NBER WORKING PAPER SERIES

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Working Paper 28424
http://www.nber.org/papers/w28424

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2021, Revised December 2021

We thank Lorenzo Caliendo, Arnaud Costinot, Dave Donaldson, Emmanuel Farhi, Cecilia Fieler, Gregor Jarosch, Pete Klenow, Sam Kortum, Andres Rodriguez-Clare, Bradley Setzler, Isaac Sorkin, Felix Tintelnot, Jose Vasquez, Ivan Werning and Daniel Xu for invaluable comments on our paper. We are also grateful for feedback from seminar participants at the Georgetown, Harvard University, MIT, University of Arizona, UBC, University of Chicago, Universidad de Chile, Universidad Mayor, UT Austin, University of Toronto; and from conference participants at the Yale University Cowles Trade Day conference, the Central Bank of Chile Heterogeneity in Macroeconomics workshop, the West Coast Trade Conference, the Econometric Society World Congress, the European Economics Association Congress, the Stanford Institute for Theoretical Economics, and the ASSA 2021 Annual Meeting. We thank the Munk School of Global Affairs and Public Policy for financial support. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. F0,F12,F16,J0,J31,J42

ABSTRACT

We develop a quantitative model in which heterogeneous firms hire heterogeneous workers in an imperfectly competitive labor market and source intermediates from suppliers in a production network. We use the model to investigate how the production network shapes three key labor market outcomes: the passthrough of firm-level productivity, demand, and cost shocks into worker earnings; the distribution of firm effects on worker earnings; and firm heterogeneity in labor shares of value-added. We establish identification of model parameters and estimate them using linked employer-employee and firm-to-firm transactions data from Chile. Reduced-form evidence based on export demand and import cost shocks support the predictions of our model regarding the passthrough of these shocks to earnings. Counterfactual simulations show that heterogeneity in network linkages explains 21% of earnings variance, while labor value-added shares are less dispersed and less negatively correlated with firm size under the observed production network than under a random network.

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1 Introduction

There is growing evidence that firms matter for worker earnings. In a survey of the empirical literature concerned with estimation of worker and firm fixed effects on earnings, Card et al. (2018) summarize that firm effects explain around 20% of the variation in worker earnings.[1] A standard explanation for this proposed by the literature is that employers are heterogeneous in some innate characteristics – productivity and amenities, for example – with this heterogeneity then passing through into differences in earnings of otherwise similar workers. At the same time, a separate, emerging literature has documented that a substantial share of firm heterogeneity is explained by differences in the *connections* that firms form with each other in a production network. For instance, using firm-to-firm transactions data for Belgium, Bernard et al. (2019) show that variation in both the number and characteristics of a firm's customers and suppliers explains more than half of the variation in firm sales. Motivated by these two facts – that network heterogeneity matters for firm heterogeneity and firm heterogeneity matters for earnings heterogeneity – we theoretically and empirically investigate the importance of the production network structure for earnings inequality.

In our theoretical framework, firms produce output using workers who are hired in imperfectly competitive labor markets as in Card et al. (2018) and Lamadon et al. (2019) and source materials (intermediate inputs) from their suppliers in a production network as in Huneeus (2019) and Lim (2019). This output is then sold to both final consumers and other customers in the firm's production network. Workers are heterogeneous in ability and firms have wage-setting power arising from workers' idiosyncratic preferences for employment at different firms.[2] The model also allows for employer amenities as in Rosen (1986) that vary at the worker-firm level and complementarities in production between worker ability and firm technology, which drive the heterogeneous sorting of workers to firms. Firms are heterogeneous in the sets of customers and suppliers they are connected to in the network, as well as in total factor productivities (TFPs), labor productivities, and buyer-seller productivities (relationship capabilities). We use this theoretical framework to shed light on three key questions.

First, how do shocks to firms pass through into changes in worker earnings? While the literature typically considers the passthrough of shocks that affect a worker's employer directly, we derive novel comparative static results characterizing how shocks to an employer's customers and suppliers affect worker earnings. We show that the extent of such indirect passthrough depends on three key elasticities – the labor supply elasticity, the labor-materials substitution

[1] Bonhomme et al. (2020) show that correcting for the well-known limited mobility bias in these estimates lowers the firm effect share of earnings variance, but increases the importance of the covariance between worker and firm effects (explaining 15% of earnings variance among US workers).

[2] Our model emphasizes employer differentiation as a source of labor market power and has been studied extensively in the literature. See Manning (2003), Sorkin (2018), Card et al. (2018), and Chan et al. (2019), just to name a few examples.

elasticity, and the price elasticity of demand – and three shares that are observable in our data – the share of a firm's sales accounted for by each of its customers, of a firm's material expenditures accounted for by each of its suppliers, and of a firm's input costs accounted for by materials.

Second, how does the distribution of firm effects on worker earnings depend on the set of firm-to-firm linkages in the production network? To develop intuition, we begin by considering a special case of our model where the elasticity of substitution between labor and materials is equal to one (Cobb-Douglas technology). In this case, the firm's profit maximization problem can be written in terms of a *value-added* production function. We show that the firm wage premium depends on value-added productivity, which in turn depends on TFP as well as a pair of endogenous statistics that summarize the demand and cost of materials that a firm faces in the production network. Hence, a firm may have a large wage premium not because it has higher innate TFP, but rather because it is connected to customers with greater demand or to suppliers that offer lower prices. With a Cobb-Douglas production function, in principle one can use production network data to infer how production network heterogeneity drives differences in value-added productivity across firms, while the approach of Lamadon et al. (2019) remains informative about the effect of productivity on wages, implying that combining the analysis of labor markets and production networks is a purely additive exercise. However, we establish that the value-added representation of the firm's profit maximization problem is valid *only* under the assumption of Cobb-Douglas technology, with the concept of value-added productivity being no longer meaningful in the more general case.[3] Our framework emphasizes a richer production function in which labor and intermediates enter more flexibly and we use this structure to provide a general characterization of how production network linkages matter for the firm effect on earnings that goes beyond the case of Cobb-Douglas technology.

Third, why do larger firms have a smaller labor share of value-added? This is a pattern that has been emphasized in the literature, in particular as being important for understanding aggregate declines in the labor share (see Autor et al. (2020) and Lamadon et al. (2021), for example). Our framework provides a new microfoundation for why this negative relationship exists: larger firms face higher costs of labor due to upward-sloping labor supply curves (i.e. there exists a positive firm-size premium on wages) and hence are more likely to choose lower labor shares of production costs if labor and materials are gross substitutes. Holding constant the ratio of cost to value-added, firms with lower labor cost shares must then also have lower labor shares of value-added. In comparison, models that assume value-added production functions implicitly abstract from firm heterogeneity in labor cost shares and hence fail to account for this as a source of heterogeneity in labor shares.

To take our model to the data, we begin by formally establishing identification of the three elasticities that are relevant for the passthrough of shocks across the production network, as high-

[3] This is a point that is similar to one made by Gandhi et al. (2017), who show that value-added cannot generally be used to identify productivity variables in a gross output production function.

lighted above. First, we identify the labor supply elasticity using the passthrough of changes in firm wage bills into changes in worker earnings, where wage bill changes are instrumented with its own lags. This extends existing approaches in the literature (as in Guiso et al. (2005) and Lamadon et al. (2019), for example) to allow for firm heterogeneity in material cost shares. Second, we develop a novel approach for identifying the labor-materials substitution elasticity. While it is well-known that this elasticity can be identified from the relationship between firms' relative expenditures on labor versus materials and the relative prices of these inputs, the literature offers little theoretical guidance as to how input prices should be aggregated when a firm pays heterogeneous wages to its workers and sources inputs from heterogeneous suppliers, as in both our model and data. Here, we show that one should aggregate input costs using two price indices: a labor price index, which corresponds exactly to the firm effect identified from the decomposition of earnings into worker and firm effects as in Bonhomme et al. (2019); and a materials price index, which is identified from a decomposition of firm-to-firm transaction values into buyer and seller effects as in Bernard et al. (2019), where seller effects reflect the marginal costs and hence output prices for every supplier of a firm. Given these price indices, which we construct from our data, identification follows by applying the instrumental variables strategy of Doraszelski and Jaumandreu (2018), where the instruments correspond to, among other things, lagged input price indices for labor and materials. Notably, this identification strategy requires linked employer-employee and firm-to-firm transaction records. Finally, we identify the demand price elasticity from the aggregate ratio of sales to profits, since this parameter governs markups in our model.

Turning to estimation, we rely on a panel dataset that combines administrative matched employer-employee records with firm-to-firm transactions data from the Chilean Internal Revenue Services. These data allow us to observe both the earnings for every employee at each firm in our data and the buyers and sellers of every firm. The highlights of our estimation results are as follows. First, we estimate a labor supply elasticity of 5.5, which is consistent with other estimates in the literature (for example, see Staiger et al. (2010), Azar et al. (2019), Kline et al. (2019), Lamadon et al. (2019), Dube et al. (2019), and Kroft et al. (2019)). Second, we estimate a price elasticity of demand equal to 4.2, which is within the range of values estimated in the literature (see Broda and Weinstein (2006), for example). Third, we estimate an elasticity of substitution between labor and materials of 1.5, indicating gross substitutability of these two inputs. We also statistically reject the hypothesis of Cobb-Douglas technology, which again cautions against the use of value-added production functions. Hence, while the Cobb-Douglas case is useful as a heuristic for developing the intuition behind the model, it remains a simplification that is unsupported by our data.[4]

[4] More fundamentally, in any particular setting, one cannot know ex ante the empirical magnitude of the labor-materials substitution elasticity. A researcher would need to estimate it and our framework shows precisely how to do this in the presence of heterogeneous labor and intermediate inputs.

Finally, we use the estimated model to provide quantitative answers to the three key questions posed above. We first provide reduced-form evidence to validate the predictions of our estimated model regarding the passthrough of firm-level shocks into changes in worker earnings. Here, we utilize transactions-level customs data for Chile to construct export demand shocks and import cost shocks using a Bartik shift-share design. We find statistically significant evidence of the passthrough of these shocks into changes in firm wage bills, average wages, and sales, both for shocks that affect a firm directly as well as indirectly through the firm's customers and suppliers. As predicted by our model, increases in demand have positive effects on earnings while increases in input costs have negative effects on earnings. These findings also contribute to the empirical literature studying the relationship between firm shocks and worker earnings (for example, Guiso et al. (2005) and Chan et al. (2021)) by extending the analysis to account for passthrough via the network.

Second, we quantify the share of earnings variance that is attributable to heterogeneity in each set of primitives in our model: worker abilities, firm productivities, firm amenities, and production network linkages. This extends the usual earnings variance decomposition into worker and firm effects by accounting for the structural dependence of firm effects on underlying primitives. Our novel finding is that network heterogeneity accounts for 21% of earnings variance, with upstream heterogeneity in matching with suppliers accounting for 12% and downstream heterogeneity in matching with customers accounting for 9%. In contrast, own-firm productivities and amenities jointly account for 12% of earnings variance. Hence, we find that heterogeneity in the production network is in fact a key driver of earnings inequality.[5]

Third, we use the model to quantify how production network heterogeneity matters for differences in labor shares of value-added and labor shares of rent across firms. We first verify that larger firms tend to have lower labor shares in our data, with an increase in log firm sales of one standard deviation associated with a three percentage point decline in the labor value-added share. Our model replicates this fact through the mechanism described above. We then simulate a counterfactual equilibrium in which production network linkages are randomized across firms. We find that this *increases* the employment-weighted standard deviation of the labor rent share across firms by 16% (from 4.6 to 5.4 percentage points) and strengthens the observed negative correlation between log firm sales and the labor rent share (from -0.28 to -0.33). This occurs because smaller firms tend to have production network linkages mainly with low-cost suppliers, whereas larger firms tend to be connected to both low- and high-cost suppliers. Hence, differences in network linkages confer a material cost advantage on smaller firms. Without this advantage under a random production network, smaller firms that already have lower material cost shares and high labor rent shares compared with larger firms reduce their dependence on materials

[5] Bernard et al. (2019) establish the relevance of network productivities for firm size. We formally show that differences in material cost shares due to heterogeneity in the production network still contribute to differences in worker earnings conditional on sales. Thus, firm size is not a sufficient statistic for the firm effect on earnings.

even further, leading to a widening of the gap in labor rent and value-added shares for small versus large firms. We thus conclude that although production network heterogeneity contributes positively to earnings inequality, it in fact dampens differences in labor shares of value-added across firms.

To our knowledge, there are only three other papers that study linked employer-employee and firm-to-firm transactions data. Adao et al. (2020) use data from Ecuador to measure the effects of international trade on individual-level factor prices, while Demir et al. (2018) study the effects of trade-induced product quality upgrading on wages in Turkey. Both of these analyses assume a market price for skill and focus on the effects of trade shocks. In contrast, we allow for imperfect competition in labor markets and use our data to speak to the role of the production network itself in shaping earnings inequality. Finally, Alfaro-Ureña et al. (2020) adopt an event study research design to examine the effects on worker earnings in Costa Rica when a local firm starts interacting with multinationals. In contrast, we use our data to address both worker-level earnings and aggregate outcomes such as earnings inequality, which requires a general equilibrium model.

The rest of our paper is organized as follows. Section 2 describes our structural model of labor markets and production networks. Section 3 then develops several theoretical results to characterize how the production network matters in relation to the three key questions posed above. In section 4, we discuss identification of the model parameters, while section 5 provides a description of our data and estimation results. In section 6, we then present our main empirical findings on how the production network matters for earnings inequality, the passthrough of firm-level shocks into changes in worker earnings, and the division of rents between firms and workers. Finally, section 7 concludes.

2 Model

The economy is populated by a set of workers Ω^L and a set of firms Ω^F. Workers are heterogeneous in a characteristic that we refer to as *ability*, denoted by a, with an exogenous measure of each ability type denoted by $L(a)$ and the set of abilities denoted by $A \subset \mathbb{R}^d_+$. The theoretical results established in this section do not require restrictions on the dimension d of the worker ability space.[6] Firms are also heterogeneous in a variety of characteristics that we specify below. While the model allows for dynamics, all meaningful economic decisions can be analyzed statically. Nonetheless, estimation of the model will involve panel data and hence in anticipation of this, we index (discrete) time by t to make explicit the variables that are allowed to vary temporally.

[6]When taking the model to data, we will assume $d = 2$ with worker ability comprised of a time-invariant and time-varying component.

2.1 Labor market

Firms and workers interact in the labor market as follows. Each firm i posts a wage $w_{it}(a)$ that is conditional on worker ability a. We take the price of final consumption that workers face as the numeraire, hence wages should be interpreted in real terms. Each worker observes all wage offers for her ability type and chooses an employer to maximize utility, where the utility of a worker with ability a employed at firm i is given by:

$$u_{it}(a) = \log w_{it}(a) + \log \tau_t + \log g_i(a) + \beta^{-1}\varepsilon_{it} \tag{2.1}$$

In addition to receiving labor income, workers are residual claimants to firm profits, which are rebated through a lump-sum income transfer τ_t that is independent of employer. Workers also derive utility from amenities $g_i(a)$ offered by firm i and have idiosyncratic preferences ε_{it} for at employment firm i, with β an inverse measure of the preference dispersion across firms.

We highlight several important features of this utility specification. First, lump-sum transfers τ_t are paid to workers in proportion to their income. This is necessary to ensure that transfers do not affect the sorting of workers across firms. Second, firms have complete information about the ability of every worker but cannot observe idiosyncratic preferences ϵ_{it}. Hence, wages are conditioned only on ability, which will imply the existence of inframarginal workers at every firm who enjoy positive rents from their employment. Third, differences in amenities $g_i(\cdot)$ allow for *vertical* differentiation across potential employers, while differences in idiosyncratic preferences ϵ_{it} introduce *horizontal* differentiation. The former rationalizes heterogeneity in compensating differentials across firms for workers of a given ability, while the latter is the source of labor market power for firms. Fourth, we follow the literature on discrete choice and assume that idiosyncratic preferences are characterized as follows.

ASSUMPTION **2.1.** *The distribution of idiosyncratic preferences across workers, $\varepsilon_t \equiv \{\varepsilon_{it}\}_{i\in\Omega^F}$, is a multivariate Gumbel distribution with cumulative distribution function:*

$$F_\epsilon(\varepsilon_t) = exp\left[-\left(\sum_{i\in\Omega^F} e^{-\frac{\varepsilon_{it}}{\rho}}\right)^\rho\right] \tag{2.2}$$

where $\rho \in (0, 1]$.

The parameter ρ controls the correlation of idiosyncratic preferences across firms: as ρ approaches zero, workers view all firms as perfect substitutes, whereas as ρ approaches one, idiosyncratic preferences across firms become independent random variables. Note also that Assumption 2.1 imposes structure on the cross-sectional distribution of ϵ_t but does not otherwise restrict its time-series properties.

Under Assumption 2.1, the probability that a worker with ability a chooses to work at firm

i is given by:

$$\mathbb{P}_{it}(a) = \left[\frac{g_i(a)\, w_{it}(a)}{I_t(a)}\right]^{\gamma} \tag{2.3}$$

where $\gamma \equiv \beta/\rho$. Hence, labor supply is more elastic when preference shocks are less dispersed or more correlated. In what follows, only γ will be of interest and not β or ρ separately. The term $I_t(a)$ is an aggregate of wage and amenity values offered by all firms in the labor market for workers of ability a, which we henceforth refer to as the *labor market index* for these workers:

$$I_t(a) \equiv \left[\sum_{i\in\Omega^F} \left[g_i(a)\, w_{it}(a)\right]^{\gamma}\right]^{\frac{1}{\gamma}} \tag{2.4}$$

Appealing to a law of large numbers, the total supply of workers of ability a for firm i can then be written as:

$$L_{it}(a) = \kappa_{it}(a)\, w_{it}(a)^{\gamma} \tag{2.5}$$

where $\kappa_{it}(a)$ is a firm-specific labor supply shifter:

$$\kappa_{it}(a) \equiv L(a)\left[\frac{g_i(a)}{I_t(a)}\right]^{\gamma} \tag{2.6}$$

We further assume that the cardinality of the set of firms Ω^F is large enough such that each firm views itself as atomistic in the labor market. In choosing wages for workers of any ability $a \in A$, each firm thus views the labor market index $I_t(a)$ as invariant to its own choices. Hence, equation (2.5) implies that every firm behaves as though it faces an upward-sloping labor supply curve with a constant elasticity γ that is common to all firms and worker ability types.[7]

2.2 Final demand

Workers use their income to finance consumption, with consumption utility derived from a constant elasticity of substitution (CES) aggregate of products produced by all firms in the economy. For a worker of ability a employed at firm i, this is given by:

$$v_{it}(a) = \left[\sum_{j\in\Omega^F} c_{ijt}(a)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2.7}$$

[7] Note that instead of arising from employer differentiation, labor market power could also stem from concentration (Berger et al. (2019), Jarosch et al. (2019)) or search frictions (Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), Taber and Vejlin (2018)). Like ours, most of these models imply that wages are a markdown below the marginal revenue product of labor (MRPL) at a firm, where the firm effect on earnings is the component of the MRPL that is common to all workers at a firm. Hence, the mechanisms that we highlight below regarding the interaction between the production network and worker earnings are relevant for a broader class of models of the labor market.

where $c_{ijt}(a)$ denotes the worker's consumption of firm j's output and $\sigma > 1$ denotes the elasticity of substitution across products. Since we take the unit price of the final consumption aggregate as the numeraire, consumption utility can also be expressed as $v_{it}(a) = w_{it}(a)\tau_t$, which corresponds to the first two terms in the utility specification (2.1).[8] Aggregate final demand for firm $i's$ output $C_{it} \equiv \sum_{j\in\Omega^F}\sum_{a\in A} c_{jit}(a) L_{jt}(a)$ is then:

$$C_{it} = E_t p_{Fit}^{-\sigma} \tag{2.8}$$

where p_{Fit} is the price of firm i's output for final sales and E_t is aggregate consumer income:

$$E_t = \sum_{i\in\Omega^F}\sum_{a\in A} w_{it}(a) L_{it}(a) + \sum_{i\in\Omega^F} \pi_{it} \tag{2.9}$$

with π_{it} denoting profit earned by firm i. Note that E_t is also equivalent to aggregate value-added in the economy.

2.3 Production technologies

Firms produce output using labor and materials.[9] Combining $L_{it}(a)$ workers of ability a with $M_{it}(a)$ units of materials at firm i produces $f[\phi_{it}(a) L_{it}(a), M_{it}(a)]$ units of output, where $\phi_{it} : A \to \mathbb{R}_+$ maps worker ability into productivity. Total output of firm i is then given as the sum of output produced by workers of all abilities:

$$X_{it} = T_{it} \sum_{a\in A} f[\phi_{it}(a) L_{it}(a), M_{it}(a)] \tag{2.10}$$

where T_{it} denotes total factor productivity (TFP) of firm i. Although the model can accommodate imperfect substitutability between workers of different abilities, the linear aggregation in equation (2.10) is necessary for the model to generate an earnings equation that is consistent with well-known reduced-form models of earnings such as those in Abowd et al. (1999) and Bonhomme et al. (2019). We will rely in particular on the identification strategy in the latter paper when taking our model to data and hence maintain the assumption of linear aggregation.

In what follows, we assume that the production function f is of the CES form:

$$f(\phi L, M) = \left[\lambda (\phi L)^{\frac{\epsilon-1}{\epsilon}} + (1-\lambda) M^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.11}$$

[8] In other words, the indirect consumption utility function is $v_{it}(a) = \frac{w_{it}(a)\tau_t}{P_t}$, where $P_t \equiv \left(\sum_{i\in\Omega^F} p_{Fit}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is the unit price of the final consumption bundle and we normalize $P_t = 1$.

[9] One can also think of certain types of capital inputs as sourced from suppliers in the production network under the label of "materials" if these inputs are chosen statically. Alternatively, it is straightforward to extend the production function to allow for a separate static capital input. See Appendix A for a formal discussion of this extension.

where ϵ denotes the elasticity of substitution between labor and materials, while λ controls the importance of labor in production relative to materials. In the limit as $\lambda \to 1$, output is produced using labor alone and the model simplifies to a version of the model studied in Lamadon et al. (2019). We also allow the labor productivity function ϕ_{it} to vary by firm and assume that this can be decomposed as follows.

Assumption **2.2.** *Productivity of ability a workers at firm i is of the form $\phi_{it}(a) = \phi_i(a)\,\omega_{it}$.*

Hence, any worker-firm complementarities are time-invariant with all time variation in labor productivity accounted for by ω_{it}, which we henceforth refer to simply as labor productivity of firm i. This assumption will be important for identification of the productivity terms.

While firms hire workers in the labor market as described in section 2.1, materials are sourced through firm-to-firm trade in the production network. We denote the set of firm i's customers and suppliers by $\Omega^C_{it} \subset \Omega^F$ and $\Omega^S_{it} \subset \Omega^F$ respectively. Where convenient for exposition, we will also describe the production network in terms of a matching function m_{ijt}, which is equal to 1 if $j \in \Omega^S_{it}$ and 0 otherwise. Materials for firm i are then aggregated by combining inputs from all of its suppliers using a CES technology:

$$M_{it} = \left[\sum_{j \in \Omega^S_{it}} \psi_{ijt}^{\frac{1}{\sigma}} \left(x_{ijt}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2.12}$$

where x_{ijt} denotes the quantity of inputs purchased by i from j and ψ_{ijt} is a relationship-specific productivity shifter. As is standard in the literature, we assume the same elasticity of substitution across products in equation (2.12) as in the consumption utility function (2.7). This simplifies the firm's profit maximization problem as it ensures that both final and intermediate demand have the same price elasticity. The total allocation of materials to a firm's workers must then be equal to total materials sourced by the firm:

$$\sum_{a \in A} M_{it}(a) = M_{it} \tag{2.13}$$

Note that the production network is not restricted to be bipartite: firms can simultaneously be buyers and sellers. However, for tractability, we treat the set of active buyer-seller relationships in the economy as an exogenous primitive of the model and do not model network formation. Since marginal costs of production are scale-dependent with imperfectly competitive labor markets, the incentive for a firm to sell to one customer depends on its existing set of customers. This violates the key assumption of constant marginal costs that is needed for tractability in existing models of endogenous production network formation (for example, Huneeus (2019) and Lim (2019)). Note, however, that treating the network as an exogenous primitive does not imply any restrictions on how the distribution of buyer-seller links is correlated with other firm primitives or how the network changes over time. For example, more productive firms may have more links and add links at a faster rate than less productive firms. As we discuss below, our identification of model parameters does not require any such restrictions.

