THE SUPPLY-SIDE EFFECTS OF MONETARY POLICY

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ABSTRACT

We propose a supply-side channel for the transmission of monetary policy. We show that in an economy with heterogeneous firms and endogenous markups, monetary shocks have first-order effects on aggregate productivity. If high-markup firms have lower pass-throughs than low-markup firms, as is consistent with the empirical evidence, then a monetary easing generates an endogenous positive "supply shock" that amplifies the positive "demand shock" on output. The result is a flattening of the Phillips curve. This effect is distinct from another mechanism discussed at length in the real rigidities literature: a monetary easing leads to a reduction in desired markups because of strategic complementarities in pricing. We derive a tractable four-equation dynamic model, disciplined by four sufficient statistics from the distribution of firms, and use it to show that a monetary easing generates a procyclical hump-shaped response in aggregate TFP and endogenous cost-push shocks in the New Keynesian Phillips curve. In the data, in line with the model's predictions, we observe both procyclical aggregate productivity and reallocations to high-markup firms during demand-driven expansions. A calibration using firm-level pass-throughs suggests that the supply-side channel is quantitatively as important as real rigidities, and amplifies both the impact and persistence of monetary shocks. Moreover, this channel becomes stronger, and the Phillips curve becomes flatter, with increases in industrial concentration.

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1 Introduction

How do demand shocks affect an economy's productivity? A common view is that they do not: aggregate productivity is taken to be orthogonal to the short-run shocks that move aggregate demand, such as monetary shocks.

Yet, aggregate productivity, as measured by labor productivity or the Solow residual, is sensitive to demand shocks. In fact, variations in monetary and fiscal policy explain between one-quarter and one-half of the observed movements in aggregate total factor productivity (TFP) at business cycle frequencies (see, e.g. Evans, 1992). This empirical finding is robust across time and across countries.[1] One interpretation of this result is that capacity utilization or external returns bias the measurement of aggregate TFP, creating an artificial correlation between measured productivity and demand shocks.

In this paper, we offer an alternative explanation. The aggregate TFP of an economy is not an exogenous primitive, but instead an endogenous outcome that depends on how resources are allocated across firms. We argue that in an economy with realistic firm heterogeneity, demand shocks *should* trigger changes in aggregate TFP. These changes do not arise from changes in technical efficiency—the technologies available to individual firms—but instead from shifts in the allocation of resources across firms.

The effect of monetary policy on the allocation of resources yields a new channel for the transmission of monetary policy, which we term *the misallocation channel*. Under conditions matching empirical patterns on firms, monetary shocks generate procyclical, hump-shaped movements in aggregate TFP consistent with empirical estimates by Evans (1992), Christiano et al. (2005) and others.[2] The endogenous "supply shock" generated by the misallocation channel complements the traditional effects of the "demand shock" on employment and output. Incorporating the misallocation channel heightens the response of output to demand shocks and flattens the Phillips curve.

This supply-side effect only appears when two conditions hold: (1) the initial cross-sectional allocation of resources is inefficient, and (2) aggregate demand shocks systematically reallocate resources from low to high marginal revenue product firms. First, if there is no initial misallocation, the marginal benefit of each input is equated across all competing uses. Therefore, starting at an efficient point, a reallocation of resources triggered by

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[1]The failed invariance of aggregate TFP to demand shocks is also observed by Hall (1990). Cozier and Gupta (1993), Evans and dos Santos (2002), and Kim and Lim (2004) extend the analysis to Canada, the G-7 countries, and South Korea.

[2]Christiano et al. (2005) estimate a positive hump-shaped response of labor productivity to monetary easing. In our one-factor model, labor productivity and aggregate TFP are the same. In Section 7, we provide our own empirical estimates of how aggregate productivity responds to identified monetary policy shocks.

a demand shock has no first-order effect on aggregate productivity. Second, even if the initial allocation of resources is distorted, if aggregate demand shocks do not differentially affect firms with different marginal revenue products, then the reallocations they induce do not systematically raise or lower aggregate productivity.

For these reasons, the misallocation channel is absent in the workhorse log-linearized New Keynesian model. First, the benchmark model, which uses a CES demand system, assumes that all firms face the same price elasticity of demand. This means that desired markups are the same for all firms. As a result, the flexible-price allocation of resources is efficient, and reallocations starting at this point do not affect aggregate productivity. Second, even starting at an equilibrium with markup dispersion, monetary policy does not differentially affect high and low marginal revenue product firms, so there is no reason to expect a monetary easing to increase productivity.

In contrast to the benchmark model, the data features substantial and persistent heterogeneity in markups across firms and systematic differences in how much firms pass through marginal cost shocks to prices. Incorporating these features of the data yields both preconditions for supply-side effects of aggregate demand shocks. First, dispersion in markups across firms means that the allocation of resources across firms is inefficient: firms with relatively high markups restrict supply and underproduce, while firms with relatively low markups overproduce. This heterogeneity in markups opens the door for reallocations caused by demand shocks to have first-order effects on aggregate productivity. Second, systematic differences in pass-through across firms cause demand shocks, like monetary policy shocks, to have differential effects on low- and high-markup firms. In particular, since low-markup firms tend to pass a higher portion of marginal cost changes into prices, a monetary easing systematically reallocates resources from low-markup to high-markup firms and therefore raises aggregate productivity.

To formally analyze these reallocations, we deviate from the CES formulation of the New Keynesian model and adopt a non-parametric generalized Kimball (1995) demand system introduced by Matsuyama and Ushchev (2017). These preferences allow us to generate downward-sloping residual demand curves of any desired shape while remaining tractable. We couple this flexible demand system with sticky prices using Calvo frictions. Our model is flexible enough to exactly match cross-sectional and time-series estimates of the firm size distribution and firm-level pass-throughs, with realistic heterogeneity in firms' price elasticities of demand and desired markups. We consider how TFP and output respond to monetary shocks in such a model. Our comparative statics do not impose any additional parametric structure on preferences and are disciplined by measurable sufficient statistics from the distribution of firms.

Our first result is that when firms' pass-throughs covary negatively with their initial markups, then a positive demand shock such as a monetary easing increases aggregate TFP. This negative relationship between markups and pass-throughs has strong empirical support across countries.[3] Intuitively, a monetary easing raises all firms' nominal marginal costs, but high-markup firms, which have lower pass-throughs, raise their prices by less than their low-markup counterparts. This triggers a reallocation toward high-markup firms and away from low-markup firms, which improves allocative efficiency. In principle, this heterogeneity in pass-throughs can be driven either by heterogeneity in desired pass-through (i.e., pass-through conditional on a price change) or heterogeneity in price-stickiness.

Our second result shows that the response of output to a monetary shock can be decomposed into distinct demand-side and supply-side effects. The demand-side effect of an expansionary shock arises from greater labor demand and employment. Intuitively, an expansionary shock increases spending, and since nominal rigidities prevent prices from rising by the same amount, this heightened demand leads to greater demand for labor, employment, and output. Real rigidities that further dampen the responsiveness of prices to increases in nominal marginal costs amplify these effects.[4] Whereas the demand-side effect raises output by raising employment, the supply-side effect boosts output by raising aggregate productivity. When markups negatively covary with pass-throughs, an expansionary shock reallocates resources to high-markup firms, thereby raising TFP and generating an endogenous "supply shock" that complements the demand shock.

A calibration of our model using firm data from Belgium (provided by Amiti et al. 2019) suggests that the misallocation channel constitutes a quantitatively important part of monetary policy transmission mechanism.[5] In a stylized one-period version of the model, we find that the misallocation channel reduces the slope of the Phillips curve by around 70% compared to a model with demand-side effects alone. As a point of comparison, we find that real rigidities flatten the Phillips curve by a similar amount.

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[3]See Berman et al. (2012) in France, Chatterjee et al. (2013) in Brazil, Li et al. (2015) in China, and Amiti et al. (2019) in Belgium. We use estimates from Amiti et al. (2019) to calibrate the empirical results presented in this paper.

[4]In this paper, when we refer to "real rigidities" we mean real rigidities caused by variable markups, such as those due to strategic complementarities in pricing, and not real rigidities caused by other forces (like decreasing returns or sticky intermediate input prices).

[5]We follow Baqaee et al. (2021) and solve a series of differential equations to back out the Kimball demand system from data on firm-level sales and pass-throughs. This approach is also preferable to using an off-the-shelf functional form for preferences, since it does not impose the counterfactual restrictions baked in by parametric families of preferences. We provide an explicit calibration exercise in Appendix G showing that off-the-shelf functional forms are incapable of simultaneously matching all the relevant sufficient statistics in the data.

Since the strength of real rigidities and the misallocation channel are governed by moments of the firm distribution, our analysis ties the strength of monetary policy to the industrial organization of the economy. In particular, we show that an increase in industrial concentration can increase the potency of both the real rigidities and misallocation channels. While the standard New Keynesian model is silent on the role of industrial concentration, in our setup increasing the Gini coefficient of firm employment from 0.80 to 0.85 flattens the Phillips curve by an additional 14%. This increase in the Gini coefficient is in line with the change in the firm employment distribution in the United States from 1978 to 2018.[6]

While we use a static model to illustrate some of the key intuitions driving our results, we also derive a fully dynamic model. We describe the movement of aggregate TFP, output, inflation, and the interest rate using a four-equation system. This augments the classic three-equation model to account for realistic firm heterogeneity and endogenous changes in allocative efficiency. Relative to the workhorse model, the Taylor rule and the Euler equation are the same but the New Keynesian Phillips curve is different. Our model features a flattened Phillips curve and endogenous cost-push shocks due to shifts in aggregate TFP. Those movements in aggregate TFP are pinned down by the fourth equation, which closes the system. These equations are all disciplined by four sufficient statistics from the firm distribution: the average markup, the average price elasticity of demand, the average desired pass-through, and the covariance of markups and desired pass-throughs.

A calibration of the dynamic model shows that the misallocation channel deepens the loss in output following a contractionary interest rate shock by an additional 44% on impact. The role of the misallocation channel also rises over time, increasing the half-life of the shock's effect on output by 31%. The net result is an increase in the cumulative output impact of the monetary shock by 65% compared to the workhorse model.

We find support for both macro- and micro-level predictions of our model. As mentioned above, at the macroeconomic level, our setup predicts procyclical, hump-shaped responses of aggregate TFP to monetary shocks. We confirm that aggregate productivity—as measured by labor productivity, the Solow residual, or the cost-based Solow residual—is procyclical in U.S. data and contracts following Romer and Romer (2004) monetary shocks, as shown by Evans (1992).[7] At the microeconomic level, our model

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[6]Whether concentration is in fact increasing for relevant market definitions or whether the Phillips curve has indeed flattened over time are topics that are beyond the scope of this paper. See e.g. Rossi-Hansberg et al. (2021), Benkard et al. (2021), Smith and Ocampo (2021) on the former, and e.g., McLeay and Tenreyro (2020), Del Negro et al. (2020), Hooper et al. (2020), Hazell et al. (2020) on the latter.

[7]We do not use capacity-utilization adjusted measures of aggregate TFP, like Basu et al. (2006) or Fernald

ties the increase in aggregate productivity during demand-driven expansions to reallo-
cations towards high-markup firms. Using Compustat data on public firms, we find
that monetary expansions cause high-markup firms to grow relative to low-markup firms
both in terms of their sales and costs. This is because firms with high markups cut their
markups relative to low-markup firms after a monetary expansion.[8] As a result, both
markup dispersion and the dispersion of firm-level revenue productivity (TFPR) fall dur-
ing demand-driven expansions.[9] Finally, in keeping with our model's predictions, we
show that multifactor productivity in NAICS4 manufacturing industries falls more after
monetary shocks in industries with higher concentration (measured as the concentration
of sales in top firms).

Other related literature. This paper contributes to the large literature on the response of
firms to monetary shocks. Our analysis is rooted in models of monopolistic competition
with staggered price setting originating in Taylor (1980) and Calvo (1983).

A strand of this literature is devoted to explaining the strength and persistence of
monetary policy shocks, which cannot be explained by nominal rigidities alone given the
frequency of price adjustment.[10] Ball and Romer (1990) introduce real rigidities, which
complement nominal rigidities to increase monetary nonneutrality.[11] A common formula-
tion of real rigidities is incomplete pass-through, where firms are slow to reflect marginal
cost shocks in their prices due to strategic complementarities in pricing. Incomplete-
ness of pass-through is documented empirically by Gopinath et al. (2010) and Gopinath
and Itskhoki (2011). Our paper complements this literature by showing that incomplete
pass-through, when paired with firm-level heterogeneity, provides another mechanism
through which monetary policy can affect output.

(2014), in our empirical exercises. This is because the exogeneity conditions used to identify utilization-
adjusted TFP—that sectoral TFP is orthogonal to oil price shocks and monetary shocks—are invalid in our
model. Indeed, our core result is that sectoral TFP is endogenous to such shocks.

[8]We document similar patterns whether we use markups estimated via the user-cost approach from
Gutiérrez and Philippon (2017) or from accounting profits; whether we use the updated Romer and Romer
(2004) series extended by Wieland and Yang (2020) or monetary shocks identified from high frequency data;
and whether we consider reallocations across all firms or within industry.

[9]When firms have constant returns to scale, as in our model, firm-level TFPR is equal to the firm's
markup. Our model's prediction of countercyclical firm-level TFPR dispersion is consistent with the
empirical findings of Kehrig (2011).

[10]This frequency has been documented by Taylor (1999) and Nakamura and Steinsson (2008) among
others.

[11]Ball and Romer (1990) has also spawned a large literature of theoretical developments on real rigidities,
which characterize the conditions under which real rigidities can generate observed levels of persistence in
monetary shocks. Eichenbaum and Fisher (2004) and Dotsey and King (2005), for example, investigate how
relaxing assumptions of constant elasticities of demand interact with other frictions to generate persistence.
Klenow and Willis (2016) compare the predictions of models where real rigidities are generated by a kinked
demand curve versus sticky intermediate prices.

In describing changes in the allocative efficiency of the economy, we also relate to a vast literature on cross-sectional misallocation, which includes Restuccia and Rogerson (2008), Hsieh and Klenow (2009), and Baqaee and Farhi (2020). For the most part, the misallocation literature is concerned with cross-country or long-run changes in misallocation, whereas we are focused on characterizing short-run changes in misallocation following nominal shocks. Some important exceptions are Cravino (2017), Baqaee and Farhi (2017), and Meier and Reinelt (2020). In an international context, Cravino (2017) shows that heterogeneity in exporters' invoicing currency and desired markups (due to local distribution costs), coupled with nominal rigidities, implies that exchange rate changes can affect domestic productivity by changing the allocation of resources. Baqaee and Farhi (2017) provide a general framework for how allocative efficiency changes in general equilibrium and apply their results to show that if price-stickiness positively covaries with markups, then monetary policy affects TFP. Meier and Reinelt (2020) provide empirical support for this covariance, and offer a microfoundation where firms have heterogeneous Calvo parameters, so firms with more rigid prices endogenously set higher markups due to a precautionary motive. Our analysis complements, and to some extent unifies, these previous analyses by showing how heterogeneity in realized pass-throughs (driven either by variable stickiness or variable desired pass-throughs) can cause nominal shocks to have effects on productivity.[12]

The differential cross-sectional response of firms to monetary policy links the slope of the Phillips curve in our analysis to moments of the firm distribution, such as industrial concentration. Here, our study is complemented by Etro and Rossi (2015), Wang and Werning (2020), Andrés and Burriel (2018), and Corhay et al. (2020) who also discuss mechanisms by which an increase in concentration may contribute to a decline in inflation and flattening of the Phillips curve; our work is unique among these in identifying the misallocation channel of monetary policy as a potential source for this effect.

Finally, our paper is also related to a recent literature on endogenous TFP movements over the business cycle (e.g., Comin and Gertler 2006, Benigno and Fornaro 2018, Anzoategui et al. 2019, and Bianchi et al. 2019). In this literature, aggregate TFP responds to the business cycle due to frictions in technology investment, adoption, and diffusion. In contrast to this body of work, the endogenous TFP movements that arise in our model are due solely to changes in the allocation of resources across firms, rather than underlying technological primitives.

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[12]Productivity shocks can also affect allocative efficiency: David and Zeke (2021) show that allocative efficiency varies over the business cycle when firms have heterogeneous exposure to aggregate shocks.
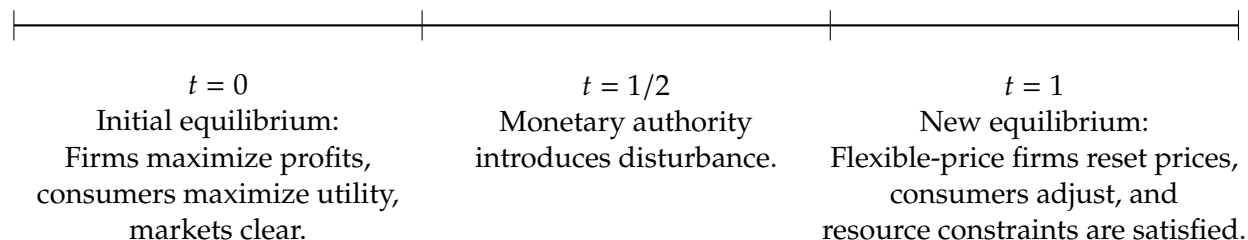
Structure of the paper. Section 2 introduces a simple one-period model and defines the equilibrium. Sections 3 and 4 describe the response of aggregate TFP and output (real GDP) to a monetary shock in the one-period model. Section 5 generalizes the static model from the previous sections to a fully dynamic setting, which yields a four-equation New Keynesian model with misallocation. Section 6 provides a quantitative illustration of the importance of the misallocation channel. Section 7 provides empirical evidence at the macro- and micro-level for the mechanisms described in the model. In Section 8, we summarize some extensions discussed in more detail in the appendices, including a version of the model with menu costs, an extension with multiple sectors, multiple factors, input-output linkages, and sticky wages, as well as an alternative micro-foundation using oligopolistic, rather than monopolistic, competition. Section 9 concludes.

2   Model Setup

We start with a simple model with a single factor, labor. To build intuition, we first develop a one-period model to highlight the mechanism driving changes in allocative efficiency. A dynamic version of the model is presented in Section 5.

The timing is shown in Figure 1. At time $t = 0$, the economy is in an equilibrium: households choose consumption and labor to maximize utility, firms choose prices to maximize profits, and markets clear. The monetary authority then introduces an unexpected disturbance in nominal marginal costs. At time $t = 1$, firms with flexible prices reset prices to maximize profits, while firms with sticky prices keep prices unchanged from the initial equilibrium. Households adjust consumption and labor to maximize utility.[13]

Figure 1: One-period model timing.

$t = 0$	$t = 1/2$	$t = 1$
Initial equilibrium:	Monetary authority	New equilibrium:
Firms maximize profits,	introduces disturbance.	Flexible-price firms reset prices,
consumers maximize utility,		consumers adjust, and
markets clear.		resource constraints are satisfied.

We describe the behavior of households, firms, and the monetary authority in turn.

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[13]We relax the one-period-ahead Calvo friction when we introduce the dynamic model in Section 5. In the infinite-horizon model each firm changes its price at a constant hazard rate.

Households. There is a population of identical consumers. Consumers' preferences over the consumption bundle Y and labor L are given by

$$u(Y,L) = \frac{Y^{1-\gamma}-1}{1-\gamma} - \frac{L^{1+\frac{1}{\zeta}}}{1+\frac{1}{\zeta}},$$

where $1/\gamma$ is the intertemporal elasticity of substitution, and ζ is the Frisch elasticity of labor supply. The consumption bundle Y consists of different varieties of goods indexed by $\theta \in [0,1]$. Consumers have homothetic preferences over these final goods, and the utility from the consumption bundle Y is defined implicitly by[14]

$$\int_0^1 \Upsilon_\theta(\frac{y_\theta}{Y})d\theta = 1.$$

Here, y_θ is the consumption of variety θ, and Υ_θ is an increasing and concave function. CES preferences are a special case of the general preferences above, when $\Upsilon_\theta = \Upsilon$ is a power function.

The representative consumer maximizes utility subject to the budget constraint

$$\int_0^1 p_\theta y_\theta d\theta = wL + \Pi,$$

where wL is labor income and Π is firm profit income. Maximization yields the inverse-demand curve for variety θ:

$$\frac{p_\theta}{P} = \Upsilon'_\theta(\frac{y_\theta}{Y}), \tag{1}$$

where the *price aggregator P* is defined as

$$P = \frac{P^Y}{\int_0^1 \Upsilon'_\theta(\frac{y_\theta}{Y})\frac{y_\theta}{Y}d\theta}, \tag{2}$$

and P^Y is the ideal price index.[15] Equation (1) shows that relative demand for a variety θ is dictated by the ratio of its price to the price aggregator P. Hence, firms compete with the

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[14]These preferences are a generalization of Kimball (1995) preferences since the aggregator function Υ_θ is allowed to vary by variety. For more information, see Matsuyama and Ushchev (2017), who refer to these as homothetic with direct implicit additivity (HDIA) preferences.

[15]The ideal price index is defined as $\min_{y_\theta}\{\int p_\theta y_\theta : Y = 1\}$. The price aggregator P, which disciplines demand curves, coincides with the ideal price index P^Y if, and only if, preferences are CES. Dividing nominal expenditure by the ideal price index P^Y yields real output Y. Changes in this ideal price index $d\log P^Y$ are first-order equivalent to changes in the consumer price index (CPI) as calculated by national statistical agencies. Therefore, changes in real output in the data are defined in a way that is consistent with $d\log Y$ in our model.

rest of the market via a single price and quantity aggregator. Equation (1) also illustrates the appeal of these preferences: we can create downward-sloping demand curves of any desired shape by choosing the aggregator Υ_θ.

Firms. Each variety is supplied by a single firm, and a firm of type θ has productivity A_θ. Firms produce using a constant returns to scale technology, so that the cost of producing an additional unit is constant at w/A_θ.

In the initial equilibrium, before the unexpected (zero-probability) monetary disturbance, each firm sets its price to maximize expected profits,

$$p_\theta^{\text{flex}} = \operatorname*{argmax}_{p_\theta} \mathbb{E}\left(p_\theta y_\theta - \frac{w}{A_\theta} y_\theta\right),$$

subject to its residual demand curve (1).

Unlike the CES demand system, which imposes that the price elasticity of demand is constant in both the time series and the cross-section of firms, we allow the price elasticity facing a firm to vary both with the firm's type θ and its position on the demand curve. We can use the inverse-demand function in (1) to solve for the price elasticity of demand facing a firm of type θ:

$$\sigma_\theta\left(\frac{y}{Y}\right) = -\frac{\partial \log y_\theta}{\partial \log p_\theta} = \frac{\Upsilon_\theta'\left(\frac{y}{Y}\right)}{-\frac{y}{Y}\Upsilon_\theta''\left(\frac{y}{Y}\right)}.$$

The profit-maximizing price p_θ^{flex} can be written as a markup μ_θ^{flex} times marginal cost. When the firm is able to change its price, the firm's desired price and markup are determined by

$$p_\theta^{\text{flex}} = \mu_\theta^{\text{flex}} \frac{w}{A_\theta}, \qquad \text{and} \qquad \mu_\theta^{\text{flex}} = \mu_\theta\left(\frac{y_\theta^{\text{flex}}}{Y}\right),$$

where the markup function is given by the Lerner formula,[16]

$$\mu_\theta\left(\frac{y}{Y}\right) = \frac{1}{1 - \frac{1}{\sigma_\theta\left(\frac{y}{Y}\right)}}. \tag{3}$$

Following Calvo (1983), we assume a firm of type θ has a probability δ_θ of being able to reset its price at time $t = 1$. These nominal rigidities are allowed to be heterogeneous across firm types. Flexible-price firms reset prices in $t = 1$ according to the optimal price and markup formulas above, and sticky-price firms keep their prices unchanged. As a

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[16]We assume that marginal revenue curves are downward-sloping, so that the optimal choice of p_θ and y_θ is unique for each firm. In terms of primitives, this requires that $x\Upsilon_\theta'''(x) + 2\Upsilon_\theta''(x) < 0$ for every x and θ.

result, the prices and markups of sticky-price firms at $t = 1$ are given by

$$p_{\theta,1}^{\text{sticky}} = p_{\theta,0} \qquad \text{and} \qquad \mu_{\theta,1}^{\text{sticky}} = \frac{w_1}{w_0}\mu_{\theta,0},$$

where the second subscript denotes the period.

A firm's desired pass-through ρ_θ is the elasticity of its optimal price with respect to its marginal cost, holding the economy-wide aggregates constant. We can express the desired pass-through of firm θ as:

$$\rho_\theta(\frac{y}{Y}) = \frac{\partial \log p_\theta^{\text{flex}}}{\partial \log mc} = \frac{1}{1 + \frac{\frac{y}{Y}\mu_\theta'(\frac{y}{Y})}{\mu_\theta(\frac{y}{Y})}\sigma_\theta(\frac{y}{Y})}. \tag{4}$$

Under CES preferences, desired markups $\mu_\theta = \mu = \sigma/(\sigma - 1)$ are constant across firms. Furthermore, desired markups do not depend on the firm's location on the demand curve. When markups do not vary as a function firm size, desired pass-through is equal to one for all firms, and firms exhibit "complete desired pass-through." For brevity, we refer to ρ_θ simply as the firm's "pass-through" instead of desired pass-through. Keep in mind, however, that this pass-through is conditional on the firm's ability to change its price. For firms that are unable to change their prices, pass-through is de facto equal to zero.

Monetary authority. At time $t = 1/2$, there is an unexpected shock to the nominal wage. We interpret this shock as a disturbance introduced by the monetary authority. We could equivalently have the monetary authority choose any other nominal variable in the economy, such as the overall price level or money supply; the nominal wage is especially convenient as it directly affects the marginal cost of every firm. In the dynamic version of the model in Section 5, changes in the nominal wage are caused by interest rate or discount factor shocks.

We say that the shock is expansionary if the nominal wage in period 1 is higher than the one in period 0, since in this case the increase in nominal marginal cost decreases markups for firms whose prices cannot adjust, and this reduction in markups boosts labor demand and hence output.

Equilibrium conditions. In equilibrium, for a given value of the nominal wage w, (1) consumers choose consumption and labor to maximize utility taking prices as given, (2) firms with flexible prices set prices to maximize profits taking other firms' prices and their residual demand curves as given, (3) firms with sticky prices produce to meet demand at fixed prices, and (4) all resource constraints are satisfied.

Notation. Throughout the rest of the paper, we use the following notation. For two variables $x_\theta > 0$ and z_θ, define the *x-weighted expectation of z* by

$$\mathbb{E}_x[z_\theta] = \frac{\int_0^1 z_\theta x_\theta d\theta}{\int_0^1 x_\theta d\theta}.$$

We write $\mathbb{E}$ to denote $\mathbb{E}_x$ when $x_\theta = 1$ for all θ. The operator $\mathbb{E}_x$ operates a change of measure by putting more weight on types θ with higher values of x_θ. We denote the sales share density by[17]

$$\lambda_\theta = \frac{p_\theta y_\theta}{\int_0^1 p_\theta y_\theta d\theta},$$

and define the *aggregate markup* to be

$$\bar{\mu} = \mathbb{E}_\lambda \left[\mu_\theta^{-1} \right]^{-1}.$$

In words, $\bar{\mu}$ is the sales-weighted harmonic average of markups.

Log-linearization around initial equilibrium. In the analysis that follows, we consider first-order perturbations around the initial equilibrium caused by a change in the nominal wage. Let $\bar{X}$ denote the value of a variable X in the initial equilibrium. We denote the deviation of $\log X$ from the initial equilibrium value $\log \bar{X}$ as $d \log X$.[18]

3 Productivity Response

In this section, we consider how aggregate productivity changes following a monetary shock. We first introduce the concept of allocative efficiency and discuss its dependence on the distribution of markups. Then, we show that, when markups are initially dispersed, the reshuffling of resources across firms following a monetary shock changes aggregate productivity.

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[17]As long as the productivity distribution is continuous, we can assume that the type distribution is uniform without loss of generality between [0,1] because we can define a firm's type by the fraction of firms whose productivity is less than that firm.

[18]$d \log X$ in our notation is the same as the lowercase x used by Galí (2015). We instead opt for $d \log X$ because we use lowercase variables to refer to firm-level variables (e.g., output y_θ and price p_θ) and uppercase variables to refer to economy-wide aggregates (e.g., aggregate output Y and labor L). Since all variables can be written as implicit functions of the wage w, more formally, $d \log X$ is a short-hand for $d \log X / d \log w \times \Delta \log w$, where $\Delta \log w$ is a small change in w and the derivatives are evaluated at the initial steady-state.

3.1 Allocative Efficiency and TFP

Define aggregate productivity A as the output produced per unit of labor in the economy, so that

$$Y = AL.$$

Since labor is the sole factor in our model economy, A is equal to both aggregate TFP and aggregate labor productivity. In a richer economy with multiple factors of production, the relevant measure of A is the distortion-adjusted (or cost-based) Solow residual (see Appendix F for an extension to multiple factors and multiple sectors).[19] Changes in aggregate output can be decomposed into changes in employment and changes in productivity:

$$d \log Y = \underbrace{d \log A}_{\substack{\text{Change in} \\ \text{aggregate productivity}}} + \underbrace{d \log L}_{\substack{\text{Change in} \\ \text{labor used}}}.$$

This change in aggregate productivity is closely linked to the distribution of markups across firms. To see why, note that aggregate productivity depends on *allocative efficiency*—the efficiency with which resources are divvied up across competing uses. In an economy with no dispersion in markups, the cross-sectional allocation of resources is efficient. However, heterogeneity in markups creates distortions in the allocation of resources across firms: firms with higher markups restrict output and are inefficiently small compared to firms with lower markups. Hence, a change in the distribution of markups can lead to change in allocative efficiency, increasing or decreasing output holding employment constant.

To characterize how aggregate productivity is related to changes in firm markups, we apply the main result from Baqaee and Farhi (2020). The change in aggregate productivity from $t = 0$ to $t = 1$ following the monetary shock is

$$d \log A = d \log \bar{\mu} - \mathbb{E}_\lambda \left[d \log \mu_\theta \right]. \tag{5}$$

Equation (5) shows that allocative efficiency increases when the (harmonic) average markup rises more than markups on average.[20] To further simplify (5), substitute in

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[19]The distortion-adjusted Solow residual weighs the change in each factor by its share of total factor costs, rather than its share in aggregate income. See Hall (1990) and Baqaee and Farhi (2020) for more information on why the distortion-adjusted Solow residual is the correct object to use in the presence of markups.

