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ABSTRACT

Asset managers face increasing pressure to only hold firms that meet net-zero carbon emissions targets. We model how these mandates incentivize firms to address the global-warming externality through investments in decarbonization capital. A firm that invests receives a lower cost of capital by an amount equal to its investments divided by its Tobin's *q*. Mandates act as a capital tax, funding a higher decarbonization-to-productive capital ratio. Due to adjustment costs, this ratio rises gradually - as does the sustainable-finance tax - until the steady state. Our model matches macro-finance moments, stock-demand elasticity, and climate-mitigation pathways. The welfare-maximizing mandate with markets approximates the planner's first-best solution.

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1 Introduction

In light of the failure of many legislative bodies to implement carbon emissions taxes, there is growing pressure on the financial sector to fulfill the 2015 Paris Agreement by keeping global temperatures within 1.5^o Celsius above pre-industrial levels. Regulations and activism are pushing asset managers toward sustainable finance mandates, whereby a fraction of their portfolios are constrained to hold firms that can meet net-zero emissions targets by 2050. Examples include the Net-Zero Managers Initiative (with 43 trillion dollars of assets under management committed) and the Network for Greening the Financial System (NGFS) (with central banks proposing climate stress tests of institutional-investor portfolios). Regulations by the European Union (and likely ones by the Security Exchange Commission) requiring disclosures of Scope 1 and 2 carbon emissions address enforceability or greenwashing concerns.

Coinciding with the pressure from these mandates, major corporations, including even energy producers, have announced plans to meet certain net-zero emissions targets. While part of these targets will be achieved with a switch to renewables, much will rely on spending enough on decarbonization measures. According to a recent Intergovernmental Panel on Climate Change (IPCC) special report (Rogelj et al. (2018)), remaining mitigation pathways to net-zero emissions require a portfolio of decarbonization measures, including negative emission technologies (NETs) such as afforestation and reforestation, soil carbon sequestration, bioenergy with carbon capture and storage (BECCs), and direct air capture (DAC).[1] While a number of these measures exist, the stock of decarbonization capital (e.g., forests and plants to do air capture) is low relative to what is needed to stabilize our climate due to externalities.

We model how these mandates incentivize firms to address the global warming externality via the accumulation of this decarbonization-capital stock. We use our model to address a number of questions. To what extent can mandates achieve first-best outcomes when it comes to mitigating global warming? How large would these mandates have to be? How

[1]One reason is that for heavy industrial sectors like cement and steel, which generate nearly 20% of global CO2 emissions, switching fuel sources is not a viable option for achieving net-zero (de Pee (2018)).

effective are mandates in competitive stock markets with arbitrageurs? How many years would it take for the economy to transition to the steady state? What are the consequences for extreme temperatures? What is the impact on economic growth, particularly during the transition? And what about the consequences for investors' portfolio performance? How much can decarbonization contribute to the goal of reaching a net-zero economy?

While there is a literature on how socially responsible investing incentivizes firms to reform via a cost-of-capital channel,[2] there is no work that we know of that can address these welfare issues. In order to conduct these calculations, we first introduce decarbonization capital into a dynamic stochastic general equilibrium model with traditional capital as the sole input of producing a homogeneous good and the sole source of carbon emissions.[3] Decarbonization capital only offsets carbon emissions and has no productive role. Both types of capital are subject to adjustment costs.

Emissions lead to extreme global temperatures, which damage economic growth (Dell, Jones, and Olken (2012), Burke, Hsiang, and Miguel (2015)).[4] The ratio of decarbonization capital to productive capital reduces the frequency of extreme temperatures and hence expected losses from global warming. Investments in decarbonization capital, which come at the expense of firm productivity, increase this ratio.

There is a high willingness-to-pay for mitigation among our households with non-expected utility ((Epstein and Zin (1989) and Weil (1990)) since disasters cause significant welfare losses (Barro (2006), Weitzman (2009), and Pindyck and Wang (2013)). But there is an

[2]The first model of green mandates and the cost-of-capital channel in a static CARA setting is Heinkel, Kraus, and Zechner (2001). Hong and Kacperczyk (2009) show how ethical investing mandates affect sin companies. Recent work (Pastor, Stambaugh, and Taylor (2020), Pedersen, Fitzgibbons, and Pomorski (2020)) model how non-pecuniary tastes of green investors influence cross-sectional asset prices in a CAPM setting. Sustainable mandates need not only be passive but also active via voting for environmentally friendly policies (Gollier and Pouget (2014), Broccardo, Hart, and Zingales (2020), and Oehmke and Opp (2020)) but exit or screens are the predominant form of mandates.

[3]Our two capital stock approach builds on Eberly and Wang (2009), who consider a general equilibrium model with two sectors of different productivity.

[4]According to the National Academy of Sciences (2016)), extreme temperatures lead to increased frequency and damage from hurricanes that make landfall (Grinsted, Ditlevsen, and Christensen (2019), Kossin et.al. (2020)). Similarly, the wildfires in the Western US states are also linked to climate change (Abatzoglou and Williams (2016)). See Bansal, Ochoa, and Kiku (2017) for the impact of higher temperature on growth stocks and Hong, Karolyi, and Scheinkman (2020) for a review of evidence on the damage of natural disasters for financial markets.

externality when it comes to mitigating the damages of emissions. Since the benefits of this mitigation only affect the aggregate risk and the market price of risk, which firms take as given, firms do not contribute to decarbonization capital in competitive markets (Hong, Wang and Yang (2020)) — i.e., there is over-accumulation of productive capital and under-accumulation of decarbonization capital.

There is a competitive stock market with a representative investor, who has access to a complete set of financial securities (e.g., all contingencies including idiosyncratic shocks are dynamically spanned) but is restricted to passively index a fixed fraction of total wealth to firms that meet sustainability guidelines. To be included in the representative investor's sustainable portfolio, otherwise ex-ante identical firms have to invest a minimally required amount on decarbonization which they otherwise would not due to externalities. Hence, a sustainable finance mandate specifies both the fraction of wealth that is restricted to the sustainable finance index and firm spending on decarbonization that is required to qualify to be part of this index. Investors also face shorting constraints due to a number of institutional reasons (see Almazan, Brown, Carlson, and Chapman (2004), Hong and Stein (2007)).

Despite being a dynamic stochastic general equilibrium model, the solution is intuitive and has a number of implications. The value of productive capital, i.e., Tobin's q, for sustainable and unsustainable firms, are endogenously determined so as to leave value-maximizing firms indifferent between being sustainable or not — the Tobin's q or stock price is the same for all firms in equilibrium. The decarbonization capital, which is unproductive and does not contribute to output, sits in the firm's assets but is not priced by markets other than through the mandate qualification mechanism.[5] The risk-free rate, stock-market risk premium, Tobin's q for aggregate productive capital, and growth rates are jointly determined. They in turn depend on the ratio of decarbonization to productive capital, which governs transition dynamics and the steady state.

In equilibrium, there is a cost-of-capital wedge between qualified and unqualified firms that equals firm decarbonization investments divided by its Tobin's q. Since firms have the same Tobin's q in equilibrium, the growth paths of both sustainable and unsustainable firms

[5]The decarbonization capital can equivalently be managed by a public sector.

are identical (path by path) over time. Sustainable firms have lower cashflows to pay out due to mitigation spending but have lower cost of capital (the expected return required by the representative investor). The cash-flow effect and the discount-rate effect have to offset each other so as to leave all firms indifferent between being a sustainable and an unsustainable firm. The lower cost of capital for sustainable firms subsidizes their decarbonization, which they would have otherwise invested in productive capital or distributed to shareholders. The benefits of this mitigation accrue to the entire economy.

Since there is a perfectly competitive and homogeneous goods market and capital is the only input, mandates act as a capital tax (i.e. a sustainable-finance tax), funding a higher decarbonization-to-productive capital ratio to mitigate global warming.[6] Due to adjustment costs, the ratio of decarbonization-to-productive capital rises gradually over time until steady-state. Since the cost of capital wedge tracks annual firm decarbonization investments, which scale with the amount of decarbonization capital in the economy, the sustainable-finance tax will also vary in the transition, tending to increase before steady state.

We focus in the paper on two types of solutions. The first is the welfare-maximizing mandate with markets — taking as given a fraction of wealth that is restricted to sustainable firms, the government announces the minimum decarbonization spending for firms to qualify that maximizes the welfare of agents given the competitive market solution. Implementing this solution only requires a sufficient fraction of wealth be restricted. When the fraction of wealth that is indexed to sustainable finance mandates is larger, all else equal, each sustainable firm needs to make less investments (i.e. qualifying standards are lower for being labeled sustainable). The second is the planner's solution or first-best solution. We compare the outcomes of a welfare-maximizing mandate with markets to that of the planner's first-best solution to address the welfare questions we posed.

In our quantitative analysis, we show that our model can simultaneously match key

[6]This is in contrast a tax on emissions (Golosov, Hassler, Krusell and Tsyviski (2014)) in traditional integrated assessment models featuring an emissions sector as the only input and a perfectly competitive goods market.

macro-finance moments (Bansal and Yaron (2004)) and realistic climate mitigation pathways as emphasized by the recent literature on integrated assessment models of a carbon tax (see, e.g., Nordhaus (2017), Jensen and Traeger (2014), Cai and Lontzek (2019), Daniel, Litterman, and Wagner (2019)), and especially by Barnett, Brock, and Hansen (2020)). In addition, our model can also match price elasticity of stock demand, in that sustainable finance mandates generate realistic stock price effects that are consistent with the literature on downward-sloping stock demand curves (i.e. cost-of-capital wedges are not unrealistic large as to violate limited arbitrage).[7]

To this end, we use estimates of the damage to GDP growth from abnormal annual country temperatures of 1.5^o Celsius (relative to pre-industrial era) to discipline our model. Such events are still uncommon, occurring in a few percent of the country-year observations but damage conditional on such an event is around minus four percentage points of GDP growth (Dell, Jones, and Olken (2012)). We then calibrate parameters governing the adjustment cost and efficiency of decarbonization capital to be consistent with reforestation, which is one of the most cost-effective forms of carbon capture (Rogelj et al. (2018), Bastin et al. (2019), and Griscom et al. (2017)).

Using the fact that the current level of decarbonization is small, we can pin down both the economic damage absent mitigation and the cost of mitigation from the first-order conditions of the planner's problem.[8] We then consider a comparative static where we increase the frequency of annual extreme temperatures. In a 1.5^o Celsius world, the frequency of extreme temperature events will rise (from a few percent of country-year observations to a much larger fraction). A small amount of mitigation will no longer be optimal. In our baseline quantitative exercise, we report the solutions (for the welfare-maximizing mandate with markets and the planner's solution) as we increase frequencies of extreme annual temperatures to one (or about 63% of country-year observations).

There are several key messages from our quantitative analysis. First, the fraction of

[7]See e,g., Shleifer (1986), Wurgler and Zhuravskaya (2002), Chang, Hong and Liskovich (2015), Kashyap, Kovrijnykh, and Pavlova (2018), and Koijen and Yogo (2019).

[8]For instance, there are some attempts already at reforestation around the world such as in US, China, Turkey, Canada and a number of countries in Europe but these are relatively small efforts.

wealth that needs to be restricted to sustainable finance companies to implement the welfare-maximizing mandate with markets solution is only a few percent since sustainable firms only have to satisfy a non-zero dividend constraint and hence can in principle dedicate all their investments toward decarbonization.[9] Estimates conservatively place sustainable finance restrictions at around 10% to 20% of wealth.[10]

Second, the welfare-maximizing mandate with markets solution can approximate the first-best solution. The steady-state ratio of decarbonization to productive capital in the planner's solution is close to 5.7%. The decarbonization-to-productive capital ratio in the market economy with the optimal mandate is around 4.6% at the steady state. Despite this short-fall, the welfare gains from the mandate solution are substantial, and close to the gains obtained by the planner's solution — almost 25% higher measured in the certainty equivalent wealth than in a purely competitive market setting. One reason is that the rise of decarbonization capital brings down the jump arrival rate of extreme temperature country-year events non-linearly, with substantial benefits for even modest increases in decarbonization capital. In our calibration, the overall benefits to temperature are broadly in line with climate-mitigation pathways connected to reforestation.

Given that the global capital stock is around 600 trillion dollars, a 4.6% decarbonization-to-productive capital ratio implies around 27.6 trillion dollars of decarbonization capital (i.e. book value of new forests) at the steady state.[11] Aggregate contributions to decarbonization capital stock each year under the welfare-maximizing mandate with markets is around 0.23% of physical capital stock in the steady state, which means spending of around 1.4 trillion dollars per year towards decarbonization. Decarbonization contributions peak at the steady state. The transition time to the steady state is about 23 years (a number that we target in

[9] It is easy to extend our model to add payout constraints. In this setting, there is then value to restricting a higher fraction of wealth. One can endogenize this fraction by assuming some regulatory costs to having more assets under management restricted.

[10] The Net-Zero Manager Initiative accounts for 7% of capital. More generally, according to US SIF Foundation in January 2019, around 38% of assets under management already undergo some type of sustainability screening (though not all of it is regarding decarbonization) and over 80% of these screens as implemented as passive portfolios.

[11] Gadzinki, Schuller and Vacchino (2018) estimate global capital stock (including both traded and non-traded assets) in 2016 to be between 500 and 600 trillion dollars.

our calibration to be consistent with the time it takes to reforest).

At 10% (20%) of wealth restricted to sustainable firms, the cost-of-capital wedge rises to 1.40% (0.70%) per annum at the steady state to compensate sustainable firms that have to spend close to 2.3% (1.2%) of their capital stock each year to qualify.[12] Even at fairly high levels of indexing to a sustainable finance mandate, these cost-of-capital wedges are not too large, which is consistent with realistic price elasticity of stock demand and limited arbitrage. The risk-free rate rises over time and the risk premium falls as the economy becomes less risky with a higher decarbonization-to-productive capital ratio in steady state.

Finally, we find that the welfare-maximizing mandate with markets based only on decarbonization (reforestation) can get us about 25% of the way towards aggregate net-zero emissions targets by 2050. Our analysis is consistent with IPCC estimates that decarbonization needs to play a significant role in meeting net-zero emissions targets by 2050. Hence, our paper also offers a new approach for analyzing climate-mitigation pathways and the net-zero economy.

2 Model

While mitigating climate disaster risk benefits the society, doing so is privately costly for the firm. We model sustainable finance mandates as portfolio restrictions on the representative agent's portfolio and examine the extent to which it encourages firms to provide risk mitigation and quantify its implications for social welfare. We use a representative-agent framework for expositional simplicity, where this agent can be interpreted as representing both public (e.g., sovereign wealth funds) and private investors.

On the demand side for financial assets, the representative agent holds and invests the entire wealth of the economy between sustainable (S) firms, unsustainable (U) firms, and the risk-free bonds. The agent has to invest an α fraction of the entire aggregate wealth in a sustainable type-S firm. The risk-averse representative agent is required to meet the sustainable investment mandate at all times when allocating assets.

[12] See Bolton and Kacperczyk (2020) for preliminary estimates of expected returns based on Scope 1+2 emissions.

On the supply side, a portfolio of S firms and a portfolio of U firms will arise endogenously in equilibrium, which we refer to as S-portfolio and U-portfolio, respectively. For a firm to qualify to be type-S, it has to spend at least a fraction m of its capital on mitigation via a portfolio of decarbonization technologies so as to reduce disaster risk. Otherwise, it is labeled a type-U for unsustainable.

