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FINANCIAL AND TOTAL WEALTH INEQUALITY WITH DECLINING INTEREST RATES

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**ABSTRACT**

Financial wealth inequality and long-term real interest rates track each other closely over the post-war period. Faced with unanticipated lower real rates, households which rely more on financial wealth must see large capital gains to afford the consumption that they planned before the decline in rates. Lower rates beget higher financial wealth inequality. Inequality in total wealth, the sum of financial and human wealth and the relevant concept for house-hold welfare, rises much less than financial wealth inequality and even declines at the top of the wealth distribution. A standard incomplete markets model reproduces the observed in-crease in financial wealth inequality in response to a decline in real interest rates because high financial-wealth households have a financial portfolio with high duration.

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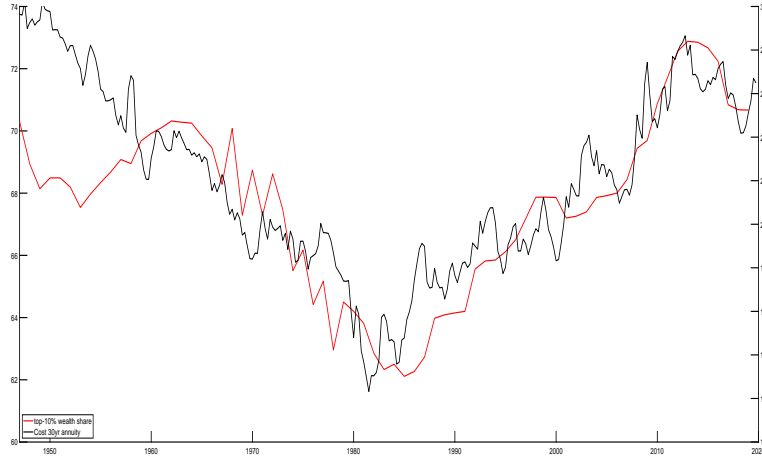
# 1 Introduction

Financial wealth inequality has increased substantially over the past several decades in the U.S. and many other countries. According to the World Inequality Database, the fraction of U.S. financial wealth held by the top-10% wealthiest households has increased from 63.0% in the 1980s to 71.9% in the 2010s, an increase of 8.9% points. The share of financial wealth held by the top-1% increased from 24.6% to 35.1% over the same period, a 10.5% point increase. Over the same period, long term nominal rates have declined dramatically. The 10-year U.S. nominal Treasury yield declined from 10.6% in the 1980s to 2.4% in the 2010s. Given that 10-year average expected inflation fell by much less than 8 percentage points over this period, and not at all since 2003, real rates declined as well. The 10-year real bond yield averaged 2.1% in 2003 before falling to 0.3% in 2016 and -0.60% in 2020. We argue that these two changes are related. Large changes in the distribution of financial wealth are to be expected if households want to finance the same consumption stream in the low rate environment as the one that they had planned prior to the onset of lower rates.

Consider a simple example to fix ideas. Figure 1 plots the cost of a savings instrument that provides \$1 of consumption in each of the next thirty years. The price of this 30-year real annuity is about \$30 in the early 1950s. The price then falls to a low of \$12.5 in 1981.Q3 when long-term real interest rates peak. As interest rates fall over the next three decades, the price of the consumption annuity more than doubles to \$29.1 in 2012.Q4. A 50-year old in 1982 who wants to spend \$10,000 per year for the next 30 years had to set aside \$125,000. In 2012, a 50-year old with the same desire needs \$291,000, or 2.5 times as much financial wealth. In contrast, a 30-year old with many more years left in the labor market is partially hedged against this real interest rate change. The market value of the 30-year old's human wealth in 2019 is much larger than that of the 30-year old in 1982 as the valuation reflects the lower interest rates. She may not need to adjust financial savings to the same extent as the 50-year old to afford the same consumption plan. This example also illustrates how inequality in total wealth, the sum of financial and human wealth, may behave very differently from that in financial wealth in the face of declining interest rates. The 30-year old has little financial wealth while the 50-year old has little human wealth. Figure 1 shows that the top wealth share and the cost of the real annuity comove strongly in the U.S. We find the same pattern in the U.K and in France.

This evidence suggests that large changes in the distribution of financial wealth are to be expected, and even desirable, in the wake of large changes in real rates. To make this point rigorously, we analyze a Bewley-style incomplete markets equilibrium economy with heterogeneous agents. The decline in long-term real rates in the model arises from a slowdown in the long-run growth rate of the economy. This slowdown is isomorphic to a decrease in the rate of time preference of all households in a stationary version of this economy. Since there is no preference heterogeneity and all households have merely become equally more patient, it is natural to ask

Figure 1: Top Inequality and the Cost of Real Annuity



*Note:* The figure plots the top-10% financial wealth share for the United States (red line). The data is annual from 1947 until 2019 from the World Inequality Database. It also plots (black line) the price of a 30-year inflation-indexed annuity which pays \$1 in real terms for the next 30 years. A dynamic affine term structure model, estimated on quarterly data from 1947:Q1-2019:Q4 and spelled out in Appendix D delivers the term structure of real bond yields. The price of the annuity equals the sum of the prices of the real zero-coupon bonds of maturities 1 through 30.

whether we can implement the same consumption allocation in the economy with low rates as the one that prevailed when rates were high. We show that the equilibrium consumption allocation in the model with high rates remains an equilibrium in the model with low rates, provided that agents' initial financial wealth is adjusted. Conversely, if the financial wealth distribution does not change in response to changes in long rates, the new equilibrium will result in large changes in the consumption distribution.

The key model ingredient behind the increase in financial wealth inequality is a positive cross-sectional correlation between financial wealth and the duration of the household's excess consumption plan. When households are ex-ante identical and labor income shocks are persistent, low-wealth households are households who experienced a recent history of bad idiosyncratic income shocks. These households tend to have a high duration of human wealth, reflecting the expected long-run mean reversion of their labor income. Their consumption plan has a low duration because of consumption smoothing. Their duration of excess consumption, consumption minus income, is low. These households have low excess consumption duration and low financial wealth. The converse is true for high-wealth households, households who have been able to accumulate financial wealth thanks to a sequence of fortunate labor income shocks. With this positive cross-sectional correlation, the wealth-weighted duration of financial wealth exceeds the equal-weighted duration, and the right tail of the financial wealth distribution grows larger when rates decline. The rise in financial top-wealth inequality is exactly what the model predicts should happen when households' consumption is fully hedged against a decline in real interest rates.

Since everyone is still able to afford the same consumption plan, nobody is worse off.

To analyze the quantitative relevance of this mechanism, we calibrate a Bewley model with ex-ante heterogeneity across households. Households belong to groups defined on age, race, gender, and education, and face income risk over the life-cycle. The income processes are calibrated using Panel Study of Income Dynamics data. We add a superstar income state to enable the model to match the financial wealth gini of the 1980s.

We calibrate the duration of financial wealth using data on the composition of households' financial portfolios from the Survey of Consumer Finances. We combine portfolio shares from the SCF with durations of major asset classes obtained from an auxiliary asset pricing model. We find that U.S. households have an equally-weighted average duration of financial wealth of 16, which is below the value-weighted (or aggregate) duration of financial wealth of 25. We also observe substantial heterogeneity in financial durations by wealth level. Low-wealth households have low financial durations, driven by their higher share of deposit-like assets, the presence of consumer debt, and lower shares of housing, private business, and stock market wealth. The reverse is true for high-wealth households. This heterogeneity in financial duration is a new empirical finding, and crucial for the response of financial inequality to interest rates.

We compute the model at a long-term real interest rate of 4.7%, the level that prevailed in the 1980s, and at a 0.1% long-term real rate, the level that prevailed in the 2010s. The interest rate change is due to an unanticipated decline in the expected growth rate of the economy. We first ask how much additional financial wealth each household would need to be able to afford the old consumption plan, as a fraction of aggregate consumption, under the new, lower interest rate. This *compensated* financial wealth distribution is a rightward shift of the original wealth distribution. While all households require more financial wealth to finance the old consumption allocation, young households require the largest compensation. Since they must save for retirement for many years, the loss in compound interest hits them particularly hard. While the wealthy see a large increase in financial wealth under the compensated distribution (as much of 38% of the increase in aggregate wealth goes to the top-1%), the top-1% and top-10% financial wealth shares and the gini nevertheless fall since the required increase in financial wealth for the young is greater still. In other words, the large human wealth of the young does not provide a large enough hedge against interest rate declines. This shows that the life-cycle aspect is a crucial addition, adding meaningfully to the intuition coming from the Bewley model with infinitely-lived households.

Next we ask what actually happens to financial wealth in the calibrated model after rates decline. When financial wealth durations are heterogeneous as in the data, we find that the model can account for the entire rise in financial wealth inequality between the 1980s and 2010s. The *repriced* financial wealth distribution exactly matches the increases in the observed financial wealth gini. It features increases in the top-10% and top-1% wealth shares that are close to the data.

Human wealth inequality is much lower, and rises by much less when rates decline. Young

agents have both high levels of human wealth and high human wealth durations, explaining the increase in human wealth inequality when rates decline. In sharp contrast to top financial wealth shares, top human wealth shares fall modestly. Total wealth inequality, which is the welfare-relevant concept, shows only a modest increase in gini and a small decline in top wealth shares. The decline in rates has not led to large increases in total wealth inequality.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 shows that the share of the top percentiles tracks the cost of an indexed annuity quite closely in the U.S., U.K., and France. Section 4 shows that the connection between low expected returns and high financial wealth inequality arises under minimal assumptions. Section 5 sets up an incomplete markets economy with aggregate uncertainty and infinitely-lived households who face idiosyncratic income risk. A first insight from this model is that one needs to use the same discount rate for the household's future labor income, financial income, and consumption to arrive at a measure of household wealth that properly aggregates. The second and main result in this section is that financial wealth becomes more concentrated in response to decline in the interest rate if financial wealth and the duration of financial wealth covary positively. This covariance condition is naturally satisfied in an incomplete markets economy with persistent labor income shocks. Section 6 quantifies the effect of an interest rate change by adding a life-cycle component to the model as well as heterogeneity across demographic groups. Section 7 concludes. Appendix A contains details on data sources and construction. Appendix B contains the proofs of the propositions. Appendix C contains some details of the calibrated model. Appendix D provides an auxiliary asset pricing model used to infer real interest rates and durations of the components of financial wealth.

## 2 Related Literature

A large strand of recent literature documents the evolution of income inequality as well as financial wealth inequality over the past century (Piketty and Saez, 2003; Piketty, 2015; Alvaredo, Chancel, Piketty, Saez and Zucman, 2018). Most of the evidence suggests that financial wealth inequality has increased in many countries over the past decades. Zucman (2019) reviews the empirical literature on the topic. Benhabib and Bisin (2018) survey economic theories of wealth inequality.

Much of the literature on wealth inequality adopts a backward-looking approach and explores the connection between past returns and current wealth. This literature has argued that high past rates of return and heterogeneity therein helps account for the increase in financial wealth inequality (Piketty and Zucman, 2015; Fagereng, Guiso, Malacrino and Pistaferri, 2020; Bach, Calvet and Sodini, 2020; Hubmer, Krusell and Smith, 2020).

But wealth is also the current value of the household's future consumption stream. Human wealth is the value of future labor income and financial wealth is the value of future consumption

minus income. We bring an asset pricing perspective to the discussion on inequality. We impute a valuation by discounting future cash flows. When rates declines, households need more wealth to finance the same consumption stream. Households that have mostly human wealth are likely to be better hedged. Households with mostly financial wealth need enough duration in their portfolio in order to finance future consumption. To keep consumption shares unchanged, a decline in real rates needs to entail a reallocation of financial wealth towards those households who rely mostly on their (current and future) financial wealth to finance future consumption.

Discount rates matter. In a simple partial equilibrium model, [Moll \(2020\)](#) explains that small discount rate-induced changes in the wealth distribution may have smaller welfare effects than cash flow-induced changes. We make a related point in a version of the Bewley-style general equilibrium model with aggregate and idiosyncratic risk. Recently, [Catherine, Miller and Sarin \(2020\)](#) show the quantitative importance of valuation effects for measures of wealth inequality. Their emphasis is on the effects of discounting social security transfers at time-varying discount rates.

[Greenwald, Lettau and Ludvigson \(2019\)](#) point to increases in the share of output accruing to profits as a key source of the rise in equity values since 1989. While we motivate our main experiment using a drop in the real risk-free rate, the decline in expected returns applies more broadly to other financial assets. This decline could arise either from a highly persistent change in the real risk free rate or to a decrease in risk premia. To the extent that economic forces have varied these quantities across time and across different financial assets, our methodology could be extended to capture these more detailed patterns. The auxiliary asset pricing model in [Appendix D](#) indeed shows declines in expected real returns not only on bonds but also on stocks and housing.

Our paper is related to recent work by [Auclert \(2019\)](#), who explores the effect of cross-sectional variation in the duration of households' financial assets for the effectiveness of monetary policy. We consider a setting with aggregate risk, we develop measures of household duration based on a no-arbitrage dynamic asset pricing model and household financial portfolios, and we assess quantitatively the extent to which households have hedged their consumption plan against interest rate innovations. In earlier work, [Doepke and Schneider \(2006\)](#) focus on the distributional consequences of inflation. Our work instead focuses on the distributional effects of changes in long-term real rates. [Gomez and Gouin-Bonenfant \(2020\)](#) study the effects of lower interest rate on the cost of raising new capital for entrepreneurs, linking the decline in interest rates to the rise in wealth inequality through a different channel.

There are important normative implications for fiscal policy. The compensated distribution that allows all households to implement their old consumption plans features substantially less financial wealth inequality than both the old distribution and the actual repriced distribution. This suggests that a financial wealth tax may be able to improve on the repriced consumption distribution. In our life-cycle model, we find that young households are hurt most by a reduction

in rates. In that respect, our model speaks to the inter-generational distribution of the burden of taxation. A large literature studies optimal labor and capital income taxation in Bewley models with idiosyncratic risk, endogenous labor supply, and capital formation (Aiyagari, 1995; Panousi and Reis, 2017; Heathcote, Storesletten and Violante, 2017; Krueger and Ludwig, 2018; Boar and Midrigan, 2020). We take labor income as given and do not model capital formation, but instead focus on the distributional implications of lower interest rates.

As an important aside, our Bewley model resolves an outstanding issue in the literature on how to compute an individual's human wealth. Lustig, Van Nieuwerburgh and Verdelhan (2013) propose using the same stochastic discount factor (SDF) that prices traded assets to discount an individual's labor income stream. Huggett and Kaplan (2016) propose using the individual's own SDF to compute human wealth. For wealth accounting, the aggregate SDF is convenient, because the aggregate value of individual wealth is consistent with market valuations. Using individual SDFs results in a wealth measure that does not aggregate.

By emphasizing total wealth (inequality), of which human wealth (inequality) forms a very significant component, our work contributes to the literature on measuring wealth (inequality). Our paper provides new and detailed statistics on the duration of financial wealth for U.S. households. Related, Kuhn, Schularick and Steins (2020) study how housing and equity portfolio shares differ across the wealth distribution and result in differing financial wealth dynamics for the middle class and the top of the financial wealth distribution. Recent work discusses the measurement of private business income and wealth (Kopczuk, 2017; Saez and Zucman, 2016; Piketty, Saez and Zucman, 2018; Smith, Yagan, Zidar and Zwick, Working Papers; Kopczuk and Zwick, 2020). In our theoretical work, we sidestep this issue by recognizing that financial wealth is the present discounted value of the future stream of consumption minus labor income. In our empirical work, we infer the duration of private business wealth from that of small stocks.

Our work is agnostic on the underlying reason for the interest rate decline, only hypothesizing a reduction in the expected rate of growth of the economy. The literature has proposed a long list of candidates for such a growth slowdown: demographics (Summers, 2014; Eggertsson and Mehrotra, 2014; Eichengreen, 2015), a productivity slowdown due to a plateau in educational attainment or diminishing technological progress (Gordon, 2017), a global saving glut and/or shortage of safe assets (Bernanke et al., 2005; Caballero, Farhi and Gourinchas, 2008), government spending that leads to depressed future aggregate demand (Mian, Straub and Sufi, 2020), a decline in competition (Gutiérrez and Philippon, 2017), a decline in desired investment due to lower relative prices of capital goods (Rachel and Smith, 2017), among others. Lower tax progressivity could lead to more saving by the rich, more aggregate wealth, and lower rates (Hubmer et al., 2020). However, Heathcote, Storesletten and Violante (2020) argue that once transfers are considered, the U.S. tax system has not become less progressive. Alternatively, a rise in income inequality could be the origin of lower interest rates. Mian et al. (2020) argue that the rich have a higher propensity to



save than the poor; [Fagereng, Blomhoff Holm, Moll and Natvik \(2019\)](#) provide empirical evidence consistent with this from Norway. This reduces aggregate demand and the real rate of interest in the wake of an exogenous increase in income inequality, for example, due to skill-biased technological change. While the interest rate is endogenous in the Bewley model of Section 5, our model features standard homothetic preferences. The model in Section 6 keeps labor income inequality constant, in order to isolate the effect of a decline in the long-run growth rate of the economy.<sup>1</sup> Our conclusions regarding the differing behavior of financial and total wealth inequality are not sensitive to the source of the decline in interest rates.

### 3 Empirical Evidence

In this section we document a strong time-series correlation between the evolution of long-term real interest rates and wealth inequality. While our focus is on the U.S. in most of the paper, this section documents that this correlation is present also in the United Kingdom and in France. This evidence suggests that households are partially hedged against changes in long real rates.

Figure 2 shows the wealth share of the top-10% of the population in the left panels and the wealth share of the top-1% of the population in the right panels. Wealth shares from the World Inequality Database. Each panel also plots the price of a thirty-year real annuity, computed from nominal yields and inflation or alternatively from an affine asset pricing model. Construction details are in Appendix A.1. The top row is for the U.S., the middle row for the U.K., and the bottom row for France. The sample is 1947-2019.<sup>2</sup>

For both inequality measures, there is a strong positive correlation between financial wealth inequality and the annuity price. Put differently, there is a strong negative association between financial wealth inequality and long-term real interest rates. Between 1947 and 1982, the top-10% (top-1%) wealth share falls from 70% (29%) to 63% (24%) in the U.S. as the annuity becomes cheaper. From 1982 until 2015, the top-10% (top-1%) wealth share rises from 63% (24%) to 73% (36%). During this period, the cost of the annuity more than doubles. There is a small decline in wealth shares from 2015 until 2019, which is expected to have reversed again in 2020.

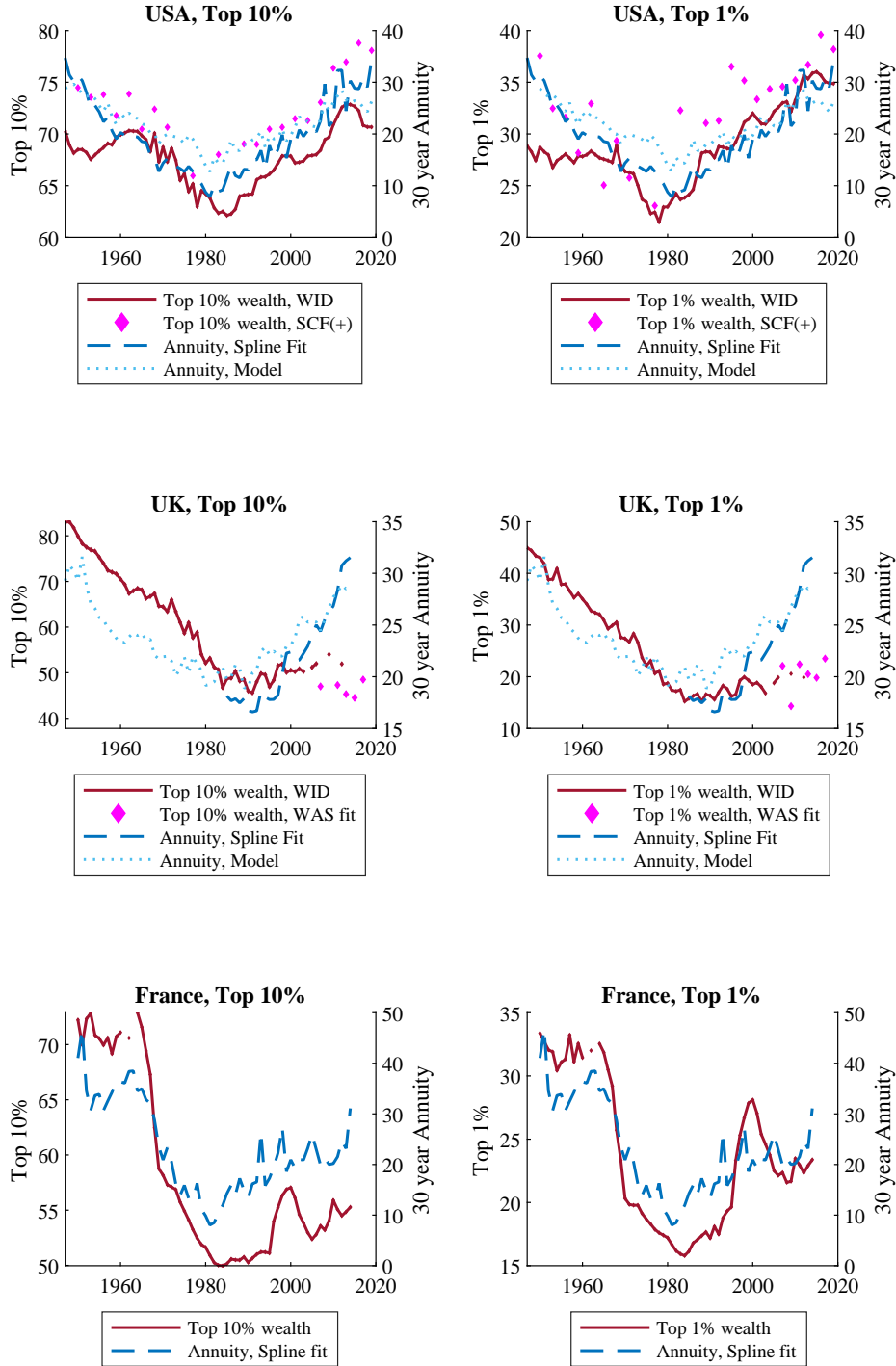
The patterns in both wealth inequality and the evolution of the cost of the annuity are similar in the UK and in France. [Rachel and Smith \(2017\)](#) show that the decline in the real rate has occurred across a broad set of developed and emerging market countries. While many other factors no doubt differ across countries, this shared trend in rates should result in a global rise in financial wealth inequality.

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<sup>1</sup>[Hubmer et al. \(2020\)](#) show that a rise in earnings risk actually lowers wealth inequality as it strengthens precautionary savings motives meaningfully for all but the richest households. A rise in top-income inequality, in contrast, can increase wealth inequality.

<sup>2</sup>For France we start our sample in 1950 since inflation was very high coming out of the WW-II, resulting in implausible real bond yield estimates.

Figure 2: Top Financial Wealth Inequality and Cost of Real Annuity



*Note:* Each panel plots a financial wealth inequality measure against a measure of the cost of a 30-year real annuity. The inequality measure in the left panels is the share of financial wealth going to the top-10% of the population. The right panels plot the share of the top-1% of the population. The wealth shares are from the World Inequality Database. Inflation and bond yield data are detailed in Appendix A.1.

## 4 Wealth Inequality and Expected Returns: A Model-free Approach

To develop an initial understanding of the relationship between financial wealth inequality and interest rates under minimal assumptions, we derive closed-form expressions using a log-linearization of the household budget constraint.

We work in a stationary version of the economy in which the aggregate endowment is constant. The hatted variables denote the stationary economy. Section 5 will show how to map these hatted variables into the corresponding variables of the stochastically growing economy. In the stationary economy, investors are computing expectations under the risk-neutral measure. Shocks to expected growth of the aggregate endowment show up as shocks to the risk-free rate.

Let  $\widehat{wc}_{t+1}^a$  denote the aggregate log wealth-consumption ratio. Given the constant aggregate endowment in the de-trended economy,  $\widehat{wc}_{t+1}^a$  is also the log price-dividend ratio of a perpetuity. The [Campbell and Shiller \(1988\)](#) log-linearization of the aggregate budget constraint around the mean aggregate log wealth-consumption ratio delivers the following expression for the real return on total wealth:

$$\widehat{r}_{t+1}^a = \rho^a \widehat{wc}_{t+1}^a + k^a - \widehat{wc}_t^a.$$

The usual aggregate consumption growth term is zero because the aggregate endowment is constant. The linearization coefficient  $\rho^a$  depends only on the mean of the log aggregate wealth-consumption ratio  $\widehat{wc}^a$ :

$$\rho^a \equiv \frac{e^{\widehat{wc}^a}}{e^{\widehat{wc}^a} + 1}, \quad k^a \equiv \log(1 + \exp(\widehat{wc}^a)) - \widehat{wc}^a \rho^a.$$

The linearization constant  $\rho^a$  captures the McCauley duration of the aggregate consumption claim. By iterating forward on the linearized return equation and imposing a TVC condition,  $\lim_{j \rightarrow \infty} (\rho^a)^j \widehat{wc}_{t+j}^a = 0$ , and taking expectations, we obtain the standard expression for the aggregate log wealth-consumption ratio as the PDV of future returns:

$$\widehat{wc}_t^a = \frac{k^a}{1 - \rho^a} - \mathbb{E}_t \sum_{j=1}^{\infty} (\rho^a)^{j-1} \widehat{r}_{t+j}^a.$$

The wealth-consumption ratio can be linked to the yield on a perpetuity  $\widehat{yp}_t^a$ :

$$\widehat{yp}_t^a = -(1 - \rho^a) \widehat{wc}_t^a + k^a = (1 - \rho^a) \mathbb{E}_t \sum_{j=1}^{\infty} (\rho^a)^{j-1} \widehat{r}_{t+j-1}^f,$$

since  $\widehat{r}_t^f = \widehat{r}_t^a$  in the detrended economy. The unconditional average yield on the perpetuity is given by:  $\widehat{yp}^a = \mathbb{E}[\widehat{r}^f]$ .<sup>3</sup> For simplicity, we assume that the risk-free rate follows an AR(1) process

<sup>3</sup>As shown by [Krueger and Lustig \(2010\)](#), the expectations hypothesis holds in the stochastically detrended economy,

with persistence  $\phi$ . Given the AR(1) process for the risk-free rate, the yield on the perpetuity can be expressed as:

$$\widehat{y}p_t^a = \widehat{y}p^a + \frac{1 - \rho^a}{1 - \rho^a \phi} \left( \widehat{r}_t^f - \mathbb{E}[\widehat{r}_t^f] \right).$$

The yield on the perpetuity governs the dynamics of the aggregate wealth-consumption ratio. When the yield increases, the wealth-consumption ratio decreases and vice versa. [Lustig et al. \(2013\)](#) estimate the wealth-consumption ratio and show that it tracks the inverse of long-term real bond yields closely.<sup>4</sup>

Aggregate consumption equals aggregate labor income plus aggregate financial income. We assume that the factor shares are constant. As a result, the aggregate human wealth-labor income ratio  $\widehat{h}y_t^a$  is identical to the aggregate wealth consumption ratio  $\widehat{w}c_t^a$ .

