \|\mathbf{M}x\| &= \sum_{t=0}^{\infty} (1 + r)^{-t} |(\mathbf{M}x)_t| = \sum_{t=0}^{\infty} (1 + r)^{-t} \left| \sum_{s=0}^{\infty} \mathbf{M}x_s \right| \\ &\leq \sum_{t=0}^{\infty} (1 + r)^{-t} \sum_{s=0}^{\infty} M_{ts} |x_s| = \sum_{s=0}^{\infty} (1 + r)^{-s} |x_s| \sum_{t=0}^{\infty} (1 + r)^{s-t} M_{ts} \\ &= \sum_{s=0}^{\infty} (1 + r)^{-s} |x_s| = \|x\| \end{aligned} \tag{A.81}$$

Note that our assumption $\mathbf{M} \geq 0$ is required here in the inequality step so that $|M_{ts}| = M_{ts}$, and the identity $\sum(1 + r)^{s-t} M_{ts} = 1$ is applied in the last line.

Since $\|\mathbf{M}x\| \leq \|x\|$, we have $\|\mathbf{M}\| \leq 1$, where $\|\mathbf{M}\|$ is the induced operator norm. Indeed, if $\alpha > 0$, we have the strict inequality $\|(1 - \alpha)\mathbf{M}\| < 1$. It then immediately follows⁵³ that the inverse $(\mathbf{I} - (1 - \alpha)\mathbf{M})^{-1}$ is uniquely defined as the Neumann series

$$(\mathbf{I} - (1 - \alpha)\mathbf{M})^{-1} = \mathbf{I} + (1 - \alpha)\mathbf{M} + (1 - \alpha)^2\mathbf{M}^2 + \dots \tag{A.82}$$

This provides both a constructive way to compute the inverse and, combined with our assumption that $\mathbf{M} \geq 0$, implies that the inverse is has all nonnegative matrices: $(\mathbf{I} - (1 - \alpha)\mathbf{M})^{-1} \geq 0$.⁵⁴

Using proposition 12 below, the extension to complete-market models is immediate, since then we have $\sum(1 + r)^{s-t} M_{ts} \leq 1$, and the same proof goes through with an inequality in the last line of (A.81).

Explosive paths. We have characterized a unique solution in the space given by our norm $\|x\| \equiv \sum_{t=0}^{\infty} (1 + r)^{-t} |x_t|$ (which includes all non-explosive solutions), but it is theoretically possible that the path of $\{dY_t\}$ in this solution will be explosive. If so, there is no non-explosive solution.

For instance, suppose that $\mathbf{M} = (1 + r)\mathbf{L}$, where \mathbf{L} is the lag operator, so that if households receive income in period t , they consume exactly that income plus interest in period $t + 1$. Then if $(1 + r)(1 - \alpha) \geq 1$, $\mathbf{I} + (1 - \alpha)\mathbf{M} + (1 - \alpha)^2\mathbf{M}^2 + \dots$ has unbounded entries: each column has entries, starting at the main diagonal, of $(1, (1 + r)(1 - \alpha), (1 + r)^2(1 - \alpha)^2, \dots)$. (Note that this particular example would require either a very low openness α or a very high rate r .) This will generally imply an explosive path for the resulting dY .

⁵³This is a standard result: for instance, the [Wikipedia article for Neumann series](#) states that for any bounded linear operator \mathbf{T} on a normed vector space, if the operator norm $\|\mathbf{T}\| < 1$, then $(\mathbf{I} - \mathbf{T})^{-1} = \mathbf{I} + \mathbf{T} + \mathbf{T}^2 + \dots$. The terms in this sequence shrink at least geometrically in norm, since $\|\mathbf{T}^k\| \leq \|\mathbf{T}\|^k$.

⁵⁴Note also that we can get (A.82), although not nonnegativity, under the weaker assumption that $\mathbf{M}^k \geq 0$ for some k , since then the argument for (A.81) implies that $\|\mathbf{M}^k\| \leq 1$, and we continue to get a geometric bound on the terms of the Neumann series (A.82).

A simple condition guarantees the existence of a non-explosive solution. Assume that

$$\sup_s \sum_t |M_{ts}| < \frac{1}{1-\alpha} \quad (\text{A.83})$$

i.e. that the sum of entries in any column of \mathbf{M} is bounded below $\frac{1}{1-\alpha}$. This is equivalent to assuming that the norm $\|\mathbf{M}\|_{\ell^1}$ of \mathbf{M} , interpreted as an operator on ℓ^1 , is below $\frac{1}{1-\alpha}$. Then it is immediate that $\|(1-\alpha)\mathbf{M}\|_{\ell^1} < 1$, and hence that the Neumann series (A.82) converges and gives a well-defined inverse in ℓ^1 , so that (assuming a summable series of shocks) the resulting $d\mathbf{Y}$ is in ℓ^1 and hence non-explosive. (Note that if (A.83) is satisfied, the assumption $\mathbf{M} \geq 0$ is unnecessary.)

Proofs of proposition 5. Starting from the international Keynesian cross (38),

$$d\mathbf{Y} = \frac{\alpha}{1-\alpha} \chi d\mathbf{Q} - \alpha \mathbf{M} d\mathbf{Q} + (1-\alpha) \mathbf{M} d\mathbf{Y}$$

we see that for $\chi = 1$, we can guess and verify the solution $d\mathbf{Y} = \frac{\alpha}{1-\alpha} d\mathbf{Q}$: substituting this into the third term on the right above, the second and third terms on the right cancel out, leaving only $d\mathbf{Y} = \frac{\alpha}{1-\alpha} d\mathbf{Q}$. This is proposition 5.

Proof of proposition 4. With the discussion above, we can solve (B.3) by inverting $\mathbf{I} - (1-\alpha)\mathbf{M}$. We find the unique solution

$$d\mathbf{Y} = (\mathbf{I} - (1-\alpha)\mathbf{M})^{-1} \left(\frac{\alpha}{1-\alpha} \chi d\mathbf{Q} - \alpha \mathbf{M} d\mathbf{Q} \right) \quad (\text{A.84})$$

For the $\chi = 1$ case this can be written as $\frac{\alpha}{1-\alpha} d\mathbf{Q} = (\mathbf{I} - (1-\alpha)\mathbf{M})^{-1} (\frac{\alpha}{1-\alpha} d\mathbf{Q} - \alpha \mathbf{M} d\mathbf{Q})$. Subtracting this special case from the general case gives

$$d\mathbf{Y} - \frac{\alpha}{1-\alpha} d\mathbf{Q} = (\mathbf{I} - (1-\alpha)\mathbf{M})^{-1} (\chi - 1) \frac{\alpha}{1-\alpha} d\mathbf{Q}$$

which, when rearranged (and expanding $(\mathbf{I} - (1-\alpha)\mathbf{M})^{-1}$ in series form), is the desired result (40).

B.4 Additional solution insights

In this section we derive several additional insights about the solution to the international Keynesian cross in some of the models introduced in section 2.2. We focus mostly on the explicit solutions in the RA-IM, TA-IM, TA-CM models, though we also show additional results regarding the portfolios in the HA-CM model.

B.4.1 The RA-IM model

As shown in appendix A.6, the iMPC matrix of the RA-IM model is given by equation (A.46)

$$\mathbf{M} = \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}}$$

where \mathbf{q} is the vector of discount factors, $\mathbf{q} = (1, (1+r)^{-1}, (1+r)^{-2}, \dots)'$. Observe that \mathbf{M} is idempotent, that is,

$$\mathbf{M}^k = \mathbf{M} \quad \text{for any } k > 0$$

Using this property in the solution to the international Keynesian cross, (40), we find

$$d\mathbf{Y} = \chi \frac{\alpha}{1-\alpha} d\mathbf{Q} + \frac{\alpha}{1-\alpha} (\chi - 1) \sum_{k \geq 1} (1-\alpha)^k \mathbf{M} d\mathbf{Q}$$

which simplifies to

$$d\mathbf{Y} = \chi \frac{\alpha}{1-\alpha} d\mathbf{Q} + (\chi - 1) \mathbf{M} d\mathbf{Q}$$

and further to

$$dY_t = \chi \frac{\alpha}{1-\alpha} dQ_t + (\chi - 1) \cdot \frac{r}{1+r} \cdot \sum_{s \geq 0} \frac{1}{(1+r)^s} dQ_s \quad (\text{A.85})$$

The second term in the RA solution (A.85) is a constant. This constant is zero in the RA-CM solution. The constant is negative precisely when $\chi < 1$ and the exchange rate depreciates, $dQ_s > 0$. It captures the influence of the real income effect wearing down on consumption. Since the RA-IM model has very low intertemporal MPCs, however, the whole term scales with $\frac{r}{1+r}$ and is typically very small.

In practice, the modest role played by the real income effect will also make it hard for the RA model to predict a recession. To illustrate this, assume that dQ_t is decreasing towards zero, with AR(1) persistence $\rho \in (0, 1)$. Then, the entire output response dY_t is negative whenever

$$\chi < (1-\alpha) \frac{1}{1 + \alpha \beta \frac{1-\rho}{1-\beta}} \equiv \chi^*$$

This is a special case of proposition 8. The threshold χ^* lies below $1-\alpha$, potentially by a lot given that standard quarterly calibrations for ρ are in the neighborhood of 0.85, while calibrations for β are in the neighborhood of 0.99. For our baseline calibration,

$$\chi^* \approx 0.14 \cdot (1-\alpha) = 0.057$$

This implies that contractionary depreciations are much less likely to be obtained in a RA-IM model than in a HA model.

B.4.2 The TA-IM model

As shown in appendix A.7, equation (A.65), the iMPC matrix of the TA-IM model is simply a convex combination of an identity matrix with the iMPC matrix of the RA-IM model,

$$\mathbf{M} = \lambda \mathbf{I} + (1-\lambda) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}}$$

Different from RA-IM, this iMPC matrix is no longer idempotent. Instead,

$$\mathbf{M}^2 = \left(\lambda \mathbf{I} + (1-\lambda) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} \right) \left(\lambda \mathbf{I} + (1-\lambda) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} \right) = \lambda^2 \mathbf{I} + (1-\lambda^2) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}}$$

and more generally

$$\mathbf{M}^k = \lambda^k \mathbf{I} + (1 - \lambda^k) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} \quad \text{for any } k > 0$$

Using this property in the solution to the international Keynesian cross, (40), we find

$$d\mathbf{Y} = \chi \frac{\alpha}{1 - \alpha} d\mathbf{Q} + \frac{\alpha}{1 - \alpha} (\chi - 1) \sum_{k \geq 1} (1 - \alpha)^k \left(\lambda^k \mathbf{I} + (1 - \lambda^k) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} \right) d\mathbf{Q}$$

which simplifies to

$$dY_t = \underbrace{\chi \frac{\alpha}{1 - \alpha} dQ_t + (\chi - 1) \cdot \frac{r}{1 + r} \cdot \sum_{s \geq 0} \frac{1}{(1 + r)^s} dQ_s}_{=dY_t^{RA-IM}} + (\chi - 1) \frac{\lambda\alpha}{1 - \lambda(1 - \alpha)} \left(dQ_t - \frac{r}{1 + r} \cdot \sum_{s \geq 0} \frac{1}{(1 + r)^s} dQ_s \right) \quad (\text{A.86})$$

Here, the first two terms are exactly the RA-IM solution. The new third term scales with λ , the share of hand-to-mouth households in the TA-IM model. It captures that these households cut their spending in response to any loss in real income with an MPC of one. However, only contemporaneous income matters for their spending, so only dQ_t enters. The denominator $1 - \lambda(1 - \alpha)$ captures the Keynesian cross that is active as the hand-to-mouth households spend less and lower everyone's real income further.

B.4.3 The TA-CM model

As shown in appendix A.7, the iMPC matrix of the TA-CM model is a diagonal matrix, with first entry equal to 0, followed by λ 's,

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 & \cdots \\ 0 & \lambda & 0 & \cdots \\ 0 & 0 & \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

This matrix has the property that

$$\mathbf{M}^k = \lambda^{k-1} \mathbf{M}$$

so that the solution to the Keynesian cross (40) simplifies to

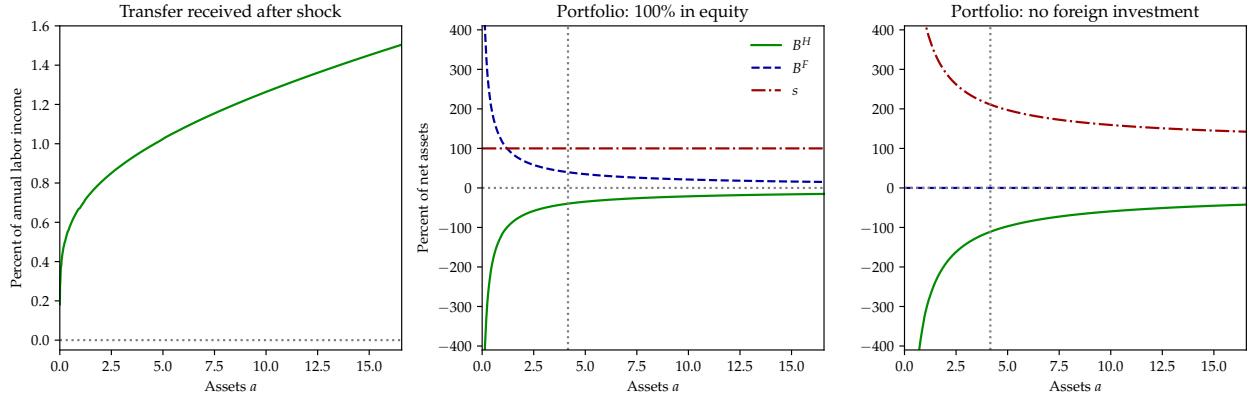
$$d\mathbf{Y} = \chi \frac{\alpha}{1 - \alpha} d\mathbf{Q} + (\chi - 1) \frac{\alpha}{1 - \lambda(1 - \alpha)} \mathbf{M} d\mathbf{Q}$$

which can be written as

$$dY_t = \begin{cases} \chi \frac{\alpha}{1 - \alpha} dQ_0 & \text{if } t = 0 \\ \chi \frac{\alpha}{1 - \alpha} dQ_t + (\chi - 1) \frac{\lambda\alpha}{1 - \lambda(1 - \alpha)} dQ_t & \text{if } t > 0 \end{cases}$$

In the first period, the TA-CM output response is exactly identical to the RA-CM response. This is because the hand-to-mouth agents are able to insure their exposure to the exchange rate shock in period 0 (see appendix A.7). Thereafter, an additional term emerges that is similar to the third term in (A.86), capturing how output dY_t is lower if $\chi < 1$ as hand-to-mouth households spend

Figure A.2: Complete-market portfolios in the HA-CM model



Note: The left panel shows the transfers HA households receive in the complete markets allocation after the shock in figure 2. The middle and right panels show two distinct sets of portfolios that can implement these transfers with domestic equity and bonds, and foreign-currency bonds.

less in response to a contemporaneous exchange rate depreciation $dQ_t > 0$. In fact, we can rewrite this response, for $t \geq 1$, as

$$dY_t = \frac{\chi\alpha(1 - \lambda(1 - \alpha)) + (\chi - 1)\lambda\alpha}{(1 - \alpha)(1 - \lambda(1 - \alpha))} dQ_t = \frac{\alpha}{1 - \alpha} \frac{\chi - (1 - \alpha)\lambda}{1 - (1 - \alpha)\lambda} dQ_t$$

which is equation (42) in the main text. Note that real income is

$$d\left(\frac{P_{Ht}}{P_t} Y_t\right) = -\frac{\alpha}{1 - \alpha} dQ_t + dY_t = \frac{\alpha}{1 - \alpha} \frac{\chi - 1}{1 - (1 - \alpha)\lambda} dQ_t$$

which is equation (41). (In the alternative TA-CM model where hand to mouth agents cannot hold any gross position in any asset at any date, these expressions also apply at $t = 0$)

B.4.4 Sustaining portfolios in HA-CM.

Figure A.2 visualizes two sets of portfolios that sustain the complete-market allocation in the HA-CM model, backed out using the procedure described in section A.8. With three assets and a single kind of shock (the exchange rate shock), many portfolios can sustain the complete markets allocation. The left panel of figure A.2 shows the transfer households receive after an exchange rate depreciation for in complete markets allocation.

The middle panel shows how these transfers can be sustained via a portfolio that assumes all agents hold a 100% position in the domestic equity market. To generate positive transfers, households, in addition, hold a positive position in foreign currency debt, and a negative one in domestic currency debt. These positions need to be large as share of net worth if net worth is low.

Similarly, the right panel shows how the complete-markets transfers can be sustained with a zero position in foreign-currency debt, by leveraging up investments in equities, as those appreciate relative to domestic-currency debt in response to a depreciation.

B.5 Proof of proposition 6

The proof follows immediately by taking the difference of the solution (40) under \mathbf{M} and $\widetilde{\mathbf{M}}$:

$$d\mathbf{Y} - d\widetilde{\mathbf{Y}} = \frac{\alpha}{1-\alpha}(\chi - 1) \sum_{k \geq 0} (1-\alpha)^k (\mathbf{M}^k - \widetilde{\mathbf{M}}^k) d\mathbf{Q} \quad (\text{A.87})$$

Since we assume that both \mathbf{M} and $\widetilde{\mathbf{M}}$ are nonnegative, $\mathbf{M} \geq \widetilde{\mathbf{M}}$ implies that $\mathbf{M}^k \geq \widetilde{\mathbf{M}}^k$, and therefore for $d\mathbf{Q} \geq 0$, the sum in (A.87) is nonnegative. It follows that the right side of (A.87) is nonnegative, so that $d\mathbf{Y} \geq d\widetilde{\mathbf{Y}}$, when $\chi > 1$, and nonpositive, so that $d\mathbf{Y} \leq d\widetilde{\mathbf{Y}}$, when $\chi < 1$.

