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ABSTRACT

Reparations is a policy proposal aiming to address the wealth gap between Black and White households. We provide a first formal analysis of the economics of reparations using a long-run model of heterogeneous dynasties with an occupational choice and bequests. Our innovation is to introduce endogenous dispersion of beliefs about risky returns, reflecting differences in dynasties' experiences with entrepreneurship over time. Feeding the exclusion of Black dynasties from labor and capital markets as driving force, the model quantitatively reproduces current and historical racial gaps in wealth, income, entrepreneurship, mobility, and beliefs about risky returns. We use the model to evaluate reparations and find that transfers eliminating the racial gap in average wealth today do not lead to wealth convergence in the long run. The logic is that century-long exclusions lead Black dynasties to enter into reparations with pessimistic beliefs about risky returns and to forego investment opportunities. We conclude by showing that entrepreneurial subsidies are more effective than wealth transfers in achieving racial wealth convergence in the long run.

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1 Introduction

Senator Booker introduced the first reparations bill in recent times to "address the fundamental injustice, cruelty, brutality, and inhumanity of slavery in the United States and the 13 American colonies between 1619 and 1865 and to establish a commission to study and consider a national apology and proposal for reparations for the institution of slavery..." While most policymakers have not explicitly endorsed wealth transfers to descendants of slaves yet, prominent cosponsors of the bill expressed their support to study reparations and make recommendations on policies.[1]

In this paper, we provide a first formal economic analysis of reparations. By design, our framework subscribes to the logic underlying reparations that racial wealth gaps do not reflect innate racial differences in ability, willingness to work, or beliefs but, instead, emerge from century-long exclusions of Black dynasties from labor and capital markets. We use this framework to evaluate the hope behind reparations. This hope is best summarized by the views of two prominent scholars, William "Sandy" Darity Jr. and Kirsten Mullen, in their discussion of reparations (https://brook.gs/3j1soQs):

> *"The wealth gap will not persist if the target of well-executed reparations is direct elimination of it."*

Motivated by this background, we ask: Will reparations today in the form of wealth transfers eliminate the racial wealth gap in the future? If not, is there a policy that is effective in eliminating this gap?

We answer these questions in three steps. We begin by developing a long-run equilibrium model with heterogeneous dynasties to quantify the sources of racial gaps in wealth, income, entrepreneurship, and mobility. The model shares two features with the wealth inequality literature (that we discuss further below). Motivated by the role of intergenerational transfers for persistence in wealth gaps, dynasties in our model choose how to allocate their resources between consumption for the current generation and wealth transmitted to descendants. Motivated by the observation that mostly entrepreneurs occupy the top of the wealth distribution, dynasties choose how to

[1] The bill, introduced in April 2019, is available at https://bit.ly/3oyNG9p. Prominent cosponsors include Vice President Kamala Harris, Senator Amy Klobuchar, Senator Bernie Sanders, and Senator Elizabeth Warren.

allocate their lifetime between labor activities and risky entrepreneurship.

The innovation of our framework relative to the wealth inequality literature is that it endogenously generates divergence of beliefs about entrepreneurial returns. Entrepreneurship takes as inputs time and capital and produces uncertain output. The return from entrepreneurship is unknown and each generation begins with a prior belief over the probability that entrepreneurial activities are successful. Dynasties who become entrepreneurs observe their investment outcome, update their beliefs, and transmit them to the next generation. Successful entrepreneurs transmit more optimistic beliefs to their descendants, unsuccessful entrepreneurs transmit more pessimistic beliefs to their descendants, while laborers do not update their beliefs lacking own entrepreneurial experiences. As a result, our model generates dispersion of expected returns from entrepreneurship, reflecting the accumulation of experiences from previous generations.[2]

We feed as driving forces to the model historical labor and capital exclusions which prohibit Black dynasties from participating in markets. By labor market exclusions, we mean both slavery taking place in our model between the Declaration of Independence in 1776 and the 13th Amendment in 1865 and lower wages for Black dynasties until today. By capital market exclusions we mean historical events such as discrimination in patenting, redlining, Jim Crow segregation laws, and exclusion from credit markets. These policies exclude Black dynasties from becoming entrepreneurs in our model until the Civil Rights Movement in the 1960s.

In the second step, we parameterize the model to evaluate its ability to account quantitatively for salient features of the data. The model is successful in accounting for the significant wealth and income dispersion in the data, both for the total population and for the population of entrepreneurs. While not targeted by the parameterization, the model matches the level of the racial wealth gap today and its evolution since the early 1900s. Additionally, the model is consistent with observed mobility patterns, where White dynasties are more likely than Black dynasties to see their children improve their rank in the income distribution. Similar to historical data, the

[2] An example we use below to illustrate model mechanisms is the Rockefeller dynasty. When asked how they manage to preserve wealth over centuries, David Rockefeller Jr., chairman of Rockefeller & Co., stated (https://cnb.cx/2YwePiE) that the family has developed a system of values, traditions and institutions that have helped the family stay together and preserve their wealth. The family meets twice per year in a forum where heirs talk about the family's direction, projects, and other family news related to careers or important milestones.

mobility gap in the model is more profound in the early 1900s than in recent times.

The model generates significant racial gaps partly because White dynasties participate more in entrepreneurial activities than Black dynasties. When White dynasties have positive experiences, they update upward their beliefs about returns from entrepreneurship and accumulate wealth over time. Black dynasties initially faced slavery and later face lower wages. They do not become entrepreneurs, which means they do not update their beliefs and do not accumulate as much wealth as White dynasties.

Confronted with historical labor and capital market exclusions, the model generates racial gaps without imposing racial differences in initial beliefs or wealth. We highlight the importance of general equilibrium for the divergence of wealth. Early on, when Black dynasties are enslaved, returns from entrepreneurship are high because assets are relatively unexploited (such as land). This makes entry to entrepreneurship worthwhile for White dynasties even if, initially, their beliefs are pessimistic and their wealth is low. As wealth in the economy accumulates, returns fall over time which dissuades Black dynasties from becoming entrepreneurs even after emancipation.

We evaluate whether model predictions concerning entrepreneurial beliefs align with available data. We first confirm the well-known observation that Black households are less likely to be entrepreneurs than White households. Using Michigan Survey data asking respondents their probability assessment over whether a diversified equity fund would increase in value, we uncover evidence that Black households are more pessimistic than White households about risky returns. Despite not targeted by the parameterization, the model generates a racial belief gap and dispersion of beliefs in the population similar to the data.

Armed with a model consistent with salient empirical observations on wealth, income, entrepreneurship, mobility, and beliefs, we proceed to assess reparations. Our first result is that, even with transfers that perfectly equate average wealth today, average wealth of Black and White dynasties diverges again in the future. The reasoning is that Black dynasties enter into the reparations era with more pessimistic beliefs about the returns from entrepreneurship and this era is characterized by a relatively low return to wealth accumulation. As a result, most Black dynasties forego investment opportunities in entrepreneurship despite increased wealth.

Strikingly, future wealth diverges even with transfers that make the average Black dynasty wealthier than the average White dynasty today. Wealth transfers are not powerful enough to change the trade-off between labor and entrepreneurship. A policy that targets precisely this trade-off is return subsidies. We show that return subsidies to Black entrepreneurs are more effective than wealth transfers in equalizing average wealth in the long run. Eliminating the wealth gap in the long run requires subsidies equal to 27 percentage points of additional return, financed with taxation of White dynasties' wealth at 100 percent above roughly 13 million dollars. Another possibility for closing the wealth gap is that Black entrepreneurs learn from others' experiences after reparations. This possibility, however, raises the question of why learning from others' experiences did not occur earlier in history, thus leading to today's gap in wealth and entrepreneurship.

This paper contributes to three literatures. Early work by Blau and Graham (1990) concludes that racial differences in intergenerational transfers account for most of the racial wealth gap.[3] Quadrini (2000) and Cagetti and De Nardi (2006) demonstrate the importance of entrepreneurship for wealth inequality, as entrepreneurs occupy most of the top of the wealth distribution. Benhabib, Bisin, and Zhu (2011) stress the role of capital income risk for the upper tail of the wealth distribution, while Benhabib, Bisin, and Luo (2019) show that accounting for both inequality and mobility requires a combination of stochastic earnings, heterogeneity in saving rates, and capital income risk. Gabaix, Lasry, Lions, and Moll (2016) show the importance of correlated returns with wealth for the fast transitions of tail inequality in the data. Our innovation relative to the wealth inequality literature is to introduce dispersion of expected returns. Different from models which treat "entrepreneurial productivity" as an exogenous process, differences in expected returns emerge endogenously in our model from accumulated entrepreneurial experiences.[4]

[3]Quantitative work on wealth inequality, such as De Nardi (2004), highlights the role of bequests and non-homothetic preferences for the emergence of large estates and the transmission of wealth across generations. Non-homothetic preferences allow models to account for the observation that households with higher lifetime income (Dynan, Skinner, and Zeldes, 2004; Straub, 2019) or higher wealth (Fagereng, Holm, Moll, and Natvik, 2019) exhibit higher saving rates.

[4]Consistent with our model that generates a positive correlation between wealth and expected return from entrepreneurship, Bach, Calvet, and Sodini (2020) use asset pricing models to document that households with higher wealth exhibit higher expected returns. While the more novel part of our model is heterogeneity in expected returns, our model is also consistent with the "scale dependence" emphasized by Gabaix, Lasry, Lions, and Moll (2016) because wealthier dynasties are more likely to be entrepreneurs and realize higher expost returns. Fagereng, Guiso,

A natural prediction of models with entrepreneurship is that wealth transfers toward poor lead to a rise in recipients' entrepreneurship rates and a convergence of wealth between recipients and non-recipients. However, Bleakley and Ferrie (2016) present historical evidence from a large wealth redistribution program, Georgia's Cherokee Land Lottery in 1832, that descendants of families who received wealth transfers did not experience higher education, income, and wealth than descendants of non-recipients. Bleakley and Ferrie (2016) conclude that financial resources play a limited role in intergenerational outcomes as opposed to other factors which may persist through family lines. This other factor in our model is beliefs about returns to entrepreneurship. If beliefs were homogeneous, a one-time reparation wealth transfer would perfectly eliminate the racial wealth gap forever. Owing to the more pessimistic beliefs of Black dynasties at the time of reparations, our model instead predicts divergence of wealth after reparations.

The second related literature concerns social capital and the transmission of culture.[5] Fogli and Veldkamp (2011) study the transition of women into the labor force in a model of learning by sampling from a small number of other women. As information about the effects of maternal employment on children accumulates, the effects of maternal employment become less uncertain, and more women enter into the labor force. Fernandez (2013) demonstrates the role of cultural transmission of beliefs about wages for women's rising labor force participation. The transmission of beliefs across generations reflects both parental beliefs and a noisy observation of aggregate labor force participation. Buera, Monge-Naranjo, and Primiceri (2011) develop and estimate a model in which a country's own and its neighbors' past experiences shape the evolution of policymakers' beliefs about the desirability of free market policies.

Our baseline learning mechanism differs from these papers because dynasties learn about risky returns only based on their own experiences. Earlier work such as Piketty (1995) highlights the role of learning from own mobility experiences for voters' attitudes on redistribution. Similar to the model of Piketty (1995) in which experimenting is unattractive and the cultural transmission model of Guiso, Sapienza, and Zingales (2008) in which learning occurs only upon participation,

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Malacrino, and Pistaferri (2020) document a positive correlation of expost returns with wealth and, consistent with our model, persistence of returns across generations.

[5] Bisin and Verdier (2011) review the literature on the intergenerational transmission of culture.

in our model heterogeneity in beliefs persists forever as some dynasties do not enter into entrepreneurship.[6] Different from models in which cultural traits are inherited or imitated, Bisin and Verdier (2001) demonstrate that parental choices of transmitting traits to children induce heterogeneity in the long-run distribution of cultural traits in the population.