Next, we turn to the dynamics of the household's wealth-consumption ratio  $\widehat{w}c_t^i$ . To focus on the valuation effects, we assume that consumption shares  $\Delta \widehat{c}_t^i$  (labor income shares) follow a random walk with drift  $\mu_i^c$  ( $\mu_i^y$ ). We assume that this is the only source of heterogeneity other than the initial shares  $\widehat{c}_0^i$ .<sup>5</sup> The household's log human wealth-income ratio is denoted by  $\widehat{h}y_t^i$ . The household's log wealth-consumption ratio equals the PDV of future consumption share growth and risk-free rates:

$$\widehat{w}c_t^i = \frac{k_i^c + \mu_i^c}{1 - \rho_i^c} + \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \Delta \widehat{c}_{t+j}^i - \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \widehat{r}_{t+j-1}^f.$$

where the linearization constants  $\rho_i^c$  and  $k_i^c$  are defined analogously to their aggregate counterparts  $\rho^a$  and  $k^a$ .

**Corollary 4.1.** The log wealth-consumption ratio (human wealth-income ratio) of household  $i$  relative to the aggregate ratio is given by:

$$\begin{aligned} \widehat{w}c_t^i - \widehat{w}c_t^a &= \frac{k_i^c + \mu_i^c}{1 - \rho_i^c} - \frac{k^a}{1 - \rho^a} + \frac{\mathbb{E}[\widehat{r}^f](\rho^a - \rho_i^c)}{(1 - \rho_i^c)(1 - \rho^a)} + \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} (\widehat{w}c_t^a - \widehat{w}c^a) \\ \widehat{h}y_t^i - \widehat{w}c_t^a &= \frac{k_i^h + \mu_i^h}{1 - \rho_i^h} - \frac{k^a}{1 - \rho^a} + \frac{\mathbb{E}[\widehat{r}^f](\rho^a - \rho_i^h)}{(1 - \rho_i^h)(1 - \rho^a)} + \frac{\phi(\rho_i^h - \rho^a)}{(1 - \rho_i^h \phi)} (\widehat{w}c_t^a - \widehat{w}c^a), \\ \widehat{w}c_t^i &= \frac{k_i^c - \mathbb{E}[\widehat{r}^f] + \mu_i^c}{1 - \rho_i^c} - \frac{1 - \rho^a \phi}{1 - \rho_i^c \phi} \frac{(\widehat{y}p_t^a - \widehat{y}p^a)}{(1 - \rho^a)}, \quad \widehat{w}c_t^a = \frac{k^a - \mathbb{E}[\widehat{r}^f]}{1 - \rho^a} - \frac{(\widehat{y}p_t^a - \widehat{y}p^a)}{(1 - \rho^a)}. \end{aligned}$$

To develop some intuition, consider a simple cross-section with only “workers” and “capital-  
and the yield on the perpetuity will only reflect future risk-free rates.

<sup>4</sup>If we allow for bond risk premia that do not vary over time, the risk premium would add an additional constant to the expression for the yield on the perpetuity.

<sup>5</sup>This assumption is not essential, but it makes the analysis that follows more tractable. The next section considers households that choose the optimal consumption path in a dynamic general equilibrium incomplete markets economy. Here, we want to focus on valuation effects.

ists." The capitalists have lower duration of their human wealth:  $\rho_w^h > \rho^a > \rho_{cap}^h$ . Assume that the duration of their consumption claim does not differ:  $\rho_w^c = \rho^a = \rho_{cap}^c$ . Their initial consumption and total wealth are also identical. Now consider a decrease in long rates that pushes up the aggregate wealth-consumption ratio. The distribution of total wealth does not change in response to  $wc_t^a$  by virtue of the equal consumption durations:  $wc_t^w - wc_t^{cap} = 0$ . However, the human wealth of the workers goes up by more than the human wealth of the capitalists:

$$\widehat{hy}_t^w - \widehat{hy}_t^{cap} = const + \left( \frac{\phi(\rho_w^h - \rho^a)}{1 - \rho_w^h \phi} - \frac{\phi(\rho_{cap}^h - \rho^a)}{1 - \rho_{cap}^h \phi} \right) (\widehat{wc}_t^a - \widehat{wc}^a),$$

because  $\rho_w^h > \rho^h > \rho_{cap}^h$ .

The financial wealth (FW) of the capitalists, which is given by the difference between their total wealth and human wealth  $\exp(\widehat{c}_t^{cap} + \widehat{wc}_t^{cap}) - \exp(\widehat{y}_t^{cap} + \widehat{hy}_t^{cap})$ , increases when rates decline ( $\widehat{wc}_t^a - \widehat{wc}^a > 0$ ):

$$FW_t^{cap} = \exp(\widehat{wc}_t^a) \left( \exp(\widehat{c}_t^{cap}) - \exp(\widehat{y}_t^{cap} + \frac{\mathbb{E}[\widehat{r}^f](\rho^a - \rho_{cap}^h)}{(1 - \rho_{cap}^h)(1 - \rho^a)} + \frac{\phi(\rho_{cap}^h - \rho^a)}{(1 - \rho_{cap}^h \phi)} (\widehat{wc}_t^a - \widehat{wc}^a)) \right)$$

The capitalists suffer a relative decline in human wealth that is offset by an increase in their financial wealth, leaving total wealth unchanged.

Next, we characterize the cross-sectional variance of total wealth assuming that all households have the same initial consumption. We use subscript  $xs$  to denote cross-sectional moments. The main result in this section is that there is a negative relationship between long rates and the cross-sectional dispersion of total wealth.

**Proposition 4.2.** If there is no initial consumption dispersion and wealth is log-normally distributed, the cross-sectional coefficient of variation of total wealth (TW) is bounded below by:

$$\frac{Std_{xs}(TW)}{\mathbb{E}_{xs}(TW)} \approx std_{xs}(w) = \left( Var_{xs} \left[ \widehat{wc}_t^i + \widehat{c}_t^i \right] \right)^{1/2} \geq \left( Var_{xs} \left[ \widehat{wc}^i \right] + Var_{xs} \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} \right] (\widehat{wc}_t^i) \right)^{1/2}.$$

The cross-sectional coefficient of variation of human wealth (HW) is bounded below by:

$$\frac{Std_{xs}(HW)}{\mathbb{E}_{xs}(HW)} \approx std_{xs}(h) = \left( Var_{xs} \left[ \widehat{hy}_t^i + \widehat{y}_t^i \right] \right)^{1/2} \geq \left( Var_{xs} \left[ \widehat{hy}^i \right] + Var_{xs} \left[ \frac{\phi(\rho_i^h - \rho^a)}{(1 - \rho_i^h \phi)} \right] (\widehat{wc}_t^i) \right)^{1/2}.$$

It immediately follows from the proposition that when long rates decline and the aggregate wealth-consumption ratio increases, the cross-sectional dispersion of total wealth and human wealth increase (at least weakly). The size of the increase in wealth inequality increases in the

cross-sectional variance of the duration of total wealth:

$$Var_{xs} \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} \right].$$

To derive results for financial wealth, we need to make an additional assumption, namely that financial and human wealth vary negatively. We use  $\alpha$  to denote the capital income share.

**Proposition 4.3.** *If there is no initial consumption dispersion, human and total wealth are log-normally distributed, and human wealth and financial wealth covary negatively, then the cross-sectional coefficient of variation of financial wealth (FW) is bounded below by:*

$$\frac{Std_{xs}(FW)}{\mathbb{E}_{xs}(FW)} \geq \sqrt{\frac{1}{(1-\alpha)^2} \left( Var_{xs} [\widehat{wc}^i] + Var_{xs} \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} \right] (\widehat{wc}_t^a)^2 \right) + \frac{\alpha^2 - 2\alpha}{(1-\alpha)^2} \left( Var_{xs} [\widehat{hy}^i] + Var_{xs} \left[ \frac{\phi(\rho_i^h - \rho^a)}{(1 - \rho_i^h \phi)} \right] (\widehat{wc}_t^a)^2 \right)}.$$

If the cross-sectional variance of total wealth is smaller than the variance of human wealth, which seems plausible when financial and human wealth are negatively correlated and given that households seek to smooth consumption, we also obtain the following bound:

$$\frac{Std_{xs}(FW)}{\mathbb{E}_{xs}(FW)} \geq std_x(a) = \left( Var_{xs} [\widehat{wc}_t^i + \tilde{c}_t^i] \right)^{1/2} \geq \left( Var_{xs} [\widehat{wc}^i] + Var_{xs} \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} \right] (\widehat{wc}_t^a)^2 \right)^{1/2}.$$

When rates go down and  $\widehat{wc}_t^a$  goes up, financial wealth inequality rises.

This section has shown that there is a tight connection between financial wealth inequality and total wealth inequality on the one hand and long rates on the other hand under minimal assumptions. When long-term real rates decline, we expect measures of inequality to increase simply because wealth is being marked-to-market and different households have different exposure to short rates. This is consistent with the evidence in Section 3. Next, we analyze this relationship in a dynamic general equilibrium model where consumption is optimally determined and interest rates are set in equilibrium.

## 5 Incomplete Markets Model

To analyze the effects of changes in discount rates on the distribution of wealth, we use a standard [Bewley \(1986\)](#) endowment economy in which ex-ante identical agents face idiosyncratic and aggregate risk. We use an endowment economy to isolate the valuation effects. We first show how to solve this model by transforming the problem into a stationary model without aggregate risk. Next, we use the model to arrive at a method of valuing individual human wealth that is consistent with aggregation. Third, we let the economy undergo a decline in the interest rate, arising

from a slowdown in expected economic growth, and show that this increases the inequality in financial wealth.

## 5.1 Endowments

Time is discrete, infinite, and indexed by  $t \in [0, 1, 2, \dots]$ . The aggregate endowment  $e$  follows the stochastic process:

$$e_t(z^t) = e_{t-1}(z^{t-1})\lambda_t(z_t)$$

where  $\lambda(z_t)$  denotes the stochastic growth rate of the aggregate endowment and  $z_t$  the aggregate state. The history of aggregate shocks is denoted by  $z^t = \{z_t, z_{t-1}, \dots\}$ . A share  $\alpha_t(z_t)$  of the aggregate endowment is financial income, the remaining  $1 - \alpha_t(z_t)$  share represents aggregate labor income.

Household labor income  $y$  follows the stochastic process:

$$y_t(s^t) = \hat{y}_t(z^t, \eta^t)(1 - \alpha_t(z_t))e_t(z^t),$$

Households are subject to idiosyncratic income shocks, whose history is denoted by  $\eta^t$ . The ratio of individual to aggregate labor income, which we refer to as the labor income share, is given by  $\hat{y}_t(z^t, \eta^t)$ . The  $\eta_t$  shocks are i.i.d. across households and persistent over time. The idiosyncratic shock process is assumed to be independent from the aggregate shock process. We use  $s^t = (z^t, \eta^t)$  to summarize the history of aggregate and idiosyncratic shocks, and  $\pi(s^t) = \pi(z^t, \eta^t)$  to denote the unconditional probability that state  $s^t$  will be realized. If the aggregate and idiosyncratic states are independently distributed, then we can decompose state transition probabilities into an aggregate and idiosyncratic component:

$$\pi(z_{t+1}, \eta_{t+1} | z^t, \eta^t) = \phi(z_{t+1} | z^t) \varphi(\eta_{t+1} | \eta^t).$$

## 5.2 Preferences

Households maximize discounted expected utility:

$$\mathbb{E}U(c) = \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{c(s^t)^{1-\gamma}}{1-\gamma},$$

where the coefficient of relative risk aversion  $\gamma > 1$ , and the subjective time discount factor  $0 < \beta < 1$ .

### 5.3 Technology

Households trade state-contingent bonds  $a_t(s^t, z_{t+1})$  at prices  $q_t(z^t, z_{t+1})$  and shares in the Lucas tree  $\sigma_t(s^t)$  at price  $v_t(z^t)$  satisfying the budget constraint:

$$c_t(s^t) + \sum_{z_{t+1}} a_t(s^t, z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(s^t) v_t(z^t) \leq W_t(s^t).$$

Household cash on hand  $W$  evolves according to:

$$W_{t+1}(s^{t+1}) = a_t(s^t, z_{t+1}) + \hat{y}_{t+1}(\eta^{t+1}, z^{t+1})(1 - \alpha(z_{t+1}))e_{t+1}(z^{t+1}) + \left( \alpha(z_{t+1})e_{t+1}(z^{t+1}) + v_{t+1}(z^{t+1}) \right) \sigma_t(s^t).$$

Households are subject to state-uncontingent and state-contingent solvency constraints:

$$\sum_{z_{t+1}} a_t(s^t, z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(s^t) v_t(z^t) \geq K_t(s^t)$$

$$a_t(s^t, z_{t+1}) + \left( \alpha(z_{t+1})e_{t+1}(z^{t+1}) + v_{t+1}(z^{t+1}) \right) \sigma_t(s^t) \geq M_t(s^t, z_{t+1})$$

where  $K$  and  $M$  denote generic borrowing limits. Incomplete risk sharing arises from two sources: the lack of an asset whose payoff depends on the idiosyncratic income shock  $\eta^t$  and the borrowing constraints.

### 5.4 Transformation into Stationary Economy

We can transform the stochastically growing economy into a stationary economy with a constant aggregate endowment following [Alvarez and Jermann \(2001\)](#); [Krueger and Lustig \(2010\)](#). To that end, define the deflated consumption allocations:

$$\hat{c}_t(s^t) = \frac{c_t(s^t)}{e_t(z^t)}, \forall s^t,$$

the deflated transition probabilities and the deflated subjective time discount factor::

$$\begin{aligned} \hat{\pi}(s_{t+1}|s^t) &= \frac{\pi(s_{t+1}|s^t) \lambda_{t+1}(z_{t+1})^{1-\gamma}}{\sum_{s_{t+1}} \pi(s_{t+1}|s^t) \lambda_{t+1}(z_{t+1})^{1-\gamma}}, \\ \hat{\beta}(s^t) &= \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t) \lambda_{t+1}(z_{t+1})^{1-\gamma}. \end{aligned}$$

Agents in the deflated economy with these preferences:

$$U(\hat{c})(s^t) = \frac{\hat{c}(s^t)^{1-\gamma}}{1-\gamma} + \sum_{s_{t+1}} \hat{\beta}(s_{t+1}, s^t) \hat{\pi}(s_{t+1}|s^t) U(\hat{c})(s_{t+1}, s^t) \quad (1)$$



rank consumption plans identically as in the original economy. Under the maintained assumption of independence of aggregate and idiosyncratic risk, the deflated aggregate transition probabilities and the deflated time discount factor are:

$$\begin{aligned}\widehat{\phi}(z_{t+1}|z^t) &= \frac{\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}}{\sum_{z_{t+1}}\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}}, \\ \widehat{\beta}(z^t) &= \beta \sum_{z_{t+1}}\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}.\end{aligned}$$

These are risk-neutral probabilities. When there is predictability in aggregate consumption growth, shocks to expected growth manifest themselves as taste shocks in the deflated economy. If aggregate growth shocks are i.i.d. over time, then the deflated time discount factor is constant and given by:

$$\widehat{\beta} = \beta \sum_{z_{t+1}}\phi(z_{t+1})\lambda_{t+1}(z_{t+1})^{1-\gamma}.$$

This i.i.d. assumption on aggregate growth shocks is the assumption we will make, noting that it can easily be relaxed. In what follows, we also assume that aggregate factor shares are constant:  $\alpha_t(z_t) = \alpha, \forall t$ . By definition, labor income shares average to one across households:

$$\sum_{\eta^t} \varphi(\eta^t|\eta_0)\widehat{y}_t(\eta^t) = 1$$

## 5.5 Equilibrium in the Stationary Economy

Agents trade a single risk-free bond and a stock. The stock yields a dividend  $\alpha$  in each period. Given initial financial wealth  $\theta_0$ , interest rates  $\widehat{R}_t$  and stock prices  $\widehat{v}_t$ , households choose consumption  $\{\widehat{c}_t(\theta_0, \eta^t)\}$ , bond positions  $\{\widehat{a}_t(\theta_0, \eta^t)\}$ , and stock positions  $\{\widehat{\sigma}_t(\theta_0, \eta^t)\}$  to maximize expected utility (1) subject to the budget constraint:

$$\widehat{c}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t)}{\widehat{R}_t} + \widehat{\sigma}_t(\eta^t)\widehat{v}_t = (1 - \alpha)\widehat{y}_t(\eta^t) + \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha),$$

and subject to borrowing constraints:

$$\frac{\widehat{a}_t(\eta^t)}{\widehat{R}_t} + \widehat{\sigma}_t(\eta^t)\widehat{v}_t \geq \widehat{K}_t(\eta^t), \quad \forall \eta^t$$

$$\widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha) \geq \widehat{M}_t(\eta^t), \quad \forall \eta^t.$$

**Definition 1.** For a given initial distribution of wealth  $\Theta_0$ , a Bewley equilibrium is a list of consumption choices  $\{\widehat{c}_t(\theta_0, \eta^t)\}$ , bond positions  $\{\widehat{a}_t(\theta_0, \eta^t)\}$ , and stock positions  $\{\widehat{\sigma}_t(\theta_0, \eta^t)\}$  as well as stock prices  $\widehat{v}_t$ , and interest rates  $\widehat{R}_t$  such that each household maximizes its expected utility,

and asset markets and goods markets clear.

$$\begin{aligned}\int \sum_{\eta^t} \varphi(\eta^t | \eta_0) \widehat{a}_t(\theta_0, \eta^t) d\Theta_0 &= 0, \\ \int \sum_{\eta^t} \varphi(\eta^t | \eta_0) \widehat{\sigma}_t(\theta_0, \eta^t) d\Theta_0 &= 1. \\ \int \sum_{\eta^t} \varphi(\eta^t | \eta_0) \widehat{c}_t(\theta_0, \eta^t) d\Theta_0 &= 1.\end{aligned}$$

In the deflated economy without aggregate risk, the return on the aggregate stock equals the risk-free rate:

$$\widehat{R}_t = \frac{\widehat{v}_{t+1} + \alpha}{\widehat{v}_t}. \quad (2)$$

The equilibrium stock price equals the present discounted value of the dividends:

$$\widehat{v}_t = \sum_{\tau=0}^{\infty} \widehat{R}_{t \rightarrow t+\tau}^{-1} \alpha,$$

discounted at the cumulative gross risk-free rate, defined as:  $\widehat{R}_{t \rightarrow t+T} = \prod_{k=0}^T \widehat{R}_{t+k}$ . Note that  $\widehat{R}_{t \rightarrow t} = \widehat{R}_t$  and define  $\widehat{R}_{t \rightarrow t-1} = 1$ .

## 5.6 Equilibrium in the Growing Economy

We can map the equilibrium in the detrended economy into an equilibrium in the stochastically growing economy.

**Proposition 5.1.** If  $\{\widehat{c}_t(\theta_0, \eta^t), \widehat{a}_t(\theta_0, \eta^t), \widehat{\sigma}_t(\theta_0, \eta^t)\}$  and  $\{\widehat{v}_t, \widehat{R}_t\}$  are a Bewley equilibrium, then  $\{c_t(\theta_0, s^t), a_t(\theta_0, s^t, z_{t+1}), \sigma_t(\theta_0, s^t)\}$  as well as asset prices  $\{v_t(z^t), q_t(z^t, z_{t+1})\}$  are an equilibrium of the stochastically growing economy with:

$$\begin{aligned}c_t(\theta_0, s^t) &= \widehat{c}_t(\theta_0, \eta^t) e_t(z^t) \\ a_t(\theta_0, s^t, z_{t+1}) &= \widehat{a}_t(\theta_0, \eta^t) e_t(z^t) \\ \sigma_t(\theta_0, s^t) &= \widehat{\sigma}_t(\theta_0, \eta^t) \\ v_t(z^t) &= \widehat{v}_t e_t(z^t) \\ q_t(z^t, z_{t+1}) &= \frac{\widehat{\phi}(z_{t+1})}{\lambda(z_{t+1})} \frac{1}{\widehat{R}_t}.\end{aligned}$$

The proof is provided in [Krueger and Lustig \(2010\)](#). The last equation implies the following

relationship between the interest rate in the growing economy and the stationary economy:

$$R_t = \left( \sum_{z_{t+1}} q_t(z^t, z_{t+1}) \right)^{-1} = \left( \sum_{z_{t+1}} \frac{\hat{\phi}(z_{t+1})}{\lambda(z_{t+1})} \right)^{-1} \hat{R}_t. \quad (3)$$

## 5.7 Wealth Accounting

What is the right discount rate when measuring household wealth? If we want a measure that can be aggregated, we have to use the same discount rate for all claims.

**Proposition 5.2.** At time 0, the financial wealth of each household equals the present discounted value of future consumption minus future labor income.

$$\theta_0 = \sum_{\tau=0}^{\infty} \sum_{\eta^\tau} \frac{\varphi(\eta^\tau)}{\hat{R}_{0 \rightarrow \tau-1}} (\hat{c}_\tau(\eta^\tau) - (1 - \alpha)\hat{y}_\tau(\eta^\tau))$$

As the proof in the appendix shows, the proposition follows easily from iterating forward on the one-period budget constraint. In this iteration, we take expectations over financial wealth in all future states using the objective probabilities of the idiosyncratic events  $\varphi(\eta^\tau)$ , and discount by the cumulative risk-free rate  $\hat{R}_{0 \rightarrow \tau-1}$ .

Aggregate financial wealth in the economy in period 0 is given by:

$$\int \theta_0 d\Theta_0 = \int (\hat{a}_{-1}(\theta_0) + \hat{\sigma}_{-1}(\theta_0)\hat{v}_0) d\Theta_0 = 0 + 1\hat{v}_0,$$

where we have used market clearing in the bond and stock markets at time 0.

Aggregating the cost of the excess consumption plan across all households, using the fact that labor income shares average to 1, and imposing goods market clearing at time 0, we get:

$$\int \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) (\hat{c}_\tau(\eta^\tau) - (1 - \alpha)\hat{y}_\tau(\eta^\tau)) d\Theta_0 = \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1} \alpha = \hat{v}_0.$$

The aggregate cost of households' excess consumption plan, or households' aggregate financial wealth, exactly equals the stock market value  $\hat{v}_0$ , the only source of net financial wealth in the economy. This result relies on market clearing:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau) (\hat{c}_\tau(\eta^\tau) - (1 - \alpha)\hat{y}_\tau(\eta^\tau)) d\Theta_0 = \alpha,$$

at each time  $t$ , because  $\int \sum_{\eta^\tau} \varphi(\eta^\tau) \hat{c}_\tau(\eta^\tau) d\Theta_0 = 1$  from market clearing, and the labor income shares sum to one as well.

The choice of the actual probability measure  $\varphi(\cdot)$  and rate  $\hat{R}$  to compute an individual's human capital, the expected present discounted value of her labor income stream, may seem arbitrary. Af-

ter all, claims to labor income are not traded in this model and markets are incomplete. The key insight is that, using any other pricing kernel to discount individual labor income and consumption streams may result in a value of aggregate financial wealth different from the value of the Lucas tree. To see this, consider using an arbitrary measure  $\psi(\eta^\tau)\varphi(\eta^\tau)$  different from the actual measure  $\varphi(\eta^\tau)$ , where the household-specific wedges satisfy  $\mathbb{E}_0[\psi_t] = 1, \forall t$ . Under this different measure, the goods markets do not clear and the labor shares do not sum to one, unless the household-specific wedges do not covary with consumption and income shares:

**Proposition 5.3.** Wealth measures aggregate if and only if the following orthogonality conditions holds for the household-specific wedges and household consumption and income:

$$Cov_0(\psi_t, \hat{c}_t) = 0, \quad Cov_0(\psi_t, \hat{y}_t) = 0.$$

For all other wedge processes  $\psi_t(\eta^\tau)$ , the resource constraint is violated:

$$\int \sum_{\eta^\tau} \psi(\eta^\tau)\varphi(\eta^\tau) (\hat{c}_\tau(\eta^\tau) - (1 - \alpha)\hat{y}_\tau(\eta^\tau)) d\Theta_0 \neq \alpha,$$

It is common in the literature to use the household's own IMRS to compute human capital (e.g., [Huggett and Kaplan, 2016](#)). The household's IMRS is a natural choice because it ties the valuation of human wealth directly to welfare. However, this approach does not lend itself to aggregation. The wedges

$$\psi(\eta^{t+1}) = \frac{u'(\hat{c}(\eta_{t+1}, \eta^t))}{u'(\hat{c}_t(\eta_0))},$$

do not satisfy the zero covariance restrictions of the proposition. Imperfect consumption insurance implies that:

$$Cov_0(\psi_t, \hat{c}_t) \leq 0, \quad Cov_0(\psi_t, \hat{y}_t) \leq 0.$$

**Proposition 5.4.** When the cross-sectional covariance between the household-specific wedges and consumption is negative ( $Cov_0(\psi_t, \hat{c}_t) \leq 0$ ), then the aggregate valuation of individual wealth is less than the market's valuation of total wealth.

Since the factor shares are constant, the consumption claim is in the span of traded assets. The value of the Lucas tree is  $\alpha$  times the value of a claim to total consumption. When aggregating, this pricing functional undervalues human wealth and total wealth. In sum, while pricing claim to consumption and labor income using the household's IMRS is sensible from a welfare perspective, this approach does not lend itself to wealth accounting and aggregation.

## 5.8 Interest Rate Decline

We now analyze the main exercise of the paper, which is to let the economy undergo an unexpected and permanent decrease in the interest rate (“MIT shock”). Since interest rates are endogenously determined, we generate this decrease through a decrease in the expected growth rate of the economy:

$$\mathbb{E}[\lambda] = \sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1}) \rightarrow \mathbb{E}[\tilde{\lambda}] = \sum_{z_{t+1}} \phi(z_{t+1})\tilde{\lambda}(z_{t+1})$$

where  $\mathbb{E}[\tilde{\lambda}] < \mathbb{E}[\lambda]$ . A lower expected growth rate manifests itself as a higher subjective time discount factor in the stationary economy, provided that the coefficient of relative risk aversion  $\gamma > 1$ , or, equivalently, the elasticity of intertemporal substitution is smaller than one:

$$\tilde{\beta} = \beta \sum_{z_{t+1}} \phi(z_{t+1})\tilde{\lambda}_{t+1}(z_{t+1})^{1-\gamma} > \hat{\beta}.$$

In the transformed incomplete markets economy, the size of the decline in the rate of time preference is governed by the EIS ( $1/\gamma$ ), exactly as the risk-free rate is determined in the representative agent economy: the larger the EIS, the smaller the effect of a decline in the expected growth rate of aggregate consumption on the risk-free rate.

In the simple case of log-normally distributed aggregate consumption growth, we obtain the following expression for the rate of time preference in the stationary economy:

$$\log \hat{\beta} = \log \beta - \gamma \mathbb{E}[\log \lambda] - \frac{1}{2} \gamma (1 - \gamma) \text{Var}[\lambda]. \quad (4)$$

Hence, the change in the transformed rate of time preference in response to the growth shock is given by:  $\frac{d \log \hat{\beta}}{d \log \mathbb{E}[\log \lambda]} = -\gamma$ .

It is natural to ask whether we can still implement the equilibrium consumption allocation  $\{\hat{c}_t(\theta_0, \eta^t)\}$  from the economy with high rates in the economy with low rates. Given that the time discount factor of all agents increased by the same amount, there should be no motive to trade away from these allocations. The following proposition shows that the old consumption allocation is indeed still an equilibrium in the low interest rate economy, provided that initial financial wealth is scaled up for every household.