2.4 Output market structure and profit maximization

We assume a market structure of monopolistic competition in output markets: each firm in the economy produces a unique product and sets prices for each of its customers taking the prices set by all other firms as given. Demand by firm i for inputs from firm j then takes the standard form implied by the CES production technology (2.12):

$$x_{ijt} = \Delta_{it} \psi_{ijt} p_{ijt}^{-\sigma} \tag{2.14}$$

where p_{ijt} is the price charged by seller j to buyer i. The term Δ_{it} is a firm-specific intermediate demand shifter that we refer to as *network demand*:

$$\Delta_{it} = E_{it}^{M} \left(Z_{it}\right)^{\sigma-1} \tag{2.15}$$

where $E_{it}^{M} \equiv Z_{it} M_{it}$ is total material cost and Z_{it} is the unit cost of materials for firm i.

Just as firms take the labor market indices $I_t(\cdot)$ as given in choosing wages, we assume that firms take the network demands of each of its customers as given in choosing prices and hence perceive a constant price elasticity of demand equal to $-\sigma$. Note that a firm's relationships with each of its customers are inherently interlinked: a reduction in the price charged to one customer increases demand and hence raises both output and marginal cost, which in turn affects the choice of prices charged to other customers. However, even though we allow firms to charge different prices to different customers, the following result establishes that it is never optimal for them to do so.[10]

Claim **1.** *The profit-maximizing price charged by a firm i to each of its customers j (including final consumers) does not vary across customers:*

$$p_{jit} = p_{it}, \, \forall j \in \Omega_{it}^{C} \cup \{F\}$$

Intuitively, each firm maximizes profits by choosing prices such that marginal revenue from each customer is equal to marginal cost. Since demand features a constant and common price elasticity of $-\sigma$, marginal revenue is proportional to price. Furthermore, even though marginal cost is increasing, it depends only on total output of the firm and hence is common across customers. As a result, each firm optimally chooses to charge a common price to each of its customers in equilibrium.

[10]Proofs of all claims and propositions are relegated to Section C of the appendix.

With this result, we can express total demand for firm i's output as:

$$X_{it} = D_{it} p_{it}^{-\sigma} \tag{2.16}$$

where D_{it} is a demand shifter for the firm given by the sum of final demand (common to all firms) and the network demands of the firm's customers:

$$D_{it} = E_t + \sum_{j \in \Omega_{it}^C} \Delta_{jt} \psi_{jit} \tag{2.17}$$

Similarly, the unit cost of materials can be expressed as:

$$Z_{it} = \left[\sum_{j \in \Omega_{it}^S} \psi_{ijt} \Phi_{jt} \right]^{\frac{1}{1-\sigma}} \tag{2.18}$$

where Φ_{it} is the *network efficiency* of firm i, an inverse measure of the firm's price:

$$\Phi_{it} \equiv p_{it}^{1-\sigma} \tag{2.19}$$

Finally, we can now write the profit-maximization problem for firm i concisely as a choice over its production inputs:

$$\pi_{it} = \max_{\{w_{it}(a), M_{it}(a)\}_{a \in A}} \left\{ D_{it}^{\frac{1}{\sigma}} X_{it}^{\frac{\sigma-1}{\sigma}} - \sum_{a \in A} w_{it}(a) L_{it}(a) - Z_{it} \sum_{a \in A} M_{it}(a) \right\} \tag{2.20}$$

subject to the labor supply curves (2.5) and production technology (2.10). Note that the relevance of the network for firm i's production decisions is summarized by the sufficient statistics $\{D_{it}, Z_{it}\}$, which we henceforth refer to as the *network characteristics* of the firm. The demand shifter D_{it} summarizes firm i's downstream connections with customers, while its unit material cost Z_{it} summarizes its upstream connections with suppliers.

2.5 Wage determination

Wages are determined by the solution to the firm's profit maximization problem. Since the price of materials is invariant with respect to worker ability, the marginal revenue product of materials must first of all be equalized across worker ability types in equilibrium, as implied by the first-order condition for (2.20) with respect to materials:

$$Z_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} T_{it} f_M(1, \nu_{it}) \tag{2.21}$$

where f_M denotes the derivative of f with respect to its second argument and ν_{it} is an endogenous variable equal to materials per efficiency unit of labor at firm i:

$$\nu_{it} = \frac{M_{it}(a)}{\phi_i(a)\,\omega_{it} L_{it}(a)} \tag{2.22}$$

The first-order condition for (2.20) with respect to $w_{it}(a)$ then allows us to express equilibrium wages as:

$$w_{it}(a) = \eta \phi_i(a)\, W_{it} \tag{2.23}$$

Equation (2.23) states the familiar result that wages are a constant markdown $\eta \equiv \frac{\gamma}{1+\gamma} \in (0,1)$ over the marginal revenue product of labor (MRPL) of the respective worker types, $\phi_i(a)\, W_{it}$. The component of wages that is common to all workers employed at firm i, W_{it}, is given by:

$$W_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} \omega_{it} T_{it} f_L(1, \nu_{it}) \tag{2.24}$$

where f_L denotes the derivative of f with respect to its first argument and we define the output markup $\mu \equiv \frac{\sigma}{\sigma-1}$ for brevity. We henceforth refer to W_{it} as the *firm effect* on earnings. Note that in the limit as labor supply becomes infinitely elastic ($\gamma \to \infty$), the markdown η approaches unity as in the benchmark with perfectly competitive labor markets.

Equilibrium output for firm i can now be characterized as follows:

$$X_{it} = T_{it} f(1, \nu_{it})\, \bar{L}_{it} \tag{2.25}$$

$$\bar{L}_{it} = (\eta W_{it})^{\gamma}\, \omega_{it} \tilde{\phi}_{it} \tag{2.26}$$

$$\tilde{\phi}_{it} \equiv \sum_{a \in A} \kappa_{it}(a)\, \phi_i(a)^{1+\gamma} \tag{2.27}$$

where $\bar{L}_{it} \equiv \sum_{a \in A} \phi_i(a)\, \omega_{it} L_{it}(a)$ is the total efficiency units of labor hired by the firm and we define the $\tilde{\phi}_{it}$ as the *sorting composite* for firm i, since this varies across firms only due to primitives that affect differential sorting of worker types across firms ($g_i(\cdot)$ and $\phi_i(\cdot)$).

Given the labor supply shifters $\kappa_{it}(\cdot)$ (which are determined by equilibrium in the labor market) and the network characteristics $\{D_{it}, Z_{it}\}$ (which are determined by equilibrium in output markets), equations (2.21), (2.24), and (2.25) define a system of three equations in the three firm-level variables $\{W_{it}, \nu_{it}, X_{it}\}$. The solution to this system determines the firm-level wage W_{it} and hence the wage of every worker employed at firm i. Firm i's total expenditure on labor and materials are then given respectively by:

$$E_{it}^{L} = \eta W_{it} \bar{L}_{it} / \omega_{it} \tag{2.28}$$

$$E_{it}^{M} = Z_{it} \nu_{it} \bar{L}_{it} \tag{2.29}$$

3 Theoretical Results

Before taking the model to data, we develop several theoretical results to characterize how the production network matters in relation to the three key questions posed in the introduction: the passthrough of firm-level shocks into worker earnings (section 3.1), the distribution of firm effects on earnings (section 3.2), and differences across firms in labor shares of value-added (section 3.3). This analysis will provide context for interpreting the empirical results that follow.

3.1 The production network and the passthrough of firm shocks into earnings

We first develop comparative static results to highlight the mechanisms through which the production network matters for the passthrough of firm-level shocks into changes in worker earnings. We first examine how wages depend on firm productivities and network characteristics. To do so, we study the system of equations defined by (2.21), (2.24) and (2.25) highlighted in section 2.5. Recall that this system determines how a firm's TFP, labor productivity, demand, and material cost affect the firm-level wage. Since $w_{it}(a)$ is endogenous only through W_{it}, this system also determines how firm productivities and network characteristics shape wages. The following proposition summarizes the key results.

Proposition **1.** *The elasticities of the firm effect W_{it} with respect to $\{T_{it}, \omega_{it}, D_{it}, Z_{it}\}$ are:*

$$\begin{aligned} \frac{\partial \log W_{it}}{\partial \log T_{it}} &= (\sigma - 1)\Gamma_{it}, & \frac{\partial \log W_{it}}{\partial \log \omega_{it}} &= (\sigma - 1)\Gamma_{it}\left(1 - s_{it}^{M}\right) + (\epsilon - 1)\Gamma_{it} s_{it}^{M} \\ \frac{\partial \log W_{it}}{\partial \log D_{it}} &= \Gamma_{it}, & \frac{\partial \log W_{it}}{\partial \log Z_{it}} &= -(\sigma - 1)\Gamma_{it} s_{it}^{M} + (\epsilon - 1)\Gamma_{it} s_{it}^{M} \end{aligned} \tag{3.1}$$

where $\Gamma_{it} \equiv \frac{1}{\gamma + \sigma\left(1 - s_{it}^{M}\right) + \epsilon s_{it}^{M}}$.

The intuition for these results can be understood as follows. First, note that the firm effect is increasing in demand ($\frac{\partial \log W_{it}}{\partial \log D_{it}} > 0$), which we refer to as the *scale effect.* This occurs because higher demand raises the output price of a firm, which translates into a higher MRPL and hence higher wages given the upward-sloping labor supply curves faced by each firm. The term Γ_{it}, which we refer to as the *scale elasticity* for firm i, summarizes the three conditions that are necessary for the existence of scale effects: (i) the labor market is imperfectly competitive ($\gamma < \infty$), so that marginal costs of labor are increasing; (ii) the output market is imperfectly competitive ($\sigma < \infty$), so that higher demand raises output prices at the level of the firm; and

(iii) labor and materials are imperfect substitutes ($\epsilon < \infty$), so that firms cannot fully escape from increasing marginal costs of labor by purchasing materials at constant marginal cost. Consequently, scale effects diminish as each of the three elasticities $\{\gamma, \sigma, \epsilon\}$ increases (unless $s_{it}^M = 0$ so that firm i does not use materials in production, in which case the scale elasticity is naturally independent of ϵ).

Second, the firm effect is increasing in TFP ($\frac{\partial \log W_{it}}{\partial \log T_{it}} > 0$), which we refer to as the *productivity effect*. Similar to an increase in demand, an improvement in TFP induces the firm to expand its scale and hire more workers at higher wages. However, this mechanism operates through an increase in the marginal product of labor instead of through an increase in the output price of a firm. Hence, the strength of the productivity effect differs from the scale elasticity of a firm by a factor of $\sigma - 1$. As σ increases, scale effects diminish but productivity effects strengthen, since a firm's sales are more sensitive to changes in production costs when products are more differentiated.

Third, like an increase in TFP, an improvement in labor productivity raises the marginal product of labor, which tends to increase wages. However, this effect is more muted compared with the case of TFP because labor productivity only affects a fraction of a firm's inputs. The strength of this effect is hence given by $(\sigma - 1)\,\Gamma_{it}\left(1 - s_{it}^M\right)$, which is equal to the productivity effect of TFP scaled by the labor share of cost $1 - s_{it}^M$. In addition, an improvement in labor productivity generates a *substitution effect*, as firms respond by increasing the ratio of labor to materials used in production. This substitution effect is captured by the term $(\epsilon - 1)\,\Gamma_{it} s_{it}^M$ and may either increase wages (if $\epsilon > 1$ so that labor and materials are substitutes) or decrease wages (if $\epsilon < 1$ so that labor and materials are complements).[11] Hence, the net effect of improvements in labor productivity on wages is ambiguous in theory. However, a sufficient condition for wages to be increasing in labor productivity is $\epsilon > 1$. We find that this condition is satisfied empirically in our estimation results described below and hence improvements in labor productivity increase wages ($\frac{\partial \log W_{it}}{\partial \log \omega_{it}} > 0$).

Finally, an increase in the cost of materials affects wages through the same channels as changes in labor productivity. On one hand, a higher cost of materials is akin to a negative productivity effect as captured by the term $-(\sigma - 1)\,\Gamma_{it} s_{it}^M$, which is equal in magnitude to the TFP productivity effect scaled by the material cost share s_{it}^M. On the other hand, an increase in material cost induces the same substitution effect as an increase in labor productivity, captured by the term $(\epsilon - 1)\,\Gamma_{it} s_{it}^M$. Hence, the net effect of changes in material cost on wages depends on the relative strength of the productivity effect (controlled by σ) and the substitution effect (controlled by ϵ). In our estimation of the model's parameters described below, we find that

[11] For a general production function f, the elasticity of substitution between workers and materials may depend on the inputs chosen and hence may vary by firm. The results in Proposition 1 still apply in this case, replacing ϵ by $\epsilon_{it} \equiv \left[\frac{\log(f_L(1,\nu_{it})/f_M(1,\nu_{it}))}{d \log \nu_{it}}\right]^{-1}$, the elasticity of substitution between labor and materials under the optimal choice of ν_{it}.

$\sigma > \epsilon$ and hence higher material costs induce lower wages ($\frac{\partial \log W_{it}}{\partial \log Z_{it}} < 0$).

Next, we characterize the propagation of shocks across firms in the production network and examine how these shocks affect wages. We highlight here the passthrough of shocks from a firm's immediate suppliers and customers, referring the reader to Appendix C.3 for a more general discussion of the passthrough of shocks from a firm's indirect suppliers and customers. In section 6.1 below, we will use our data to test the model's predictions regarding the passthrough of demand and material cost shocks into changes in worker earnings, both for shocks that affect a firm directly as well as for shocks that affect a firm indirectly through its suppliers and customers.

Proposition **2.** *The first-order effect of shocks to $\nu \in \{T, \omega, D, Z\}$ for an immediate supplier $j \in \Omega^S_{it}$ of a firm i on the firm effect W_{it} is characterized by:*

$$\frac{\partial \log W_{it}}{\partial \log v_{jt}} = \frac{\partial \log p_{jt}}{\partial \log v_{jt}} \times \frac{\partial \log Z_{it}}{\partial \log p_{jt}} \times \frac{\partial \log W_{it}}{\partial \log Z_{it}} \tag{3.2}$$

where $\frac{\partial \log Z_{it}}{\partial \log p_{jt}} = s^{mat}_{ijt} \equiv \frac{R_{ijt}}{E^M_{it}}$, $\frac{\partial \log W_{it}}{\partial \log Z_{it}}$ is given by Proposition 1, and the passthrough of shocks to the supplier's output price is given by:

$$\frac{\partial \log p_{jt}}{\partial \log T_{jt}} = -\left(\gamma + 1 - s^M_{jt} + \epsilon s^M_{jt}\right)\Gamma_{jt} \qquad \frac{\partial \log p_{jt}}{\partial \log \omega_{jt}} = -\left(\gamma + 1\right)\left(1 - s^M_{jt}\right)\Gamma_{jt} \tag{3.3}$$
$$\frac{\partial \log p_{jt}}{\partial \log D_{jt}} = \left(1 - s^M_{jt}\right)\Gamma_{jt} \qquad \frac{\partial \log p_{jt}}{\partial \log Z_{jt}} = \left(\gamma + \epsilon\right) s^M_{jt}\Gamma_{jt}$$

Intuitively, the first-order effect of a change in the output price p_{jt} of supplier j on the buyer's material cost Z_{it} is given by the share of firm i's material expenditures accounted for by j, s^{mat}_{ijt}. The change in Z_{it} then affects the firm-level wage W_{it} as in Proposition 1. The passthrough of shocks into the supplier's output price can then be understood as follows. First, improvements in either TFP or labor productivity reduce production costs and hence lower the output price of the supplier. Second, an increase in demand raises the supplier's output price due to the scale effects associated with upward-sloping labor supply curves. Note that the strength of this effect is increasing in the labor share of cost for the firm, $1 - s^M_{jt}$, since scale effects operate only via the labor market and not through the market for materials. Finally, an increase in the supplier's material cost is partially passed through to its output price, with the degree of passthrough increasing in extent to which the supplier relies on material inputs. In sum, since $\frac{\partial \log W_{it}}{\partial \log Z_{it}} < 0$ under our estimated model parameters described below, an increase in TFP or labor productivity for an immediate supplier of firm i *raises* wages at firm i, whereas an increase in demand or material cost for an immediate supplier *lowers* wages at firm i.

Proposition **3.** *The first-order effect of shocks to $\nu \in \{T, \omega, D, Z\}$ for an immediate customer*

$j \in \Omega_{it}^{C}$ of a firm i on the firm effect W_{it} is characterized by:

$$\frac{\partial \log W_{it}}{\partial \log v_{jt}} = \frac{\partial \log \Delta_{jt}}{\partial \log v_{jt}} \times \frac{\partial \log D_{it}}{\partial \log \Delta_{jt}} \times \frac{\partial \log W_{it}}{\partial \log D_{it}} \tag{3.4}$$

where $\frac{\partial \log D_{it}}{\partial \log \Delta_{jt}} = s_{jit}^{sales} \equiv \frac{R_{ijt}}{R_{jt}}$, $\frac{\partial \log W_{it}}{\partial \log D_{it}}$ is given by Proposition 1 and the passthrough of shocks to the supplier's network demand is given by:

$$\frac{\partial \log \Delta_{jt}}{\partial \log T_{jt}} = (\gamma + \epsilon)(\sigma - 1)\Gamma_{jt} \qquad \frac{\partial \log \Delta_{jt}}{\partial \log \omega_{jt}} = (\gamma + 1)(\sigma - \epsilon)\left(1 - s_{jt}^{M}\right)\Gamma_{jt} \tag{3.5}$$

$$\frac{\partial \log \Delta_{jt}}{\partial \log D_{jt}} = (\gamma + \epsilon)\Gamma_{jt} \qquad \frac{\partial \log \Delta_{jt}}{\partial \log Z_{jt}} = (\gamma + \sigma)(\sigma - \epsilon)\left(1 - s_{jt}^{M}\right)\Gamma_{jt}$$

Similar to the intuition for Proposition 2, the first-order effect of a change in the network demand Δ_{jt} for customer j on the seller's demand D_{it} is given by the share of firm i's sales accounted for by firm j, s_{jit}^{sales}. The change in D_{it} then affects the firm-level wage W_{it} as in Proposition 1. The passthrough of shocks into the supplier's network demand can then be understood as follows. First, an increase in TFP or demand for the customer leads the firm to expand in scale and hence to demand more material inputs from its suppliers. Second, an improvement in labor productivity has similar scale effects on material demand, but also exerts a substitution effect as the customer adjusts the ratio of materials to labor used in production. As discussed above, the net of the scale and substitution effects is determined by the sign of $\sigma - \epsilon$, which we find to be greater than zero in our estimates. Finally, an increase in material cost also exerts both scale and substitution effects, although the scale effect in this case arises from the fact that demand for inputs from an individual supplier is increasing in the overall cost of materials for the buyer. In sum, since $\frac{\partial \log W_{it}}{\partial \log D_{it}} > 0$, an increase in TFP, labor productivity, demand, or material cost for an immediate customer of firm i *raises* wages at firm i.

3.2 The production network and firm effect on earnings

3.2.1 Case with Cobb-Douglas technology

Next, we discuss in greater detail how the production network shapes the firm effect on earnings. We first examine the case of the model with Cobb-Douglas technology ($\epsilon = 1$). This case admits closed-form solutions and hence is useful for providing a more transparent discussion of the key mechanisms. At the same time, we highlight below properties of the model that obtain *only* under Cobb-Douglas technology, which underscores the importance of proper identification of the labor-materials substitution elasticity ϵ.

Under Cobb-Douglas technology, the firm's profit maximization problem (2.20) can be rewritten by first solving out for the optimal choice of material inputs:

$$\max_{\{w_{it}(a)\}_{a\in A}} \left\{ A_{it}\tilde{X}_{it}^{1-\alpha} - \sum_{a\in A} w_{it}\left(a\right) L_{it}\left(a\right) \right\} \tag{3.6}$$

$$\text{s.t. } \tilde{X}_{it} = \sum_{a\in A} \phi_{it}\left(a\right) L_{it}\left(a\right) \tag{3.7}$$

The term $A_{it}\tilde{X}_{it}^{1-\alpha}$ is equal to nominal value-added for firm i, $VA_{it} \equiv p_{it}X_{it} - Z_{it}M_{it}$, where $\alpha \equiv \frac{1}{\sigma\lambda+(1-\lambda)} > 0$ reflects curvature in value-added arising from imperfectly elastic demand ($\sigma < \infty$) and A_{it} is a composite term that can be interpreted as *value-added productivity*:

$$A_{it} \equiv \text{const.} \times T_{it}^{\frac{\sigma-1}{\sigma\lambda+1-\lambda}} \omega_{it}^{\frac{\lambda(\sigma-1)}{\sigma\lambda+1-\lambda}} D_{it}^{\frac{1}{\sigma\lambda+1-\lambda}} Z_{it}^{-\frac{(1-\lambda)(\sigma-1)}{\sigma\lambda+1-\lambda}} \tag{3.8}$$

Equations (3.6) and (3.7) represent the firm's profit maximization problem in terms of a value-added production function, as in Lamadon et al. (2019). Evidently, value-added depends on gross TFP, labor productivity, demand via the production network and final consumers (downstream), and material costs via the production network (upstream). The firm effect W_{it} can then be solved for explicitly as:

$$W_{it} = \text{const.} \times A_{it}^{\frac{\sigma\lambda+1-\lambda}{\gamma+\sigma\lambda+1-\lambda}} \tilde{\phi}_{it}^{-\frac{1}{\gamma+\sigma\lambda+1-\lambda}} \tag{3.9}$$

This special case of the model allows for several important takeaways.

First, from equation (3.8), demand and supply shocks in the network that operate through $\{D_{it}, Z_{it}\}$ act as shifters of value-added productivity A_{it}. In this sense, the introduction of production networks provides a microfoundation for value-added productivity that is relevant for earnings through equation (3.9). Second, it is immediately obvious from equations (3.8) and (3.9) that without further information, identification of A_{it} alone does not allow one to separately identify the components of A_{it} (and hence of the firm effect W_{it}) that stem from TFP, labor productivity, and network characteristics. Hence, the value-added approach naturally leaves open the question of how heterogeneity in production network linkages – both downstream and upstream – shapes earnings inequality, which we examine in section 6.2.

Finally, note that the above discussion only applies when $\epsilon = 1$. When this condition does not hold, the firm's profit maximization problem generally does not admit the representation in equations (3.6) and (3.7), and hence the concept of value-added productivity is no longer meaningful.[12] Furthermore, as highlighted in section 3.2.2, heterogeneity in material cost Z_{it} is relevant for earnings inequality even conditional on size and sorting whenever $\epsilon \neq 1$. In our empirical estimates below, we find that ϵ is statistically different from 1, while labor shares of cost exhibit non-trivial variation across firms in contradiction with the Cobb-Douglas implication of

[12]The value-added representation is valid for any ϵ in the limit as $\sigma \to \infty$. This case is not empirically relevant since it corresponds to perfect competition in output markets.

a common labor share for all firms.[13] Hence, while the Cobb-Douglas case is useful as a heuristic for developing the intuition behind the model, it remains a simplification that is unsupported by our data.

3.2.2 General case

In the general case of the model where production technologies are not necessarily Cobb-Douglas, we can gain further insight into the relationship between network linkages and the firm effect on earnings by first expressing the latter as follows:

$$\log W_{it} = \text{const.} + \frac{1}{1+\gamma}\log R_{it} - \frac{1}{1+\gamma}\log\tilde{\phi}_{it} - \frac{1}{1+\gamma}\log\left(1 - s_{it}^{M}\right) \tag{3.10}$$

where $R_{it} \equiv p_{it}X_{it}$ is firm i's sales and $s_{it}^{M} \equiv \frac{E_{it}^{M}}{E_{it}^{M}+\frac{1}{\eta}E_{it}^{L}}$ is firm i's material share of cost adjusted for markdowns on wages.

Now, it is well-known that workers at larger firms tend to earn higher wages, although the extent to which this is true varies across countries and industries (see Oi and Idson (1999) for a survey of the literature). In our framework, however, firms with identical sales can have different firm effects on earnings through two channels.

First, such differences may stem from heterogeneity in primitives $\{g_i(\cdot), \phi_i(\cdot)\}$ that affect the heterogeneous sorting of workers to firms through the sorting composite $\tilde{\phi}_{it}$. For example, conditional on sales, a firm may pay low wages if its employees are compensated for by good amenities. Second, differences in firm effects conditional on sales may arise from heterogeneity in material cost shares, s_{it}^{M}. Under profit-maximizing behavior by firms, these shares are determined by the cost of materials Z_{it} relative to the cost of labor adjusted for labor productivity W_{it}/ω_{it}:

$$s_{it}^{M} = \frac{(1-\lambda)\,Z_{it}^{1-\epsilon}}{\lambda\left(W_{it}/\omega_{it}\right)^{1-\epsilon} + (1-\lambda)\,Z_{it}^{1-\epsilon}} \tag{3.11}$$

Thus, a firm's upstream network connections with its suppliers matters directly for the firm effect conditional on sales and sorting composites. For instance, if labor and materials are substitutes ($\epsilon > 1$), a firm may pay low wages conditional on sales if it has access to cheap materials that can be used to substitute for labor. On the other hand, if labor and materials are complements ($\epsilon < 1$), lower material costs are associated with higher wages instead. Only in the special case of the model with Cobb-Douglas technology ($\epsilon = 1$) is the firm effect independent of $\frac{Z_{it}}{W_{it}/\omega_{it}}$ conditional on R_{it} and $\tilde{\phi}_{it}$, since material cost shares are exogenous and equal to $1-\lambda$ for every firm in this scenario.

