THE MACROECONOMICS OF FINANCIAL SPECULATION

Alp Simsek

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The Macroeconomics of Financial Speculation
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ABSTRACT

I review the literature on financial speculation driven by belief disagreements from a macroeconomics perspective. To highlight unifying themes, I develop a stylized macroeconomic model that embeds several mechanisms. With short-selling constraints, speculation can generate overvaluation and speculative bubbles. Leverage can substantially inflate speculative bubbles and leverage limits depend on perceived downside risks. Shifts in beliefs about downside tail scenarios can explain the emergence and the collapse of leveraged speculative bubbles. Speculative bubbles are related to rational bubbles, but they match better the empirical evidence on the predictability of asset returns. Even without short-selling constraints, speculation induces pro- cyclical asset valuation. When speculation affects the price of aggregate assets, it also influences macroeconomic outcomes such as aggregate consumption, investment, and output. Speculation in the boom years reduces asset prices, aggregate demand, and output in the subsequent recession. Macroprudential policies that restrict speculation in the boom can improve macroeconomic stability and social welfare.

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A Dynamic link to most recent draft is available at
https://www.dropbox.com/s/l44eqwxdfz02tt1/speculationMacro_public.pdf

1. Introduction

The U.S. housing boom-bust cycle of the 2000s was a vivid reminder of the dangers of *financial speculation*—trading of financial assets by investors with heterogeneous beliefs. In the upswing, optimistic investors purchased homes, often with borrowed money, with the hope that house prices would continue to appreciate. When the downswing finally arrived, many optimists found themselves in financial ruin. Financial institutions that lent to optimists against their housing collateral suffered heavy losses too. In contrast, pessimistic investors and institutions that had gambled on the collapse of the bubble—with the help of financial innovation—achieved substantial riches. More troubling, financial speculation damaged not only the optimists but also the broader macroeconomy. As house prices declined, the economy sunk into a deep recession that hurt many individuals who hadn't speculated on the housing bubble.

This episode is by no means an exception: Empirical studies find that financial speculation is widespread and regularly associated with macroeconomic instability. Asset price and credit booms often feature substantial trading volume—suggesting a key role for speculation (Hong and Stein (2007); DeFusco et al. (2017)). These booms often end with recessions and financial crises (e.g., Jordà et al. (2015); Krishnamurthy and Muir (2017)), and the downturn is often predictable—suggesting a key role for distorted beliefs (Baron and Xiong (2017); Greenwood et al. (2020)).

These facts revived an old tradition, going back to Keynes (1936); Minsky (1977); Kindleberger (1978), that emphasizes the importance of financial speculation and distorted beliefs for finance and macroeconomics. A large literature shows that speculative trading that results from *belief disagreements* (or heterogeneously distorted beliefs) provides a natural explanation for asset price booms (e.g., Miller (1977); Harrison and Kreps (1978); Scheinkman and Xiong (2003)) and credit booms (e.g., Geanakoplos (2010); Simsek (2013a)). When this type of speculation concerns large markets, such as aggregate house prices, it can also influence macroeconomic outcomes—aggregate consumption, investment, output, and the severity of recessions (e.g., Caballero and Simsek (2020)). In this paper, I review and extend the *macroeconomic* lessons from this emerging literature on *financial speculation with disagreements*. I take a *theoretical* perspective and illustrate the mechanisms by which speculation affects asset prices and business cycles (see the concluding section for a brief discussion of empirical evidence). To highlight unifying themes, I develop a stylized macroeconomic model that embeds many channels studied by different strands of the literature. My analysis synthesizes the following lessons, each covered in a separate section.

First, for risky assets subject to short-selling restrictions, financial speculation can generate *overvaluation*—the marginal investor (who determines the asset price) is more optimistic than the average investor. Overvaluation increases aggregate wealth and consumption (and typically also investment) in the short run. For an open economy, overvaluation also increases current account deficits and drains foreign assets, which subsequently exerts downward pressure on aggregate wealth and consumption (Section 3).

Second, overvaluation can trigger *speculative asset price bubbles* in which the asset price can exceed even the most optimistic investor's valuation. Optimistic investors purchase the asset to sell to an even more optimistic investor in the future. Speculative bubbles help explain the large trading volume associated with overvaluation episodes (Section 4).

Third, leverage can substantially amplify overvaluation and speculative bubbles. Moreover, with collateral constraints, leverage limits are *endogenous* and depend on *perceived downside risks.* In particular, shifts in (lenders') beliefs about downside tail scenarios can explain the emergence as well as the collapse of *leveraged speculative bubbles* (Section 5).

Fourth, overvaluation and (leveraged) speculative bubbles generate similar macroeconomic effects as *rational bubbles*—an alternative mechanism for high asset valuations studied by a large macroeconomics literature. Compared to rational bubbles, the overvaluation mechanism is more consistent with the empirical evidence on the predictability of asset returns following asset price booms. Overvaluation and speculative bubbles also make more precise testable predictions and do not require restrictive assumptions on the economic environment (Section 6).

Fifth, even without short-selling constraints, financial speculation can induce *procyclical asset valuation* through its impact on investors' *wealth dynamics.* These speculative dynamics typically increase the risky asset valuation (or reduce the risk premium) in good times, and decrease the valuation in bad times. This helps explain the excess asset price volatility and the countercyclical risk premium observed in practice (Section 7).

Sixth, *financial speculation can exacerbate aggregate demand contractions.* With nominal rigidities, speculative wealth dynamics not only make asset prices more volatile, but they also create excessive fluctuations in aggregate demand and output. In particular, speculation that takes place in the boom lowers asset prices, consumption, investment, and output after the economy transitions to recession (as long as monetary policy is somewhat constrained). Speculation exacerbates the recession even without the standard financial frictions, although speculation would cause a larger contraction in demand (and could induce a financial crisis) when the recession features financial frictions. Macroprudential policies that restrict speculation in the boom mitigate the subsequent recession and improve macroeconomic stability. These policies can increase social welfare even according to the standard (non-paternalistic) Pareto criterion, since they internalize *aggregate demand externalities* (Section 8).

Seventh, policies that restrict speculation can also increase the speculators' own welfare (even without any externalities). Empirical evidence suggests that the speculators on average suffer large welfare losses according to the *objective* belief—the belief that will be realized in the data. However, these losses typically go undetected by the standard Pareto criterion that respects individuals' subjective beliefs. An alternative *belief-neutral welfare criterion* can identify pure speculation as socially wasteful *even when the planner does not know the objective belief* (Section 9).

Throughout, I take investors' beliefs and disagreements as given. I abstract from heterogeneous information and interpret disagreements as resulting mostly from psychological biases that heterogeneously distort investors' beliefs (see Remark 1). *Extrapolation bias* in particular plays a central role in the historical narratives of asset price bubbles. Most bubbles start with good news about fundamentals—what Kindleberger (1978) calls a "displacement"—which then induces euphoria and overreaction. While extrapolation can explain asset price booms even when investors share the same beliefs, disagreements amplify the price impact of extrapolation and help explain the large trading volume and leverage that usually accompany these episodes. Overall, extrapolation and speculation provide highly complementary accounts of asset price bubbles as well as procyclical asset valuation (see Sections 4 and 7 for further discussion).

A caveat on the modeling approach is in order. My goal is to highlight and connect several mechanisms on the macroeconomics of financial speculation. This requires me to adopt a *stylized* model with stark assumptions. A more standard macroeconomic model would be useful to assess the quantitative strength of each mechanism, but it would struggle to illustrate a large number of mechanisms. Even with a stylized model, however, it is impossible to cover all of the related ideas from the literature. In each section, I formally illustrate a few key ideas. I then often conclude with "related mechanisms" and "related literature." In the former, I discuss ideas from specific strands of the literature without delving into details. In the latter, I highlight broader connections with the literature.

2. A macroeconomic model of speculation

In this section, I describe a macroeconomic model that allows for belief disagreements and speculation. I also characterize the equilibrium in a (common-belief) benchmark setting without speculation. In subsequent sections, I use variations of this model to illustrate the mechanisms by which speculation affects asset prices and macroeconomic outcomes. The model builds upon ingredients from Blanchard (1985), Simsek (2010) (Chapter 2), and Caballero and Simsek (2020).

Consider an infinite horizon economy in discrete time, $t \in \{0, 1, ...\}$. There are two factors, labor and capital. The supply of both factors is constant and normalized to one. There is no investment but there might be depreciation. Specifically, a unit of capital in period t becomes $1-\delta$ units at the start of the next period where $\delta \geq 0$ denotes the depreciation rate. To keep the supply of capital constant, δ new units of capital are injected into the economy in every period.

The productivity of labor and capital are given by $e_t n$ and e_t, respectively. The factors are fully substitutable and potential output is given by

$$Y_t^* = e_t (n+1) . \tag{1}$$

I start with a neoclassical setting in which there are no nominal rigidities and factor markets are competitive. This implies output is at its potential and factors earn their marginal products:

$$Y_t = Y_t^* = e_t (n+1) \text{ and } w_t = ne_t, r_t = e_t. \tag{2}$$

Here, Y_t, w_t, and r_t denote actual output, the wage, and the rental rate of capital, respectively. In Section 8, I introduce nominal rigidities and modify the supply side to allow for demand recessions.

The demand side features a perpetual youth structure. At the beginning of every period, a mass one of new investors ("young investors") are born endowed with all of the labor and the newly injected capital (δ units). They can be thought of as working and starting new companies. Investors that were born in previous periods ("old investors") do not work and spend out of their accumulated savings. For simplicity, investors never die.

Importantly, investors disagree about the productivity growth, z_{t+1}, which follows

$$e_{t+1} = z_{t+1} e_t \text{ with } z_{t+1} \in \{H, L\} \text{ and } H > L. \tag{3}$$

Here, H (resp. L) corresponds to a high growth (resp. low growth) state. To simplify the portfolio problem, I focus on extreme beliefs: Investors either have "optimistic" beliefs and think $z_{t+1} = H$ will be realized with certainty; or they have "pessimistic" beliefs and think $z_{t+1} = L$. Investors' beliefs are *dogmatic*—formally, investors know each others' beliefs and they *agree to disagree* (see Remark 1 for other interpretations).

To illustrate speculative bubbles, I allow investors' beliefs to change after the first period of their life (see Section 4). There are four exogenous investor types denoted by the superscript $i \in \{o, p, mo, mp\}$. The types $\{o, p\}$ correspond to young investors ("optimists" and "pessimists"), and the types $\{mo, mp\}$ correspond to old investors ("mature optimists" and "mature pessimists"). Young investors are born with a randomly assigned type and are optimistic with probability $\alpha \in [0, 1]$. As I will specify later, young optimists can transition into either mature optimists or mature pessimists.

Investors trade two types of financial assets. There is a (potentially) risky asset that represents claims to (nondepreciated) capital. I let P_t denote the (ex-dividend) price of the asset in period t. I also define the *normalized price*, $p_t = P_t / e_t$. This notation will facilitate the analysis since I focus on equilibria in which allocations scale with productivity, e_t. The asset's (gross) return between periods t and $t+1$ is given by

$$R_{t,t+1} = \frac{(1-\delta)(r_{t+1} + P_{t+1})}{P_t} \qquad \text{where } P_t = p_t e_t. \tag{4}$$

There is also a risk-free asset with gross return denoted by R_t^f. I will start with a small open economy case in which the risk-free asset is elastically supplied (by foreigners) at a fixed rate. I will eventually consider the closed economy case in which the risk-free asset is in zero net supply at an endogenous rate. Labor and future capital injections by newborns are nontradable.

Type i investors choose how much to spend c_t^i, and what fraction of their portfolio to invest in the risky asset, ω_t^i (with the remaining fraction invested in the risk-free asset) to maximize time-separable log utility:

$$\begin{aligned} V_t^i\left(a_t^i\right) &= \max_{c_t^i, \omega_t^i} \log c_t^i + \beta E_t^i\left[V_{t+1}^i\left(a_{t+1}^i\right)\right], \qquad (5)\\ \text{s.t. } a_{t+1}^i &= \left(a_t^i - c_t^i\right)\left(R_{t,t+1}\omega_t^i + R_t^f\left(1-\omega_t^i\right)\right) \\ \text{with } \omega_t^i &\in \left[\underline{\omega}_t, \overline{\omega}_t\right] \text{ and } a_{t+1}^i \geq 0. \end{aligned}$$

Here, a_t^i denotes the total wealth of type i investors before they spend. For young investors, $a_t^o = \alpha (w_t + \delta P_t)$ and $a_t^p = (1-\alpha)(w_t + \delta P_t)$. The lower and upper bounds for the portfolio weights, $\underline{\omega}_t$ and $\overline{\omega}_t$, correspond to exogenous short-selling and leverage limits, respectively. I also require the investors to obtain a nonnegative portfolio return in each state—this captures a collateral constraint and helps illustrate the endogenous leverage limit (see Section 5.2).

In view of log utility, investors optimally consume a constant fraction of their wealth, $c_t^i = (1-\beta) a_t^i$ (regardless of asset returns). Therefore, aggregate consumption denoted by C_t is also a constant fraction of aggregate wealth:

$$C_t = (1-\beta) \sum_i a_t^i. \tag{6}$$

Since investors have extreme beliefs, they perceive no risk. Therefore, they invest in the asset that they believe generates a higher return, as much as allowed by the constraints. When the nonnegative wealth constraint does not bind ($a_{t+1}^i > 0$), investors' optimal portfolio weight satisfies:[1]

$$\begin{cases} \omega_t^i = \overline{\omega}_t & \text{if } E_t^i [R_{t,t+1}] > R_t^f \\ \omega_t^i \in [\underline{\omega}_t, \overline{\omega}_t] & \text{if } E_t^i [R_{t,t+1}] = R_t^f \\ \omega_t^i = \underline{\omega}_t & \text{if } E_t^i [R_{t,t+1}] < R_t^f \end{cases}. \tag{7}$$

The last step is to specify the market clearing conditions. I consider both open and closed economy cases. For an open economy, I assume foreigners cannot trade domestic capital and let F_t denote the country's (ex-dividend) net risk-free foreign asset position. With this notation, the asset and goods market clearing conditions are

$$\sum_i \omega_t^i \beta a_t^i = P_t \text{ and } \sum_i \beta a_t^i = P_t + F_t, \tag{8}$$

$$Y_t = C_t + F_t - R_{t-1}^f F_{t-1}. \tag{9}$$

The first line says that the amount of saving in the risky asset is equal to its supply. In addition, the total amount of saving in all assets is equal to the total amount of asset supply, which includes the country's foreign asset position. The second line says that output is equal to the sum of consumption and the net investment in foreign assets.

For the baseline setting (until Section 7), I focus on a small open economy, which helps to illustrate how speculation affects risky asset prices and aggregate demand while keeping the interest rate and investors' endowments exogenous. Specifically, investors can borrow from or lend to foreigners at an exogenous and constant rate R^f. In this case, F_t is unconstrained and Eqs. $(8-9)$ hold with

$$R_t^f = R^f \text{ and } Y_t = Y_t^* \text{ for each } t. \tag{10}$$

[1] When $a_{t+1}^i = 0$ for some continuation states, Eq. (7) holds with the *endogenous* leverage or short-selling limits that I characterize in Section 5.2.

I assume the exogenous interest rate satisfies $R^f > H(1-\delta)$, which ensures the investors' problem has a well-defined solution, and $\beta R^f < L$, which ensures aggregate wealth normalized by productivity remains bounded.

Remark 1 (Sources of heterogeneous beliefs)**.** *The distinctive feature of speculative episodes is high trading volume. I generate speculative trading by assuming* ***heterogeneous dogmatic beliefs*** *in the sense that investors do not think other investors (or prices) have information about fundamentals that they haven't already incorporated. These beliefs can emerge from various sources: investors can have* ***heterogeneous prior beliefs*** *(e.g., Morris et al. (1995)), they might* ***interpret information heterogeneously*** *(e.g., Harris and Raviv (1993); Kandel and Pearson (1995); Banerjee and Kremer (2010)), they might* ***neglect the information in other investors' beliefs*** *(e.g., Eyster et al. (2019)), or their beliefs might be* ***heterogeneously distorted by behavioral forces*** *such as* ***overconfidence*** *(e.g., Odean (1998); Daniel and Hirshleifer (2015)),* ***experience effects*** *(e.g., Malmendier and Nagel (2011); Malmendier et al. (2020)),* ***motivated reasoning*** *(e.g., Brunnermeier and Parker (2005); Bénabou and Tirole (2016)), and* ***social interactions*** *(e.g., Burnside et al. (2016); Bailey et al. (2018)).*[2] *For simplicity, I assume beliefs are common knowledge and abstract from asymmetric information. A literature on no-trade theorems shows that asymmetric information by itself cannot induce trade among rational agents with common priors (e.g., Milgrom and Stokey (1982); Sebenius and Geanakoplos (1983)).*

Macroeconomic outcomes. I now characterize the equilibrium. For the baseline (small open economy) setting, output is proportional to productivity $Y_t = Y_t^* = e_t(n+1)$ [see (2)]. I focus on equilibria in which the normalized price is constant $p_t = p$, so that the asset price is also proportional to productivity, $P_t = pe_t$.

To characterize the remaining variables, let $c_t = C_t/e_t$ and $f_t = F_t/e_t$ denote the normalized consumption and foreign assets. Appendix A.1 derives the dynamics for these variables as follows:

$$c_t = (1-\beta)\left(1+n+p+\frac{R^f f_{t-1}}{z_t}\right), \tag{11}$$

$$p + f_t = \beta\left(1+n+p+\frac{R^f f_{t-1}}{z_t}\right). \tag{12}$$

The first equation captures the *wealth effect on consumption* [see (6)]. The initial aggregate wealth (in normalized terms) is the sum of output, $1+n$, the domestic asset, p, and the initial foreign assets, $\frac{R^f f_{t-1}}{z_t}$. The second equation captures the evolution of aggregate wealth [see (8)].

