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ABSTRACT

The flow of resources across sectors to their best use, with concomitant entry and exit, is central to the functioning and welfare properties of a market economy. Nevertheless, most industrial organization research, including applications to competition policy, undertakes partial equilibrium analysis in a single sector, often with a fixed number of firms. This article examines competition policy in a simple, multi-sector, general equilibrium model with free entry and exit. Even partial equilibrium analysis yields some lessons, such as that accounting for free entry often makes strengthening competition policy more rather than less attractive. When admitting flows between sectors, familiar prescriptions readily reverse. But such results may be partially offset or overturned yet again when incorporating free entry and exit in nontargeted sectors. Finally, the analysis of efficiencies also changes qualitatively with free entry because even fixed costs are fully borne by consumers in equilibrium.

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1. Introduction

In bygone days, leading economic theorists such as Lerner (1934) explained how the presence of market power in multiple sectors undermined "particular [i.e., partial equilibrium] analysis"—specifically, rendering the economy-wide level of markups irrelevant; only their "deviations" mattered. For Lipsey and Lancaster (1956), this problem of "degrees of monopoly," which was so well known that they felt no need "to review the voluminous literature on this controversy," helped motivate *The General Theory of the Second Best*. Subsequently, however, this perspective has largely vanished from industrial organization economics, including its application to competition policy, even as concerns about significant market power in many sectors of the economy have recently escalated.

Examining these earlier, informal intuitions in a simple general equilibrium model reveals important respects in which Lerner's claim is correct, readily reversing competition policy prescriptions derived from partial equilibrium analysis. However, complete analysis that explicitly incorporates free entry and exit partly offsets and can more than overturn conclusions in the spirit of Lerner in some important settings, but not in others. Even in single-sector partial equilibrium analysis, entry and exit are first-order concerns. For example, accounting for entry and exit can make tougher competition policy more rather than less attractive. Moreover, the analysis of efficiencies, notably fixed costs or others not fully passed on to consumers, changes qualitatively because consumers ultimately bear all costs in a free-entry equilibrium.

The consideration of effects across sectors in a general equilibrium setting with free entry should not be regarded as a subtle refinement of standard analysis but rather as central to our understanding of competition policy. After all, even in elementary economics instruction or cocktail party conversations among the uninitiated that invoke Adam Smith, the concept of resources flowing across sectors, with concomitant entry and exit, is appreciated to be a central feature of a market economy, not some incidental effect of occasional importance. Hence, it is foundational to examine competition policy in a multi-sector, general equilibrium model with free entry. Moreover, the effects examined here can be quite large as a practical matter if indeed many sectors exhibit substantial market power. Therefore, however strong may be the need to employ simpler rules or shortcuts in everyday application, such proxies must be chosen in light of the results of such an investigation.

Section 2 presents a simple general equilibrium model. A representative individual allocates a given income across goods in multiple sectors. The utility function allows for arbitrary substitution across sectors and any degree of preference for variety within each sector. Firms in each sector are symmetric, with cost functions taking a general form that may differ across sectors. Entry is free.

The model takes a reduced-form approach to the analysis of competition, as in Mankiw and Whinston (1986). Rather than specifying some particular mode of competitive interaction, a fairly general formulation assumes only that, ceteris paribus, a greater number of firms in a sector reduces price in that sector. Competition policy is also modeled in a general, reduced-form manner, assuming only that, ceteris paribus, stronger competition policy in a sector reduces that sector's price. The analysis will address the effect of strengthening competition policy in a targeted sector on social welfare, taken to be the representative individual's utility.

To build intuition and align more closely with prior work, section 3 begins with a simplified version of the model in which general equilibrium effects on prices, quantities, and the number of firms in nontargeted sectors are set to the side. In doing so, an outside good—familiar in much work in industrial organization (e.g., Lancaster 1980 and Berry 1994)—is employed to absorb expenditure outflows from and inflows to the targeted sector. In this limited setting, tougher competition policy raises welfare more (or reduces it less) than when the planner directly adjusts the number of firms (such as through an entry tax or subsidy) by the same degree as that induced by changing competition policy. Also, in the common case in which the equilibrium number of firms is socially excessive, tougher competition policy improves welfare more when the effect on entry is taken into account than when it is ignored, contrary to conventional wisdom that views the possibility of entry as rendering competition enforcement less important.

Section 4 returns to the core model and analyzes it in two stages. First, general equilibrium effects associated with changing prices and quantities in nontargeted sectors are considered while still holding the number of firms in those sectors fixed. Because the latter implies profits or losses, which are inconsistent with free-entry equilibrium, the device of an entry tax or subsidy is used to close the model. Here, the results support some of the implications of Lerner (1934). Tougher competition policy tends to raise (lower) welfare when the markup in the targeted sector is "high" ("low")—in a sense made precise in the analysis—relative to the markups in the rest of the economy (a result that holds whether or not general equilibrium effects on prices are taken into account). Thus, when the targeted sector's markup is low, conventional policy prescriptions reverse.

The core intuition for this result is straightforward. On one hand, stronger competition policy tends to raise welfare in the targeted sector: because the initial price exceeds marginal cost, induced inflows of expenditures reduce deadweight loss. However, inflows to the targeted sector are outflows from other sectors, so when those sectors are also distorted, deadweight loss rises there. Moreover, although the effect from each other sector of the economy may be small, the relevant aggregate of outflows from those sectors is the same magnitude as the inflows to the targeted sector. Hence, welfare effects in other sectors that are traditionally ignored are of the same order as effects in the targeted sector (and, in this case of the model, of opposite sign).

Section 4 then analyzes the full general equilibrium model with free entry and exit. Allowing the number of firms in all sectors to adjust freely overturns the foregoing multi-sector result in an important sense but also may preserve its welfare effects depending on how variety is valued in different sectors. The former point arises because, although the direct effect of resource outflows is to raise deadweight loss, the concomitant reduction in profits (from lost sales at prices in excess of marginal cost) induces exit, which in turn reduces resource use. If, for example, a nontargeted sector involves homogeneous goods, this might seem to be the end of the story, with exit-related efficiency gains offsetting outflow-related efficiency losses, leaving only the effects in the targeted sector and in other nontargeted sectors. There is, however, a further general equilibrium price effect (from the decline in demand for that sector's goods) that contributes further to overall efficiency. However, when variety in such a nontargeted sector is valuable, exit as such reduces welfare. Indeed, if the number of firms in the initial equilibrium was socially optimal, the resource savings from exit is fully offset by the loss in variety, so the overall welfare effect from the outflow is negative and to precisely the same extent as in the analysis that held the number of firms fixed.

A greater understanding of the full model can be obtained by reflecting on the underlying sources of welfare effects in general equilibrium with free entry. Transfers from consumers to producers are moot if one thinks of firms as owned by individuals, but in any event profits equal zero in a free-entry equilibrium, so all costs are ultimately borne directly by individuals. Note further that marginal cross-sector reallocations—the focus here—have no direct effect on utility as a consequence of the individual's envelope condition. Much of what remains are general equilibrium effects, including those due to entry and exit. These involve pecuniary externalities that ordinarily are taken to be welfare-irrelevant, but here we have an economy that is distorted in every sector (price exceeds marginal cost) and entry decisions in each sector are distorted as well (due to business-stealing effects and the incomplete appropriability of the inframarginal consumer surplus generated by increased variety). Most of the welfare impacts in the full model, therefore, are attributable to effects that are omitted from conventional analysis.

Section 5 extends the model to incorporate efficiencies. A change in competition policy is allowed to alter firms' cost functions as well as their competitive interaction. The conventional focus on variable cost efficiencies and the extent to which they are passed on to consumers obviously changes qualitatively in a free-entry equilibrium because consumers ultimately bear all costs, including those that are fixed. Indeed, when markups in all sectors are equal and the number of firms is held constant, the only welfare effect is the change in total costs, regardless of whether they are fixed or variable. Finally, a concluding section comments on implications of the analysis for competition policy.

The most relevant prior literature constitutes the aforementioned brief, informal discussions by Lerner (1934) and others over a half-century ago. As reflected by Lipsey and Lancaster (1956), economic theorists were concerned about policy prescriptions in a world with multiple distortions. Relatedly, it was understood that, although pecuniary externalities due to general equilibrium effects do not affect social welfare in an otherwise undistorted economy, they can have first-order effects in the presence of other distortions. See Scitovsky (1954). The importance of this longstanding theoretical concern is reinforced by contemporary work suggesting that substantial market power may indeed be present in many sectors of the economy. See, for example, De Loecker, Eeckhout, and Unger (2020) and Hall (2018), as well as the discussions in Basu (2019) and Syverson (2019).

In this light, it is notable that the modern field of industrial organization, including that on competition policy, typically engages in one-sector, partial equilibrium analysis. See, for example, the texts and surveys by Tirole (1988), Motta (2004), Whinston (2006), and Kaplow and Shapiro (2007). In that setting, Spence (1976), Dixit and Stiglitz (1977), Mankiw and Whinston (1986), and others have examined how the number of firms in free-entry equilibrium differs from the social optimum. These analyses take as given the state of competitive interaction and hence do not analyze the implications for competition policy, including those of efficiency effects of such policy.[1]

2. Model

There are I sectors, indexed by i. The utility function of the representative individual is

$$U \equiv u(X^1, \ldots, X^I) \qquad (1)$$

where $X^i(x^i, n^i)$ is the aggregator (subutility function) for sector i, x^i is the quantity purchased from each (symmetric) firm in sector i, and n^i is the number of firms in sector i, sometimes

[1] There is, however, some interesting literature on dynamic merger policy that considers largely different issues. See Gowrisankaran (1999) and Mermelstein et al. (2020).

referred to as varieties and taken to be substitutes within each sector. The representative individual chooses the x^i to maximize U subject to the budget constraint

$$\sum_{i=1}^{I} p^i x^i n^i = y \qquad (2)$$

taking as given the prices p^i, the n^i, and income y.