[13] As we discuss in Appendix B, the assumption of Cobb-Douglas technology also implies complete passthrough of changes in firm value-added per worker to changes in worker earnings, a prediction that is strongly rejected by the empirical literature. See Berger et al. (2019) and Kline et al. (2019), for example. In contrast, our model implies incomplete passthrough outside of the Cobb-Douglas case.

Consequently, even though firms with higher TFP T_{it} and demand D_{it} may have both larger size and greater firm effects on earnings, firm size is in general not a sufficient statistic for the firm effect on earnings. Even if one accounts for differences in sorting through $\tilde{\phi}_{it}$ as in Lamadon et al. (2019), differences in material cost shares due to heterogeneity in the production network still contribute to differences in worker earnings conditional on sales (as long as $\epsilon \neq 1$). This further implies that understanding the determinants of heterogeneity in firm size (as examined by Bernard et al. (2019), for example) is complementary but not equivalent to understanding the determinants of heterogeneity in worker earnings.

3.3 The production network and labor shares of value-added

Our final set of theoretical results characterize how the production network shapes differences in labor shares of value-added across firms. In our framework, value-added is equal to the sum of a firm's wage bill and profits. Hence, the labor share of value-added depends only on the wage bill to profit ratio, which can be expressed as:

$$\frac{E_{it}^L}{\pi_{it}} = \underbrace{\left(\frac{E_{it}^L + E_{it}^M}{\pi_{it}}\right)}_{=\sigma-1} \times \left(\frac{E_{it}^L}{E_{it}^L + E_{it}^M}\right) \tag{3.12}$$

The first term on the right-hand side is the firm's cost to profit ratio, which under CES markups is equal to $\sigma - 1$ for every firm, while the second term is the firm's labor share of production cost. Hence, differences in labor cost shares uniquely determine differences in labor value-added shares. For example, to the extent that larger firms rely more on materials relative to labor, firm size will be negatively correlated with labor shares of value-added. An immediate corollary of this result is that models which assume homogeneity across firms in both markups and labor cost shares (such as the model in Lamadon et al. (2019)) will also predict homogeneous labor value-added shares.

What then determines the labor share of production cost? Combining equations (2.21), (2.24), (2.28), and (2.29), we can express the ratio of labor to material cost for a firm i as:

$$\frac{E_{it}^L}{E_{it}^M} = \eta \left(\frac{W_{it}/\omega_{it}}{Z_{it}}\right)^{1-\epsilon} \tag{3.13}$$

Hence, the labor share of production cost varies across firms only in relation to differences in the productivity-adjusted cost of labor relative to the cost of materials. Heterogeneity in the production network matters directly for the cost of materials Z_{it} and also affects the cost of labor W_{it} as discussed in section 3.2 above. Furthermore, the sign of the relationship between relative input expenditures and relative costs depends critically on whether the labor-materials substitution elasticity ϵ is greater or less than one. We will return to these points below when we consider identification of ϵ and how the network matters for the relationship between firm size and labor value-added shares in our data.

4 Identification of Model Parameters

We now turn towards identification of the model's parameters. In section 4.1, we first impose additional assumptions needed for identification. We then discuss identification results for each model parameter of interest in section 4.2. As we move toward connecting the model with worker-level and firm-level data, we now explicitly index individual workers by m.

4.1 Assumptions for identification

We impose three additional sets of assumptions: (i) functional form assumptions (section 4.1.1); (ii) assumptions regarding stochastic processes and orthogonality of shocks (section 4.1.2); and (iii) a steady-state assumption (section 4.1.3).

4.1.1 Functional form assumptions

Assumption **4.1.** *The ability of worker m at time t, a_{mt}, is comprised of a permanent (time-invariant) component $\bar{a}_m$ and a transient (time-varying) component $\hat{a}_{mt}$. The labor productivity function takes the following form:*

$$\log \phi_i (a_{mt}) = \theta_i \log \bar{a}_m + \log \hat{a}_{mt} \tag{4.1}$$

and the firm amenity function depends only on permanent worker ability, $g_i (a_{mt}) = g_i (\bar{a}_m)$.

We refer to the parameter θ_i as the *production complementarity* of firm i. The distinction between permanent and transient worker ability follows Lamadon et al. (2019) and is important for the identification of worker-firm interaction effects on worker earnings. Note that there are two sources of worker-firm sorting in the model: workers of different abilities may have different productivity levels in different firms through $\phi_i (a_{mt})$ and may value the amenities of these firms differently through $g_i (a_{mt})$. Assumption 4.1 restricts these determinants of sorting to be time-invariant, which facilitates identification based on earnings at the worker-firm-time level.

Assumption **4.2.** *Log relationship-specific productivity between buyer i and seller j at time t is given by:*

$$\log \psi_{ijt} = \log \psi_{it} + \log \psi_{jt} + \log \tilde{\psi}_{ijt} \tag{4.2}$$

where ψ_{it} denotes the relationship capability of firm i and $\log \tilde{\psi}_{ijt}$ is a residual.

This assumption will be important for decomposing observed firm-to-firm transactions into buyer and seller effects, which we will then use to construct firm-specific material prices Z_{it}.

4.1.2 Stochastic processes and orthogonality conditions

The sources of heterogeneity in the model can now be summarized as follows: workers are heterogeneous with respect to $\chi^L_{mt} \equiv \{\bar{a}_m, \hat{a}_{mt}\}$, firms are heterogeneous with respect to $\chi^F_{it} \equiv \{T_{it}, \omega_{it}, \psi_{it}, \theta_i, g_i(\cdot)\}$, and buyer-seller matches are heterogeneous with respect to $\chi^M_{ijt} \equiv \left\{\tilde{\psi}_{ijt}\right\}$. We now specify stochastic processes for these variables and describe the orthogonality conditions that characterize them.

Assumption **4.3.** *Log transient worker ability,* $\log \hat{a}_{mt}$*, follows a stationary mean-zero stochastic process that is independent of permanent worker ability* $\bar{a}_m$*.*

Stationarity of transient ability implies that we can treat the supply of workers of each ability type, $\{L(a)\}_{a\in A}$, as time-invariant, which is consistent with the steady-state assumption that we impose below. Note that all mean differences in ability across workers are captured by differences in permanent ability.

Assumption **4.4.** *Time-varying firm productivities* $\{T_{it}, \omega_{it}, \psi_{it}\}$ *follow stationary first-order Markov processes:*

$$\log T_{it} = F^T(\log T_{i,t-1}) + \xi^T_{it} \tag{4.3}$$

$$\log \omega_{it} = F^\omega(\log \omega_{i,t-1}) + \xi^\omega_{it} \tag{4.4}$$

$$\log \psi_{it} = F^\psi(\log \psi_{i,t-1}) + \xi^\psi_{it} \tag{4.5}$$

where the Markov innovations $\left\{\xi^T_{it}, \xi^\omega_{it}, \xi^\psi_{it}\right\}$ *are iid across both firms and time.*

The Markov structure of firm productivities follows well-known papers in the literature on production function estimation such as Olley and Pakes (1996) and Doraszelski and Jaumandreu (2018). As described below, we adopt the approach in the latter paper to estimate parameters of the production function and hence adopt this Markov structure. This will not be required otherwise for identification of the productivity variables themselves. Stationarity of the Markov processes also implies that the cross-sectional distribution of firm characteristics χ^F_{it} is time-invariant, which is consistent with the steady-state assumption that we impose below.

Assumption **4.5.** *Relationship productivity residuals* $\tilde{\psi}_{ijt}$ *are iid across firm pairs and time.*

As with Assumption 4.2, this will be important for decomposing observed firm-to-firm transactions into buyer and seller effects. Note that this does not imply that relationship productivities ψ_{ijt} are serially uncorrelated. Instead, persistence of ψ_{ijt} is allowed for through the Markov structure of firm relationship capabilities ψ_{it} in Assumption 4.4.

Assumption **4.6.** *The stochastic processes for worker characteristics* χ^L_{mt}*, firm characteristics* χ^F_{it}*, and firm-to-firm characteristics* χ^M_{ijt} *are mutually independent.*

Together with the conditions imposed in Assumption 4.1, independence of the stochastic processes for worker and firm characteristics ensures that residual worker earnings due to transient ability shocks are uncorrelated with the characteristics of the worker's firm. This is the same as the orthogonality assumption imposed in Lamadon et al. (2019). Note also that independence of firm characteristics and relationship productivity residuals does not imply that firms match at random, only that they do not match based on the residual $\tilde{\psi}_{ijt}$.

4.1.3 Steady-state

The last assumption that we impose for identification is that the data are characterized by a steady-state of the model in which general equilibrium terms do not vary over time.

Assumption **4.7.** *Aggregate income, E_t, and the labor market indices, $I_t(\cdot)$, are time invariant.*

This is equivalent to the restriction that there are no aggregate (economy-wide) shocks in the model and implies that the labor supply shifters $\kappa_{it}(\cdot)$ are time-invariant.[14] As discussed in section 4.2.2, this assumption will be important for the identification of firm effects on earnings.

4.2 Identification results

The parameters of the model that we seek to identify – denoted by Θ – can now be summarized as: (i) the labor supply elasticity, γ; (ii) production function parameters, $\{\sigma, \epsilon\}$; (iii) worker abilities for every worker m, $\{\bar{a}_m, \hat{a}_{mt}\}$; (iv) firm productivity parameters for every firm i, $\{T_{it}, \omega_{it}, \psi_{it}, \theta_i\}$; (v) amenity values for every firm i and worker m, $g_i(\bar{a}_m)$; and (vi) relationship productivity residuals for every buyer-seller firm pair ij, $\tilde{\psi}_{ijt}$. We now describe identification of each of these parameters.

4.2.1 Labor supply elasticity

We identify the labor supply elasticity γ from the passthrough of firm-level changes in wage bills E_{it}^L to worker-level wages w_{mt}. Here, we follow Lamadon et al. (2019) and allow for measurement error in firm wage bills, such that wage bills in the data $\ddot{E}_{it}^L$ are related to wage bills in the model E_{it}^L as follows:

$$\log E_{it}^L = \log \ddot{E}_{it}^L + e_{it}^L \tag{4.6}$$

where e_{it}^L denotes an MA(k) measurement error given by $e_{it}^L = \sum_{s=0}^{k} \delta^{L,s} u_{i,t-s}^L$ for some weights $\delta^{L,s}$ and mean-zero shocks u_{it}^L that are iid across firms and time. We allow the shocks u_{it}^L to be correlated with the firm productivity innovations specified in equations (4.3)-(4.5) only contemporaneously, so that $\mathbb{E}\left[\xi_{it}^x u_{is}^L\right] \neq 0$ only if $s = t$, for all $x \in \{T, \omega, \psi\}$ and $i \in \Omega^F$.

[14] The Lamadon et al. (2019) model allows for multiple regions of production with aggregate shocks within each region, but the identification strategy rules out economy-wide aggregate shocks across all regions. Since we abstract from multiple production regions, our steady-state assumption is equally restrictive.

Combining equations (2.23), (2.28), and (4.1), we can then express the wage for worker m at firm i as:

$$\log w_{imt} = \theta_i \log \bar{a}_m - \frac{1}{1+\gamma} \log \tilde{\phi}_i + \frac{1}{1+\gamma} \log \ddot{E}_{it}^L + \frac{1}{1+\gamma} e_{it}^L + \log \hat{a}_{mt} \tag{4.7}$$

Under Assumption 4.1, the worker-firm productivity term $\theta_i \log \bar{a}_m$ is time-invariant, while under Assumption 4.7, the sorting composite $\tilde{\phi}_i$ is time-invariant. Restricting attention to workers that do not change employers between t and $t+1$ (stayers), we can then take first-differences of equation (4.7) and write:

$$\Delta \log w_{imt} = \frac{1}{1+\gamma} \Delta \log \ddot{E}_{it}^L + \Delta \log \hat{a}_{mt} + \frac{1}{1+\gamma} \Delta e_{it}^L \tag{4.8}$$

Equation (4.8) implies that the change in a firm's wage bill is a sufficient statistic for all firm-level shocks that matter for changes in the earnings of stayers at the firm. Since the labor supply elasticity γ controls the extent of imperfect competition in the labor market and mediates the extent of rent-sharing between a firm and its employees, the passthrough of changes in wage bills to changes in wages is informative about the magnitude of γ. In particular, stronger passthrough implies greater labor market power and a smaller value of γ. Equation (4.7) also makes clear why identification of γ should rely only on stayers: the change in earnings for a worker that switches employers between t and $t+1$ is driven not only by rent-sharing but also by changes in permanent firm characteristics $\left\{\theta_i, \tilde{\phi}_i\right\}$ and hence cannot be used to identify γ.

Note that in the absence of measurement error, the residual in equation (4.8) contains only worker-level shocks ($\Delta \log \hat{a}_{mt}$). However, with measurement error in wage bills, the unobserved error term in equation (4.8) contains a component that is potentially correlated with observed changes in the wage bill since $\mathbb{E}\left[\Delta \log \ddot{E}_{it}^L \Delta \log e_{it}^L\right] \neq 0$. To address this, note that $\mathbb{E}\left[\log \ddot{E}_{is}^L \Delta e_{it}^L\right] = 0$ for all $s < t-k-1$ since Δe_{it}^L depends only on measurement error shocks u_{it}^L in periods $\{t-k-1, \cdots, t\}$. Hence, under Assumption 4.6, lagged changes in wage bills $\log \Delta \ddot{E}_{is}^L$ for any $s < t-k-1$ are valid instruments for $\log \Delta \ddot{E}_{it}^L$ in identifying γ from equation (4.8).[15] As highlighted above, this argument does not require any restrictions on how network linkages are distributed across firms or how these linkages change over time. Hence, although we do not model the endogenous formation of network linkages, our identification strategy for γ is robust to allowing for such endogeneity.

To provide context, we point out that identification of γ from equation (4.8) resembles the passthrough analysis in Guiso et al. (2005) and Lamadon et al. (2019), but with two key differences. First, both of these papers construct the firm-level shock of interest using changes

[15] This requires serial correlation in $\Delta \ddot{E}_{it}^L$ to be non-zero with at least $k+2$ lags, which is consistent with the Markov processes for firm productivities specified in Assumption 4.4.

in value-added, whereas our model motivates using changes in wage bills instead.[16] In Guiso et al. (2005), the relevance of value-added is based on a reduced-form model of worker earnings and hence is not structurally derived. Lamadon et al. (2019) go one step further and model the structural relationship between firm and worker outcomes, but because intermediate inputs are absent from their model, the wage bill is a constant fraction of value-added for any given firm. Hence, the passthrough coefficient $\frac{1}{1+\gamma}$ in equation (4.7) can be identified from either changes in value-added or the wage bill. In contrast, with both imperfect competition in output markets and the existence of intermediate inputs, the wage bill is no longer proportional to firm value-added and identification stems from changes in the former instead of the latter.[17]

Second, Lamadon et al. (2019) model the underlying shock to firm value-added as stemming from changes in firm TFPs. In contrast, our framework implies that changes in the wage bill for firm i may stem from either shocks to firm i's own primitives (such as TFP or labor productivity) or from shocks to primitives for other firms that firm i is connected to both directly and indirectly in the production network upstream and downstream. Hence, both types of shocks are useful for identification of the labor supply elasticity in our framework. We leverage this below by constructing an alternative set of instruments for wage bill changes using shocks in the production network.

4.2.2 Worker and firm wage effects

Next, we discuss identification of the worker and firm effects in the earnings equation (4.7). We first follow Lamadon et al. (2019) and rewrite this as:

$$\log \tilde{w}_{imt} = \underbrace{\theta_i \log \bar{a}_m}_{\text{worker-firm interaction}} + \underbrace{\log \bar{W}_i}_{\text{firm FE}} + \underbrace{\log \hat{a}_{mt}}_{\text{residual}} \tag{4.9}$$

where $\bar{W}_i \equiv \frac{1}{\eta}\left(\bar{E}_i^L/\tilde{\phi}_i\right)^{\frac{1}{1+\gamma}}$ is a time-invariant firm effect that depends on both $\tilde{\phi}_i$ and the firm's mean wage bill over time $\bar{E}_i^L$, while $\log \tilde{w}_{imt} \equiv \log w_{imt} - \frac{1}{1+\gamma}\left(\log E_{it}^L - \log \bar{E}_i^L\right)$ is worker earnings residualized by the innovation in its employer's wage bill. Equation (4.9) is of the same form as the reduced-form model of earnings in Bonhomme et al. (2019), who show that the

[16] There are also subtle differences in the assumptions placed on the stochastic processes for firm-level shocks. Guiso et al. (2005) assume that log value-added follows an AR(1) process with innovations comprised of a unit root process plus an MA(1) process. Lamadon et al. (2019) make the same assumptions as Guiso et al. (2005) but constrain the AR(1) coefficient to be zero. In contrast, we allow for non-linear first-order Markov processes in firm primitives that determine firm wage bills (Assumption 4.4) and MA(k) measurement errors in wage bills, but consider only stationary processes for firm and worker shocks (which is necessary for the steady-state described in Assumption 4.7 to exist).

[17] In appendix E.1, we document our estimates of the labor supply elasticity γ using value-added shocks instead of wage bill shocks and show that we obtain different results. Hence, the distinction is both theoretically and empirically relevant.

model implies the following restriction:

$$\mathbb{E}\left[\frac{1}{\theta_j}\left(\log\tilde{w}_{jm,t+1}-\log\bar{W}_j\right)-\frac{1}{\theta_i}\left(\log\tilde{w}_{im,t}-\log\bar{W}_i\right)|m\in M_{t,t+1}^{i\to j}\right]=0 \tag{4.10}$$

where the expectation is taken over the set of workers $M_{t,t+1}^{i\to j}$ that move from firm i at time t to firm j at time $t+1$.

In theory, equation (4.10) gives $\left|\Omega^F\right|^2$ moment conditions for identification of $2\left|\Omega^F\right|$ parameters. Intuitively, changes in earnings that accompany changes in employers are informative about the firm-specific determinants of earnings $\left\{\theta_i,\bar{W}_i\right\}$. In practice, however, we follow Bonhomme et al. (2019) and restrict the firm effects $\left\{\theta_i,\bar{W}_i\right\}$ to vary only by K clusters of firms, so that $\theta_i=\theta_{k(i)}$ and $\bar{W}_i=\bar{W}_{k(i)}$, where we refer to $k(i)$ as the *earnings cluster* of firm i. Although not strictly necessary for identification, this reduces the dimension of the parameter set that needs to be estimated and ameliorates the well-known limited mobility bias issue. We discuss the clustering procedure in more detail below.

Given identification of $\left\{\theta_{k(i)},\bar{W}_{k(i)}\right\}$, permanent worker ability is then identified from:

$$\log\bar{a}_m=\mathbb{E}\left[\frac{\log\tilde{w}_{imt}-\log\bar{W}_{k(i)}}{\theta_{k(i)}}\right] \tag{4.11}$$

and transient worker ability is identified as the residual in earnings given identification of all other determinants of earnings. Furthermore, the time-varying firm effect W_{it} can be recovered from:

$$\log W_{it}=\log\bar{W}_{k(i)}+\frac{1}{1+\gamma}\left(\log E_{it}^L-\log\bar{E}_i^L\right) \tag{4.12}$$

Note that even though the time-invariant firm effect $\bar{W}_{k(i)}$ is restricted to vary only by cluster, the full time-varying firm effect W_{it} is firm-specific.

4.2.3 Amenities

Just as we restrict production complementarities θ_i to vary only by firm cluster for purposes of estimation, we impose a similar restriction on the firm amenity function:

$$g_i(a)=\tilde{g}_i\bar{g}_{k(i)}(\bar{a}) \tag{4.13}$$

This is the same decomposition of amenities as in Lamadon et al. (2019). The component $\bar{g}_{k(i)}(\bar{a})$ allows for worker-firm variation in amenities but restricts this to be the same for firms within a cluster, again for the purpose of reducing dimensionality. Variation in amenities across firms within a cluster is then accounted for by $\tilde{g}_i$.

In Appendix E.3, we show that the cluster-ability component of amenities can be identified from:

$$\bar{g}_k(\bar{a}) = (\bar{a})^{-\theta_k} [\Lambda_{kt}(\bar{a})]^{\frac{1}{\gamma}} \tag{4.14}$$

where $\Lambda_{kt}(\bar{a})$ is the share of workers of permanent ability $\bar{a}$ that are employed by firms in cluster k. Since a firm with a high value of amenities is able to attract a large share of workers at a lower wage, the amenities component $\bar{g}_k(\bar{a})$ is intuitively identified from cluster-ability level employment shares after controlling for the relevant determinants of earnings heterogeneity across cluster-ability groups, namely permanent worker abilities $\bar{a}$ and production complementarities θ_k. Similarly, the firm-specific component of amenities can be identified from:

$$\tilde{g}_i = \frac{1}{W_{it}} \left(\frac{\bar{\Lambda}_{it}}{\bar{\Lambda}_{k(i)t}} \right)^{\frac{1}{\gamma}} \tag{4.15}$$

where $\bar{\Lambda}_{it}$ and $\bar{\Lambda}_{k(i)t}$ denote the shares of employment across all worker types accounted for by firm i and cluster $k(i)$ respectively. The firm-specific component of amenities is hence intuitively identified from within-cluster employment shares $\frac{\bar{\Lambda}_{it}}{\bar{\Lambda}_{k(i)t}}$ controlling for the relevant determinant of earnings heterogeneity within clusters, namely the firm-level wage W_{it}.

4.2.4 Firm relationship capability and relationship-specific productivity

The value of sales from firm j to firm i is given by the product of the buyer's network demand, the seller's network productivity, and relationship-specific productivity:

$$R_{ijt} = \Delta_{it} \Phi_{jt} \psi_{ijt} \tag{4.16}$$

Using equation (4.2), we can write this in log terms as:

$$\log R_{ijt} = \gamma \log \eta + \underbrace{\log \tilde{\Delta}_{it}}_{\text{buyer effect}} + \underbrace{\log \tilde{\Phi}_{jt}}_{\text{seller effect}} + \log \tilde{\psi}_{ijt} \tag{4.17}$$

where $\tilde{\Delta}_{it} \equiv \Delta_{it}\psi_{it}$ and $\tilde{\Phi}_{it} \equiv \Phi_{it}\psi_{it}$. Under Assumption 4.2, the assignment of buyers to sellers is independent of $\tilde{\psi}_{ijt}$ and hence $\mathbb{E}\left[\log \tilde{\Delta}_{it} \log \tilde{\psi}_{ijt}\right] = \mathbb{E}\left[\log \tilde{\Phi}_{it} \log \tilde{\psi}_{ijt}\right] = 0$.[18] The buyer effect $\tilde{\Delta}_{it}$ is thus identified from purchases by firm i from all its suppliers controlling for total sales by these suppliers, while the seller effect $\tilde{\Phi}_{jt}$ is identified from sales by firm j to all its customers controlling for total expenditures by these customers. This follows Bernard et al. (2019). Note that this only requires that buyer-seller relationships are not selected based on residual relationship productivities $\tilde{\psi}_{ijt}$, but does not otherwise rule out selection based on firm-level primitives such as TFP or labor productivity since these are absorbed by the

[18] Bernard et al. (2019) find strong evidence in support of this assumption using Belgian firm-to-firm transactions data.

buyer and seller effects. Furthermore, since matching in intermediate input markets can occur many-to-many (each seller can have several buyers at once and each buyer can have several sellers), this identification strategy only requires cross-sectional moments. This is in contrast with identification of the worker and firm earnings effects in equation (4.9), which requires movements of workers across firms over time.

To separately identify network demand Δ_{it}, network efficiency Φ_{it}, and relationship capability ψ_{it} from the buyer and seller effects in equation (4.17), note from equations (2.8) and (2.14) that the share of a firm's total sales that come from the network (i.e. excluding final sales) can be expressed as:

$$s_{it}^{net} = \frac{\psi_{it} \sum_{j \in \Omega_{it}^C} \tilde{\Delta}_{jt} \tilde{\psi}_{jit}}{E_t + \psi_{it} \sum_{j \in \Omega_{it}^C} \tilde{\Delta}_{jt} \tilde{\psi}_{jit}} \tag{4.18}$$

Solving for ψ_{it}, we obtain:

$$\psi_{it} = E_t \left(\frac{s_{it}^{net}}{1 - s_{it}^{net}} \right) \frac{1}{\sum_{j \in \Omega_{it}^C} \tilde{\Delta}_{jt} \tilde{\psi}_{jit}} \tag{4.19}$$

Firm relationship capability ψ_{it} is therefore identified (up to a normalizing constant) from observable network sales shares s_{it}^{net} and terms $\left\{ \tilde{\Delta}_{jt}, \tilde{\psi}_{ijt} \right\}$ that are identified from equation (4.17). Intuitively, a higher value of ψ_{it} increases sales only within the network but not to final consumers. Network demands and efficiencies are then easily recovered from the buyer and seller effects $\left\{ \tilde{\Delta}_{it}, \tilde{\Phi}_{it} \right\}$.

