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Haelim Anderson
Kinda Cheryl Hachem
Simpson Zhang

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ABSTRACT

We study financial stability with constraints on central bank intervention. We show that a forced reallocation of liquidity across banks can achieve fewer bank failures than a decentralized market for interbank loans, reflecting a pecuniary externality in the decentralized equilibrium. Importantly, this reallocation can be implemented through the issuance of clearinghouse loan certificates, such as those issued in New York City during the Panic of 1873. With a new dataset constructed from archival records, we demonstrate that the New York Clearinghouse issued loan certificates to member banks in the way our model suggests would have helped resolve the panic.

Haelim Anderson
Center for Financial Research
Federal Deposit Insurance Corporation
550 17th St NW
Washington, DC 20429
handerson@fdic.gov

Kinda Cheryl Hachem
University of Virginia
Darden School of Business
100 Darden Boulevard
Charlottesville, VA 22906
and NBER
hachemk@darden.virginia.edu

Simpson Zhang
Office of the Comptroller of the
Currency (OCC) - Treasury
400 7th St SW
Washington, DC 20219
Simpson.Zhang@occ.treas.gov

1 Introduction

While the advent of deposit insurance has dramatically reduced runs on banks in the traditional sector, shadow banks engaging in maturity transformation remain highly vulnerable. The preservation of the banking system during an aggregate liquidity crunch then requires either suspensions of convertibility or monetary injections by the central bank. Suspensions can undermine investor confidence and cause market disruptions, making them a nuclear option. The 2007-09 financial crisis, widely characterized as a run on the shadow banking system, was instead resolved with a massive expansion of the monetary base. However, weary of the implications for bailout expectations and bank risk-taking, lawmakers have since imposed restrictions on how central banks can respond to future financial crises.[1] This paper investigates the scope for mitigating bank failures without resorting to suspension or monetary injections. If such policies exist, they would make the restrictions on central banks credible and provide a powerful tool against bailout expectations.

The setting for our analysis is the Panic of 1873. This was the first major financial crisis of the U.S. National Banking Era and its magnitude necessitated a strong response in order to contain the damage. Since the Federal Reserve did not yet exist, the banking system had to resolve financial crises without central bank intervention. The New York Clearinghouse (NYCH) took the lead in New York City, the center of the financial system. The Panic of 1873 led to the first, large-scale use of a novel instrument devised by the NYCH: the issuance of clearinghouse loan certificates to member banks. These certificates were collateralized notes that members of the clearinghouse could use instead of cash to settle payment obligations with each other, then driven principally by check clearing. Members paid in loan certificates were entitled to cash at a later date. The certificates did not increase the total amount of cash in the system nor did they circulate as money in the general public. Instead, clearinghouse loan certificates provided a mechanism for reallocating cash across banks and their activities. Our paper studies this reallocation, its value-added relative to a decentralized interbank market, and its role in resolving the Panic of 1873. The lessons transcend the historical episode as regulations mandating the central clearing of credit derivatives after the

[1] Regulations were passed after the financial crisis to restrict the scope of central bank lending. In the U.S., for example, the Dodd-Frank Act imposes restrictions on the Fed's powers to lend to non-bank financial institutions. The Treasury Secretary must approve Fed loans to non-banks, and banks are limited from using discount window loans to channel funds to non-bank affiliates. In 2017, lawmakers attempted to further restrict lending to non-banks in the Financial CHOICE Act, though the act did not ultimately pass. The latest battle over the Fed's emergency lending authority occurred in December 2020 during negotiations for the Consolidated Appropriations Act.

2007-09 financial crisis have led to a revival of clearinghouses in modern financial systems.

We start by constructing a new dataset from archival records of the NYCH to shed light on the usage of clearinghouse loan certificates. These records include the daily clearing and settlement payments of all member banks through the NYCH, the amount of loan certificates applied for and received by banks, and the interest payments among banks. With this information, we can characterize the magnitude of liquidity shortages and the amount of liquidity transfers across banks.

We document that banks experiencing the most pressure on cash reserves (specie and legal tender) because of check settlement received the most loan certificates from the NYCH. Recipient banks were also experiencing much larger deposit outflows than non-recipient banks during the crisis, so, in the absence of loan certificates, they would not have had enough cash to honor both clearing obligations and deposit withdrawals without liquidating large amounts of assets, most likely call loans. The call loan market was a critical source of funding for stock-brokers, thus liquidations of call loans by banks in need of cash would have had potentially deleterious effects on stock prices. We also document that both recipient and non-recipient banks emerged from the Panic of 1873 with cash-to-deposit ratios that were similar to the ratios they had before the crisis began, despite experiencing differential deposit outflows during the panic. Liquidity thus appears to have been successfully redistributed across the members of the NYCH.

The empirical findings raise an important theoretical question: were clearinghouse loan certificates simply substituting for a decentralized interbank market, which by all accounts did not exist in New York in 1873, or do loan certificates improve outcomes over and above what such markets can achieve? We develop a model to answer this question. The NYCH provides a blueprint for the design and rollout of clearinghouse loan certificates, so, if such certificates also correct market failures, then we have identified an alternative policy tool that can be used to manage liquidity crises with less central bank intervention.

Banks in the model borrow short and lend long, subjecting themselves to runs by patient depositors (i.e., those who do not need to withdraw early but may choose to do so). Whether an otherwise patient depositor runs on his bank depends on whether the bank can withstand a run by all of its depositors by liquidating assets or borrowing from other banks. We begin with a benchmark where banks can borrow cash from each other in a Walrasian interbank market at an endogenous market clearing interest rate. We then introduce clearinghouse loan certificates to study whether there exists a centralized reallocation of liquidity that improves social welfare.

When the total amount of cash in the system exceeds the withdrawal needs of impatient depositors (i.e., those who experience liquidity shocks and have no choice but to withdraw early), the Walrasian market efficiently redistributes liquidity across banks and achieves an equilibrium with no bank runs. In contrast, when total cash is lower than the amount that needs to be withdrawn by impatient depositors, an equilibrium with no runs cannot be achieved. We show that the measure of banks that fails in an equilibrium with runs is increasing in the interbank rate. This introduces a pecuniary externality because individual banks do not internalize the effect of their net borrowing on the interest rate in the interbank market. The higher the interbank rate, the more expensive it is to obtain additional cash. The marginal bank that was preventing a run by borrowing on the interbank market can no longer do so profitably; the amount it needs to borrow is simply too high to be fully repaid at the higher interest rate. The minimum level of cash reserves that a bank must have in order to be run-proof thus rises, as does the measure of banks that fails.

Next, we explore whether a social planner can use clearinghouse loan certificates to achieve a better outcome. We model specifically the certificates designed by the NYCH, which built on an existing network of bilateral exposures between banks. In 1873, these exposures were payment obligations stemming from check-clearing activity, but the same logic would apply to obligations stemming from derivatives trading. We show that loan certificates reduce bank failures and improve social welfare relative to a decentralized interbank market if (i) most of the cross-sectional variation in cash holdings of banks comes from variation in their bilateral exposures and (ii) the average exposure is high. Then, the clearinghouse should allocate to banks above a cash threshold enough loan certificates to cover the payment obligations stemming from their exposures, e.g., checks owed to other banks. Banks below this threshold receive no loan certificates and must use cash to pay checks owed, while receiving less overall cash from other banks as payment for checks owing. This constitutes a forced reallocation of liquidity from failing banks (and their depositors) to the rest of the system. In turn, the interest rate on loan certificates can be set below the borrowing rate that prevails in the decentralized equilibrium, allowing more banks to fend off runs. The measure of failed banks falls and total welfare rises.

Among surviving banks, the welfare-improving allocation issues more loan certificates to banks that owe more checks. This is consistent with the empirical patterns we documented using the archival records of the NYCH. We conclude from our theoretical model that there exists an allocation of loan certificates that improves welfare and that this allocation is roughly in line with the allocation implemented by the NYCH during the Panic of 1873. Calibrating the model to historical data, we find that social welfare with loan certificates is 2% higher than the welfare with a decentralized interbank market. A welfare improvement of 2% is notable since it fills almost half of the gap between the decentralized equilibrium and the first best. Our calibration also reveals that the total amount of cash in the banking system at the onset of the panic was too low for any reallocation mechanism to have completely eliminated bank failures. Since none of the members of the NYCH failed during the Panic of 1873, other policies must have been responsible for driving bank failures down to zero. We attribute the lack of bank failures to partial suspension of convertibility rather than the suppression of bank-level information. Suspension entailed a welfare loss to individual depositors and for plausible parameters reduced aggregate welfare relative to a system of only loan certificates despite eliminating bank failures. However, information suppression without suspension would have been disastrous for a banking system at the calibrated parameters, triggering a system-wide run due to the paucity of cash.

Related Literature This paper contributes to the literature on interbank markets and liquidity provision. Diamond and Dybvig (1983) present the classical model of bank runs driven by coordination issues among depositors. Allen and Gale (2000) show that interbank markets can help mitigate bank runs if the banking system is sufficiently liquid. With excess demand for liquidity, however, the interbank market can breed contagion. Freixas, Parigi, and Rochet (2000) show that a central bank can act as a coordinating device to solve liquidity shortages in payment networks.[2] See also Allen, Carletti, and Gale (2009) who show that interbank markets feature excessive price volatility without a central bank, Bluhm (2018) who shows that interbank markets increase total lending but act as channels for financial contagion, and Hachem and Song (2021) who show that interbank market power subverts the effectiveness of liquidity regulation.

[2] In Freixas, Martin, and Skeie (2011), the central bank provides a different form of coordination, setting state-contingent interest rates that select the best equilibrium from a continuum of Pareto-ranked ex ante equilibria. In the case where the aggregate state always requires interbank trade, the interest rate that incentivizes ex ante liquidity holdings is too high to achieve optimal deposit contracts (see also Farhi, Golosov, and Tsyvinski (2009)). This externality differs from ours; it does not operate through the marginal bank that prevents a run by borrowing on the interbank market. Robatto (2019) studies central bank interventions when pecuniary externalities affect liquidity-constrained banks, but the mechanism does not involve run-proofing or operate through an interbank market.

Several papers have also documented how interbank markets became frozen or highly stressed during the 2007-09 financial crisis (e.g., Afonso, Kovner, and Schoar (2011), Cornett, McNutt, Strahan, and Tehranian (2011), Acharya and Merrouche (2012), di Patti and Sette (2016)). We analyze the scope for interbank lending during the Panic of 1873 and conclude that shortfalls in aggregate liquidity would have prevented a decentralized interbank market from effectively resolving the panic. Clearinghouse loan certificates provided an alternative mechanism to redistribute liquidity across banks and would have functioned better than a decentralized market had the latter existed, especially in the absence of a partial suspension of convertibility. Given the paucity of cash reserves relative to the demands of impatient depositors, however, the measure of bank failures could not have been driven down to zero without a partial suspension.

We also contribute to the literature on banking panics during the National Banking Era and the actions of the NYCH to fight these crises. Tallman and Moen (2012), Gorton and Tallman (2016a), and Gorton and Tallman (2016b) provide overviews of the banking panics. Anderson, Paddrik, and Wang (2016) analyze financial network structures after the passage of the National Banking Act in the 1860s and the vulnerability of the banking system to financial shocks. Anderson and Bluedorn (2017) and Calomiris and Carlson (2017) highlight the importance of network effects and financial spillovers from New York City banks during the Panics of 1884 and 1893, respectively. Our paper represents the first analysis of the Panic of 1873 using detailed loan certificate data, as well as the first rigorous theoretical modeling of crisis responses by the New York Clearinghouse.

The rest of the paper is organized as follows. Section 2 introduces the historical background, describes our data, and presents empirical evidence on the impact of clearinghouse loan certificates. Section 3 presents our theoretical model and derives predictions about the ability of loan certificates to improve on a decentralized interbank market. Section 4 estimates the welfare gains from loan certificates using our historical data. Section 5 compares loan certificates with other interventions used by the New York Clearinghouse. Section 6 concludes. All proofs are collected in Appendix A.

2 Empirical Evidence

Before the creation of the Federal Reserve System in 1913, the New York Clearinghouse (NYCH) was the main authority in place for responding to banking panics in New York City. We first provide some background on the Clearinghouse and its response to the Panic of 1873. We then discuss our new archival record dataset and use it to study the effectiveness of the NYCH's innovative response: the issuance of clearinghouse loan certificates.

2.1 Historical Background

2.1.1 The NYCH during the Panic of 1873

The New York Clearinghouse was an association of all of the major banks in New York City. Clearinghouses emerged in various cities during the 1850s to facilitate the exchange of checks. In normal times, the function of the clearinghouse was to net interbank payments between parties so that they would not need to be settled bilaterally. Meeting in a single place and settling balances with only one other party (the clearinghouse) dramatically simplified the check-clearing process.[3]

During banking crises, member banks within each clearinghouse tended to act cooperatively and the clearinghouse became the de facto leader in liquidity management for its city. The Panic of 1873 was the first major banking crisis of the National Banking Era. It originated from failures of major financial institutions, such as Jay Cooke & Co., that had made bad investments into the massive railroad construction bubble. These failures sent stocks tumbling on September 18th and caused pandemonium throughout Wall Street. In the following days, many more institutions failed and banks experienced runs by depositors. The magnitude of the crisis necessitated a strong response by the NYCH in order to contain the damage.