2.1 Firm Production and K Capital Accumulation

The firm's output at t, Y_t, is proportional to its capital stock, K_t, which we refer to as productive capital and is the only factor of production:

$$Y_t = AK_t \,, \tag{1}$$

where $A > 0$ is a constant that defines productivity for all firms. This is a version of widely-used AK models in macroeconomics and finance. All firms start with the same level of initial capital stock K_0 and have the same production and capital accumulation technology. Additionally, they are subject to the same shocks (path by path).

That is, there is no idiosyncratic shock in our model. This simplifying assumption makes our model tractable and allows us to focus on the impact of the investment mandate on equilibrium asset pricing and resource allocation. Despite being identical in all aspects, some firms choose to be sustainable while others remain unsustainable in equilibrium.

Investment. Let I_t denote the firm's investment. As in Pindyck and Wang (2013), the firm's productive capital stock, K_t, evolves as:

$$dK_t = \Phi(I_{t-}, K_{t-})dt + \sigma K_{t-} d\mathcal{B}_t - (1 - Z)K_{t-} d\mathcal{J}_t \;. \tag{2}$$

As in Lucas and Prescott (1971) and Jerrmann (1998), we assume that $\Phi(I, K)$, the first term in (2), is homogeneous of degree one in I and K, and thus can be written as

$$\Phi(I, K) = \phi(i)K \;, \tag{3}$$

where $i = I/K$ is the firm's investment-capital ratio and $\phi(\,\cdot\,)$ is increasing and concave. This specification captures the idea that changing capital stock rapidly is more costly than changing it slowly. As a result, installed capital earns rents in equilibrium so that Tobin's q, the ratio between the value and the replacement cost of capital exceeds one.

The second term captures continuous shocks to capital, where $\mathcal{B}_t$ is a standard Brownian motion and the parameter σ is the diffusion volatility (for the capital stock growth). This $\mathcal{B}_t$ is the source of shocks for the standard AK models in macroeconomics. This diffusion shock is common to all firms. Had we introduced an additional shock that is idiosyncratic across firms, our solution would remain unchanged as firms can perfectly hedge idiosyncratic shocks at no cost and our aggregation results remain valid.

Jump shocks. The firm's K capital stock is also subject to an aggregate jump shock. We capture this jump effect via the third term, where $\mathcal{J}_t$ is a (pure) jump process with an endogenously determined arrival rate, which we denote by $\lambda_{t-} > 0$, which we discuss in detail later. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at t. Examples of jumps include hurricanes or wildfires (and related extreme temperatures) that destroy physical and housing capital stock.

When a jump arrives ($d\mathcal{J}_t = 1$), it permanently destroys a stochastic fraction $(1 - Z)$ of the firm's capital stock K_{t-}, as Z is the recovery fraction where $Z \in (0, 1)$. (For example, if a shock destroyed 15 percent of capital stock, we would have $Z = .85$.) There is no limit to the number of these jump shocks.[13] If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears. We assume that the cumulative distribution function (cdf) and probability density function (pdf) for the recovery fraction, Z, conditional on a jump arrival at any time t, are time invariant. Let $\Xi(Z)$ and $\xi(Z)$ denote the cdf and pdf of Z, respectively.

We use **boldfaced** notations for aggregate variables. Before discussing the endogenous jump arrival rate λ_{t-}, we first introduce emissions, emission removals, and the dynamics of decarbonization capital stock $\mathbf{N}$.

[13]Stochastic fluctuations in the capital stock have been widely used in the growth literature with an AK technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (disaster) jumps.

2.2 Aggregate Emissions, Emission Removals, and Decarbonization Capital Stock N

We assume that the aggregate emissions $\mathbf{E}$ is proportional to $\mathbf{K}$:

$$\mathbf{E}_{t-} = \mathbf{e}\mathbf{K}_{t-} \,, \tag{4}$$

where $\mathbf{e} > 0$ is a constant. That is, aggregate emissions increases linearly with the size of the production sector of the economy, which is measured by the aggregate capital stock $\mathbf{K}$ or equivalently GDP ($A\mathbf{K}$).

Similarly, we assume that the aggregate emission removals $\mathbf{R}$ is proportional to the decarbonization capital stock $\mathbf{N}$:

$$\mathbf{R}_{t-} = \tau\mathbf{N}_{t-} \,, \tag{5}$$

where $\tau > 0$ is a constant. Equations (4) and (5) state that both aggregate emissions and carbon removals are given by an "AK"-type of technology.

Let $\mathbf{X}_t$ denote the aggregate mitigation spending. The aggregate decarbonization capital stock $\mathbf{N}$ evolves as follows:

$$\frac{d\mathbf{N}_t}{\mathbf{N}_{\mathbf{t}-}} \;=\; \omega(\mathbf{X}_{t-}/\mathbf{N}_{\mathbf{t}-})dt + \sigma d\mathcal{B}_t - (1-Z)d\mathcal{J}_t \,. \tag{6}$$

In (6), $\omega(\mathbf{X}_{t-}/\mathbf{N}_{\mathbf{t}-})$ is the rate at which aggregate mitigation spending $\mathbf{X}_{t-}$ increases $d\mathbf{N}_t/\mathbf{N}_{t-}$. We assume that $\omega(\,\cdot\,)$ is increasing and concave as we do for $\phi(i)$. This specification captures the idea that changing the decarbonization capital stock rapidly is more costly than changing it slowly.

We further assume that the growth rate $d\mathbf{N}_t/\mathbf{N}_{t-}$ for decarbonization capital stock $\mathbf{N}_{t-}$ is subject to the same diffusion and jump shocks as the growth rate of capital stock K, dK_t/K_{t-}, path by path (e.g., for each realized jump and recovery fraction Z). This explains why the last two terms in (6) take the same form as those in (2).

Let $\mathbf{n}_{t-}$ denote the decarbonization stock $\mathbf{N}_{t-}$ scaled by $\mathbf{K}_{t-}$:

$$\mathbf{n}_{t-} = \frac{\mathbf{N}_{t-}}{\mathbf{K}_{t-}} \,. \tag{7}$$

Using Ito's lemma, we obtain the following dynamics for $\mathbf{n}_t$:

$$\frac{d\mathbf{n}_t}{\mathbf{n}_{t-}} = [\omega(\mathbf{x}_{t-}/\mathbf{n}_{t-}) - \phi(\mathbf{i}_{t-})]\, dt\,. \tag{8}$$

Since the two types of capital stock are subject to the same jump-diffusion shocks, there is no uncertainty for the dynamics of $\mathbf{n}_t$. Next, we describe the distribution for the recovery fraction Z.

2.3 Mitigation and Externality

Since global warming is expected to increase the frequency of disasters, we assume that the jump arrival rate λ_{t-} increases with the aggregate emissions $\mathbf{E}_{t-}$ and decreases with the aggregate emissions removals $\mathbf{R}_{t-}$. As $\mathbf{E}_{t-} = \mathbf{e}\mathbf{K}_{t-}$ and $\mathbf{R}_{t-} = \tau\mathbf{N}_{t-}$ (see equations (4) and (5)), we may write λ_{t-} as a function that is increasing in $\mathbf{K}_{t-}$ and decreasing in $\mathbf{N}_{t-}$.

The pre-jump expected damage over a small dt period is $\lambda_{t-}\mathbb{E}(1-Z)\mathbf{K}_{t-}dt$, where $\mathbb{E}(\,\cdot\,)$ is the expectation operator. We further make the following homogeneity assumption: the expected damage doubles if we simultaneously double both the size of the productive sector ($\mathbf{K}_{t-}$) and the size of the protective sector ($\mathbf{N}_{t-}$). This boils down to assuming that λ_{t-} is homogeneous of degree zero in $\mathbf{K}_{t-}$ and $\mathbf{N}_{t-}$, which means λ_{t-} is simply a function of the pre-jump scaled *aggregate* decarbonization stock $\mathbf{n}_{t-} = \mathbf{N}_{t-}/\mathbf{K}_{t-}$. It is useful to make the dependence of λ_{t-} on $\mathbf{n}_{t-}$ explicit: $\lambda_{t-} = \lambda(\mathbf{n}_{t-})$. Intuitively, increasing $\mathbf{n}$ lowers the jump arrival rate, $\lambda'(\mathbf{n}) < 0$. Additionally, the marginal impact of $\mathbf{N}$ on the change of λ decreases as $\mathbf{N}$ increases, i.e., $\lambda''(\mathbf{n}) > 0$.

As disaster shocks are aggregate and disaster damages are only curtailed by *aggregate* decarbonization stock $\mathbf{N}$, absent mandates or other incentive programs, firms have no incentives to mitigate on their own as the economy is competitive and their own mitigation spending have no impact on the *aggregate* mitigation spending (Hong, Wang, and Yang (2020)).

2.4 Sustainable Investment Mandates

Let $\mathbf{1}_t^S$ be an indicator function describing the status of a firm at t. To qualify as a sustainable (S) firm at t, the firm has to spend at least M_t at t on disaster risk mitigation, which contributes to the reduction of aggregate risk. That is, $\mathbf{1}_t^S = 1$ if and only if the firm's mitigation spending X_t satisfies:

$$X_t \geq M_t \,. \tag{9}$$

Otherwise, $\mathbf{1}_t^S = 0$ and the firm is unsustainable (U).

To preserve our model's homogeneity property, we assume that the mandated mitigation spending is proportional to firm size K_t for given $\mathbf{n_t}$:

$$M_t = m(\mathbf{n_t})K_t \,, \tag{10}$$

where m_t is the minimal level of mitigation per unit of the firm's capital stock to qualify a firm to be sustainable. That is, it is cheaper for a firm (with smaller K_t) to qualify as a sustainable firm. Later, we endogenize the S-firm qualification threshold, $m(\mathbf{n_t})$, to maximize the representative agent's utility.

The investment mandate α creates the inelastic demand for S firms. In equilibrium, the remaining $1-\alpha$ fraction is invested in the U-portfolio so that the agent has no investment in the risk-free bonds in equilibrium.

2.5 Optimal Firm Mitigation

Each firm can choose to be either a sustainable (S) or a unsustainable firm (U). We assume that a firm's mitigation is observable and contractible. While spending on aggregate risk mitigation yields no monetary payoff for the firm, doing so allows it to be included in the S-portfolio.

A value-maximizing firm chooses whether to be sustainable or unsustainable depending on which strategy yields a higher value. Let Q_t^j denote the the market value of a type-j firm at t, where $j = \{S, U\}$. By exploiting our model's homogeneity property, we conjecture and verify that the equilibrium value of a type-j firm at time t must satisfy:

$$Q_t^j = q^j(\mathbf{n_t})K_t^j \,, \tag{11}$$

where q^j is Tobin's average q for a type j-firm for given $\mathbf{n_t}$.

In equilibrium, as mitigation spending has no direct benefit for the firm, if the firm chooses to be U, i.e., $\mathbf{1}_t^S = 0$, it will set $X_t = 0$. Moreover, even if a firm chooses to be a S firm, it has no incentive to spend more than M_t, i.e., (9) always binds for a type-S firm.

As we later verify, the equilibrium expected rate of return for a type-j firm, which we denote by $r^j(\mathbf{n_t})$, is a function of $\mathbf{n_t}$. A type-j firm maximizes its present value:

$$\max_{I^j, X^j} \mathbb{E}\left(\int_0^\infty e^{-r^j(\mathbf{n_t})t} CF^j(\mathbf{n_t})dt\right) \tag{12}$$

subject to the standard transversality condition specified in the Appendix A. In equation (12), $CF^j(\mathbf{n_t})$ is the firm's cash flow at t, which is given by

$$CF^S(\mathbf{n_t}) = AK_t^S - I_t^S(\mathbf{n_t}) - X_t^S(\mathbf{n_t}) \quad \text{and} \quad CF^U(\mathbf{n_t}) = AK_t^U - I_t^U(\mathbf{n_t}) \,, \tag{13}$$

as an unsustainable firm spends nothing on mitigation.

Since I_t and X_t are both proportional to K_t, spending on X_t effectively reduces the productivity of firms. Hence, X_t can be broadly interpreted as spending on various decarbonization measures.

2.6 Dynamic Consumption and Asset Allocation

The representative agent makes all the consumption and asset allocation decisions. We thus use individual and aggregate variables for the agent interchangeably. For example, the aggregate wealth, $\mathbf{W}_t$, is equal to the representative agent's wealth, W_t. Similarly, the aggregate consumption, $\mathbf{C}_t$, is equal to the representative agent's consumption, C_t.

The representative agent has the following investment opportunities: (a) the S portfolio which includes all the sustainable firms; (b) the U portfolio which includes all other firms that are unsustainable; (c) the risk-free asset that pays interest at a constant risk-free interest rate r determined in equilibrium; and (d) actuarially fair insurance claims for disasters with every possible recovery fraction Z (and also for diffusion shocks.)

Type-S and type-U portfolios. The S and U portfolios include all the S and U firms, respectively. Let $\mathbf{Q}_t^S$ and $\mathbf{Q}_t^U$ denote the aggregate market value of the S portfolio firm and the U portfolio at t, respectively. Similarly, Let $\mathbf{D}_t^S$ and $\mathbf{D}_t^U$ denote the aggregate dividend of the S portfolio firm and the U portfolio at t, respectively.

We conjecture and then verify that the cum-dividend return for the type-n portfolio is given by

$$\frac{d\mathbf{Q}_t^j + \mathbf{D}_{t-}^j dt}{\mathbf{Q}_{t-}^j} = r^j(\mathbf{n_{t-}})dt + \sigma d\mathcal{B}_t - (1-Z)\left(d\mathcal{J}_t - \lambda(\mathbf{n_{t-}})dt\right), \tag{14}$$

where $r^j(\mathbf{n_{t-}})$ is the endogenous expected cum-dividend return for a type-j firm in equilibrium for given $\mathbf{n}$. In equation (14), the diffusion volatility is equal to σ as in equation (2). The third term on the right side of equation (14) is a jump term capturing the effect of disasters on return dynamics. Both the diffusion volatility and jump terms are martingales (and this is why $r^j(\mathbf{n_{t-}})$ is the expected return.) Note that the only difference between the S- and U-portfolio is the expected return. The diffusion and jump terms are the same as those in the capital evolution dynamics given in equation (2).

Disaster risk insurance (DIS). We define DIS as follows: a DIS for the survival fraction in the interval $(Z, Z+dZ)$ is a swap contract in which the buyer makes insurance payments $p(Z)dZ$, where $p(Z)$ is the equilibrium insurance premium payment, to the seller and in exchange receives a lump-sum payoff if and only if a shock with survival fraction in $(Z, Z+dZ)$ occurs. That is, the buyer stops paying the seller if and only if the defined disaster event occurs and then collects one unit of the consumption good as a payoff from the seller. The DIS contract is priced at actuarially fairly terms so that investors earn zero profits.