[20]To get Equation (5) from Baqaee and Farhi (2020), note that Baqaee and Farhi (2020) Theorem 1 shows that the change in allocative efficiency (in an economy with arbitrary input-output linkages) is given by $d \log A = -\tilde{\Lambda} d \log \Lambda - \tilde{\lambda} d \log \mu$, where Λ and $\tilde{\Lambda}$ are vectors of sales- and cost-based factor Domar weights and $\tilde{\lambda}$ is a vector of cost-based Domar weights for firms. We get (5) by imposing that labor is the sole factor

$d \log \bar{\mu} = -\mathbb{E}_{\lambda \mu^{-1}} \left[d \log(\lambda_\theta / \mu_\theta) \right]$ to get

$$d \log A = \mathbb{E}_\lambda \left[(\bar{\mu}/\mu_\theta - 1) \, d \log \mu_\theta \right] - \mathbb{E}_\lambda \left[(\bar{\mu}/\mu_\theta) \, d \log \lambda_\theta \right] = Cov_\lambda \left[-(\bar{\mu}/\mu_\theta), d \log(\lambda_\theta / \mu_\theta) \right].$$

Noting that the ratio of a firm's sales share (λ_θ) to its markup (μ_θ) is proportional to its variable costs yields Lemma 1.

Lemma 1. *Following a monetary shock, the response of aggregate TFP at $t = 1$ is proportional to the (sales-weighted) covariance of inverse markups with changes in firms' variable costs:*

$$d \log A = Cov_\lambda \left[-(\bar{\mu}/\mu_\theta), d \log Costs_\theta \right] = Cov_\lambda \left[-(\bar{\mu}/\mu_\theta), d \log wl_\theta \right]. \tag{6}$$

Lemma 1 shows that allocative efficiency increases when resources are reallocated to high-markup firms. Since labor is the sole factor of production and production has constant returns to scale, this is equivalent to a reallocation of the total wage bill to high-markup firms. Intuitively, since high-markup firms are inefficiently small in the initial equilibrium, a reallocation of resources to these firms alleviates misallocation and increases TFP.

A corollary of Lemma 1 is that when markups are initially identical, a monetary shock has no first-order effect on aggregate productivity.

Corollary 1. *If $\mu_\theta = \mu$ in the initial equilibrium, then following a monetary shock, the response of aggregate TFP at $t = 1$ is*

$$d \log A = 0,$$

regardless of changes in markups $d \log \mu_\theta$.

Corollary 1 can also be seen as a consequence of the envelope theorem: when markups are identical across firms, the initial cross-sectional allocation of resources is efficient. Hence, changes in aggregate TFP due to reallocations are zero to a first order. As a result, in models with no initial markup dispersion, the response of aggregate TFP to monetary shocks is zero to a first-order, regardless of how markups respond.

Relationship to relative price adjustments. Whether output and costs are reallocated to high-markup firms following a monetary shock depends on the degree to which high- and low-markup firms adjust their prices following the shock. We can rearrange (6) to

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(so that $\tilde{\Lambda}_L = 1$ and the change in the sales-based Domar weight of labor is simply $d \log \Lambda_L = -d \log \bar{\mu}$) and that there are no markups downstream of firms (so that $\tilde{\lambda} = \lambda$).

get:

$$d \log A = Cov_\lambda \left[(\bar{\mu}/\mu_\theta), \sigma_\theta \, d \log(p_\theta/P) \right] = \bar{\mu} Cov_\lambda \left[\sigma_\theta / \mathbb{E}_\lambda \left[\sigma_\theta \right], d \log \mu_\theta \right].$$

That is, resources are reallocated to firms that reduce their prices, and hence markups, relative to other firms.

How the price charged by firm type θ changes following the monetary shock depends on firm θ's price-stickiness (δ_θ) and the degree to which the firm adjusts its markup conditional on a price change (ρ_θ), as well as the changes in prices set by all other firms (which enters the price aggregator P). The final result requires us to solve a fixed point for the change in the price aggregator, and hence the details are relegated to Appendix A. The next section characterizes the TFP response in terms of these primitives.

3.2 Productivity Response: Two Mechanisms

Proposition 1 characterizes the response of aggregate TFP to a monetary shock in terms of primitives: firms' price-stickiness, pass-throughs, elasticities of demand, and markups in the initial equilibrium.

Proposition 1. *Following a monetary shock, the response of aggregate TFP at $t = 1$ is*

$$\frac{d \log A}{d \log w} = \underbrace{\kappa_\rho Cov_\lambda \left[\rho_\theta, \sigma_\theta | \text{flex} \right]}_{\substack{\text{Reallocation due to} \\ \text{heterogeneous} \\ \text{pass-through}}} + \underbrace{\kappa_\delta Cov_\lambda \left[\sigma_\theta, \delta_\theta \right]}_{\substack{\text{Reallocation due to} \\ \text{heterogeneous} \\ \text{price-stickiness}}}, \tag{7}$$

where $Cov_\lambda \left[\rho_\theta, \sigma_\theta | \text{flex} \right]$ is the covariance of pass-throughs and elasticities for the subset of flexible-price firms,[21] and κ_ρ and κ_δ are positive constants

$$\kappa_\rho = \frac{\bar{\mu} \mathbb{E}_\lambda [\delta_\theta] \mathbb{E}_\lambda \left[1 - \delta_\theta \right]}{\mathbb{E}_\lambda \left[[\delta_\theta \rho_\theta + (1 - \delta_\theta)] \sigma_\theta \right]}, \qquad \kappa_\delta = \frac{\bar{\mu} \mathbb{E}_{\lambda\delta} \left[\rho_\theta \right]}{\mathbb{E}_\lambda \left[[\delta_\theta \rho_\theta + (1 - \delta_\theta)] \sigma_\theta \right]}.$$

A first glance reveals that the response of aggregate TFP is nonzero only when markups are dispersed. If markups μ_θ and thus elasticities $\sigma_\theta = \mu_\theta/(\mu_\theta - 1)$ are equal across firms, both covariance terms in Equation (7) are zero.

However, dispersion in markups is not sufficient for monetary policy to affect aggregate productivity. It must also be the case that markups (or equivalently, σ_θ) covary systematically with either desired pass-throughs or price-stickiness. Either of these two

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[21]This is equivalent to $Cov_{\lambda\delta} \left[\rho_\theta, \sigma_\theta \right]$.

mechanisms cause *realized* pass-throughs to covary with the level of the markups, and as long as realized pass-through covaries negatively with the level of markups, an increase in nominal marginal costs will result in productivity-increasing reallocations.[22] To build more intuition, we now consider each of the two mechanisms mentioned above in isolation.

Mechanism I: heterogeneous pass-through.   If price-stickiness is homogeneous across firms ($\delta_\theta = \delta$), then the second covariance term in Proposition 1 is zero, and the productivity response depends on the covariance between desired pass-throughs ρ_θ and elasticities σ_θ alone:

Corollary 2. *If price stickiness is homogeneous across firms ($\delta_\theta = \delta$), then*

$$\frac{d \log A}{d \log w} = \kappa_\rho Cov_\lambda \left[\rho_\theta, \sigma_\theta\right].$$

If markups negatively covary with pass-throughs, then $\frac{d \log A}{d \log w} > 0$.

One of the salient reasons why markups may covary negatively with pass-throughs is related to firm size.

Definition 1. *Marshall's strong second law of demand* requires that desired markups are increasing in quantity and desired pass-throughs are decreasing in quantity. That is,[23]

$$\mu'(\frac{y}{Y}) > 0 \qquad \text{and} \qquad \rho'(\frac{y}{Y}) < 0.$$

If Marshall's strong second law holds, then markups are increasing and pass-throughs are decreasing in firm size. This guarantees that monetary expansions will raise aggregate productivity. Marshall's second law of demand has strong empirical support (see, for example, empirical estimates of pass-throughs by firm size from Amiti et al. 2019).[24]

While Marshall's strong second law is sufficient for a supply-side channel of monetary policy to operate, it is not necessary. Markups and pass-throughs may be correlated for

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[22]For concreteness, in this paper, we interpret increases in nominal marginal cost $d \log w > 0$ to be the consequence of monetary easing. However, the basic intuition will apply to other kinds of demand shocks as well, since other shocks to aggregate demand will also raise nominal marginal costs, and hence lead to productivity-increasing reallocations.

[23]Marshall's strong second law of demand is equivalent to requiring that the individual marginal revenue curve be log-concave. See Melitz (2018) for more information.

[24]Oligopolistic competition models, such as Atkeson and Burstein (2008), also satisfy Marshall's strong second law of demand. In Appendix I, we show that our results can also be derived under such a framework.

reasons unrelated to firm size, such as quality or nicheness (e.g. as shown empirically by Chen and Juvenal, 2016 or Auer et al., 2018).

To understand the intuition for Corollary 2, consider an expansionary shock ($d \log w > 0$). The higher nominal wage increases marginal costs, leading flexible-price firms to increase their prices. The optimal price satisfies

$$d \log p_\theta^{\text{flex}} = (1 - \rho_\theta) d \log P + \rho_\theta d \log w, \tag{8}$$

where $d \log P$ is the change in the price aggregator defined in Equation (2). The optimal price of high pass-through firms moves closely with marginal cost. Firms with low pass-through instead exhibit "pricing-to-market" behavior: they place less weight on their own marginal cost and more weight on the expected price of their competitors, summarized by the price aggregator. Sticky-price firms, of course, cannot adjust their prices after observing the nominal wage shock.

Following an increase in the nominal wage, flexible-price firms shrink and sticky-price firms, whose prices are kept artificially low, expand[25]:

$$d \log(\frac{y_\theta^{\text{flex}}}{Y}) = -\sigma_\theta \rho_\theta \left(d \log w - d \log P \right) < 0, \quad \text{and} \quad d \log(\frac{y_\theta^{\text{sticky}}}{Y}) = \sigma_\theta d \log P > 0.$$

If $Cov_\lambda [\rho_\theta, \sigma_\theta] > 0$, then among flexible-price firms, firms with high markups and low pass-throughs raise their prices by less in response to an expansionary shock. As a result, a greater share of resources is directed towards high-markup firms and this boosts allocative efficiency.

Note that this mechanism disappears when prices are either fully flexible or fully rigid (in both cases, $\kappa_\rho = 0$). When prices are fully flexible, there is complete pass-through of marginal cost shocks into prices in general equilibrium despite the fact that, in partial equilibrium, pass-through is incomplete. This is because in the absence of nominal rigidities, the fixed-point in (8) results in full adjustment of all prices to marginal costs. On the other hand, when prices are fully rigid, relative prices cannot change, so again, there are no reallocations from changes in nominal marginal costs.

Mechanism II: heterogeneous price-stickiness. Now consider the case where pass-through is homogeneous, but price-stickiness is not.

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[25]Due to nominal rigidities, the price aggregator P will move more slowly than the nominal wage, so generically $\frac{d \log P}{d \log w} \in [0, 1]$.

Corollary 3. *If desired pass-through is homogeneous across firms ($\rho_\theta = \rho$),[26] then*

$$\frac{d \log A}{d \log w} = \kappa_\delta Cov_\lambda \left[\sigma_\theta, \delta_\theta \right].$$

(9)

If high-markup firms have higher price-stickiness, then $\frac{d \log A}{d \log w} > 0$.

Consider an expansionary shock ($d \log w > 0$). If high markups firms are less likely to adjust prices, low-markup firms will tend to increase their prices more on average than high-markup firms. This causes high-markup firms to expand relative to low-markup firms in response to a monetary expansion. This also compresses the markup distribution and increase aggregate TFP.

The mechanism by which heterogeneous price-stickiness can result in endogenous aggregate TFP changes was previously pointed out in Baqaee and Farhi (2017) and has been recently analyzed by Meier and Reinelt (2020). Meier and Reinelt (2020) show that in a CES model with heterogeneous Calvo parameters, firms with greater price rigidity endogenously set higher markups due to a precautionary motive; this generates a reduction in markup dispersion following expansionary shocks.

Although our model allows for the possibility that price-stickiness varies systematically with firm type, we do not pursue this in our quantitative application. When we quantify the model, we assume there is no variation in price-stickiness and instead focus on heterogeneity in desired pass-through only. This is because there is more robust empirical support for Marshall's strong second law of demand than for a systematic correlation between markups and price stickiness.[27]

4   Output Response and the Phillips Curve

In the previous section, we showed that aggregate TFP responds to monetary shocks. In this section, we show how monetary shocks are transmitted to output, taking into account these movements in aggregate productivity. We show that the change in output can be decomposed into three channels: (1) nominal rigidities (as in a CES economy with sticky prices), (2) real rigidities due to imperfect pass-through (which arise from strategic complementarities in pricing), and (3) the endogenous response of aggregate TFP caused by the misallocation channel.

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[26]Homogeneous desired pass-throughs are generated when the Kimball aggregator takes the form, $\Upsilon(x) = -\text{Ei}(-Ax^{\rho-1})$ where $\text{Ei}(x) = \int_{-x}^\infty \frac{e^{-t}}{t} dt$ is the exponential integral function.
[27]For example, Goldberg and Hellerstein (2011) find that larger firms, who presumably have higher markups, also have more flexible prices.

This section is organized as follows. We first characterize the response of output to a monetary shock. Then, we characterize the slope of the Phillips curve and formalize two channels—real rigidities and the misallocation channel—which flatten the slope of the Phillips curve relative to the benchmark model. Finally, to gain intuition, we compute the slope of the Phillips curve in a few simple example economies.

4.1 Output Response

Proposition 2 describes the response of output to a monetary shock.

Proposition 2. *Following a shock to the nominal wage $d \log w$, the response of output at $t = 1$ is*

$$d \log Y = \underbrace{\frac{1}{1 + \gamma\zeta} d \log A}_{\text{Supply-side effect}} - \underbrace{\frac{\zeta}{1 + \gamma\zeta} \mathbb{E}_\lambda [d \log \mu_\theta]}_{\text{Demand-side effect}}, \tag{10}$$

where

$$\frac{\mathbb{E}_\lambda [d \log \mu_\theta]}{d \log w} = - \underbrace{\mathbb{E}_\lambda [1 - \delta_\theta]}_{\text{Nominal rigidities}} - \underbrace{\frac{\mathbb{E}_\lambda [\delta_\theta(1 - \rho_\theta)] \mathbb{E}_\lambda [\sigma_\theta(1 - \delta_\theta)]}{\mathbb{E}_\lambda [[\delta_\theta\rho_\theta + (1 - \delta_\theta)] \sigma_\theta]}}_{\text{Real rigidities}}. \tag{11}$$

Equation (10) breaks down the response of output into a supply-side and demand-side effect. The demand-side effect of an expansionary shock arises from the average reduction in markups, which increases labor demand (and employment). The supply-side effect is due to changes in aggregate TFP and arises from changes in the economy's allocative efficiency.

Equation (11) further decomposes the demand-side effect into the effect of sticky prices and the effect of real rigidities. The first is the standard New Keynesian channel: nominal rigidities prevent sticky-price firms from responding to the marginal cost shock. As a result, markups fall for a fraction $\mathbb{E}_\lambda [\delta_\theta]$ of firms. This reduction in the markups of sticky-price firms boosts labor demand, employment, and ultimately output.

This sticky price effect in (11) is exacerbated by real rigidities, which arise from imperfect pass-through. When pass-through is incomplete, flexible-price firms increase prices less than one-for-one with the marginal cost shock. As a result, the markups of flexible-price firms also fall. Together, the reduction in the markups of both sticky-price and flexible-price firms increase labor demand, which spurs employment and output.

The supply-side effect is concerned with the productivity of these resources. Returning to (10), we find that when aggregate TFP increases following an expansionary shock

($d \log A / d \log w > 0$), the endogenous positive "supply shock" complements the effects of the positive "demand shock" on output.

Interestingly, whereas the demand-side effect is increasing in the size of the elasticity of labor supply ζ, the supply-side effect is decreasing in ζ. In fact, the supply-side effect is strongest when labor is inelastically supplied ($\zeta = 0$). On the other hand, as the Frisch elasticity of labor supply approaches infinity, the supply side effect becomes irrelevant for output. This is because when the Frisch is infinite, the reallocations that boost productivity and raise the market share of high-markup firms are exactly cancelled out by reductions in labor supply, which contracts due to the expansion of high-markup firms.

4.2   The Misallocation Channel and the Phillips Curve

How does incorporating this supply-side effect change the slope of the Phillips curve? We now construct the Phillips curve—the relationship between the change in output and change in the price level generated by a demand shock—in the model and show that the misallocation channel flattens its slope.[28]

We can directly get the slope of the wage Phillips curve by rearranging the output response in Proposition 2. To get the price Phillips curve, we use the relationship between the consumer price index P^Y, the nominal wage, and average markups,

$$d \log P^Y = d \log w + \mathbb{E}_\lambda \left[d \log \mu_\theta \right].$$

Both are presented in Proposition 3.

Proposition 3. *For the static model, the wage Phillips curve is given by*

$$d \log w = (1 + \gamma \zeta) \frac{1}{\left[\frac{d \log A}{d \log w} - \zeta \mathbb{E}_\lambda \left[\frac{d \log \mu_\theta}{d \log w} \right] \right]} d \log Y.$$

The price Phillips curve is given by

$$d \log P^Y = (1 + \gamma \zeta) \frac{1 + \mathbb{E}_\lambda \left[\frac{d \log \mu_\theta}{d \log w} \right]}{\left[\frac{d \log A}{d \log w} - \zeta \mathbb{E}_\lambda \left[\frac{d \log \mu_\theta}{d \log w} \right] \right]} d \log Y.$$

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[28]In the data, this relationship between the output gap (or unemployment) and inflation is confounded by other shocks that affect output or prices independently. For example, Fratto and Uhlig (2014) show that wage and price markup shocks play an important role in inflation dynamics, thus affecting the empirical Phillips curves constructed from aggregate data. The dynamic version of the model micro-founds some of these confounds because in that model demand shocks generate endogenous markup (cost-push) shocks that raise output and lower inflation.

By increasing the denominator, the supply-side effect flattens both the price and wage Phillips curves. We can further quantify the degree to which real rigidities and the misallocation channel each flatten the Phillips curve relative to the CES baseline. To do this, we calculate the flattening of the Phillips curve due to real rigidities by dividing the slope of the Phillips curve due to sticky-prices alone by the slope of the Phillips curve due to sticky prices and real rigidities. Since real rigidities flatten the Phillips curve, this ratio is greater than one. If this quantity is, say, 1.5, this means that incorporating real rigidities decreases the responsiveness of output to the price level by 50%. Similarly, we calculate the flattening of the Phillips curve due to misallocation channel by comparing the slope of the Phillips curve due to sticky prices and real rigidities by the slope of the Phillips curve where we account for changes in allocative efficiency. As long as $\frac{d \log A}{d \log w} > 0$, this quantity is also greater than one.

Proposition 4 presents the flattening of the price Phillips curve due to each channel. For simplicity, we present the case where pass-throughs are heterogeneous and price-stickiness is constant across firms (the general version is Proposition 6 in Appendix A).

Proposition 4. *Suppose $\delta_\theta = \delta$ for all firms. The flattening of the price Phillips curve due to real rigidities, compared to nominal rigidities alone, is*

$$Flattening\ due\ to\ real\ rigidities = 1 + \frac{\mathbb{E}_\lambda\left[\sigma_\theta\right]\mathbb{E}_\lambda\left[1-\rho_\theta\right]}{\delta Cov_\lambda\left[\rho_\theta, \sigma_\theta\right] + \mathbb{E}_\lambda\left[\rho_\theta\right]\mathbb{E}_\lambda\left[\sigma_\theta\right]}. \tag{12}$$

The flattening of the price Phillips curve due to the misallocation channel is

$$Flattening\ due\ to\ the\ misallocation\ channel = 1 + \frac{\bar{\mu}}{\zeta}\frac{\delta Cov_\lambda\left[\rho_\theta, \sigma_\theta\right]}{\delta Cov_\lambda\left[\rho_\theta, \sigma_\theta\right] + \mathbb{E}_\lambda\left[\sigma_\theta\right]}. \tag{13}$$

In Equation (12), we see that the flattening of the price Phillips curve due to real rigidities increases as average pass-throughs fall. The flattening due to real rigidities in (12) is also decreasing in price flexibility δ. As price flexibility increases, the price aggregator moves more closely with shocks to marginal cost; hence the "pricing-to-market" effect from incomplete pass-throughs is less powerful.

The flattening of the price Phillips curve due to the misallocation channel depends positively on covariance of the pass-throughs and elasticities. When $Cov_\lambda\left[\rho_\theta, \sigma_\theta\right] = 0$, there is no allocative efficiency effect on the slope of the Phillips curve. Equation (13) also shows that the flattening due to misallocation is decreasing in the Frisch elasticity ζ. Finally, since the expansion of high-markup firms relative to low-markup firms occurs only for flexible-price firms, the misallocation channel is stronger relative to real rigidities when price flexibility is higher (δ closer to one).

To cement intuition, we now calculate the change in allocative efficiency and the slope of the Phillips curve in three simple benchmark economies: an economy with CES preferences, an economy with real rigidities but a representative firm, and an economy with two firm types.

CES Example. We obtain the CES benchmark by setting $\Upsilon_\theta(x) = x^{\frac{\sigma-1}{\sigma}}$, where $\sigma > 1$. Under CES, desired markups for all firms are fixed at $\mu = \frac{\sigma}{\sigma-1}$, and all firms exhibit complete desired pass-through of cost shocks to price ($\rho = 1$).

Since desired markups are uniform, the initial allocation of the economy is efficient. By Corollary 1, $d \log A = 0$. Applying Proposition 3, the slope of the price Phillips curve is

$$d \log P^Y = \frac{1 + \gamma\zeta}{\zeta} \frac{\delta}{1 - \delta} d \log Y.$$

This is the traditional New Keynesian Phillips Curve.[29] Nominal rigidities, captured by the Calvo parameter $\delta < 1$, flatten the Phillips curve. As δ approaches one, prices become perfectly flexible and the Phillips curve becomes vertical.

Representative Firm Example. We now relax the assumption of CES preferences, but consider an economy with a representative firm: all firms have the same price-stickiness ($\delta_\theta = \delta$), the same residual demand curve $\Upsilon'_\theta = \Upsilon'$, and productivity level ($A_\theta = 1$).

The homogeneous firms in this economy have identical markups, $\mu_\theta = \mu$, and pass-throughs, $\rho_\theta = \rho$. By deviating from CES, however, we allow firms' desired pass-throughs to be incomplete, i.e., $\rho < 1$.

Since markups are uniform, the cross-sectional allocation of resources across firms in the initial equilibrium is efficient. Applying Corollary 1, we have $d \log A = 0$. Unlike the CES case, incomplete pass-throughs imply that flexible-price firms will not fully adjust prices to reflect increases in marginal cost from a monetary shock. Compared to the CES economy, prices in this economy are slower to respond, and hence, the slope of the price Phillips curve is flatter:

$$d \log P^Y = \frac{1 + \gamma\zeta}{\zeta} \frac{\delta}{1 - \delta} \rho d \log Y.$$

In particular, Proposition 4 implies that the amount of flattening due to the real rigidities channel is ρ^{-1}. That is, the amount of flattening is decreasing in the average desired pass-through ρ.

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[29]See, for example, Galí (2015). Section 4.2 can be replicated exactly from Galí (2015) pg. 63 by setting $\beta = 0$ and assuming constant returns to scale.

Two Type Example.   We now allow for heterogeneous firms of two types: high- and low-markup firms. High- and low-markup firms differ in their markups and pass-throughs, and we denote them with subscripts H and L.

Following Lemma 1, the change in aggregate TFP following a nominal shock is

$$d \log A = d \log \bar{\mu} - \mathbb{E}_\lambda \left[d \log \mu_\theta \right] = \lambda_H \left(1 - \frac{\bar{\mu}}{\mu_H} \right) \left(d \log l_H - d \log l_L \right),$$

where l_H and l_L are employment by H and L firms. Reallocations in inputs (labor in our single factor model) across high- and low-markup firms, paired with the initial distribution of sales and markups, determines the change in aggregate TFP. In particular, aggregate TFP increases if the growth in employment at high-markup firms outpaces the growth of employment at low-markup firms.[30]

For simplicity, we again impose homogeneous price-stickiness ($\delta_H = \delta_L = \delta$). Proposition 3 implies that the price Phillips curve is

$$d \log P^Y = \frac{1 + \gamma \zeta}{\zeta} \frac{\delta}{1 - \delta} \frac{\delta (\sigma_L - \sigma_H)(\rho_L - \rho_H) + \left(\lambda_L^{-1} \sigma_H + \lambda_H^{-1} \sigma_L \right) \left(\lambda_H \rho_H + \lambda_L \rho_L \right)}{\delta \left(1 + \frac{\bar{\mu}}{\zeta} \right) (\sigma_L - \sigma_H)(\rho_L - \rho_H) + \left(\lambda_L^{-1} \sigma_H + \lambda_H^{-1} \sigma_L \right)} d \log Y.$$

This price Phillips curve is flatter than the CES economy if $\rho_L > \rho_H$, i.e., if low-markup firms have higher pass-throughs than high-markup firms. An increase in the covariance of elasticities and pass-throughs, $(\sigma_L - \sigma_H)(\rho_L - \rho_H)$, further flattens the Phillips curve.

4.3   Discussion

Before moving onto the dynamic version of the model, we discuss some of implications and extensions of the results in this section.

First, unlike the standard model, our model links the slope of the Phillips curve to the industrial organization of the economy, via statistics like the covariance of pass-throughs and price-elasticities. This means that industrial concentration plays a role in shaping the Phillips curve. We consider this effect quantitatively in Section 6, where we illustrate the effect of increasing industrial concentration on the Phillips curve slope.

Second, the results in Sections 3 and 4 can also be derived in models of oligopolistic competition that are populated by a discrete number of firms instead of a continuum of infinitesimal firms in monopolistic competition. As discussed above, the nested CES model of Atkeson and Burstein (2008) generates markups and pass-throughs that conform

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[30]See Section 7 for supporting empirical evidence.

with Marshall's strong second law of demand, and hence yields similar implications (we show this in Appendix I). In the body of the paper we focus on the monopolistic competition model because monopolistic competition is much more tractable in a fully dynamic environment.

5   Four-Equation Dynamic Model

We now present a general dynamic model that generalizes the workhorse three-equation model presented in Galí (2015) to account for imperfect pass-through and endogenous aggregate productivity. Households choose consumption and leisure to maximize discounted future utility,

$$
\max_{\{Y_t, L_t\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t Z_t u(Y_t, L_t),
$$

where the per-period utility function is as in Section 2, and β is the discount factor, and Z_t is a discount factor shifter. We allow for the possibility that there may be unanticipated shocks to the discount factor, as in Krugman (1998).

Each firm sets its price to maximize discounted future profits, subject to a Calvo friction: firm i's profit-maximization problem is

$$
\max_{p_{i,t}} \mathbb{E}\left[\sum_{k=0}^\infty \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})} (1-\delta_i)^k y_{i,t+k}(p_{i,t} - \frac{w_{t+k}}{A_i}) \right],
\tag{14}
$$

where δ_i is the Calvo parameter and $y_{i,t+k}$ is the quantity firm i sells in period $t+k$ if it last set its price in period t. The solution to the firms' maximization problem describes how prices move in the economy. We use this to describe the movement of inflation, output, and aggregate TFP by aggregating across firms. For expositional simplicity, we present a version with homogeneous price-stickiness across firms.

As in Galí (2015), we log-linearize around the no-inflation steady state. The model is closed by the actions of the monetary authority, which we assume follow a Taylor rule.

5.1   The New Keynesian Model with Misallocation

Proposition 5 is the main result in this section, characterizing the equilibrium allocation up to a first-order approximation.

Proposition 5. *Changes in aggregates are described by the following four-equation system:*

$$d \log i_t = \phi_\pi d \log \pi_t + \phi_y d \log Y_t + v_t, \qquad \text{(Taylor rule)}$$

$$d \log Y_t = d \log Y_{t+1} - \frac{1}{\gamma}(d \log i_t - d \log \pi_{t+1} + \epsilon_t), \qquad \text{(Euler equation)}$$

$$d \log \pi_t = \beta d \log \pi_{t+1} + \varphi \mathbb{E}_\lambda[\rho_\theta] \frac{1 + \gamma\zeta}{\zeta} d \log Y_t - \alpha d \log A_t, \qquad \text{(Phillips curve)}$$

$$d \log A_t = \frac{1}{\kappa_A} d \log A_{t-1} + \frac{\beta}{\kappa_A} d \log A_{t+1} + \frac{\varphi}{\kappa_A} \frac{1 + \gamma\zeta}{\zeta} \bar{\mu} \frac{Cov_\lambda[\rho_\theta, \sigma_\theta]}{\mathbb{E}_\lambda[\rho_\theta]} d \log Y_t, \qquad \text{(TFP)}$$

where $d \log i_t$ is the nominal interest rate, $d \log \pi_t = d \log P_t^Y - d \log P_{t-1}^Y$ is inflation, ϕ_π and ϕ_y are policy parameters, v_t is a monetary policy shock, $1/\gamma$ is the intertemporal elasticity of substitution, $\epsilon_t = d \log Z_{t+1} - d \log Z_t$ is a discount rate shock, $\varphi = \frac{\delta}{1-\delta}(1 - \beta(1-\delta))$, $\alpha = \frac{\varphi}{\bar{\mu}}\left(\mathbb{E}_\lambda[\rho_\theta]\left(1 + \frac{\bar{\mu}}{\zeta}\right) - 1\right)$, and $\kappa_A = 1 + \beta + \varphi\left[1 + \frac{Cov_\lambda(\rho_\theta, \sigma_\theta)}{\mathbb{E}_\lambda[\sigma_\theta]}\left(1 + \frac{\bar{\mu}}{\zeta}\right)\right]$.

The Taylor rule and Euler equation are the same as the workhorse three-equation model. Differences arise in the last two equations. We discuss the last two equations in turn.

Consider the New Keynesian Phillips Curve (NKPC) with misallocation. We note two key differences: first, in the standard NKPC, the coefficient on $d \log Y_t$ is $\varphi \frac{1+\gamma\zeta}{\zeta}$.[31] In the NKPC with misallocation, this coefficient is multiplied by $\mathbb{E}_\lambda[\rho_\theta]$. Imperfect pass-through moderates the response of prices to nominal shocks, and hence flattens the NKPC, as in the static version of the model. Second, changes in aggregate TFP enter the Phillips curve as endogenous, negative cost-push shocks, given by $\alpha d \log A_t$.[32] This means that procyclical movements in aggregate TFP dampen the response of inflation to an expansionary shock, similarly flattening the inflationary effects of a monetary expansion.

The path of aggregate TFP is pinned down by the final equation in Proposition 5. When markups covary negatively with pass-throughs, changes in output $d \log Y$ are positively associated with aggregate TFP. Note that, unlike the standard New Keynesian model's equations, which are first-order difference equations, aggregate TFP follows a second-order difference equation. As a result, the augmented four-equation model can generate hump-shaped impulse responses to monetary shocks.

Proposition 5 also generalizes the static model presented in Sections 2–4 as shown by the following corollary.

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[31] See, e.g., Galí (2015) with constant returns.