On September 20th, following the closure of the stock market, the NYCH committee met and authorized the issuance of $10 million in clearinghouse loan certificates, which we describe in more detail below. An additional $10 million in clearinghouse loan certificates was authorized a few days later (September 22nd).[4] The NYCH also implemented a reserve pooling arrangement on September 20th. Under the arrangement, the reserves of the member banks were mutualized into one pool. If the reserves of a bank fell dangerously low, those of the other members were assessed and reserves were directly provided to the troubled bank from the pool. Unlike loan certificates,

[3]Clearinghouses also helped mitigate counterparty risks involved in check clearing. If a bank is unable to settle its obligations, then the banks it owes might also be unable to settle what they owe to other banks, etc. The netting of positions that occurs when a clearinghouse is the counterparty to all trades helps prevent such contagion. Similar arguments were used to move credit derivatives into central clearing after the 2007-09 financial crisis.

[4]Such decisions required unanimous approval. Due to the extent of the crisis in 1873, cooperation was not an issue. However, disagreements among banks did arise in later crises which hindered recovery efforts.

which continued to be used in later crises, the reserve pooling arrangement was abandoned in 1873. We will return to this point in Section 4.2.

Two other measures were taken by the NYCH in response to the Panic of 1873. First, the clearinghouse began suppressing bank-specific balance sheet information on September 20th, publishing instead only the aggregate balance sheet across all members in order to avoid revealing the weakest banks. Second, on September 22nd, the NYCH decided to partly suspend the convertibility of deposits into cash to limit the drain of cash reserves. Country banks holding deposits at NYCH members could continue to withdraw, but individual depositors could not.

In total, the NYCH responded to 5 banking crises during the National Banking Era. It issued clearinghouse loan certificates and suspended convertibility to at least some degree during the major crises (1873, 1893, 1907) and was able to avoid suspension during the minor ones (1884 and 1890). Of all the actions taken by the NYCH over the course of these crises, loan certificates and their role in crisis resolution are the least studied and the least well understood and hence the focus of our paper. We revisit the other actions (reserve pooling, information suppression, and partial suspension of convertibility) later in the paper.

2.1.2 Clearinghouse Loan Certificates

Clearinghouse loan certificates were collateralized notes that members of the NYCH could use instead of cash reserves (specie and legal tender) to settle obligations with each other during the check-clearing process. To obtain loan certificates, a member bank had to apply to the clearinghouse loan committee, submitting some of its loans and bonds for examination as collateral. Upon accepting the collateral, the clearinghouse would issue loan certificates to the applicant amounting to no more than 75% of the assessed value of the collateral. The applicant also agreed to pay an interest rate on any loan certificates that it used during check clearing. The NYCH set an interest rate of 7% when it introduced loan certificates during the Panic of 1873. This interest rate was high enough that banks would want to pay off their loan certificates quickly after the crisis terminated, but not so high that it was unaffordable.

For all intents and purposes, clearinghouse loan certificates functioned as forced loans during the check-clearing process. When receiving payment in the form of loan certificates, the accepting bank was effectively lending the value of those certificates to the paying bank. The difference from ordinary loans was that banks could not refuse to accept loan certificates in lieu of cash reserves during check clearing.[5]

By reducing the pressure on banks' cash reserves, loan certificates also helped maintain the call loan market, which was a critical source of liquidity for the stock exchange. Stock-brokers used call loans for margin purchases and for the daily settlement of transactions on the exchange. The member banks of the NYCH were the primary funding source for the call loan market during the National Banking Era.[6] Call loans, as the name suggests, were callable on demand by the lender. This was rare in normal times, as most call loans were rolled over daily. However, during a banking crisis, banks with insufficient cash would be forced to call in their loans, with potentially deleterious effects on stock prices.

2.2 Data and Summary Statistics

We use various sources, including archival materials from the New York Clearinghouse, to study how clearinghouse loan certificates helped with bank liquidity management.

We obtain information about clearinghouse loan certificates from the minutes of the NYCH committee. When the Panic of 1873 started, the NYCH appointed a subcommittee of member bank officers to oversee the issuance of loan certificates. The minutes of this committee include the identities of banks who applied for loan certificates, the amount of loan certificates requested, the dates of certificate issues, and the dates of cancellation (repayment) of the certificates. The NYCH also separately tabulated the amount of interest paid and received by each bank in relation to loan certificates on November 1st, December 1st, and January 1st. None of this information was made public during the panic.

Two additional archival materials from the NYCH are useful for our analysis. First, an internal document compiled by the NYCH summarizes the deposits at each member bank on October 21st, 1873. This date falls within the period where bank-specific information was being withheld from the public, providing us with a unique snapshot of the conditions of member banks. Second, we

[5] In transactions where loan certificates were used, the NYCH acted only as a guarantor of the final repayment of the certificates. If member banks were exposed to losses arising from unpaid loan certificates, the members of the NYCH would share these losses based on their relative capital.

[6] New York City banks used the call loan market to invest deposits from interior banks into other parts of the country. Call loans enabled New York City banks to profit from the typically positive spread between the call loan interest rate and the interest paid on deposits. The interest on these loans and their perceived safety and liquidity during normal times made them attractive investments for New York City banks.

obtained daily ledgers of the New York Clearinghouse. These ledger books feature daily records of payments between major banks that were cleared through the NYCH. In 1873 the NYCH cleared checks twice a day, and we have information for both the morning and afternoon clearings.

Finally, we collected balance sheet information for the member banks of the NYCH to examine their conditions prior to the Panic of 1873. For national banks, this information comes from national bank examination records and the September 1873 call reports. The call reports provide balance sheet information for all national banks on the same date, whereas the examination reports were filed at various dates by OCC bank examiners who visited each bank once or twice a year. That being said, the examination reports are still useful because they contain more detailed information about bank loan books than the call reports (e.g., the amount of unsecured loans, the amount of loans payable on demand, and the amount of loans secured by real estate). For state banks, we collected balance sheet information from the Annual Report of the Superintendent of the Banking Department of the State of New York. The state banking department made quarterly calls to investigate the conditions of state banks and published this information in its annual report.

Table 1 reports summary statistics separately for banks that received loan certificates and banks that did not. The last column reports the same statistics for all banks together. The statistics are based on the September call report right before the crisis. Of the 61 member banks in the NYCH, our data sources recover balance sheet information for all but two of them.

On the whole, the member banks of the NYCH were liquid and solvent. On average, they held 10% of their total assets as cash reserves, specifically specie (i.e., gold and silver recognized as lawful money) and legal tender notes. These reserves amounted to 16% of total deposits, where we define total deposits to include retail deposits as well as institutional deposits recorded as "due tos." The banks in the sample also held about 25% of their loan book in the form of call loans. Moreover, they held a large amount of equity, almost 30% of their total assets.

Comparing banks that received loan certificates to banks that did not, three differences are statistically significant at the 5% level. First, recipient banks tended to be less liquid. On average, their ratio of cash reserves to total assets was 2.66 percentage points lower than non-recipient banks. Recipient banks also tended to be less well capitalized. On average, their capital ratio was 9.43 percentage points lower than non-recipient banks. Lastly, recipient banks tended to take in more deposits from outside banks, as measured by due-tos. Institutional deposits are generally flightier than retail deposits, so a higher incidence of due-tos among recipient banks may have made them more susceptible to runs in the absence of crisis mitigation policies by the NYCH.

2.3 Liquidity Reallocation During the Panic of 1873

This section provides empirical evidence that clearinghouse loan certificates helped stabilize the banking sector by reallocating liquidity across banks.

We first examine the relationship between balances due to the clearinghouse (i.e., balances owed by banks as part of the check-clearing process) and the daily issuance of clearinghouse loan certificates to individual members. We look at the period from September 22nd to September 30th, since this is when most loan certificates were issued.[7] Table 2 presents the results. Issuance of loan certificates is highly correlated with balances due to the clearinghouse; the correlation coefficient is 0.8 and statistically significant. In other words, banks that were experiencing the most pressure on cash reserves because of check settlement received the most loan certificates from the NYCH.

The allocation of loan certificates to these banks would have freed up their cash in the event of large depositor withdrawals. Banks were vulnerable to cash drains arising from both check-clearing activities and depositor withdrawals. By allowing recipient banks to settle their clearing balances without using specie or legal tender, loan certificates would have helped economize on cash reserves.

Table 3 presents average deposit growth for recipient and non-recipient banks during the Panic of 1873, which lasted from September 20th to December 6th. We use the NYCH's internal summary of deposits on October 21st to divide the panic into two periods. From September 20th to October 21st, the currency premium defined in Gorton and Tallman (2018) was positive. From October 21st to December 6th, the currency premium was effectively zero.[8] The most intense part of the panic thus occurred in the first month. While the entire banking system experienced large deposit outflows during this month, the first column of Table 3 shows that banks that received loan certificates were experiencing much greater deposit outflows than non-recipient banks.[9] The ability of loan certificates to free up the cash reserves of recipient banks would have therefore been particularly helpful at the beginning of the panic.

[7]Following their introduction on September 20th, 30% of loan certificates were issued by the clearinghouse on September 22nd. Total issuance peaked on October 2nd, after which very few new loan certificates were issued.

[8]The currency premium became zero on October 24th (Gorton and Tallman (2018)).

[9]The difference is statistically significant at the 1% level, in contrast to the difference later in the panic (second column of Table 3) which is not statistically significant at standard levels.

Implicit in our discussion is the assumption that recipient banks actually used (circulated) the loan certificates they were granted. Studying the policies of the NYCH in 1884, Goehring, Tallman, and Van Horn (2019) argue that this assumption can be verified by examining the interest paid and received by each bank in relation to loan certificates. Interest was only paid on circulated loan certificates, so the interest data make it possible to compute the amount of loan certificates in actual circulation. Table 4 summarizes average interest payments to and from the NYCH during and immediately following the Panic of 1873. A payment to the clearinghouse means that the bank making the payment used loan certificates to clear checks. A payment from the clearinghouse means that the bank receiving the payment accepted loan certificates while clearing checks. As a benchmark against which to compare actual interest payments, Goehring, Tallman, and Van Horn (2019) propose computing the interest payments that should have been observed had all loan certificates circulated. We find these payments to be exactly equal to the actual payments, indicating that recipient banks used all of their loan certificates in 1873.

Finally, we compare the liquidity positions of recipient and non-recipient banks at the onset and conclusion of the panic. Table 5 reports the ratio of cash reserves to deposits on September 20th and December 6th. Although recipient banks experienced much larger deposit outflows in between these dates (Table 3), both recipient and non-recipient banks emerged from the Panic of 1873 with cash-to-deposit ratios that were similar to the ratios they had before the crisis began.[10] Liquidity thus appears to have been successfully redistributed across the members of the NYCH.

3 Theoretical Model

In this section, we develop a theoretical model of liquidity reallocation among banks that allows us to analyze the effectiveness of the NYCH's policies during the Panic of 1873. We first present a decentralized model of interbank lending with no clearinghouse to establish a benchmark against which clearinghouse intervention can be compared. We then introduce loan certificates of the type issued by the NYCH to study whether there exists a centralized reallocation of liquidity that improves social welfare. To the best of our knowledge, there was no formal, decentralized interbank market in New York in 1873, hence any welfare improvements predicted by our model are likely a

[10]Formally, there is no statistically significant difference between the first and second columns in Table 5 for each group of banks.

conservative estimate of the impact of the NYCH's intervention.

3.1 Decentralized Interbank Market

We consider an economy with three dates, $t = 0, 1, 2$, and a continuum of banks, $i \in [0, 1]$. Each bank i is endowed with cash holdings c_i and loans outstanding $\tilde{z}$ at $t = 0$. Loans pay $x \in (0, 1)$ per unit if liquidated at $t = 1$ and $(1 + r_z)$ per unit if held until $t = 2$. Naturally, $r_z > 0$ so a bank i only liquidates loans if it needs more cash than c_i at $t = 1$.

Cash is used to pay short-term liabilities at $t = 1$, namely depositors who want to withdraw funds before loans have matured. Cash can also be used to make additional loans (i.e., if bank i needs less cash than c_i at $t = 1$). To increase its loans by an amount z_i at $t = 1$, bank i must pay z_i units of cash upfront plus an additional adjustment cost, $\frac{1}{2}\zeta z_i^2$, due at the end of $t = 2$. Note that banks that liquidate loans will not make additional loans since $x < 1$. Thus:

$$z_i l_i = 0$$

where $l_i \in [0, \tilde{z}]$ denotes the amount of loans liquidated by bank i at $t = 1$. The total amount of loans held by bank i at the end of $t = 1$ is $(\tilde{z} - l_i + z_i)$.