Preferences. We use the Duffie and Epstein (1992) continuous-time version of the recursive preferences developed by Epstein and Zin (1989) and Weil (1990), so that the representative agent has homothetic recursive preferences given by:

$$V_t = \mathbb{E}_t\left[\int_t^\infty f(C_s, V_s)ds\right], \tag{15}$$

where $f(C,V)$ is known as the normalized aggregator given by

$$f(C,V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\chi}}{((1-\gamma)V)^{\chi-1}} \,. \tag{16}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and we let $\chi = (1-\psi^{-1})/(1-\gamma)$. Unlike expected utility, recursive preferences as defined by (15) and (16) disentangle risk aversion from the EIS. An important feature of these preferences is that the marginal benefit of consumption is $f_C = \rho C^{-\psi^{-1}}/[(1-\gamma)V]^{\chi-1}$, which depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

If $\gamma = \psi^{-1}$ so that $\chi = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C,V) = \frac{\rho\, C^{1-\gamma}}{1-\gamma} - \rho V. \tag{17}$$

This more flexible utility specification is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the large follow-up long-run risk literature.

Wealth dynamics. Let W_t denote the representative agent's wealth. Let H_t^S and H_t^U denote the dollar amount invested in the S and U portfolio, respectively. Let H_t denote the agent's wealth allocated to the market portfolio at t. That is, $H_t = H_t^S + H_t^U$. The dollar amount, $(W_t - H_t)$ is the dollar amount invested in the risk-free asset. For disasters with recovery fraction in $(Z, Z+dZ)$, $\delta_t(Z, \mathbf{n_{t-}})W_t dt$ gives the total demand for the DIS over time period $(t, t+dt)$.

The agent accumulates wealth as:

$$\begin{aligned} dW_t = {} & [r(\mathbf{n_{t-}})\,(W_{t-} - H_{t-}) - C_{t-}]\,dt + \left(r^S(\mathbf{n_{t-}})H_{t-}^S + r^U(\mathbf{n_{t-}})H_{t-}^U\right)dt + \sigma H_{t-} d\mathcal{B}_t \\ & - (1-Z)\,H_{t-}(d\mathcal{J}_t - \lambda(\mathbf{n_{t-}})dt) - \left(\int_0^1 \delta(Z,\mathbf{n_{t-}})p(Z,\mathbf{n_{t-}})dZ\right) W_{t-}dt + \delta(Z,\mathbf{n_{t-}})W_{t-}d\mathcal{J}_t \,. \end{aligned} \tag{18}$$

The first term in (18) is the interest income from savings in the risk-free asset minus consumption. The second term is the expected return from investing in the S and U portfolios. Note that the expected returns are different: $r^S(\mathbf{n})$ and $r^U(\mathbf{n})$ for the S and U portfolios, respectively. The third and fourth terms are the diffusion and jump martingale terms for the stock market portfolio. Note that the stochastic (shock) components of the returns (diffusion and jumps) for the two portfolios are identical path by path. The fifth term is the total DIS premium paid by the consumer before the arrival of jumps and captures the financial hedging cost. The last term describes the DIS payments by the DIS seller to the household when a jump occurs.

The total market capitalization of the economy, $\mathbf{Q}_t$, is given by

$$\mathbf{Q}_t = q^S(\mathbf{n_t})\mathbf{K}_t^S + q^U(\mathbf{n_t})\mathbf{K}_t^U \,. \tag{19}$$

Let π_t^S and π_t^U denote the fraction of total wealth W_t allocated to the S and U portfolio at time t, respectively. That is, $H_t^S = \pi_t^S H_t$, $H_t^U = \pi_t^U H_t$, and the remaining fraction $1 - \left(\pi_t^S + \pi_t^U\right)$ of W_t is allocated to the risk-free asset.

In equilibrium, the investment mandate requires that the total capital investment in the S portfolio has to be at least an α fraction of the total stock market capitalization $\mathbf{Q}_t$:

$$H_t^S \geq \alpha \mathbf{Q}_t \,. \tag{20}$$

The total stock market capitalization $\mathbf{Q}_t$ depends on the mandate. We later derive a closed-form expression for the relation between $\mathbf{Q}_t$ and α.

Let $\mathbf{Y}_t$, $\mathbf{C}_t$, $\mathbf{I}_t$, and $\mathbf{X}_t$ denote the aggregate output, consumption, investment, and mitigation spending, respectively. Adding across all type-S and U firms, we obtain the aggregate resource constraint:

$$\mathbf{Y}_t = \mathbf{C}_t + \mathbf{I}_t + \mathbf{X}_t \,. \tag{21}$$

2.7 Competitive Equilibrium with Mandates

We define the competitive equilibrium subject to the investment mandate as follows: (1) the representative agent dynamically chooses consumption and asset allocation among the

S portfolio, the U portfolio, and the risk-free asset subject to the investment mandate given in (20); (2) each firm chooses its status (S or U), and investment policy I to maximizes its market value; (3) all firms that choose sustainable investment policies are included in the S portfolio and all remaining firms are included in the U portfolio; and (4) all markets clear.

The market-clearing conditions include (i) the net supply of the risk-free asset is zero; (ii) the representative agent's demand for the S portfolio is equal to the total supply by firms choosing to be sustainable; (iii) the representative agent's demand for the U portfolio is equal to the total supply by firms choosing to be brown; (iv) the net demand for the DIS of each possible recovery fraction Z is zero; and (v) the goods market clears, i.e., the resource constraint given in (21) holds.

Because the risk-free asset and all DIS contracts are in zero net supply, the agent's entire wealth W_t is invested in the S and U portfolios.

3 Equilibrium Solution

In this section, we solve for the equilibrium solution with the sustainable finance mandate. First, we introduce the investment mandate at the firm level.

3.1 Sustainability Investment Mandate

For a firm to be sustainable at t, it is required to spend the minimal required m_t fraction of its productive capital stock K_t. We assume that m_t is a function of $\mathbf{n}_t$ to preserve our model's homogeneity property:

$$x_t^S = \frac{X_t^S}{K_t^S} = m(\mathbf{n}_t). \tag{22}$$

Any additional spending on mitigation is suboptimal as it yields no further benefit to the firm. All other firms spend nothing on mitigation and hence are unsustainable, i.e., $x_t^U = 0$. The fraction of total wealth allocated to meet the sustainability investment mandate is α.

Next, we consider the firm's decision problem when it takes the sustainability mandate $\{m_t : t \geq 0\}$ as given.

3.2 Firm Optimization

We solve for optimal investment policies for both types of firms. The firm's objective (12) implies that $\int_0^s e^{-\int_0^t r^j(\mathbf{n}_v)dv} CF^j(\mathbf{n}_t)dt + e^{-\int_0^s r^j(\mathbf{n}_v)dv} Q_s^j$ is a martingale under the physical measure. We obtain the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} r^j(\mathbf{n})Q^j(K^j,\mathbf{n}) \quad = \quad & \max_{I^j} \; CF^j(\mathbf{n}) + \left(\Phi(I^j,K^j)Q_K^j(K^j,\mathbf{n}) + \frac{1}{2}(\sigma K^j)^2 Q_{KK}^j(K^j,\mathbf{n})\right) \\ & + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right] \mathbf{n} Q_{\mathbf{n}}^n(K^j,\mathbf{n}) + \lambda(\mathbf{n})\mathbb{E}\left[Q^j(ZK^j,\mathbf{n}) - Q^j(K^j,\mathbf{n})\right] , \end{aligned} \tag{23}$$

where $r^j(\mathbf{n})$ is the cost of capital and $CF^j(\mathbf{n})$ is the cash flow for a type-j firm given by (13). The preceding equation takes the aggregate decarbonization stock $\mathbf{n}$, aggregate mitigation spending $\mathbf{x}$, and aggregate investment $\mathbf{i}$ as given. In (23), $\mathbb{E}[\,\cdot\,]$ is the conditional expectation operator with respect to the distribution of recovery fraction Z. The last term depends on the scaled aggregate decarbonization capital stock $\mathbf{n}$ and has the same effect on all firms.

By using our model's homogeneity property, $Q_t^j = q^j(\mathbf{n_t})\mathbf{K_t}$, we obtain the following

$$r^j(\mathbf{n})q^j(\mathbf{n}) = \max_{i^j} \; cf^j(\mathbf{n}) + g(i^j)q^j(\mathbf{n}) + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right] \mathbf{n} q^{j\prime}(\mathbf{n}) , \tag{24}$$

where $g(i(\mathbf{n}))$ is the expected firm growth rate:

$$g(i(\mathbf{n})) = \phi(i(\mathbf{n})) - \lambda(\mathbf{n})(1 - \mathbb{E}(Z)) , \tag{25}$$

and $cf^j(\mathbf{n}) = CF^j(\mathbf{n})/K^j$ is the scaled cash flow for a type-j firm. As $x^S(\mathbf{n}) = m(\mathbf{n})$ and $x^U(\mathbf{n}) = 0$, we have $cf^S(\mathbf{n}) = A - i^S(\mathbf{n}) - m(\mathbf{n})$ for a type-S firm and $cf^U(\mathbf{n}) = A - i^U(\mathbf{n})$ for a type-U firm.

The investment FOC for both types of firms implied by (24) is the following well known condition in the q-theory literature:

$$q^j(\mathbf{n}) = \frac{1}{\phi'(i^j(\mathbf{n}))} . \tag{26}$$

A type-j firm's marginal benefit of investing is equal to its marginal q, $q^j(\mathbf{n})$, multiplied by $\phi'(i^j(\mathbf{n}))$. Equation (26) states that this marginal benefit, $q^j(\mathbf{n})\phi'(i^j(\mathbf{n}))$, is equal to one, the marginal cost of investing at optimality. The homogeneity property implies that a firm's marginal q is equal to its average q (Hayashi, 1982).

3.3 Representative Agent's Optimization

In the appendix, we show that both the optimal risk-free asset holding and the jump hedging demand $\delta(Z, \mathbf{n})$ are zero for all Z and $\mathbf{n}$ in equilibrium. Additionally, the fraction of total wealth allocated to the S-portfolio, which we denote by $\pi^S = H^S/(H^S + H^U) = H^S/W$, is equal to the fraction of wealth mandated to invest in the S portfolio: $\pi^S = \alpha$. The remaining $1 - \pi^S$ fraction of total wealth is allocated to the U-portfolio. That is, $H_t^S = \alpha W_t = \mathbf{Q}_t^S = \alpha \mathbf{Q}_t$, $H_t^U = (1-\alpha)W_t = \mathbf{Q}_t^U = (1-\alpha)\mathbf{Q}_t$, and $W_t = \mathbf{Q}_t = \mathbf{Q}_t^S + \mathbf{Q}_t^U$.

To ease exposition, here we only highlight the FOC with respect to consumption and the following associated simplified HJB equation for the agent's value function, $V(W, \mathbf{n})$:

$$0 = \max_C \; f(C,V) + \left[\left(r^S(\mathbf{n})\alpha + r^U(\mathbf{n})(1-\alpha)\right) W - C\right] V_W(W,\mathbf{n}) + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right] \mathbf{n} V_{\mathbf{n}}(W,\mathbf{n})$$
$$+ \frac{\sigma^2 W^2 V_{WW}(W,\mathbf{n})}{2} + \lambda(\mathbf{n}) \int_0^1 \left[V(ZW,\mathbf{n}) - V(W,\mathbf{n})\right] \xi(Z) dZ \,. \tag{27}$$

The FOC for consumption C is the standard condition:

$$f_C(C,V) = V_W(W,\mathbf{n}) \,. \tag{28}$$

3.4 Market Equilibrium

The equilibrium risk-free rate $r(\mathbf{n})$, the expected returns ($r^S(\mathbf{n})$ and $r^U(\mathbf{n})$) for the S and U portfolios, Tobin's average q for all firms are all functions of $\mathbf{n}$.

As a firm can choose being either sustainable or not, it must be indifferent between the two options at all time. That is, in equilibrium, all firms have the same Tobin's q, which in equilibrium is also Tobin's $\mathbf{q}$ for the aggregate economy:

$$q^S(\mathbf{n}) = q^U(\mathbf{n}) = \mathbf{q}(\mathbf{n}) \,. \tag{29}$$

Equations (26) and (29) imply that all firms also have the same equilibrium investment-capital ratio, which is also the aggregate $\mathbf{i}(\mathbf{n})$ for given $\mathbf{n}$:

$$i^S(\mathbf{n}) = i^U(\mathbf{n}) = \mathbf{i}(\mathbf{n}) \,. \tag{30}$$

As a result, the cash flows difference between a U and an S firm is exactly the mitigation spending:

$$cf^U(\mathbf{n}) - cf^S(\mathbf{n}) = m(\mathbf{n})\,, \tag{31}$$

where $cf^U(\mathbf{n}) = A - i(\mathbf{n})$.

Since each S firm spends $m(\mathbf{n})K_t^S$ units on mitigation and all firms are of the same size, we have the following relation between the scaled mitigation $m(\mathbf{n})$ at the firm level and scaled mitigation at the aggregate level $\mathbf{x}(\mathbf{n}) = \mathbf{X}(\mathbf{n})/\mathbf{K}$:

$$m(\mathbf{n}) = \frac{\mathbf{x}(\mathbf{n})}{\alpha} \geq \mathbf{x}(\mathbf{n})\,. \tag{32}$$

The mitigation spending mandate for a firm, $m(\mathbf{n})$, is larger than the aggregate scaled mitigation, $\mathbf{x}(\mathbf{n})$, as only an α fraction of firms are sustainable.

In equilibrium, the aggregate consumption is equal to the aggregate dividend:

$$\mathbf{c}(\mathbf{n}) = \mathbf{cf}(\mathbf{n}) = A - \mathbf{i}(\mathbf{n}) - \mathbf{x}(\mathbf{n})\,. \tag{33}$$

Equilibrium risk-free rate $r(\mathbf{n})$ and expected market return $r^M(\mathbf{n})$ for a given n. Building on Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we calculate the aggregate stock-market risk premium, $r^M(\mathbf{n}) - r(\mathbf{n})$, by using

$$r^M(\mathbf{n}) - r(\mathbf{n}) = \gamma\sigma^2 + \lambda(\mathbf{n})\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right]\,.$$

The risk-free rate is

$$r(\mathbf{n}) = \frac{c(\mathbf{n})}{\mathbf{q}(\mathbf{n})} + \phi(\mathbf{i}(\mathbf{n})) + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n}))\right]\frac{\mathbf{n}\mathbf{q}'(\mathbf{n})}{\mathbf{q}(\mathbf{n})} - \gamma\sigma^2 - \lambda(\mathbf{n})\mathbb{E}\left[(1-Z)Z^{-\gamma}\right]\,. \tag{34}$$

Aggregate $\mathbf{i}(\mathbf{n})$, $\mathbf{q}(\mathbf{n})$, and $\mathbf{c}(\mathbf{n})$ for a given $\mathbf{x}(\mathbf{n})$ process. For a given $\mathbf{x}(\mathbf{n})$ process, we obtain the aggregate scaled investment $\mathbf{i}(\mathbf{n})$ by solving

$$\begin{aligned} 0 \;=\; & \frac{(A - \mathbf{i}(\mathbf{n}) - \mathbf{x}(\mathbf{n}))\,\phi'(\mathbf{i}(\mathbf{n})) - \rho}{1-\psi^{-1}} + \phi(\mathbf{i}(\mathbf{n})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\ & + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right]\left(\frac{\psi}{1-\psi}\frac{\mathbf{n}\mathbf{q}'(\mathbf{n})}{\mathbf{q}(\mathbf{n})} - \frac{1}{1-\psi}\frac{\mathbf{n}\mathbf{i}'(\mathbf{n}) + \mathbf{n}\mathbf{x}'(\mathbf{n})}{A - \mathbf{i}(\mathbf{n}) - \mathbf{x}(\mathbf{n})}\right)\,, \end{aligned} \tag{35}$$

where $\mathbf{q}(\mathbf{n})$ is given by

$$\mathbf{q}(\mathbf{n}) = \frac{1}{\phi'(\mathbf{i}(\mathbf{n}))}\,. \tag{36}$$

Welfare, optimal mitigation, and equilibrium investment. In Appendix C, we show that the welfare measure per unit of capital, $b(\mathbf{n}) = u(\mathbf{n}) \times q(\mathbf{n})$, satisfies the following ODE:

$$\begin{aligned} 0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}-\mathbf{x}}{b(\mathbf{n})}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i})+(\omega(\mathbf{x}/\mathbf{n})-\phi(\mathbf{i}))\frac{\mathbf{n}b'(\mathbf{n})}{b(\mathbf{n})} \\ & -\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]. \end{aligned} \tag{37}$$

The FOC for the welfare-maximizing level of $\mathbf{x}$ is given by

$$b(\mathbf{n})^{-\psi^{-1}}b'(\mathbf{n}) = (A-\mathbf{i}-\mathbf{x})^{-\psi^{-1}}\frac{\rho}{\omega'(\mathbf{x}/\mathbf{n})}. \tag{38}$$

The FOC for the optimal investment is

$$b(\mathbf{n})^{1-\psi^{-1}} = (A-\mathbf{i}-\mathbf{x})^{-\psi^{-1}}\frac{\rho}{\phi'(\mathbf{i})}. \tag{39}$$

At the steady state, the drift of $\mathbf{n}$ is zero. Let $\mathbf{i}^*$ and $\mathbf{x}^*$ denote the corresponding steady-state investment-capital ratio and mitigation spending, respectively. We have

$$\omega(\mathbf{x}^*/\mathbf{n}^*) - \phi(\mathbf{i}^*) = 0 \tag{40}$$

and

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}^*-\mathbf{x}^*}{b(\mathbf{n}^*)}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}^*)-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n}^*)}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]. \tag{41}$$

Summary. The steady state is an endogenously determined boundary that satisfies (38), (39), (40), and (41). For the transition path, we solve the ODE (37) together with the FOCs (38) and (39) subject to the boundary conditions for the steady state given above.

