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REPARATIONS AND PERSISTENT RACIAL WEALTH GAPS

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ABSTRACT

We present a first formal, dynamic, economic analysis of reparations using a long-run model of heterogeneous dynasties with an occupational choice and bequests. Our innovation is to introduce endogenous dispersion of beliefs about risky returns, reflecting differences in dynasties' investment experiences over time. Feeding the exclusion of Black dynasties from labor and capital markets as driving force, the model quantitatively reproduces current and historical racial gaps in wealth, income, entrepreneurship, mobility, and beliefs about risky returns. We evaluate whether different forms of reparations compensate for historical exclusions, which means restoring the long-run outcomes we would have observed in the absence of exclusions. We find that direct transfers that eliminate the racial gap in average wealth today do not lead to long-run wealth convergence, which is the long-run outcome without exclusions. The logic is that century-long exclusions lead Black dynasties to enter into reparations with pessimistic beliefs about risky returns and to forego investment opportunities. We show that investment subsidies are more effective than wealth transfers in eliminating the racial wealth gap.

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1 Introduction

In April 2019, Senator Booker introduced a bill to "address the fundamental injustice, cruelty, brutality, and inhumanity of slavery in the United States and the 13 American colonies between 1619 and 1865 and to establish a commission to study and consider a national apology and proposal for reparations for the institution of slavery..." While most policymakers have not explicitly endorsed wealth transfers to descendants of slaves yet, prominent cosponsors of the bill expressed their support to study reparations and make policy recommendations.¹

In this paper, we provide a first formal, dynamic, economic analysis of reparations. We begin by clarifying how to evaluate reparation policies. Our framework subscribes to the logic underlying reparations that persistent racial wealth gaps do not reflect innate differences in ability, preferences, or beliefs but, instead, emerge from century-long exclusions of Black dynasties from labor and capital markets. Our criterion when evaluating reparations is that they compensate appropriately for historical exclusions only if they restore the outcomes we would have observed in a world without these exclusions. Reparation policies today that do not eliminate the racial wealth gap in the future do not compensate appropriately because, in the absence of exclusions, outcomes for Black and White dynasties are identical. Using this criterion, we ask: Will reparations today in the form of direct wealth transfers eliminate racial wealth gaps in the future? If not, is there a policy that is effective in eliminating racial gaps?

We answer these questions in three steps. We begin by developing a long-run equilibrium model with heterogeneous dynasties to quantify the sources of racial gaps in wealth, income, entrepreneurship, and mobility. The model shares two features with the wealth inequality literature (that we discuss further below). Motivated by the role of intergenerational transfers for persistence in wealth gaps, dynasties in our model choose how to allocate their resources between consumption for the current generation and wealth transmitted to descendants. Motivated by the observation that mostly entrepreneurs occupy the top of the wealth distribution, dynasties choose how to allocate their lifetime between labor and risky investment activities.

¹The bill is available at https://bit.ly/3oyNG9p. Prominent cosponsors include Vice President Harris and Senators Klobuchar, Sanders, and Warren.

The innovation of our framework relative to the wealth inequality literature is that it generates endogenously divergence of beliefs about risky investment returns. Entrepreneurship uses time and capital as inputs and produces uncertain output. The true return from investment is unknown and each generation begins with a prior belief over the objective probability that investment activities are successful. Dynasties who become capitalists observe their investment outcome, update their beliefs, and transmit them to the next generation. Successful capitalists transmit more optimistic beliefs to their descendants, unsuccessful capitalists transmit more pessimistic beliefs to their descendants, while laborers do not update their beliefs lacking own investment experiences. As a result, our model generates dispersion of expected risky returns, reflecting differences in the accumulation of investment experiences from previous generations.²

We feed as driving forces to the model historical labor and capital exclusions which prohibit Black dynasties from participating in markets. By labor market exclusions, we mean both slavery taking place in our model between the Declaration of Independence in 1776 and the 13th Amendment in 1865 and lower wages for Black dynasties until today. By capital market exclusions, we mean historical events such as discrimination in patenting, redlining, Jim Crow segregation laws, and exclusion from credit markets. These policies exclude Black dynasties from becoming capitalists in our model until the Civil Rights Movement in the 1960s.

In the second step, we parameterize the model to evaluate its ability to account quantitatively for salient features of the data. The model is successful in accounting for the significant dispersion of wealth and income observed in the data, both for the total population and for the population of entrepreneurs. While not targeted by our parameterization, the model matches the racial wealth gap today and its evolution since the early 1900s. In addition, the model is consistent with observed mobility patterns, where White dynasties are more likely than Black dynasties to see their children exceed their rank in the income distribution. Similar to the data, the mobility gap in the model is more profound in the early 1900s than in recent times.

²An example we use to illustrate model mechanisms is the Rockefeller dynasty. When asked how they manage to preserve wealth over centuries, David Rockefeller Jr., chairman of Rockefeller & Co., stated (https://cnb.cx/2YwePiE) that the family has developed a system of values, traditions and institutions that have helped the family stay together and preserve their wealth. The family meets twice per year in a forum where heirs talk about the family's direction, projects, and other news related to careers or important milestones.

The model generates significant racial wealth gaps because Black dynasties earn lower wages than White dynasties which, in turn, leads to racial differences in holdings of both safe and risky assets. When White dynasties have positive investment experiences, they update upward their beliefs about risky returns and accumulate wealth over time. Black dynasties initially faced slavery and later face lower wages. They do not become capitalists, which means that they do not update their beliefs and do not accumulate as much wealth as White dynasties. To corroborate this mechanism, we show that the model is consistent with the observed contribution of racial gaps in holdings of risky assets to the total wealth gap in the Survey of Consumer Finances (SCF), when risky assets include investments such as public equity, own business assets, and real estate.

Confronted with historical labor and capital market exclusions, the model generates racial gaps without imposing differences in preferences, initial beliefs, or initial wealth. We highlight the importance of general equilibrium for the divergence of wealth. Early on, when Black dynasties are enslaved, investment returns are high because assets are relatively unexploited (such as land). This makes risky investments worthwhile for White dynasties even if, initially, their beliefs are pessimistic and their wealth is low. As wealth in the economy accumulates, returns fall over time which dissuades Black dynasties from becoming capitalists even after emancipation.³

We evaluate whether model predictions concerning beliefs about risky returns align with available data. We first confirm the known observation that Black households are less likely to be entrepreneurs than White households. Using Michigan Survey data asking respondents their probability assessment over whether a diversified equity fund would increase in value, we present new evidence that Black households are more pessimistic than White households about risky returns. Despite not targeted by the parameterization, the model generates a racial belief gap and dispersion of beliefs similar to the data.

Armed with a model consistent with salient observations on wealth, income, entrepreneurship, mobility, and beliefs, we proceed to assess reparations. We evaluate reparation policies in terms of whether they achieve equal representation of Black dynasties in wealth, which is the long-run outcome we would observe in the absence of historical exclusions. We are interested in the effects of

³Consistent with our mechanism, Kuvshinov and Zimmermann (2021) document a decline in U.S. expected risky returns since 1890 and Schmelzing (2020) documents a decline in long-term U.S. yields since the late 18th century.

reparations on wealth because, in our model, more than half of the racial welfare gap is accounted for by differences in wealth as opposed to differences in wages. To separate the wealth effects of reparations from the wealth effects of different wages, we assume that labor market policies are enacted and permanently close the racial wage gap at the same time when reparations are given.

Our first result is that, with transfers that eliminate the racial gap in average wealth today, average wealth of Black and White dynasties diverges again in the future, Black dynasties are strongly underrepresented at the top of the wealth distribution, and the racial welfare gap persists.⁴ Historical labor and capital market exclusions lead Black dynasties to enter into the reparations era with more pessimistic beliefs about risky returns than White dynasties. Since this era is also characterized by a relatively low return to wealth accumulation, most Black dynasties forego investment opportunities despite increased wealth.

Racial outcomes differ in the long run, even with larger transfers that make the average Black dynasty significantly wealthier than the average White dynasty today. Wealth transfers are not powerful in changing the trade-off between labor and capital activities. A policy that targets directly this trade-off is investment subsidies, which are more effective than wealth transfers in compensating Black dynasties for historical exclusions. A subsidy equal to 27 percentage points of additional return, financed with taxation of White dynasties' wealth at 100 percent above roughly 17 million dollars, eliminates racial gaps in the long run. Another possibility for closing racial gaps is that, after reparations, Black capitalists update their beliefs by learning from others' experiences. This possibility, however, raises the question of why learning from others' experiences did not occur earlier in history, thus leading to today's gap in wealth and entrepreneurship.

This paper contributes to three literatures. Early work by Blau and Graham (1990) concludes that racial differences in intergenerational transfers account for most of the racial wealth gap.⁵

⁴We calculate transfers to Black dynasties that total 10 trillion dollars and consider either progressive or proportional wealth taxes on White dynasties that finance these transfers. Wealth transfers is the most commonly discussed reparation policy (Darity and Mullen, 2020). The long-run equalization of wealth is viewed as a goal from proponents of reparations. For example, Darity and Mullen in their discussion of reparations (https://brook.gs/3j1soQs) argue that "The wealth gap will not persist if the target of well-executed reparations is direct elimination of it."

⁵Quantitative work on wealth inequality, such as De Nardi (2004), highlights the role of bequests and non-homothetic preferences for the emergence of large estates and the transmission of wealth across generations. Non-homothetic preferences allow models to account for the observation that households with higher lifetime income (Dynan, Skinner, and Zeldes, 2004; Straub, 2019) or higher wealth (Fagereng, Holm, Moll, and Natvik, 2019) exhibit higher saving rates.

Quadrini (2000) and Cagetti and De Nardi (2006) demonstrate the importance of entrepreneurship for wealth inequality, as entrepreneurs occupy most of the top of the wealth distribution. Benhabib, Bisin, and Zhu (2011) stress the role of capital income risk for the upper tail of the wealth distribution, while Benhabib, Bisin, and Luo (2019) show that accounting for both inequality and mobility requires a combination of stochastic earnings, heterogeneity in saving rates, and capital income risk. Gabaix, Lasry, Lions, and Moll (2016) show the importance of correlated returns with wealth for the fast transitions of tail inequality in the data. Our innovation relative to the wealth inequality literature is to introduce dispersion of expected returns. Different from models which treat entrepreneurial productivity as an exogenous process, differences in expected returns emerge endogenously in our model from accumulated investment experiences.⁶

A natural prediction of models with occupational choice is that wealth transfers toward poor lead to a rise in recipients' entrepreneurship rates and a convergence of wealth between recipients and non-recipients. However, Bleakley and Ferrie (2016) present historical evidence from a large wealth redistribution program, Georgia's Cherokee Land Lottery in 1832, that descendants of families who received wealth transfers did not experience higher education, income, and wealth than descendants of non-recipients. Bleakley and Ferrie (2016) conclude that financial resources play a limited role in intergenerational outcomes as opposed to other factors which may persist through family lines. This other factor in our model is beliefs about risky investment returns. If beliefs were homogeneous, a one-time wealth transfer would perfectly eliminate the racial wealth gap forever. Owing to the more pessimistic beliefs of Black dynasties at the time of reparations, our model instead predicts divergence of wealth after transfers.

The second related literature concerns social capital and the transmission of culture. Fogli and Veldkamp (2011) study the transition of women into the labor force in a model of learning by sampling from a small number of other women. As information about the effects of maternal

⁶Consistent with our model that generates a positive correlation between wealth and expected returns, Bach, Calvet, and Sodini (2020) use asset pricing models to document that households with higher wealth exhibit higher expected returns. While a novel part of our model is heterogeneity in expected returns, the model is also consistent with "scale dependence" as emphasized by Gabaix, Lasry, Lions, and Moll (2016) because wealthier dynasties are more likely to be capitalists and realize higher expost returns. Fagereng, Guiso, Malacrino, and Pistaferri (2020) document a positive correlation of expost returns with wealth and, consistent with our model, persistence of returns across generations.

employment on children accumulates, the effects of maternal employment become less uncertain and more women enter into the labor force. Fernandez (2013) demonstrates the role of cultural transmission of beliefs about wages for women's rising labor force participation. The transmission of beliefs across generations reflects both parental beliefs and a noisy observation of aggregate labor force participation. Buera, Monge-Naranjo, and Primiceri (2011) develop and estimate a model in which a country's own and its neighbors' past experiences shape policymakers' beliefs about the desirability of free market policies.