Each bank i serves a unique set of depositors of measure one. The set of depositors in bank $i \in [0, 1]$ is denoted by $\{ij\}_{j \in [0,1]}$. Each depositor in the set $\{ij\}_{j \in [0,1]}$ has 1 unit of deposits in bank i at $t = 0$. Bank i's equity at $t = 0$ can then be defined as $c_i + \tilde{z} - 1$, where $c_i + \tilde{z} \geq 1$. A depositor is entitled to 1 unit if he withdraws his funds at $t = 1$ and $(1 + r)$ units if he waits until $t = 2$, where $r \in [0, r_z)$. A fraction $\rho \in (0, 1)$ of depositors at each bank will experience a liquidity shock that forces them to withdraw at $t = 1$. The remaining depositors can choose whether to withdraw at $t = 1$ or $t = 2$. The action of depositor $ij \in [0, 1] \times [0, 1]$ is represented by the date at which he withdraws, i.e., $a_{ij} \in \{1, 2\}$. Without loss of generality, depositors can be ordered so that $a_{ij} = 1$ for $ij \in [0, 1] \times [0, \rho]$ and $a_{ij} \in \{1, 2\}$ for $ij \in [0, 1] \times [\rho, 1]$. Bank i experiences a run if $a_{ij} = 1$ for all depositors in the set $\{ij\}_{j \in [\rho,1]}$.

At $t = 1$, after observing depositor withdrawal decisions, banks borrow and lend cash among each other on a Walrasian interbank market. The interest rate charged on interbank loans, r_b, is determined in equilibrium via market clearing. Denote by Δ_i the net borrowing of bank i on the interbank market. If $\Delta_i < 0$, then bank i is a net lender on the interbank market. Market clearing requires:

$$\int_0^1 \Delta_i di = 0 \tag{1}$$

so that there is no excess demand or excess supply of interbank loans.

Interbank loans are repaid at the end of $t = 2$. There is no uncertainty in the model after the realization of depositor liquidity shocks. Therefore, when a bank borrows on the interbank market, it knows whether or not it will be able to repay the loan. We assume banks can only take out loans that they know they can repay alongside withdrawals at $t = 2$.[11]

The timing of events in the model can be summarized as follows:

- Date $t = 0$: Each bank i begins with endowments of cash c_i and loans $\tilde{z}$.

- Date $t = 1$: Depositors learn their liquidity shocks and decide whether or not to withdraw from their banks, with full information about the bank's portfolio (more on this below). After observing the decisions of its depositors, each bank i chooses liquidations l_i, additional loans z_i, and net interbank borrowing Δ_i. Banks with insufficient cash to pay depositors after these choices fail.

- Date $t = 2$: Solvent banks obtain returns from unliquidated loans and repay any interbank loans they borrowed or receive payment for any interbank loans they lent. They also repay depositors who did not withdraw at $t = 1$.

3.2 Equilibrium

An equilibrium is a set of actions for the banks $\{l_i, z_i, \Delta_i\}_{i \in [0,1]}$ that maximizes each bank i's profit, a set of actions for the depositors $\{a_{ij}\}_{ij \in [0,1] \times [0,1]}$ that maximizes each depositor ij's profit, and an interest rate r_b such that the interbank market clears (Eq. (1)).

Since depositors act simultaneously and before each bank i chooses $\{l_i, z_i, \Delta_i\}$, each depositor $ij \in [0, 1] \times [\rho, 1]$ must have beliefs about the actions of banks and other depositors when choosing a_{ij}. As is common in bank-run models, there could be multiple equilibria based on depositor

[11] We thus abstract away from additional adverse selection issues which could reduce the efficiency of the decentralized interbank market. Recall our goal here is a benchmark against which to compare loan certificates, so abstracting away from ingredients that reduce efficiency of the interbank market will make it harder, not easier, to find a welfare improvement from loan certificates.

beliefs. In choosing among different equilibria, we assume that depositors are conservative and decide whether or not to withdraw at $t = 1$ based on the worst case scenario that all other depositors in their bank withdraw at $t = 1$. Formally:

Definition 1. *A conservative equilibrium is an equilibrium such that, for each bank $i' \in [0,1]$, fixing $\{l_i, z_i, \Delta_i\}_{i \neq i'}$ and $\{a_{ij}\}_{i \neq i'}$, depositor $i'j' \in [0,1] \times [\rho, 1]$ withdraws at $t = 1$ if and only if he prefers to withdraw at $t = 1$ when $a_{i'j} = 1$ for all $i'j \neq i'j'$.*

We now characterize the conservative equilibria of our model:

Proposition 1. *Consider $c_i + (1 + r_z)\,\tilde{z} \geq 1 + r$ for all i.*

1. *If $\int_0^1 c_i di \geq \rho$, then there exists a conservative equilibrium where $a^*_{ij} = 2$ for all depositors $ij \in [0,1] \times [\rho, 1]$.*

2. *If $\int_0^1 c_i di < \rho$, then a conservative equilibrium will involve runs on a positive measure of banks, namely any bank i with cash endowment*

$$c_i < \bar{c}\,(r^*_b) \equiv 1 - \frac{(1 + r_z)\,\tilde{z}}{1 + r^*_b} \tag{2}$$

where the interbank rate r^*_b solves

$$\int_{\{i | c_i \geq \bar{c}(r^*_b)\}} (\rho - c_i)\,di = 0 \tag{3}$$

and existence of the conservative equilibrium requires parameters such that $r^*_b \in \left(r_z, \frac{1+r_z}{x} - 1\right)$.

Proposition 1 shows that there exists a conservative equilibrium without runs if and only if the total amount of cash in the system, $\int_0^1 c_i di$, is at least as large as the fraction ρ of depositors who experience liquidity shocks at $t = 1$ (i.e., impatient depositors). When $\int_0^1 c_i di$ is below ρ, there is not enough cash in the system to pay off all the impatient depositors. Since the interbank market only redistributes liquidity instead of creating it, there must be some banks that cannot meet the needs of their impatient depositors. The patient depositors of these banks realize that their bank will be insolvent and thus all choose to run at $t = 1$. In contrast, when total cash exceeds ρ, the interbank market is effective at redistributing liquidity across banks and there is a conservative equilibrium where no banks face runs. The scope for welfare-improving interventions will therefore be highest when $\int_0^1 c_i di < \rho$. We analyze the potential improvement in social welfare at low liquidity levels next.

3.3 Social Welfare

We consider a social welfare function that puts equal weight on all agents. We model the recipients of bank loans as firms who engage in production. This can include stock-brokers who intermediate between banks and producers of goods and services. A firm who has loans $(\tilde{z} - \ell + z)$ at $t = 1$ is able to generate output $f(\tilde{z} - \ell + z)$ at $t = 2$, where $f'(\cdot) > 0$ with $f(0) = 0$ and $f(\tilde{z}) \geq (1 + r_z)\tilde{z}$.

If $\int_0^1 c_i di \geq \rho$ as in the first part of Proposition 1, no banks fail and an additional amount of loans z_a is made, with $z_a = \int_0^1 c_i di - \rho$ if the adjustment cost parameter ζ is not too high. Social welfare in the benchmark model with a decentralized interbank market is then:

$$\mathcal{W}_b^{(1)} = \int_0^1 c_i di + f(\tilde{z} + z_a) - \frac{\zeta}{2} z_a^2 \tag{4}$$

Now consider $\int_0^1 c_i di < \rho$ as in the second part of Proposition 1. Banks with initial cash $c_i < \overline{c}(r_b^*)$ experience runs and liquidate all loans. Banks with initial cash $c_i \geq \overline{c}(r_b^*)$ do not experience runs and do not liquidate, but even the most liquid among these banks do not make additional loans because the scarcity of aggregate cash bids up the interbank rate and makes lending on the interbank market more profitable, i.e., $r_b^* > r_z$. Social welfare is therefore:

$$\mathcal{W}_b^{(2)} = \int_0^1 c_i di + x\tilde{z} + [f(\tilde{z}) - x\tilde{z}] \int_{\{i|c_i \geq \overline{c}(r_b^*)\}} di \tag{5}$$

Eq. (5), with $\overline{c}(r_b^*)$ as defined in Eq. (2), clearly highlights that social welfare is decreasing in the market-clearing interest rate r_b^*. Therefore, a mechanism which successfully lowers the interbank rate could improve social welfare.

The intuition is as follows. An individual bank does not internalize the effect of its net borrowing on the interest rate in the interbank market. This imposes a pecuniary externality because the measure of banks that fails in a conservative equilibrium is increasing in the interbank rate when $\int_0^1 c_i di < \rho$ (see Proposition 1). The higher the interbank rate, the more expensive it is to get additional cash to pay depositors at $t = 1$. The marginal bank that was preventing a run by borrowing on the interbank market can no longer do so profitably; the amount it needs to borrow given its endowed cash holdings is simply too high to be fully repaid at the higher interest rate. The minimum level of endowed cash that a bank must have in order to be run-proof thus rises, as does the measure of banks that fails. The lower the interbank rate, the lower the measure of banks that fails. Such an externality opens the door for centralized intervention, which we consider in Section 3.4.

For comparison, it will be useful to define the maximum level of welfare when $\int_0^1 c_i di < \rho$, assuming surviving banks do not liquidate loans.[12] Social welfare in Eq. (5) is maximized when the measure of banks that survives is maximized. The highest possible measure of surviving banks is $\frac{\int_0^1 c_i di}{\rho}$ when $\int_0^1 c_i di < \rho$ and surviving banks do not liquidate, leading to a maximum welfare level of

$$\mathcal{W}_{\max}^{(2)} = \int_0^1 c_i di + x\tilde{z} + [f(\tilde{z}) - x\tilde{z}] \frac{\int_0^1 c_i di}{\rho} \tag{6}$$

Eq. (6) abstracts from the feasibility of so many banks surviving when depositors behave as in Definition 1, i.e., not running if and only if their bank can survive a run by all of its depositors. If (i) $\rho \geq 1 - x\tilde{z}$ and (ii) $c_i = \rho$ for all $i \in S$, where $S \subset [0,1]$ and $|S| = \frac{\int_0^1 c_i di}{\rho}$, and $c_i = 0$ for all $i \notin S$, then $\mathcal{W}_{\max}^{(2)}$ is also feasible as a conservative equilibrium; each bank $i \in S$ could liquidate enough loans to survive a run, in which case runs and liquidations do not occur for those banks. An immediate implication is that a social planner could improve on the decentralized equilibrium by reallocating cash so that each bank has cash holdings of either 0 or ρ. If $\rho < 1 - x\tilde{z}$, then the planner would also specify a set of (off-equilibrium) transfers that eliminate runs. We present the formal planning problem in Appendix B. It corresponds to the use of reserve pooling, wherein the cash reserves of banks are pooled and redistributed.

3.4 Loan Certificates

We now explore whether a social planner can use loan certificates to achieve a better outcome than the decentralized interbank market when $\int_0^1 c_i di < \rho$. The model is similar to Section 3.1, except

[12] The case of $\int_0^1 c_i di > \rho$ is innocuous; a social planner cannot do better than $W_b^{(1)}$ as in Eq. (4), and, for ζ not too high, he will also choose the same z_a. We therefore focus on $\int_0^1 c_i di < \rho$. The assumption on loan liquidations is an outcome in the decentralized equilibrium, which we impose here to obtain a conservative benchmark for comparison. This benchmark may differ from the second-best level of welfare, which is discussed separately in Appendix B.

that loan certificates are introduced as an alternative to the interbank market. Specifically, the planner allocates a maximum amount of loan certificates $\widehat{k}_i$ to bank i at $t = 0$, which bank i can use at $t = 1$ to meet certain obligations with other banks. Loan certificates incur an interest rate r_k if used, and both principal and interest must be repaid by solvent banks at $t = 2$.[13] The planner sets the allocation of loan certificates $\{\widehat{k}_i\}_{i\in[0,1]}$ and the interest rate r_k, understanding the best responses of banks and their depositors.

As discussed in Section 2, the loan certificates issued by the NYCH during the Panic of 1873 were connected to the check-clearing process. They could only be used in the settlement of obligations that involved the transfer of cash from one bank to another, not obligations that involved the withdrawal of cash out of the banking system. To model this constraint on loan certificates, we introduce the notation ν_i for check-clearing obligations and decompose bank i's cash holdings at the beginning of $t = 1$ into three components:

$$c_i \equiv \widetilde{c}_i - \nu_i + \overline{\nu}$$

where $\widetilde{c}_i$ is cash reserves (specie and legal tender), ν_i is cash outflows associated with checks owed to other banks, and $\overline{\nu}$ is cash inflows associated with checks owing from other banks. To simplify the exposition, we assume that bank i owes the same amount of checks ν_i to each bank $i' \neq i$. The aggregate amount owed by bank i to all other banks is $\int_{i'\neq i} \nu_i di'$, which is also equal to ν_i. The total amount owed to bank i from all other banks is $\overline{\nu} \equiv \int \nu_{i'} di'$. Checks are cleared before any depositor withdraws at $t = 1$. In the model of Section 3.1, only cash could be used to settle checks, hence c_i was the amount of cash brought by bank i into $t = 1$. Now, loan certificates exist as an alternative to cash for check settlement. In particular, loan certificates can be used to pay ν_i and must be accepted in lieu of $\overline{\nu}$, helping banks to preserve cash reserves $\widetilde{c}_i$ for depositors who withdraw from the system at $t = 1$. The preservation occurs because the loan certificate defers the final settlement (in cash and at the interest rate r_k) to $t = 2$, after depositor withdrawal decisions have been made. For simplicity, when a bank uses a loan certificate, we assume the loan certificate is given in equal proportions to all other banks.