A firm that operates in sector i, producing x^i, incurs costs $c^i(x^i)$, which include any fixed costs. Profits of a firm in sector i are

$$\pi^i = p^i x^i - c^i(x^i) \qquad (3)$$

with $\pi^i = 0$ in free-entry equilibrium.

The state of competition is determined in part by competition policies γ^i applied in each sector i. Competition itself is modeled in a fairly general, abstract, reduced-form manner, following the approach employed by Mankiw and Whinston (1986) that enables the model to capture many types of competitive interactions which, moreover, may differ across sectors. Price in each sector i is given by some function

$$p^i = f^i(n^i, \gamma^i; \boldsymbol{p}^{\setminus i}, \boldsymbol{n}^{\setminus i}) \qquad (4)$$

where $\boldsymbol{p}^{\setminus i}$ and $\boldsymbol{n}^{\setminus i}$ are vectors of prices and numbers of firms in all sectors except sector i. This function is the reduced-form representation of how the competitive interaction among firms—with their given cost functions, along with the demand function (which is implicit in the representative individual's utility maximization problem)—determines price in sector i for a given number of firms and a prevailing competition policy in that sector.

In the analysis that follows, we will assume that $f_1^i < 0$ (where subscripts indicate partial derivatives): a larger number of firms n^i reduces p^i, ceteris paribus. The rate at which additional firms reduce price and how high price is to begin with are unrestricted, so this formulation is consistent with a wide range of intensities of competition. The use of a strict inequality, which is for expositional convenience, does rule out (but can still approximate) Bertrand competition with homogeneous goods when marginal cost is constant as well as perfect coordination that is impervious to entry.

A key feature is that f^i is a function of—and thus p^i directly depends on—the competition policy γ^i in sector i. As a convention, it is assumed that $f_2^i < 0$, which is to say that "stronger" competition policy γ^i in sector i reduces p^i, ceteris paribus.[2] This is described as a convention because, as far as the model is concerned, γ^i is merely some instrument that, when increased, reduces p^i; if an instrument of interest instead increased p^i, one could simply redefine it to have the opposite sign. Furthermore, as will become apparent in the analysis that follows, which is confined to comparative statics, it is only necessary that this property hold locally. Hence, the analysis can readily encompass an instrument that, when increased, reduces price in some ranges and increases price in other ranges. It can likewise be applied to instruments that have differently signed effects in different sectors.

[2] This assumption is relaxed in section 5, which introduces efficiencies.

Throughout the analysis that follows, attention is confined to cases in which utility functions, cost functions, and competitive interactions in each sector are such that an equilibrium for the economy exists, it is symmetric within each sector (justifying the use of the single variable x^i to denote the quantity per firm in sector i), and comparative statics exercises are well defined. Relatedly, no integer constraint will be imposed on the number of firms in each sector.[3] Finally, for ease of exposition, interpretations of various conditions will assume that price exceeds marginal cost and that lower prices in a sector increase expenditures in that sector, even accounting for induced exit. (For other cases, it will usually be obvious which interpretations would change and how.)

The analysis will be conducted in two phases. Section 3 examines the effects of strengthening competition policy in a single sector, denoted as sector j, setting aside the effects on prices, quantities, and the number of firms in other sectors. This simpler analysis builds intuition and corresponds more closely to conventional analysis that employs (sometimes implicitly) an outside good that absorbs all inflows and outflows of expenditures to and from the sector under analysis, which allows partial equilibrium analysis. Section 4 removes this device so that the analysis will include reallocations across sectors and resulting general equilibrium effects on prices as well as induced entry and exit in other sectors.

3. Analysis: Single Sector with an Outside Good

This section undertakes analysis confined to the single sector j in which competition policy γ^j will be strengthened. Because we wish to take as fixed the prices, quantities, and number of firms in sectors $i \neq j$ ($\boldsymbol{p}^{\setminus j}$, $\boldsymbol{x}^{\setminus j}$, and $\boldsymbol{n}^{\setminus j}$), it is necessary to introduce an outside good z that absorbs what otherwise would be flows into or out of sectors $i \neq j$ as a consequence of changes in expenditures in sector j.[4] Accordingly, we will use the modified utility function

$$\widetilde{U} \equiv u(X^1, \dots, X^I) + \lambda z \qquad (5)$$

where λ will (from the individual's optimization problem) equal the marginal utility of income. The budget constraint for this formulation (which takes $p^z = 1$) is:

$$\sum_{i=1}^{I} p^i x^i n^i + z = y \qquad (6)$$

Note that the envelope theorem applied to the representative individual's maximization problem in the unrestricted, multi-sector version of the model implies that this restricted model yields the

[3] For analysis of the integer constraint in the case of homogeneous goods, see Mankiw and Whinston (1986).

[4] The presentation that follows implicitly takes $z = 0$ as the initial value and allows z to be negative. This formulation enables partial equilibrium analysis, such as in Mankiw and Whinston (1986, page 50 and note 5) and, more broadly, analysis of consumer demand that focuses on a single sector (for example, Lancaster 1980 and Berry 1994).

same welfare effects for sector j as would the broader version, except for the general equilibrium effects arising from changing prices, quantities, and the number of firms in other sectors.[5]

We are now ready to determine the effect of changing competition policy in sector j on social welfare, which is here taken to be the representative individual's utility.[6] That is, we wish to evaluate the derivative of expression (5):

$$\left.\frac{d\widetilde{U}}{d\gamma^j}\right|_{\setminus j} = u_j\left(X_1^j \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} + X_2^j \left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j}\right) + \lambda \left.\frac{dz}{d\gamma^j}\right|_{\setminus j} \qquad (7)$$

where the notation "$|_{\setminus j}$" indicates (really, reminds us) that prices, quantities, and the number of firms in all sectors $i \neq j$ ($\boldsymbol{p}^{\setminus j}$, $\boldsymbol{x}^{\setminus j}$, and $\boldsymbol{n}^{\setminus j}$) do not change, and subscripts indicate partial derivatives. The analysis in the Appendix demonstrates:

Proposition 1. *In the one-sector model with an outside good (in which prices, quantities, and the number of firms in all other sectors do not change), the effect of strengthening competition policy (raising γ^j) on social welfare is given by*

$$\left.\frac{d\widetilde{U}}{d\gamma^j}\right|_{\setminus j} = \left(p^j - \frac{dc^j}{dx^j}\right)\lambda n^j \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} + \left(\frac{X_2^j}{X_1^j} - \frac{x^j}{n^j}\right)\lambda p^j n^j \left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j} \qquad (8)$$

The first term on the right side of expression (8) is the welfare effect of tougher competition policy in reducing deadweight loss and the second term is the direct welfare effect of the induced reduction in the number of firms. Regarding the first term, $n^j\, dx^j/d\gamma^j\, |_{\setminus j} > 0$ is the total increase in the number of units consumed in sector j from remaining firms. Raising γ^j by construction reduces p^j for a given n^j, which in turn raises x^j. In addition, the reduction in p^j is taken to induce exit, which has two further effects on x^j.[7] First, by reducing competition, the fall in p^j is mitigated (but not fully offset), which lessens the aforementioned increase in x^j. Second, exit as such increases x^j because goods in each sector are symmetric substitutes: at least some of the purchases from the exiting firm are reallocated to the remaining firms in sector j (keeping in mind that x^j indicates the per-firm quantity in sector j). The parenthetical expression is simply the difference between price and marginal cost in sector j, taken to be

[5] In other words, when undertaking the multi-sector analysis in section 4, one could first confine analysis to sector j and then examine the outflows from sectors $i \neq j$—both their direct effects and their indirect, general equilibrium effects. Because of the representative individual's envelope condition, the resulting welfare effects will be a consequence of pecuniary externalities (notably, on firms whose sales are at prices above their marginal costs and on the representative individual, whose inframarginal consumer surplus is external to firms) and, relatedly, general equilibrium effects—all consequences that are ignored in the individual's maximization problem that takes prices and the number of firms in all sectors as given.

[6] In the free-entry equilibria examined in much of the analysis, profits are zero. Also, in the cases in which exit or entry are restricted, any losses or profits are offset by subsidies or taxes that, in turn, are taken to be financed by the representative individual. See note 11.

[7] Attention in the interpretive exposition is confined to cases in which the fall in p^j, despite the concomitant rise in x^j, reduces firms' profits in sector j at the initial n^j; that is, before the increase in γ^j, prices were not at or above sector j's joint profit-maximizing price.

positive, which measures the deadweight loss per marginal unit not consumed and hence the increase in welfare due to the rise in total consumption in sector j. Finally, this is all multiplied by λ, the marginal utility of income, converting dollars to utils.

The second term, as mentioned, is the welfare effect directly due to the exit of firms in sector j that arises from the tougher competition policy reducing the price associated with any given number of firms. All of the term that precedes $dn^j/d\gamma^j\,|_{\backslash j}$ constitutes the representative individual's value of variety: what is obtained when taking the derivative of utility with respect to n^j while simultaneously adjusting (reducing) x^j so as to keep constant total consumption in sector j, $x^j n^j$ (and making a further substitution using the individual's first-order condition for x^j). Equivalently, the parenthetical term by itself equals that value divided by the simple derivative of utility with respect to x^j—so that component is the ratio of the value of variety to the value of quantity.[8] The second term as a whole indicates the fall in utility as a direct consequence of the reduction in variety.[9]

The simplest case is when variety as such has no value—that is, homogeneous goods—in which event (without loss of generality) we can write $X^j\left(x^j, n^j\right) = x^j n^j$, which implies that

$$\frac{X_2^j}{X_1^j} = \frac{x^j}{n^j} \qquad (9)$$

Then the second term in expression (8) equals zero, so the full welfare effect is given by the reduction in deadweight loss. Aside from their effect on price, and therefore quantity (already reflected in $dx^j/d\gamma^j\,|_{\backslash j}$, which is the total derivative associated with this policy experiment), additional firms add no value, so their exit causes no welfare loss.[10] In this case, the first term by itself indicates the welfare effect of stronger competition policy, and it is unambiguously positive.