[32] We find that $\alpha > 0$ when $\mathbb{E}_\lambda[\rho_\theta] > \frac{\bar{\mu}^{-1}\zeta}{1+\bar{\mu}^{-1}\zeta}$. The reciprocal of the average markup $\bar{\mu}^{-1}$ is bounded above by 1, and estimates of the Frisch elasticity place ζ between 0.1 and 0.4. Average pass-through is greater than 0.5, which suggests that $\alpha > 0$ holds nearly always.

Corollary 4. *Suppose output, aggregate TFP, and the price level are in steady state at $t = 0$ (i.e., $d \log Y_0 = d \log A_0 = d \log P_0^Y = 0$). When the discount factor $\beta = 0$, the effect of shocks on impact are the same as the static results from Proposition 1 and Proposition 2.*

5.2 Solution Strategy

We provide a high-level walk-through of the derivation to highlight the key intuitions; the detailed derivation is in Appendix B. The derivation of the Euler equation is standard, so we focus instead on the Phillips curve and the TFP equations. Start with the firm maximization problem described in Equation (14). The optimal reset price $p_{i,t}^*$ for profit maximization satisfies

$$\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1 + r_{t+j})}(1 - \delta_i)^k y_{i,t+k}\left[\frac{d y_{i,t+k}}{d p_{i,t}} \frac{p_{i,t}^*}{y_{i,t+k}} \frac{p_{i,t}^* - \frac{w_{t+k}}{A_i}}{p_{i,t}^*} + 1\right]\right] = 0. \tag{15}$$

We log-linearize this equation around the perfect foresight zero inflation steady state. Note that the steady state is characterized by a constant discount factor such that $\frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})} = \beta^k$.

With some manipulation, the log-linearization of Equation (15) yields,

$$d \log p_{i,t}^* = [1 - \beta(1 - \delta_i)] \sum_{k=0}^{\infty} \beta^k (1 - \delta_i)^k \left[\rho_i d \log w_{t+k} + (1 - \rho_i) d \log P_{t+k}\right]. \tag{16}$$

When prices are fully flexible, this simplifies to the static optimality condition:

$$d \log p_{i,t}^* = (1 - \rho_i) d \log P_t + \rho_i d \log w_t.$$

Compared to the case without nominal rigidities, a firm with sticky prices is forward looking and incorporates expected future prices and marginal costs into its reset price today. Just as in the completely flexible benchmark, firms with high pass-throughs are more responsive to expected changes in their own marginal costs, while firms with low pass-throughs are more responsive to expected changes in the economy's price aggregator.

Rewrite Equation (16) recursively, and for each firm type θ, as

$$d \log p_{\theta,t}^* = [1 - \beta(1 - \delta_\theta)]\left[\rho_\theta d \log w_t + (1 - \rho_\theta) d \log P_t\right] + \beta(1 - \delta_\theta) d \log p_{\theta,t+1}^*.$$

The price level of a firm of type θ at time t is equal to the firm's reset price with probability

δ_θ, or else pinned at the last period price with probability $(1 - \delta_\theta)$. In expectation,

$$\mathbb{E}\left[d \log p_{\theta,t}\right] = \delta_\theta \mathbb{E}\left[d \log p^*_{\theta,t}\right] + (1 - \delta_\theta)\mathbb{E}\left[d \log p_{\theta,t-1}\right]$$

Combining the above two equations and rearranging yields a second-order difference equation for the expected price of firm type θ,

$$\mathbb{E}[d \log p_{\theta,t} - d \log p_{\theta,t-1}] - \beta\mathbb{E}[d \log p_{\theta,t+1} - d \log p_{\theta,t}]$$
$$= \varphi\left[-\mathbb{E}[d \log p_{\theta,t}] + \rho_\theta d \log w_t + (1 - \rho_\theta)d \log P_t\right], \quad (17)$$

where
$$\varphi = \frac{\delta_\theta}{1 - \delta_\theta}(1 - \beta(1 - \delta_\theta)).$$

We are almost finished. Equation (17) pins down the movement of the price of a firm with type θ over time and, by extension, the movement in the firm's markup over time. Our aggregate variables, such as the consumer price index, aggregate TFP, labor supply, and output, can all be recovered by manipulating this expression and averaging over firm types.

For instance, by taking the sales-weighted expectation of both sides in Equation (17), we can recover the movement of the consumer price index.[33] We get,

$$d \log \pi_t - \beta d \log \pi_{t+1} = \varphi\left[\mathbb{E}_\lambda\left[\rho_\theta\right](d \log w_t - d \log P_t) + (d \log P_t - d \log P^Y_t)\right]. \quad (18)$$

The objects that remain—the difference between the price aggregator $d \log P_t$ and the nominal wage $d \log w_t$, and the difference between the aggregator $d \log P_t$ and the consumer price index $d \log P^Y_t$ —depend solely on the average markup and the distribution of markups. In particular, the following identities allow us to express these endogenous objects in terms of output and aggregate TFP:

$$d \log P_t - d \log P^Y_t = \bar{\mu}^{-1} d \log A_t, \quad (19)$$

$$d \log P^Y_t - d \log w_t = \frac{1}{\zeta}\left[d \log A_t - (1 + \gamma\zeta)d \log Y_t\right]. \quad (20)$$

Equation (19) can be derived by log-linearizing and rearranging the expression for the price aggregator in (2),[34] and (20) comes from rearranging (2) for the average change in

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[33]The CPI price index, log linearized around the steady state, is $\mathbb{E}_\lambda\left[\mathbb{E}\left[d \log p_\theta\right]\right] = d \log P^Y$.

[34]In particular, $d \log P_t = d \log P^Y_t - \mathbb{E}_\lambda\left[\left(1 - \frac{1}{\sigma_\theta}\right)d \log(\frac{y_{\theta,t}}{Y})\right] = d \log P^Y_t - \mathbb{E}_\lambda\left[\mu_\theta^{-1}\right]\mathbb{E}_{\lambda\mu^{-1}}\left[d \log(\frac{y_{\theta,t}}{Y})\right] = d \log P^Y_t + \bar{\mu}^{-1}d \log A_t$.

markups. Substituting these identities into (18) yields the NKPC with misallocation in Proposition 5.

The Endogenous TFP equation can also be derived by rearranging (17). In particular, from Lemma 1, we have

$$d \log A_t = d \log \bar{\mu} - \mathbb{E}_\lambda [d \log \mu_\theta] = \bar{\mu} \left(\mathbb{E}_{\sigma\lambda} [d \log \mu_{\theta,t}] - \mathbb{E}_\lambda [d \log \mu_{\theta,t}] \right). \tag{21}$$

The changes in markups can in turn be derived from (17) by subtracting changes in marginal cost (the nominal wage) from changes in prices. This yields a second-order difference equation for the change in markups for each firm-type. Taking sales-weighted averages over these markup changes and rearranging yields expressions for the two terms on the right-hand side of (21).

5.3 Discussion

The model presented in Proposition 5 provides a tractable, four-equation system that can be used to calibrate economies with realistic heterogeneity in markups and pass-throughs. This model incorporates real rigidities and the misallocation channel while being parsimoniously governed by four objects from the firm distribution: the average sales-weighted elasticity $\mathbb{E}_\lambda [\sigma_\theta]$, the average sales-weighted pass-through $\mathbb{E}_\lambda [\rho_\theta]$, the covariance of elasticities and pass-throughs $Cov_\lambda [\sigma_\theta, \rho_\theta]$, and the aggregate markup $\bar{\mu}$. We present one such calibration in the section below.

The second-order difference equation for aggregate TFP generates hump-shaped patterns for $d \log A$ and potentially other aggregate variables. Empirical estimates of the impulse response of labor productivity to monetary shocks (see, e.g., Christiano et al. 2005) also exhibit this shape. TFP follows a second-order equation because changes in prices, and hence changes in markups, are second-order difference equations. The backward looking part comes from the Calvo friction, and the forward looking part comes from expectations.

Although for the purposes of our discussion, we have focused on monetary policy shocks, other disturbances in the Euler equation like discount factor shocks or fiscal policy shocks trigger similar reallocations across firms and hence also affect aggregate TFP.

6   Calibration

We now calibrate both the static and dynamic versions of the model to assess the quantitative importance of the misallocation channel. This section is organized as follows. First, we describe how to calibrate the model without relying on an off-the-shelf functional form for the Kimball aggregator. Second, we calibrate the model using empirical pass-through estimates from Amiti et al. (2019) with Belgian firm-level data. We then report results from this calibration exercise: we compute the flattening of the Phillips curve due to real rigidities and misallocation in the static model and discuss comparative statics with respect to the Frisch elasticity and the degree of industrial concentration. At the end of the section, we turn to the dynamic model, where we present impulse response functions following a monetary policy shock.

6.1   Non-parametric Calibration Procedure

It may be tempting to use an off-the-shelf functional form for the Kimball aggregator and tune parameters to match moments from the data. However, there is no guarantee that parametric specifications of preferences are able to match the relevant features of the data required for generating correct aggregate properties.[35] Instead, we follow Baqaee et al. (2021) and back out the shape of the Kimball aggregator non-parametrically from the data. We summarize this approach below.

We assume that Υ_θ take the form,

$$\Upsilon_\theta(\frac{y_\theta}{Y}) = \Upsilon(B_\theta \frac{y_\theta}{Y}).$$

Hence, firms differ in their productivities A_θ and taste shifters B_θ. Allowing for taste-shifters is important since, in practice, two firms that charge the same price in the data can have very different sales and taste-shifters allow us to accommodate this possibility.[36]

We order firms by their size and let $\theta \in [0,1]$ be firm θ's centile in the size distribution. Baqaee et al. (2021) show that, in the cross-section, markups and sales must satisfy the

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[35]As an example, see Section 8 for a discussion of the unsuitability of the popular parametric family of preferences considered by Klenow and Willis (2016) for our application.

[36]If there were no taste-shifters, then one could identify the residual demand curve by simply plotting price against quantity in the cross-section. In practice, this is impracticable because the prices firms report are not directly comparable to one-another.

following differential equation[37]

$$\frac{d \log \mu_\theta}{d\theta} = (\mu_\theta - 1)\frac{1 - \rho_\theta}{\rho_\theta}\frac{d \log \lambda_\theta}{d\theta}. \tag{22}$$

Given data on sales shares λ_θ and pass-throughs ρ_θ, we can use this differential equation to solve for markups μ_θ up to a boundary condition. We choose the boundary condition to target a given value of the (harmonic) sales-weighted average markup, $\bar{\mu}$. We can then use $\sigma_\theta = 1/(1 - 1/\mu_\theta)$ to recover price-elasticities. The distributions of pass-throughs, markups, price elasticities, and sales shares are the sufficient statistics we need to calibrate the model.[38]

6.2 Data and Parameter Values

We follow Baqaee et al. (2021) to implement this procedure (we refer interested readers to Appendix A of that paper for details). To calibrate the model, we need data on pass-throughs ρ_θ and the sales density λ_θ. For pass-throughs, we use estimates of (partial equilibrium) pass-throughs by firm size for manufacturing firms in Belgium from Amiti et al. (2019).[39] We interpolate between their point estimates with smooth splines and assume that pass-throughs go to 1 for the smallest firms (they find that the average pass-through for the smallest 75% of firms is already 0.97). Figure 2 shows the pass-through ρ_θ and log sales share density $\log \lambda_\theta$ as a function of θ. Pass-throughs are strictly decreasing with firm size, which means that Marshall's strong second law holds.

To compute the distribution of markups and elasticities from this data using equation (22), we must take a stance on the average markup. We assume that the average markup $\bar{\mu} = \mathbb{E}_\lambda\left[\mu_\theta^{-1}\right]^{-1} = 1.15$, in line with estimates from micro-data.[40]

[37]This follows from combining the following two differential equations: $\frac{d \log \lambda_\theta}{d\theta} = \frac{\rho_\theta}{\mu_\theta - 1}\frac{d \log(A_\theta B_\theta)}{d\theta}$, and $\frac{d \log \mu_\theta}{d\theta} = (1 - \rho_\theta)\frac{d \log(A_\theta B_\theta)}{d\theta}$. The first differential equation uses the fact that the firm of type $\theta + d\theta$ will have lower "taste-adjusted" price, $\log p_{\theta+d\theta} - \log p_\theta = \rho_\theta d \log(A_\theta B_\theta)/d\theta$, and higher sales $d \log \lambda_{\theta+d\theta} - \log \lambda_\theta = (\sigma_\theta - 1)\rho_\theta d \log(A_\theta B_\theta)/d\theta$, with $\sigma_\theta - 1 = 1/(\mu_\theta - 1)$. The second differential equation uses the fact that the relationship of desired markups to productivity is $d \log \mu_\theta/d \log(A_\theta B_\theta) = 1 - \rho_\theta$.

[38]Our calibration imposes that markups and pass-throughs vary only as a function of market share. In Appendix J, we characterize how arbitrary noise in markups and pass-throughs unrelated to firm size affects the strength of the TFP response. We show that typical noise that moves markups and pass-throughs in the same direction will result in a stronger negative correlation between markups and pass-throughs and thus magnify the TFP response.

[39]Amiti et al. (2019) use exchange rate shocks as instruments for changes in marginal cost and control for the portion of price changes due to competitors' prices. This identifies the partial equilibrium pass-through by firm size (under assumptions consistent with our model).

[40]The resulting markup function μ_θ is shown in Figure G.1 in Appendix G. The markup distribution we recover is consistent with direct estimates from the litearture. Konings et al. (2005) use micro-evidence

Figure 2: Pass-through ρ_θ and sales share density $\log \lambda_\theta$ for Belgian manufacturing firms ordered by type θ.

(a) Pass-through ρ_θ (b) Log sales share density $\log \lambda_\theta$
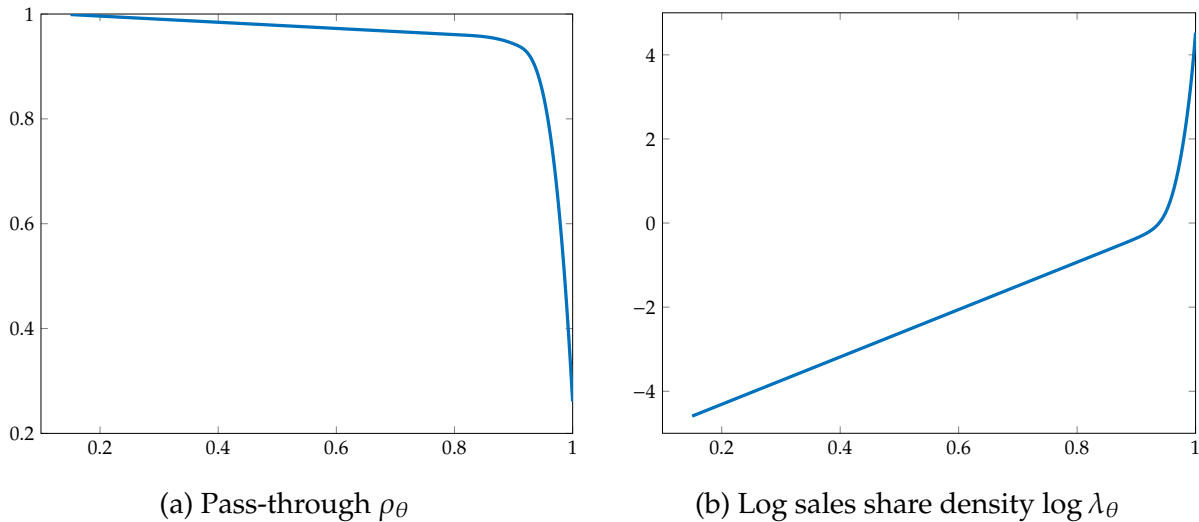
This choice of the average markup, as well as the remaining parameter values, are listed in Table 1: We set the intertemporal elasticity of substitution $\gamma = 1$, which is a focal point for empirical estimates, and set the Frisch elasticity $\zeta = 0.2$ in line with recent estimates (see, for example, Chetty et al. 2011, Martinez et al. 2018, Sigurdsson 2019). For calibrating the static model, we consider a time period of approximately one quarter. Given an average price duration of about six months (Nakamura and Steinsson 2008), this means $\delta_\theta = \delta = 0.5$.

For the calibration of the dynamic model, we choose the coefficients on the Taylor rule, ϕ_π and ϕ_y, to match the calibration of the standard New Keynesian model given in Galí (2015). We also match Galí (2015) by setting the discount factor $\beta = 0.99$, corresponding to a 4% annual interest rate. We assume that monetary disturbances follow an AR(1) process $v_t = \rho_v v_{t-1} + \epsilon_t$, and set $\rho_v = 0.7$, indicating strong persistence to the interest rate shock, and set the size of the initial interest rate shock to 25 basis points. We keep the Calvo parameter in the dynamic model the same as the static model.

We first present estimates from the static model, and discuss comparative statics with respect to the Frisch elasticity and the degree of industrial concentration. Then, we present impulse response functions from the dynamic model.

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to estimate price-cost margins in Bulgaria and Romania, and find that average price-cost margins range between 5-20% for nearly all sectors. In the working paper version of Amiti et al. (2019), they report that small firms in their calibration have a markup of around 14%, and large firms have markups of around 30%. These micro-estimated average markups are also broadly in line with macro estimates from Gutiérrez and Philippon (2017) and Barkai (2020), who estimate average markups on the order of 10-20%, but lower than estimates by De Loecker et al. (2020), who estimate the average markup for public firms at 61%.

Table 1: Parameters for empirical calibration.

Parameter	Description	Estimate
Static model:		
$\bar{\mu}$	(Harmonic) average markup	1.15
$1/\gamma$	Intertemporal elasticity of substitution	1
ζ	Frisch elasticity	0.2
δ	Calvo friction	0.5
Dynamic model:		
δ	Calvo friction (quarterly)	0.5
ϕ_y	Taylor rule coefficient on output gap	0.5 / 4
ϕ_π	Taylor rule coefficient on inflation gap	1.5
β	Discount factor	0.99
ϵ_0	Initial interest rate shock	25bp
ρ_v	Shock persistence	0.7

6.3   Results from Static Model

Table 2 reports the estimated flattening of the Phillips curve due to real rigidities and the misallocation channel (as given by Proposition 4). We find that the misallocation channel is quantitatively important: compared to the real rigidities channel, which flattens the wage Phillips curve by 27% and the price Phillips curve by 73%, the misallocation channel flattens both Phillips curves by 71%. This means that including supply-side effects increases the responsiveness of output to a monetary shock by 71%.

Table 2: Estimates of Phillips curve flattening due to real rigidities and the misallocation channel.

| Flattening	Wage Phillips curve	CPI Phillips curve
Real rigidities	1.27	1.73
Misallocation channel	1.71	1.71

To highlight the key forces at play in this calibration, we consider how these estimates change as we vary the Frisch elasticity, the degree of industrial concentration, the average markup, and the level of price-stickiness.

The Frisch elasticity.   The discussion following Propositions 1 and 2 shows that the misallocation channel should be more important for lower values of the Frisch elasticity of labor supply. This intuition is confirmed in Figure 3, where we plot the slope of the

Phillips curve as a function of the Frisch elasticity. The flattening of the Phillips curve due to real rigidities does not depend on the Frisch elasticity. However, the flattening due to the misallocation channel increases dramatically as the Frisch elasticity approaches zero.

The introduction of the misallocation channel—and its increased strength at low Frisch elasticities—may help explain the discrepancy between micro-evidence on the Frisch elasticity and those required to explain the slope of the Phillips curve in traditional models. Evidence accumulated from quasi-experimental studies suggests that the labor supply elasticity is on the order of 0.1-0.4. In order to match the slope of the Phillips curve that the model with real rigidities and misallocation predicts at $\zeta = 0.2$, the model with nominal rigidities alone would require $\zeta \approx 1$. Incorporating the misallocation channel allows us to generate more monetary non-neutrality at lower levels of the Frisch elasticity.

Figure 3: Decomposition of Phillips curve slope, varying the Frisch elasticity ζ.
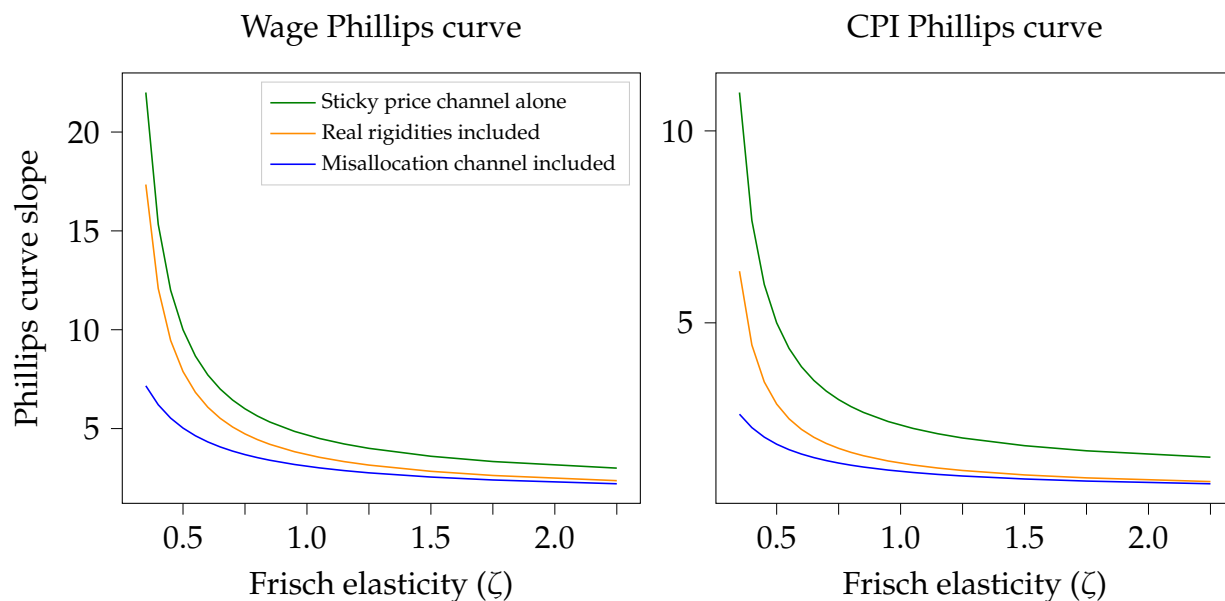
Industrial concentration. Our analysis explicitly links the slope of the Phillips curve to characteristics of the firm distribution. A natural question, then, is how varying that firm distribution will affect the strength of the real rigidities and misallocation channels.

In order to illustrate the role of industrial concentration, we consider counterfactual firm distributions. To do so, we use a beta distribution for firm productivities, A_θ.[41] We choose the shape parameters of the beta distribution, $a = 0.14$ and $b = 15.7$, to match the

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[41]We choose the beta distribution since, as a bounded distribution, it allows us to remain within the range of productivities for which we have estimated the Kimball aggregator.

Gini coefficient of firm employment in the Belgian data and the slope of the price Phillips curve in our baseline calibration.

We then perturb the distribution by scaling a and b by a constant. Scaling the parameters of the beta distribution preserves the mean of the distribution while decreasing the variance, hence decreasing the concentration of firm employment. In Figure 4, we plot the slope of the Phillips curve against the Gini coefficient as we scale the parameters of the beta distribution. As the distribution in productivity becomes less concentrated, the employment distribution becomes more equal, and the Gini coefficient falls. As expected, the slope of the Phillips curve under nominal rigidities alone (as in the CES demand system) is unchanged as we vary the concentration of employment over this range. However, the strength of real rigidities and the misallocation channel do depend on the firm size distribution: the strength of both channels increases as we increase concentration.

To put these numbers into context, our model predicts that increasing the Gini coefficient from 0.80 to 0.85 flattens the price Phillips curve by an additional 14%. This experiment is in line with the increase in the Gini coefficient in firm employment from 1978 to 2018 in the United States (measured using the Census Business Dynamics Statistics, see Appendix K). Increasing the Gini coefficient from 0.72 to 0.86 (the increase in the Gini coefficient in the retail sector over the same period) flattens the price Phillips curve by 41%.

Figure 4: The slope of the Phillips curve, and its decomposition, as a function of the Gini coefficient of the employment distribution.
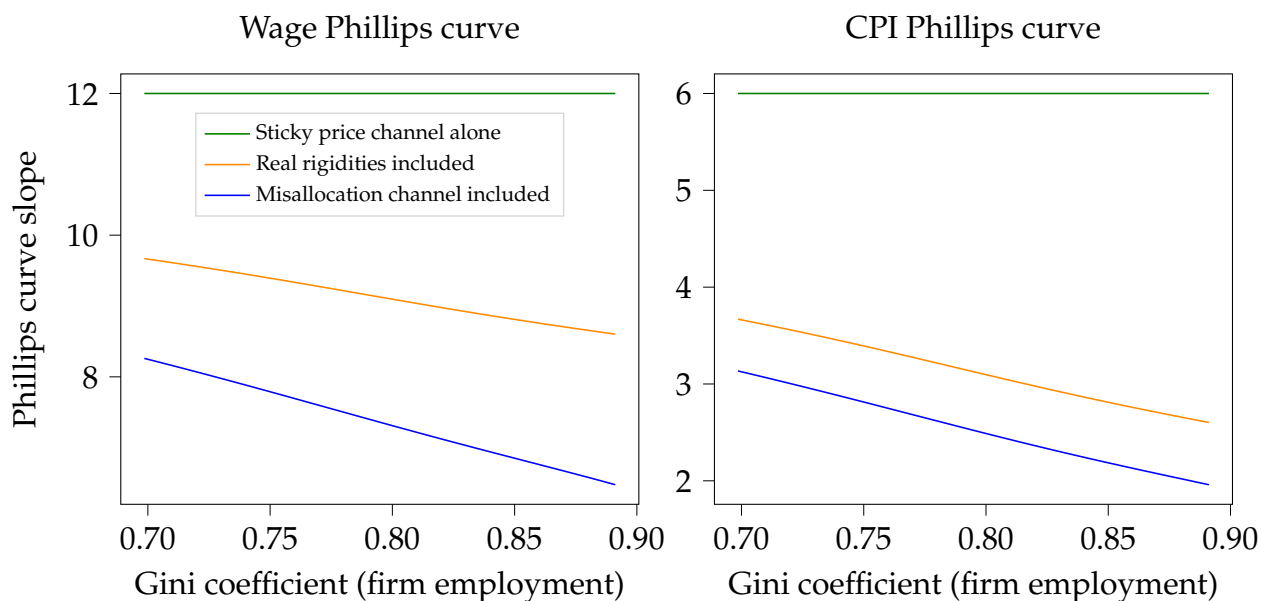
Other parameters. We show how the estimated slope of the Phillips curve changes as we vary the average markup $\bar{\mu}$ and the price-stickiness δ in Appendix C. We briefly summarize the results: Increasing the average markup $\bar{\mu}$ has no effect on the flattening due to real rigidities, but increases the flattening due to misallocation channel linearly. This means that our estimates of the misallocation channel are conservative relative to those implied by the average markups estimated by De Loecker et al. (2020). Increasing the price flexibility parameter δ increases the flattening of the price Phillips curve due to the misallocation channel and decreases the flattening due to real rigidities, for reasons explained after the statement of Proposition 4.

6.4   Results from Dynamic Model

Figure 5 shows the impulse response functions of aggregate variables following a persistent, 25 basis point (100bp annualized) shock to the interest rate in the dynamic model. We compare the benchmark heterogeneous firm model to a homogeneous firm model, which has real rigidities but no misallocation channel, and a CES model, which has neither real rigidities nor the misallocation channel.

In the CES and homogeneous firms case, aggregate TFP does not react to the monetary shock, as implied by Corollary 1. In contrast, when firms have heterogeneous markups, the dispersion in TFPR across firm types increases by around 15 basis points following the contractionary shock, and the response of aggregate TFP is procyclical and hump-shaped.[42] The fall in aggregate TFP dampens the extent of disinflation following the contractionary monetary shock and deepens the immediate response of output to the shock.

We quantify how the misallocation channel affects real output in Table 3. We find that the contraction in output in the full model is 44% deeper on impact than in the homogeneous firm model. The persistence of the shock's effect on real output also increases: while the CES and homogeneous firm models feature a constant half-life of just under two quarters, the misallocation channel increases the half-life of the shock by 31% to about 2.6 quarters.[43] Table 3 also reports the cumulative impact on output of the monetary shock in the different models. The misallocation channel increases the cumulative output impact

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[42]Under constant returns to scale, like our model, changes in TFPR are equal to changes in firm markups: $\Delta \log \mathrm{TFPR} = \Delta \log p_\theta y_\theta - \Delta \log l_\theta = \Delta \log \mu_\theta$. (Foster et al. (2008) discuss the relationship between TFPR and "physical productivity"—TFPQ, or A_θ in our setting.) For comparison, Kehrig (2011) finds that TFPR dispersion increases about 10% during recessions and increased over 20% from 2007 to the trough of the recession in 2009.

[43]Due to the second-order difference equation in aggregate TFP, the full model no longer features a constant half-life.

Figure 5: Impulse response functions (IRFs) following a 25bp monetary shock. Green, orange, and blue IRFs indicate the CES, homogeneous firms, and heterogeneous firms models respectively.
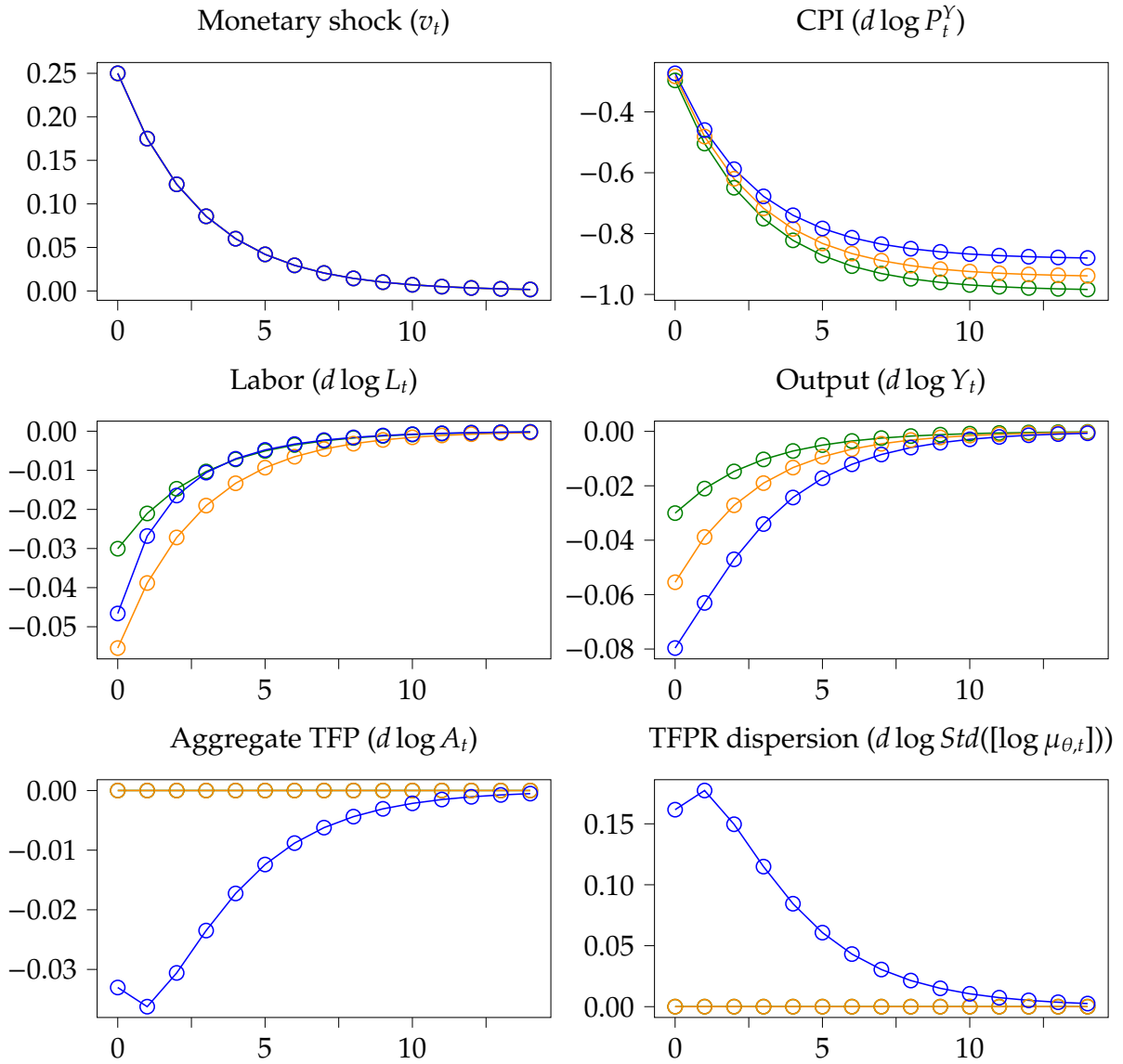
of monetary shocks by around 65 log points, which is similar to the real rigidities channel.