In general, the normalized consumption and foreign assets are stochastic and depend on the realization of growth, $z_t \in \{H, L\}$. The condition $\beta R^f < L$ ensures the normalized aggregate wealth remains bounded for all feasible paths. Occasionally, I focus on paths in which the growth shock is constant, $z_t = z \in \{H, L\}$ for each t. Along these paths, the normalized consumption and

[2] See Hirshleifer (2001); Barberis and Thaler (2003) for a review of the psychological forces that distort investors' beliefs and affect their investment decisions in practice.

foreign assets converge to constants given by,

$$
\begin{aligned}
c &= (1-\beta)\frac{1+n-\left(R^f/z-1\right)p}{1-\beta R^f/z}, && (13)\\
f &= \frac{\beta(1+n)-(1-\beta)p}{1-\beta R^f/z}. && (14)
\end{aligned}
$$

I refer to these allocations as a balanced growth path (BGP).

Importantly, Eqs. $(11-14)$ imply that, when there is an equilibrium with a constant normalized asset price p, this price is a *sufficient statistic for macroeconomic outcomes* (for a given path of realized growth rates, $\{z_t\}$). In subsequent analysis, I characterize p for different cases and refer to these equations to discuss the macroeconomic effects that are common across specifications.

Common-belief benchmark. I start with common beliefs, which provide a benchmark for the main analysis with disagreements. Suppose all investors think $z_{t+1} = z \in \{H, L\}$. Suppose also that there is no exogenous leverage limit $\overline{\omega}_t = \infty$ (which allows the aggregate risk-free asset position to be negative). Using Eqs. $(2), (3), (4), (7), (10)$, there is an equilibrium with a constant normalized asset price given by

$$
p(z) \equiv \frac{z(1-\delta)}{R^f - z(1-\delta)}. \qquad (15)
$$

Here, $z(1-\delta)$ denotes the net growth rate of earnings, which combines the productivity growth and depreciation. With common beliefs, the normalized price is determined by the standard Gordon growth formula. As expected, the price is higher when all investors are optimistic than when they are all pessimistic, $p(H) > p(L)$.

3. Disagreements and overvaluation

I next turn to my main focus, belief disagreements and speculation. In environments with restricted short selling, disagreements can create *overvaluation*, as first emphasized by Miller (1977). I illustrate this mechanism and discuss its macroeconomic effects on aggregate consumption and investment.

Short selling—borrowing an asset to sell—enables an investor to take a negative position on an asset she does not own. In practice, short selling real estate is typically not possible, and short selling financial assets can be quite costly. Derivatives markets can make short selling easier, but they usually do not eliminate short-selling constraints (see Reed (2013) for a review of short selling and its relationship with the options market). Short selling is also frequently banned or restricted by regulators (see, e.g., Bris et al. (2007)).

To investigate the effect of these constraints, consider (for simplicity) the extreme case in which short selling is prohibited, $\underline{\omega}_t^i = 0$. Suppose also that investors' beliefs are persistent over time: young optimists (pessimists) become mature optimists (pessimists). Finally, suppose the young optimists' wealth (even without leverage) is sufficient to purchase the entire asset supply,

$$\beta\alpha\left(n+\delta p\left(H\right)\right)>p\left(H\right). \tag{16}$$

It is then easy to verify that there is an equilibrium with a constant normalized price given by:

$$p\left(H\right)=\frac{H\left(1-\delta\right)}{R^{f}-H\left(1-\delta\right)}. \tag{17}$$

Even though only a fraction of the economy is optimistic, asset prices and macroeconomic outcomes behave *as if all investors are optimistic.* At this price, pessimists would like to short sell the asset; but they are constrained.

In general, short-selling restrictions imply that pessimists' beliefs are not fully reflected in the market price. As long as optimists have sufficient wealth, the marginal investor is an optimist and the asset price exceeds the valuation that would obtain absent short-selling constraints.[3] This qualitative insight applies under less extreme restrictions on short selling, e.g., when short selling is possible but costlier than buying (see Atmaz et al. (2019)).

Remark 2 (Comparison with the objective valuation)**.** *So far, I haven't specified "the objective belief"—the belief that will be realized in the data on average. This belief does not matter for most of my analysis—the allocations and prices are determined by investors' subjective beliefs. However, the objective belief is important for mapping the results to empirical evidence. To match the evidence on asset price booms generating predictably low expected returns (e.g., Greenwood et al. (2020)), I focus on scenarios in which overvaluation raises the price above the objective valuation—the valuation according to the objective belief. In comparative static exercises (in this section and subsequent sections), I take the objective belief as the pessimistic belief so that the objective valuation is $p\left(L\right)$. This assumption simplifies the exposition although it is stronger than necessary.*

Remark 3 (Market selection hypothesis)**.** *The objective belief also matters for the dynamics of investors' wealth over time. The market selection hypothesis suggests that investors with "less correct" beliefs should eventually be driven out of the market as they consistently lose money (see, e.g., Friedman (1953); Sandroni (2000); Blume and Easley (2006, 2010)). This hypothesis does not apply in my model since new investors (with possibly incorrect beliefs) enter the market in view of the overlapping generations structure. Recent research has uncovered several additional reasons this hypothesis is unlikely to apply in practice, e.g., irrational investors might be unable to pledge their future* ***non-financial*** *wealth (e.g., Cao (2018)), they might have a* ***greater propensity to save*** *(e.g., Yan (2008)), and their savings might* ***endogenously*** *increase as their wealth share declines—due to the increase in their perceived return from speculation (e.g., Borovička (2020)).*

Macroeconomic effects. Next consider the *macroeconomic* effects of overvaluation. To fix ideas, suppose initially all investors are pessimistic ($\alpha=0$), the normalized asset price is given by $p\left(L\right)$,

[3] In this model, absent any exogenous short-selling and leverage limits, the asset price is the wealth-weighted harmonic average of optimists' and pessimists' valuation (see Eq. (33) in Section 7).

and the economy is on a BGP with $z = L$ [see $(13 - 14)$]. In period 0, the share of young optimists, α, permanently increases to a sufficiently high level that the normalized price increases to $p(H)$. Suppose also that pessimists have the objective belief and (accordingly) focus on a path in which the growth rate is realized to be low, $z_t = z = L$ for each t [cf. Remark 2]. What happens to aggregate consumption in the short run (in period 0)? How about in the longer run (as $t \to \infty$)?

Eqs. $(11 - 12)$ show that overvaluation initially increases aggregate wealth $(p + f_0)$ and consumption (c_0) (the initial foreign asset position f_{-1} is determined by history). Subsequently, the country converges to a new BGP where consumption and foreign assets are given by Eqs. $(13 - 14)$. Thus, overvaluation eventually reduces foreign assets (f). Overvaluation also eventually reduces aggregate wealth $(p + f)$ and consumption (c) as long as the parameters satisfy, $R^f > z = L$. As I discuss in Section 6, this is the condition for the economy to be dynamically efficient.

Intuitively, an increase in asset prices raises investors' (perceived) wealth and induces them to spend more. The country finances greater consumption by running a current account deficit, which lowers its normalized foreign asset position. This typically lowers aggregate wealth and consumption in subsequent periods (unless the economy is dynamically inefficient and features too high savings to begin with). Investors raise their spending in the hope that future productivity will be high. If these hopes are not realized, aggregate wealth and spending eventually declines.

Even though overvaluation is driven by optimists, its short run wealth effects apply to all investors that are exposed to assets (or that hold assets when overvaluation starts). For instance, since a young optimist and a young pessimist are both endowed with δ units of capital, overvaluation increases their initial spending by the same amount, $c_t^o/\alpha = c_t^p/(1-\alpha) = (1-\beta)(n + \delta p(H))e_t$. Since investors choose different portfolios, their fortunes diverge in subsequent periods depending on the realization of uncertainty—a natural consequence of speculation.

Related mechanism: Limits to arbitrage. In my analysis, pessimists are unable to undo the overvaluation due to short-selling restrictions. A related literature emphasizes more general *limits to arbitrage*—such as fundamental risk, endogenous price volatility, or capital constraints—that prevent professional arbitrageurs from correcting "mispricing" (see, e.g., De Long et al. (1990a); Shleifer and Vishny (1997); Lamont and Thaler (2003a)). This suggests the overvaluation mechanism can apply *even without short-selling constraints*. However, the alternative limits do not necessarily generate a bias toward high valuations, so these versions of the argument require an additional source of optimism. A related literature emphasizes dynamic mechanisms by which professional arbitrageurs might actually *exacerbate* the overvaluation, e.g., arbitrageurs might predict investor sentiment (e.g., De Long et al. (1990b)) or they might "ride the bubble" due to a lack of synchronization among each other (e.g., Abreu and Brunnermeier (2003)). Brunnermeier and Nagel (2004) present empirical evidence that suggests hedge funds amplified the overvaluation of the U.S. Internet stocks in the late 1990s.

Related mechanism: Investment. While I focus on consumption, overvaluation can also affect investment. The precise channels are subtle and depend on the firm managers' beliefs and incentives as well as the firms' financial constraints (see, e.g., Morck et al. (1990); Blanchard et al. (1993); Stein (1996); Baker et al. (2003); Panageas (2005); Bolton et al. (2006)). Consider two extreme cases. If the firm managers maximize the firm's *current* market value (e.g., because they cater to investors' beliefs or have a short time horizon), then overvaluation increases investment through a standard Q-theory mechanism—regardless of the managers' own beliefs. In the other extreme case in which the managers maximize the firm's *long-term* market value *and* have the objective belief, then the managers should in principle ignore the overvaluation when evaluating investment projects. However, overvaluation can increase investment even in this latter case by relaxing the firm's financial frictions and reducing its cost of capital. Empirically, the literature finds that overvaluation *has* increased investment in specific episodes such as the Japanese stock market bubble of the late 1980s (e.g., Chirinko and Schaller (2001)) and the U.S. Internet bubble of the late 1990s (e.g., Gilchrist et al. (2005)), as well as more systematically (e.g., Polk and Sapienza (2008)). As I discuss in the concluding section, overvaluation has also increased the residential investment in the U.S. housing boom of the 2000s.

Related literature. A large finance literature empirically investigates short selling and typically finds support for the overvaluation mechanism I discuss in this section (e.g., Chen et al. (2002); Diether et al. (2002); Jones and Lamont (2002); Nagel (2005); Boehme et al. (2006); Yu (2011)). The literature has also applied this mechanism (or its variants) to explain specific overvaluation episodes such as the U.S. Internet bubble (e.g., Ofek and Richardson (2003); Lamont and Thaler (2003b)) or the U.S. housing boom (e.g., Piazzesi and Schneider (2009); Nathanson and Zwick (2018)), and to shed light on more general empirical asset pricing puzzles (e.g., Hong and Sraer (2016); Chu et al. (2016)).

In related work, Guzman and Stiglitz (2020) present a model that generates similar macroeconomic effects as in this section. While their model does not feature short-selling constraints, disagreements nonetheless increase agents' expected income ("pseudo-wealth"), which increases their spending in the short run (and reduces it in the longer run) for appropriate utility functions.

4. Speculative bubbles

The analysis so far does not (necessarily) generate a large trading volume—a common feature of asset price booms. A literature initiated by Harrison and Kreps (1978) shows that in a dynamic setting belief disagreements can induce speculative trading, which substantially increases the degree of overvaluation. In particular, the asset price can exceed the *present discounted valuation of all investors*—a situation which Scheinkman and Xiong (2003) refer to as a *speculative bubble*. I next illustrate the speculative bubble mechanism. In this and the next section, I abstract from macroeconomic outcomes and focus on characterizing how different specifications of beliefs or the credit environment affect the normalized asset price. Given the impact on the asset price (relative to the appropriate benchmark), Eqs. $(11-14)$ describe the effect on macroeconomic outcomes.

Consider the same assumptions as in the previous section with the difference that young optimists become mature pessimists (and young pessimists remain mature pessimists). Therefore, young optimists are the only optimistic investors in the economy. Moreover, young optimists know they will become pessimistic in the next period (they think "this time is different").

With these assumptions, there is an equilibrium with *exactly the same* asset price as in the previous section—even though optimists are less optimistic than before. In particular, young optimists' present discounted (or buy-and-hold) valuation is given by $p^o_{pdv} = \frac{H(1-\delta)}{R^f - L(1-\delta)}$, which is strictly smaller than the equilibrium price, $p(H) = \frac{H(1-\delta)}{R^f - H(1-\delta)}$. *Even though optimists expect high growth for only one period, the asset is priced as if it will have high growth for all future periods.*

Intuitively, young optimists do not follow a buy-and-hold strategy. Instead, they buy the asset to sell to future young optimists. The asset price reflects this *resale option value.* This value drives up the price of the asset beyond the present discounted valuation of all investors in the economy. The difference between the price and the marginal investor's valuation reflects a *speculative bubble.*

Importantly, the equilibrium features a large trading volume: young optimists hold the asset for one period and sell to future young optimists. The turnover in each period is 100%. This speculative trading creates a large resale option value that can considerably exacerbate overvaluation.

Related mechanism: Other reasons for resale. In this model, the resale option value is driven by future investors that enter the market. However, the resale option value can also result from other modeling ingredients. Harrison and Kreps (1978) assume investors' *relative* optimism flips over time. Scheinkman and Xiong (2003) generate this type of flipping endogenously by assuming investors are *overconfident* and weight their own signals relatively more than others' (see also Morris (1996)). The common element is that optimists who buy the asset think they will be able to sell it to another investor in a future period. In a complementary review, Xiong (2013) discusses this literature on speculative bubbles in greater detail and highlights its *empirical* success in accounting for the historical bubbles associated with trading frenzies.

Related mechanism: Extrapolative bubbles. I take investors' beliefs as exogenously given. A recent literature explains asset price bubbles associated with high trading volume using models with *endogenous* beliefs (e.g., DeFusco et al. (2017); Barberis et al. (2018); Bordalo et al. (2020a)). This literature typically relies on extrapolative (or diagnostic) beliefs to generate optimism and overvaluation, and an additional ingredient to generate disagreements and trading volume. For instance, Barberis et al. (2018) assume investors' beliefs in a bubble waver over time between a "growth" signal (that suggests to buy) and a "value" signal (that suggests to sell). As the bubble grows, these signals become stronger and wavering induces large *endogenous* disagreements. This literature complements my analysis and helps match empirically relevant features of asset price bubbles—such as the dynamics of the relationship between prices and trading volume.

5. Leveraged speculative bubbles

The analysis so far assumes borrowers have sufficient resources and do not need leverage to undertake their desired investments. In practice, leverage plays an important role in most asset price booms. A recent literature endogenizes leverage limits in environments with financial speculation through *collateral constraints*. The endogenous leverage limit depends on *perceived downside risks*. In particular, shifts in (lenders') beliefs about downside tail scenarios can explain the emergence as well as the collapse of leveraged speculative bubbles, as emphasized by Geanakoplos (2010); Simsek (2013a). I next illustrate how leveraged bubbles form, starting with exogenous leverage limits, and then considering the case with endogenous leverage limits.

Throughout this section, I assume there is no depreciation or injection of new capital, $\delta = 0$. This ensures that young investors' wealth, $\alpha\left(w_t + \delta P_t\right)$, is exogenous to asset prices. I discuss the case with endogenous wealth at the end of the section.

5.1. Bubbles with exogenous leverage limits

Consider the same assumptions as in Section 4 (which lead to speculative bubbles) with two differences. First, investors face an exogenous leverage limit, $\omega_t^i \leq \overline{\omega}$ for some $\overline{\omega} \geq 1$ (and they cannot short sell, $\omega_t^i \geq 0$).[4] Second, the parameters satisfy

$$p\left(H\right) > \beta\alpha n\overline{\omega} > p\left(L\right) \text{ where } p\left(z\right) = \frac{z}{R^f - z}. \tag{18}$$

In particular, young optimists' wealth, even if leveraged up to the limit, is not sufficient to purchase the asset supply at the optimistic valuation.

Under these conditions, there is an equilibrium in which young optimists are constrained and the normalized asset price is given by

$$p_t = p\left(\overline{\omega}\right) \equiv \beta\alpha n\overline{\omega} \text{ for each } t. \tag{19}$$

The asset price is determined by young optimists' levered wealth. At this price, young optimists think the asset is underpriced and would like to increase their investment, but they are constrained by the leverage limit. Pessimists (all other investors) think the asset is overpriced but are constrained by the short selling limit (see Appendix A.4.1).

Importantly, the equilibrium asset price is increasing in the leverage limit. Leverage enables optimists to exert greater influence on the asset price, which in turn leads to more overvaluation and a larger speculative bubble.

Likewise, it is easy to see that allowing for short selling—borrowing the asset—has a similar effect *but in the opposite direction*. Suppose young investors (and only young investors) face a more relaxed short selling restriction, $\omega_t^i \geq \underline{\omega}$ for some $\underline{\omega} \leq 0$. Then, under appropriate parametric

[4] I also assume the exogenous leverage limit is tighter than the endogenous limit that I characterize later in the section (see Appendix A.4.1 for the precise condition).

restrictions, the equilibrium price is given by [cf. Eq. (19)]

$$p_t = \beta n \left(\alpha \overline{\omega} + (1-\alpha) \underline{\omega} \right). \tag{20}$$

Relaxing short-selling constraints (lowering $\underline{\omega}$) enables pessimists to wield a greater influence on the price, which reduces the speculative bubble.

5.2. Collateral constraints and endogenous leverage limits

Next consider the same setup but without an exogenous leverage limit, $\overline{\omega}_t = \infty$. For simplicity, I focus on the case without short selling, $\omega_t^i \geq 0$ and discuss the case with short selling in Remark 5. I first illustrate *the collateral constraint* that leads to an *endogenous leverage limit.* I then analyze how endogenous leverage affects speculative bubbles.