B.6 Effect of complete markets on output response to depreciation

In this section we show that, when $\chi < 1$, a depreciation under complete markets is always less contractionary than under incomplete markets. This follows from an application of proposition 6 to the following result.

Proposition 12. *Regardless of heterogeneity (RA, TA, HA), complete markets always reduces the entries of the \mathbf{M} matrix: $\mathbf{M}^{CM} \leq \mathbf{M}^{IM}$. This implies that when $\chi < 1$, the output response under incomplete markets lies below that with complete markets, $d\mathbf{Y}^{IM} \leq d\mathbf{Y}^{CM}$.*

We prove the proposition with a series of auxiliary results. Our main proof strategy is to show that the complete-market correction term $\widehat{\mathbf{M}}$ in (A.74) is weakly negative, $\widehat{\mathbf{M}} \leq 0$, in the HA economy. We treat the RA and TA models below. Once this is established, it will follow that $\mathbf{M}^{CM} \leq \mathbf{M}^{IM}$. The ordering of the output responses is then a direct consequence of proposition 6.

Proof that $\widehat{\mathbf{M}} \leq 0$. Our first lemma characterizes the steady state value function in the heterogeneous-agent economy.

Lemma 1. *The steady-state value function $V(a^p, e)$ in (9) is strictly concave in a^p .*

Proof. This is standard and follows from the strict concavity of flow utility and the convexity of the constraints. In particular, the solution to problem (8), starting from some initial state (a^p, e_0) , can be written in the form of a sequence of state-contingent policies $a'(a^p, e^t)$ and $c(a^p, e^t)$, which are functions of the sequence $e^t = (e_0, \dots, e_t)$ of endowment realizations so far. We have $V(a^p, e_0) = \mathbb{E} [\sum_{t=0}^{\infty} \beta^t u(c(a^p, e^t))]$. For any convex combination $\zeta a_0^p + (1-\zeta)a_1^p$ of some a_0^p and a_1^p , one feasible plan starting at $\zeta a_0^p + (1-\zeta)a_1^p$ is to do the corresponding convex combination $\zeta a'(a_0^p, e^t) + (1-\zeta)a'(a_1^p, e^t)$ and $\zeta c(a_0^p, e^t) + (1-\zeta)c(a_1^p, e^t)$ of all state-contingent policies. This plan puts a lower bound on the value function, so we have

$$\begin{aligned} V(\zeta a_0^p + (1-\zeta)a_1^p, e_0) &\geq \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(\zeta c(a_0^p, e^t) + (1-\zeta)c(a_1^p, e^t)) \right] \\ &> \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t (\zeta u(c(a_0^p, e^t)) + (1-\zeta)u(c(a_1^p, e^t))) \right] = \zeta V(a_0^p, e_0) + (1-\zeta)V(a_1^p, e_0) \end{aligned}$$

□

Lemma 2. $MPC(a^p, e) \equiv \partial c(a^p, e) / \partial a^p$ satisfies $0 < MPC(a^p, e) \leq 1$.

Proof. Note that from (9), if we define the continuation value $\mathbb{W}(a', e) \equiv \beta \mathbb{E}_t[V((1+r)a', e')|e]$, then the non-constant part of the steady-state optimization problem reduces to

$$\begin{aligned} V(a^p, e) &= \max_{c, a'} u(c) + \mathbb{W}(a', e) \\ \text{s.t. } c + a' &= a^p + e \frac{W}{P} N (\equiv y(a^p, e)) \\ a' &\geq 0 \end{aligned}$$

If $a' \geq 0$ does not bind, the first-order condition is $u'(c) = W'(a', e)$. Dropping the constant e for economy of notation, and considering an increase $dy = da^p$ in incoming resources, we have

$$\begin{aligned} u''(c)dc &= \mathbb{W}''(a')da' \\ dc + da' &= dy \end{aligned}$$

and it immediately follows that $MPC \equiv dc/dy = \frac{1}{\frac{u''(c)}{u''(c) + \mathbb{W}''(a')}} \frac{1}{u''(c)}$. We know that $u'' < 0$, and $\mathbb{W}'' < 0$ follows from strict concavity $V'' < 0$ proved in the previous lemma. In this case $0 < MPC < 1$.

In the alternative case where the borrowing constraint does bind and $a' = 0$ locally, then we have $MPC = 1$. \square

Lemma 3. $MPC_t(a^p, e) \geq 0$.

Proof. Recall that $MPC_t(a^p, e) = \frac{\partial \mathcal{E}_t(a^p, e)}{\partial a^p}$, and note that we can recursively write (for $t > 0$)

$$\mathcal{E}_t(a^p, e) = \mathbb{E}[\mathcal{E}_{t-1}((1+r)a'(a^p, e), e')|e]$$

Differentiating both sides, we have

$$MPC_t(a^p, e) = (1+r) \cdot MPS(a^p, e) \cdot \mathbb{E}[MPC_{t-1}((1+r)a'(a^p, e), e')|e] \quad (\text{A.88})$$

where $MPS(a^p, e) \equiv \frac{\partial a'(a^p, e)}{\partial a^p} = 1 - MPC(a^p, e)$ must be nonnegative by the previous lemma. Through induction on (A.88), starting from the base case $MPC_0(a^p, e) = MPC(a^p, e) > 0$, the result follows. \square

Lemma 4. Suppose there is a shock dx to labor income at date s . The response $\alpha^s(a^p, e) \equiv da_0^p(a^p, e)/dx$ is nonpositive, and strictly negative when $s = 0$.

Proof. Applying the envelope theorem to (9) in period s , we have $V'_s(a^p, e) = u'(c_s(a^p, e))$, and then if there is a shock $e \cdot d \frac{W_s}{P_s} N_s = dx$ to labor income at date s , the effect on the marginal value function is

$$dV'_s(a^p, e) = u''(c(a^p, e)) \cdot MPC(a^p, e) \cdot dx < 0$$

Then, in previous periods $t < s$, the envelope theorem implies that $dV_t(a^p, e) = \beta \mathbb{E}[dV_{t+1}((1+r)a', e')|e]$, and differentiating with respect to a^p , we get

$$dV'_t(a^p, e) = \beta(1+r) \cdot MPS(a^p, e) \cdot \mathbb{E}[dV'_{t+1}((1+r)a', e')|e] \quad (\text{A.89})$$

which is the same recursion as (A.88), except with an additional β . Through induction on (A.89), starting from the base case $dV'_s(a^p, e) < 0$, we have $dV'_0(a^p, e) \leq 0$ (with strict negativity if $s = 0$),

which implies that $dc_0(a^p, e) \geq 0$ via the envelope condition. It then follows from equation (A.69) that $da_0^{p,s}(a^p, e) < 0$. \square

Proposition 13. *The complete-market correction term $\widehat{\mathbf{M}}$ in (A.74) satisfies $\widehat{\mathbf{M}} \leq 0$, and also $\widehat{M}_{0,0} < 0$.*

Proof. Given a shock to labor income in period s , the $\widehat{M}_{t,s}$ is obtained by evaluating (A.71) given the da_0^p induced by the shock. Lemma 4 shows that $da_0^p \leq 0$, while lemma 3 shows that $MPC_t \geq 0$. Since the other factors in (A.71) are clearly nonnegative, (A.71) must be nonpositive. For $t = s = 0$, we have $da_0^p < 0$ and $MPC_0 = MPC > 0$ by the same lemmas, so (A.71) is strictly negative. \square

Proof of proposition 12. The result is clearly true for RA, comparing equation (A.46) for IM to $\mathbf{M} = 0$ for CM. For TA, comparing (A.65) for IM to (A.64) for CM, we have:

$$\mathbf{M}^{TA-IM} - \mathbf{M}^{TA-CM} = (1 - \lambda) \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} + \lambda \begin{pmatrix} 1 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

which is clearly positive. Finally, for HA, comparing (A.67) and (A.74), we have:

$$\mathbf{M}^{HA-IM} - \mathbf{M}^{HA-CM} = \left(1 - \frac{1}{\mu}\right) \mathbf{m}^{cap} \mathbf{q}' - \frac{1}{\mu} \widehat{\mathbf{M}}$$

The first term is positive, since $\mathbf{q} \geq 0$ and $\mathbf{m}^{cap} \geq 0$ since each m_t^{cap} is a weighted average of MPC_t s, which lemma 3 shows are non-negative. The second term, $\widehat{\mathbf{M}}$, is weakly negative by proposition 13. Hence, $\mathbf{M}^{IM} - \mathbf{M}^{CM} \geq 0$, which completes the proof.

B.7 Proof of propositions 7 and 8

We start with the proof of proposition 7. Equation (39) shows that the consumption response is given by

$$d\mathbf{C} = \frac{\alpha}{1 - \alpha} (\chi - 1) \sum_{k \geq 0} (1 - \alpha)^k \mathbf{M}^{k+1} d\mathbf{Q}$$

with real income moving according to

$$d\frac{\mathbf{P}_H \mathbf{Y}}{\mathbf{P}} = \frac{\alpha}{1 - \alpha} (\chi - 1) \sum_{k \geq 0} (1 - \alpha)^k \mathbf{M}^k d\mathbf{Q}$$

Since $\mathbf{M} \geq 0$ by assumption 1, it immediately follows that when $\chi < 1$, $d\mathbf{C} \leq 0$ and $d\frac{\mathbf{P}_H \mathbf{Y}}{\mathbf{P}} \leq 0$. This proves proposition 7.

To prove proposition 8, we compute present values of the output solution (40), that is, we multiply the solution by \mathbf{q}' ,

$$PV(d\mathbf{Y}) = \mathbf{q}' d\mathbf{Y} = \frac{\alpha}{1 - \alpha} \mathbf{q}' d\mathbf{Q} + \frac{\alpha}{1 - \alpha} (\chi - 1) \sum_{k \geq 0} (1 - \alpha)^k \mathbf{q}' \mathbf{M}^k d\mathbf{Q}$$

As $\mathbf{q}' \mathbf{M} = \mathbf{q}'$ by proposition 11, we have that

$$PV(d\mathbf{Y}) = \frac{\chi - (1 - \alpha)}{1 - \alpha} PV(d\mathbf{Q}) \tag{A.90}$$

This shows that if $\chi < 1 - \alpha$, the present value of output to a depreciation $d\mathbf{Q} \leq 0$ is (weakly) negative; and strictly so if $PV(d\mathbf{Q}) < 0$.

We can similarly derive the present value of the consumption and net export responses $d\mathbf{C}$ and $d\mathbf{N}\mathbf{X}$. Applying \mathbf{q}' to $d\mathbf{Y} = \frac{\alpha}{1-\alpha}\chi d\mathbf{Q} + (1-\alpha)d\mathbf{C}$ gives $\mathbf{q}'d\mathbf{Y} = \frac{\alpha}{1-\alpha}\chi \mathbf{q}'d\mathbf{Q} + (1-\alpha)\mathbf{q}'d\mathbf{C}$, and combining with (A.90), we obtain the result

$$PV(d\mathbf{C}) = \frac{\chi - 1}{1 - \alpha} \mathbf{q}'d\mathbf{Q} \quad (\text{A.91})$$

For net exports, writing (A.78) in vector form and taking present values gives $\mathbf{q}'d\mathbf{N}\mathbf{X} = \frac{\alpha}{1-\alpha}(\chi - 1)\mathbf{q}'d\mathbf{Q} - \alpha\mathbf{q}'d\mathbf{C}$, which we combine with (A.91) to get

$$\mathbf{q}'d\mathbf{N}\mathbf{X} = 0$$

which is just rewriting the intertemporal budget constraint of the home economy.

The impact response of output is given by

$$dY_0 = \frac{\alpha}{1-\alpha}dQ_0 + \frac{\alpha}{1-\alpha}(\chi - 1) \sum_{k \geq 0} (1-\alpha)^k [\mathbf{M}^k d\mathbf{Q}]_0 \quad (\text{A.92})$$

Here, the first entry of matrix $\mathbf{M}^k d\mathbf{Q}$ is denoted by $[\mathbf{M}^k d\mathbf{Q}]_0$. Because $d\mathbf{Q} \geq 0$ and $\mathbf{M} \geq 0$, we know that

$$[\mathbf{M}^k d\mathbf{Q}]_0 \geq M_{0,0}^k dQ_0$$

Thus, if $\chi < 1$, we have

$$dY_0 \leq \frac{\alpha}{1-\alpha}dQ_0 + \frac{\alpha}{1-\alpha}(\chi - 1) \frac{1}{1 - (1-\alpha)M_{0,0}}dQ_0$$

or simplified

$$dY_0 \leq \frac{\alpha}{1-\alpha} \frac{\chi - (1-\alpha)M_{0,0}}{1 - (1-\alpha)M_{0,0}} dQ_0$$

Now, since (A.92) is monotonically increasing in χ , with $dY_0 = \frac{\alpha}{1-\alpha}dQ_0 > 0$ for $\chi = 1$ and $dY_0 \leq 0$ for $\chi = (1-\alpha)M_{0,0}$, we know that there must exist a threshold $\chi^* \in [(1-\alpha)M_{0,0}, 1)$, such that $dY_0 < 0$ for any $\chi < \chi^*$.

C Appendix to section 4

C.1 International Keynesian cross with monetary shocks

In this section, we derive the international Keynesian cross with monetary policy shocks (44),

$$d\mathbf{Y} = (1-\alpha)\mathbf{M}^r d\mathbf{r} + \frac{\alpha}{1-\alpha}\chi d\mathbf{Q} - \alpha\mathbf{M}d\mathbf{Q} + (1-\alpha)\mathbf{M}d\mathbf{Y}$$

We proceed just like we did for the derivation of the international Keynesian cross (38). We begin with the goods market clearing condition (33),

$$Y_t = (1-\alpha) \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{P_{Ht}}{\mathcal{E}_t} \right)^{-\gamma} C^*$$

Linearizing the condition gives (A.80),

$$d\mathbf{Y} = \frac{\alpha}{1-\alpha} \chi d\mathbf{Q} + (1-\alpha) d\mathbf{C} \quad (\text{A.93})$$

As before, consumption C_t here is described by the intertemporal consumption function (31),

$$C_t = \mathcal{C}_t \left(\left\{ \frac{P_{Hs}}{P_s} Y_s \right\}_{s=0}^{\infty}, \{r_s\}_{s=0}^{\infty} \right)$$

Let \mathbf{M} and \mathbf{M}^r denote the Jacobians of \mathcal{C} with respect to real income and real interest rates. To first order, therefore,

$$d\mathbf{C} = -\frac{\alpha}{1-\alpha} \mathbf{M} d\mathbf{Q} + \mathbf{M} d\mathbf{Y} + \mathbf{M}^r d\mathbf{r} \quad (\text{A.94})$$

where the first two terms are exactly as in (A.79). Substituting (A.94) into (A.93) yields (44).

C.2 Proof of proposition 9

Proof of the non-linear neutrality result for $\eta = \gamma = 1$. We start by proving the neutrality result non-linearly. To do so, we state an important property that comes out of the closed economy result in Werning (2015) based on its proof in Appendix A of Auclert, Rognlie and Straub (2020). The consumption function that we introduce in Section 4, $C_t = \mathcal{C}_t(r_t, \mathcal{Y}_t)$, where we abbreviate real income by $\mathcal{Y}_t \equiv \frac{P_{Ht}}{P_t} Y_t$, has the following property,

$$\mathcal{C}_t(\{r_s, \mathcal{Y}_s\}) = \mathcal{Y}_t \cdot \mathcal{C}_t \left(\left\{ (1+r_s) \cdot \frac{\mathcal{Y}_s}{\mathcal{Y}_{s+1}} - 1, 1 \right\} \right) \quad (\text{A.95})$$

In particular, this implies that if an aggregate Euler equation relationship between \mathcal{Y}_t and r_t holds, that is, $\frac{1}{\mathcal{Y}_s} = \frac{1+r_s}{1+r_{ss}} \cdot \frac{1}{\mathcal{Y}_{s+1}}$, this simplifies to

$$\mathcal{C}_t(\{r_s, \mathcal{Y}_s\}) = \mathcal{Y}_t \cdot C_{ss} \quad (\text{A.96})$$

where $C_{ss} = \mathcal{C}_t(r_{ss}, 1)$ denotes steady state consumption (normalized to 1 in our model).

With this in mind, we now prove the non-linear equivalence result with $\eta = \gamma = 1$, that is, $\mathcal{Y}_t = Y_t^{RA}$ in response to an arbitrary monetary policy shock $\{r_t\}$. We begin by deriving C_t^{RA} , Y_t^{RA} and the path of aggregate real income $\mathcal{Y}_t^{RA} \equiv \frac{P_{Ht}}{P_t} Y_t^{RA}$ in the RA model.