The amount of loan certificates used by bank i to pay ν_i before depositor withdrawals at the

[13] Loan certificates must be repaid by solvent banks, regardless of whether the bank that accepted them survives to $t = 2$. In the historical setting, we can think of payments to failed banks as going to the clearinghouse instead.

beginning of $t = 1$ is denoted by k_i, where

$$k_i \in \left[0, \min\left\{\nu_i, \widehat{k}_i + \overline{k}\right\}\right] \tag{7}$$

and $\overline{k} \equiv \int k_{i'} di'$ denotes the amount of loan certificates used by other banks to pay bank i the obligations $\overline{\nu}$. As loan certificates can be recirculated during the check-clearing process, receiving more loan certificates from others allows a bank to utilize loan certificates beyond its initial allocation $\widehat{k}_i$. The $\overline{k}$ thus represents an equilibrium of the loan certificate exchange process, whereby the loan certificates received by a bank can affect the amount of loan certificates it distributes to other banks.[14]

In addition to choosing the initial allocation of loan certificates and the interest rate r_k, the planner can impose restrictions on the recirculation of loan certificates, i.e., the upper bound on k_i in (7) can be made stricter for some banks. The following proposition shows that loan certificates can achieve a better outcome than the decentralized equilibrium in Proposition 1 when total cash in the system is low:

Proposition 2. *Consider $\widetilde{c}_i = \widetilde{c}$ for all $i \in [0,1]$ and hence $\int_0^1 c_i di = \widetilde{c}$. If $\overline{\nu} > \rho - \widetilde{c}$, then there exists an allocation of loan certificates $\left\{\widehat{k}_i^*\right\}_{i\in[0,1]}$ and an interest rate r_k^* that achieves higher welfare than the decentralized equilibrium when $\rho > \widetilde{c}$. This allocation involves:*

$$\widehat{k}_i^* = \begin{cases} 0 & if \quad c_i < \overline{c}\left(r_k^*\right) \\ \nu_i & if \quad c_i \geq \overline{c}\left(r_k^*\right) \end{cases} \tag{8}$$

and r_k^* solving:

$$\int_{\left\{i|c_i \geq \overline{c}\left(r_k^*\right)\right\}} \left(\rho - c_i\right) di = \int_{\left\{i|c_i < \overline{c}\left(r_k^*\right)\right\}} \min\{\overline{k}, c_i\} di > 0 \tag{9}$$

Any bank i with $c_i < \overline{c}\left(r_k^*\right)$ experiences a run. However, $r_k^* < r_b^*$ so there are fewer runs than in Proposition 1. The welfare-improving allocation implements $\Delta_i = 0$ for all i and imposes that any bank with $c_i < \overline{c}\left(r_k^*\right)$ uses cash before recirculated loan certificates during check-clearing.

[14] Our concept of a payment equilibrium is analogous to the payment equilibrium concept presented in the system of payments literature, for instance in Eisenberg and Noe (2001). This literature shows that there exists a unique payment equilibrium and that this payment equilibrium can be derived as the limit of a repeated series of payment transaction steps among the banks.

The condition $\widetilde{c}_i = \widetilde{c}$ at the beginning of Proposition 2 captures an environment where most of the variation in initial cash holdings across banks comes from variation in check-clearing obligations. Then, as long as there is enough aggregate check-clearing activity, i.e., $\overline{\nu} > \rho - \widetilde{c}$, Proposition 2 says that the planner can reduce bank failures and improve social welfare by allocating to banks above the cash threshold $\overline{c}\left(r_k^*\right)$ enough loan certificates to cover their checks owed to other banks. Banks below this threshold receive no loan certificates from the clearinghouse and must use cash reserves before loan certificates received from other banks to pay checks owed. The result is a reallocation of cash reserves away from failing banks (and their depositors) towards the rest of the system. In turn, the interest rate on loan certificates can be set below the borrowing rate that prevails in the decentralized equilibrium, allowing more banks to fend off runs. The measure of failed banks falls and total welfare rises.

Next, we compare the social welfare generated by the loan certificates in Proposition 2 to the welfare from reserve pooling in Eq. (6):

Lemma 1. *Consider $\overline{\nu} > \rho - \widetilde{c} > 0$ as in Proposition 2.*

- *Case 1 (Unconstrained): If $\overline{c}\left(r_k^*\right) \leq \overline{\nu} + \widetilde{c} - \rho$, then loan certificates will transfer all the cash of failing banks to run-proof banks, achieving the same level of welfare as reserve pooling.*

- *Case 2 (Constrained): If $\overline{c}\left(r_k^*\right) > \overline{\nu} + \widetilde{c} - \rho$, then some cash will remain with failing banks under the loan certificate arrangement in Proposition 2, making the welfare improvement over the decentralized equilibrium smaller than the welfare improvement achieved by reserve pooling.*

The solution r_k^* to Eq. (9) is increasing in ρ, so the unconstrained case applies for ρ not too large above $\widetilde{c}$. Alternatively, fix any value of $\rho > \widetilde{c}$ and consider an increase in check-clearing obligations from ν_i to $\nu_i + \varepsilon$ at all banks i, where $\varepsilon > 0$. The unconstrained case will apply at higher ε.

The unconstrained case in Lemma 1 introduces a condition $\overline{\nu} \geq \rho - \widetilde{c} + \overline{c}\left(r_k^*\right)$ which is stricter than the condition $\overline{\nu} > \rho - \widetilde{c}$ in Proposition 2. The condition in Proposition 2 determines whether loan certificates can provide higher social welfare than the interbank market, while the condition in Lemma 1 determines whether the loan certificates can achieve the same welfare as reserve pooling. Overall, Lemma 1 highlights that loan certificates are as capable as reserve pooling when all banks are highly involved in check-clearing activity. The intuition follows from the fact that loan certificates are connected to the check-clearing process. With more checks owed, failing banks have to transfer more cash towards run-proof banks without receiving more cash in return for checks owing. This achieves the same outcome as reserve pooling when the volume of check-clearing activity is high enough that failing banks have to transfer all of their cash.

4 Quantitative Analysis

We now use historical data to parameterize the model developed in Section 3. We then obtain some plausible estimates of the welfare gains from loan certificates during the Panic of 1873 relative to a hypothetical interbank market with decentralized trade.

4.1 Parameterization

We consider a range of 0.068 to $c_{\max}$ for the cash levels of the NYCH member banks, where the 1873 level of $c_{\max}$ is 0.311. Our model normalizes deposits at each bank to 1 so the range of cash levels captures that the least liquid banks in the NYCH held 6.8% of their deposits as cash reserves while the most liquid banks held 31.1%. These ratios trim the sample at the 7th and 93rd percentiles to exclude outliers. We use a uniform distribution for simplicity, with density $f(c_i) = \frac{1}{c_{\max}-0.068}$ at each cash level $c_i \in [0.068, c_{\max}]$. The average cash ratio (which is also the aggregate cash level) implied by the uniform distribution at $c_{\max} = 0.311$ is $\widetilde{c} = 0.189$. For the decomposition of cash holdings, $c_i \equiv \widetilde{c} - \nu_i + \overline{\nu}$, we assume $c_{\max} = \widetilde{c} + \overline{\nu}$, i.e., the most liquid banks owe no checks. Then, $\nu_i = c_{\max} - c_i$ and $\overline{\nu} = \frac{c_{\max}-0.068}{2}$ with $\widetilde{c} = \frac{0.068+c_{\max}}{2}$. Notice that a higher (lower) value of $c_{\max}$ would capture a system with more (less) total cash. We will vary $c_{\max}$ from its 1873 level in counterfactual computations and quantify the effect on welfare.

During the period we study, depositors were paid an average of 2% on their deposits, so we set $r = 0.02$ for the deposit interest rate. The average call loan rate in the fall of 1873 was 10%, with call loans accounting for 25% of bank lending. We assume that the interest rate on other loans was roughly equal to the deposit rate after chargeoffs are taken into account. The average interest rate on bank loans is then $r_z = 0.04$. We set the output function for recipients of bank loans to $f(z) = (1 + r_z) z$. The loan liquidation value is set to $x = 0.75$, which is to say 75% of the face value of bank loans could be recovered on demand. This is consistent with full recovery on call loans (by definition of being callable) and a recovery rate of two-thirds for other types of lending. We consider $\zeta \to \infty$ for the loan adjustment cost to capture the difficulty of finding additional profitable lending opportunities during a financial crisis. We then set loans outstanding to $\tilde{z} = 1 - 0.068 = 0.932$, which ensures $c_i + \tilde{z} \geq 1$ for all i and reflects the fact that the total amount of investments was roughly equal to the total amount of deposits among the banks.

A key parameter for our analysis is the fraction of early deposit withdrawals ρ. The NYCH set an interest rate of 7% on outstanding loan certificates, so we calibrate ρ to get $r_k^* = 0.07$ from Proposition 2. This gives $\rho = 0.212$. While there is no direct data on ρ, the calibrated value is reasonable given the composition of bank deposits. Around 64% of total deposits in the NYCH members in 1873 were individual checking deposits. Assuming a withdrawal rate of 0.2 for individual depositors based on the typical withdrawals in 1872, the calibrated ρ implies a withdrawal rate of 0.23 by institutional depositors, i.e., country banks, which is consistent with greater withdrawal pressures from banks in the interior of the country during the crisis.

4.2 Welfare Gains from Loan Certificates

Figure 1(a) compares the welfare with loan certificates to the welfare under reserve pooling and the welfare that could have been achieved with a decentralized interbank market. We also plot the "first best" level of welfare, measured as $\widetilde{c} + f(\tilde{z})$.

In the baseline parameterization of Section 4.1 with $c_{\max} = 0.311$, loan certificates are able to achieve the same welfare as reserve pooling, i.e., the unconstrained case in Lemma 1 applies. This suggests that it was redundant for the NYCH to explicitly adopt reserve pooling in 1873, providing a new perspective on the subsequent abandonment of the reserve pooling arrangement. Social welfare with loan certificates is 2% higher than the welfare with a decentralized interbank market in the baseline parameterization, reflecting that the market clearing interbank rate, $r_b^* = 0.094$, would have exceeded the 7% interest rate on loan certificates. A welfare improvement of 2% is notable since it fills almost half of the gap between the decentralized market and the first best.

Notice that the average cash ratio $\widetilde{c}$ is below the calibrated ρ when $c_{\max} = 0.311$. Thus, the banking system during the Panic of 1873 was in the range of parameters where loan certificates alone would not have been able to completely prevent bank failures (recall Proposition 2, where banks with $c_i < \bar{c}(r_k^*)$ experience a run when $\widetilde{c} < \rho$).[15] When total cash in the system is below ρ, there is simply no redistribution of cash across banks that allows them all to become run-proof. An important difference between the analysis in Proposition 2 and the historical episode is that none of the members of the NYCH actually failed during the Panic of 1873. This suggests that additional policies pursued by the NYCH helped drive failures down to zero, despite possibly introducing costs of their own. We consider these policies in Section 5. Among surviving banks, however, the welfare-improving allocation in Proposition 2 issues more loan certificates to banks that owe more checks. This is consistent with Table 2, where we found that banks with higher amounts due to the NYCH received more loan certificates. Our model thus shows that there is an allocation of loan certificates that improves welfare over a decentralized market and that, on the intensive margin, this allocation is in line with the allocation implemented by the NYCH during the Panic of 1873.

The horizontal axis in Figure 1(a) varies $c_{\max}$ to analyze the welfare effect of loan certificates under different liquidity scenarios for the banking system. We keep all other parameters as in Section 4.1, including $\rho = 0.212$, and use Proposition 2 to solve for r_k at each value of $c_{\max}$. The welfare gains from loan certificates relative to the interbank market are largest at low $c_{\max}$ (and hence low $\widetilde{c}$) even though at these values the constrained case in Lemma 1 applies and loan certificates do not achieve the same welfare as reserve pooling. As $c_{\max}$ increases to bring $\widetilde{c}$ equal to or above ρ, all mechanisms for redistributing cash (interbank market, loan certificates, reserve pooling) are able to achieve the first best level of welfare.

5 Other Interventions by the NYCH

We now consider the additional policies introduced by the NYCH during the Panic of 1873 – information suppression and partial suspension – and assess their potential welfare effects.

5.1 Information Suppression

Starting on September 20th, the NYCH stopped publishing weekly balance sheet information about individual banks, reporting instead the aggregate across its members. The NYCH made this deci-

[15] As a robustness check, we shifted the cash distribution rightward, from c_i to $c_i + 0.05$ for each i, to consider a somewhat broader definition of liquid assets than cash reserves. The gap between the new $\widetilde{c}$ and the recalibrated ρ widens, i.e., the latter increases more than one-for-one with the former to justify the same solution $r_k^* = 0.07$, bolstering the welfare gain from loan certificates relative to a decentralized interbank market.

sion on behalf of all member banks. Had the decision to suppress bank-level information been left up to each bank individually, the lack of a coordination mechanism could have caused information suppression to unravel.