**Proposition 5.5.** If the allocations and asset market positions  $\{\hat{c}_t(\theta_0, \eta^t), \hat{a}_t(\theta_0, \eta^t), \hat{v}_t(\theta_0, \eta^t)\}$  and asset prices  $\{\hat{v}_t, \hat{R}_t\}$  are a Bewley equilibrium in the economy with  $\hat{\beta}$  and natural borrowing limits  $\{\hat{K}_t(\eta^t)\}$ ,

$$\hat{K}_t(\eta^t) = \sum_{\tau=t}^{\infty} \hat{R}_{t \rightarrow \tau}^{-1} \sum_{\eta^\tau | \eta^t} \varphi(\eta^\tau | \eta^t) (1 - \alpha) \hat{y}_\tau(\eta^\tau),$$

then the allocations and asset market positions  $\{\hat{c}_t(\tilde{\theta}_0, \eta^t), \hat{a}_t(\tilde{\theta}_0, \eta^t), \hat{v}_t(\tilde{\theta}_0, \eta^t)\}$  and asset prices

$\{\tilde{v}_t, \tilde{R}_t\}$  will be an equilibrium of the economy with  $\tilde{\beta}$  and natural borrowing limits  $\{\tilde{K}_t(\eta^t)\}$ ,

$$\tilde{K}_t(\eta^t) = \sum_{\tau=t}^{\infty} \tilde{R}_{t \rightarrow \tau-1}^{-1} \sum_{\eta^\tau | \eta^t} \varphi(\eta^\tau | \eta^t) (1 - \alpha) \hat{y}_\tau(\eta^\tau),$$

asset prices are given by

$$\tilde{\beta} \tilde{R}_t = \hat{\beta} \hat{R}_t, \text{ and } \tilde{v}_t = \sum_{\tau=0}^{\infty} \tilde{R}_{t \rightarrow t+\tau}^{-1} \alpha,$$

and every household's initial wealth is adjusted as follows:

$$\tilde{\theta}_0 = \theta_0 \frac{\sum_{\tau=0}^{\infty} \tilde{R}_{0 \rightarrow \tau}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) (\hat{c}_\tau(\eta^\tau) - (1 - \alpha) \hat{y}_\tau(\eta^\tau))}{\sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) (\hat{c}_\tau(\eta^\tau) - (1 - \alpha) \hat{y}_\tau(\eta^\tau))}.$$

The proof is in the appendix. Aggregate financial wealth undergoes an adjustment equal to the ratio of the price of two perpetuities:

$$\frac{\sum_{\tau=0}^{\infty} \tilde{R}_{0 \rightarrow \tau}^{-1}}{\sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau}^{-1}} = \frac{\tilde{v}_0}{\hat{v}_0}.$$

Intuitively, with lower interest rates, all asset prices are higher than in the high-rate economy. The Lucas tree becomes more valuable. A fraction  $1 - \alpha$  of this tree reflects aggregate human wealth, the remaining fraction is aggregate financial wealth. Each individual's financial wealth adjustment differs, and depends on the expected discounted value of the same future excess consumption plan discounted at different rates. The higher one's expected future excess consumption, the larger the initial financial wealth adjustment needed to implement the old equilibrium allocation.

To a first-order approximation, i.e., for a small change in the interest rate, the adjustment in initial financial wealth needed for agents to keep their initial consumption plan is given by the duration of their planned consumption in excess of labor income. This is the duration households will need in their financial assets and liabilities in order to be fully hedged.

**Characterizing Interest Rate Sensitivity Using Duration of Excess Consumption** Define the duration of a household's excess consumption plan at time 0, following the realization of the idiosyncratic labor income shock  $\eta_0$ , as follows:

$$D^{c-y}(\theta_0, \eta_0) = \frac{\sum_{\tau=0}^{\infty} \sum_{\eta^\tau | \eta_0} \tau \hat{R}_{0 \rightarrow \tau}^{-1} \varphi(\eta^\tau | \eta_0) (\hat{c}_\tau(\eta^\tau | \eta_0) - (1 - \alpha) \hat{y}_\tau(\eta^\tau | \eta_0))}{\sum_{\tau=0}^{\infty} \sum_{\eta^\tau | \eta_0} \varphi(\eta^\tau | \eta_0) \hat{R}_{0 \rightarrow \tau}^{-1} (\hat{c}_\tau(\eta^\tau | \eta_0) - (1 - \alpha) \hat{y}_\tau(\eta^\tau | \eta_0))}$$

The duration measures the sensitivity of the cost of its excess consumption plan to a change in the interest rate. This is the duration households need in their financial asset portfolio in order to be

fully hedged against interest rate risk.<sup>6,7</sup> In our endowment economy, aggregate consumption is fixed. We are interested in the valuation effects of interest rate changes.

The duration of the excess consumption claim equals the value-weighted difference of the duration of the consumption claim and that of the labor income claim:

$$D^{c-y} = \frac{P_0^c}{P_0^{c-y}} D^c - \frac{P_0^y}{P_0^{c-y}} D^y.$$

where  $P_0^{c-y} = \theta_0$  is household financial wealth,  $P_0^y$  is human wealth, and  $P_0^c$  is total household wealth, the sum of financial and human wealth. Households with a high positive duration of excess consumption face a large increase in the cost of their consumption plan when interest rates go down, insofar that this increased cost is not offset fully by the increase in their human wealth.

The duration of the aggregate excess consumption claim, the aggregate duration for short, equals:

$$D^a = \frac{\sum_{\tau=0}^{\infty} \tau \widehat{R}_{0 \rightarrow \tau}^{-1}}{\sum_{\tau=0}^{\infty} \widehat{R}_{0 \rightarrow \tau}^{-1}}$$

This is the duration of a claim to aggregate consumption minus aggregate labor income, or equivalently to aggregate financial income. It is the duration of a perpetuity in the stationary economy. Recall that  $\widehat{v}_0 = v_0 = \alpha \sum_{\tau=0}^{\infty} \widehat{R}_{0 \rightarrow \tau}^{-1}$  denotes aggregate financial wealth.

**Proposition 5.6.** The aggregate duration equals the wealth-weighted average duration of households' excess consumption claims:

$$D^a = \int D^{c-y}(\theta_0, \eta_0) \frac{\theta_0}{v_0} d\Theta_0.$$

The proof follows directly from the definition of the household specific duration measure and market clearing.

The next proposition shows that, when households that are richer than average tend to have excess consumption plans of higher duration, then the (equally-weighted) average household's excess consumption plan duration is smaller than the aggregate duration, the duration of the aggregate excess consumption.

**Proposition 5.7.** If  $cov(\theta_0, D^{c-y}(\theta_0)) > 0$  then

$$\int D^{c-y}(\theta_0, \eta_0) d\Theta_0 \leq D^a.$$

<sup>6</sup>Note that households in the detrended economy's equilibrium face a deterministic path for risk-free rates, and do not anticipate interest rate shocks.

<sup>7</sup>Auclert (2019) was the first to conduct this type of duration analysis in a model with endogenous labor supply to gauge the effects of monetary policy on consumption.

Given this condition, lower interest rates increase financial wealth inequality. In other words, if all households are perfectly hedged in their portfolio, then wealth inequality should increase.

The proof follows from recognizing the following relationship between (cross-sectional) expectations and covariances:

$$D^a = \mathbb{E} \left[ \frac{\theta_0}{\nu_0^a} D^{c-y}(\theta_0, \eta_0) \right] = \mathbb{E} [D^{c-y}(\theta_0, \eta_0)] + cov \left[ \frac{\theta_0}{\nu_0}, D^{c-y}(\theta_0, \eta_0) \right],$$

In this class of Bewley models, agents with low financial wealth have encountered a bad history of labor income shocks. If labor income is highly persistent, their labor income is low today and in the near future relative to labor income in the distant future (because of mean-reversion). This pattern makes the duration of their labor income stream high. But since the household is smoothing consumption inter-temporally,  $D^c < D^y$ . As a result, low-wealth agents tend to have low duration of their excess consumption plan. Conversely, rich agents have high labor income and high excess consumption duration. Consumption smoothing is the force that makes the covariance assumption satisfied in a Bewley model where the only source of heterogeneity is income shock realizations. It follows immediately that, under the stated covariance restriction, the increase in the cost of the excess consumption plan for the average household is smaller than the aggregate (per capita) wealth increase. Put differently, financial wealth inequality should increase when rates go down if households want to afford their old consumption plans.

Low-financial wealth households in a Bewley model have high-duration human wealth, which provides a natural interest rate hedge. High financial-wealth households have low-duration human wealth and need to increase financial wealth by more when rates decline to be able to afford the old consumption plan.

The insights of this normative proposition apply more broadly. The covariance condition can be verified in a richer model with ex-ante heterogeneity across households, like the one discussed in the next section. It can also be tested in the data, with the additional observation that households' financial portfolios may not have the same duration as their excess consumption-plans. In other words, real-world households may not be fully hedged, unlike the households in the Bewley model.

Next, we measure the actual duration of the household's financial assets in the data, denoted  $D^{fin}$ , which can differ from the duration of the excess consumption claim  $D^{c-y}$ . If they differ, the household is not hedged. We use a calibrated version of the Bewley model with overlapping generations to assess how well these households are really hedged against interest rate risk.



## 6 Calibrated Model with Ex-ante Heterogeneity

The previous section showed that in a Bewley model a rise in financial wealth inequality is required when interest rates decline when agents are fully hedged. In this section, we aim to quantify this effect in a model with realistic heterogeneity. It introduces overlapping generations of finitely-lived agents, generating heterogeneity by age. We also allow for additional heterogeneity by gender, race, and educational attainment. We investigate how the financial, human, and total wealth distributions change with low versus high interest rates.

### 6.1 Calibration

#### 6.1.1 Income Process

The income process consists of a regular component and a superstar component.

**Regular Income Component** The income process for household  $i$  in group  $g$  of age  $a$  at time  $t$  is standard in the literature and given by:

$$\log \left( y_{t,a}^{i,g} \right) = m_t + \chi' X_t^i + z_t^i, \quad (5)$$

$$z_{t+1}^i = \alpha_i + \varepsilon_{t+1}^i + v_{t+1}^i, \quad (6)$$

$$\varepsilon_{t+1}^i = \rho \varepsilon_t^i + \eta_{t+1}^i, \quad (7)$$

where  $m_t$  is a year-fixed effect and  $X_t^i$  is a vector of household characteristics that includes a cubic function of age, an indicator variable for education that is one for college completion, an indicator variable for race that is one for whites, and an indicator variable for gender that is one when male. The vector also includes the interaction of the age function with the groups dummies. Thus, there are eight groups with their own deterministic income profile: the interactions of college and non-college (C/N), white and non-white (W/O), and male and female (M/F).

The stochastic income component  $z_t^i$  contains a household-fixed effect  $\alpha^i$ , a persistent component  $\varepsilon_{t+1}^i$ , and an i.i.d. component  $v_{t+1}^i$ . We have:  $\mathbb{E}[z^i] = \mathbb{E}[\alpha^i] = E[v^i] = \mathbb{E}[\varepsilon^i] = 0$  and  $Var[v^i] = \sigma_v^2$ ,  $Var[\eta^i] = \sigma_\eta^2$ ,  $Var[\alpha^i] = \sigma_\alpha^2$ , and  $Var[\varepsilon_0^i] = \sigma_{\varepsilon,0}^2$ . Note that the income risk parameters are common across groups. The parameters are estimated by GMM using PSID data from 1970 until 2017, as detailed in Appendix A.2. Figure A5 in that appendix plots the deterministic income profile of the different groups.

The literature typically estimates (5)-(7) on labor income for white males between ages 25 and 55. We deviate from this practice in three ways, all of which are important for our purposes. First, we consider a broader income concept. Second, we consider the entire life-cycle from age 18 to 80. Third, we focus on households rather than individuals.

First, from the model’s perspective, the relevant notion of income includes transfers. It is the risk in this income that the household is hedging by trading in financial markets (borrowing and saving). To that end, we measure income in the data as income from wages and salaries, the labor income component of proprietor’s income, and government transfers (unemployment benefits, social security, other government transfers), and private defined-benefit pension income.<sup>8</sup> Obtaining consistent data on the various components of transfers is involved because successive waves of the PSID use different variable codes for the same concepts. Appendix A.2 provides the details. Catherine et al. (2020) also focuses on after-transfer income.

Second, we are interested in the entire life-cycle. We start at age 18 and go until age 80. Because our income concept includes transfers such as unemployment benefits and retirement income from public or private defined-benefit pension plans, we do not have to model labor force participation decisions or retirement decisions. Our approach captures the average decisions made in the data. For example, we do not need to make the assumption that retirement starts at age 65, that income in retirement is some constant fraction of pre-retirement income, or that income risk disappears in retirement. We can let the data speak on these issues. Since our income concept includes income from part-time work, it captures income earned by students, for example. We assign to students the educational achievement they will attain even before they have completed their education, so that they are classified in the correct group.

Third, we focus on households, aggregating income across its adult members. This absolves us from having to model demographic changes such as getting married, getting divorced, getting widowed. We simply follow households identified by the head of household as designated in the data. If women who get married are less likely to be the head of households than men who get married, then the income profile for women will be lower than that for men since there will be fewer two-earner households headed by women than by men.

**Superstar Income Component** To help the model match the level of wealth inequality in the high-interest rate regime, we enrich the income process in (5)-(7) with a superstar income state. This state has a high income level  $Y^{sup}$ . Households enter in this state with probability  $p_{12}^{sup}$  when they are in the normal income state, and return to the normal state with probability  $p_{21}^{sup}$  when they currently are in the superstar income state. The income level  $Y^{sup}$  is chosen to match the wealth gini in the 1980s exactly, which requires a value equal to 75 times average income. The transition probability parameters  $p_{12}^{sup} = 0.0002$  and  $p_{21}^{sup} = 0.975$  are taken from Boar and Midrigan (2020). There is about a 1% probability of entering in the superstar income state over one’s life-time. Conditional on entering, the state has an expected duration of 40 years.

In the computations, we discretize the stochastic income process  $z$ , with the extra superstar state, as a markov chain. Since the groups differ by deterministic income profile (but not by risk

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<sup>8</sup>In future versions, we plan to also subtract taxes using NBER taxsim software.

parameters), we simulate agents from all eight demographic groups in numbers proportional to their empirical frequency.

### 6.1.2 Mortality Risk

Households face mortality risk which differs by gender, and which is calibrated to the data. For simplicity, we assume that households in each age-gender group share mortality risk within their cohort.<sup>9</sup>

### 6.1.3 Financial Asset

We model a stationary economy with a single risk-free assets. As the previous section explained, having only safe assets is without much loss of generality since a model with aggregate risk in total income maps into a stationary economy without aggregate risk as long as the idiosyncratic and aggregate risk are uncorrelated. The presence of aggregate risk in the growing economy affects the time discount factor and hence the equilibrium risk-free rate in the stationary economy. As before, we envision that the source of the decline in interest rates is a decline in the expected growth rate of the aggregate endowment (GDP).

The risk-free asset is long-lived, modeled as a zero-coupon bond. Therefore, its duration equals its maturity. Agents start life at age 18 with zero financial assets.

### 6.1.4 Duration of Financial Wealth

The real world's counter-part to the model's financial asset is a portfolio of various financial and real assets that households own. As Table 1 shows, household assets consist of (i) cash, deposits, and money market instruments, (ii) stocks held directly and indirectly in mutual funds and pension accounts, (iii) real estate, (iv) private business wealth, and (v) fixed income assets (directly and indirectly held). Household liabilities consist of mortgage, student, and consumer debt. The duration of a financial portfolio is the weighted average duration of the components of the financial portfolio, where the weights are the portfolio weights  $\omega(k)$  of the various financial assets  $k$ :

$$D_{t,i}^{fin} = \sum_k \omega_t^i(k) D_t(k). \quad (8)$$

To measure the duration of each of the components of financial wealth,  $D_t(k)$ , we build a rich asset pricing model, detailed in Appendix D. It prices bonds of various maturities, both nominal and real, the aggregate stock market, several cross-sectional stock market factors including small stocks, and aggregate housing wealth. For these assets, the model provides a McCauley duration

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<sup>9</sup>This is implemented as a tontine system, where all agents of a certain age pool resources to eliminate mortality risk. Our results are not sensitive to this assumption.

in each quarter from 1947.Q1 until 2019.Q4. We use the durations for the 1980s, averaged across the 40 quarters in that decade.

We use the model-implied duration of the aggregate stock market to proxy for the duration of households' directly- and indirectly-held stocks. We use the duration of small stocks to proxy for the duration of household business wealth. We use the duration of owner-occupied housing wealth to measure the duration of households' real estate assets. For cash and deposits, we assume a duration of 0.25 years. For fixed income, we assume a duration of 4 years.<sup>10</sup>

For student debt, we assume a duration of 4.5 years. Student loans are typically 10 year annuities. At an interest rate of 5.8%, the average rate on outstanding student loans in 2017, the duration is 4.56. At higher the interest rates that prevailed in the 1980s, the duration would be slightly smaller. For consumer debt, we assume a duration of 1 year. Much of this debt is revolving debt, while some of it is 24-month personal loans. The personal loans are amortizing.<sup>11</sup> For mortgage debt, we obtain data for the Bloomberg-Barclays Aggregate MBS Index. It is a representative portfolio of all outstanding U.S. pass-through mortgage-backed securities. The average McCauley duration of this representative mortgage portfolio in 1989 and 1990 was 5.2 years. Most mortgage debt in the U.S. is 30-year fixed-rate mortgages. The reasons for this much lower duration than 30 are several: amortization, high interest rates, and prepayment.<sup>12</sup> The resulting durations are reported in the first column of Table 1.

Next, we collect data from the Survey of Consumer Finances (SCF) on household portfolio shares, the  $\omega_i^j(k)$  in (8). The wealth-weighted portfolio weights are reported in the remaining columns of Table 1, first when pooling all agents ("All") and then for each group separately. The details are in Appendix A.3.

We calculate the duration for each household separately, combining the household-level portfolio weights with the asset-specific durations listed in the first column. The last two rows of Table 1 report the wealth-weighted (aggregate) and equally-weighted (average) financial duration among households. When pooling all households, the average financial duration is 15.96. This is the value we use in the version of the model below with homogenous financial duration.

As shown in Proposition 5.7, if the aggregate duration exceeds the average duration of financial wealth, then a reduction in interest rates leads to a rise in financial wealth inequality. This condition, which results from a positive cross-sectional covariance between duration of financial wealth and financial wealth itself, is clearly satisfied in the U.S. data. It occurs because richer

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<sup>10</sup>For reference, the maturity of outstanding U.S. Treasury marketable securities averages 62 months between 2000 and 2020. The duration is strictly smaller than the maturity since bonds pay coupons. For example, if the coupon rate is 4.65% and the bond pays semi-annual coupons, then the duration is 4.5 years. Other corporate and international bonds and loans held by U.S. households tends to have somewhat lower duration than U.S. Treasuries because there are fewer long-term bonds and coupons are higher.

<sup>11</sup>We exclude auto debt since we also exclude vehicles from assets. The reason is that our consumption measure includes durable consumption.

<sup>12</sup>The average maturity of the outstanding MBS portfolio in 1989-1990 was 9.8 years and the average coupon rate was 9.35%.

Table 1: Duration of the Household Financial Wealth Portfolio 1980s

	Duration	Portfolio Shares								
		All	MWC	MWN	MOC	MON	FWC	FWN	FOC	FON
<b>Assets</b>										
Cash and Deposits	0.25	11.51	9.33	12.34	9.40	9.66	14.50	23.08	12.55	5.55
Equities	28.70	11.52	17.25	7.32	7.92	1.78	13.57	6.41	3.16	2.07
Real Estate	14.70	48.38	38.60	50.09	52.81	95.58	55.50	61.27	102.25	109.27
Private Business Wealth	61.10	24.37	26.33	26.77	37.33	14.17	6.94	7.65	2.52	0.96
Fixed Income	4.00	17.62	21.00	15.65	12.92	14.70	20.04	11.57	6.04	7.04
<b>Liabilities</b>										
Mortgage Debt	5.20	12.03	11.09	11.10	18.25	33.48	9.26	8.86	15.73	20.83
Student Debt	4.50	0.27	0.26	0.15	0.29	0.40	0.92	0.23	4.33	1.58
Other Debt	1.00	1.10	1.16	0.92	1.83	2.01	0.38	0.88	6.47	2.48
<b>Aggregate Duration</b>		25.39	26.97	25.88	32.40	22.05	16.60	15.56	16.67	16.36
<b>Average Duration</b>		15.96	20.40	17.19	15.84	15.15	12.61	10.63	11.33	10.55

*Note:* The column “Duration” reports the duration of the asset, again averaged over all quarters in the 1980s. For Equities, Private Business Wealth, and Real Estate, the durations are computed from the asset pricing model in Appendix D, averaging across the 40 quarters in the 1980s. The column “All” reports the wealth-weighted average or aggregate portfolio weights. Liabilities receive negative portfolio weights. The next eight columns also report wealth-weighted average portfolio shares, but for the eight demographic groups. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college. These weights are based on the 1989 SCF.

households tend to hold more private business wealth, equities, and housing wealth, which are long-duration assets, hold fewer short-duration assets (cash), and hold less debt (negative duration). The goal of this section is to analyze how much financial wealth inequality the model can generate given the observed decline in real interest rates.

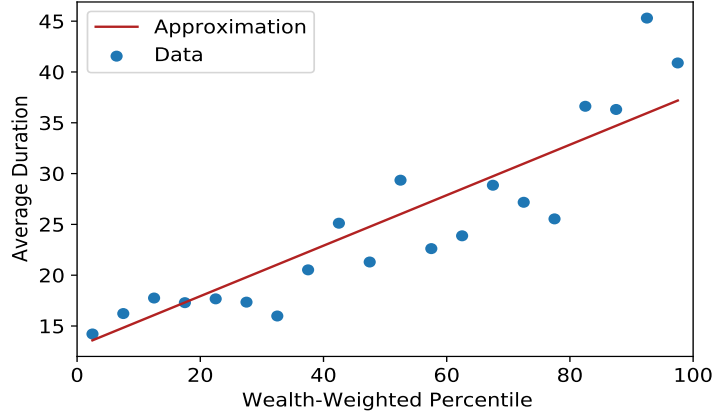
In the data, financial duration is strongly correlated with the level of financial wealth. Using the SCF data, the dots in Figure 3 plots the average duration by wealth group. The first wealth group contains the lowest-wealth households who collectively own 5% of financial wealth. The next group contains the next-lowest-wealth households who combined own the next 5% of financial wealth. Together with the first group, they account for 10% of total financial wealth. The last group contains the richest households who together hold 5% of financial wealth. Because of the unequal distribution of financial wealth, the last group is the smallest in terms of number of members. The figure shows that wealthier households hold longer-duration financial portfolios.

We consider a version of the model with heterogeneous duration of financial wealth. For this version, we assume that the duration of household  $i$  in group  $g$  is given by:

$$D_{i,g}^{fin} = D_g^{LB} + (D_g^{UB} - D_g^{LB})Lorenz_i \tag{9}$$

where  $Lorenz_i$  is the value of the Lorenz curve at household  $i$ , equal to the share of total wealth held by households in that group with wealth less than or equal to that of household  $i$ . This

Figure 3: Duration by Net Worth, SCF Data



*Note:* This plot compares durations in the data against the model approximation (9). The scatterplot displays average durations from the Survey of Consumer Finances, binned by net wealth. Households are sorted by their net wealth and allocated to different bins in such a way that the share of total wealth held by each bin is equal. Data are from the SCF survey in 1989. The solid red line displays the model’s approximation (9).

specification is equivalent to a linear interpolation over Figure 3 by group, varying from duration  $D_g^{LB}$  at the least wealthy household in the group to duration  $D_g^{UB}$  at the most wealthy household in the group. We choose  $D_g^{LB}$  and  $D_g^{UB}$  to exactly match both the average duration and the aggregate duration that we found in the data, reported in the last two rows in Table 1. We do this separately for each demographic group, resulting in eight pairs of parameters  $(D_g^{LB}, D_g^{UB})$ . To illustrate this procedure, the red line in Figure 3 shows the model-implied duration heterogeneity when pooling all households.

### 6.1.5 Preferences

Households have CRRA preferences with risk aversion  $\gamma$  equal to 2. We set  $\beta = 1/R$ . They maximize life-time expected utility subject to a budget constraint and subject to a natural borrowing constraint. The Bellman equations for this model are in Appendix C.

### 6.1.6 Size of Decline in Real Yields

According to the auxiliary asset pricing model in Appendix D, the ten-year real bond yield averaged 4.70% in the 40 quarters of the 1980s decade and 0.12% in the 2010s decade.<sup>13</sup> The asset pricing model shows similarly large declines in expected returns on stocks and on real estate, as

<sup>13</sup>The asset pricing model matches the available data on Treasury Inflation-Indexed Securities over the period for which they are available. The model-implied yield changes are similar for real bonds of different maturities. The one-quarter real bond yield fell from 4.21% in the 1980s to -0.84% in the 2010s. The five-year real bond yield fell from 4.65% in the 1980s to -0.31% in the 2010s. The thirty-year real bond yield fell from 4.76% to 1.40%.

shown in Table 2. In other words, the decline in expected returns was broad-based.

Table 2: Expected Real Returns Decade Averages

Asset	1980s	2010s	Decline
Aggregate stock market	8.0%	2.0%	6.0%
Small stocks	3.6%	3.2%	0.4%
Housing wealth	8.2%	4.8%	3.6%
Ten-year real bond yield	3.7%	0.1%	3.6%

*Note:* The table reports model-implied real expected real returns and average them over the 40 quarters in the 1980s and the 40 quarters of the 2010s. The model is described in Appendix D.

We calibrate the model to a decline in real rates of 4.6%. In the stationary model, interest rates must be adjusted for growth. The adjusted risk-free rates decline from 2.7% to -1.8%. This change in  $R$  is the result of an unexpected and permanent decline in the expected aggregate growth rate of the economy (an MIT shock). We rely on the case of log-normally distributed aggregate consumption growth. The decline in expected aggregate growth needed to generate this decline in the risk-free rate is governed by the EIS  $1/\gamma$ , as shown in eqn. 4. In the case of  $\gamma = 2$ , we need a decline in  $\mathbb{E}[\log \lambda]$  of 2.25% to get a decline in  $\hat{r}$  of 4.5%. Following Proposition 5.5, we also adjust the discount factor to preserve the relation  $\tilde{\beta}\tilde{R} = \hat{\beta}\hat{R} = 1$ .

To study the impact of the change in interest rates, we first simulate the model to generate an initial draw from the model’s stationary distribution. We then change the interest rate, re-solve the model at the new interest rate, and simulate forward 50 periods (years). To isolate the effect of the rate change, we subtract out the results of the simulation with the same idiosyncratic shock realizations under the old interest rate. We do not clear the bond market in this exercise. As a result, when interest rates decline, the economy produces excess savings. We rebate those savings to households so as to keep the total resources of the economy unchanged before and after the interest rate change. Appendix C explains the details.