4.2.5 Demand price elasticity

In Appendix E.5, we show that the price elasticity of demand σ is identified from the following moment condition:

$$\sigma = \mathbb{E} \left[\frac{R_{it}}{R_{it} - \frac{1}{\eta} E_{it}^L - E_{it}^M} \right] \tag{4.20}$$

Intuitively, σ controls the extent of product differentiation and hence determines the sales to profit ratio that appears on the right-hand side of (4.20), where profits in the denominator are adjusted for markdowns on wages. We thus identify σ from firm sales R_{it} and input expenditures $\left\{ E_{it}^L, E_{it}^M \right\}$. Note that as the markdown η approaches unity, σ is identified from the population average sales-profit ratio, as in standard CES production models with perfectly competitive labor markets.

4.2.6 Labor-materials substitution elasticity and labor productivity

In Appendix E.6, we show that a firm's relative expenditure on materials versus labor can be expressed as:

$$\log \frac{E_{it}^{M}}{E_{it}^{L}} = \log \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \right] + (\epsilon - 1) \log \frac{W_{it}}{Z_{it}} + (1-\epsilon) F^{\omega} \left(\log \omega_{i,t-1} \right) + (1-\epsilon) \xi_{it}^{\omega} \quad (4.21)$$

This is the standard relationship between relative factor expenditures and relative factor prices implied by cost minimization under CES technologies, except that W_{it} and Z_{it} are not simple averages of wages and input prices across a firm's workers and suppliers respectively. Instead, W_{it} is identified from the decomposition of worker earnings into worker and firm effects as discussed in section 4.2.2, while $Z_{it} = \left(\sum_{j \in \Omega_i^S} \tilde{\Phi}_{jt} \psi_{it} \tilde{\psi}_{ijt} \right)^{\frac{1}{1-\sigma}}$ can be constructed using only identified terms discussed in sections 4.2.4 and 4.2.5.[19]

With the first-order Markov structure of productivity innovations in Assumption 4.4, identification of the labor-materials substitution elasticity ϵ follows the strategy in Doraszelski and Jaumandreu (2018), which uses lagged values of input expenditures and factor prices as instruments for $\log \frac{W_{it}}{Z_{it}}$ and a control function in lagged factor prices and expenditures to control for the lagged labor productivity term $F^{\omega} \left(\log \omega_{i,t-1} \right)$. Given a value for ϵ, labor productivities are then easily recovered as residuals in the relationship between relative input expenditures and prices, $\omega_{it} = \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \left(\frac{E_{it}^{L}}{E_{it}^{M}} \right) \right]^{\frac{1}{\epsilon-1}} \left(\frac{W_{it}}{Z_{it}} \right)$. Since identification of W_{it} and Z_{it} is robust to endogenous selection of buyer-seller linkages on firm-level primitives (excluding $\tilde{\psi}_{ijt}$), identification of ϵ and ω_{it} is robust to such selection as well.

Note that the relevant factor prices are firm-specific, instead of market-specific as is commonly assumed in the production function estimation literature: W_{it} due to imperfect competition in the labor market and Z_{it} due to heterogeneity in the production network. Furthermore, as pointed out in Doraszelski and Jaumandreu (2018), the weight on labor in the production function λ is not separately identified from the average level of labor productivity ω_{it} across firms. This is intuitive, since both λ and ω_{it} control the productivity of labor relative to materials. Hence, in what follows we set λ to an arbitrary constant in the interval $(0, 1)$ without any loss of generality.[20]

[19] Although we allow for measurement error in firm wage bills in section 4.2.1, we follow the production function estimation literature and assume that *relative* input costs are observed without error, so that $\frac{E_{it}^{M}}{E_{it}^{L}} = \frac{\ddot{E}_{it}^{M}}{\ddot{E}_{it}^{L}}$.

[20] This can be seen from the production function (2.10). Output for a given worker ability type (omitting firm, time, and ability indices) can be written as $X = \tilde{T} \left[(\phi \tilde{\omega} L)^{\frac{\epsilon-1}{\epsilon}} + M^{\frac{\epsilon}{\epsilon-1}} \right]^{\frac{\epsilon}{\epsilon-1}}$, where $\tilde{T} \equiv (1-\lambda)^{\frac{1}{\epsilon-1}} T$ and $\tilde{\omega} \equiv \left(\frac{\lambda}{1-\lambda} \right)^{\frac{1}{\epsilon-1}} \omega$. Hence, the production function is parameterized in terms of $\tilde{T}$ and $\tilde{\omega}$ instead of $\{\lambda, T, \omega\}$ separately.

4.2.7 Firm TFP

We have now established identification of all parameters of interest Θ except for firm TFPs, T_{it}. For this, we require (at least) $\left|\Omega^F\right|$ moment conditions. We construct these using the time-varying firm effects W_{it}, identification of which is given by equation (4.12). Note that one can write these firm effects in general as:

$$W_{it} = F_i \left(T_t | \Theta_{-T} \right) \tag{4.22}$$

where $T_t \equiv \{T_{it}\}_{i \in \Omega^F}$, $\Theta_{-T} \equiv \Theta \setminus T_t$, and $\{F_i\}_{i \in \Omega^F}$ is a set of *known* functions that depend on the structural relationships of the model. Given identification of all other parameters Θ_{-T}, equation (4.22) hence provides a set of moments for exact identification of firm TFPs.

We choose this approach because it ensures that the model replicates the firm effects on earnings that we estimate from the data, which in turn guarantees that the model matches observed earnings for a given worker conditional on also replicating the worker's observed choice of employer. This allows us to examine changes in earnings under various counterfactual scenarios with the confidence that the baseline model provides a good fit to observed earnings. Note that in the limit of our model without intermediates ($\lambda \to 1$), $\log W_{it}$ is linear in $\log T_{it}$ and hence identification is trivial. With intermediates, however, the functions F_i are in general defined implicitly and involve complex non-linearities, which hence requires a numerical solution for the TFP vector.[21]

5 Estimation of Model Parameters

We first describe the data that we use to estimate the model in section 5.1. Our estimation results are presented in section 5.2 and the fit of the model to data is assessed in section 5.3.

5.1 Data Sources and Sample Selection

To implement estimation of the model's parameters, we use four administrative datasets from the Internal Revenue Service (IRS, or SII for its acronym in Spanish) in Chile. These datasets cover the entire formal private sector in Chile. Below, we describe the data sources, sample selection, and key variables.

First, we use a matched employer-employee dataset (IRS tax affidavits 1887 and 1879) that reports annual earnings from each job that a worker has from 2005-2018. Earnings include wages, salaries, bonuses, tips, and other sources of labor income deemed taxable by the IRS. As earnings are reported net of social security payments, we adjust the earnings measure to include

[21] Due to these features of the F_i functions, establishing a unique solution for T_t given a vector of firm effects W_t is not trivial. Nonetheless, we have explored the potential for multiplicity by varying the initial guess for the TFP vector and never find multiplicity to occur in practice.

these payments. Second, we use a database from the civil registry that has the year of birth of each individual who is alive in 2018. We merge this dataset with the employer-employee dataset using workers' unique tax IDs to measure the age of every worker. Third, we use a firm-to-firm dataset (IRS tax form 3323 and 3327) that is based on value-added tax (VAT) records from 2005-2017. Each firm in this dataset reports the full list of its intermediate buyers and suppliers, as well as the total gross value of transactions with each buyer and supplier. As reporting occurs semi-annually, we aggregate this data to the annual level to make it consistent with the other datasets. Since this dataset reports transactions gross of taxes, we measure transactions net of taxes by using the flat value-added tax rate of 19% that was in effect in Chile during the sample period. Finally, we use an administrative dataset (IRS tax form 29) that contains a set of firm balance sheet characteristics from 2005-2018. We use this dataset to measure total sales and material cost for each firm. Firms in each of the datasets above are assigned a unique tax ID that is consistent across datasets, which facilitates the merging of these datasets. In what follows, we define a firm as a tax ID.[22]

For the firm-to-firm dataset, we impose the following sample restrictions. We drop relationships involving firms that do not report value-added or employment, or firms that report negative value-added, sales, or materials. Next, we implement an iterative procedure that drops firms that have only one relationship, as in Bernard et al. (2019), which is required for the decomposition of firm-to-firm transaction values into buyer and seller effects. After imposing these sample restrictions, the dataset includes 32 million firm-to-firm-year observations and 17 million observations of unique firm pairs. This corresponds to 593 (923) thousand supplier-year (buyer-year) observations and 195 (289) thousand unique suppliers (buyers). We refer to this restricted dataset as the *baseline firm-to-firm dataset*.

For the employer-employee dataset, we impose the following sample restrictions following the criteria of Lamadon et al. (2019). In each year, we start with all individuals aged 25-60 who are linked to at least one employer. We identify links using only information on labor contracts (tax affidavit 1887). Next, we drop firms that have missing or negative value-added, sales, or materials in the balance sheet data (tax form 29). Then, we keep for each worker the firm that pays the highest earnings in a given year. Since we do not have hours worked or a direct measure of full-time employment, we follow the literature by including workers for whom annual earnings are above a minimum threshold (Song et al., 2019). We set the threshold equal to 32.5% of the national average of earnings in order to make our estimates comparable to the cross-country study of earnings inequality in Bonhomme et al. (2020). After imposing these sample restrictions, the dataset includes 42 (2) million worker-year (firm-year) observations and 6,497 (488) thousand unique workers (firms). We refer to this restricted dataset as the *baseline*

[22]As all tax forms are reported at the headquarter-level, plant-level information is not available. Furthermore, while it is possible that a firm has several tax IDs, information that allows us to observe firm ownership is not available.

employer-employee dataset. Finally, for both the employer-employee and firm-to-firm datasets, we transform nominal variables to 2015 real dollars.

Starting from the baseline employer-employee dataset, we define two subsamples that we will use in different parts of the paper. The first, which we refer to as the *stayers* sample, restricts the baseline sample to workers observed with the same employer for at least 8 consecutive years. This restriction is needed to allow for a flexible specification of how worker's earnings evolve over time. We also omit the first and last years of these spells to avoid concerns over workers exiting and entering employment during the year, confounding the measure of annual earnings. The stayers sample is also restricted to firms with at least 10 stayers every year which helps to ensure sufficient sample size to perform the analyses at the firm level. The stayers sample includes 6,571 (603) thousand spells and 725 (6) thousand unique workers (firms).

The second, which we refer to as the *movers* sample, restricts the baseline to workers observed at multiple firms over time. In other words, the firm that pays a worker her greatest earnings in a given year is not the same firm in all years. Following previous work and motivated by concerns about limited mobility bias, we also restrict the movers sample to firms with at least two movers (Lamadon et al., 2019). Finally, as in the previous literature (Abowd et al., 1999; Lamadon et al., 2019), we restrict this sample to firms that belong to the largest connected set of firms, which in our dataset represents 99.9% of workers. The movers sample includes 40 (1.4) million worker-year (firm-year) observations and 6,184 (201) thousand unique workers (firms).

Finally, for the purpose of estimating the elasticity of substitution between labor and materials, we merge the baseline employer-employee and the baseline firm-to-firm dataset using the unique tax IDs discussed above. We implement this merge at the firm-year level and thus exclude in the merged dataset the set of firms that do not have information in either the employer-employee or the firm-to-firm dataset. The sample includes 126 thousand firm-year observations and 48 thousand unique firms. We refer to this merged dataset as the *baseline firm-level dataset*.

Appendix Table A.1 compares the size of the three employer-employee datasets, the firm-to-firm dataset and the firm dataset we use throughout the paper. Detailed summary statistics of these samples are provided in Appendix Table A.2. The samples are broadly similar. The most noticeable differences are that the stayers sample has older, higher-earning workers and higher labor share, as well as larger firms in terms of employment and degree (number of suppliers and buyers). Nonetheless, the firms in the stayers sample are broadly similar to the firms in the baseline employer-employee dataset in terms of value-added per worker, materials share of sales, and intermediate sales as a share of total sales.

5.2 Estimation results

In this section we present the estimation results for each of the parameters in the model. Henceforth, we follow Lamadon et al. (2019) in removing age and year effects from measured wages.

Specifically, we assume that measured wages $\ddot{w}_{mt}$ are related to model wages w_{mt} through $\log \ddot{w}_{mt} = \beta'_\Upsilon \Upsilon_{mt} + \log w_{mt}$, where Υ_{mt} is a vector of year and cubic age effects. We estimate β_Υ via OLS and construct $\log w_{mt} = \log \ddot{w}_{mt} - \beta'_\Upsilon \Upsilon_{mt}$ as our measure of wages for subsequent steps of the estimation.

5.2.1 Labor supply elasticity

We estimate the labor supply elasticity γ using equation (4.8) applied to the stayers sample, with lagged values of $\Delta \log \ddot{E}^L_{it}$ as instruments. We assume an MA(1) process for the measurement errors e^L_{it}, which implies that lagged changes in wage bills $\Delta \log \ddot{E}^L_{is}$ for any $s < t - 2$ are valid instruments. Hence, we use a cubic polynomial of instruments with 3 to 5 lags of wage bill changes and choose the specification with the highest F-statistic. We do not use lags above 5 in order to avoid reducing the sample size available for implementing the estimation.

Our preferred specification based on the criterion above is shown in Column 1 of Table 1, which uses 3, 4 and 5 lags of wage bill changes as instruments. We find that the passthrough elasticity of changes in firm wage bills to changes in worker earnings is around 0.15, which implies a labor supply elasticity of $\gamma = 5.5$.[23] For comparison, we also report estimation results that we obtain from other specifications. In Column 2, we use wage bill changes with the minimum instruments allowed by the MA(1) process – 3 lags of wage bill changes (with a cubic polynomial) – and find that the passthrough elasticity increases to 0.18 ($\gamma = 4.6$). The estimates reported in Columns 1-2 are in line with estimates from the literature of the passthrough elasticity from firm shocks to worker earnings.[24] Finally, in Column 3, we report the OLS estimate that ignores measurement error in wage bills. We find that the passthrough elasticity is substantially larger at 0.27. This implies $\gamma = 2.7$, which is half of our preferred estimate.

[23]For robustness, we also estimate γ using the difference-in-difference estimator proposed by Lamadon et al. (2019). Results obtained using this approach are discussed in Appendix E.1. We find the same estimate using this alternative approach.

[24]For example, in a review of the literature, Card et al. (2018) report values for this elasticity between 0.10 and 0.15. Lamadon et al. (2019) in particular estimate a passthrough elasticity of 0.15. Note that these estimates rely on different sources of variation. Whereas we use changes in wage bills (as justified by our model), Card et al. (2018) review estimates using value added per worker while Lamadon et al. (2019) use changes in value added.

Table 1: Estimation of labor supply elasticity (γ)

	$\Delta \log w_{imt}$		
	(1)	(2)	(3)
$\Delta \log \tilde{E}^L_{it}$	0.155	0.177	0.268
	(0.006)	(0.007)	(0.001)
γ	5.5	4.6	2.7
Strategy	GMM	GMM	OLS
Instruments Accumulated Lags	5	3	
First Stage F-Stat	2325	1426	
Number of Observations	2,507,868	2,507,868	2,507,868

Notes: This table presents results from the passthrough regression based on equation (4.8). All GMM strategies use different instruments of cubic polynomials of lags of wage bill and is implemented in two stages with a robust weighting matrix used to compute standard errors. Column 1 (our preferred specification) uses changes of wage bill lagged for 3, 4 and 5 periods as instruments. Column 2 uses changes of wage bill lagged for 3 periods as instruments. Column 3 estimates the model with OLS, which ignores measurement error on the wage bill. Standard errors are shown in parentheses.

5.2.2 Worker and firm effects on earnings

To estimate the worker and firm effects in the earnings equation (4.9), we use the movers sample. We first follow Bonhomme et al. (2019) and assign each firm in our data to one of ten earnings clusters via a K-means clustering algorithm that targets moments of the within-firm distribution of residualized earnings $\tilde{w}_{imt}$.[25] This groups together firms whose earnings distributions are the most similar, which is motivated by the restriction that the firm-level determinants of these earnings – the firm fixed effect $\log \bar{W}_i$ and production complementarity θ_i – do not vary within a cluster. With the cluster assignment in hand, we then estimate $\left\{\log \bar{W}_i, \theta_i\right\}$ by cluster via limited information maximum likelihood, based on the moment condition (4.10).[26]

Our results are presented in Table 2, where clusters are sorted according to the firm fixed effect $\log \bar{W}_i$. We observe a positive correlation between $\log \bar{W}_i$ and θ_i, indicating that firms with higher wage premia are also those where workers of higher ability are more productive.[27] In addition, the estimates that we obtain for θ_i are indicative of strong production complementarities. For example, they imply that workers in the top 2% of the permanent ability distribution are around 40% more productive when employed at firms in the highest $\bar{W}_i$ earnings cluster than at firms in the lowest $\bar{W}_i$ cluster.

[25] Appendix E.2 provides more details including diagnostics of the clustering procedure and robustness of our results with respect to the number of clusters.

[26] We thank Lamadon et al. (2019) for providing the code for this step of the estimation procedure.

[27] This positive correlation is also document in Lamadon et al. (2019) using US data.

Table 2: Estimates of firm fixed effects and production complementarities

Cluster	1	2	3	4	5	6	7	8	9	10
$\log \bar{W}_i$	0	0.25	0.61	0.89	1.06	1.24	1.50	1.69	1.80	1.92
θ_i	1	1.13	1.42	1.66	1.77	1.91	2.19	2.37	2.44	2.26

Notes: This table presents estimates of the firm fixed effect $\log \bar{W}_i$ and production complementarities θ_i in the earnings Equation (4.9) using the movers sample. Clusters are sorted in ascending order of the firm fixed effect, $\log \bar{W}_i$. Note that $\log \bar{W}_i$ and θ_i are normalized to zero and one respectively for firms in the first earnings cluster.

We also use our estimates to perform a preliminary decomposition of the variance of log worker earnings, which will inform the counterfactual simulations that we examine below. We follow Lamadon et al. (2019) and base this exercise on the following transformation of components in the earnings equation (4.9):

$$\log w_{imt} = x_m + \bar{f}_i + \hat{f}_{it} + \iota_{im} + \log \hat{a}_{mt} \tag{5.1}$$

where the transformed components are defined as follows:

$$\begin{aligned} x_m &\equiv \bar{\theta} \left(\log \bar{a}_m - \log \bar{a}\right), & \bar{f}_i &\equiv \log \eta \bar{W}_i + \theta_i \log \bar{a} \\ \iota_{im} &\equiv \left(\theta_i - \bar{\theta}\right) \left(\log \bar{a}_m - \log \bar{a}\right) & \hat{f}_{it} &= \log W_{it} - \log \bar{W}_i \end{aligned} \tag{5.2}$$

This transformation separates the worker-firm interaction effect $\theta_i \log \bar{a}_m$ into worker and firm components, thus facilitating interpretation of the variance decomposition that follows. Here, $\log \bar{a}$ and $\bar{\theta}$ denote the average values of $\log \bar{a}_m$ and θ_i respectively, where both averages are calculated at the worker-level. Intuitively, x_m is a measure of productivity for worker m when employed at the average firm, $\bar{f}_i$ is the time-averaged firm effect on earnings when matched with the average worker, $\hat{f}_{it}$ accounts for time-variation in the firm effect, and ι_{im} captures non-linear interactions between worker and firm effects. Hence, the variance of log earnings can be decomposed as:

$$\begin{aligned} \text{var}\left(\log w_{imt}\right) = & \underbrace{\text{var}\left(x_m\right)}_{\text{1. worker effect var.}} + \underbrace{\text{var}\left(\bar{f}_i\right)}_{\text{2. firm fixed effect var.}} + \underbrace{\text{var}\left(\hat{f}_{it}\right)}_{\text{3. time-varying firm effect var.}} \\ & + \underbrace{2\text{cov}\left(x_m, f_{it}\right)}_{\text{4. sorting cov.}} + \underbrace{\text{var}\left(\iota_{im}\right) + 2\text{cov}\left(\iota_{im}, x_m + f_{it}\right)}_{\text{5. interactions}} + \underbrace{\text{var}\left(\log \hat{a}_{mt}\right)}_{\text{6. residual}} \end{aligned} \tag{5.3}$$

where $f_{it} \equiv \bar{f}_i + \hat{f}_{it}$ and all variances are computed at the worker-level.

Column 1 of Table 3 presents the shares of log earnings variance accounted for by each component in equation (5.3). Unsurprisingly, the variance of the worker effect accounts for the majority (57.0%) of earnings variance. However, we also find that firms play an important role. The variance of the total (transformed) firm effect f_{it} accounts for 10.8% of the log earnings variance, with most of this share accounted for by cross-sectional heterogeneity rather than variation over time. The sorting of workers to firms is even more important, with the sorting covariance explaining 19.8% of the log earnings variance. The interaction term, on the other hand, explains little.[28] Note that in the context of the structural model, the firm effect is endogenously determined by firm primitives, including network connections. In the counterfactual simulations below, we expand on this in detail to quantify the contributions of each primitive to earnings inequality.

For comparison, Table 3 also presents variance decomposition results obtained under three alternative approaches. In Column 2, we estimate the earnings equation (4.9) without residualizing worker earnings by firm innovations in wage bills, which eliminates the time-varying component of the firm effect $\hat{f}_{it}$. This has a negligible effect on the variance shares of the remaining components. In Column 3, we repeat the approach in Column 2 but set $\theta_i = 1$ for all firms, which eliminates production complementarities and hence the interaction term in equation (5.3). This leads to a slightly smaller share for the firm effect. In Column 4, we repeat the approach in Column 3 but estimate the earnings equation without grouping firms into earnings clusters, which is equivalent to the approach in Abowd et al. (1999). This increases the share of the firm effect and decreases the share of the sorting covariance, which is qualitatively similar to the effects of firm clustering reported in Bonhomme et al. (2019, 2020).

[28]For comparison, Lamadon et al. (2019) find the following variance shares using US data: worker effect variance, 71.6%; fixed firm effect variance, 4.3%; time-varying firm effect variance, 0.3%; sorting covariance, 13.0%; interactions, 0.9%; and residual, 10.0%. Hence, we find a slightly larger role for firm effects and sorting.

Table 3: Earnings variance decomposition results

share of earnings variance explained by:		(1)	(2)	(3)	(4)
1.	worker effect variance	57.0	56.6	56.8	58.7
2.	firm fixed effect variance	10.3	10.2	7.8	12.3
3.	time-varying firm effect variance	0.5	-	-	-
4.	sorting covariance	19.8	20.5	19.9	14.4
5.	interactions	-2.0	-2.1	-	-
6.	residual	14.4	14.8	15.5	14.6
	time-varying firm effects	yes	no	no	no
	production complementarities	yes	yes	no	no
	firm clustering	yes	yes	yes	no

Notes: This table presents variance decomposition results for worker earnings using the movers sample. Column 1 is based on the model-consistent specification in Equation (5.3). Columns 2-4 successively remove time-varying firm effects $\hat{f}_{it}$, production complementarities θ_i, and firm clustering respectively from the estimation procedure. Thus, Column 4 corresponds to estimates following the approach from Abowd et al. (1999), whereas Column 1-3 corresponds to estimate following the approach from Lamadon et al. (2019) and Bonhomme et al. (2019).

5.2.3 Amenities

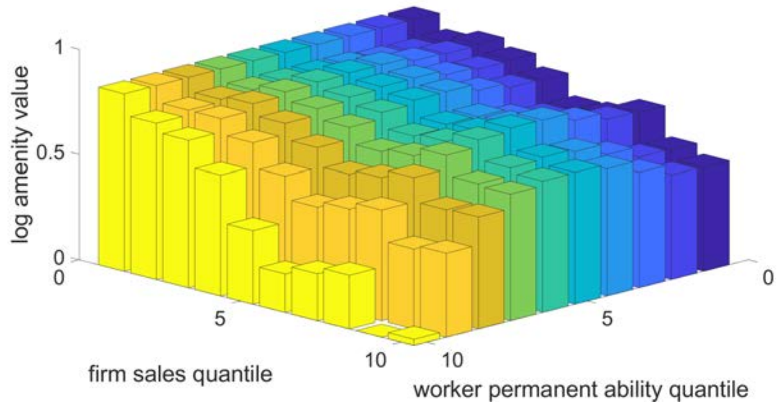
Amenities $g_i(\bar{a})$ are identified for each value of permanent worker ability $\bar{a}$ from equations (4.14) and (4.15). However, to reduce the dimension of the parameters that we estimate, we average log amenity values across workers in each of 50 quantiles of $\bar{a}$. These estimates are shown in Figure 1, where we further average log amenity values by deciles of firm sales and worker permanent ability for presentation purposes. We highlight two observations. First, for a given worker type, we find lower amenity values at larger firms. Second, this negative relationship is stronger for workers of higher ability.