Our baseline learning mechanism differs from these papers because dynasties learn about risky returns only based on their own experiences. Earlier work such as Piketty (1995) highlights the role of learning from own mobility experiences for voters' attitudes on redistribution. Similar to the model of Piketty (1995) in which experimenting is unattractive and the cultural transmission model of Guiso, Sapienza, and Zingales (2008) in which learning occurs only upon participation, in our model heterogeneity in beliefs is persistent as some dynasties do not enter into entrepreneurship. We extend the baseline model to discuss learning from others' experiences and show that strengthened networks close racial gaps.

Finally, our work contributes to the racial gaps literature. Darity and Frank (2003) narrate century-long exclusions of Black dynasties from labor and capital markets and offer proposals for the implementation of reparations.⁸ Aliprantis, Carroll, and Young (2019) and Ashman and Neumuller (2020) use quantitative models to show how racial income gaps generate racial wealth gaps. Given differences in labor earnings, these models generate divergence of wealth after one-time transfers. In our model, instead, wealth diverges after one-time transfers even if we eliminate forever the labor earnings gap. Hsieh, Hurst, Jones, and Klenow (2019) demonstrate that removing labor market exclusions of Black workers increases aggregate productivity by improving the allocation of talent across occupations. Like these authors, our model does not feature differential

⁷In our model, learning depends, directly, on own investment experiences and, indirectly, on the aggregate risky return. In their study of perceptions of intergenerational mobility, Alesina, Stantcheva, and Teso (2018) offer evidence that individuals who have experienced upward mobility in their own life are more optimistic about mobility. In the context of forming inflation expectations, Malmendier and Nagel (2016) present evidence that individuals put more weight on personal experiences than on other available historical data, especially following periods of volatile inflation.

⁸We confront quantitatively the model with historical evidence on racial gaps in wealth, wages, and mobility from Higgs (1982) and Margo (1984), Margo (2016), and Collins and Wanamaker (2017).

changes in innate abilities by race over time and removes labor market exclusions at the time of reparations. Different from Hsieh, Hurst, Jones, and Klenow (2019) who do not consider wealth accumulation, our interest lies in how the racial wealth gap emerged from historical events and how it will evolve after reparations.

Our conclusion that reparations in the form of transfers do not eliminate the racial wealth gap in the long run is reminiscent of the conclusion of Loury (1977) for labor market policies aiming to equalize racial outcomes. Loury (1977) argues that equal opportunity policies may not completely eliminate racial inequality because labor market outcomes also depend on accumulated social capital and networks that disadvantage Black households. The parallel we draw with Loury (1977) is that equalizing wealth in our model does not suffice to eliminate the racial wealth gap in the future because, at the time of reparations, Black dynasties have accumulated fewer positive investment experiences from their network.

2 Model

We present the model and characterize its equilibrium. We then discuss key mechanisms through an example.

2.1 Environment

The economy is populated by a continuum of heterogeneous dynasties indexed by $\iota \in [0,1]$. The horizon is infinite and periods $t=1,2,\ldots$ represent the economic life for a generation within a dynasty. We denote by Φ_t the distribution of dynasties in period t.

Demographics. Dynasty ι in period t has size $N_{\iota t}$. The evolution of $N_{\iota t}$ is given by:

$$N_{\iota t+1} = (1 + n_{\iota t+1}) N_{\iota t}, \tag{1}$$

where $n_{\iota t+1}$ is the population growth rate of dynasty ι between periods t and t+1. The total population is $N_t = \int N_{\iota t} d\Phi_t$ and the population growth rate is $1 + n_{t+1} \equiv N_{t+1}/N_t$.

Technology. The model features an occupational choice between labor and capital, motivated by the observation that the majority of households at the top of the wealth distribution are

entrepreneurs. Each generation within a dynasty is endowed with one unit of time. Generations allocate fraction $1 - k_{it}$ of their lifetime to a safe technology which we call labor and fraction k_{it} to a risky technology which we call capital or entrepreneurship. The choice of time $k_{it} \in [0, 1]$ is continuous.

The safe technology produces labor income from working and non-labor income from saving in a risk-free asset. Dynasties who allocate fraction $1 - k_{\iota t}$ of their time to the safe technology earn income:

$$(z_{\iota t} + i_t a_{\iota t})(1 - k_{\iota t}), \tag{2}$$

where z_{it} is the wage and i_t is the safe return on assets a_{it} , both taken as given by dynasties.

Operating the risky technology requires time and, thus, dynasties who allocate time k_{it} to entrepreneurial activities forego labor income. Entrepreneurship is risky as k_{it} is chosen before an idiosyncratic investment shock is realized. Allocating time k_{it} produces capital income:

$$r_t a_{\iota t} k_{\iota t}$$
, if $e_{\iota t} = G$,
 0 , if $e_{\iota t} = B$. (3)

Capital income depends on the realization of an idiosyncratic event e_{tt} . If the dynasty's experience is good, $e_{tt} = G$, entrepreneurship yields a net return r_t per unit of assets invested a_{tt} . If the dynasty's experience is bad, $e_{tt} = B$, entrepreneurship yields a net return of zero. The return r_t is determined in equilibrium and also taken as given.

Beliefs. Our modeling innovation is to introduce heterogeneity in beliefs about risky investment returns. Dynasties do not know the objective probability of a good experience, which we denote by $q^* = \mathbb{P}(e_{it} = G)$. This probability is common across dynasties and constant over time. Dynasties learn about q^* from events they experience when they are capitalists.

Each dynasty ι begins period t with a prior belief, $\pi_{\iota\iota}(q)$, over the probability that risky investment activities are successful, q^* . The belief induces a subjective expectation of a good event, $\mathbb{E}_{\iota\iota}q^* = \int q\pi_{\iota\iota}(q)\mathrm{d}q$. As we illustrate below, this expectation partly determines the choice to become a capitalist.

Capitalists, $k_{it} > 0$, update their prior belief following their experiences using Bayes' rule:

$$\pi_{\iota t+1}(q) = \begin{cases} \pi_{\iota t}(q) \frac{q}{\mathbb{E}_{\iota t} q^*}, & \text{if } e_{\iota t} = G, \\ \pi_{\iota t}(q) \frac{1-q}{\mathbb{E}_{\iota t} (1-q^*)}, & \text{if } e_{\iota t} = B. \end{cases}$$

$$(4)$$

Following a good experience, $e_{tt} = G$, the posterior belief that q^* equals q, $\pi_{tt+1}(q)$, equals the prior belief, $\pi_{tt}(q)$, multiplied by the likelihood of experiencing a good event, q, divided by the probability of occurrence of a good event, $\mathbb{E}_{tt}q^*$. Dynasties with good experiences increase their belief about probabilities that exceed their prior mean of a good experience. Conversely, following a bad experience, $e_{tt} = B$, dynasties lower their belief about probabilities that exceed their prior mean of a good experience.

Capitalists, $k_{tt} > 0$, pass their posterior beliefs to their children who begin the next period with prior π_{tt+1} . Laborers, $k_{tt} = 0$, do not accumulate risky investment experiences and, therefore, do not update their prior, $\pi_{tt+1} = \pi_{tt}$. Beliefs in our model are martingale, $\mathbb{E}_{tt}\pi_{tt+1}(q) = \pi_{tt}(q)$. Therefore, beliefs converge to the truth in the long run, $\lim_{t\to\infty} \pi_{tt}(q) = 0$ for $q \neq q^*$, but only conditional on being a capitalist.

Similar to the learning assumption of Piketty (1995) in his model of income mobility, in our model dynasties learn only from their own experiences if they become capitalists. We think this is a natural benchmark, partly because it allows the model to generate persistence in expected returns and wealth. In Section 4.3 we consider alternative assumptions under which dynasties also learn from the experiences of other dynasties, irrespective of whether they themselves are capitalists.

Timing. The timing of events in each period is:

- 1. Dynasty ι begins period t with state $(z_{\iota t}, a_{\iota t}, \pi_{\iota t}(q), n_{\iota t+1}, T_{\iota t})$, where $z_{\iota t}$ is the wage, $a_{\iota t}$ is assets, $\pi_{\iota t}(q)$ is the prior belief about the probability the good event is q, $n_{\iota t+1}$ is the growth rate of the size of the dynasty, and $T_{\iota t}$ is transfers.
- 2. Dynasties choose the fraction of time spent on capital activities $k_{\iota t}$ before $e_{\iota t}$ is realized.
- 3. Dynasties experience e_{it} and realize income y_{it} .
- 4. Dynasties choose consumption c_{it} and transmit assets a_{it+1} and posterior beliefs π_{it+1} to the next generation it+1.

Preferences and budget. The model is analytically tractable when each generation has preferences over their own consumption c_{it} and over assets bequeathed per child a_{it+1} . The utility function is:

$$U = \frac{(c_{\iota t} - \bar{c}_t)^{1-\gamma} - 1}{1-\gamma} + \beta^{\gamma} \frac{a_{\iota t+1}^{1-\gamma} - 1}{1-\gamma},\tag{5}$$

where parameter $\gamma \geq 0$ governs the curvature of the utility function for consumption and bequests and the discount factor $\beta > 0$ governs the preference for bequests relative to consumption.

Preferences are non-homothetic with $\bar{c}_t > 0$ denoting the subsistence level of consumption. We motivate non-homothetic preferences with the observation that households with higher lifetime income (for example, Dynan, Skinner, and Zeldes, 2004; Straub, 2019) and wealth (for example, Fagereng, Holm, Moll, and Natvik, 2019) exhibit higher saving rates, leading to increased wealth inequality (for example, De Nardi and Fella, 2017). In our quantitative analysis, \bar{c}_t allows the model to generate the low wealth share held by the bottom half of the population.

The budget constraint of each generation is:

$$c_{\iota t} + (1 + n_{\iota t+1})a_{\iota t+1} = y_{\iota t}(k_{\iota t}, e_{\iota t}) + (1 - \delta)a_{\iota t}, \tag{6}$$

where $(1 + n_{\iota t+1})a_{\iota t+1}$ are resources saved to transfer $a_{\iota t+1}$ to each member of the next generation and δ is the depreciation rate of assets. Income $y_{\iota t}$ is a function of the allocation of time $k_{\iota t}$ and the realization of experience $e_{\iota t}$:

$$y_{tt}(k_{tt}, e_{tt}) = \begin{cases} T_{tt} + (z_{tt} + i_t a_{tt})(1 - k_{tt}) + r_t a_{tt} k_{tt}, & \text{if } e_{tt} = G, \\ T_{tt} + (z_{tt} + i_t a_{tt})(1 - k_{tt}), & \text{if } e_{tt} = B, \end{cases}$$

$$(7)$$

where $T_{\iota t}$ are transfers.

Interpretation of wealth and risky investments. Consistent with our model in which wealth derives from both safe and risky investments, we interpret a_{tt} as net worth from all assets minus liabilities, including public equity, business assets, bonds, durables, and housing. Our modeling of risky investments is consistent with two aspects of the data that we discuss below. Similar to our model in which risky investment activities amplify the racial gap in average wealth, in Section 3.3.1 we use the SCF to document that holdings of private business assets and public equity are

quantitatively the most important drivers for the racial wealth gap. Further, in Section 3.3.5, we document the existence of a racial gap in beliefs about returns on a diversified equity fund, which directly maps to expected returns $\mathbb{E}_{tt}q^*r_t$.