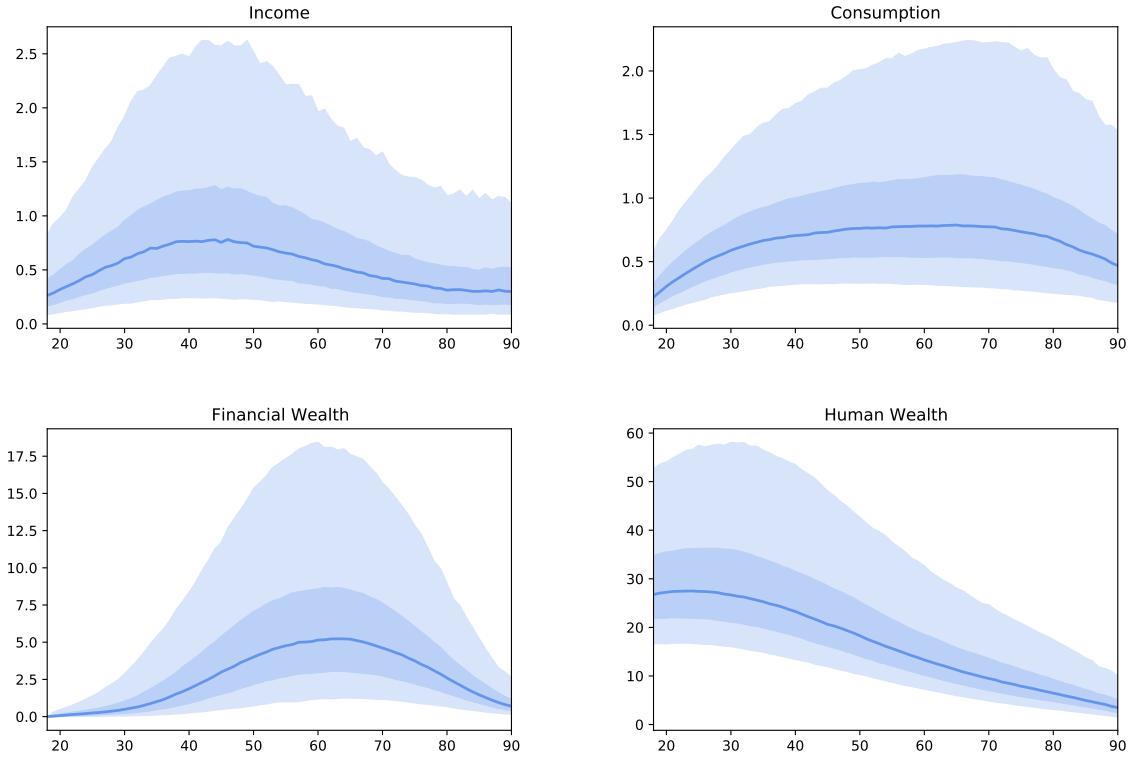
## 6.2 High Interest-Rate Regime

We begin by describing the properties of the model in its stationary distribution under the high interest rate regime. Figure 4 displays the life cycle profiles of income, consumption, financial wealth, and human wealth. The plot is for all agents in all groups, and the axes are normalized such that average income is 1 in the population. Income inequality is increasing over the life cycle because of the accumulation of income shocks and because of the increase in average income over the life cycle profile. The income inequality drops after retirement but is still nonzero since agents have heterogeneous retirement income and still face some income risk.

The top-right panel shows that both the level and dispersion of consumption are rising over the life cycle, with dispersion falling in retirement when income risk reduces. This is consistent

with the data which show that consumption inherits the hump-shaped profile from income (e.g., [Krueger and Perri, 2006](#)).

Figure 4: Life Cycle Profiles



*Note:* This figure plots the life cycle profiles by age for the all agents of all groups combined. The axes are normalized so that the average income across all agents of all ages is equal to unity. The center line displays the median, while the dark and light bands represent 66.7% and 95% percentile bands. Although agents in the model have a maximum age of 100, we truncate the plot at age 90 due the relatively small sample of agents surviving past this age.

Financial wealth in the bottom left panel increases in preparation for retirement, and is subsequently run down during retirement. Financial wealth inequality rises and falls over the life cycle.

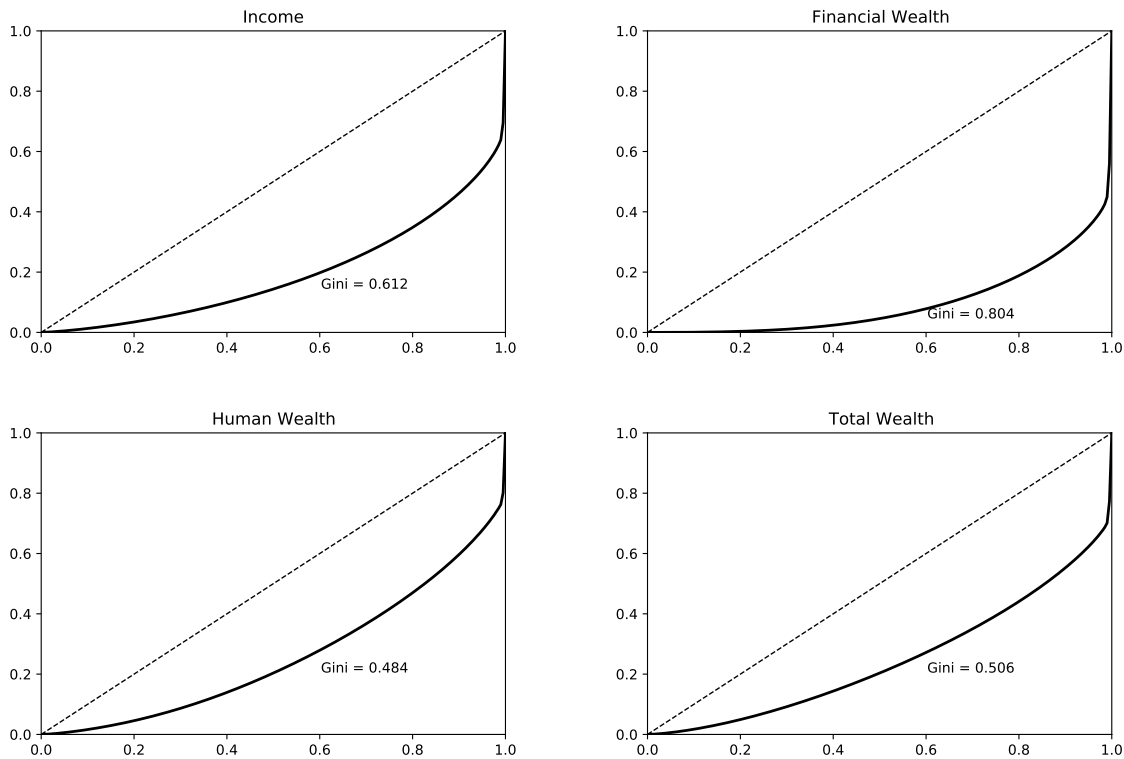
Human wealth in the bottom right panel is decreasing in age. There are two effects at play. Human wealth rises as the households' highest-earning periods are brought closer to the present. Human wealth falls due to the overall decrease in the remaining periods of work. The latter effect dominates. Total wealth consists almost exclusively of human wealth when young. As households age and accumulate financial wealth, a larger share of total wealth becomes financial wealth. However, human wealth remains a large component of total wealth throughout the life-cycle.

Figure 5 displays the Lorenz curves for consumption and wealth for all households (in all



groups), and reports the gini coefficients. The model generates a gini coefficient for (after-transfer) household income of 0.612. Consumption inequality closely tracks income inequality and has a gini coefficient of 0.575. Financial wealth is much more unequally distributed than human wealth or total wealth. The gini coefficients of human and total wealth are 0.484 and 0.506, compared to the gini of financial wealth of 0.804. The low total wealth inequality arises from (i) the importance of human wealth in total wealth, and (ii) the negative cross-sectional correlation between financial and human wealth.

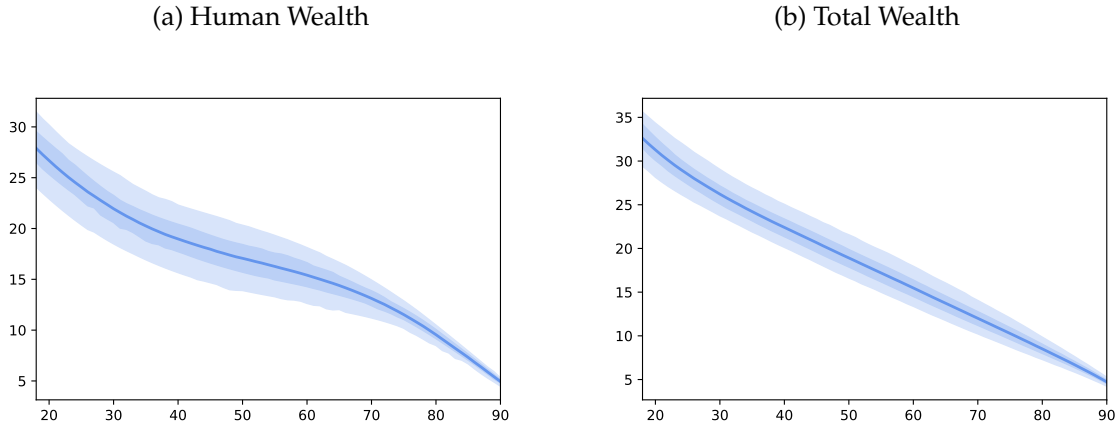
Figure 5: Lorenz Curves



Note: This figure plots the Lorenz curve for each variable, obtained from a long simulation of the model.

Figure 6 displays the duration of human and total wealth by age. Human wealth represents a claim on lifetime income whereas total wealth represents a claim on lifetime consumption. Both of these durations are similar because of the importance of human wealth in total wealth. These durations are high when young, around 30, and drop rapidly as age increases, since there are fewer years of life remaining to earn labor/pension income. The duration of financial wealth is fixed at 15.96 in this exercise and not shown.

Figure 6: Wealth Durations



*Note:* This figure plots the durations of labor income (human) wealth (left panel) and consumption (right panel). The plots display durations computed for many agents simulated from the stationary equilibrium of the model. The economy is normalized so that the average income is equal to unity. The center line displays the median, while the dark and light bands represent 66.7% and 95% percentile bands.

### 6.3 Change to Low Interest Rates

In this section we apply the main experiment of an unanticipated, permanent decline in the real interest rate from 4.7% to 0.1% in the growing economy, corresponding to a decline from 2.7% to -1.8% in the stationary economy. Before turning to the response of households' actual wealth portfolios, we first note that agents' prior consumption plans may no longer be budget feasible. Thus, even if financial wealth were unchanged, the change in interest rates could have large effects on lifetime consumption and welfare.

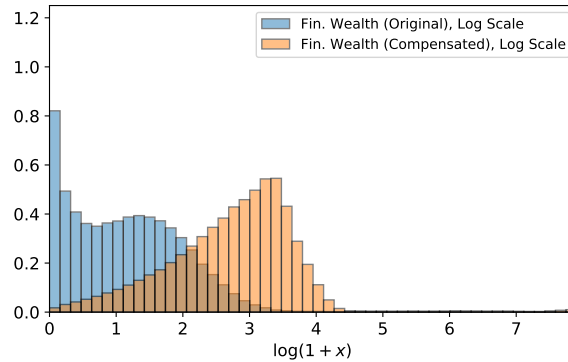
#### 6.3.1 The Compensated Wealth Distribution

To establish an intuitive baseline that is consistent with the theoretical analysis in Proposition 5.5 of Section 5.8, we compute the change in financial wealth that would be required to maintain the household's prior consumption plan in the high interest-rate economy. We refer to the counterfactual wealth allocation in which "fully hedged" households receive this financial wealth as the *compensated* financial wealth distribution (defined as  $\tilde{\theta}$  in the theory above).

The resulting distribution of financial wealth, alongside the original (pre-shock) distribution, is displayed in Figure 7. To ensure that the full distribution is visible, we display transformed variables  $\log(1 + x)$  on the x-axis.<sup>14</sup> This comparison shows two major differences between the pre-shock and compensated distribution. First, the compensated distribution is shifted substantially to the right. Households in this economy mostly save ( $c_t < y_t$ ) earlier in life before dissaving

<sup>14</sup>Because many agents have zero financial wealth, a standard log transform would be inappropriate in this context.

Figure 7: Histogram, Compensated vs. Original Financial Wealth Distribution



*Note:* This plot displays the distribution of financial wealth under the stationary distribution and under the compensated distribution drawn from the stationary distribution of the economy. The x-axis displays a transformation  $\log(1 + x)$  of the original data. Each distribution is top coded at the top 0.1% of the pre-shock wealth distribution.

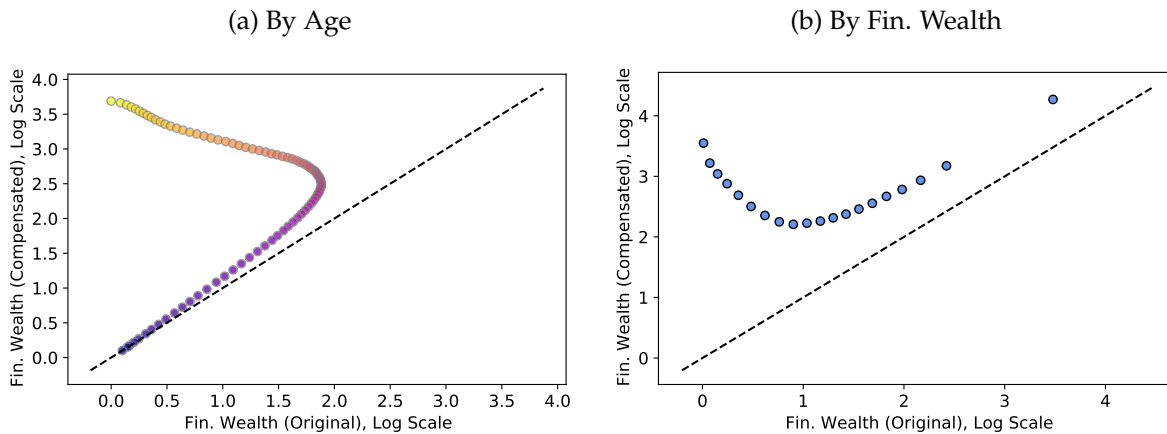
( $c_t > y_t$ ) in old age. When rates are much lower, households lose much of the effect of compound interest on their retirement savings. As a result, the aggregate amount of financial wealth in the compensated distribution exceeds the pre-shock total by 327%. As can be seen from the plot, this rightward shift extends up to the very top, implying that even the wealthiest individuals must be compensated with additional financial assets to attain their old consumption plans. Indeed, more than one third (37%) of new financial wealth accrues to top-1% financial wealth holders under the compensated distribution.

Second, although the wealthiest gain under this compensated distribution, the financial wealth gini falls substantially in the compensated distribution, as the less wealthy gain proportionally more. Visually, while the original high interest-rate distribution of financial wealth is heavily right-skewed, the compensated distribution is actually left-skewed. Quantitatively, the share of financial wealth held by the top-1% decreases from 54% in the baseline economy to 41% in the compensated economy.

To see why inequality falls in the compensated distribution, we can turn to Figure 8. Panel (a) compares the original (horizontal axis) and compensated financial wealth distributions (vertical axis) by age. The youngest agents (light/yellow) in the top left have close to zero financial wealth in the original distribution, but require the most financial wealth in the compensated distribution. As households age, their actual wealth initially increases, but their compensated wealth falls. Finally, late in life, both actual and compensated wealth fall rapidly toward zero, with the actual and compensated distributions close to coinciding for these older households.

This result is perhaps surprising, since the young have virtually their entire asset portfolio invested in human wealth. Because human wealth has a very long duration (left panel of Figure

Figure 8: Scatterplots, Compensated vs. Original Financial Wealth Distribution

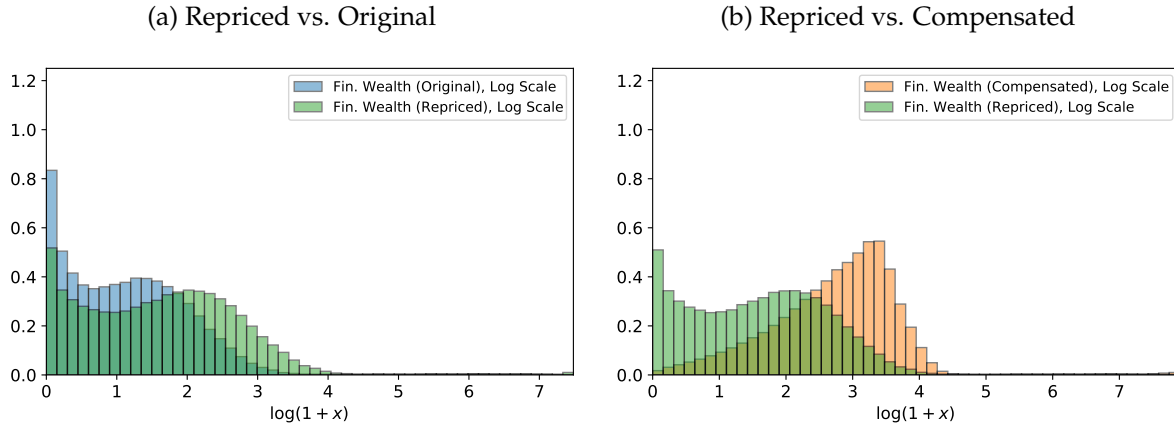


*Note:* Panel (a) plots the distribution of original financial wealth against the distribution of compensated financial wealth by age. Each dot represents one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Both variables are plotted using the transform  $\log(1 + x)$ . The dashed line represents equality between the original and compensated distributions. Panel (b) plots the same distribution by bins of original financial wealth in place of age.

6), it is well-hedged against interest rate changes. The key challenge the young face in a low interest rate environment, however, is not from their current portfolio, but their future portfolios. Due to the life cycle profile of income, the young plan to save during middle age, then dissave during retirement. Under a low interest rate, the young will be unable to accumulate enough interest on their future savings, making their original consumption plans unattainable without large infusions of financial wealth today. In contrast, older agents have already benefited from the higher rate of return in accumulating their retirement assets, while the oldest are dissaving, consuming principal rather than interest. These households are less affected by the loss of high-return investment opportunities, and require little compensation.

Panel (b) aggregates over ages to present the total compensation required for various levels of pre-shock financial wealth. The lowest levels of financial wealth mix young agents who have not begun saving with old agents who are spending down assets late in life. As a result, this group mixes over agents requiring the largest and smallest amounts of compensation. Quantitatively, the young make up a disproportionate share of this group and dominate the aggregate result, so that the least wealthy agents in this economy require the most compensation, measured as the vertical distance from the dot to the dashed 45-degree line. As wealth increases, we move toward the middle-aged individuals in the economy, who require a non-zero level of compensation, but less than those at the bottom of the wealth distribution. Finally, the wealthiest agents in the top bin, whose wealth is more driven by their income realizations than by demographics, also require a strictly positive level of compensation, but less than that of the least wealthy.

Figure 9: Histograms, Repriced Financial Wealth Distribution



*Note:* This plot displays the distribution of financial wealth under the repriced distribution, compared to the original distribution and compensated distribution. All distributions are drawn from the stationary distribution of the economy. The x-axis displays a transformation  $\log(1+x)$  of the original data. Each distribution is top coded at the top 0.1% of the pre-shock wealth distribution.

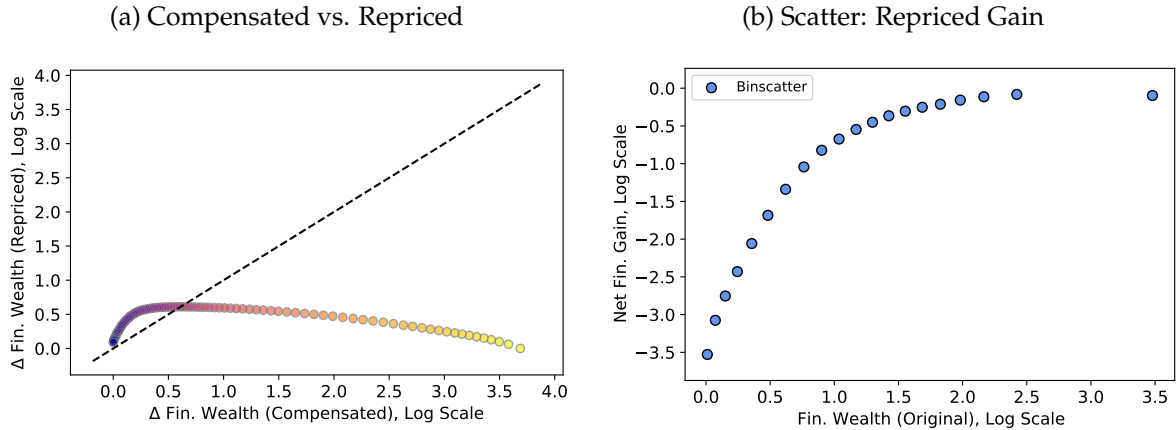
### 6.3.2 The Repriced Wealth Distribution - Homogenous Financial Duration

Having computed the financial wealth distribution required to keep consumption plans constant, we can compare it to the financial wealth distributions that actually results under low interest rates. We refer to this distribution as the *repriced* distribution. Unlike the compensated distribution, it depends on the duration of financial wealth. To begin, we consider an economy in which all households hold financial portfolios with the same duration, which we set to 15.96 following our results in Table 1. We implement assets with this duration as zero coupon bonds with the same maturity, and compute the actual change in financial asset values following our assumed change in interest rates.<sup>15</sup> The repriced distribution is displayed in Figure 9. Panel (a) shows that repricing shifts the financial wealth distribution to the right. Lower interest rates increase aggregate financial wealth by 105%. While this is a large amount, it is insufficient to compensate all agents. Panel (b) shows that the repriced distribution largely fails to reproduce the compensated distribution for all but the wealthiest agents.

Further comparisons can be seen in Figure 10, which plot the effects by age in Panel (a) and by wealth in Panel (b). Panel (a) shows that repricing delivers virtually no additional financial wealth to the young, despite their large need for compensating transfers. In contrast, the old are, if anything, slightly over-hedged, receiving more wealth under repricing than needed to afford their former consumption plan. These are the points above the 45-degree line. Panel (b) reinforces this

<sup>15</sup>While the duration is sufficient to compute the change in portfolio value as the shock size approaches zero, for large shocks this local approximation breaks down, making the exact timing of the cash flows relevant for the change in portfolio value. Using zero-coupon assets eliminates this complexity.

Figure 10: Scatterplots, Repriced Financial Wealth Distribution



*Note:* This plot displays the distribution of financial wealth under the repriced distribution, compared to the compensated distribution. Panel (a) displays the change in financial wealth relative to the original distribution for the compensated (x-axis) and repriced (y-axis) distributions. Both axes display a transformation  $\log(1 + x)$  of the original data. Each dot represents one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Panel (b) displays original financial wealth on the x-axis and the net financial gain (repriced minus compensated wealth) on the y-axis. The x-axis displays the transform  $\log(1 + x)$ , while the y-axis displays the difference in transformed values. Each dot represents one bin from the original wealth distribution. All distributions are drawn from the stationary distribution of the economy.

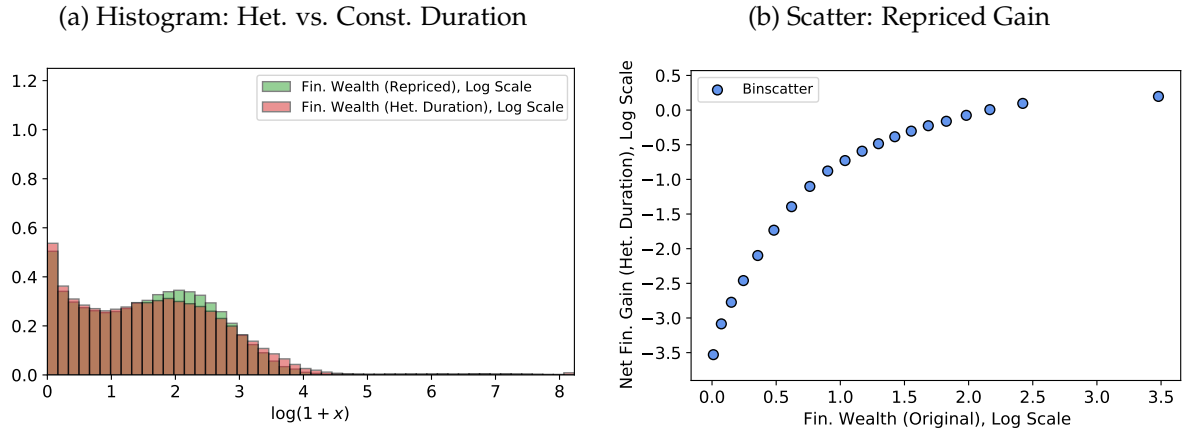
finding, showing that only the wealthiest are approximately hedged. The least wealthy experience a large net loss from the interest rate change, as repricing fails to appropriately compensate these households.

### 6.3.3 The Repriced Wealth Distribution - Heterogeneous Financial Duration

Next, we consider the role of duration heterogeneity. While our results in Figures 9 and 10 assumed a constant economy-wide duration, Figure 3 showed that duration varies widely across the wealth distribution, with wealthier agents holding longer-duration portfolios. As a result, these households will receive, on average, larger increases in wealth from repricing following a decline in interest rates. To investigate the quantitative importance of this heterogeneous repricing, we extend the model to allow for variation in durations, computed from equation (9).

The repriced distribution under heterogeneous financial durations is plotted in Figure 11. Panel (a) compares the histogram of the repriced financial wealth distributions under heterogeneous and homogenous durations. Incorporating heterogeneous durations causes the distribution to spread out, with less density in the middle, but more at both the upper and lower ends. While the difference might appear modest, the log scale of the x-axis implies large wealth gains for the top of the financial wealth distribution. Under heterogeneous durations, financial wealth thus increases more following a drop in interest rates — by 216%, more than twice the 105% increase un-

Figure 11: Repriced Distribution with Duration Heterogeneity



*Note:* This plot displays the distribution of repriced financial wealth from the model with duration heterogeneity. Panel (a) displays a histogram of repriced wealth under the baseline model and heterogeneous duration models, respectively. Panel (b) compares the net financial gain (repriced minus compensated wealth) for the heterogeneous duration economy, binned by original financial wealth. The x-axis displays a transformation  $\log(1 + x)$  of the original data, while the y-axis in Panel (b) displays the difference of these transformations. All distributions are drawn from the stationary distribution of the economy. Each distribution in Panel (a) is top coded at the top 0.1% of the pre-shock wealth distribution.

der equal durations. Panel (b) shows that heterogeneous financial duration results in the wealthy being even more “over-hedged” while the poor are even more under-hedged than under homogeneous financial durations.

The decline in interest rates increases financial wealth inequality. The share of financial wealth held by the top 1% rises from 54% to 66% following the drop in rates.

### 6.3.4 Financial and Total Wealth Inequality

Our model’s combined implications for inequality following a fall in interest rates are summarized in Table 3. Each row of the table displays a different statistic measuring inequality. The first two columns display the statistics from the data. We take the 1980s to be the period preceding the interest rate decline, and the 2010s to be the period following the interest rate decline. These measures are computed from the SCF+, as detailed in Appendix A.3.6. The next four columns display the results from the model. They report results for the initial pre-shock, high-interest rate distribution, and results for the various distributions after the interest rate decline: the compensated, the repriced with homogenous duration, and the repriced with heterogeneous duration distributions.

The top panel of Table 3 shows that our model is able to produce a realistic level of financial wealth inequality for the pre-shock period, matching the 0.804 gini and coming close to matching the top-10% wealth share. It overstates the top-1% financial wealth share.