Since we are considering the Cole-Obstfeld case $\eta = 1$, the CPI (4) is

$$P_t = P_{Ht}^{1-\alpha} \mathcal{E}_t^\alpha$$

and the real exchange rate (6) is

$$Q_t = \frac{\mathcal{E}_t}{P_t} = \frac{\mathcal{E}_t}{P_{Ht}^{1-\alpha} \mathcal{E}_t^\alpha} = \left(\frac{\mathcal{E}_t}{P_{Ht}} \right)^{1-\alpha}$$

In particular, the relative price of home goods in units of the CPI is

$$\frac{P_{Ht}}{P_t} = \frac{P_{Ht}}{P_{Ht}^{1-\alpha} \mathcal{E}_t^\alpha} = \left(\frac{P_{Ht}}{\mathcal{E}_t} \right)^\alpha = (Q_t)^{-\frac{\alpha}{1-\alpha}}$$

Home output is therefore given by

$$Y_t^{RA} = (1 - \alpha) Q_t^{\frac{\alpha}{1-\alpha}} C_t^{RA} + \alpha Q_t^{\frac{1}{1-\alpha}} \quad (\text{A.97})$$

Observe that if the Backus-Smith condition $C_t^{RA} = Q_t$ holds, then (A.97) implies that aggregate real income is simply given by

$$\mathcal{Y}_t^{RA} = \frac{P_{Ht}}{P_t} Y_t^{RA} = Q_t^{-\frac{\alpha}{1-\alpha}} Y_t^{RA} = C_t^{RA} \quad (\text{A.98})$$

Moreover, aggregate consumption satisfies the Euler equation (25),

$$\frac{1}{C_t^{RA}} = \frac{1 + r_t}{1 + r_{ss}} \frac{1}{C_{t+1}^{RA}} \quad (\text{A.99})$$

Combining (A.98) and (A.99), we see that \mathcal{Y}_t^{RA} satisfies the same Euler equation,

$$\frac{1}{\mathcal{Y}_t^{RA}} = \frac{1 + r_t}{1 + r_{ss}} \frac{1}{\mathcal{Y}_{t+1}^{RA}} \quad (\text{A.100})$$

To verify that $Y_t^{RA}, \mathcal{Y}_t^{RA}, C_t^{RA}$ are identical in the HA model, we need to show that

$$C_t = \mathcal{C}_t \left(\left\{ r_s, \mathcal{Y}_s^{RA} \right\} \right) = C_t^{RA} \quad (\text{A.101})$$

But (A.101) follows directly from property (A.95) of the consumption function, which simplifies to (A.96) here given (A.100). Since the other aggregate equations of the model are the same, the result holds under any monetary policy rule and applies to all aggregate prices and quantities. This concludes our proof. \square

Proof to first-order for general η, γ such that $(1 - \alpha) \eta + \gamma = 2 - \alpha$. We proceed in two steps. First we prove a helpful lemma.

Lemma 1. *For our heterogeneous-agent model with $\sigma = 1$, we have that*

$$\mathbf{M}^r = -(\mathbf{I} - \mathbf{M}) \mathbf{U} \quad (\text{A.102})$$

Proof to lemma 1. This result is the differential version of (A.95). To see this, construct, for any given path $\{r_t\}$ a path of real income $\{\mathcal{Y}_t\}$ defined recursively by

$$\mathcal{Y}_t = \frac{1 + r_{ss}}{1 + r_t} \mathcal{Y}_{t+1}$$

To first order, this equation implies that

$$d\mathcal{Y} = -\mathbf{U} dr \quad (\text{A.103})$$

where

$$\mathbf{U} \equiv \begin{pmatrix} 1 & 1 & 1 & \cdots \\ 0 & 1 & 1 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (\text{A.104})$$

Now we linearize (A.95). We find

$$d\mathbf{C} = \mathbf{M}^r d\mathbf{r} + \mathbf{M} d\mathbf{Y} = d\mathbf{Y}$$

Substituting $d\mathbf{Y}$ with (A.103), this can be restated as

$$\mathbf{M}^r d\mathbf{r} - \mathbf{M} \mathbf{U} d\mathbf{r} = -\mathbf{U} d\mathbf{r}$$

As this holds for an arbitrary path $d\mathbf{r}$, we find

$$\mathbf{M}^r - \mathbf{M} \mathbf{U} = -\mathbf{U}$$

which is equivalent to (A.102). \square

We use lemma 1 to restate the generalized Keynesian cross (44), now with arbitrary η, γ as

$$d\mathbf{Y} = \left(\frac{\alpha}{1-\alpha} \chi + 1 - \alpha \right) d\mathbf{Q} - \mathbf{M} d\mathbf{Q} + (1-\alpha) \mathbf{M} d\mathbf{Y} \quad (\text{A.105})$$

where $d\mathbf{Q}$ continues to be given by $d\mathbf{Q} = -\mathbf{U} d\mathbf{r}$ (see section 4.1) as the implied real exchange rate response to the monetary policy shock. Solving (A.105) as in appendix B.3, we find

$$d\mathbf{Y} = d\mathbf{Y}^{RA-CM} + \alpha (\chi - (2 - \alpha)) (\mathbf{I} - (1 - \alpha) \mathbf{M})^{-1} d\mathbf{Q}$$

Following the same steps as in section B.7, this allows us to sign the magnitude of the output response relative to the RA solution. For example, in response to monetary easing, inducing an exchange rate depreciation $d\mathbf{Q} \geq 0$, a value $\chi < 2 - \alpha$ results in an output response $d\mathbf{Y}$ that lies below the RA model's output response. \square

C.3 Proof of proposition 10

Proof. Linearizing (23) around the steady state (when $t > 0$), we obtain

$$dnfa_t = (1 + r_{ss}) dnfa_{t-1} + dNX_t$$

Iterating forward, this implies that for any $t \geq 0$,

$$dnfa_t = \sum_{s=1}^{\infty} (1 + r_{ss})^{-s} dNX_{t+s}$$

Substituting equation (A.78) for dNX_t , this becomes

$$dnfa_t = \frac{\alpha}{1-\alpha} (\chi - 1) \left(\sum_{s=1}^{\infty} (1 + r_{ss})^{-s} dQ_{t+s} \right) - \alpha \left(\sum_{s=1}^{\infty} (1 + r_{ss})^{-s} dC_{t+s} \right)$$

In the case where $dQ_{t+s} = 0$ for all $s \geq 1$, this simplifies to

$$\sum_{s=1}^{\infty} (1 + r_{ss})^{-s} dC_{t+s} = \frac{1}{\alpha} dnfa_t \quad (\text{A.106})$$

the desired expression for the present value of remaining consumption in terms of the NFA.

For all $t + s$ where $dQ_{t+s} = 0$, we have $dY_{t+s} = (1 - \alpha)dC_{t+s}$, and therefore from (A.106) we get

$$\sum_{s=1}^{\infty} (1 + r_{ss})^{-s} dY_{t+s} = \frac{1 - \alpha}{\alpha} d\text{nfat}_t$$

□

Proof. We also prove the statement here that the present value of the output response to monetary policy is negative whenever $\chi < 1 - \alpha$. To do so, consider equation (A.105) and take present values on both sides. Since \mathbf{M} preserves present values, we find

$$PV(d\mathbf{Y}) = \left(\frac{\alpha}{1 - \alpha} \chi + 1 - \alpha - 1 \right) PV(d\mathbf{Q}) + (1 - \alpha) PV(d\mathbf{Y})$$

and, simplifying,

$$PV(d\mathbf{Y}) = \frac{\chi - (1 - \alpha)}{1 - \alpha} PV(d\mathbf{Q})$$

This shows that the present value is negative, due to “stealing demand from the future”, whenever $\chi < 1 - \alpha$. □

C.4 Representative-agent incomplete-market (RA-IM) model

The RA-IM model is a special case of the derivations in appendices C.1–C.2, with specific \mathbf{M} and \mathbf{M}^r . Following the same steps as in appendix B.4, we find that, for general σ , we have

$$\begin{aligned} \mathbf{M} &= \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} \\ \mathbf{M}^r &= -\frac{1}{\sigma} (\mathbf{I} - \mathbf{M}) \mathbf{U} \end{aligned}$$

The solution to the international Keynesian cross is

$$d\mathbf{Y} = (1 - \alpha) \mathbf{M}^r d\mathbf{r} + \frac{\alpha}{1 - \alpha} \chi d\mathbf{Q} - \alpha \mathbf{M} d\mathbf{Q} + (1 - \alpha) \mathbf{M} d\mathbf{Y}$$

With pure monetary shocks, we have $d\mathbf{Q} = -\mathbf{U} d\mathbf{r}$, so using the relation between \mathbf{M}^r and \mathbf{M} this rewrites as

$$d\mathbf{Y} = \left(\frac{1 - \alpha}{\sigma} (\mathbf{I} - \mathbf{M}) + \frac{\alpha}{1 - \alpha} \chi - \alpha \mathbf{M} \right) d\mathbf{Q} + (1 - \alpha) \mathbf{M} d\mathbf{Y}$$

or

$$d\mathbf{Y} = \left(\left(\frac{1 - \alpha}{\sigma} + \frac{\alpha}{1 - \alpha} \chi \right) \mathbf{I} - \left(\alpha + \frac{1 - \alpha}{\sigma} \right) \mathbf{M} \right) d\mathbf{Q} + (1 - \alpha) \mathbf{M} d\mathbf{Y}$$

Next note that, using the fact that $\mathbf{M}^k = \mathbf{M}$ for $k \geq 1$, we have

$$(\mathbf{I} - (1 - \alpha) \mathbf{M})^{-1} = \mathbf{I} + \frac{1 - \alpha}{\alpha} \mathbf{M}$$

so, the solution for the RA-IM model with general σ is

$$\begin{aligned}
d\mathbf{Y} &= \left(\mathbf{I} + \frac{1-\alpha}{\alpha} \mathbf{M} \right) \left(\left(\frac{1-\alpha}{\sigma} + \frac{\alpha}{1-\alpha} \chi \right) \mathbf{I} - \left(\alpha + \frac{1-\alpha}{\sigma} \right) \mathbf{M} \right) d\mathbf{Q} \\
&= \left(\left(\frac{1-\alpha}{\sigma} + \frac{\alpha}{1-\alpha} \chi \right) \mathbf{I} + \left(-\left(\alpha + \frac{1-\alpha}{\sigma} \right) + \frac{1-\alpha}{\alpha} \left(\frac{1-\alpha}{\sigma} + \frac{\alpha}{1-\alpha} \chi \right) - \frac{1-\alpha}{\alpha} \left(\alpha + \frac{1-\alpha}{\sigma} \right) \right) \mathbf{M} \right) d\mathbf{Q} \\
&= \left(\left(\frac{1-\alpha}{\sigma} + \frac{\alpha}{1-\alpha} \chi \right) \mathbf{I} + \left(\chi - \alpha - (1-\alpha) \left(1 + \frac{1}{\sigma} \right) \right) \mathbf{M} \right) d\mathbf{Q} \\
&= \underbrace{\left(\frac{1-\alpha}{\sigma} + \frac{\alpha}{1-\alpha} \chi \right) d\mathbf{Q}}_{d\mathbf{Y}^{RA-CM}} + \frac{1}{1-\alpha} \left(\chi - \left(1 + \frac{1-\alpha}{\sigma} \right) \right) \mathbf{M} d\mathbf{Q}
\end{aligned}$$

It immediately follows that, if $\chi = 1 + \frac{1-\alpha}{\sigma}$ (in particular, when $\sigma = 1$ and $\chi = 2 - \alpha$), the solution collapses to the RA-CM model's solution. Moreover, when $d\mathbf{Q} > 0$, we have $\mathbf{M} d\mathbf{Q} > 0$ (since $\mathbf{M} > 0$), so $d\mathbf{Y}^{HA-IM} < d\mathbf{Y}^{RA-CM}$ if and only if $\chi < 1 + \frac{1-\alpha}{\sigma}$. This completes the extension of Proposition 9 to the comparison between RA-CM and RA-IM.

C.5 Taylor rules and productivity shocks

In this section, we explore how the HA and RA models respond to shocks when the central bank follows a Taylor rule based on producer prices, as in [Galí and Monacelli \(2005\)](#). We first revisit the neutrality results for exchange rate depreciations and monetary policy, and then consider the response to productivity shocks as well.

Exchange rate depreciations. A depreciation $dQ_t \geq 0$, $d\iota_t^* \geq 0$, affects the demand for home goods and thus PPI inflation $\pi_{Ht} = \pi_{wt}$ through the Phillips curve (18). CPI inflation is then determined by

$$\pi_t = \pi_{Ht} + \frac{\alpha}{1-\alpha} (dQ_t - dQ_{t-1}) \quad (\text{A.107})$$

and the real interest rate path by

$$dr_t = d\iota_t - \pi_{t+1} = \phi \pi_{Ht} - \pi_{Ht+1} - \frac{\alpha}{1-\alpha} (dQ_{t+1} - dQ_t) \quad (\text{A.108})$$

As the Phillips curve involves endogenous variables, that themselves depend on dr_t , this situation is significantly less tractable than the case with a fixed real interest rate. Still, we can make progress by focusing on AR(1) shocks to ι_t^* , with some fixed persistence $\rho \in (0, 1)$. We show below that, in this case, there still exists a value for χ ,

$$\chi = 1 - \alpha \frac{1 + \zeta \varphi}{1 + \zeta (\varphi + \sigma)} \in (1 - \alpha, 1), \quad \zeta \equiv \frac{\kappa_w (\phi - \rho)}{(1 - \rho) (1 - \beta \rho)} > 0$$

for which the responses of all aggregate variables, such as output, employment, and consumption are independent of \mathbf{M} , and hence the same for the RA and the HA model.⁵⁵

Monetary policy shocks. The path of real rates is now no longer just a function of the path of monetary policy shocks ϵ_t ; it also depends on the response of inflation, and thereby also on the endogenous response of aggregate variables. However, the international Keynesian cross (44) and $d\mathbf{Q} = -\mathbf{U} d\mathbf{r}$ still hold, and thus, for $\chi = 2 - \alpha$, the response of aggregates to the shock is still

⁵⁵The response is also the same in the incomplete markets RA and the TA model.

independent of \mathbf{M} *conditional* on a given real interest rate path. This is why our neutrality result for monetary policy goes through unchanged for the Taylor rule $\iota_t = r_{ss} + \phi\pi_{Ht} + \epsilon_t$.

Productivity shocks. The exact same logic applies to the case of productivity shocks Z_t . Those shocks affect home prices according to $P_{Ht} = \mu W_t / Z_t$. This causes shifts in wage inflation via the Phillips curve (18) and thus shifts in real interest rates via (A.108). Since the responses in RA and HA models to changes in real interest rates are still identical when $\chi = 2 - \alpha$, the responses to productivity shocks in both models are also identical for this choice of χ .

Proof of the result on exchange rate depreciations with a Taylor rule. We focus on AR(1) shocks here, such that $dQ_t = \rho dQ_{t-1}$ for some $\rho \in (0, 1)$. The output response with a Taylor rule is determined by the following system of equations. The first is the international Keynesian cross (44),

$$d\mathbf{Y} = - (1 - \alpha) (\mathbf{I} - \mathbf{M}) \mathbf{U} d\mathbf{r} + \alpha \frac{\chi}{1 - \alpha} d\mathbf{Q} - \alpha \mathbf{M} d\mathbf{Q} + (1 - \alpha) \mathbf{M} d\mathbf{Y} \quad (\text{A.109})$$

simplified using (A.102). The second is the Phillips curve for wage inflation (18), in linearized terms given by

$$\pi_{wt} = \kappa_w (\varphi dN_t + \sigma dC_t) + \beta \pi_{wt+1} \quad (\text{A.110})$$

The third is the determination of the real interest rate through the Taylor rule (A.108),

$$dr_t = d\iota_t - \pi_{t+1} = \phi \pi_{wt} - \pi_{wt+1} - \frac{\alpha}{1 - \alpha} (dQ_{t+1} - dQ_t) \quad (\text{A.111})$$

and the forth is the determination of the real exchange rate as

$$dQ_t = \sum_{s \geq t} \frac{di_s^* - dr_s}{1 + r} \quad (\text{A.112})$$

We guess and verify that all variables are exponentially decaying with the same persistence ρ . In this case,

$$d\mathbf{r} = k \cdot d\mathbf{i}^*$$

with an unknown $k = \frac{dr_0}{di_0^*}$. From (A.112), we then get that

$$d\mathbf{Q} = \frac{1}{1 - \rho} (d\mathbf{i}^* - d\mathbf{r}) = \frac{k^{-1} - 1}{1 - \rho} d\mathbf{r} \quad (\text{A.113})$$

So we can rewrite (A.109) as

$$d\mathbf{Y} = \left(\alpha \frac{\chi}{1 - \alpha} - (1 - \alpha) \frac{k}{1 - k} \right) d\mathbf{Q} - \mathbf{M} \left(\alpha - (1 - \alpha) \frac{k}{1 - k} \right) d\mathbf{Q} + (1 - \alpha) \mathbf{M} d\mathbf{Y} \quad (\text{A.114})$$

Rearranging,

$$d\mathbf{Y} - \left(\alpha \frac{\chi}{1 - \alpha} - (1 - \alpha) \frac{k}{1 - k} \right) d\mathbf{Q} = (1 - \alpha) \mathbf{M} \left(d\mathbf{Y} - \frac{\alpha - (1 - \alpha) \frac{k}{1 - k}}{1 - \alpha} d\mathbf{Q} \right)$$

we see that the solution is independent of \mathbf{M} precisely if and only if

$$\alpha \frac{\chi}{1 - \alpha} - (1 - \alpha) \frac{k}{1 - k} = \frac{\alpha - (1 - \alpha) \frac{k}{1 - k}}{1 - \alpha}$$

which is equivalent to

$$\chi = 1 - (1 - \alpha) \frac{k}{1 - k}$$

For $k = 0$, we recover neutrality at $\chi = 1$. For $k \rightarrow \infty$, the monetary response dominates the output response, so the threshold converges to $\chi \rightarrow 2 - \alpha$. In case the neutrality result holds, the output response is given by

$$d\mathbf{Y} = \left(\frac{\alpha}{1 - \alpha} - \frac{k}{1 - k} \right) d\mathbf{Q}$$