2.2 Dynasty Optimization

We solve the dynasty problem backwards. In the last stage, the solution for consumption and assets, given income level y_{tt} , is:

$$c_{tt} = \bar{c}_t + \omega_{tt+1} \left(y_{tt} + (1 - \delta) a_{tt} - \bar{c}_t \right), \tag{8}$$

$$a_{it+1} = \frac{1 - \omega_{it+1}}{1 + n_{it+1}} (y_{it} + (1 - \delta)a_{it} - \bar{c}_t), \tag{9}$$

with weight $\omega_{\iota t+1} \equiv \frac{(1+n_{\iota t+1})^{\frac{1-\gamma}{\gamma}}}{(1+n_{\iota t+1})^{\frac{1-\gamma}{\gamma}}+\beta}$. The solutions are constant elasticity of substitution demand functions, augmented to account for subsistence consumption. Each generation allocates a fraction $\omega_{\iota t+1}$ of their resources net of subsistence consumption to consumption above this subsistence level. Remaining resources are passed to the next generation in the form of assets. Because each generation is succeeded by $1+n_{\iota t+1}$ members, an increase in population growth reduces the resources transferred per member of the next generation. The consumption weight $\omega_{\iota t+1}$ varies with population growth when $\gamma \neq 1$. With log utility, $\gamma \to 1$, the weight is independent of population growth and is inversely related to the discount factor, $\omega_{\iota t+1} = 1/(1+\beta)$.

Working backwards, we solve for the allocation of time $k_{\iota\iota}$ conditional on the optimal choice of consumption and assets in the last stage of the period. Dynasties maximize expected utility:

$$V = \max_{k_{\iota t}} \int \left(q U^*(y_{\iota t}(k_{\iota t}, G)) + (1 - q) U^*(y_{\iota t}(k_{\iota t}, B)) \right) \pi_{\iota t}(q) dq, \tag{10}$$

where U^* is indirect utility given income $y_{\iota t} = y_{\iota t}(k_{\iota t}, e_{\iota t})$. The expectation is formed under the probability distribution $\pi_{\iota t}$. The solution for time allocated to capital is:

$$k_{tt} = \begin{cases} 0, & \text{if } r_{t}a_{tt} \leq z_{tt} + i_{t}a_{tt}, \text{ or } \mathbb{E}_{tt}q^{*}r_{t}a_{tt} \leq z_{tt} + i_{t}a_{tt}, \\ \left(1 + \frac{T_{tt} + (1 - \delta)a_{tt} - \bar{c}_{t}}{z_{tt} + i_{t}a_{tt}}\right) \left(\frac{\left[\frac{\mathbb{E}_{tt}q^{*}}{1 - \mathbb{E}_{tt}q^{*}}\left(\frac{r_{t}a_{tt}}{z_{tt} + i_{t}a_{tt}} - 1\right)\right]^{\frac{1}{\gamma}} - 1}{\left[\frac{\mathbb{E}_{tt}q^{*}}{1 - \mathbb{E}_{tt}q^{*}}\left(\frac{r_{t}a_{tt}}{z_{tt} + i_{t}a_{tt}} - 1\right)\right]^{\frac{1}{\gamma}} - 1 + \frac{r_{t}a_{tt}}{z_{tt} + i_{t}a_{tt}}}\right), \text{ else,} \end{cases}$$

$$(11)$$

with $k_{it} = 1$ if the expression in the last line exceeds one.

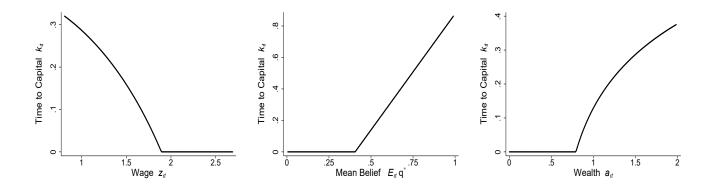


Figure 1: Policy Functions

Figure 1 presents the policy functions for k_{tt} . Our model includes two channels affecting the decision to become a capitalist. The first is wealth, captured by a_{tt} , and the second is relative factor returns, captured by the expected return $\mathbb{E}_{tt}q^*r_t$ relative to the wage z_{tt} . Dynasties with sufficiently high wage z_{tt} , low assets a_{tt} , or pessimistic beliefs about returns $\mathbb{E}_{tt}q^*r_t$ become laborers, $k_{tt} = 0$. Conditional on becoming a capitalist, k_{tt} increases in assets and expected returns and decreases in the wage.

2.3 Labor and Capital Market Exclusions

We treat labor and capital market exclusions as exogenous driving forces and feed them to the model in a time-varying way that we specify in the quantitative part of our analysis. In modelling these exclusions, we distinguish between Black dynasties b and White dynasties w. Black dynasties are a fraction ϕ_t of the population. We denote by Φ_t^h the distribution of dynasties $h \in \{b, w\}$ conditional on race.

By labor market exclusion we refer, collectively, to slavery and the racial wage gap. Slavery, which we indicate by $\chi_t^{\ell} = 1$, forces Black dynasties to be laborers, consume subsistence consumption, and not transfer resources to their children:

$$k_{\iota\iota}^b = 0, \quad c_{\iota\iota}^b = \bar{c}_t, \quad a_{\iota\iota+1}^b = 0.$$
 (12)

During slavery, production of Black dynasties in excess of subsistence consumption is expropriated and evenly distributed among White dynasties:

$$(1 - \phi_t)T_{\iota t}^w = \chi_t^\ell \phi_t \int (z_{\iota t} - \bar{c}_t) d\Phi_t^b.$$
(13)

The racial wage gap means that Black dynasties draw wages from a distribution with a lower mean than the mean of the distribution of White dynasties, $\mathbb{E}z_{tt}^b < \mathbb{E}z_{tt}^w$.

Capital market exclusions, which we indicate by $\chi_t^k = 1$, capture events such as discrimination in patenting, redlining, Jim Crow segregation laws, and exclusion from credit markets.⁹ In our model, capital market exclusions prohibit Black dynasties from becoming capitalists:

$$k_{\iota\iota}^b = 0. (14)$$

2.4 Equilibrium

The return to risky investments, r_t , adjusts to clear the asset market: ¹⁰

$$N_t \int (1 + n_{\iota t+1}) a_{\iota t+1}(\cdot, r_t) d\Phi_t = \bar{A}_{t+1}/r_t^{\alpha}.$$
 (15)

The left-hand side of this equation is total desired assets given a return r_t . Desired assets increase in r_t because a higher return induces more dynasties to become capitalists and to transfer more assets to their children. The supply of assets has to meet a limit on investment opportunities, given by the right-hand side of equation (15). Parameter $\alpha \geq 0$ governs the magnitude of the decline in returns as wealth accumulates. In the limiting case with $\alpha = 0$, assets are in fixed supply (for example, land) and r_t adjusts to make assets equal to the exogenous constant \bar{A}_{t+1} . In the other limiting case with $\alpha \to \infty$, the return to risky investment is exogenous as in a small open economy. For intermediate cases, $0 < \alpha < \infty$, the equilibrium return and assets are jointly determined as in standard closed-economy general equilibrium models.

We conclude this section with the definition of equilibrium. Given an initial distribution over assets, beliefs, and population size by race, $\Phi_1^h(a_1, \pi_1, N_1)$, exogenous dynasty sequences $\{z_{it}, e_{it}, n_{it+1}\}_{it}$, and exogenous aggregate sequences $\{\bar{A}_t, i_t, \chi_t^\ell, \chi_t^k\}_t$, an equilibrium is a sequence

⁹Cook (2014) uses patent records matched with census data and other survey data from the U.S. Patent Office to document a decline in Black patents in areas with higher incidence of race riots and segregation laws between 1870 and 1940. The example of the Boyd dynasty we reference in Section 2.5 fits well with Cook's evidence on missing Black patents. Fairlie, Robb, and Robinson (2020) present evidence from the Kauffman Firm Survey that Black startups face more difficulty raising external capital. Even controlling for credit scores and net worth, Black entrepreneurs are significantly more likely than White entrepreneurs to report fear of denial as the reason for not applying for loans.

 $^{^{10}}$ The demand for safe assets is perfectly elastic, implying an exogenous return i_t . In our quantitative results we set i_t to a constant but the results are not sensitive to feeding in the time-varying return from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019).

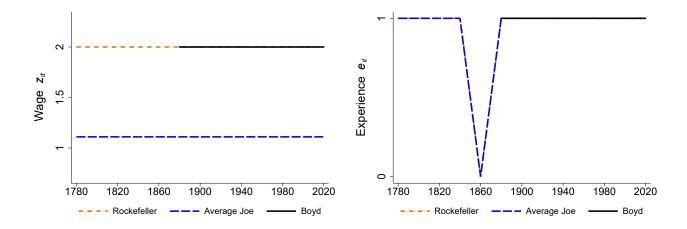


Figure 2: Wages and Entrepreneurial Experiences

of dynasty choices and beliefs $\{k_{\iota t}, y_{\iota t}, c_{\iota t}, a_{\iota t+1}, \pi_{\iota t+1}\}_{\iota t}$ and a return to risky investments $\{r_t\}_t$ such that: (i) dynasties maximize their expected utility in equation (10) under uncertainty about $e_{\iota t}$ and maximize their utility in equation (5) after $e_{\iota t}$ is realized; (ii) beliefs are consistent with Bayes' rule in equation (4); (iii) the expropriation rule in equation (13) is satisfied; and (iv) the asset market in equation (15) clears.

2.5 Illustrative Example

The goal of the example is to show the mechanism by which labor market exclusions generate wealth divergence before reparations and to highlight the role of equilibrium effects in amplifying this divergence. Figure 2 presents historical realizations for three illustrative dynasties. The first, which we label the *Rockefeller* dynasty with the orange dot-dashed line, is White and always has high wage z_{it} . The second, which we label the *Average Joe* dynasty with the blue dashed line, is White and has lower z_{it} . The third, which we label the *Boyd* dynasty with the black solid line, has as high z_{it} as Rockefeller but is Black and hence enslaved between 1780 and 1860. All dynasties are identical in terms of their potential investment experiences e_{it} in the right panel. We feed the realizations of z_{it} and e_{it} into the analytical solutions of Section 2.2 to generate the dynasties' evolution of occupational choice, beliefs, and wealth.¹¹

¹¹While some readers may be familiar with Rockefeller, we suspect the story of Henry Boyd is not well known among economists. Boyd was born into slavery in 1802 in Kentucky and later was apprenticed out to a cabinetmaker. Boyd was very skilled and earned money to buy his freedom. He moved to Cincinnati in 1826 as a free man, but faced discrimination finding a job. His first job was to unload iron and eventually he was promoted to janitor.