Recall that investors' portfolio return is required to be nonnegative in all states, $R_{t,t+1}\omega_t^i + R^f \left(1 - \omega_t^i\right) \geq 0$ [see (5)]. Since there are only two assets, this requirement can be interpreted as *a collateral constraint that rules out default*:

$$R^f \left(\omega_t^i - 1 \right) \leq R_{t,t+1} \omega_t^i. \tag{21}$$

Specifically, suppose borrowers can walk away from the loan. To prevent default, lenders ask borrowers to post their assets as collateral. Finally, suppose lenders require the loan to be fully safe (I discuss the case with default risk at the end of the section). These ingredients imply the collateral constraint in (21): borrowers' promised risk-free payment must be lower than the value of the collateral *in all states*.

In equilibrium, the lowest growth state, $z_{t+1} = L$, leads to the smallest return $R_{t,t+1} = \frac{e_t L (1+p_{t+1})}{e_t p_t}$ [see (4)]. Using this condition, Eq. (21) implies the *endogenous leverage limit*:

$$\omega_t^i \leq \overline{\omega}_t^{end} \equiv \frac{p_t}{p_t - \frac{1}{R^f} L \left(1 + p_{t+1}\right)}. \tag{22}$$

This limit has a natural interpretation. The denominator is *the required downpayment*—the minimum amount the borrower must spend out of her own pocket to purchase the asset at the equilibrium price. This downpayment is determined by the difference between the asset's price, p_t, and *its pledgeable value,* $\frac{1}{R^f} L \left(1 + p_{t+1}\right)$—the value lenders can recover if the low growth state is realized. The leverage limit is the ratio of the asset price to the required downpayment.

Importantly, the leverage limit is determined by the (lenders') beliefs about the downside tail scenario—state L. An increase in L increases the pledgeable value, reduces the required downpayment (or margin), and loosens the leverage limit. Conversely, a decline in L, e.g., due to an increase in uncertainty, increases the required downpayment and tightens the leverage limit.

Remark 4 (Alternative Collateral Constraints)**.** *I assume the pledgeable value incorporates next period's earnings,* e_{t+1} *(as in Fostel and Geanakoplos (2014)). The literature has also considered* *alternative collateral constraints where the pledgeable value is determined only by the asset price in the next period, P_{t+1} (e.g., Kiyotaki and Moore (1997)). In my context, this distinction makes little difference. Having said this, my assumption is arguably more appropriate for financial assets where it would be difficult for the borrower to hide (or abscond with) the earnings.*

Remark 5 (Endogenous Short Selling Limit)**.** *While I focus on endogenous leverage limits, the model also features a collateral constraint for short positions that implies an endogenous short-selling limit (see Appendix A.4.2):*

$$\omega_t^i \geq \overline{\omega}_t^{end} \equiv -\frac{p_t}{\frac{1}{R^f} H\left(1+p_{t+1}\right) - p_t}. \tag{23}$$

*Intuitively, since a short position is more likely to default when the asset price increases, its required downpayment is determined by the difference between the valuation in the high growth state, $\frac{1}{R^f} H\left(1+p_{t+1}\right)$, and the current price, p_t. Thus, the short selling limit is determined by **upside risks** and is tightened by an increase in H—as opposed to a decline in L (see also Simsek (2013a)).*

Remark 6 (Endogenous Limits and Market Incompleteness)**.** *In my model, collateral constraints by themselves do not generate market incompleteness. In fact, when both leverage and short selling are allowed, the equilibrium in each period is as if the markets are complete (see Appendix A.4.2). The main reason is that investors receive no nontradable endowments in future periods (they only receive endowments when they are born) so their future wealth is equal to their financial wealth. Thus, the analysis is equivalent to allowing investors to choose their future wealth (and consumption) subject to obtaining nonnegative wealth in each state [see problem* (5)*]. With two continuation states and two assets, this replicates the complete market outcomes. The endogenous limits in* (22) *and* (23) *extend to more realistic (incomplete market) scenarios in which investors also have nonfinancial wealth, e.g., when they receive nontradable labor income in future periods (see Cao (2018)).*

Next consider how endogenous leverage affects the speculative bubble (without short selling). I conjecture an equilibrium in which the normalized price p and the young optimists' *endogenous* leverage limit are constant, $\overline{\omega}_t^{end} \equiv \overline{\omega}^{end}(p)$, as in Section 5.1. Combining Eqs. (22) and (19), the price is the solution to

$$p = \beta\alpha n + \frac{1}{R^f} L\left(1+p\right), \text{ which gives } p = \frac{\beta\alpha n R^f + L}{R^f - L}. \tag{24}$$

This price is an equilibrium as long as the parameters satisfy $p < p\left(H\right)$ (see Appendix A.4.2).

Eq. (24) captures an amplification mechanism that can lead to a large speculative bubble even when borrowers are constrained. The asset price is determined by borrowers' available liquidity, which consists of their own wealth, $\beta\alpha n$, and the asset's pledgeable value, $\frac{1}{R^f} L\left(1+p\right)$. The pledgeable value is increasing in not only the exogenous downside tail scenario (L) but also in the endogenous price in *the next period* (p). With loose collateral constraints (high perceived L), speculation increases the price in the next period, which leads to even looser collateral constraints in the current period, and so on.

The broader idea is that, even if lenders are not optimistic about an asset, they are willing to lend against the asset *as long as they think future optimists will come along* and *those optimists will be relatively unconstrained.* Therefore, small changes in the credit environment or lenders' *higher order* beliefs (here, their beliefs about *future investors'* beliefs) can have a large impact on the extent of mispricing and on the size of the speculative bubble.[5]

Related mechanism: Net worth channel. The amplification mechanism in this section is from Simsek (2010). It is easy to extend the model to capture mechanisms that further relax optimists' borrowing constraints in asset price booms. When $\delta > 0$, young optimists are initially endowed with some asset, which introduces a standard amplification mechanism that operates through borrowers' net worth, $\alpha(w_t + \delta P_t)$ (e.g., Kiyotaki and Moore (1997)).

Related mechanism: Default risk and the type of disagreements. In this model, I assume lenders require the loans to be fully safe. Fostel and Geanakoplos (2015) show that in models with two continuation states (such as the current model) this assumption is without loss of generality. Loosely speaking, lending a larger amount exposes lenders to the asset's risk, but with two states this exposure can be replicated by purchasing an appropriate amount of the asset. With multiple continuation states, this result no longer applies. In Simsek (2013a), I develop a model with a continuum of states in which the equilibrium typically *does* feature default. The model shows that in general there is not a fixed leverage limit, and the equilibrium leverage and default risk depend on *the type of disagreements.* Disagreements about upside risks are conducive to leveraged asset price booms. In this case, borrowers take on larger and riskier loans in equilibrium, because lenders largely share the borrowers' views about downside risks and do not charge high interest rate spreads. In contrast, disagreements about downside risks lead to lower leverage and asset prices. In this case, while borrowers have access to larger and riskier loans, they choose smaller and safer loans—similar to the above model—because lenders perceive a greater default risk and charge high interest rate spreads. This result clarifies that, when borrowers and lenders have different views about downside risks, the equilibrium leverage is mostly driven by the *lenders'* beliefs about downside tail scenarios.

Related literature. A growing literature uses *endogenous* collateral constraints as in this section to shed light on several important macroeconomic issues such as the global leverage cycle (e.g., Fostel and Geanakoplos (2008)), international capital flows (e.g., Phelan and Toda (2019); Fostel et al. (2019)), financial innovation and securitization (e.g., Fostel and Geanakoplos (2012); Broer (2018)) as well as issues in corporate finance (e.g., Geerolf (2018); Gong and Phelan (2019)) and household finance (e.g., Bailey et al. (2019)). See Fostel and Geanakoplos (2014) for a review of

[5] To illustrate the role of higher order beliefs, consider a scenario in which young optimists' beliefs are the same as before but the pessimists' beliefs are different. Specifically, pessimists believe if state L is realized then all future investors—including "young optimists"—will hold the pessimistic belief. In this scenario, the pledgeable value of the asset is given by $p(L)$. Consequently, the asset price is lower and given by $p = \beta \alpha n + p(L) = \beta \alpha n + \frac{L}{R^f - L}$.

this literature.

6. A comparison with rational bubbles

So far, I have discussed how belief disagreements can generate overvaluation and (leveraged) speculative bubbles. A macroeconomics literature emphasizes *rational bubbles* as an alternative mechanism that can generate high asset valuations in settings with overlapping generations or financial frictions (see Samuelson (1958); Diamond (1965); Tirole (1985) for seminal contributions). In this section, I derive rational bubbles in my model and compare them with overvaluation and speculative bubbles.

To fix ideas, consider the common belief benchmark from Section 2 with the pessimistic belief $z = L$. Rational bubbles are possible when investment opportunities are limited, which I ensure with two assumptions. Suppose $\delta > 0$, so that investment in physical capital depreciates; and $R^f \leq L$, so that investment in foreign assets generates a lower return than the growth rate of the economy. For simplicity, I make the stronger assumption $R^f = L$ (which leads to a BGP equilibrium with a stationary bubble). As before, I also assume $R^f \in (H(1-\delta), L/\beta)$.

In this case, the benchmark allocations are still an equilibrium [characterized by Eq. (15) and Eqs. $(11-14)$]. However, there can also be equilibria that feature rational bubbles. Suppose investors can invest in a bubble asset that yields zero dividends in all periods. Let B_t denote the price of the bubble asset. Appendix A.5 verifies that there is an equilibrium with a positive bubble that grows at the same rate as the economy, $B_t = be_t$ with some $b > 0$. In this equilibrium, the bubble is fairly priced because its gross return is $\frac{be_{t+1}}{be_t} = L = R^f$. The normalized risky asset price is given by $p(L)$ as in the benchmark [see (15)]. Intuitively, the bubble has a positive value, even though it pays no dividends, as it helps address the asset shortage that results from limited investment opportunities.

The macroeconomic effects of a rational bubble are described by Eqs. $(11-12)$ (with $z_t = L$) after replacing p with $p(L) + b$. I obtain,

$$c_t = (1-\beta)\left(1 + n + p(L) + b + \frac{R^f f_{t-1}}{L}\right), \tag{25}$$

$$p(L) + b + f_t = \beta\left(1 + n + p(L) + b + \frac{R^f f_{t-1}}{L}\right). \tag{26}$$

Consequently, the normalized consumption and foreign assets converge to the BGP allocations in $(13-14)$ (with $z = L$) after replacing p with $p(L) + b$.

How does the emergence of a rational bubble affect macroeconomic outcomes? Suppose the economy is initially on a BGP without a bubble ($b = 0$). In period 0, a rational bubble with $b > 0$ unexpectedly forms. Eqs. $(25-26)$ imply that the aggregate variables follow *the same dynamics* as in the analysis of overvaluation in Section 3. The difference is that in this case the parameters satisfy, $R^f = L$ (whereas previously I focused on the case, $R^f > L$). With these parameters, the bubble leaves the normalized BGP consumption unchanged [see (13)]. Intuitively, the economy is dynamically inefficient and saves *too much* in foreign assets with low return. A rational bubble absorbs the excess savings and improves efficiency. Consequently, the bubble can increase consumption in the longer run as well as in the short run.

Importantly, this analysis highlights that rational bubbles generate similar *macroeconomic* effects as overvaluation and (leveraged) speculative bubbles. Put differently, rational bubbles can capture certain macroeconomic aspects of overvaluation in reduced form, while remaining agnostic about the underlying investor psychology.

On the other hand, overvaluation and speculative bubbles have several important advantages relative to rational bubbles. First, the overvaluation mechanism is more consistent with the empirical literature documenting that asset price and credit booms are associated with high crash risk and generate *predictably* low expected returns (e.g., Baron and Xiong (2017); Greenwood et al. (2019)). This return predictability is at odds with the rational bubble model in which all assets—including the bubble asset—generate a fair return; but it can be reconciled with overvaluation as long as the objective belief is not too optimistic (see Remark 2).

Second, by specifying investors' beliefs (and psychology) explicitly, overvaluation and speculative bubbles make more precise testable predictions. The equilibrium (absent rational bubbles) is often uniquely determined given investors' beliefs. In contrast, rational bubbles (when they are feasible) are associated with multiple equilibria. For instance, the allocation in (25) is an equilibrium for a continuum of bubble sizes b. This multiplicity makes the model less useful for empirical analysis.

In fact, the multiplicity of equilibria creates challenges even for basic comparative statics. Relying on a particular equilibrium selection, Galí (2014) shows that an increase in the interest rate can have the surprising effect of *increasing* the size of the bubble (after the first period). In contrast, the comparative statics of speculative bubbles typically do not require an equilibrium selection and are consistent with conventional wisdom. For example, the interest rate reduces the size of overvaluation in Section 3, given by $p(H) - p(L) = \frac{R^f (H-L)(1-\delta)}{\left(R^f - H(1-\delta)\right)\left(R^f - L(1-\delta)\right)}$; as well as the size of the speculative bubble in Section 4, given by $p(H) - p^o_{pdv} = \frac{H(1-\delta)(H-L)(1-\delta)}{\left(R^f - H(1-\delta)\right)\left(R^f - L(1-\delta)\right)}$.

Finally, rational bubbles require rather restrictive conditions on asset maturities or the economic environment (see, e.g., Campbell et al. (1997); Santos and Woodford (1997)). For instance, the model in this section does not admit a rational bubble when there is no depreciation, $\delta = 0$. In this case, capital is infinitely lived and its price ($p(L) = \frac{L}{R^f - L}$) becomes unbounded when the interest rate is sufficiently low to allow for a rational bubble ($R^f \leq L$). In contrast, speculative bubbles are feasible more generally as long as investors' beliefs satisfy the appropriate assumptions.

Related mechanism: Risk shifting bubbles. Another strand of the literature emphasizes bubbles driven by agency frictions and risk shifting, e.g., Allen and Gorton (1993); Allen and Gale (2000); Allen et al. (2019). In these models, some "borrowers" receive funds from "lenders" to invest in risky assets. With debt contracts and limited liability, borrowers have an incentive to take on excessive risks, which can drive up risky asset prices. Risk-shifting bubbles provide a complementary explanation for asset price booms but they have somewhat different macroeconomic (and welfare) implications than speculative and rational bubbles (see Barlevy (2012) for a discussion). Moreover, like rational bubbles, these bubbles have trouble generating the disappointing returns that are an empirical regularity (the "lenders" are aware that they receive a low expected return due to risk shifting).

Related literature. My analysis only scratches the surface of the vast theoretical literature on asset price bubbles. See Martin and Ventura (2018) for a review that focuses on rational bubbles, and Brunnermeier and Oehmke (2013) for a review that contrasts different theories of bubbles.

7. Speculative wealth dynamics and procyclical valuation

So far, I have focused on the effects of speculation in environments with short-selling constraints. A large finance literature shows that speculation affects asset prices by shaping investors' *wealth dynamics*, even without short-selling constraints (e.g., Detemple and Murthy (1994)). The key idea is that optimists become more dominant after the realization of good states, whereas pessimists become more dominant in bad states. These speculative wealth dynamics help explain the excess asset price volatility as well as the procyclical asset valuation (or the countercyclical risk premium) observed in the data (e.g., Cochrane (2011); Shiller (2014)). In this section, I demonstrate the procyclical valuation mechanism. The next section shows procyclical valuation also exacerbates demand recessions.

Throughout this section, I make three assumptions (for the general case, $\delta \geq 0$). First, I switch to a closed economy, by modifying (9) so that $R_t^f \equiv R_t^{f*}$ is endogenous and

$$F_t = 0 \text{ and } Y_t = Y_t^* \text{ for each } t. \tag{27}$$

The closed economy assumption doesn't play an important role beyond ensuring that foreign assets are not a state variable (so the model has a single state variable, optimists' wealth share). Second, investors face no exogenous limits on short selling or leverage ($\underline{\omega}_t = -\infty, \overline{\omega}^t = \infty$) so that markets are effectively complete (see Remark 6). The mechanisms also apply with exogenous portfolio restrictions, but complete markets simplify the analysis. Finally, I focus on the case in which investors have the same belief when young and old: optimism and pessimism are persistent.

I next characterize the equilibrium in three steps. I first establish a key relationship between output and asset prices. I then extend the analysis of the common-belief benchmark to this case. Finally, I consider the case with disagreements and establish the main results in this section.

Output-asset price relation in a closed economy. In a closed economy, there is a tight relationship between output and risky asset prices that I refer to as *the output-asset price relation.*

Specifically, Eqs. $(6),(8),(9)$ (and $F_t = 0$) imply

$$Y_t = C_t = (1-\beta)(Y_t + P_t) = (1/\beta - 1) P_t. \tag{28}$$

As before, consumption is determined by the wealth effect. In a closed economy, aggregate wealth comes from the current income (Y_t) and the domestic asset price (P_t). In addition, output is equal to consumption (since there is no domestic or foreign investment). Solving the equation, output is proportional to the domestic asset price.

Combining the output-asset price relation with the assumption that output is at its potential, $Y_t^* = e_t(1+n)$, the normalized price (P_t/e_t) is constant and given by

$$p_t = p^* = \frac{n+1}{1/\beta - 1}. \tag{29}$$

Intuitively, the domestic asset price (or aggregate wealth) must be sufficiently high to ensure that output is at its potential. Since the normalized output is constant $(n+1)$, the required level of the normalized asset price is also constant. I refer to p^* as the *potential asset price*. Therefore, in this section beliefs or speculation do not affect the output or the asset price *in equilibrium* (their effects are absorbed by the equilibrium interest rate). This feature will change in the next section, where output is not necessarily at its potential due to nominal rigidities.