Finally, our work contributes to the racial gaps literature. Darity and Frank (2003) narrate century-long exclusions of Black dynasties from labor and capital markets and offer practical proposals on the implementation of reparations.[7] Recent work by Aliprantis, Carroll, and Young (2019) uses a model with heterogeneous agents to quantify racial differences and concludes that one-time wealth transfers have transitory effects on the wealth gap unless they also address the racial gap in labor earnings. In our model, instead, wealth diverges after reparations even if we eliminate forever the labor earnings gap. Hsieh, Hurst, Jones, and Klenow (2019) study the allocation of talent and economic growth in a model of occupational choice. Like these authors, our model does not feature differential changes in innate abilities by race over time and our counterfactuals remove exclusions preventing Black dynasties from participating in labor markets. Different from Hsieh, Hurst, Jones, and Klenow (2019) who do not consider wealth accumulation, our interest lies in how the racial wealth gap emerged from historical events and how it will evolve in the future after reparations.

Our conclusion that reparations in the form of transfers do not eliminate the racial wealth gap in the long run is reminiscent of the conclusion of Loury (1977) for labor market policies aiming to equalize racial outcomes. Loury (1977) argues that equal opportunity policies may not completely eliminate racial inequality because labor market outcomes also depend on accumulated social capital and networks that disadvantage Black households. The parallel we draw with Loury (1977) is that equalizing wealth in our model does not suffice to eliminate the racial wealth gap

[6]In our model, learning depends, directly, on own entrepreneurial experiences and, indirectly, on the aggregate risky return. In their study of perceptions of intergenerational mobility, Alesina, Stantcheva, and Teso (2018) offer evidence that individuals who have experienced upward mobility in their own life are more optimistic about mobility. In the context of forming inflation expectations, Malmendier and Nagel (2016) present evidence that individuals put more weight on personal experiences than on other available historical data, especially following periods of highly volatile inflation. Giuliano and Spilimbergo (2014) present evidence that individuals who experienced a recession when young believe that success in life depends more on luck than on effort.

[7]We confront quantitatively the model with historical evidence on racial gaps in wealth, wages, and mobility from Higgs (1982) and Margo (1984), Margo (2016), and Collins and Wanamaker (2017).

in the future because, at the time of reparations, Black dynasties have accumulated fewer positive investment experiences from their network.

2 Model

We first present the model and characterize its equilibrium. We then discuss key mechanisms through an example.

2.1 Environment

The economy is populated by a continuum of heterogeneous dynasties indexed by $\iota \in [0, 1]$. The horizon is infinite and periods $t = 1, 2, \ldots$ represent the length of economic life for a generation within a dynasty. We denote by Φ_t the distribution of dynasties in period t.

Demographics. Dynasty ι in period t has size $N_{\iota t}$. The evolution of $N_{\iota t}$ is given by:

$$N_{\iota t+1} = (1 + n_{\iota t+1})N_{\iota t}, \tag{1}$$

where $n_{\iota t+1}$ is the population growth rate of dynasty ι between periods t and $t + 1$. The total population is $N_{t+1} = \int N_{\iota t+1} \mathrm{d}\Phi_t$ and the population growth rate is $1 + n_{t+1} \equiv N_{t+1}/N_t$.

Technology. The model features an occupational choice between labor and entrepreneurship, motivated by the observation that the majority of households at the top of the wealth distribution are entrepreneurs. Each generation within a dynasty is endowed with one unit of time. Generations allocate fraction $1 - k_{\iota t}$ of their lifetime to a safe technology which we call labor and fraction $k_{\iota t}$ to a risky technology which we call entrepreneurship. The choice of time $k_{\iota t} \in [0, 1]$ is continuous.

The safe technology produces labor income from working and capital income from investing in a risk-free asset. Dynasties who allocate fraction $1 - k_{\iota t}$ of their time to the safe technology earn total income:

$$(z_{\iota t} + i_t a_{\iota t})(1 - k_{\iota t}), \tag{2}$$

where $z_{\iota t}$ is labor productivity and i_t is the safe return on assets $a_{\iota t}$.

Operating the risky technology requires time and, thus, dynasties who allocate time $k_{\iota t}$ to entrepreneurship forego labor income. Entrepreneurship entails risk as the choice of $k_{\iota t}$ is made before the entrepreneurial shock is realized. Allocating $k_{\iota t}$ to entrepreneurship produces capital income:

$$\begin{aligned} r_t a_{\iota t} k_{\iota t}, \quad &\text{if} \quad e_{\iota t} = G, \\ 0, \quad &\text{if} \quad e_{\iota t} = B. \end{aligned} \tag{3}$$

The output of entrepreneurship depends on the realization of an idiosyncratic shock $e_{\iota t}$. If the dynasty's experience is good, $e_{\iota t} = G$, entrepreneurship yields a net return r_t per unit of asset invested $a_{\iota t}$. If the dynasty's experience is bad, $e_{\iota t} = B$, entrepreneurship yields a net return of zero. The return r_t is determined in equilibrium and taken as given by entrepreneurs.

Beliefs. Our modeling innovation is to introduce heterogeneity in beliefs about returns from entrepreneurship. Dynasties do not know a priori the objective probability of a good experience, which we denote by $q^* = \mathbb{P}(e_{\iota t} = G)$. This probability is common across dynasties and constant over time. Dynasties learn about q^* over time from experiencing a sequence of events when they are entrepreneurs.

Each dynasty ι enters period t with a prior belief $\pi_{\iota t}(q) = \mathbb{P}_{\iota t}(q^* = q)$ over the probability that entrepreneurial activities are successful. The belief induces a subjective expectation of a good event, $\mathbb{E}_{\iota t} q^* = \int q \pi_{\iota t}(q) \mathrm{d}q$. As we illustrate below, this expectation partly determines the decision to become an entrepreneur.

Dynasties who allocate a positive fraction of their time in entrepreneurship, $k_{\iota t} > 0$, update their prior belief following their experiences using Bayes' rule:

$$\pi_{\iota t+1}(q) = \begin{cases} \pi_{\iota t}(q) \frac{q}{\mathbb{E}_{\iota t} q^*}, & \text{if} \quad e_{\iota t} = G, \\ \pi_{\iota t}(q) \frac{1-q}{\mathbb{E}_{\iota t}(1-q^*)}, & \text{if} \quad e_{\iota t} = B. \end{cases} \tag{4}$$

Following a good experience, $e_{\iota t} = G$, the posterior belief that q^* equals q, $\pi_{\iota t+1}(q)$, equals the prior belief, $\pi_{\iota t}(q)$, multiplied by the likelihood of experiencing a good event, q, divided by the probability of occurrence of a good event, $\mathbb{E}_{\iota t} q^*$. Thus, dynasties with good experiences increase their belief about probabilities that exceed their prior mean of a good experience. Conversely, following a bad experience, $e_{\iota t} = B$, dynasties lower their belief about probabilities that exceed their prior mean of a good experience.

Entrepreneurs, $k_{\iota t} > 0$, pass their posterior beliefs to their children who begin next period with prior $\pi_{\iota t+1}(q)$. Laborers, $k_{\iota t} = 0$, do not accumulate additional experiences and, therefore, do not update their prior, $\pi_{\iota t+1}(q) = \pi_{\iota t}(q)$. Beliefs in our model are martingale, $\mathbb{E}_{\iota t}\pi_{\iota t+1}(q) = \pi_{\iota t}(q)$. Thus, beliefs converge to the truth in the long run, $\lim_{t\to\infty} \pi_{\iota t}(q) = 0$ for $q \neq q^*$, but only conditional on always participating in entrepreneurship.

Similar to Piketty (1995), the baseline assumption is that dynasties learn only from their own experiences if they enter into entrepreneurship. We think this is a natural benchmark, partly because it allows the model to generate persistence in expected returns and persistence in accumulated wealth across generations. In Section 4.4 we consider alternative assumptions under which dynasties also learn from the experiences of other dynasties, irrespective of whether they enter into entrepreneurship.

Timing. The timing of events in each period is:

1. Dynasty ι enters period t with state $(z_{\iota t}, a_{\iota t}, \pi_{\iota t}(q), n_{\iota t+1}, \tau_{\iota t})$, where $z_{\iota t}$ is labor productivity, $a_{\iota t}$ is assets, $\pi_{\iota t}(q)$ is the prior belief about the probability the good event is q, $n_{\iota t+1}$ is the growth rate of the size of the dynasty, and $\tau_{\iota t}$ is transfers.
2. Dynasties choose entrepreneurial time $k_{\iota t}$ before uncertainty about $e_{\iota t}$ is resolved.
3. Dynasties experience $e_{\iota t}$ and realize income $y_{\iota t}$.
4. Dynasties choose consumption $c_{\iota t}$ and transmit assets $a_{\iota t+1}$ and posterior beliefs $\pi_{\iota t+1}$ to the next generation $\iota t + 1$.

Preferences and budget. The model retains analytical tractability by considering each generation's preferences over their own consumption $c_{\iota t}$ and over assets bequeathed per child $a_{\iota t+1}$. The utility function is:

$$U = \frac{(c_{\iota t} - \bar{c}_t)^{1-\gamma} - 1}{1-\gamma} + \beta^{\gamma} \frac{a_{\iota t+1}^{1-\gamma} - 1}{1-\gamma}, \tag{5}$$

where parameter $\gamma \geq 0$ governs the curvature of the utility function with respect to consumption and bequests and the discount factor $\beta > 0$ governs the preference for bequests relative to consumption.

Preferences are non-homothetic with $\bar{c}_t > 0$ denoting the subsistence level of consumption. We motivate non-homothetic preferences with the observation that households with higher lifetime income (for example, Dynan, Skinner, and Zeldes, 2004; Straub, 2019) and wealth (for example, Fagereng, Holm, Moll, and Natvik, 2019) exhibit higher saving rates leading to more wealth inequality (for example, De Nardi and Fella, 2017). In our quantitative results, a relatively high value for $\bar{c}_t$ allows the model to generate a wealth distribution that matches the observed distribution in terms of the low share of wealth held by the bottom half of the population.

The budget constraint of each generation is given by:

$$c_{\iota t} + (1 + n_{\iota t+1})a_{\iota t+1} = y_{\iota t}(k_{\iota t}, e_{\iota t}), \tag{6}$$

where $(1+n_{\iota t+1})a_{\iota t+1}$ is total resources saved in order to transfer $a_{\iota t+1}$ to each member of the next generation. Income $y_{\iota t}$ is a function of the allocation of time $k_{\iota t}$ and the realization of experience $e_{\iota t}$:

$$y_{\iota t}(k_{\iota t}, e_{\iota t}) = \begin{cases} \tau_{\iota t} + (1-\delta)a_{\iota t} + (z_{\iota t} + i_t a_{\iota t})(1 - k_{\iota t}) + r_t a_{\iota t} k_{\iota t}, & \text{if } e_{\iota t} = G, \\ \tau_{\iota t} + (1-\delta)a_{\iota t} + (z_{\iota t} + i_t a_{\iota t})(1 - k_{\iota t}), & \text{if } e_{\iota t} = B. \end{cases} \tag{7}$$

In this definition of income, $\tau_{\iota t}$ is transfers and δ is the depreciation rate of assets.