Given the linear production function, this is equivalent to the employment response, $d\mathbf{N} = d\mathbf{Y}$. The aggregate consumption response is

$$d\mathbf{C} = -\frac{k}{1 - k} d\mathbf{Q}$$

Substituting these formulas into the linearized wage Phillips curve (A.110), we find that

$$\pi_{wt} = \frac{1}{1 - \beta\rho} \kappa_w (\varphi dN_t + \sigma dC_t) = \frac{1}{1 - \beta\rho} \kappa_w \left(\varphi \left(\frac{\alpha}{1 - \alpha} - \frac{k}{1 - k} \right) - \sigma \frac{k}{1 - k} \right) dQ_t$$

and therefore that

$$dr_t = \left\{ \frac{\phi - \rho}{1 - \beta\rho} \kappa_w \left(\varphi \left(\frac{\alpha}{1 - \alpha} - \frac{k}{1 - k} \right) - \sigma \frac{k}{1 - k} \right) + \frac{\alpha}{1 - \alpha} (1 - \rho) \right\} dQ_t$$

Comparing this with (A.113), we find an equation for $k/(1 - k)$,

$$(1 - \rho) \frac{k}{1 - k} = \frac{\phi - \rho}{1 - \beta\rho} \kappa_w \left(\varphi \left(\frac{\alpha}{1 - \alpha} - \frac{k}{1 - k} \right) - \sigma \frac{k}{1 - k} \right) + \frac{\alpha}{1 - \alpha} (1 - \rho)$$

We solve this equation for $k/(1 - k)$

$$\begin{aligned} \left(1 - \rho + \frac{\phi - \rho}{1 - \beta\rho} \kappa_w (\varphi + \sigma) \right) \frac{k}{1 - k} &= \frac{\phi - \rho}{1 - \beta\rho} \kappa_w \varphi \frac{\alpha}{1 - \alpha} + \frac{\alpha}{1 - \alpha} (1 - \rho) \\ \left(1 - \rho + \frac{\phi - \rho}{1 - \beta\rho} \kappa_w (\varphi + \sigma) \right) \frac{k}{1 - k} &= \frac{\alpha}{1 - \alpha} \left(1 - \rho + \frac{\phi - \rho}{1 - \beta\rho} \kappa_w \varphi \right) \\ \frac{k}{1 - k} &= \frac{\alpha}{1 - \alpha} \cdot \frac{1 + \frac{\kappa_w(\phi - \rho)}{(1 - \rho)(1 - \beta\rho)} \varphi}{1 + \frac{\kappa_w(\phi - \rho)}{(1 - \rho)(1 - \beta\rho)} (\varphi + \sigma)} \end{aligned}$$

Note that the solution lies in $k \in (0, \alpha)$. The neutrality threshold is then given by

$$\chi = 1 - \alpha \frac{1 + \frac{\kappa_w(\phi - \rho)}{(1 - \rho)(1 - \beta\rho)} \varphi}{1 + \frac{\kappa_w(\phi - \rho)}{(1 - \rho)(1 - \beta\rho)} (\varphi + \sigma)} \in (1 - \alpha, 1)$$

D Appendix to section 5

D.1 Non-homothetic demand

The Bellman equation (9) that incorporates the preferences in (47) reads

$$\begin{aligned}
 V_t(a^p, e) &= \max_{c_F, c_H, a'} u \left(\left[\alpha^{1/\eta} (c_F - \underline{c})^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_H^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \right) \\
 &\quad - v(N_t) + \beta \mathbb{E}_t [V_{t+1}((1+r_t)a', e')] \\
 \text{s.t.} \quad &\frac{P_{Ft}}{P_t} c_F + \frac{P_{Ht}}{P_t} c_H + a' = a^p + \frac{W_t}{P_t} N_t \\
 &a' \geq 0
 \end{aligned}$$

Notice that, by relabeling $\tilde{c}_F \equiv c_F - \underline{c}$, this is equivalent to

$$\begin{aligned}
 V_t(a^p, e) &= \max_{\tilde{c}_F, c_H, a'} u \left(\left[\alpha^{1/\eta} (\tilde{c}_F)^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_H^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \right) \\
 &\quad - v(N_t) + \beta \mathbb{E}_t [V_{t+1}((1+r_t)a', e')] \\
 \text{s.t.} \quad &\frac{P_{Ft}}{P_t} \tilde{c}_F + \frac{P_{Ht}}{P_t} c_H + a' = a^p + \frac{W_t}{P_t} N_t - \frac{P_{Ft}}{P_t} \underline{c} \\
 &a' \geq 0
 \end{aligned}$$

or, defining $\tilde{c} = \left[\alpha^{1/\eta} (\tilde{c}_F)^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_H^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}$, as simply

$$\begin{aligned}
 V_t(a^p, e) &= \max_{\tilde{c}, a'} u(\tilde{c}) - v(N_t) + \beta \mathbb{E}_t [V_{t+1}((1+r_t)a', e')] \\
 \text{s.t.} \quad &\tilde{c} + a' = a^p + \frac{W_t}{P_t} N_t - \frac{P_{Ft}}{P_t} \underline{c} \\
 &a' \geq 0
 \end{aligned}$$

where P_t is the standard CES price index (4). Hence, this is the standard consumption-saving problem, only with a modified income process that subtracts $-\frac{P_{Ft}}{P_t} \underline{c}$ to real income in every state of the world. The policy functions $\tilde{c}_t(a^p, e)$, $a_{t+1}(a^p, e)$ for this problem, as well as the aggregates \tilde{C}_F and A_{t+1} of policies integrated against the time varying distribution, can be obtained from the sequence of inputs $\{r_t^p, w_t N_t, \frac{P_{Ft}}{P_t} \underline{c}\}$ using standard tools. Since every agent's policy for $c_{Ft}(a^p, e) = \underline{c} + \tilde{c}_{Ft}(a^p, e)$, it follows that aggregate spending on foreign goods is simply

$$\begin{aligned}
 C_{Ft} &= \underline{c} + \tilde{C}_{Ft} \\
 &= \underline{c} + \alpha \left(\frac{P_{Ft}}{P_t} \right)^{-\eta} \tilde{C}_t
 \end{aligned} \tag{A.115}$$

while aggregate spending on domestic goods is

$$C_{Ht} = (1-\alpha) \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} \tilde{C}_t \tag{A.116}$$

Note that equations (A.115) and (A.116) only require the standard CES prices index P_t , which is the price index of an infinitely wealthy agent, and do not require the ideal price indices for agents at different points in the distribution (a^p, e) .

D.2 Unequal incidence of aggregate shocks

As in [Auclert and Rognlie \(2018\)](#), we assume that the real labor income of a household with productivity e , denoted $\text{inc}_t(e)$, is

$$\text{inc}_t(e) = \frac{W_t}{P_t} N_t \frac{e^{1+\zeta \log\left(\frac{W_t}{P_t} N_t / \frac{W_{ss}}{P_{ss}} N_{ss}\right)}}{\int \tilde{e}^{1+\zeta \log\left(\frac{W_t}{P_t} N_t / \frac{W_{ss}}{P_{ss}} N_{ss}\right)} d\tilde{e}} \quad (\text{A.117})$$

Elasticity to aggregate shocks. Taking logs and differentiating this equation with respect to aggregate income $\frac{W_t}{P_t} N_t$ around the steady state gives

$$d \log \text{inc}_t(e) = \left(\zeta \log e + 1 - \zeta \left(\int \tilde{e} \log \tilde{e} d\tilde{e} \right) \right) d \log \left(\frac{W_t}{P_t} N_t \right) \quad (\text{A.118})$$

Hence, the elasticity of individual income to aggregate income $d \log \text{inc}_t(e) / d \log \left(\frac{W_t}{P_t} N_t \right)$ is increasing in $\zeta \log e$. When $\zeta < 0$, the income of low-productivity households increases more than the income of high-productivity households in booms and falls more in recessions.

Calibration. We use information from [Blanco et al. \(2024\)](#) to calibrate ζ . The authors provided us with estimates for b_i from a regression

$$\Delta \log \text{inc}_{jit} = a_i + b_i \Delta \log \left(\frac{W_t}{P_t} N_t \right) + \epsilon_{jit} \quad (\text{A.119})$$

where j indicates a worker, and i the decile of income. The equation is estimated for each income decile i separately using administrative employer-employee data from Argentina. $\Delta \log \text{inc}_{jit}$ is the annual growth rate of real income for worker j in income decile i and $\Delta \log \frac{W_t}{P_t} N_t$ is the average annual growth of real income for all workers. The income deciles are defined using income in 2000-2001 and the estimation is done on the 2000-2006 period.

In our model, we can aggregate equation (A.118) across workers within income decile i to get

$$d \log \text{inc}_{it} \equiv \sum_{e \in i} d \log \text{inc}_t(e) = \left(\zeta \sum_{e \in i} \log e + 1 - \zeta \left(\int \tilde{e} \log \tilde{e} d\tilde{e} \right) \right) d \log \left(\frac{W_t}{P_t} N_t \right) \quad (\text{A.120})$$

Our strategy is to find $\zeta < 0$ such that the standard deviation across income deciles of the elasticity in the model $\hat{b}_i \equiv \zeta \sum_{e \in i} \log e + 1 - \zeta \left(\int \tilde{e} \log \tilde{e} d\tilde{e} \right)$ is identical to the standard deviation of the elasticity in the data b_i ⁵⁶.

⁵⁶Equation (A.120) can be written as $d \text{inc}_{it} = \left(\frac{\text{inc}_i}{\frac{W_t}{P_t} N_t} \hat{b}_i \right) d \left(\frac{W_t}{P_t} N_t \right)$. The identity $\sum_i \text{inc}_{it} = \frac{W_t}{P_t} N_t$ implies that $\sum_i \left(\frac{\text{inc}_i}{\frac{W_t}{P_t} N_t} \hat{b}_i \right) = 1$. In the data, this identity does not hold so we multiply the empirical estimates for b_i by a constant to ensure that $\sum_i \left(\frac{\text{inc}_i}{\frac{W_t}{P_t} N_t} b_i \right) = 1$. This manipulation influences the levels of b_i but has very little impact on our estimates for ζ since we target the standard deviation of b_i across income deciles.

D.3 Delayed substitution model

We introduce delayed substitution as a modification of the household side of the model, both in the domestic economy and in the rest of the world. The basic idea is to assume that each household has only an iid probability $1 - \theta$ in each period of being able to adjust the composition of its consumption bundle \mathbf{x} . With probability θ , the household is required to keep the same consumption bundle \mathbf{x} , and consume each good in proportion to it, i.e. $c_k = x_k c$, where x_k is the amount of k in the consumption bundle and c is total consumption.

First we work out the analytics, in a general static problem where consumption is some constant-returns-to-scale aggregate of many goods, of the optimal bundle given the prices of each good. Then we show how this problem can be embedded in the dynamic incomplete markets problem that the households in our model solve, and work out the first-order equations characterizing household behavior. Finally, we specialize the consumption side to the nested CES of home and foreign goods in our model, and derive the consequences.

Static problem: the optimal bundle in terms of prices. Suppose that a household has preferences over aggregate consumption goods using some constant-returns-to-scale aggregator F

$$c = F(c_1, \dots, c_K) \quad (\text{A.121})$$

We define a *bundle* $\{x_k\}$ as anything that gives a total consumption of one

$$1 = F(x_1, \dots, x_K) \quad (\text{A.122})$$

and say that the price index $P(\{x_k\}, \{P_k\})$ corresponding to this bundle is just its cost:

$$P(\{x_k\}, \{P_k\}) = \sum_k P_k x_k \quad (\text{A.123})$$

The cost of the bundle that minimizes (A.123) subject to (A.122) is the ordinary price index P .

Consider the optimal bundle $\{\hat{x}_k\}$ given some prices $\{P_k\}$, i.e. the solution to the problem of minimizing price (A.123) subject to (A.122). The Lagrangian is

$$\sum_k P_k \hat{x}_k - \lambda (F(\hat{x}_1, \dots, \hat{x}_K) - 1)$$

and first-order conditions are

$$P_k = \lambda F_k \quad (\text{A.124})$$

(Note that since F is constant-returns-to-scale, the marginal cost λ equals the conventional price index P .)

Log-differentiating gives

$$\begin{aligned} d \log P_k &= d \log \lambda + d \log F_k \\ &= d \log \lambda + \sum_{k'} \frac{F_{kk'} \hat{x}_{k'}}{F_k} d \log \hat{x}_{k'} \end{aligned} \quad (\text{A.125})$$

Log-differentiating the condition $F(\hat{x}_1, \dots, \hat{x}_K) = 1$ gives

$$\sum_k \frac{F_k \hat{x}_k}{F} d \log \hat{x}_k = 0 \quad (\text{A.126})$$

We note that $\sum_{k'} \frac{F_{kk'} \hat{x}_{k'}}{F_k} = 0$ by Euler's identity: a proportional shift in all \hat{x}_k does not change any partial derivatives. This means that the matrix $\mathbf{F} \equiv [\frac{F_{kk'} \hat{x}_{k'}}{F_k}]_{kk'}$ has a null vector of ones. By the envelope theorem, we also know that $d\lambda = \sum_k dP_k \hat{x}_k$, or that $d \log \lambda = \sum_k \alpha_k d \log P_k$, where $\alpha_k \equiv P_k \hat{x}_k / P$ is the share of good k in the bundle. So then we need to find the solution \hat{x} to

$$\mathbf{F} d \log \hat{\mathbf{x}} = d \log \mathbf{P} - (\alpha' d \log \mathbf{P}) \mathbf{1} \quad (\text{A.127})$$

such that (A.126) holds, i.e. $\sum_k \frac{F_{kk'} \hat{x}_k}{F} d \log \hat{x}_k = 0$. Given one such solution $d \log \hat{\mathbf{x}}$ to (A.127), we can find exactly one that satisfies (A.126), since $\mathbf{1}$ is in the null-space of \mathbf{F} .⁵⁷ Let us denote the linear map from $d \log \mathbf{P}$ to this solution $d \log \hat{\mathbf{x}}$ by the matrix \mathbf{G} :

$$d \log \hat{\mathbf{x}} = \mathbf{G} d \log \mathbf{P} \quad (\text{A.128})$$

Incomplete markets dynamic problem. Now consider a household who is solving a generalization of the standard incomplete markets problem in (9), where consumption within each period t is given by an aggregator (A.121) of goods with prices $\mathbf{P}_t = \{P_{kt}\}$. We assume that the household picks a bundle $\mathbf{x} \equiv \{x_k\}$ of goods giving an aggregate of 1 as in (A.122), and must consume in proportion to that bundle until it receives a Calvo option to reset the bundle, which has probability $1 - \theta$ in each period. (Since the bundle has aggregate value 1, consumption of each good in each period is given by $c_{ikt} = x_k c_{it}$.)

The value function given an inherited \mathbf{x} at date t is then

$$\begin{aligned} V_t(a, e, \mathbf{x}) &= \max_{E, a'} U\left(E \cdot \frac{P_t}{\bar{P}(\mathbf{x}, \mathbf{P}_t)}\right) + \beta \theta \mathbb{E}_t [V_{t+1}(a', e', \mathbf{x})] + \beta(1 - \theta) \mathbb{E} \left[\max_{\mathbf{x}'} V_{t+1}(a', e', \mathbf{x}') \right] \\ &E + a' = (1 + r_t^p) a + w n_t(e) \\ &a' \geq \underline{a} \end{aligned} \quad (\text{A.129})$$

where the choice is made over expenditure in real units deflated at the ordinary price index P_t , but consumption may be less because the actual cost of consumption at the fixed bundle x is $\bar{P}(\mathbf{x}, \mathbf{P}_t)$.

Note that the envelope condition for x , which appears only in the objective (A.129), is

$$\begin{aligned} \frac{\partial V_t(a, e, \mathbf{x})}{\partial \mathbf{x}} &= -U'(c_t)c_t \frac{\partial \log \bar{P}(\mathbf{x}, \mathbf{P}_t)}{\partial \mathbf{x}} + \beta \theta \mathbb{E}_t \left[\frac{\partial V_{t+1}(a', e', \mathbf{x})}{\partial \mathbf{x}} \right] \\ &= -U'(c_t)c_t \mathbf{P}_t + \beta \theta \mathbb{E}_t \left[\frac{\partial V_{t+1}(a', e', \mathbf{x})}{\partial \mathbf{x}} \right] \end{aligned} \quad (\text{A.130})$$

where we use $\frac{\partial \log \bar{P}(\mathbf{x}, \mathbf{P}_t)}{\partial \mathbf{x}} = \mathbf{P}_t$ from (A.123) to simplify and denote real consumption, i.e. the aggregate in (A.121), by c_t . Recursively expanding this out and subsuming the states a and e into the subscript $i \equiv (a, e)$, we get

$$\frac{\partial V_{it}(\mathbf{x}_{it})}{\partial \mathbf{x}} = \mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta \theta)^s U'(c_{it+s}) c_{it+s} \mathbf{P}_{t+s} \quad (\text{A.131})$$

⁵⁷In more detail: assuming that $\mathbf{F} \equiv [\frac{F_{kk'} \hat{x}_{k'}}{F_k}]_{kk'}$ otherwise has full rank, i.e. its rank is $K - 1$, then we note that $\alpha' \mathbf{F} = 0$, so that the range of \mathbf{F} is the space orthogonal to α . The right side of (A.127) is always orthogonal to α . There is therefore always a one-dimensional space of solutions to (A.127), with the difference between any two solutions given by a multiple of $\mathbf{1}$, and exactly one of these solutions will satisfy (A.126) as well.