We next calculate the costs of capital for S and U firms.

Cost-of-capital wedge. It is helpful to use $\theta^j(\mathbf{n})$ to denote the wedge between the expected return for a type-j firm, $r^j(\mathbf{n})$, and the aggregate stock-market return, $r^M(\mathbf{n})$, and write for $j = \{S, U\}$,

$$r^j(\mathbf{n}) = r^M(\mathbf{n}) + \theta^j(\mathbf{n}). \tag{42}$$

As an α fraction of the total stock market is the S portfolio and the remaining $1-\alpha$ fraction is the U portfolio, we have

$$r^M(\mathbf{n}) = \alpha \cdot r^S(\mathbf{n}) + (1-\alpha) \cdot r^U(\mathbf{n})\,. \tag{43}$$

Using (24) for both S- and U-portfolios, we obtain

$$\theta^U(\mathbf{n}) \;\; = \;\; \frac{\mathbf{x}(\mathbf{n})}{\mathbf{q}(\mathbf{n})} = \frac{\alpha m(\mathbf{n})}{q(\mathbf{n})} > 0\,. \tag{44}$$

Equation (44) states that investors demand a higher rate of return to invest in U firms than in the aggregate stock market. The expected return wedge between the U-portfolio and the market portfolio is equal to $\theta^U(\mathbf{n})$, which is equal to the aggregate mitigation spending $\mathbf{X}(\mathbf{n})$ divided by aggregate stock market value $\mathbf{Q}(\mathbf{n})$. This ratio $\mathbf{x}(\mathbf{n})/\mathbf{q}(\mathbf{n})$ can be viewed as a "tax" on the unsustainable firms by investors in equilibrium.

Substituting (42) into (43) and using (44), we obtain:

$$\theta^S(\mathbf{n}) = -\frac{1-\alpha}{\alpha}\,\theta^U(\mathbf{n}) = -\frac{1-\alpha}{\alpha}\frac{\mathbf{x}(\mathbf{n})}{\mathbf{q}(\mathbf{n})} = -\,(1-\alpha)\,\frac{m(\mathbf{n})}{q(\mathbf{n})} < 0\,. \tag{45}$$

The cost-of-capital difference between U and S firms is given by

$$r^U(\mathbf{n}) - r^S(\mathbf{n}) = \theta^U(\mathbf{n}) - \theta^S(\mathbf{n}) = \frac{1}{\alpha}\frac{\mathbf{x}(\mathbf{n})}{\mathbf{q}(\mathbf{n})} = \frac{m(\mathbf{n})}{\mathbf{q}(\mathbf{n})}\,. \tag{46}$$

By being sustainable, a firm lowers its cost of capital from $r^U(\mathbf{n})$ to $r^S(\mathbf{n})$ by $r^U(\mathbf{n}) - r^S(\mathbf{n})$. To enjoy this benefit, the firm spends $m(\mathbf{n})$ on mitigation. To make it indifferent between being sustainable and not, the cost-of-capital wedge is given by $r^U(\mathbf{n}) - r^S(\mathbf{n}) = m(\mathbf{n})/\mathbf{q}(\mathbf{n})$, the ratio between the firm's mitigation spending, $m(\mathbf{n})K$, and its market value, $\mathbf{q}(\mathbf{n})K$.

4 Welfare-Maximizing Mandate with Markets versus the Planner's Solution

We focus on two types of solutions: the welfare-maximizing mandate with markets versus the planner's solution.

4.1 Welfare-Maximizing Mandate with Markets

For a given level of α, we endogenize the criterion at the firm level characterized by the scaled mitigation threshold $M_t = m(\mathbf{n_t})K_t$, for a firm to qualify as a sustainable firm. Specifically, at time 0, the planner announces $\{M_t; t \geq 0\}$ and commits to the announcement with the goal of maximizing the representative agent's utility given in equation (15) taking into account that the representative agent and firms take the mandate as given and optimize in competitive equilibrium.[14] Since no firm spends more than M_t to qualify as an S firm, the equilibrium aggregate mitigation spending satisfies:

$$\mathbf{X_t} = \alpha M_t. \tag{47}$$

Comment. In our model, the representative agent represents investors in the whole economy including both the private and public sectors. We may also interpret our representative-agent model as one with heterogeneous agents where an α fraction of them are sustainable investors, who have investment mandates (e.g., large asset managers and sovereign wealth funds), and the remaining $1-\alpha$ fraction do not. The sustainable investors group has inelastic demand for sustainable firms and moreover they do not lend their shares out for other investors to short sustainable firms.

4.2 Planner's (First-Best) Solution

We contrast the welfare-maximizing mandate with markets to the planner's solution where the planner chooses aggregate $\mathbf{C}$, $\mathbf{I}$, and $\mathbf{X}$ to maximize the representative agent's utility defined in (15)-(16).

As our model features the homogeneity property, it is convenient to work with scaled variables at both aggregate and individual levels. We use lower-case variables to denote the corresponding upper-case variables divided by contemporaneous capital stock. For example, at the firm level, $i_t = I_t/K_t$, $\phi_t = \Phi_t/K_t$, and $x_t = X_t/K_t$. Similarly, at the aggregate level, $\mathbf{i}_t = \mathbf{I}_t/\mathbf{K}_t$, $\mathbf{x}_t = \mathbf{X}_t/\mathbf{K}_t$. For consumers, $c_t = \mathbf{c_t} = \mathbf{C}_t/\mathbf{K}_t$.

[14] Broadly speaking, our mandate choice is related to the optimal fiscal and monetary policy literature (e.g., Lucas and Stokey, 1983) in macroeconomics. See Ljungqvist and Sargent (2018) for a textbook treatment.

Let $V(\mathbf{K},\mathbf{N})$ denote the representative agent's value function. As in Hong, Wang, and Yang (2020), the following Hamilton-Jacobi-Bellman (HJB) equation characterizes the planner's optimization problem:

$$
\begin{aligned}
0 \quad = \quad & \max_{\mathbf{C},\mathbf{i},\mathbf{X}} f(\mathbf{C},V) + \phi(\mathbf{i})\mathbf{K}V_K + \omega(\mathbf{x}/\mathbf{n})\mathbf{N}V_N + \frac{\mathbf{K}^2 V_{KK} + 2\mathbf{N}\mathbf{K}V_{NK} + \mathbf{N}^2 V_{NN}}{2}\sigma^2 \\
& +\lambda(\mathbf{n})\int_0^1 \left[V\left(Z\mathbf{K}, Z\mathbf{N}\right) - V(\mathbf{K},\mathbf{N})\right]\xi(Z)dZ\,, \qquad (48)
\end{aligned}
$$

subject to the following aggregate resource constraint at all t:

$$
A\mathbf{K_t} \quad = \quad \mathbf{C_t} + \mathbf{i}_t\mathbf{K_t} + \mathbf{x}_t\mathbf{K_t}\,. \qquad (49)
$$

The first-order condition (FOC) for the scaled investment $\mathbf{i}$ is

$$
f_{\mathbf{C}}(\mathbf{C},V) = \phi'(\mathbf{i})V_K(\mathbf{K},\mathbf{N})\ . \qquad (50)
$$

The first-order condition (FOC) for the scaled aggregate mitigation spending $\mathbf{x}$ is

$$
f_{\mathbf{C}}(\mathbf{C},V) = \omega'(\mathbf{x}/\mathbf{n})V_N(\mathbf{K},\mathbf{N})\ , \qquad (51)
$$

if the solution is strictly positive, $\mathbf{x} > 0$. Otherwise, $\mathbf{x} = 0$ as mitigation cannot be negative.

The representative agent's value function takes the following homothetic form:

$$
V(\mathbf{K},\mathbf{N}) = \frac{1}{1-\gamma}(b(\mathbf{n})\mathbf{K})^{1-\gamma}\,, \qquad (52)
$$

where $b(\mathbf{n})$ is a function measuring the agent's certainty-equivalent wealth and is endogenously determined.

Substituting (52) into the FOCs (50)-(51) and the HJB equation (48) and simplifying, we obtain the following two equations for optimal policies

$$
\begin{aligned}
b(\mathbf{n})^{1-\psi^{-1}} \quad &= \quad (A-\mathbf{i}-\mathbf{x})^{-\psi^{-1}}\rho\left[\frac{\mathbf{n}}{\omega'(\mathbf{x}/\mathbf{n})} + \frac{1}{\phi'(\mathbf{i})}\right]\ , \qquad (53) \\
\frac{b(\mathbf{n}) - \mathbf{n}b'(\mathbf{n})}{b'(\mathbf{n})} \quad &= \quad \frac{\omega'(\mathbf{x}/\mathbf{n})}{\phi'(\mathbf{i})}\,, \qquad (54)
\end{aligned}
$$

and the following ODE:

$$
\begin{aligned}
0 \quad = \quad & \frac{\rho}{1-\psi^{-1}}\left[\frac{A-\mathbf{i}-\mathbf{x}}{\rho\left[\frac{\mathbf{n}}{\omega'(\mathbf{x}/\mathbf{n})} + \frac{1}{\phi'(\mathbf{i})}\right]} - 1\right] + \phi(\mathbf{i}) - \frac{\gamma\sigma^2}{2} + \frac{\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})}{1 + \frac{\omega'(\mathbf{x}/\mathbf{n})}{\mathbf{n}\phi'(\mathbf{i})}} \\
& + \frac{\lambda(\mathbf{n})}{1-\gamma}\left[\int_0^1 \left[\xi(Z)Z^{1-\gamma}\right]dZ - 1\right]\,. \qquad (55)
\end{aligned}
$$

At the fist-best steady state $\mathbf{n^{FB}}$, we have

$$\omega(\mathbf{x^{FB}}/\mathbf{n^{FB}}) - \phi(\mathbf{i^{FB}}) = 0\,, \tag{56}$$

and

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\frac{A - \mathbf{i^{FB}} - \mathbf{x^{FB}}}{\rho\left(\frac{\mathbf{n^{FB}}}{\omega'(\mathbf{x^{FB}}/\mathbf{n^{FB}})} + \frac{1}{\phi'(\mathbf{i^{FB}})}\right)} - 1\right] + \phi(\mathbf{i^{FB}}) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n}^{FB})}{1-\gamma}\left[\int_0^1 \left[\xi(Z)Z^{1-\gamma}\right] dZ - 1\right]. \tag{57}$$

Summary. The first-best steady state is an endogenously determined boundary that satisfies (53), (54), (56), and (57). For the transition path, we solve the ODE (55) together with the FOCs (53) and (54) subject to the boundary conditions for the steady state given above.

5 Quantitative Analysis

In this section, we operationalize our model. First, we specify various functional forms in our model. Second, we calibrate our model and choose parameter values based on a variety of moments from the data. Finally, we describe our quantitative results and findings.

5.1 Functional Form Specifications for the Model

Capital accumulation processes for K and $\mathbf{N}$. As in Pindyck and Wang (2013), we specify the investment-efficiency function $\phi(i)$ as

$$\phi(i) = i - \frac{\eta_K\, i^2}{2}\,, \tag{58}$$

where η_K measures the degree of adjustment costs.

We assume that the controlled drift function for decarbonization stock $\mathbf{N}$ takes the same form as that for capital stock K:

$$\omega(x/\mathbf{n}) = (x/\mathbf{n}) - \frac{\eta_{\mathbf{N}}\,(x/\mathbf{n})^2}{2}\,, \tag{59}$$

where $\eta_{\mathbf{N}}$ measures the degree of adjustment costs for decarbonization capital. Note that $x/\mathbf{n} = X/\mathbf{N}$ is the firm mitigation spending X scaled by $\mathbf{N}$, which is analogous to the firm's investment scaled by capital stock: $i = I/K$.

Conditional damage and disaster arrival rate. We define disasters as events that cause the temperature to be 1.5^o Celsius higher than the historical normal level. As in Barro (2006) and Pindyck and Wang (2013), we model the stochastic damage upon the arrival of a disaster by assuming that the stochastic recovery fraction, $Z \in (0,1)$, of capital stock is governed by the following cdf:

$$\Xi(Z) = Z^{\beta} \,, \tag{60}$$

where $\beta > 0$ is a constant. To ensure that our model is well defined (and economically relevant moments are finite), we require $\beta > \max\{\gamma - 1, 0\}$. That is, the damage caused by a disaster follows a fat-tailed power-law function (Gabaix, 2009).

Decarbonization capital can ameliorate the damage of extreme weather to economic growth by reducing the frequencies of these extreme events. Specifically, we use the following specification for the disaster arrival rate $\lambda(\mathbf{n})$:

$$\lambda(\mathbf{n}) = \lambda_0(1 - \mathbf{n}^{\lambda_1}) \,, \tag{61}$$

where $\lambda_0 > 0$, and $0 < \lambda_1 < 1$.

For a given $\mathbf{n}$, the expected aggregate growth rate, $\mathbf{g}$, is

$$\mathbf{g} = \phi(\mathbf{i}) - \lambda(\mathbf{n})\mathbb{E}(\mathbf{1} - \mathbf{Z}) = \phi(\mathbf{i}) - \frac{\lambda(\mathbf{n})}{\beta + \mathbf{1}} = \phi(\mathbf{i}) - \lambda(\mathbf{n})\ell \,. \tag{62}$$

5.2 Baseline Calibration

Our calibration exercise is intended to highlight the importance of mitigation for welfare analysis. Our model has ten parameters in total. We summarize the values of these ten parameters for our baseline analysis in Table 1.