Turning to the last column, corresponding to the repriced distribution under heterogeneous

Table 3: Inequality, Model Comparison

	Data		Model			
	Before	After	Initial	Comp	Repriced	Het Dur
Gini FW	0.804	0.873	0.804	0.640	0.804	0.871
Top-10% share FW	68.6%	77.4%	71.5%	55.1%	71.5%	80.7%
Top-1% share FW	31.7%	37.2%	54.4%	40.8%	54.4%	66.0%
Gini HW	–	–	0.484	0.516	0.516	0.516
Top-10% share HW	–	–	40.3%	38.4%	38.4%	38.4%
Top-1% share HW	–	–	23.7%	17.3%	17.3%	17.3%
Gini TW	–	–	0.506	0.542	0.510	0.544
Top-10% share TW	–	–	44.7%	42.5%	41.2%	45.2%
Top-1% share TW	–	–	29.7%	23.5%	22.6%	27.9%

*Note:* Top 1%, Top 10% financial wealth shares as well as financial wealth gini coefficients are estimated using SCF surveys. For the Before period we use the average values in 1983 and 1989. For the After period we use the average values in 2010, 2013 and 2016. More details on the computations are provided in Appendix A.3.6. For model results, the columns represent the pre-shock wealth distribution (“Initial”), the compensated distribution (“Comp”), the repriced distribution with homogeneous duration (“Repriced”), and the repriced distribution with heterogeneous duration (“Het Dur”).

durations, we see that this model can explain nearly all of the 0.069 increase in the financial wealth gini over the intervening period, even though this was not a target of the calibration. The model also produces an increase in the top-10% financial wealth share of 9.2% points, which is close to the observed increase in the data of 8.8% points. The model produces a large increase in the top-1% financial wealth share of 11.6% point increase. This overstates the 5.5% point increase in the SCF+ measure but is close to the 10.5% points increase in the WID measure listed in the opening paragraph of the paper.<sup>16</sup> In short, lower expected returns on financial assets explain the entire rise in financial wealth inequality in the data.

At the same time, the top panel of Table 3 shows that a compensated distribution, allowing agents to afford their prior consumption plans, would have corresponded to a major *decrease* in inequality, with top wealth shares and the gini coefficient all falling substantially compared to their pre-shock levels. This suggests that the actual allocations in the data failed to fully compensate younger and less wealthy individuals, leaving them less well off than they were prior to the rate shock. Abstracting from incentive effects—which may well be very important—progressive financial wealth taxation would help move the economy under the repriced distribution closer to that under the compensated distribution.

<sup>16</sup>As noted in Section 3, the rise in the top-1% financial wealth share in the United States was even larger, at 12% points, when measured between 1982 and 2015 according to the World Inequality Database. This is nearly identical to what the model produces. The SCF+ generates an increase in the top-1% financial wealth share of 7.2% between the 1983 and 2016 surveys. The WID generates a 8.9% point increase in the top-10% share between the 1980s and the 2010s, which is nearly identical to the 8.8% point increase in the top-10% share in the SCF+ over the same period. Hence, the disagreement between data sources is concentrated in the top-1% only.



Turning to the center panel of Table 3, we observe that all three human inequality indicators are much lower than their financial wealth inequality counterparts in the initial distribution. Lower interest rates modestly increase the human wealth gini from 0.484 to 0.516. Younger households own most of the human wealth, and have a high duration of human wealth. The interest rate decline generates the largest increase for the highest-human wealth households, explaining the rise in human wealth inequality. However, the 0.032 increase in the human wealth gini is less than half as large as the 0.067 increase in the financial wealth gini. The model predicts a substantial decline in the top-1% human wealth share, which spills over to a modest decline in the top-10% share. The top percentile of human wealth contains many households who currently are in the superstar income state. Since that state arrives at random times in the life-cycle, ends with 2.5% probability each period, and certainly ends at death, the human wealth duration of the superstars is lower than that of typical young households.<sup>17</sup> Hence, a decline in interest rates lowers the top-1% human wealth share.

The bottom panel of Table 3 reports on total wealth inequality, where total wealth is the sum of financial and human wealth. Since human wealth is by far the largest component of total wealth for most households, the total wealth gini (0.506) is close to the human wealth gini (0.484) and much lower than the financial wealth gini (0.804). When interest rates decline, the total wealth gini rises by 0.038, substantially less than the rise in the financial wealth gini. The differences are even larger at the top of the wealth distribution. The top-10% total wealth share increase by only 0.5% points, far less than the 9.2% point rise in the corresponding financial wealth share. The top-1% total wealth share falls by 1.8% points compared to the 11.6% point increase in the top-1% financial wealth share. The behavior of the top total wealth percentile in response to an interest rate decline depends on the composition of wealth of the households in that top percentile. These households are made up of two clusters. Some have most of their wealth in financial wealth. These are older households who have saved for a long time and households who transitioned out of the superstar state. The second cluster are households who currently are in the superstar state. They have much lower ratios of financial to total wealth.<sup>18</sup> The wealth dynamics of the former cluster are governed by the dynamics of the top-1% financial wealth share—which increases sharply,—while the wealth dynamics of the second cluster are governed by the dynamics of the top-1% human wealth share—which fall sharply. The effect of the second cluster dominates, and on net, there is a modest decline in the total wealth share of the top-1%. The main take-away is that total wealth inequality does not rise nearly as much when rates decline. Since consumption is ultimately what matters to the households in the model, and total wealth is the present value of consumption, the

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<sup>17</sup>The average human wealth duration of households in the superstar state is 12.0 compared to 17.5 for those not in the superstar state. Intuitively, the exit rate acts as an additional discount rate which lowers the duration. Moreover, when younger agents enter in the superstar state, it pulls forward their income profile, again lowering its duration.

<sup>18</sup>The first cluster has an average ratio of financial to total wealth of 0.95 whereas the second cluster has an average ratio of 0.30.

most relevant measure of wealth inequality has changed little.

Finally, we note that the total wealth gini under the repriced (Het Dur) and compensated distributions are nearly identical. That implies that the total wealth distribution that enables households to afford their old consumption plans after the rate change has the same level of inequality as the actual distribution. However, the former faces less top-wealth inequality than the latter.

## 7 Conclusion

A persistent decline in real interest rates, like the one experienced in much of the world between the 1980s and the 2010s, naturally leads to a rise in financial wealth inequality. Households whose wealth is predominantly made up of financial rather than human wealth, and particularly those with short-maturity assets, must increase savings to be able to afford the same consumption plan. We show how a standard incomplete markets Bewley model predicts that a decline in rates increases financial wealth inequality. We quantify the effect of declining rates on inequality by adding rich heterogeneity by age, race, gender, and education. Once the positive correlation between financial wealth and financial wealth duration is taken into account, the model with declining interest rates explains all of the increase in financial wealth inequality. Human wealth inequality is much lower than financial wealth inequality, and increases by much less when rates decline. Since human wealth represents a majority of total wealth, the effect of lower rates on total wealth inequality is small, and even reverses at the top of the total wealth distribution. While most households have been made worse off by the decline in interest rates, due to imperfectly hedged portfolios of human and financial wealth, the costs have fallen disproportionately on young and low-wealth households.

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## A Data Appendix

### A.1 Inequality Data

The top wealth shares presented in 2 are from the World Inequality Database. The data for the U.S. are available until 2019, for the U.K. until 2012, and for France until 2014.

We construct the price of a real 30 year annuity by estimating the historical real yield curve for each country. Letting  $y_t^r(h)$  denote the real yield at maturity  $h$  at time  $t$  the cost of the annuity is calculated as:

$$\sum_{h=1}^{30} \frac{1}{(1 + y_t^r(h))^h}$$

Due to varying availability of data and for robustness, we use three different approaches to estimate the real yield curve that lead to broadly consistent estimates.

First, for the UK post 1985 we use historical time series of real yields of various maturities available from the Bank of England. We fit a spline through these points and construct the real yield curve directly.

Second, for the U.S. and France we use the time series of historical nominal yields and inflation provided by Global Financial Data, augmented with data from the Macroeconomic History database constructed by [Jordà, Schularick and Taylor \(2017\)](#), to estimate real yields at different maturities and then fit a spline through the estimated real yields to construct the real yield curve. We construct real yields for each year by estimating an AR(1) process for inflation on a rolling sample of 50 years of past data, and then subtracting forecasted inflation from nominal yields at all available maturities. Those are 3-month treasury yields and 10-year government bond yields for all periods, as well as 30-year government bond yields for later years.

Third, for the U.K. and U.S. we also use model estimates of the real yield curve. The U.S. estimates are from the model in Section D. The U.K. estimates are from a similar model estimated for the U.K. in [Jiang, Lustig, Van Nieuwerburgh and Xiaolan \(2021\)](#).

### A.2 Income Data

#### A.2.1 Data Source: PSID

The Panel Study of Income Dynamics (PSID) is a household panel survey that began in 1968. The PSID was originally designed to study the dynamics of income and poverty. Thus, the original 1968 PSID sample was drawn from two independent samples: an over-sample of 1,872 low income families from the Survey of Economic Opportunity (the “SEO sample”) and a nationally representative sample of 2,930 families designed by the Survey Research Center at the University of Michigan (the “SRC sample”). A total of approximately 500 post-1968 immigrant families were

added in 1997/1999 to update the PSID by adding a representative sample of recent immigrants to the United States: this sample is called the 1997 PSID Immigrant Refresher Sample. A total of 615 post-1997 immigrant families were added in 2017 to update the PSID by adding a representative sample of recent immigrants to the United States: this sample is called the 2017 PSID Immigrant Refresher Sample. We use data from the SRC sample starting in 1970 and ending with the 2017 wave.

## A.2.2 PSID Income variables

We construct the following income variables: *labinc2f* is labor income excluding transfers but including the labor part of business and farm income for both head and eventual spouse, *transf* which are total households transfer (including Social Security Income and other transfers). These two variables are then summed to *labinc3f* which is our measure of total household income for both head and eventual spouse. Here we detail the construction of these variables.<sup>19</sup>

### **labinc2f**

- 1970 - 1993.
  - Total labor income of head, including wages and salaries, labor part of business income and farm income (1993:V23323).
  - Spouse’s total labor income, including labor part of business income and farm income (1993:V23324)
- 1993 - 2017
  - Reference Person’s total labor (including wages and other labor) excluding Farm and Unincorporated Business Income, (2017:ER71293)
  - Labor Part of Business Income from Unincorporated Businesses (2017:ER71274)
  - Reference Person’s and Spouse’s/Partner’s Income from Farming (2017:ER71272)
  - Wife’s Labor Income, Excluding Farm and Unincorporated Business Income (2017:ER71321)
  - Wife’s Labor Part of Business Income from Unincorporated Businesses (2017:ER71302)

Note that farm’s income includes both labor and asset portions of income.

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<sup>19</sup>Note that PSID variables tickers changed in each survey so here in order to indicate a specific ticker we define it as follow (YYYY:Ticker).



## transf

- 1970-1993
  - Total Transfer Income of Head and Wife/“Wife” (1993:V22366)
  - Total Transfer Income of Others (1993:V22397)
- 1994-2003
  - Head’s and Wife’s Total Transfer Income, Except Social Security (2017:ER71391)
  - Other Total Transfer Income, Except Social Security (2017:ER71419)
  - Total Family Income from Social Security (1994:ER4152)
- 2004-2017
  - Head’s and Wife’s Total Transfer Income, Except Social Security (2017:ER71391)
  - Other Total Transfer Income, Except Social Security (2017:ER71419)
  - Reference Person’s Income from Social Security (2017:ER71420)
  - Spouse’s/Partner’s Income from Social Security (2017:ER71422)
  - Others Income from Social Security (2017:ER71424)

**labinc3f** We then construct *labinc3f* by summing *labinc2f* and total family transfers *transf*.

Figure A1 plots the three variables described above averaged across all households. All variables are deflated to 2016 dollars using the CPI index.

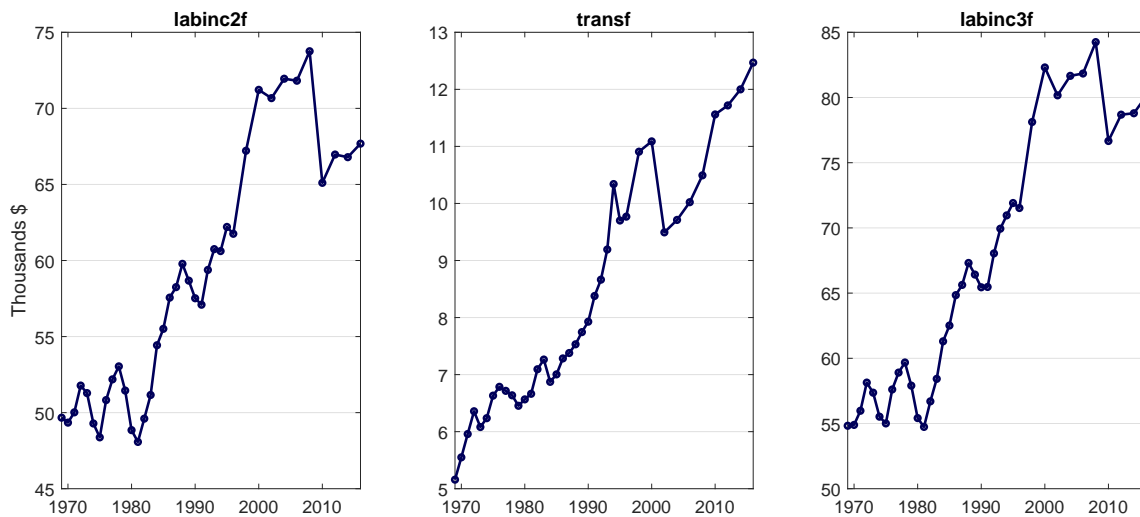


Figure A1: PSID average

We then split the sample in different cohorts: those age from 20 to 40 (Young), 40 to 60 (Middle) and 60 to 80 (Elderly). Figure A2 plots the same set of variables for these cohorts.

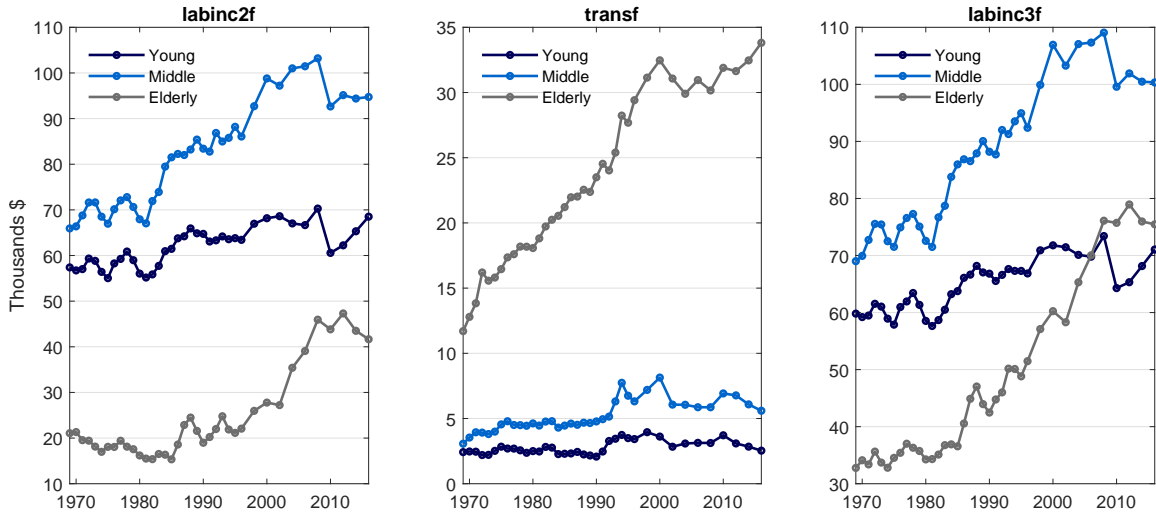


Figure A2: PSID by Cohorts

### A.2.3 Aggregation: NIPA vs PSID

We compare the PSID aggregates to the NIPA table aggregates from NIPA Table 2.1. We use NIPA *Wages and salaries* and compare to *labinc2f*. We then use the Census data on US number of households to compute *Wages and salaries* per households (note that our PSID measures are at the household level). In Figure A3, the left-hand side plot exhibits the nominal amount (thousands \$ per households) for the NIPA aggregates, the PSID simple average and PSID weighted average using the longitudinal weights. The right-hand side plot exhibits the real variables (deflated using the CPI index). The variables are indexed such that the index are equal to 100 in 2000. While the two definitions of income (NIPA *Wages and salaries* vis-a-vis our measure *labinc2f*) are not strictly identical, we find that they evolve quite closely over our sample.

### A.2.4 Group Definitions

Our groups are defined based on gender, race and education. Here we detail the variables used from the PSID. **Sex.** We use the sex of the head of the household (2017:ER66018).

**Race.** We use the variable race (2017:ER70882). We only have an indicator function if the head is white and zero for all other races.

**Education.** We use a measure of years completed of education (2017:ER34548). The question in the survey is: "What is the highest grade or year of school that (you/he/she) has completed?". We make the following assumption: Education is based on highest level of educational achievement with perfect foresight. So, income of an 18 year old who goes to college later should be part of the college income profile. We define an individual to be college educated if they have 16 years of schooling or more. This definition is consistent with [Heathcote, Perri and Violante \(2010\)](#). Before

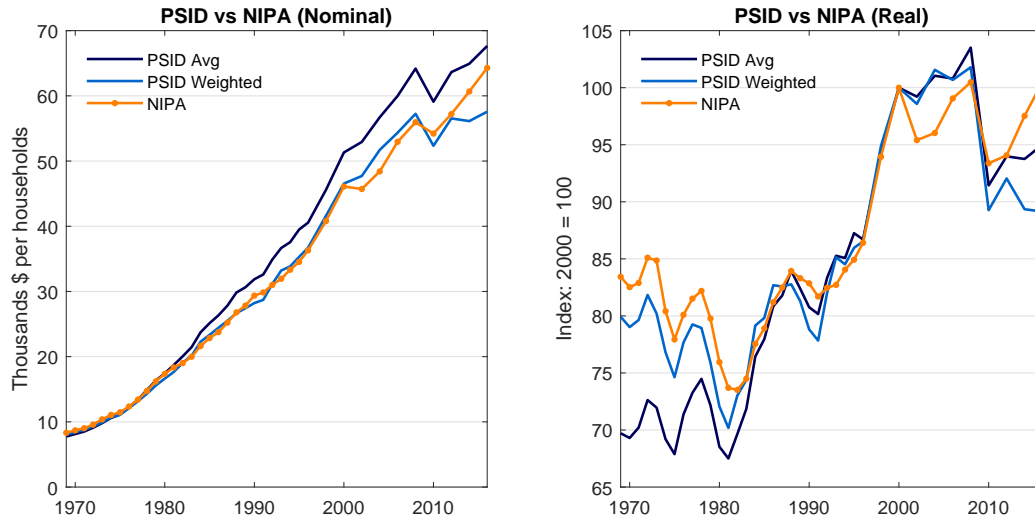


Figure A3: PSID vs NIPA Income

1975, we use the variable (1975:V4198).

Based on the above variables we measure the labor income for different groups. Figure A4 plots the variable *labinc3f* averaged across all households in each group.

### A.2.5 Estimating Income Process

We estimate the income profile for different groups following Meghir and Pistaferri (2004). The income process for household  $i$  in group  $g$  of age  $a$  at time  $t$  is given by (5)-(7). The estimation proceeds in two steps. In the first step, we estimate the year-fixed effects and the coefficients on the deterministic income profile  $\chi$  from (5). In the second stage, we estimate the risk parameters using the residuals  $z_{it}$  from the first step. This estimation is done by GMM as detailed below.

Figure A5 plots the deterministic income profile of the different groups, evaluated at the 2016 year-fixed effects. The graph plots the expected income profile for the average person in each group who is 18 years old in 2016, expressed in thousands of 2016 dollars.

Figure A6 plots the income at different age, expressed relative to the level at age 18.

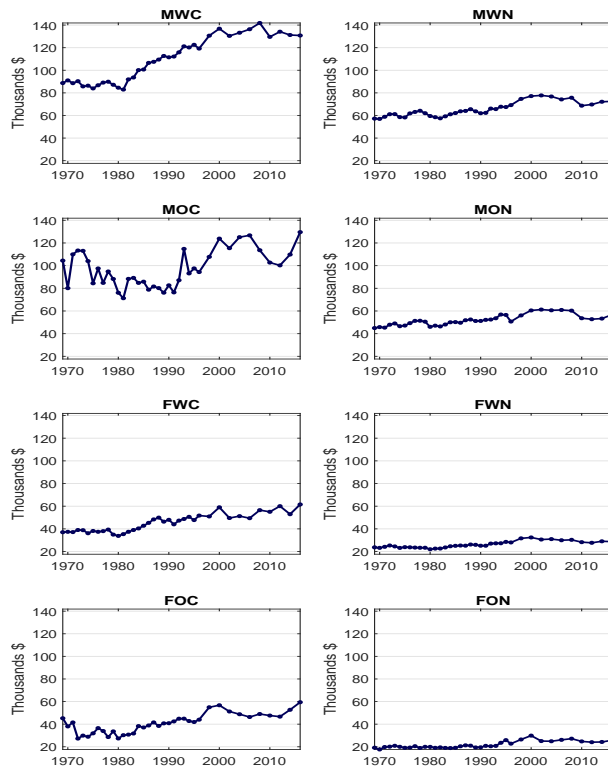


Figure A4: Labor Income by Group

Note: Average total labor income (*labinc2f*) of each group. Variables are in 2016 thousands dollars. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college.

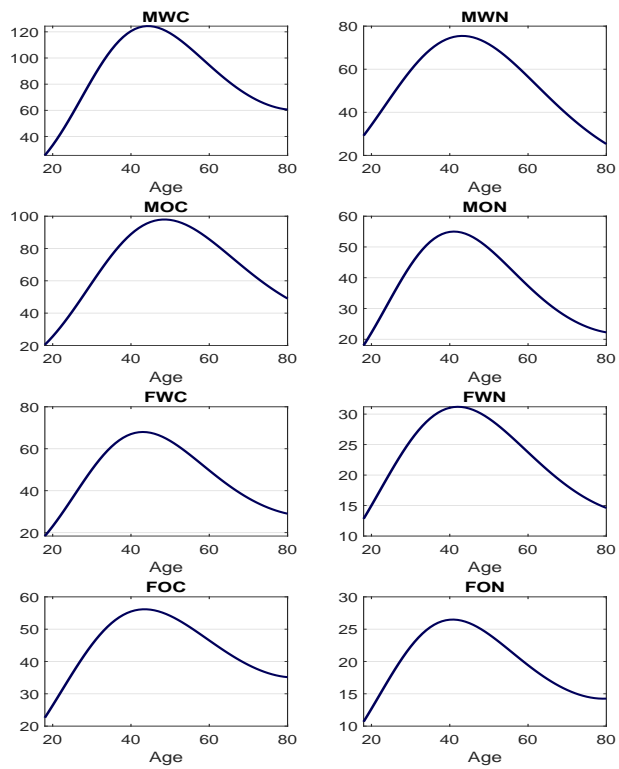


Figure A5: Income Profile by Group

*Note:* This figure displays the life cycle income profile of households within different groups. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college. We use the 2016 year fixed effects. The figure is in thousands of 2016 dollars. The model is estimated according to Equation (5)-(7) on PSID data from 1970 to 2017.

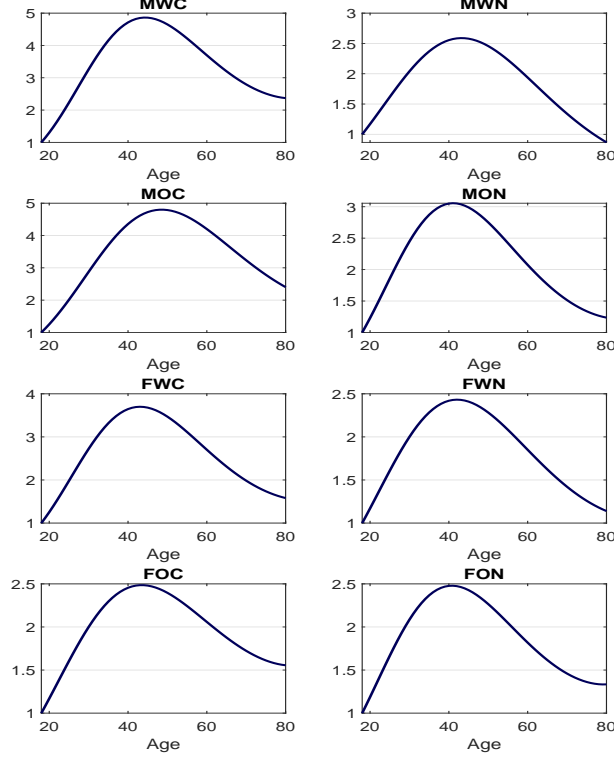


Figure A6: Income Profile by Group

Using Equation (5)-(7), and define  $j$  as equal to the age of the households minus the minimum age (18), we find that:

$$E[z_j^i, z_{j+h}^i] = \sigma_\alpha^2 + E[\varepsilon_j^{i2}] + \sigma_v^2 \quad \text{if } h = 0 \quad (10)$$

$$E[z_j^i, z_{j+h}^i] = \sigma_\alpha^2 + \rho^h E[\varepsilon_j^{i2}] \quad \text{if } h > 0 \quad (11)$$

$$E[\varepsilon_j^{i2}] = \rho^{2j} \sigma_{\varepsilon_0}^2 + \sum_{k=1}^j \rho^{2(j-k)} \sigma_\eta^2 \quad (12)$$

We then use a GMM estimation to estimate  $\theta = (\rho, \sigma_v, \sigma_\eta, \sigma_\alpha, \sigma_{\varepsilon_0})$ . We use a Minimum Distance Estimator, where the weighting matrix is the identity matrix.

**Sample Selection.** We use PSID data from 1970 to 2017. We only include households whose head is 18 to 80 years old. We only include households which were in the survey for three or more periods. We exclude households with zero or negative income. In each year, we trim the top 2.5% of households by their income.

We pool all households together, after removing group-specific year-fixed effects and cubic age-profiles, and estimate the idiosyncratic risk parameters  $\theta$ . The point estimates are displayed

in Panel A in Table A1. These are the parameters used in the main text.

Table A1: Idiosyncratic Risk Parameter Estimates

	$\rho$	$\sigma_{\eta}^2$	$\sigma_v^2$	$\sigma_{\alpha}^2$	$\sigma_{\varepsilon_0}^2$	N. Obs.
All	0.950	0.023	0.195	0.066	0.194	10638

Note:  $\rho, \sigma_{\eta}^2, \sigma_v^2, \sigma_{\alpha}^2, \sigma_{\varepsilon_0}^2$  are estimated using Equation Equation (5)-(7). Data runs from 1970 to 2017.