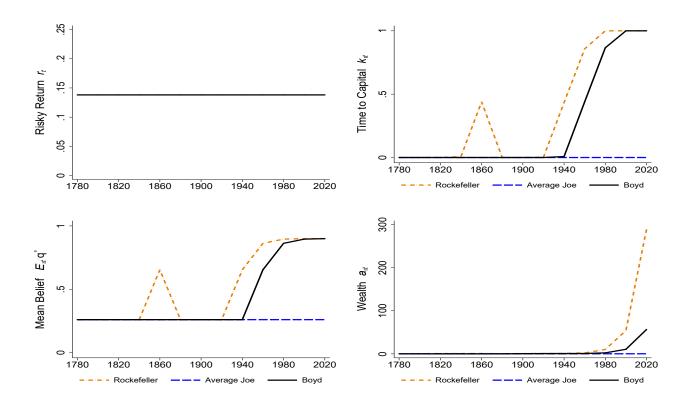


Figure 3: Illustrative Example in Partial Equilibrium

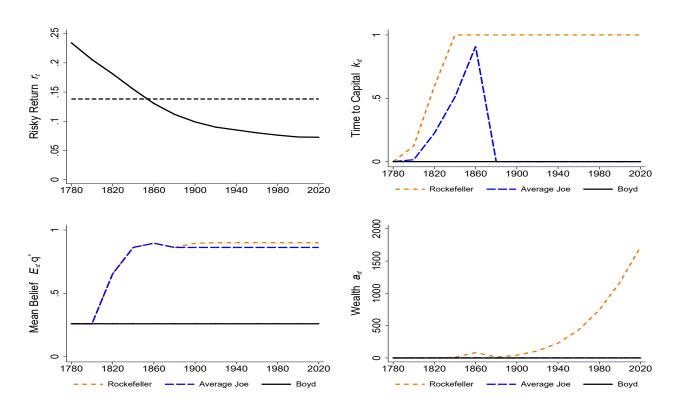


Figure 4: Illustrative Example in General Equilibrium

Figure 3 narrates the history of the dynasties in partial equilibrium. By partial equilibrium, we mean the case with $\alpha \to \infty$ and, thus, a constant risky return r_t . Beginning with the Rockefeller dynasty, by 1840 it has accumulated sufficient wealth to become capitalists. In 1860, during the Civil War, Rockefeller has a negative experience which discourages entrepreneurship until 1940. Thereafter, the Rockefeller dynasty continues investing with positive experiences, resulting in their beliefs converging to the truth and significant accumulation of wealth. Average Joe differs from Rockefeller because the dynasty never accumulates enough wealth to make entrepreneurship worthwhile. Average Joe remains a laborer, does not update beliefs, and ends up with low wealth. Boyd is initially enslaved and prohibited from building wealth. In partial equilibrium, Boyd eventually catches up with Rockefeller, becomes a capitalist, updates beliefs toward the truth, and accumulates wealth.

The evolution of occupational choice and wealth is different in general equilibrium, as shown in Figure 4. By general equilibrium, we mean the case with $\alpha < \infty$ and, thus, a declining risky return r_t .¹² General equilibrium generates different predictions as slavery early on coincides with higher return while the abolition of slavery coincides with lower return. Owing to the high initial return, Rockefeller becomes a capitalist earlier in general equilibrium than in partial equilibrium. Good experiences encourage continued investments and wealth accumulation throughout history. Given high initial returns, even Average Joe becomes a capitalist. However, its wealth and the return are sufficiently low to discourage the dynasty from further investment activity following the bad experience of 1860. The Boyd dynasty never catches up with Rockefeller in general equilibrium, despite having the same wage. The return on risky investments is low after the abolition of slavery in general equilibrium, discouraging Boyd from becoming a capitalist and from accumulating wealth.

According to historical accounts, one day a white carpenter showed up drunk at work and Boyd substituted. Impressing his boss, Boyd earned enough money as a carpenter to buy the freedom of two of his siblings and open up his own business. Unable to patent his bed frame invention, he stamped bedsteads with his name and eventually partnered with a white business man. While Boyd expanded his business, arsonists burned his shop three times and companies denied him insurance resulting in business closure in 1862. Boyd's estate was teared down in the early 1900s in Cincinnati and turned into a garage. See https://bit.ly/2Ny8AII for some original sources on these historical accounts.

 $^{^{12}}$ Kuvshinov and Zimmermann (2021) document a decline in expected risky returns for many countries, including the United States since 1890. The 2 percentage points decline in r_t during the 20th century in Figure 4 is quantitatively consistent with the decline in expected returns documented by these authors.

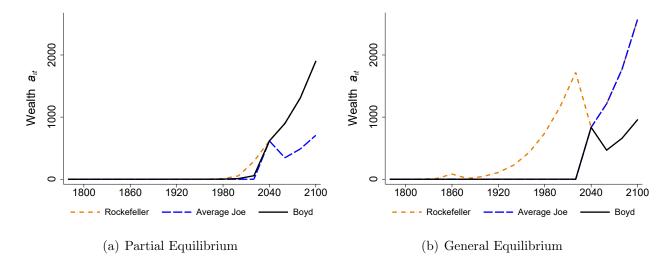


Figure 5: Wealth Transfers in the Illustrative Example

Equilibrium effects, where during slavery returns on wealth accumulation are higher than in recent times, discourage Black dynasties from becoming capitalists. An example of an asset captured by the general equilibrium model is land which was initially unexploited and offered high returns. The absence of early positive investment experiences for Black dynasties leads them to have more pessimistic beliefs about risky investments than White dynasties and causes a more persistent racial divergence of wealth than if returns were constant.

We conclude by previewing the logic of why one-time transfers do not lead to wealth convergence in the future even if they eliminate the racial wealth gap today. Figure 5 shows the evolution of wealth for the three dynasties following a redistribution of wealth that eliminates wealth inequality in 2040. In partial equilibrium, Boyd accumulates as much wealth as Rockefeller after reparations. In general equilibrium, Boyd's wealth falls behind White dynasties' wealth beginning in 2060. This result can be understood in terms of the policy function for the choice of capital in equation (11), which we summarize as $k(z, a, \mathbb{E}q^*; r)$. Despite having equal wage z and assets a with Rockefeller in 2040, relatively pessimistic beliefs $\mathbb{E}q^*$ and the relatively low return r lead Boyd to choose labor and forego the opportunity to invest. By contrast, given optimistic beliefs, White dynasties continue to be capitalists. As a result, wealth for the two types of dynasties diverges forever.

3 Quantitative Results

We first describe inputs to the model and the parameterization strategy. Next, we evaluate the ability of the model to account for racial gaps in wealth, income, entrepreneurship, mobility, and beliefs, all of which are not targeted by the parameterization. Finally, we present welfare analyses and counterfactuals when removing labor and capital market exclusions and changing structural features of the economy.

3.1 Model Inputs

A model period is twenty years. We begin in 1780, roughly corresponding to the Declaration of Independence. Slavery ends in 1860, $\chi_t^{\ell} = 0$, the model period closest to the time when Congress passed the 13th Amendment abolishing slavery. We remove capital market restrictions on Black dynasties in 1960, $\chi_t^k = 0$, aligning with the start of the Civil Rights Movement. The last historical model period is 2020 and we consider reparations in 2040.

We use tables from the Census between 1780 and 2000 for the population of Black and White dynasties, N_t^b and N_t^w . The fraction of Black dynasties in 1780 is $\phi_1 = 0.21$. We present annualized population growth in the first two column of Table 1. The White population grows more until the early 1900s and the Black population grows faster after that.¹³

The wage of dynasty ι belonging to race h in period t is:

$$z_{tt}^{h} = \max\{Z_t \mu_t^h \theta_{tt}, \bar{c}_t\},\tag{16}$$

where Z_t is the aggregate component, μ_t^h is the race component, and θ_{it} is the idiosyncratic component of wages. Wages are adjusted so that they do not fall below the subsistence level of consumption \bar{c}_t . The idiosyncratic component follows the autoregressive process:

$$\log \theta_{it} = \rho_t \log \theta_{it-1} + \sigma_t \varepsilon_{it}, \tag{17}$$

 $^{^{13}}$ For periods 2020 and after, we extrapolate population figures using the average annual growth rate of Non-Hispanic Whites between 1980 and 2000 and set $1 + n = 1.0007^{20}$ for both Black and White dynasties. Our model population excludes American Indian, Hispanics, and Asian individuals, or individuals who identify with multiple races. In our model, every Black dynasty is enslaved until 1860. According to Census sources, roughly 90 percent of the Black population in the United States was enslaved by 1860.

Table 1: Population and Wages

	Populatio	n Growth		Wag	es	
	White	Black	Agg. Growth	Racial Gap	Persistence	Dispersion
	$(1+n^w)^{\frac{1}{20}}-1$	$(1+n^b)^{\frac{1}{20}}-1$	$(1+g)^{\frac{1}{20}}-1$	μ^b	ho	σ
1780	3.40	2.81	0.39	0.28	0.40	0.71
1800	3.06	2.89	0.39	0.28	0.40	0.71
1820	2.99	2.45	1.00	0.28	0.40	0.71
1840	3.23	2.20	1.27	0.28	0.40	0.71
1860	2.40	1.98	1.40	0.28	0.40	0.71
1880	2.19	1.48	1.68	0.30	0.40	0.71
1900	1.74	0.85	1.32	0.32	0.40	0.71
1920	1.09	1.04	0.82	0.35	0.40	0.71
1940	1.39	1.93	2.29	0.38	0.40	0.71
1960	0.64	1.71	2.28	0.44	0.35	0.63
1980	0.07	1.35	2.01	0.57	0.32	0.67
2000	0.07	0.07	1.43	0.62	0.58	0.85
2020	0.07	0.07	1.43	0.65	0.58	0.85

Table 1 presents the time series for population growth and for parameters of the wage process. Growth rates are in percent and annualized. The last measured population growth rates are from 1980 to 2000 and beyond 2000 we use the population growth for Non-Hispanic Whites between 1980 and 2000. For the wage process we set missing values to their closest available estimates.

where ρ_t governs the persistence across generations, σ_t governs the cross-sectional dispersion of wages, and the shock ε_{tt} is drawn from a standard normal distribution. Persistence ρ_t and dispersion σ_t are common across dynasties.

The last four columns of Table 1 present the evolution of the four parameters of the wage process. The aggregate component of wages $Z_t = (1+g_t)Z_{t-1}$, grows at rate g_t which we set equal to the growth of GDP per capita from the Maddison project (Bolt and van Zanden, 2020). For the race component, $\mu_t^w = 1$ for White dynasties and μ_t^b for Black dynasties is presented in the fourth column of the table. We take the racial gap in wages from Margo (2016) who uses labor force participation rates for agricultural workers and urban workers to compute the Black-White

relative income starting from 1870.¹⁴ We set the persistence ρ_t from the intergenerational mobility study of Aaronson and Mazumder (2008) who match men in the Census to synthetic parents in the prior generation starting from 1940. Finally, we use the American Community Survey starting in 1940 to estimate the cross-sectional dispersion σ_t .¹⁵

All dynasties begin with zero initial wealth, $a_{t1} = 0$. We feed into the model an asset limit which evolves as $\bar{A}_t = \bar{A}_1(1+g)^{t-1}(1+n)^{t-1}$, where the initial value \bar{A}_1 is a parameter. We grow \bar{A}_t at a rate equal to the growth rate of wages g and population n allowing the model to asymptote to a balanced growth path with constant returns and factor shares.

3.2 Parameterization

Table 2 presents our parameter values. The upper panel shows parameter values chosen externally without solving the model. The lower panel shows parameter values chosen such that model-generated moments match their data analogs. Table 3 presents the targeted moments.¹⁶

Preferences are logarithmic, $\gamma = 1$. We use the inverse of the labor share from national income and product accounts to set $\alpha = 1.77$. The annual rate of depreciation, $1 - (1 - \delta)^{1/20} = 0.05$, equals the average annual depreciation rate of private fixed assets in national income and product accounts. Finally, for the safe return we pick i = 0.02 using the average return on safe assets from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019).

The distribution of beliefs in 1780 is Beta $\left(\frac{b\bar{q}_{\iota 1}}{1-\bar{q}_{\iota 1}},b\right)$. The prior mean, $\bar{q}_{\iota 1}=\mathbb{E}_{\iota 1}q^*$, is heterogeneous across dynasties ι . Heterogeneity in initial expected returns allows the model to generate significant wealth inequality as optimistic dynasties become capitalists before pessimistic dynasties. Initial expected returns are drawn from a uniform distribution, $\bar{q}_{\iota 1} \sim \mathrm{U}(0,\bar{q})$. Given $\bar{q}_{\iota 1}$, parameter b governs the dispersion of beliefs and is common across dynasties. Initial beliefs vary

¹⁴The first observations in Margo (2016) are for 1870, 1900, and 1940. We interpolate linearly to set the racial wage gap in 1880 and 1920. We use the 1870 observation for the racial wage gap in model period 1860 and before. The more recent data after 1940 come from Census records. Aizer, Boone, Lleras-Muney, and Vogel (2020) study the role of defense production during the second World War for closing the racial wage gap after 1940. Derenoncourt and Montialoux (2021) study the role of expansions of the federal minimum wage for closing the racial earnings and income gap during the Civil Rights era. Bayer and Charles (2018) document that, despite the narrowing of the racial gap in median earnings between 1940 and 1970, the median Black man's rank in the total earnings distribution did not improve significantly.