Now suppose that we are maximizing over feasible \hat{x}_{it} , i.e. solving the problem which has Lagrangian

$$V_{it}(\hat{x}) - \lambda (F(\hat{x}_1, \dots, \hat{x}_K) - 1) \quad (\text{A.132})$$

where for simplicity we suppress the i and t subscripts on \hat{x} . The first-order condition with respect to each \hat{x}_k is

$$\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s} P_{k,t+s} = \lambda F_k \quad (\text{A.133})$$

In a steady state, with constant prices $P_{k,t+s} = P_k$, this simplifies to $P_k = (\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s})^{-1} \lambda F_k$, which is the same as the static first-order condition (A.124) but with an extra factor rescaling λ . The constraint (A.122) is also the same as in the static problem. Therefore the steady-state solution is the same (with a bundle independent of i), and while λ equaled the conventional price index P in the static problem, it now equals P times an extra factor:

$$P \left(\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s} \right) = \lambda \quad (\text{A.134})$$

Now consider linearizing (A.133) around the steady-state prices P and bundle \hat{x} . We get

$$P_k \mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s d(U'(c_{it+s}) c_{it+s}) + \mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s} dP_{k,t+s} = \lambda F_k (d \log \lambda + d \log F_k) \quad (\text{A.135})$$

and then, noting that $\sum_k dF_k \hat{x}_k = 0$ and $\sum_k P_k \hat{x}_k = P$, we can sum this weighted by \hat{x}_k to obtain

$$P \mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s d(U'(c_{it+s}) c_{it+s}) + \mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s} \sum_k \hat{x}_k dP_{k,t+s} = \lambda d \log \lambda \quad (\text{A.136})$$

Now, if we multiply both sides of (A.136) by F_k , use $P_k = PF_k$, and subtract it from (A.135), the first term on both sides cancels and we get

$$\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s} \left(dP_{k,t+s} - F_k \sum_{k'} \hat{x}_{k'} dP_{k',t+s} \right) = \lambda F_k d \log F_k$$

Now, divide both sides by $P_k = PF_k$, we get

$$\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s} \left(d \log P_{k,t+s} - \sum_{k'} \alpha_{k'} d \log P_{k',t+s} \right) = \frac{\lambda}{P} d \log F_k$$

and finally, using (A.134), we get

$$d \log F_k = \frac{\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s} (d \log P_{k,t+s} - \sum_{k'} \alpha_{k'} d \log P_{k',t+s})}{\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s}}$$

for each k , which in vector form (using $\mathbf{F} \equiv [\frac{F_{kk'} \hat{x}_{k'}}{F_k}]_{kk'}$ from the static problem) can be stacked as

$$\mathbf{F} d \log \hat{\mathbf{x}} = \frac{\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s} (d \log \mathbf{P}_t - (\alpha' d \log \mathbf{P}_t) \mathbf{1})}{\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta\theta)^s U'(c_{it+s}) c_{it+s}} \quad (\text{A.137})$$

The solution $\hat{\mathbf{x}}$ is whatever satisfies (A.137) and also satisfies $\sum_k \frac{F_k \hat{x}_k}{F} d \log \hat{x}_k = 0$.

In the static problem, we obtained the matrix \mathbf{G} that mapped $d \log \mathbf{P}$ to $d \log \hat{\mathbf{x}}$ in order to solve $F d \log \hat{\mathbf{x}} = d \log \mathbf{P} - (\alpha' d \log \mathbf{P}) \mathbf{1}$, subject to the same condition $\sum_k \frac{F_k \hat{x}_k}{F} d \log \hat{x}_k = 0$, for a single change in log prices $d \log \mathbf{P}$. Using linearity, we can combine this with (A.137) and move \mathbf{G} inside the numerator to obtain

$$\begin{aligned} d \log \hat{x}_{it} &= \frac{\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta \theta)^s U'(c_{it+s}) c_{it+s} \mathbf{G} d \log \mathbf{P}_t}{\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta \theta)^s U'(c_{it+s}) c_{it+s}} \\ &= \frac{\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta \theta)^s U'(c_{it+s}) c_{it+s} \cdot d \log \hat{x}_{t+s}^{static}}{\mathbb{E}_{it} \sum_{s=0}^{\infty} (\beta \theta)^s U'(c_{it+s}) c_{it+s}} \end{aligned} \quad (\text{A.138})$$

where we define $d \log \hat{x}_{t+s}^{static} = \mathbf{G} d \log \mathbf{P}_{t+s}$ to be the *statically* optimal bundle given prices at time $t+s$, ignoring the substitution friction, and restore the explicit i and t subscripts.

(A.138) is our primary result: the log change around the aggregate steady state in optimum bundle for individual i at time t , given the substitution friction, is a weighted average of the log change around the aggregate steady state in future static optimum bundles, with the weight on each future date $t+s$ being $(\beta \theta)^s U'(c_{it+s}) c_{it+s}$.

Special cases (RA-CM and log preferences). (A.138) is somewhat complex, but there are two special cases where it simplifies dramatically. First, if there is a representative agent and complete markets, then in the aggregate steady state, consumption c_{it+s} in (A.138) is constant over time. Second, if $U(c) = \log c$, then $U'(c)c = 1$. In both cases, (A.138) collapses to just

$$d \log \hat{x}_t = (1 - \beta \theta) \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \theta)^s d \log \hat{x}_{t+s}^{static} \quad (\text{A.139})$$

These correspond to the cases we will consider in this paper: log preferences for the heterogeneous agents in the domestic economy, and a representative agent in foreign economies. Note that even in the former case, the i subscripts now disappear: all agents want to set the same bundle.⁵⁸

Evolution of aggregate bundle. Note that in (A.129), the bundle \mathbf{x} only enters the problem in $\bar{P}(\mathbf{x}, \mathbf{P}_t)$, which is second-order in \mathbf{x} around the steady-state optimum. Heterogeneity in \mathbf{x} therefore has a second-order impact on total consumption c_{it+s} of each individual.

Disregarding these second-order terms, in period t , aggregate consumption of good k is

$$\begin{aligned} C_{kt} &= \int \left((1 - \theta) \sum_{s=0}^{\infty} \theta^s \hat{x}_{kt-s} c_{it} \right) di = \left((1 - \theta) \sum_{s=0}^{\infty} \theta^s \hat{x}_{kt-s} \right) \int c_{it} di \\ &= \underbrace{\left((1 - \theta) \sum_{s=0}^{\infty} \theta^s \hat{x}_{kt-s} \right)}_{\equiv x_{kt}} C_t \end{aligned}$$

⁵⁸In the general case, without the simplification (A.139), for each agent type i in the state space, $\mathbb{E}_{it} (\beta \theta)^s U'(c_{it+s}) c_{it+s}$ can be calculated recursively for increasing s using the law of iterated expectations. With this, we can implement (A.138) and can calculate how, for each i , $d \log \hat{x}_{it}$ depends on the path of \hat{x}_{t+s}^{static} . (Note that this only depends on i and s , not on t , due to translation invariance.) Similar tricks are needed to aggregate below into an effective average bundle $d \log \mathbf{x}_t$ at each date, since the choice of $d \log \hat{x}_{it}$ is no longer independent of expected consumption c_{it+s} .

where $\int di$ denotes aggregation over the idiosyncratic state space $i = (s, a)$ (using the distribution of agents from the aggregate steady state). For each i , the mass $(1 - \theta)\theta^s$ of agents whose bundle were last updated in period $t - s$ consume $\hat{x}_{kt-s}c_{it}$ of good k . Aggregating over all s , factoring out, and defining $x_{kt} \equiv (1 - \theta) \sum_{s=0}^{\infty} \theta^s \hat{x}_{kt-s}$ to be the “average” bundle at date t , we have simply $C_{kt} = x_{kt}C_t$.

In log deviations from steady state, we have $d \log C_{kt} - d \log C_t = d \log x_{kt}$, where stacking and log-linearizing the definition of \mathbf{x}_t immediately yields

$$d \log \mathbf{x}_t = (1 - \theta) \sum_{s=0}^{\infty} \theta^s d \log \hat{\mathbf{x}}_{kt-s} \quad (\text{A.140})$$

Summarizing the system. We can rewrite (A.139) and (A.140) in AR(1) form as

$$d \log \hat{\mathbf{x}}_t = (1 - \beta\theta) d \log \hat{\mathbf{x}}_t^{static} + \beta\theta \mathbb{E}_t d \log \hat{\mathbf{x}}_{t+1} \quad (\text{A.141})$$

$$d \log \mathbf{x}_t = (1 - \theta) d \log \hat{\mathbf{x}}_t + \theta d \log \mathbf{x}_{t-1} \quad (\text{A.142})$$

After calculating the statically optimal bundles $d \log \hat{\mathbf{x}}_t^{static}$, one can iterate backward on (A.141) to obtain all $d \log \hat{\mathbf{x}}_t$, and then iterate forward on (A.142) to obtain all $d \log \mathbf{x}_t$.

Application to our nested CES case. Suppose that the consumption aggregator function F takes the form

$$F(c_H, c_{F1}, \dots, c_{Fn}) = \left((1 - \alpha)^{1/\eta} c_H^{(\eta-1)/\eta} + \alpha^{1/\eta} c_F(c_{F1}, \dots, c_{Fn})^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}$$

where

$$c_F(c_{F1}, \dots, c_{Fn}) = \left(\frac{1}{n} \sum_{i=1}^n c_{Fi}^{(\gamma-1)/\gamma} \right)^{\gamma/(\gamma-1)}$$

and the steady-state prices are assumed to be $P_H = 1$ and $P_{Fi} = \frac{1}{n}$ for all i .

It follows directly from standard CES demand that the statically optimal quantities in a bundle (i.e. the most efficient way to achieve a value $F = 1$) obey

$$d \log \hat{x}_H^{static} = -\eta d \log (P_H/P) \quad (\text{A.143})$$

$$d \log \hat{x}_F^{static} = -\eta d \log (P_F/P) \quad (\text{A.144})$$

$$d \log \hat{x}_{Fi}^{static} = d \log \hat{x}_F^{static} - \gamma d \log (P_{Fi}/P_F) \quad (\text{A.145})$$

where $d \log P_F = \frac{1}{n} \sum d \log P_{Fi}$ and $d \log P = (1 - \alpha)d \log P_H + \alpha d \log P_F$ are the standard price indices. We interpret the limit $n \rightarrow \infty$ as the case in this paper, with a continuum of foreign countries.

Specializing to the two sources of demand for home country goods in our paper, we can combine (A.143) and (A.141) to obtain

$$d \log \hat{x}_{Ht} = -(1 - \beta\theta)\eta d \log (P_{Ht}/P_t) + \beta\theta d \log \hat{x}_{Ht+1} \quad (\text{A.146})$$

and, looking from the perspective of a foreign country demanding the home good (which to it is one of a continuum of home goods), we can combine (A.145) and (A.141), and use the fact that there are no aggregate shocks affecting foreign countries (so that the first term on the right in

(A.145) is zero) to obtain

$$d \log \hat{x}_{Ht}^* = -(1 - \beta\theta)\gamma d \log(P_{Ht}/\mathcal{E}_t) + \beta\theta d \log \hat{x}_{Ht+1}^* \quad (\text{A.147})$$

Both also satisfy (A.142), i.e. $d \log x_{Ht} = (1 - \theta)d \log \hat{x}_{Ht} + \theta d \log x_{Ht-1}$ and $d \log x_{Ht}^* = (1 - \theta)d \log \hat{x}_{Ht}^* + \theta d \log x_{Ht-1}^*$, where $C_{Ht} = x_{Ht}C_t$ and $C_{Ht}^* = x_{Ht}^*C_t^*$.

Note that one feature of this model is that the elasticities are not just time-dependent, but also shock-dependent: $d \log \hat{x}_{Ht}$ and $d \log \hat{x}_{Ht}^*$ are forward-looking, and they are therefore of greater magnitude when the shocks to $d \log \frac{P_{Ht}}{P_t}$ and $d \log \frac{P_{Ht}}{\mathcal{E}_t}$ are more persistent. This leads to a greater response of $d \log x_{Ht}$ and $d \log x_{Ht}^*$ to persistent shocks, and it can explain, for instance, why permanent tariff changes can have different effects on export volumes than shocks to exchange rates due to capital flows, as estimated in [Fitzgerald and Haller \(2018\)](#) and [Cavallo, Gopinath, Neiman and Tang \(2021\)](#).

Combining delayed substitution with non-homothetic demand. One has to assume that the consumption bundle x_F is defined in terms of the consumption of foreign goods net of the subsistence level, i.e. $\tilde{c}_F \equiv c_F - \underline{c}$. This way, the consumption aggregator $F(c_H, \tilde{c}_F)$ is constant-returns-to-scale and the derivations in this section go through. The aggregate consumption of foreign goods by home agents is then given by $C_{Ft} = \underline{c} + x_{Ft}\bar{C}_t$.

D.4 Calibrating the income process

This appendix provides details on the calibration of the income process.

From the GRID project ([Guvenen et al. 2022](#)), we obtain estimates for Mexico of the standard deviation and autocorrelation of order 1 for the residual log annual income of females and males aged 25 to 55 years old. The average estimates for the years 2005 to 2019 are 1.1 for the standard deviation and 0.78 for the autocorrelation.

We then adjust these series for progressive income taxation, using estimates from [De Magalhaes et al. \(2022\)](#). The authors estimate a tax function of the form

$$\log \text{income}^{\text{post-tax}} = \text{cst} + (1 - \phi) \log \text{income}^{\text{pre-tax}} \quad (\text{A.148})$$

In table 6, the authors report an average estimate of $\phi = 0.24$ for Mexico, which implies a standard deviation of log annual post-tax income of 0.84.

We estimate the persistence and standard deviation of our quarterly income process by simulated method of moments. Specifically, we simulate a quarterly AR(1) for log income with persistence ρ_e and standard deviation of innovations $\tilde{\sigma}_e$. We then average the quarterly series at the annual level and search for the parameters $\rho_e, \tilde{\sigma}_e$ such that the standard deviation and autocorrelation of order 1 of the annual averages match the estimates from the data. The parameter reported in table 2, denoted σ_e , represents the standard deviation of quarterly log income.

We then approximate the quarterly AR(1) with a Markov chain following [Rouwenhorst \(1995\)](#) with a grid of 7 points for productivity e . For the non-homothetic model, we rescale income e using

$$e^{\text{rescaled}} = e(1 - \mu\underline{c}) + \mu\underline{c} \quad (\text{A.149})$$

so that households with the lowest productivity and without assets can still afford the subsistence level \underline{c} at wage $w = 1/\mu$. To ensure that the level of risk remains the same with and without

non-homothetic demand, we adjust the volatility of innovations to productivity so that the cross-sectional dispersion in productivity e remains the same after the rescaling.

D.5 Calibrating openness and price pass-through

This appendix provides data from a representative set of countries that experienced a large depreciation. This includes Mexico, which we use as our main calibration target, as well as eight other countries with a depreciation episode studied in [Burstein and Gopinath \(2015\)](#).

Calibrating α . We start by providing recent data on the import/GDP ratio from the IMF International Financial Statistics in the top panel of Table [A.1](#). The import-GDP ratio informs the choice of α in our benchmark model, or of the aggregate $\frac{C_F}{C}$ in our quantitative model with non-homothetic demand. This justifies our calibration to $\alpha = 0.4$ for Mexico.

Calibrating θ_F . To calibrate price stickiness parameters, we use information from the country's large devaluation episode to inform our choice of exchange rate pass-through. For this exercise, we proceed as follows. We start from the equations describing the dynamics of import prices P_{Ft} in response to an exchange rate change, [\(50\)](#). Note that this equation delivers price dynamics as a pure function of the exchange rate path \mathcal{E}_t and parameters θ_F and r , independently of the rest of the model. In particular, [\(50\)](#) implies that the linearized price dynamics of $p_{Ft} = \log P_{Ft}$ in response to an impulse to the exchange rate of $e_t = \log \mathcal{E}_t$ are

$$p_{Ft} - p_{Ft-1} = \frac{\left(1 - \frac{1}{1+r}\theta_F\right) (1 - \theta_F)}{\theta_F} (e_t - p_{Ft}) + \frac{1}{1+r} (p_{Ft+1} - p_{Ft}) \quad (\text{A.150})$$

We conceptualize the exchange rate depreciations experienced by each country in our case study as a one-time permanent shock to the exchange rate, from its initial level of $e_{-1} = 0$ to $e_t = \bar{e}$ for $t \geq 0$. Though stylized, this provides a useful approximation to the behavior of the nominal exchange rate in these episodes (see e.g. [Burstein, Eichenbaum and Rebelo 2005](#), figure 1). It is easy to verify that the solution for p_{Ft} under this particular path for e_t in [\(A.150\)](#) is:

$$p_{Ft} = \bar{e} (1 - \theta_F^t) \quad (\text{A.151})$$

Equation [\(A.151\)](#) delivers a simple way to back out the Calvo price rigidity coefficient as a function of the import price pass-through as of date t ,

$$\theta_F = \left(1 - \frac{p_{Ft}}{\bar{e}}\right)^{\frac{1}{t}} \quad (\text{A.152})$$

To perform this calculation for each of our countries, we need a measure of the pass-through to the *retail price of imported goods*, p_{Ft}/\bar{e} at some date t following the depreciation at $t = 0$. [Burstein, Eichenbaum and Rebelo \(2005\)](#) (henceforth BER) measured pass-through at 24 months, corresponding to $t = 8$, but only for dock prices, tradable retail prices and nontradable retail prices. We convert this information into a measure of the retail price of imported goods following BER's framework. Specifically, we assume that (log) traded goods prices p_{Tt} are made up of imported goods prices and local goods, whose price is well proxied by the price of non-traded goods, so that

$$p_{Tt} = (1 - \phi) p_{Ft} + \phi p_{Nt} \quad (\text{A.153})$$

	Mexico	Argentina	Brazil	Korea	Thailand	Finland	Sweden	Italy	UK
Latest data	2019	2019	2019	2019	2019	2019	2019	2019	2019
Imports/GDP	40%	15%	14%	37%	51%	40%	44%	29%	32%
Depreciation year	1994	2001	1998	1997	1997	1992	1992	1992	1992
Dock PT	107%	87%	126%	60%	68%	116%	76%	63%	141%
Tradable retail PT	82%	36%	36%	30%	28%	64%	29%	32%	22%
Nontradable PT	42%	10%	20%	14%	28%	26%	25%	27%	58%
Imported retail PT	122%	63%	52%	46%	28%	102%	34%	38%	-15%
Implied θ_F	0.00	0.78	0.91	0.93	0.96	0.00	0.95	0.94	1.00
dY_0	- 0.12	- 0.03	- 0.01	0.01	0.03	- 0.12	0.02	0.00	0.05
$\sum_{t=0}^3 dY_t$	- 0.24	- 0.11	- 0.04	0.02	0.13	- 0.24	0.08	0.01	0.16

Notes: data on dock and retail pass-through are taken from Table 7.5 in [Burstein and Gopinath \(2015\)](#), using the ratio of the increase in dock import prices and retail prices at 24 month to the trade weighted exchange rate at 24 month. Data on imports are taken from the IMF International Financial Statistics. The bottom two rows replicate the exercise of Table 3 for these countries.