2.2 Dynasty Optimization

We solve the dynasty problem backwards. In the last stage, we solve for consumption and assets given income level $y_{\iota t}$:

$$c_{\iota t} = \bar{c}_t + \omega_{\iota t+1}\big(y_{\iota t} - \bar{c}_t\big), \tag{8}$$

$$a_{\iota t+1} = \big(1 - \omega_{\iota t+1}\big)\frac{y_{\iota t} - \bar{c}_t}{1 + n_{\iota t+1}}, \tag{9}$$

where weight $\omega_{\iota t+1} \equiv \frac{(1+n_{\iota t+1})^{\frac{1-\gamma}{\gamma}}}{(1+n_{\iota t+1})^{\frac{1-\gamma}{\gamma}}+\beta}$. The solutions are constant elasticity of substitution demand functions, augmented to account for the subsistence consumption. Each generation allocates a fraction $\omega_{\iota t+1}$ of income net of subsistence consumption to consumption above this subsistence level. The remaining income is passed to the next generation in the form of assets. Because

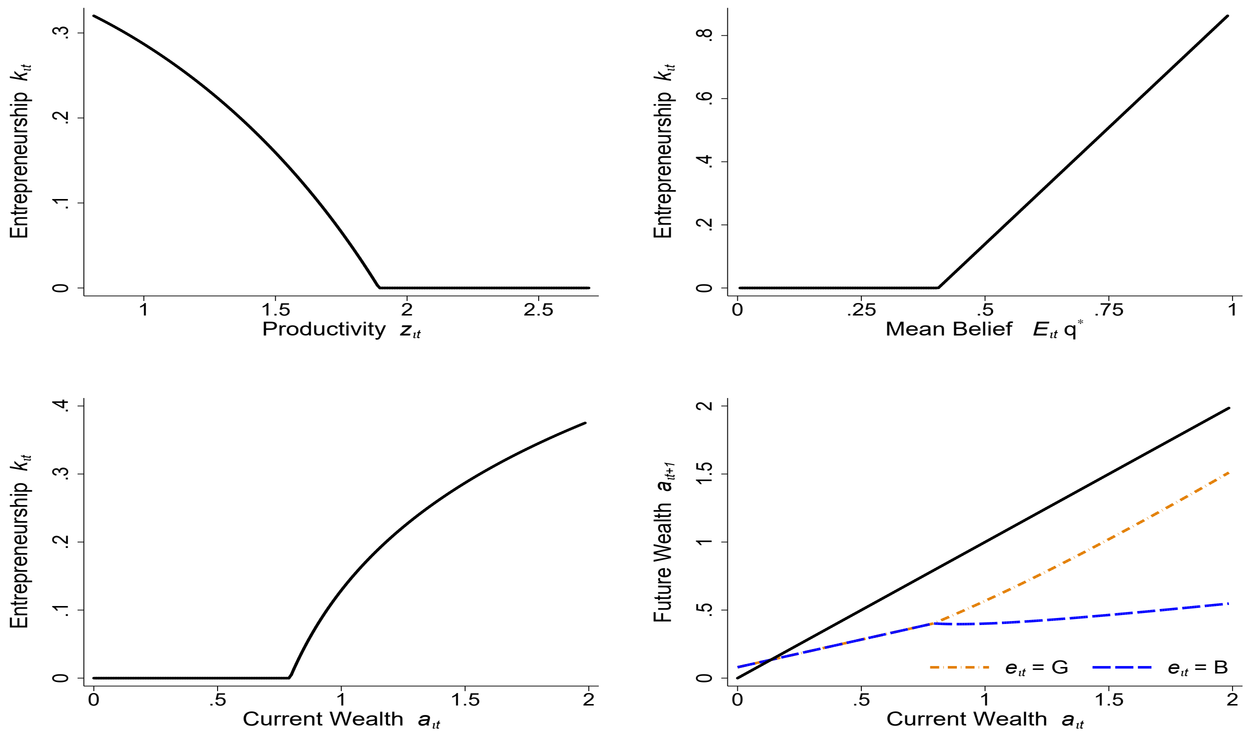


Figure 1: Policy Functions

each generation is succeeded by $1 + n_{\iota t+1}$ members, an increase in population growth reduces the resources transferred per member of the next generation. The consumption weight $\omega_{\iota t+1}$ varies with population growth when $\gamma \neq 1$. With log utility, $\gamma \to 1$, the weight is independent of population growth and is inversely related to the discount factor β.

Working backwards, we solve for the allocation of time $k_{\iota t}$ conditional on the optimal choice of consumption and assets in the last stage of the period. Dynasties maximize expected utility:

$$V = \max_{k_{\iota t}} \int \Big(qU^*(y_{\iota t}(k_{\iota t}, G)) + (1-q)U^*(y_{\iota t}(k_{\iota t}, B)) \Big) \pi_{\iota t}(q) \mathrm{d}q, \tag{10}$$

where U^* is the indirect utility given income $y_{\iota t} = y_{\iota t}(k_{\iota t}, e_{\iota t})$. The expectation is formed under the probability distribution $\pi_{\iota t}$. The solution for time allocated to entrepreneurship is:

$$k_{\iota t} = \begin{cases} 0, \quad \text{if} \quad r_t a_{\iota t} \leq z_{\iota t} + i_t a_{\iota t}, \quad \text{or} \quad \mathbb{E}_{\iota t} q^* r_t a_{\iota t} \leq z_{\iota t} + i_t a_{\iota t}, \\ \left(1 + \frac{\tau_{\iota t} + (1-\delta) a_{\iota t} - \bar{c}_t}{z_{\iota t} + i_t a_{\iota t}}\right) \left(\frac{\left[\frac{\mathbb{E}_{\iota t} q^*}{1 - \mathbb{E}_{\iota t} q^*} \left(\frac{r_t a_{\iota t}}{z_{\iota t} + i_t a_{\iota t}} - 1\right)\right]^{\frac{1}{\gamma}} - 1}{\left[\frac{\mathbb{E}_{\iota t} q^*}{1 - \mathbb{E}_{\iota t} q^*} \left(\frac{r_t a_{\iota t}}{z_{\iota t} + i_t a_{\iota t}} - 1\right)\right]^{\frac{1}{\gamma}} - 1 + \frac{r_t a_{\iota t}}{z_{\iota t} + i_t a_{\iota t}}} \right), \quad \text{else,} \end{cases} \tag{11}$$

where $k_{\iota t} = 1$ if the expression in the last line exceeds one.

Figure 1 presents the policy functions. Beginning with the policy functions for $k_{\iota t}$, our model captures two channels shaping the decision of becoming an entrepreneur. The first is wealth, captured by $a_{\iota t}$, and the second is relative prices, captured by the expected return $\mathbb{E}_{\iota t} q^* r_t$ relative to labor productivity $z_{\iota t}$. Dynasties with sufficiently high labor productivity $z_{\iota t}$, low assets $a_{\iota t}$, or pessimistic beliefs about returns $\mathbb{E}_{\iota t} q^* r_t$ become laborers $k_{\iota t} = 0$. Conditional on becoming an entrepreneur, $k_{\iota t}$ increases in assets and expected returns and decreases in labor productivity.

Moving to the policy function for transferred assets $a_{\iota t+1}$ in the last panel, we distinguish two regions of the state space. For sufficiently low assets $a_{\iota t}$, dynasties become laborers. The policy function for transferred assets $a_{\iota t+1}$ has a slope less than one and cuts the 45 degree black solid line once from above. For sufficiently high assets $a_{\iota t}$, dynasties become entrepreneurs and, thus, their income depends on the realization of $e_{\iota t}$. The policy function for good experiences (orange dash-dot line) is convex because the returns from entrepreneurship are high. In the long-run distribution of the model, entrepreneurs move between the good and bad state (blue dash line) which induces stationarity in the distribution of assets.

2.3 Labor and Capital Market Exclusions

We treat labor and capital market exclusions as driving forces and feed them to the model in a time-varying way that we specify in the quantitative part of our analysis. In modelling these exclusions, we distinguish between Black dynasties b and White dynasties w. Black dynasties are fraction ϕ_t of the population. We denote by Φ_t^h the distribution of dynasties $h \in \{b, w\}$ conditional on race.

By labor market exclusion we refer, collectively, to slavery and the racial wage gap. Slavery, which we indicate by $\chi_t = 1$, forces Black dynasties to be laborers and consume subsistence consumption and excludes them from transferring resources to their children:

$$k_{\iota t}^b = 0, \quad c_{\iota t}^b = \bar{c}_t, \quad a_{\iota t+1}^b = 0. \tag{12}$$

During slavery, the production of Black dynasties in excess of subsistence consumption is expropriated and evenly distributed to White dynasties:

$$(1 - \phi_t)\tau_{\iota t}^w = \chi_t \phi_t \int (z_{\iota t} - \bar{c}_t) \mathrm{d}\Phi_t^b. \tag{13}$$

The racial wage gap means that Black dynasties draw labor productivity from a distribution featuring a lower mean than the mean of the distribution of White dynasties:

$$\mathbb{E}z^b_{\iota t} < \mathbb{E}z^w_{\iota t}. \tag{14}$$

Finally, capital market exclusions capture historical events such as discrimination in patenting, redlining, Jim Crow segregation laws, and exclusion from credit markets.[8] In our model, this is a constraint prohibiting Black dynasties from becoming entrepreneurs:

$$k^b_{\iota t} = 0. \tag{15}$$

2.4 Equilibrium

The return to risky investments, r_t, adjusts to clear the asset market:[9]

$$N_t \int (1 + n_{\iota t+1}) a_{\iota t+1}(\cdot, r_t) \mathrm{d}\Phi_t = \bar{A}_{t+1}/{r_t}^{\alpha}. \tag{16}$$

The left-hand side of this equation is total desired assets under a return r_t. Desired assets are increasing in r_t because a higher r_t induces more dynasties to become entrepreneurs and to transfer more assets to their children. The supply of assets has to meet a limit on investment opportunities, given by the right-hand side of equation (16). The limit is parameterized by $\alpha \geq 0$ which governs the magnitude of the decline in returns as wealth accumulates. In the limiting case with $\alpha = 0$, assets are in fixed supply (for example, land) and r_t adjusts to make assets equal to the exogenous constant $\bar{A}_{t+1}$. In the other limiting case with $\alpha \to \infty$, the return to risky investment is exogenous as in a small open economy. For intermediate cases, $0 < \alpha < \infty$, the equilibrium return and assets are jointly determined as in standard dynamic general equilibrium closed-economy models.

We conclude this section with the definition of equilibrium. Given an initial distribution over assets, beliefs, and population size by race, $\Phi^h_1(a_1, \pi_1, N_1)$, dynasty sequences $\{z_{\iota t}, e_{\iota t}, n_{\iota t+1}\}_{\iota t}$, and aggregate sequences $\{\bar{A}_t, i_t, \chi_t, \bar{k}^b_t\}_t$, an equilibrium is a sequence of dynasty choices and beliefs

[8] Cook (2014) uses patent records matched with census data and other survey data from the U.S. Patent Office to document a decline in Black patents in areas with higher incidence of race riots and segregation laws between 1870 and 1940. The example of the Boyd dynasty we reference in Section 2.5 fits well with Cook's evidence on missing Black patents.

[9] The demand for safe assets is perfectly elastic, implying an exogenous return to safe assets, i_t. In our quantitative results we set i_t to a constant but the results are not sensitive to feeding a time varying i_t.

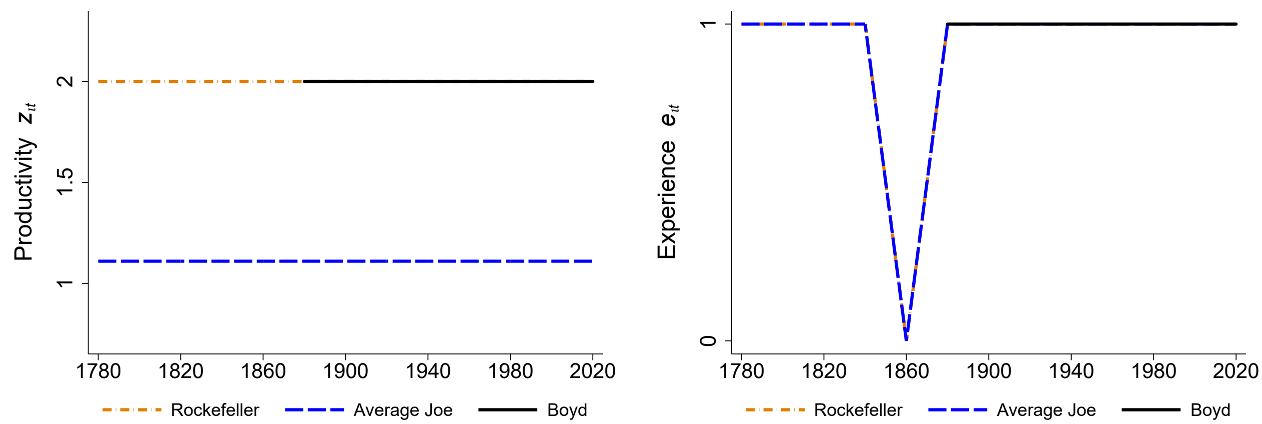


Figure 2: Labor Productivity and Entrepreneurial Experiences

$\{k_{\iota t}, y_{\iota t}, c_{\iota t}, a_{\iota t+1}, \pi_{\iota t+1}\}_{\iota t}$ and a risky return $\{r_t\}_t$ such that: (i) dynasties maximize their expected utility in equation (10) under uncertainty about $e_{\iota t}$ and maximize their utility in equation (5) after $e_{\iota t}$ is realized; (ii) beliefs are consistent with Bayes' rule in equation (4); (iii) the expropriation rule in equation (13) is satisfied; and (iv) the asset market in equation (16) clears.