To characterize the effect of information suppression on welfare independently of loan certificates, consider an environment where (i) banks trade in a decentralized interbank market as in Section 3.1 and (ii) depositors know the distribution from which the cash endowments c_i are drawn but not the value of c_i for any bank i. The following proposition shows that information suppression alone, even if coordinated by an entity like the NYCH, cannot reduce bank failures when total cash in the system is low:

Proposition 3. *Consider a decentralized interbank market with $\widetilde{c} < \rho$.*

1. *For a fixed interbank rate, there are (weakly) fewer bank failures if information is suppressed than if it is not.*
2. *If there exists an equilibrium with information suppression where patient depositors do not run (i.e., $a_{ij} = 2$ for all $ij \in [0,1] \times [\rho, 1]$), then the measure of banks that fails is exactly the same as in Proposition 1 but social welfare may be higher.*

The first part of Proposition 3 compares bank failures with and without information suppression for the same interbank rate. With information suppression, depositors cannot distinguish strong (i.e., run-proof) banks from weak ones. Instead, all banks appear the same to patient depositors, hence patient depositors at all banks make the same decision. Ex post, patient depositors would only want to have run on weak banks, so, if there are enough strong banks in the system, depositors may prefer not to run ex ante. This sort of cross-subsidization of weak banks by strong ones is what would permit information suppression to achieve fewer bank failures for a fixed interbank rate.

Importantly though, the interbank rate is not fixed. The second part of Proposition 3 establishes that the interbank interest rate will adjust in equilibrium to deliver exactly the same measure of bank failures as in Proposition 1. Intuitively, there would be more banks with low cash participating in the interbank market if information suppression decreased the threshold level of cash below which banks fail. This would imply an increase in loan demand on the interbank market, pushing up the interbank rate and increasing the threshold. Unlike loan certificates, information suppression offers no mechanism by which to extract cash from failing banks for use by the rest of the system. As a result, information suppression does not allow fewer banks to fail when the interbank rate is endogenously determined in a decentralized market.

Even though information suppression does not reduce the measure of bank failures, the second part of Proposition 3 reveals that welfare may still increase relative to the decentralized market without information suppression. The welfare gains come from fewer loans being liquidated at $t = 1$. The key to this result is that information suppression defers some bank failures until $t = 2$ if there are enough strong banks in the system to convince patient depositors to wait until $t = 2$ to withdraw. Specifically, banks with $c_i \in (\rho - x\widetilde{z}, \overline{c}\,(r_b^*))$ now only need to liquidate a fraction of their loans at $t = 1$ to repay impatient depositors. The unliquidated fraction then earns the interest rate r_z, which can be used to partly repay the patient depositors at $t = 2$. These depositors are not fully repaid (i.e., they do not get $1 + r$ at $t = 2$), but they do get more than if they ran at $t = 1$ and forced the bank to liquidate all of its loans $\widetilde{z}$.

Figure 1(b) plots welfare under information suppression for different values of $c_{\max}$. There is a value of $c_{\max}$, call it $c^*_{\max}$, at which welfare jumps discontinuously. For any $c_{\max} < c^*_{\max}$, there does not exist an equilibrium with information suppression where patient depositors do not run. The cash distribution is not strong enough to convince patient depositors that the average bank in the system can withstand a run, hence all depositors withdraw from all banks at $t = 1$. This leads to much lower welfare than without information suppression where banks towards the top of the same cash distribution did not experience runs.

For any $c_{\max} > c^*_{\max}$, there exists an equilibrium with information suppression where patient depositors do not run; Figure 1(b) plots the welfare in that equilibrium. As $c_{\max}$ is increased above $c^*_{\max}$, a decentralized interbank market achieves higher welfare with information suppression than without. Suppressing information also achieves higher welfare than replacing the decentralized market with loan certificates. The welfare gains reflect fewer liquidations at $t = 1$, as discussed above in relation to Proposition 3. As $c_{\max}$ is increased further, the system moves to $\widetilde{c} \geq \rho$ and the first best level of welfare is attained in all the models we consider.

The important lesson from Figure 1(b) is that the 1873 level of $c_{\max}$ was well below $c^*_{\max}$. Thus, information suppression alone would have been disastrous for the banking system. The system-wide run induced by information suppression at low $c_{\max}$ renders any mechanism for redistributing cash across banks moot. No bank can lend without liquidating the principal of the interbank loan from $\widetilde{z}$ and no amount of interbank lending changes the run probability for the recipient bank; the recipient's liquidations just decrease by the amount that the lender liquidates to fund the interbank loan. While the mechanism through which cash is redistributed (decentralized market versus loan certificates) can affect the value of $c^*_{\max}$, the 1873 level of $c_{\max}$ is low enough that it does not matter which mechanism we consider alongside information suppression to conclude that another policy was responsible for the lack of bank failures during the Panic of 1873. The final policy of the NYCH, partial suspension of convertibility, is discussed next.

5.2 Suspension of Convertibility

On September 22nd, the NYCH partially suspended the convertibility of deposits, preventing individual depositors in New York City from withdrawing while still allowing withdrawals from country banks. We inferred a withdrawal rate of 0.23 for country banks at the end of Section 4.1, with country banks accounting for around 36% of total deposits in the NYCH members. All else constant then, the suspension policy of the NYCH would have decreased the fraction of early withdrawals to $\rho = 0.36 \times 0.23 = 0.083$, putting the banking system in a situation where the average cash ratio $\widetilde{c} = 0.189$ was high enough to achieve a conservative equilibrium without any bank failures as long as there existed a mechanism to redistribute liquidity across banks. Loan certificates provided this mechanism in the absence of a formal, decentralized interbank market. No banks fail under partial suspension, and banks with less cash receive more loan certificates, in line with the summary statistics in Section 2.

The NYCH's suspension policy differs from the one that achieves the first best in Diamond and Dybvig (1983) when the fraction of impatient depositors is known. In particular, the NYCH was restricting withdrawals below the true fraction of impatient depositors, imposing a cost on individual depositors who really needed access to their funds at $t = 1$. Denote by κ the welfare loss incurred by impatient depositors per dollar of deposits that they cannot withdraw at $t = 1$. The NYCH's suspension policy would have improved aggregate welfare relative to loan certificates alone if and only if $\kappa < 0.225$. That is, the average individual depositor in New York City who needed to withdraw deposits early but could not would have had to incur a welfare loss of no more than 22.5% of the value of those deposits.

The currency premium in New York City reached 5% by September 29th, meaning that some depositors were getting cash from brokers at this much of a discount against their deposits. At face value, this motivates $\kappa = 0.05$, in which case the combination of partial suspension and loan certificates brings welfare just shy of its first best level and constitutes a 2% improvement relative to just loan certificates. However, the currency premium is a lower bound for κ in our model. It would suffice for 20% of individual depositors to not have had access to cash brokers to bring the weighted average κ above 0.225. Sprague (1910, p. 57) also notes that "there were wide differences both in supply and demand from hour to hour, and especially high rates were regularly paid for currency in quantity." It is therefore possible that partial suspension of convertibility reduced aggregate welfare despite eliminating bank failures.

Had there existed a formal, decentralized interbank market in New York City in 1873 and loan certificates not been implemented, the suspension policy would have improved aggregate welfare if and only if $\kappa < 0.393$. Since a decentralized equilibrium would have involved more bank failures than loan certificates in the absence of suspension, the welfare gain from suspension is higher when the mechanism for redistributing liquidity is a decentralized interbank market. The welfare cost can then also be higher before the suspension policy becomes overall welfare-reducing.

6 Conclusion

In the aftermath of the 2007-09 financial crisis, lawmakers began curtailing the ability of central banks to create new, stigma-free lending facilities that inject emergency liquidity, especially when it comes to shadow banks. At the same time, regulations mandating the central clearing of credit derivatives have led to a revival of clearinghouses in modern financial systems.

This paper has studied how financial stability can be achieved in the absence of monetary injections by a central bank. We showed theoretically that a forced reallocation of liquidity across banks can achieve fewer bank failures than a decentralized market for interbank loans due to a pecuniary externality in the decentralized equilibrium. We also showed that a forced reallocation of liquidity can be implemented through the issuance of clearinghouse loan certificates, such as those issued by the New York Clearinghouse during the Panic of 1873.

We created an extensive, hand-collected dataset from a variety of archival sources to study this novel feature of the NYCH's response. We demonstrated that the NYCH issued loan certificates to member banks in the way our model suggests it should have, helping to resolve the panic. Our findings bring to light a role for clearinghouse loan certificates in the management of future liquidity crises. The optimal design of these certificates, especially in environments with multiple clearinghouses, is a promising avenue for future work.

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Table 1:
Summary Statistics, NYCH Members

	Non-Recipient Banks	Recipient Banks	All Banks
Cash / Total Assets	11.18	8.52	9.70
	(4.88)	(3.29)	(4.25)
Cash / Total Deposits	20.08	12.43	15.80
	(8.87)	(4.78)	(7.82)
Call Loans / Total Loans	26.07	23.69	24.74
	(23.19)	(18.90)	(20.74)
Investment / Total Assets	66.68	62.13	64.13
	(5.50)	(11.40)	(9.48)
Equity / Total Liabilities	34.53	25.10	29.26
	(7.07)	(8.58)	(9.19)
Due From Banks / Total Assets	4.46	5.27	4.91
	(2.54)	(3.52)	(3.13)
Due To Banks / Total Liabilities	5.67	19.23	13.25
	(6.12)	(19.77)	(16.67)
Loan Certificate Volume / Total Deposits	0	13.09	7.32
	(0)	(7.67)	(8.68)
Total Assets	$6,164,669	$8,870,033	$7,677,838
	(5,824,123)	(8,345,285)	(7,408,112)
Obs.	26	33	59

Notes: Recipient refers to whether or not the bank received clearinghouse loan certificates during the panic. Data are averages over the indicated NYCH members just before the panic. All ratios are expressed as percentages. Cash refers to cash reserves (i.e., specie and legal tender). Call loans are loans to stock-brokers. Investment is the sum of loans, bonds, and stocks. "Due froms" are interbank deposits due from other banks. "Due tos" are interbank deposits due to other banks. Loan certificate volume is the aggregate value of loan certificates taken out during the Panic of 1873. Total assets is the average value of total assets. Total deposits are the sum of retail deposits and institutional deposits (due-tos). Standard deviations are in parentheses. Sources: Office of the Comptroller of Currency, Annual Report of the Superintendent of the Banking Department of the State of New York, and authors' calculations.

Table 2:
Check Clearing and Loan Certificates, Correlations

	Due to CH	Due from CH
Issuance	0.799***	-0.138
	(0.00)	(0.235)
Obs.	76	76

Notes: This table computes Pearson correlations between the volume of clearinghouse loan certificates and the volume of debit and credit payments with the clearinghouse from September 22nd to September 30th. Among 61 banks in the clearinghouse, 33 banks received loan certificates. Loan certificates were issued 76 times because some banks received loan certificates multiple times in this window. p-values are in parentheses. ***p<0.01, **p<0.05, *p<0.1. Sources: NYCH minutes and authors' calculations.

Table 3:
Deposit Growth During the Panic, Recipient and Non-Recipient Banks

	9/20-10/21	10/21-12/6
Non-Recipient Banks	-13.92	22.34
	(16.12)	(34.48)
Recipient Banks	-32.76	13.24
	(29.07)	(25.69)

Notes: This table provides information on deposit growth for banks that received clearinghouse loan certificates and banks that did not during the periods 9/20/1873 to 10/21/1873 and 10/21/1873 to 12/6/1873. The deposit information for 10/21/1873 was not made public. Standard deviations are in parentheses. Sources: Deposit information for Sept 20th and Dec 6th comes from the Commercial and Financial Chronicles; Deposit information for Oct 21st comes from the NYCH Loan Certificate Committee minutes.

Table 4:
Average Interest Payments, Recipient and Non-Recipient Banks

Non-Recipient Banks		Recipient Banks	
Paid to the CH	Received from the CH	Paid to the CH	Received from the CH
0	$5,240.17	$8,877.30	$4,748.47
(0)	(5,754.94)	(11,448.72)	(3,301.75)

Notes: Average value of interest payments made to and received from the clearinghouse. Data presented separately for banks that received clearinghouse loan certificates and banks that did not receive clearinghouse loan certificates. Standard deviations are in parentheses. Sources: NYCH minutes and authors' calculations.