Table 3: Effect of monetary policy shock on output.

Model	Output effect at $t = 0$	Half life	Cumulative output impact
CES	-0.030	1.95	-0.10
Homogeneous Firms	-0.055	1.95	-0.18
Heterogeneous Firms	-0.080	2.56	-0.31

Figure 6 shows the covariance between firms' initial markups and their change in markups (left) and change in total input costs (right). Following Lemma 1, Figure 6 shows the reallocation of inputs to low-markup firms that generates the fall in TFP. This is a directly testable prediction of the model that we return to in Section 7.

Figure 6: Covariance of firms' initial markups with the change in markups and change in costs following a 25bp monetary shock. The contractionary shock leads high-markup firms to increase their markups relative to low-markup firms (left), causing a reallocation of resources away from high-markup firms (right).

$$Cov_\lambda(\mu_\theta, d\log\mu_\theta) \qquad Cov_\lambda(\mu_\theta, d\log\text{Costs}_\theta)$$
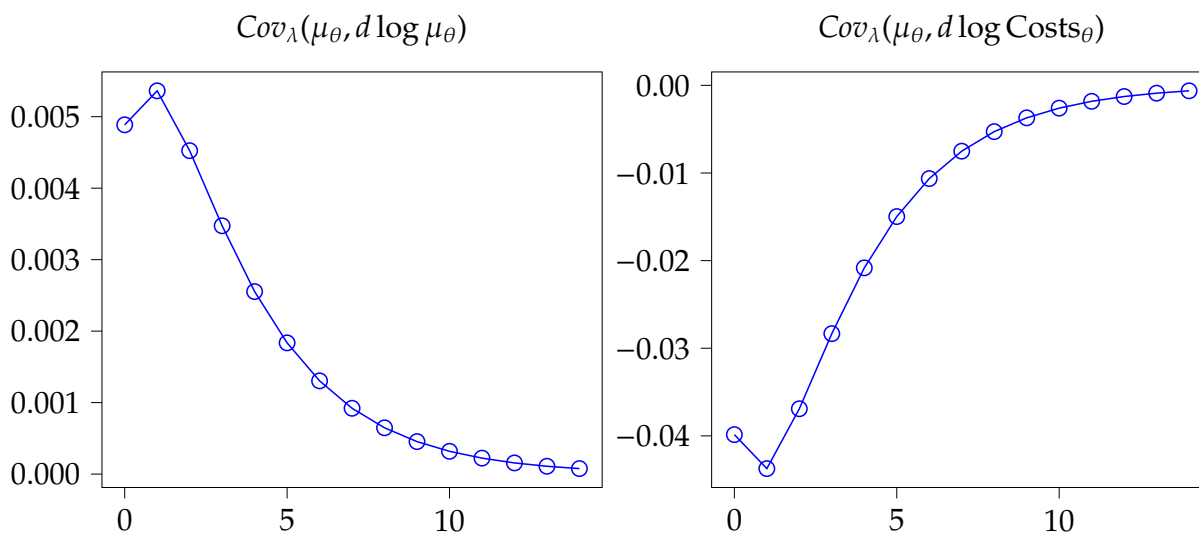
We provide additional calibration results in Appendix C. In particular, we report the change in sales shares for firms at different percentiles of the size distribution. The sales shares of small firms are about one order of magnitude more volatile than output, whereas the sales shares of the largest firms are less volatile than output. In Appendix E, we show that results are quantitatively similar when monetary policy is implemented via changes in money supply (with a cash-in-advance constraint) rather than an interest rate rule. All

in all, our results suggest that the misallocation channel is as powerful as the real rigidities channel in affecting the transmission of monetary policy.

7 Empirical Evidence

In this section, we provide some empirical evidence in support of the reallocation mechanism described in this paper. We first present macro-level evidence on the cyclicality of aggregate TFP and its response to exogenous monetary shocks. We then show, at the micro-level, that contractionary monetary shocks lead high-markup firms to increase their markups relative to low-markup firms, leading to a reallocation of inputs away from high-markup firms. Finally, we provide evidence that the contraction in productivity following monetary shocks is greater in more concentrated industries, as in Figure 4.

Table 4: Procyclical aggregate productivity.

$\%\Delta TFP$	Labor productivity			Solow residual			Cost-based Solow residual		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Unemp.	-0.355**			-0.465**			-0.477**		
	(0.126)			(0.141)			(0.142)		
Recession		-0.878**			-2.114**			-2.082**	
		(0.394)			(0.414)			(0.500)	
$\%\Delta GDP$			0.221**			0.209*			0.354**
			(0.087)			(0.106)			(0.097)
Period	1948-2020			1948-2020			1961-2014		

Notes: Unemp. is the average unemployment rate in year $t + 1$, $\%\Delta GDP$ is real GDP growth from year $t - 1$ to t, and *Recession* = 1 if any quarter in the year is marked an NBER recession. Robust standard errors in parentheses. * indicates significance at 10%, ** at 5%.

Macro-level evidence. Table 4 shows the unconditional association between various measures of aggregate productivity and the business cycle. We use three different measures of aggregate productivity—labor productivity, the Solow residual, and the cost-based Solow residual (see Hall, 1990)—and three different measures of the business cycle—the unemployment rate, NBER recession dates, and real GDP growth. For all measures, we find that productivity covaries significantly with business cycle indicators.[44]

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[44]We use measures of labor productivity and the Solow residual for the U.S. business sector provided by the Federal Reserve Bank of San Francisco for the period 1948-2020. To calculate cost-based Solow

Figure 7: Local projection of a contractionary Romer and Romer (2004) shock on aggregate productivity. The shaded region indicates robust standard errors, and dashed lines are 95% confidence intervals.

(a) Labor productivity

(b) Solow residual

(c) Cost-based Solow residual

(d) Output
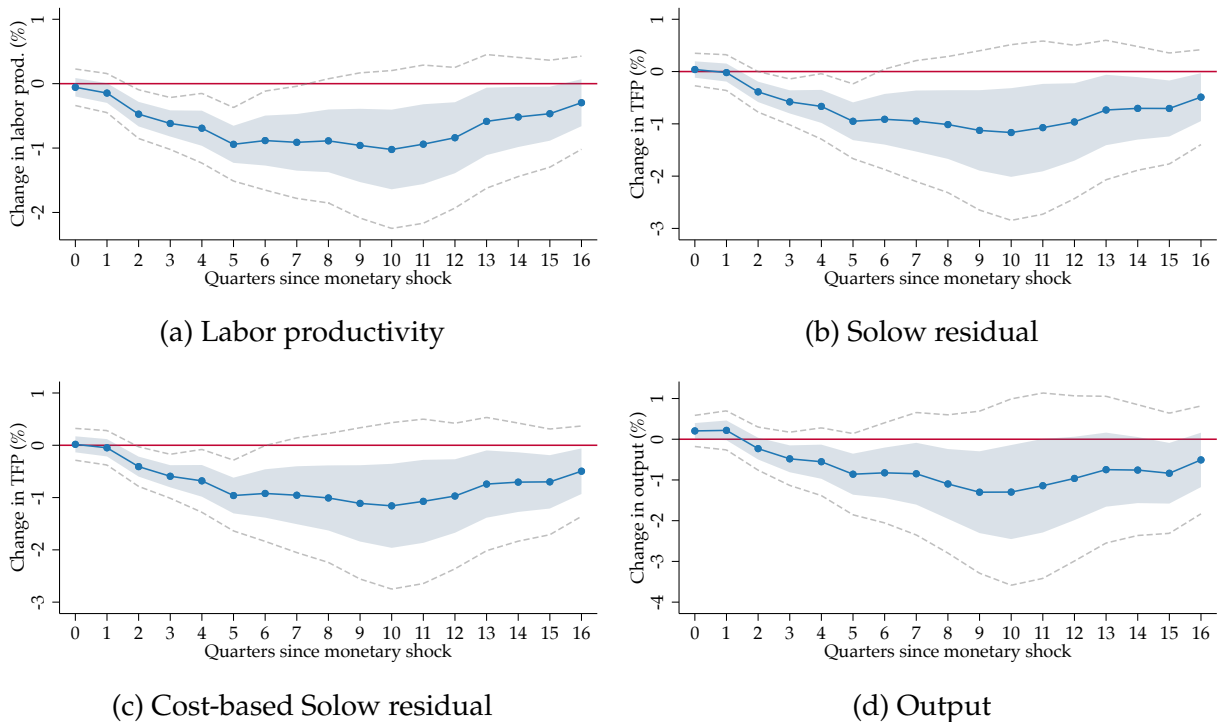
We do not use utilization-adjusted TFP (e.g. Basu et al., 2006; Fernald, 2014) in Table 4. This is because these series are identified using the assumption that sectoral productivity is orthogonal to monetary shocks, and this exogeneity condition fails in our model.

In Figure 7, we show that aggregate productivity falls after contractionary monetary policy shocks, using the extended version of Romer and Romer (2004) monetary shock series constructed by Wieland and Yang (2020) for 1969–2007. Following a contractionary shock, there is a significant negative contraction in aggregate productivity for two to six quarters. The magnitude of the decline in aggregate productivity is roughly half of the effect on output. These movements in aggregate productivity are a little larger than those predicted by our model, which suggests that allocative effects explain part (but perhaps not all) of the procyclical movements of aggregate productivity.

Micro-level evidence. In our model, the procyclical movements of aggregate TFP appears due to endogenous reallocations towards high-markup firms. We now turn to

residual, we use the aggregate markup, estimated using sales and accounting profits of Compustat firms from 1961-2014, to estimate input cost shares.

micro-level evidence on these reallocations. To do so, we use estimates of markups for publicly traded firms in Compustat and explore how the growth of high- and low-markup firms change following monetary shocks. Of course, this exercise must be interpreted with caution since measuring markups accurately at high frequency is challenging and Compustat is not a representative sample of all US producers. Nevertheless, our empirical results are supportive of the basic mechanism underlying the misallocation channel.
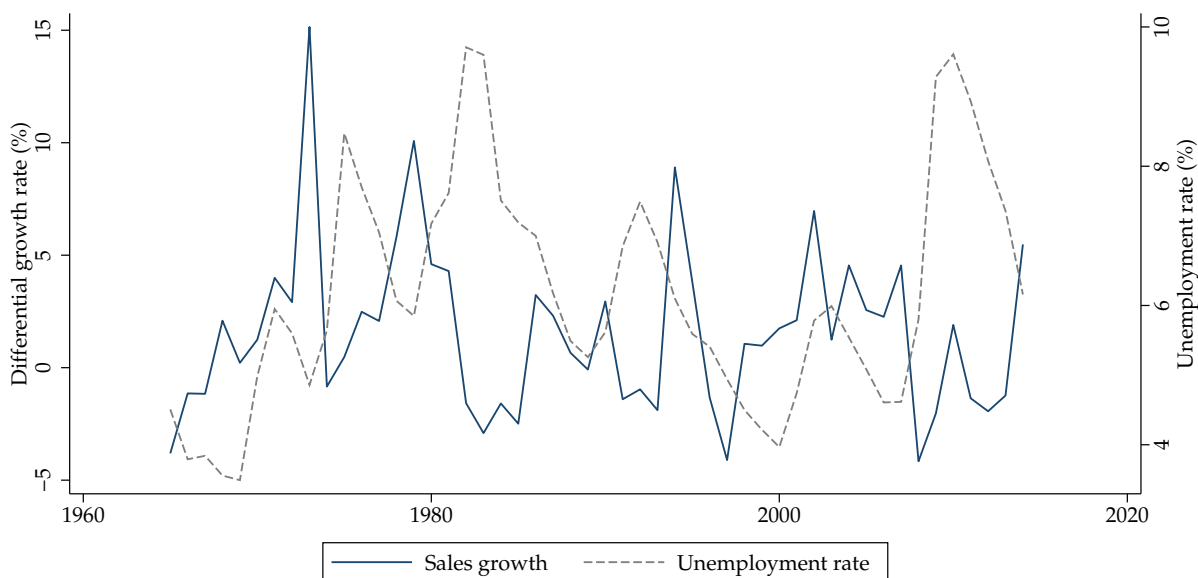
Figure 8: Procyclical reallocations to high-markup firms. A firm is categorized as high-markup (low-markup) if its markup is above (below) median in year t. The solid line shows the difference in sales growth of high- and low-markup firms from t to $t + 1$.

Before looking at identified responses to monetary shocks, we start with an unconditional association. Figure 8 shows the difference in the sales growth of high- and low-markup firms from 1965-2015. We use accounting profits to estimate firm markups in year t and split public firms into high-markup (above median) and low-markup (below median) groups.[45] We then calculate the difference in the sales growth of both groups from year t to $t + 1$.[46] As shown in Figure 8, the differential growth rate shows substantial variance over the sample and is correlated with the business cycle (here, captured by the unemployment rate).

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[45]Specifically, we assume operating income minus depreciation is profit and infer the markup by assuming firms have constant returns to scale. We use accounting profits in Figure 8 since that allows us to plot the series for the longest sample.

[46]For each year t, we limit our analysis to firms in the sample in both years t and $t + 1$. The high-markup and low-markup group are constructed in year t by comparing each firm's markup to the median markup in that year. The differential growth rate is then calculated as the growth rate of total sales for the high-markup group minus the growth rate of total sales for the low-markup group.

For a more convincing test of the model, we consider the response of firm markup changes and reallocations across firms to identified exogenous monetary shocks. For our baseline estimate of firm markups, we follow the user-cost approach of Gutiérrez and Philippon (2017) and Gutiérrez (2017). This approach measures each firm's capital stock and user-cost of capital to estimate factor costs and back out markups.[47] Appendix D describes the data sources and assumptions underlying our markup estimation procedure in more detail.

We then estimate the following local projections:

$$Cov_\lambda(\mu, d\log\mu_{t\to t+h}) = a + \sum_{k=0}^{4} b_k \cdot \text{MonetaryShock}_{t-k} + \sum_{k=1}^{4} c_k \cdot Cov_\lambda(\mu, d\log\mu_{t-k\to t}) + \epsilon_t,$$

$$Cov_\lambda(\mu, d\log\text{Costs}_{t\to t+h}) = \tilde{a} + \sum_{k=0}^{4} \tilde{b}_k \cdot \text{MonetaryShock}_{t-k} + \sum_{k=1}^{4} \tilde{c}_k \cdot Cov_\lambda(\mu, d\log\text{Costs}_{t-k\to t}) + \epsilon_t,$$

where $Cov_\lambda(\mu, d\log\mu_{t\to t+h})$ is the sales-weighted covariance between markups at time t and the change in markups from time t to time $t + h$, $Cov_\lambda(\mu, d\log\text{Costs}_{t\to t+h})$ is the sales-weighted covariance between markups at t and the change in total costs (which are measured as sales divided by the firm's markup), and MonetaryShock_t is the (extended) Romer and Romer (2004) shock in quarter t.[48] As shown in Figure 6, our model predicts that a contractionary shock leads to relative increases in the markups of high-markup firms ($Cov_\lambda(\mu, d\log\mu) > 0$) and a reallocation of resources toward low-markup firms ($Cov_\lambda(\mu, d\log\text{Costs}) < 0$).

Figure 9 shows estimates of b_0 and $\tilde{b}_0$ for ten quarters following a monetary shock. As the top panel shows, a contractionary shock leads high-markup firms to increase their markups relative to low-markup firms; the result is a reallocation of resources away from high-markup firms and toward low-markup firms. In the bottom panel, we estimate a panel version of the above specifications across 3-digit NAICS industries with industry fixed effects. Both the direction and magnitude of the impulse responses are similar, suggesting that within-sector reallocations play an important role.[49]
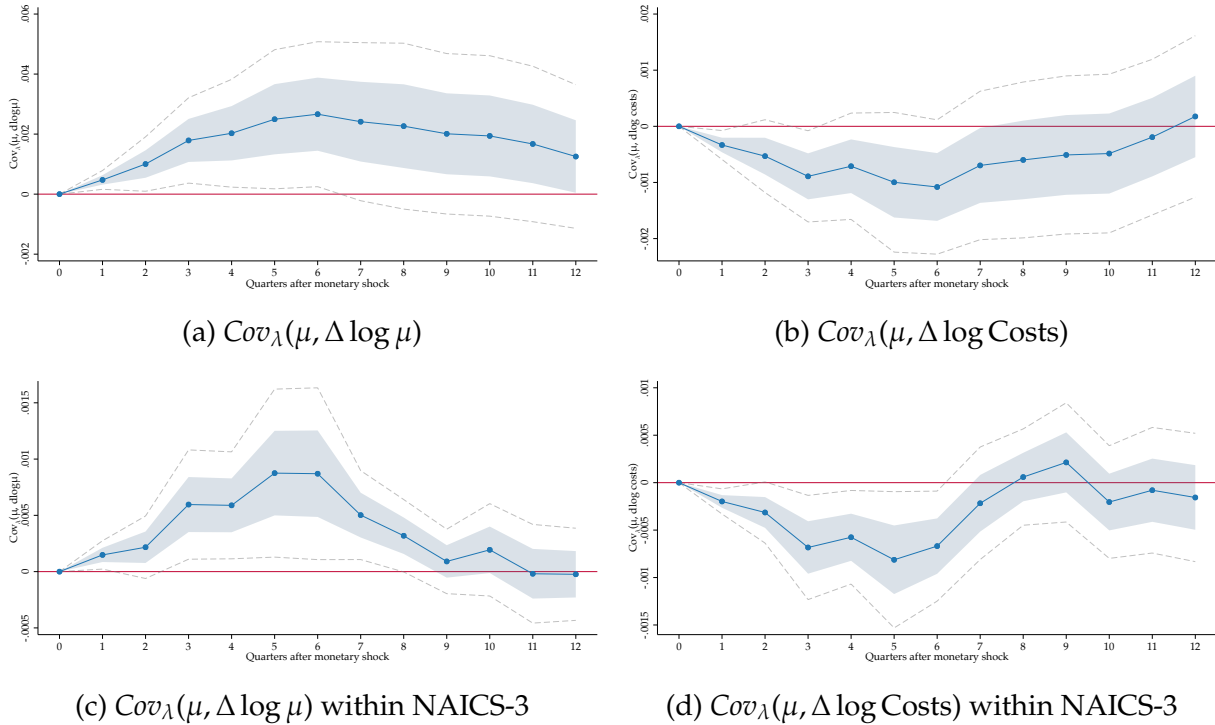
In Appendix D, we show that these results are robust to using firm accounting profits to measure markups (Figure D.1) instead. Our results are also robust to including intangible

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[47]The user-cost approach is also used by Foster et al. (2008) and by De Loecker et al. (2020) as a robustness check.

[48]As above, we limit the covariance to firms that report earnings in both quarter t and $t + h$. Sales in quarter t are used to weight the covariances.

[49]In magnitudes, we find that the ratio of the covariance of markups with the change in markups to the response of output in the model and in the data. The covariance of markups with the change in costs is somewhat smaller in the data than predicted by the model.

Figure 9: Local projection of Romer and Romer monetary shocks (extended series) on $Cov_\lambda(\mu, d\log\mu)$ and $Cov_\lambda(\mu, d\log \text{Costs})$.

(a) $Cov_\lambda(\mu, \Delta\log\mu)$

(b) $Cov_\lambda(\mu, \Delta\log\text{Costs})$

(c) $Cov_\lambda(\mu, \Delta\log\mu)$ within NAICS-3

(d) $Cov_\lambda(\mu, \Delta\log\text{Costs})$ within NAICS-3

capital when estimating markups (Figure D.2). Our results are also robust to using alternative monetary shock series by Gorodnichenko and Weber (2016) and Miranda-Agrippino and Ricco (2021) (Figure D.3).

Cross-sector evidence. Our analysis suggests that industrial concentration may play a role in how productivity responds to monetary shocks. All things being equal, higher industrial concentration is likely to be accompanied by greater heterogeneity in pass-through and hence a greater response of productivity to monetary shocks.

To see whether this prediction is borne out in the data, we use annual estimates of multifactor productivity across manufacturing NAICS4 industries from the Bureau of Labor Statistics and data on the concentration of sales from the Economic Census of Manufacturing. We estimate the following local projection:

$$\Delta\log\text{MultifactorProductivity}_{i,t} =$$
$$\beta\left(\text{Concentration}_i \times \text{MonetaryShock}_t\right) + \gamma_0\text{Concentration}_i + \gamma_1\text{MonetaryShock}_t +$$

$$\sum_{k=1}^{2} \gamma_k^p \log \text{MultifactorProductivity}_{i,t-k} + \sum_{k=1}^{2} \gamma_k^m \text{MonetaryShock}_{t-k} + \delta_i + \alpha_t + \epsilon_{i,t}, \quad (23)$$

where i is the NAICS4 industry, t is the year, δ_i are industry fixed effects, and α_t are year fixed effects.[50] The coefficient of interest is β, which indicates whether multifactor productivity in concentrated industries is differentially responsive to the monetary shock. Our analysis predicts that a contractionary monetary shock leads to a greater reduction in multifactor productivity in concentrated industries, and hence $\beta < 0$.

Table 5 shows the estimated coefficient β using three measures of industrial concentration—the sales share of the industry's top eight, twenty, and fifty firms in the 2002 Economic Census for Manufacturing—and using the extended Romer and Romer (2004) shock series. For all three measures, we observe that the estimated $\beta < 0$, which suggests that the productivity effects of a monetary shock are more pronounced in concentrated industries.

Table 5: Differential response of industry multifactor productivity to monetary shocks in concentrated manufacturing industries.

	$\Delta \log \text{MultifactorProductivity}_{i,t}$		
	(1)	(2)	(3)
Top 8 Firms Share$_i$ × MonetaryShock$_t$	-0.0185** (0.00906)		
Top 20 Firms Share$_i$ × MonetaryShock$_t$		-0.0183** (0.00762)	
Top 50 Firms Share$_i$ × MonetaryShock$_t$			-0.0176** (0.00699)
Industry FEs	Yes	Yes	Yes
Year FEs	Yes	Yes	Yes
N	1634	1634	1634

Notes: The sales share of top 8, 20, and 50 firms is from the 2002 Economic Census for Manufacturing. Monetary shocks are from the extension of the Romer and Romer (2004) shock series by Wieland and Yang (2020). * indicates significance at 10%, ** at 5%.

In Appendix D, we show that these results are robust to instead using concentration data from the 2007 Economic Census of Manufacturing (Table D.1) and to using an alternative monetary shock series from Gorodnichenko and Weber (2016) (Table D.2).

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[50]We include concentration and the lags of the monetary shock for completeness, but since these terms are collinear with the time fixed effects, the corresponding coefficients cannot be separately identified.

8 Extensions

Before concluding, we summarize some extensions that are developed in detail in the appendices.

Multiple sectors, multiple factors, input-output linkages, and sticky wages. The model we use in the main text of the paper is deliberately stylized for clarity. In particular, it is missing some ingredients that are quantitatively important for how output responds to monetary shocks, but that are unrelated to the mechanism this paper studies. In particular, our model has only one sector and only one factor of production, labor. In Appendix F, we show how to extend the model to have a general production network structure, with multiple sectors and multiple factors. As an example, in Appendix F.1 we consider an economy with two factors (labor and capital), a firm sector, and a "labor union" sector that generates sticky wages. The intuition underlying the supply-side effects of a monetary shock are unchanged in this extension compared to the model presented in the main text, and we find that the misallocation channel remains similar in magnitude.

Variation in markups and pass-throughs unrelated to size. In our calibrations, we assume that markups and pass-throughs, at the initial equilibrium, only vary as a function of firm size. Whilst markups and pass-throughs do vary as a function of firm size (e.g. see Burstein et al., 2020 or Amiti et al., 2019), in practice, firm markups and pass-throughs also vary for reasons unrelated to size, such as firm-specific shifters in demand curves, quality differences, or markup dispersion inherited from previous periods. In Appendix J, we show how our baseline results change if there is variation in markups and pass-throughs unrelated to size. We show that the supply-side effects of monetary policy are strengthened if the excess variation in markups is negatively correlated with the excess variation in pass-throughs, and weakened if this correlation is positive. When excess variation in markups and pass-throughs are orthogonal, then the presence of the noise does not affect the strength of supply-side effects of monetary policy relative to our benchmark calibration.

Menu cost model. The price rigidities in the main text of the paper take the form of Calvo frictions. In Appendix H, we construct and simulate a quantitative model with menu costs instead. In that version of the model, idiosyncratic productivity shocks generate large, frequent, and symmetric price changes in the model, as observed in the data (Bils and Klenow 2004). We first calibrate the model under standard CES preferences, and then

replace those preferences with the Kimball demand system estimated in the Belgian data. In response to a money supply shock, as in our baseline results, the Kimball calibration generates a procyclical TFP response and significantly more nonneutrality in output. As in the Calvo model, this occurs because, conditional on a price change, high-markup firms adjust prices by less in response to a monetary expansion. However, in the menu cost model, high-markup firms also endogenously choose to keep their prices unchanged for longer due to strategic complementarities, and this creates an additional channel that strengthens the misallocation channel.

Oligopoly calibration. In the main text of the paper, we model a continuum of firms in monopolistic competition where the positive covariance between price elasticities and pass-throughs appears because of the shape of the residual demand curve. An alternative micro-foundation for this covariance is an oligopoly model like the one in Atkeson and Burstein (2008). In Appendix I, we develop a static oligopoly version of our model and compute the flattening of the Phillips curve due to real rigidities and the misallocation channel. The results are qualitatively and quantitatively similar to the monopolistic competition calibration in Section 6.

Klenow and Willis (2016) calibration. In the main text of the paper, we caution against using off-the-shelf functional forms for preferences. We illustrate this by calibrating our model with the commonly used Klenow and Willis (2016) specification in Appendix G. We show that to match the observed relationship between pass-through and firm-size, with near complete pass-through for small firms and very incomplete pass-through for large firms, large firms must have markups that are on the order of $10,000\%$. Under standard calibrations, which do not produce astronomically large markups for large firms, the implied pass-through function does not vary much as a function of firm-size. Therefore, under standard calibrations, these preferences fail to capture the cross-sectional covariance between pass-throughs and markups, and hence imply counterfactually small supply-side effects.

9   Conclusion

We analyze the transmission of aggregate demand shocks, like monetary policy shocks, in an economy with heterogeneous firms, variable desired markups and pass-throughs, and sticky prices. In contrast to the benchmark New Keynesian model, where the envelope theorem renders reallocations irrelevant to output, we find that in this richer model

aggregate demand shocks have quantitatively significant effects on aggregate output and productivity via reallocations.

These results accord with evidence at both the micro level, where previous studies document that dispersion in plant- and firm-level revenue productivity is countercyclical, and at the macro level, where previous studies document procyclical movements in aggregate TFP. We link these pieces of evidence and show how monetary shocks can generate both effects.

While we focus on heterogeneous markups in product markets, it is possible that similar distortions could exist in inputs markets. Specifically, if firms have heterogeneous and variable monopsony power in the labor market, then TFP would be procyclical if firms with relatively high markdowns reduce their markdowns during booms and raise them during recessions. Finally, our analysis is purely positive, and we leave the normative implications for optimal policy for future work.

References

Alvarez, F., H. L. Bihan, and F. Lippi (2016). The real effects of monetary shocks in sticky price models: A sufficient statistic approach. *American Economic Review 106*(10), 2817–2851.

Amiti, M., O. Itskhoki, and J. Konings (2019). International shocks, variable markups, and domestic prices. *The Review of Economic Studies 86*(6), 2356–2402.

Andrés, J. and P. Burriel (2018). Inflation and optimal monetary policy in a model with firm heterogeneity and bertrand competition. *European Economic Review 103*, 18–38.

Anzoategui, D., D. Comin, M. Gertler, and J. Martinez (2019). Endogenous technology adoption and r&d as sources of business cycle persistence. *American Economic Journal: Macroeconomics 11*(3), 67–110.

Atkeson, A. and A. Burstein (2008). Pricing-to-market, trade costs, and international relative prices. *American Economic Review 98*(5), 1998–2031.

Auer, R. A., T. Chaney, and P. Sauré (2018). Quality pricing-to-market. *Journal of International Economics 110*, 87–102.

Autor, D., D. Dorn, L. F. Katz, C. Patterson, and J. V. Reenen (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics 135*(2), 645–709.

Ball, L. and D. Romer (1990). Real rigidities and the non-neutrality of money. *Review of Economic Studies 57*(2), 183–203.

Baqaee, D. R. and E. Farhi (2017). Productivity and misallocation in general equilibrium. Technical Report 24007, National Bureau of Economic Research.

Baqaee, D. R. and E. Farhi (2018). Macroeconomics with heterogeneous agents and input-output networks. Technical Report 24684, National Bureau of Economic Research.

Baqaee, D. R. and E. Farhi (2020). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics 135*(1), 105–163.

Baqaee, D. R., E. Farhi, and K. Sangani (2021). The darwinian returns to scale. Technical Report 27139, National Bureau of Economic Research.

Barkai, S. (2020). Declining labor and capital shares. *The Journal of Finance 75*(5), 2421–2463.

Basu, S., J. G. Fernald, and M. S. Kimball (2006). Are technology improvements contractionary? *American Economic Review 96*(5), 1418–1448.