Common-belief benchmark in a closed economy. Next consider the benchmark in which all investors think $z_{t+1} = z \in \{H, L\}$. There is an equilibrium with an endogenous and constant interest rate denoted by $R^{f*}(z)$. In Appendix A.6, I characterize this equilibrium and show Eq. (15) still applies:

$$p(z) \equiv \frac{z(1-\delta)}{R^{f*}(z) - z(1-\delta)}. \tag{30}$$

The asset price is still determined by the Gordon growth formula but with the endogenous interest rate. Combining this equation with Eq. (29), I solve for the equilibrium interest rate:

$$R^{f*}(z) = z(1-\delta)\left(1 + \frac{1}{p^*}\right) = z(1-\delta)\frac{n+1/\beta}{n+1}. \tag{31}$$

In a closed economy, shocks to asset valuations affect the equilibrium interest rate instead of the equilibrium price. For instance, an optimism shock that changes the common belief from $z = L$ to $z = H$ increases the interest rate while leaving the asset price unchanged. The feature that the normalized price remains constant is extreme (driven by the assumption that there is a single risky asset in positive supply) but the effect on the interest rate applies quite generally.[6]

[6] In a more realistic scenario with multiple assets, belief changes for an asset would affect its equilibrium price due to relative valuation effects.

Equilibrium asset price with disagreements. I next consider my preferred setup with belief disagreements: a constant fraction (α) of young investors are optimists; and young optimists (pessimists) become mature optimists (pessimists).

Recall that in this section I assume no exogenous limits on short selling or leverage. However, the model still features the endogenous limits that I characterize in Section 5.2. In equilibrium, optimists and pessimists are both against their respective endogenous limits. Consequently, the market clearing price is determined by optimists' and pessimists' relative purchasing powers.

Appendix A.6 formalizes this argument and shows that the asset price is a particular *wealth-weighted average* of investors' valuations. To state the result, let $\alpha_t \in (0,1)$ denote the *optimists' wealth share* defined by:

$$\alpha_t = \frac{\sum_{i \in \{o,mo\}} a_t^i}{\sum_i a_t^i}. \tag{32}$$

The equilibrium price is then given by

$$\begin{aligned} P_t &= \mathcal{P}\left(P_t\left(H\right), P_t\left(L\right) | \alpha_t\right) \equiv \left(\frac{\alpha_t}{P_t\left(H\right)} + \frac{1-\alpha_t}{P_t\left(L\right)}\right)^{-1}, \\ \text{where } P_t\left(z'\right) &= \left(1-\delta\right) \frac{r_{t+1,z'} + P_{t+1,z'}}{R_t^f} \text{ for } z' \in \left\{H, L\right\}. \end{aligned} \tag{33}$$

Here, $P_{t+1,z'}$ is the equilibrium price in period $t+1$ if state z' is realized; and $P_t\left(z'\right)$ is the valuation of the asset under the extreme belief that state z' will be realized for sure. Hence, $P_t\left(H\right)$ and $P_t\left(L\right)$ correspond to optimists' and pessimists' valuations, respectively. The function $\mathcal{P}\left(\cdot\right)$ is the wealth-weighted harmonic average of these two valuations.[7]

Speculative wealth dynamics. Next consider the dynamics of optimists' wealth share, α_t. Since (currently alive) optimists and pessimists perceive no risk, they take extreme positions that imply their wealth declines to zero if the state on which they bet is not realized. Consequently, (all) optimists' wealth share also follows extreme dynamics:

$$\alpha_{t,z} = \alpha_z \text{ where } \alpha_L \equiv \frac{\alpha\left(n+\delta p^*\right)}{n+1+p^*} < \alpha_H \equiv 1 - \frac{\left(1-\alpha\right)\left(n+\delta p^*\right)}{n+1+p^*}. \tag{34}$$

Optimists' wealth share depends only on the most recent state realization. If $z = L$ is realized, then old optimists lose all of their wealth. In this case, $\alpha_t \equiv \alpha_L$ is equal to the wealth share of young optimists. If instead $z_t = H$ is realized, then old pessimists lose all of their wealth. In this case, $\alpha_t \equiv \alpha_H$ is equal to one minus the wealth share of young pessimists. This also implies $\alpha_H > \alpha_L$: optimists' wealth share is larger after a high growth realization.

[7] It is easy to check that this harmonic average is also the equilibrium price that would obtain in a "representative investor" setting in which the investor believes the probability of the high growth state is $\alpha_t \in (0,1)$. Hence, the asset is priced as if there is a representative investor whose degree of optimism is proportional to optimists' wealth share.

Procyclical asset valuation. Recall that the asset price and the rental rate are given by, respectively, $P_t = p^* e_t$ and $r_{t+1} = e_{t+1}$ [see (29) and (2)]. Combining these expressions with Eqs. (33) and (34), there is an equilibrium in which the interest rate is a function of the most recent state realization, $R^{f*}_{t,z} = R^{f*}_z$, with

$$R^{f*}_z = \mathcal{P}\left(R^{f*}(H), R^{f*}(L) | \alpha_z\right) \text{ for } z \in \{H, L\}. \tag{35}$$

Recall that $R^{f*}(H)$ and $R^{f*}(L)$ denote the interest rate with common optimism and common pessimism [see (31)]. With disagreements, the equilibrium interest rate in each state z is a weighted average of $R^{f*}(H)$ and $R^{f*}(L)$. The weight on the interest rate with common optimism, $R^{f*}(H)$, is determined by optimists' wealth share, α_z.

Combining Eqs. (34) and (35) establishes the main result in this section: the equilibrium risk-free interest rate is procyclical, $R^{f*}_H > R^{f*}_L$. Intuitively, good state realizations vindicate optimists and increase their wealth share, whereas bad state realizations do the opposite. Consequently, risky assets' valuation increases (the risk premium declines) in good times and their valuation decreases in bad times. In equilibrium, procyclical asset valuation translates into procyclical interest rates.

Extensions. These results require *the relative optimism* of investors to be *somewhat (though not necessarily fully) persistent across aggregate booms and busts.* This is consistent with empirical evidence: using survey data, Giglio et al. (forthcoming) find that investors' beliefs feature quite large and persistent individual heterogeneity over time. A natural question is whether the results are robust to allowing for "flipping" of beliefs as in the speculative bubbles literature. In fact, the analysis can accommodate an arbitrary amount of flipping of beliefs *as long as it takes place within optimists (or pessimists).* For instance, it is easy to envision a version of the model in which optimists speculate among each other on upside states, e.g., $H' > H$. This could drive a speculative bubble as in the previous section. The realization of a low growth state L would reduce optimists' wealth share as a group and reduce asset valuations.

The results still apply when investors are subject to exogenous portfolio restrictions. For instance, consider the model in Section 3 in which short selling is prohibited. Recall that optimists invest in the risky asset whereas pessimists invest in the risk-free asset. With these portfolios, the realization of the low growth state would lower optimists' wealth share (and in general, asset prices as well). The main difference is that optimists' wealth share would decline less than in this section with complete markets. Portfolio restrictions reduce financial speculation, which mitigates but does not overturn the mechanisms.

Related mechanism: Extrapolation and diagnostic beliefs. In this model, speculation induces extrapolative dynamics for asset valuations even though individual investors do not extrapolate. This connects the analysis to a large literature emphasizing extrapolative or diagnostic beliefs (and related psychological frictions) as a key driver of asset price and macroeconomic fluctuations (see, e.g., Cutler (1990); Barsky and De Long (1993); Barberis et al. (1998); Hong and Stein

(1999); Fuster et al. (2012); Greenwood and Shleifer (2014); Shiller (2015); Hirshleifer et al. (2015); Barberis et al. (2015); Bordalo et al. (2018, 2020b, 2021); Maxted (2020)). While the two mechanisms have complementary effects on asset prices, they make distinct predictions for investors' portfolios. Unlike speculation, extrapolation (without additional ingredients) does not necessarily induce high trading volume or leverage.

Related literature. A large finance literature develops the speculative wealth dynamics mechanism further to analyze a number of issues such as excessive asset price volatility, time-varying risk premium, and high trading volume (e.g., Dumas et al. (2009); Xiong and Yan (2010); Bhamra and Uppal (2014); Atmaz and Basak (2018); Martin and Papadimitriou (2019)), option prices (e.g., Buraschi and Jiltsov (2006)), and financial innovation (e.g., Zapatero (1998); Kubler and Schmedders (2012)).

8. Speculation and demand recessions

In recent work, Caballero and Simsek (2020) show how speculation not only exacerbates asset price fluctuations but can also worsen demand recessions. Moreover, appropriate macroprudential policies intended to discipline speculation in the boom can improve macroeconomic stability. These policies can generate *Pareto* improvements in welfare, since they internalize aggregate demand externalities. In this section, I illustrate these results and I discuss closely related mechanisms including investment hangover, financial frictions, and deleveraging.

Model with nominal rigidities and interest rate policy. Consider the setup in the previous section but with one key difference: output is not necessarily at its potential due to nominal rigidities. As a consequence, the risk-free rate, R_t^f, and output, Y_t, are *both* endogenous and determined by *the interest rate policy of the central bank*.

Specifically, there are New Keynesian firms with fully sticky prices they never change (for simplicity). The utilization of labor and capital by these firms is endogenous and denoted by $\eta_t^l, \eta_t^k \in [0,1]$, so that $Y_t = \eta_t^l e_t n + \eta_t^k e_t$. For either factor, utilization can be increased for free until it is equal to one but cannot be increased further. With these assumptions, actual output is determined by *aggregate demand*, $Y_t = C_t$, and it is either at or below its potential, $Y_t \leq Y_t^*$ (see Appendix A.7.2 for details). I define *relative output* as

$$y_t \equiv \frac{Y_t}{Y_t^*} \leq 1,$$

and I refer to $y_t - 1$ as the *output gap*. The case with a negative output gap, $y_t < 1$, captures a demand recession. To simplify the analysis, I also make appropriate assumptions (relegated to Appendix A.7.2) that ensure there are no pure profits and a demand recession reduces both factors' returns proportionally:

$$w_t = y_t e_t n \text{ and } r_t = y_t e_t. \tag{36}$$

This setup allows for equilibria in which outcomes scale with productivity but otherwise does not play a significant role.[8]

Since prices are fully sticky, the real risk-free interest rate is the same as the nominal rate. I assume the central bank sets the interest rate to close the output gap, i.e., to achieve $y_t = 1$. However, the interest rate has a soft lower bound, denoted by $\underline{R}^f$. This could capture the zero lower bound but also other frictions (such as fixed or stabilized exchange rates or concerns with banks' financial health) that make it difficult to cut interest rates below a certain level. I allow the central bank to reduce the interest rate below $\underline{R}^f$. These interest rate cuts are subject to unmodeled costs and used only if the output gap is sufficiently low.[9]

Formally, the interest rate follows

$$R_t^f = \max\left(\tilde{R}_t^{f*}, \underline{R}^f\left(1 + u\left(y_t - 1\right)\right)\right) \text{ with } u = 1. \tag{37}$$

Here, $\tilde{R}_t^{f*}$ is defined recursively as the interest rate that closes the current output gap given the policy in future periods. As long as this rate is sufficiently large, $\tilde{R}^{f*} \geq \underline{R}^f$, conventional policy stabilizes output, $y_t = 1$. When the required rate is below the lower bound, $\tilde{R}^{f*} < \underline{R}^f$, the equilibrium features a demand recession, $y_t < 1$. In this case, the central bank cuts interest rates in proportion to the output gap (due to unmodeled costs), with intensity captured by the parameter, $u > 0$. I focus on the special case, $u = 1$, which leads to particularly simple expressions (although this simplification is not necessary for the qualitative results). I next characterize the equilibrium by following the same steps from Section 7.

Output-asset price relation with nominal rigidities. With nominal rigidities, the output-asset price relation in (28) still applies. Likewise, the potential (normalized) asset price is still given by p^* [see (29)]. However, the equilibrium price p_t can be below this level. Moreover, the price ratio $\frac{p_t}{p^*}$, which I refer to as *the relative asset price*, determines the relative output,

$$y_t = \frac{p_t}{p^*} \leq 1. \tag{38}$$

Using this relation, I rewrite the interest rate policy as [see (37)]

$$R_t^f = \max\left(\tilde{R}_t^{f*}, \underline{R}^f \frac{p_t}{p^*}\right). \tag{39}$$

[8] Alternatively, I could assume there is no labor, $n = 0$. In this case, the results apply regardless of how the earnings from capital are distributed between pure profits and the rental rate of capital—as long as the risky asset is a claim on both types of earnings.

[9] I view the policy in this region as capturing not only costly interest rate cuts (e.g., negative interest rates) but also unconventional policies such as large-scale asset purchases (LSAPs) in reduced form. Caballero and Simsek (forthcoming) develop a model in which LSAPs mitigate demand recessions (and they operate through a similar mechanism as conventional monetary policy), but they are not free and the optimal LSAP typically does not fully close the output gap.

Common-belief benchmark with nominal rigidities. Consider the benchmark in which all investors think $z_{t+1} = z \in \{H, L\}$. There is an equilibrium with a constant normalized asset price, $p(z) \leq p^*$, and a constant interest rate, $R^f(z)$. To characterize this equilibrium, consider the common-belief interest rate from the previous section without nominal rigidities [see (31)]:

$$R^{f*}(z) = z(1-\delta)\left(1 + \frac{1}{p^*}\right).$$

If $R^{f*}(z) \geq \underline{R}^f$, the equilibrium is the same as before, $R^f(z) = R^{f*}(z)$ and $p(z) = p^*$. The interest rate lower bound does not bind and the monetary policy replicates the potential outcomes.

If instead $R^{f*}(z) < \underline{R}^f$, the equilibrium is different. The interest rate is below its lower bound and there is a demand recession. The recession lowers not only the output but also the earnings (as well as asset prices) in future periods, $r_{t+1} = \frac{p(z)}{p^*} Le_{t+1}$ [see (36)]. This additional effect leads to a slightly modified asset pricing equation [cf. (30)]:

$$p(z) = \frac{z(1-\delta)\frac{p(z)}{p^*}}{R^f(z) - z(1-\delta)}.$$

Combining this with the policy rule in (39), I solve for the equilibrium as[10]

$$\frac{p(z)}{p^*} = \frac{R^{f*}(z)}{\underline{R}^f} < 1 \text{ and } R^f(z) = R^{f*}(z) < \underline{R}^f. \tag{40}$$

Eqs. (38) and (40) illustrate that, when the interest rate policy is constrained, asset valuations affect the severity of the demand recession. For instance, a decline in the perceived growth rate z reduces not only the interest rate but also the equilibrium asset price and output. Since speculation induces procyclical asset valuation, this analysis suggests speculation can also exacerbate demand recessions, as I verify.

Equilibrium with disagreements and nominal rigidities. Consider the main focus with belief disagreements. As before, the equilibrium depends on optimists' wealth share, $\alpha_t \in (0,1)$ [see (32)]. Moreover, Eqs. (33) and (34) also apply in this context. The asset price is still given by a wealth-weighted (harmonic) average of optimists' and pessimists' valuations captured by the function $\mathcal{P}(\cdot)$. Optimists' wealth share still follows extreme dynamics, which leads to $\alpha_{t,z} \equiv \alpha_z$ with $\alpha_L < \alpha_H$.

Recall that, absent nominal rigidities, the equilibrium interest rate is procyclical, $R_L^{f*} < R_H^{f*}$ [see (35)]. This suggests that, with nominal rigidities, the interest rate is more likely to violate the lower bound in the low growth state than in the high growth state. Under appropriate parametric conditions, there is in fact an equilibrium in which the low growth state features a demand recession,

[10] The interest rate is the same as in Section 7, $R^f(z) = R^{f*}(z)$, even though there is a demand recession. However, this does not correspond to the output gap-stabilizing rate ("rstar") in this context, $\tilde{R}_t^{f*} \neq R^{f*}(z)$ [see (37)]. Since the demand recession reduces earnings and asset prices in future periods, it also reduces the "rstar", $\tilde{R}_t^{f*} = \frac{p(z)}{p^*} R^{f*}(z) < R^{f*}(z)$.

$R_L^f < \underline{R}^f, p_L < p^*$, and the high growth state features potential outcomes, $R_H^f > \underline{R}^f, p_H = p^*$ (and the interest rate and the normalized price remain constant within states).[11]

Appendix A.7.1 characterizes this equilibrium and shows that the relative price in the low growth state, p_L/p^*, solves

$$\frac{p_L}{p^*}\underline{R}^f = \mathcal{P}\left(\frac{R^{f*}(H)}{p_L/p^*}, R^{f*}(L)\,|\alpha_L\right). \tag{41}$$

The left side is an increasing function of p_L/p^*; whereas the right side is a decreasing function of p_L/p^*. The equilibrium corresponds to the intersection. Importantly, *an increase in the wealth-share of optimists in the recession state, α_L, increases the relative asset price, p_L/p^*, and mitigates the demand recession.*

Amplification of demand recessions. Combining this analysis with optimists' wealth dynamics illustrates the main insight in this section: *speculation exacerbates demand recessions.* In particular, Eq. (34) shows that speculation in the period before the recession reduces optimists' wealth share in the recession to its lowest possible level, $\alpha_{t,L} = \alpha_L$. Eqs. (41) and (38) imply this lower wealth share for optimists reduces asset prices and output in the recession. The decline in output reduces the income of all investors, including the new investors that have not (yet) speculated. Thus, speculation can create macroeconomic damage that extends beyond the speculators.

Macroprudential policy in the boom. This analysis also suggests that *macroprudential policy* that restricts speculation in the boom can improve macroeconomic stability. Suppose the economy is currently in period 0 with state $z_0 = H$. Consider extreme macroprudential policy that bans leverage in period 0: $\omega_0^i \leq 1$ (less extreme leverage limits or other restrictions on risk taking also work). To simplify the exposition, suppose there is no macroprudential policy starting period 1 onward, and that there is no interest rate lower bound in period 0.

With these assumptions, if $z_1 = L$ is realized in period 1, then optimists' wealth share is

$$\tilde{\alpha}_{1,L} = \alpha_L + \alpha_H \frac{(1-\delta)(1+p^*)}{n + (1-\delta)(1+p^*)} > \alpha_L. \tag{42}$$

Consequently, the equilibrium asset price in the recession is greater than in the case without macroprudential policy, $\tilde{p}_{1,L} > p_{1,L}$ (see Appendix A.7.1 for the derivation).