Preferences parameters. We choose consensus values for the coefficient of relative risk aversion, $\gamma = 3$, and the time rate of preferences, $\rho = 5\%$ per annum. Estimates of the EIS ψ in the literature vary considerably, ranging from a low value near zero to values as high as two.[15] We choose $\psi = 1.5$ which is larger than one, as in Bansal and Yaron (2004) and

[15] Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. Guvenen (2006) reconciles the conflicting evidence on the elasticity of intertemporal substitution from a macro perspective.

Table 1: Parameter Values

Parameters	Symbol	Value
elasticity of intertemporal substitution	ψ	1.5
time rate of preference	ρ	5%
coefficient of relative risk aversion	γ	3
productivity for K	A	14%
adjustment parameter for K	$\eta_{\mathbf{K}}$	9
diffusion volatility for $\mathbf{N}$ and K	σ	14%
power-law exponent	β	24
jump arrival rate with no mitigation	λ_0	1
adjustment cost parameter for $\mathbf{N}$	$\eta_{\mathbf{N}}$	12
mitigation technology parameter	λ_1	0.3

All parameter values, whenever applicable, are continuously compounded and annualized.

the long-run risk literature for asset-pricing purposes.

Parameters for productive capital accumulation process. We set the productivity parameter to $A = 14\%$ per annum and the capital adjustment parameter $\eta_{\mathbf{K}} = 9$ to primarily target an average q of 1.87 and an average growth rate of $g = 3.8\%$ per annum in the pre-climate-change sample when the disaster arrival rate is low. The estimates of A and $\eta_{\mathbf{K}}$ are in the range of estimates reported in Eberly, Rebelo, and Vincent (2012). We set the annual diffusion volatility at $\sigma = 14\%$ primarily to target a historical stock market risk premium of 6% per annum (Mehra and Prescott, 1985).

Parameters for disaster arrival and conditional damage function. We calibrate the parameter (λ_0) describing the arrival rate of extreme temperature disasters (i.e., abnormal temperatures above 1.5^o Celsius) and damages conditioned on arrival (β) using a set of panel regressions documenting the adverse effects of exogenous annual changes in temperature (i.e., weather shocks) for economic growth (Dell, Jones, and Olken (2012)).[16]

[16]This panel regression approach initially focused on how weather affects crop yields (Schenkler and Roberts (2009)) by using location and time fixed effects. But it is now applied to many other contexts

First, we calibrate β as follows. For the median country, a 1.5^o Celsius abnormal temperature over one year results in a 4% lower GDP growth rate. Conditional on a jump arrival, the expected fractional capital loss, ℓ, is given by

$$\ell = \mathbb{E}(1 - Z) = \frac{1}{\beta + 1}\,. \tag{63}$$

To match this moment, we set the power-law parameter to $\beta = 24$ as the implied expected fractional capital loss is $\ell = 1/(\beta + 1) = 1/25 = 4\%$. Second, using Dell, Jones, and Olken (2012), we infer that the jump arrival rate is $\lambda_0 = 0.05$ per annum in the pre-climate-change sample.

Parameters for decarbonization capital adjustment and its benefits to mitigation. As discussed in the Introduction, reforestation has the potential to keep global temperatures from breaching the 1.5^o Celsius barrier assuming that we can roughly double the size of forests. This adjustment is likely to take two to three decades (Bastin et al. (2019), and Griscom et al. (2017)). In order to match this gradual adjustment, we set $\eta_{\mathbf{N}}$ =12 for our baseline analysis. We then calibrate the efficiency parameter of decarbonization capital stock (λ_1) in reducing the disaster arrival rate. Since there has only been negligible attempts at reforestation, we determine the value of λ_1 by using the planner's FOCs for mitigation and investment by targeting a small amount of mitigation ($\mathbf{x} = 0.003\%$) and a low level of scaled decarbonization stock ($\mathbf{n} = 0.05\%$) in the data.[17] Doing so yields a value of $\lambda_1 = 0.3$.

The first column ($\lambda_0 = 0.05$) in Table 2 reports the pre-climate-change steady-state equilibrium when extreme temperature events are uncommon for the economy to which we calibrate our parameters. Targeting Tobin's average q at 1.87, the expected annual growth rate at $\mathbf{g} = 3.8\%$, and the annual stock market risk premium at 5.93%, we obtain an annual

including economic growth and productivity. The main idea is that extreme annual temperature fluctuations are plausibly exogenous shocks that causally trace out the impact of higher temperatures on output. Burke, Hsiang, and Miguel (2015) find that the effects of temperature on growth is nonlinear. But we stay with the linear specification from Dell, Jones and Olken (2012) in this paper.

[17] We do not count existing forest as part of the decarbonization capital stock since these are old forests and not new ones planted with the purpose of addressing climate change. We can think of these new forests as carbon capture plants of comparable efficiency. Or we can alternatively set n_0 using a book value for current forests. The conclusions would be qualitatively similar.

Table 2: THE EFFECT OF λ_0 IN THE STEADY STATE UNDER MANDATES.

variable	notation	$\lambda_0 = 0.05$	$\lambda_0 = 0.5$	$\lambda_0 = 1$
scaled mitigation spending	$\mathbf{x}$	0.003%	0.09%	0.23%
scaled decarbonization stock	$\mathbf{n}$	0.05%	1.57%	4.59%
scaled aggregate investment	$\mathbf{i}$	5.16%	4.73%	4.35%
Tobin's q	$\mathbf{q}$	1.869	1.740	1.644
scaled aggregate consumption	$\mathbf{c}$	8.83%	9.19%	9.42%
expected GDP growth rate	$\mathbf{g}$	3.78%	2.30%	1.09%
(real) risk-free rate	r	2.58%	1.27%	0.21%
stock market risk premium	$r^M - r$	5.93%	6.31%	6.60%
transition time from $\mathbf{n} = 0$ to $\mathbf{n}_{0.99}$	$\mathbf{t}_{0.99}$	3.15	11.35	22.64

$\mathbf{n}_{0.99}$ is the 99% level of that in the steady state: $\mathbf{n}_{0.99} = 0.99 \times n^*$ and $\mathbf{t}_{0.99}$ is the transition time from 0 to $\mathbf{n}_{0.99}$.

risk-free rate of 2.6%, an investment-capital ratio of $\mathbf{i} = 5.2\%$ per annum, and the aggregate consumption-capital ratio of $\mathbf{c} = 8.8\%$ per annum.

5.3 Steady States Under Welfare-Maximizing Mandate with Markets

To analyze the effects of various key parameter values, we conduct comparative statics for the steady-state solution.

Varying disaster arrival rate λ_0. Now consider how the steady-state equilibrium outcomes change as we increase λ_0. We focus our discussion on the effect of increasing λ_0 from 5% to 1, which is the value of λ_0 for our baseline.[18] Mitigation rises from $\mathbf{x} = 0.003\%$ to 0.23% per annum. Since the physical capital stock is 600 trillion dollars, the aggregate contribution to decarbonization stock is about 1.4 trillion dollars per year. The ratio of decarbonization to physical capital stock $\mathbf{n}$ is 4.6%, which means the aggregate decarbonization capital stock $\mathbf{N}$ is about 27.6 trillion dollars or the book value of new forests dedicated toward decarbonization in steady-state.

Our model generates mitigation spending that is in line with projections for the cost

[18]The results for $\lambda_0 = 0.5$ are between those for the $\lambda = 5\%$ and $\lambda = 1$ scenarios.

of decarbonization (Gates (2021)). Of course, our model can be applied to other forms of decarbonization capital accumulation such as direct carbon capture. We calibrate our model using reforestation simply because it is currently the most feasible and well understood form of decarbonization capital. Other decarbonization methods are less mature and the costs of these methods are similar to if not more expensive than reforestation.[19]

As a result of mitigation, aggregate investment is modestly lower at 4.35% as is Tobin's q at 1.644. The expected growth rate is still positive at 1.09% per annum as a result of mitigation, down from 3.78% absent global warming. The market risk premium increases from 5.93% to 6.60%, and the real risk-free rate falls to 0.21% from 2.6% per annum. From the time that mitigation starts, it takes 23 years to transition to the 99% of the steady state.

Varying decarbonization capital adjustment parameter $\eta_{\mathbf{N}}$. Next, we examine in Table 3 how the equilibrium outcomes change as we change the decarbonization capital adjustment parameter $\eta_{\mathbf{N}}$. The middle column ($\eta_{\mathbf{N}} = 12$) summarizes our baseline case results. As we increase $\eta_{\mathbf{N}}$ to 14 from 12, the annual mitigation spending $\mathbf{x}$ falls to 0.10% and the steady-state decarbonization to physical capital ratio $\mathbf{n}$ falls to 1.62% corresponding to a drop of $\mathbf{N}$ to 9 trillion dollars. Aggregate investment, Tobin's q, consumption $\mathbf{c}$ are hardly changed from our baseline case. However, the expected growth rate $\mathbf{g}$ decreases significantly to 0.62%. The reason is that the higher adjustment cost leads to lower levels of optimal mitigation and steady-state decarbonization capital. The risk-free rate now approaches zero and the stock market risk premium increased slightly. The transition time to the 99% of the steady state is now 38.07 years due to the higher adjustment cost.

When we decrease $\eta_{\mathbf{N}}$ to 9, the scaled mitigation spending increases to 0.33% and the steady state $\mathbf{n}$ is higher at 7.60% since adjustment costs are lower. Again, investment, Tobin's q, and consumption $\mathbf{c}$ are hardly changed compared to our baseline case. The growth rate $\mathbf{g}$ is higher as is the risk-free rate. Moreover, the transition time to the steady

[19] According to estimates from a McKinsey Sustainability report (de Pee (2018)), decarbonization using carbon capture of just the heavy industries that account for around 20% of the global carbon emissions will cost around 20 trillion dollars up to 2050 (or around 1 trillion dollars per year just for heavy sectors) to be net-zero.

Table 3: THE EFFECT OF $\eta_{\mathbf{N}}$ IN THE STEADY STATE UNDER MANDATES.

	notation	$\eta_{\mathbf{N}} = 9$	$\eta_{\mathbf{N}} = 12$	$\eta_{\mathbf{N}} = 14$
scaled mitigation spending	$\mathbf{x}$	0.33%	0.23%	0.10%
scaled decarbonization stock	$\mathbf{n}$	7.60%	4.59%	1.62%
scaled aggregate investment	$\mathbf{i}$	4.38%	4.35%	4.29%
Tobin's q	$\mathbf{q}$	1.651	1.644	1.629
scaled aggregate consumption	$\mathbf{c}$	9.28%	9.42%	9.61%
expected GDP growth rate	$\mathbf{g}$	1.36%	1.09%	0.62%
(real) risk-free rate	r	0.46%	0.21%	-0.20%
stock market risk premium	$r^M - r$	6.52%	6.60%	6.73%
transition time from $\mathbf{n} = 0$ to $\mathbf{n}_{0.99}$	$\mathbf{t}_{0.99}$	16.73	22.64	38.07

$\mathbf{n}_{0.99}$ is the 99% level of that in the steady state: $\mathbf{n}_{0.99} = 0.99 \times n^*$ and $\mathbf{t}_{0.99}$ is the transition time from 0 to $\mathbf{n}_{0.99}$.

state falls only slightly to 16.73 years since the targeted steady-state decarbonization level $\mathbf{n}$ is also higher. We view the $\eta_{\mathbf{N}}$ results as being quite pertinent since adjustment costs of decarbonization capital has significant effects on the steady-state accumulation level and hence welfare outcomes.

Varying damages from extreme temperatures $\ell = 1/(\beta+1)$ conditional on arrival. Next, we consider in Table 4 how varying the conditional damage $\ell = 1/(\beta + 1)$ changes equilibrium outcomes. Recall that β measures the damage to the economy from extreme weather in the absence of mitigation. Our baseline of the conditional damage of $\ell = 4\%$ (as $\beta = 24$) is informed by existing empirical studies based on historical data on extreme temperature events. We are extrapolating from these estimates. Needless to say, there is bound to be uncertainty regarding these estimates. In this vein, it is instructive to consider values for β that are higher and lower than our baseline. A higher $\beta = 49$ corresponds to less damage absent mitigation (about $\ell = 2\%$ conditioned on an arrival of an extreme 1.5^o Celsius event). A lower $\beta = 11.5$ corresponds to a higher conditional damage of $\ell = 8\%$.

When $\ell = 2\%$, the optimal annual mitigation spending $\mathbf{x}$ is lower at 0.08% and so is the steady-state $\mathbf{n}$ at 1.41%, though these are still quite significant amounts. Again, the effects on investment, Tobin's q, and consumption are limited. The expected growth rate $\mathbf{g}$ is not

Table 4: THE EFFECT OF CONDITIONAL DAMAGE $\ell = 1/(1+\beta)$ IN THE STEADY STATE UNDER MANDATES.

	notation	$\ell = 2\%$	$\ell = 4\%$	$\ell = 8\%$
scaled mitigation spending	$\mathbf{x}$	0.08%	0.23%	0.67%
scaled decarbonization stock	$\mathbf{n}$	1.41%	4.59%	16.32%
scaled aggregate investment	$\mathbf{i}$	4.75%	4.35%	3.70%
Tobin's q	$\mathbf{q}$	1.747	1.644	1.500
aggregate consumption/dividends	$\mathbf{c}$	9.17%	9.42%	9.63%
expected GDP growth rate	$\mathbf{g}$	2.29%	1.09%	-0.27%
(real) risk-free rate	r	1.47%	0.21%	-2.35%
stock market risk premium	$r^M - r$	6.07%	6.60%	8.50%
transition time from $\mathbf{n} = 0$ to $\mathbf{n}_{0.99}$	$\mathbf{t}_{0.99}$	9.58	22.64	74.30

$\mathbf{n}_{0.99}$ is the 99% level of that in the steady state: $\mathbf{n}_{0.99} = 0.99 \times n^*$ and $\mathbf{t}_{0.99}$ is the transition time from 0 to $\mathbf{n}_{0.99}$.

surprisingly higher at 2.29% and so is the risk-free rate (at 1.47%) even though mitigation $\mathbf{x}$ and decarbonization capital $\mathbf{n}$ are much lower in equilibrium. The transition to the 99% of the steady state is now much faster (it takes about 9.58 years.)

When $\ell = 9\%$ (as $\beta = 11.5$), optimal mitigation $\mathbf{x}$ increases significantly to 0.67% per annum and the steady-state $\mathbf{n}$ is now 16.32%. The growth rate $\mathbf{g}$ is negative at -0.27% and the risk-free rate is significantly more negative at -2.35%. It also takes much longer to reach the 99% of the steady state at 74.30 years.

Overall, our quantitative analysis suggests that our main conclusions are fairly robust across a number of perturbations. The one outcome that is very sensitive is transition time to the steady state, which depends on $\eta_{\mathbf{N}}$ and the severity of the conditional damage ℓ. A conservative estimate of a higher adjustment cost for decarbonization capital suggests that it may take up to 38 years to reach steady state should the economy start today to accumulate decarbonization capital stock.

5.4 Transition Dynamics and Comparison to Planner's Outcomes

In this subsection, we discuss the transition dynamics. We also highlight the extent to which welfare-maximizing mandates (Section 4.1) can attain the planner's first-best outcomes

(Section 4.2).