Bauer, M. D. and E. T. Swanson (2022). A reassessment of monetary policy surprises and high-frequency identification. Technical Report 29939, National Bureau of Economic Research.

Benigno, G. and L. Fornaro (2018). Stagnation traps. *The Review of Economic Studies 85*(3), 1425–1470.

Benkard, C. L., A. Yurukoglu, and A. L. Zhang (2021). Concentration in product markets. Technical Report 28745, National Bureau of Economic Research.

Berman, N., P. Martin, and T. Mayer (2012). How do different exporters react to exchange rate changes? *The Quarterly Journal of Economics 127*(1), 437–492.

Bianchi, F., H. Kung, and G. Morales (2019). Growth, slowdowns, and recoveries. *Journey of Monetary Economics 101*, 47–63.

Bils, M. and P. J. Klenow (2004). Some evidence on the importance of sticky prices. *Journal of Political Economy 112*(5), 947–985.

Burstein, A., V. M. Carvalho, and B. Grassi (2020). Bottom-up markup fluctuations. Technical report, National Bureau of Economic Research.

Burstein, A. T. (2006). Inflation and output dynamics with state-dependent pricing decisions. *Journal of monetary Economics 53*(7), 1235–1257.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journey of Monetary Economics 12*(3), 383–398.

Chatterjee, A., R. Dix-Carneiro, and J. Vichyanond (2013). Multi-product firms and exchange rate fluctuations. *American Economic Journal: Economic Policy 5*(2), 77–110.

Chen, N. and L. Juvenal (2016). Quality, trade, and exchange rate pass-through. *Journal of International Economics 100*, 61–80.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011). Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review 101*(3), 471–475.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the

dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Claus, J. and J. Thomas (2001). Equity premia as low as three percent? evidence from analysts' earnings forecasts for domestic and international stock markets. *The Journal of Finance 56*(5), 1629–1666.

Comin, D. and M. Gertler (2006). Medium-term business cycles. *American Economic Review 96*(3), 523–551.

Corhay, A., H. Kung, and L. Schmid (2020). Q: Risk, rents, or growth? Technical report, Working Paper.

Cozier, B. and R. Gupta (1993). Is productivity exogenous over the business cycle? some canadian evidence on the solow residual. Technical report, Bank of Canada.

Cravino, J. (2017). Exchange rates, aggregate productivity and the currency of invoicing of international trade. Technical report.

David, J. and D. Zeke (2021, January). Risk-taking, capital allocation and optimal monetary policy. Technical report.

De Loecker, J., J. Eeckhout, and G. Unger (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics 135*(2), 561–644.

Del Negro, M., M. Lenza, G. E. Primiceri, and A. Tambalotti (2020). What's up with the phillips curve? Technical Report 27003, National Bureau of Economic Research.

Dotsey, M. and R. G. King (2005). Implications of state-dependent pricing for dynamic macroeconomic models. *Journey of Monetary Economics 52*(1), 213–242.

Eichenbaum, M. and J. Fisher (2004). Evaluating the calvo model of sticky prices. Technical report, National Bureau of Economic Research.

Etro, F. and L. Rossi (2015). New-keynesian phillips curve with bertrand competition and endogenous entry. *Journal of Economic Dynamics and Control 51*, 318–340.

Evans, C. L. (1992). Productivity shocks and real business cycles. *Journey of Monetary Economics 29*(2), 191–208.

Evans, C. L. and F. T. dos Santos (2002). Monetary policy shocks and productivity measures in the g-7 countries. *Portuguese Economic Journal 1*(1), 47–70.

Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity. Technical report, Federal Reserve Bank of San Francisco.

Foster, L., J. Haltiwanger, and C. Syverson (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review 98*(1), 394–425.

Fratto, C. and H. Uhlig (2014). Accounting for post-crisis inflation and employment: A retro analysis. Technical Report 20707, National Bureau of Economic Research.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new*

Keynesian framework and its applications. Princeton University Press.

Goldberg, P. and R. Hellerstein (2011). How rigid are producer prices? Technical Report 407, Federal Reserve Bank of New York.

Gopinath, G. and O. Itskhoki (2011). In search of real rigidities. *NBER Macroeconomics Annual 25*(1), 261–310.

Gopinath, G., O. Itskhoki, and R. Rigobon (2010). Currency choice and exchange rate pass-through. *American Economic Review 100*(1), 304–336.

Gorodnichenko, Y. and M. Weber (2016). Are sticky prices costly? evidence from the stock market. *American Economic Review 106*(1), 165–99.

Gutiérrez, G. (2017). Investigating global labor and profit shares. Working Paper.

Gutiérrez, G. and T. Philippon (2017). Declining competition and investment in the u.s. Technical Report 23583, National Bureau of Economic Research.

Hall, R. E. (1990). Invariance properties of solow's productivity residual. In P. Diamond (Ed.), *Growth, Productivity, Unemployment: Essays to Celebrate Bob Solow's Birthday*, pp. 71–112. Cambridge: MIT Press.

Hazell, J., J. Herreno, E. Nakamura, and J. Steinsson (2020). The slope of the phillips curve: Evidence from us states. Technical Report 28005, National Bureau of Economic Research.

Hooper, P., F. S. Mishkin, and A. Sufi (2020). Prospects for inflation in a high pressure economy: Is the phillips curve dead or is it just hibernating? *Research in Economics 74*(1), 26–62.

Hsieh, C.-T. and P. J. Klenow (2009). Misallocation and manufacturing tfp in china and india. *The Quarterly Journal of Economics 124*(4), 1403–1448.

Kehrig, M. (2011). The cyclicality of productivity dispersion. Technical Report CES-WP-11-15, US Census Bureau Center for Economic Studies.

Kim, S. and H. Lim (2004). Does solow residual for korea reflect pure technology shocks? Technical report, Seoul, Korea.

Kimball, M. S. (1995). The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit and Banking 27*(4), 1241–77.

Klenow, P. J. and J. L. Willis (2016). Real rigidities and nominal price changes. *Economica 83*(331), 443–472.

Konings, J., P. V. Cayseele, and F. Warzynski (2005). The effects of privatization and competitive pressure on firms' price-cost margins: Micro evidence from emerging economies. *The Review of Economics and Statistics 87*(1), 124–134.

Krugman, P. R. (1998). It's baaack: Japan's slump and the return of the liquidity trap. *Brookings papers on economic activity 1998*(2), 137–205.

Levy, D., M. Bergen, S. Dutta, and R. Venable (1997). The magnitude of menu costs: Direct evidence from large u.s. supermarket chains. *The Quarterly Journal of Economics 112*(3), 791–825.

Li, H., H. Ma, and Y. Xu (2015). How do exchange rate movements affect chinese exports? a firm-level investigation. *Journal of International Economics 97*(1), 148–161.

Martinez, I. Z., E. Saez, and M. Siegenthaler (2018). Intertemporal labor supply substitution? evidence from the swiss income tax holidays. Technical Report 24634, National Bureau of Economic Research.

Matsuyama, K. and P. Ushchev (2017). Beyond ces: Three alternative classes of flexible homothetic demand systems.

McLeay, M. and S. Tenreyro (2020). Optimal inflation and the identification of the phillips curve. *NBER Macroeconomics Annual 34*(1), 199–255.

Meier, M. and T. Reinelt (2020, June). Monetary policy, markup dispersion, and aggregate tfp. *ECB Working Paper No. 2427*.

Melitz, M. J. (2018). Competitive effects of trade: Theory and measurement. *Review of World Economics 154*(1), 1–13.

Midrigan, V. (2011). Menu costs, multiproduct firms, and aggregate fluctuations. *Econometrica 79*(4), 1139–1180.

Miranda-Agrippino, S. and G. Ricco (2021). The transmission of monetary policy shocks. *American Economic Journal: Macroeconomics 13*(3), 74–107.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *The Quarterly Journal of Economics 123*(4), 1415–1464.

Nakamura, E. and J. Steinsson (2010). Monetary non-neutrality in a multisector menu cost model. *The Quarterly Journal of Economics 125*(3), 961–1013.

Restuccia, D. and R. Rogerson (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics 11*(4), 707–720.

Romer, C. D. and D. H. Romer (2004). A new measure of monetary shocks: Derivation and implications. *American Economic Review 94*(4), 1055–1084.

Rossi-Hansberg, E., P.-D. Sarte, and N. Trachter (2021). Diverging trends in national and local concentration. *NBER Macroeconomics Annual 35*(1), 115–150.

Sigurdsson, J. (2019). Labor supply responses and adjustment frictions: A tax-free year in iceland. Technical Report 3278308, SSRN.

Smith, D. and S. Ocampo (2021). The evolution of us retail concentration. Working paper.

Taylor, J. B. (1980). Aggregate dynamics and staggered contracts. *Journal of Political Economy 88*(1), 1–23.

Taylor, J. B. (1999). Staggered price and wage setting in macroeconomics. *Handbook of*

Macroeconomics 1, 1009–1050.

Wang, O. and I. Werning (2020). Dynamic oligopoly and price stickiness. Technical Report 27536, National Bureau of Economic Research.

Wieland, J. F. and M.-J. Yang (2020). Financial dampening. *Journal of Money, Credit and Banking 52*(1), 79–113.

Appendix A Proofs for Static Model

In this appendix, we provide derivations for the one-period model. We start by expressing the full set of equilibrium conditions as a function of endogenous equilibrium objects and exogenous parameters. We then characterize the response of firm markups, quantities, and sales shares in response to a nominal wage shock. By aggregating across firms, we derive the changes in aggregate variables, which yields a fixed point in changes in output.

A.1 Equilibrium conditions

Note that we can express each firm's output y_θ and price p_θ as a function of the sales density λ_θ and markup μ_θ:

$$y_\theta = \frac{\bar{\mu} L \lambda_\theta A_\theta}{\mu_\theta} \quad \text{and} \quad \frac{p_\theta}{w} = \frac{\mu_\theta}{A_\theta}. \tag{24}$$

This allows us to write all equilibrium conditions in terms of the endogenous equilibrium variables $\lambda_\theta, \mu_\theta, P, \bar{\mu}, L$, and Y, and the exogenous parameters $A_\theta, \delta_\theta, w, \zeta$, and γ. We restate the equilibrium conditions from the main text.

1. *Consumers maximize utility.* Consumer demand for each good is given by the inverse-demand curve in Equation (1), which becomes

$$\frac{\mu_\theta}{A_\theta} = wP\Upsilon'_\theta(\frac{\bar{\mu} L \lambda_\theta A_\theta}{\mu_\theta Y}), \tag{25}$$

with the constraint on utility from the consumption bundle, Y, given in Section 2, expressed as,

$$1 = \mathbb{E}\left[\Upsilon_\theta(\frac{\bar{\mu} L \lambda_\theta A_\theta}{\mu_\theta Y})\right]. \tag{26}$$

The aggregate markup is

$$\bar{\mu} = \mathbb{E}_\lambda\left[\mu_\theta^{-1}\right]^{-1} = \frac{wL + \Pi}{wL}. \tag{27}$$

Finally, the consumer maximizes overall utility from the consumption bundle and labor supply such that

$$\bar{\mu} L^{1+\frac{1}{\zeta}} = Y^{1-\gamma}. \tag{28}$$

2. *Firms maximize profits.* Flexible-price firms set markups according to the Lerner

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formula,

$$\mu_\theta^{\text{flex}} = \mu_\theta(\bar\mu \frac{L\lambda_\theta A_\theta}{\mu_\theta Y}), \tag{29}$$

while the markups of sticky-price firms move due to changes in marginal cost that are out of the firm's control,

$$\mu_\theta^{\text{sticky}} = \frac{w_{t-1}}{w_t} \frac{A_{\theta,t}}{A_{\theta,t-1}} \mu_{\theta,t-1}. \tag{30}$$

3. *Markets clear.* The price aggregator satisfies

$$\frac{1}{P} = \frac{Y}{wL} \mathbb{E}\left[\frac{L\lambda_\theta A_\theta}{\mu_\theta Y} \Upsilon'_\theta(\frac{\bar\mu L\lambda_\theta A_\theta}{\mu_\theta Y})\right]. \tag{31}$$

A.2   Response to monetary policy shocks

We consider a shock to the nominal wage given by $d\log w$. All other exogenous primitives (A_θ, δ_θ, ζ, and γ) are unchanged by the shock. First, we characterize changes in firm-level markups $d\log\mu_\theta$, quantities $d\log y_\theta$, and sales shares $d\log\lambda_\theta$.

Markups.   The markup of a firm of type θ satisfies

$$d\log\mu_\theta = d\log p_\theta - d\log w. \tag{32}$$

The prices of sticky-price firms do not change, hence

$$d\log\mu_\theta^{\text{sticky}} = -d\log w. \tag{33}$$

For flexible-price firms,

$$d\log p_\theta^{\text{flex}} = d\log\mu_\theta(\frac{y_\theta}{Y}) + d\log w \tag{34}$$

$$= \frac{\frac{y_\theta}{Y}\mu'_\theta(\frac{y_i}{Y})}{\mu_\theta} d\log(\frac{y_\theta}{Y}) + d\log w \tag{35}$$

$$= \frac{1-\rho_\theta}{\rho_\theta} \frac{1}{\sigma_\theta} d\log(\frac{y_\theta}{Y}) + d\log w \tag{36}$$

$$= (1-\rho_\theta)d\log P + \rho_\theta d\log w. \tag{37}$$

This yields,

$$d\log\mu_\theta^{\text{flex}} = (1-\rho_\theta)(d\log P - d\log w). \tag{38}$$

Quantities. We have

$$d\log(\frac{y_\theta}{Y}) = -\sigma_\theta \left(d\log p_\theta - d\log P\right) \tag{39}$$

$$= -\sigma_\theta \left(d\log \mu_\theta + d\log w - d\log P\right). \tag{40}$$

This yields,

$$d\log(\frac{y_\theta^{\text{sticky}}}{Y}) = \sigma_\theta \left(d\log P\right), \tag{41}$$

$$d\log(\frac{y_\theta^{\text{flex}}}{Y}) = \sigma_\theta \rho_\theta \left(d\log P - d\log w\right). \tag{42}$$

Sales shares. We have

$$d\log \lambda_\theta = d\log(\frac{y_\theta}{Y}) + d\log \mu_\theta + d\log Y - d\log \bar{\mu} - d\log L. \tag{43}$$

This yields,

$$d\log \lambda_\theta^{\text{flex}} = (d\log Y - d\log \bar{\mu} - d\log L) + (1 + (\sigma_\theta - 1)\rho_\theta)(d\log P - d\log w), \tag{44}$$

$$d\log \lambda_\theta^{\text{sticky}} = (d\log Y - d\log \bar{\mu} - d\log L) - d\log w + \sigma_\theta d\log P. \tag{45}$$

Now, we aggregate across firms to get the change in aggregate variables, $d\log \bar{\mu}$, $d\log P$, $d\log Y$, and $d\log L$.

Aggregate markup. We start with the equation for the change in the aggregate markup,

$$d\log \bar{\mu} = \mathbb{E}_{\lambda \mu^{-1}} \left[d\log \mu_\theta\right] - \mathbb{E}_{\lambda \mu^{-1}} \left[d\log \lambda_\theta\right]. \tag{46}$$

We use

$$\mathbb{E}_{\lambda \mu^{-1}} \left[d\log \lambda_\theta\right] = (d\log Y - d\log \bar{\mu} - d\log L)$$
$$+ \mathbb{E}_{\lambda \mu^{-1}} \left[\delta_\theta \left[(1 + (\sigma_\theta - 1)\rho_\theta)(d\log P - d\log w)\right]\right]$$
$$+ \mathbb{E}_{\lambda \mu^{-1}} \left[(1 - \delta_\theta)\left[-d\log w + \sigma_\theta d\log P\right]\right], \tag{47}$$

which simplifies to

$$\mathbb{E}_{\lambda \mu^{-1}} \left[d\log \lambda_\theta\right] = (d\log Y - d\log L - d\log \bar{\mu})$$
$$+ \mathbb{E}_{\lambda \mu^{-1}} \left[\delta_\theta \left(1 + (\sigma_\theta - 1)\rho_\theta\right) + (1 - \delta_\theta)\sigma_\theta\right] d\log P$$

$$+ \, \mathbb{E}_{\lambda\mu^{-1}} \left[(-1 - \delta_\theta (\sigma_\theta - 1) \rho_\theta) \right] d\log w, \quad (48)$$

and

$$\mathbb{E}_{\lambda\mu^{-1}} \left[d\log \mu_\theta \right] = \mathbb{E}_{\lambda\mu^{-1}} \left[\delta_\theta \left[(1 - \rho_\theta)(d\log P - d\log w) \right] \right] + \mathbb{E}_{\lambda\mu^{-1}} \left[(1 - \delta_\theta) \left[-d\log w \right] \right] \quad (49)$$

$$= \mathbb{E}_{\lambda\mu^{-1}} \left[\delta_\theta (1 - \rho_\theta) \right] d\log P - \mathbb{E}_{\lambda\mu^{-1}} \left[1 - \delta_\theta \rho_\theta \right] d\log w. \quad (50)$$

Combining yields,

$$d\log Y - d\log L = -\mathbb{E}_{\lambda\mu^{-1}} \left[\left[\delta_\theta \rho_\theta + (1 - \delta_\theta) \right] \sigma_\theta \right] d\log P + \mathbb{E}_{\lambda\mu^{-1}} \left[\delta_\theta \sigma_\theta \rho_\theta \right] d\log w. \quad (51)$$

Price aggregator. We have

$$d\log P = -d\log Y + d\log L + d\log \bar{\mu} + d\log w - \mathbb{E}_\lambda \left[\left(1 - \frac{1}{\sigma_\theta} \right) d\log(\frac{y_\theta}{Y}) \right]. \quad (52)$$

Using

$$\mathbb{E}_\lambda \left[\left(1 - \frac{1}{\sigma_\theta} \right) d\log(\frac{y_\theta}{Y}) \right] = \mathbb{E}_\lambda \left[\delta_\theta \left(1 - \frac{1}{\sigma_\theta} \right) \sigma_\theta \rho_\theta \left(d\log P - d\log w \right) \right] \quad (53)$$

$$+ \, \mathbb{E}_\lambda \left[(1 - \delta_\theta) \left(1 - \frac{1}{\sigma_\theta} \right) \sigma_\theta \left(d\log P \right) \right] \quad (54)$$

$$= \mathbb{E}_\lambda \left[\left[1 + \delta_\theta \left(\rho_\theta - 1 \right) \right] \left(\sigma_\theta - 1 \right) \right] d\log P - \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \left(\sigma_\theta - 1 \right) \right] d\log w, \quad (55)$$

we get

$$d\log P = \frac{ -\left(d\log Y - d\log L - d\log \bar{\mu} \right) + \mathbb{E}_\lambda \left[1 + \delta_\theta \rho_\theta \left(\sigma_\theta - 1 \right) \right] d\log w }{ 1 + \mathbb{E}_\lambda \left[(\sigma_\theta - 1) \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right] }. \quad (56)$$

Output. We have

$$d\log Y = d\log L + d\log \bar{\mu} - \mathbb{E}_\lambda \left[d\log \mu_\theta \right] \quad (57)$$

$$= d\log L + d\log \bar{\mu} - \mathbb{E}_\lambda \left[\delta_\theta (1 - \rho_\theta) \right] d\log P + \mathbb{E}_\lambda \left[1 - \delta_\theta \rho_\theta \right] d\log w. \quad (58)$$

Labor. We have

$$d\log \bar{\mu} + \left(1 + \frac{1}{\zeta} \right) d\log L = (1 - \gamma) d\log Y, \quad (59)$$

which we rearrange to get

$$d \log L = \frac{\zeta(1-\gamma)}{1+\zeta} d \log Y - \frac{\zeta}{1+\zeta} d \log \bar{\mu}. \tag{60}$$

Solving the fixed point. We have a system of four equations in four unknowns, $d \log Y$, $d \log L$, $d \log \bar{\mu}$, and $d \log P$:

$$d \log Y = d \log L + d \log \bar{\mu} - \mathbb{E}_\lambda \left[\delta_\theta (1 - \rho_\theta) \right] d \log P + \mathbb{E}_\lambda \left[1 - \delta_\theta \rho_\theta \right] d \log w \tag{61}$$

$$d \log L = \frac{\zeta(1-\gamma)}{1+\zeta} d \log Y - \frac{\zeta}{1+\zeta} d \log \bar{\mu} \tag{62}$$

$$d \log Y - d \log L = -\mathbb{E}_{\lambda \mu^{-1}} \left[\left[\delta_\theta \rho_\theta + (1 - \delta_\theta) \right] \sigma_\theta \right] d \log P + \mathbb{E}_{\lambda \mu^{-1}} \left[\delta_\theta \sigma_\theta \rho_\theta \right] d \log w \tag{63}$$

$$d \log P = \frac{-(d \log Y - d \log L - d \log \bar{\mu}) + \mathbb{E}_\lambda \left[1 + \delta_\theta \rho_\theta (\sigma_\theta - 1) \right] d \log w}{1 + \mathbb{E}_\lambda \left[(\sigma_\theta - 1) \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]}. \tag{64}$$

Rearranging yields,

$$d \log P = \frac{\mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} d \log w. \tag{65}$$

Proof of Proposition 1. Using our equations for $d \log P$ and $d \log Y - d \log L$, we solve for the response of aggregate TFP

$$d \log A = d \log Y - d \log L \tag{66}$$

$$= -\mathbb{E}_{\lambda \mu^{-1}} \left[\left[\delta_\theta \rho_\theta + (1 - \delta_\theta) \right] \sigma_\theta \right] d \log P + \mathbb{E}_{\lambda \mu^{-1}} \left[\delta_\theta \sigma_\theta \rho_\theta \right] d \log w \tag{67}$$

$$= \left[\mathbb{E}_{\lambda \mu^{-1}} \left[\delta_\theta \sigma_\theta \rho_\theta \right] - \frac{\mathbb{E}_{\lambda \mu^{-1}} \left[\left[\delta_\theta \rho_\theta + (1 - \delta_\theta) \right] \sigma_\theta \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} \right] d \log w \tag{68}$$

$$= \bar{\mu} \left[\mathbb{E}_\lambda \left[\delta_\theta \rho_\theta (\sigma_\theta - 1) \right] - \left[\frac{\mathbb{E}_\lambda \left[\left[\delta_\theta \rho_\theta + (1 - \delta_\theta) \right] (\sigma_\theta - 1) \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} \right] \right] d \log w \tag{69}$$

$$= \bar{\mu} \frac{\mathbb{E}_\lambda \left[1 - \delta_\theta \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right] - \mathbb{E}_\lambda \left[\sigma_\theta (1 - \delta_\theta) \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} d \log w \tag{70}$$

$$= \bar{\mu} \frac{\mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right] - \mathbb{E}_\lambda \left[\delta_\theta \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right] - \mathbb{E}_\lambda \left[\sigma_\theta \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \right] + \mathbb{E}_\lambda \left[\sigma_\theta \delta_\theta \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} d \log w \tag{71}$$

$$= \bar{\mu} \frac{\mathbb{E}_\lambda \left[\delta_\theta \right] (1 - \mathbb{E}_\lambda \left[\delta_\theta \right]) \mathbb{E}_{\lambda\delta} \left[\rho_\theta \sigma_\theta \right] + \mathbb{E}_{\lambda\delta} \left[\rho_\theta \right] (\mathbb{E}_\lambda \left[\sigma_\theta \delta_\theta \right] \mathbb{E}_\lambda \left[\delta_\theta \right] - \mathbb{E}_\lambda \left[\sigma_\theta \right] \mathbb{E}_\lambda \left[\delta_\theta \right])}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} d \log w \tag{72}$$

$$= \bar{\mu} \frac{\mathbb{E}_\lambda \left[\delta_\theta \right] (1 - \mathbb{E}_\lambda \left[\delta_\theta \right]) Cov_{\lambda\delta} \left[\rho_\theta, \sigma_\theta \right] + \mathbb{E}_{\lambda\delta} \left[\rho_\theta \right] Cov_\lambda \left[\sigma_\theta, \delta_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} d \log w, \tag{73}$$

which concludes the proof of Proposition 1.

Proof of Proposition 2.   We use the change in firm markups to calculate

$$\mathbb{E}_\lambda \left[d\log \mu_\theta \right] = \mathbb{E}_\lambda \left[\delta_\theta (1 - \rho_\theta) \right] d\log P - \mathbb{E}_\lambda \left[1 - \delta_\theta \rho_\theta \right] d\log w \tag{74}$$

$$= \left[\frac{\mathbb{E}_\lambda \left[\delta_\theta (1 - \rho_\theta) \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} - \mathbb{E}_\lambda \left[1 - \delta_\theta \rho_\theta \right] \right] d\log w \tag{75}$$

$$= \left[\mathbb{E}_\lambda \left[\delta_\theta (1 - \rho_\theta) \right] \left(\frac{\mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} - 1 \right) - \mathbb{E}_\lambda \left[1 - \delta_\theta \right] \right] d\log w \tag{76}$$

$$= \left[-\frac{\mathbb{E}_\lambda \left[\delta_\theta (1 - \rho_\theta) \right] \mathbb{E}_\lambda \left[\sigma_\theta (1 - \delta_\theta) \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \left[(1 - \delta_\theta) + \delta_\theta \rho_\theta \right] \right]} - \mathbb{E}_\lambda \left[1 - \delta_\theta \right] \right] d\log w, \tag{77}$$

which yields Equation (11). We have

$$d\log L = \frac{\zeta (1 - \gamma)}{1 + \zeta} d\log Y - \frac{\zeta}{1 + \zeta} d\log \bar{\mu}, \tag{78}$$

and

$$d\log A = d\log \bar{\mu} - \mathbb{E}_\lambda \left[d\log \mu_\theta \right], \tag{79}$$

which yields,

$$\frac{1 + \gamma \zeta}{1 + \zeta} d\log Y = \frac{1}{1 + \zeta} d\log A - \frac{\zeta}{1 + \zeta} \mathbb{E}_\lambda \left[d\log \mu_\theta \right]. \tag{80}$$

Rearranging yields Equation (10), which concludes the proof of Proposition 2.

General form for Phillips curve flattening.   Proposition 3 follows immediately from dividing Equation (10) by $d\log w$ and rearranging.

Proposition 6 generalizes Proposition 4 to the case where both price-stickiness and pass-throughs are allowed to be heterogeneous.

Proposition 6. *The flattening of the price Phillips curve due to real rigidities, compared to nominal rigidities alone, is*

$$\frac{\textit{Phillips curve slope w/ nominal rigidities only}}{\textit{Phillips curve slope w/ real rigidities}}$$

$$= 1 + \frac{1}{\mathbb{E}_\lambda \left[1 - \delta_\theta \right]} \frac{\mathbb{E}_\lambda \left[\delta_\theta (1 - \rho_\theta) \right] \mathbb{E}_\lambda \left[\sigma_\theta (1 - \delta_\theta) \right]}{\mathbb{E}_\lambda \left[\delta_\theta \right] \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \sigma_\theta \right] + \mathbb{E}_\lambda \left[\delta_\theta \rho_\theta \right] \mathbb{E}_\lambda \left[\sigma_\theta (1 - \delta_\theta) \right]}. \tag{81}$$

The flattening of the price Phillips curve due to the misallocation channel is

$$\frac{\textit{Phillips curve slope w/ real rigidities}}{\textit{Phillips curve slope w/ misallocation}}$$

$$= 1 + \frac{\bar{\mu}}{\zeta} \frac{\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[1 - \delta_\theta\right] Cov_{\lambda\delta}[\rho_\theta, \sigma_\theta] + \mathbb{E}_{\lambda\delta}\left[\rho_\theta\right] Cov_\lambda[\sigma_\theta, \delta_\theta]}{\mathbb{E}_\lambda\left[1 - \delta_\theta\right] \mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right] + \mathbb{E}_\lambda\left[1 - \delta_\theta\rho_\theta\right] \mathbb{E}_\lambda\left[\sigma_\theta(1 - \delta_\theta)\right]}. \quad (82)$$

Proof. The flattening due to the misallocation channel is,

Flattening due to the misallocation channel $\qquad\qquad\qquad\qquad\qquad\qquad (83)$

$$= \frac{\frac{d \log A}{d \log w} - \zeta\mathbb{E}_\lambda\left[\frac{d \log \mu_\theta}{d \log w}\right]}{-\zeta\mathbb{E}_\lambda\left[\frac{d \log \mu_\theta}{d \log w}\right]} \quad (84)$$

$$= 1 + \frac{\frac{d \log A}{d \log w}}{-\zeta\mathbb{E}_\lambda\left[\frac{d \log \mu_\theta}{d \log w}\right]} \quad (85)$$

$$= 1 + \frac{1}{\zeta} \frac{\kappa_\rho Cov_{\lambda\delta}[\rho_\theta, \sigma_\theta] + \kappa_\delta Cov_\lambda[\sigma_\theta, \delta_\theta]}{\frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda[\sigma_\theta(1-\delta_\theta)]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]} + \mathbb{E}_\lambda\left[1 - \delta_\theta\right]} \quad (86)$$

$$= 1 + \frac{\bar{\mu}}{\zeta} \frac{\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[1 - \delta_\theta\right] Cov_{\lambda\delta}[\rho_\theta, \sigma_\theta] + \mathbb{E}_{\lambda\delta}\left[\rho_\theta\right] Cov_\lambda[\sigma_\theta, \delta_\theta]}{\mathbb{E}_\lambda\left[\delta_\theta(1 - \rho_\theta)\right] \mathbb{E}_\lambda\left[\sigma_\theta(1 - \delta_\theta)\right] + \mathbb{E}_\lambda\left[1 - \delta_\theta\right] \mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta + (1 - \delta_\theta)\right]\sigma_\theta\right]} \quad (87)$$

$$= 1 + \frac{\bar{\mu}}{\zeta} \frac{\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[1 - \delta_\theta\right] Cov_{\lambda\delta}[\rho_\theta, \sigma_\theta] + \mathbb{E}_{\lambda\delta}\left[\rho_\theta\right] Cov_\lambda[\sigma_\theta, \delta_\theta]}{\mathbb{E}_\lambda\left[1 - \delta_\theta\rho_\theta\right] \mathbb{E}_\lambda\left[\sigma_\theta(1 - \delta_\theta)\right] + \mathbb{E}_\lambda\left[1 - \delta_\theta\right] \mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right]}. \quad (88)$$

The flattening due to real rigidities is,

Flattening due to real rigidities $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (89)$

$$= \left(\underbrace{\frac{1 - \mathbb{E}_\lambda\left[1 - \delta_\theta\right] - \frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda[\sigma_\theta(1-\delta_\theta)]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]}}{-\zeta\left[-\mathbb{E}_\lambda\left[1 - \delta_\theta\right] - \frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda[\sigma_\theta(1-\delta_\theta)]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]}\right]}}_{\text{Slope with real rigidities}} \underbrace{\frac{1 - \mathbb{E}_\lambda\left[1 - \delta_\theta\right]}{\zeta\mathbb{E}_\lambda\left[1 - \delta_\theta\right]}}_{\text{Slope in CES model}} \right)^{-1} \quad (90)$$

$$= \frac{\mathbb{E}_\lambda\left[\delta_\theta\right]\mathbb{E}_\lambda\left[1 - \delta_\theta\right] + \frac{\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda[\sigma_\theta(1-\delta_\theta)]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]}}{\mathbb{E}_\lambda\left[\delta_\theta\right]\mathbb{E}_\lambda\left[1 - \delta_\theta\right] - \frac{\mathbb{E}_\lambda[1-\delta_\theta]\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda[\sigma_\theta(1-\delta_\theta)]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]}} \quad (91)$$

$$= 1 + \frac{1}{\mathbb{E}_\lambda\left[1 - \delta_\theta\right]\mathbb{E}_\lambda\left[\delta_\theta\right]} \frac{\mathbb{E}_\lambda\left[\delta_\theta(1 - \rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1 - \delta_\theta)\right]}{\mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right] + \mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\right]\mathbb{E}_\lambda\left[\sigma_\theta(1 - \delta_\theta)\right]}. \quad (92)$$

Setting $\delta_\theta = \delta$ in both equations yields Proposition 4.