2.5 Illustrative Example

We use an example to provide insights into the mechanics of the model. The goal of the example is to show how labor and capital market exclusions generate wealth divergence before reparations and to highlight the role of equilibrium effects in amplifying this divergence. In addition, we preview the logic of why a reparation policy redistributing wealth may not eliminate the racial wealth gap in the long run.

Figure 2 presents historical events of three illustrative dynasties beginning in 1780. The first, which we label the *Rockefeller* dynasty with the orange dash-dotted line, is White and always has high labor productivity $z_{\iota t}$. The second, which we label the *Average Joe* dynasty with the blue dash line, is White but has lower $z_{\iota t}$. The third, which we label the *Boyd* dynasty with the black solid line, is Black and has as high $z_{\iota t}$ as Rockefeller but is enslaved between 1780 and 1860. All dynasties are identical in terms of their potential entrepreneurial experiences $e_{\iota t}$ in the right panel. We feed the realizations of $z_{\iota t}$ and $e_{\iota t}$ shown in Figure 2 into the analytical solutions of Section

2.2 to generate the dynasties' evolution of entrepreneurship, beliefs, and wealth.[10]

Figure 3 narrates the history of the three dynasties in partial equilibrium. By partial equilibrium, we mean the case with $\alpha \to \infty$ and, thus, a constant risky return r_t. Beginning with the Rockefeller dynasty, by 1840 it has accumulated sufficient wealth to allocate time to entrepreneurship. In 1860 Rockefeller is hit by a negative experience which discourages entrepreneurship until wealth is rebuild by 1940. After then, the Rockefeller dynasty continues investing with positive experiences, resulting in their beliefs converging to the truth and significant accumulation of wealth. Average Joe differs from Rockefeller because the dynasty never accumulates enough wealth to make entry into entrepreneurship worthwhile. Average Joe remains a laborer forever, does not update beliefs, and ends up with low wealth. Boyd is initially enslaved, which prohibits from building wealth. However, in partial equilibrium Boyd eventually catches up with Rockefeller, enters entrepreneurship, updates beliefs toward the truth, and accumulates wealth.

The evolution of entrepreneurship and wealth is different in general equilibrium, as shown in Figure 4. By general equilibrium, we mean the case with $\alpha < \infty$ and, thus, a declining risky return r_t. General equilibrium generates different predictions because slavery early on coincides with higher return while the abolition of slavery coincides with lower return. Owing to high initial return, Rockefeller enters into entrepreneurship sooner in general equilibrium than in partial equilibrium. Good experiences encourage continued entrepreneurship and wealth accumulation throughout history. Given high initial returns, even the dynasty of Average Joe enters entrepreneurship. However, its wealth and the return are sufficiently low to discourage the dynasty from further entrepreneurial activity following the bad experience of 1860. The Boyd dynasty never catches up with Rockefeller in general equilibrium, despite being equally productive as Rockefeller.

[10] While some readers may be familiar with Rockefeller, we suspect the story of Henry Boyd is not well known among economists. Boyd was born into slavery in 1802 in Kentucky. He was apprenticed out to a cabinetmaker. Boyd was very skilled and earned money to buy his freedom. He moved to Cincinnati in 1826 as a free man, but faced discrimination finding a job. His first job was to unload iron and eventually he was promoted to a janitor. According to historical accounts, one day a white carpenter showed up drunk at work and Boyd substituted. Impressing his boss, Boyd earned enough money as a carpenter to buy the freedom of two of his siblings and open up his own business. Unable to patent his bed frame invention, he stamped bedsteads with his name and eventually partnered with a white business man. While Boyd expanded his business, arsonists burned his shop three times and companies denied him insurance resulting in business closure in 1862. Boyd's estate was teared down sometime in the early 1900s in Cincinnati and turned into a garage. See https://bit.ly/3iULZBY and https://bit.ly/2Ny8AII for some original sources on these historical accounts.

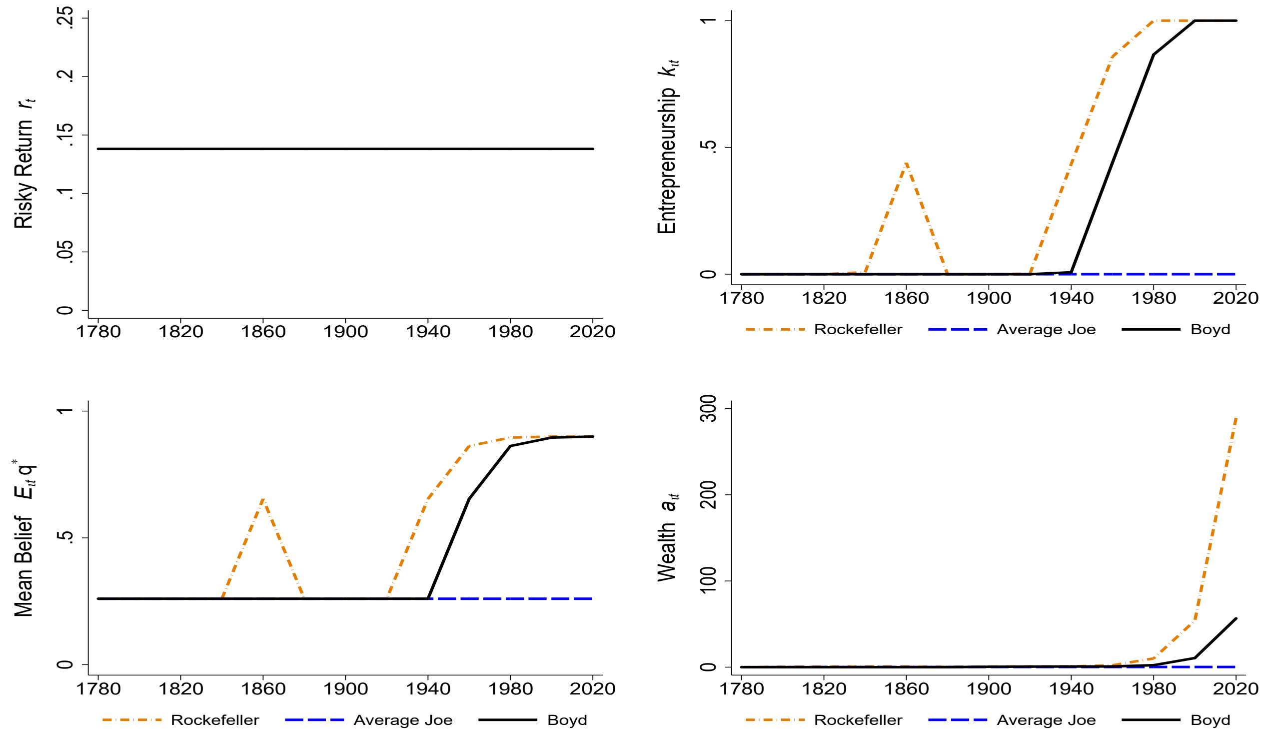


Figure 3: Tale of Three Dynasties in Partial Equilibrium

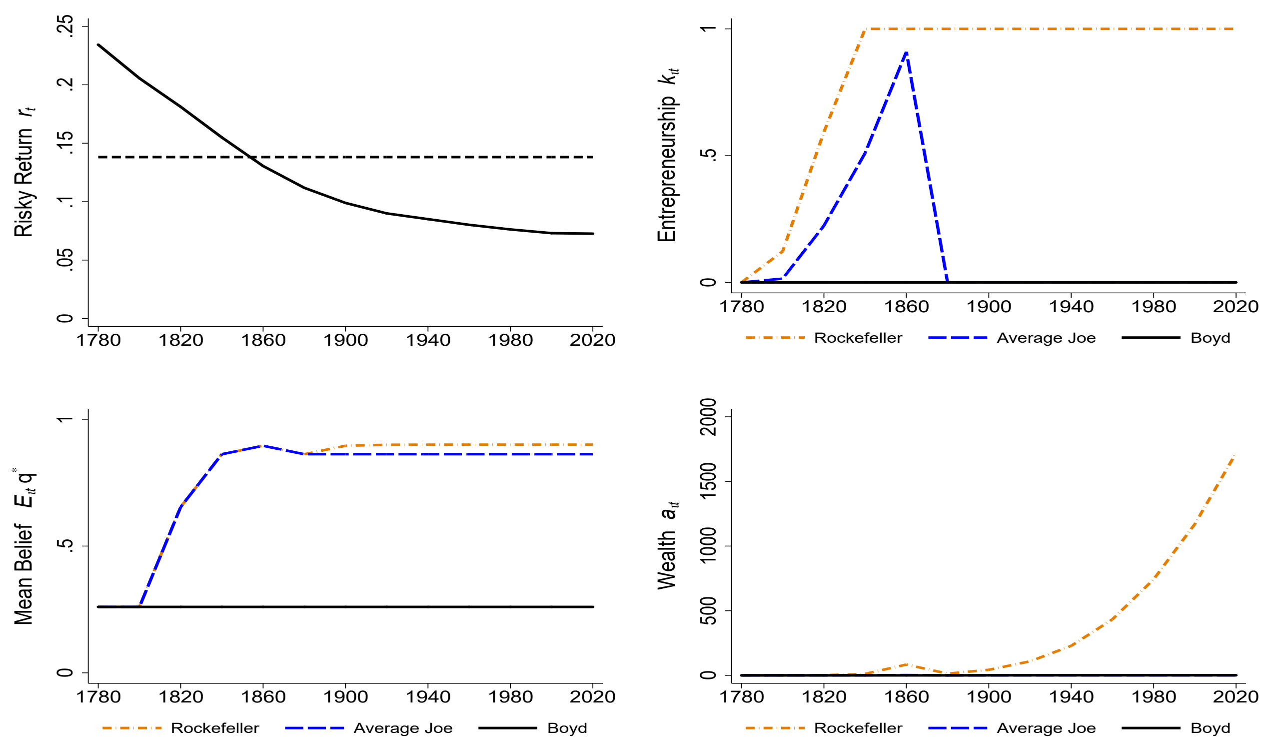


Figure 4: Tale of Three Dynasties in General Equilibrium

The risky return is low after the abolition of slavery in general equilibrium, discouraging Boyd from entering entrepreneurship and accumulating wealth.

There are two insights we draw from this example. Equilibrium effects, where during slavery returns on wealth accumulation are higher than in recent times, discourage Black dynasties from becoming entrepreneurs. An example of an asset captured by the general equilibrium model is land which was initially unexploited and offered high returns. The absence of entrepreneurial experiences feeds back into more pessimistic beliefs about entrepreneurship and causes wealth to diverge more persistently than if returns were constant.

The second lesson is that reparations in the form of transfers which equalize wealth today need not close the wealth gap in the long run. At the time of reparations, dynasties start with different beliefs due to different historical experiences. Given optimistic beliefs, Rockefeller continues to be an entrepreneur and eventually accumulates again significant wealth. By contrast, given pessimistic beliefs, Boyd does not become an entrepreneur and accumulate lower wealth in the long run.

3 Quantitative Results

We begin this section by describing inputs to the model and the parameterization strategy. Next, we evaluate the ability of the model to account for racial gaps in wealth, income, entrepreneurship, mobility, and beliefs, all of which were not targeted by the parameterization. We conclude by presenting counterfactual outcomes when removing the exclusion of Black dynasties from labor and capital markets and changing features of the model economy.