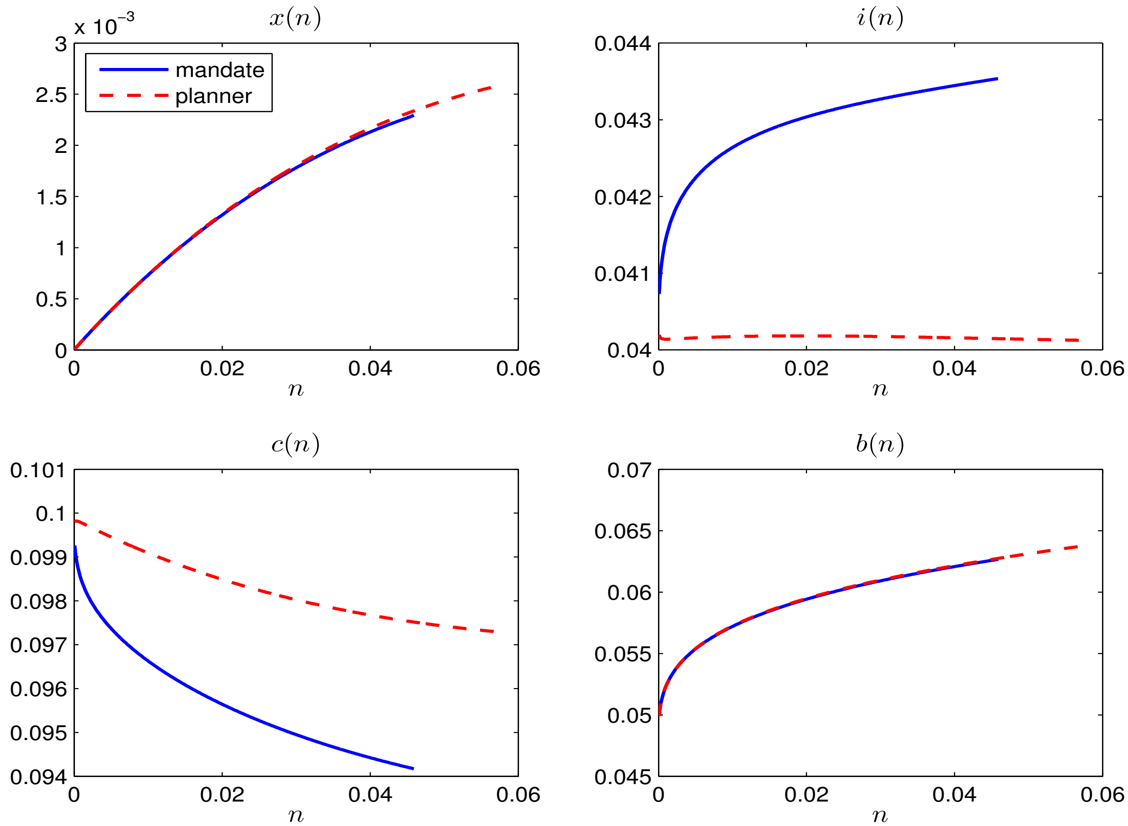

Figure 1: This figure plots the aggregate mitigation spending $\mathbf{x}$, aggregate investment $\mathbf{i}$, aggregate consumption $\mathbf{c}$ and the aggregate welfare measure b as functions of the scaled decarbonization capital stock $\mathbf{n}$. The parameters values are reported in Table 1.

Mitigation, consumption, investment and welfare gains under mandates versus planner's solution. In Figure 1, we examine the transition dynamics for the optimal mitigation $\mathbf{x}$, investment $\mathbf{i}$, consumption $\mathbf{c}$ and certainty-equivalent welfare measure b. All these aggregates are dependent on the underlying state variable $\mathbf{n}$ — the ratio of decarbonization capital to physical capital. For all four panels, the blue lines indicate the optimal mandate solution and the red lines describe the planner's solution.

Panel A shows that the two solutions track each other closely up to the level of $\mathbf{n} = 4.59\%$. The planner's solution peaks at a higher value and is also higher in the steady state. That is, the mitigation spending under the optimal mandate is only materially below the first-best when $\mathbf{n}$ is sufficiently high. This is intuitive as the marginal return of mitigation is quite high when $\mathbf{n}$ is not too high.

Panel B shows that investment $\mathbf{i}$ is higher under the mandate than the planner's solution, whereas Panel C shows that consumption $\mathbf{c}$ is lower under the mandate than the planner's solution. Note that this is to a large extent expected as the sum of $\mathbf{i}$, $\mathbf{c}$, and $\mathbf{x}$ is the constant productivity A. As risk mitigation is a public good, $\mathbf{n} = 0$ is the competitive markets outcome with no mandate. Therefore, mandates move all three of these policies from the market solution towards the planner's solution. However, the mandate solution does not track well the planner's solution.

Nonetheless, as Panel D shows, the welfare measure $b(\mathbf{n})$ for the mandate solution is quite close to that for the planner's solution. This is good news for the usefulness of mandates in incentivizing firms to reform and contribute to decarbonization. In fact, the welfare gains are massive. Under the unmitigated competitive market solution, the certainty equivalent wealth measure $b(0)$ is 0.0498. The certainty equivalent at the steady state is 0.0627 under the mandate's solution and 0.0638 under the planner's solution. We thus obtain a 26% gain in welfare with an optimal mandate than without. The magnitudes are large as the world (based on the estimates we use from the literature) at 1.5^o Celsius with no mitigation is dismal.

Extreme temperature event arrivals $\lambda(\mathbf{n})$ and the aggregate growth rate $\mathbf{g}(\mathbf{n})$. In Figure 2, we examine how the extreme temperature event arrival rate $\lambda(\cdot)$ and the expected aggregate growth rate $\mathbf{g}(\cdot)$ vary with $\mathbf{n}$. Panel A shows that the disaster arrival rate $\lambda(\cdot)$ falls with $\mathbf{n}$, as we expect. In competitive markets (and hence $\mathbf{n} = 0$), the arrival rate is $\lambda(0) = 1$ at 1.5^o Celsius with no mitigation. As the society builds up the decarbonization capital, the disaster arrival rate falls and approaches around 0.6 per annum at the steady state: $\lambda(\mathbf{n}^*) = 0.6$ where $\mathbf{n}^* = 4.59\%$.

The high jump arrival rate in competitive markets implies that growth $\mathbf{g}(0)$ is low at around -0.42%. Note that this result depends on the assumptions of our production economy absent mitigation in the pre-climate-change period. (We have chosen a modest productivity

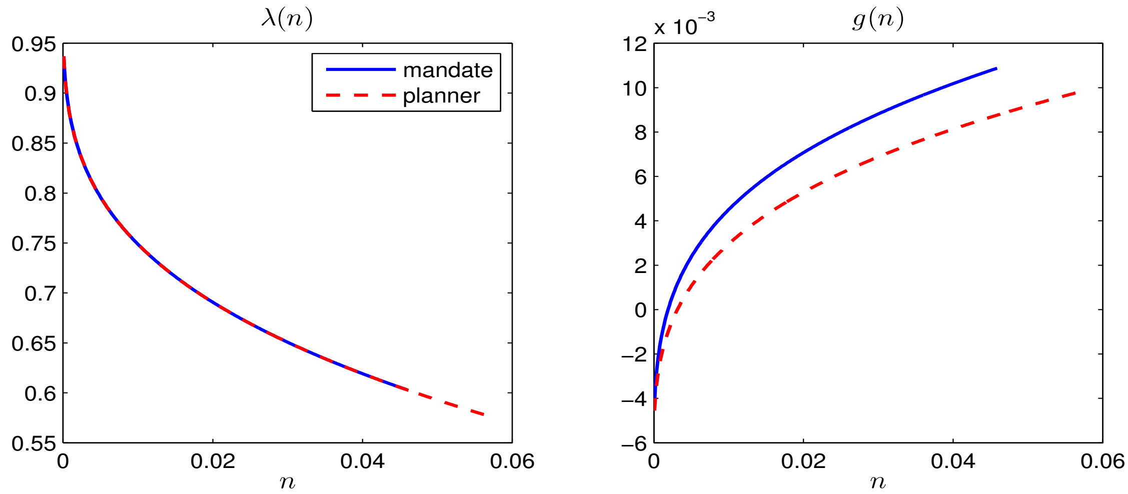

Figure 2: This figure plots the jump arrival rate $\lambda(\cdot)$ and the expected aggregate growth rate $\mathbf{g}(\cdot)$ as functions of the scaled decarbonization capital stock $\mathbf{n}$. The parameters values are reported in Table 1.

scenario where the expected growth rate is about 4%.) Hence, a 1.5^o Celsius world if unmitigated will lead to low growth. As the society accumulates the decarbonization capital, the growth rate increases. For the mandate solution, the expected growth rate at the steady-state is positive at 1.09% per annum as we have discussed above. The planner's steady-state growth rate is lower.

The difference of growth rates between the two solutions is because the planner solution emphasizes de-risking in building up more decarbonization capital at the expense of investing in productive capital. Despite a lower growth rate, the welfare is higher in the planner's solution because the planner fully fixes the under-provision of risk mitigation (a public good).

Next, in Figure 3, we plot the transition path of $\mathbf{n}_t$ over time t. We see that the society reaches a higher steady state under the planner's solution than under the mandate.

5.5 Asset Prices

In Figure 4, we report how key asset pricing variables, the risk-free rate r, the stock market risk premium rp, and Tobin's average q, vary with $\mathbf{n}$. At $\mathbf{n} = 0$, the unmitigated competitive market equilibrium has a negative interest rate of around -1.1%, a market risk premium of

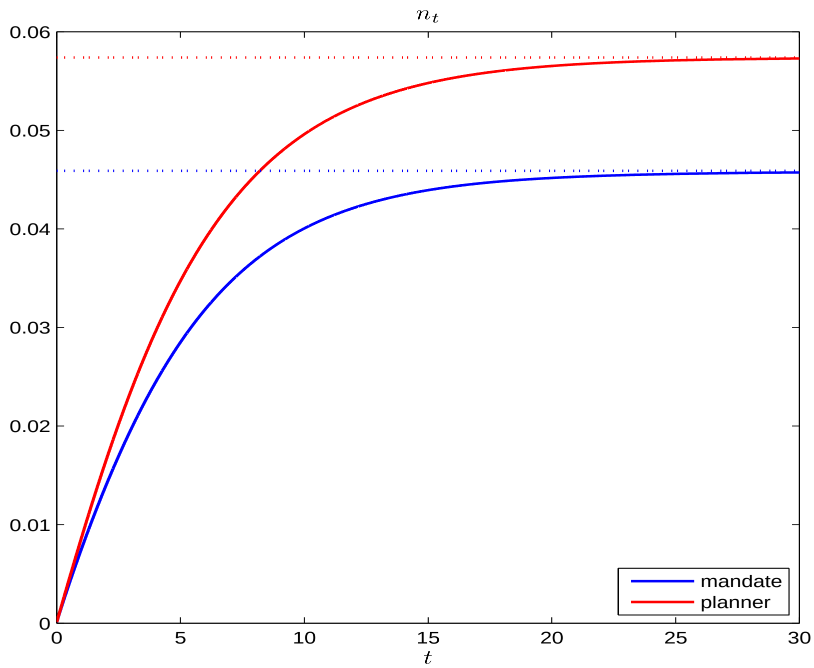

Figure 3: This figure plots the transition path of $\mathbf{n}_t$ over time. The parameters values are reported in Table 1.

around 7% per annum, and Tobin's q of 1.58. As $\mathbf{n}$ increases, the risk-free rate increases, the risk premium falls, and Tobin's q rises.

In Figure 5, we analyze the costs of accumulating decarbonization capital to firms and investors. We consider three investment mandate levels: $\alpha = 0.1, 0.2, 0.3$. For these three levels of α, our mandate solution can all be implemented. Naturally when α is lower, each firm needs to spend more to qualify for the sustainable portfolio but it also gets compensated with a larger cost-of-capital wedge in equilibrium.

The blue solid line depicts the solution when 10% of wealth is indexed to sustainable mandates ($\alpha = 0.1$). The qualifying standard m increases with $\mathbf{n}$, peaking at 2.3% per annum at the steady state. That is, a firm would need to spend 2.23% of its capital on decarbonization to qualify for the sustainable portfolio at the steady state. The sustainable firms get compensated for their contributions with a significant cost-of-capital wedge $r^U(\mathbf{n}) - r^S(\mathbf{n})$ of over 1.4% per annum at the steady state.

Interestingly, the optimal ramp-up schedules of both $m(\mathbf{n})$ and cost-of-capital wedge

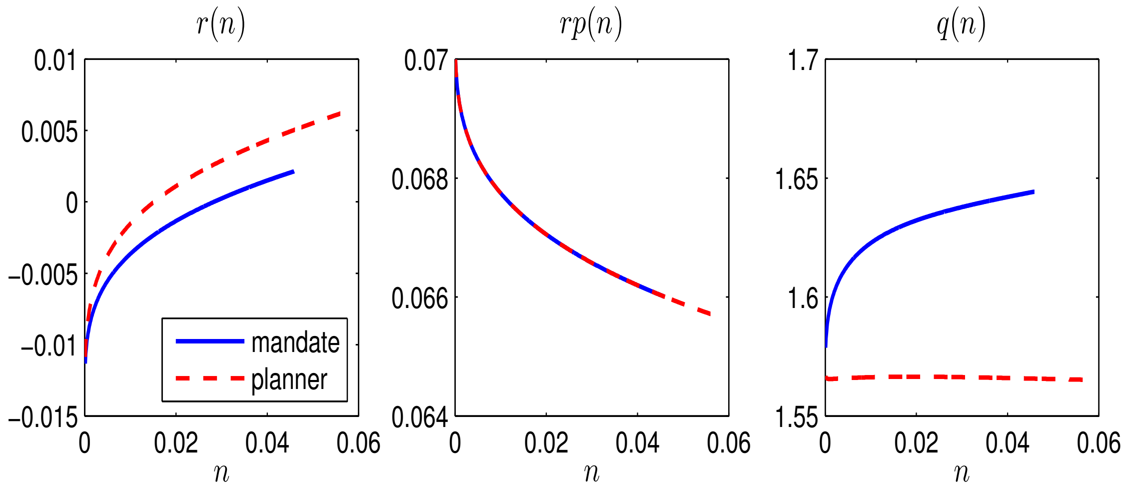

Figure 4: This figure plots the equilibrium interest rate r, stock market risk premium rp, and Tobin's q as functions of the scaled decarbonization capital stock $\mathbf{n}$ for both planner's solution and the solution under competitive markets with mandates. The parameters values are reported in Table 1.