Appendix B   Proofs for Dynamic Model

Firms choose reset prices to maximize future discounted profits,

$$\max_{p_{i,t}} \mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}(p_{i,t} - \frac{w_{t+k}}{A_i})\right]. \tag{93}$$

The first order condition yields,

$$\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}\left[\frac{dy_{i,t+k}}{dp_{i,t}}\frac{p_{i,t}^*}{y_{i,t+k}}\frac{p_{i,t}^* - \frac{w_{t+k}}{A_i}}{p_{i,t}^*} + 1\right]\right] = 0. \tag{94}$$

Using $\sigma_{i,t} = -\frac{p_i}{y_{i,t}}\frac{dy_{i,t}}{dp_i}$,

$$\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}\left(-\sigma_{i,t+k}\left(1 - \frac{w_{t+k}}{p_{i,t}^* A_i}\right) + 1\right)\right] = 0. \tag{95}$$

Rearranging gives us

$$\frac{p_{i,t}^* A_i}{w_t} = \frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}\left(-\sigma_{i,t+k}\frac{w_{t+k}}{w_t}\right)\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}\left(1 - \sigma_{i,t+k}\right)\right]}. \tag{96}$$

We now log-linearize around a perfect foresight, no-inflation steady state. This steady state is characterized by a constant discount factor such that $\left[\prod_{j=0}^{k-1}(1+r_{t+j})\right]^{-1} = \beta^k$. After removing all second-order terms, we get:

$$\frac{p_{i,t}^* A_i}{w_t} = \frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k y_{i,t+k}\sigma_{i,t+k}\left(d\log\left(\frac{w_{t+k}}{w_t}\right) + 1\right)\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k y_{i,t+k}\left(\sigma_{i,t+k} - 1\right)\right]} \tag{97}$$

$$= \frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k y_{i,t}\sigma_{i,t}(1 + d\log(y_{i,t+k}\sigma_{i,t+k}))\left(d\log\left(\frac{w_{t+k}}{w_t}\right) + 1\right)\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k y_{i,t}\left(\sigma_{i,t} - 1\right)(1 + d\log(y_{i,t+k}(\sigma_{i,t+k} - 1)))\right]} \tag{98}$$

$$= \mu_{i,t}\frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k\right] + \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k d\log\left(\frac{w_{t+k}}{w_t}\right)\right] + \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k d\log(y_{i,t+k}\sigma_{i,t+k})\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k\right] + \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k(1-\delta_i)^k d\log(y_{i,t+k}(\sigma_{i,t+k} - 1))\right]}. \tag{99}$$

Rearranging, we get

$$d \log \mu_{i,t}^* = \frac{\frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log\left(\frac{w_{t+k}}{w_t}\right)\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k\right]} + \frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log(y_{i,t+k} \sigma_{i,t+k})\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k\right]} - \frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log(y_{i,t+k}(\sigma_{i,t+k}-1))\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k\right]}}{1 + \frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log(y_{i,t+k}(\sigma_{i,t+k}-1))\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k\right]}}$$

(100)

$$= \frac{\Gamma^{-1}\left[\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log\left(\frac{w_{t+k}}{w_t}\right)\right] + \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log(\mu_{i,t+k})\right]\right]}{1 + \Gamma^{-1} \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log(y_{i,t+k}(\sigma_{i,t+k}-1))\right]},$$

(101)

where $\Gamma = \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k\right] = [1 - \beta(1-\delta_i)]^{-1}$. We multiply both sides by the denominator and remove second order terms to get

$$d \log \mu_{i,t}^* = \Gamma^{-1}\left[\mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log\left(\frac{w_{t+k}}{w_t}\right)\right] + \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log(\mu_{i,t+k})\right]\right].$$

(102)

At the time of a price reset, we know that

$$\mu_{i,t} = \mu_i\left(\frac{y_{i,t}}{Y_t}\right).$$

(103)

Then,

$$d \log\left(\mu\left(\frac{y_{i,t+k}}{Y_{t+k}}\right)\right) = \frac{\frac{y_{i,t}}{Y_t} \mu_i'\left(\frac{y_{i,t}}{Y_t}\right)}{\mu_{i,t}} d \log\left(\frac{y_{i,t+k}}{Y_{t+k}}\right)$$

(104)

$$= \frac{1 - \rho_{i,t}}{\rho_{i,t}} \frac{1}{\sigma_{i,t}} d \log\left(\frac{y_{i,t+k}}{Y_{t+k}}\right)$$

(105)

$$= \frac{1 - \rho_{i,t}}{\rho_{i,t}} (d \log P_{t+k} - d \log w_{t+k} - d \log \mu_{i,t+k})$$

(106)

$$= \frac{1 - \rho_{i,t}}{\rho_{i,t}} \left(d \log P_{t+k} - d \log w_{t+k} - d \log \mu_{i,t}^* + d \log \frac{w_{t+k}}{w_t}\right),$$

(107)

where in the last line, we use the fact that the change in the markup $d \log \mu_{i,t+k}$ includes changes that occur at the time of the price change ($d \log \mu_{i,t}^*$) and subsequent changes due to the shifts in the nominal wage.

Plugging this in yields,

$$\frac{1}{\rho_{i,t}} d \log \mu_{i,t}^* = \Gamma^{-1}\left[\frac{1}{\rho_{i,t}} \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k d \log\left(\frac{w_{t+k}}{w_t}\right)\right] + E\left[\sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k \frac{1-\rho_{i,t}}{\rho_{i,t}} (d \log P_{t+k} - d \log w_{t+k})\right]\right].$$

(108)

Finally, since $d \log \mu_{i,t}^* = d \log p_{i,t}^* - d \log w_t$, we get

$$d \log p_{i,t}^* = [1 - \beta(1 - \delta_i)] \sum_{k=0}^{\infty} \beta^k (1 - \delta_i)^k [\rho_i d \log w_{t+k} + (1 - \rho_i) d \log P_{t+k}]. \tag{109}$$

We can also write this equation recursively as

$$d \log p_{i,t}^* = (1 - \beta(1 - \delta_i)) [\rho_i d \log w_t + (1 - \rho_i) d \log P_t] + \beta(1 - \delta_i) p_{i,t+1}^*, \tag{110}$$

or in terms of firm types as,

$$d \log p_{\theta,t}^* = (1 - \beta(1 - \delta_\theta)) [\rho_\theta d \log w_t + (1 - \rho_\theta) d \log P_t] + \beta(1 - \delta_\theta) p_{\theta,t+1}^*. \tag{111}$$

Now that we have a recursive formulation for the optimal reset price, we can solve for the movement in the expected price for firms of type θ. Here, we use $\mathbb{E}$ to indicate the expectation over a continuum of identical firms of type θ, some of which will have the opportunity to change their prices and the remainder of which will not. The expected price for a firm of type θ follows,

$$\mathbb{E}[d \log p_{\theta,t+1}] = \delta_\theta d \log p_{\theta,t+1}^* + (1 - \delta_\theta) d \log p_{\theta,t}, \tag{112}$$

since with probability δ_θ the firm is able to change its price to the optimal reset price at time $t + 1$. Combining this with the recursive formula for optimal reset prices above, we get

$$\mathbb{E}[d \log p_{\theta,t} - d \log p_{\theta,t-1}] - \beta \mathbb{E}[d \log p_{\theta,t+1} - d \log p_{\theta,t}]$$
$$= \frac{\delta_\theta}{1 - \delta_\theta} (1 - \beta(1 - \delta_\theta)) [-\mathbb{E}[d \log p_\theta] + \rho_\theta d \log w_t + (1 - \rho_\theta) d \log P_t]. \tag{113}$$

We can then aggregate this equation over firm types to get the modified New Keynesian Phillips curve and to get the Endogenous TFP equation.

New Keynesian Phillips curve with misallocation. We list a few identities that will be helpful in the subsequent derivations. The first four are derived in the main text, and the latter two can be formed be rearranging the above.

$$d \log P_t - d \log P_t^Y = \bar{\mu}^{-1} d \log A_t \tag{114}$$
$$d \log P_t^Y - d \log w_t = \mathbb{E}_\lambda [d \log \mu_\theta] \tag{115}$$

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$$d \log A_t = d \log \bar{\mu}_t - \mathbb{E}_\lambda \left[d \log \mu_{\theta,t} \right] \tag{116}$$

$$d \log Y_t = \frac{1}{1 + \gamma \zeta} \left(d \log A_t - \zeta \mathbb{E}_\lambda \left[d \log \mu_{\theta,t} \right] \right) \tag{117}$$

$$-\mathbb{E}_\lambda \left[d \log \mu_{\theta,t} \right] = \left(\frac{1 + \gamma \zeta}{\zeta} \right) d \log Y_t - \frac{1}{\zeta} d \log A_t \tag{118}$$

$$d \log w_t - d \log P_t = \frac{1 + \gamma \zeta}{\zeta} d \log Y_t - \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) d \log A_t. \tag{119}$$

We now take the sales-weighted expectation of Equation (113) to get:

$$d \log \pi_t - \beta d \log \pi_{t+1} = \varphi \left[-d \log P_t^Y + \mathbb{E}_\lambda \left[\rho_\theta \right] d \log w_t + (1 - \mathbb{E}_\lambda \left[\rho_\theta \right]) d \log P_t \right] \tag{120}$$

$$= \varphi \left[\left(d \log P_t - d \log P_t^Y \right) + \mathbb{E}_\lambda \left[\rho_\theta \right] (d \log w_t - d \log P_t) \right] \tag{121}$$

$$= \varphi \left[\left(\bar{\mu}^{-1} d \log A_t \right) + \mathbb{E}_\lambda \left[\rho_\theta \right] \left(\frac{1 + \gamma \zeta}{\zeta} d \log Y_t - \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) d \log A_t \right) \right] \tag{122}$$

$$= \varphi \mathbb{E}_\lambda \left[\rho_\theta \right] \frac{1 + \gamma \zeta}{\zeta} d \log Y_t + \varphi \left[\frac{1}{\bar{\mu}} - \mathbb{E}_\lambda \left[\rho_\theta \right] \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) \right] d \log A_t, \tag{123}$$

which concludes the proof.

Endogenous TFP equation. Start by subtracting $\mathbb{E} \left[d \log w_t - d \log w_{t-1} \right] - \beta \mathbb{E} \left[d \log w_{t+1} - d \log w_t \right]$ from both sides of Equation (113). This yields,

$$\mathbb{E} \left[d \log \mu_{\theta,t} - d \log \mu_{\theta,t-1} \right] - \beta \mathbb{E} \left[d \log \mu_{\theta,t+1} - d \log \mu_{\theta,t} \right]$$
$$= - \left[\mathbb{E} \left[d \log w_t - d \log w_{t-1} \right] - \beta \mathbb{E} \left[d \log w_{t+1} - d \log w_t \right] \right]$$
$$+ \varphi \left[-\mathbb{E} \left[d \log \mu_{\theta,t} \right] + (\rho_\theta - 1) d \log w_t + (1 - \rho_\theta) d \log P_t \right]. \tag{124}$$

We can write

$$d \log A_t = d \log \bar{\mu} - \mathbb{E}_\lambda \left[d \log \mu_\theta \right] = \bar{\mu} \left(\frac{\mathbb{E}_\lambda \left[\sigma_\theta d \log \mu_{\theta,t} \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \right]} - \mathbb{E}_\lambda \left[d \log \mu_\theta \right] \right). \tag{125}$$

Now, we take Equation (125) and (1) multiply all terms by σ_θ, take the sales-weighted expectation, and divide by $\mathbb{E}_\lambda [\sigma_\theta]$; (2) take the sales-weighted expectation of (125); and multiply (1) − (2) by $\bar{\mu}$. This yields,

$$(d \log A_t - d \log A_{t-1}) - \beta (d \log A_{t+1} - d \log A_t) \tag{126}$$

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$$= \varphi \left[-d \log A_t + \bar{\mu} \left(1 - \frac{\mathbb{E}_\lambda \left[\sigma_\theta \rho_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \right]} - (1 - \mathbb{E}_\lambda \left[\rho_\theta \right]) \right) (d \log P_t - d \log w_t) \right] \qquad (127)$$

$$= \varphi \left[-d \log A_t + \bar{\mu} \left(\frac{Cov_\lambda \left[\rho_\theta, \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \right]} \right) (d \log w_t - d \log P_t) \right] \qquad (128)$$

$$= \varphi \left[-d \log A_t + \bar{\mu} \left(\frac{Cov_\lambda \left[\rho_\theta, \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \right]} \right) \left(\frac{1 + \gamma \zeta}{\zeta} d \log Y_t - \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) d \log A_t \right) \right] \qquad (129)$$

$$= \varphi \left[- \left(1 + \bar{\mu} \left(\frac{Cov_\lambda \left[\rho_\theta, \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \right]} \right) \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) \right) d \log A_t + \bar{\mu} \frac{Cov_\lambda \left[\rho_\theta, \sigma_\theta \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \right]} \frac{1 + \gamma \zeta}{\zeta} d \log Y_t \right],$$
$$(130)$$

which concludes the proof.

Appendix C   Empirical Calibration: Additional Results

In this appendix, we provide additional results from our calibration exercise. C.1 provides additional comparative statics from the calibration of the static model as we change the average markup and the degree of price-stickiness. C.2 shows additional impulse responses for the dynamic calibration of a 25bp interest rate shock.

Our procedure for extracting pass-throughs over the firm distribution from estimates provided by Amiti et al. (2019) is described in Appendix A of Baqaee et al. (2021). We refer interested readers to that appendix.

C.1   Static model: Additional results

We vary the average markup $\bar{\mu}$ from just over one to 1.60 in Figure C.1. We do so by re-calculating markups of all firms according to the differential equation in Equation (22) according to the boundary condition implied by $\bar{\mu}$. As expected, the average markup does not affect the CES or real rigidities models, but the strength of the misallocation channel increases in $\bar{\mu}$. This reflects the dependence of the productivity response on $\bar{\mu}$.

In Figure C.2, we vary the degree of price-stickiness between zero (complete rigidity) and one (complete flexibility). We find that the flattening of the price Phillips curve due to real rigidities increases as the price becomes more rigid, and the flattening of the price Phillips curve due to the misallocation channel decreases as the price becomes more rigid. These comparative statics match the intuitions provided in the main text (see the discussion of Proposition 4).

Figure C.1: Decomposition of Phillips curve slope, varying the average markup $\bar{\mu}$.
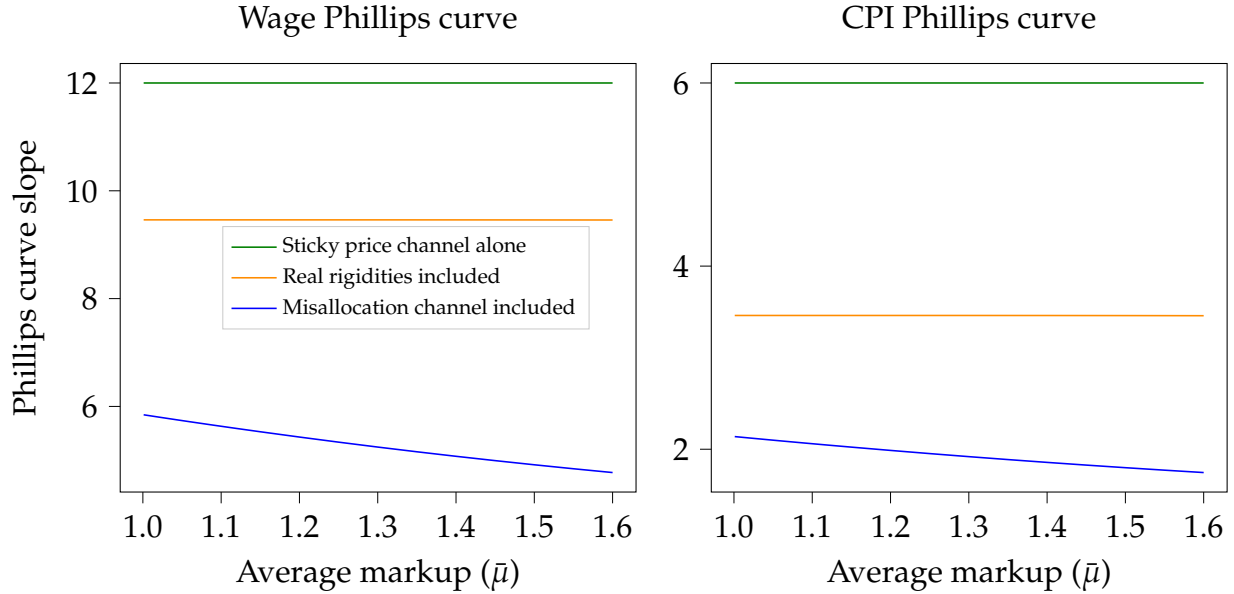
Figure C.2: Decomposition of Phillips curve slope, varying the degree of price-stickiness δ.
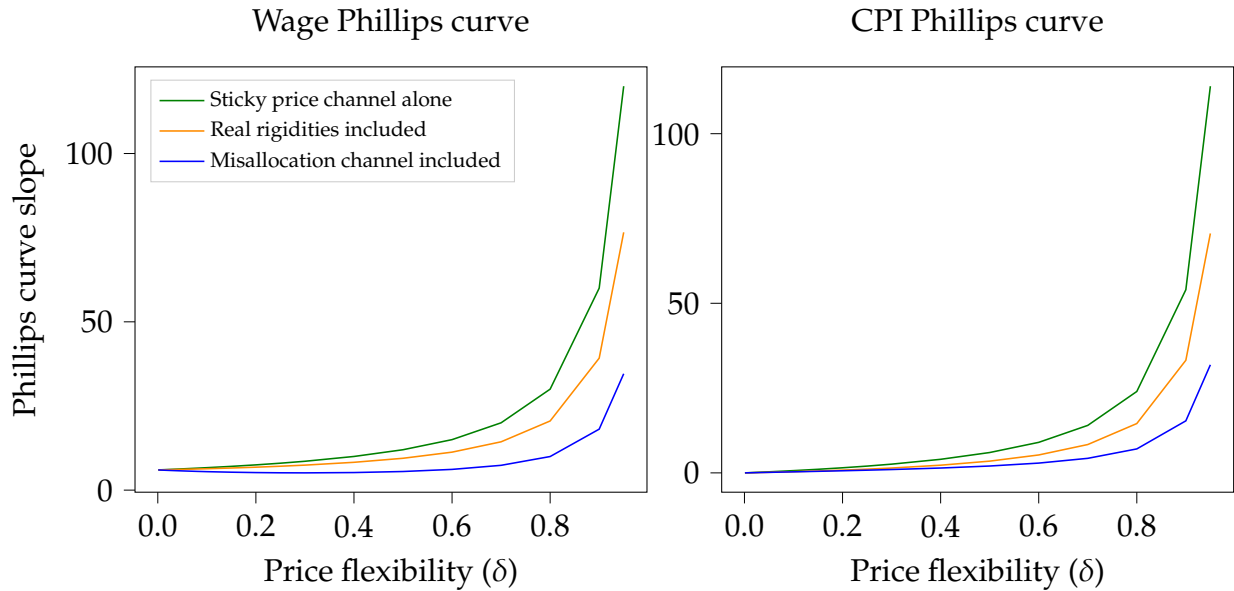
C.2 Dynamic model: Additional results

Figure C.3 shows the impulse response of the nominal interest rate and inflation following the 25bp contractionary monetary policy shock calibrated in the main text. The nominal interest rate differs across models since the monetary authority responds to the contemporaneous output and inflation gap. Compared to the CES and homogeneous firm models,

the full model predicts less deflation following the shock.

Figure C.3: Impulse response functions (IRFs) following a 25bp monetary shock. Green, orange, and blue IRFs indicate the CES, homogeneous firms, and heterogeneous firms models respectively.
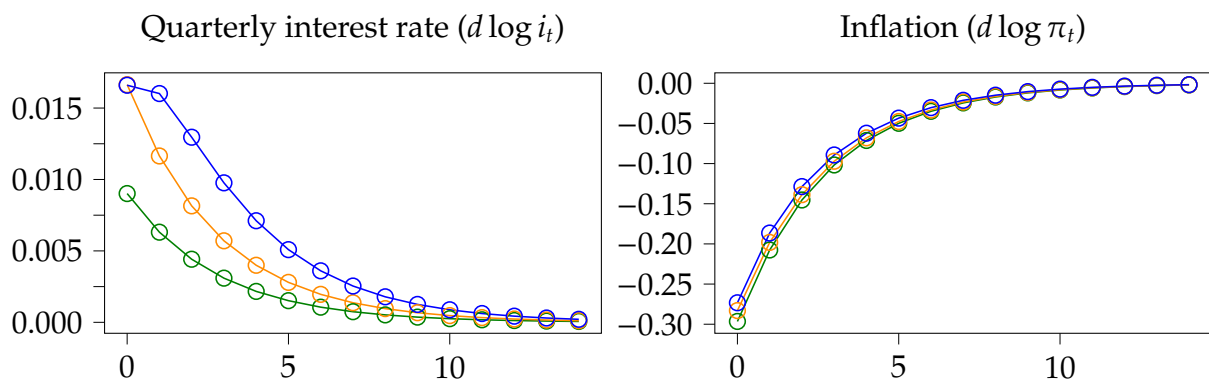
Figure C.4 shows the change in sales shares of different firm types following the 25bp contractionary monetary policy shock calibrated in the main text. The contractionary shock leads to an expansion in the sales of smaller firms and a contraction in the sales of larger firms.

Figure C.4: Change in sales shares following a 25bp contractionary monetary policy shock by firm type. In the legend, $d \log \lambda_j$ refers to the change in the sales share of a firm at the j'th percentile of cumulative sales.
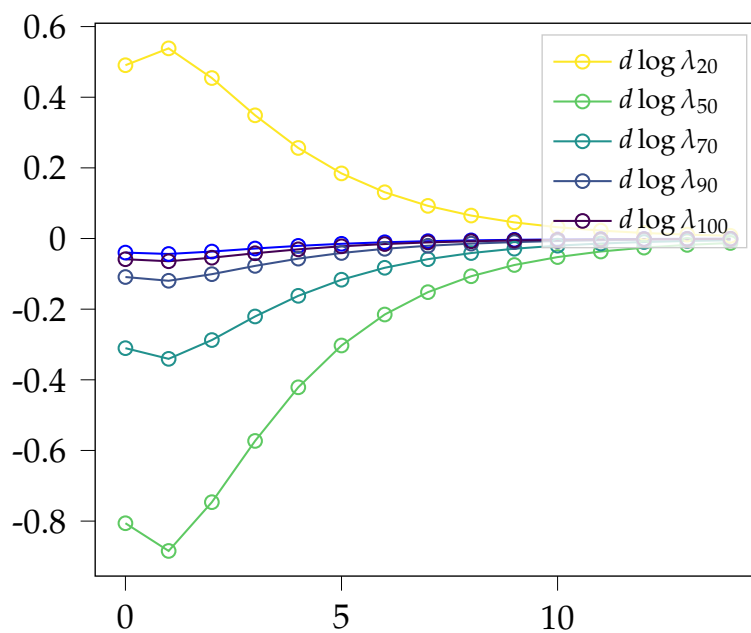
Appendix D   Details of Empirical Evidence

This appendix describes the data and procedures used in Section 7. First, section D.1 describes how we construct firm-level markup data. Then, section D.2 provides additional detail for the estimation of procyclical reallocations to high-markup firms.

D.1   Estimates of Markups

We construct firm-level estimates of markups using data from Compustat, which includes all public firms in the U.S. We exclude Farm and Agriculture (SIC codes 0100-0999), Construction (SIC codes 1500-1799), Financials (SIC codes 6000-6999), Real Estate (SIC codes 5300-5399), Utilities (SIC codes 4900-4999), and other (SIC codes 9000-9999). We also exclude firm-year observations with assets less than 1 million, negative revenues, negative book or market value, or missing year, assets, or book liabilities. Our analysis is over the period from 1965-2015. Firm-level markups are estimated using two approaches: (1) accounting profits (AP), (2) user cost (UC). We broadly use the same approaches described in Baqaee and Farhi (2020); the following text provides a brief overview.

D.1.1   Accounting Profits Approach

The accounting profits approach estimates accounting profits as operating income before depreciation minus depreciation. Operating income before depreciation comes directly from Compustat. For depreciation, we use the industry-level depreciation rate from the BEA's investment series. BEA depreciation rates are better than the Compustat depreciation measures, since the latter are influenced by accounting rules and tax incentives. Markups are estimated as:

$$\text{Accounting Profits}_i = \left(1 - \frac{1}{\mu_i}\right)\text{Sales}_i.$$

D.1.2   User Cost Approach

The user-cost approach accounts for the user cost of capital more carefully. We rely on replication files from Gutiérrez and Philippon (2017) provided by German Gutierrez. We assume that the operating surplus of each firm consists of payments of capital and rents:

$$OS_{i,t} - r_{i,t}K_{i,t} = \left(1 - \frac{1}{\mu_i}\right)\text{Sales}_i,$$

where $OS_{i,t}$ is operating income after depreciation and minus income taxes, $r_{i,t}$ is the user-cost of capital to firm i, and $K_{i,t}$ is the quantity of capital used by firm i. Following Gutiérrez and Philippon (2017), the user cost of capital is given by

$$r_{i,t} = r_t^f + RP_{j,t} - (1 - \delta_{j,t})E[\Pi_{j,t+1}],$$

where r_t^f is the risk-free rate, $RP_{j,t}$ is the industry-level capital risk premium, $\delta_{j,t}$ is the industry-level BEA depreciation rate, and $E[\Pi_{j,t+1}]$ is the expected growth in the relative price of capital. For the risk-free rate, we use the yield on the 10-year TIPS starting in 2003 and the 10-year yield on nominal Treasuries minus the average nominal-TIPS spread before 2003. Following Gutiérrez (2017), we calculate the industry-level risk premium from equity risk premia as in Claus and Thomas (2001). Due to limitations on this data, markup estimates from the user cost approach are limited to 1985-2015. We assume expected capital gains are equal to realized capital gains, measured as the growth in the relative price of capital compared to the PCE deflator. Finally, for a measure of the capital stock, we use either net property, plant, and equipment (UC1) or net property, plant, and equipment plus intangibles (UC2).

D.2 Reallocations to High-Markup Firms

Figure D.1: Robustness including accounting profits markups.

(a) $Cov_\lambda(\mu, \Delta \log \mu)$

(b) $Cov_\lambda(\mu, \Delta \log \text{Costs})$

(c) $Cov_\lambda(\mu, \Delta \log \mu)$ within NAICS-3

(d) $Cov_\lambda(\mu, \Delta \log \text{Costs})$ within NAICS-3
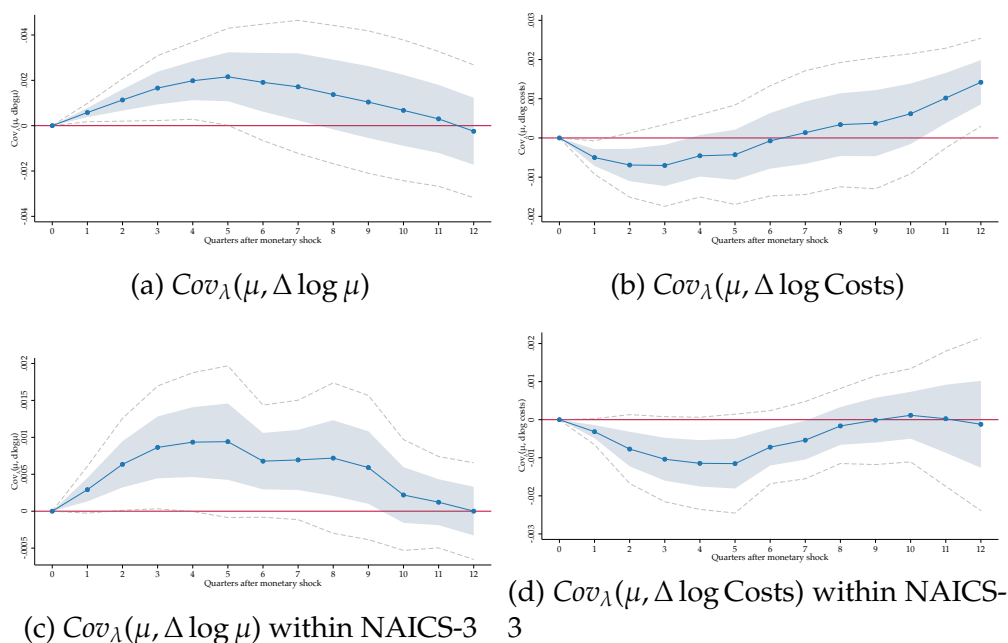
Figure D.2: Robustness including intangible capital in user cost estimates.
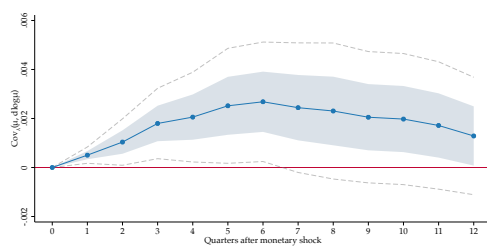
(a) $Cov_\lambda(\mu, \Delta \log \mu)$
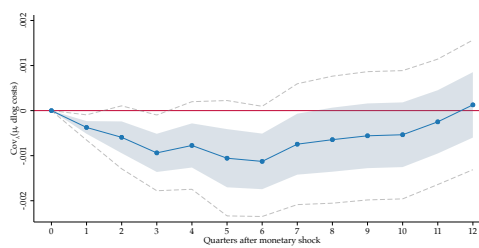
(b) $Cov_\lambda(\mu, \Delta \log \text{Costs})$
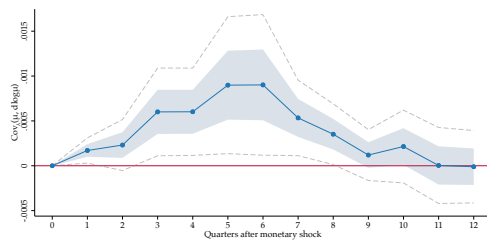
(c) $Cov_\lambda(\mu, \Delta \log \mu)$ within NAICS-3
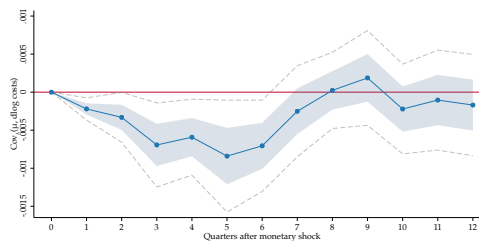
(d) $Cov_\lambda(\mu, \Delta \log \text{Costs})$ within NAICS-3

Table D.1: Differential response of industry multifactor productivity to monetary shocks in concentrated manufacturing industries.