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[8] Appendix expression ($A6$) presents this equivalent formulation for the second term:

$$\left(u_j X_2^j - \lambda p^j x^j\right) \frac{dn^j}{d\gamma^j}\bigg|_{\backslash j}$$

The first term in parentheses is the value in utils of adding (for a constant x^j) another variety, which includes the value of inframarginal units of consumption, whereas the second term values variety using price, which is the representative individual's marginal valuation for quantity. This formulation is suggestive of the variety externality explanation associated with Spence (1976) and Mankiw and Whinston (1986): gross social value (ignoring production costs) is indicated by the former whereas a firm contemplating entry only receives (as revenue) the latter.

[9] Expression (8) can also be written as

$$\frac{d\tilde{U}}{d\gamma^j}\bigg|_{\backslash j} = \left[\mathcal{L}^j \frac{dx^j}{d\gamma^j}\bigg|_{\backslash j} + \left(\frac{X_2^j}{X_1^j} - \frac{x^j}{n^j}\right)\frac{dn^j}{d\gamma^j}\bigg|_{\backslash j}\right]\lambda p^j n^j$$

where $\mathcal{L}^j$ is the Lerner index for sector j. Furthermore, if one substitutes using the first-order condition for x^j, $\lambda p^j n^j = u_j X_1^j$, this version registers the two effects on utility in terms of the marginal utility of raising x^j.

[10] This term equaling zero in the homogeneous goods case is consistent with the fact that exit, ceteris paribus, raises social welfare, as explained, for example, in Mankiw and Whinston (1986). The marginal exiting firm earned zero profits, so its costs were just covered by its revenue. But its sales are now absorbed by other firms (as noted in the text, exit increases $n^j\, dx^j/d\gamma^j\,|_{\backslash j}$, keeping in mind that x^j is the quantity per remaining firm), so the welfare gain (the difference between price and marginal cost on those new sales) is captured by the first term.

More generally, when variety has some positive value we have a tradeoff. To explore this further, it is useful to decompose competition policy's direct effect, in lowering p^j for a given n^j, and its effect on exit, the reduction in n^j. This decomposition does not directly correspond to the two terms in expression (8) because, as explained, exit itself influences $dx^j/d\gamma^j\,|_{\backslash j}$—both because the reduction in the number of firms itself raises p^j and thus reduces x^j, and because exit induces the representative individual to reallocate at least some of the purchases in sector j from the marginal exiting firm to the remaining firms.

To explore this further, we can directly determine competition policy's effect when the number of firms n^j is held constant. To do so, we will consider the following policy experiment: as the social planner raises γ^j, which itself induces exit, the planner will simultaneously increase an entry subsidy, denoted σ^j (and initially set equal to zero), at a rate that keeps n^j fixed. The Appendix derives the following result:

Corollary 1.1. *In the one-sector model with an outside good (in which prices, quantities, and the number of firms in all other sectors do not change), when there is also a subsidy on entry set to keep the number of firms n^j constant, the effect of strengthening competition policy (raising γ^j) on social welfare is given by*

$$\left.\frac{d\widetilde{U}}{d\gamma^j}\right|_{n^j,\backslash j} = \left(p^j - \frac{dc^j}{dx^j}\right)\lambda n^j \left.\frac{dx^j}{d\gamma^j}\right|_{n^j,\backslash j} \qquad (10)$$

where the notation $|_{n^j,\backslash j}$ indicates that the pertinent derivatives hold the number of firms in sector j constant (in addition to prices, quantities, and the number of firms in other sectors not changing, as before).

This expression, in accord with the foregoing explanation, is similar to the first term in Proposition 1, with the difference that here we have $dx^j/d\gamma^j\,|_{n^j,\backslash j}$, reflecting the difference in the policy experiment. When n^j is held constant, the only effect on x^j is through strengthened competition policy reducing p^j (which it does by definition), and here since n^j is in fact held constant, this is the sole effect of the policy. That reduction in p^j raises x^j, which raises welfare by reducing deadweight loss. Much conventional analysis of competition policy—which ignores cross-sector effects and induced entry and exit—accords with this formulation.

This policy experiment can also be used to answer another question: What is the relationship between the optimal setting of competition policy and the degree to which the free-entry equilibrium (in light of the value of variety) involves socially excessive or insufficient entry? In the present setting, note that the given free-entry equilibrium will depend on the prevailing competition regime, γ^j, so any statement must be relative to some such baseline. In this one-sector model with an outside good, the answer is that optimal competition policy will be stronger than that associated with an equilibrium number of firms equal to what a social planner would choose.

The reasoning has to do with the difference in experiments. Prior literature (e.g., Mankiw and Whinston 1986) asks whether a social planner would prefer to have more or fewer firms than the number that arises in free-entry equilibrium; there, the planner is assumed able to raise or lower the number of firms but must take as given their competitive interaction (which determines price and quantity as a function of the number of firms). Here, we consider the optimal strength of competition policy, which operates through changing that competitive interaction but leaves the number of firms endogenous. It is natural to ask what, if any, adjustment to competition policy would be optimal if the existing regime happened to generate a free-entry equilibrium, $n^j|_{\gamma^j,\backslash j}$, with the (conditionally) optimal number of firms: that is, at the equilibrium, the planner would wish neither to encourage nor to discourage entry. This answer, it turns out, is embedded in the foregoing analysis.

Corollary 1.2. *In the one-sector model with an outside good (in which prices, quantities, and the number of firms in all other sectors do not change), when competition policy γ^j is such that the resulting number of firms, n^j, is socially optimal (in the free-entry equilibrium), the effect of strengthening competition policy (raising γ^j) on social welfare is positive and is given by*

$$\left.\frac{d\widetilde{U}}{d\gamma^j}\right|_{\backslash j} = \left(p^j - \frac{dc^j}{dx^j}\right)\lambda n^j \left.\frac{dx^j}{d\gamma^j}\right|_{n^j,\backslash j} \qquad (11)$$

even though the number of firms in sector j, n^j, is not held fixed.

This result follows immediately from the derivation of Corollary 1.1. There, the policy experiment involved strengthening competition policy and simultaneously increasing an entry subsidy σ^j so as to keep the number of firms n^j fixed. Now, if the number of firms is optimal (conditional on γ^j), we could perform that same marginal experiment while instantaneously reducing the subsidy σ^j back to zero. The latter would have no effect on welfare because it only marginally reduces n^j, which was taken to be at its optimum. More precisely, we can write

$$\left.\frac{d\widetilde{U}}{d\gamma^j}\right|_{\backslash j} = \left.\frac{d\widetilde{U}}{d\gamma^j}\right|_{n^j,\backslash j} + \left.\frac{d\widetilde{U}}{dn^j}\right|_{\backslash j} \left.\frac{dn^j}{d\gamma^j}\right|_{\backslash j} \qquad (12)$$

The statement that n^j is optimal means that $d\widetilde{U}/dn^j\,|_{\backslash j} = 0$. Therefore, the total effect on welfare of strengthening competition policy is the same as the effect of strengthening competition policy while employing a subsidy to hold n^j fixed; hence, the same expression. Note further that, whatever is the value of $d\widetilde{U}/dn^j\,|_{\backslash j}$, the welfare effect of strengthening competition policy will always be greater than that of an entry tax that directly reduces n^j to the same degree because the first term on the right side of expression (12) is positive.

Why the difference, and why in particular is this expression necessarily positive under the stated conditions rather than equal to zero? When the planner directly pushes down the number of firms n^j (say, by an entry tax, that is, a negative σ^j), the price p^j rises due to the reduction in competition, and that latter effect reduces welfare because price exceeds marginal cost. When the planner instead indirectly induces an equivalent reduction in n^j by strengthening competition policy (raising γ^j), exit is instead associated with a lower p^j. In this setting (which ignores other sectors, as well as possible efficiencies, which is to say, induced changes in the cost function, $c^j(x^j)$), generating the same decline in the number of firms but through a lower price rather than a higher one is better for social welfare.

The preceding analysis teaches another lesson: strengthening competition policy tends to increase welfare more (less) when free entry is taken into account the more socially excessive (insufficient) is the free-entry number of firms n^j. This follows from expression (12) when n^j is not socially optimal, so that $d\widetilde{U}/dn^j\,|_{\setminus j} \neq 0$. For example, if sector j involves homogeneous goods (or nearly so), the combined effect of induced exit—through reduced variety as well as (let us suppose) increased quantity per firm—will be to raise welfare. Hence, the conventional wisdom that the prospect of entry as such renders tougher competition policy less valuable is incorrect in important settings (again, ignoring other sectors and efficiencies).

4. Analysis: Multiple Sectors

For the multi-sector analysis, we dispense with the outside good z and return to the utility function in expression (1). Likewise, the budget constraint (2) does not involve expenditures on the outside good. It will be helpful to conduct this analysis in two stages: first, the number of firms in each sector $i \neq j$ will be held fixed (through the device of entry subsidies/taxes), and second, the full general equilibrium results with free entry and exit in all sectors will be derived.

4.1. Holding $\boldsymbol{n}^{\setminus j}$ (or $\boldsymbol{n}$) fixed

To implement the first stage, we will employ a modification to the budget constraint analogous to that used in deriving Corollary 1.1, where we held n^j fixed. Even though we are now in a multi-sector model in which prices and quantities will adjust in all sectors (notably, including sectors $i \neq j$), we wish at this stage to hold the $n^{\setminus j}$ constant. To implement this restriction, we will use entry subsidies σ^i in each sector $i \neq j$, set so as to hold the $n^{\setminus j}$ constant. Hence, in each sector $i \neq j$ the entry condition is

$$p^i x^i - c^i\left(x^i\right) + \sigma^i = 0 \qquad (13)$$

whereas, in sector j we will allow free entry:

$$p^j x^j - c^j\left(x^j\right) = 0 \qquad (14)$$

The resulting budget constraint is[11]

$$\sum_{i=1}^{I} p^i x^i n^i = y - \sum_{i \neq j} \sigma^i n^i \qquad (15)$$

The Appendix demonstrates:

[11] Note that this construction is equivalent to allowing a social planner to directly hold the $n^{\setminus j}$ constant, while adding economy-wide profits to the representative individual's budget constraint. When the subsidies are positive—so as to keep firms operating in a sector when that would not otherwise be profitable—the profits resulting when the planner simply forces the firms to continue producing would be negative, and in just the amount of the requisite subsidy.