### A.3 Portfolio Shares

The Survey of Consumer Finances (SCF) is a statistical survey of the balance sheet, pension, income and other demographic characteristics of families in the United States. We use data from the Summary Extract Data – that is, the extract data set of summary variables used in the Federal Reserve Bulletin. It includes data from the triennial surveys beginning in 1989.<sup>20</sup>

#### A.3.1 Variables

We collect the following variable:

**Total Financial Assets.** This includes:

1. All types of transaction account (liquid assets)
2. Certificates of deposit
3. Directly held pooled investment funds (exc. money mkt funds)
4. Savings bonds
5. Directly held stocks
6. Directly held bonds (excl. bond funds savings bonds)
7. Cash value of whole life insurance
8. Other managed assets
9. Quasi-liquid retirement accounts
10. Other misc. financial assets

**Cash & Deposits** This includes all types of transaction account (liquid assets) and certificated of deposits. The list of variables are:

<sup>20</sup>The SCF Flow Chart provides information on how variables are constructed <https://www.federalreserve.gov/econres/files/networth%20flowchart.pdf>. The code on how different variables in the Summary Extract Data are constructed can be found here: <https://www.federalreserve.gov/econres/files/bulletin.macro.txt>

1. Money market accounts
2. Checking accounts (excl. money market)
3. Savings accounts
4. Call accounts
5. Prepaid cards
6. Certificates of deposit

**Equities (direct & indirect).** Total value of financial assets held by household that are invested in stock. That includes:

1. directly-held stock
2. Stock mutual funds: full value if described as stock mutual fund, 1/2 value of combination mutual funds
3. RAs/Keoghs invested in stock: full value if mostly invested in stock, 1/2 value if split between stocks/bonds or stocks/money market, 1/3 value if split between stocks/bonds/money market
4. Other managed assets with equity interest (annuities, trusts, MIAs): full value if mostly invested in stock, 1/2 value if split between stocks/MFs & bonds/CDs, or "mixed/diversified", 1/3 value if "other"
5. Thrift-type retirement accounts invested in stock: full value if mostly invested in stock, 1/2 value if split between stocks and interest earning assets

The allocation rules for mixed investments in 3), 4), and 5) do not apply to 2004 since new questions in 2004 directly ask the share of stock in those assets.

**Real Estate.** The real estate variable includes:

1. Primary residence
2. Residential property excluding primary residence (e.g., vacation homes)

**Private Business Wealth.** Businesses (with either an active or nonactive interest). Businesses include both actively and nonactively-managed business(es). Value of active business(es) calculated as net equity if business(es) were sold today, plus loans from the household to the business(es), minus loans from the business(es) to the household not previously reported, plus value of personal assets used as collateral for business(es) loans that were reported earlier. Value of nonactive business(es) is calculated as the market value of the business(es).



**Fixed Income.** Fixed income is calculated as the residual of Total financial assets minus Cash & Deposits and Equity (direct & indirect).

**Mortgage Debt.** This includes:

1. Debt secured by prim. resid. (mortgages, home equity loans, HELOCs)
2. Debt secured by other residential property

**Student Debt.** Total value of education loans held by household. This includes education loans that are currently in deferment and loans in scheduled repayment period. We exclude installment loans: these are mostly student loans (which we accounts for separately), vehicle loans (which we do not account as debt as vehicles are part of consumption).

**Consumer and Other Debt.** This includes:

1. Other lines of credit (not secured by resid. real estate)
2. Credit card balances after last payment

**Net Wealth.** We calculate net wealth for each household as the difference between total assets (Cash & Deposits, Equities (direct & indirect), Real Estate, Private Business Wealth and Fixed Income) and total liabilities (Mortgage Debt, Student Debt and Consumer and Other Debt).

### A.3.2 Groups

From the SCF data, we extract the sex of the reference person, the education attainment and the race. Using the education attainment we divide the sample into households with college degree and households without college degree. We only include households older than 25 years old. Table A2 provides information on the different groups.

Table A2: Summary Statistics by Group

Groups	Population Share (%)	Median Age	Median NW	Average NW	Std NW	Negative NW	Zero NW
MWC	16.82	42	292.69	830.41	3434.78	3.44	0.00
MWN	41.20	46	114.33	310.68	1181.03	4.68	3.32
MOC	2.61	41	87.95	458.48	1771.98	10.41	1.02
MON	12.26	42	28.48	91.57	279.31	8.56	12.92
FWC	3.62	51	167.92	321.48	645.08	8.17	0.00
FWN	14.12	63	54.90	138.66	341.28	6.36	5.89
FOC	0.79	40	20.54	97.58	159.10	31.34	5.89
FON	8.57	49	0.37	35.94	112.66	10.79	33.73
All	100.00	46	87.95	325.97	1657.30	6.20	6.75

*Note:* Groups Information. SCF 1989. Column 1 is in percentage. Column 3, 4 and 5 in 2016 thousands dollars, Column 6 and 7 are in percentage. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college.

### A.3.3 Holdings

We compute the holdings for each of the assets and liabilities for each household. Table A3 shows summary statistics for the distribution of asset holdings. Note that Private Business Wealth is a measure net of loans from the business to the households and hence may also be a negative number.

Table A3: Portfolio Holdings

	Mean	std	Min	25%	50%	75%	90%	95%	Max
Cash and Deposits	37.51	241.24	0.00	0.75	4.11	22.09	82.90	158.70	67785
Equities	37.54	327.22	0.00	0.00	0.00	5.00	52.28	142.83	103857
Real Estate	157.69	292.74	0.00	0.00	84.02	199.78	382.76	560.13	65343
Private Business Wealth	79.45	1226.34	-1334.98	0.00	0.00	0.00	9.34	186.71	258164
Fixed Income	57.43	459.29	-0.00	0.00	3.73	27.63	103.06	223.49	171190
Mortgage Debt	39.22	96.42	0.00	0.00	0.00	50.41	123.23	192.31	30004
Student Debt	0.87	5.57	0.00	0.00	0.00	0.00	0.00	3.73	166
Other Debt	3.58	45.20	-0.00	0.00	0.00	1.68	5.88	11.20	3780
Net Wealth	325.95	1657.20	-3034.51	9.58	87.94	272.60	645.92	1208.02	290271

*Note:* Data are based on SCF 1989 and are reported in 2016 thousands dollars. Note that Private Business Wealth is a measure net of loans from the business to the households. For this reason some observations are negative.

### A.3.4 Financial Duration

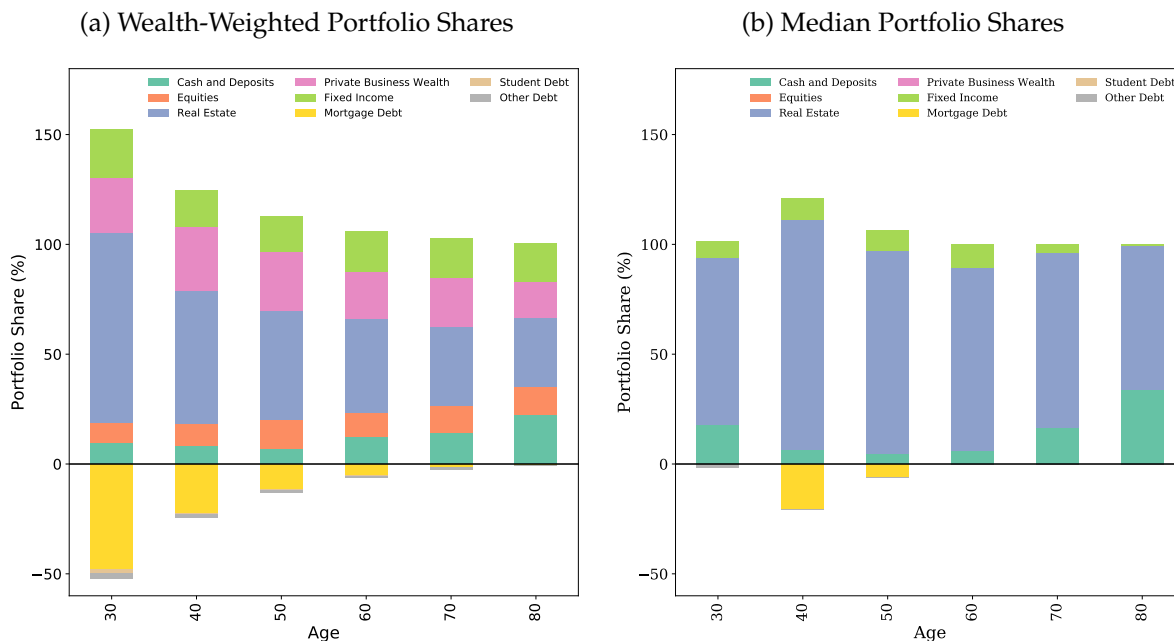
For the purpose of our duration calculation, we exclude households with zero net-wealth. These households account for roughly 7% of the population (see Table A2). We then compute household's portfolio share in each asset by dividing the dollar holdings in the asset by the households net wealth. Using the portfolio shares, we compute the durations of the household's financial portfolio by multiplying the asset duration of an asset (assets durations are reported in the first column of Table 1) by the portfolio share of that asset, and summing over all assets in the portfolio. We trim household financial durations by excluding the top and bottom 2.5% of observations. In the last row of Table 1, we report the average duration, by averaging over all households (using the SCF sampling weights). Similarly, we compute average durations by group by averaging durations among the households in a group (using the SCF sampling weights).

Table 1 also reports value-weighted portfolio shares for each asset. They are obtained by summing dollar holdings of an asset among all households (households in a group) by the total dollar holdings of all assets among all households (households in a group). Aggregate durations are then obtained by multiplying the value-weighted portfolio weights for each asset by the duration of that asset, and summing over assets. They are reported in the last but one row of Table 1.

Figure A7 shows portfolio shares by age in the 1989 SCF. We bundle households into different cohort groups: 25-35, 35-45, 45-55, 55-65, 65-75, 75-85. Figure A7a uses the value-weighted port-

folio shares. Figure A7b plots the median portfolio share in each asset category, and then rescales the resulting shares so that they sum to 100%.

Figure A7: Portfolio Shares by Cohorts



Note: Portfolio shares by age in the 1989 SCF. We bundle households into different cohort groups: 25-35, 35-45, 45-55, 55-65, 65-75, 75-85. The top panel uses the value-weighted portfolio shares. The bottom panel uses the median portfolio share in each asset category, and then rescales the resulting shares so that they sum to 100%. We exclude households with zero net wealth as the portfolio shares are undefined.

Figure A8 provides further information on the distribution of durations across households. Figure A8a plots the average duration by cohort. We bundle households into cohort groups and estimate the average duration. Figure A8b bundles households in wealth-weighted percentile and estimate the average duration of households in each bin.<sup>21</sup> Figure A8c and A8d rank households according to their wealth and income percentile, respectively, and estimate the average duration of each group.

Figure A8e and Figure A8f also rank households according to their wealth and income. Then plot the average duration of each group against the average net-wealth or income.

We also evaluate more formally the correlation between financial duration and some covariates of interest. First, we regress household financial duration on household position in the Lorenz Curve. To calculate households' positions, we rank households by their net-wealth, then calculate the cumulative sum of net-wealth and divide by the aggregate net-wealth. We then add a dummy for each group, a quadratic function of age and the log of household income. We exclude house-

<sup>21</sup>Households are ranked according to their net-wealth and allocated to different bins. Each bin is designed such that the share of total wealth held by the households in each bin is the same across different bins.

holds with zero net-wealth and trim the bottom/top 2.5% of households ranked by their duration. The regression estimate take into account survey weights. Table A4 reports the estimation results.

Table A4: Determinants of Household-level Financial Duration

	(1)	(2)	(3)	(4)	(5)
Lorenz	0.23*** (25.85)		0.19*** (19.70)	0.23*** (23.32)	0.19*** (16.34)
MWC		9.98*** (16.23)	5.82*** (8.79)	3.66*** (5.33)	1.60** (2.30)
MWN		6.72*** (12.43)	4.80*** (9.00)	3.78*** (6.75)	2.31*** (4.05)
MOC		5.41*** (4.68)	3.29*** (3.03)	1.20 (1.08)	-0.33 (-0.30)
MON		4.67*** (6.79)	4.19*** (6.21)	2.85*** (4.07)	1.72** (2.47)
FWC		2.14** (2.35)	-0.096 (-0.11)	-0.79 (-0.89)	-2.10** (-2.38)
FWN		0.21 (0.36)	-0.65 (-1.15)	0.45 (0.74)	-0.25 (-0.42)
FOC		0.86 (0.70)	-0.11 (-0.09)	-1.18 (-1.02)	-2.47** (-2.20)
Age				0.077 (1.23)	-0.025 (-0.41)
Age Squard				-0.0023*** (-4.26)	-0.0010* (-1.92)
Log-Income					1.87*** (9.90)
Constant	13.2*** (72.62)	10.5*** (21.07)	10.0*** (20.50)	13.0*** (7.32)	-3.73 (-1.44)
Observations	13784	13784	13784	13784	13784
R <sup>2</sup>	0.087	0.064	0.118	0.160	0.170
Adjusted R <sup>2</sup>					

Note: Data based on SCF 1989. T-stats in parentheses (\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ )

### A.3.5 Financial Duration - Robustness

To make sure our results are robust we replicate Figure 3 using different samples and different duration estimates for business wealth.

Figure A9a includes households with negative net-worth and households with zero income (that we excluded from the original sample in Figure 3), Figure A9b uses a different trimming: trim the bottom/top 1%. Figure A9c plots the median duration instead of the average duration,

Figure A9d uses as duration of business wealth, the same duration used for equity (28.7).

### A.3.6 Wealth Shares, Income Shares, and Gini Coefficients

We estimate the net-wealth shares held by the top-10% and top-1%. We also estimate gini coefficients. We use the SCF+ database developed by Kuhn et al. (2020) in order to have a longer time series of wealth and income. We slightly modify their definition of total financial net-wealth by subtracting vehicles and other non-financial wealth.

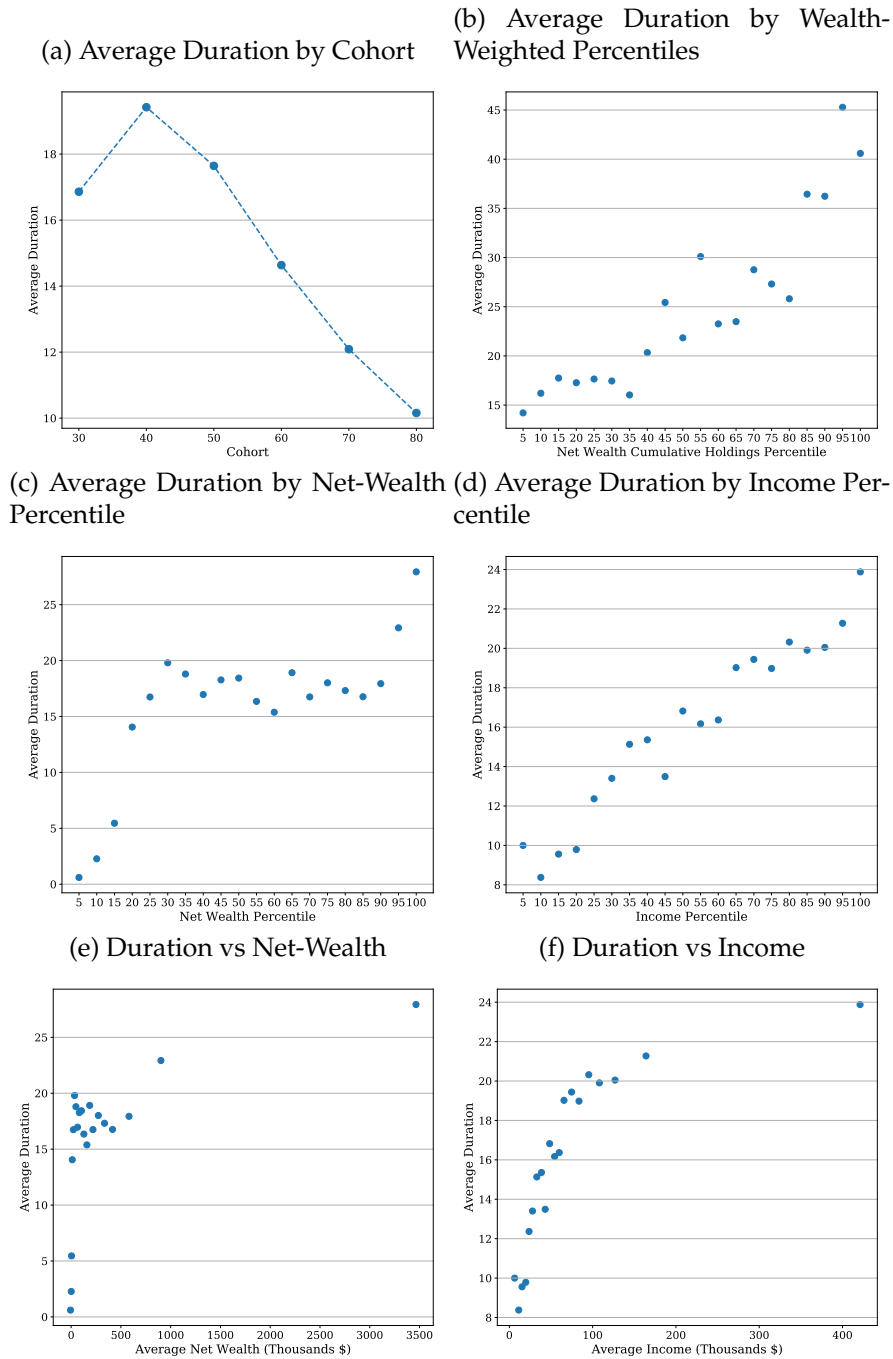
Figure A10 plots the top shares and the gini coefficient for financial (net) wealth. Table A5 computes averages for these moments, computed over all surveys in the 1980s and all surveys in the 2010s, for both financial wealth and income. The income moments in this table are from the SCF. We define household income as SCF total household income minus capital income.

Table A5: Summary Statistics Wealth and Income Inequality in SCF

	SCF		WID					
	1980s	2010s	1980s	2010s				
Wealth: Top 1 Share (%)	28.8	37.2	25.3	35.1				
Wealth: Top 10 Share (%)	67.6	77.3	63.2	71.8				
Wealth: gini ( $\times 100$ )	79.4	87.2	77.8	83.6				
	SCF		WID		PSID		PSID (ex transf.)	
	1980s	2010s	1980s	2010s	1980s	2010s	1980s	2010s
Income: Top 1 Share (%)	11.5	18.3	12.2	18.6	6.4	9.5	8.1	11.8
Income: Top 10 Share (%)	36.3	45.5	36.3	45.1	29.2	34.3	35.4	41.7
Income: gini ( $\times 100$ )	48.2	56.1	48.7	57.9	42.8	47.8	56.9	62.7

*Note:* Shares and Gini coefficients estimated using the SCF+ developed by Kuhn et al. (2020), the WID database and the PSID. We use our income variable *labinc3f* from the PSID as well the income variable excluding transfers (*labinc2f*). From the SCF+ we use the total income variable excluding capital gain. SCF+ 1980s average over the surveys in 1977, 1983 and 1989.

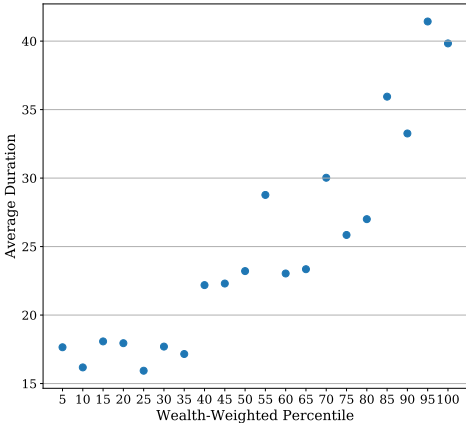
Figure A8: Distribution of Durations



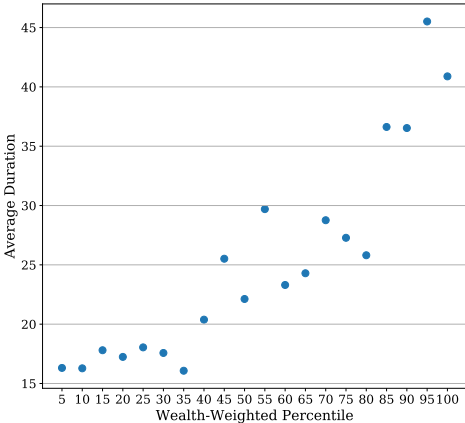
Note: Data are based on SCF 1989. We exclude households with zero net wealth (as their portfolio shares would be undetermined) and we trim the data based on households' overall duration: we exclude the top/bottom 2.5%. We only include households with positive income. Panel (a) plots the average duration by cohort. We bundle households into cohort groups and estimate the average duration. Panel (b) bundles households in wealth-weighted percentile and estimate the average duration of households in each bin. Panel (c) and Panel (d) rank households according to their wealth and income percentile, respectively, and estimate the average duration of each group. Then plot the average duration of each group against the average net-wealth (Panel (e)) or income (Panel (f)).

Figure A9: Duration by Net Worth, Robustness

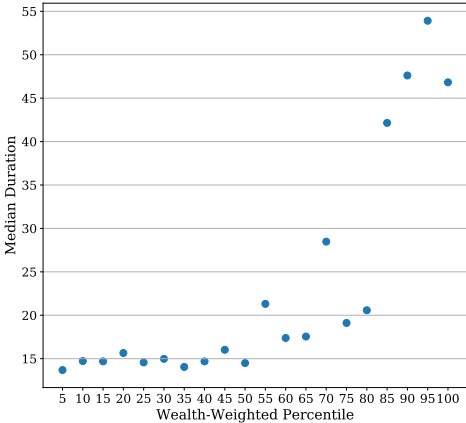
(a) Include Negative Wealth and Zero Income



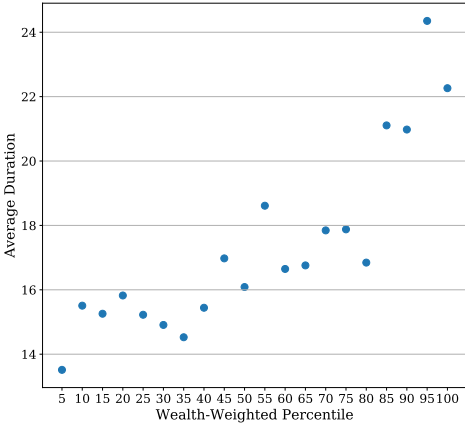
(b) Trim bottom/top 1%



(c) Median Duration

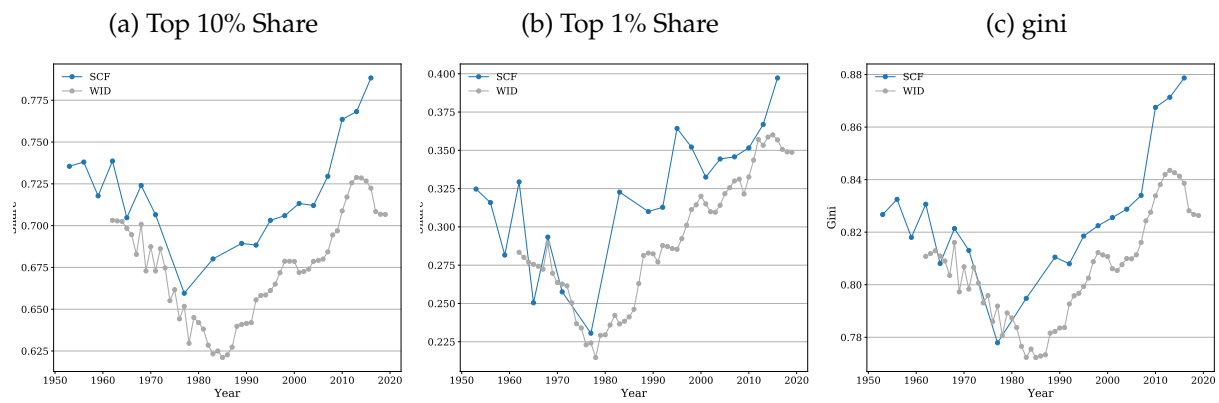


(d) Modified Business Wealth Duration



Note: Data are based on SCF 1989. We always exclude households with zero net wealth (as their portfolio shares would be undetermined). Figure A9a includes households with negative net-worth and households with zero income (that we excluded from the original sample in Figure 3), Figure A9b uses a different trimming: trim the bottom/top 1%. Figure A9c plots the median duration instead of the average duration, Figure A9d uses as duration of business wealth, the same duration used for equity (28.7).

Figure A10: Financial Wealth Inequality in the SCF+



Note: Data are based on SCF+ database developed by [Kuhn et al. \(2020\)](#) and WID database.