	$\Delta \log \text{MultifactorProductivity}_{i,t}$		
	(1)	(2)	(3)
Top 8 Firms Share$_i$ × MonetaryShock$_t$	-0.0155*		
	(0.00909)		
Top 20 Firms Share$_i$ × MonetaryShock$_t$		-0.0156**	
		(0.00748)	
Top 50 Firms Share$_i$ × MonetaryShock$_t$			-0.0155**
			(0.00702)
Industry FEs	Yes	Yes	Yes
Year FEs	Yes	Yes	Yes
N	1634	1634	1634

Notes: The sales share of top 8, 20, and 50 firms is from the 2007 Economic Census for Manufacturing. Monetary shocks are from the extension of the Romer and Romer (2004) shock series by Wieland and Yang (2020). * indicates significance at 10%, ** at 5%.

Table D.2: Differential response of industry multifactor productivity to monetary shocks in concentrated manufacturing industries.

	$\Delta \log \text{MultifactorProductivity}_{i,t}$		
	(1)	(2)	(3)
Top 8 Firms Share$_i$ × MonetaryShock$_t$	-0.127**		
	(0.0383)		
Top 20 Firms Share$_i$ × MonetaryShock$_t$		-0.120**	
		(0.0343)	
Top 50 Firms Share$_i$ × MonetaryShock$_t$			-0.117**
			(0.0331)
Industry FEs	Yes	Yes	Yes
Year FEs	Yes	Yes	Yes
N	1204	1204	1204

Notes: The sales share of top 8, 20, and 50 firms is from the 2002 Economic Census for Manufacturing. Monetary shocks are from Gorodnichenko and Weber (2016). * indicates significance at 10%, ** at 5%.

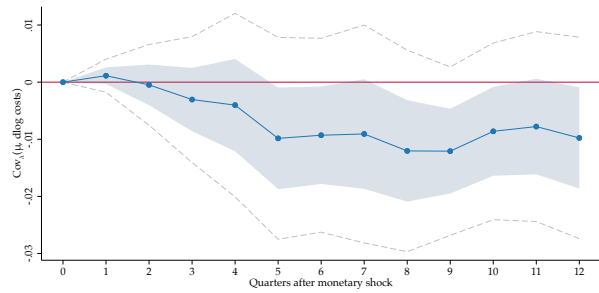Figure D.3: Robustness using other monetary shock series.
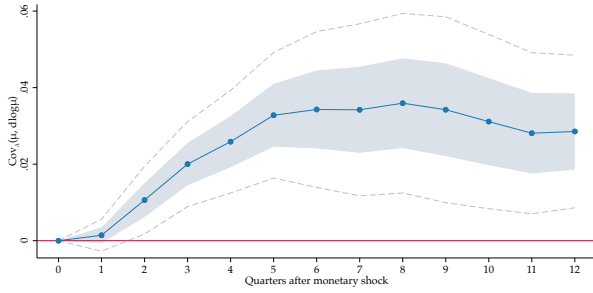
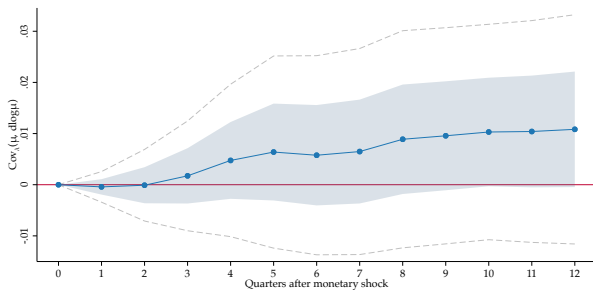(a) $Cov_\lambda(\mu, \Delta \log \mu)$: Gorodnichenko and Weber (2016)
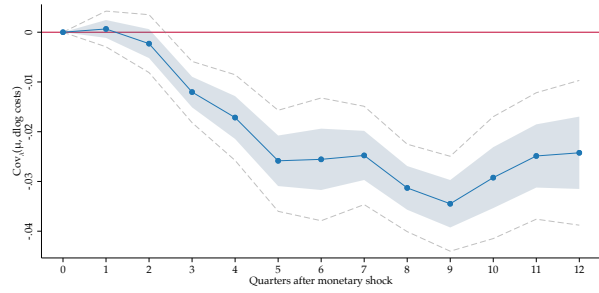
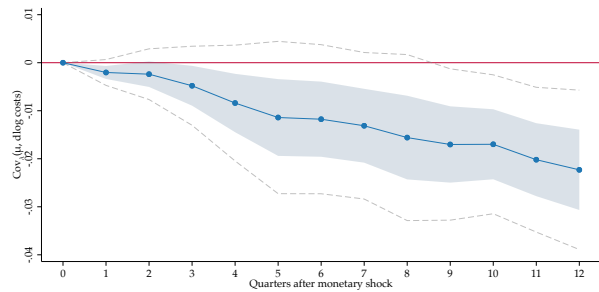(b) $Cov_\lambda(\mu, \Delta \text{Costs})$: Gorodnichenko and Weber (2016)
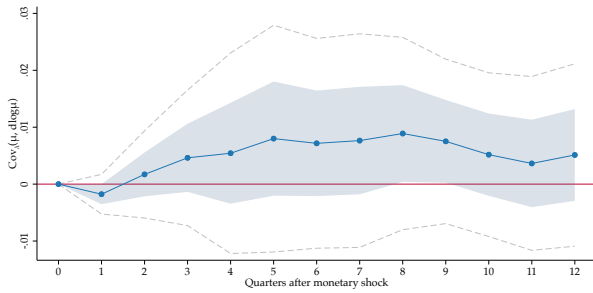
(c) $Cov_\lambda(\mu, \Delta \log \mu)$: Miranda-Agrippino and Ricco (2021)

(d) $Cov_\lambda(\mu, \Delta \text{Costs})$: Miranda-Agrippino and Ricco (2021)

(e) $Cov_\lambda(\mu, \Delta \log \mu)$: Bauer and Swanson (2022)

(f) $Cov_\lambda(\mu, \Delta \text{Costs})$: Bauer and Swanson (2022)

Appendix E   Dynamic Calibration: Money Supply Shock

Suppose the monetary shock takes the form of an exogenous shock to the money supply, rather than the interest rate rule. We calibrate the impulse response functions for the dynamic model, as in Section 6.4, for such a shock.

Money supply is linked to real variables via a cash-in-advance constraint, so that

$$d \log M = d \log P^Y + d \log Y. \tag{131}$$

As in Galí (2015), we assume that the money supply follows an exogenous AR(1) process,

$$\Delta d \log M_t = \rho_m \Delta d \log M_{t-1} + \epsilon_t^m. \tag{132}$$

where $\Delta d \log M_t = d \log M_t - d \log M_{t-1}$ and ϵ_t^m is white noise. We choose $\rho_m = 0.5$ and calibrate impulse response functions for an expansionary money supply shock where $\epsilon_t^m = 0.25$ for $t = 0$ and zero in all subsequent periods.

Figure E.1 shows the response of output to the money supply shock, and Figure E.2 shows the response of other variables. Like an interest rate shock, the money supply shock generates procyclical aggregate TFP and countercyclical dispersion in firm-level TFPR. Real rigidities and the misallocation channel both increase the responsiveness of output to the monetary shock.

Figure E.1: Impulse response function of output following an expansionary money supply shock.
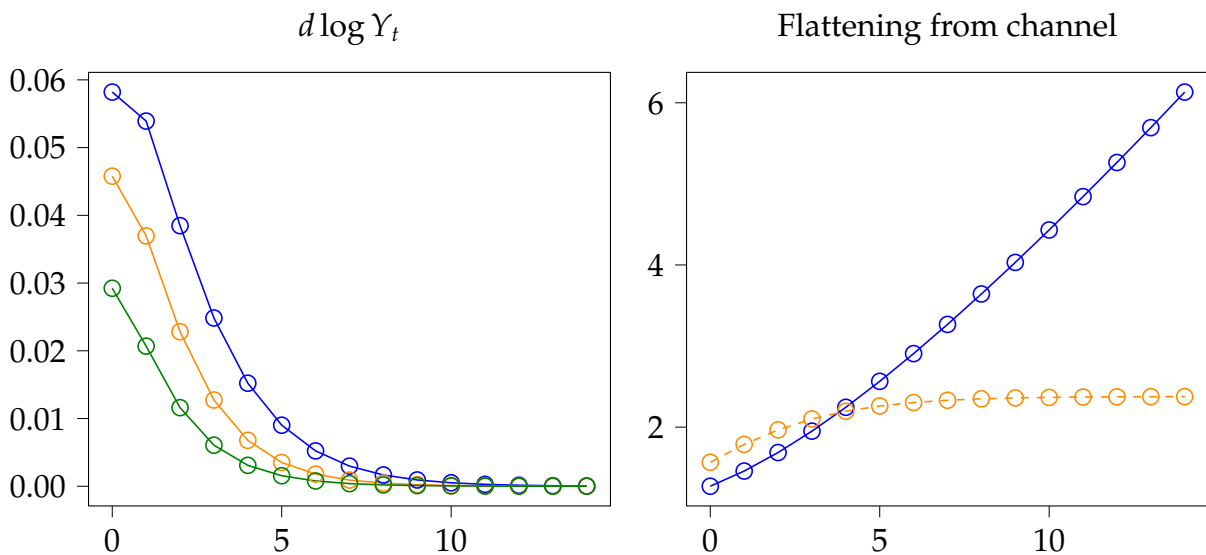
The effects on output are summarized in Table E.1. The misallocation channel increases the half-life of the shock by 35% and increases the total output impact by 60% compared to the model with real rigidities alone.

Table E.1: Effect of exogenous money supply shock on output. The cumulative output impact is calculated as in Alvarez et al. (2016).

Model	Output effect at $t = 0$	Half life	Cumulative output impact
CES	0.029	1.67	0.074
Homogeneous Firms	0.046	1.99	0.132
Heterogeneous Firms	0.058	2.69	0.212

Figure E.2: Impulse response functions (IRFs) following an expansionary money supply shock. Green, orange, and blue IRFs indicate the CES, homogeneous firms, and heterogeneous firms models respectively.
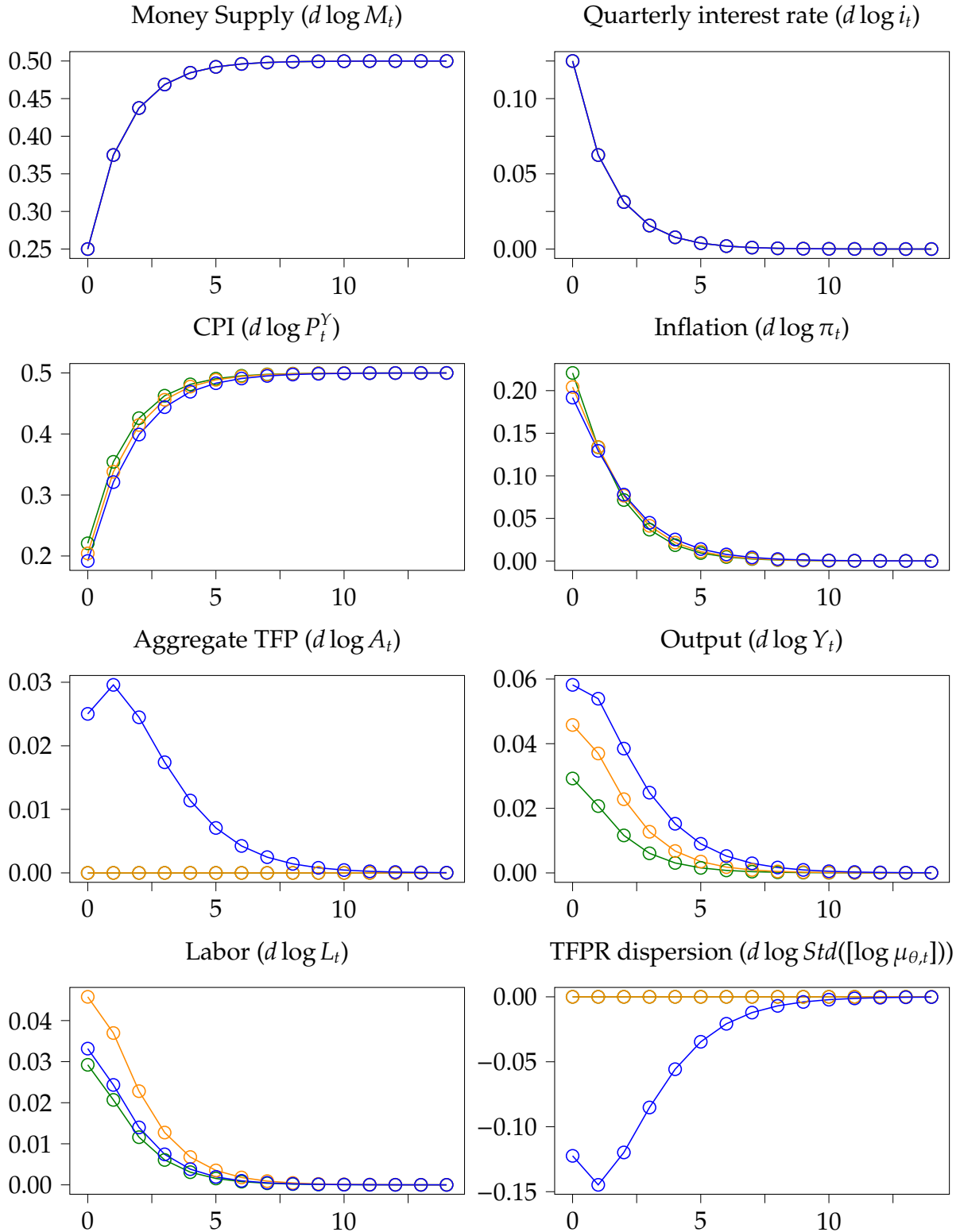
Appendix F Multiple Sectors, Multiple Factors, and Sticky Wages

In this appendix, we provide an extension of the model to multiple sectors and multiple factors, following the general network production structure provided by Baqaee and Farhi (2018). We use Ω to refer to the revenue-based input-output matrix,

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \tag{133}$$

where Ω_{ij} is share of producer i's costs spent on good j as a fraction of producer i's total revenue. Similarly, the cost-based input-output matrix,

$$\tilde{\Omega}_{ij} = \frac{p_j x_{ij}}{\sum_l p_l x_{il}}, \tag{134}$$

describes producer i's spending on good j as a fraction of producer i's total costs. The revenue-based Leontief inverse matrix and cost-based Leontief inverse matrix are defined as,

$$\Psi = (1 - \Omega)^{-1}, \tag{135}$$

$$\tilde{\Psi} = (1 - \tilde{\Omega})^{-1}. \tag{136}$$

Some additional notation: We use $\tilde{\Lambda}_f$ and Λ_f to refer to the share of factor f as a fraction of nominal GDP and as a fraction of total factor costs, respectively, and use $\lambda_{\mathcal{I}}$ to refer to the sales share of sector $\mathcal{I}$. The parameter ζ_f is the elasticity of factor f to its real price (or wage, in the case of labor), and $\gamma_f \zeta_f$ is the elasticity of factor f to income. The parameter $\theta_{\mathcal{I}}$ is the elasticity of substitution between inputs for sector $\mathcal{I}$. We use the notation of the covariance operator $Cov_{\Omega^{(j)}}$ as defined in Baqaee and Farhi (2018).

We can now derive the aggregate productivity and markup of any sector $\mathcal{I}$ just as in the one-sector model:

$$d\log A_{\mathcal{I}} = \mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}\left[\mu_\theta^{-1}\right] \frac{\mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[\delta_\theta]\left(1 - \mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[\delta_\theta]\right)Cov_{\frac{\lambda}{\lambda_{\mathcal{I}}}\delta}[\rho_\theta, \sigma_\theta] + \mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}\delta}[\rho_\theta]Cov_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[\sigma_\theta, \delta_\theta]}{\mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}\left[[\delta_\theta\rho_\theta + (1 - \delta_\theta)]\sigma_\theta\right]}$$

$$\cdot \left[\sum_{\mathcal{J}} \tilde{\Omega}_{\mathcal{I}\mathcal{J}} d\log \frac{p_{\mathcal{J}}}{P} + d\log P\right]. \tag{137}$$

$$d \log \mu_I = - \left[\frac{\mathbb{E}_{\frac{\lambda}{\lambda_I}}\left[\delta_\theta(1-\rho_\theta)\right] \mathbb{E}_{\frac{\lambda}{\lambda_I}}\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_{\frac{\lambda}{\lambda_I}}\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]} + \mathbb{E}_{\frac{\lambda}{\lambda_I}}\left[1-\delta_\theta\right] \right] \left[\sum_{\mathcal{J}} \tilde{\Omega}_{IJ} \, d \log \frac{p_{\mathcal{J}}}{P} + d \log P \right]$$
$$+ \, d \log A_I. \quad (138)$$

The remaining aggregation equations follow directly from Baqaee and Farhi (2018). The change in output is:

$$d \log Y = \frac{1}{\sum_f \tilde{\Lambda}_f \frac{1+\gamma_f\zeta_f}{1+\zeta_f}} \left[\sum_I \tilde{\lambda}_I \left(d \log A_I - d \log \mu_I\right) - \frac{1}{1+\zeta_f} \sum_f \tilde{\Lambda}_f d \log \Lambda_f \right]. \quad (139)$$

The change in the sales share of sector $\mathcal{K}$ is:

$$d \log \lambda_{\mathcal{K}} = \sum_I \left(\delta_{\mathcal{K}I} - \lambda_I \frac{\Psi_{I\mathcal{K}}}{\lambda_{\mathcal{K}}} \right) d \log \mu_I$$
$$+ \sum_{\mathcal{J}} (\theta_{\mathcal{J}}-1)\lambda_{\mathcal{J}}\mu_{\mathcal{J}}^{-1} Cov_{\tilde{\Omega}^{(\mathcal{J})}} \left(\sum_I \check{\Psi}_{(I)} \left(d \log A_I - d \log \mu_I\right), \frac{\Psi_{(\mathcal{K})}}{\lambda_{\mathcal{K}}} \right)$$
$$- \sum_{\mathcal{J}} (\theta_{\mathcal{J}}-1)\lambda_{\mathcal{J}}\mu_{\mathcal{J}}^{-1} Cov_{\tilde{\Omega}^{(\mathcal{J})}} \left(\sum_g \frac{\check{\Psi}_{(g)}}{1+\zeta_g} \left(d \log \Lambda_g + (\gamma_g\zeta_g - \zeta_g) \log Y\right), \frac{\Psi_{(\mathcal{K})}}{\lambda_{\mathcal{K}}} \right). \quad (140)$$

The change in the share of income going to factor f is:

$$d \log \Lambda_f = - \sum_I \lambda_I \frac{\Psi_{If}}{\Lambda_f} d \log \mu_I + \sum_{\mathcal{J}} (\theta_{\mathcal{J}}-1)\lambda_{\mathcal{J}}\mu_{\mathcal{J}}^{-1} Cov_{\tilde{\Omega}^{(\mathcal{J})}} \left(\sum_I \check{\Psi}_{(I)} \left(d \log A_I - d \log \mu_I\right), \frac{\Psi_{(f)}}{\Lambda_f} \right)$$
$$- \sum_{\mathcal{J}} (\theta_{\mathcal{J}}-1)\lambda_{\mathcal{J}}\mu_{\mathcal{J}}^{-1} Cov_{\tilde{\Omega}^{(\mathcal{J})}} \left(\sum_g \frac{\check{\Psi}_{(g)}}{1+\zeta_g} \left(d \log \Lambda_g + (\gamma_g\zeta_g - \zeta_g) \log Y\right), \frac{\Psi_{(f)}}{\Lambda_f} \right). \quad (141)$$

Factor and sector prices follow:

$$d \log \frac{w_f}{P} = \frac{1}{1+\zeta_f} d \log \Lambda_f + \frac{1+\gamma_f\zeta_f}{1+\zeta_f} d \log Y, \quad (142)$$

$$d \log \frac{p_I}{P} = - \sum_{\mathcal{K}} \check{\Psi}_{I\mathcal{K}} \left(d \log A_{\mathcal{K}} - d \log \mu_{\mathcal{K}}\right) + \sum_f \check{\Psi}_{If} d \log \frac{w_f}{P}. \quad (143)$$

To illustrate the results, we consider a simple example with two factors (capital and labor) and sticky wages.

F.1 Example: Two factors and sticky wages

We apply the multiple factor and multiple sector model above. Consider an economy with two factors, labor and capital. Labor is elastic, with a Frisch elasticity of 0.2, as in the model considered in the main text, while capital is inelastic. We allow for sticky wages by introducing a "labor union sector": this sector buys all labor, and then supplies labor to firms in the industry sector at a price which is subject to nominal rigidities.

The industry sector consists of firms in monopolistic competition who use capital and labor provided by the labor union to produce varieties. Just as in the main text, firms in the industry sector have heterogeneous productivities and endogenous markups and pass-throughs; we use the same parameters and objects from the firm distribution given in the main text for this calibration. Additionally, we set the share of labor to $\tilde{\Lambda}_L = 2/3$ and the share of capital to $\tilde{\Lambda}_K = 1/3$. We allow both the elasticity of substitution between labor and capital used by firms in the industry sector, denoted $\theta_{\mathcal{I}}$, and the degree of wage-stickiness, denoted δ_w, to vary across calibrations.

We show the results of this model in Figure F.1. The plot shows the change in aggregate productivity in the firm sector, $(d \log A_{\mathcal{I}})$, the change in output $(d \log Y)$, the change in the shares of income to labor and capital $(d \log \lambda_W$ and $d \log \Lambda_K)$, and the real price of labor and capital $(d \log p_{W/P}$ and $d \log r/P)$ following a shock to the price level $(d \log P)$.[51]

One immediate implication of this exercise is that the productivity response in the firm sector is independent of frictions upstream, such as sticky wages or complementarity in inputs. As a result, the importance of the misallocation channel in transmitting monetary shocks is robust to the addition of wage rigidities or deviating from Cobb-Douglas production. Furthermore, note that the cyclicality of labor's share of income is, in general, ambiguous. With sufficiently rigid wages, it is possible to make the labor share countercyclical (and the share of income accruing to profits and capital procyclical).

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[51]We focus on the labor share and the real wage of the labor union sector, since these are the labor share and real wage that would be observed.

Figure F.1: Response to shock to price level ($d \log P$) in one period model with capital, labor, and sticky wages. The degree of wage-stickiness varies along the x-axis, from complete rigidity (zero) to complete flexibility (one). Lines indicate calibrations with different elasticities of substitution between capital and labor.
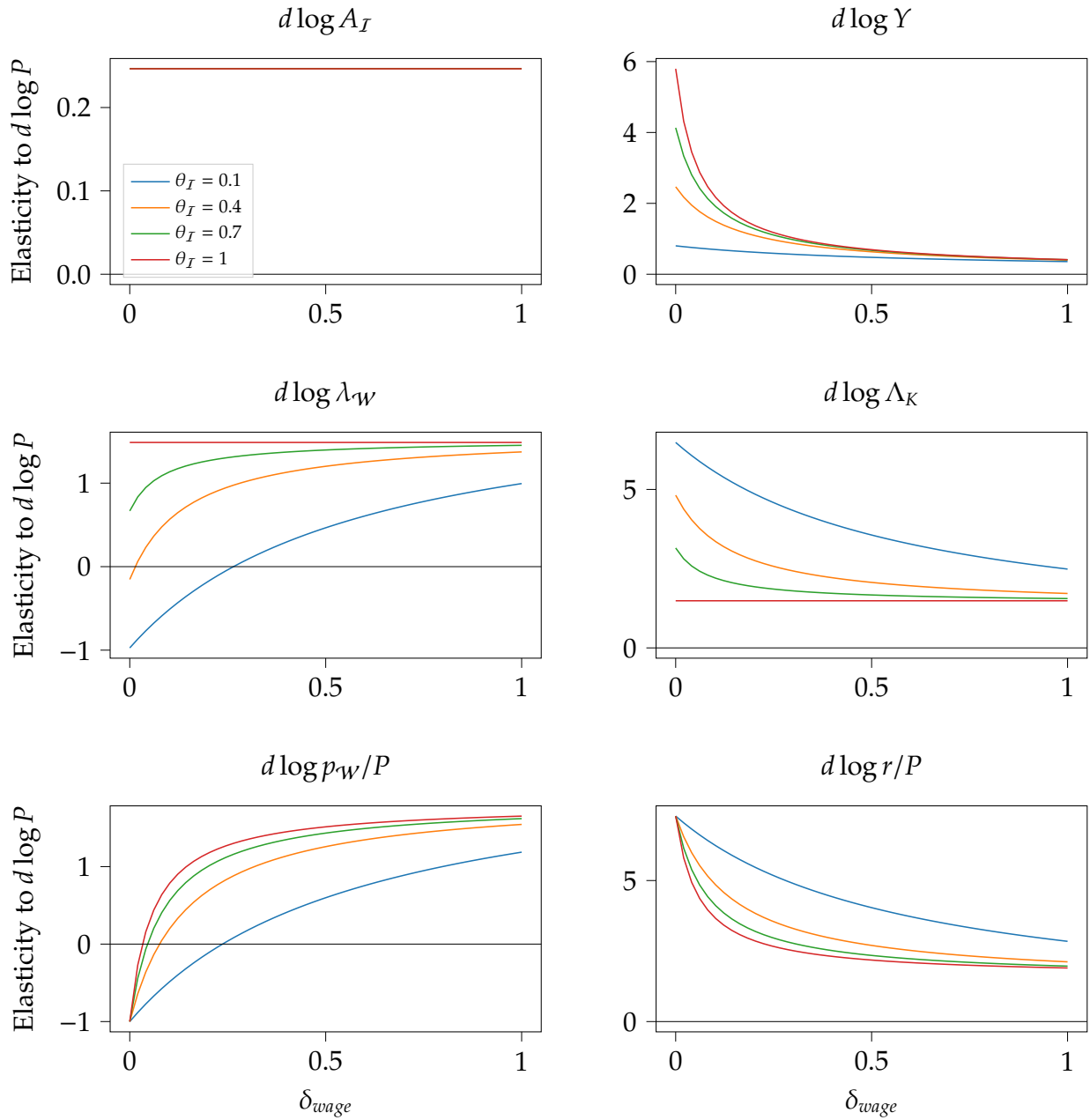
Appendix G   Klenow-Willis Calibration

Under Klenow and Willis (2016) preferences, the markup and pass-through functions are

$$\mu_\theta = \mu\left(\frac{y_\theta}{Y}\right) = \frac{1}{1 - \frac{1}{\sigma}\left(\frac{y_\theta}{Y}\right)^{\frac{\epsilon}{\sigma}}}, \tag{144}$$

$$\rho_\theta = \rho\left(\frac{y_\theta}{Y}\right) = \frac{1}{1 + \frac{\epsilon}{\sigma - \left(\frac{y_\theta}{Y}\right)^{\frac{\epsilon}{\sigma}}}} = \frac{1}{1 + \frac{\epsilon}{\sigma}\mu_\theta}. \tag{145}$$

where the parameters σ and ϵ are the elasticity and superelasticity (i.e., the rate of change in the elasticity) that firms would face in a symmetric equilibrium. This functional form imposes a maximum output of $(y_\theta/Y)^{\max} = \sigma^{\frac{\sigma}{\epsilon}}$, at which markups approach infinity.

Unfortunately, these preferences are unable to match the empirical distribution of firm pass-throughs without counterfactually large markups. To see why, note that the pass-through function $\rho(\cdot)$ is strictly decreasing, and that the maximum pass-through admissible (for a firm with $y_\theta/Y = 0$) is

$$\rho^{\max} = \frac{1}{1 + \epsilon/\sigma}.$$

Amiti et al. (2019) estimate the average pass-through for the smallest 75% of firms in ProdCom is 0.97. In order to match the nearly complete pass-through for small firms, we must choose ϵ/σ to be around $0.01 - 0.03$.

This makes it difficult, however, to match the incomplete pass-throughs estimated for the largest firms. To match a pass-through of $\rho_\theta = 0.3$ with $\epsilon/\sigma \in [0.01, 0.03]$, for example, we need a markup of $\mu_\theta \in [78, 233]$ for the largest firms. In contrast, our non-parametric procedure matches the pass-through distribution with moderate markups for the largest firms, shown in Figure G.1. Importantly, since markups and pass-throughs depend on the elasticity of $\Upsilon'(\cdot)$, incorporating additional modeling elements (such as demand shifters correlated with firm productivity) does not avoid the counterfactual properties shown here.

Rather than attempting to match the empirical pass-through distribution, suppose we used a set of parameters from the literature. We adopt the calibration from Appendix D of Amiti et al. (2019): $\sigma = 5$, $\epsilon = 1.6$, and firm productivities are drawn from a Pareto distribution with shape parameter equal to 8.[52] The simulated distributions of firm pass-throughs and sales shares are shown in Figure G.2. Over the range of drawn productivities,

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[52]We calibrate the model by drawing 1000 firms and finding a fixed point in output. Since the Pareto distribution is unbounded, we could theoretically draw firms with zero pass-throughs and infinite sales shares; the simulated distributions are bounded away from these extremes.

Figure G.1: Firm markups μ_θ estimated using nonparametric approach with $\bar{\mu} = 1.15$.
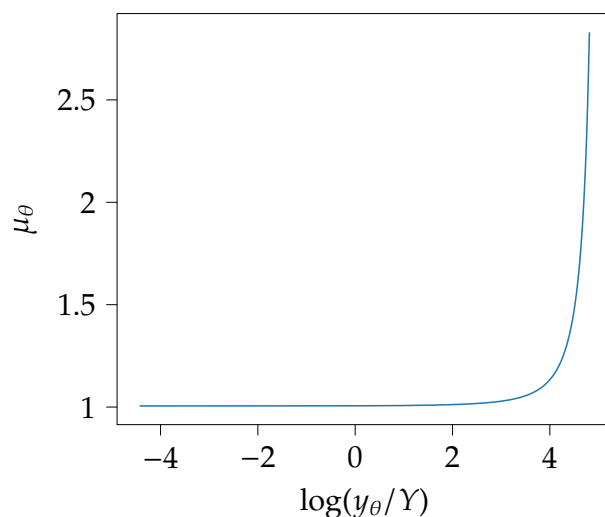
we see little variation in pass-through. Figure G.3 shows the response of output to an interest rate shock, calibrated with the same parameters as in Section 6.4. We find that the parametric specification dramatically understates the misallocation channel, compared to the nonparametric approach adopted in the main text.