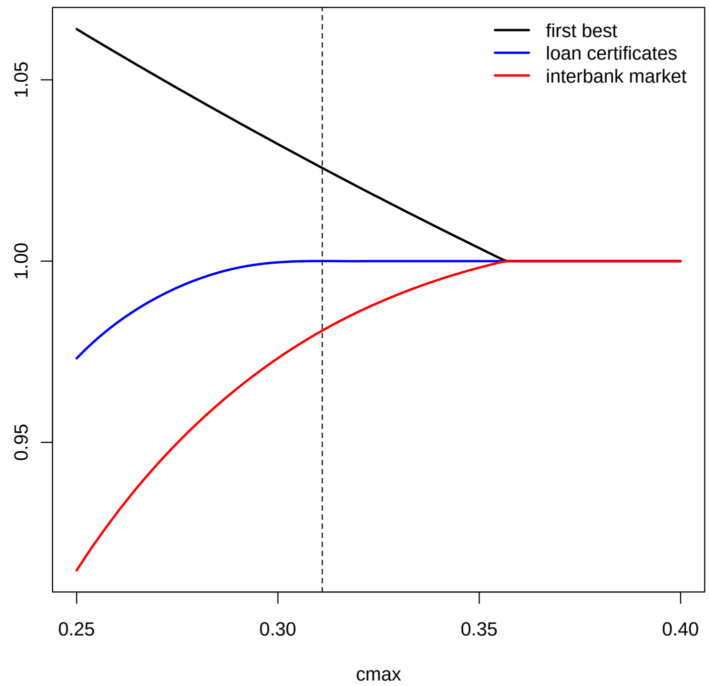
Table 5:
Ratio of Cash Reserves to Deposits, Recipient and Non-Recipient Banks

	9/20/1873	12/6/1873
Non-Recipient Banks	0.332	0.339
	(0.137)	(0.192)
Recipient Banks	0.263	0.278
	(0.083)	(0.107)

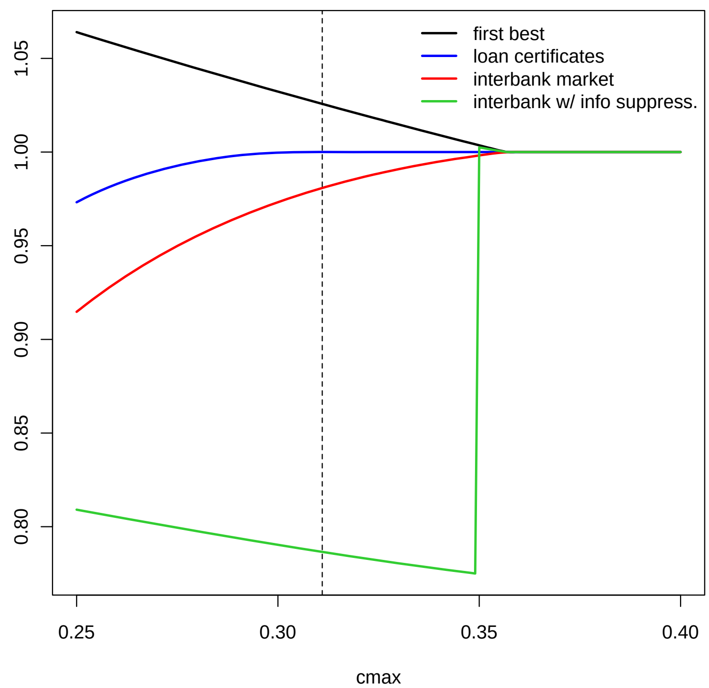
Notes: Ratio of cash reserves (specie and legal tender) to deposits, reported separately for banks that received clearinghouse loan certificates and banks that did not receive clearinghouse loan certificates. Standard deviations are in parentheses. Sources: Commercial and Financial Chronicles, authors' calculations.

Figure 1:
Welfare Relative to Reserve Pooling

(a) Loan Certificates



(b) Information Suppression



Notes: The dashed vertical line indicates the 1873 level of c_{max}.

Appendix A – Proofs

Proof of Proposition 1

Let ω_i denote the fraction of depositors that withdraw from bank i at $t = 1$. Fixing ω_i, the optimization problem of bank i is given by:

$$V_i(\omega_i) \equiv \max_{\ell_i, z_i, \Delta_i} \left\{ \begin{array}{l} c_i + x\ell_i - z_i + \Delta_i - \omega_i - \frac{1}{2}\zeta z_i^2 \\ +(1+r_z)(\tilde{z} - \ell_i + z_i) - (1+r_b)\Delta_i - (1+r)(1-\omega_i) \end{array} \right\}$$

$$s.t.$$

$$c_i + x\ell_i - z_i + \Delta_i \geq \omega_i$$

The Lagrangian (with multiplier $\lambda_i \geq 0$ on the constraint) is:

$$\begin{aligned} \mathcal{L} \quad = \quad & c_i + x\ell_i - z_i + \Delta_i - \omega_i - \frac{1}{2}\zeta z_i^2 \\ & +(1+r_z)(\tilde{z} - \ell_i + z_i) - (1+r_b)\Delta_i - (1+r)(1-\omega_i) \\ & +\lambda_i [c_i + x\ell_i - z_i + \Delta_i - \omega_i] \end{aligned}$$

F.O.C. wrt ℓ_i:

$$\frac{\partial \mathcal{L}}{\partial \ell_i} = x(1+\lambda_i) - (1+r_z)$$

F.O.C. wrt z_i:

$$\frac{\partial \mathcal{L}}{\partial z_i} = r_z - \lambda_i - \zeta z_i$$

F.O.C. wrt Δ_i:

$$\frac{\partial \mathcal{L}}{\partial \Delta_i} = \lambda_i - r_b$$

Consider the various cases for ℓ_i:

1. If $\ell_i \in (0, \tilde{z})$, then $\lambda_i = \frac{1+r_z}{x} - 1 > 0$, where the inequality follows from the assumptions of $r_z > 0$ and $x \in (0,1)$ in the main text. Subbing λ_i into the F.O.C. for z_i:

$$\frac{\partial \mathcal{L}}{\partial z_i} = -(1+r_z)\left(\frac{1}{x} - 1\right) - \zeta z_i < 0$$

Therefore $z_i = 0$. Now sub λ_i into the F.O.C. for Δ_i:

$$\frac{\partial \mathcal{L}}{\partial \Delta_i} = \frac{1+r_z}{x} - (1+r_b)$$

- If $1 + r_b > \frac{1+r_z}{x}$, then the bank sets Δ_i as low as possible (i.e., lend as much as possible on the interbank market). With $\lambda_i > 0$, complementary slackness requires

$$\Delta_i = \omega_i - c_i - x\ell_i \tag{A.1}$$

  so Δ_i is minimized by setting $\ell_i = \tilde{z}$, which contradicts $\ell_i \in (0, \tilde{z})$.

- If $1 + r_b < \frac{1+r_z}{x}$, then the bank sets Δ_i as high as possible (i.e., borrow as much as possible on the interbank market). From Eq. (A.1), Δ_i is maximized by setting $\ell_i = 0$, which contradicts $\ell_i \in (0, \tilde{z})$.

- If $1 + r_b = \frac{1+r_z}{x}$, then Δ_i is indeterminate. Rearranging Eq. (A.1):

$$\ell_i = \frac{\omega_i - c_i - \Delta_i}{x}$$

  We thus need

$$\Delta_i \in (\omega_i - c_i - x\tilde{z}, \omega_i - c_i)$$

  for $\ell_i \in (0, \tilde{z})$ to be satisfied.

2. If $\ell_i = 0$, then $\lambda_i < \frac{1+r_z}{x} - 1$.

- If $\lambda_i > r_z$, then $z_i = 0$ and $\lambda_i > 0$ so

$$\Delta_i = \omega_i - c_i$$

  The above satisfies the F.O.C. for Δ_i if (and only if) $\lambda_i = r_b$, so this case requires $r_b \in \left(r_z, \frac{1+r_z}{x} - 1\right)$.

- If $\lambda_i < r_z$, then consider an interior solution for z_i:

$$z_i = \frac{r_z - \lambda_i}{\zeta}$$

Note that $\lambda_i < r_z$ is stricter than $\lambda_i < \frac{1+r_z}{x} - 1$. There are two subcases:

– If $\lambda_i > r_b$, then the bank sets Δ_i as high as possible. Moreover, the constraint in the optimization problem holds with equality:

$$\lambda_i = r_z - \zeta\left(\Delta_i + c_i - \omega_i\right)$$

The highest possible Δ_i is the one that delivers $\lambda_i = r_b$, so

$$\Delta_i = \omega_i - c_i + \frac{r_z - r_b}{\zeta}$$

$$z_i = \frac{r_z - r_b}{\zeta}$$

This case requires $r_b < r_z$.

– If $\lambda_i < r_b$, then the bank sets Δ_i as low as possible, where

$$\Delta_i \geq \omega_i - c_i + \frac{r_z - \lambda_i}{\zeta}$$

Note that this case requires $\lambda_i < \min\left\{r_b, r_z\right\}$. The lowest possible Δ_i is the one that makes the above hold with equality at $\lambda_i = \min\left\{r_b, r_z\right\}$, namely

$$\Delta_i = \omega_i - c_i + \frac{r_z - \min\left\{r_b, r_z\right\}}{\zeta}$$

If $r_b < r_z$, then $\lambda_i = r_b$ and we get the results of the previous bullet. If $r_b > r_z$, then $\lambda_i = r_z$ and we get $\Delta_i = \omega_i - c_i$ with $z_i = 0$.

3. If $\ell_i = \tilde{z}$, then $\lambda_i > \frac{1+r_z}{x} - 1$. Note that this implies $\lambda_i > r_z$ so $z_i = 0$. Moreover, with $\lambda_i > 0$, the constraint in the optimization problem holds with equality. We can thus write

$$\Delta_i = \omega_i - c_i - x\tilde{z}$$

The above satisfies the F.O.C. for Δ_i if (and only if) $\lambda_i = r_b$, so this case requires $1+r_b > \frac{1+r_z}{x}$.

Putting things together:

1. If $r_b < r_z$, then

$$\ell_i = 0$$

$$z_i = \frac{r_z - r_b}{\zeta}$$

$$\Delta_i = \omega_i - c_i + \frac{r_z - r_b}{\zeta}$$

$$V_i(\omega_i) = (r_z - r_b)\,\tilde{z} + (1 + r_b)(c_i + \tilde{z} - 1) + \frac{(r_z - r_b)^2}{2\zeta} + (r_b - r)(1 - \omega_i)$$

If $r_b \geq r$, then V_i^* is (weakly) decreasing in ω_i. In the worst-case scenario of $\omega_i = 1$,

$$V_i(1) = (r_z - r_b)\,\tilde{z} + (1 + r_b)(c_i + \tilde{z} - 1) + \frac{(r_z - r_b)^2}{2\zeta} > 0$$

so the bank is run-proof and thus $\omega_i = \rho$, i.e., only depositors that really need to withdraw money out of the banking system at $t = 1$ do. If instead $r_b < r$, then V_i^* is increasing in ω_i, i.e., it is less costly for the bank to cover a run by borrowing on the interbank market and repaying the interest rate r_b than it is to pay patient depositors the interest rate r if they do not run. We therefore need $V_i(\rho) \geq 0$, or equivalently,

$$(1 + r_b)\,c_i + (1 + r_z)\,\tilde{z} - (1 + r) + \frac{(r_z - r_b)^2}{2\zeta} + (r - r_b)\,\rho \overset{?}{\geq} 0$$

a sufficient condition for which is $c_i + (1 + r_z)\,\tilde{z} \geq 1 + r$ for all i. Under this condition, $\omega_i = \rho$ regardless of the sign of $r_b - r$. This is true for all banks so the interbank market clearing condition pins down

$$r_b = \max\left\{ r_z - \zeta\left(\int_0^1 c_i di - \rho\right), 0\right\}$$

The case $r_b < r_z$ is thus valid if and only if $\int_0^1 c_i di > \rho$ (i.e., aggregate cash holdings are sufficient to cover all depositors that really need to withdraw at $t = 1$).

2. If $r_b \in \left(r_z, \frac{1+r_z}{x} - 1\right)$, then

$$\ell_i = 0$$

$$z_i = 0$$

$$\Delta_i = \omega_i - c_i$$

$$V_i(\omega_i) = (r_z - r_b)\,\tilde{z} + (1 + r_b)(c_i + \tilde{z} - 1) + (r_b - r)(1 - \omega_i)$$

In the worst-case scenario of $\omega_i = 1$,

$$V_i(1) = (r_z - r_b)\,\tilde{z} + (1 + r_b)(c_i + \tilde{z} - 1)$$

The bank is run-proof if and only if $c_i \geq \bar{c}(r_b)$, where $\bar{c}(\cdot)$ is as defined in Eq. (2). If $c_i \geq \bar{c}(r_b)$, then $\omega_i = \rho$ with ℓ_i, z_i, and Δ_i as above. If $c_i < \bar{c}(r_b)$, then the bank experiences a run at $t = 1$ (i.e., $\omega_i = 1$) and fails with $\ell_i = 1$, $z_i = 0$, and $\Delta_i = 0$. The interbank market clearing condition is given by Eq. (3). The equilibrium value of r_b is pinned down by Eq. (3), with $\bar{c}(\cdot)$ is as defined in Eq. (2). More on this below.