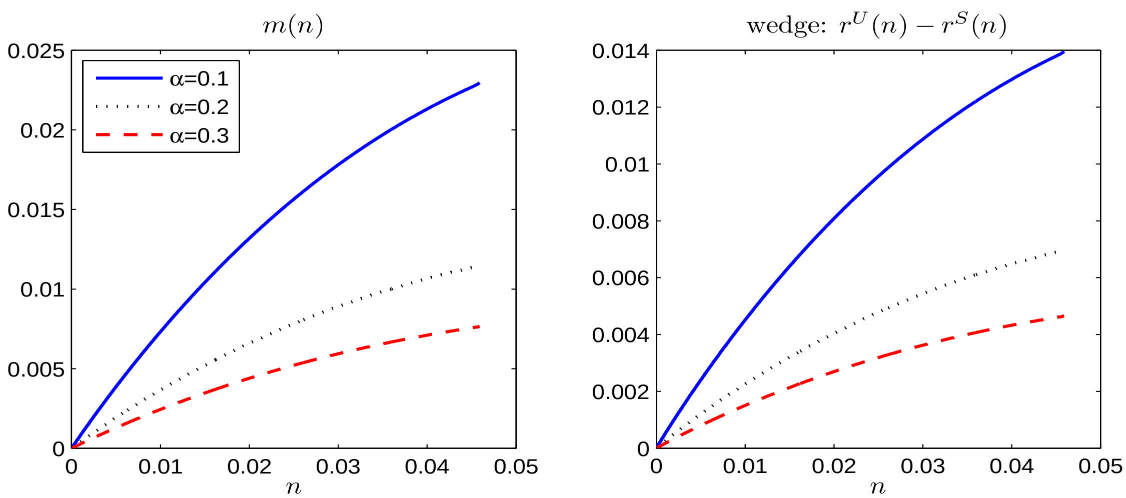

Figure 5: This figure plots the mitigation spending mandate $m(\mathbf{n})$ and the cost-of-capital wedge $r^U(\mathbf{n}) - r^S(\mathbf{n})$ as functions of scaled decarbonization capital stock $\mathbf{n}$. The parameters values are reported in Table 1.

$r^U(\mathbf{n}) - r^S(\mathbf{n})$ are non-linear. As we increase α from 0.1 to 0.2 (the grey dotted line) and 0.3 (the red dotted line), the qualification standard falls and so do the cost-of-capital wedges. The non-linearity discussed above remains however. Current estimates have sustainable finance mandates α in the range of 10% to 20%. Moreover, our model clearly predicts that as decarbonization $\mathbf{n}$ ramps up, qualification standards start rising and the cost to investors also rise.

5.6 Connecting to Net-zero Emissions Targets

Finally, we show that our calibration implies that a sustainable finance solution based on reforestation can meet a significant fraction of net-zero emissions targets. First, $\mathbf{e} = \mathbf{E}/\mathbf{K}$ in equation (4) is approximately 0.067 since net emissions is about 40 billion metric tons and there is 600 trillion dollars of aggregate productive capital stock $\mathbf{K}$. Second, $\tau = \mathbf{R}/\mathbf{N}$ in equation (5) is 0.36. We obtain this value by dividing the new forests' aggregate net absorption of carbon per annum (which we place at 10 billion metric tons following Bastin et al. (2019), and Griscom et al. (2017)) by the book value of forests in aggregate (which based on our calculations is 27.6 trillion dollars) (i.e. .36=10/27.6).

To achieve net-zero emissions, i.e. $\mathbf{E} - \mathbf{R} = \mathbf{e} - \tau\mathbf{n} = 0$, we need to roughly target $\mathbf{n} = 0.18 = 0.067/0.36$. The steady-state $\mathbf{n}$ from reforestation in our baseline is 4.59%. Hence, sustainable finance mandates can get us about 25% of the way towards net-zero through the funding of reforestation pathways. Of course, society also needs to pursue other strategies as well to achieve a net-zero economy.

6 Conclusion

Sustainable finance mandates have grown significantly in the last decade in lieu of government failures to address climate disaster externalities. Firms that spend enough resources on mitigation of these externalities qualify for sustainable finance mandates. These mandates incentivize otherwise ex-ante identical unsustainable firms to become sustainable for a lower cost of capital. We present and solve a dynamic stochastic general equilibrium model featuring the gradual accumulation of nonproductive but protective decarbonization capital to study the welfare consequences. The model is highly tractable, including a simple formula that characterizes the cost-of-capital wedge between sustainable and unsustainable firms as the tax rate on firm value to subsidize mitigation. There are a number of testable implications that can be taken to the data. The model is also useful for quantitative analysis of both transition dynamics and the steady state.

References

Abatzoglou, J.T. and Williams, A.P., 2016. Impact of anthropogenic climate change on wildfire across western US forests. *Proceedings of the National Academy of Sciences*, 113(42), pp.11770-11775.

Almazan, A., Brown, K.C., Carlson, M. and Chapman, D.A., 2004. Why constrain your mutual fund manager?. *Journal of Financial Economics*, 73(2), pp.289-321.

Attanasio, O.P., and Vissing-Jørgensen (2003), A. and . Stock-market participation, intertemporal substitution, and risk-aversion. *American Economic Review*, 93(2), pp.383-391.

Bansal, R., Ochoa, M. and Kiku, D., 2017. Climate change and growth risks (No. w23009). National Bureau of Economic Research.

Bansal, R. and Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4), pp.1481-1509.

Barnett, M., Brock, W. and Hansen, L.P., 2020. Pricing uncertainty induced by climate change. The *Review of Financial Studies*, 33(3), pp.1024-1066.

Barro, R.J., 2006. Rare Disasters and Asset Markets in the Twentieth Century. *Quarterly Journal of Economics*, 121, pp. 823-866.

Barro, R.J., and Jin, T., 2011. On the size distribution of macroeconomic disasters. *Econometrica*, 79(5), pp.1567-1589.

Bastin, J.F., Finegold, Y., Garcia, C., Mollicone, D., Rezende, M., Routh, D., Zohner, C.M. and Crowther, T.W., 2019. The global tree restoration potential. *Science*, 365(6448), pp.76-79.

Bolton, P. and Kacperczyk, M., 2020. Global pricing of carbon transition risks. Imperial University Working Paper.

Broccardo, E., Hart, O.D. and Zingales, L., 2020. Exit vs. voice (No. w27710). National Bureau of Economic Research.

Burke, M., Hsiang, S.M. and Miguel, E., 2015. Global non-linear effect of temperature on economic production. *Nature*, 527(7577), pp.235-239.

Cai, Y. and Lontzek, T.S., 2019. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6), pp.2684-2734.

Chang, Y.C., Hong, H. and Liskovich, I., 2015. Regression discontinuity and the price effects of stock market indexing. *Review of Financial Studies*, 28(1), pp.212-246.

Daniel, K.D., Litterman, R.B. and Wagner, G., 2019. Declining CO2 price paths. *Proceedings of the National Academy of Sciences*, 116(42), pp.20886-20891.

Dell, M., Jones, B.F. and Olken, B.A., 2012. Temperature shocks and economic growth: Evidence from the last half century. American Economic Journal: Macroeconomics, 4(3), pp.66-95.

de Pee, A., Pinner, D., Roelofsen, O., Somers, K., Speelman, E.,, and Witteveen, M., 2018. Decarbonization of industrial sectors: the next frontier. *McKinsey Sustainability Report*, Mckinsey and Company.

Duffie, D. and Epstein, L.G., 1992. Stochastic differential utility. *Econometrica*, pp.353-394.

Eberly, J., Rebelo, S. and Vincent, N., 2012. What explains the lagged-investment effect? *Journal of Monetary Economics*, 59(4), pp.370-380.

Eberly, J.C. and Wang, N., 2009, March. Reallocating and pricing illiquid capital: Two productive trees. In AFA 2010 Atlanta Meetings Paper.

Epstein, L.G. and Zin, S.E., 1989. Substitution, risk aversion, and the temporal behavior of consumption. *Econometrica*, 57(4), pp.937-969.

Gabaix, X., 2009. Power laws in economics and finance. *Annual Review Economics*, 1(1), pp.255-294.

Gadzinski, G., Schuller, M. and Vacchino, A., 2018. The global capital stock: finding a proxy for the unobservable global market portfolio. *Journal of Portfolio Management*, 44(7), pp.12-23.

Gates, B., 2021. *How to Avoid a Climate Disaster: The Solutions We Have and the Breakthroughs We Need.* Knopf.

Gollier, C. and Pouget, S., 2014. The " washing machine": Investment strategies and corporate behavior with socially responsible investors.

Golosov, M., Hassler, J., Krusell, P. and Tsyvinski, A., 2014. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1), pp.41-88.

Grinsted, A., Ditlevsen, P. and Christensen, J.H., 2019. Normalized US hurricane damage estimates using area of total destruction, 1900-2018. *Proceedings of the National Academy of Sciences*, 116(48), pp.23942-23946.

Griscom, B.W., Adams, J., Ellis, P.W., Houghton, R.A., Lomax, G., Miteva, D.A., Schlesinger, W.H., Shoch, D., Siikamki, J.V., Smith, P. and Woodbury, P., 2017. Natural climate solutions. *Proceedings of the National Academy of Sciences*, 114(44), pp.11645-11650.

Guvenen, F., 2006. Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. *Journal of Monetary Economics*, 53(7), pp.1451-1472.

Hall, R.E., 1988. Intertemporal substitution in consumption. *Journal of Political Economy*, 96(2), pp.339-357.

Heinkel, R., Kraus, A. and Zechner, J., 2001. The effect of green investment on corporate behavior. *Journal of Financial and Quantitative Analysis*, pp.431-449.

Hong, H. and Kacperczyk, M., 2009. The price of sin: The effects of social norms on markets. *Journal of Financial Economics*, 93(1), pp.15-36.

Hong, H., Karolyi, G.A. and Scheinkman, J.A., 2020. Climate finance. *Review of Financial Studies*, 33(3), pp.1011-1023.

Hong, H. and Stein, J.C., 2007. Disagreement and the stock market. *Journal of Economic Perspectives*, 21(2), pp.109-128.

Hong, H. Wang N., and Yang, J., 2020. Mitigating disaster risks in the age of climate change. NBER Working Paper.

Jensen, S. and Traeger, C.P., 2014. Optimal climate change mitigation under long-term growth uncertainty: Stochastic integrated assessment and analytic findings. *European Economic Review*, 69, pp.104-125.

Jermann, U. J., 1998. Asset pricing in production economies. *Journal of Monetary Economics*, 41(2), pp.257-275.

Kappen, Georg et.al., 2020, The staggering value of forest — and how to save them. *BCG Report*, June 9, 2020.

Kashyap, A.K., Kovrijnykh, N., Li, J. and Pavlova, A., 2018. The benchmark inclusion subsidy (No. w25337). National Bureau of Economic Research.

Koijen, R.S. and Yogo, M., 2019. A demand system approach to asset pricing. *Journal of Political Economy*, 127(4), pp.1475-1515.

Lucas, Jr, R. E. and Prescott, E. C., 1971. Investment under uncertainty. *Econometrica*, 39: 659-681.

Lucas Jr, R. E. and Stokey, N. L., 1983. Optimal fiscal and monetary policy in an economy without capital. Journal of monetary Economics, 12(1), pp.55-93.

Matos, P., 2020. ESG and Responsible Institutional Investing Around the World: *A Critical Review*, CFA Institute Working Paper.

Mehra, R. and Prescott, E.C., 1985. The equity premium: A puzzle. Journal of monetary Economics, 15(2), pp.145-161.

National Academy of Sciences (2016), *Attribution of extreme weather events in the context of climate change*. Washington, D.C.: The National Academies Press.

Nordhaus, W.D., 2017. Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7), pp.1518-1523.

Oehmke, M. and Opp, M.M., 2020. A theory of socially responsible investment, LSE Working Paper.

Pastor, L., Stambaugh, R.F. and Taylor, L.A., 2019. Sustainable Investing in Equilibrium (No. w26549). *Journal of Financial Economics*, forthcoming.

Pedersen, L.H., Fitzgibbons, S. and Pomorski, L., 2020. Responsible investing: The ESG-efficient frontier. *Journal of Financial Economics*, forthcoming.

Pindyck, R. S., and Wang, N., 2013. The economic and policy consequences of catastrophes. *American Economic Journal: Economic Policy*, 5(4), pp.306-339.

Rietz, T. A., 1988. The equity risk premium: a solution. *Journal of Monetary Economics*, 22(1): 117-131.

Rogelj, J., D. Shindell, K. Jiang, S. Fifita, P. Forster, V. Ginzburg, C. Handa, H. Kheshgi, S. Kobayashi, E. Kriegler, L. Mundaca, R. Seferian, and M.V.Vilario, 2018: Mitigation Pathways Compatible with 1.5^oC in the Context of Sustainable Development. In: *Global Warming of* 1.5^oC*. An IPCC Special Report on the impacts of global warming of* 1.5^oC *above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty* [Masson-Delmotte, V., P. Zhai, H.-O. Prtner, D. Roberts, J. Skea, P.R. Shukla, A. Pirani, W. Moufouma-Okia, C. P an, R. Pidcock, S. Connors, J.B.R. Matthews, Y. Chen, X. Zhou, M.I. Gomis, E. Lonnoy, T. Maycock, M. Tignor, and T. Waterfield (eds.)].

Schlenker, W. and Roberts, M.J., 2009. Nonlinear temperature effects indicate severe damages to US crop yields under climate change. *Proceedings of the National Academy of Sciences*, 106(37), pp.15594-15598.

Shleifer, A., 1986. Do demand curves for stocks slope down?. *Journal of Finance*, 41(3), pp.579-590.

Stokey, N.L. and Rebelo, S., 1995. Growth effects of flat-rate taxes. *Journal of Political Economy*, 103(3), pp.519-550.

Weil, P., 1990. Nonexpected utility in macroeconomics. *Quarterly Journal of Economics*, 105(1), pp.29-42.

Wurgler, J. and Zhuravskaya, E., 2002. Does arbitrage flatten demand curves for stocks?. *Journal of Business*, 75(4), pp.583-608.

Appendices

A Firm Value Maximization

Using the standard dynamic programming, we obtain the following HJB equation for Q^j given aggregate decarbonization stock $\mathbf{n}$, aggregate mitigation spending $\mathbf{x}$, and aggregate investment $\mathbf{i}$:

$$\begin{aligned} r^j(\mathbf{n})Q^j(K^j,\mathbf{n}) &= \max_{I^j,X^j} AK^j - I^j - X^j + \left(\Phi(I^j,K^j)Q^j_K(K^j,\mathbf{n}) + \frac{1}{2}(\sigma K^j)^2 Q^j_{KK}(K^j,\mathbf{n})\right) \\ &\quad + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})]\,\mathbf{n}Q^n_{\mathbf{n}}(K^j,\mathbf{n}) + \lambda(\mathbf{n})\mathbb{E}^{\mathbf{n}}\left[Q^j(ZK^j,\mathbf{n}) - Q^j(K^j,\mathbf{n})\right]. \end{aligned} \tag{A.64}$$

And then substituting $Q^j(K^j,\mathbf{n}) = q^j(\mathbf{n})K^j$ into (A.64), we obtain

$$r^j(\mathbf{n})q^j(\mathbf{n}) = \max_{i^j,x^j} A - i^j - x^j + \phi(i^j)q^j(\mathbf{n}) + \lambda\left[\mathbb{E}^{\mathbf{n}}(Z) - 1\right]q^j(\mathbf{n}) + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})]\,\mathbf{n}q^{j\prime}(\mathbf{n})\,. \tag{A.65}$$

The FOC for investment implied by (A.65) is

$$q^j(\mathbf{n}) = \frac{1}{\phi'(i^j)}\,, \tag{A.66}$$

which is the standard Tobin's q formula (e.g., Lucas and Prescott, 1971; Hayashi, 1982). As $x^U \geq 0$ and $x^S \geq m$, the optimal mitigation spending is $x^U = 0$ for a type-U firm and $x^S = m$ for a type-S firm as no firm wants to spend more than it has to on mitigation.