## B Proofs

### B.1 Proof of Corollary 4.1

*Proof.* Next, we analyze the household's wealth-consumption ratio. A [Campbell and Shiller \(1988\)](#) log-linearization of the return equation around the mean log wealth-consumption ratio  $wc^i$  for household  $i$  delivers the following expression for the log returns on a claim to household  $i$ 's consumption stream:

$$\widehat{r}_{t+1}^i = \Delta \widehat{c}_{t+1}^i + \rho_i^c \widehat{wc}_{t+1}^i + k_i^c - \widehat{wc}_t^i, \quad (13)$$

where the linearization coefficient  $\rho_i^c$  depends only on the mean of the log wealth-consumption ratio  $wc^i$ :  $\rho_i^c \equiv \frac{e^{\widehat{wc}^i}}{e^{\widehat{wc}^i} + 1}$ . By taking unconditional averages of the return in [Equation 13](#), we obtain:

$$(1 - \rho_i^c) \widehat{wc}^i = \mu_i^c - \mathbb{E}[\widehat{r}_{t+1}^i] + k_i^c,$$

where  $\mu_i^c$  denotes the household's average consumption growth rate. We obtain the following expression for the average wealth-consumption ratio:

$$\widehat{wc}^i = \frac{\mu_i^c - \mathbb{E}[\widehat{r}_{t+1}^i]}{1 - \rho_i^c} + \frac{k_i^c}{1 - \rho_i^c}.$$

By iterating forward on the linearized return equation and imposing a no-bubble condition,

$$\lim_{j \rightarrow \infty} (\rho_i^c)^j \widehat{wc}_{t+j}^i = 0,$$

we obtain an expression for the log wealth-consumption ratio:

$$\widehat{wc}_t^i = \frac{k_i^c}{1 - \rho_i^c} + \left[ \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \Delta \widehat{c}_{t+j}^i \right] - \left[ \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \widehat{r}_{t+j}^i \right].$$

We take expectations at time  $t$  to obtain:

$$\widehat{wc}_t^i = \frac{k_i^c}{1 - \rho_i^c} + \mathbb{E}_t \left[ \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \Delta \widehat{c}_{t+j}^i \right] - \mathbb{E}_t \left[ \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \widehat{r}_{t+j}^i \right].$$

In our baseline model, the expected one period returns on the consumption-claim have to be equal to the return on the aggregate consumption claim. As shown in [section 5](#), in this incomplete markets economy, to compute measures of wealth that can be aggregated, we discount all consumption claims using the discount rate on the aggregate consumption claim. If the expectations hypothesis holds, then the household's log wealth-consumption ratio is determined by the

PDV of future consumption growth and risk-free rates:

$$\widehat{wc}_t^i = \frac{k_i^c}{1 - \rho_i^c} + \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \Delta \widehat{c}_{t+j}^i - \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \widehat{r}_{t+j-1}^f.$$

As a result, we obtain the following expression for :

$$\widehat{wc}_t^i = \frac{k_i^c + \mu_i^c}{1 - \rho_i^c} - \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^c)^{j-1} \widehat{r}_{t+j-1}^f.$$

Using the autoregressive process for the risk-free rate, we obtain the following expression for the log wealth-consumption ratio:

$$\widehat{wc}_t^i = \frac{k_i^c - \mathbb{E}[\widehat{r}^f] + \mu_i^c}{1 - \rho_i^c} + \frac{\widehat{r}_t^f - \mathbb{E}[\widehat{r}^f]}{1 - \rho_i^c \phi}.$$

Hence, we obtain the following expression for the household's consumption wealth ratio in deviation from the average:

$$\widehat{wc}_t^i - \widehat{wc}^i = \frac{1 - \rho^a \phi}{1 - \rho_i^c \phi} (\widehat{wc}_t^a - \widehat{wc}^a).$$

As a result, we have that the log household wealth-consumption ratio can be stated as:

$$\widehat{wc}_t^i = \frac{k_i^c - E[\widehat{r}^f] + \mu_i^c}{1 - \rho_i^c} - \frac{1 - \rho^a \phi}{1 - \rho_i^c \phi} \frac{(\widehat{yp}_t^a - \widehat{yp}^a)}{(1 - \rho^a)}$$

By the same token, the aggregate wealth consumption ratio can be stated as:

$$\widehat{wc}_t^a = \frac{k^a - E[\widehat{r}^f]}{1 - \rho^a} - \frac{(\widehat{yp}_t^a - \widehat{yp}^a)}{(1 - \rho^a)}$$

We can apply the same approach to derive an expression for the log human wealth-income ratio. The household's log human wealth-income ratio is determined by the PDV of future income growth and risk-free rates:

$$\widehat{hy}_t^i = \frac{k_i^h}{1 - \rho_i^h} + \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^h)^{j-1} \Delta \widehat{y}_{t+j}^i - \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^h)^{j-1} \widehat{r}_{t+j-1}^f.$$

Given the AR(1) process for the risk-free rate, when log household income  $i$  follows a random walk with drift  $\mu_i^h$ , the household's log human wealth-income ratio can be expressed as:

$$\widehat{hy}_t^i - \widehat{wc}_t^a = \frac{k_i^h + \mu_i^h}{1 - \rho_i^h} - \frac{k^a}{1 - \rho^a} + \frac{\mathbb{E}[\widehat{r}^f](\rho^a - \rho_i^h)}{(1 - \rho_i^h)(1 - \rho^a)} + \frac{\phi(\rho_i^h - \rho^a)}{(1 - \rho_i^h \phi)} (\widehat{wc}_t^a - \widehat{wc}^a).$$

□

## B.2 Proof of Proposition 4.2

*Proof.* In the economy without consumption dispersion, the cross-sectional standard deviation of log household wealth is bounded below by:

$$\sigma \left[ \widehat{wc}_t^i + \widehat{c}_t^i \right] \geq \left( \text{Var}_x \left[ \widehat{wc}_t^i \right] + \text{Var}_x \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} \right] (\widehat{wc}_t^a) \right)^{1/2}.$$

As the wealth/consumption ratio increases, the lower bound on the cross-sectional standard deviation of wealth increases. As interest rates declines and  $\widehat{wc}_t^a$  increases, the cross-sectional variance of the log wealth distribution increases. We know that the household's log wealth-consumption ratio can be stated as:

$$\widehat{wc}_t^i - \widehat{wc}_t^a = \widehat{wc}^i - \widehat{wc}^a + \frac{\mathbb{E}[\widehat{r}^f](\rho^a - \rho_i^c)}{(1 - \rho_i^c)(1 - \rho^a)} + \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} (\widehat{wc}_t^a - \widehat{wc}^a).$$

Hence, the cross-sectional variance of log wealth is given by the cross-sectional variance of consumption, plus the cross-sectional variance of wealth-consumption and the cross-sectional covariance:

$$\begin{aligned} \text{Var}_x \left[ \widehat{wc}_t^i + \widehat{c}_t^i \right] &= \text{Var}_x \left[ \widehat{c}_t^i \right] + \text{Var}_x \left[ \widehat{wc}_t^i \right] + 2\text{Cov}_x \left[ \widehat{wc}_t^i, \widehat{c}_t^i \right], \\ &= \text{Var}_x \left[ \widehat{c}_t^i \right] + \text{Var}_x \left[ \widehat{wc}^i \right] + 2\text{Cov}_x \left[ \widehat{wc}^i, \widehat{c}_t^i \right] \\ &\quad + 2\text{Cov}_x \left[ \widehat{wc}^i, \widehat{c}_t^i \right] + 2\widehat{wc}_t^a \text{Cov}_x \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)}, \widehat{wc}^i \right] \\ &\quad + \text{Var}_x \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} \right] (\widehat{wc}_t^a)^2. \end{aligned}$$

Assume that there is no dispersion in initial consumption shares. Then the cross-sectional variance of log wealth is given by:

$$\text{Var}_x \left[ \widehat{wc}_t^i + \widehat{c}_t^i \right] = \text{Var}_x \left[ \widehat{wc}^i \right] + 2\widehat{wc}_t^a \text{Cov}_x \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)}, \widehat{wc}^i \right] + \text{Var}_x \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)} \right] (\widehat{wc}_t^a)^2.$$

Since  $\mu_i^c$  is the only source of heterogeneity, this implies that  $\text{Cov}_x \left[ \frac{\phi(\rho_i^c - \rho^a)}{(1 - \rho_i^c \phi)}, \widehat{wc}^i \right] \geq 0$ , because  $\rho_i^c$  depends on  $\mu_i^c$ .

We can use the cumulant-generating function to back out the moments of wealth in levels. Let

$w$  denote the log of wealth. The cross-sectional coefficient of variation of wealth is given by:

$$\left( \frac{\mathbb{E}_x[\exp 2w] - (\mathbb{E}_x[\exp w])^2}{(\mathbb{E}_x[\exp w])^2} \right)^{1/2} = \sqrt{\left[ \exp \left( \sum_{j=2}^{\infty} 2^{j-1} \kappa_{j,t+1}(w_{t+1}) / j! \right) - 1 \right]},$$

where  $\kappa_{j,t+1}(w_{t+1})$  denotes the  $j$ -th cumulant.

We can use the cumulant-generating function to back out the moments of wealth in levels. Let  $w$  denote the log of wealth. The first moment of wealth in levels is given by:

$$\mathbb{E}_x[\exp w] = \exp(\kappa_{1,t+1}(w_{t+1}) + \sum_{j=2}^{\infty} \kappa_{j,t+1}(w_{t+1}) / j!),$$

where  $\kappa_{j,t+1}$  denotes the  $j$ -th cumulant of the log wealth distribution. Similarly, the second moment of wealth in levels is given by:

$$\mathbb{E}_x[\exp 2w] = \exp(2\kappa_{1,t+1}(w_{t+1}) + \sum_{j=2}^{\infty} 2^j \kappa_{j,t+1}(w_{t+1}) / j!).$$

As a result, the cross-sectional variance of wealth is

$$\begin{aligned} \mathbb{E}_x[\exp 2w] - (\mathbb{E}_x[\exp w])^2 &= \exp \left( 2\kappa_{1,t+1}(w_{t+1}) + \sum_{j=2}^{\infty} 2^j \kappa_{j,t+1}(w_{t+1}) / j! \right) \\ &\quad - \exp \left( 2\kappa_{1,t+1}(w_{t+1}) + 2 \sum_{j=2}^{\infty} \kappa_{j,t+1}(w_{t+1}) / j! \right). \end{aligned}$$

As a result, the cross-sectional variance of wealth is given by:

$$\begin{aligned} \mathbb{E}_x[\exp 2w] - (\mathbb{E}_x[\exp w])^2 &= \exp \left( 2\kappa_{1,t+1}(w_{t+1}) + 2 \sum_{j=2}^{\infty} \kappa_{j,t+1}(w_{t+1}) / j! \right) \\ &\quad \left[ \exp \left( \sum_{j=2}^{\infty} 2^{j-1} \kappa_{j,t+1}(w_{t+1}) / j! \right) - 1 \right]. \end{aligned}$$

Hence, if we scale the cross-sectional variance of wealth by the cross-sectional mean we obtain:

$$\left( \frac{\mathbb{E}_x[\exp 2w] - (\mathbb{E}_x[\exp w])^2}{(\mathbb{E}_x[\exp w])^2} \right)^{1/2} = \sqrt{\left[ \exp \left( \sum_{j=2}^{\infty} 2^{j-1} \kappa_{j,t+1}(w_{t+1}) / j! \right) - 1 \right]}$$

In the case of log-normal wealth distribution, all of the higher-order cumulants drop out ( $\kappa_{j,t} = 0, k > 2$ ). □

### B.3 Proof of proposition 4.3

*Proof.* The cross-sectional variance of financial wealth  $A = W - H$  can be stated as:

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} = \frac{\mathbb{E}_x(W)^2 Var_x(W)}{\mathbb{E}_x(A)^2 \mathbb{E}_x(W)^2} + \frac{\mathbb{E}_x(H)^2 Var_x(H)}{\mathbb{E}_x(A)^2 \mathbb{E}_x(H)^2} - 2 \frac{\mathbb{E}_x(H)\mathbb{E}_x(W)}{\mathbb{E}_x(A)^2} Cov_x(W, H).$$

$1 - \alpha$  denotes the capital share. The previous equation can be restated as follows:

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} = \frac{1}{(1 - \alpha)^2} \frac{Var_x(W)}{\mathbb{E}_x(W)^2} + \frac{\alpha^2}{(1 - \alpha)^2} \frac{Var_x(H)}{\mathbb{E}_x(H)^2} - 2 \frac{\alpha}{(1 - \alpha)^2} Cov_x(W, H).$$

Note that  $Cov_x(W, H) = Cov_x(A, H) + Var_x(H)$ . We assume that  $Cov_x(A, H) < 0$ . As a result, using  $Cov_x(W, H) \geq Var_x(H)$ , we obtain:

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} \geq \frac{1}{(1 - \alpha)^2} \frac{Var_x(W)}{\mathbb{E}_x(W)^2} + \frac{\alpha^2}{(1 - \alpha)^2} \frac{Var_x(H)}{\mathbb{E}_x(H)^2} - 2 \frac{\alpha}{(1 - \alpha)^2} Var_x(H).$$

If human wealth and financial wealth covary negatively, the cross-sectional variance of financial wealth is bounded below (approximately) by the following expression

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} \geq \frac{1}{(1 - \alpha)^2} Var_x(w) + \frac{\alpha^2 - 2\alpha}{(1 - \alpha)^2} Var_x(h).$$

provided that  $Cov_x(A, H) < 0$ . If  $Var_x(w_t) \leq Var_x(h_t)$  for all  $t$ , which seems plausible given that households seek to smooth consumption, we also obtain:

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} \geq Var_x(w).$$

□

### B.4 Proof of proposition 5.2

*Proof.* The one-period budget constraint:

$$\widehat{c}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t)}{\widehat{R}_t} + \widehat{\sigma}_t(\eta^t)\widehat{v}_t = (1 - \alpha)\widehat{y}_t(\eta^t) + \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha),$$

can be restated, using equation (2), as:

$$\widehat{c}_t(\eta^t) - (1 - \alpha)\widehat{y}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha)}{\widehat{R}_t} = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha). \quad (14)$$

Rewriting (14) one period later:

$$\widehat{c}_{t+1}(\eta^{t+1}) - (1 - \alpha)\widehat{y}_{t+1}(\eta^{t+1}) + \frac{\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha)}{\widehat{R}_{t+1}} = \widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha).$$

Multiply this equation by  $\varphi(\eta_{t+1}|\eta^t)$  and sum across all states  $\eta_{t+1}$  to obtain:

$$\begin{aligned} & \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{c}_{t+1}(\eta^{t+1}) - (1 - \alpha)\widehat{y}_{t+1}(\eta^{t+1}) + \frac{\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha)}{\widehat{R}_{t+1}} \right) \\ &= \widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha), \end{aligned}$$

where we used the fact that  $\sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) = 1$  on the right-hand side. Next, substitute this expression back into (14) to obtain:

$$\begin{aligned} & \widehat{c}_t(\eta^t) - (1 - \alpha)\widehat{y}_t(\eta^t) + \widehat{R}_t^{-1} \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{c}_{t+1}(\eta^{t+1}) - (1 - \alpha)\widehat{y}_{t+1}(\eta^{t+1}) \right) \\ & + \widehat{R}_{t \rightarrow t+1}^{-1} \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha) \right) = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha). \end{aligned}$$

Define financial wealth, scaled by the aggregate endowment, as:

$$\widehat{\theta}_t = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha).$$

Continuing the forward substitution, we end up with the following expression:

$$\widehat{\theta}_t = \sum_{\tau=t}^{\infty} \widehat{R}_{t \rightarrow \tau-1}^{-1} \sum_{\eta^\tau|\eta^t} \varphi(\eta^\tau|\eta^t) (\widehat{c}_\tau(\eta^\tau) - (1 - \alpha)\widehat{y}_\tau(\eta^\tau)).$$

where  $\varphi(\eta^t|\eta^t) = 1$ . Financial wealth must equal the cost of the household's excess consumption plan, where excess refers to the part not paid for with labor income. Noting that  $e_0 = 1$  so that  $\widehat{\theta}_0 = \theta_0$ , writing this expression at time zero:

$$\theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0 \rightarrow \tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) (\widehat{c}_\tau(\eta^\tau) - (1 - \alpha)\widehat{y}_\tau(\eta^\tau))$$

recovers the statement of the proposition. □

### B.5 Proof of Proposition 5.3

*Proof.* We note that the cross-sectional expectation of the product can be decomposed in the standard way:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau) \psi(\eta^\tau) (\hat{c}_\tau(\eta^\tau)) d\Theta_0 = \mathbb{E}_0[\psi_\tau c_\tau] = \text{Cov}_0[\psi_\tau, c_\tau] + \mathbb{E}_0[\psi_\tau] \mathbb{E}_0[c_\tau].$$

If the orthogonality condition is satisfied, then the following result obtains:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau) \psi(\eta^\tau) (\hat{c}_\tau(\eta^\tau)) d\Theta_0 = \mathbb{E}_0[\psi_\tau c_\tau] = \mathbb{E}_0[\psi_\tau] \mathbb{E}_0[c_\tau] = \mathbb{E}_0[c_\tau] = 1,$$

because  $\mathbb{E}_0[\psi_t] = 1$ . □

### B.6 Proof of Proposition 5.4

*Proof.* This inequality  $0 \geq \text{Cov}(\psi_t, \hat{c}_t)$  directly implies that the following inequalities obtain:

$$\begin{aligned} \int \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \psi(\eta^\tau) \hat{c}_\tau(\eta^\tau) d\Theta_0 &\leq \int \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \hat{c}_\tau(\eta^\tau) d\Theta_0 = \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1}, \\ \int \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \psi(\eta^\tau) \hat{y}_\tau(\eta^\tau) d\Theta_0 &\leq \int \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \hat{y}_\tau(\eta^\tau) d\Theta_0 = \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1}. \end{aligned}$$

As a result, this new measure implies an aggregate value of individual wealth that falls short of total wealth,  $\sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1}$ . Note that even though this claim to total consumption is itself not traded, the Lucas tree is a claim to  $\alpha$  of the same cash flow stream. The market value of the Lucas tree is  $\alpha \sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1}$ , and hence the value of total wealth has to be  $\sum_{\tau=0}^{\infty} \hat{R}_{0 \rightarrow \tau-1}^{-1}$ . □

### B.7 Proof of proposition 5.5

*Proof.* An unconstrained household's Euler equation in the high-growth economy is given by:

$$1 = \hat{\beta} \hat{R}_t \sum_{\eta^{t+1}} \varphi(\eta_{t+1} | \eta^t) \frac{u'(\hat{c}(\eta_{t+1}, \eta^t))}{u'(\hat{c}_t(\eta^t))}.$$

This Euler equation is satisfied because the allocations and prices constitute a Bewley equilibrium in the high-growth economy. This household's Euler equation in the new economy with lower interest rates is still satisfied at the old consumption allocation. This can be seen by plugging in the new equilibrium interest rates:

$$\tilde{R}_t \tilde{\beta} = \hat{\beta} \hat{R}_t,$$

to recover the unconstrained household's Euler equation in the low-growth economy:

$$1 = \tilde{\beta} \tilde{R}_t \sum_{\eta_{t+1}} \phi(\eta_{t+1} | \eta_t) \frac{u'(\hat{c}(\eta^t, \eta_{t+1}))}{u'(\hat{c}(\eta^t))}.$$

We allocate the following amount of financial wealth at time 0 to ensure the household can afford the same consumption plan:

$$\tilde{\theta}_0(\theta_0, \eta_0) = \sum_{\tau=0}^{\infty} \tilde{R}_{0 \rightarrow \tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) (\hat{c}_\tau(\eta^\tau) - (1 - \alpha)\hat{y}_\tau(\eta^\tau)).$$

Aggregating this initial financial wealth across households:

$$\int \tilde{\theta}_0 d\Theta_0 = \alpha \sum_{\tau=0}^{\infty} \tilde{R}_{0 \rightarrow \tau}^{-1} = \tilde{v}_0,$$

where we have used the goods market clearing condition and the definition of labor income shares. The last equation shows that the new allocation of initial financial wealth uses up all aggregate financial wealth in the economy. Finally, note that the natural borrowing constraints are not binding in the high-growth economy. They remain non-binding in the low-growth economy because consumption is nonnegative. Hence, the allocations are feasible, and they satisfy the sufficient conditions for optimality.  $\square$



## C Life-Cycle Model Details

Each agent in the life-cycle model with age  $j$ , portfolio of financial assets  $\{a_{k,t}\}$ , and idiosyncratic labor income state  $z$  solves the Bellman equation:

$$V_j(a_t; z_t) = \max_{a_{t+1}} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta s_j \mathbb{E}_t [V_{j+1}(a_{t+1}; z_{t+1})] \quad (15)$$

subject to the budget constraint:

$$c_t \leq y_t + \sum_{k=0}^K (q_k + \delta_k) s_j^{-1} a_{k,t} - q_k a_{k,t+1} \quad (16)$$

where  $y$  is after-tax income as specified in equations (5) and (6),  $s_j$  is the probability of surviving to age  $j + 1$ ,  $q_k$  and  $\delta_k$  are the prices and cash flows, respectively, of the set of risk free financial assets available to the household. The term  $s_j^{-1}$  in the budget constraint (16) represents that households enter an annuity or tontine system in which surviving households receive the assets of households in their age cohort who died, proportional to their asset holdings. This assumption ensures a sufficiently strong savings motive for older households in the absence of a bequest motive.

We can generalize the problem through some convenient variable substitutions. First, we can simplify the asset structure. In a stationary equilibrium, without aggregate shocks or changes to the interest rate, the specific form of the financial assets is arbitrary, although it will be relevant for repricing assets following an interest rate shock. As a result, we can define  $x$  to be the start-of-period value of the entire portfolio, including both its cash flow and continuation value:

$$\theta_t = \sum_{k=0}^K (q_k + \delta_k) a_{k,t}.$$

By no arbitrage, we have

$$\frac{q_k + \delta_k}{q_k} = R$$

for all  $k$ , which implies

$$\sum_{k=0}^K q_k a_{k,t+1} = \sum_{k=0}^K (q_k + \delta_k) R^{-1} a_{k,t+1} = R^{-1} \theta_{t+1}.$$

Substituting now yields the simplified the budget constraint

$$c_t \leq y_t + \theta_t - R^{-1} \theta_{t+1}. \quad (17)$$

Under a constant interest rate, the problem can therefore be solved as if the agents held one-period debt with face value  $\theta$  in each period, allowing us to use a single solution to characterize economies with portfolios over many possible assets.

**Compensated Distribution.** To compute the compensated distribution under a change from interest rate  $R$  to  $\tilde{R}$ , we first compute total wealth under the original and new interest rates:

$$\begin{aligned}\Omega_t &= \sum_{\tau=0}^{\infty} R^{-\tau} c_{t+\tau} \\ \tilde{\Omega}_t &= \sum_{\tau=0}^{\infty} \tilde{R}^{-\tau} c_{t+\tau}.\end{aligned}$$

We next compute human wealth under the original and new interest rates:

$$\begin{aligned}Y_t &= \sum_{\tau=0}^{\infty} R^{-\tau} y_{t+\tau} \\ \tilde{Y}_t &= \sum_{\tau=0}^{\infty} \tilde{R}^{-\tau} y_{t+\tau}.\end{aligned}$$

The implied amount of financial wealth that makes the original consumption plan affordable is therefore

$$\begin{aligned}\theta_t^{comp} &= \tilde{\Omega}_t - \tilde{Y}_t \\ &= \theta_t + (\tilde{\Omega}_t - \Omega_t) + (\tilde{Y}_t - Y_t)\end{aligned}$$

where  $\theta_t$  is pre-shock financial wealth.

**Repriced Distribution.** To compute the repriced distribution following a change from interest rate  $R$  to  $\tilde{R}$ , we will need to specify the specific asset structure. We assume that agents hold zero coupon bonds with maturity  $m$ , which implies  $q_m = R^{-m}$ . At the moment of the interest rate change, the repriced (post-shock) financial wealth  $\theta_t^{repriced}$  is related to pre-shock financial wealth  $\theta_t$  according to the formula

$$\theta_t^{repriced} = \left( \frac{\tilde{q}_m}{q_m} \right) \theta_t = \left( \frac{\tilde{R}}{R} \right)^{-m} \theta_t \quad (18)$$

for a household with bonds of maturity (duration)  $m$ . For our computations, we set  $m$  equal to financial wealth duration, and apply (18).

## D Affine Asset Pricing Model

This appendix develops a reduced-form asset pricing model. The asset pricing model is used for three main purposes. First, to compute long-term real bonds yields and the cost of a 30-year real annuity. Second, to compute the McCauley duration of the aggregate stock market, small stocks, and real estate wealth in a manner that is consistent with the history of bond and stock prices. Third, the model delivers the price and duration of a claim to aggregate consumption and to aggregate labor income.

The asset pricing model in the class of exponentially-affine SDF models. A virtue of the reduced-form model is that it can accommodate a substantial number of aggregate risk factors. We argue that it is important to go beyond the aggregate stock and bond markets to capture the risk embedded in households' financial asset portfolios as well as the aggregate risk in consumption and labor income claims. Similar models are estimated in [Lustig et al. \(2013\)](#); [Jiang, Lustig, Van Nieuwerburgh and Xiaolan \(2019\)](#); [Gupta and Van Nieuwerburgh \(2021\)](#).

### D.1 Setup

#### D.1.1 State Variable Dynamics

Time is denoted in quarters. We assume that the  $N \times 1$  vector of state variables follows a Gaussian first-order VAR:

$$z_t = \Psi z_{t-1} + \Sigma^{\frac{1}{2}} \varepsilon_t, \quad (19)$$

with shocks  $\varepsilon_t \sim i.i.d. \mathcal{N}(0, I)$  whose variance is the identity matrix. The companion matrix  $\Psi$  is a  $N \times N$  matrix. The vector  $z$  is demeaned. The covariance matrix of the innovations to the state variables is  $\Sigma$ ; the model is homoscedastic. We use a Cholesky decomposition of the covariance matrix,  $\Sigma = \Sigma^{\frac{1}{2}} \Sigma^{\frac{1}{2}'}$ , which has non-zero elements only on and below the diagonal. The Cholesky decomposition of the residual covariance matrix allows us to interpret the shock to each state variable as the shock that is orthogonal to the shocks of all state variables that precede it in the VAR. We discuss the elements of the state vector and their ordering below. The (demeaned) one-quarter bond nominal yield is one of the elements of the state vector:  $y_{t,1}^{\$} = y_{0,1}^{\$} + e'_{yn} z_t$ , where  $y_{0,1}^{\$}$  is the unconditional average 1-quarter nominal bond yield and  $e_{yn}$  is a vector that selects the element of the state vector corresponding to the one-quarter yield. Similarly, the (demeaned) inflation rate is part of the state vector:  $\pi_t = \pi_0 + e'_{\pi} z_t$  is the (log) inflation rate between  $t - 1$  and  $t$ . Lowercase letters denote logs.

### D.1.2 Stochastic Discount Factor

The nominal SDF  $M_{t+1}^{\$} = \exp(m_{t+1}^{\$})$  is conditionally log-normal:

$$m_{t+1}^{\$} = -y_{t,1}^{\$} - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1}. \quad (20)$$

Note that  $y_{t,1}^{\$} = -\mathbb{E}_t[m_{t+1}^{\$}] - 0.5\text{Var}_t[m_{t+1}^{\$}]$ . The real log SDF  $m_{t+1} = m_{t+1}^{\$} + \pi_{t+1}$  is also conditionally Gaussian. The innovations in the vector  $\varepsilon_{t+1}$  are associated with a  $N \times 1$  market price of risk vector  $\Lambda_t$  of the affine form:

$$\Lambda_t = \Lambda_0 + \Lambda_1 z_t. \quad (21)$$

The  $N \times 1$  vector  $\Lambda_0$  collects the average prices of risk while the  $N \times N$  matrix  $\Lambda_1$  governs the time variation in risk premia. Asset pricing amounts to estimating the market prices of risk  $(\Lambda_0, \Lambda_1)$ . We specify the moment conditions to identify the market prices of risk below.

### D.1.3 State Vector Elements

The state vector contains the following  $N = 20$  variables, in order of appearance: (1) GDP price inflation, (2) real GDP growth, (3) the nominal short rate (3-month nominal Treasury bill rate), (4) the spread between the yield on a five-year Treasury note and a three-month Treasury bill, (5) the log price-dividend ratio on the CRSP value-weighted stock market, (6) the log real dividend growth rate on the CRSP stock market. Elements 7, 9, 11, and 13 are the log price-dividend ratios on the first size quintile of stocks (small), the first book-to-market quintile of stocks (growth), the fifth book-to-market quintile of stocks (value), and a listed infrastructure index (infra). Elements 8, 10, 12, and 14 are the corresponding log real dividend growth rates. Element 15 is the log price-dividend ratio on housing wealth, element 16 is log real dividend growth on housing wealth. Finally, the state vector contains the log change in the consumption/GDP ratio  $\Delta cx$  in 17th, the log change in the log labor income/GDP ratio  $\Delta lx$  in 18th, the log level of the consumption/GDP ratio  $cx$  in 19th, and the log level of the labor income/GDP ratio  $lx$  in 20th position.

$$z_t = \left[ \pi_t, x_t, y_{t,1}^{\$}, y_{t,20}^{\$} - y_{t,1}^{\$}, pd_t^m, \Delta d_t^m, pd_t^{small}, \Delta d_t^{small}, \right. \\ pd_t^{growth}, \Delta d_t^{growth}, pd_t^{value}, \Delta d_t^{value}, pd_t^{infra}, \Delta d_t^{infra} \\ \left. pd_t^{hw}, \Delta d_t^{hw}, \Delta cx_{t+1}, \Delta lx_{t+1}, cx_{t+1}, lx_{t+1} \right]'. \quad (22)$$

This state vector is observed at quarterly frequency from 1947.Q1 until 2019.Q4 (292 observations). This is the longest available time series for which all variables are available. We use the average of daily Constant Maturity Treasury yields within the quarter. All dividend series are

deseasonalized by summing dividends across the current month and past 11 months. The infrastructure stock index is measured as the value-weighted average of the eight relevant Fama-French industries (Aero, Ships, Mines, Coal, Oil, Util, Telcm, Trans).

Dividend growth on housing wealth is measured as housing services consumption growth from the Bureau of Economic analysis Table 2.3.5. The price-dividend ratio is the ratio of owner-occupied housing wealth from the Financial Accounts of the United States Table B.101.h divided by housing services consumption. We subtract inflation from dividend growth on housing wealth and we also subtract 0.6% per quarter to reflect the fact that the size of the housing stock is growing and we are only interested in the rental price change. The resulting real rental growth rate is 1.7% per year, which is in line with (and still on the higher end of the numbers reported in) the literature.