Proposition 2. *In the multi-sector model, when there is also a subsidy on entry set to keep the number of firms in all sectors $i \neq j$ constant, the effect of strengthening competition policy (raising γ^j) in sector j on social welfare is given by*

$$\left.\frac{dU}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} = \sum_{i=1}^{I}\left(p^i - \frac{dc^i}{dx^i}\right)\lambda n^i \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} + \left(\frac{X_2^j}{X_1^j} - \frac{x^j}{n^j}\right)\lambda p^j n^j \left.\frac{dn^j}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} \qquad (16)$$

This expression resembles expression (8) in Proposition 1 for the one-sector model with an outside good. The latter term, indicating the effect of reduced variety in sector j, is identical except that the change in n^j only holds $\boldsymbol{n}^{\setminus j}$ fixed, whereas in Proposition 1 the corresponding derivative also took the prices ($\boldsymbol{p}^{\setminus j}$) and quantities ($\boldsymbol{x}^{\setminus j}$) in all other sectors to be unchanged. Consider the case in which tougher competition policy in sector j (raising γ^j), which reduces p^j, causes expenditures to flow out of all other sectors and the resulting fall in demand has the effect of reducing prices in those sectors ($\boldsymbol{p}^{\setminus j}$).[12] This general equilibrium adjustment will dampen the outflows from sectors $i \neq j$, which will reduce demand in the targeted sector j and thereby dampen the fall in p^j, and, accordingly, the fall in n^j. So in this basic case, the reduction in variety in sector j will be moderated.

Now compare the first terms of expressions (8) and (16). The most obvious difference is that we now have a summation of effects on deadweight loss, covering all sectors and not just the targeted sector j. This core difference will be elaborated momentarily. The other difference is that the change in quantity demanded (now, again, for all sectors and not just in sector j) is likewise the general equilibrium price effect because now the prices ($\boldsymbol{p}^{\setminus j}$) and quantities ($\boldsymbol{x}^{\setminus j}$) in all other sectors adjust. In the case just noted, the price effect in sector j of toughening competition policy (γ^j) in that sector will likewise be moderated. Regarding sector j as a whole, the general equilibrium effects moderate both the reduction in deadweight loss and the reduction in variety, leaving us with a tradeoff qualitatively similar to that in the one-sector model.[13]

Before further elaboration of the first term in Proposition 2, it is helpful to state the following analogue to Corollary 1.1.[14]

Corollary 2.1. *In the multi-sector model, when there is also a subsidy on entry set to keep the number of firms in all sectors (including sector j) constant, the effect of strengthening competition policy (raising γ^j) in sector j on social welfare is given by*

[12] Obviously, throughout, complements involve inflows and signs reverse.

[13] Note further that, if general equilibrium effects due to the adjustment of prices in other sectors were ignored—more precisely, if the p^i for all $i \neq j$ were held constant—but the quantities x^i in sectors $i \neq j$ were still allowed to adjust, the expression for the effect of competition policy on welfare would be the same as that in Proposition 2 (and the associated corollaries that follow), except that the derivatives for this policy experiment would reflect that $\boldsymbol{p}^{\setminus j}$ was also held constant. This would constitute a partial equilibrium exercise with multiple sectors.

[14] The derivation is the same as that for Proposition 2 except that the change in n^j is omitted and, relatedly, in manipulating the individual's budget constraint, the free-entry condition in sector j is replaced with a subsidy (σ^j) that adjusts to hold n^j constant.

$$\left.\frac{dU}{d\gamma^j}\right|_{\boldsymbol{n}} = \sum_{i=1}^{I}\left(p^i - \frac{dc^i}{dx^i}\right)\lambda n^i \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}} \qquad (17)$$

Comparing this expression to Proposition 2's expression (16), we obviously no longer have the effect of changing variety in sector j, and also the derivative here reflects that all of the n^i, including n^j, are held constant. There are still general equilibrium effects due to adjustments in prices and quantities in other sectors embodied in this derivative.

This summation, like that in the first term of expression (16) in Proposition 2, indicates that deadweight loss changes not only in sector j, where x^j rises so deadweight loss falls, but also in all other sectors $i \neq j$. Taking the simple case just above in which x^i falls for all $i \neq j$, the implication is that deadweight loss rises in all other sectors as a consequence of forgone purchases at prices that are in excess of marginal cost. Hence, focusing solely on deadweight loss, we have a cross-sector tradeoff. Moreover, even if the effect in each sector $i \neq j$ is small, the aggregate effects in all such sectors will be of the same order as the effects in the targeted sector j because the total expenditure outflows from sectors $i \neq j$ necessarily equal the inflow to sector j.

To explore this tradeoff further, we can develop a more intuitive condition for when tougher competition policy in a given sector reduces total deadweight loss in the economy. In doing so, it is helpful to define the Lerner index (introduced in footnote 9) as

$$\mathcal{L}^i \equiv \frac{p^i - \frac{dc^i}{dx^i}}{p^i} \qquad (18)$$

Further analysis in the Appendix shows:

Corollary 2.2. *In the multi-sector model, when there is also a subsidy on entry set to keep the number of firms in all sectors (including sector j) constant,*

$$sign\left(\left.\frac{dU}{d\gamma^j}\right|_{\boldsymbol{n}}\right) = sign\left(\mathcal{L}^j - \sum_{i\neq j}\alpha^i\mathcal{L}^i\right) \qquad (19)$$

where

$$\alpha^i \equiv -\frac{p^i n^i \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}}}{p^j n^j \left.\frac{dx^j}{d\gamma^j}\right|_{\boldsymbol{n}}} \qquad (20)$$

The interpretation of these weights is that the α^i, for $i \neq j$, indicate the relative outflows of expenditures from each of the other sectors, which collectively fund the inflow into sector j. (These α^i are related to diversion ratios that are sometimes used in competition analysis.) Note that, for substitutes in the relevant sense, $\alpha^i > 0$ (x^i and x^j move in opposite directions, and there is a negative sign in the definition of α^i). Also, $\alpha^j = -1$.

The right side of expression (19) indicates that strengthening competition policy in sector j reduces (raises) total deadweight loss when the Lerner index (which indicates marginal deadweight loss) in that sector is greater (less) than a knock-out weighted average of the Lerner index in all other sectors. This statement is in accord with intuition in light of the aforementioned description of the weights. As price falls in sector j, deadweight loss is reduced in that (distorted) sector as expenditures flow in. However, those expenditures necessarily flow out of other sectors that, in general, are also distorted. Hence, marginal deadweight loss rises in sectors $i \neq j$. The magnitude of that aggregate is given by the Lerner index in each other sector, weighted by the extent of the outflow from the corresponding sector.

Corollary 2.2 states that, in a world in which there are multiple sectors and general equilibrium effects—but the number of firms in all sectors $\boldsymbol{n}$ is held fixed—the welfare-maximizing prescription is to strengthen (weaken) competition policy in sectors with relatively "high" ("low") markups. The use of the quotation marks reflects that there is not, in general, a single value of the Lerner index, $\mathcal{L}^*$, such that it is optimal to raise (lower) γ^j whenever $\mathcal{L}^j$ is greater (less) than $\mathcal{L}^*$. This reflects that the knock-out weighted-average Lerner index both omits a different Lerner index depending on the sector under consideration and, moreover, the α^i weights, defined in expression (20), depend on which sector is targeted.

It was convenient to develop the analysis of multi-sector deadweight loss for the version of the competition policy experiment in which we fixed n^i for all i. Returning to the experiment that was the subject of Proposition 2, in which free entry was allowed (only) in sector j, the overall welfare effect of competition policy also included the effect of induced exit in reducing variety in sector j. The first term of that condition would yield precisely the same expression in terms of the comparison of Lerner indexes, the only adjustment being that, in defining the α^i, the derivatives of the x^i would reflect the corresponding policy experiment (allowing free entry in sector j).

Reflection on Corollary 2.2's condition—which refers to the case in which the number of firms is held fixed in all sectors—provides another important insight. Specifically, it is suggestive of Lerner's (1934) important but largely neglected claim that the *level* of the markups in an economy is irrelevant; more precisely, if all markups are in the same proportion, there is no distortion. To prove this point in the present model (and, as per Corollary 2.2, with entry subsidies that hold the number of firms in all sectors constant), it suffices to demonstrate that the weights in the other sectors sum to one, that is, $\sum_{i \neq j} \alpha^i = 1$ when $\mathcal{L}^i = \mathcal{L}$ for all i. That is done in the Appendix, which establishes:

Corollary 2.3. *In the multi-sector model, when there is also a subsidy on entry set to keep the number of firms in all sectors (including sector j) constant and, moreover, if the Lerner indexes in all sectors are equal (that is, there exists $\mathcal{L}$ such that $\mathcal{L}^i = \mathcal{L}$ for all i), then strengthening competition policy (raising γ^j) in sector j has no effect on social welfare—that is, $dU/d\gamma^j\big|_{\boldsymbol{n}} = 0$.*

4.2. Free Entry and Exit in All Sectors

Let us now move to the second stage of the analysis: general equilibrium with free entry and exit in all sectors. The analysis of this case uses methods and formulations similar to those already employed, especially for Proposition 2, which differed only in holding the $\boldsymbol{n}^{\setminus j}$ fixed using an entry subsidy. As we will see, however, the results diverge in important ways. As a preview, note that most conventional effects are absent. After all, gains (losses) to consumers are offset by losses (gains) to firms in the first instance, the representative individual's reallocation of consumption across sectors has no direct effect on utility due to the envelope condition, and all firms have zero profits in equilibrium when there is free entry. Therefore, welfare impacts arise primarily as a consequence of pecuniary externalities due to general equilibrium effects as well as firms' entry and exit decisions.