3.1 Model Inputs

A model period is twenty years. We begin our model in 1780, roughly corresponding to the Declaration of Independence. Slavery $\chi_t = 1$ in our model ends in 1860, the model period closest to the time when Congress passed the 13th amendment abolishing slavery. In 1960 we remove capital market restrictions on Black dynasties, $k_t^b = 0$, aligning with the start of the Civil Rights Movement. The last historical model period is 2020 and we consider reparations in 2040.

Table 1: Population and Labor Productivity

	Population Growth		Labor Productivity			
	White	Black	Agg. Growth	Racial Gap	Persistence	Dispersion
	$(1+n^w)^{\frac{1}{20}}-1$	$(1+n^b)^{\frac{1}{20}}-1$	$(1+g)^{\frac{1}{20}}-1$	μ^b	ρ	σ
1780	3.40	2.81	0.39	0.28	0.40	0.71
1800	3.06	2.89	0.39	0.28	0.40	0.71
1820	2.99	2.45	1.00	0.28	0.40	0.71
1840	3.23	2.20	1.27	0.28	0.40	0.71
1860	2.40	1.98	1.40	0.28	0.40	0.71
1880	2.19	1.48	1.68	0.30	0.40	0.71
1900	1.74	0.85	1.32	0.32	0.40	0.71
1920	1.09	1.04	0.82	0.35	0.40	0.71
1940	1.39	1.93	2.29	0.38	0.40	0.71
1960	0.64	1.71	2.28	0.44	0.35	0.63
1980	0.07	1.35	2.01	0.57	0.32	0.67
2000	0.07	0.07	1.43	0.62	0.58	0.85
2020	0.07	0.07	1.43	0.65	0.58	0.85

Table 1 presents the time series of population growth and parameters of the labor productivity process. In the first three columns, percent growth rates are annualized. The last measured population growth rates are from 1980 to 2000 and beyond 2000 we use the population growth for Non-Hispanic Whites between 1980 and 2000. For the labor productivity process we set lacking values to their earliest available estimates.

We use tables from the Census between 1780 and 2000 for the population of Black and White dynasties, N_t^b and N_t^w. The fraction of Black dynasties in 1780 is $\phi_1 = 0.21$. We present the evolution of annualized population growth in the first two column of Table 1. White population grows more until the early 1900s and Black population grows more after that.[11]

Labor productivity of dynasty ι belonging to race h in period t is:

$$z_{\iota t}^h = \max\{Z_t \mu_t^h \theta_{\iota t}, \bar{c}_t\}, \tag{17}$$

[11]For periods 2020 and after, we extrapolate population figures using the average annual growth rate of Non-Hispanic Whites between 1980 and 2000 and set $1+n = 1.0007^{20}$ for both Black and White dynasties. Our model population excludes American Indian, Hispanics, and Asian individuals, or individuals who identify with multiple races. In our model, every Black dynasty is enslaved until 1860. According to Census sources, roughly 90 percent of the Black population in the United States was enslaved by 1860.

where Z_t is the aggregate component, μ_t^h is the race component, and $\theta_{\iota t}$ is the idiosyncratic component of productivity. Labor productivity is adjusted such that it does not fall below the subsistence level of consumption $\bar{c}_t$. The idiosyncratic component follows the process:

$$\log \theta_{\iota t} = \rho_t \log \theta_{\iota t-1} + \sigma_t \varepsilon_{\iota t}, \tag{18}$$

where ρ_t governs the persistence across generations, σ_t governs the cross-sectional dispersion of labor productivity, and the innovation $\varepsilon_{\iota t}$ follows a standard normal distribution. Persistence ρ_t and dispersion σ_t are common across dynasties. Despite that, the model can account for the racial income mobility gap partly due to the race component μ_t^h.

The last four columns of Table 1 present the evolution of the four parameters of labor productivity. The aggregate component of productivity $Z_t = (1 + g_t)Z_{t-1}$, grows at rate g_t which we equate to the growth of GDP per capita from the Maddison project (Bolt and van Zanden, 2020). For the race component, $\mu_t^w = 1$ for White dynasties and μ_t^b for Black dynasties is presented in the fourth column of the table. We take the race gap in wages from the calculations of Margo (2016) who uses labor force participation rates for rural agricultural workers and urban workers to compute the Black-White relative income starting from 1870.[12] We set the persistence ρ_t from the historical study of intergenerational mobility of Aaronson and Mazumder (2008) who match men in the Census to synthetic parents in the prior generation starting from 1940. Finally, we use the American Community Survey starting in 1940 to estimate the cross-sectional dispersion σ_t.[13]

Finally, all dynasties begin with no wealth, $a_{\iota 1} = 0$. We feed into the model an asset limit which evolves as $\bar{A}_t = \bar{A}_1(1+g)^{t-1}(1+n)^{t-1}$, where the initial value $\bar{A}_1$ is a parameter. We grow $\bar{A}_t$ at a rate equal to the growth rate of productivity g and population n allowing the model to asymptote to a balanced growth path with constant risky returns and factor shares.

[12]The first observations in Margo (2016) are for 1870, 1900, and 1940. We interpolate linearly to set the racial wage gap in 1880 and 1920. We use the 1870 observation for the racial wage gap in model period 1860 and before. The more recent data after 1940 come from Census records. Aizer, Boone, Lleras-Muney, and Vogel (2020) study the role of defense production during the second World War for closing the racial wage gap after 1940. Derenoncourt and Montialoux (2021) study the role of expansions of the federal minimum wage for closing the racial earnings and income gap during the Civil Rights era.

[13]Our estimates are dispersion of log income and control for age and race fixed effects.

3.2 Parameterization

Table 2 presents our parameter values. The upper panel shows parameter values chosen externally without solving the model. The lower panel shows parameter values chosen such that model-generated variables match their analogs in the data. Table 3 presents the targeted moments used to estimate the parameters in the lower panel of Table 2.

Preferences are logarithmic, $\gamma = 1$. We use the inverse of the labor share from national income and product accounts to set $\alpha = 1.77$. The annual rate of depreciation, $1 - (1 - \delta)^{1/20} = 0.05$, equals the average annual depreciation rate of private fixed assets in national income and product accounts. Finally, for the safe return we pick $i = 0.02$ using the average return on safe assets from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019).

The distribution of beliefs in 1780 is Beta $\left(\frac{b\bar{q}_{\iota 1}}{1-\bar{q}_{\iota 1}}, b\right)$. The prior mean of the Beta distribution is $\bar{q}_{\iota 1} = \mathbb{E}_{\iota 1} q^*$ and indexed by dynasty ι. Heterogeneity in initial expected returns allows the model to generate wealth inequality as optimistic dynasties enter into entrepreneurship earlier than pessimistic dynasties. Initial expected returns are drawn from $\bar{q}_{\iota 1} \sim \mathrm{U}(0, \bar{q})$. Given $\bar{q}_{\iota 1}$, parameter b of the Beta distribution governs the dispersion of beliefs which is common across dynasties. Initial beliefs vary within group, but not across groups meaning that Black and White dynasties draw from the same initial distribution.

Along with parameters of initial beliefs, $\bar{q}$ and b, we pick the discount factor β, the probability of a good event q^*, the initial asset limit $\bar{A}_1$, and subsistence levels of consumption $\bar{c}_t$ to target six moments. Table 3 shows that we target the level of wealth relative to income, the labor share, the risky return, the share of wealth accruing to the top 10 and 50 percent of the wealth distribution, and the entrepreneurship rate.[14]

The model matches these targets with (annualized) discount factor $\beta^{1/20} = 0.98$ and probability of successful entrepreneurship $q^* = 0.91$. The parameter estimates favor dispersed initial beliefs around a pessimistic mean ($\bar{q}/2 = 0.15$). These initial beliefs imply that few, initially

[14]Average wealth to income equals household net worth over GDP from the Flow of Funds. The risky return is taken from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) and equals the total return on assets (including housing) minus inflation. The shares of wealth accruing to the top 10 and 50 percent and the entrepreneurship rate are calculated from the Survey of Consumer Finances between 2010 and 2019. For the entrepreneurship rate, we define an individual as entrepreneur, $k > 0$, if their business assets exceed average net worth in the survey.

Table 2: Model Parameters

Parameter		Value	Source/Logic
curvature of utility	γ	1.00	log preferences
returns to scale	α	1.77	inverse of labor share
depreciation rate (annual)	$1-(1-\delta)^{1/20}$	0.05	NIPA fixed assets
safe return (annual)	$(1+i)^{1/20}-1$	0.02	Jorda et al (2019)
discount factor (annual)	$\beta^{1/20}$	0.98	match targets
probability of success	q^*	0.91	match targets
initial asset limit	$\bar{A}_1$	0.23	match targets
subsistence consumption	$\bar{c}_t$	$0.81Z_t$	match targets
shape Beta distribution	$\bar{q}$	0.30	match targets
shape Beta distribution	b	1.20	match targets

Table 2 presents values of model parameters. The upper panel shows parameters values chosen externally. The lower panel shows parameter values chosen such that model-generated variables match their analogs in the data.

Table 3: Model Targets

Target		Data	Model
wealth / lifetime income	$\int a \mathrm{d}\Phi / \int y \mathrm{d}\Phi$	0.25	0.25
labor share	$\int z(1-k) \mathrm{d}\Phi / \int y \mathrm{d}\Phi$	0.56	0.56
risky return	$q^* r$	0.07	0.07
top 10% wealth share	$\int a \mathbb{I}(\Phi(a) \geq 0.9) \mathrm{d}\Phi / \int a \mathrm{d}\Phi$	0.76	0.76
top 50% wealth share	$\int a \mathbb{I}(\Phi(a) \geq 0.5) \mathrm{d}\Phi / \int a \mathrm{d}\Phi$	0.99	0.98
entrepreneurship rate	$\int \mathbb{I}(k > 0) \mathrm{d}\Phi$	0.03	0.03

Table 3 presents the moments targeted to estimate the parameters in the lower panel of Table 2.

more optimistic, dynasties end up being entrepreneurs. Given the high probability of success q^*, entrepreneurs hold a significant fraction of wealth in the economy and the model matches the high concentration of wealth at the top. Finally, the value of $\bar{c}_t$ implies a strong non-homotheticity in preferences, helping the model account for the observation that the bottom half of the population owns almost none of the economy's wealth.[15]

3.3 Comparing the Model with the Data

In this section we evaluate the ability of the model to generate current and historical racial gaps in outcomes such as wealth, income, entrepreneurship, mobility, and beliefs about risky returns.

3.3.1 Wealth and Income Concentration

Table 4 compares the concentration of wealth and income generated by the model to the concentration observed in the data. The measure of concentration is the share of wealth and income held by households above selected percentiles of their respective distributions. The data come from the Survey of Consumer Finances (SCF) between 2010 and 2019, corresponding to model period 2020.

Our parameterization strategy targets directly the share of wealth held by the top 10 and top 50 percent of the wealth distribution. As shown in Table 4, the model accounts almost perfectly for the wealth concentration in the data up to the fifth percentile. The model also performs well in terms of matching the observed income concentration up to the fifth percentile. Above the fifth percentile, the model overestimates wealth and income concentration relative to the data. An alternative strategy is to target higher percentiles than the top 10 and top 50 in our parameterization. In that case, the model would underestimate the concentration at lower percentiles. We prefer targeting relatively lower percentiles of concentration because matching concentration statistics around these percentiles is more important for the reparations policy.[16]

[15]We parameterize $\bar{c}_t = 0.81 Z_t$, ensuring that subsistence consumption per capita grows at rate g over time. With this parameterization, $\bar{c}$ equals 49 percent of average labor productivity z in 2020.

[16]Our estimates of the share of wealth held by the top 1 percent and top 0.1 percent are compatible with other estimates in the literature. Saez and Zucman (2016) estimates for these shares are 38 and 20 percent using IRS data. Using the same data, but a different capitalization approach, Smith, Zidar, and Zwick (2020) estimates are 30 and 14 percent. For our SCF data, these authors report a share of 13 percent for the top 0.1 percent.