Intuitively, macroprudential policy ensures that old optimists have *positive wealth* in the recession—as opposed to zero wealth as in the earlier analysis. *This increases optimists' wealth share and raises asset prices and output in the recession.*

[11] The exact parametric condition is given by

$$\mathcal{P}\left(R^{f*}(H), R^{f*}(L)\,|\alpha(L)\right) < \underline{R}^f < \mathcal{P}\left(R^{f*}(H), \frac{R^{f*}(L)^2}{\underline{R}^f}|\alpha(H)\right).$$

Here, the lower bound is R_L^{f*} from the previous section and the upper bound is a slight modification of R_H^{f*} [cf. (35)]. Hence, the condition says $R_L^{f*} < \underline{R}^f < R_H^{f*}$ up to a slight modification.

In Appendix A.7.1, I show that macroprudential policy reduces the current interest rate, $\tilde{R}^f_{0,H} < R^f_{0,H}$. Intuitively, restricting the optimists' risk taking enables the pessimists to wield a greater influence than before. This reduces the asset valuation and aggregate demand in the boom. The central bank reacts by cutting rates to ensure output is at its potential. Thus, macroprudential policy is less desirable in a recession when the interest rate policy is constrained and cannot undo the side effects of restricted risk taking on aggregate demand.

Extensions. Caballero and Simsek (2020) use a related model to establish the results in this section more generally (without the extreme assumptions on beliefs or policies). They also present a formal welfare analysis and show that restricting speculation in the boom can increase social welfare according to the standard Pareto criterion (that is, the planner evaluates each investor's expected utility with her own belief). Intuitively, the planner improves welfare in view of *aggregate demand externalities*. In the boom period before the recession, optimists and pessimists do not internalize that their speculation reduces asset prices and output in the recession. This induces optimists to take on socially excessive risks that can be corrected by (procyclical) macroprudential policy.

As in Section 7, these results also apply with short-selling constraints—these constraints reduce but do not eliminate speculation. The analysis in this section raises an intriguing possibility: the planner can improve welfare *by restricting short selling in a demand recession*. In fact, it is straightforward to construct a version of the model in which tightening the short-selling constraints in state L increases not only asset prices as in Section 3 *but also output*. Intuitively, pessimists that take on short positions in a demand recession induce negative aggregate demand externalities that can be corrected by short-selling restrictions. In practice, governments occasionally do introduce short-selling restrictions during recessions and crises associated with low asset prices (see, e.g., Beber and Pagano (2013); Boehmer et al. (2013)). This model provides one rationale for these policies, although these benefits should be weighed against potential (unmodeled) costs of short-selling restrictions on market liquidity or price discovery.

Related mechanism: Investment dynamics and hangover. While I focus on consumption, speculation also affects investment dynamics. Caballero and Simsek (2020) develop a variant of the model in which output is increasing in asset prices not only through a wealth effect on consumption but also because high asset prices increase investment through a standard Q-theory mechanism. Consequently, speculation in the boom lowers asset prices, investment, and consumption once the economy transitions to recession—consistent with what happens in a typical recession.

Rognlie et al. (2018) develop a complementary "investment hangover" mechanism by which speculation in the boom years could further reduce investment and asset prices in the recession. Motivated by the overbuilding of homes in the run-up to the Great Recession, they consider a model with the key feature that (housing) capital has diminishing returns. An excess supply of capital at the onset of the recession reduces (housing) investment and exacerbates the demand recession. While Rognlie et al. (2018) use optimism in the ex-ante boom period to motivate excess investment, recall that speculation in the boom also increases asset valuations and investment [see Section 3]. Thus, speculation would exacerbate excess investment and contribute to the investment hangover mechanism once the economy transitions to recession.

Related mechanism: Financial frictions. In the model in this section, a negative shock induces levered optimists to make losses, which in turn reduces asset prices and economic activity. A large literature on financial frictions also emphasizes the role of leverage and asset price feedback in causing or amplifying economic slowdowns (e.g., Shleifer and Vishny (1992); Kiyotaki and Moore (1997); Holmström and Tirole (1998); Bernanke et al. (1999); Caballero and Krishnamurthy (2001); Gertler and Kiyotaki (2010); Adrian and Shin (2010); He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)). This literature typically focuses on the role of low asset prices in tightening borrowing constraints, whereas I emphasize aggregate demand as the main channel by which low asset prices create macroeconomic damage.

The mechanisms in the financial frictions literature naturally complement the results in this section. In fact, as Caballero and Simsek (2020) emphasize, optimists in the model can be viewed more broadly as "high-valuation investors" that capture risk-tolerant investors (e.g., banks or institutional investors) and actual optimists.[12] Moreover, introducing financial frictions could reinforce the aggregate demand mechanism. For instance, if optimists and banks take similar positions in the boom, then a decline in asset prices in the recession driven by the reduction in optimists' wealth share would also exacerbate banks' distress. This could lead to a credit crunch (and it can trigger a financial crisis) which would further reduce aggregate demand (both consumption and investment). Furthermore, financial frictions could lead to fire-sale externalities that would strengthen the case for macroprudential policy (see, e.g., Lorenzoni (2008); Davila and Korinek (2016)).

Related mechanism: Debt hangover and deleveraging. In my model, low asset prices reduce aggregate consumption through a wealth effect. A decline in asset prices can reduce households' or firms' spending also by exacerbating their *deleveraging*, according to a strand of the financial frictions literature (e.g., Iacoviello (2005); Eggertsson and Krugman (2012); Guerrieri and Lorenzoni (2017)). To see how the deleveraging mechanism interacts with speculation, consider an extension with another group of agents that are endowed with a fraction of the capital but do not buy or sell. Suppose some of these agents ("the borrowers") have a strong motive to take on debt and spend (e.g., due to impatience) and their debt limits depend on the value of their capital (e.g., due to collateral constraints). In the boom period with high asset prices, the borrowers would increase their debt. When the recession arrives and asset prices decline, the borrowers could be forced to cut their spending to pay back some of their debt. As before, speculation in the boom (among the investors) would reduce asset prices in the recession, which would lead to greater deleveraging and a more severe recession. Hence, the deleveraging mechanism would leave the analysis qualita-

[12] See Caballero and Farhi (2018); Kekre and Lenel (2020); Caballero and Simsek (forthcoming) for models of demand recessions with heterogeneous risk tolerance.

tively unchanged. Its main effect would be to strengthen the output-asset price relation in (38).[13] In addition, deleveraging would expand the scope of macroprudential policy. While I focus on policies that restrict the optimists' (or high-valuation investors') risk taking in the boom, Korinek and Simsek (2016); Farhi and Werning (2016) emphasize that policies that restrict the borrowers' leverage in the boom can also internalize aggregate demand externalities and improve welfare.

Related mechanism: Prudential monetary policy. In this section, I have focused on macroprudential policy as a potential solution to the macroeconomic instability caused by speculation. Policymakers often argue that macroprudential policy in practice might be insufficient to deal with financial excesses. This raises the question of whether prudential monetary policy (PMP)—that is, raising the interest rate beyond the level that stabilizes the output gap ("rstar")—could be useful for disciplining speculation in the boom. Caballero and Simsek (2021) address this issue using a variant of the model in this section in which macroprudential policy is imperfect. They establish conditions under which small doses of PMP can achieve similar macroeconomic benefits to directly tightening leverage limits. Intuitively, PMP reduces the asset price in the boom, which softens the asset price crash when the economy transitions to recession. Since optimists are levered and exposed to asset prices, softening the crash (typically) improves their wealth share in the recession (α_t)—similar to tightening leverage limits. While PMP is costlier than macroprudential policy, since it reduces aggregate demand in the boom, these costs are second order if the economy initially features efficient factor utilization and the planner increases the interest rate by a small amount.[14]

Related literature. A recent literature incorporates nominal rigidities into the analysis of *rational* bubbles and finds that the collapse of the bubble can trigger a demand recession when the interest rate policy is constrained or unreactive (e.g., Asriyan et al. (forthcoming); Hanson and Phan (2017); Biswas et al. (2020); Galí (forthcoming)). My analysis shares many common elements with this literature: As I have argued in Section 6, rational and speculative bubbles generate similar macroeconomic effects. However, there are also differences, e.g., policies that restrict speculation in the boom will not necessarily mitigate the subsequent demand recession if the bubble is rational. In general, the optimal policy response to bubbles will depend (among other things) on what causes the high asset valuation and on the likelihood of a price crash (see Dávila and Walther (2020); Krishnamurthy and Li (2020) for recent analyses of prudential policy with distorted beliefs).

More broadly, the mechanisms in this section are related to a New Keynesian literature that emphasizes how shocks that affect asset prices can induce demand-driven business cycles, e.g., "news shocks" (Beaudry and Portier (2006)), "noise shocks" (Lorenzoni (2009)), "confidence shocks"

[13] In fact, this relation can be thought of as capturing in reduced form not only the consumption wealth effect but also the many other channels by which asset prices influence aggregate demand.

[14] In recent work, Farhi and Werning (2020) find that PMP is even more desirable when agents *extrapolate* recent price changes. By lowering the asset price in the boom, PMP mitigates agents' optimism that results from extrapolation. More broadly, a lively literature investigates the costs and benefits of PMP, e.g., Woodford (2012); Stein (2013); Borio (2014); Svensson (2017); Gourio et al. (2018). For historical perspectives on the role of PMP and macroprudential policies in prominent bubbles, see Brunnermeier and Schnabel (2015).

(Ilut and Schneider (2014)), and "uncertainty shocks" (Basu and Bundick (2017)). Speculation *endogenously* exacerbates asset price fluctuations and demand recessions. In recent work, L'Huillier et al. (2021) show diagnostic or extrapolative beliefs also amplify demand recessions (see Section 7 for further discussion of the similarities between extrapolation and speculation). While I focus on demand recessions, Bigio and Zilberman (2019) show that speculation can amplify the business cycle further through its impact on the *supply side*. In their model, firms make hiring decisions *before* observing the productivity shock for the period in which labor will be employed. Investors' beliefs and speculation can then affect labor demand and employment even without nominal rigidities.

9. Policy implications with a belief-neutral welfare criterion

My analysis so far provides one rationale for restricting speculation in economic booms based on the negative externalities that the speculators induce on the rest of the society. However, speculation arguably also hurts the speculators. A large finance literature finds that individual investors that trade frequently tend to have poor investment performance (relative to investors with a well-diversified portfolio) due to excessive risks, transaction costs, and poor timing to buy or sell (e.g., Barber and Odean (2013)). For instance, Barber et al. (2009) find that individual investors' trading losses in the Taiwan stock market amount to 2% of Taiwan's GDP over a five year period. These findings suggest that the speculators on average suffer large welfare losses according to the *objective* belief. However, these losses typically go undetected by the standard Pareto criterion that respects individuals' subjective beliefs. The literature proposed an alternative *belief-neutral criterion* that identifies the welfare losses from speculation in many applied settings *even when the planner does not know the objective belief* (e.g., Brunnermeier et al. (2014)). In this section, I illustrate the belief-neutral criterion and how it (typically) strengthens the case for regulating speculation.

Consider the model in Section 7 in which speculative wealth dynamics induce procyclical asset valuation. In this model, individual optimists and pessimists take extremely risky positions that lower their wealth (and consumption) to zero if the state they bet on is not realized (see (34)). To investigate how these speculative wealth dynamics affect welfare, consider the special case with $n = \delta = 0$ so that the future generations' wealth is zero and there is effectively a single generation. In this case, the equilibrium is in fact Pareto efficient, even though it deviates considerably from perfect risk sharing. Intuitively, optimists and pessimists are "consenting adults" that are willing to gamble all of their wealth in pursuit of high expected returns. However, while both investor types expect a high return according to their own beliefs, *at most one type* has a high expected return according to the objective belief. Arguably, the equilibrium features a *collective* form of irrationality even though it is Pareto efficient. A long literature has recognized this shortcoming of the Pareto criterion in detecting inefficiencies in environments with heterogeneous beliefs (see, e.g., Starr (1973); Harris (1978); Hammond (1981); Mongin (2016); Gilboa et al. (2004); Weyl (2007)).

Brunnermeier et al. (2014) develop a *belief-neutral welfare criterion* that detects the collective irrationality that results from speculation (see Gilboa et al. (2014); Gayer et al. (2014); Blume et al. (2018); Heyerdahl-Larsen and Walden (2019) for related criteria). The belief-neutral criterion does not require the planner to know the objective belief: *the planner is not more informed than the investors*. Instead, the planner evaluates investors' expected utilities *according to a common belief* and requires welfare comparisons to be *robust* to the choice of this belief from a large set of reasonable beliefs (e.g., the convex hull of all investors' beliefs). Formally, the planner rules an allocation as belief-neutral efficient (resp. inefficient) if it is Pareto efficient (resp. Pareto inefficient) under every reasonable belief.

In the model of Section 7, *regardless of the common belief*, the Pareto efficient allocation features common weights on the market portfolio $\omega_t^i = 1$ for each i—which ensures perfect risk sharing. Consequently, this allocation is belief-neutral efficient, whereas *the equilibrium is belief-neutral inefficient*. Intuitively, investors choose heterogeneous portfolio weights because they speculate on their belief disagreements. This leads to imperfect risk sharing and low social welfare according to any common belief.

In general, the welfare analysis is more complicated since investors trade for multiple reasons, e.g., risk sharing as well as speculation. Nonetheless, Brunnermeier et al. (2014) show that the belief-neutral criterion can also detect the costs of speculation in these richer settings. In fact, the literature has applied the belief-neutral criterion to measure the costs and benefits of new financial assets (e.g., Simsek (2013b); Posner and Weyl (2013)), and to evaluate policies that restrict speculative trading such as financial transaction taxes (e.g., Dávila (2020)) and leverage limits (e.g., Heimer and Simsek (2019)).[15]

The belief-neutral criterion strengthens the case for macroprudential policies that restrict the speculation that takes place in booms. There is, however, one caveat to this conclusion: Overvaluation and speculation can occasionally be associated with *positive* externalities. Recall that, when the economy is dynamically inefficient, speculative bubbles that absorb savings can increase consumption in the long run as well as in the short run (Sections 4 and 6). Recall also that overvaluation and speculative bubbles often increase investment (Section 3). While the investment effects can lower welfare further, by misallocating capital, this is not a foregone conclusion in view of potential spillovers. For instance, if a speculative bubble forms in nonresidential capital, it can also increase R&D investment. There is indeed some evidence that high stock valuations increase the value of patents (e.g., Haddad et al. (2020)) and lead to greater R&D investment and patent production (e.g., Dang and Xu (2018)). By stimulating innovation, speculation can internalize knowledge spillovers as well as other positive externalities. These concerns seem especially relevant when speculative bubbles form in new industries, e.g., the U.S. Internet bubble of the late 1990s.

The possibility of positive externalities provides only a weak rationale for allowing unrestricted speculation. If there are positive spillovers from investment, they should arguably be internalized by *targeted* policy interventions (such as investment subsidies). Even if targeted policies are not avail-

[15] For a counterview, see Duffie (2014), who provides philosophical and practical challenges to regulating speculative trading driven by belief disagreements.

able, it is unclear that a speculative bubble—driven by investors' beliefs and disagreements—will properly internalize the externalities. As Keynes (1936, p.159) noted "when the capital development of a country becomes a by-product of the activities of a casino, the job is likely to be ill-done."

10. Conclusion

In this paper, I review the literature on financial speculation driven by belief disagreements from a *macroeconomics* perspective. For financial assets subject to short-selling constraints, speculation can generate overvaluation and speculative bubbles by making the marginal investor more optimistic than average. Leverage can substantially inflate speculative bubbles by increasing optimists' purchasing power. Leveraged speculative bubbles are more likely when lenders as well as borrowers are optimistic about downside risks, and the bubble can collapse when lenders begin to think downside tail scenarios are possible. Overvaluation and (leveraged) speculative bubbles have similar macroeconomic effects as rational bubbles, but they better match the empirical evidence on the predictability of asset returns and make more precise testable predictions. Even without short-selling constraints, speculation induces procyclical asset valuation, since speculative wealth dynamics make optimists more dominant after good shocks while reducing their influence following bad shocks. When speculation concerns aggregate assets, its valuation effects also influence macroeconomic outcomes. During the boom, overvaluation and speculative bubbles raise consumption and investment in the short run while draining foreign assets. During the bust, low valuations as well as the investment hangover reduce aggregate demand and exacerbate the recession as long as monetary policy is somewhat constrained. Financial frictions and deleveraging in the recession reinforce the macroeconomic damage that results from speculation. Macroprudential policies that restrict speculation in the boom can mitigate the subsequent demand recession. These policies can improve social welfare by internalizing aggregate demand externalities, and they can also improve the speculators' own welfare according to a belief-neutral criterion.

In this review, I take a *theoretical* perspective and focus on illustrating mechanisms. A growing empirical literature analyzes and typically finds support for the key channels that underlie my analysis. For instance, an empirical finance literature generally supports the overvaluation mechanism, using case studies (e.g., Xiong and Yu (2011); Xiong (2013)) as well as more systematic analyses (see, e.g., Yu (2011)). More recently, Ma et al. (2021) provide evidence that lenders' beliefs about downside tail scenarios drive the credit supply, consistent with my model. Likewise, recent empirical studies confirm that risky asset prices substantially influence aggregate demand due to, e.g., housing wealth effects (e.g., Mian et al. (2013); Guren et al. (forthcoming)), stock wealth effects (e.g., Chodorow-Reich et al. (forthcoming); Majlesi et al. (2020)), or credit spreads' impact on financial frictions (e.g., Gilchrist and Zakrajšek (2012)). Central banks typically respond to a decline in risky asset prices by cutting the interest rate, as in my model (e.g., Pflueger et al. (2020); Cieslak and Vissing-Jorgensen (forthcoming)).