The basic difference in the setup is that we now omit the subsidy from the entry condition in all sectors (so there is free entry and exit) and, correspondingly, from the budget constraint. We are now back to the budget constraint of

$$\sum_{i=1}^{I} p^i x^i n^i = y \qquad (21)$$

and the free-entry condition in every sector of

$$p^i x^i - c^i\left(x^i\right) = 0 \qquad (22)$$

The Appendix derives the following:

Proposition 3. *In the multi-sector model with free entry in all sectors, the effect of strengthening competition policy (raising γ^j) in sector j on social welfare is given by*

$$\frac{dU}{d\gamma^j} = \sum_{i=1}^{I}\left[\left(p^i - \frac{dc^i}{dx^i}\right)\lambda n^i \frac{dx^i}{d\gamma^j} + \left(\frac{X_2^i}{X_1^i} - \frac{x^i}{n^i}\right)\lambda p^i n^i \frac{dn^i}{d\gamma^j}\right] \qquad (23)$$

This condition bears superficial resemblance to prior results. One difference throughout is that each derivative here takes nothing as fixed and hence is the full derivative in general equilibrium with free entry. As explained in connection with Proposition 2, where in a basic case (in which goods in all sectors $i \neq j$ are substitutes in the relevant sense with those in sector j), the general equilibrium price effects in other sectors (the $p^{\setminus j}$ fall) dampen effects in sector j. Now we have, in addition, that the reduction in the number of firms in other sectors (the $n^{\setminus j}$ also fall, due to the reduction in demand) mitigates this dampening effect (it mitigates the fall in price and also reduces variety, both of which make expenditures in other sectors less attractive), so the net, overall offset in sector j is attenuated relative to what it was before.

The two types of effects, which these derivatives on the right side of expression (23) weight, are as before. The sum of the first terms across sectors indicates the overall change in deadweight loss, which could be interpreted in terms of how the weighted average of the Lerner indexes, $\mathcal{L}^i$, in all sectors $i \neq j$ compare to $\mathcal{L}^j$ (where now the α^i weights in an analogue to expression (20) would instead be defined using the full derivatives, without the n^i being fixed). The sum of the second terms across sectors indicates the overall change in utility due to changes in the number of varieties in all sectors. In our simple case, variety falls in every sector.

Alternatively, one can interpret this expression sector by sector, noting that the two terms for each sector i are the same as the two terms in Proposition 1's expression (8) for the one-sector model of sector j (except for the derivatives being for a different policy experiment). Hence, we can think of first determining the net welfare effect in each sector due to effects in that sector on deadweight loss and on variety, and then sum these effects across all sectors—or, perhaps, as with Corollary 2.2, we could compare the net effect in the targeted sector j with the total effects in all other sectors.

Such interpretations, however, are potentially misleading. One must carefully account for the fact that the pertinent derivatives now embody all of the general equilibrium effects with free entry. Moreover, even though the *expressions* for sector j and for sectors $i \neq j$ are the same, they are not in substance symmetric, particularly regarding the first (deadweight loss) term in the summation. We have already encountered this point with regard to Proposition 2 and its corollaries: increasing γ^j, in a typical case that has been the focus of prior discussion, causes the representative individual's expenditures to flow *into* sector j but *out of* sectors $i \neq j$. When the $n^{\setminus j}$ were held constant, this meant (in our benchmark case) that $dx^i/d\gamma^j\big|_{\boldsymbol{n}} < 0$ for $i \neq j$, so that the outflow from sectors $i \neq j$ caused deadweight loss to rise. However, with free entry and exit, it is possible—and in important cases true—that $dx^i/d\gamma^j > 0$ for some or all $i \neq j$, so that outflows from some or all sectors $i \neq j$ cause deadweight loss to fall.

A sharp illustration, which more broadly illuminates the interpretation of Proposition 3, involves the case of homogeneous goods. Suppose that, for some sector $k \neq j$, goods are homogeneous. As shown after Proposition 1 (see expression 9), this implies that $X_2^k/X_1^k = x^k/n^k$, so the second term for sector k equals zero. It might appear from our earlier analysis that this leaves a positive first term, indicating a rise in deadweight loss due to expenditures flowing out of sector k, which we are assuming is a sector in which price exceeds marginal cost. However, in typical cases—and allowing for free entry and exit in sector k—the outflow from sector k instead causes overall deadweight to fall, and to an extent that is reflected by this first term, but with $dx^k/d\gamma^j > 0$.

To see why, suppose further that, for this homogeneous goods sector, $c^k(x^k) = F^k + \varphi^k x^k$: that is, there is a fixed cost and constant marginal cost for each firm in sector k. Taking again the case in which raising γ^j reduces expenditures in sector k, we have a lower price, p^k. In addition to inducing exit—which has no direct on welfare because sector k goods are homogeneous, implying that the second term in (23) for sector k equals zero—it must also be true that average cost falls: the free-entry condition, $p^k x^k - c^k(x^k) = 0$, implies that $p^k = c^k(x^k)/x^k$, and, as mentioned, p^k falls. That, in turn, implies that $dx^k/d\gamma^j > 0$.

Taken together, free entry has in a sense reversed the result from Proposition 2 with regard to sector k. When n^k was held fixed, the reduction in expenditures in sector k simply reduced sales that had been made at a price in excess of marginal cost, so deadweight loss rose. Now, with free entry and exit, we have exit in sector k, which involves a savings in fixed costs. Because firms (which are excessive in number with homogenous goods) that still operate now produce more output, production is overall more efficient in this sector. Note in particular that, the greater the excess of price over marginal cost (the larger is $p^k - dc^k/dx^k$), ceteris paribus, the greater was the excessive incentive for entry. Here, the excess of price over marginal cost does not translate into profits because they are fully dissipated by entry. Summing up the welfare effects associated with sector k in the case of homogeneous goods: the movement of expenditures from sector k to sector j leaves the representative individual indifferent as a consequence of the envelope condition, leaves firms in sector k indifferent because they earn zero profits regardless as a consequence of free entry, and avoids wasted resources associated with fixed costs in sector k as a consequence of induced exit. As per the preceding discussion, this latter effect, at the margin, is indicated by the excess of price over marginal cost in sector k because that profit margin is precisely what induces entry and is consumed, in terms of fixed costs, in the process.

One way to drive home the lesson from the homogeneous goods case is to state:

Corollary 3.1. *In the multi-sector model with free entry in all sectors, if the goods in each sector $i \neq j$ are homogeneous, the effect of strengthening competition policy (raising γ^j) in sector j on social welfare is given by*

$$\frac{dU}{d\gamma^j} = \sum_{i=1}^{I}\left(p^i - \frac{dc^i}{dx^i}\right)\lambda n^i \frac{dx^i}{d\gamma^j} + \left(\frac{X_2^j}{X_1^j} - \frac{x^j}{n^j}\right)\lambda p^j n^j \frac{dn^j}{d\gamma^j} \qquad (24)$$

Taking the concrete case in which goods in all sectors $i \neq j$ are substitutes for the goods in sector j and, moreover, have a cost function of the form in the preceding example, we have that all of the terms in the summation are positive, contributing to an increase in welfare. The fact that this expression is almost identical to expression (16) in Proposition 2 yet has almost opposite implications drives home the point that the pertinent derivatives must be interpreted with care. It can be very misleading to examine a world that ignores effects that arise in general equilibrium with free entry.

To gain further intuition about homogeneous goods in sectors $i \neq j$, one can rewrite the corresponding terms in the summation by substituting for $dx^i/d\gamma^j$ from a differentiated free-entry condition (22) to yield the following:

$$\frac{dU}{d\gamma^j} = \left(p^j - \frac{dc^j}{dx^j}\right)\lambda n^j \frac{dx^j}{d\gamma^j} + \left(\frac{X_2^j}{X_1^j} - \frac{x^j}{n^j}\right)\lambda p^j n^j \frac{dn^j}{d\gamma^j} + \sum_{i\neq j}\left(-\lambda x^i n^i \frac{dp^i}{d\gamma^j}\right) \qquad (25)$$

The first two terms, for the targeted sector j, correspond to the full welfare effect in the one-sector model with an outside good, except of course that here we instead have multi-sector general equilibrium derivatives.

The third term sums the effects associated with the homogeneous goods sectors $i \neq j$. Each $x^i n^i \, dp^i/d\gamma^j$ constitutes the fall in expenditures in such a sector specifically as a consequence of the general equilibrium reduction in p^i (that is, not including the more direct substitution effect of expenditures flowing out of this sector on account of the reduction in p^j). The preceding minus sign indicates a welfare gain, which λ converts from dollars to utils. The more direct effect from expenditures flowing out of each sector $i \neq j$ does not appear because it is associated with two equal and offsetting forces: a loss in sales that were at a price in excess of marginal cost (a welfare loss, via a reduction in profits) and the induced exit (saving the same amount of resources and hence an offsetting welfare gain). The remaining effect, measured by the third term, reflects that the general equilibrium reduction in price causes both a net null effect as producer surplus becomes consumer surplus and also additional induced exit that saves further resources (a social gain), which has a magnitude equal to the profit reduction from this price effect. As a consequence, the contrast with Proposition 2 is even starker. When $\boldsymbol{n}^{\setminus j}$ was held fixed, the outflow from sectors $i \neq j$ was an unmitigated welfare loss. With free entry and exit in the homogeneous goods case, not only is this loss fully offset but also there is a further general equilibrium effect on price that induces further exit, yielding an overall welfare gain rather than a welfare loss.

Having shown that the expenditure outflows from other sectors are beneficial for sectors in which goods are homogeneous, let us now consider the matter more broadly. To stake out another reference point along the continuum of possibilities, consider the case in which, in every sector $i \neq j$, the value of variety is such that the free-entry equilibrium number of firms just equals the socially optimal number of firms (in the sense discussed above, in connection with Corollary 1.2). In this instance, results analogous to those in Proposition 2 and Corollaries 2.1 and 2.2 are fully restored, even though we now allow the number of firms to be endogenous, that is, we have free entry and exit in all sectors.