Note that amenities can also be interpreted as residuals in employment shares that are not explained by observed wages. Therefore, our procedure for estimating amenities allows the model to fit the observed share of each worker type employed at each firm earnings cluster. The fit to shares constructed at the cluster level is shown in Figure 2, from which we observe the sorting of high-ability workers to firms with high wage premia (large values of $\bar{W}_i$).

5.2.4 Firm relationship capability and relationship-specific productivity

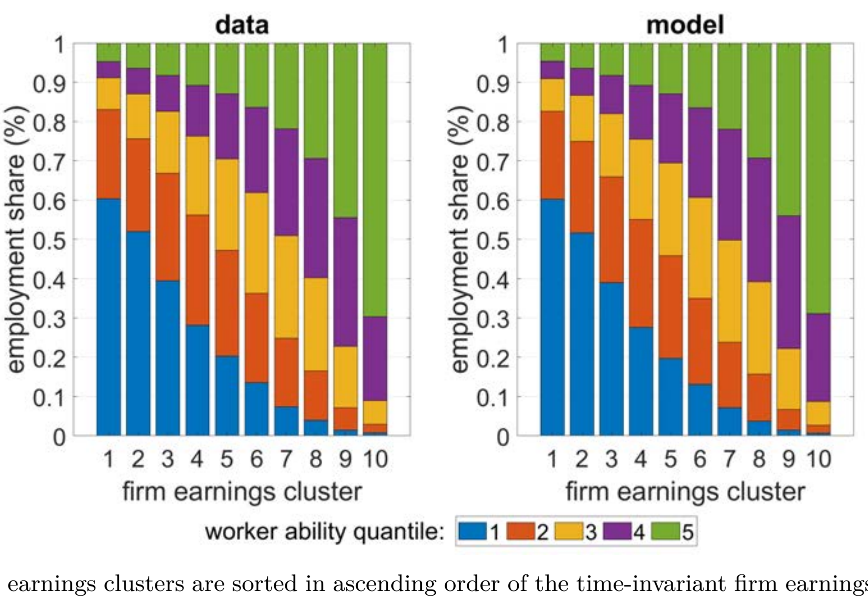
We estimate firm buyer effects $\tilde{\Delta}_{it}$, firm seller effects $\tilde{\Phi}_{it}$, and residual relationship productivities $\tilde{\psi}_{ijt}$ via a two-way fixed effects OLS regression based on equation (4.17). Details of the implementation are discussed in Appendix E.4. Of the total variance in log transaction values across all relationships, we find that 11.8% is explained by the buyer effect, 33.6% by the seller effect, -0.5% by the covariance of the seller and buyer effect and the remaining 55.1% by residual relationship productivity. Therefore, both firm-specific and relationship-specific characteristics

Figure 1: Distribution of Amenities



Notes: This figure shows the joint distribution of amenity estimates $\log g_i(\bar{a})$ by deciles of firm sales and worker permanent ability. Values are normalized for presentation purposes such that: (i) average log amenities within the smallest decile of firm sales are equal across deciles of worker permanent ability, and (ii) the smallest value of mean log amenities across sales-ability quantiles is equal to zero.

Figure 2: Model fit to employment shares by firm earnings cluster and worker ability



Notes: Firm earnings clusters are sorted in ascending order of the time-invariant firm earnings effect, $\bar{W}_i$.

are important determinants of variation in firm-to-firm sales.

With estimates of these effects in hand, we then recover firm relationship capabilities ψ_{it} using equation (4.19) and data on network sales shares s_{it}^{net}. As described in section 4.2.4, this approach only identifies ψ_{it} up to a constant. Hence, we calibrate the overall level of ψ_{it} to match the aggregate ratio of gross output to value-added in the sample.

5.2.5 Demand price elasticity

We estimate σ using the sample moment analog of the adjusted sales-profit ratio population moment in equation (4.20). Pooling observations across years, we find an average value of $\sigma = 4.2$, which implies an output markup of around 31%. We also investigate an alternative method for estimating σ, by choosing this parameter to match the aggregate profit share of sales directly in the model simulations. Using this approach, we find a similar estimate of $\sigma = 3.6$. These estimates are in line but on the low end of the range of typical values estimated in the literature, which is to be expected given that we constrain the product substitution elasticity to be same across all goods in the economy.[29]

5.2.6 Labor-materials substitution elasticity and labor productivity

We implement the approach in Doraszelski and Jaumandreu (2018) to estimate the labor-materials substitution elasticity ϵ and labor productivities ω_{it} from equation (4.21), which we repeat here for convenience:

$$\log \frac{E_{it}^M}{E_{it}^L} = \log \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \right] + (\epsilon - 1) \log \frac{W_{it}}{Z_{it}} + (1-\epsilon) F^{\omega} (\log \omega_{i,t-1}) + (1-\epsilon) \xi_{it}^{\omega} \tag{5.4}$$

Following the production function estimation literature, we adopt a control function approach to control for $F^{\omega}(\cdot)$, approximating this non-parametrically using a cubic polynomial in $\log \frac{E_{it-1}^M}{E_{it-1}^L}$ and $\log \frac{W_{it-1}}{Z_{it-1}}$. To instrument for $\log \frac{W_{it}}{Z_{it}}$, we use polynomials of one-period lagged input expenditures and factor prices of labor and materials, $\{E_{it-1}^M, E_{it-1}^L, W_{it-1}, Z_{it-1}\}$. For all factor prices, we use the estimated values of W_{it} and Z_{it}, as described in sections 4.2.2 and 4.2.6. Since there are many potential instruments available, we implement estimation using all possible combinations of the instruments and vary the order of the polynomials used. We then choose the specification that delivers a first-stage F-statistic greater than 10 and a p-value of the Hansen J test above 0.1. If there is more than one specification that satisfies these criteria, we choose the one with the highest F-statistic.

Table 4 presents our results. Our preferred specification based on the criteria above is shown

[29] For example, Broda and Weinstein (2006) find an average value of $\sigma = 4$ across SITC-3 product categories, estimated using trade data for the US between 1990 and 2001, and report that estimates of σ increase when using data at higher levels of disaggregation.

in Column 1. This specification uses quadratic polynomials in $\{E^M_{it-1}, E^L_{it-1}\}$ as instruments and delivers an estimate of $\epsilon = 1.5$ (s.e.=0.058). This implies that labor and materials are substitutes in the production function ($\epsilon > 1$), a result that holds with statistical significance. For comparison, we also present the estimates of ϵ that we obtain under other specifications. In Column 2, we use cubic polynomials in $\{E^M_{it-1}, E^L_{it-1}, W_{it-1}, Z_{it-1}\}$ as instruments instead of applying the instrument selection criteria discussed above. With this specification, we find that $\epsilon = 1.0$ (s.e.=0.027). In Column 3, we estimate W_{it} using the wage model and estimation strategy in Abowd et al. (1999), which does not address the issue of limited mobility bias. Applying the instrument selection criteria above, we use a linear polynomial in $\{E^M_{it-1}, E^L_{it-1}, W_{it-1}, Z_{it-1}\}$ as instruments and find $\epsilon = 1.6$ (s.e.=0.094), which is not statistically different from our preferred estimate in Column 1. Finally, in Column 4 we follow the standard approach in the literature of using average firm wages instead of the model-consistent firm-level wage W_{it}. Our instrument set in this case is comprised of quadratic polynomials in $\{W_{it-1}, Z_{it-1}\}$. We find $\epsilon = 1.05$ (s.e.=0.043), which is not statistically different from one. Note that this specification is not consistent with our theory since the model-consistent price index of labor is W_{it} and not the average wage.[30]

Despite differences in the estimates of ϵ across the four specifications in Table 4, we find in all cases that $\sigma > \epsilon$, which holds with statistical significance (recall from section 5.2.5 that we estimate $\sigma = 4.2$). This result has important implications for the counterfactual exercises that we study below, since from Proposition 1, it implies that reductions in material input costs Z_{it} have *positive* effects on wages.

[30] The previous literature has estimated a value of ϵ below one (Doraszelski and Jaumandreu, 2018; Oberfield and Raval, 2019). Doraszelski and Jaumandreu (2018) use the ratio of the average cost of labor to the average input cost as the main right-hand side variable. Thus, our estimates, which are based on constructed price indices, are not strictly comparable. Nevertheless, in Column 4, we move closer to the empirical specification in Doraszelski and Jaumandreu (2018) by using the average wage instead of our model-based labor price index. Our estimate of ϵ falls and becomes more similar to their estimates. However, some important differences remain. In particular, we do not observe the average intermediate input price in our production network dataset (since we only observe transaction values) and as such, we cannot replicate their exact specification. A further difference is that their sample is restricted to the manufacturing sector, whereas our sample spans all sectors.

Table 4: Estimation of Elasticity of Substitution between Materials and Labor (ϵ)

	$\log E^M/E^L$			
	(1)	(2)	(3)	(4)
$\log W/Z$	0.553	0.023	0.623	
	(0.058)	(0.027)	(0.094)	
$\log \bar{w}/Z$				0.052
				(0.043)
ϵ	1.55	1.02	1.62	1.05
Model for Wage Component	BLM	BLM	AKM	Average
Instruments	$\{E^M_{it-1}, E^L_{it-1}\}$	$\{E^M_{it-1}, E^L_{it-1}, W_{it-1}, Z_{it-1}\}$	$\{E^M_{it-1}, E^L_{it-1}, W_{it-1}, Z_{it-1}\}$	$\{W_{it-1}, Z_{it-1}\}$
Instrument Polynomial	Quadratic	Cubic	Linear	Quadratic
First Stage F-Stat	130	45	84	186
Hansen's J Test	0.121	0.000	0.379	0.003
Number of Observations	44,967	44,967	44,967	44,967

Notes: This table presents the results of the estimation of ϵ outlined in Section 4.2.6 by using the baseline firm-level dataset described in section 5.1. In particular, the table presents estimates of equation (5.4). Column 1, which is our preferred result because it is consistent with our theory and also satisfy the selection criteria of instruments that delivered an F-stat above 10 and a p-value of Hansen's J test above 0.1, presents estimates of equation (5.4). Column 2 presents estimates of equation (5.4) including all available instruments: expenditure and input prices with a cubic polynomial. Column 3 and 4 presents estimates from equation (5.4) but using different measures of labor price. Column 3 uses the AKM wage model to estimate W_{it} and Column 4 uses the average wage of the firm, as in Doraszelski and Jaumandreu (2018). Note that Column 4 presents a Hansen's J test p-value lower than 0.1. That happens because across all specifications using the average wage as the price index of labor, that is the highest Hansen's J test p-value achieved. All specifications are estimated with a two-stage GMM. A robust weighting matrix is used. Standard errors are shown in parentheses.

5.2.7 Firm TFPs

We estimate firm TFPs by fitting the reduced-form estimates of firm-level wages W_{it} as specified in equation (4.22). Since this requires numerical solution of a non-linear set of equations, we first perform a secondary clustering procedure to further reduce the dimensionality of the parameter space. We do this also in anticipation of the counterfactual simulations discussed below, which will require numerical solution of the general equilibrium model and hence necessitates a reduction in dimensionality from the 29 thousand firms in the baseline firm-level sample. Therefore, within each earnings cluster k, we again cluster firms into k' subclusters via a K-means clustering algorithm targeting the other primitives $\{\omega_{it}, \psi_{it}, \tilde{g}_i\}$ that have been estimated at the firm-level. For our baseline results, we use $k' = 10$ subclusters for a total of 100 firm cluster-subcluster pairs that we henceforth simply refer to as firm *groups*.

Table 5 shows the correlation matrix of our TFP estimates, other estimated firm primitives, and observed sales. These are computed at the firm group level, weighted by the number of firms in each group. We highlight four observations. First, we find a negative correlation between TFP and labor productivity, although TFP is positively correlated with the product $T\omega$.[31]

[31] We estimate similar standard deviations for log TFP and labor productivity, both around 1.8.

Note that one can interpret this product as a total measure of labor productivity and T as the productivity of material inputs alone. Hence, firms that use labor more efficiently also tend to tend to be firms that use materials more efficiently. Second, we find positive correlation between production complementarities θ and both TFP and labor productivity. Therefore, higher-ability workers tend to be more productive at firms that are also inherently more productive. Third, we find negative correlation between relationship productivity ψ and TFP, but positive correlation between ψ and ω. This implies that firms that are more productive do not necessarily have more productive relationships. Fourth, firm-level amenities are negatively correlated with all productivity primitives except ψ, which is consistent with Figure 1. Finally, T, $T\omega$, and θ are all positively correlated with sales, which is reassuring.

Table 5: Correlation Matrix of Firm Characteristics

	$\log \omega$	$\log T\omega$	$\log \psi$	θ	$\log \tilde{g}$	$\log R$
$\log T$	-0.59	0.30	-0.82	0.13	-0.08	0.46
$\log \omega$		0.60	0.41	0.36	-0.15	-0.03
$\log T\omega$			-0.32	0.56	-0.25	0.42
$\log \psi$				-0.18	0.05	-0.59
θ					-0.87	0.77
$\log \tilde{g}$						-0.60

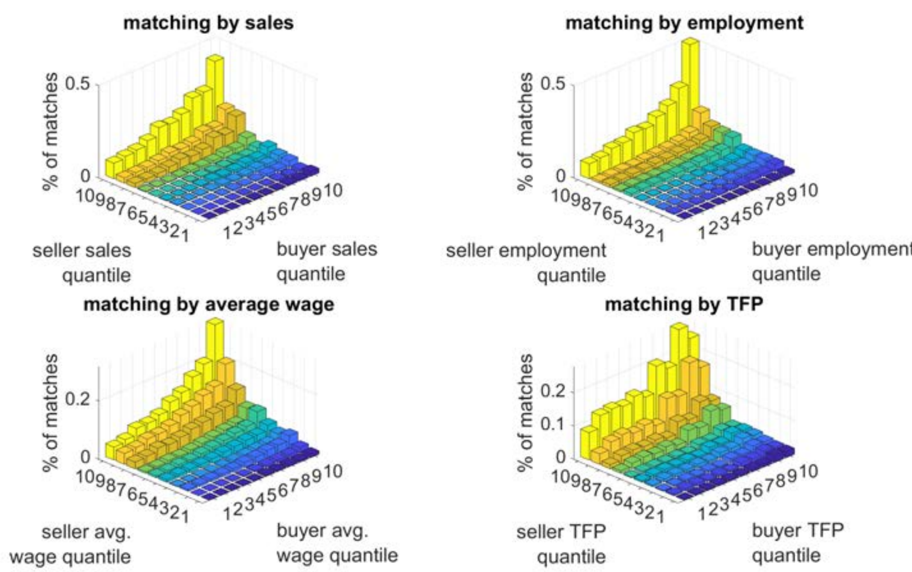
Notes: Correlations are computed at the firm group level weighted by the number of firms in each group.

5.2.8 The network

To quantify the importance of the production network for worker earnings, we measure the fraction of potential buyer-seller linkages between each pair of 100×100 firm groups in each year. We then take the average of this across years as our measure of the network $\{m_{ijt}\}_{i,j\in\Omega^F}$ in the counterfactual simulations that we study below. Figure 3 shows the patterns of matching that we observe in the network in terms of sales, employment, average wages, and TFP, where we further bin firms by decile on each variable for presentation purposes. We highlight two features of the network that are particularly important for understanding the effects of network heterogeneity on earnings inequality.

First, firms with larger sales, employment, average wages, and estimated TFPs tend to have more customers and suppliers. This dimension of heterogeneity in network connections is substantial. For example, firms in the largest TFP decile have around 3 times the number of suppliers as firms in the smallest TFP decile and 14 times the number of customers. These differences in the extensive margin of the network therefore *amplify* differences in own-firm characteristics, suggesting that network heterogeneity contributes positively to differences in mean earnings across firms and therefore to earnings inequality overall. Second, we observe negative assortative matching on sales, employment, TFP, and degree (number of customers or suppliers). For example, the average TFP of a firm's customers or suppliers is negatively correlated with the firm's own TFP. This stems from the fact that more productive firms sell to firms with both high and low TFP, whereas less productive firms sell mainly to high TFP firms. This heterogeneity in the intensive margin of the network therefore *dampens* heterogeneity in own-firm characteristics and suggests that network heterogeneity has a negative effect on earnings inequality. Hence, heterogeneity in the production network overall does not necessarily make firms more different from each other and does not mechanically induce greater earnings inequality. Rather, how network heterogeneity affects earnings inequality is a quantitative question.[32]

Figure 3: Firm-to-firm matching in the production network



Notes: Each subfigure shows the fraction of all potential buyer-seller relationships that are formed between each buyer-seller firm decile pair, where deciles are computed in terms of the indicated firm-level outcome.

5.3 Model fit

Table 6 shows the fit of the estimated model to key moments in the data. There are several important takeaways. First, the model matches observed aggregate factor shares well (panel (a)). This is by construction: the value-added share of sales is targeted through the mean of ψ_{ijt} (section 5.2.4), the aggregate labor share of value-added is targeted through σ (section 5.2.5),

[32]Negative matching on sales, employment, and degree has been documented in various other firm-to-firm datasets. See for example Bernard et al. (2018, 2019), Huneeus (2019), and Lim (2019). We also find weakly *positive* assortative matching on average wages in the data, which is consistent with findings reported by Demir et al. (2018).

and the labor-material cost ratio is targeted through the labor-materials substitution elasticity ϵ and labor productivities ω_{it} (section 5.2.6).

Second, since we estimate amenities by fitting observed employment shares (section 5.2.3) and firm TFP by fitting the firm effects in the earnings equation (section 5.2.7), the model closely replicates the observed earnings distribution (panel (b)). This is despite the fact that our amenities estimates are at a level coarser than the worker-firm level, which implies that the model does not perfectly replicate the observed assignment of individual workers to firms. In the counterfactual exercises below, we also show that the simulated model provides a good fit to the components of earnings variance in equation (5.3).

Finally, the model provides a reasonable fit to the dispersion in firm-level outcomes such as sales and wage bills, although the predicted moments have slightly lower variance (panel (c)). This is to be expected given that we restrict the estimates of firm production complementarities and TFP to vary only at the earnings cluster and group level respectively, thereby effectively abstracting from within-cluster variation. The model also slightly overpredicts the firm size-wage premium, which as discussed above is due to mismatch on sales rather than earnings. The correlation between sales and network statistics such as out-degree (number of customers) and in-degree (number of suppliers), on the other hand, matches more closely with the data.

Table 6: Fit of the model to aggregate, worker, and firm moments

(a) Aggregate			**(b) Worker earnings**			**(c) Firm-level**		
	data	model		data	model		data	model
labor share of VA	0.24	0.24	s.d.	0.75	0.80	sales, s.d.	1.60	1.27
VA share of sales	0.39	0.39	Gini coeff.	0.48	0.49	wage bill, s.d.	1.66	1.30
labor/material cost	0.13	0.15	90/10 ratio	7.10	7.37	mean wage, s.d.	0.57	0.50
			75/25 ratio	2.91	2.76	corr(sales, avg. wage)	0.53	0.73
			75/50 ratio	1.81	1.79	corr(sales, out-degree)	0.53	0.70
			50/25 ratio	1.61	1.54	corr(sales, in-degree)	0.78	0.75

Notes: Empirical moments are averages over years in 2005-2010. All variables are in logs except those expressed in shares or ratios.

6 Empirical Findings

6.1 Reduced-form Passthrough Evidence

Before using our estimated model to study the determinants of earnings inequality, we first provide reduced-form evidence to validate the model's predictions regarding the passthrough of firm-level shocks into worker earnings.

6.1.1 Deriving testable predictions

We begin by deriving testable predictions from the model that can be evaluated empirically. We focus here on the passthrough of shocks to demand D_{it} and material cost Z_{it}. Even though these variables are endogenous in our model, we will leverage below additional data on exports and imports by Chilean firms to construct plausibly exogenous shifters of demand and material costs.

First, consider the own-firm passthrough of demand shocks into earnings described in Proposition 1. Since the scale elasticities $\Gamma_{it} \equiv \frac{1}{\gamma+\sigma\left(1-s_{it}^{M}\right)+\epsilon s_{it}^{M}}$ vary by firm in theory, the model predicts firm-specific passthrough coefficients, which are difficult to validate empirically. However, our estimates of $\{\gamma, \sigma, \epsilon\}$ and the observed variation in material cost shares s_{it}^{M} imply that variation in Γ_{it} should be minimal in practice: the estimated scale elasticity evaluated at the average material cost share is $\bar{\Gamma} = 13\%$, while the interquartile range for the scale elasticity is $[12.8\%, 13.2\%]$. Hence, in what follows, we approximate the scale elasticity for each firm as $\Gamma_{it} \approx \bar{\Gamma}$. We also denote by $\hat{Y}_{it}$ the marginal log change in a firm-level variable Y_{it}. The own-firm passthrough of material cost shocks can then be expressed as:

$$\hat{W}_{it}^{Z,own} \approx \beta^{Z,own} s_{it}^{M} \hat{Z}_{it} \tag{6.1}$$

where $\hat{W}_{it}^{Z,own}$ denotes the marginal log change in the firm-level wage for firm i in response to a marginal log change in the firm's own material cost, $\hat{Z}_{it}$. Recall that the passthrough coefficient is given by equation (3.1) as $\beta^{Z,own} \equiv -\left(\sigma-\epsilon\right)\bar{\Gamma}$, which is negative under our estimated parameter values. Similarly, we can approximate the own-firm passthrough of demand shocks as:

$$\hat{W}_{it}^{D,own} \approx \beta^{D,own} \hat{D}_{it} \tag{6.2}$$

where the passthrough coefficient is given by $\beta^{D,own} \equiv \bar{\Gamma} > 0$. We summarize these results regarding own-passthrough with the following pair of testable predictions.

Prediction 1. *An exogenous increase in material cost for a firm weighted by its material cost share lowers the earnings of its workers ($\beta^{Z,own} < 0$).*

Prediction 2. *An exogenous increase in demand for a firm raises the earnings of its workers ($\beta^{D,own} > 0$).*

Next, combining the results of Propositions 1 and 2, the passthrough of material cost shocks from a firm's immediate suppliers can be approximated as:

$$\hat{W}_{it}^{Z,sup} \approx \beta^{Z,sup} s_{it}^{M} \hat{Z}_{it}^{sup} \tag{6.3}$$

with the passthrough coefficient given by $\beta^{Z,sup} \equiv -\left(\gamma+\epsilon\right)\left(\sigma-\epsilon\right)\bar{\Gamma}^{2}$. Here, we define $\hat{Z}_{it}^{sup}$ as the *supplier material cost shock* for firm i:

$$\hat{Z}_{it}^{sup} \equiv \sum_{j \in \Omega_{it}^{S}} s_{ijt}^{mat} s_{jt}^{M} \hat{Z}_{jt} \tag{6.4}$$

Similarly, combining the results of Propositions 1 and 3, the passthrough of demand shocks from immediate customers can be approximated as:

$$\hat{W}_{it}^{D,cus} \approx \beta^{D,cus} \hat{D}_{it}^{cus} \tag{6.5}$$

with the passthrough coefficient given by $\beta^{D,cus} \equiv (\gamma + \epsilon) \bar{\Gamma}^2$. Here, we define $\hat{D}_{it}^{cus}$ as the *customer demand shock* for firm i:

$$\hat{D}_{it}^{cus} \equiv \sum_{j \in \Omega_{it}^{C}} s_{jit}^{sales} \hat{D}_{jt} \tag{6.6}$$

We summarize these results regarding passthrough from suppliers and customers with the following pair of testable predictions.

Prediction 3. *An exogenous increase in the supplier material cost shock for a firm weighted by its material cost share lowers the earnings of the firm's workers ($\beta^{Z,sup} < 0$).*

Prediction 4. *An exogenous increase in the customer demand shock for a firm raises the earnings of its workers ($\beta^{D,cus} > 0$).*

We now turn towards testing the theoretical predictions described above using our data. This first requires the construction of firm-level demand shocks $\hat{D}_{it}$ and material cost shocks $\hat{Z}_{it}$. To accomplish this, we rely on firms that participate directly in exporting and importing. Although we do not explicitly account for international trade in the model, we adopt this approach solely to leverage foreign demand and cost shocks as sources of exogenous variation in testing the model's predictions. For this purpose, we first define an international trade market m as a product-by-foreign-country pair. We then construct the following Bartik shift-share measure of export demand:

$$\hat{\Delta}_{it}^{X} \equiv \sum_{m \in \Omega_{i1}^{M,X}} s_{mi1}^{X} \hat{s}_{mt}^{I} \tag{6.7}$$

where $\Omega_{i1}^{M,X}$ denotes the markets in which firm i actively exports in the first year of our sample, s_{mi1}^{X} denotes the share of firm i's exports accounted for by market m in the first year of our sample, and $\hat{s}_{mt}^{I}$ denotes the annual log change in market m's share of world imports from all source countries excluding Chile within the corresponding product category. Intuitively, if a Chilean firm initially exports to markets that subsequently become more important sources of demand for imports from countries other than Chile, we interpret this as an increase in export demand for the Chilean firm.