¹⁵Our estimates are dispersion of log income and control for age and race fixed effects.

¹⁶We have confirmed that parameters are identified locally by examining the sensitivity of the targeted moments to parameter variations around their baseline values shown in Table 2.

Table 2: Model Parameters

Parameter		Value	Source/Logic
curvature of utility	γ	1.00	log preferences
returns to scale	α	1.77	inverse of labor share
depreciation rate (annual)	$1 - (1 - \delta)^{1/20}$	0.05	NIPA fixed assets
safe return (annual)	$(1+i)^{1/20} - 1$	0.02	Jorda et al (2019)
discount factor (annual)	$\beta^{1/20}$	0.98	match targets
probability of success	q^*	0.91	match targets
initial asset limit	$ar{A}_1$	0.23	match targets
subsistence consumption	$ar{c}_t$	$0.81Z_{t}$	match targets
shape Beta distribution	$ar{q}$	0.30	match targets
shape Beta distribution	b	1.20	match targets

Table 2 presents values of model parameters. The upper panel shows parameters values chosen externally. The lower panel shows parameter values chosen such that model-generated variables match their analogs in the data.

Table 3: Model Targets

Target		Data	Model
wealth / lifetime income	$\int a \mathrm{d}\Phi / \int y \mathrm{d}\Phi$	0.25	0.25
labor share	$\int z(1-k)\mathrm{d}\Phi/\int y\mathrm{d}\Phi$	0.56	0.56
risky return	q^*r	0.07	0.07
top 10% wealth share	$\int_{0.9}^{1} a \mathrm{d}\Phi / \int a \mathrm{d}\Phi$	0.76	0.76
top 50% wealth share	$\int_{0.5}^{1} a \mathrm{d}\Phi / \int a \mathrm{d}\Phi$	0.99	0.98
entrepreneurship rate	$\int \mathbb{I}(k>0) \mathrm{d}\Phi$	0.03	0.03

Table 3 presents the moments targeted to estimate the parameters in the lower panel of Table 2.

within group, but Black and White dynasties draw from the same initial distribution.

Along with parameters for initial beliefs, \bar{q} and b, we pick the discount factor β , the objective probability of a good experience q^* , the initial asset limit \bar{A}_1 , and subsistence levels of consumption \bar{c}_t to target six moments. Table 3 shows that we target the wealth to income ratio, the labor share, the risky return, the share of wealth accruing to the top 10 and 50 percent of the wealth distribution, and the entrepreneurship rate.¹⁷

The model matches these targets with (annualized) discount factor $\beta^{1/20} = 0.98$ and probability of successful entrepreneurship $q^* = 0.91$. The parameter estimates favor dispersed initial beliefs around a pessimistic mean ($\bar{q}/2 = 0.15$). These beliefs imply that few, initially more optimistic, dynasties end up being capitalists. Given the high probability of success q^* , capitalists hold a significant fraction of wealth in the economy and the model matches the high concentration of wealth at the top. Finally, the subsistence consumption value of \bar{c}_t implies strong non-homotheticity in preferences, helping the model account for the observation that the bottom half of the population owns almost none of the economy's wealth.¹⁸

3.3 Comparing the Model to the Data

In this section we evaluate the ability of the model to generate current and historical racial gaps in outcomes such as wealth, income, entrepreneurship, mobility, and beliefs about risky returns.

3.3.1 Wealth and Income

Table 4 compares the concentration of wealth and income between the model and the data. The measure of concentration is the share of wealth and income held by households above selected percentiles of their respective distributions. The data come from the SCF between 2010 and 2019, corresponding to model period 2020.

The model accounts almost perfectly for the wealth concentration in the data up to the 5th

 $^{^{17}}$ Average wealth to income equals household net worth over GDP from the Flow of Funds. The risky return is taken from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) and is adjusted for inflation. The shares of wealth accruing to the top 10 and 50 percent and the entrepreneurship rate are calculated from the Survey of Consumer Finances between 2010 and 2019. For the entrepreneurship rate, we define an individual as entrepreneur, k > 0, if their business assets exceed average net worth in the survey.

¹⁸We parameterize $\bar{c}_t = 0.81Z_t$, ensuring that subsistence consumption per capita grows at rate g over time. With this parameterization, \bar{c} equals 49 percent of the average wage in 2020.

Table 4: Wealth and Income Concentration

	Wealth		Income	
(2020, share of top)	Data	Model	Data	Model
50 percent	0.99	0.98	0.86	0.89
20 percent	0.87	0.86	0.62	0.71
10 percent	0.76	0.76	0.47	0.60
5 percent	0.64	0.67	0.37	0.52
1 percent	0.37	0.53	0.20	0.40
0.1 percent	0.14	0.25	0.07	0.19

Table 4 presents the share of wealth and income held by households above selected percentiles of their respective distributions. The data moments are calculated from the SCF between 2010 and 2019.

Table 5: Racial Gap in Wealth and Income

	Wealth Ratio		Incom	e Ratio
(2020, Black/White)	Data	Model	Data	Model
Mean	0.15	0.17	0.45	0.36
99 percentile	0.13	0.19	0.25	0.28
90 percentile	0.19	0.41	0.51	0.58
50 percentile	0.10	0.06	0.57	0.59

Table 5 presents the Black to White ratio of wealth and income at the mean and selected percentiles of the corresponding distributions. The data moments are calculated from the SCF between 2010 and 2019.

Table 6: Decomposition of Racial Wealth Gap

(2020)	Asset Type	Portfolio Weight	Contribution to Wealth Gap
1. Model	Risky	0.58	0.70
2. Data	Risky	0.58	0.60
3. Data	Public Equity	0.25	0.26
4. Data	Private Equity	0.33	0.34
5. Data	Owner-Occupied Housing	0.18	0.15
6. Data	Bonds	0.18	0.18

Table 6 shows for various types of assets their contribution to the racial wealth gap defined in equation (18) and their portfolio weights. The data entries come from the SCF between 2010 and 2019.

percentile. The model also performs well in terms of matching the observed income concentration up to the 5th percentile. Above that percentile, the model overestimates wealth and income concentration relative to the data. An alternative strategy is to target higher percentiles than the 10th and 50th in our parameterization. In that case, the model would underestimate the concentration at lower percentiles. We prefer targeting lower percentiles because matching concentration around these percentiles is more important for analyzing reparations policies. ¹⁹

We next assess the ability of the model to generate racial gaps in wealth and income not targeted by the parameterization. In Table 5 we report ratios of average variables for Black households relative to White households. In the SCF, the ratio of average wealth is 0.15 and the ratio of average income is 0.45. These large racial gaps corroborate previous findings in the literature such as in recent work of Kuhn, Schularick, and Steins (2020). The model is successful in generating the large racial differences in average wealth and income observed in the data. Similar to the data, the model also generates a lower ratio for median wealth than average wealth and a higher ratio of median income than average income.

Wealth in the SCF is net worth from all assets minus liabilities, which includes public equity, business assets, real estate, owner-occupied housing, bonds, durables, and other savings accounts. Consistent with this definition, we interpret a_{tt} in the model as encompassing all these forms of wealth. To assess the relative importance of different types of assets in accounting for the racial gap in total wealth, we decompose the racial wealth gap as follows:

$$Gap = 1 - \frac{\text{Wealth}^b}{\text{Wealth}^w} = \sum_{j} \frac{\text{Asset}_j^w}{\text{Wealth}^w} \times \left(1 - \frac{\text{Asset}_j^b}{\text{Asset}_j^w}\right),\tag{18}$$

where wealth is the sum over all assets j. Equation (18) decomposes the gap between White and Black wealth into gaps arising from different asset classes, where the asset-specific gaps are weighted by the portfolio weights of White households.

The first row of Table 6 applies this decomposition to the model. Wealth in our model is invested in risky assets, $k_{tt}a_{tt}$, and safe assets, $(1 - k_{tt})a_{tt}$. The portfolio weight on risky assets

¹⁹Our estimates of the share of wealth held by the top 1 percent and top 0.1 percent are consistent with other estimates in the literature. Saez and Zucman (2016) estimates for these shares are 38 and 20 percent using IRS data. Using the same data, but a different capitalization approach, Smith, Zidar, and Zwick (2020) estimates are 30 and 14 percent. For our SCF data, these authors report a share of 13 percent for the top 0.1 percent.

Table 7: Evolution of Racial Wealth Ratio

	Wealth Ratio (Black/White)		
	Data	Model	
1900	0.04	0.02	
2020	0.15	0.17	

Table 7 presents the Black to White ratio of average wealth in 1900 and 2020. The data entry for 1900 comes from Higgs (1982) and Margo (1984). The data entry for 2020 comes from the SCF between 2010 and 2019.

for White households is 58 percent. The contribution of risky assets to the total wealth gap is 70 percent. The second row performs the same decomposition in the SCF. Risky investments in our model correspond to investments in public equity, real estate investment excluding own housing, and private business assets.²⁰ We obtain a portfolio weight of 58 percent in the data, exactly matching the portfolio weight of risky assets in the model. Further, similar to the model, we find that the racial gap in risky assets accounts for more than half of the total gap in the data.

The other rows repeat these decompositions for subcategories of assets to assess their relative importance in accounting for the total wealth gap in the data. Rows 3 and 4 present the two largest subcategories of risky assets, investments in public equity and investments in private equity which includes assets used in private businesses and real estate activities excluding owner-occupied housing. Rows 5 and 6 present the two largest subcategories of safe assets, investments in owner-occupied housing and bonds. Among the largest types of assets, we find that private equity is the most important asset and owner-occupied housing is the least important asset for the racial gap in total wealth.

3.3.2 Wealth Convergence

We next assess the ability of the model to generate a realistic speed of wealth convergence across race. Matching the historical speed of convergence lends credibility to the model when analyzing how reparations affect the future evolution of the racial wealth gap. Table 7 presents the ratio of Black to White wealth in 1900 and 2020. The ratio for the early period comes from the historical

²⁰In the SCF, we split asset classes with an ambiguous asset mix (non-money-market mutual funds, quasi-liquid individual retirement accounts, and pensions) between stocks and bonds in proportion to their unambiguous shares.

evidence of Higgs (1982), who uses Georgia tax assessment records to measure wealth by race, and Margo (1984) who complements this analysis with data from Arkansas, Kentucky, Louisiana, North Carolina, and Virginia. The historical data show a slow convergence of Black wealth, with the wealth ratio increasing from 4 cents on the dollar in 1900 to 15 cents in 2020. Our model is successful in generating such a slow convergence, with the wealth ratio transitioning from 2 cents on the dollar to 17 cents.

3.3.3 Entrepreneurship

The model generates significant racial wealth and income gaps partly because more White dynasties are capitalists than Black dynasties. Are such differences in entrepreneurship rates present in the data? Panel A of Table 8 shows differences in entrepreneurship rates by race. Quantitatively, the model generates a 3.6 percentage points gap in the entrepreneurship rate, close to the 2.8 percentage points gap in the data.²¹ Additionally, in panel B of Table 8, the model performs well in matching the observed distribution of wealth and income within entrepreneurs. For example, the top 1 percent of entrepreneurs hold roughly 20 percent of entrepreneurial wealth in the data, compared to 23 percent in the model.