Corollary 3.2. *In the multi-sector model with free entry in all sectors, when competition policy γ^j is such that the resulting number of firms n^i in each sector $i \neq j$ is socially optimal (in the free-entry equilibrium), the effect of strengthening competition policy (raising γ^j) in sector j on social welfare is given by*

$$\frac{dU}{d\gamma^j} = \sum_{i=1}^{I} \left(p^i - \frac{dc^i}{dx^i}\right) \lambda n^i \frac{dx^i}{d\gamma^j}\bigg|_{\boldsymbol{n}^{\setminus j}} + \left(\frac{X_2^j}{X_1^j} - \frac{x^j}{n^j}\right) \lambda p^j n^j \frac{dn^j}{d\gamma^j}\bigg|_{\boldsymbol{n}^{\setminus j}} \qquad (26)$$

even though the number of firms in sectors $i \neq j$, $\boldsymbol{n}^{\setminus j}$, is not held fixed.

The demonstration follows the logic of Corollaries 1.1 and 1.2, while using the method for Proposition 2 that is applicable to the multi-sector model. First, employ an entry subsidy σ^i set to keep n^i constant for all $i \neq j$. That yields Proposition 2 and its corollaries. Second, remove those subsidies. If the n^i in sectors $i \neq j$ were all socially optimal, then removal of the σ^i, which causes the respective n^i to fall, has no effect on welfare. Hence, the characterization for the welfare effect of the policy change is the same.

It is worth elaborating on the intuition for this special case. When we hold the n^i constant, we have the simple increase in deadweight loss as expenditures flow out of sectors $i \neq j$. Next, as with the homogeneous goods case, we have exit, which saves production costs and provides an offset to the reduction in deadweight loss. Finally, however, when variety is valuable, that production cost savings from exit comes at some expense to the utility from variety that the exiting firms had provided. In this special case in which the number of firms in each sector $i \neq j$ is optimal in the initial equilibrium, the latter two effects are equal (and opposite), so we are back to just the expression for the increase in deadweight loss (that is, for sectors whose goods are substitutes with those in the targeted sector j).[15]

[15] As explained previously, when the $n^{\setminus j}$ are held constant, this means (in our benchmark case with substitutes) that $dx^i/d\gamma^j\big|_{\boldsymbol{n}^{\setminus j}} < 0$ for $i \neq j$, so the outflow from each sector $i \neq j$ causes deadweight loss to rise.

With the results from the special case of variety being optimal in equilibrium and from the case of homogeneous goods in mind, we can see more generally how effects in each sector $i \neq j$ contribute to the overall impact on welfare of strengthening competition policy in sector j.[16] For any sector $i \neq j$ that is homogeneous, the outflow of expenditures (continuing to focus on the basic case of substitutes) raises welfare. As the value of variety increases from zero, at some point the outflow will have no net welfare consequences: the effects associated with the two terms for the sector in Proposition 3's (expression 23's) summation will be of equal magnitude but opposite sign. (Note that if this condition held in every sector $i \neq j$, the conventional approach that confines analysis to the targeted sector j would then be valid, as long as the appropriate general equilibrium derivatives were employed.) From that point, as the value of variety increases still further, the outflows will increasingly reduce welfare. For the particular value of variety such that the initial equilibrium has the socially optimal number of firms, the naïve expression given by ignoring entry and exit is precisely correct (as long as one interprets the $dx^i/d\gamma^j$ correctly). Past that point, the welfare loss from the outflow of expenditures is even greater than this.

Finally, note that the aggregate effects from sectors $i \neq j$ will, in general, depend on which sector j is targeted (which γ^j is being raised). To be sure, if all sectors were fully symmetric with each other, the results would be the same for assessing competition policy in any sector. However, such symmetry does not hold even approximately in actual economies. Furthermore, the first term in Proposition 3's expression (23) is weighted by $dx^i/d\gamma^j$ and the second term by $dn^i/d\gamma^j$, so the weight on each effect in each sector $i \neq j$ depends on which sector j is targeted. This point is also clear from the expression (20) for the α^i (adjusted for the present experiment). Relatedly, the sign of the effect of competition policy on welfare in the case examined in Corollary 2.2 makes clear how, in addition to the weights, it also matters which sector j is targeted because that determines which $\mathcal{L}^j$ is compared to the knock-out weighted average of the $\mathcal{L}^i$, for $i \neq j$ (which excludes $\mathcal{L}^j$ from the summation).

In summary, even though the expressions for deadweight loss and variety are similar across all of these cases that differ regarding what (if anything) is held constant, because the pertinent derivatives reflect qualitatively different exercises, the relative magnitudes differ and the signs of the terms (notably, the deadweight loss terms) can reverse. Just as Proposition 2 and its corollaries show how moving from one sector to many changes results dramatically, Proposition 3 shows that taking into account free entry and exit in these other sectors produces notably different outcomes even from that more encompassing multi-sector benchmark.

5. Production Efficiency

Competition policy often affects production efficiency, which is to say that it may alter firms' cost functions. Often this is posed as a tradeoff: stronger competition policy—perhaps restricting exclusive dealing, limiting cooperation through joint ventures, or prohibiting mergers—comes at the expense of productive efficiency. Further inquiry may be made regarding the extent to which efficiencies involve marginal (variable) costs versus fixed costs and, relatedly, the extent to which changes in costs are passed on to consumers. On the other hand, productive efficiency may be complementary to tougher competition policy, such as when

[16] The statements in the text are rough and intuitive, with implicit ceteris paribus restrictions.

competitive pressure spurs cost minimization and innovation. This section extends this article's model to incorporate these possibilities, although for ease of exposition the interpretation will discuss cases involving tradeoffs.

Two modifications are necessary. First, firms' cost functions will now be written as $\hat{c}^i(x^i, \gamma^i)$. That is, we will allow the strength of competition policy in any sector i to shift in an arbitrary (but identical) manner the cost functions of all firms in sector i (but not the cost functions of firms in other sectors). The analysis and discussion that follows will highlight when and how it matters whether changes involve marginal costs, fixed costs (really, any inframarginal costs), or both.

Second, the competitive interaction function in expression (4) will now be written as

$$p^i = \hat{f}^i(n^i, \gamma^i; \boldsymbol{p}^{\setminus i}, \boldsymbol{n}^{\setminus i}) \qquad (27)$$

Expression (4) in the original model, in showing how competitive interactions in sector i (along with other factors) determine price, took the cost function of firms in sector i as given. Because we are now contemplating that changing competition policy may change firms' cost functions, we will indicate that there is now a different function, $\hat{f}^i$, mapping competition policy and the number of firms in sector i to that sector's price, p^i. For example, if stronger competition policy raises marginal cost, we would usually suppose that a higher price will result than otherwise, ceteris paribus. Accordingly, we do not assume here that $\hat{f}_2^i < 0$: if increasing γ^i raises costs enough and in an appropriate way, the direct effect may be to raise p^i rather than to lower it.

Much of the additional insight from incorporating efficiencies can be gleaned from the one-sector model with an outside good. The analysis to follow will accordingly use the modifications introduced in section 3. To keep clear that efficiencies are now taken into account in the cost and price functions when conducting our policy experiments, it is useful to restate expression (5) for the representative individual's utility using the notation

$$\hat{U} \equiv u(X^1, \ldots, X^I) + \lambda z \qquad (28)$$

The condition for free entry in sector j, previously expression (14), is now

$$\hat{\pi}^j = p^j x^j - \hat{c}^j\left(x^j, \gamma^j\right) = 0 \qquad (29)$$

If one repeats section 3's analysis leading to Proposition 1, mutatis mutandis, the result is:

Proposition 4. *When production efficiency effects are introduced in the one-sector model with an outside good (in which prices, quantities, and the number of firms in all other sectors do not change), the effect of strengthening competition policy (raising γ^j) on social welfare is given by*

$$\left.\frac{d\hat{U}}{d\gamma^j}\right|_{\setminus j} = \left(p^j - \hat{c}_1^j\right)\lambda n^j \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} + \left(\frac{X_2^j}{X_1^j} - \frac{x^j}{n^j}\right)\lambda p^j n^j \left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j} - \lambda n^j \hat{c}_2^j \qquad (30)$$

Here, subscripts of the cost function are used to denote derivatives with respect to its two arguments.

The first two terms in expression (30) are the same as the (only) two terms in expression (8) in Proposition 1. Their interpretations, however, may differ qualitatively. Before, raising γ^j unambiguously reduced p^j, which increased x^j, thereby reducing deadweight loss in sector j. Now, by contrast, the opposite is possible: if $\hat{c}^j$ rises in a manner that, despite the tougher competition policy, leads to a higher rather than a lower p^j, x^j falls rather than rises. Here, effects on marginal cost may typically be most pertinent. In addition, raising costs, whether marginal or fixed, tends to induce exit, a fall in n^j—beyond that induced by tougher competition policy pushing down price (which, as noted, it may no longer do in any event)—which itself causes p^j to rise. However, as discussed in section 3, any reduction in n^j itself tends to increase x^j due to the reallocation of at least some purchases from exiting firms to those that remain (keeping in mind that x^j is per-firm output in sector j).

The third term is new. It indicates how the increase in each firm's costs, $\hat{c}_2^j$, raises the total costs of the n^j firms and thus reduces welfare. Note that the pertinent cost derivative, $\hat{c}_2^j$, denotes the rise in the total costs of producing x^j, without any distinction between marginal costs and fixed costs. In this regard, recall that the experiment affects the free-entry equilibrium in which firms' profits are zero; hence, all production costs are borne by the representative individual. With free entry, revenue per firm $p^j x^j$ equals total costs per firm $\hat{c}^j(x^j, \gamma^j)$, so the equilibrium price p^j equals average cost, $\hat{c}^j / x^j$. In this sense, all costs are fully passed on.