Aggregate consumption is measured as non-durables plus services plus durable services consumption. Durable services consumption is constructed as the depreciation rate (20%) multiplied by the stock of durables. The stock of durables itself is computed using the perpetual inventory method. This series is divided by nominal GDP and logs are taken. Aggregate labor income is measured as wages and salaries plus business income (Proprietors' income with inventory valuation and capital consumption adjustments) plus transfer income (Personal current transfer receipts) minus taxes (Personal current taxes and Contributions for government social insurance, domestic). This series is divided by nominal GDP and logs are taken. Real consumption growth can then be written as the sum of real GDP growth plus the change in the consumption/GDP ratio:

$$\Delta c_{t+1}^a = x_{t+1} + \Delta cx_{t+1}$$

and similar for labor income growth.

All state variables are demeaned with the observed full-sample mean. The first 18 equations of the VAR are estimated by OLS equation by equation. We recursively zero out all elements of the companion matrix  $\Psi$  whose t-statistic is below 2.2. The resulting point estimates for  $\Psi$  and  $\Sigma^{\frac{1}{2}}$  are reported below.

The dynamics of  $cx$  are pinned down by the dynamics of  $\Delta cx$ :

$$cx_{t+1} = cx_t + \Delta cx_{t+1} = (e_{cx} + e_{cxgr}\Psi)' z_t + e_{cxgr}\gamma^{\frac{1}{2}}\varepsilon_{t+1}$$

Therefore the 19th row of  $\Psi$  is identical to the 17th row, except that  $\Psi(19,19) = \Psi(17,19) + 1$ . Similarly, the 20th row of  $\Psi$  is identical to the 18th row, except that  $\Psi(20,20) = \Psi(18,20) + 1$ . The innovations to the 19th and 20th row are not independent innovations but determined by the innovations that precede it. Essentially, the level variables  $cx$  and  $lx$  are only added to the VAR to enforce cointegration between consumption and GDP and between labor income and GDP. As a result of this cointegration, the aggregate consumption and labor income claims will have the same aggregate risk as the GDP claim.

## D.2 Estimation

### D.2.1 Bond Pricing

In this setting, nominal bond yields of maturity  $\tau$  are affine in the state variables:

$$y_{t,\tau}^{\$} = -\frac{1}{\tau}A_{\tau}^{\$} - \frac{1}{\tau} \left( B_{\tau}^{\$} \right)' z_t.$$

The scalar  $A^{\$}(\tau)$  and the vector  $B_{\tau}^{\$}$  follow ordinary difference equations (ODE) that depend on the properties of the state vector and on the market prices of risk. Real bond yields are also exponentially affine with coefficients that follow their own ODEs. We will price the cross-section of nominal and real bond yields (price levels), putting more weight on matching the time series of one- and twenty-quarter nominal bond yields since those yields are part of the state vector  $z_t$ . We also fit the dynamics of 20-quarter nominal bond risk premia (price changes).

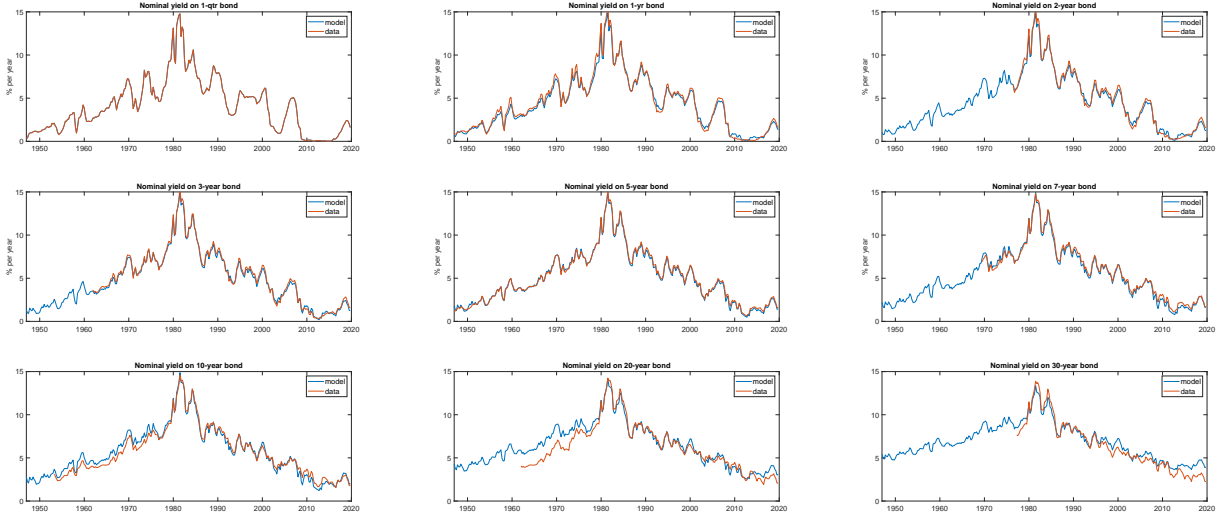
Figure D1 plots the nominal bond yields on bonds of maturities 1 quarter, 1 year, 5 years, and 10 years. Those are the most relevant horizons for the private equity cash-flows. The model matches the time series of bond yields in the data closely. It matches nearly perfectly the 1-quarter and 5-year bond yield which are part of the state space. Figure D2 shows that the model also does a good job matching real bond yields. The top panels of Figure D3 show the model's implications for the average nominal (left panel) and real (right panel) yield curves at longer maturities. These long-term yields are well behaved. The bottom right panel shows a decomposition of the yield on a five-year nominal bond into the five-year real bond yield, annual expected inflation over the next five years, and the five-year inflation risk premium. The importance of these components fluctuates over time. The bottom left panel shows that the model matches the dynamics of the nominal bond risk premium, defined as the expected excess return on five-year nominal bonds. The compensation for interest rate risk varies substantially over time, both in data and in the model.

### D.2.2 Equity and Real Estate Pricing

The VAR contains both the log price-dividend ratio and log dividend growth for five equity risk factors (the aggregate stock market, small stocks, growth stocks, value stocks, and infrastructure stocks) and residential real estate. Together these two time-series imply a time-series for log stock returns. The VAR implies linear dynamics for the expected excess stock return, or equity risk premium, for each equity risk factor. We chose market prices of risk to match these dynamics (price changes).

The price of a stock equals the present-discounted value of its future cash-flows. By value-additivity, the price of the aggregate stock index,  $P_t^m$ , is the sum of the prices to each of its future cash-flows  $D_t^m$ . These future cash-flow claims are the so-called market dividend strips or zero-

Figure D1: Dynamics of the Nominal Term Structure of Interest Rates



Note: The figure plots the observed and model-implied 1-, 4-, 20-, 40-quarter nominal bond yields.

coupon equity (Wachter, 2005). Dividing by the current dividend  $D_t^m$ :

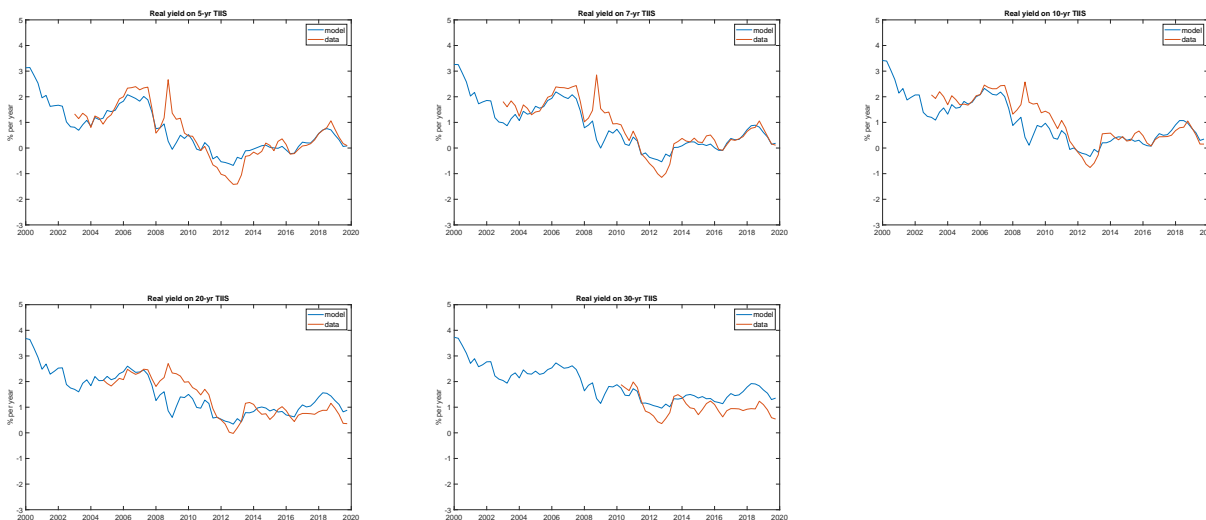
$$\frac{P_t^m}{D_t^m} = \sum_{\tau=1}^{\infty} P_{t,\tau}^d \quad (23)$$

$$\exp\left(\overline{p\bar{d}} + e'_{pd^m} z_t\right) = \sum_{\tau=0}^{\infty} \exp\left(A_{\tau}^m + B_{\tau}^{m'} z_t\right), \quad (24)$$

where  $P_{t,\tau}^d$  denotes the price of a  $\tau$ -period dividend strip divided by the current dividend. The log price-dividend ratio on each dividend strip,  $p_{t,\tau}^d = \log(P_{t,\tau}^d)$ , is affine in the state vector and the coefficients  $(A_{\tau}^m, B_{\tau}^{m'})$  follow an ODE. Since the log price-dividend ratio on the stock market is an element of the state vector, it is affine in the state vector by assumption. Equation (24) restates the present-value relationship from equation (23). It articulates a non-linear restriction on the coefficients  $\{(A_{\tau}^m, B_{\tau}^{m'})\}_{\tau=1}^{\infty}$  at each date (for each state  $z_t$ ), which we impose in the estimation (price levels). Analogous present value restrictions are imposed for each of the other traded equity factors, whose price-dividend ratios and dividend growth rates are also included in the state vector.

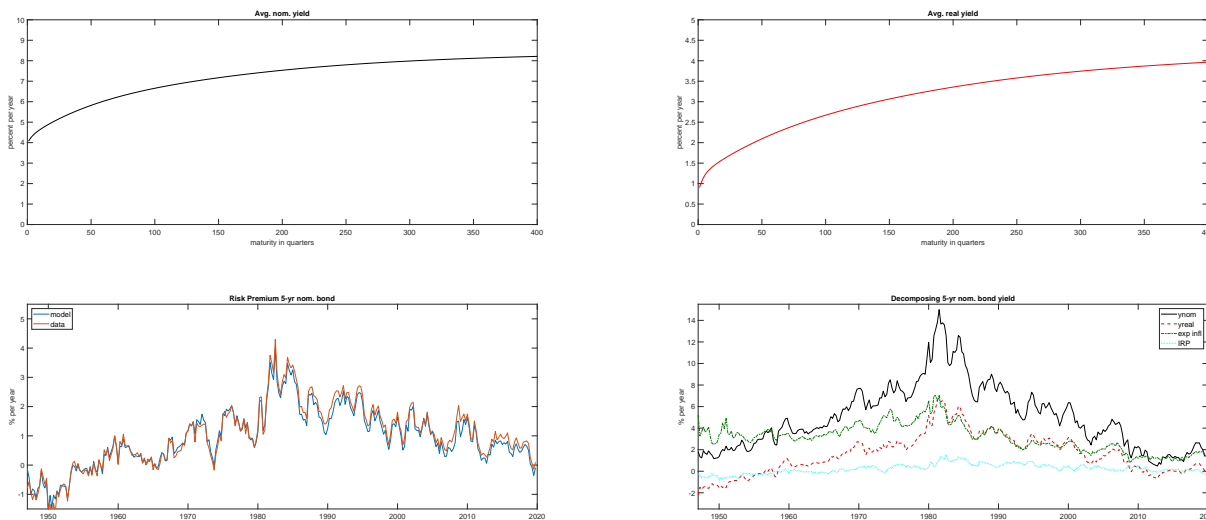
If dividend growth were unpredictable and its innovations carried a zero risk price, then dividend strips would be priced like real zero-coupon bonds. The strips' dividend-price ratios would equal yields on real bonds with the coupon adjusted for deterministic dividend growth. All variation in the price-dividend ratio would reflect variation in the real yield curve. In reality, the dynamics of real bond yields only account for a small fraction of the variation in the price-dividend ratio, implying large prices of risk associated with shocks to dividend growth that are orthogonal

Figure D2: Dynamics of the Real Term Structure of Interest Rates



Note: The figure plots the observed and model-implied 20-, 28-, 40-, and 80-quarter real bond yields.

Figure D3: Long-term Yields and Bond Risk Premia



Note: The top panels plot the average bond yield on nominal (left panel) and real (right panel) bonds for maturities ranging from 1 quarter to 400 quarters. The bottom left panel plots the nominal bond risk premium in model and data. The bottom right panel decomposes the model's five-year nominal bond yield into the five-year real bond yield, the five-year inflation risk premium and the five-year real risk premium.

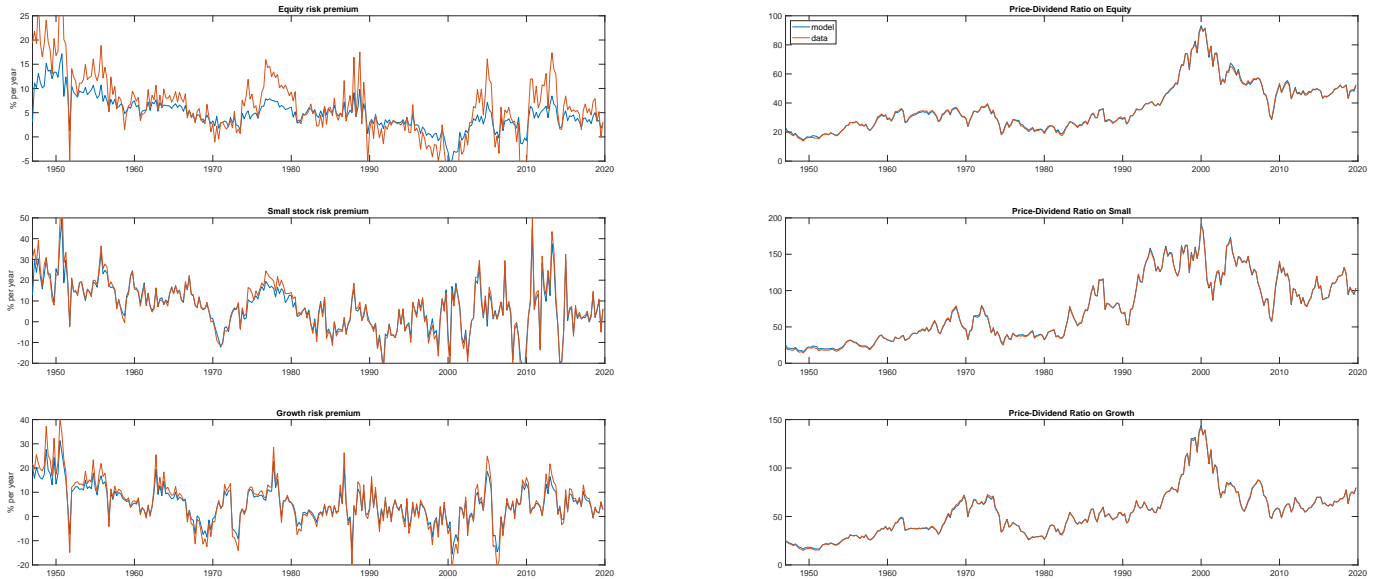
to shocks to bond yields.

Figures D4 and D5 show the equity risk premium, the expected excess return, in the left panels and the price-dividend ratio in the right panels. The various rows cover the five equity indices and the we price. The dynamics of the risk premia in the data are dictated by the VAR. The model chooses the market prices of risk to fit these risk premium dynamics as closely as possible.



The price-dividend ratios in the model are formed from the price-dividend ratios on the strips of maturities ranging from 1 to 3600 quarters, as explained above. The figure shows an excellent fit for price-dividend levels and a good fit for risk premium dynamics. Some of the VAR-implied risk premia have outliers which the model does not fully capture. This is in part because the good deal bounds restrict the SDF from becoming too volatile and extreme. We note large level differences in valuation ratios across the various stock factors, as well as big differences in the dynamics of both risk premia and price levels, which the model is able to capture well.

Figure D4: Equity Risk Premia and Price-Dividend Ratios (part 1)



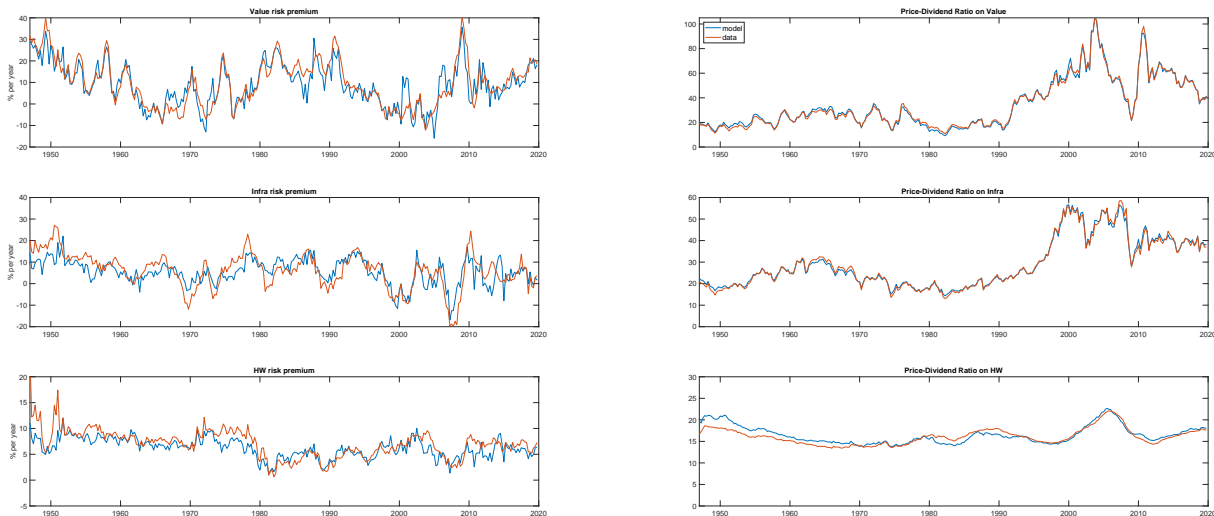
*Note:* The figure plots the observed and model-implied equity risk premium on the overall stock market, small stocks, and growth stocks in the left panels, as well as the corresponding price-dividend ratio in the right panels. The model is the blue line, the data are the red line.

### D.2.3 Pricing Claims to Aggregate Consumption and Labor Income

Shocks to the growth rate in consumption/GDP (labor/income) ratio are priced only to the extent that they are correlated with other priced sources of risk. The innovation to the change in the consumption/GDP (labor income/GDP) ratio that is orthogonal to all prior shocks is not priced.

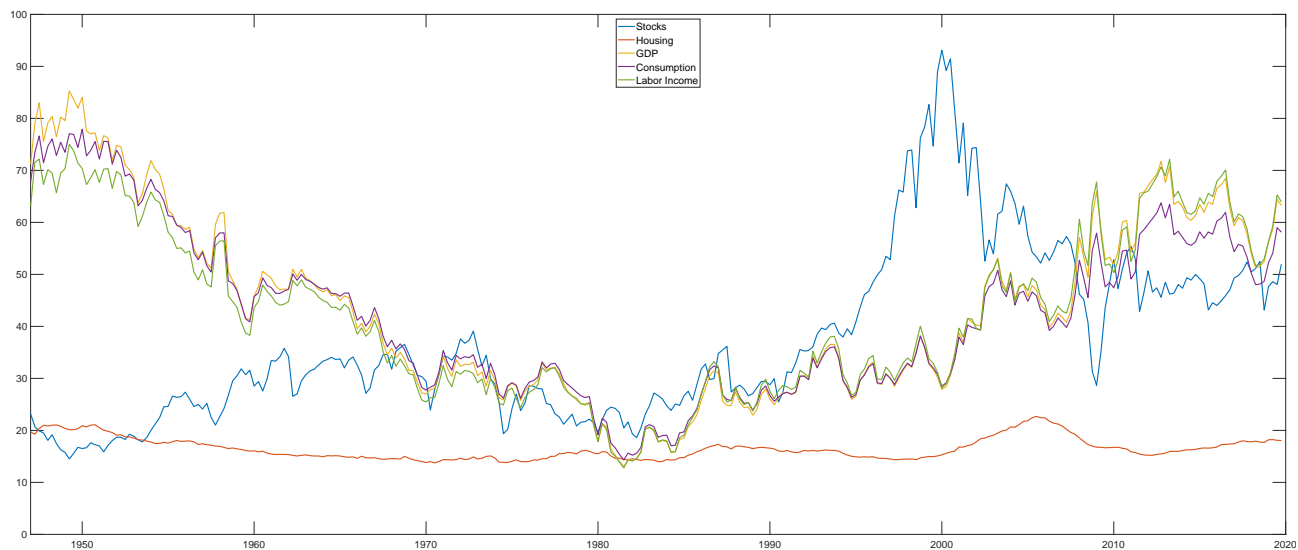
Figure D6 plots the annual price-dividend ratios on the claims to GDP, aggregate consumption, and aggregate labor income. It contrasts these valuation ratios to that on the aggregate stock market.

Figure D5: Equity Risk Premia and Price-Dividend Ratios (part 2)



*Note:* The figure plots the observed and model-implied equity risk premium on value stocks, infrastructure stocks, and housing wealth in the left panels, as well as the corresponding price-dividend ratio in the right panels. The model is the blue line, the data are the red line.

Figure D6: Valuation Ratios



*Note:* The figure plots the annual price-dividend ratios on the aggregate stock market and claims to GDP, aggregate consumption, and aggregate labor income.

## D.2.4 Cash-flow Duration

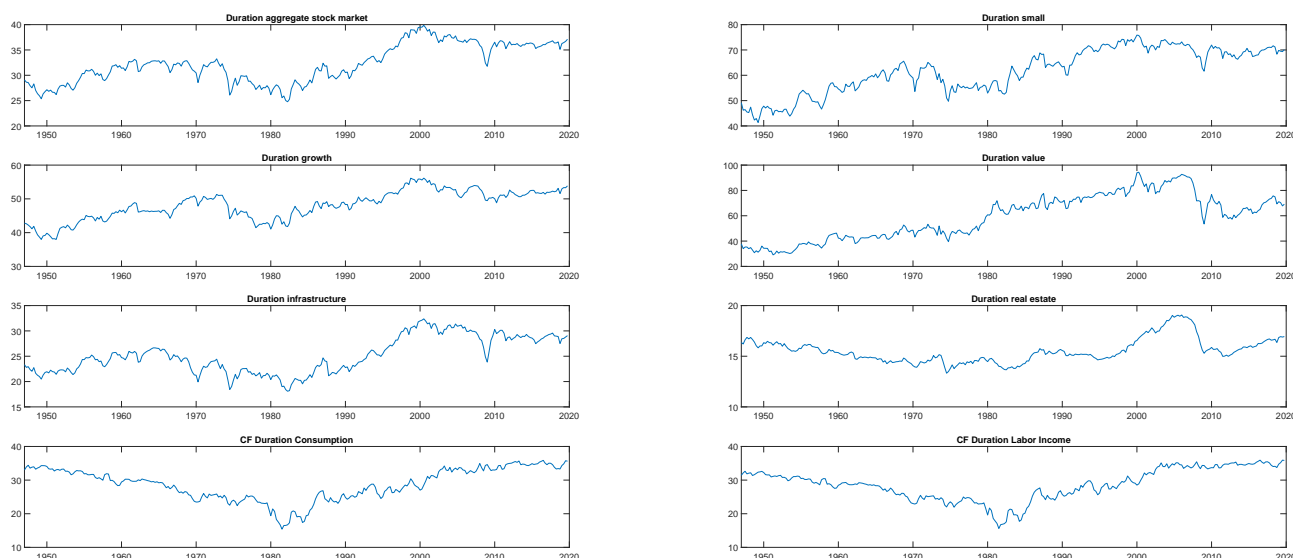
The (McCauley) duration is the weighted average time for an investor to receive cash flows. For the aggregate stock market, this measure is computed as follows:

$$D_t^{CF,m} = \sum_{\tau=1}^{\infty} w_{t,\tau} \tau, \quad w_{t,h} = \frac{P_{t,\tau}^d}{P_t^m} = \frac{\exp(A_\tau^m + B_\tau^{m'} z_t)}{\exp(\bar{p}\bar{d} + e'_{pd^m} z_t)}$$

where  $P_{t,\tau}^d$  is the price-dividend ratio of a  $\tau$ -period dividend strip. Since durations are usually expressed in years while time runs in quarters in our model, we have to divide by 4. This duration concept is defined similarly for any other equity index, real estate, GDP, consumption, and labor income claims. Note that for a nominal or real zero-coupon bond of maturity  $\tau$ ,  $D_t^{CF} = \tau$ .

Figure D7 The figure plots the model-implied time series of cash-flow durations on the overall stock market, small stocks, growth stocks, value stocks, infrastructure stocks, housing wealth, the GDP claim, the aggregate consumption claim, and the aggregate labor income claim.

Figure D7: Cash-Flow Duration



*Note:* The figure plots the model-implied time series of cash-flow durations on the overall stock market, small stocks, growth stocks, value stocks, infrastructure stocks, housing wealth, the GDP claim, the aggregate consumption claim, and the aggregate labor income claim. The duration is expressed in years.

## D.2.5 Market Price of Risk Estimates

The market prices of risk are pinned down by the moments discussed in the main text. Here we report and discuss the point estimates. Note that the prices of risk are associated with the orthogonal VAR innovations  $\varepsilon \sim \mathcal{N}(0, I)$ . Therefore, their magnitudes can be interpreted as (quarterly)

Sharpe ratios. The constant in the market price of risk estimate  $\widehat{\Lambda}_0$  is:

0.11	0.00	-0.36	0.05	0.00	0.43	0.00	-0.01	0.00	0.12	0.00	0.25	0.00	0.25	0.00	2.74	0.00	0.00	0.00
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The matrix that governs the time variation in the market price of risk is estimated to be  $\widehat{\Lambda}_1 =$ :

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19.3	16.7	-31.8	-250.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37.8	15.8	0.0	3.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0	19.6	0.9
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23.3	3.8	-29.7	-160.2	-0.9	12.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	3.7	0.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-5.0	-1.5	1.6	-20.9	0.6	2.0	-0.5	8.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.0	-4.1	0.2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.1	34.7	-23.8	13.9	0.9	-11.1	-0.1	1.8	-1.1	16.7	0.0	0.0	0.0	0.0	0.0	0.0	0.9	1.2	-0.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	33.8	-11.0	34.0	0.8	-15.5	0.1	-0.9	-0.3	0.4	-2.0	0.6	0.0	0.0	0.0	0.0	-1.5	1.3	17.6
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10.5	42.9	-39.3	-20.9	7.6	-10.7	1.0	-2.3	-5.9	-3.2	0.5	1.7	-4.4	0.1	0.0	0.0	3.4	3.7	-5.2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8.8	29.2	-109.1	66.7	5.6	1.8	-2.2	-5.8	-0.3	-3.2	0.9	0.2	-3.2	0.0	1.3	0.4	0.1	-0.2	-14.7
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0