3. If $r_b > \frac{1+r_z}{x} - 1$, then

$$\ell_i = \tilde{z}$$

$$z_i = 0$$

$$\Delta_i = \omega_i - c_i - x\tilde{z}$$

$$V_i(\omega_i) = (1 + r_b)(c_i + x\tilde{z} - 1) + (r_b - r)(1 - \omega_i)$$

In the worst-case scenario of $\omega_i = 1$,

$$V_i(\omega_i) = (1 + r_b)(c_i + x\tilde{z} - 1)$$

The bank is run-proof if and only if $c_i + x\tilde{z} \geq 1$, in which case all run-proof banks have $\Delta_i = \rho - (c_i + x\tilde{z}) < 0$. In words, all run-proof banks are net lenders on the interbank market, which violates the market clearing condition. Therefore, we can rule out $r_b > \frac{1+r_z}{x} - 1$.

To summarize, there exists a conservative equilibrium with no runs if $\int_0^1 c_i di > \rho$. If instead $\int_0^1 c_i di < \rho$, then a conservative equilibrium, if it exists, involves runs on some banks. To determine existence in this case, return to Eq. (3), with $\bar{c}(\cdot)$ is as defined in Eq. (2). This pins down the equilibrium interbank rate r_b^* as the solution to

$$\int_{\left\{i|c_i\geq 1-\frac{(1+r_z)\tilde{z}}{1+r_b^*}\right\}} (\rho - c_i)\, di = 0 \tag{A.2}$$

Parameter conditions such that the solution to Eq. (A.2) satisfies $r_b^* \in \left(r_z, \frac{1+r_z}{x} - 1\right)$ are necessary and sufficient for the existence of a conservative equilibrium when $\int_0^1 c_i di < \rho$. By way of example, consider c_i uniformly distributed over the unit interval. Then Eq. (A.2) implies

$$r_b^* = \frac{(1+r_z)\,\tilde{z}}{2\,(1-\rho)} - 1 \tag{A.3}$$

so existence requires $\frac{\tilde{z}}{2(1-\rho)} \in \left(1, \frac{1}{x}\right)$, where $\rho > \frac{1}{2} = \int_0^1 c_i di$. ■

Proof of Proposition 2

The optimization problem of bank i in the model with loan certificates is:

$$V_i(\omega_i) \equiv \max_{\ell_i, z_i, \Delta_i, k_i} \left\{ \begin{array}{l} \widetilde{c} - (\nu_i - k_i) + \left(\overline{\nu} - \overline{k}\right) + x\ell_i - z_i + \Delta_i - \omega_i - \frac{1}{2}\zeta z_i^2 \\ + (1+r_z)(\tilde{z} - \ell_i + z_i) - (1+r_b)\,\Delta_i - (1+r)(1-\omega_i) - (1+r_k)\left(k_i - \overline{k}\right) \end{array} \right\}$$

$$s.t.$$

$$\widetilde{c} - (\nu_i - k_i) + \left(\overline{\nu} - \overline{k}\right) + x\ell_i - z_i + \Delta_i \geq \omega_i$$

$$k_i \in [0, k_i^{\max}]$$

where $k_i^{\max} \leq \min\left\{\nu_i, \widehat{k}_i + \overline{k}\right\}$. The Lagrangian (with multipliers $\lambda_i \geq 0$, $\mu_i^0 \geq 0$, and $\mu_i^1 \geq 0$ on the constraints) is:

$$\begin{aligned} \mathcal{L} = {} & \widetilde{c} - (\nu_i - k_i) + \left(\overline{\nu} - \overline{k}\right) + x\ell_i - z_i + \Delta_i - \omega_i - \frac{1}{2}\zeta z_i^2 \\ & + (1+r_z)(\tilde{z} - \ell_i + z_i) - (1+r_b)\,\Delta_i - (1+r)(1-\omega_i) - (1+r_k)\left(k_i - \overline{k}\right) \\ & + \lambda_i\left[\widetilde{c} - (\nu_i - k_i) + \left(\overline{\nu} - \overline{k}\right) + x\ell_i - z_i + \Delta_i - \omega_i\right] + \mu_i^0 k_i + \mu_i^1\left[k_i^{\max} - k_i\right] \end{aligned}$$

F.O.C. wrt ℓ_i:

$$\frac{\partial \mathcal{L}}{\partial \ell_i} = x(1+\lambda_i) - (1+r_z)$$

F.O.C. wrt z_i:

$$\frac{\partial \mathcal{L}}{\partial z_i} = r_z - \lambda_i - \zeta z_i$$

F.O.C. wrt Δ_i:

$$\frac{\partial \mathcal{L}}{\partial \Delta_i} = \lambda_i - r_b$$

F.O.C. wrt k_i:

$$\frac{\partial \mathcal{L}}{\partial k_i} = \mu_i^0 - \mu_i^1 + \lambda_i - r_k$$

We restrict attention to combinations of r_k and $\left\{\widehat{k}_i\right\}_{i \in [0,1]}$ that implement $\Delta_i = 0$ for all i (i.e., all interbank trade is conducted through loan certificates) in equilibrium. We then show that there exists such a combination where welfare is higher than in the decentralized equilibrium.

Notice that $\Delta_i = 0$ satisfies the F.O.C. for $\Delta_i \in \mathbb{R}$ if (and only if) $\lambda_i = r_b$. The remaining conditions are:

$$\frac{\partial \mathcal{L}}{\partial \ell_i} \overset{sign}{=} r_b - \left(\frac{1 + r_z}{x} - 1\right)$$

$$\frac{\partial \mathcal{L}}{\partial z_i} \overset{sign}{=} \frac{r_z - r_b}{\zeta} - z_i$$

$$\frac{\partial \mathcal{L}}{\partial k_i} = \mu_i^0 - \mu_i^1 + r_b - r_k$$

In the equilibrium we are considering, no one transacts at the interest rate r_b. However, a value of r_b must still be specified in case a bank were to deviate and use the interbank market off equilibrium, e.g., in the evaluation of run-proofness. Specifying a latent value of r_b effectively selects an equilibrium from a continuum of possible equilibria.

We set the latent interbank rate at $r_b = r_k$ and conjecture $r_k \in \left(r_z, \frac{1+r_z}{x} - 1\right)$, in which case $\ell_i = 0$ and $z_i = 0$. Moreover, $\widetilde{c} - (\nu_i - k_i) + \left(\overline{\nu} - \overline{k}\right) + \Delta_i = \omega_i$ by complementary slackness with $\lambda_i = r_b > 0$. Bank i's maximized value is then

$$V_i(\omega_i) = (1 + r_z)\,\tilde{z} + (1 + r_k)\,(\widetilde{c} - \nu_i + \overline{\nu}) - (1 + r) - (r_k - r)\,\omega_i$$

where we recall

$$c_i \equiv \widetilde{c} - \nu_i + \overline{\nu} \tag{A.4}$$

The bank is run-proof if and only if $V_i(1) \geq 0$, or equivalently $c_i \geq \overline{c}(r_k)$, where $\overline{c}(\cdot)$ is as defined in Eq. (2). Thus, we focus on $\Delta_i = 0$ and

$$\widetilde{c} - (\nu_i - k_i) + \left(\overline{\nu} - \overline{k}\right) = \rho \tag{A.5}$$

for all i such that $c_i \geq \overline{c}(r_k)$. Banks that are not run-proof fail at $t = 1$ and are precluded from interbank borrowing, i.e., they also have $\Delta_i = 0$.

With $r_b = r_k$, bank i is indifferent between any $k_i \in [0, k_i^{\max}]$ so we proceed with $k_i = k_i^{\max}$.[16] Consider $\widehat{k}_i = 0$ if $c_i < \overline{c}(r_k)$ and

$$k_i^{\max} = \begin{cases} \max\left\{\overline{k} - c_i, 0\right\} & if \quad c_i < \overline{c}(r_k) \\ \min\left\{\nu_i, \widehat{k}_i + \overline{k}\right\} & if \quad c_i \geq \overline{c}(r_k) \end{cases}$$

where we note

$$\overline{k} - c_i \equiv \nu_i - \left(\widetilde{c} + \left(\overline{\nu} - \overline{k}\right)\right)$$

from Eq. (A.4). In words, any bank i with $c_i < \overline{c}(r_k)$ receives no initial allocation of loan certificates and must use its available cash, $\widetilde{c} + \left(\overline{\nu} - \overline{k}\right)$, to cover its check-clearing obligations ν_i before it is allowed to use any recirculated loan certificates $\overline{k}$. Then,

$$\overline{k} \equiv \int_0^1 k_i di = \int_{\{i|c_i<\overline{c}(r_k)\}} \max\left\{\overline{k} - c_i, 0\right\} di + \int_{\{i|c_i\geq\overline{c}(r_k)\}} \min\left\{\nu_i, \widehat{k}_i + \overline{k}\right\} di \tag{A.6}$$

where Eq. (A.5) implies

$$\min\left\{\nu_i, \widehat{k}_i + \overline{k}\right\} = \rho + \overline{k} - c_i \tag{A.7}$$

for all i such that $c_i \geq \overline{c}(r_k)$. Subbing Eq. (A.7) into (A.6) delivers

$$\int_{\{i|c_i\geq\overline{c}(r_k)\}} (\rho - c_i)\, di = \int_{\{i|c_i<\overline{c}(r_k)\}} \min\left\{c_i, \overline{k}\right\} di \tag{A.8}$$

which pins down r_k conditional on the initial allocations $\widehat{k}_i$ for $c_i \geq \overline{c}(r_k)$.

[16] Alternatively, we could have set $r_b = r_k + \epsilon$, with $\epsilon > 0$ arbitrarily small, in which case the cutoff for run-proofness would be arbitrarily close to $\overline{c}(r_k)$ and the F.O.C. for k_i would deliver $k_i = k_i^{\max}$ without indifference. Setting instead $r_b = r_k - \epsilon$ would deliver $k_i = 0$ for all i, i.e., the environment in Proposition 1 with $r_b = r_b^*$, and will be ruled out with $r_k < r_b^*$ as derived below.

Social welfare takes the same form as Eq. (5), but with r_k in place of r_b^*, where r_b^* solves (3). The right-hand side of Eq. (A.8) is strictly positive if loan certificates are issued, i.e., if $\overline{k} > 0$, in which case Eqs. (3) and (A.8) imply

$$\int_{\{i|c_i \geq \overline{c}(r_k)\}} (\rho - c_i)\, di > \int_{\{i|c_i \geq \overline{c}(r_b^*)\}} (\rho - c_i)\, di \tag{A.9}$$

Recall $\overline{c}'(\cdot) > 0$ from Eq. (2). Then, $r_k < r_b^*$ from (A.9) and it follows immediately from Eq. (5) that welfare is higher than in the decentralized equilibrium. Also recall from Proposition 1 that $r_b^* < \frac{1+r_z}{x} - 1$ if $\int_0^1 c_i di < \rho$, provided a conservative equilibrium exists. Thus, Eq. (A.8) delivers $r_k < \frac{1+r_z}{x} - 1$ for any parameters where a conservative equilibrium exists, verifying our initial conjecture about r_k. Verification of $r_k > r_z$ follows trivially from $\int_0^1 c_i di < \rho$.

It only remains to find initial allocations $\widehat{k}_i$ for $c_i \geq \overline{c}(r_k)$ such that $\overline{k} > 0$. Use Eq. (A.4) to rewrite Eq. (A.7) as

$$\min\left\{\nu_i, \widehat{k}_i + \overline{k}\right\} = \left(\rho + \overline{k} - \overline{\nu} - \widetilde{c}\right) + \nu_i \tag{A.10}$$

The planner can satisfy Eq. (A.10) for all $c_i \geq \overline{c}(r_k)$ by setting an initial allocation of loan certificates

$$\widehat{k}_i = \begin{cases} 0 & if \quad c_i < \overline{c}(r_k) \\ \nu_i & if \quad c_i \geq \overline{c}(r_k) \end{cases} \tag{A.11}$$

and an interest rate r_k such that

$$\overline{k} = \overline{\nu} + \widetilde{c} - \rho \tag{A.12}$$

Notice that $\overline{k} > 0$ will require $\overline{\nu} > \rho - \widetilde{c}$. Subbing Eqs. (A.11) and (A.12) into the definition of $\overline{k}$ in Eq. (A.6) delivers

$$\int_{\{i|c_i < \overline{c}(r_k)\}} \min\{\nu_i, \rho\}\, di = \rho - \widetilde{c} \tag{A.13}$$

The left-hand side of Eq. (A.13) is positive, strictly increasing in r_k, and ranges from 0 to at most $\overline{\nu}$. Therefore, with $\overline{\nu} > \rho - \widetilde{c} > 0$, there is a unique solution for r_k.