Similarly, we construct the following shift-share measure of import cost shocks:

$$\hat{p}^I_{it} \equiv - \sum_{m \in \Omega^{M,I}_{i1}} s^I_{im1} \hat{s}^X_{mt} \tag{6.8}$$

where $\Omega^{M,I}_{i1}$ denotes the markets in which firm i actively imports in the first year of our sample, s^I_{im1} denotes the share of firm i's imports accounted for by market m in the first year of our sample, and $\hat{s}^X_{mt}$ denotes the annual log change in market m's share of world exports to all destination countries excluding Chile within the corresponding product category. Here, we interpret the change in market m's share of world exports as reflecting changes in its production costs. Hence, if a Chilean firm initially imports from markets that subsequently become more important suppliers of exports to countries other than Chile, we interpret this as a decline in the cost of imports for the Chilean firm.

In Appendix F, we describe a simple extension of the model in which the shocks $\hat{\Delta}^X_{it}$ and $\hat{p}^I_{it}$ constructed above are valid exogenous shifters of demand D_{it} for exporting firms and of material costs Z_{it} for importing firms respectively. Specifically, these are related by:

$$\hat{D}^X_{it} \equiv s^{sales}_{Xit} \hat{\Delta}^X_{it} \tag{6.9}$$

$$\hat{Z}^I_{it} \equiv s^{mat}_{iIt} \hat{p}^I_{it} \tag{6.10}$$

where $\hat{D}^X_{it}$ denotes the change firm i's demand arising from a change in its export demand, $\hat{Z}^I_{it}$ denotes the change in firm i's material cost arising from a change in the cost of its imports, s^{sales}_{Xit} is the share of firm i's sales accounted for by exports, and s^{mat}_{iIt} is the share of firm i's expenditure on materials accounted for by imports.[33] Using these firm-level demand and material cost shocks, we then construct the supplier material cost shock and customer demand shock as specified in equations (6.4) and (6.6).

To test the theoretical passthrough predictions described in section 6.1.1, we then estimate the following regression:

$$\hat{Y}_{it} = \underbrace{\alpha^{Z,own} s^M_{it} \hat{Z}_{it}}_{\text{own cost}} + \underbrace{\alpha^{D,own} \hat{D}_{it}}_{\text{own demand}} + \underbrace{\alpha^{Z,sup} s^M_{it} \hat{Z}^{sup}_{it}}_{\text{supplier cost}} + \underbrace{\alpha^{D,cus} \hat{D}^{cus}_{it}}_{\text{customer demand}} + f_{\text{ind}(i)} + \varepsilon_{it} \tag{6.11}$$

where $\hat{Y}_{it}$ is the log change in an outcome of interest and $f_{\text{ind}(i)}$ is an industry fixed effect corresponding to the industry ind (i) of firm i. The residual ε_{it} accounts for changes in the firm effect W_{it} arising from shocks other than $\left\{\hat{D}^X_{it}, \hat{Z}^I_{it}\right\}_{i \in \Omega^F}$ – for example, fluctuations in TFP and labor productivities – which by construction are orthogonal to the regressors. The residual also

[33] Note that s^{sales}_{Xit} and s^{mat}_{iIt} are analogous to first-stage regression coefficients in an instrumental variables specification where $\hat{\Delta}^X_{it}$ and $\hat{p}^I_{it}$ serve as instruments for $\hat{D}_{it}$ and $\hat{Z}_{it}$ respectively.

captures passthrough of shocks via indirect customer and supplier relationships, which we treat as negligible for the purposes of estimation. Hence, we estimate the α coefficients in equation (6.11) via ordinary least-squares, using long differences between 2011 and 2016.

Note that the theoretical passthrough coefficients discussed in section 6.1.1 relate to the effect of demand and material cost shocks on the firm effect W_{it}. However, demand and material cost shocks also affect firm wage bills only through changes in the firm effect (see equation (2.28)). Since we can measure wage bills directly in the data whereas firm effects must be estimated, we test the model's predictions regarding passthrough using changes in firm wage bills as the main outcome of interest. Our estimated structural model then predicts the following signs of the regression coefficients in equation (6.11): $\alpha^{Z,own} < 0$ (Prediction 1); $\alpha^{D,own} > 0$ (Prediction 2); $\alpha^{Z,sup} < 0$ (Prediction 3); and $\alpha^{D,cus} > 0$ (Prediction 4).

Column (1) of Table 7 shows our reduced-form estimates of the passthrough coefficients in equation (6.11) treating $\hat{Y}_{it}$ as the log change in firm i's wage bill. For comparison, we also include results for specifications where the outcome of interest is the log change in a firm's average wage (column (2)) and the log change in a firm's sales (column (3)). We highlight the following takeaways. First, we estimate a negative and statistically significant effect of own cost shocks on the firm wage bill (row A, column (1)), which is consistent with Prediction 1. Own cost shocks also have a negative effect on a firm's average wage and sales (row A, columns (2) and (3)), although the former effect is not found to be statistically significant. Second, we find suggestive evidence in favor of Prediction 2, estimating positive effects of demand shocks on average wages and sales (row B, columns (2) and (3)), although these estimates are imprecise. Third, in support of Prediction 3, we estimate a negative and statistically significant effect of supplier cost shocks on the firm wage bill (row C, column (1)). We also find negative effects of supplier cost shocks on average wages and firm sales. Finally, we find a positive and significant effect on wage bills arising from demand shocks to a firm's customers (row D, column (1)), which is consistent with Prediction 4.[34] In sum, we find evidence of cost and demand shock passthrough into changes in earnings that is broadly consistent with the predictions of our estimated structural model.

[34]We also note that evidence of positive passthrough from both own demand shocks and customer demand shocks has been documented by Dhyne et al. (2021) using data from Belgium.

Table 7: Reduced-form passthrough estimates

	(1) Wage Bill	(2) Average Wage	(3) Sales
A. own cost shock, $\alpha^{Z,own}$	-0.35	-0.10	-0.41
	(0.15)	(0.09)	(0.17)
B. own demand shock, $\alpha^{D,own}$	-0.03	0.11	0.10
	(0.13)	(0.08)	(0.15)
C. supplier cost shock, $\alpha^{Z,sup}$	-0.18	-0.16	-0.69
	(0.11)	(0.07)	(0.14)
D. customer demand shock, $\alpha^{D,cus}$	0.13	0.04	0.15
	(0.05)	(0.03)	(0.06)
sector fixed effects	yes	yes	yes
N	63,967	63,967	58,448

Notes: This table presents our estimates of the passthrough coefficients in equation (6.11) for different outcome variables $\hat{Y}_{it}$. Each of the outcome variables in columns (1)-(3) are measured in logs. Standard errors are shown in parentheses.

6.2 The sources of earnings inequality

The variance decomposition of earnings presented in Table 3 provides evidence that firms are quantitatively important in shaping earnings inequality. Since the firm effect f_{it} in this decomposition is determined endogenously by model primitives, however, the question of *why* firms matter for earnings inequality requires further analysis. In particular, what explains the variance of the firm effect (which accounts for 10.8% of earnings variance) and the covariance between the worker and firm effects (which accounts for 19.8%)? We shed light on these questions by using the estimated structural model to further decompose the firm effects f_{it} in the earnings equation (5.1). These effects are endogenously determined by firm primitives, including network connections, and matter for earnings inequality through both $\text{var}\left(f_{it}\right)$ and the sorting covariance $\text{cov}\left(x_m, f_{it}\right)$.

To begin, note that all heterogeneity in worker earnings w_{imt} is accounted for in the model by heterogeneity in the following primitives: (i) the extensive and intensive margins of the production network, $\{m_{ijt}, \psi_{ijt}\}$; (ii) firm productivities, $\{T_{it}, \omega_{it}\}$; (iii) production complementarities, θ_i; (iv) amenities, $g_i\left(\cdot\right)$; (v) permanent worker ability, $\bar{a}_m$; and (vi) transient worker ability, $\hat{a}_{mt}$. To quantify the contribution of each set of primitives to earnings inequality, we then simulate counterfactual equilibria of the model in which each dimension of heterogeneity is eliminated by replacing the relevant parameters by their corresponding means across firms or workers. For example, to quantify the importance of heterogeneity in supplier matching for earnings inequality, we replace the observed network $\{m_{ijt}, \psi_{ijt}\}$ with a counterfactual network $\left\{\hat{m}^S_{ijt}, \hat{\psi}^S_{ijt}\right\}$ that is randomized across suppliers while holding constant the total supplier count and mean relationship productivity across suppliers for each firm:

$$\hat{m}^S_{ijt} = \frac{1}{|\Omega^F|} \sum_{j \in \Omega^F} m_{ijt}, \qquad \log \hat{\psi}_{ijt} = \frac{1}{|\Omega^F|} \sum_{j \in \Omega^F} \log \psi_{ijt} \tag{6.12}$$

We follow an analogous procedure to quantify the importance of heterogeneity in customer matching and in the other model primitives listed above.[35]

Note that eliminating heterogeneity in a given set of primitives Θ not only removes the contribution to earnings variance arising from $\text{var}(\Theta)$ but also from the covariance between Θ and all other sets of primitives. Therefore, the reduction in earnings variance that arises from eliminating heterogeneity in Θ cannot be attributed to Θ alone. To address this, we simulate counterfactuals by eliminating *all* possible combinations of heterogeneity in the primitives listed above and then compute the Shapley value for each primitive in terms of its effect on earnings variance.[36] Intuitively, this provides an average measure of the reduction in earnings variance when heterogeneity in one set of primitives is eliminated under all possible combinations of heterogeneity in the remaining primitives. This procedure can hence be viewed as a generalization of the reduced-form variance decomposition exercise described in section 4.2.2 that allows us to quantify the share of variance accounted for by high-dimensional objects such as the production network. The formal definition of the Shapley value is provided in Appendix G.

To provide context, note that this exercise is similar in spirit to the structural counterfactuals in Lamadon et al. (2019), which examine how the firm effect variance and sorting covariance depend on heterogeneity in firm TFP and amenities. The key difference is that we expand the set of firm primitives to include heterogeneous production network linkages. This allows us to quantify the contribution of network heterogeneity to the components of earnings variance relative to other dimensions of heterogeneity such as TFP. The inclusion of the network in the set of primitives also introduces a technical challenge: unlike the model in Lamadon et al. (2019), which is log-linear in TFP T_{it} and the sorting composite $\tilde{\phi}_i$, the dependence of the firm effect f_{it} on the underlying firm primitives in our model does not admit an additively separable representation. This arises from the non-linearities in the input-output structure of the network. Hence, we require an iterative numerical solution procedure. Since this is not computationally

[35] In each counterfactual simulation, we also hold constant the aggregate ratio of gross output to value-added by recalibrating the grand mean of relationship productivity ψ_{ijt} (see section 5.2.4). This keeps the overall importance of materials relative to labor constant as various dimensions of primitive heterogeneity are eliminated.

[36] To illustrate, consider two univariate primitives, Θ_A and Θ_B, and suppose the variance of earnings in the baseline can be expressed as $\text{var}(\Theta_A) + \text{var}(\Theta_B) + 2\text{cov}(\Theta_A, \Theta_B)$. The change in earnings variance from eliminating heterogeneity in Θ_A relative to the baseline is $\delta_{A1} = \text{var}(\Theta_A) + 2\text{cov}(\Theta_A, \Theta_B)$. The change in earnings variance from eliminating heterogeneity in Θ_A relative to the equilibrium in which heterogeneity in Θ_B has already been eliminated is $\delta_{A2} = \text{var}(\Theta_A)$. The Shapley contribution of Θ_A to earnings variance is then $\frac{\delta_{A1}+\delta_{A2}}{2} = \text{var}(\Theta_A) + \text{cov}(\Theta_A, \Theta_B)$. The Shapley approach is therefore equivalent to splitting the covariance equally between Θ_A and Θ_B in this univariate linear case, but generalizes the variance decomposition of the form in equation (5.3) to cases where Θ_A is high-dimensional (for example, the production network) and where the dependence of earnings is not linear in primitives.

feasible at the level of individual workers and firms, we group workers into 50×50 quantiles by permanent and transient ability and aggregate firms to the group level as described in section 5.2.7. The details of the numerical solution algorithm are provided in Appendix H.

Table 8 presents our findings. The first two rows compare the results of the earnings variance decomposition in equation (5.3) from the raw data (as documented in Column 1 of Table 3) and in the baseline of our simulated model. There are two reasons for potential discrepancies between these two sets of values. First, the empirical decomposition uses data at the level of individual workers and only aggregates firms only by earnings cluster, whereas the model simulations aggregate both workers and firms into groups (as described in the preface to this section). Second, given the restriction on amenities $g_i(\cdot)$ described in section 4.2.3, the model does not perfectly rationalize employment shares of different worker ability types within an earnings cluster. Nonetheless, we see that the model provides a good fit to the empirical variance decomposition shares.

Column 1 shows the share of total earnings variance accounted for by each set of model primitives (rows a-g). Similar to the results from the preliminary decomposition in Table 3, we find that permanent worker ability (row a) accounts for just over half of log earnings variance, while transient worker ability (row b) accounts for 13.8%. Firm-specific primitives (rows c-g) account for the remaining 32.4% of earnings variance. In particular, we find network heterogeneity to be a key driver of earnings inequality: heterogeneity in upstream connections with suppliers accounts for 11.9% of log earnings variance (row c), while heterogeneity in downstream connections with customers explains 8.6% (row d). Network heterogeneity overall therefore explains approximately 60% of the firm primitive share of earnings variance (20.5% out of 32.4%). The remaining firm-specific primitives (rows e-g) jointly account for 11.9% of log earnings variance, with more important roles for TFP, labor productivity, and production complementarity than for amenities.

Our finding that network heterogeneity explains the majority of the firm primitive share of earnings variance mirrors the results in Bernard et al. (2019), who report that more than half of the variance in log firm sales is explained by heterogeneity in network connections. Hence, our findings are consistent with evidence that network connections matter for firm-level outcomes in general and highlight the importance of accounting for network heterogeneity, since failing to do so will load heterogeneity in networks onto other factors like TFP.

To provide further insight into the role of each set of primitives in shaping earnings inequality, we also report in Columns 2-5 of Table 8 the share of each component of earnings (as defined in equation (5.1)) accounted for by various model primitives.[37] Naturally, worker ability accounts for almost all of the variance in the worker effect (Column 2).[38] Interestingly, our results reveal

[37]Since the model-based variance decomposition is purely cross-sectional, we do not distinguish between the fixed component $\bar{f}_i$ and the innovation component $\hat{f}_{it}$ of the firm effect f_{it}.

[38]Given the definition of the worker effect in (5.2), changes in firm primitives such as TFP and labor produc-

that network heterogeneity explains almost all of the variance of the firm effect (Column 3). However, this heterogeneity matters less for sorting (Column 4). This indicates that good network connections are important for how much a firm pays its workers overall, but less so for determining the types of workers that sort to a firm. Finally, we see that network heterogeneity is not important for interactions (Column 5), although this component contributes little to overall earnings variance to begin with.

Table 8: Earnings variance decomposition results

	(1) earnings variance	(2) worker effect variance	(3) firm effect variance	(4) sorting covariance	(5) interactions
share of earnings variance (data)	100	57.0	10.8	19.8	-2.0
share of earnings variance (model)	100	52.5	9.8	20.8	3.1
of which:					
a. worker permanent ability, $\bar{a}_m$	53.8	48.6	-1.5	4.1	2.6
b. worker transient ability, $\hat{a}_{mt}$	13.8	-	-	-	-
c. supplier network, $\{m_{ijt}, \psi_{ijt}\}_{j\in\Omega_{it}^S}$	11.9	0.9	7.9	2.7	0.4
d. customer network, $\{m_{jit}, \psi_{jit}\}_{j\in\Omega_{it}^C}$	8.6	-0.1	6.7	1.5	0.4
e. firm productivities, $\{T_{it}, \omega_{it}\}$	6.1	7.5	-4.3	3.3	-0.5
f. production complementarities, θ_i	4.6	-4.0	-2.7	8.6	2.6
g. amenities, $g_i(\cdot)$	1.2	-0.4	3.6	0.5	-2.6

Notes: The first two rows show results from the earnings variance decomposition of equation (5.3) in the data and baseline model simulation. Values in the first row are the same as in Column 1 of Table 3. Subsequent rows show the share of earnings variance (Column 1) and each component of earnings variance (Columns 2-5) that are accounted for by each set of model primitives. Variance shares are computed using the Shapley approach described in Appendix G. Units are in percentage points.

6.3 The distribution of worker and firm rents

Finally, in addition to earnings, we examine how the production network shapes the division of rents between firms and workers. We first treat the profit earned by firm i as our measure of rent for the firm:

$$\mathbb{R}_{it}^{firm} = \pi_{it} \tag{6.13}$$

For workers, we follow Lamadon et al. (2019) in defining the rent $\mathbb{R}_{it}^{worker}(a)$ for a worker of ability a employed at firm i as the reduction in earnings at firm i that would make the worker indifferent from employment at her second-best choice of firm. As shown in Lamadon et al. (2019), the rent for the average worker at firm i is proportional to the average wage $\bar{w}_{it}$ paid to

tivity also affect the worker effect through the term $\bar{\theta}$, which is computed as an average across *workers* rather than *firms*. Hence, changes in firm primitives that affect the allocation of workers across firms also directly change the worker effect.

workers at the firm:

$$\bar{\mathbb{R}}_{it}^{worker} \equiv \frac{\sum_{a \in A} \mathbb{R}_{it}^{worker}(a)}{\sum_{a \in A} L_{it}(a)} = \frac{1}{1+\gamma} \bar{w}_{it} \tag{6.14}$$

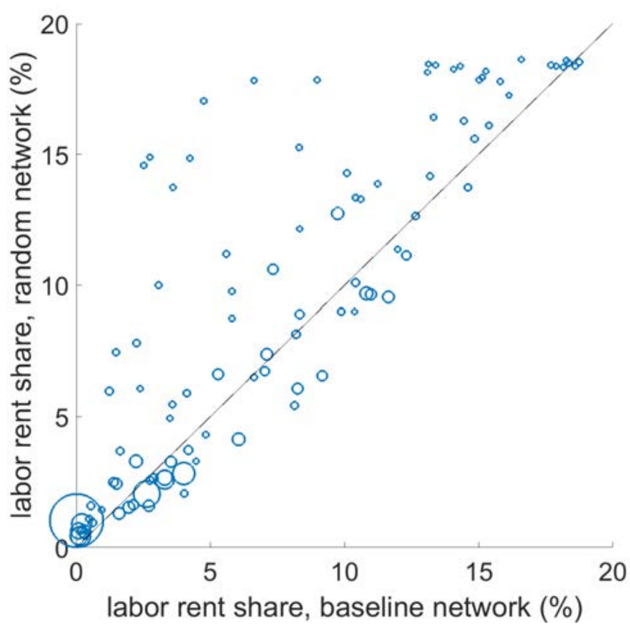
Total worker rents at firm i are then proportional to the wage bill of the firm:

$$\mathbb{R}_{it}^{worker} = \frac{1}{1+\gamma} E_{it}^{L} \tag{6.15}$$

Note that in the limit as $\gamma \to \infty$, there is no horizontal differentiation of employers based on workers' idiosyncratic preference shocks and hence worker rent approaches zero.

Figure 4 shows the labor rent share $s_{it}^{\mathbb{R}} \equiv \frac{\mathbb{R}_{it}^{worker}}{\mathbb{R}_{it}^{worker} + \mathbb{R}_{it}^{firm}}$ for each group of firms in our sample under the baseline equilibrium (x-axis) and under a counterfactual equilibrium in which the production network is completely randomized as described above (y-axis), where the size of each marker in the figure is proportional to the share of sales accounted for by each firm group. Note that this figure is also informative about variation in labor shares of value-added, since labor rent and value-added shares are related through $s_{it}^{\mathbb{R}} = \frac{E_{it}^{L}/VA_{it}}{1+\gamma-\gamma\left(E_{it}^{L}/VA_{it}\right)}$ where $VA_{it} \equiv E_{it}^{L} + \pi_{it}$ is value-added for firm i. We highlight two key takeaways.

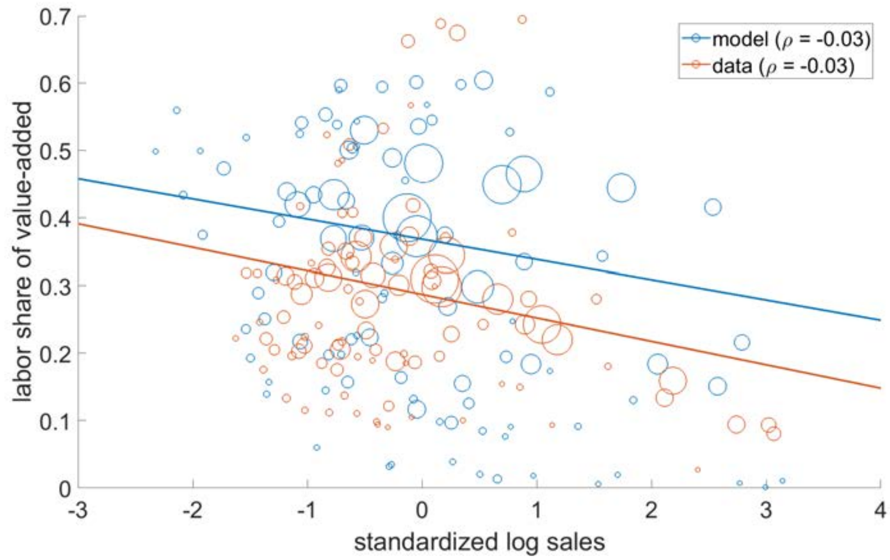
Figure 4: Labor share of rent under baseline network versus random network



Notes: Each marker in the figure represents a firm group, with the size of each marker proportional to the share of sales accounted for by each firm group. The labor rent share for a firm i is computed as $\frac{\mathbb{R}_{it}^{worker}}{\mathbb{R}_{it}^{worker} + \mathbb{R}_{it}^{firm}}$, where $\mathbb{R}_{it}^{firm}$ and $\mathbb{R}_{it}^{worker}$ are as defined by equations (6.13) and (6.15) respectively.

First, larger firms tend to have lower labor rent shares compared with smaller firms. This pattern is shown directly in Figure 5, where both our model and data indicate that a one standard deviation increase in log firm sales is associated with a decline in the labor value-added share by three percentage points. As indicated in the introduction, this pattern is also consistent with recent empirical findings in the literature that have documented a negative relationship between firm size and labor value-added shares.

Figure 5: Relationship between firm size and labor share of value-added



Notes: Each marker in the figure represents a firm group, with the size of each marker proportional to the share of employment accounted for by each firm group. Log sales are standardized by subtracting the mean of log sales and dividing by the standard deviation of log sales. The labor rent share for a firm i is computed as $\frac{\mathbb{R}_{it}^{worker}}{\mathbb{R}_{it}^{worker}+\mathbb{R}_{it}^{firm}}$, where $\mathbb{R}_{it}^{firm}$ and $\mathbb{R}_{it}^{worker}$ are as defined by equations (6.13) and (6.15) respectively. The coefficient ρ denotes the estimated slope from a regression of the labor value-added share on standardized log sales at the firm group level, where each group is weighted by the share of employment accounted for by the group.

As discussed in section 3.3, our model implies a tight link between labor shares of value-added and labor shares of cost (equation (3.12)), where the latter depends on the productivity-adjusted cost of labor relative to materials, $\frac{W_{it}/\omega_{it}}{Z_{it}}$ (equation (3.13)). In our data, we find that larger firms tend to spend a smaller fraction of their input costs on labor relative to materials and hence have lower labor rent and value-added shares in the baseline. On one hand, larger firms tend to have higher productivity-adjusted labor costs, with the employment-weighted correlation between $\log R_{it}$ and $\log \frac{W_{it}}{\omega_{it}}$ equal to 0.24. This partially reflects the positive firm-size premium on wages and stems from the fact that larger firms must pay higher wages to attract more workers (conditional on amenities). This tends to generate a negative relationship between firm size and labor cost shares (given that we estimate $\epsilon > 1$, so that labor and materials are gross substitutes). On the other hand, we find that larger firms also tend to have higher material input costs, with the employment-weighted correlation between $\log R_{it}$ and $\log Z_{it}$ equal to 0.28. This is because even though smaller firms have fewer production network linkages than larger firms, they also tend to have better relationship capabilities and to match with suppliers that have lower costs on average.[39] This pattern tends to generate a positive relationship between firm size and labor cost shares.