Table A.1: Imported price pass-through and openness for selected countries

Following BER, we assume $\phi = 1/2$, and use equation (A.153) to back out p_{Ft} .⁵⁹

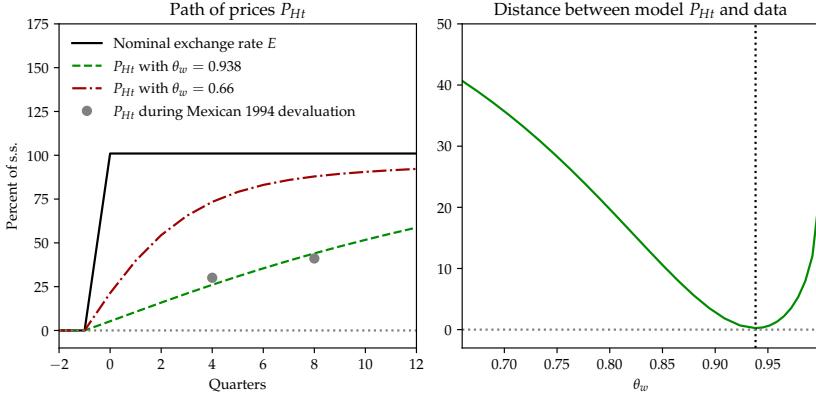
The bottom row of Table A.1 reports the result of this exercise. The first two rows report p_{Dt}/\bar{e} , p_{Tt}/\bar{e} and p_{Nt}/\bar{e} for $t = 8$ quarters. The next row reports p_{Ft}/\bar{e} backed out from (A.153), and the final row reports the implied quarterly θ_F from equation (A.152). As can be seen, the devaluations suggest a lot of heterogeneity in imported price pass-through in each episode. In Mexico, this procedure infers full price pass-through, given the large movements in tradable retail prices and limited movement in nontradable prices. This is consistent with the large amount of price pass-through observed at the dock in that episode. In other episodes tradable retail prices move a lot less, so our procedure infers much more limited import price pass-through. The range of values that we obtain for the pass-through of exchange rates into import prices is consistent with estimates from [Campa and Goldberg \(2005\)](#) for OECD countries.

Calibrating θ_w . We also use information from Table A.1 to calibrate the wage stickiness parameter θ_w . We use information from the time path of non-traded good prices, which in our model correspond to p_{Ht} , in order to discipline that parameter. Note that, in contrast to θ_F , which is identified directly from the price pass-through data, θ_w depends on the entire structure of the model, and in particular on the relationship between monetary policy and domestic economic activity. Moreover, the path of p_{Ht} does not separately identify the stickiness of wages θ_w and the stickiness of prices θ_H . We therefore follow the standard in the literature and set θ_H to imply a price duration of 3 quarters. We then find θ_w to match the path of prices in response to the pure devaluation shock as described above.

Figure A.3 shows the outcome of that exercise. The left panel plots the path of prices P_{Ht} after the devaluation in the model and in our data, and the right panel plots the sum of square distance

⁵⁹We obtain similar results if we assume that the retail price of imported goods is a mix $p_{Ft} = (1 - \phi) p_{Dt} + \phi p_{Nt}$ of dock prices and non-traded goods prices.

Figure A.3: Calibrating θ_w



Note: The left panel shows the impulse response of home goods prices P_{Ht} following a permanent change in the nominal exchange rate E_t . The data is taken from Table 7.5 in [Burstein and Gopinath \(2015\)](#). The right panel plots the sum of square distance between model and data at $t = 4$ and $t = 8$, for different values of the calvo parameter θ_w .

between model and data at $t = 4$ and $t = 8$. When wages are relatively flexible ($\theta_w = 0.66$), home goods prices increase too fast in response to the devaluation shock relative to the path of prices that we observed in the Mexican devaluation. Our model infers that wages are stickier than this, with a calvo parameter equal to $\theta_w = 0.938$.

We use the calibration in this section to consider the effect of depreciations in other countries than our benchmark of Mexico, which featured high openness and full import pass-through. We recalibrate our model to hit their import-GDP ratio and their degree of import price pass-through, but keeping the MPCs the same. The bottom two rows of the table illustrates that the effects of the same exchange rate shock are very heterogeneous across countries. Countries with lower import price pass-through have much less of an immediate impact on output, since the real income effect is really muted. Openness has a non-monotonic effect, since the immediate effect of an exchange rate depreciation is not as large in an economy that is more closed, but the general equilibrium effect of any open international position is much larger, through the logic of [Proposition 10](#).

D.6 Currency mismatch in balance sheets

To incorporate currency mismatch in the net foreign asset position, we proceed as follows. We expand the setting in [section 2](#) by allowing agents to invest in *long-duration* domestic-currency and foreign-currency assets, modeled as bonds with nominal coupons that exponentially decay at a rate δ and with prices \mathcal{Q}_t and \mathcal{Q}_t^* respectively, where δ is calibrated to empirical duration data. We then consider an incomplete markets scenario where all agents hold an equal portfolio with gross positions in these assets, and another where this portfolio is held by the government.

Direct investment. We first assume that agents hold these long-duration assets directly. Suppressing the choice of domestic and foreign nominal one-period bonds for simplicity (we will assume that these are not held by anyone in equilibrium), the budget constraint in [\(1\)](#), for an agent in state $(s, \Lambda, \Lambda^*, e)$ becomes

$$P_t c + \mathcal{P}_t s' + \mathcal{Q}_t \Lambda' + \mathcal{E}_t \mathcal{Q}_t^* \Lambda^{*'} = (\mathcal{P}_t + D_t) s + (1 + \delta \mathcal{Q}_t) \Lambda + \mathcal{E}_t (1 + \delta \mathcal{Q}_t^*) \Lambda^* + e W_t N_t$$

where Λ, Λ^* are the number of domestic-currency and foreign-currency asset coupons held at the beginning of the period.

No arbitrage now implies, in addition to the equations in (5), that

$$1 + \iota_t = \frac{1 + \delta \mathcal{Q}_{t+1}}{\mathcal{Q}_t}, \quad 1 + \iota_t^* = \frac{1 + \delta \mathcal{Q}_{t+1}^*}{\mathcal{Q}_t^*} \quad (\text{A.154})$$

which gives the valuation equation for the bond prices $\mathcal{Q}_t, \mathcal{Q}_t^*$.

The real value of end-of-period assets a'_t , and the real value of beginning-of-period assets including returns a_t^p , are now defined respectively as

$$a'_t \equiv \frac{\mathcal{P}_t s' + \mathcal{Q}_t \Lambda' + \mathcal{E}_t \mathcal{Q}_t^* \Lambda'^*}{P_t}, \quad a_t^p \equiv \frac{(\mathcal{P}_t + D_t) s + (1 + \delta \mathcal{Q}_t) \Lambda + \mathcal{E}_t (1 + \delta \mathcal{Q}_t^*) \Lambda^*}{P_t} \quad (\text{A.155})$$

and given these definitions, the problem can still be written in the form (9).

In the steady state, we now have

$$a_{-1}^p = (1 + r_{ss}) p_{ss} s_{-1}^H + (1 + i_{ss}) \mathcal{Q}_{ss} \Lambda_{-1} + \underbrace{(1 + i_{ss}^*) \mathcal{Q}_{ss}^* \Lambda_{-1}^*}_{\equiv f_Y Y} \quad (\text{A.156})$$

We continue to calibrate the steady state of the model so that agents have total value of beginning-of-period assets $a_{-1}^p = (1 + r_{ss}) p_{ss}$ (so that in particular, in the aggregate we have $nfa_{-1} = 0$) and hold all of domestic stocks $s_{-1}^H = 1$. However, we now allow for a gross currency mismatch, where the country holds a share f_Y of foreign currency assets in excess of foreign currency liabilities, relative to its GDP, with all agents holding the country portfolio. This investment is offset by foreign direct investment in domestic nominal bonds. When $f_Y < 0$, as in the data, the country has borrowed in foreign currency and invested in domestic bonds.

After a depreciation induced by a change in the path of ι_t^* for $t \geq 0$, the country experiences an adverse valuation effect to its liabilities. We use equation (A.154) to calculate the induced new bond prices $\mathcal{Q}_0, \mathcal{Q}_0^*$ (note that when $\delta > 0$, the increase in foreign interest rates reduces the present value of liabilities in foreign currency term), and (7) to calculate the new real exchange rate Q_0 . Together with our calibration of f_Y , this determines a_0^p via equation (A.155) and therefore the magnitude of the valuation effect.

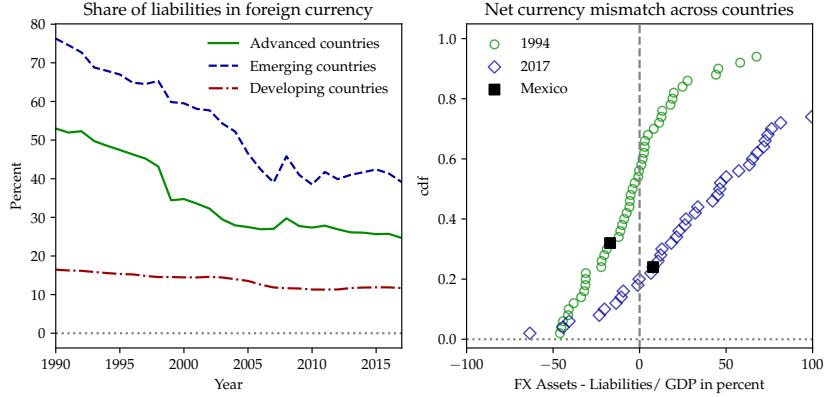
Investment through the government balance sheet. With the assumption of equal portfolios made above, the valuation effects are distributed in the population according to their holdings of assets. One possibility to quantify the effect of alternative distribution rules is to assume different exogenous portfolio distributions, for instance taken from the data as in [Auclert and Rognlie \(2018\)](#). Here, we follow a simpler route, which is to assume that the government holds the country's gross currency exposure, and rebates it according to various schemes. We add a government, with B_{t-1} coupons in real debt, Λ_{t-1}^G coupons in domestic nominal assets and Λ_{t-1}^{G*} coupons in foreign nominal assets. Its budget constraint is

$$-P_t B_t + \mathcal{Q}_t \Lambda_t^G + \mathcal{E}_t \mathcal{Q}_t^* \Lambda_t^{G*} = P_t T_t - (1 + r_{t-1}) P_t B_{t-1} + (1 + \delta \mathcal{Q}_t) \Lambda_{t-1}^G + \mathcal{E}_t (1 + \delta \mathcal{Q}_t^*) \Lambda_{t-1}^{G*} \quad (\text{A.157})$$

where T_t are aggregate taxes. We distribute those taxes by modifying the household budget constraint (9) to read

$$\frac{P_{Ft}}{P_t} c_F + \frac{P_{Ht}}{P_t} c_H + a' = a^p + e \frac{W_t}{P_t} N_t - T_t \frac{e^{1-\lambda}}{\mathbb{E}[e^{1-\lambda}]}$$

Figure A.4: Calibrating f_Y



Note: The left panel shows the fraction of liabilities denominated in foreign currency as a share of total liabilities, on average across advanced, emerging and developing countries, respectively. The right panel shows the distribution, in a set of 50 countries, of the difference between foreign currency assets and liabilities (“gross currency mismatch”), as a share of GDP. Source: [Bénétrix et al. \(2020\)](#), updating an earlier study by [Lane and Shambaugh \(2010\)](#).

$\lambda = 1$ represents lump-sum taxes, while $\lambda = 0$ are proportional taxes. The higher λ , the more regressive the tax system is.

In steady state, we assume that $B = T = 0$ but the government holds a gross position $f_Y Y$ in domestic assets, financed by borrowing in foreign assets. After the aggregate shock occurs, government debt evolves according to the fiscal rule

$$B_t = \rho_B \left(B_{t-1} + f_Y Y \left[\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{1 + \delta Q_t^*}{Q_{t-1}^*(1 + \pi_t)} - \frac{1 + \delta Q_t}{Q_{t-1}(1 + \pi_t)} \right] \right)$$

where the terms in brackets represent the valuation effects from gross positions after the aggregate shock, which are positive at $t = 0$ and null for $t > 0$. When $\rho_B = 0$, through (A.157), the government immediately must adjust taxes to shore up its balance sheet loss from foreign liabilities after a depreciation. When $\rho_B > 0$, the government builds up debt and taxes later, which mitigates the immediate effect on spending.

Calibrating δ and f_Y . To calibrate the coupon δ , we note that the duration of a bond with price (A.154) is given by

$$D = \frac{1 + i}{1 + i - \delta}$$

We calibrate δ to hit a liability duration of $D = 18$ quarters, as implied by [Doepke and Schneider \(2006\)](#)’s estimates for the U.S.

We calibrate f_Y to data from [Lane and Shambaugh \(2010\)](#) and [Bénétrix et al. \(2020\)](#). [Lane and Shambaugh \(2010\)](#) documented aggregate foreign currency exposures for 1994 to 2004 for a sample of 117 countries; [Bénétrix et al. \(2020\)](#) subsequently updated their data to 2017 for a sample of 50 countries. These studies measure foreign currency exposure as the difference between county i ’s gross foreign currency assets and gross foreign currency liabilities.⁶⁰ The right panel of figure

⁶⁰[Lane and Shambaugh \(2010\)](#)’s headline measure of currency exposure for country i at time t , FX_{it}^{AGG} ,

	Baseline	Gross positions	Gov, lump-sum	Gov, proportional tax	Gov, deficit-finance
dY_0	0.31	0.23	0.03	0.16	0.19
$\sum_{t=0}^3 dY_t$	1.38	1.17	0.86	1.05	1.08

Note: this table replicates the exercise from table 4 in our quantitative model with a static trade elasticity.

Table A.2: Balance sheet effects with static trade elasticity $\chi = 1$

A.4 shows the distribution of these currency exposures, normalized by GDP, from the most recent study by Bénétix et al. (2020). As emphasized by these authors, countries have dramatically reduced the aggregate currency mismatch in their balance sheets since the 1990s: for instance, while Mexico used to have around 25% more foreign currency liabilities than assets as a share of its GDP, it now has around 5% more foreign currency assets than liabilities (the left panel illustrates that this has tended to happen via a reduction in the fraction of the share of liabilities that are in foreign currency.) In the latest 2017 data, only three countries in the dataset have foreign currency liabilities exceeding assets by more than 40% of GDP: Tunisia (-63%), Egypt (-44%) and Sri Lanka (-40%). In our exercise of section 5.6, we set $f_Y = -50\%$. This calibration therefore represents an upper bound on the size of valuation effects.

Balance sheet effects with static trade elasticity We now perform a similar exercise than in section 5.6 but in the quantitative model with a static trade elasticity equal to $\chi = 1$. The results, reported in table A.2, show that currency mismatch by itself does not cause a contraction in output following a depreciation. Thus, it is the interaction of currency mismatch and the real income channel that causes contractions.

D.7 Quantitative model with alternative Taylor rules

Figure A.5 replicates figure 8 with a Taylor rule based on producer prices, as in Galí and Monacelli (2005). The results show that the contraction is even larger when monetary policy targets producer prices because the real interest rate increases more after the depreciation. This additional contractionary monetary policy shock reduces consumption and output further relative to our quantitative model.