3.3.4 Income Mobility

We next assess the ability of the model to generate historical patterns of income mobility by race. The evidence comes from Collins and Wanamaker (2017) who link individual census records between 1910 and 1930 to derive historical mobility statistics and use National Longitudinal Survey of Youth data for the more recent period. Entries in Table 9 denote upward rank mobility probabilities by decile d:

Upward Rank Mobility(d) =
$$\mathbb{P}(\operatorname{rank}(y_t) > \operatorname{rank}(y_{t-1}) | \operatorname{rank}(y_{t-1}) \in d)$$
. (19)

²¹Fairlie and Meyer (2000) document that the that self-employment rate is lower for Black than White men and the ratio of self-employment rates has remained remarkable stable since the early 1900s. Bogan and Darity (2008) argue that, while much of the literature tries to account for the entrepreneurship gap by appealing to cultural differences, the comparison of Black with immigrant groups suggest a more important role for discriminatory practices, institutions, and legislation restricting Black entrepreneurship. Our model is compatible with a more elaborate version of the Bogan and Darity (2008) view, as the historical exclusions they describe endogenously lead to differences in beliefs about entrepreneurship.

Table 8: Entrepreneurship

	White		Bl	lack
A. Fraction entrepreneurs	Data	Model	Data	Model
(percent)	3.1	3.6	0.3	0.0
B. Entrepreneurs	We	ealth	Income	
Share of top	Data	Model	Data	Model
50 percent	0.88	0.96	0.89	0.98
20 percent	0.66	0.84	0.68	0.87
10 percent	0.51	0.68	0.52	0.71
5 percent	0.39	0.52	0.40	0.54
1 percent	0.20	0.23	0.19	0.24
0.1 percent	0.06	0.06	0.06	0.07

Table 8 presents statistics of entrepreneurship. The first panel shows the share of households who are entrepreneurs and the second panel shows the share of wealth and income held by entrepreneurs above selected percentiles of their respective distributions. The data entries come from the SCF between 2010 and 2019. In the SCF we define an individual as entrepreneur, k > 0, if their business assets exceed average net worth in the survey.

This index measures the probability that a child exceeds their parent's rank in the income distribution for a parent belonging to decile d. Rank is measured in the total population, including both Black and White dynasties.

Beginning with the upper panel of Table 9, the model is successful in generating a racial mobility gap in the latter period. For example, starting at the third decile, 76 percent of White children exceed their parents' rank whereas only 54 percent of Black children exceed it. The observed gap of 22 percentage points is close to the 17 percentage points gap generated by the model. In the lower panel of Table 9, the model is also successful in generating even larger mobility gaps observed during the early 1900s. For example, again at the third decline, 59 percent of White children exceeded their parents' rank whereas only 31 percent of Black children exceeded it. The observed gap of 28 percentage points is close to the 32 percentage points gap generated by the model.

Two features of the model allow it to generate mobility patterns similar to the patterns in the data. First, only White dynasties become entrepreneurs and, given the high returns from

Table 9: Upward Rank Mobility

(1990)	Da	ita	Mod	lel
Decile d	White	Black	White	Black
1	0.97	0.80	0.88	0.82
3	0.76	0.54	0.67	0.50
5	0.58	0.51	0.51	0.38
7	0.36	_	0.39	0.27
9	0.31	—	0.24	0.16
(1930)	Da	ta	Model	
Decile d	White	Black	White	Black
1	0.90	0.68	0.86	0.59
3	0.59	0.31	0.65	0.33
5	0.38		0.50	0.22
7	0.28		0.39	0.13
9	0.16		0.25	

Table 9 presents probabilities of upward rank mobility at selected deciles of the income distribution. Data entries are from Collins and Wanamaker (2017). Missing entries indicate no or very few observations to calculate probabilities.

entrepreneurship, entrepreneurs are more likely to surpass laborers in the income distribution. Second, despite persistence ρ_t and dispersion σ_t being common across dynasties in the wage process (17), Black dynasties are less likely to move upward in the total population's income distribution because they draw wages from a lower mean than the average dynasty in the population, as shown by μ_t^h in equation (16). Since the racial wage gap was larger in the 1900s, Black dynasties were less likely to move upward in the 1900s than today.

3.3.5 Beliefs about Risky Returns

We compare model predictions for beliefs about risky returns to measures of beliefs in survey responses from the University of Michigan Surveys of Consumers. The question we use is:

Think about a diversified stock fund which holds stock in many different companies engaged in a wide variety of activities. Suppose someone were to invest one thousand dollars in such a mutual fund. What do you think is the chance this one thousand

dollar investment will increase in value, so that it is worth more than one thousand dollars one year from now?

We use Michigan Survey responses to this question between June 2002 and December 2015. We merge these data to microdata samples, available through the Inter-University Consortium for Political and Social Research, which contain the respondent's race. We restrict our sample to include Black and White individuals between 25 and 65 years of age. The cleaned dataset contains 42,756 observations, of which roughly 10 percent identifies as Black.²²

The response to the Michigan Survey question is a probability assessment that a diversified equity fund increases in value. The difficulty in comparing beliefs in the model to survey responses is that respondents in the data may have a different benchmark return in their mind than zero (say, because of inflation) and that dynasties in our model have the option to invest in a safe technology featuring growth. Our solution is to choose a benchmark return in the model, called \bar{r} , such that the model-generated average probability assessment that risky investments exceed this benchmark \bar{r} matches the average probability assessment of 0.51 from the survey. Formally, for each dynasty we calculate the probability $P_t(\pi_{tt}, \bar{r})$ that the economy-wide risky return exceeds \bar{r} , evaluated under their model-generated belief π_{tt} in 2020:

$$P_t(\pi_{\iota t}, \bar{r}) = \int_{q; qr_t > \bar{r}} \pi_{\iota t}(q) \mathrm{d}q. \tag{20}$$

The model-generated average probability assessment equals $\int P_{\iota t} d\Phi = 0.51$ for $\bar{r} = 0.006$.

Having targeted the average response in the total population, we evaluate the ability of the model to generate dispersion of beliefs. Table 10 summarizes our results. In the first row, the mean probability of successful investment in the total population is the same in the model and the data by construction. The second and third rows show the mean probability for White and Black households. The racial gap in mean beliefs is 12 percentage points in the model, compared to 7 percentage points in the data. At the same time, the model generates a dispersion of beliefs in the total population as large as measured in the data. The model replicates almost perfectly

²²Responses to this question were studied by Dominitz and Manski (2011) for the period between June 2002 and August 2004. The authors find nearly identical results to the results we report in Table 10. Dominitz and Manski (2011) also report similar results for their Survey of Economic Expectations. Since the Survey of Economic Expectations only has 85 Black respondents, we focus on the Michigan Survey.

Table 10: Beliefs about Risky Returns

Probability of Successful Investment	Data	Model
Mean	0.51	0.51
Mean, White	0.51	0.53
Mean, Black	0.44	0.41
Std Deviation	0.29	0.28
Std Deviation, White	0.29	0.30
Std Deviation, Black	0.29	0.16

Table 10 presents statistics on the probability assessment of successful investments. Data entries denote means and standard deviations of the probability of an increase in the value of a diversified stock fund as reported in the Michigan Survey. The data contain 42,756 observations, out of which roughly 10 percent identifies as Black.

the dispersion of beliefs within White households, but underestimates the dispersion within Black households.²³

3.4 Racial Welfare Gap

We calculate the racial welfare gap and decompose it into a component reflecting differences in wages and a component reflecting differences in wealth. Our welfare gap is the tax $\mathcal{T}_{\iota t}$ that would make a White dynasty with $(z_{\iota t}^w, a_{\iota t}^w, \mathbb{E}_{\iota t}^w q^*)$ indifferent to being a Black dynasty with $(z_{\iota t}^b, a_{\iota t}^b, \mathbb{E}_{\iota t}^b q^*)$. Formally, we find the tax that solves:

$$V(z_{tt}^b, a_{tt}^b, \mathbb{E}_{tt}^b q^*) = \hat{V}(z_{tt}^w, a_{tt}^w, \mathbb{E}_{tt}^w q^*; \mathcal{T}_{tt}), \tag{21}$$

where V is the maximized expected utility in equation (10) that takes into account the optimal occupational choice and the allocation of resources between consumption and intergenerational transfers after the realization of the idiosyncratic investment shock. The value \hat{V} is the same value function but under tax \mathcal{T}_{tt} which subtracts resources from the income side of the budget constraint in equation (6).

 $^{^{23}}$ Consistent with the model in which the probability assessment P depends on variables that differ by race, Table 10 shows mean probabilities by group without controls. A regression of the probability assessment on a race dummy controlling for a host of observables (age, time, sex, marital status, region, education, number of children, number of adults, and income) produces an estimated coefficient for the race dummy equal to 5 percentage points with a standard error of 0.5. We corroborate our finding of a racial gap in beliefs about risky returns by finding that Black households are also more pessimistic in terms of expectations of five-year aggregate growth between 1990 and 2015.

Table 11: Racial Welfare Gaps

(relative to mean wealth, 2020)	Baseline	No Wages	No Wealth
Mean	0.14	0.11	0.10
99 percentile	1.26	1.25	1.07
95 percentile	0.18	0.06	0.14
90 percentile	0.12	0.04	0.09
75 percentile	0.07	0.02	0.05
50 percentile	0.03	0.01	0.02

Table 11 presents statistics of the welfare difference \mathcal{T} in equation (21) and decompositions of the welfare difference between the component due to differences in wages and differences in assets.

In calculating $\mathcal{T}_{\iota t}$, we compare White and Black dynasties who are identical in terms of history of wage and investment shocks $\{\varepsilon_{\iota j}, e_{\iota j}\}_{j=1}^{t}$. In the absence of historical exclusions, the two dynasties would thus be identical.²⁴ When both White and Black dynasties are laborers, the solution for the racial welfare gap for dynasties ι :

$$\mathcal{T}_{tt} = (z_{tt}^w + (1 + i_t - \delta)a_{tt}^w) - (z_{tt}^b + (1 + i_t - \delta)a_{tt}^b). \tag{22}$$

For laborers, the welfare gap equals the difference in available resources for consumption and intergenerational transfers in the budget constraint (6). Since both races have the same preferences and production technologies, equalizing resources leads to identical consumption and intergenerational transfers and eliminates welfare differences.

Table 11 presents the racial welfare gap at various percentiles of the welfare gap distribution. The gaps are annualized for ease of interpretation. In the first column, the median gap equals 0.03 times average wealth in the economy. Applying an average wealth of 750,000 dollars leads to a median welfare gap of 22,500 dollars per year. The mean welfare gap is 105,000 dollars. What explains this difference between the median and the mean? As the first column shows, the distribution of welfare gaps is significantly dispersed with the welfare gaps at the top exceeding the welfare gaps at the middle of the distribution.

²⁴More precisely, the dynasties are identical up to population differences. However, we eliminate population growth differences in Table 1 starting in 2000 so that the two dynasties are identical in the absence of exclusions starting in 2000.

To understand the relative importance of wages and wealth in accounting for the racial welfare gap, the second column presents the same statistics when we equalize wages and the third column presents the same statistics when we equalize wealth between Black and White dynasties ι that share the same history of shocks. The mean gap falls from 0.14 to 0.11 when we equalize wages and to 0.10 when we equalize wealth. The reductions in racial gaps do not add up to the total welfare gap, partly because wages and wealth covary positively in the cross section and partly because dynasties have different beliefs which we do not change. For the mean welfare gap, we find a larger contribution of wealth than wages. However, wages become the dominant source of the welfare gap for most of the distribution as wealth differences matter most at the top of the distribution.

It is instructive to compare our results to Brouillette, Jones, and Klenow (2021) who construct a measure of consumption-equivalent welfare for Black and White dynasties. The first difference between the two approaches is the sources of welfare differences. Relative to them, we incorporate utility from intergenerational transfers to descendants. Relative to us, they account for life expectancy, mortality, and leisure differences. The second difference is that we have an equilibrium framework that expresses arguments of the utility function (consumption and intergenerational transfers) as a function of primitives, while Brouillette, Jones, and Klenow (2021) change directly inputs to the utility function (for example, consumption) in measuring welfare differences. While our approach comes with added complexity, it has the conceptual advantage of being able to account for optimal responses with respect to changes in policies such as reparations.