We have thus found a combination of r_k and $\left\{\widehat{k}_i\right\}_{i\in[0,1]}$ that implements $\Delta_i = 0$ for all i and achieves higher welfare than the decentralized equilibrium. By way of example, consider $\widetilde{c} = \frac{1}{2}$ and

ν_i uniformly distributed over the unit interval, with $\rho > \frac{1}{2}$. Then, $\overline{\nu} = \frac{1}{2}$ and Eq. (A.13) implies

$$r_k = \begin{cases} 2\rho\left(1 + r_z\right)\tilde{z} - 1 & if \quad \rho \in \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right] \\ \frac{(1+r_z)\tilde{z}}{\sqrt{1-\rho^2}} - 1 & if \quad \rho \in \left(\frac{1}{\sqrt{2}}, 1\right) \end{cases}$$

which is lower than the r_b^* in Eq. (A.3) in the $\rho > \frac{1}{2}$ region. ■

Proof of Lemma 1

Recall Eq. (A.13), which pins down r_k under the loan certificate arrangement in Proposition 2:[17]

$$\int_{\{i|c_i<\overline{c}(r_k)\}} \min\left\{\nu_i, \rho\right\} di = \rho - \widetilde{c}$$

Suppose the parameters are such that the solution to Eq. (A.13) satisfies $\overline{c}\left(r_k\right) \leq \overline{\nu} + \widetilde{c} - \rho$. Then, using Eq. (A.4), the solution must also satisfy $\overline{c}\left(r_k\right) \leq c_i + \nu_i - \rho$ for all i. This implies $\nu_i > \rho$ for any i with $c_i < \overline{c}\left(r_k\right)$, simplifying Eq. (A.13) to

$$\int_{\{i|c_i\geq\overline{c}(r_k)\}} di = \frac{\widetilde{c}}{\rho}$$

The social welfare with loan certificates (given by Eq. (5) with r_k in place of r_b^*) then simplifies to

$$\int_0^1 c_i di + x\tilde{z} + \left[f\left(\tilde{z}\right) - x\tilde{z}\right]\frac{\widetilde{c}}{\rho}$$

which is exactly the welfare in Eq. (6) when c_i is determined by Eq. (A.4).

Using Eq. (A.12), $\overline{c}\left(r_k\right) \leq \overline{\nu} + \widetilde{c} - \rho$ can also be expressed as $\overline{c}\left(r_k\right) \leq \overline{k}$. Any bank i with $c_i < \overline{c}\left(r_k\right)$ thus has

$$\nu_i - \left(\widetilde{c} + \left(\overline{\nu} - \overline{k}\right)\right) \equiv \overline{k} - c_i > 0$$

which is to say it uses all of its cash and then some recirculated loan certificates to cover its check-clearing obligations. We call this the "unconstrained case" because check-clearing obligations are such that all of the cash holdings of failing banks can be transferred towards run-proof banks.

[17] Recall that Eq. (A.13) is equivalent to Eq. (9) with c_i as per Eq. (A.4) and $\overline{k}$ as per Eq. (A.12).

If instead $\overline{c}(r_k) > \overline{\nu} + \widetilde{c} - \rho$, then $\overline{c}(r_k) > \overline{k}$ and any bank i with $c_i \in (\overline{k}, \overline{c}(r_k))$ has

$$\nu_i - \left(\widetilde{c} + \left(\overline{\nu} - \overline{k}\right)\right) \equiv \overline{k} - c_i < 0$$

which is to say it covers all of its check-clearing obligations with cash and still has some cash left over, i.e., it does not use any recirculated loan certificates. We call this the "constrained case" because check-clearing obligations are such that not all of the cash holdings of failing banks can be transferred towards run-proof banks.

Both sides of Eq. (A.13) are increasing in ρ, with the right-hand side increasing one-for-one and the left-hand side increasing less than one-for-one. The left-hand side is also increasing in r_k, thus Eq. (A.13) defines r_k increasing in ρ. The unconstrained case, $\overline{c}(r_k) \leq \overline{\nu} + \widetilde{c} - \rho$, will therefore require ρ not too high. For the example considered at the end of the proof of Proposition 2, that is, $\widetilde{c} = \frac{1}{2} < \rho$ and ν_i uniformly distributed over the unit interval, the unconstrained case corresponds to $\rho \in \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$.

Alternatively, consider an increase in check-clearing obligations from ν_i to $\nu_i + \varepsilon$ at all banks i, where $\varepsilon > 0$. From Eq. (A.4), c_i is unchanged. The left-hand side of Eq. (A.13) is then increasing in ε, which means the solution r_k is decreasing in ε. The condition for the unconstrained case, $\overline{c}(r_k) \leq \overline{\nu} + \widetilde{c} - \rho$, is therefore easier to satisfy for higher ε. ■

Proof of Proposition 3

Consider the model with a decentralized interbank market and no loan certificates.

If $r_b \in \left(r_z, \frac{1+r_z}{x} - 1\right)$, then:

$$V_i(\omega_i) = (r_z - r_b)\,\widetilde{z} + (1 + r_b)(c_i + \widetilde{z} - 1) + (r_b - r)(1 - \omega_i)$$

from the proof of Proposition 1. This is the maximized value of bank i at $t = 2$ if it honors withdrawals ω_i at $t = 1$.

With information suppression, $\omega_i = \phi$ for all $i \in [0, 1]$, where $\phi \in [\rho, 1]$ is to be determined in equilibrium. If $\phi \in (\rho, 1)$, then patient depositors are playing a mixed strategy where they withdraw at $t = 1$ with probability $\frac{\phi - \rho}{1 - \rho} \in (0, 1)$.

Bank i survives beyond $t = 1$ if and only if $V_i(\phi) \geq 0$, or equivalently:

$$c_i \geq \widetilde{c}(r_b) \equiv \overline{c}(r_b) - \frac{(r_b - r)(1-\phi)}{1+r_b} \tag{A.14}$$

where $\overline{c}(\cdot)$ is as defined in Eq. (2). We can see from Eq. (A.14) that $\widetilde{c}(r_b) \leq \overline{c}(r_b)$ for the same interest rate r_b, with strict inequality if and only if $\phi < 1$.

Let r_b^s denote the equilibrium interbank rate with information suppression. Market clearing pins down r_b^s as the solution to:

$$\int_{\{i|c_i \geq \widetilde{c}(r_b^s)\}} (\phi - c_i)\, di = 0 \tag{A.15}$$

Suppose there exists an equilibrium with $\phi = \rho$. Comparing Eq. (3) and (A.15), it must be the case that $\widetilde{c}(r_b^s) = \overline{c}(r_b^*)$, where r_b^* is the equilibrium interbank rate without information suppression. Comparing Eq. (2) and (A.14), we then conclude $\overline{c}(r_b^s) > \overline{c}(r_b^*)$ and $r_b^s > r_b^*$.

Next, define:

$$\widehat{c} \equiv \phi - x\widetilde{z}$$

A sufficient condition for $\widehat{c} < \widetilde{c}(r_b^s)$ is:

$$\phi < 1 - \frac{(1-x)(1+r_z)\widetilde{z}}{1+r}$$

Suppose this sufficient condition is satisfied. Then, banks with $c_i \in (\widehat{c}, \widetilde{c}(r_b^s))$ only have to liquidate $\ell_i = \frac{\phi - c_i}{x} < \widetilde{z}$ to satisfy depositor withdrawals at $t = 1$. Social welfare is then:

$$\begin{aligned}
\mathcal{W}_s^{(2)} &= \int_0^1 c_i di + x\widetilde{z} + [f(\widetilde{z}) - x\widetilde{z}] \int_{\{i|c_i \geq \widetilde{c}(r_b^s)\}} di \\
&\quad + \underbrace{\left(\frac{(1-\phi)(1+r_z)}{x} - 1\right)}_{\text{positive iff } \phi < 1 - \frac{x}{1+r_z}} \underbrace{\int_{\{i|c_i \in (\widehat{c}, \widetilde{c}(r_b^s))\}} (c_i + x\widetilde{z} - \phi)\, di}_{\text{positive by definition of } \widehat{c}}
\end{aligned}$$

If there exists an equilibrium with $\phi = \rho$, then $\mathcal{W}_s^{(2)}$ exceeds $\mathcal{W}_b^{(2)}$ as defined in Eq. (5) if $\rho < \min\left\{1 - \frac{x}{1+r_z}, 1 - \frac{(1-x)(1+r_z)\widetilde{z}}{1+r}\right\}$. ■

Appendix B – Welfare Under Reserve Pooling

Consider $\int_0^1 c_i di < \rho$. Some loans will have to be liquidated to repay depositors who receive liquidity shocks, so it follows immediately that $z_i = 0$ for all i, i.e., it cannot be optimal for the planner to make additional loans. Social welfare is then

$$\int_0^1 c_i di + x\tilde{z} + \left[f\left(\tilde{z}\right) - x\tilde{z}\right] \times |S|$$

where S is the set of run-proof banks. We constrain the planner to not liquidate loans among run-proof banks, as noted in Section 3.3. The best he can do is then

$$|S| = \frac{\int_0^1 c_i di}{\rho}$$

It remains to show that this is achievable given depositor behavior.

Consider the allocation $c_i = \rho$ for all $i \in S$ and $c_i = 0$ for all $i \notin S$. As in the proof of Proposition 1, let ω_i denote the fraction of depositors that withdraw from bank i at $t = 1$. Suppose $\omega_i = \rho$ for all $i \in S$ then consider a deviation to $\omega_{i'} = 1$ for one bank $i' \in S$. Bank i' is run-proof, that is, this deviation can be ruled out, if

$$\left(1 + r_z\right)\left(\tilde{z} - \ell_{i'}\right) - T_{3i'} \geq 0$$

$$\rho + x\ell_{i'} + T_{1i'} = 1$$

where $T_{1i'}$ and $T_{3i'}$ are transfers between bank i' and other banks $i \in S$.[18] Transfers must satisfy

$$T_{1i'} + \frac{\int_0^1 c_i di}{\rho} \times T_{1i} = 0$$

$$T_{3i'} + \frac{\int_0^1 c_i di}{\rho} \times T_{3i} = 0$$

with

$$\left(1 + r_z\right)\left(\tilde{z} - \ell_i\right) - T_{3i} - \left(1 + r\right)\left(1 - \rho\right) \geq 0$$

[18] These transfers (and any liquidations) are all off equilibrium if the deviation is ultimately ruled out.

$$x\ell_i + T_{1i} = 0$$

The program reduces to finding values of $\ell_{i'}$ and $T_{3i'}$ that satisfy

$$T_{3i'} \leq (1+r_z)\left(\tilde{z} - \ell_{i'}\right)$$

$$T_{3i'} \geq \left[(1+r)(1-\rho) - (1+r_z)\tilde{z}\right]\frac{\int_0^1 c_i di}{\rho} + \frac{1+r_z}{x}\left(1-\rho-x\ell_{i'}\right)$$

$$\ell_{i'} \leq \frac{1-\rho}{x}$$

$$\ell_{i'} \geq \frac{1}{x}\left(1-\rho-x\tilde{z}\frac{\int_0^1 c_i di}{\rho}\right)$$

The first inequality is the solvency condition for i' and the second inequality is the solvency condition for each $i \in S$ (where $i \neq i'$) after the final transfers. The third and fourth inequalities are the conditions $\ell_i \geq 0$ and $\ell_i \leq \tilde{z}$, respectively.

The bounds on $T_{3i'}$ define a non-empty set if and only if

$$x\tilde{z} \geq \left(1 - \frac{1-\frac{x(1+r)}{1+r_z}}{1+\frac{\rho}{\int_0^1 c_i di}}\right)(1-\rho) \tag{B.1}$$

which is also sufficient for the bounds on $\ell_{i'}$ to define a non-empty set with $\ell_{i'} \in [0, \tilde{z}]$. Thus, for parameters satisfying (B.1), which is a weaker condition than $\rho \geq 1 - x\tilde{z}$, there is a reserve pooling arrangement that achieves the welfare in Eq. (6).

Now return to the restriction that loans are not liquidated among run-proof banks in Eq. (6). We derive a sufficient condition for this to be efficient. Consider an allocation $c_i = \rho - x\ell$ for all $i \in \widetilde{S}$ and $c_i = 0$ for all $i \notin \widetilde{S}$, where $\widetilde{S} \subset [0,1]$ and $\left|\widetilde{S}\right| = \frac{\int_0^1 c_i di}{\rho - x\ell}$ with $\ell \geq 0$. Social welfare is at most

$$\mathcal{W} = \int_0^1 c_i di + x\tilde{z} + \left[f(\tilde{z}-\ell) - x(\tilde{z}-\ell)\right]\frac{\int_0^1 c_i di}{\rho - x\ell}$$

where

$$\frac{\partial \mathcal{W}}{\partial \ell} = \left(\frac{x}{\rho - x\ell}\left[f(\tilde{z}-\ell) - x(\tilde{z}-\ell)\right] - \left[f'(\tilde{z}-\ell) - x\right]\right)\frac{\int_0^1 c_i di}{\rho - x\ell}$$

and

$$\frac{\partial^2 \mathcal{W}}{\partial \ell^2} = \frac{2x}{\rho - x\ell} \frac{\partial \mathcal{W}}{\partial \ell} + f''(\tilde{z} - \ell) \frac{\int_0^1 c_i di}{\rho - x\ell}$$

Therefore, $\ell = 0$ is indeed optimal if $f''(\cdot) \leq 0$ and $\frac{\partial W}{\partial \ell}\big|_{\ell=0} \leq 0$, i.e.,

$$f(\tilde{z}) \leq x\tilde{z} + \rho \left(\frac{f'(\tilde{z})}{x} - 1 \right) \tag{B.2}$$

For any parameterization satisfying both (B.1) and (B.2), the welfare in Eq. (6) is also the second-best level of welfare.