Empirically, we find that most of the variation in $\frac{W_{it}/\omega_{it}}{Z_{it}}$ across firms is driven by differences in $\frac{W_{it}}{\omega_{it}}$ rather than differences in Z_{it}. Hence, the positive covariance between firm size and $\frac{W_{it}}{\omega_{it}}$ dominates the positive covariance between firm size and Z_{it}, such that larger firms tend to have higher productivity-adjusted costs of labor relative to materials compared with smaller firms. Consequently, larger firms have lower labor cost and rent shares, with the employment-weighted correlation between $\log R_{it}$ and $s_{it}^{\mathbb{R}}$ equal to -0.28.

The second key takeaway from Figure 4 is that eliminating heterogeneity in the production network *increases* both the dispersion of the labor rent share across firms as well as the negative correlation between firm size and the labor rent share. Specifically, randomizing the production network increases the employment-weighted standard deviation of the labor rent share across firms by 16% (from 4.6 to 5.4 percentage points), while the correlation between $\log R_{it}$ and $s_{it}^{\mathbb{R}}$ falls from -0.28 to -0.33. As discussed above, this occurs because production network heterogeneity confers a material cost advantage on smaller firms, leading these firms to have higher material shares of cost and lower labor shares of rent and value-added in the baseline. When the production network is randomized, this cost advantage disappears, causing a decline in the labor rent share for larger firms and an increase in the labor rent share for smaller firms. In sum, while production network heterogeneity contributes positively to earnings inequality across workers, it in fact dampens both inequality in labor rent shares across firms as well as the negative relationship between firm size and labor rent shares.

7 Conclusion

Matched employer-employee and firm-to-firm transactions datasets have attracted substantial interest from researchers in recent years, but these have largely been studied in isolation from each other. We have argued in this paper that the ability to link these two rich sources of data offer novel and important insights into a fundamental set of economic questions, and have provided a unified quantitative framework linking a theory of labor markets with a theory of production networks to uncover these insights.

Our analysis establishes that production network linkages matter for the passthrough of firm-level demand and cost shocks into changes in worker earnings, with positive demand shocks

[39] Such patterns of negative assortative matching between firms have been documented in various other contexts. See for example Bernard et al. (2018).

to a firm's customers having positive effects on the wage bill of the firm and negative input cost shocks to a firm's suppliers having negative effects on wage bills. Heterogeneity in network linkages also matters for earnings inequality, explaining 21% of log earnings variance in total, with upstream heterogeneity accounting for 12% and downstream heterogeneity accounting for 9%. Finally, firm heterogeneity in labor shares of rent and value-added are less dispersed and less negatively correlated with firm size under the observed heterogeneity in the production network than under a purely random production network.

We conclude with four potential directions for future research on the interaction between workers and production networks. First, there is growing evidence that worker outsourcing is a key driver of increases in earnings inequality (Goldschmidt and Schmieder (2017)). However, there are as yet no studies documenting such evidence where both worker flows between firms and firm-to-firm linkages are simultaneously observed. The ability to observe these jointly will allow for a refinement of the definition of outsourcing and hence of the study of its effects on worker earnings. For example, worker transitions between linked buyers and sellers may differ fundamentally in both cause and effect from worker transitions between unrelated firms.

Second, there is growing interest among both policymakers and researchers in understanding the effects of automation on worker outcomes. It is natural to view these effects as arising from the substitution of labor by inputs such as industrial robots. For example, Acemoglu and Restrepo (2020) estimate the effects of increased robot usage on employment and wages in US labor markets, finding robust negative effects. More recent theoretical work by Jackson and Kanik (2020) develops a model of robot-labor substitution that accounts for production network linkages between firms. A quantitative study of the mechanisms highlighted by this literature using matched employer-employee and firm-to-firm transactions data is therefore likely to yield important insights.

Third, an emerging literature has emphasized the importance of production network linkages for determining optimal industrial policy (Liu (2019)). However, this literature has largely focused on outcomes in product markets such as sales and aggregate output, while abstracting from labor market frictions. The framework that we have developed in this paper offers a natural starting point for the extension of such policy analyses to consider implications for heterogeneous workers, in a context with imperfect competition in labor markets and production network linkages.

Finally, while we consider in this paper how changes in the production network structure affect worker earnings, there is also nascent evidence that worker flows between firms shape the formation of network linkages. For example, Patault and Lenoir (2020) document using French data that movements of sales managers across firms induce the formation of new buyer-seller relationships. This evidence points toward the need for a better understanding of the economic determinants of both worker transitions and firm-to-firm relationship formation, which linked employer-employee and firm-to-firm transactions data are well-suited to examine.

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A Model Extension with Capital Inputs

Suppose that firms produce output using capital in addition to labor and materials with a production function of the following form:

$$X_{it} = T_{it} K_{it}^{\alpha} F\left[\left\{\phi_{it}(a) L_{it}(a), M_{it}(a)\right\}_{a\in A}\right]^{1-\alpha} \tag{A.1}$$

where α is the capital share of cost. Suppose also that capital is available at a price r_{it} that may vary across firms due to differences in access to capital markets. The firm's profit maximization problem can now be written as:

$$\max_{K_{it},\{w_{it}(a),M_{it}(a)\}_{a\in A}} \left\{ D_{it}^{\frac{1}{\sigma}} X_{it}^{\frac{\sigma-1}{\sigma}} - \sum_{a\in A} w_{it}(a) L_{it}(a) - Z_{it} \sum_{a\in A} M_{it}(a) - r_{it} K_{it} \right\} \tag{A.2}$$

subject to the production function (A.1) and labor supply curves (2.5). The first-order condition for this problem with respect to the capital input is:

$$\alpha D_{it}^{\frac{1}{\sigma}} X_{it}^{\frac{\sigma-1}{\sigma}} = r_{it} K_{it} \tag{A.3}$$

Using this to substitute for the choice of capital, we can rewrite the profit maximization problem as a choice over wages and material inputs alone, as in the original problem (2.20):

$$\max_{\{w_{it}(a),M_{it}(a)\}_{a\in A}} \left\{ D_{it}^{\frac{1}{\tilde\sigma}} \tilde X_{it}^{\frac{\tilde\sigma-1}{\tilde\sigma}} - \sum_{a\in A} w_{it}(a) L_{it}(a) - Z_{it} \sum_{a\in A} M_{it}(a) \right\} \tag{A.4}$$

$$\text{s.t. } \tilde X_{it} = \tilde T_{it} F\left[\left\{\phi_{it}(a) L_{it}(a), M_{it}(a)\right\}_{a\in A}\right] \tag{A.5}$$

where $\tilde\sigma \equiv \sigma(1-\alpha)+\alpha$ and $\tilde T_{it} \equiv (1-\alpha)^{\frac{\tilde\sigma}{\tilde\sigma-1}} \left(\frac{\alpha}{r_{it}}\right)^{\frac{\alpha}{1-\alpha}} T_{it}^{\frac{1}{1-\alpha}}$. Hence, the firm's problem with capital is isomorphic to the problem without capital if one replaces σ with $\tilde\sigma$ and T_{it} with $\tilde T_{it}$. Note that the introduction of capital lowers the effective price elasticity of demand (since $\tilde\sigma < \sigma$ for any $\alpha \in (0,1)$), while differences in capital prices r_{it} can be viewed as differences in effective productivity.

B Value-added per worker and wages

We discuss here the implications of the model for the passthrough of changes in value-added per worker to changes in worker earnings. For brevity, consider a simplified version of our model with no heterogeneity in worker ability. The production function (2.10) can then generally be represented as:

$$X_{it} = T_{it} \omega_{it} L_{it} f(1, \nu_{it}) \tag{B.1}$$

where we have utilized the property that f is homogeneous of degree one. Wages are given by:

$$w_{it} = \eta W_{it} \tag{B.2}$$

while value-added is equal to the difference between sales and material costs:

$$VA_{it} = p_{it}X_{it} - Z_{it}M_{it} \tag{B.3}$$

Since $\nu_{it} \equiv \frac{M_{it}}{\omega_{it}L_{it}}$, value-added per worker is then:

$$VAPW_{it} = p_{it}T_{it}\omega_{it}f\left(1,\nu_{it}\right) - Z_{it}\nu_{it}\omega_{it} \tag{B.4}$$

Eliminating Z_{it} using the firm's first-order condition for materials (2.21) and again utilizing the property of f being homogeneous of degree one, we can rewrite this as:

$$VAPW_{it} = \frac{\sigma-1}{\sigma}p_{it}T_{it}\omega_{it}f_L\left(1,\nu_{it}\right) + \frac{1}{\sigma}p_{it}T_{it}\omega_{it}f\left(1,\nu_{it}\right) \tag{B.5}$$

Finally, substituting the firm's first-order condition with respect to labor (2.24), we obtain:

$$VAPW_{it} = W_{it}\left[1+\left(\frac{1}{\sigma-1}\right)\frac{f\left(1,\nu_{it}\right)}{f_L\left(1,\nu_{it}\right)}\right] \tag{B.6}$$

Hence, value-added per worker is not proportional to the firm effect W_{it} except in three special cases of the model: (i) no output market power ($\sigma\to\infty$); (ii) no materials in production ($\nu_{it}=0$); and (iii) Cobb-Douglas technology (so that $\frac{f(1,\nu_{it})}{f_L(1,\nu_{it})}$ is independent of ν_{it}). An immediate corollary is that the assumption of Cobb-Douglas technology in our model implies complete passthrough of changes in value-added per worker to changes in worker earnings, which is strongly rejected by the empirical literature (as in Berger et al. (2019) and Kline et al. (2019), for example).

C Proofs of Claims and Propositions

C.1 Proof of Claim 1

Omitting time subscripts for brevity, the profit-maximization problem for a firm i can be written generally as:

$$\max_{\{p_{ji}\}_{j\in\Omega_i^C\cup\{F\}}} \left\{\sum_{j\in\Omega_i^C\cup\{F\}_i} p_{ji}x_{ji} - C\left[X_i|l_i\left(\cdot\right),Z_i\right]\right\} \tag{C.1}$$

$$\text{s.t. } x_{ji} = \Delta_j\psi_{ji}p_{ji}^{-\sigma} \tag{C.2}$$

$$X_i = \sum_{j\in\Omega_i^C\cup\{F\}} x_{ji} \tag{C.3}$$

where $\psi_{Fi}=1$. Here, $C\left[X_i|l_i\left(\cdot\right),Z_i\right]$ denotes the total cost of producing X_i units of output given the labor supply functions $l_i\left(\cdot\right)$ and material input cost Z_i. The latter depends on the prices charged by suppliers of firm i, which firm i takes as given in the problem above. Importantly, the total production cost for firm i depends only on total output of the firm X_i and not on how this output is allocated to each customer.

The first-order condition for the profit-maximization problem with respect to p_{ji} is then:

$$(1-\sigma)\,\Delta_j\psi_{ji}p_{ji}^{-\sigma} = -\sigma C'\left[X_i|l_i\left(\cdot\right),Z_i\right]\Delta_j\psi_{ji}p_{ji}^{-\sigma-1} \tag{C.4}$$

Solving for the optimal price yields:

$$p_{ji} = \frac{\sigma}{\sigma-1}C'\left[X_i|l_i\left(\cdot\right),Z_i\right] \tag{C.5}$$

Note that the right-hand side of (C.5) does not vary by customer j. Hence, the optimal prices set by firm i do not vary by customer and are equal to the standard CES markup over the firm's marginal cost. The existence of imperfect competition in the labor market implies that marginal cost is not constant, but this does not break the standard CES markup result.

C.2 Proof of Proposition 1

In what follows, we omit firm and time subscripts for brevity and all derivatives of the production function f are evaluated at $\{\phi L, M\} = \{1,\nu\}$. Totally differentiating (2.21), (2.24), and (2.25) for a given firm, we obtain:

$$\hat{W} + \frac{1}{\sigma}\hat{X} - \left(\frac{f_{LM}\nu}{f_L}\right)\hat{\nu} = \frac{1}{\sigma}\hat{D} + \hat{T} + \hat{\omega} \tag{C.6}$$

$$\frac{1}{\sigma}\hat{X} - \left(\frac{f_{MM}\nu}{f_M}\right)\hat{\nu} = \frac{1}{\sigma}\hat{D} + \hat{T} - \hat{Z} \tag{C.7}$$

$$-\gamma\hat{W} + \hat{X} - \left(\frac{f_M v}{f}\right)\hat{\nu} = \hat{T} + \hat{\omega} + \hat{\hat{\phi}} \tag{C.8}$$

Solving for $\left\{\hat{W},\hat{X},\hat{\nu}\right\}$, we obtain:

$$\hat{W} = \Gamma\hat{D} + (\sigma-1)\,\Gamma\hat{T} - (\sigma-\epsilon)\,\epsilon_m\Gamma\hat{Z} + \left[\sigma - 1 - (\sigma-\epsilon)\,\epsilon_M\right]\Gamma\hat{\omega} - \Gamma\hat{\hat{\phi}} \tag{C.9}$$

$$\hat{X} = (\gamma+\epsilon\varepsilon_M)\,\Gamma\hat{D} + \sigma\left(\gamma+\epsilon\epsilon_m+1-\varepsilon_M\right)\Gamma\hat{T} - \sigma\left(\gamma+\epsilon\right)\varepsilon_M\Gamma\hat{Z} + \sigma\left(1-\varepsilon_M\right)\left(1+\gamma\right)\Gamma\hat{\omega} + \sigma\left(1-\varepsilon_M\right)\Gamma\hat{\hat{\phi}} \tag{C.10}$$

$$\hat{v} = \epsilon\Gamma\hat{D} + \epsilon\left(\sigma-1\right)\Gamma\hat{T} - \epsilon\left(\gamma+\sigma\right)\Gamma\hat{Z} - \epsilon\left(1+\gamma\right)\Gamma\hat{\omega} - \epsilon\Gamma\hat{\hat{\phi}} \tag{C.11}$$

where $\varepsilon_M \equiv \frac{f_M\nu}{f}$ denotes the elasticity of f with respect to materials and $\Gamma \equiv \left[\gamma+\sigma\left(1-\varepsilon_M\right)+\epsilon\varepsilon_M\right]^{-1}$.

Now from equations (2.28) and (2.29), we can express the material share of cost (adjusted for markdowns on wage) for the firm as:

$$s^M \equiv \frac{E^M}{\frac{1}{\eta}E^L + E^M} = \frac{Z\omega\nu}{W + Z\omega\nu} \tag{C.12}$$

Then, from the first-order conditions (2.24) and (2.21), relative factor prices can be expressed as:

$$\frac{Z}{W/\omega} = \frac{f_M}{f_L} \tag{C.13}$$

Using the result that $f = f_M \nu + f_L$ for a homogeneous of degree one function f then implies:

$$\varepsilon_M = \frac{Z\omega\nu}{W + Z\omega\nu} \tag{C.14}$$

Finally, comparing equations (C.12) and (C.14) implies:

$$\varepsilon_M = s^M \tag{C.15}$$

so that the elasticity of f with respect to materials is equal to the material share of cost in equilibrium. The coefficients on the right-hand side of equation (C.9) then establish the comparative static results described in Proposition 1.

C.3 Proof of Propositions 2 and 3

In what follows, we omit time subscripts for brevity and denote by $\hat{Y}$ the vector of changes in a firm-specific variable $\hat{Y}_i$ for all firms. We begin by deriving an expression for marginal changes in demand shifters, $\hat{D}$. Totally differentiating equation (2.17) gives:

$$\hat{D} = S^{sales}\hat{\Delta} \tag{C.16}$$

where we have used the result that the share of firm i's sales accounted for by firm j can be expressed using (2.8), (2.17) and (4.16) as:

$$s^{sales}_{jit} \equiv \frac{R_{jit}}{\sum_{k\in\Omega^C_{it}\cup\{F\}} R_{kit}} = \frac{\Delta_j}{D_i} \tag{C.17}$$

Recall also that we are assuming no changes in general equilibrium variables and hence $\hat{\Delta}_{Ft} = 0$. Totally differentiating (2.15) and using (2.29), we obtain:

$$\hat{\Delta} = \gamma\hat{W} + \sigma\hat{Z} + \hat{\nu} + \hat{\omega} \tag{C.18}$$

Then, taking the ratio of the first-order conditions for the profit-maximization problem (2.24)-(2.21) and totally differentiating gives:

$$\hat{W} - \hat{Z} = \epsilon^{-1}\hat{\nu} + \hat{\omega} \tag{C.19}$$

Combining (C.16), (C.18), and (C.19), we then obtain the following expression for marginal changes in demand shifters:

$$\hat{D} = S^{sales}\left[(\gamma+\epsilon)\,\hat{W} + (\sigma-\epsilon)\,\hat{Z} + (1-\epsilon)\,\hat{\omega}\right] \tag{C.20}$$

Next, we derive an expression for marginal changes in material costs, $\hat{Z}$. Totally differentiating equation (2.18) gives:

$$\hat{Z} = -\frac{1}{\sigma-1}S^{mat}\hat{\Phi} \tag{C.21}$$

where we have used the result that the share of firm i's input expenditures accounted for by firm j can be expressed using (2.18) and (4.16) as:

$$s^{mat}_{ijt} \equiv \frac{R_{ijt}}{\sum_{k\in\Omega^S_{it}} R_{ikt}} = \frac{\Phi_{jt}\psi_{ijt}}{Z^{1-\sigma}_{it}} \tag{C.22}$$

Then, from (2.16) and (2.19), we can express marginal changes in network productivities as:

$$\hat{\Phi} = \frac{\sigma-1}{\sigma}\left(\hat{X}-\hat{D}\right) \tag{C.23}$$

Hence, combining (C.21) and (C.23), we obtain the following expression for marginal changes in material costs:

$$\hat{Z} = \frac{1}{\sigma}S^{mat}\left(\hat{D}-\hat{X}\right) \tag{C.24}$$

Now equations (C.9)-(C.11), (C.20), and (C.24) define a linear system in $\left\{\hat{W},\hat{X},\hat{\nu},\hat{D},\hat{Z}\right\}$, given changes in TFP $\hat{T}$ and labor productivity $\hat{\omega}$. Recall that we are assuming no changes in general equilibrium variables and hence $\hat{\kappa}(\cdot)=0$. Eliminating $\hat{X}$ and $\hat{\nu}$ from this system, we can write the remaining equations as:

$$\hat{W} = H^{WT}\hat{T} + H^{W\omega}\hat{\omega} + H^{WD}\hat{D} + H^{WZ}\hat{Z} \tag{C.25}$$

$$\hat{D} = S^{sales}\left[H^{DT}\hat{T} + H^{D\omega}\hat{\omega} + H^{DD}\hat{D} + H^{DZ}\hat{Z}\right] \tag{C.26}$$

$$\hat{Z} = S^{mat}\left[H^{ZT}\hat{T} + H^{Z\omega}\hat{\omega} + H^{ZZ}\hat{Z} + H^{ZD}\hat{D}\right] \tag{C.27}$$

where the H matrices are all $\left|\Omega^F\right| \times \left|\Omega^F\right|$ diagonal matrices. The matrices summarizing the dependence of $\left\{\hat{W},\hat{D},\hat{Z}\right\}$ on productivity shocks $\left\{\hat{T},\hat{\omega}\right\}$ have i^{th}-diagonal elements given by:

$$\begin{array}{ll} H^{WT}_i = (\sigma-1)\,\Gamma_i & H^{W\omega}_i = \left[(\sigma-1)-(\sigma-\epsilon)\,s^M_i\right]\Gamma_i \\ H^{DT}_i = (\gamma+\epsilon)\,(\sigma-1)\,\Gamma_i & H^{D\omega}_i = (1+\gamma)\,(\sigma-\epsilon)\left(1-s^M_i\right)\Gamma_i \\ H^{ZT}_i = -\left[\gamma+1-s^M_i+\epsilon s^M_i\right]\Gamma_i & H^{Z\omega}_i = -(1+\gamma)\left(1-s^M_i\right)\Gamma_i \end{array} \tag{C.28}$$

while the matrices summarizing the interrelation between $\left\{\hat{W},\hat{D},\hat{Z}\right\}$ have i^{th}-diagonal elements given by:

$$\begin{array}{lll} H^{WT}_i = (\sigma-1)\,\Gamma_i & H^{WD}_i = \Gamma_i & H^{WZ}_i = -(\sigma-\epsilon)\,s^M_i\Gamma_i \\ H^{DT}_i = (\gamma+\epsilon)\,(\sigma-1)\,\Gamma_i & H^{DD}_i = (\gamma+\epsilon)\,\Gamma_i & H^{DZ}_i = (\sigma-\epsilon)\,(\gamma+\sigma)\left(1-s^M_i\right)\Gamma_i \\ H^{ZT}_i = -\left[\gamma+1-s^M_i+\epsilon s^M_i\right]\Gamma_i & H^{ZD}_i = \left(1-s^M_i\right)\Gamma_i & H^{ZZ}_i = (\gamma+\epsilon)\,s^M_i\Gamma_i \end{array} \tag{C.29}$$

Note that all the coefficients in equations (C.25)-(C.27) depend only on $\{\gamma,\sigma,\epsilon\}$, network shares $\left\{S^{sales},S^{mat}\right\}$, and material shares of cost S^M.

To derive expressions for the passthrough coefficients in Propositions 2 and 3, we proceed as follows. First, a shock $\hat{v}_{jt}$ received by firm i's supplier j leads to a change in the price p_{jt} charged by the supplier. Since $\hat{p} = \frac{1}{1-\sigma}\hat{\Phi}$, the elasticities in equation (3.3) follow from equations (C.10) and (C.26). The change in p_{jt} then affects firm i's material input cost Z_{it}. Totally differentiating equation (2.18) shows that this elasticity is equal to the share of firm i's material cost accounted for by supplier j, s^{mat}_{ijt}. In turn, the change in Z_{it} affects the firm-level wage at firm i with elasticity given by Proposition 1.

Second, a shock $\hat{v}_{jt}$ received by firm i's customer j leads to a change in the network demand Δ_{jt} of the customer. Combining equations () gives the elasticities specified in equation (3.5). The change in Δ_{jt} then affects firm i's demand D_{it}. Totally differentiating equation (2.17) shows that this elasticity is equal to the share of firm i's sales accounted for by customer j, s^{sales}_{jit}. In turn, the change in D_{it} affects the firm-level wage at firm i with elasticity given by Proposition 1.

One can also characterize the passthrough of shocks from a firm's indirect suppliers and customers. To illustrate, we describe here the passthrough of shocks via second-degree relationships. For example, a shock $\hat{v}_{kt}$ to a *second-degree supplier* k of firm i affects firm k's output price p_{kt}, which affects the material cost Z_{jt} and output price p_{jt} of supplier j, and consequently the material cost Z_{it} and firm effect W_{it} of firm i. Following the above logic, the elasticity of this effect is given by:

$$\frac{\partial \log W_{it}}{\partial \log v_{kt}} = \frac{\partial \log p_{kt}}{\partial \log v_{kt}} \times s^{mat}_{jkt} \times \frac{\partial \log p_{jt}}{\partial \log Z_{jt}} \times s^{mat}_{ijt} \times \frac{\partial \log W_{it}}{\partial \log Z_{it}}, \qquad \forall k \in \Omega^S_{jt},\ j \in \Omega^S_{it} \tag{C.30}$$

where each of the elasticities on the right-hand side have been characterized above. Similarly, a shock $\hat{v}_{kt}$ to a second-degree customer k of firm i affects firm k's network demand Δ_{kt}, which affects demand D_{jt} and network demand Δ_{jt} of customer j, and consequently the demand D_{it} and firm effect W_{it} of firm i. The elasticity of this effect is given by:

$$\frac{\partial \log W_{it}}{\partial \log v_{kt}} = \frac{\partial \log \Delta_{kt}}{\partial \log v_{kt}} \times s^{sales}_{kjt} \times \frac{\partial \log \Delta_{jt}}{\partial \log D_{jt}} \times s^{sales}_{jit} \times \frac{\partial \log W_{it}}{\partial \log D_{it}}, \qquad \forall k \in \Omega^C_{jt},\ j \in \Omega^C_{it} \tag{C.31}$$

where agian each of the elasticities on the right-hand side have been characterized above.

D Data Details

Table A.1: Overview of Sample Sizes

Panel A: Firm-to-Firm Dataset		**Links**		**Suppliers**		**Buyers**
Sample	**Unique**	**Observation-Years**	**Unique**	**Observation-Years**	**Unique**	**Observation-Years**
Baseline	16,831,546	31,743,495	194,615	592,622	289,344	923,155
Panel B: Employer-Employee Dataset		**Workers**		**Firms**		
Sample	**Unique**	**Observation-Years**	**Unique**	**Observations-Years**		
Baseline	6,496,849	41,954,008	487,504	2,315,927		
Movers	6,183,692	40,130,960	200,592	1,378,554		
Stayers: Complete Spells	953,865	8,472,302	64,670	602,622		
Stayers: 10 Stayers per Firm	724,957	6,571,483	5,726	61,823		
Panel C: Firm Dataset		**Firms**				
Sample	**Unique**	**Observations-Years**				
Baseline	47,685	125,726				

Notes: This table provides an overview of the samples used throughout the paper.