The mechanisms I discuss played a central role in the U.S. housing boom-bust cycle of the

2000s. Regarding the root causes of the housing boom, many prominent analysts have emphasized the optimism of a fraction of investors (e.g., Reinhart and Rogoff (2009); Shiller (2015)) as well as more widely held optimism about downside risks in house prices (e.g., Gennaioli and Shleifer (2020)). Short-selling constraints were also relevant during the boom, as colorfully portrayed by Lewis (2011). These are exactly the ingredients necessary for a leveraged speculative bubble. A growing empirical literature provides evidence that leveraged speculation has indeed amplified the U.S. housing boom-bust cycle (see, e.g., Haughwout et al. (2011); Chinco and Mayer (2016); Albanesi et al. (2017); Mian and Sufi (forthcoming); Gao et al. (2020); Bayer et al. (2020)). For instance, Mian and Sufi (forthcoming) find that areas that received greater credit in the boom years featured greater speculative trading activity driven by a small group of investors ("optimists" in the model) as well as greater belief disagreements between homebuyers and the rest of the population. Consistent with the model, these areas experienced a greater boom-bust cycle in house prices. Likewise, Gao et al. (2020) find that speculation has exacerbated not only the housing cycle *but also the macroeconomic cycle—through the channels I emphasize in this review.* Their evidence suggests, by reducing house prices in the bust, speculation has contributed to the macroeconomic decline through its impact on households' wealth and spending (see also Mian and Sufi (2014)), as well as on housing investment (see also Rognlie et al. (2018)). The decline in house prices has arguably created further damage through its impact on financial institutions' balance sheets (see Gertler and Gilchrist (2018)).

Finally, while I focus on the implications for business cycles, speculation can also shed light on some important macroeconomic trends. The literature suggests speculation can be a major driver of financial innovation (Simsek (2013b)), and might have contributed to the extraordinary growth of the financial sector in recent decades (Heimer and Simsek (2019)). Speculation—unleashed by financial innovation—also helps explain the recent downward trend in interest rates (Iachan et al. (forthcoming)). I leave the *long-run* macroeconomics of financial speculation for future work.

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A. Online Appendix: Not for Publication

A.1. Omitted derivations for Section 2

Consider the open economy model without nominal rigidities described in Section 2. Recall that for this case I assume the parameters satisfy $R^f > H(1-\delta)$ and $\beta R^f < L$. I first show that, in any equilibrium with a constant normalized price $p_t = p$, the normalized consumption and foreign assets follow the dynamics in Eqs. $(11-12)$. I also derive Eqs. $(13-14)$ that describe the normalized consumption and foreign assets in a BGP. I then consider the common-belief benchmark and show that there is an equilibrium with a constant normalized price given by Eq. (15).

First note that consumption is proportional to aggregate wealth. Specifically, Eqs. (6) and (8) imply that

$$C_t = \frac{1-\beta}{\beta}(P_t + F_t). \tag{A.1}$$

Here, $P_t + F_t$ denotes the aggregate wealth at the end of the period. Using the resource constraint in (9), foreign assets follow $F_t = Y_t - C_t + R^f_{t-1}F_{t-1}$. After substituting this expression and solving for consumption, I obtain,

$$C_t = (1-\beta)\left(Y_t + P_t + R^f F_{t-1}\right). \tag{A.2}$$

Hence, consumption is also proportional to aggregate wealth at the beginning of the period. This wealth consists of output, Y_t, the value of domestic assets, P_t, and the initial value of foreign assets, $R^f_{t-1}F_{t-1}$.

Next note that output is proportional to productivity, $Y_t = Y_t^* = (1+n)e_t$ [see (2)]. I will focus on equilibria in which the normalized asset price is constant, $p_t = p$, so that the asset price is also proportional to productivity, $P_t = pe_t$. To characterize the remaining allocations, let $c_t = C_t/e_t$ and $f_t = F_t/e_t$ denote the normalized consumption and foreign assets, respectively. Substituting the normalized variables into Eqs. $(A.1)$ and $(A.2)$, and using the dynamics of productivity $e_t = e_{t-1}z_t$ [see (3)], the normalized consumption is given by (11),

$$c_t = \frac{1-\beta}{\beta}(p + f_t) = (1-\beta)\left(1 + n + p + \frac{R^f f_{t-1}}{z_t}\right).$$

Here, the second equality also implies that the dynamics of normalized aggregate wealth is given by (12),

$$p + f_t = \beta\left(1 + n + p + \frac{R^f f_{t-1}}{z_t}\right).$$

In general, the evolution of normalized aggregate wealth (and foreign assets) depends on the realization of uncertainty $z_t \in \{H, L\}$. The parametric condition $\beta R^f < L$ ensures that the normalized aggregate wealth remains bounded along all possible paths. Occasionally, I focus on paths in which the growth shock remains constant, $z_t = z \in \{H, L\}$ for each t. Along these paths, Eq. (12) implies that the normalized foreign assets converge to a constant given by,

$$f = \frac{\beta(1+n) - (1-\beta)p}{1 - \beta R^f/z},$$

Substituting this expression into (11), normalized consumption also converges to a constant given by,

$$c = \frac{1-\beta}{\beta}(p + f) = (1-\beta)\frac{1 + n - \left(R^f/z - 1\right)p}{1 - \beta R^f/z}.$$

This proves Eqs. $(13-14)$ that describe the BGP allocations.

Next consider the benchmark case in which all investors think $z_{t+1} = z \in \{H, L\}$ and there is no exogenous leverage limit $\overline{\omega}_t = \infty$. Since beliefs are common (and leverage is allowed), Eq. (7) implies all investors are indifferent between the two assets, $E_t^i [R_{t,t+1}] = R^f$. Substituting this into (4), I obtain,

$$P_t = \frac{(1-\delta) E_t^i [r_{t+1} + P_{t+1}]}{R^f} = \frac{(1-\delta)(r_{t+1} + P_{t+1})}{R^f}. \tag{A.3}$$

Here, the second equality uses the observation that beliefs are extreme and feature no uncertainty. Substituting $P_{t+1} = pe_t, P_{t+1} = pe_{t+1}, r_{t+1} = e_{t+1}$ [see (2)], and the expected productivity growth $E_t^i [e_{t+1}] = e_t z$ [see (3)], I obtain Eq. (15),

$$p_t \equiv p(z) = \frac{(1-\delta) z}{R^f - (1-\delta) z}.$$

The normalized price is constant over time and is well defined in view of the parametric restriction, $R^f > H(1-\delta)$. Given this price, the remaining allocations follow the dynamics in $(11-12)$. This completes the characterization of equilibrium for the common-belief benchmark.

A.2. Omitted derivations for Section 3

Consider the case with persistent disagreements analyzed in Section 3. Most of the analysis is described in the main text. Here, I verify that the conjectured equilibrium price (17) corresponds to an equilibrium.

In the conjectured equilibrium, optimists invest in the asset and they are unconstrained. Therefore, Eq. (7) implies optimists are indifferent between the two assets, $E_t^o [R_{t,t+1}] = R^f$. This leads to Eq. $(A.3)$ with the optimistic belief $i = o$. Following the same steps as in the common-belief benchmark, I obtain Eq. (17),

$$p(H) = \frac{(1-\delta) H}{R^f - (1-\delta) H}.$$

Given this equilibrium price, condition (16) ensures optimists' wealth is sufficient to purchase the entire risky asset supply. Therefore, optimists purchase all of the asset and invest the remaining fraction of their wealth in the risk-free asset.

It remains to check the optimality of pessimists' portfolios. In the conjectured equilibrium, pessimists do not invest in the risky asset, $\omega_t^p = \underline{\omega}_t^p = 0$. They invest all of their wealth in the risk-free asset. Eq. (7) implies this is optimal as long as $E_t^p [R_{t,t+1}] \leq R_t^f$. Using Eq. (4), I calculate the expected return along the conjectured equilibrium according to pessimistic belief to obtain:

$$E_t^p [R_{t,t+1}] = \frac{e_t (1-\delta) L (1 + p(H))}{e_t p(H)} < R^f.$$

Here, the inequality follows from the definition of $p(H)$. Hence, pessimists believe the asset is overpriced and delivers a lower return than the risk-free rate. This proves the optimality of their portfolios and completes the characterization of equilibrium.

A.3. Omitted derivations for Section 4

Consider the belief structure for the speculative bubbles analyzed in Section 4: that is, young optimists are the only optimistic investors in the economy. The analysis is mostly described in the main text. In particular, given the belief structure and condition (16), the equilibrium from Section 3 is still an equilibrium. It remains to check that the young optimists' present discounted (or buy-and-hold) valuation is given by, $p_{pdv}^o = \frac{H(1-\delta)}{R^f - L(1-\delta)}$.

Using Eq. $(A.3)$, the present discounted valuation of an investor is given by:

$$P_{t,pdv}^{i} = \lim_{N\to\infty} \sum_{n=1}^{N} \frac{(1-\delta)^{n} E_{t}^{i}[r_{t+1}]}{(R^{f})^{n}} + \frac{(1-\delta)^{N} E_{t}^{i}[P_{t+N}]}{(R^{f})^{N}}.$$

In the equilibria I consider, the price grows at the same rate as the productivity and the productivity grows at rate $z \in \{H, L\}$. Combining this with the parametric condition, $R^{f} > (1-\delta) H$, the price term disappears from the limit and I have:

$$P_{t,pdv}^{i} = \sum_{n=1}^{\infty} \frac{(1-\delta)^{n} E_{t}^{i}[r_{t+n}]}{(R^{f})^{n}}. \tag{A.4}$$

Next recall that $r_{t+1} = e_{t+1} = z_{t+1} e_{t}$ [see (2) and (3)]. Note also that young optimists in period t $(i = o)$ perceive $z_{t+1} = H$ and $z_{t+n} = L$ for each $n \geq 2$. Combining these observations with $(A.4)$, I obtain,

$$P_{t,pdv}^{o} = \frac{(1-\delta) e_{t} H / R^{f}}{1 - \frac{L(1-\delta)}{R^{f}}} = \frac{(1-\delta) e_{t} H}{R^{f} - L(1-\delta)}.$$

This establishes that the normalized valuation is given by, $p_{pdv}^{o} = P_{t,pdv}^{o} / e_{t} = \frac{H(1-\delta)}{R^{f} - L(1-\delta)}$. This is strictly less than the equilibrium price, $p(H) = \frac{H(1-\delta)}{R^{f} - H(1-\delta)}$. Thus, the price exceeds the present discounted valuation for all investors, illustrating the speculative bubble (see the main text for further discussion).

A.4. Omitted derivations for Section 5

Consider the belief structure for the speculative bubbles analyzed in Sections 4 and 5: young optimists are the only optimistic investors in the economy. Consider also the special case, $\delta = 0$. The main difference from Section 4 is that young investors' wealth is limited, which implies the leverage limit affects the size of the speculative bubble.

A.4.1. Exogenous leverage limits

Consider the case with an exogenous leverage limit and no short selling analyzed in Section 5.1 (I discuss the case with limited short selling at the end of the section). Suppose the parameters satisfy the following conditions

$$\begin{aligned} p(H) &> \beta n \alpha \overline{\omega} > p(L) \text{ where } p(z) = \frac{z}{R^{f} - z}, \\ \overline{\omega} &\leq \overline{\omega}^{end} = \frac{\beta \alpha n R^{f} + L}{\beta \alpha n (R^{f} - L)}. \end{aligned}$$

Here, the first line replicates condition (18) from the main text. The second condition says the exogenous leverage limit is tighter than the endogenous leverage limit (along the BGP) that I characterize subsequently (see Eq. $(A.7)$). This condition ensures the exogenous leverage limit binds.

Under these conditions, I conjecture a BGP equilibrium in which the normalized asset price is given by (19) from the main text,

$$p(\overline{\omega}) = \beta n \alpha \overline{\omega}.$$

In this equilibrium, young optimists are against their leverage limit, $\omega_{t}^{o} = \overline{\omega}$, and all other investors (young and old pessimists) are against the short-selling constraint, $\omega_{t}^{i} = 0$.

To verify the equilibrium price, note that $a_t^o = \alpha n e_t$ (since $\delta = 0$). Using this along with $\omega_t^o = \overline{\omega}$ and $\omega_t^i = 0$ for other investors, the asset market clearing condition implies

$$P_t = \beta n \alpha \overline{\omega} e_t.$$

This verifies that the normalized asset price is constant and given by (19).

To verify the optimality of the equilibrium portfolios, consider investors' expected return from the risky asset. Young optimists' expected return satisfies,

$$E_t^o \left[R_{t,t+1}\right] = \frac{He_t \left(1 + p\left(\overline{\omega}, \underline{\omega}\right)\right)}{p\left(\overline{\omega}, \underline{\omega}\right) e_t} > H \frac{1 + p\left(H\right)}{p\left(H\right)} = R^f.$$

Here, the first equality uses Eq. (4) and substitutes the equilibrium allocations. The inequality follows since $1/p\left(\overline{\omega}\right) < 1/p\left(H\right)$ [see (18)]. The last equality follows since $p\left(H\right) = \frac{H}{R^f - H}$. Since $E_t^o\left[R_{t,t+1}\right] > R^f$, young optimists optimally choose the maximum allowed leverage, $\omega_t^o = \overline{\omega}$.

Likewise, other investors' (young and old pessimists') expected return satisfies,

$$E_t^i \left[R_{t,t+1}\right] = \frac{Le_t \left(1 + p\left(\overline{\omega}, \underline{\omega}\right)\right)}{p\left(\overline{\omega}, \underline{\omega}\right) e_t} < L \frac{1 + p\left(L\right)}{p\left(L\right)} = R^f,$$

where the inequality follows since $1/p\left(\overline{\omega}\right) < 1/p\left(L\right)$ and the last equality follows since $p\left(L\right) = \frac{L}{R^f - L}$. Since $E_t^i\left[R_{t,t+1}\right] < R^f$, other (pessimistic) investors optimally choose $\omega_t^i = 0$. This establishes the optimality of the portfolios and completes the characterization of equilibrium.

Exogenous leverage and short selling limits. Next consider the case with limited short selling discussed in the main text. Specifically, young investors (and only them) are allowed to engage in limited short selling: $\omega_t^i \geq \underline{\omega}$ where $\underline{\omega} \leq 0$ for $i \in \{o, p\}$ and $\omega_t^i \geq 0$ for other investors. The above analysis generalizes to this case as long as the parameters satisfy the following generalization of condition (18),

$$p\left(H\right) > \beta n \left(\alpha \overline{\omega} + \left(1 - \alpha\right) \underline{\omega}\right) > p\left(L\right),$$

and the limits $\overline{\omega}, \underline{\omega}$ are sufficiently tight (in particular, tighter than the corresponding endogenous limits along the equilibrium path). Under these conditions, all of the above steps still hold and imply a BGP equilibrium in which the price is given by Eq. (20),

$$p = \beta n \left(\alpha \overline{\omega} + \left(1 - \alpha\right) \underline{\omega}\right).$$

In this equilibrium, young optimists are against their leverage limit, $\omega_t^o = \overline{\omega}$, and all other investors (young and old pessimists) are against their respective short-selling limits, $\omega_t^p = \underline{\omega}$ and $\omega_t^{mp} = 0$.

A.4.2. Endogenous leverage limits

Next consider the case with collateral constraints analyzed in Section 5.2. I first derive the endogenous and short selling limits. I then show that in this model these limits replicate the complete market outcomes. Finally, I consider the case with endogenous leverage (and no short selling) and complete the characterization of equilibrium.

Endogenous leverage and short selling limits. As described in the main text, the collateral constraint implies the leverage limit in (22),

$$\omega_t^i \leq \overline{\omega}_t^{end} \equiv \frac{p_t}{p_t - \frac{1}{R^f} L \left(1 + p_{t+1}\right)}.$$

To derive the short-selling limit, consider a short position, $\omega_t^i \leq 0$. For a short position, the nonnegative portfolio return condition, $R_{t,t+1}\omega_t^i + R^f \left(1 - \omega_t^i\right) \geq 0$ implies the following collateral constraint [cf. (21)]:

$$\left(R_{t,t+1} - R^f\right)\left(-\omega_t^i\right) \leq R^f. \tag{A.5}$$

A short position makes losses when the risky asset generates a lower return than the risk-free asset, $R_{t,t+1} < R^f$. Moreover, the short position is collateralized by cash. The expression says that, for a dollar put up as collateral in a short position, the loss from the position (the left side) must be lower than the value of collateral (the right side) *in all states.*

Eq. $(A.5)$ illustrates that the collateral constraint for a short position is most likely to bind when the asset generates the highest possible return. In equilibrium, this obtains when the high growth state is realized, $z_{t+1} = H$, which leads to the return $R_{t,t+1} = \frac{e_t H(1+p_{t+1})}{e_t p_t}$ [see (4)]. After substituting this into the collateral constraint, I obtain the endogenous short limit in (23),

$$\omega_t^i \geq \underline{\omega}_t^{end} \equiv -\frac{p_t}{\frac{1}{R^f} H \left(1 + p_{t+1}\right) - p_t}.$$

Equivalence with complete market outcomes. I next verify the claim in Remark 6 that (when there is disagreement) the endogenous limits in (22) and (23) lead to complete markets outcomes in every period.

First consider the equilibrium with complete markets. Note that optimists (resp. pessimists) think the state H (resp. L) will be realized with certainty. Therefore, with complete markets, they endogenously choose positions that generate zero wealth if the other state L (resp. H) is realized, that is, $a^o_{t+1,L} = a^p_{t+1,H} = 0$.

Next consider the equilibrium with the endogenous leverage limits in (22) and (23) (and no exogenous limits). In a period in which optimists and pessimists have both positive wealth, the equilibrium price p_t lies between their one-period valuations,

$$p_t \in \left(\frac{1}{R^f} L \left(1 + p_{t+1}\right), \frac{1}{R^f} H \left(1 + p_{t+1}\right)\right).$$

Given this price, the optimistic and the pessimistic expectations of the risky return satisfy, $E_t^o\left[R_{t,t+1}\right] > R^f > E_t^p\left[R_{t,t+1}\right]$ [see (4)]. Consequently, optimists and pessimists are both at their respective limits [see (7)],

$$\omega_t^o = \overline{\omega}_t^{end} \text{ and } \omega_t^p = \underline{\omega}_t^{end}.$$

Recall that, by definition, the leverage limit induces a zero portfolio return in the low growth state L and the short selling limit induces a zero return in the high growth state H. This implies $a^o_{t+1,L} = a^p_{t+1,H} = 0$—the same outcome as in the equilibrium with complete markets.