Table 4: Wealth and Income Concentration

	Wealth		Income	
(2020, share of top)	Data	Model	Data	Model
50 percent	0.99	0.98	0.86	0.89
20 percent	0.87	0.86	0.62	0.71
10 percent	0.76	0.76	0.47	0.60
5 percent	0.64	0.67	0.37	0.52
1 percent	0.37	0.53	0.20	0.40
0.1 percent	0.14	0.25	0.07	0.19

Table 4 presents the share of wealth and income held by households above selected percentiles of their respective distributions. The data moments are calculated from the SCF between 2010 and 2019.

Table 5: Racial Gap in Wealth and Income

	Wealth Ratio		Income Ratio	
(2020, White/Black)	Data	Model	Data	Model
Mean	6.7	5.7	2.2	2.9
99 percentile	7.5	5.3	4.0	3.8
90 percentile	5.3	2.5	2.0	1.8
50 percentile	9.7	17.5	1.8	1.8

Table 5 presents the White to Black ratio of wealth and income at the mean and selected percentiles of the corresponding distributions. The data moments are calculated from the SCF between 2010 and 2019.

Table 6: Evolution of Racial Wealth Gap

	Wealth Ratio (White/Black)	
	Data	Model
1900	28.2	42.5
2020	6.7	5.7

Table 6 presents the White to Black ratio of average wealth in 1900 and 2020. The data entry for 1900 comes from Higgs (1982) and Margo (1984). The data entry for 2020 comes from the SCF between 2010 and 2019.

We now assess the ability of the model to generate racial gaps in wealth and income not targeted by the parameterization. In Table 5 we report ratios of average variables for White relative to Black households. For example, in the SCF the ratio of average wealth is 6.7 and the ratio of average income is 2.2. The model is successful in generating such a large average difference in wealth and income. Similar to the data, the model also generates a larger gap for median wealth than average wealth and a smaller gap of median income than average income.

Finally, we assess whether the model generates a reasonable speed of wealth convergence across race. Matching the historical speed of convergence lends credibility to the model when analyzing the effects of reparations on the evolution of the racial wealth gap. Table 6 presents the evolution of the racial wealth gap over time in the model and the data. The gap for the early period comes from the historical evidence of Higgs (1982) and Margo (1984).[17] Our model successfully captures the speed of convergence of wealth. The historical data show a slow convergence of Black wealth from almost 3.5 cents on a dollar to 15 cents. Our model generates such a slow convergence, transitioning from 2.3 cents on a dollar to 18 cents.

3.3.2 Entrepreneurship

The model generates significant racial wealth and income gaps, including at the top of the distributions, partly because more White dynasties become entrepreneurs than Black dynasties. Are such differences in entrepreneurship rates observed in the data? Panel A of Table 7 shows significant differences in entrepreneurship rates by race. Quantitatively, the model generates a 3.6 percentage points gap in entrepreneurship rate, close to the 2.8 percentage points gap in the data.[18]

Additionally, in panel B of Table 7, the model performs well in matching the observed distribution of wealth and income within those dynasties who become entrepreneurs. For example, the top 1 percent of entrepreneurs hold roughly 20 percent of wealth and income in the data, as

[17] Higgs (1982) uses tax assessment records from Georgia to measure the racial wealth gap and Margo (1984) complements this analysis with data from Arkansas, Kentucky, Louisiana, North Carolina, and Virginia.

[18] Fairlie and Meyer (2000) document that the that self-employment rate is lower for Black than White men and the ratio of self-employment rates has remained remarkable stable since the early 1900s. Bogan and Darity (2008) argue that, while much of the literature tries to account for the entrepreneurship gap by appealing to cultural differences, the comparison of Black with immigrant groups suggest a more important role for discriminatory practices, institutions, and legislation restricting Black entrepreneurship. Our model is compatible with a more elaborate version of the Bogan and Darity (2008) view, because the historical exclusions they describe endogenously lead to differences in beliefs about entrepreneurship.

Table 7: Entrepreneurship

	White		Black	
A. Fraction entrepreneurs	Data	Model	Data	Model
(percent)	3.6	3.1	0.3	0.0
B. Entrepreneurs	Wealth		Income	
Share of top ...	Data	Model	Data	Model
50 percent	0.88	0.96	0.89	0.97
20 percent	0.66	0.84	0.68	0.86
10 percent	0.51	0.68	0.52	0.70
5 percent	0.39	0.52	0.40	0.53
1 percent	0.20	0.23	0.19	0.23
0.1 percent	0.06	0.06	0.06	0.07

Table 7 presents statistics of entrepreneurship in the data and the model. The first panel shows the share of White and Black households who are entrepreneurs and the second panel shows the share of wealth and income held by entrepreneurs above selected percentiles of their respective distributions. The data entries come from the SCF between 2010 and 2019. In the SCF we define an individual as entrepreneur, $k > 0$, if their business assets exceed average net worth in the survey.

opposed to 23 percent in the model.

3.3.3 Income Mobility

We next assess the ability of the model to generate current and historical patterns of income mobility by race. The historical evidence comes from Collins and Wanamaker (2017) who link individual census records between 1910 and 1930 to derive mobility statistics.[19] Entries in Table 8 denote upward rank mobility probabilities by decile d:

$$\text{Upward Rank Mobility}(d) = \mathbb{P}(\text{rank}(y_t) > \text{rank}(y_{t-1}) | \text{rank}(y_{t-1}) \in d). \tag{19}$$

This index measures the probability that a child exceeds their parent's rank in the income distribution, conditional on a parent belonging to decile d. Rank is measured in the total population, including both Black and White dynasties.

Beginning with the upper panel of Table 8, the model is successful in generating a racial mobility gap in modern times. For example, starting at the third decile, the model predicts that

[19]For the more recent period, the authors use National Longitudinal Survey of Youth data.

Table 8: Upward Rank Mobility

(1990)	Data		Model	
Decile d	White	Black	White	Black
1	0.97	0.80	0.87	0.81
3	0.76	0.54	0.65	0.49
5	0.58	0.51	0.49	0.38
7	0.36	—	0.38	0.25
9	0.31	—	0.24	0.18
(1930)	Data		Model	
Decile d	White	Black	White	Black
1	0.90	0.68	0.85	0.58
3	0.59	0.31	0.62	0.31
5	0.38	—	0.48	0.22
7	0.28	—	0.38	0.13
9	0.16	—	0.25	—

Table 8 presents probabilities of upward rank mobility at selected deciles of the income distribution. Data entries are from Collins and Wanamaker (2017). Missing entries indicate no or very few observations to calculate probabilities.

65 percent of White children exceed their parents' rank whereas only 49 percent of Black children exceed the parents' rank. The gap of 16 percentage points is close to the 22 percentage points gap in the data. In the lower panel of Table 8, the model is also successful in generating the even larger mobility gaps observed during the early 1900s. For example, starting again at the third decline, 59 percent of White children exceeded their parents' rank whereas only 31 percent of Black children exceeded the parents' rank. The gap of 28 percentage points is close to the 30 percentage points gap in the data.

Two features of the model allow it to generate mobility patterns similar to the patterns in the data. First, only White dynasties become entrepreneurs and, given the high returns from entrepreneurship, entrepreneurs are more likely to surpass laborers in the income distribution. Second, Black dynasties are less likely to move upward in the distribution of the total population's income distribution because they draw wages from a lower mean than the average dynasty in the population, as shown by the term μ_t^h in equation (18). Since the racial wage gap was larger in the 1900s, Black dynasties were less likely to move upward in the 1900s than today.

3.3.4 Beliefs about Risky Returns

We next compare the predictions of the model for beliefs about risky returns to measures of beliefs in survey responses from the University of Michigan Surveys of Consumers. The survey question we use is:

> *Think about a diversified stock fund which holds stock in many different companies engaged in a wide variety of activities. Suppose someone were to invest one thousand dollars in such a mutual fund. What do you think is the chance this one thousand dollar investment will increase in value, so that it is worth more than one thousand dollars one year from now?*

We use Michigan Survey responses to this question between June 2002 and December 2015 in the *Time Series Demographics Data*. We merge these data to microdata samples, available through the Inter-University Consortium for Political and Social Research, which contain information on race. We restrict our sample to include Black and White individuals between 25 and 65 years of age. The cleaned dataset contains 42,521 observations, out of which roughly 10 percent identifies as Black.[20]

The response to the Michigan Survey question is a probability assessment that a diversified equity fund increases in value. The difficulty in comparing beliefs between the data and the model is that respondents in the data may have a different benchmark return in their mind than zero (say, because of inflation) and that dynasties in our model have the option of investing in a safe technology featuring growth. Our process is to choose a benchmark return in the model, $\bar{r}$, such that the model-generated average probability assessment that risky investments exceed this benchmark $\bar{r}$ matches the average probability assessment of 0.51 from the survey. Formally,

[20] Responses to this question were studied by Dominitz and Manski (2011) for the period between June 2002 and August 2004. The authors find nearly identical results in the survey to the results we report in Table 9. Dominitz and Manski (2011) also report similar results for their Survey of Economic Expectations. Since the Survey of Economic Expectations only has only 85 Black respondents for a short period of time, we prefer to use the Michigan Survey.

Table 9: Beliefs about Risky Returns

Probability of Successful Investment	Data	Model
Mean	0.51	0.51
Mean, White	0.51	0.53
Mean, Black	0.44	0.41
Std Deviation	0.29	0.28
Std Deviation, White	0.29	0.30
Std Deviation, Black	0.29	0.16

Table 9 presents statistics of the probability assessment of successful investments. Data entries denote means and standard deviations of the reported probability of an increase in the value of a diversified stock fund as reported in the Michigan Survey. The data contain 42,521 observations, out of which roughly 10 percent identifies as Black.

for each dynasty we calculate the probability $P_t(\pi_{\iota t}, \bar{r})$ that the economy-wide return exceeds $\bar{r}$, evaluated under our model-generated belief $\pi_{\iota t}$ in the last period:

$$P_t(\pi_{\iota t}, \bar{r}) = \sum_{q: q r_t > \bar{r}} \pi_{\iota t}(q). \tag{20}$$

The model-generated average probability assessment equals $\int P_{\iota t} \mathrm{d}\Phi = 0.51$ for $\bar{r} = 0.006$.

Having targeted the average response in the total population, we now evaluate the ability of the model to generate dispersion of beliefs. Table 9 summarizes our results. In the first row, the mean probability of successful investment in the total population align perfectly between the model and the data by construction. The second and third rows show the mean probability for White and Black households. The racial gap in mean beliefs is 12 percentage points in the model, as opposed to 7 percentage points in the data. At the same time, the model generates a dispersion of beliefs in the total population as large as measured in the data. The model replicates almost perfectly the dispersion of beliefs within White households, but underestimates the dispersion within Black households.[21]

[21] The racial gap in beliefs is a robust feature of the data along various dimensions. Regressions of probability assessments on a race dummy uncover a similar gap to our 7 percentage points estimate in Table 9, when controlling for a host of variables (age, time, sex, marital status, region, education, number of children, number of adults, and income). Additionally, we corroborate our racial gap of beliefs about returns by studying the racial gap in expectations for the aggregate economy over a five year horizon. Black households are also more pessimistic in terms of expectations of five-year aggregate growth between 1990 and 2015.

Table 10: Counterfactual History

(2020, White over Black)	Wealth Ratio	Income Ratio	Entrepreneurship Gap
A. All Differences	5.7	2.9	3.6
− Labor Exclusion	2.3	1.7	3.3
− Capital Exclusion	5.7	2.9	3.6
− Demographics	5.5	2.9	3.6
B. No Differences	1.0	1.0	0.0
+ Labor Exclusion	5.5	2.9	3.6
+ Capital Exclusion	2.1	1.7	3.2
+ Demographics	0.7	0.8	-0.9
C. Baseline	5.7	2.9	3.6
Perfect Information	5.9	2.9	14.3
$\beta^{1/20} = 0.95$	4.0	2.2	2.8
$q^* = 0.70$	4.0	2.1	2.5
$\bar{c} = 0.50Z$	4.1	2.6	7.6
$\gamma = 3$	4.6	2.5	0.7

Table 10 presents counterfactual outcomes when we remove historical events driving racial differences in the model (panel A), when we add historical events driving racial differences in the model (panel B), and when we change features of the model environment relative to the baseline (panel C).