D.8 Quantitative model with representative agent

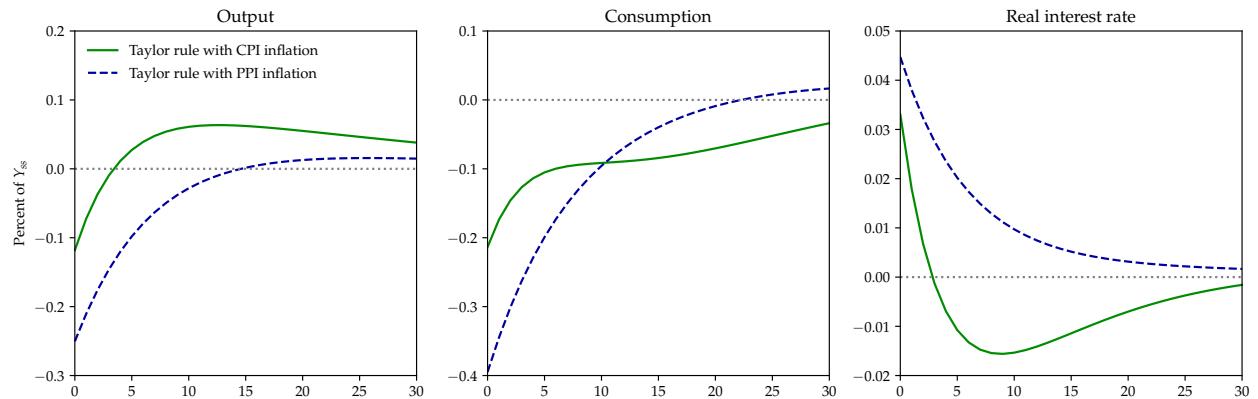
Figure A.6 replicates figure 8 with a representative agent. The results show that the depreciation generates a boom in the model with a representative agent whereas it generates a contraction in the model with heterogeneous agents.

E Alternative models

This appendix presents extensions of our baseline HA-IM model. Appendix E.1 derives a version of the model with dollar currency pricing and shows that it dampens expenditure switching by

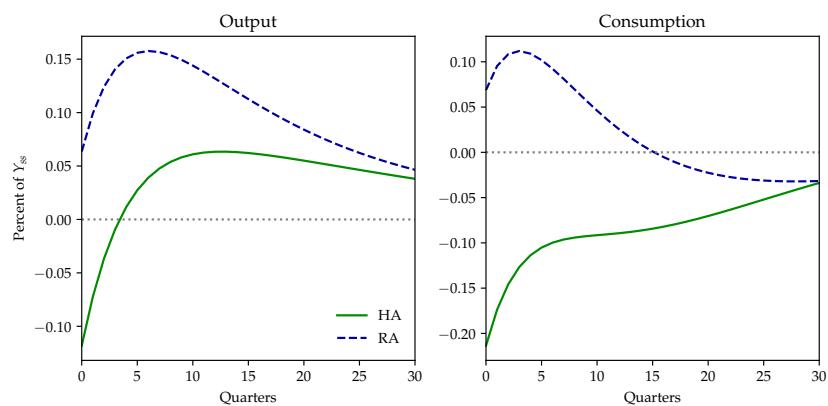
is normalized by the sum of assets and liabilities, but the supplementary data in both Lane and Shambaugh (2010) and Bénétix et al. (2020) report measures normalized by GDP, which correspond exactly to our f_Y .

Figure A.5: Contractionary depreciations with alternative Taylor rules



Note: impulse response in the quantitative model to the shock to i_t^* from figure 2.

Figure A.6: Contractionary depreciations with representative agent



Note: impulse response in the quantitative model to the shock to i_t^* from figure 2.

foreign households but also stimulates the profits of exporters. As a result, dollar currency pricing can either increase or lower the response of output to a depreciation.

Next, we present three extensions of our baseline model, which we show can be reinterpreted as versions of our baseline model with different parameters. Appendix E.2 adds produced non-tradable goods in addition to tradable goods. Appendix E.3 adds imported intermediate goods. Both of these can be directly reinterpreted as our baseline model with an appropriate reparameterization of the openness parameter α and the elasticity of substitution between home and foreign goods η . Appendix E.4 considers a tradable-nontradable model of a commodity exporter, which takes as given the price of exports. We show that in the standard case where a fixed quantity of tradables (commodities) is being produced each period, akin to a fixed endowment of tradable goods, this can be reinterpreted as our model with dollar currency pricing.

Finally, E.5 presents a version of the model with UIP deviations. We derive a new interpretation to i^* shocks, and allow for a potential feedback effect from changes in the net foreign asset position to the real exchange rate.

E.1 Dollar currency pricing

We have seen that the degree of expenditure switching crucially influences whether a depreciation is expansionary in the HA model. One reason for a weaker expenditure switching channel is the prevalence of dollar (or dominant) currency pricing (DCP). With DCP, exports are invoiced in dollars. This means that export prices do not immediately adjust in response to exchange rate fluctuations (Gopinath, 2016), limiting the response of export demand (Gopinath et al., 2020).

To explore the effects of DCP for our baseline model, we replace equation (17) with $P_{Ht}^* = \bar{P}_H^*$. Hence, all exports are invoiced in dollars, and for simplicity these dollar prices are fixed.⁶¹ This influences our analysis in two ways. First, it lowers the trade elasticity χ from $\eta(1 - \alpha) + \gamma$ to simply $\eta(1 - \alpha)$: the volume of export demand no longer responds to a depreciation. We refer to this as the “standard effect” of DCP. Second, domestic firms’ markups on exports are now endogenous to the exchange rate: after a depreciation, markups increase, raising profits via equation (15).⁶² These profits are earned by domestic shareholders, generating a positive effect on spending. We refer to this as the “profit effect” of DCP. In an economy in which firms perfectly hedge exchange rate fluctuations, the profit effect would be absent.

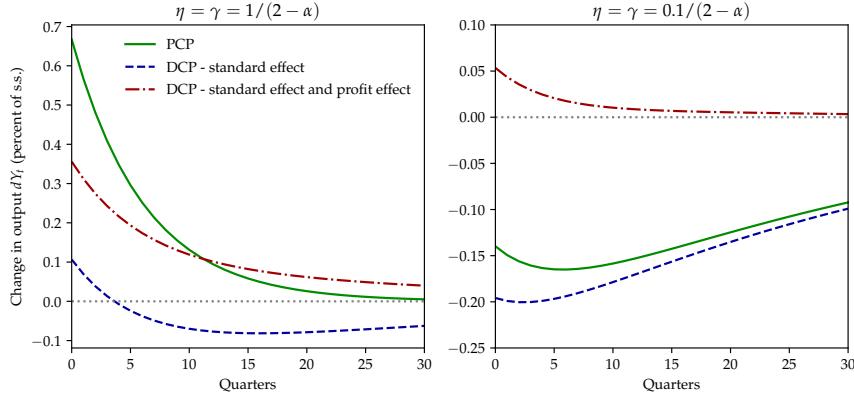
To investigate the role of the two effects of DCP, figure A.7 compares the output response to a depreciation under PCP to the responses under DCP with (i) only the standard effect and (ii) both effects. The left panel shows the case of larger elasticities $\eta = \gamma = 1/(2 - \alpha)$, chosen to give $\chi = 1$. Here, the standard DCP effect causes a large reduction in the output response, as it effectively sets γ to zero. Elasticities are already small in the right panel so that the standard effect of DCP reduces the output response only mildly.

In both panels, the profit effect is positive for output, as asset owners spend some of the additional profit earned on exports. In the version of the HA model here, the profit effect is sufficiently large to push the output response above zero even with small elasticities. The reason for this is that the model considered here has relatively large MPCs out of capital gains. A two-account model, such as that considered in Kaplan et al. (2018) or Auclert et al. (2024a) would meaningfully reduce the profit effect. We leave a thorough analysis of this effect for future research.

⁶¹In section 5 we relax this assumption by allowing for dynamic adjustment of the dollar price.

⁶²Barbiero (2021) empirically documents these foreign-exchange-induced variations in the profits of French firms that price in foreign currency.

Figure A.7: Capital outflows under dollar currency pricing



Note: impulse responses to the shock to t_t^* from figure 2. PCP corresponds to producer currency pricing, DCP to dollar currency pricing. The standard (reduced expenditure switching) and the profit effect are discussed in the main text.

To shed further analytical light on the two effects, consider a one-time depreciation dQ_0 . For any given agent i , the depreciation causes a reduction in real wage income of $\frac{1}{\mu}e_{i0}$, where e_{i0} is the idiosyncratic productivity of agent i at date 0, and it raises real dividend income by $\frac{1}{\mu}\frac{a_{i0}}{A_{ss}}$ where $\frac{a_{i0}}{A_{ss}}$ is the wealth owned by agent i relative to mean wealth. Let us define the *net exchange rate exposure* NXE_i of agent i by

$$NXE_i \equiv \left(\frac{a_{i0}}{A_{ss}} - e_{i0} \right) \cdot \frac{1}{\mu} \quad (\text{A.158})$$

We show below that the impact output response is then given by

$$dY_0 = \frac{\alpha}{1-\alpha} \chi dQ_0 + \alpha \text{Cov} (MPC_i, NXE_i) dQ_0 + (1-\alpha) \sum_{s \geq 0} M_{0,s} dY_s \quad (\text{A.159})$$

Compared with (38), we see that the real income channel is now given by the *cross-sectional covariance* of MPCs and net exchange rate exposures. In our model, this covariance is endogenously negative, since firms' shareholders tend to be richer and have lower MPCs than agents who predominantly rely on labor income. Our model thus provides a micro-founded counterpart to [Díaz-Alejandro \(1963\)](#) and [Krugman and Taylor \(1978\)](#), who previously discussed this mechanism in the context of IS-LM models. We regard measuring net exchange rate exposures such as (A.158) in the data, and analyzing their aggregate implications using equation (A.159), as a very promising avenue for future research.

In commodity exporting countries, exchange rate depreciations also create a profit effect: they raise the domestic price at which commodity exporting firms sell, so that depreciations redistribute from workers, whose real income falls, to the owners of these firms. Appendix E.4 proves that there is, in fact, a formal analogy: by reinterpreting α and χ , a model with produced nontradables and endowed tradables (e.g. commodities) is exactly equivalent to the model with DCP and fixed dollar prices studied here.

Derivation of equation (A.159). Consider a one-time depreciation dQ_0 . This affects real income in period s by

$$d \left(\frac{P_{Hs}}{P_s} Y_s \right) = -\frac{\alpha}{1-\alpha} 1_{\{s=0\}} dQ_0 + dY_s$$

The effects on labor income is then simply

$$d \left(\frac{W_s}{P_s} N_s \right) = \frac{1}{\mu} d \left(\frac{P_{Hs}}{P_s} Y_s \right) = -\frac{\alpha}{1-\alpha} 1_{\{s=0\}} \frac{1}{\mu} dQ_0 + \frac{1}{\mu} dY_s$$

while the effect on dividends (15) is given by

$$dD_s = -\frac{\alpha}{1-\alpha} 1_{\{s=0\}} \left(1 - \frac{1}{\mu} \right) dQ_0 + \left(1 - \frac{1}{\mu} \right) dY_s + \frac{\alpha}{1-\alpha} 1_{\{s=0\}} dQ_0 \quad (\text{A.160})$$

where the last term captures increased markups on exports that emerge with DCP. Following the steps in section C.1, we see that, by definition of $M_{t,s}$, the total date-0 consumption response to $\{dY_s\}$ is given by $\sum_{s \geq 0} M_{0,s} dY_s$.

To obtain the date-0 consumption response to dQ_0 (the real income channel at date 0), denote by MPC_i agent i 's date-0 MPC out of a transitory date-0 transfer. Note that agent i 's exposure to dQ_0 depends on its initial income state e_{i0} (its share of labor income) and initial share of wealth $\frac{a_{i0}}{A_{ss}}$, multiplied by the aggregate changes in labor income and dividends respectively. We collect these terms in an object we call *net exchange rate exposure* NXE_i ,

$$\frac{\alpha}{1-\alpha} NXE_i \equiv \frac{a_{i0}}{A_{ss}} \cdot \frac{1}{\mu} \frac{\alpha}{1-\alpha} - e_{i0} \cdot \frac{1}{\mu} \frac{\alpha}{1-\alpha} = \left(\frac{a_{i0}}{A_{ss}} - e_{i0} \right) \cdot \frac{1}{\mu} \frac{\alpha}{1-\alpha}$$

The total date-0 consumption response is then

$$dC_0 = \frac{\alpha}{1-\alpha} \mathbb{E} [MPC_i \cdot NXE_i] \cdot dQ_0 + \sum_{s \geq 0} M_{0,s} dY_s$$

Observe that net exchange rate exposures average to zero, $\mathbb{E}[NXE_i] = 0$, so that $\mathbb{E}[MPC_i \cdot NXE_i] = \text{Cov}(MPC_i, NXE_i)$. Substituting dC_0 into the linearized goods market clearing condition (A.77), we find (A.159).

International Keynesian cross with DCP. We can also derive a version of the (generalized) international Keynesian cross (44) with DCP. The main differences with the derivation in appendix C.1 are that expenditure switching by foreign households is absent, as if $\gamma = 0$, but there is an additional term entering dividends, as in (A.160),

$$d\mathbf{D} = \frac{\alpha}{1-\alpha} \frac{1}{\mu} d\mathbf{Q} + \left(1 - \frac{1}{\mu} \right) d\mathbf{Y}$$

The change in the ex-post return at date 0 then becomes

$$dr_0 = \mathbf{J}'_r d\mathbf{r} + \mathbf{J}'_d \left(\left(1 - \frac{1}{\mu} \right) d\mathbf{Y} + \frac{\alpha}{1-\alpha} \frac{1}{\mu} d\mathbf{Q} \right)$$

Following the same steps as before, we then end up at

$$d\mathbf{Y} = \underbrace{(1-\alpha)(\mathbf{C}_r + \mathbf{C}_v\mathbf{J}'_r) d\mathbf{r}}_{\text{intertemporal subst. + valuation}} + \underbrace{\alpha\eta d\mathbf{Q}}_{\text{expenditure switching}} - \underbrace{\alpha\left(\frac{1}{\mu}\mathbf{C}_Y - \frac{1}{\mu}\mathbf{C}_v\mathbf{J}'_d\right) d\mathbf{Q}}_{\text{real income}} + \underbrace{(1-\alpha)\left(\frac{1}{\mu}\mathbf{C}_Y + \left(1 - \frac{1}{\mu}\right)\mathbf{C}_v\mathbf{J}'_d\right) d\mathbf{Y}}_{\text{multiplier effect}}$$

which is almost the same expression as before, except for reduced expenditure switching and real income channels. To interpret this equation, we define \mathbf{M}^r, \mathbf{M} as before, but also define $\mathbf{M}^w \equiv \mathbf{C}_Y$ as the (intertemporal) MPCs out of labor income, and $\mathbf{M}^D \equiv \mathbf{C}_v\mathbf{J}'_d$ as the (intertemporal) MPCs out of dividends. Then, the international Keynesian cross decomposition for the DCP model is

$$d\mathbf{Y} = \underbrace{(1-\alpha)\mathbf{M}^r d\mathbf{r}}_{\text{int. subst + valuation}} + \underbrace{\alpha\eta d\mathbf{Q}}_{\text{D expenditure switching}} - \underbrace{\frac{\alpha}{\mu}(\mathbf{M}^w - \mathbf{M}^D) d\mathbf{Q}}_{\text{real income}} + \underbrace{(1-\alpha)\mathbf{M} d\mathbf{Y}}_{\text{Multiplier effect}} \quad (\text{A.161})$$

E.2 Nontradable goods

We first add nontradable goods to the model. Instead of (3), assume that household consumption is now an aggregate between tradable goods and (home-produced) nontradable goods,

$$c = \left[\phi^{1/\zeta} c_T^{(\zeta-1)/\zeta} + (1-\phi)^{1/\zeta} c_{H,NT}^{(\zeta-1)/\zeta} \right]^{\zeta/(\zeta-1)} \quad (\text{A.162})$$

where the tradable bundle is a mix of imported tradables and home-produced tradable goods,

$$c_T = \left[\alpha^{1/\eta} c_F^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_{H,T}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \quad (\text{A.163})$$

Here, ϕ is the tradable share, while $1-\alpha$ is home bias *within* tradables; ζ is the elasticity of substitution between tradables and nontradables (which is plausibly quite low), while η is the elasticity of substitution between home and foreign goods within tradables. For this section, we assume that the production functions for tradables and nontradables are identical, so that they always have the same price, and that all that matters is the sum of $c_{H,T}$ and $c_{H,NT}$.

With this demand system, total demand for home goods coming domestic residents is

$$c_H \equiv c_{H,T} + c_{H,NT} = \left((1-\alpha) \left(\frac{P_H}{P_T} \right)^{-\eta} \phi \left(\frac{P_T}{P} \right)^{-\zeta} + (1-\phi) \left(\frac{P_H}{P} \right)^{-\zeta} \right) c \quad (\text{A.164})$$

At the steady state where all prices are 1, the overall home and foreign shares of consumption are therefore

$$\frac{c_H}{c} = (1-\alpha)\phi + (1-\phi) \equiv 1 - \bar{\alpha}; \quad \frac{c_F}{c} = \alpha\phi \equiv \bar{\alpha} \quad (\text{A.165})$$

In response to a shock to prices around the steady state, we log-linearize and find that this relative demand changes by

$$\hat{c}_H - \hat{c} = -\frac{(1-\alpha)\phi}{(1-\alpha)\phi + (1-\phi)} (\eta(\hat{p}_H - \hat{p}_T) + \zeta(\hat{p}_T - \hat{p})) - \frac{1-\phi}{(1-\alpha)\phi + (1-\phi)} \zeta(\hat{p}_H - \hat{p}) \quad (\text{A.166})$$

Noting that $\hat{p} = \phi\hat{p}_T + (1 - \phi)\hat{p}_H$, we can write

$$\begin{aligned}\hat{p}_H - \hat{p}_T &= \phi^{-1}(\hat{p}_H - \hat{p}) \\ \hat{p}_T - \hat{p} &= -\phi^{-1}(1 - \phi)(\hat{p}_H - \hat{p})\end{aligned}$$

and substitute these into (A.166) to obtain

$$\begin{aligned}-\frac{\hat{c}_H - \hat{c}}{\hat{p}_H - \hat{p}} &= \frac{(1 - \alpha)\phi}{(1 - \alpha)\phi + (1 - \phi)}(\eta\phi^{-1} - \zeta\phi^{-1}(1 - \phi)) + \frac{1 - \phi}{(1 - \alpha)\phi + (1 - \phi)}\zeta \\ &= \frac{(1 - \alpha)\eta + (1 - \phi)\alpha\zeta}{(1 - \alpha) + (1 - \phi)\alpha} \equiv \bar{\eta}\end{aligned}\tag{A.167}$$

Note that the elasticity $\bar{\eta}$ in (A.167) is a weighted average of the primitive elasticities η and ζ .