Despite these differences, we highlight some similarities between our results and Brouillette, Jones, and Klenow (2021). The authors calculate a 35 percent gap in consumption equivalents for the recent period, translating to a mean welfare gap of roughly 45,000 dollars. Our mean welfare gap is almost twice as large because we take into account the tail of the wealth distribution that introduces a significant deviation between mean and median welfare gap. Our welfare gap aligns with the estimate of Brouillette, Jones, and Klenow (2021) around the 75 percentile of the welfare gap distribution.

Table 12: Counterfactual History

(2020, Black over White)	Wealth Ratio	Income Ratio	Entrepreneurship Gap
A. All Differences	0.17	0.36	-3.6
 Labor Exclusion 	0.43	0.60	-3.3
 Capital Exclusion 	0.17	0.36	-3.6
- Demographics	0.18	0.37	-3.6
B. No Differences	1.00	1.00	0.0
+ Labor Exclusion	0.18	0.37	-3.6
+ Capital Exclusion	0.47	0.62	-3.2
+ Demographics	1.37	1.28	0.9
C. Baseline	0.17	0.36	-3.6
$\beta^{1/20} = 0.95$	0.25	0.47	-2.7
$q^* = 0.70$	0.25	0.48	-2.3
$\bar{c} = 0.50Z$	0.24	0.39	-7.6
$\gamma = 3$	0.20	0.40	-0.6

Table 12 presents counterfactual outcomes when we remove historical events driving racial differences in the model (panel A), when we add historical events driving racial differences in the model (panel B), and when we change features of the model environment relative to the baseline (panel C).

3.5 How Historical Events Shape Today's Racial Gaps

We conclude this section by evaluating the effects of historical events and features of the model economy on current racial gaps. Table 12 summarizes our results. Panel A shows the racial gap in wealth, income, and the entrepreneurship rate when we remove historical events. Panel B shows racial gaps when we add historical events starting from no racial differences. Panel C shows racial gaps when we change features of the model economy.

The top panel shows that removing the history of labor market exclusions results in a ratio of average wealth of 0.43 as opposed to 0.17 in our baseline model with these exclusions, an income ratio of 0.60 as opposed to 0.36, and a gap in entrepreneurship rate of 3.3 percentage points as opposed to 3.6. Without labor market exclusions, we would also observe gaps today because capital market exclusions prohibit Black dynasties from becoming entrepreneurs. On the other hand, removing only capital market exclusions does not affect current outcomes. Labor market

exclusions on their own put Black dynasties at significant disadvantage in terms of becoming capitalists and accumulating wealth, making the incremental effect of capital market exclusions in our model zero.

The middle panel confirms these conclusions by adding historical events one at a time. If there were only demographic differences, Black dynasties in our model would achieve a higher wealth, income, and entrepreneurship rate than White dynasties by 2020. Following Table 1, in early periods the population growth of White dynasties is higher than the population growth of Black dynasties, making it more costly for White dynasties to transfer wealth across generations on a per capita basis.

In the lower panel we assess the role of structural parameters for racial gaps. A lower discount factor β is associated with a higher wealth and income ratio, as dynasties have a lower willingness to bequeath wealth. A lower probability of successful investments q^* is also associated with a higher wealth and income ratio, as White capitalists realize lower income from risky investments. A lower level of subsistence consumption \bar{c} is associated with higher wealth and income ratios, as a smaller fraction of Black dynasties are concentrated at the bottom of the distribution where \bar{c} tends to bind. Finally, wealth and income ratios increase under a higher curvature in the utility function, $\gamma = 3$, because increased risk aversion induces more dynasties to choose safe labor over risky investment activities.

4 Reparations

We begin by studying the effects of wealth transfers toward Black dynasties. Then, we analyze the alternative policy of subsidizing Black risky investments and the role of learning from others for wealth convergence.

4.1 Wealth Transfers

We first discuss the financing of wealth transfers and their size. Next, we discuss our assumptions on the future racial wage gap. We then present the effects of wealth transfers on various outcomes by race.

4.1.1 Transfers and Financing

Wealth transfers to Black dynasties are financed by a one-time unanticipated tax on White dynasties' wealth. The wealth tax rate function, Λ , for White dynasties is:

$$\Lambda(a_{\iota}^{w}) = \begin{cases} \lambda & \text{if } a_{\iota}^{w} < a^{*}, \\ (1-\lambda)\frac{a_{\iota}^{w} - a^{*}}{a_{\iota}^{w}} & \text{if } a_{\iota}^{w} \ge a^{*}. \end{cases}$$
(23)

We consider two forms of financing which differ in their progressivity.

- 1. Progressive wealth taxation corresponds to $\lambda = 0$ and $a^* > 0$. Wealth below a^* is not taxed and wealth above a^* is taxed at a 100 percent rate.
- 2. Proportional wealth taxation corresponds to $\lambda > 0$ and $a^* \to \infty$. All wealth is taxed at a proportional rate λ .

We consider transfers τ to Black dynasties resulting in average wealth of Black dynasties being a multiple m of average wealth of White dynasties. For any multiple m, we solve for the tax parameter (either λ or a^*) such that every Black dynasty receives a lump sum equal to τ :

$$\int (a_t^b + \tau) d\Phi^b = m \int (1 - \Lambda(a_t^w)) a_t^w d\Phi^w, \quad \text{such that} \quad \phi \tau = (1 - \phi) \int \Lambda(a_t^w) a_t^w d\Phi^w, \tag{24}$$

where ϕ is the share of Black dynasties in the population.²⁵

Our baseline policy is m=1, so that average wealth is equalized between Black and White dynasties at the time of reparations. This is a natural policy because, in the absence of historical exclusions, average wealth is equal for Black and White dynasties. The size of the required transfers is $\tau=0.75$ relative to the average wealth in the economy. Given average household wealth of 750,000 dollars, we obtain a transfer of 562,500 dollars per Black dynasty. Using the Census count of 18 million Black households, aggregate reparation transfers equal roughly 10 trillion dollars. To finance these wealth transfers, the progressive policy taxes at 100 percent all White wealth exceeding 113 times average wealth, roughly 84 million dollars. The proportional tax rate to finance these transfers is 13 percent.

 $^{^{25}}$ Every Black dynasty receives the same transfer τ . We have experimented with two alternative transfer functions, proportional transfers and transfers that generate the same dispersion of wealth between Black and White dynasties. Both give similar evolutions of the racial wealth gap.

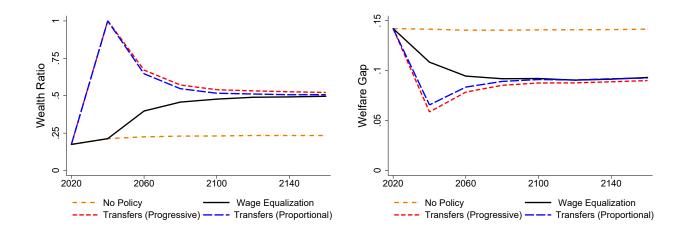


Figure 6: Evolution of Racial Wealth and Welfare Gaps

The left panel of Figure 6 shows the wealth ratio under no policy changes (orange dot-dashed line), after eliminating the racial wage gap in 2040 (black solid line), after eliminating the racial wage gap and providing wealth transfers financed with progressive wealth taxes in 2040 (red dashed line), and after eliminating the racial wage gap and providing wealth transfers financed with proportional wealth taxes in 2040 (blue long-dashed line). The right panel presents the average welfare gap under the same scenarios.

Our approach is to calculate the tax and transfer system that eliminates the wealth gap between Black and White dynasties today. Consistent with our approach, Darity and Mullen (2020) propose as criterion the equalization of average wealth and arrive at 8 trillion dollars. Despite its simplicity, our approach results in total wealth transfers of similar magnitude to other estimates in the literature based on the present discounted value of foregone earnings from slavery. For example, Neal (1990) calculates wealth transfers of 1.4 trillion dollars in 1983. Applying the average real return, $i\frac{\int (1-k_t)a_t d\Phi}{\int a_t d\Phi} + q^*r\frac{\int k_t a_t d\Phi}{\int a_t d\Phi} = 0.05$, and adding an inflation rate of 0.02 yields 12 trillion dollars.

4.1.2 Wage Equalization

Average wealth differences between Black and White dynasties persist forever in the presence of racial differences in wages. To separate the effects of reparations from the effects of different wages on wealth, we assume that, at the time of reparations, labor market policies are enacted that permanently close the racial wage gap. Formally, we set the racial component of wages to $\mu_t^b = 1$ in equation (16) for all periods starting in 2040.

The left panel of Figure 6 presents the evolution of the ratio of average Black to average White

wealth. To examine the effect of reparations on the racial wealth gap, we require a benchmark of where the economy converges in the absence of transfers. This benchmark is the racial wealth ratio when we permanently equalize mean wages between Black and White dynasties. Even without any policies we would observe further convergence of the wealth ratio, which tends to about 0.25 as shown by the orange dot-dashed line. Eliminating wage differences increases the wealth ratio which now converges to roughly 0.5 in the black solid line. Thus, the value of 0.5 is the long-run wealth ratio absent reparations to which we compare reparations.

4.1.3 Effects of Transfers on Wealth and Welfare Gaps

The left panel of Figure 6 shows the evolution of the wealth ratio after transfers toward Black dynasties financed either with progressive taxes (red dashed line) or proportional taxes (blue long-dashed line). The transfers are enacted in addition to the wage equalization policy. By construction, the racial wealth ratio is one in 2040 under our baseline policy which calculates the tax and transfer system that equalizes average wealth. However, under both financing systems, the wealth ratio reverts back to a value of approximately 0.5, which is the long-run wealth ratio in the absence of wealth transfers. Thus, the long-run effect of one-time wealth transfers on the racial wealth gap is zero.

The logic of why transfers do not lead to wealth convergence in the future follows the example of Section 2.5. Following centuries of exclusions from labor and capital markets, Black dynasties enter the reparations era with pessimistic beliefs about investment returns. Despite having equal mean wages and wealth, the lower expected returns lead Black dynasties to forego risky investments and to continue to be laborers. Over time, the economy converges to the same outcome observed without reparations.

The right panel of Figure 6 shows the evolution of the racial welfare gap. By eliminating the wage gap, the welfare gap declines from 0.14 to 0.09. After wealth transfers to Black dynasties, the welfare gap decreases even more in the short run. However, similar to the wealth gap, after few periods the welfare gap tends to the same value we would observe without transfers.

Next, we explore distributional effects of wealth transfers. We define the representation index

in period t:

$$R_t(p) = \frac{\int_p^1 \mathbb{I}(h_t = b) a_{tt} d\Phi_t}{\phi_t \int_p^1 a_{tt} d\Phi_t},$$
(25)

where p denotes the percentile of the wealth distribution, $\mathbb{I}(h_t = b)$ is an indicator that selects Black dynasties, and ϕ_t is the Black population share. The index captures Black representation in wealth. The numerator equals wealth of Black dynasties in the top p percentiles of the total wealth distribution. The integral in the denominator equals wealth of all dynasties in the top ppercentiles. We obtain $R_t(0) = 1$ when average wealth is equalized across race. Black dynasties are represented equally, $R_t(p) = 1$ for all p, when the Black wealth share equals the Black population share at all percentiles of the wealth distribution. The long-run outcome we obtain in the absence of historical exclusions is $R_t(p) = 1$. When Black households are underrepresented in wealth, $R_t(p) < 1$.

Figure 7 shows representation at percentiles of the wealth distribution in 2040, 2060, 2100, and the long run. After reparation transfers in 2040, average wealth is equalized and so representation equals 1 at the zero percentile. The transfers lead to an overrepresentation of Black dynasties between the 50th and the 75th percentile because every Black dynasty receives the same transfer τ which exceeds the wealth of the median White dynasty. Consistent with the divergence of wealth and welfare we documented before, over time the representation of Black dynasties tends to values below one. As the last panel demonstrates, in the long run Black dynasties do not have greater representation than in the absence of transfers. Wage equalization improves Black representation, but is still significantly lower than the one without historical exclusions.