Table A.2: Descriptive Satistics of Datasets

Dataset	**Employer-Employee**			**Firm**	**Firm-to-Firm**
Panel A: Worker Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Log Worker Earnings (Log US)	9.36	9.38	9.74	9.17	9.22
Median Log Worker Earnings (Log US)	9.25	9.27	9.66	9.02	9.10
Mean Worker Age	40.2	40.1	42.6	39.3	39.8
Median Worker Age	39.4	39.4	42.6	38.5	39.0
Panel B: Firm Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Number of Workers	9	20	281	27	12
Median Number of Workers	2	4	94	7	2
Mean Wage Bill per Worker (US)	10,199	11,145	7,833	9,440	8,306
Median Wage Bill per Worker (US)	6,943	8,323	6,672	7,103	5,490
Mean Value Added per Worker (US)	56,315	58,610	50,077	49,604	50,091
Median Value Added per Worker (US)	23,424	25,659	26,583	23,389	18,771
Mean Log Value Added (Log US)	11.0	11.8	14.6	12.2	10.9
Median Log Value Added (Log US)	11.0	11.7	14.8	12.1	10.9
Mean Labor Share	0.49	0.45	0.70	0.42	0.49
Median Labor Share	0.32	0.34	0.21	0.34	0.32
Panel C: Production Network Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Number of Suppliers	67	67	306	67	35
Median Number of Suppliers	36	36	208	36	19
Mean Number of Buyers	80	80	580	80	34
Median Number of Buyers	8	8	59	8	4
Mean Materials Share of Sales	0.58	0.58	0.55	0.58	0.57
Median Materials Share of Sales	0.61	0.61	0.60	0.61	0.61
Mean Intermediate Share of Sales	0.40	0.40	0.45	0.40	0.38
Median Intermediate Share of Sales	0.38	0.38	0.50	0.38	0.33

Notes: This table provides descriptive statistics of all the samples used in the paper.

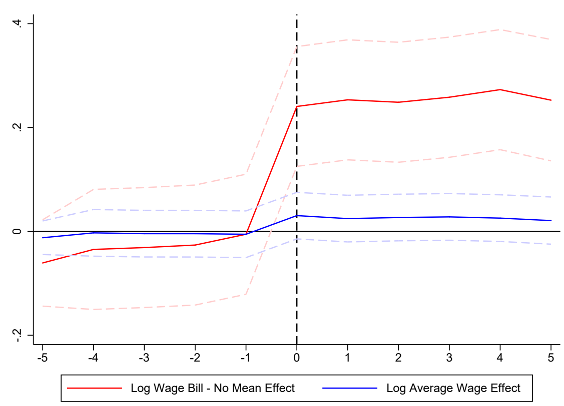
E Estimation Details

E.1 Labor supply elasticity

For robustness, we also follow Lamadon et al. (2019) and estimate γ using a difference-in-difference approach (DiD). For this, we follow a three step procedure. First, for each year, we order firms according to log changes of the wage bill of the firm. Second, we identify the treatment when firms have log changes of their wage bill above the median of log changes of wage bill across firms each year. Finally, we plot difference in wage bill of treated and control firms both at each year ($t = 0$) and years before ($t < 0$) and after ($t > 0$). We perform this step for each calendar year and weight firms by the number of workers.

Results are presented Figure A.1. By construction, the treatment and control groups differ in the wage bill from period $t = -1$ to $t = 0$. On average, firms in the treatment group face an increase of 21 log points growth in their wage bill relative to firms in the control group. The effect of the treatment appears to be permanent in levels up to 5 years after the treatment. Figure A.1 also shows the effect on the average earnings of firms. On average, firms in the treatment group face an increase of 3.25 log points of their average earnings relative to firms in the control group. Once again, the effect of the treatment appears to be permanent in levels up to 5 years after the treatment. Finally, firms in the treatment and control group do not experience statistically significant differences up to 5 years before the treatment, for both the wage bill and the average earnings. Through the lens of a DiD design, these results imply a passthrough rate of firms shocks of around 0.155 ($= 0.0325/0.21$). From equation (4.7), this implies a labor supply elasticity of $\hat{\gamma} = 5.5$, which is the same as our preferred estimate documented in the main text.

Figure A.1: Difference-in-difference Estimate of passthrough of Firm Shocks to Worker Earnings



Notes: This figure presents the results from the Lamadon et al. (2019) difference-in-difference approach to estimating passthrough of wage bill shocks to worker wages.

E.2 Worker and firm wage effects

To estimate the Bonhomme et al. (2019) decomposition of worker earnings from equation (4.9), we first cluster firms using a k-means clustering algorithm into $K = 10$ groups. We use a weighted K-means algorithm with 100 randomly generated starting values. We use firms' empirical distributions of log earnings on a grid of 10 percentiles of the overall log-earnings distribution. Second, we use these K groups as the relevant firm identifier in the Bonhomme et al. (2019) estimation approach. This procedure yields estimates of the firm fixed effect $\bar{W}_i$ and the worker-firm production complementarity θ_i for every firm $i \in \Omega^F$, as well as the permanent and transient components of ability for every worker.

To assess robustness of our results to the number of clusters used, Table A.3 documents the share of variance of wages accounted for by the firm fixed effect $\bar{W}_i$. We implement this for the basic model of Abowd et al. (1999) and also the basic version of the model of Bonhomme et al. (2019) with only firm and worker fixed effects for different levels of K (thus, excluding interactions and time-varying firm effects). First, one can see that the basic version of the model of Bonhomme et al. (2019) implies a role for the firm fixed effect that is significantly lower than the model of Abowd et al. (1999), consistent with previous literature that has found that addressing the limited mobility bias inherent in estimates of Abowd et al. (1999) decreases the share of the variance accounted for by the firm fixed effect (Bonhomme et al., 2020). Second, as one increases K from 10 to 50, the share of the variance of wages accounted for the firm fixed effects increases only 0.7 percentage points from 7.8 to 8.5%. At least with this piece of evidence, this implies that the limited mobility bias does not represent a substantially bigger problem for $K = 50$ than what it represents for $K = 10$.

Table A.3: Share of Log Earnings Variance Accounted for by the Firm Fixed Effect

Estimation Strategy	Number of Clusters	Firm Fixed EffectShare
AKM		12.3
BLM	10	7.8
BLM	50	8.5

Notes: This table documents the share of the log of earnings variance accounted for by the firm fixed effect. It is documented for the estimation strategy of Abowd et al. (1999) (row 1), for the estimation strategy of Bonhomme et al. (2019) with $K = 10$ clusters (row 2) and the estimation strategy of Bonhomme et al. (2019) with $K = 50$ clusters (row 3). Note that this table documents the version of the wage models following the estimation strategy of Lamadon et al. (2019) documented in Section 5.2.2 without interactions and without time-varying firm effects. Thus, the share documented in row 2 corresponds to the same one document in row 2 and column 3 of Table 3.

To further assess whether clustering with $K = 10$ or $K = 50$ makes a difference, we document how much clusters account for the variance of firm-level characteristics. Tables A.4-A.5 document the share of the variance of variables accounted for by within-cluster variation. Table A.4 shows the within-cluster share of variance of variables in levels, whereas Table A.5 shows the same evidence for variables in ratios. Although there is substantial heterogeneity across firms that the clustering procedure of Bonhomme et al. (2019) does not account for, this result does not vary significantly if one uses $K = 10$ or $K = 50$ clusters.

Table A.4: Within Clusters Share of Total Variance of Variables in Levels

Number Clusters	Total Sales	Materials	Wage Bill	Employment	Number of Buyers	Number of Suppliers	Firm-to-Firm Sales
10	79	90	67	88	90	85	95
50	74	86	62	84	88	81	92

Notes: This table documents the share of the variance of each variable accounted for by the within cluster variance. It is implemented for $K = 10$ and $K = 50$ and for variables in levels.

Table A.5: Within Clusters Share of Total Variance of Variables in Ratios

Number Clusters	Wage Bill/Sales	Materials/Sales	Materials/Wage Bill	Sales/Employment	Wage Bill/Employment	Materials/Employment
10	96	97	95	92	26	99
50	95	97	95	90	21	98

Notes: This table documents the share of the variance of each variable accounted for by the within cluster variance. It is implemented for $K = 10$ and $K = 50$ and for variables in ratios.

E.3 Amenities

To estimate firm amenities, we begin with the labor supply equation (2.5). It will be useful for the exposition to write this explicitly in terms of permanent and transient worker abilities:

$$\frac{L_{it}\left(\bar{a},\hat{a}\right)}{L\left(\bar{a},\hat{a}\right)} = \frac{\left[g_i\left(\bar{a}\right)w_{it}\left(\bar{a},\hat{a}\right)\right]^{\gamma}}{\sum_{j\in\Omega^F}\left[g_j\left(\bar{a}\right)w_{jt}\left(\bar{a},\hat{a}\right)\right]^{\gamma}} \tag{E.1}$$

where note that under Assumption 4.1, amenity values only vary across workers in relation to permanent ability $\bar{a}$. Next, consider the equilibrium wage equation (2.23). Under assumption 4.1, we can write this as:

$$w_{it}\left(\bar{a},\hat{a}\right) = \eta\bar{a}^{\theta_i}\hat{a}W_{it} \tag{E.2}$$

The average wage paid by firm i to workers with permanent ability $\bar{a}$ is hence:

$$\bar{w}_{it}\left(\bar{a}\right) = \eta\bar{a}^{\theta_i}\mathbb{E}\left[\hat{a}\right]W_{it} \tag{E.3}$$

where $\mathbb{E}\left[\hat{a}\right]$ denotes the average value of transient ability. Under Assumptions 4.1 and 4.3, this mean does not depend on permanent ability of the worker or the identity of the firm. Combining (E.2) and (E.3), we then have:

$$w_{it}\left(\bar{a},\hat{a}\right) = \bar{w}_{it}\left(\bar{a}\right)\frac{\hat{a}}{\mathbb{E}\left[\hat{a}\right]} \tag{E.4}$$

Substituting this into (E.1) and using the decomposition of amenities in equation (4.13), we obtain:

$$\frac{L_{it}\left(\bar{a},\hat{a}\right)}{L\left(\bar{a},\hat{a}\right)} = \frac{\left[\tilde{g}_i\bar{g}_{k(i)}\left(\bar{a}\right)\bar{a}^{\theta_{k(i)}}W_{it}\right]^{\gamma}}{\sum_j\left[\tilde{g}_j\bar{g}_{k(j)}\left(\bar{a}\right)\bar{a}^{\theta_{k(j)}}W_{jt}\right]^{\gamma}} \tag{E.5}$$

Now notice that the employment share of workers of ability $\{\bar{a},\hat{a}\}$ varies across firms only in relation to permanent ability $\bar{a}$. This is a direct implication of Assumption 4.1, which implies that workers do not sort to firms based on transient ability $\hat{a}$. Therefore, the share of workers of permanent ability $\bar{a}$ employed by firm i is also given by equation (E.5). Summing this (E.5) across all firms within cluster k, we can similarly express the share of workers of permanent ability $\bar{a}$ that are employed by firms in cluster k as:

$$\Lambda_{kt}(\bar{a}) = \frac{\sum_{i \in k} \left[\tilde{g}_i \bar{g}_k(\bar{a}) \bar{a}^{\theta_k} W_{it}\right]^{\gamma}}{\sum_j \left[\tilde{g}_j \bar{g}_{k(j)}(\bar{a}) \bar{a}^{\theta_{k(j)}} W_{jt}\right]^{\gamma}} \tag{E.6}$$

Next, note that for each value of permanent ability $\bar{a}$, equilibrium outcomes are invariant to scaling $g_i(\bar{a})$ by a constant for all firms i. Therefore, we are allowed to choose one normalization of amenity values for each permanent worker ability type $\bar{a}$. For this, we choose $\sum_j \left[\tilde{g}_j \bar{g}_{k(j)}(\bar{a}) \bar{a}^{\theta_{k(j)}} W_{jt}\right]^{\gamma} = 1$. Furthermore, mean differences in amenity values can be loaded onto either $\tilde{g}_i$ or $\bar{g}_{k(i)}(\bar{a})$. Hence, we are allowed to choose one normalization of the values for $\tilde{g}_i$ for each firm cluster. For this, we choose $\sum_{i \in k} [\tilde{g}_i W_{it}]^{\gamma} = 1$. With these normalizations, equations (4.14) and (4.15) follow immediately.

E.4 Firm relationship capability and relationship-specific productivity

To estimate equation (4.17), firms must have multiple connections. To identify seller fixed effects, each seller needs to have at least two buyers. Similarly, to identify buyer fixed effects, each buyer needs to have at least two sellers. In the data, some firms have either one supplier or one seller. Hence, we implement the aforementioned restriction using an iterative approach known as "avalanching". Specifically, we first drop firms with one supplier or seller. Doing this may result in additional firms that have one supplier or seller, hence in the next step, we drop these firms as well. We continue this process until firms are no longer dropped from the sample. The algorithm takes three iterations to converge in practice and reduces the sample size of firm-to-firm linkages from a total of 32 million transactions to 31.7 million transactions, that is, a reduction of 1% of transactions. Hence, the avalanching algorithm has little impact on our sample size. Bernard et al. (2019) report that avalanching also eliminates around 1% of firm-to-firm links in the production network for Belgium.

E.5 Product substitution elasticity

To derive equation (4.20), first note that the share of firm profits in total sales can be expressed as:

$$\frac{\pi_{it}}{R_{it}} = \frac{1}{\sigma}\left[1 + \frac{(\sigma - 1)(1 - \eta)}{1 + \eta \frac{E_{it}^M}{E_{it}^L}}\right] \tag{E.7}$$

Solving for σ and using the fact that $\pi_{it} = R_{it} - E_{it}^L - E_{it}^M$ gives equation (4.20). Hence, we estimate σ using the sample average of the right-hand side of (4.20), which is observable given our estimate of the labor supply elasticity γ and data on firm sales, labor costs, and material costs.

E.6 Labor-materials substitution elasticity and labor productivity

To derive equation (4.21), first note that under the CES production function specified in equation (2.11), a firm's relative expenditure on materials versus labor inputs can be expressed using equations (2.24), (2.21), (2.28), and (2.29) as:

$$\log \frac{E_{it}^M}{E_{it}^L} = \log \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \right] + (\epsilon - 1) \log \frac{W_{it}}{Z_{it}} + (1-\epsilon) \log \omega_{it} \tag{E.8}$$

Using the Markov process for labor productivity (4.4) to substitute for ω_{it} in (E.8) then gives equation (4.21).

For estimation of ϵ using equation (4.21), we follow the approach in Doraszelski and Jaumandreu (2018). To control for $F^\omega(\omega_{i,t-1})$, we first rearrange the $t-1$ version of equation (E.8) to write:

$$\log \omega_{i,t-1} = \frac{1}{\epsilon - 1} \log \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \right] - \frac{1}{\epsilon - 1} \log \frac{E_{i,t-1}^M}{E_{i,t-1}^L} + \log \frac{W_{i,t-1}}{Z_{i,t-1}} \tag{E.9}$$

$$\equiv G \left(\log \frac{E_{i,t-1}^M}{E_{i,t-1}^L}, \log \frac{W_{i,t-1}}{Z_{i,t-1}} \right) \tag{E.10}$$

Substituting this into (4.21), we obtain:

$$\log \frac{E_{it}^M}{E_{it}^L} = \log \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \right] + (\epsilon - 1) \log \frac{W_{it}}{Z_{it}} + H \left(\log \frac{E_{i,t-1}^M}{E_{i,t-1}^L}, \log \frac{W_{i,t-1}}{Z_{i,t-1}} \right) \tag{E.11}$$

$$+ (1-\epsilon) \xi_{it}^\omega \tag{E.12}$$

where $H(\cdot,\cdot) \equiv (1-\epsilon) F^\omega [G(\cdot,\cdot)]$. Hence, we control for the term H using a third-degree polynomial in lagged relative expenditures $\log \frac{\tilde{E}_{i,t-1}^M}{\tilde{E}_{i,t-1}^L}$ and lagged relative input prices $\log \frac{\tilde{W}_{i,t-1}}{Z_{i,t-1}}$. We then instrument for relative input prices at date t using all available lags of logged input expenditures and constructed prices from dates $t-1$ and earlier.

E.7 Firm TFP

We choose values for TFP T_{it} to fit the estimated firm-level wages W_{it} as specified in equation (4.22). We do this using an iterative numerical procedure that is similar in spirit to the equilibrium solution algorithm described in section H:

1. Compute $\left\{ \tilde{\phi}_{it} \right\}_{i \in \Omega^F}$ from (2.27), using (2.4), (2.6), and the estimated firm-level wages $\{W_{it}\}_{i \in \Omega^F}$.
2. Guess E_t.
   - (a) Guess $\{D_{it}, Z_{it}\}_{i \in \Omega^F}$.
   - (b) Compute the values of $\{T_{it}\}_{i \in \Omega^F}$ implied by equation (H.1), given the estimated firm-level wages $\{W_{it}\}_{i \in \Omega^F}$.
   - (c) Compute new guesses of $\{D_{it}\}_{i \in \Omega^F}$ from (2.17) and $\{Z_{it}\}_{i \in \Omega^F}$ from (2.18).

(d) Iterate on steps (a)-(c) until convergence.

3. Compute a new guess of E_t from (2.9), using (2.5), (2.20), and (2.23).

4. Iterate on steps 1-2 until convergence.

In practice, we also add as an outer loop iterations over guesses of mean relationship productivity (as described in section 5.2.4) and the product substitution elasticity σ (as described in section 5.2.5), targeting the aggregate value-added share of gross output and labor share of value-added respectively.

F Construction of export demand and import cost shocks

To construct export demand shocks, suppose that Chilean exporter i sells to a set of export markets $\Omega_{it}^{M,X}$, with each market comprised of a representative customer with exogenous network demand Δ_{mt}^{F}. Then, we can write the firm's demand as:

$$D_{it} = E_t + \sum_{j \in \Omega_{it}^{C,D}} \Delta_{jt} \psi_{jit} + \sum_{m \in \Omega_{it}^{M,X}} \Delta_{mt}^{F} \psi_{mit}^{F} \tag{F.1}$$

where $\Omega_{it}^{C,D}$ now denotes the set of firm i's *domestic* customers and ψ_{mit}^{F} accounts for firm heterogeneity in export demand from each export market m. Then, differentiating (F.1) with respect to $\left\{\Delta_{mt}^{F}\right\}_{m \in \Omega_{it}^{M,X}}$ allows us to write:

$$\hat{D}_{it} = s_{Xit}^{sales} \sum_{m \in \Omega_{it}^{M,X}} s_{mit}^{X} \hat{\Delta}_{mt}^{F} \tag{F.2}$$

Now let E_{mt}^{I} denote the total value of imports by market m from all countries excluding Chile in year t. Under the assumption that foreign customers have the same CES preferences as consumers in Chile, imports by market m are given by:

$$\log E_{mt}^{I} = (1-\sigma) \log Z_{mt}^{F} + \log \Delta_{mt}^{F} \tag{F.3}$$

where Z_{mt}^{F} is the CES price index that market m faces for its non-Chilean imports. We further suppose that these price indices can be decomposed as $Z_{mt}^{F} = \tau_m z_{h(m)t}^{F}$, where τ_m captures static differences in import costs across markets, while $z_{h(m)t}^{F}$ captures time-varying differences in import costs that are equal for all importing countries within the product category $h(m)$ to which market m belongs. Hence, taking first differences of equation (F.3), we obtain:

$$\hat{E}_{mt}^{I} = \delta_{h(m)t} + \hat{\Delta}_{mt}^{F} \tag{F.4}$$

where $\delta_{h(m)t} \equiv (1-\sigma) \hat{z}_{h(m)t}$ is a product-year fixed effect that is orthogonal to $\hat{\Delta}_{mt}^{F}$ by assumption. Now we assume that the expectation of $\hat{\Delta}_{mt}^{F}$ across countries is zero, so that there is no aggregate growth in demand within a product category. The log change in market m's network demand is then equal to the log change in E_{mt}^{I} relative to the corresponding change in world imports of product category $h(m)$. Given this, equation (6.9) follows immediately from (F.2).

Similarly, to construct import cost shocks, suppose that Chilean importer i purchases imported materials from a set of markets $\Omega_{it}^{M,I}$, with each market comprised of a representative supplier that charges price p_{mt}^{X}. Then, we can write the firm's material input cost as:

$$Z_{it} = \left[\sum_{s \in \Omega_{it}^{S,D}} (p_{st})^{1-\sigma} + \sum_{m \in \Omega_{it}^{M,I}} \left(p_{mt}^{X} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{F.5}$$

Differentiating this with respect to $\left\{ p_{mt}^{X} \right\}_{m \in \Omega_{it}^{M,I}}$ and replacing contemporaneous import shares with initial import shares as before then gives equation (6.10).

G A Shapley value approach for model counterfactuals

In the counterfactual exercises studied in section 6.2, we deal with interdependencies between model primitives in shaping outcomes of interest using the following approach. Let Θ denote the estimated vector of values for all model primitives and let $X(\Theta)$ denote the value of some equilibrium outcome X under this parameter vector. Now, define some N subsets of the parameter vector $\{\theta_n\}_{n=1}^{N}$ such that $\Theta = \cup_{n=1}^{N} \theta_n$ and denote $\mathcal{N} \equiv \{1, \cdots, N\}$. We are interested in computing values of outcome X under known counterfactual values $\hat{\theta}_n$ for each subset of the parameter vector. Therefore, let $\hat{\Theta}_S \equiv \left\{ \cup_{n \in S} \hat{\theta}_n \right\} \cup \{ \cup_{n \notin S} \theta_n \}$ denote the parameter vector under counterfactual values for parameter subsets in S for some $S \subseteq \mathcal{N}$. We define the Shapley value X_n for parameter subset n in relation to outcome X as follows:

$$X_n = \sum_{S \subseteq \mathcal{N} \setminus \{n\}} \frac{|S|! \left(N! - |S|! - 1 \right)}{N!} \left[X\left(\hat{\Theta}_{S \cup \{n\}} \right) - X\left(\hat{\Theta}_S \right) \right] \tag{G.1}$$

For example, suppose that X is the variance of log earnings across all workers, θ_n is the estimated vector of firm TFPs, and $\hat{\theta}_n$ is a counterfactual vector of firm TFPs with each value equal to the mean of θ_n across firms. Then, we measure the contribution of TFP heterogeneity to earnings variance as $-\frac{X_n}{X(\Theta)}$. By construction of the Shapley value, these measures sum to one across all $n \in \mathcal{N}$.

H Solution Algorithm

We solve numerically for an equilibrium of the model using the following solution algorithm.

1. Guess E_t.
   (a) Guess $\left\{ \Delta_{it}, \Phi_{it}, \tilde{\phi}_{it} \right\}_{i \in \Omega^F}$.
   (b) Compute $\{D_{it}\}_{i \in \Omega^F}$ from (2.17) and $\{Z_{it}\}_{i \in \Omega^F}$ from (2.18).
   (c) Solve for $\{W_{it}, \nu_{it}, X_{it}\}_{i \in \Omega^F}$ from (2.24), (2.21), and (2.25).
   (d) Compute new guesses of $\{\Delta_{it}\}_{i \in \Omega^F}$ from (2.15), $\{\Phi_{it}\}_{i \in \Omega^F}$ from (2.19), and $\left\{ \tilde{\phi}_{it} \right\}_{i \in \Omega^F}$ from (2.27).

(e) Iterate on steps (a)-(d) until convergence.

2. Compute a new guess of E_t from (2.9), using (2.5), (2.20), and (2.23).

3. Iterate on steps 1-2 until convergence.

Note that step 1(c) involves numerical solution of a system in $\{W_{it}, \nu_{it}, X_{it}\}$. This system can be reduced to one in firm-level wages alone:

$$W_{it}^{\gamma+\epsilon}\left[\lambda\left(W_{it}/\omega_{it}\right)^{1-\epsilon}+(1-\lambda)\,Z_{it}^{1-\epsilon}\right]^{\frac{\sigma-\epsilon}{1-\epsilon}}\tilde{\phi}_{it}=\frac{\lambda}{\mu^{\sigma}\eta^{\gamma}}D_{it}T_{it}^{\sigma-1}\omega_{it}^{\epsilon-1} \tag{H.1}$$

which has a unique solution for W_{it} given $\left\{D_{it}, Z_{it}, \tilde{\phi}_{it}\right\}$. Solutions for ν_{it} and X_{it} are then easy to recover given W_{it}.