Similar analysis allows us to restate Corollary 1.1, which addresses the case in which an entry subsidy is used to hold n^j fixed:

Corollary 4.1. *When production efficiency effects are introduced in the one-sector model with an outside good (in which prices, quantities, and the number of firms in all other sectors do not change), and when there is also a subsidy on entry set to keep the number of firms n^j constant, the effect of strengthening competition policy (raising γ^j) on social welfare is given by*

$$\left.\frac{d\hat{U}}{d\gamma^j}\right|_{n^j,\backslash j} = \left(p^j - \hat{c}_1^j\right)\lambda n^j \left.\frac{dx^j}{d\gamma^j}\right|_{n^j,\backslash j} - \lambda n^j \hat{c}_2^j \qquad (31)$$

Regarding the first term in expression (31), even holding n^j fixed rather than allowing it to fall (as a consequence of tougher competition policy and also due to higher costs inducing exit), we still have the possibility that p^j rises rather than falls, so that x^j may fall rather than rise. Moreover, even when p^j falls, so that x^j rises, it is still possible that welfare falls on account of the second term. Recall as well that this term measures the rise in total costs, which may primarily involve fixed costs.

Finally, we can state the analogue to section 3's Corollary 1.2, for when the free-entry equilibrium involves the optimal number of firms in sector j (rather than using the entry subsidy to hold n^j fixed).

Corollary 4.2. *When production efficiency effects are introduced in the one-sector model with an outside good (in which prices, quantities, and the number of firms in all other sectors do not change), and when competition policy γ^j is such that the resulting number of firms, n^j, is socially optimal (in the free-entry equilibrium), the effect of strengthening competition policy (raising γ^j) on social welfare is given by*

$$\left.\frac{d\hat{U}}{d\gamma^j}\right|_{\setminus j} = \left(p^j - \hat{c}_1^j\right)\lambda n^j \left.\frac{dx^j}{d\gamma^j}\right|_{n^j,\setminus j} - \lambda n^j \hat{c}_2^j \qquad (32)$$

even though the number of firms in sector j, n^j, is not held fixed.

Consider the implications for marginal versus fixed cost reductions of the earlier lesson (drawn from Corollary 1.2) that it is better for social welfare to reduce n^j through tougher competition policy than through the planner doing so directly (via an entry tax). Specifically, compare a competition-policy-induced increase in marginal costs and a competition-policy-induced increase in fixed costs where each involves an identical increase in total costs, $\hat{c}^j(x^j, \gamma^j)$, at the initial x^j; that is, the final term in Proposition 4's expression (30) (and in the associated corollaries), which contains $\hat{c}_2^j$, has the same value. Regarding the first term in expression (30), supposing as is typical that marginal costs are passed on to some degree (and more so than fixed costs), the price p^j will fall less when it is marginal costs that rise, so x^j will rise less and, accordingly, deadweight loss will fall less—which captures the standard intuition. However, Proposition 4 does not hold n^j fixed, and under the stated conditions n^j will be higher (it will not fall as much), contributing to variety, creating the usual tradeoff. Under the conditions of Corollary 4.2, the first term remains positive (precisely the analogue to Corollary 1.2) as long as the increase in marginal cost does not reverse the sign of the price effect. More broadly, and quite intuitively, we can see that the greater concern for changes in marginal rather than fixed costs—which we have seen has no direct effect when the change in total costs is the same, due to free entry, as indicated by the last term—is particularly apt when products are homogeneous so that variety has no value. The overall welfare cost of higher marginal rather than fixed costs decreases as variety becomes more valuable.[17]

To conclude our treatment of efficiencies, return briefly to the multi-sector version of the model (with no outside good). Section 4's presentation of this case will not be restated because the analysis and results are each modified in an analogous manner. Because competition policy in sector j is taken to influence only the cost functions of firms in sector j, only the sector j components of each of the conditions change directly. Note, however, that because the sign of the sector j effects may reverse, in which case expenditures may instead flow out of sector j and into sectors $i \neq j$, the sign of all of those other terms may change as well.

To take one simple illustration of how results may differ even when efficiencies do not reverse the sign of the price effect in sector j, reconsider Corollaries 2.1 and 2.2, involving the case in which entry subsidies are set to keep constant the number of firms in all sectors (including sector j). Expression (19) indicates that raising γ^j raises welfare if and only if sector j's Lerner index, $\mathcal{L}^j$, exceeds a knock-out weighted average of the Lerner index in all other sectors. If, however, we suppose that raising γ^j raises costs, we have—in addition to dampening the reduction in p^j and hence the extent of the reallocation of expenditures across sectors, which expression (19) assesses—the increase in total production costs in sector j. Consider as well that

[17] It follows from the discussion in the text that— because the difference between marginal and fixed cost changes that alter total costs by the same amount is confined to the price effect—one could to that extent simply reinterpret what it means to move γ^j by a stated amount or, equivalently, to view the pass-through of changes in marginal costs as tantamount to a correspondingly smaller (or larger) "real" change in γ^j.

the cost increase may involve fixed as well as variable costs. If all of the cost increase involved fixed costs, we might have roughly the same cross-sector expenditure flows as before except that, now, in offsetting the rise in deadweight loss due to the outflows from sectors $i \neq j$, the inflows to sector j would not only reduce deadweight loss in that sector through the rise in x^j but also would expend resources to cover sector j firms' higher fixed costs. Indeed, under Corollary 2.3's assumptions that $\mathcal{L}^{\mathrm{i}} = \mathcal{L}$ for all i and that the number of firms in all sectors is held constant, the only effect of strengthening competition policy on welfare is the increase in sector j firms' costs, be they fixed or variable. The same result would hold if the number of firms was endogenous but variety was optimal in all sectors.

Finally, keep in mind that restating the cost function as $\hat{c}^i(x^i, \gamma^i)$ encompasses the possibility that more intense competition improves production efficiency rather than reducing it. That is, we may have $\hat{c}_2^j < 0$. Clearly, this modification would reverse each of the foregoing statements regarding the welfare consequences of competition policy's effects on production efficiency, including the prospect that lower fixed costs would produce utility gains to the representative individual.

6. Conclusion

Analysis of a simple, multi-sector, general equilibrium model with free entry substantially reorients our understanding of the welfare effects of competition policy. Some factors vanish or are relevant in qualitatively different ways while other channels—some of which are of the same order of magnitude but of opposite sign—are activated. Moreover, the results differ greatly across models with only one sector, with multiple sectors but with the number of firms held fixed, and with multiple sectors and free entry. Familiar shortcuts not only omit factors (which may be small in most of the other sectors but are large in aggregate) but can produce highly misleading prescriptions, such as when results regarding the welfare impacts associated with nontargeted sectors reverse sign once entry and exit are taken into account. The effect of cost efficiencies on individuals' welfare also changes qualitatively. These phenomena are importantly explained by free entry and exit, the utility-maximization envelope condition, and pecuniary externalities of first-order importance that are due to markups in multiple sectors as well as firms' entry and exit decisions not being socially optimal—which, taken together, modify or nullify some welfare effects and generate others.

The model employed here is indeed simple, notably in imposing symmetry across firms within each sector, although the representation of individuals' preferences (demand), firms' costs, competitive interactions, and competition policy were general. A natural next step would be to substitute a more particular model with heterogeneous firms in the sector being targeted by competition policy, and for some sectors and interventions to employ a dynamic model in which firm heterogeneity arises endogenously. Although these and other extensions are necessary for many applications, they would be unlikely to extinguish the key forces identified here.

A different sort of extension would examine spillovers more broadly. In the present model, because variety contributes to utility but the inframarginal benefit is not appropriated by firms, there is a positive externality to entry (in addition to the negative, business-stealing externality). This might be interpreted as a stand-in for other sources of spillovers, such as when R&D or learning by doing benefits other firms in a sector. One could also contemplate negative externalities, such as involved with rent-seeking.

To connect research to policy, we would like to identify sufficient statistics for welfare analysis. Moreover, because it is impractical for a competition agency, especially in analyzing a specific case, to perform a multi-sector general equilibrium analysis, it would be particularly useful to develop simplified rules, proxies, and other shortcuts to facilitate practical application. Perhaps empirical investigation can suggest the approximate magnitude of average cross-market effects, and that estimate might be used as a surrogate for a more complete analysis. In addition, when particular sectors might be more closely related (as substitutes or complements) to the targeted sector, an appropriately reweighted average of other sectors' effects might be employed.

Even though most rules and case-specific analyses will be circumscribed, we would like to know—or at least have a rough sense of—the full impact of competition policy across the economy. Cross-sector effects with concomitant impacts on entry and exit are central features of how an economy operates. The present analysis indicates that these phenomena may substantially influence overall welfare assessments, including by reversing their sign. Ignoring these forces is all the more difficult to justify in light of recent evidence on the existence of significant market power in many sectors of the economy. In any event, the primary aim of the present analysis is conceptual: to identify and understand the welfare effects of competition policy in general equilibrium with free entry.