As all firms have the same Tobin's q in equilibrium, we have $i^S(\mathbf{n}) = i^U(\mathbf{n}) = \mathbf{i}(\mathbf{n})$ and

$$\mathbf{q}(\mathbf{n}) = \frac{A - \mathbf{i} - m + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})]\,\mathbf{n}\mathbf{q}'(\mathbf{n})}{r^S - g(\mathbf{i})} = \frac{A - \mathbf{i} + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})]\,\mathbf{n}\mathbf{q}'(\mathbf{n})}{r^U - g(\mathbf{i})}\,. \tag{A.67}$$

As in the steady state, $\omega(\mathbf{x}^*/\mathbf{n}^*) - \phi(\mathbf{i}^*) = 0$, we have

$$\mathbf{q}(\mathbf{n}^*) = \frac{A - \mathbf{i}^* - m}{r^S - g(\mathbf{i}^*)} = \frac{A - \mathbf{i}^*}{r^U - g(\mathbf{i}^*)}\,. \tag{A.68}$$

B Household's Optimization Problem

Using the same procedure as in Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we can show that both the optimal risk-free asset holding and the jump hedging demand for all levels of Z are zero in equilibrium. Therefore, we may rewrite the household's wealth dynamics given by (18) as follows

$$\begin{aligned} dW_t &= W_{t-}\left[\left[r(\mathbf{n_{t-}}) + (r^S(\mathbf{n_{t-}}) - r(\mathbf{n_{t-}}))\pi^S_{t-} + (r^U(\mathbf{n_{t-}}) - r(\mathbf{n_{t-}}))(1 - \pi^S_{t-})\right]dt + \sigma d\mathcal{B}_t\right] \\ &\quad - W_{t-}\left[(1 - Z)\left(d\mathcal{J}_t - \lambda(\mathbf{n_{t-}})dt\right)\right] - C_{t-}dt\,, \end{aligned} \tag{B.69}$$

where $\pi^S = H^S/(H^S + H^U) = H^S/W$.

The post-jump wealth is $W^{\mathcal{J}} = W - (1-Z)W = ZW$. And by using the standard dynamic programming method, we may use the following HJB equation to characterize $J(W, \mathbf{n})$:

$$
\begin{aligned}
0 = \max_{C,\pi^S} \; & \left[r(\mathbf{n})W + \left((r^S(\mathbf{n}) - r(\mathbf{n}))\pi^S + (r^U(\mathbf{n}) - r(\mathbf{n}))(1-\pi^S) + \lambda(\mathbf{n})(1-\mathbb{E}(Z))\right) W - C\right] V_W(W, \mathbf{n}) \\
& + f(C, V) + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right] \mathbf{n} V_{\mathbf{n}}(W, \mathbf{n}) + \frac{\sigma^2 W^2 V_{WW}(W, \mathbf{n})}{2} + \lambda(\mathbf{n}) \int_0^1 \left[V(ZW, \mathbf{n}) - V(W, \mathbf{n})\right] \xi(Z) dZ \,,
\end{aligned}
\tag{B.70}
$$

subject to $\pi^S \geq \alpha$. And the FOC for consumption C is the standard condition given by (28). Because the S- and the U-portfolio have exactly the same (diffusion and jump) risk exposures with probability one, the optimality for π^S is positive infinity if $r^S > r^U$ as we can see from (27). This is not an equilibrium. In equilibrium, $r^S \leq r^U$ and $\pi^S = \alpha$ holds, which implies the agent's value function satisfies the HJB equation (27). We later pin down the equilibrium relation between r^S and r^U.

Let $V_t = V(W_t, \mathbf{n}_t)$ denote the household's value function. We show that

$$
V(W, \mathbf{n}) = \frac{1}{1-\gamma} (u(\mathbf{n})W)^{1-\gamma} \,, \tag{B.71}
$$

where $u(\mathbf{n})$ is determined endogenously. Substituting (B.71) into the FOC (28) yields the following linear consumption rule:

$$
C(W, \mathbf{n}) = \rho^{\psi} u(\mathbf{n})^{1-\psi} W \,. \tag{B.72}
$$

C Market Equilibrium

First, a sustainable firm spends minimally on mitigation: $x^S = \frac{X^S}{K^S}$. Second, in equilibrium, the household invests all wealth in the stock market and holds no risk-free asset, $H = W$ and $W = \mathbf{Q}^S + \mathbf{Q}^U$, and has zero disaster hedging position, $\delta(Z, \mathbf{n}) = 0$ for all Z. Third, the representative agent's (dollar amount) investment in the S portfolio is equal to the total market value of sustainable firms, $\pi^S = \alpha$ and (dollar amount) investment for the U portfolio is equal to the total market value of unsustainable firms, $\pi^U = 1 - \alpha$. Finally, goods market clears.

By using the preceding equilibrium conditions together with $H = W = \mathbf{Q}^S + \mathbf{Q}^U = q^S(\mathbf{n})\mathbf{K}^S + q^U(\mathbf{n})\mathbf{K}^U = q(\mathbf{n})(\mathbf{K}^S + \mathbf{Q}^U) = q(\mathbf{n})\mathbf{K}$, $W^{\mathcal{J}} = ZW$ and $\pi^S = \alpha$, we obtain

$$
\alpha r^S(\mathbf{n}) + (1-\alpha) r^U(\mathbf{n}) = r(\mathbf{n}) + \gamma \rho^2 \sigma^2 + \lambda(\mathbf{n}) \mathbb{E}\left[(1-Z)(Z^{-\gamma} - 1)\right] = r^M(\mathbf{n}) \,. \tag{C.73}
$$

Using $\alpha r^S(\mathbf{n}) + (1-\alpha) r^U(\mathbf{n}) = r^M(\mathbf{n})$, $\mathbf{x} = \alpha m(\mathbf{n})$, and (A.67), we obtain

$$
\begin{aligned}
& \frac{A - \mathbf{i} - \mathbf{x} + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right] \mathbf{n}\mathbf{q}'(\mathbf{n})}{r^M(\mathbf{n}) - g(\mathbf{i})} \\
= \; & \frac{\alpha(A - \mathbf{i} - m(\mathbf{n}) + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right] \mathbf{n}\mathbf{q}'(\mathbf{n})) + (1-\alpha)(A - \mathbf{i} + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right] \mathbf{n}\mathbf{q}'(\mathbf{n}))}{\alpha r^S(\mathbf{n}) + (1-\alpha) r^U(\mathbf{n}) - g(\mathbf{i})} \\
= \; & \frac{\alpha \mathbf{q}(\mathbf{n})(r^S(\mathbf{n}) - g(\mathbf{i})) + (1-\alpha)\mathbf{q}(\mathbf{n})(r^U(\mathbf{n}) - g(\mathbf{i}))}{\alpha(r^S(\mathbf{n}) - g(\mathbf{i})) + (1-\alpha)(r^U(\mathbf{n}) - g(\mathbf{i}))} = \mathbf{q}(\mathbf{n}) \,.
\end{aligned}
\tag{C.74}
$$

And then by solving

$$\mathbf{q}(\mathbf{n}) = \frac{\mathbf{A} - \mathbf{i} + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})]\, \mathbf{n}\mathbf{q}'(\mathbf{n})}{\mathbf{r}^{\mathbf{M}}(\mathbf{n}) + \theta^{\mathbf{U}} - \mathbf{g}(\mathbf{i})} = \frac{\mathbf{A} - \mathbf{i} + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})]\, \mathbf{n}\mathbf{q}'(\mathbf{n})}{\mathbf{r}^{\mathbf{U}}(\mathbf{n}) - \mathbf{g}(\mathbf{i})} = \frac{\mathbf{A} - \mathbf{i} - \mathbf{x} + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})]\, \mathbf{n}\mathbf{q}'(\mathbf{n})}{\mathbf{r}^{\mathbf{M}}(\mathbf{n}) - \mathbf{g}(\mathbf{i})}, \tag{C.75}$$

we obtain $(A - \mathbf{i} + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})]\, \mathbf{n}\mathbf{q}'(\mathbf{n}))\theta^U(\mathbf{n}) = \mathbf{x}(r^U(\mathbf{n}) - g(\mathbf{i}))$ and $\theta^U(\mathbf{n}) = \mathbf{x}/\mathbf{q}(\mathbf{n}) = \alpha m(\mathbf{n})/q(\mathbf{n})$ as shown in (44).

In addition, the optimal consumption rule given in (B.72) implies

$$c(\mathbf{n}) = \frac{C}{\mathbf{K}} = \frac{C}{W} q(\mathbf{n}) = \rho^{\psi} u(\mathbf{n})^{1-\psi} \mathbf{q}(\mathbf{n}). \tag{C.76}$$

And then substituting c given by (C.76) and the value function given in (B.71) into the HJB equation (27), we obtain

$$\begin{aligned} 0 &= \frac{1}{1-\psi^{-1}} \left(\frac{c(\mathbf{n})}{\mathbf{q}(\mathbf{n})} - \rho \right) + \left(\alpha r^S(\mathbf{n}) + (1-\alpha) r^U(\mathbf{n}) - \frac{c(\mathbf{n})}{\mathbf{q}(\mathbf{n})} + \lambda(\mathbf{n})(1 - \mathbb{E}(Z)) \right) \\ &\quad + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})] \frac{\mathbf{n} u'(\mathbf{n})}{u(\mathbf{n})} - \frac{\gamma \sigma^2}{2} + \frac{\lambda(\mathbf{n})}{1-\gamma} \left[\mathbb{E}(Z^{1-\gamma}) - 1 \right] \\ &= \frac{1}{1-\psi^{-1}} \left(\frac{c(\mathbf{n})}{\mathbf{q}(\mathbf{n})} - \rho \right) + \left(r^M(\mathbf{n}) - \frac{c(\mathbf{n})}{\mathbf{q}(\mathbf{n})} + \lambda(\mathbf{n})(1 - \mathbb{E}(Z)) \right) \\ &\quad + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})] \frac{\mathbf{n} u'(\mathbf{n})}{u(\mathbf{n})} - \frac{\gamma \sigma^2}{2} + \frac{\lambda(\mathbf{n})}{1-\gamma} \left[\mathbb{E}(Z^{1-\gamma}) - 1 \right]. \end{aligned} \tag{C.77}$$

By using (C.75) and the goods market clear condition, we obtain

$$\frac{c(\mathbf{n})}{\mathbf{q}(\mathbf{n})} = r^M(\mathbf{n}) - g(\mathbf{i}) - [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})] \frac{\mathbf{n}\mathbf{q}'(\mathbf{n})}{\mathbf{q}(\mathbf{n})}. \tag{C.78}$$

And then by substituting it into (C.77) and combining $c(\mathbf{n}) = A - \mathbf{i}(\mathbf{n}) - \mathbf{x}(\mathbf{n})$, we have

$$\begin{aligned} 0 &= \frac{1}{1-\psi^{-1}} \left(\frac{A - \mathbf{i}(\mathbf{n}) - \mathbf{x}(\mathbf{n})}{\mathbf{q}(\mathbf{n})} - \rho \right) + \phi(\mathbf{i}(\mathbf{n})) - \frac{\gamma \sigma^2}{2} + \frac{\lambda(\mathbf{n})}{1-\gamma} \left[\mathbb{E}(Z^{1-\gamma}) - 1 \right] \\ &\quad + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})] \left(\frac{\mathbf{n} q'(\mathbf{n})}{q(\mathbf{n})} + \frac{\mathbf{n} u'(\mathbf{n})}{u(\mathbf{n})} \right) \\ 0 &= \frac{1}{1-\psi^{-1}} \left(\frac{A - \mathbf{i}(\mathbf{n}) - \mathbf{x}(\mathbf{n})}{\mathbf{q}(\mathbf{n})} - \rho \right) + \phi(\mathbf{i}(\mathbf{n})) - \frac{\gamma \sigma^2}{2} + \frac{\lambda(\mathbf{n})}{1-\gamma} \left[\mathbb{E}(Z^{1-\gamma}) - 1 \right] \\ &\quad + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})] \left(\frac{\psi}{1-\psi} \frac{\mathbf{n} q'(\mathbf{n})}{q(\mathbf{n})} - \frac{1}{1-\psi} \frac{\mathbf{n}\mathbf{i}'(\mathbf{n}) + \mathbf{n}\mathbf{x}'(\mathbf{n})}{A - \mathbf{i}(\mathbf{n}) - \mathbf{x}(\mathbf{n})} \right), \end{aligned} \tag{C.79}$$

which implies (35).

And then by substituting it into (C.77) and combining $r^M(\mathbf{n}) = r(\mathbf{n}) + \gamma\sigma^2 + \lambda(\mathbf{n})\mathbb{E}\left[(1-Z)(Z^{-\gamma} - 1)\right]$, we obtain the risk-free rate is

$$r(\mathbf{n}) = \frac{c(\mathbf{n})}{\mathbf{q}(\mathbf{n})} + \phi(\mathbf{i}(\mathbf{n})) + [\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n}))] \frac{\mathbf{n}\mathbf{q}'(\mathbf{n})}{\mathbf{q}(\mathbf{n})} - \gamma\sigma^2 - \lambda(\mathbf{n})\mathbb{E}\left[(1-Z)Z^{-\gamma}\right]. \tag{C.80}$$

In addition, under the steady state the risk-free rate satisfies

$$r(\mathbf{n}^*) = \rho + \psi^{-1}\phi(\mathbf{i}(\mathbf{n}^*)) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \lambda(\mathbf{n}^*)\mathbb{E}\left[\left(Z^{-\gamma} - 1\right) + \left(\psi^{-1} - \gamma\right) \left(\frac{1 - Z^{1-\gamma}}{1-\gamma} \right) \right] \tag{C.81}$$

Welfare-maximizing mandate. As the household's value function is given by (B.71) and $W = \mathbf{q}(\mathbf{n})\mathbf{K}$ in equilibrium, we may write

$$V = \frac{1}{1-\gamma}(u(\mathbf{n})W)^{1-\gamma} = \frac{1}{1-\gamma}(u(\mathbf{n}) \times q(\mathbf{n})\mathbf{K})^{1-\gamma} = \frac{1}{1-\gamma}(b(\mathbf{n})\mathbf{K})^{1-\gamma}, \tag{C.82}$$

where $b(\mathbf{n}) = q(\mathbf{n}) \times u(\mathbf{n})$ is proportional to the certainty equivalent wealth (welfare) per unit capital. And then substituting $b(\mathbf{n}) = q(\mathbf{n}) \times u(\mathbf{n})$ into (C.79) and using $c(\mathbf{n}) = \frac{C}{\mathbf{K}} = \frac{C}{W}q(\mathbf{n}) = \rho^{\psi}u(\mathbf{n})^{1-\psi}q(\mathbf{n})$, we obtain the ODE given in (37) for $b(\mathbf{n})$. Immediately, by following ODE (37) we have the FOC for $\mathbf{x}$ to maximize welfare $b(\mathbf{n})$ is given by (38). And then substituting the good market clearing conditions $c(\mathbf{n}) = A - \mathbf{i}(\mathbf{n}) - \mathbf{x}(\mathbf{n})$ under equilibrium into the optimal consumption rule (C.76), and recalling $b(\mathbf{n}) = q(\mathbf{n}) * u(\mathbf{n})$ we obtain the optimal investment satisfying (39). Finally, the welfare-maximizing mandate is given by (32) for given α by following the welfare-maximizing mitigation obtained above.