3.4 How Historical Events Shape Today's Racial Gaps

We conclude this section by evaluating the effects of historical events and features of the model economy on current racial gaps. Table 10 summarizes our results. Panel A shows the racial gap in wealth, income, and the entrepreneurship rate when we remove historical events. Panel B shows racial gaps when we add historical events starting from no racial differences. Panel C shows racial gaps when we change features of the model economy.

The top panel shows that removing the history of labor market exclusions results in a ratio of average wealth of 2.3 as opposed to 5.7 in our baseline model with these exclusions, an income ratio of 1.7 down from 2.9 in the baseline, and a difference in entrepreneurship rate of 3.3 percentage points as opposed to 3.6 in the baseline. Without labor market exclusions, we would also observe gaps today because capital market exclusions prohibit Black dynasties from becoming entrepreneurs. On the other hand, removing only capital market exclusions does not affect current outcomes. Labor market exclusions on their own put Black dynasties at significant disadvantage in terms of becoming entrepreneurs and accumulating wealth, making the incremental effect of capital market exclusions in our model zero.

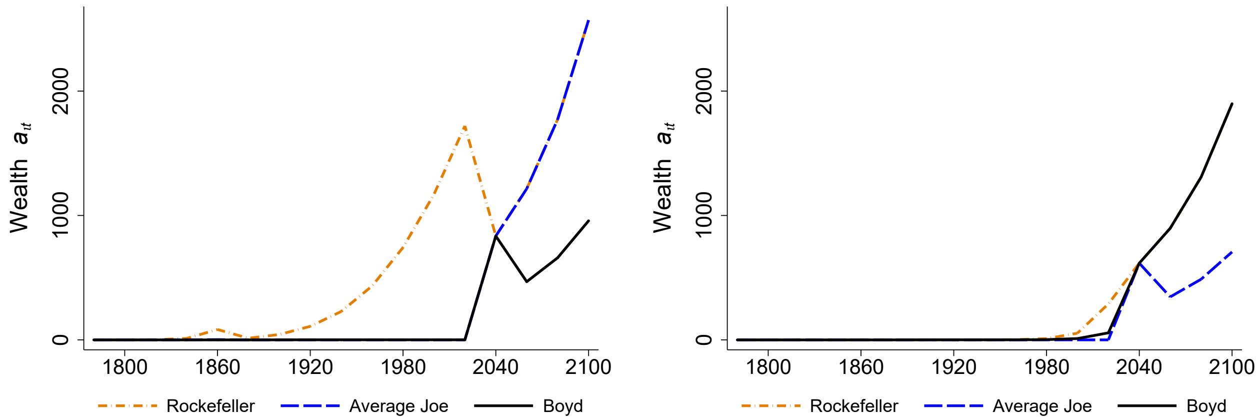
The middle panel confirms these conclusions by adding historical events one at a time. If there were only demographic differences, Black dynasties in our model would achieve a higher wealth, income, and entrepreneurship rate than White dynasties by 2020. As Table 1 shows, in early periods the population growth of White dynasties is higher than the population growth of Black dynasties. Therefore, it is more costly for White dynasties to transfer wealth across generations on a per capita basis.

In the lower panel we assess the role of model elements for racial gaps. With perfect information about the probability of success, the model would generate a much larger gap in the entrepreneurship rate with roughly 20 percent of White dynasties becoming entrepreneurs as opposed to 6 percent of Black dynasties.[22] In the next row, a lower discount factor β is associated with smaller racial gaps, as White dynasties transfer less wealth across generations. Similarly, a lower probability of success q^* is associated with smaller racial gaps. Black dynasties are more likely to be concentrated at the bottom of the distribution and, thus, are affected more by the level of subsistence consumption $\bar{c}$. This means a lower $\bar{c}$ is associated with lower wealth and income gaps. At the same time, White dynasties close to $\bar{c}$ are more likely to forego labor and the entrepreneurship gap increases. Finally, racial gaps decline under a higher curvature in the utility function, $\gamma = 3$, because increased risk aversion makes more dynasties choose labor over entrepreneurship.

4 Reparations

We begin this section by analyzing the effects of wealth transfers toward Black dynasties. We then discuss the alternative policy of subsidizing Black entrepreneurship and the role of learning from others for wealth convergence.

[22] In terms of targeted moments in Table 3 the model misses the concentration of wealth and income at the top because entrepreneurship is more widespread among the population.



(a) General Equilibrium (b) Partial Equilibrium

Figure 5: Wealth Redistribution in the Illustrative Example

Figure 5 shows the evolution of wealth in the example of Section 2.5 under a one-time policy that unexpectedly equalizes wealth in 2040. For all dynasties, labor productivity $z_{\iota t}$ and experience $e_{\iota t}$ are always constant at their 2020.

4.1 Wealth Redistribution in the Illustrative Example

We use the example of Section 2.5 to preview the logic of why one-time, unanticipated, wealth transfers do not lead to wealth convergence in the future even if they completely eliminate the wealth gap today. Figure 5 shows the evolution of wealth for the three dynasties following a redistribution of wealth that eliminates wealth inequality in 2040. The left panel performs the experiment in general equilibrium and the right panel performs it in partial equilibrium.

In general equilibrium, Boyd's wealth falls behind White dynasties' wealth immediately after reparations. This result can be understood in terms of the policy function for entrepreneurship in equation (11), which we summarize succinctly as $k(z, a, \mathbb{E}q^*; r)$. Despite having equal labor productivity z and assets a with Rockefeller in 2040, pessimistic beliefs $\mathbb{E}q^*$ and the relatively low return r (shown in Figure 4) lead Boyd to choose labor and forego the opportunity to invest. By contrast, given optimistic beliefs, the Rockefeller dynasty continues entrepreneurship despite their one-time reduction in wealth.

The right panel of Figure 5 conducts the same experiment in partial equilibrium. Here, Boyd's wealth catches up forever with Rockefeller's. In partial equilibrium, Boyd enters the reparation regime with relatively optimistic beliefs and the return is relatively high. Thus, Boyd becomes an entrepreneur and accumulates as much wealth as Rockefeller after reparations.

The example illustrates the difficulty in closing the racial wealth gap using wealth transfers. While differences in some state variables, such as labor productivity and wealth, can be eliminated with policies, centuries of exclusions are also associated with a gap in experiences and beliefs about risky investments. These differences imply that next generations of Black dynasties will forego investment opportunities and fall behind in terms of wealth accumulation.

4.2 Wealth Redistribution in the Quantitative Model

We next consider wealth transfers taking place in 2040 in the quantitative model. Because we want to isolate only effects coming from wealth transfers, at the time of reparations we permanently close the gap in wages z between Black and White dynasties by setting $\mu_t^b = 1$ for all periods beginning in 2040. If we did not close the wage gap, wealth would diverge mechanically after reparations due to differences in labor earnings.

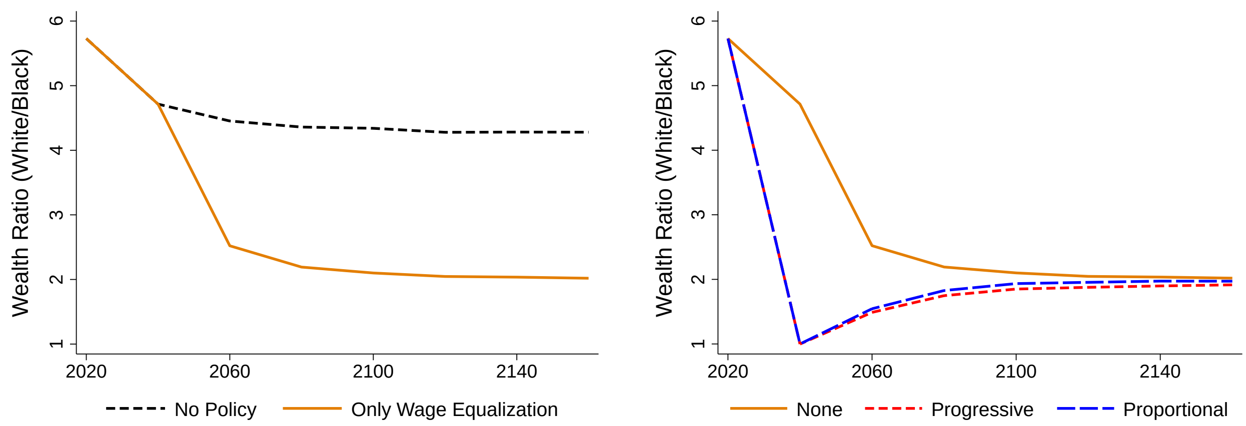
Transfers to Black dynasties are financed by a one-time and unexpected tax that falls on White dynasties' wealth. The wealth tax rate function, Λ, for White dynasties is given by:

$$\Lambda(a_\iota^w) = \begin{cases} \lambda & \text{if } a_\iota^w < a^*, \\ (1-\lambda)\frac{a_\iota^w - a^*}{a_\iota^w} & \text{if } a_\iota^w \geq a^*. \end{cases} \tag{21}$$

We consider two forms of financing which differ in their degree of progressivity.

1. Progressive wealth taxation corresponds to $\lambda = 0$ and $a^* > 0$. Wealth below a^* is not taxed and wealth above a^* is taxed at a 100 percent rate.
2. Proportional wealth taxation corresponds to $\lambda > 0$ and $a^* \to \infty$. All wealth is taxed at a proportional rate λ.

We consider wealth transfers to Black dynasties resulting in average wealth of White dynasties being a multiple m of average wealth of Black dynasties. Our baseline policy experiment features $m = 1$, so that average wealth is perfectly equalized between Black and White dynasties. For $m < 1$, transfers to Black dynasties make them wealthier than White dynasties. For any desired multiple m, we solve for tax parameters (either λ or a^*) such that every Black dynasty receives a



(a) Wage Equalization

(b) Wealth Transfers

Figure 6: Racial Wealth Before and After Reparations

The left panel of Figure 6 shows the wealth ratio under no policy changes relative to our baseline model (black dashed line) and the wealth ratio after eliminating the racial wage gap in 2040 (orange solid line). The right panel adds to the wealth ratio under no wage gap (orange solid line) the wealth ratios under reparations eliminating the wealth gap in 2040 and are financed either with progressive (red dashed line) or with proportional taxes (blue dashed line).

lump sum wealth transfer equal to τ:

$$m\int(a_\iota^b+\tau)\mathrm{d}\Phi^b=\int(1-\Lambda(a_\iota^w))a_\iota^w\mathrm{d}\Phi^w,\quad \text{such that}\quad \phi\tau=(1-\phi)\int\Lambda(a_\iota^w)a_\iota^w\mathrm{d}\Phi^w, \tag{22}$$

where ϕ is the fraction of Black dynasties in the population.

Figure 6 presents our baseline results for the evolution of the wealth ratio. To examine the effect of wealth transfers on the racial wealth gap, we require a benchmark of where the economy converges in the absence of any transfers. This benchmark is the racial wealth gap if we permanently equalize wages across groups. In the left panel we present the evolution of the wealth gap under no policy and under the policy of wage equalization. In 2020, the model is still transitioning and, thus, even without any policy we would observe some convergence of the wealth ratio (from roughly 6 in 2020 to roughly 5). Eliminating wage differences reduces the wealth ratio which now converges to roughly 2. Thus, the value of 2 is the long-run wealth ratio absent reparations to which we compare reparations to.