Define the *consumption aggregator* function $\bar{c}(c_F, c_H)$ to maximize c subject to (A.162), (A.163) and $c_H = c_{H,T} + c_{H,NT}$. It is immediate that this has constant returns to scale. We can view total foreign and home consumption (as calculated above) as optimizing this function subject to prices P_F and P_H .

For steady-state $P_F = P_H = P = 1$, we found c_F and c_H in (A.165), with $\bar{\alpha} = \alpha\phi$ the steady-state foreign share and $1 - \bar{\alpha}$ the home share. In (A.167) we calculated the local elasticity of substitution of the consumption aggregator function, $\bar{\eta}$.

To first order in aggregate shocks, therefore, our model remains the same when nontradables are introduced; we need only replace the openness parameter α and elasticity of substitution between home and foreign goods η with their counterparts $\bar{\alpha}$ and $\bar{\eta}$ in (A.165) and (A.167).⁶³ The two implications of this equivalence mapping are the following.

First, the import-to-GDP ratio is now $\frac{c_F}{c} = \alpha\phi = \bar{\alpha}$. Hence, even in the presence of nontradables, it is appropriate to calibrate $\bar{\alpha}$ to that ratio.

Second, $\bar{\eta}$ is a weighted average of η (elasticity between home and foreign within tradables, which could be relatively high) and ζ (the elasticity between nontradable and tradable, which could plausibly be much lower), with a larger weight on ζ when the nontradable share is higher. Hence, $\bar{\eta}$ itself could plausibly be relatively low.

E.3 Imported intermediates

We now return to the consumption basket in (3), but change the production structure to allow for imported intermediate goods. Specifically, suppose that the continuum of firms in each country now produce an intermediate good X using the technology $X = ZN$, and that the final good Y in each country is a CES aggregate of the country's own intermediate good and the foreign intermediate good.

Concretely, for the home country, suppose that production of the final good is given by

$$Y = \left[\phi^{1/\zeta} X_F^{(\zeta-1)/\zeta} + (1 - \phi)^{1/\zeta} X_H^{(\zeta-1)/\zeta} \right]^{\zeta/(\zeta-1)}\tag{A.168}$$

where X_H is the home country's demand for the home intermediate, and X_F is the home country's demand for imported intermediates. Suppose further that X_F (analogous to c_F) is a CES aggregate

⁶³The problem of allocating within c_F between different countries' varieties is unchanged; the elasticity there remains γ .

of each other country's intermediate, with elasticity ν . As before, normalize all prices and quantities at the steady state to 1, and assume that foreign prices and quantities do not change. Note that a country's total value added, or GDP, equals its X .

It follows that total demand for the home country's intermediate X is

$$X = (1 - \phi) \left(\frac{P_H^X}{P_H} \right)^{-\zeta} Y + \phi \left(\frac{P_H^X}{\mathcal{E}} \right)^{-\nu} Y^* \quad (\text{A.169})$$

where demand for Y is the same as before

$$Y = (1 - \alpha) \left(\frac{P_H}{P} \right)^{-\eta} C + \alpha \left(\frac{P_H}{\mathcal{E}} \right)^{-\gamma} C^* \quad (\text{A.170})$$

Equations (A.11) and (A.12) continue to hold, replacing Y by X and P_H by P_H^X . Totally differentiating (A.169), we get

$$\begin{aligned} dX = & -(1 - \phi)(1 - \alpha)\eta(dP_H - dP) - (1 - \phi)\alpha\gamma(dP_H - d\mathcal{E}) \\ & -(1 - \phi)\zeta(dP_H^X - dP_H) - \phi\nu(dP_H^X - d\mathcal{E}) + (1 - \phi)(1 - \alpha)dC \end{aligned} \quad (\text{A.171})$$

As in appendix B.1, linearizing the CPI equation, we have $dP = (1 - \alpha)dP_H + \alpha d\mathcal{E}$. Linearizing the price index corresponding to (A.168), we get $dP_H = (1 - \phi)dP_H^X + \phi d\mathcal{E}$.

Writing all the relative prices in (A.171) in terms of the real exchange rate $dQ = d\mathcal{E} - dP$, we have

$$\begin{aligned} dP_H - dP &= -\frac{\alpha}{1 - \alpha}dQ \\ dP_H - d\mathcal{E} &= -\frac{1}{1 - \alpha}dQ \\ dP_H^X - dP_H &= \frac{\phi}{1 - \phi}(dP_H - d\mathcal{E}) = -\frac{\phi}{1 - \phi}\frac{1}{1 - \alpha}dQ \\ dP_H^X - d\mathcal{E} &= \frac{1}{1 - \phi}(dP_H - d\mathcal{E}) = -\frac{1}{1 - \phi}\frac{1}{1 - \alpha}dQ \end{aligned}$$

and can plug this into (A.171) to obtain

$$dX = \left((1 - \phi)\alpha\eta + (1 - \phi)\frac{\alpha}{1 - \alpha}\gamma + \phi\zeta\frac{1}{1 - \alpha} + \frac{\phi}{1 - \phi}\nu\frac{1}{1 - \alpha} \right) dQ + (1 - \phi)(1 - \alpha)dC \quad (\text{A.172})$$

If we define $\bar{\alpha} \equiv 1 - (1 - \phi)(1 - \alpha)$, and $\bar{\chi} \equiv \frac{1 - \bar{\alpha}}{\bar{\alpha}} \left((1 - \phi)\alpha\eta + (1 - \phi)\frac{\alpha}{1 - \alpha}\gamma + \phi\zeta\frac{1}{1 - \alpha} + \frac{\phi}{1 - \phi}\nu\frac{1}{1 - \alpha} \right)$, then (A.172) becomes just

$$dX = \frac{\bar{\alpha}}{1 - \bar{\alpha}}\bar{\chi}dQ + (1 - \bar{\alpha})dC \quad (\text{A.173})$$

which is identical to equation (A.80) in appendix B.2, but with $\bar{\alpha}$, $\bar{\chi}$, and dX replacing α , χ , and dY . With these substitutions, the International Keynesian Cross remains the same, and our analysis in the main body of the paper goes through. The two implications of this equivalence mapping are the following.

First, $\bar{\alpha} = \alpha + \phi - \alpha\phi$, while the import-to-GDP ratio is $\alpha + \phi$. Hence, provided α and ϕ are not too large, $\bar{\alpha}$ is close to the import-to-GDP ratio, though an ideal calibration would subtract the reexported good-to-GDP ratio $\alpha\phi$.

Second, the trade elasticity $\bar{\chi}$ is now a more complex amalgam of four primitive elasticities: substitution between home and foreign final goods η , substitution between different countries' final goods γ , substitution between home and foreign intermediates ζ , and substitution between different countries' intermediates ν .

E.4 Commodity exporter model

As our last alternative model, we consider a model of a commodity exporter, who takes as given the price of tradable goods. We set this up as in [Uribe and Schmitt-Grohé \(2017\)](#), by assuming that the economy possesses a constant stream of tradable goods Y^T that it can sell in the world market at fixed prices. Vice versa, there are non-tradable goods that the economy does not export. We describe the main changes in this economy relative to the one in section 2 and argue that this model is identical to the DCP model in section E.1 in which dollar prices of exports are fully rigid.

Households. Domestic households are assumed to behave as in section 2, except that they consume tradable and non-tradable goods, rather than foreign and domestic goods. P_{Tt} is the price of tradables and P_{Nt} is the price of non-tradables. The utility function $u(c_T, c_N)$ is the same as before, with c_T and c_N entering a CES basket with elasticity η and a consumption share of tradables of α , analogous to (3). The CPI is analogous to (4), individual demands analogous to (10)–(11). Foreign households elastically buy or sell tradables at a fixed dollar price $P_T^* = 1$.

Production. Non-tradables are produced using the linear production function

$$Y_{Nt} = Z_N N_{Nt}$$

and sold by a continuum of firms charging flexible prices at a markup μ . N_{Nt} is labor demand by non-tradable producers. Tradables are produced by the Leontief production function

$$Y_{Tt} = Z_T \min\{N_{Tt}, L\} \quad (\text{A.174})$$

where $L > 0$ is a fixed factor the country is endowed with, such as the land on which natural resources can be found. Again we assume Y_{Tt} is sold by a continuum of firms charging flexible prices at a markup μ . N_{Tt} is labor demand by tradable producers.

We assume here that (A.174) is Leontief in line with the idea that tradables are basically an endowment of the economy, $Y_{Tt} = Z_T L = \text{const}$. The only reason why we do not outright assume that Y_{Tt} is an endowment is that in a heterogeneous-agent context, it matters *whose* endowment Y_{Tt} is. (A.174) provides us with a simple way to split the proceeds from selling Y_{Tt} into labor and profit income.

Rest of the model. All the remaining model ingredients are identical. For example, all firms' dividends (tradable and non-tradable alike)

$$D_t = \frac{P_{Nt} Y_{Nt} - W_t N_{Nt}}{P_t} + \frac{\mathcal{E}_t P_T^* Y_{Tt} - W_t N_{Tt}}{P_t}$$

are capitalized and traded, just like domestic firms' dividends before. Unions and wage stickiness, notation for exchange rates, market structure, and monetary policy are all identical.

Market clearing for non-tradable goods is given by

$$Y_{Nt} = (1 - \alpha) \left(\frac{P_{Nt}}{P_t} \right)^{-\eta} C_t \quad (\text{A.175})$$

essentially (33) without the second term, as non-tradable goods are not exported. We normalize all prices $\mathcal{E}_{ss}, Q_{ss}, P_{ss}, P_{Nss}, P_{Tss}$ to 1, and quantities $C_{ss} = 1, Y_{Nss} = 1 - \alpha, Y_{Tss} = \alpha$ in the steady state of the model.

Consumption function. We can write consumption as function of real labor income and dividends $C_t = C_t \left(\left\{ \frac{W_s}{P_s} N_s, D_s \right\} \right)$ just like before.

Model analysis and equivalence to DCP model. Define real GDP as

$$Y_t \equiv \frac{P_{Tss}}{P_{ss}} Y_{Tt} + \frac{P_{Nss}}{P_{ss}} Y_{Nt} = Y_T + Y_{Nt}$$

We can write dividends as

$$D_t = \frac{P_{Nt} Y_t - W_t N_t}{P_t} + \frac{\mathcal{E}_t P_T^* - P_{Nt}}{P_t} \alpha \quad (\text{A.176})$$

which is identical to (15) in the case of DCP with fully rigid dollar export prices, where $C_H^* = \alpha$ and $P_H^* = P_T^* = 1$. Just like before, aggregate labor income is given by

$$\frac{W_t}{P_t} N_t = \frac{1}{\mu} \frac{P_{Nt}}{P_t} Y_t \quad (\text{A.177})$$

Rewriting (A.175), we find that

$$Y_t = (1 - \alpha) \left(\frac{P_{Nt}}{P_t} \right)^{-\eta} C_t + \alpha \quad (\text{A.178})$$

which is identical to the goods market clearing condition (33) with DCP. Given that (A.176)–(A.178) are the same as in the model with DCP (section E.1), and the consumption function is unchanged, this proves that the model with tradable and non-tradable goods is isomorphic to the DCP model.

International Keynesian cross. Due to the equivalence with the DCP model, we can derive an international Keynesian cross decomposition for the commodity exporter model that is analogous to (A.161). For a similar decomposition of the RA consumption response in a version of the commodity exporter model see [Bianchi and Coulibaly \(2023\)](#).

E.5 UIP deviations

We consider a version of our model in which we allow for deviations in the UIP condition. In particular, we assume that households cannot directly hold positions in foreign bonds. Instead, there exist foreign intermediaries that can trade in both foreign and domestic bond markets. Denote the end-of-period positions of these intermediaries by b_t^I . We assume that foreign intermediaries have an imperfectly elastic demand for domestic bonds, that is,

$$b_t^I = \frac{1}{\Gamma} \left[(1 + \iota_t) \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right) - (1 + \iota_t^*) \right]$$

Such imperfect elasticity can be microfounded by assuming that there is only a limited number of foreign intermediaries (of similar measure as the small open economy itself) and that foreign intermediaries face limited commitment ([Gabaix and Maggiori 2015](#)), risk ([Itskhoki and Mukhin 2021](#)) or adjustment costs ([Alvarez, Atkeson and Kehoe 2009](#), [Fanelli and Straub 2021](#)). These microfoundations are identical for our purposes.

In addition to foreign intermediaries, we also allow for noise traders with exogenous demand

ξ_t for domestic bonds as in [Gabaix and Maggiori \(2015\)](#) and [Itskhoki and Mukhin \(2021\)](#). Together, foreign intermediaries and noise traders hold the inverse of the country's net foreign asset position,

$$b_t^I + \xi_t = -\text{nfa}_t$$

Rearranging, this implies that the UIP condition (5) no longer holds whenever $\Gamma > 0$

$$(1 + \iota_t) \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right) = 1 + \iota_t^* - \Gamma (\xi_t + \text{nfa}_t) \quad (\text{A.179})$$

The dependence on the NFA in (A.179) captures the idea that the country has to pay a premium when it is a net borrower $\text{nfa}_t < 0$, in terms of a greater interest rate ι_t . The dependence on ξ_t captures the idea that noise shocks can also move exchange rates.⁶⁴

In the limit where $\Gamma \rightarrow 0$, we recover the UIP condition (5). On the other hand if, as in [Itskhoki and Mukhin \(2021\)](#), we simultaneously assume $\Gamma \rightarrow 0$ but $\Gamma \xi_t \not\rightarrow 0$, we obtain a version of (A.179) with exogenous UIP shocks

$$(1 + \iota_t) \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right) = 1 + \iota_t^* - \Gamma \xi_t$$

Observe that those shocks enter in exactly the same way as the world interest rate shocks $1 + \iota_t^* = \frac{1}{\beta^*} \frac{\mathcal{B}_t}{\mathcal{B}_{t+1}}$ that we introduced via discount factor shocks \mathcal{B}_t in section 2. In that sense, our analysis for ι_t^* shocks carries over to exogenous UIP shocks, by simply redefining $\iota_t^* \equiv \iota_t^* - \Gamma \xi_t$.

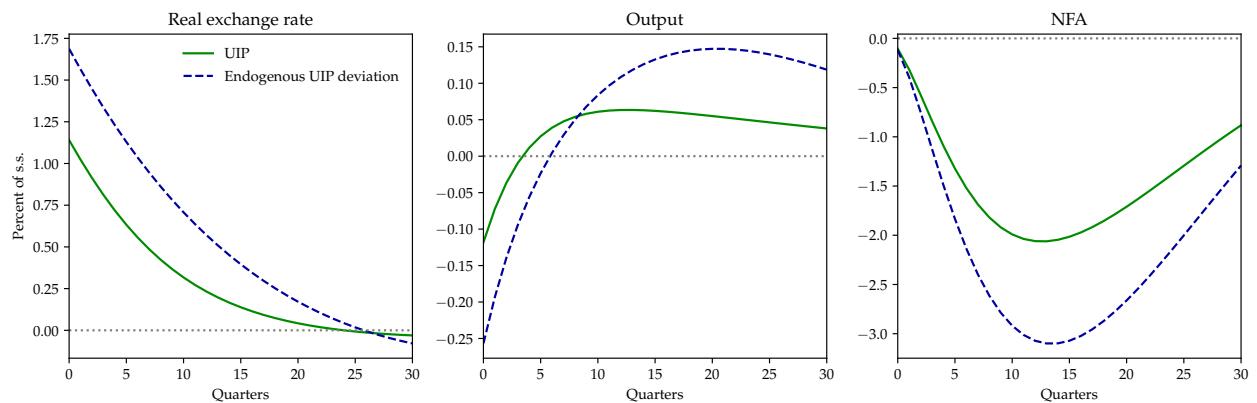
Endogenous UIP deviations. We now study the effects of endogenous UIP deviations (A.179) with $\xi_t = 0$, as in [Gabaix and Maggiori \(2015\)](#) and [Itskhoki and Mukhin \(2021\)](#). The UIP condition becomes

$$(1 + \iota_t) \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right) = 1 + \iota_t^* - \Gamma \cdot \text{nfa}_t$$

Figure A.8 simulates an ι_t^* shock in our quantitative model with endogenous UIP deviations. As before, the shock depreciates the exchange rate and leads to increased domestic interest rates ι_t . As the NFA declines due to greater import prices, however, foreign intermediaries require even higher domestic interest rates ι_t , captured by a more positive UIP deviation $-\Gamma \text{nfa}_t$. This amplifies the exchange rate depreciation and ultimately worsens the contractionary effects of the depreciation. Thus, endogenous UIP deviations amplify contractionary depreciations, especially in countries with high Γ . This presents another reason why interest rates may be procyclical in open economies, especially in emerging markets prone to having greater UIP deviations.

⁶⁴An alternative way to think of noise shocks is as movements in risk premia.

Figure A.8: Depreciations with endogenous UIP deviations



Note: impulse response to the shock to i_t^* from figure 2. The green line shows the impulse in the quantitative model (UIP) while the blue dotted line shows the impulse in the model with endogenous UIP deviations.