Could larger wealth transfers lead to long-run convergence? The first row of Table 13 repeats the evolution of the wealth ratio for the baseline policy which equalizes wealth between Black and White dynasties. In the second row, reparation transfers target a wealth ratio of m = 3, which means that we transfer to Black dynasties enough wealth to make them three times as wealthy as White dynasties. Under m = 3, the transfer per Black dynasty is 1.5 million dollars and the progressive tax starts at wealth above 17 million. While the divergence of wealth is slower, the wealth ratio asymptotes again toward roughly 0.5, as with m = 1.

 $^{^{26}}$ The long-run outcomes refer to the median outcome over the last 300 model periods from a simulation of 600

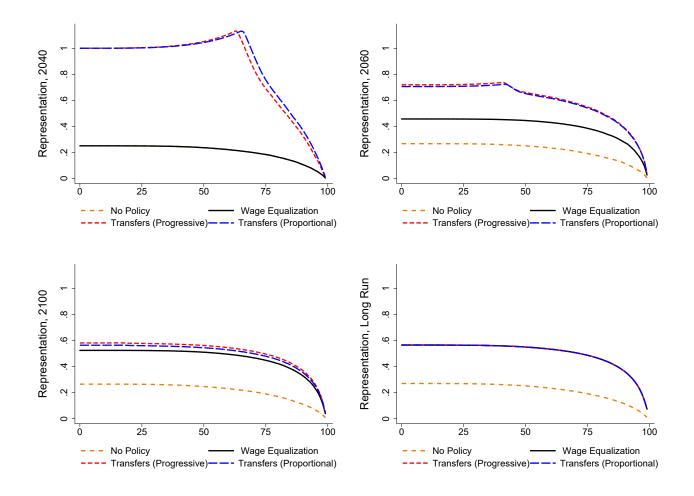


Figure 7: Representation in Wealth: Transfers

Figure 7 shows representation in equation (25) as a function of the percentile of the wealth distribution in 2020, 2040, 2100, and the long run. In each panel, we demonstrate the index under no policy changes (orange dot-dashed line), after eliminating the racial wage gap in 2040 (black solid line), after eliminating the racial wage gap and providing wealth transfers financed with progressive wealth taxes in 2040 (red dashed line), and after eliminating the racial wage gap and providing wealth transfers financed with proportional wealth taxes in 2040 (blue long-dashed line)

4.2 Investment Subsidies

What policy can eliminate the racial wealth gap in the long run? Following centuries of labor and capital market exclusions, Black dynasties enter into reparations with pessimistic beliefs, making their entrepreneurship rates relatively inelastic to wealth transfers. This suggests that more effective policies directly target the trade-off between labor and capital. Instead of giving a transfer, we now use collected tax revenues to finance an investment subsidy for Black capitalists. model periods.

Table 13: Wealth Transfers versus Investment Subsidies

	(relative to	mean wealth)	Wealth Ratio (Black/White)		
	Transfer	Financing	2040	2100	∞
1.	$\tau = 0.75$	$a^* = 113$	1.00	0.54	0.52
2.	$\tau = 2.01$	$a^* = 23$	3.00	0.70	0.52
3.	s = 0.22	$a^* = 113$	0.24	0.59	0.85
4.	s = 0.27	$a^* = 23$	0.32	0.96	1.00

Table 13 presents the wealth ratio in selected periods under four reparation programs. The two top rows consider wealth transfers while the two bottom rows consider investment subsidies. Both transfers and subsidies are financed with progressive taxes. In row 1 transfers eliminate the racial wealth gap in 2040 (m = 1) and in row 2 the average wealth of Black dynasties is three times (m = 3) higher than the average wealth of White dynasties in 2040. In rows 3 and 4 we use the same revenues as collected in rows 1 and 2 to finance a subsidy s which is the additional annualized return on successful risky investment activities.

We calculate the subsidy s that exhausts the revenues collected by taxing the wealth of White dynasties:

$$\phi q^*(r+s) \int a_i^b k_i^b d\Phi^b = (1-\phi) \int \Lambda(a_i^w) a_i^w d\Phi^w.$$
(26)

The two bottom rows of Table 13 summarize our results when investment subsidies are financed with progressive taxes. Tax revenues finance an investment subsidy of 22 percentage points in row 3 and an investment subsidy of 27 percentage points in row 4. The smaller subsidy generates a racial wealth ratio of 0.24 in 2040. While the policy does not eliminate the wealth gap at the time of the subsidy, it incentivizes risky investments by Black dynasties. Investment subsidies are more powerful than wealth transfers in increasing the wealth ratio in the long run, with the wealth ratio tending to 0.85. In row 4, a 27 percentage points subsidy toward Black capitalists eliminates the racial wealth gap in the long run. Such subsidies effectively offset the pessimistic beliefs of Black dynasties at the time of reparations and reset the conditions of Black dynasties in 2040 to the same conditions of White dynasties.

Figure 8 shows Black representation following subsidies toward risky investment activities. Compared to Figure 7 for wealth transfers, subsidies do not increase the representation of Black dynasties in 2040 as much as transfers. Consistent with the evolution of the average wealth gap, subsidies are more powerful than transfers in generating equal representation of Black dynasties

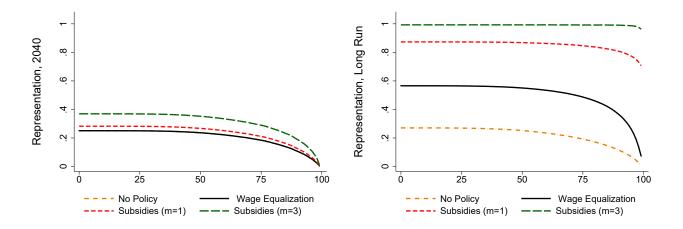


Figure 8: Representation in Wealth: Subsidies

Figure 8 shows representation in equation (25) as a function of the percentile of the wealth distribution in 2020, 2040, 2100, and the long run. In each panel, we demonstrate the index under no policy changes (orange dot-dashed line), after eliminating the racial wage gap and providing investment subsidies financed with progressive wealth taxes in 2040 (red dashed line), and after eliminating the racial wage gap and providing larger investment subsidies financed with progressive wealth taxes in 2040 (green long-dashed line).

in the long run. In fact, with m=3, Black dynasties are equally represented in wealth in the long run.

4.3 Learning from Others' Experiences

An alternative possibility for the convergence of wealth is that Black dynasties learn from others' experiences after reparations. To understand the importance of networks for the speed of learning and the convergence of wealth, we let dynasties learn from others' experiences. Our formulation of learning from others is that dynasties observe outcomes of other dynasties around them, which we label friends. Every dynasty has a number of own-race friends F^o and a number of cross-race friends F^c . The number of friends is the number of additional experiences that each dynasty potentially observes in a given period, in addition to their own experience. Dynasties observe the entrepreneurial experiences of friends who are entrepreneurs, but do not learn from friends who are laborers. As in our baseline model, dynasties do not learn from aggregate variables.²⁷

²⁷We formalize learning as follows. Friends of dynasty ι are denoted f_{ι} . To construct the set of friends, we order dynasties along a unit circle and denote a dynasty's location by $\iota \in \mathbb{I} \equiv [0,1]$. Dynasties in the interval $\mathbb{I}^w \equiv [0,\phi)$ are White, while dynasties in the interval $\mathbb{I}^b \equiv [\phi,1]$ are Black. Similarly, every dynasty has an identity $\iota^h \in [0,1]$ along a race-specific unit circle, where $\iota^w \equiv \frac{\iota}{\phi}$ for every $\iota \in \mathbb{I}^w$ and $\iota^b \equiv \frac{\iota - \phi}{1 - \phi}$ for every $\iota \in \mathbb{I}^b$. Friends are drawn

Table 14: Learning from Others' Experiences After Reparations

	Friends		Wealth Ratio (Black/White)		
	F^o	F^c	2040	2100	∞
1.	0	0	1.00	0.54	0.52
2.	1	0	1.00	0.53	0.52
3.	2	0	1.00	0.53	0.55
4.	4	0	1.00	0.53	0.63
5.	10	0	1.00	0.52	0.94
6.	1	1	1.00	0.55	1.00

Table 14 presents the wealth ratio under wealth transfers financed with progressive taxes. Each dynasty has a number of own-race friendships F_o and a number of cross-race friendships F_c . The total number of friends equals the number of additional experiences that each dynasty observes in a given period, in addition to their own experience.

Table 14 summarizes our results when transfers are financed with progressive taxes. The first row repeats the baseline results without learning from others. In row 2, we allow dynasties to potentially observe the experience of one additional dynasty from their own race, $F^o = 1$. The next three rows increase the number of friends from a dynasty's own race F^o to 2, 4, and 10. When dynasties learn from the experiences of others, they update their beliefs at a faster rate toward the objective probability of success in entrepreneurship q^* . As the table shows, the wealth ratio increases as we increase the number of friends. To eliminate the racial wealth gap in the long run, the model requires more than 10 additional observations. As the wealth ratio in 2100 demonstrates, the speed of convergence is in all cases quite slow.

Another possibility is that dynasties learn from dynasties of the other race. In row 6 of Table 14, the racial wealth gap is eliminated after reparations if Black dynasties can observe at least one White dynasty. We interpret this result as showing the importance of interracial networks. To the extent that networks are amenable to policy or technological change, increased learning from others' experiences can complement reparations in closing the racial wealth gap.²⁸

from the set of dynasties who are within distance d on the circle. By construction, the friendships are not mutual. Dynasties draw F^o friends along the circle specific to their race, $f_\iota^o \sim U([\iota^h, \iota^h + d]^{F^o})$, and F^c friends along the circle for the cross-race, $f_\iota^c \sim U([\iota^h, \iota^h + d]^{F^c})$. For White dynasties, the identities of friends along the unit circle are given by the union $f_\iota = \phi f_\iota^o \cup (\phi + (1 - \phi) f_\iota^c)$, while for Black dynasties, friends are $f_\iota = \phi f_\iota^c \cup (\phi + (1 - \phi) f_\iota^c)$.

²⁸An example of such a policy is the Gates Scholarship, which is geared toward outstanding minority students

5 Conclusion

In this paper we develop a dynamic, quantitative, framework for analyzing the economics of reparations. The model features heterogeneous dynasties, an occupational choice, bequests, and endogenous dispersion of beliefs about risky returns. Feeding historical events which exclude Black dynasties from labor and capital markets as driving force, the model is consistent with salient features of the data such as current and historical racial gaps in wealth, income, entrepreneurship, and mobility. Methodologically, our model thus contributes to the wealth inequality literature by highlighting the importance of divergent beliefs about risky returns.

We reach a negative answer to the first question we pose "will reparations today in the form of direct wealth transfers eliminate the racial wealth gap in the future?" Transfers eliminating the racial wealth gap today do not generate convergence of wealth, welfare, and equal representation in the long run. Thus, wealth transfers do not compensate appropriately for historical exclusions. The logic of the divergence result is that century-long exclusions lead Black dynasties to enter into the reparations era with pessimistic beliefs about risky returns and forego risky investment opportunities. We corroborate this model prediction by presenting survey evidence suggesting significant racial gaps in beliefs about risky returns.

We reach a positive answer to the second question we pose "is there a policy that is effective in eliminating racial gaps?" We show that investment subsidies toward Black dynasties can achieve convergence of racial outcomes in the long run. Thus, investment subsidies can compensate appropriately for historical exclusions. We also highlight the possibility that Black dynasties may learn faster after reparations if information networks strengthen compared to the past. Stronger network can complement other forms of reparations to eliminate racial gaps in the future.

from low-income backgrounds by the Bill & Melinda Gates Foundation. The Foundation organizes a Summer Institute and uses an online platform to enable current and former recipients to directly connect with and learn from one another.

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