Endogenous leverage and speculative bubbles. Next consider the case with the endogenous leverage limit, $\overline{\omega}_t^{end}$ (and no short selling, $\underline{\omega} = 0$). Suppose the parameters satisfy,

$$\frac{\beta\alpha n R^f + L}{R^f - L} < p(H) = \frac{H}{R^f - H} \tag{A.6}$$

Under this condition, I conjecture a BGP equilibrium in which the price and the leverage limit are both constant, $p_t = p$ and $\overline{\omega}_t^{end} = \overline{\omega}^{end}$, and the leverage limit binds.

In this equilibrium, our analysis with an exogenous leverage limit applies after replacing $\overline{\omega}$ with $\overline{\omega}^{end}$ (see Section 5.1 and Appendix A.4.1). Combining Eqs. (22) and (19), the asset price is given by (24),

$$p = \frac{\beta\alpha n R^f + L}{R^f - L}.$$

Substituting this into (22), I obtain the endogenous leverage limit along the equilibrium path,

$$\overline{\omega}^{end} = \frac{1}{1 - \frac{1}{R^f} L \frac{1+p}{p}} = \frac{\beta\alpha n R^f + L}{\beta\alpha n (R^f - L)}. \tag{A.7}$$

The analysis with an exogenous leverage limit implies this is an equilibrium as long as the price satisfies, $p = \beta\alpha n \overline{\omega}^{end} \in (p(L), p(H))$ [see (18)]. The equilibrium price always satisfies, $p > p(L) = \frac{L}{R^f - L}$. Intuitively, since lenders perceive no uncertainty, they are always willing to extend credit up to their own valuations. Thus, lenders' valuations provide a lower bound on the equilibrium price. In view of condition $(A.6)$, the equilibrium price also satisfies, $p < p(H)$. This completes the characterization of equilibrium with endogenous leverage.

A.5. Omitted derivations for Section 6

Consider the rational bubbles analyzed in Section 6. I first describe how to extend the open economy setup in Section 2 to allow for the bubble asset. I then characterize the equilibrium for the case analyzed in the main text. For simplicity, I consider a representative-investor setting (or common beliefs) and drop the investor superscript i.

Suppose, in addition to the risky asset, there is a distinct bubble asset that yields zero dividends in all periods. Let B_t denote the price of the bubble asset. Thus, the one-period return of the bubble asset is given by [cf. (4)]

$$R^b_{t,t+1} = \frac{B_{t+1}}{B_t}. \tag{A.8}$$

The representative investor can invest in the bubble asset as well as the risky and the risk-free assets. Therefore, her problem (5) is modified so that she chooses, $c_t, \omega_t, \omega_t^b$ subject to the budget constraint

$$a_{t+1} = (a_t - c_t)\left(R_{t,t+1}\omega_t + R^b_{t,t+1}\omega_t^b + R_t^f\left(1 - \omega_t - \omega_t^b\right)\right). \tag{A.9}$$

For simplicity, suppose there are no exogenous short selling or leverage restrictions. Finally, the market clearing conditions are modified [cf. (8)]:

$$\omega_t \beta a_t = P_t, \omega_t^b \beta a_t^i = B_t \text{ and } \beta a_t = P_t + B_t + F_t.$$

Hence, there is a separate market clearing condition for the bubble, and the market clearing condition for all assets also reflects the bubble. The rest of the model is unchanged.

Next consider the equilibrium for the case analyzed in the main text. Specifically, suppose the representative investor has the pessimistic belief, $z_{t+1} = L$, and the risk-free return satisfies, $R^f = L$. Consider the path in which the realized growth is the same as what investors expect, $z_t = L$ for each t. With these assumptions, I conjecture an equilibrium in which the bubble asset satisfies $B_t = be_t$ with a constant $b > 0$, the risky asset price is given by the same expression as before (15) with $z = L$, and the normalized consumption and aggregate wealth follow the dynamics in $(25-26)$.

To verify the conjecture, note that the investor perceives no uncertainty. Then, Eqs. $(A.8)$ and $(A.9)$ along with no-arbitrage require,

$$R^b_{t,t+1} = R_{t,t+1} = R^f.$$

The requirement on the risky asset return (and $z_{t+1} = L$) implies that the normalized risky asset price is the same as before, $p(L) = \frac{(1-\delta)L}{R^f-(1-\delta)L}$. Using Eq. $(A.8)$ and the parametric condition $R^f = L$, the requirement on the bubble asset return becomes,

$$R^b_{t,t+1} = \frac{B_{t+1}}{B_t} = R^f = L.$$

The conjectured BGP satisfies this requirement because $B_t = be_t$ and $e_{t+1} = e_t L$.

Finally, following the same steps as in Section A.1, aggregate consumption satisfies [cf. $(A.1)$ and $(A.2)$],

$$C_t = \frac{1-\beta}{\beta}\left(P_t + B_t + F_t\right) = (1-\beta)\left(Y_t + P_t + B_t + R^f F_{t-1}\right)$$

Substituting the normalized prices, $p(L) = P_t/e_t, b = B_t/e_t$, and allocations, $1+n = Y_t/e_t, c_t = C_t/e_t, f_t = F_t/e_t$, and using the dynamics $e_t = e_{t-1}L$, the normalized consumption and aggregate wealth follow the dynamics in $(25-26)$. Using these dynamics, normalized consumption and aggregate wealth converge to the BGP allocations in $(25-26)$ (with $z = L$) after replacing p with $p(L) + b$, that is:

$$c = (1-\beta)\frac{1+n-\left(R^f/L-1\right)(p(L)+b)}{1-\beta R^f/L} = 1+n, \tag{A.10}$$

$$f = \frac{\beta(1+n)-(1-\beta)(p(L)+b)}{1-\beta R^f/L} = \frac{\beta}{1-\beta}(1+n) - (p(L)+b). \tag{A.11}$$

Here, the second equality in each line substitutes $R^f = L$ to simplify the expressions. This completes the characterization of the equilibrium with rational bubbles.

A.6. Omitted derivations for Section 7

In this appendix, I complete the characterization of equilibrium for the closed economy model analyzed in Sections 7. Recall that the interest rate is endogenous, $R^f_t \equiv R^{f*}_t$, and the goods market equilibrium condition is given by (27),

$$F_t = 0 \text{ and } Y_t = Y^*_t = (1+n)\,e_t \text{ for each } t.$$

Recall that there are no exogenous short selling or leverage constraints ($\underline{\omega}_t = -\infty, \overline{\omega}^t = \infty$). Finally, beliefs are persistent: young optimists (resp. pessimists) become mature optimists (resp. pessimists). I next characterize the equilibrium. Throughout, I use the notation $x_t(z)$ to refer to a variable in period t under the belief that the next period's state will be $z_{t+1} = z$; and I use the notation $x_{t,z}$ to refer to a variable in period t and state $z_t = z$.

Output-asset price relation in a closed economy. To derive the output asset price relation, first note that Eqs. $(A.1 - A.2)$ hold in this setting after substituting $F_t = F_{t-1} = 0$, that is:

$$C_t = \frac{1-\beta}{\beta} P_t = (1-\beta)(Y_t + P_t).$$

As before, consumption is proportional to both end-of-period and beginning-of-period aggregate wealth. Substituting $C_t = Y_t$ into these expressions, I obtain the output asset price relation (28),

$$Y_t = (1/\beta - 1) P_t.$$

Setting output equal to its potential, $Y_t^* = e_t(1+n)$, this also implies the potential asset price is constant and given by (29),

$$p_t = p^* = \frac{n+1}{1/\beta - 1}.$$

Common-belief benchmark in a closed economy. Next consider a benchmark case in which all investors have the same belief. I conjecture an equilibrium with a constant interest rate, $R^{f*}(z)$, and a constant normalized price, $p(z)$. To characterize this equilibrium, first note that Eq. $(A.3)$ from Appendix A.1 still applies but with the endogenous interest rate,

$$P_t = \frac{(1-\delta)(r_{t+1} + P_{t+1})}{R_t^f}. \tag{A.12}$$

Evaluating this with $r_{t+1} = e_{t+1}$ and $e_{t+1} = e_t z$, I solve for the equilibrium price as [see Eq. (30)],

$$p(z) = \frac{(1-\delta) z}{R^f(z) - (1-\delta) z}.$$

Combining this with (29), I also solve for the equilibrium interest rate [see Eq. (31)],

$$R^{f*}(z) = z(1-\delta)\left(\frac{1+p^*}{p^*}\right) = z(1-\delta)\frac{n+1/\beta}{n+1}.$$

Hence, in this model, changes in beliefs (e.g., an increase or decrease in the perceived growth rate z) affect the equilibrium interest rate as opposed to the equilibrium price.

Disagreements and procyclical asset valuation. I next turn to the main focus with disagreements. I characterize the equilibrium in three steps. I first show that, given optimists' wealth share $\alpha_t \in (0,1)$ [defined in (32)], the equilibrium price in period t is characterized by Eq. (33). Second, I establish that optimists' wealth share follows the dynamics in (34). Finally, I characterize the dynamic equilibrium and establish (35).

Recall that I assume there are no exogenous leverage and short selling limits. Following the same steps as in Appendix A.4.2, the endogenous leverage limits are given by [see (22) and (23)]:

$$\begin{aligned} \overline{\omega}_t^{end} &= \frac{P_t}{P_t - P_t(L)} > 1 \text{ and } \underline{\omega}_t^{end} = -\frac{P_t}{P_t(H) - P_t} < 0 \qquad \text{(A.13)} \\ \text{with } P_t(L) &= \frac{(1-\delta)(r_{t+1,L} + P_{t+1,L})}{R_t^f}, P_t(H) = \frac{(1-\delta)(r_{t+1,H} + P_{t+1,H})}{R_t^f}. \end{aligned}$$

Here, $P_{t,L}$ and $P_{t,H}$ correspond to the worst case (pessimistic) and the best case (optimistic) valuations, respectively. The analysis in Appendix A.4.2 also implies that optimists and pessimists are at their respective limits,

$$\omega_t^o = \overline{\omega}_t^{end} \text{ and } \omega_t^p = \underline{\omega}_t^{end}. \tag{A.14}$$

Next note that, combining Eqs. (32) and (8), the risky asset market clearing condition (for the closed economy) can be written as,

$$\sum_i \omega_t^i \alpha_t^i = \omega_t^o \alpha_t + \omega_t^p (1 - \alpha_t) = 1. \tag{A.15}$$

Hence, the risky asset market clears when investors' wealth-weighted average portfolio weight is equal to one. Combining this with Eqs. $(A.13)$ and $(A.14)$, I obtain

$$\frac{P_t}{P_t - P_t(L)}\alpha_t + \frac{P_t}{P_t - P_t(H)}(1 - \alpha_t) = 1.$$

After rearranging terms, I obtain

$$P_t = \left(\alpha_t \frac{1}{P_t(H)} + (1 - \alpha_t)\frac{1}{P_t(L)}\right)^{-1} = \mathcal{P}(P_t(H), P_t(L) | \alpha_t).$$

This establishes Eq. (33): the asset price is equal to the wealth-weighted harmonic average of optimists' and pessimists' valuations.

Next consider the dynamics of optimists' wealth share. First suppose a low growth state is realized, $z_t = L$. Optimists' portfolio weight in $(A.14)$ implies that old optimists' wealth is zero. Therefore, optimists' wealth share, α_t, is equal to young optimists' wealth share. Using this observation, I characterize:

$$\begin{aligned} \alpha_{t,L} = \frac{a_t^o}{\sum_i a_t^i} &= \frac{\alpha(w_t + \delta P_t)}{w_t + r_t + P_t} \\ &= \frac{e_t \alpha (n + \delta p_t)}{e_t(n + 1 + p_t)} \\ &= \frac{\alpha(n + \delta p^*)}{n + 1 + p^*} \equiv \alpha_L. \end{aligned} \tag{A.16}$$

Here, the second line substitutes the wage and the rental rate from (2). The last line uses the observation that the equilibrium features a constant normalized price, $p_t = p^*$ [see (29)].

Next suppose a high growth state is realized, $z_t = H$. In this case, pessimists' portfolio weight in $(A.14)$ implies that old pessimists' wealth is zero. Therefore, pessimists' wealth share, $1 - \alpha_t$, is equal to young pessimists' wealth share. Following similar steps, I obtain:

$$\begin{aligned} \alpha_t = 1 - \frac{a_t^p}{\sum_i a_t^i} &= 1 - \frac{(1-\alpha)(w_t + \delta P_t)}{w_t + r_t + P_t} \\ &= 1 - \frac{(1-\alpha)(n + \delta p^*)}{n + 1 + p^*} \equiv \alpha_H. \end{aligned} \tag{A.17}$$

This establishes the dynamics in (34). Note also that $\alpha_H > \alpha_L$: optimists have greater wealth in the high growth state than in the low growth state.

Finally, consider the dynamic equilibrium. Combining $P_t = p^* e_t$ and $r_{t+1} = e_{t+1}$ with (33), the optimistic and the pessimistic valuations in $(A.13)$ are given by,

$$P_t(H) = \frac{(1-\delta) H (1+p^*) e_t}{R_t^f} \text{ and } P_t(L) = \frac{(1-\delta) L (1+p^*) e_t}{R_t^f}.$$

Substituting this into Eq. (33), and using $P_t = p^* e_t$ along with the linear homogeneity of the harmonic average, I obtain:

$$\begin{aligned} R_t^f &= \mathcal{P}\left(\frac{(1-\delta) H (1+p^*)}{p^*}, \frac{(1-\delta) L (1+p^*)}{p^*} | \alpha_t \right) \\ &= \mathcal{P}\left(R^{f*}(H), R^{f*}(L) | \alpha_t \right). \end{aligned}$$

Here, the second line substitutes the definition of the common belief benchmark interest rate $R^{f*}(z)$ [see (31)]. Combining this with the dynamics in (34) completes the characterization of equilibrium. In particular, the equilibrium interest rate is a function of the most recent realization, $R_{t,z}^{f*} = R_z^{f*}$, with R_z^{f*} given by (35).

A.7. Omitted derivations for Section 8

In this appendix, I first complete the characterization of the closed economy model with nominal rigidities analyzed in Sections 8. I then present the New Keynesian microfoundations for nominal rigidities.

A.7.1. Equilibrium with nominal rigidities and demand recessions

Recall that the setup is similar to the closed economy model analyzed in Section 7 with three differences. First, since production firms have sticky nominal prices, output is determined by aggregate demand can be less than potential (see Section A.7.2),

$$y_t = \frac{Y_t}{Y_t^*} \leq 1.$$

Here, y_t denotes relative output. Second, under appropriate assumptions (see Section A.7.2), factor returns satisfy (36),

$$w_t = y_t e_t n \text{ and } r_t = y_t e_t.$$

Finally, the interest rate policy follows (37),

$$R_t^f = \max\left(\tilde{R}_t^{f*}, \underline{R}^f y_t \right).$$

Here, $\tilde{R}_t^{f*}$ is the interest rate that closes the current output gap given the policy in future periods. I next characterize the equilibrium.

Output-asset price relation with nominal rigidities. As described in the main text, the output asset price relation (28) still applies. In this context, this relation implies (38), which says that the relative output is equal to the relative asset price,

$$y_t = \frac{p_t}{p^*} \leq 1.$$

Using this relation, the interest rate policy becomes (39),

$$R_t^f = \max\left(\tilde{R}_t^{f*}, \underline{R}^f \frac{p_t}{p^*} \right).$$

Common-belief benchmark with nominal rigidities. Next consider the benchmark in which all investors think $z_{t+1} = z \in \{H, L\}$. I conjecture an equilibrium in which the asset price, $p(z) \leq p^*$, and the interest rate, $R^f(z)$, are both constant.

To characterize this equilibrium, let $R^{f*}(z) = z(1-\delta)\frac{1+p^*}{p^*}$ denote the common-belief interest rate without nominal rigidities [see (31)]. If $R^{f*}(z) \geq \underline{R}^f$, the interest rate policy is unconstrained and replicates the potential outcomes, that is, $R^f(z) = R^{f*}(z)$ and $p(z) = p^*$. Consider the other case $R^{f*}(z) < \underline{R}^f$. In this case, the interest rate policy is constrained and there is a demand recession. To characterize the equilibrium outcomes, first note that the price still satisfies Eq. $(A.12)$,

$$P_t = \frac{(1-\delta)(r_{t+1} + P_{t+1})}{R_t^f}.$$

Note that Eqs. (36) and (38) imply $r_{t+1} = \frac{p(z)}{p^*} L e_{t+1}$ and the conjectured equilibrium features $P_t = p(z)e_t$ and $R_t^f = R^f(z)$. Combining these observation gives (30),

$$p(z) = \frac{z(1-\delta)\frac{p(z)}{p^*}}{R^f(z) - z(1-\delta)}.$$

After rearranging this expression, I obtain,

$$R^f(z) = R^{f*}(z) = z(1-\delta)\frac{1+p^*}{p^*}.$$

Combining it with the policy rule in (39), I further obtain,

$$\frac{p(z)}{p^*} = \frac{R^{f*}(z)}{\underline{R}^f} < 1.$$

This proves (40) and completes the characterization of equilibrium with common beliefs.