Appendix

Proof of Proposition 1:

To analyze the derivative in expression (7), we will perform some manipulations. First, differentiate the budget constraint (6):

$$\left.\frac{dp^j}{d\gamma^j}\right|_{\setminus j} x^j n^j + \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} p^j n^j + \left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j} p^j x^j + \left.\frac{dz}{d\gamma^j}\right|_{\setminus j} = 0 \qquad (A1)$$

Using this equation to substitute for $dz/d\gamma^j|_{\setminus j}$ in expression (7) yields

$$\left.\frac{d\tilde{U}}{d\gamma^j}\right|_{\setminus j} = u_j\left(X_1^j \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} + X_2^j \left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j}\right) - \left.\frac{dp^j}{d\gamma^j}\right|_{\setminus j} \lambda x^j n^j - \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} \lambda p^j n^j - \left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j} \lambda p^j x^j \qquad (A2)$$

The condition for free entry in sector j is

$$\pi^j = p^j x^j - c^j(x^j) = 0 \qquad (A3)$$

If we differentiate this expression and then (for convenience in the next step) multiply through by λn^j, we have:

$$\left.\frac{dp^j}{d\gamma^j}\right|_{\setminus j} \lambda x^j n^j + \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} \lambda p^j n^j - \frac{dc^j}{dx^j}\left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} \lambda n^j = 0 \qquad (A4)$$

We can add the left side of expression ($A4$) (which equals zero) to the right side of expression ($A2$):

$$\left.\frac{d\tilde{U}}{d\gamma^j}\right|_{\setminus j} = u_j\left(X_1^j \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} + X_2^j \left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j}\right) - \left.\frac{dp^j}{d\gamma^j}\right|_{\setminus j} \lambda x^j n^j - \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} \lambda p^j n^j - \left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j} \lambda p^j x^j$$

$$+\left.\frac{dp^j}{d\gamma^j}\right|_{\setminus j} \lambda x^j n^j + \left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} \lambda p^j n^j - \frac{dc^j}{dx^j}\left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} \lambda n^j \qquad (A5)$$

Simplifying (the terms that cancel represent changes in payments from the individual to firms; i.e., from consumers to producers) and then grouping terms accordingly yields

$$\left.\frac{d\tilde{U}}{d\gamma^j}\right|_{\setminus j} = \left(u_j X_1^j - \frac{dc^j}{dx^j}\lambda n^j\right)\left.\frac{dx^j}{d\gamma^j}\right|_{\setminus j} + \left(u_j X_2^j - \lambda p^j x^j\right)\left.\frac{dn^j}{d\gamma^j}\right|_{\setminus j} \qquad (A6)$$

Finally, using the representative individual's first-order condition for the choice of x^j, which is that $u_j X_1^j = \lambda p^j n^j$, we can substitute in the preceding expression and rearrange terms slightly to produce expression (8) in Proposition 1.■

Proof of Corollary 1.1:

With the posited entry subsidy, firms' free-entry condition in sector j (14) becomes

$$p^j x^j - c^j\left(x^j\right) + \sigma^j = 0 \qquad (A7)$$

Differentiating this expression and isolating the effect on σ^j yields

$$\left.\frac{d\sigma^j}{d\gamma^j}\right|_{n^j,\backslash j} = p^j \left.\frac{dx^j}{d\gamma^j}\right|_{n^j,\backslash j} + x^j \left.\frac{dp^j}{d\gamma^j}\right|_{n^j,\backslash j} - \frac{dc^j}{dx^j}\left.\frac{dx^j}{d\gamma^j}\right|_{n^j,\backslash j} \qquad (A8)$$

where the notation $|_{n^j,\backslash j}$ indicates that the pertinent derivatives hold the number of firms in sector j constant (in addition to prices, quantities, and the number of firms in other sectors not changing, as before). Individuals' budget constraint for this experiment is

$$\sum_{i=1}^{I} p^i x^i n^i + z = y - \sigma^j n^j \qquad (A9)$$

reflecting that the representative individual must fund the subsidy σ^j for each of the n^j firms.[18] If one now parallels the derivation used for Proposition 1—taking the pertinent derivative of the representative individual's utility for this policy experiment, differentiating this budget constraint, making appropriate substitutions including through use of the first-order condition for x^j, and rearranging terms—the result is expression (10) in Corollary 1.1.■

Proof of Proposition 2:

It is convenient to use the entry condition (13) in sectors $i \neq j$ to substitute for σ^i in the budget constraint (15), to yield

$$\sum_{i=1}^{I} p^i x^i n^i = y + \sum_{i \neq j}\left(p^i x^i - c^i\left(x^i\right)\right) n^i \qquad (A10)$$

which can be rewritten as

$$p^j x^j n^j = y - \sum_{i \neq j} c^i\left(x^i\right) n^i \qquad (A11)$$

[18] See the discussion in note 11.

Further substitution for $p^j x^j$ using the free-entry condition in sector j (14) yields

$$\sum_{i=1}^{I} c^i(x^i) n^i = y \qquad (A12)$$

(Note that this expression also states the resource constraint for this economy.[19]) We can now evaluate the policy experiment of raising γ^j, holding $\boldsymbol{n}^{\setminus j}$ fixed:

$$\left.\frac{dU}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} = \sum_{i=1}^{I} u_i X_1^i \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} + u_j X_2^j \left.\frac{dn^j}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} \qquad (A13)$$

Next, differentiate our revised budget constraint $(A12)$:

$$\sum_{i=1}^{I} \frac{dc^i}{dx^i} \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} n^i + c^j(x^j) \left.\frac{dn^j}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} = 0 \qquad (A14)$$

On the left side, we can substitute $p^j x^j$ for $c^j(x^j)$ using the free-entry condition for sector j (14), multiply both sides by λ (the marginal utility of income), and then subtract the left side of this expression from the right side of expression $(A13)$ for the welfare effect. This yields

$$\left.\frac{dU}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} = \sum_{i=1}^{I} \left(u_i X_1^i - \frac{dc^i}{dx^i} \lambda n^i\right) \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} + \left(u_j X_2^j - \lambda p^j x^j\right) \left.\frac{dn^j}{d\gamma^j}\right|_{\boldsymbol{n}^{\setminus j}} \qquad (A15)$$

Now, make use of the representative individual's first-order condition, $u_i X_1^i = \lambda p^i n^i$, to substitute for the first component in the first term's parentheses, and, in the second term's parentheses, both multiply and divide each component by the appropriate side of this first-order condition (for sector j). This produces expression (16) in Proposition 2.■

Proof of Corollary 2.2.

In Corollary 2.1's expression (17) for welfare effects, multiply and divide by p^i and substitute as follows:

$$\left.\frac{dU}{d\gamma^j}\right|_{\boldsymbol{n}} = \sum_{i=1}^{I} \mathcal{L}^i \lambda p^i n^i \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}} \qquad (A16)$$

[19] Hence, what follows is the equivalent to forming the Lagrangian for the planner's problem of setting γ^j to maximize U subject to the economy's resource constraint and the representative individual selecting the x^i to maximize utility.

Next, we can use the ratios defined in expression (20) to rewrite this expression as

$$\left.\frac{dU}{d\gamma^j}\right|_{\boldsymbol{n}} = -\lambda p^j n^j \left.\frac{dx^j}{d\gamma^j}\right|_{\boldsymbol{n}} \sum_{i=1}^{I} \alpha^i \mathcal{L}^i \qquad (A17)$$

The sign of the aggregate welfare effect is[20]

$$sign\left(\left.\frac{dU}{d\gamma^j}\right|_{\boldsymbol{n}}\right) = sign\left(-\sum_{i=1}^{I} \alpha^i \mathcal{L}^i\right) = sign\left(\mathcal{L}^j - \sum_{i\neq j} \alpha^i \mathcal{L}^i\right) \qquad (A18)$$

which establishes Corollary 2.2.■

Proof of Corollary 2.3:

To prove the requisite property of the weights, we can proceed as follows. The budget constraint for this setting, after substituting for the entry condition with the subsidy, is the same as that given in expression $(A12)$ above. Differentiating yields

$$\sum_{i=1}^{I} \frac{dc^i}{dx^i} \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}} n^i = 0 \qquad (A19)$$

Next, we can add and subtract $p^i n^i \, dx^i/d\gamma^j\big|_{\boldsymbol{n}}$ to each term in this summation and group the terms as follows:

$$\sum_{i=1}^{I} \left(p^i + \left(-p^i + \frac{dc^i}{dx^i}\right)\right) n^i \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}} = 0 \qquad (A20)$$

Note that the term in the inner parentheses equals $-p^i \mathcal{L}^i$. Making that substitution and factoring out p^i yields

$$\sum_{i=1}^{I} \left(1 - \mathcal{L}^i\right) p^i n^i \left.\frac{dx^i}{d\gamma^j}\right|_{\boldsymbol{n}} = 0 \qquad (A21)$$

If we now take the case in which $\mathcal{L}^i = \mathcal{L}$ for all i, we can divide both sides by $1 - \mathcal{L}$; furthermore, we can divide both sides by $p^j n^j dx^j/d\gamma^j\big|_{\boldsymbol{n}}$ and substitute using our expression (20) for α^i to produce

[20] The analysis assumes that $dx^j/d\gamma^j\big|_{\boldsymbol{n}} > 0$ (i.e., that x^j is not a Giffen good), as stipulated in section 2.

$$\sum_{i=1}^{I}\left(-\alpha^i\right) = 0 \qquad (A22)$$

Using the fact that $\alpha^j = -1$, this demonstrates that $\sum_{i\neq j}\alpha^i = 1$, which establishes Corollary 2.3.■

Proof of Proposition 3:

Using the free-entry condition (22) to substitute into the budget constraint (21) produces the same result as before (expression $A12$):

$$\sum_{i=1}^{I} c^i\left(x^i\right)n^i = y \qquad (A23)$$

We can now evaluate the policy experiment of raising γ^j in this setting.

$$\frac{dU}{d\gamma^j} = \sum_{i=1}^{I} u_i\left[X_1^i\frac{dx^i}{d\gamma^j} + X_2^i\frac{dn^i}{d\gamma^j}\right] \qquad (A24)$$

Next, differentiate our revised budget constraint ($A23$):

$$\sum_{i=1}^{I}\left(\frac{dc^i}{dx^i}\frac{dx^i}{d\gamma^j}n^i + c^i\left(x^i\right)\frac{dn^i}{d\gamma^j}\right) = 0 \qquad (A25)$$

On the left side, we can substitute $p^i x^i$ for $c^i\left(x^i\right)$ using the free-entry condition (22) for sector i, multiply both sides by λ, and then subtract the left side of this expression from the right side of expression ($A24$). This yields

$$\frac{dU}{d\gamma^j} = \sum_{i=1}^{I}\left[\left(u_iX_1^i - \frac{dc^i}{dx^i}\lambda n^i\right)\frac{dx^i}{d\gamma^j} + \left(u_iX_2^i - \lambda p^i x^i\right)\frac{dn^i}{d\gamma^j}\right] \qquad (A26)$$

Now, we can again make use of the representative individual's first-order condition, $u_iX_1^i = \lambda p^i n^i$, to substitute for the first component in the first term's parentheses, and, in the second term's parentheses we can both multiply and divide each component by the appropriate side of this first-order condition. This yields expression (23) of Proposition 3.■

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