The right panel of Figure 6 adds to the wage equalization policy (with the orange solid line), wealth transfers toward Black dynasties financed either with progressive (red dashed line) or proportional taxes (blue dashed line). By design, the racial wealth ratio is one in 2040. After

Table 11: Wealth Transfers

	(relative to average wealth)		Wealth Ratio			Output Change (percent)		
	Transfer	Financing	2040	2100	∞	2040	2100	∞
1.	$\tau = 0.75$	$a^* = 112$	1.0	1.9	1.9	-4.3	-4.3	0.0
2.	$\tau = 2.23$	$a^* = 16$	0.3	1.3	1.9	-16.6	-16.5	0.0
3.	$\tau = 0.75$	$\lambda = 0.13$	1.0	1.9	1.9	-2.9	-2.1	0.0
4.	$\tau = 2.23$	$\lambda = 0.38$	0.3	1.6	1.9	-8.9	-7.0	0.0

Table 11 presents the wealth ratio and changes in output relative to the benchmark (with only wage equalization) in selected periods under four different wealth transfer systems. In the top two rows transfers are financed with progressive taxation and in the two bottom rows transfers are financed with proportional taxation. In rows 1 and 3 transfers eliminate the racial wealth gap in 2040 ($m = 1$), while in rows 2 and 4 transfers make average wealth of White dynasties $m = 0.3$ times average wealth of Black dynasties in 2040.

then we observe convergence toward a ratio of roughly 2 with both financing systems. This value approximates the same long-run level the ratio would tend to absent wealth transfers. In that sense, the effect of one-time wealth transfers on the future racial wealth gap is zero.[23]

Could larger wealth transfers lead to convergence? Table 11 presents the evolution of the wealth ratio and output over time under four policies.[24] The two top rows show outcomes under progressive taxation, while the two bottom rows show outcomes under proportional financing. In rows 1 and 3 we repeat our baseline experiment in which we perfectly eliminate the racial wealth gap in 2040, corresponding to a multiple of $m = 1$. In rows 2 and 4 we target a wealth ratio of $m = 0.3$, which means that we transfer to Black dynasties enough wealth to make them more than 3 times wealthier than White dynasties on average. While the divergence of wealth is now slower, remarkably the wealth ratio asymptotes again toward roughly 2.

What is the output cost of such wealth transfers? Taxing wealth in our economy entails output losses relative to the benchmark as it shifts the occupational composition of the population

[23]The size of the required transfers and taxes is documented in Table 11. The value of transfers is $\tau = 0.75$ relative to average wealth in the economy. Applying an average household wealth of 800,000 dollars, we obtain a transfer of 600,000 dollars per Black dynasty. If there are roughly 20 million Black households, this aggregates to roughly 12 trillion dollars of reparation transfers. The progressive policy imposes a 100 percent tax rate for wealth above 112 times the average wealth, roughly 90 million dollars. The proportional tax rate imposes a flat 13 percent tax on all White wealth. In the larger wealth transfers we consider next, the size of transfer is more than 1.5 million dollars, the progressive tax is imposed on wealth above 13 million, and the flat rate is 38 percent.

[24]The long-run outcomes are averages over the last 250 model periods from a simulation of 600 model periods.

away from entrepreneurship toward labor. For our baseline experiments in rows 1 and 3 that perfectly equalize wealth, we obtain output losses during the transition of roughly 4 percent under progressive taxation and 3 percent under proportional taxation relative to the benchmark. Larger wealth transfers require higher taxes, which translates in larger output losses from redistribution. For example, with progressive taxation the output loss is as high as 17 percent in the transition. As the wealth ratio converges to the benchmark value without wealth transfers, the output loss converges to zero.

4.3 Subsidies in the Quantitative Model

What policy can eliminate the racial wealth gap in the long run? Owing to Black dynasties' pessimistic beliefs at the time of reparations, wealth transfers are not effective in incentivizing entry into entrepreneurship. This suggests that more effective policies should directly target the trade-off between labor and entrepreneurship. Instead of giving a wealth transfer, we now use collected tax revenues to finance a return subsidy for Black entrepreneurs. The goal of the policy is not to eliminate the wealth gap at the time of the subsidy, but to encourage entry into entrepreneurship which allows Black dynasties to build wealth in the long run. We calculate the subsidy s such that it exhausts the revenues collected by taxing wealth of White dynasties, accounting for the endogenous response of Black entrepreneurship k_ι^b:

$$\phi q^*(r+s)\int a_\iota^b k_\iota^b \mathrm{d}\Phi^b = (1-\phi)\int \Lambda(a_\iota^w)a_\iota^w \mathrm{d}\Phi^w. \tag{23}$$

Table 12 summarizes our results for the case in which subsidies are financed with progressive wealth taxes. In row 1 we repeat our results with transfers that equalize wealth in 2040 and in row 3 we repeat our results with larger wealth transfers. In rows 2 and 4, we present policies under which the revenues raised in rows 1 and 3 respectively are used to subsidize returns to Black entrepreneurship. These subsidies equal 22 and 27 percentage points (annualized). In row 2, the subsidy generates a racial wealth ratio of 4 in 2040. Entrepreneurial subsidies are a more powerful instrument to reduce the wealth gap in the long run, as the wealth ratio tends to 1.3 which is below 1.9 with wealth transfers. The output losses are also somewhat smaller with entrepreneurial subsidies during the transition.

Table 12: Subsidies to Black Entrepreneurs

	Transfer	Financing	Wealth Ratio 2040	2100	∞	Output Change (percent) 2040	2100	∞
1.	$\tau = 0.75$	$a^* = 112$	1.0	1.9	1.9	-4.3	-4.3	0.0
2.	$s = 0.22$	$a^* = 112$	4.0	1.7	1.3	-3.3	-4.0	0.1
3.	$\tau = 2.23$	$a^* = 16$	0.3	1.3	1.9	-16.6	-16.5	0.0
4.	$s = 0.27$	$a^* = 16$	2.9	0.9	1.0	-13.5	-14.3	0.2

Table 12 presents the wealth ratio and changes in output relative to the benchmark in selected periods under four different policies. Rows 1 and 3 repeat the results of Table 12 in which transfers are financed with progressive wealth taxes and lead to a White to Black wealth ratio of $m = 1$ and $m = 0.3$ in 2040. Rows 2 and 4 use the revenues raised in rows 1 and 3 to subsidize Black entrepreneurs in 2040. The subsidy s represents the additional annualized return on successful entrepreneurial activities.

In row 4 of Table 12, a 27 percentage points subsidy toward Black entrepreneurs eliminates the racial wealth gap in the long run. Such subsidies effectively reset the conditions of Black dynasties in 2040 to the initial conditions of White dynasties. The policy generates a significant decline in output in the short run due to reduced investing by White dynasties. However, it also generates a small output increase in the long run as total entrepreneurial activity increases.

4.4 Learning from Others' Experiences

An alternative possibility for the convergence of wealth is that Black dynasties learn from others' experiences after reparations. To understand the importance of networks for the speed of learning and the convergence of wealth, we now allow dynasties to learn from others' experiences. Our formulation of learning from others is that dynasties observe outcomes of other dynasties around them, which we label "friends." Every dynasty has a number of own-race friends F^o and a number of cross-race friends F^c. The number of friends equals the number of additional experiences that each dynasty potentially observes in a given period, in addition to their own experience. When friends are entrepreneurs dynasties observe the entrepreneurial experiences of friends. Dynasties do not learn about entrepreneurial outcomes from friends who are laborers. As in our baseline model, we maintain the assumption that dynasties do not learn from aggregate variables.[25]

[25]We formalize learning as follows. Friends of dynasty ι are denoted f_ι. To construct the set of friends, we order dynasties along a unit circle and denote a dynasty's location by $\iota \in \mathbb{I} \equiv [0, 1]$. Dynasties in the interval $\mathbb{I}^w \equiv [0, \phi)$

Table 13: Learning from Others' Experiences After Reparations

	Friends		Wealth Ratio		
	F^o	F^c	2040	2100	∞
1.	0	0	1.0	1.9	1.9
2.	1	0	1.0	1.9	1.9
3.	2	0	1.0	1.9	1.8
4.	4	0	1.0	1.9	1.5
5.	10	0	1.0	1.9	1.0
6.	1	1	1.0	1.8	1.0

Table 13 presents the wealth ratio under wealth transfers financed with progressive taxes. Each dynasty has a number of own-race friendships F_o and a number of cross-race friendships F_c. The total number of friends equals the number of additional experiences that each dynasty observes in a given period, in addition to their own experience.

Table 13 summarizes our results for the case in which transfers are financed with progressive wealth taxes. The first row repeats the baseline results without learning from others. In row 2, we allow dynasties to potentially observe the experience of one additional dynasty from their own group. The next three rows increase the number of friends from a dynasty's own group F^o to 2, 4, and 10. When dynasties learn from the experiences of others, they update their beliefs at a faster rate toward the objective probability of success in entrepreneurship q^*. As the table shows, wealth diverges to a lower ratio as we increase the number of friends. To completely eliminate the racial wealth gap in the long run, the model requires 10 additional observations. As the 2100 values of the wealth ratio demonstrate, the speed of convergence is, however, in all cases quite slow.

Another possibility is that dynasties learn from dynasties of the other race. In row 6 of the table, the racial wealth gap is eliminated after reparations if Black dynasties can learn from at least one White dynasty. We interpret this result as showing the importance of expanding networks outside one's own group. To the extent that networks of friends are amenable to policy

are White, while dynasties in the interval $\mathbb{I}^b \equiv [\phi, 1]$ are Black. Similarly, every dynasty has an identity $\iota^h \in [0,1]$ along a race-specific unit circle, where $\iota^w \equiv \frac{\iota}{\phi}$ for every $\iota \in \mathbb{I}^w$ and $\iota^b \equiv \frac{\iota-\phi}{1-\phi}$ for every $\iota \in \mathbb{I}^b$. Friends are drawn from the set of dynasties who are within distance d on the circle. By construction, friendships are not mutual. Dynasties draw F^o friends along the circle specific to their race, $f^o_\iota \sim U\left([\iota^h, \iota^h + d]^{F^o}\right)$, and F^c friends along the circle for the cross-race, $f^c_\iota \sim U\left([\iota^h, \iota^h + d]^{F^c}\right)$. For White dynasties, the identities of friends along the unit circle are given by the union $f_\iota = \phi f^o_\iota \cup (\phi + (1-\phi) f^c_\iota)$, while for Black dynasties, friends are $f_\iota = \phi f^c_\iota \cup (\phi + (1-\phi) f^o_\iota)$.

or technological change, increased learning from others' experiences can complement reparations toward closing the racial wealth gap.[26]

5 Conclusion

In this paper we develop a framework for analyzing the economics of reparations. The model features heterogeneous dynasties, an occupational choice, bequests, and endogenous dispersion of beliefs about risky returns. Feeding historical events which exclude Black dynasties from labor and capital markets as driving force, we find that the model is consistent with salient features of the data such as current and historical racial gaps in wealth, income, entrepreneurship, and mobility. Methodologically, our model thus contributes to the wealth inequality literature which highlights the role of entrepreneurship and heterogeneity in returns.

We reach a negative answer to the first question we pose "will reparations today in the form of direct wealth transfers eliminate the racial wealth gap in the future?" Even with transfers which completely eliminate the racial wealth gap today, wealth diverges again in the long run. The logic is that century-long exclusions lead Black dynasties to enter into the reparations era with pessimistic beliefs about risky returns and forego investment opportunities in entrepreneurship. We corroborate this model prediction by presenting survey evidence suggesting significant racial gaps in beliefs about risky returns.

We reach a positive answer to the second question we pose "is there a policy that is effective in eliminating this gap?" We show that subsidies toward Black entrepreneurs are more effective than wealth transfers in achieving racial wealth convergence in the long run. We also highlight the possibility that Black dynasties may learn faster after reparations if networks strengthen compared to the past. Stronger network effects can complement reparations to eliminate the racial wealth gap in the future.

[26] An example of such a policy is the Gates Scholarship, which is geared toward outstanding minority students from low-income backgrounds by the Bill & Melinda Gates Foundation. The Foundation organizes a Summer Institute and uses an online platform to enable current and former recipients to directly connect with and learn from one another.

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