Speculation and demand recessions. Next consider the case with disagreements. Since the model is similar to the one analyzed in Section 7, most of the analysis in Appendix A.6 still applies. In particular, Eqs. $(A.13 - A.15)$ still hold. Combining these expressions, the asset price is still characterized by Eq. (33), which I replicate for ease of exposition,

$$\begin{aligned} P_t &= \left(\alpha_t \frac{1}{P_t(H)} + (1-\alpha_t)\frac{1}{P_t(L)}\right)^{-1} = \mathcal{P}(P_t(H), P_t(L) | \alpha_t) \\ \text{with } P_t(L) &= \frac{(1-\delta)(r_{t+1,L} + P_{t+1,L})}{R_t^f}, P_t(H) = \frac{(1-\delta)(r_{t+1,H} + P_{t+1,H})}{R_t^f}. \end{aligned}$$

Likewise, Eq. (34) that describes optimists' wealth dynamics remains unchanged. To see this, first suppose the low growth state is realized. Then, I have the following analogue of Eq. $(A.16)$,

$$\begin{aligned}\alpha_{t,L} &= \frac{a_t^o}{\sum_i a_t^i} = \frac{\alpha\left(w_t + \delta P_t\right)}{w_t + r_t + P_t} \\ &= \frac{e_t \alpha\left(n\frac{p_t}{p^*} + \delta p_t\right)}{e_t\left(n\frac{p_t}{p^*} + \frac{p_t}{p^*} + p_t\right)} \\ &= \frac{\alpha\left(n + \delta p^*\right)}{n + 1 + p^*} = \alpha_L.\end{aligned}$$

Here, the second line substitutes the wage and the rental rate using Eqs. (36) and (38). Likewise, I have the following analogue of Eq. $(A.17)$,

$$\begin{aligned}\alpha_t = 1 - \frac{a_t^p}{\sum_i a_t^i} &= 1 - \frac{(1-\alpha)\left(w_t + \delta P_t\right)}{w_t + r_t + P_t} \\ &= 1 - \frac{(1-\alpha)\left(n\frac{p_t}{p^*} + \delta p_t\right)}{n\frac{p_t}{p^*} + \frac{p_t}{p^*} + p_t} \\ &= 1 - \frac{(1-\alpha)\left(n + \delta p^*\right)}{n + 1 + p^*} = \alpha_H.\end{aligned}$$

Since the demand recession lowers prices and factor returns proportionally, it leaves the wealth share dynamics unchanged [see (34)].

Next consider the dynamic equilibrium. Suppose the parameters satisfy:

$$\mathcal{P}\left(R^{f*}(H), R^{f*}(L)\,|\alpha_L\right) < \underline{R}^f < \mathcal{P}\left(R^{f*}(H), \frac{R^{f*}(L)^2}{\underline{R}^f}|\alpha_H\right). \tag{A.18}$$

Under this condition, I conjecture an equilibrium in which the low growth state features a demand recession, $R_L^f < \underline{R}^f, p_L < p^*$, and the high growth state features potential outcomes, $R_H^f > \underline{R}^f, p_H = p^*$ (and the interest rate and the normalized price remain constant within states).

To characterize this equilibrium, I let $p_z(z')$ denote the normalized asset valuation in state z according to an investor who believes state z' will be realized in the next period with certainty. For instance, $p_L(H)$ denotes the valuation by an optimist (who believe $z' = H$ will be realized) in the low growth state $z = L$. Using (36), I calculate:

$$\begin{aligned}p_z(z') &= \frac{(1-\delta)\,z'\left(\frac{p_{z'}}{p^*} + p_{z'}\right)}{R_z^f} \\ &= \frac{p_{z'}}{R_z^f} z'(1-\delta)\,\frac{1+p^*}{p^*} \\ &= \frac{p_{z'}}{R_z^f} R^{f*}(z') \text{ for } z, z' \in \{H, L\}.\end{aligned} \tag{A.19}$$

Here, the last line substitutes the definition of $R^{f*}(z')$,—"rstar" for the common-belief benchmark [see (31)].

Next, I use Eq. (33) to characterize the normalized equilibrium price in each state in terms of the normalized valuations by optimists and pessimists:

$$\begin{aligned} p_H = p^* &= \mathcal{P}\left(p_H\left(H\right), p_H\left(L\right)|\alpha_H\right) \\ p_L &= \mathcal{P}\left(p_L\left(H\right), p_L\left(L\right)|\alpha_L\right). \end{aligned} \tag{A.20}$$

Here, recall that $\mathcal{P}\left(x, y|\tilde{\alpha}\right) = \left(\frac{\tilde{\alpha}}{x} + \frac{1-\tilde{\alpha}}{y}\right)^{-1}$.

$$\mathcal{P}\left(R^{f*}\left(H\right), R^{f*}\left(L\right)|\alpha_L\right) < \underline{R}^f < \mathcal{P}\left(R^{f*}\left(H\right), \frac{R^{f*}\left(L\right)^2}{\underline{R}^f}|\alpha_H\right)$$

Finally, I combine $(A.19)$ and $(A.20)$, and use the linear homogeneity of the function $\mathcal{P}\left(\cdot\right)$, to obtain:

$$\begin{aligned} R^f_H &= \mathcal{P}\left(R^{f*}\left(H\right), \frac{p_L}{p^*}R^{f*}\left(L\right)|\alpha_H\right) \\ R^f_L = \frac{p_L}{p^*}\underline{R}^f &= \mathcal{P}\left(\frac{R^{f*}\left(H\right)}{p_L/p^*}, R^{f*}\left(L\right)|\alpha_L\right). \end{aligned} \tag{A.21}$$

Here, the second line also uses the optimal monetary policy $\frac{R^f_t}{\underline{R}^f} = \frac{p_L}{p^*}$ [see (39)]. The second line proves Eq. (41) in the main text that describes the asset price in the recession. The first line obtains a similar expression for the interest rate in the boom.

I next show that, under condition $(A.18)$, this system has a unique solution that satisfies $p_L < p^*$ and $R^f_H > \underline{R}^f$. First consider Eq. (41) that characterizes p_L. The left side is an increasing function of $\frac{p_L}{p^*}$ and the right side is a decreasing function of $\frac{p_L}{p^*}$. Under condition $(A.18)$, there is a unique solution that satisfies $\frac{p_L}{p^*} \in \left(\frac{R^{f*}(L)}{\underline{R}^f}, 1\right)$. Given this solution, Eq. (41) characterizes R^f_H. Moreover, the solution satisfies

$$R^f_H > \mathcal{P}\left(R^{f*}\left(H\right), \frac{R^{f*}\left(L\right)^2}{\underline{R}^f}|\alpha_H\right) > \underline{R}^f.$$

Here, the first inequality follows from $\frac{p_L}{p^*} > \frac{R^{f*}(L)}{\underline{R}^f}$ and the second inequality follows from condition $(A.18)$. This verifies the conjecture and completes the characterization of equilibrium.

Macroprudential policy in the boom. I next consider how macroprudential policies in the boom affect this equilibrium and establish Eq. (42). Specifically, suppose the economy is currently in period 0 with state $z_0 = H$ and the planner implements macroprudential policy that bans leverage, $\omega^i_0 \leq 1$. Finally, I also make two simplifying assumptions: starting period 1 onward there is no macroprudential policy, and in period 0 there is no interest rate lower bound.

First consider the equilibrium in period 0. It is easy to check that the extreme leverage ban implies all investors (optimists and pessimists) choose the same portfolios, $\omega^i_0 = 1$. With these allocations, optimists are constrained (they would like to increase their positions on the risky asset) and pessimists are unconstrained.

In particular, the (normalized) asset price in period 0 is equal to the pessimists' valuation. Using similar steps as before, I calculate this valuation as [see $(A.19)$],

$$\tilde{p}_0 = \frac{\left(1-\delta\right)L\left(\frac{p_{1,L}}{p^*} + p_{1,L}\right)}{R^f_0} = \frac{p_{1,L}}{R^f_0}R^{f*}\left(L\right).$$

Since the interest rate does not bind in period 0 (by assumption), I also obtain:

$$\tilde{p}_0 = p^* \text{ and } \tilde{R}_0^f = R_{0,1,L} \equiv \frac{p_{1,L}}{p^*} R^{f*}(L). \tag{A.22}$$

Here, $R_{0,1,L}$ denotes the asset's realized return when the low state is realized. In equilibrium, the interest rate is equal to the asset's return in the low state and pessimists are indifferent to invest.

Note also that the interest rate with macroprudential policy satisfies $\tilde{R}_0^f \leq R^{f*}(L) < \underline{R}^f$, where the second inequality follows from condition $(A.18)$. In contrast, the interest rate without macroprudential policy (characterized earlier) satisfies $R_0^f = R^f(H) > \underline{R}^f$. This proves that macroprudential policy reduces the interest rate in period 0.

Next consider the equilibrium in period 1 when state L is realized. The equilibrium depends on optimists' wealth share. Unlike before, mature optimists' wealth is not zero and contributes to optimists' wealth share. In fact, recall that mature optimists hold symmetric positions as mature pessimists in period 0 (in view of the extreme leverage constraint, $\omega_0^i \leq 1$). Therefore, their wealth in period 1 is given by $\alpha_H(1-\delta)P_t$—their wealth share in period 0 multiplied by the nondepreciated part of capital. Using this observation, I calculate:

$$\begin{aligned}
\tilde{\alpha}_{1,L} = \frac{\sum_{i \in \{o,mo\}} a_{t,L}^i}{\sum_i a_{t,L}^i} &= \frac{\alpha(w_{t,L} + \delta P_{t,L}) + \alpha_H(1-\delta)P_{t,L}}{w_{t,L} + r_{t,L} + P_{t,L}} \\
&= \frac{\frac{p_{1,L}}{p^*}\left[\alpha(n + \delta p^*) + \alpha_H(1-\delta)p^*\right]}{\frac{p_{1,L}}{p^*}\left[n + 1 + p^*\right]} \\
&= \alpha_L + \alpha_H \frac{(1-\delta)p^*}{n+1+p^*}
\end{aligned}$$

This proves Eq. (42). Intuitively, mature optimists' wealth remains largely intact except for the fact that it is diluted by the inflow of new wealth endowed to the young generation in period 1.

To characterize the equilibrium asset price (in period 1 and state L), note that the equilibrium in subsequent periods is unchanged. Therefore, the normalized asset price still satisfies the following version of Eq. (33):

$$\tilde{p}_{1,L} = \mathcal{P}\left(p_L(H), p_L(L) | \tilde{\alpha}_{1,L}\right).$$

Substituting $p_L(H)$ and $p_L(L)$ from Eq. $(A.19)$, I obtain the following version of Eq. (41):

$$\frac{\tilde{p}_{1,L}}{p^*}\tilde{R}_{1,L}^f = \mathcal{P}\left(R^{f*}(H), \frac{p_L}{p^*}R^{f*}(L) | \tilde{\alpha}_{1,L}\right). \tag{A.23}$$

There are two cases to consider. First suppose the right hand side of $(A.23)$ is greater than $\underline{R}^f$. In this case, the solution features $\tilde{p}_{1,L} = p^*$ and $\tilde{R}_{1,L}^f > \underline{R}^f$: that is, macroprudential policy fully eliminates the demand recession. Next suppose the right hand side is smaller than $\underline{R}^f$. In this case, the solution features $\tilde{R}_{1,L}^f < \underline{R}^f$ and $\tilde{p}_{1,L} < p^*$. Using the optimal monetary policy $\tilde{R}_{1,L}^f = \frac{\tilde{p}_{1,L}}{p^*}\underline{R}^f$ [see (39)], I obtain

$$\left(\frac{\tilde{p}_{1,L}}{p^*}\right)^2 \underline{R}^f = \mathcal{P}\left(R^{f*}(H), \frac{p_L}{p^*}R^{f*}(L) | \tilde{\alpha}_{1,L}\right).$$

This uniquely pins down $\frac{\tilde{p}_{1,L}}{p^*} \in (0,1)$. Moreover, the earlier price, $\frac{p_L}{p^*}$, solves the same equation after substituting α_L for $\tilde{\alpha}_{1,L}$ [see (41)]. Since the function, $\mathcal{P}(\cdot|\alpha)$, is increasing in α and $\tilde{\alpha}_{1,L} > \alpha_L$, this implies $\frac{\tilde{p}_{1,L}}{p^*} > \frac{p_L}{p^*}$. Hence, in either case, macroprudential policy increases the asset price and mitigates the demand recession.

A.7.2. New Keynesian microfoundations for nominal rigidities

Finally, I present the microfoundations that ensure output is determined by demand. I also specify the assumptions (on the distribution of output across factors and firms) that ensure the factor returns satisfy (36).

As before, there is one unit of capital and labor supplied inelastically up to one unit. To introduce nominal rigidities, suppose there is a continuum of measure one of production firms, denoted by ν. Each firm chooses its demand for labor, $l_t(\nu)$, and capital, $k_t(\nu)$; and its utilization rate for each factor, $\eta_t^l(\nu), \eta_t^k(\nu) \in [0,1]$. For either factor, the utilization rate can be increased for free up to one and it cannot be increased further. The firm's output is given by,

$$Y_t(\nu) = \eta_t^l(\nu)\, l_t(\nu)\, e_t n + \eta_t^k(\nu)\, k_t(\nu)\, e_t. \tag{A.24}$$

The production firms are monopolistically competitive and sell their output to a competitive final good sector. This sector produces the consumption good according to the CES technology,

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}, \tag{A.25}$$

for some $\varepsilon > 1$. This implies a production firm's demand is given by,

$$Y_t(\nu) \leq q_t(\nu)^{-\varepsilon} Y_t, \text{ where } q_t(\nu) = Q_t(\nu)/Q_t. \tag{A.26}$$

Here, $q_t(\nu)$ denotes the firm's relative price, which depends on its nominal price, $Q_t(\nu)$, as well as the ideal nominal price index, $Q_t = \left(\int Q_t(\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}$. I write the demand constraint as an inequality because a production firm can in principle refuse to meet the demand for its goods.

The key friction is that firms have a preset nominal price that is the same across firms, $Q_t(\nu) = Q$. Thus, the relative price of a firm is fixed and equal to one, $q_t(\nu) = 1$. I also assume the aggregate demand (consumption) is weakly below the potential output, $C_t \leq Y_t^* = e_t(n+1)$ (which is the case for the equilibrium in the main text).

With these assumptions, it is easy to check there is an equilibrium in which the final good firm's output is equal to aggregate demand, $Y_t = C_t$, and individual production firms find it optimal to meet this demand, $Y_t(\nu) = Y_t$ [see $(A.26)$]. Hence, these assumptions are sufficient to ensure output is determined by aggregate demand.

However, the setup has an unappealing feature: since labor and capital are supplied inelastically (up to one unit), the equilibrium wage and the rental rate is zero and all output accrues to production firms as pure profits. To obtain a less extreme distribution of output, I assume the government implements appropriate subsidies that ensure there are no pure profits and output accrues to factors according to their relative productivity.[16]

Formally, for every unit of labor and capital that each production firm hires, the government subsidizes the firm by $n\tau_t$ and τ_t units, respectively. These factor subsidies are financed by taxes that extract firms' profits. Specifically, the government imposes a lump-sum tax on each production firm determined by the

[16] Alternatively, I could abstract from labor, $n = 0$, and interpret the risky asset as a claim to both the rental rate of capital and pure profits. In this case, the results in the main text hold without any assumptions on subsidies.

final good output, Y_t. Finally, the government chooses the level of the linear subsidy, τ_t, to break even,

$$\tau_t \left(n \int_\nu l_t(\nu)\, d\nu + \int_\nu k_t(\nu)\, d\nu \right) = Y_t. \tag{A.27}$$

With these assumptions, a production firm chooses $l_t(\nu), k_t(\nu)$ and $\eta_t^l(\nu), \eta_t^k(\nu) \in [0,1]$ to solve,

$$\begin{aligned} &\max Y_t(\nu) - (w_t - n\tau_t)\, l_t(\nu) - (R_t - \tau_t)\, k_t(\nu) - Y_t, \\ &\text{s.t.} Y_t(\nu) = \eta_t^l(\nu)\, l_t(\nu)\, e_t n + \eta_t^k(\nu)\, k_t(\nu)\, e_t \leq Y_t. \end{aligned} \tag{A.28}$$

Here, the second line captures the supply and demand constraints in $(A.24)$ and $(A.26)$ (with $q_t(\nu) = 1$). There are also standard labor and capital market clearing conditions.

I conjecture a symmetric equilibrium (across production firms) in which the factor prices are determined by the subsidies,

$$w_t = n\tau_t \text{ and } r_t = \tau_t. \tag{A.29}$$

At these factor prices, firms are indifferent to hire labor or capital. In equilibrium, they hire all of the labor and capital, $l_t(\nu) = k_t(\nu) = 1$. Substituting these features, the firm's problem becomes

$$\begin{aligned} &\max_{\eta_t^l(\nu), \eta_t^k(\nu) \leq 1} Y_t(\nu) - Y_t, \\ &\text{s.t.} Y_t(\nu) = \eta_t^l(\nu)\, e_t n + \eta_t^k(\nu)\, e_t \leq Y_t. \end{aligned}$$

Since aggregate demand satisfies, $Y_t = C_t \leq Y_t^* = e_t(n+1)$, the firm can adjust utilization rates $\eta_t^l(\nu), \eta_t^k(\nu)$ (costlessly) to meet the demand. Therefore, the firm's demand constraint binds, $Y_t(\nu) = Y_t$. Note also that the firm makes zero profits after paying the lump-sum taxes.

Finally, substituting $l_t(\nu) = k_t(\nu) = 1$ into Eq. $(A.27)$, the equilibrium level of the linear subsidy satisfies $\tau_t = \frac{Y_t}{n+1}$. Substituting this into $(A.29)$, and using $Y_t^* = e_t(n+1)$, I obtain Eq. (36) from the main text,

$$w_t = y_t e_t n \text{ and } r_t = y_t e_t, \text{ where } y_t = \frac{Y_t}{Y_t^*}.$$

In sum, the equilibrium level of output is determined by demand ($Y_t = C_t \leq Y_t^*$). With appropriate subsidies and taxes, this output is distributed to factors in proportion to their relative productivity (and the production firms make zero pure profits).