Figure G.2: Pass-through ρ_θ and sales share density $\log \lambda_\theta$ for Klenow and Willis (2016) calibration.

(a) Pass-through ρ_θ

(b) Log sales share $\log \lambda_\theta$
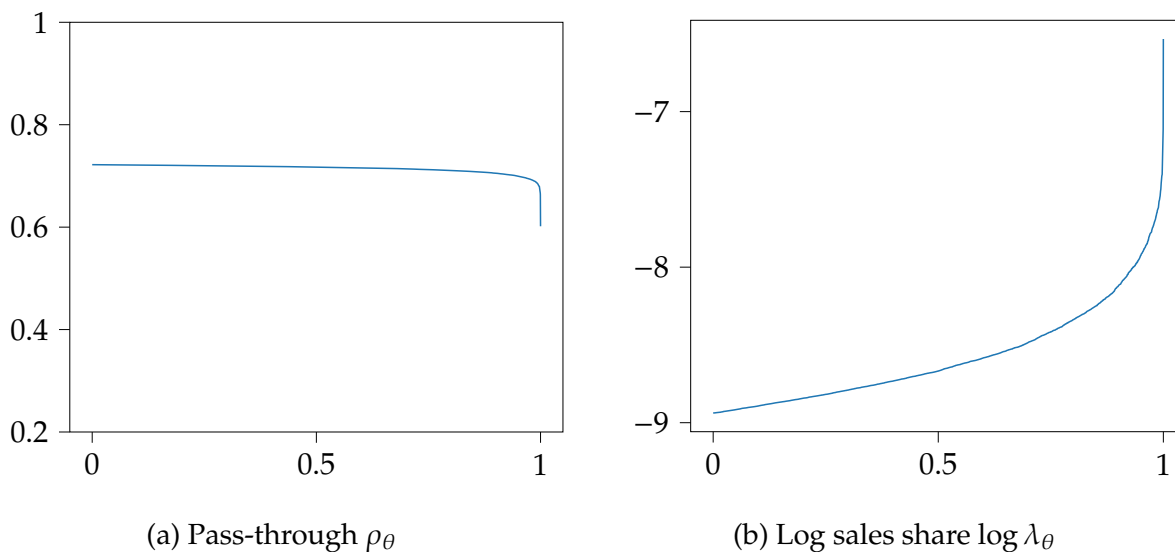
Figure G.3: Impulse response function of output following a monetary policy shock, calibrated using Klenow and Willis (2016) preferences. The real rigidities model IRF and full model IRF coincide in the left panel.
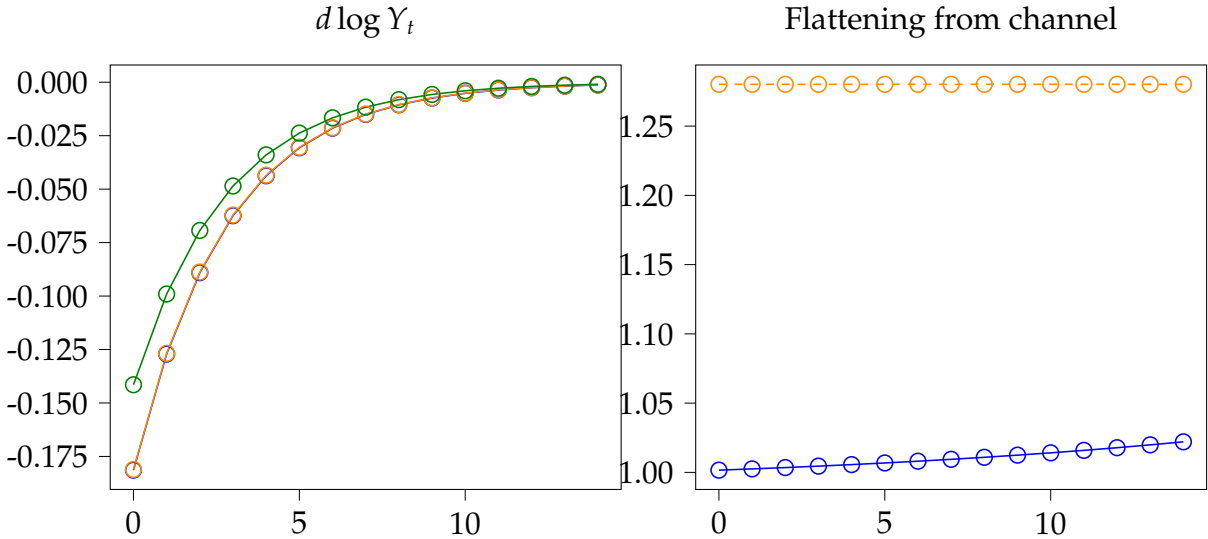
Appendix H   Menu Cost Calibration

In our baseline model, price rigidities take the form of Calvo frictions. An alternative is to use menu costs, which are incurred by firms that choose to change their prices. In this appendix, we calibrate a version of the model where firms face menu costs and idiosyncratic productivity shocks. The calibration yields results that are quantitatively similar to those in our baseline calibration while matching empirical evidence on firm price changes documented by the literature.[53]

The strategy for the calibration is as follows. We start by calibrating a model with a CES demand system, menu costs, and idiosyncratic productivity shocks in line with recent work. We consider how this economy responds to an MIT shock to the money supply. Then, we replace the CES demand system with the Kimball demand system estimated from the Belgian data and simulate the economy's response to the money supply shock keeping all other factors constant.

Following the insight by Midrigan (2011) that fat-tailed productivity shocks are required to generate sufficient nonneutrality in menu costs models, we choose a fat-tailed, symmetric productivity process. Figure H.1 compares the shock process we use to a normal distribution. The standard deviation of productivity shocks is 0.025 log points. To preserve the steady-state distribution of firm productivity levels, we assume that firms that receive productivity shocks to points outside the productivity grid exit and are replaced by new firms at the productivity grid boundary.

We choose menu costs to generate a mean frequency of price adjustment in steady-state of 11% per month, in line with Nakamura and Steinsson (2008).[54] The result is menu costs that are 2% of monthly steady-state revenue, which means that firms spend about 0.24% of annual revenue on menu costs. This cost is moderate relative to Levy et al. (1997), who measure menu costs equal to 0.7% of revenue, and Midrigan (2011), who sets menu costs to 0.34% of annual revenue. We use fine grids to discretize both prices and productivities: the price grid consists of 2,800 points with spacing of 0.001 log price points, and the productivity grid consists of 71 points with spacing of 0.02 log productivity points. Our results do not change significantly if we further discretize the grids.

The remaining parameters are set in line with our baseline calibration. The Frisch elasticity is set to $\zeta = 0.2$; the intertemporal elasticity of substitution is set to $\gamma = 1$; the elasticity of substitution in the CES model is set to $\sigma = 5$, corresponding to a static profit-

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[53]This calibration is also fully nonlinear and hence is not limited to first-order effects captured in the log-linearized model.

[54]Nakamura and Steinsson (2008) estimate the median frequency of nonsale price changes is 9–12% per month.

maximizing markup of 1.25%; and the magnitude of the money supply shock is set to 4 basis points.[55]

As in the data, our model generates large, frequent, and symmetric price changes in steady state (Bils and Klenow 2004). The steady-state median price change is 0.079 log points in the CES calibration, which is close to the median regular price change in BLS data of 0.07 log points reported by Midrigan (2011). When we instead apply the Kimball aggregator from the Belgian data, the median price change is moderately smaller at 0.033 log points.

To simulate the response of the economy to an MIT shock to money supply, we use the algorithm in Burstein (2006). We conjecture that the distribution of prices and productivities T periods after the shock is identical to the steady-state distribution, but with all prices increased by the size of the money shock. Given a set of conjectured path of wages, output, and the price aggregator, we calculate the pricing decisions of firms by backward iteration from period T. Then, we use forward iteration from the initial steady-state distribution to calculate the distribution of firms across the price-productivity grid in each period and the resulting path of all aggregates. We iterate this procedure until firms' pricing decisions from backward iteration and the path of aggregates are mutually consistent.

Figure H.2 shows the response of the CES and Kimball economies to the money supply shock. Menu costs generate significant non-neutrality: 8–20% of the initial money shock loads on real output. Compared to the CES economy, the Kimball economy has less inflation and a greater output effect on impact. The procyclical increase in aggregate productivity accounts for half of the output response. Notably, the CES model still generates a procyclical, albeit much smaller, response of aggregate productivity to the money shock. This response is driven by inherited dispersion in markups in the steady-state.

Unlike the calibration in the main text, where the Calvo friction was assumed to be identical across firm types, the frequency of price adjustment in the menu cost model is endogenous. Accordingly, the aggregate productivity effect in this menu cost model may be driven by differences across firms in both the extensive margin and intensive margin of price adjustment.

To investigate the correlation between firms' initial markups and the extensive margin of price changes, Figure H.3 plots the percent of firms changing their price t months after the shock separately for small firms (defined as the below the 70th percentile of firm

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[55]Note that most menu cost calibrations (e.g., Midrigan 2011 and Nakamura and Steinsson 2010) assume an infinite Frisch elasticity. Our menu cost calibration with an infinite Frisch elasticity generates larger non-neutrality in employment and output. The share of the output response from changes in productivity remains quantitatively significant.
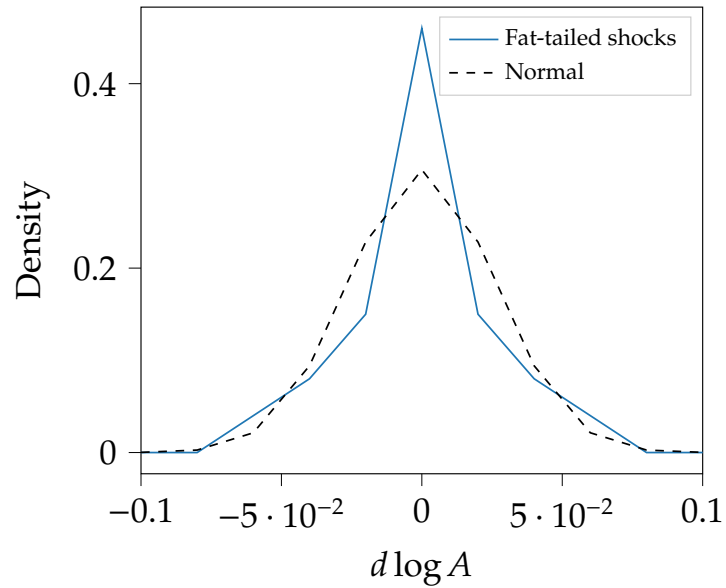
Figure H.1: Fat-tailed productivity shocks.

Figure H.2: Impulse response functions (IRFs) following a 4bp money supply shock in the menu cost model. Green and blue IRFs indicate the CES and Kimball models respectively.
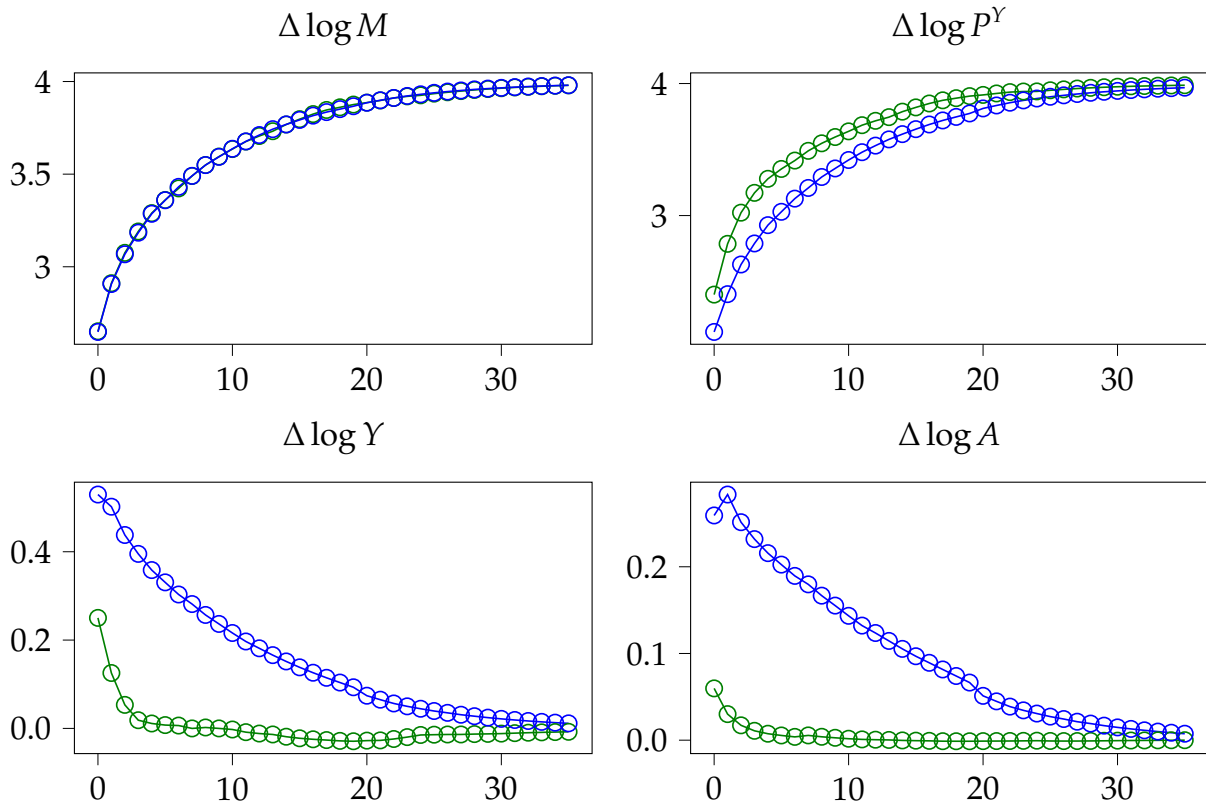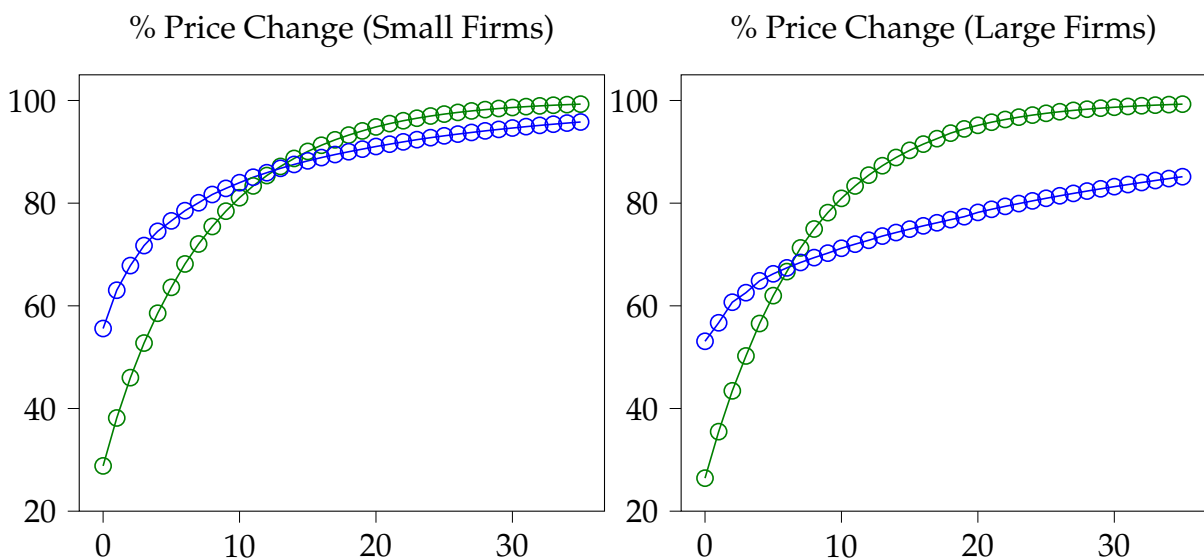
Figure H.3: Extensive margin of price changes across large (above 70th percentile pro-
ductivity) and small firms. Green and blue lines indicate the CES and Kimball models
respectively.

productivity) and large firms (above the 70th percentile of firm productivity). Unlike in
the CES calibration, in the Kimball calibration there are clear differences in the likelihood
of a price change between small and large firms. About the same fraction of large firms
as small firms change their price on impact, but a smaller fraction of large firms change
their prices in subsequent periods. Intuitively, large firms are unwilling to pay the menu
cost while others' prices are low due to strategic complementarities. As a result, the
prices of large firms are endogenously more rigid, their markups fall by more after the
expansionary money supply shock, and the reallocation of resources to these high-markup
firms leads to an increase in aggregate productivity.

Appendix I   Oligopoly Calibration

An alternative to using the monopolistic competition framework is analyzing monetary
policy through the lens of oligopoly. We describe the model set up first, and then show
our calibrated results. We find that both qualitatively and quantitatively, the misallocation
channel behaves similarly to the model with monopolistic competition.

I.1 Model Setup

We show how Propositions 1 and 2 can be rederived in an environment with oligopolistic competition. To do so, we adopt the nested CES model of Atkeson and Burstein (2008). Assume that there is a continuum of sectors indexed by $\mathcal{I}$. The representative agent has Cobb-Douglas preferences across sectors. There is a finite number of heterogenous firms in each sector. The representative agent has CES preferences with an elasticity $\sigma_{\mathcal{I}}$ over varieties within a sector. We denote by γ and ζ the income and Frisch elasticities of labor supply.

Each firm $i \in \mathcal{I}$ has a probability δ_i of being able to change its price, and a probability $1 - \delta_i$ of its price remaining fixed. The realizations are independent across firms. It will simplify the analysis to assume that when the firms that get to change their price make their pricing decision, they do not know which other firms will get to change their prices. We assume throughout that firms take the prices of inputs and other firms as given (Bertrand competition). Let λ_i be the sales share of firm i and $\lambda_{\mathcal{I}}$ be the sales share of sector $\mathcal{I}$.

Desired pass-through is given by

$$\rho_i^{flex} = 1 - s_i \frac{\sigma_{\mathcal{I}} - 1}{\sigma_{\mathcal{I}}},$$

where $s_i = \lambda_i / \lambda_{\mathcal{I}}$ is the market share of firm i. Hence, larger firms will have lower desired pass-throughs.

Lemma 2 (Effective pass-through). *The effective expected equilibrium pass-through of firm i is given by*

$$\rho_i = 1 - \frac{1}{1 + \delta_i \frac{1-\rho_i^{flex}}{1-s_i} s_i} \left[\delta_i \frac{1 - \rho_i^{flex}}{1 - s_i} \frac{\sum_{j \in \mathcal{I}} s_j \frac{1-\delta_j}{1+\delta_j \frac{1-\rho_j^{flex}}{1-s_j} s_j}}{1 - \sum_{j \in \mathcal{I}} \frac{\delta_j \frac{1-\rho_j^{flex}}{1-s_j} s_j}{1+\delta_j \frac{1-\rho_j^{flex}}{1-s_j} s_j}} + (1 - \delta_i) \right].$$

This is how much the price of firm i is expected to change in response to an aggregate shock to nominal marginal cost, taking into account the nominal rigidities and the responses of other firms in the sector. Effective expected equilibrium pass-through of firm i is increasing in desired pass-through. Note that when there are no nominal rigidities, effective equilibrium pass-through is complete. Define the sectoral markup $\mu_{\mathcal{I}}$ and the aggregate markup μ to be market-share weighted harmonic averages.

Proposition 7 (TFP in Oligopoly Model). *Following a monetary shock, the response of aggregate TFP at $t = 1$ is*

$$d \log A = - \sum_I \lambda_I \sigma_I Cov_{\frac{\lambda_i}{\lambda_I}} \left(1 - \frac{\mu_i^{-1}}{\mu_I^{-1}}, \mathbb{E}\left[d \log \mu_i\right] \right) - Cov_{\lambda_I} \left(1 - \frac{\mu_I^{-1}}{\mu^{-1}}, \mathbb{E}\left[d \log \mu_I\right] \right),$$

where

$$\mathbb{E}\left[d \log \mu_i\right] = (1 - \rho_i) d \log w$$

and

$$\mathbb{E}\left[d \log \mu_I\right] = -\mathbb{E}_{\frac{\lambda_i}{\lambda_I}}\left[1 - \rho_i\right] + \sigma_I \mu_I Cov_{\frac{\lambda_i}{\lambda_I}}\left(\mu_i^{-1}, \rho_i\right) d \log w.$$

The first set of summands in $d \log A$ are changes in allocative efficiency due to reallocations within sectors, and the second set of summands are changes in allocative efficiecy due to reallocations across sectors. If sectoral markups are the same across all sectors, the second set of summands in $d \log A$ drop out.

Proposition 8 (Output in Oligopoly Model). *Following a monetary shock, the response of aggregate output at $t = 1$ is*

$$\frac{d \log Y}{d \log w} = \rho \left[d \log A - \frac{\zeta}{1 + \zeta} \sum_I \frac{\lambda_I \mu_I^{-1}}{\mu^{-1}} \mathbb{E}\left[\frac{d \log \mu_I}{d \log w}\right] \right].$$

Using these expressions we can recover the price and wage Phillips curve, and calibrate the amount of flattening due to the misallocation channel and due to real rigidities respectively.

I.2 Calibration

To calibrate the model, we follow Amiti et al. (2019) and set the elasticity of substitution across sectors to one, and the elasticity within sectors to 10. We draw firm productivities from a Pareto distribution with shape parameter equal to 8.[56]

We order firms by market share within sector, and plot the markups and pass-throughs of firms in Figure I.1.[57] The markups and pass-throughs generated by the nested CES model satisfy Marshall's strong second law of demand: markups are increasing in firm

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[56]These parameters are chosen by Amiti et al. (2019) to match moments of the empirical distribution. We refer readers to Appendix D of their paper for more detail.

[57]If we instead plot markups and pass-throughs against firm market shares, we exactly replicate Figure A3 from Amiti et al. (2019).

Figure I.1: Markups μ_i and pass-throughs ρ_i for firms in the oligopoly calibration, ordered by market share.

(a) Markup μ_i (b) Pass-through ρ_i
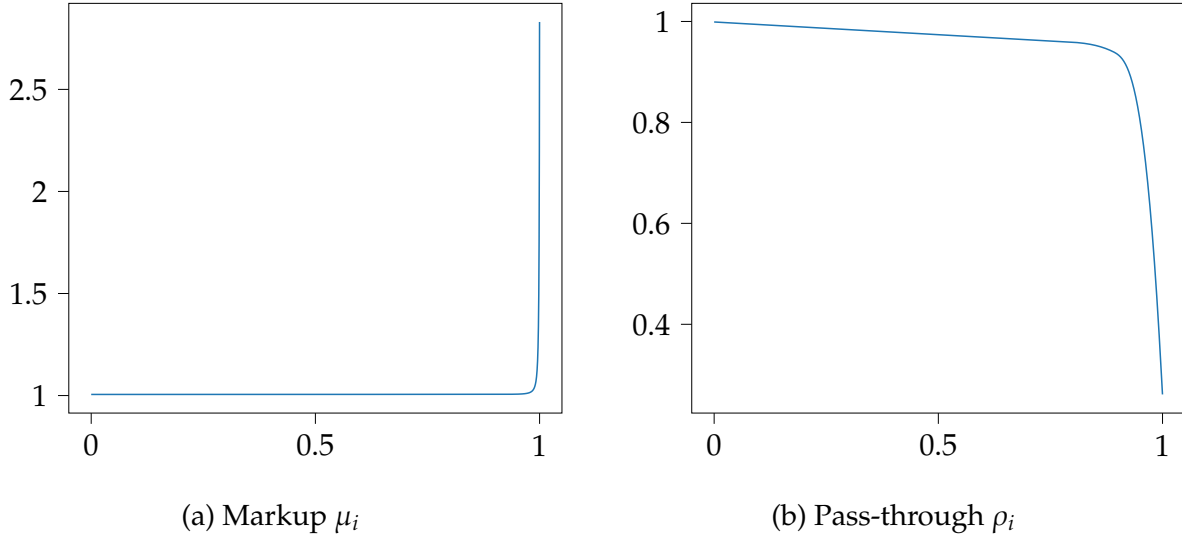
productivity, and pass-throughs are decreasing in productivity. Both the markup and pass-through function are quantitatively similar to the ones we derived for the monopolistic competition version of the model used in the main text.

We calculate the slope of the wage and price Phillips curves in a one-period setting, mirroring the timing of the one-period model presented in the main text. The flattening of the Phillips curves due to real rigidities and the misallocation channel are presented in Table I.1. In this setting, as in the setting with monopolistic competition, we find that the misallocation channel is quantitatively important: the misallocation channel flattens both the wage and price Phillips curves by 31%, compared to real rigidities, which flatten the wage Phillips curve by 17% and the price Phillips curve by 42%.

Table I.1: Estimates of Phillips curve flattening due to real rigidities and the misallocation channel in oligopoly calibration.

Flattening	Wage Phillips curve	Price Phillips curve
Real rigidities	1.17	1.42
Misallocation channel	1.31	1.31

Appendix J   Variation in markups and pass-throughs

The calibration in the main text assumes that firm markups and pass-throughs are vary only as a function of firm size. In practice, other factors unrelated to firm size may also influence markups and pass-throughs, however. Suppose that we allow the demand elasticity and desired pass-throughs of a firm i to vary due to factors unrelated to firm size,

$$\sigma_i = \underbrace{\mathbb{E}\left[\sigma_i|\lambda_i\right]}_{\sigma_\lambda} + \epsilon_i,$$

$$\rho_i = \underbrace{\mathbb{E}\left[\rho_i|\lambda_i\right]}_{\rho_\lambda} + \nu_i,$$

where ϵ_i and ν_i are orthogonal to λ_i (and hence to σ_λ and ρ_λ), but may be correlated with each other ($\mathbb{E}[\epsilon_i\nu_i] \neq 0$). We can microfound this by perturbing the Kimball aggregator by firm. We consider how this flexibility changes the sales-weighted elasticity, sales-weighted pass-through, and covariance of elasticities and pass-throughs, which are sufficient to determine the model's results.

Introducing variation unrelated to firm size does not change the sales-weighted average elasticity and pass-through, due to the law of iterated expectations,

$$\mathbb{E}_\lambda\left[\sigma_i\right] = \mathbb{E}\left[\mathbb{E}\left[\lambda_i\sigma_i|\lambda_i\right]\right]/\mathbb{E}\left[\lambda_i\right]$$

$$= \mathbb{E}\left[\lambda_i\sigma_\lambda\right]/\mathbb{E}\left[\lambda_i\right]$$

$$= \mathbb{E}_\lambda\left[\sigma_\lambda\right].$$

The covariance of elasticities and pass-throughs may change, however:

$$Cov_\lambda\left[\sigma_i, \rho_i\right] = Cov_\lambda\left(\sigma_\lambda + \epsilon_i, \rho_\lambda + \nu_i\right)$$

$$= Cov_\lambda\left(\sigma_\lambda, \rho_\lambda\right) + Cov_\lambda\left(\epsilon_i, \nu_i\right)$$

$$= Cov_\lambda\left(\sigma_\lambda, \rho_\lambda\right) + \underbrace{\sqrt{Var_\lambda\left(\epsilon_i\right)Var_\lambda\left(\nu_i\right)}\,Corr_\lambda\left(\epsilon_i, \nu_i\right)}_{\text{Bias}}.$$

Whether the bias attenuates or magnifies the supply-side effects in the model depends on the correlation between ϵ_i and ν_i, and the magnitude of the bias is bounded by the sales-weighted variance of both errors.

For example, consider the case where the consumer bundle aggregator includes de-

mand shifters B_i (i.e., $\Upsilon_i(\cdot) = B_i\Upsilon(\cdot)$):

$$\int_0^1 B_i\Upsilon(\frac{y_i}{Y})di = 1.$$

Suppose we perturb B_i for some firm i away from one, and hold $B_j = 1$ for all $j \neq i$. To a first order, the changes in the elasticity and pass-through of firm i are,

$$\frac{d\log\sigma_i}{d\log B_i} = \frac{\partial\log\sigma(\frac{y}{Y})}{\partial\log\frac{y}{Y}}\rho_i\sigma_i$$

$$\frac{d\log\rho_i}{d\log B_i} = \frac{\partial\log\rho(\frac{y}{Y})}{\partial\log\frac{y}{Y}}\rho_i\sigma_i$$

Under Marshall's strong second law, $\frac{\partial\log\sigma(\frac{y}{Y})}{\partial\log\frac{y}{Y}} < 0$ and $\frac{\partial\log\rho(\frac{y}{Y})}{\partial\log\frac{y}{Y}} < 0$, hence $Corr(\epsilon_i, v_i) > 0$, and the supply-side effects are magnified, rather than attenuated.

More generally, we can bound the bias in the supply-side effects using the result from from Proposition 1 (assuming $\delta_i = \delta$ across firms):

$$d\log A = \bar{\mu}\left(\frac{\delta(1-\delta)Cov_\lambda[\sigma_i, \rho_i]}{(1-\delta)\mathbb{E}_\lambda[\sigma_i] + \delta(Cov_\lambda[\sigma_i, \rho_i] + \mathbb{E}_\lambda[\sigma_i]\mathbb{E}_\lambda[\rho_i])}\right)d\log w.$$

The true supply-side effect, $d\log A^{true}$ (calculated using $Cov_\lambda[\sigma_i, \rho_i]$) is related to the supply-side effect calculated using variation due to sales share alone, $d\log A$ (calculated using $Cov_\lambda[\sigma_\lambda, \rho_\lambda]$), by

$$\frac{d\log A^{true}}{d\log A} = 1 + \frac{1 - d\log A}{d\log A + \frac{Cov_\lambda(\sigma_\lambda, \rho_\lambda)}{\sqrt{Var_\lambda(\epsilon_i)Var_\lambda(v_i)}Corr_\lambda(\epsilon_i, v_i)}}.$$

To illustrate, suppose 90% of variation in elasticities and pass-throughs comes from sales share, and 10% from other factors. For the calibration exercise given in the main paper, we find $\frac{d\log A^{true}}{d\log A} \in (0.69, 1.27)$; i.e., if variation not due to sales share in elasticities and pass-throughs is perfectly negatively correlated, the supply-side effect is attenuated by 31%, and if this variation is perfectly positively correlated, the supply-side effect is magnified by 27%.

Appendix K   Gini coefficient in US data

We use Business Dynamic Statistics (BDS) data from the US Census to calculate the Gini coefficient in firm employment. Figure K.1 shows the Lorenz curve in employment for the firm distribution in 2018. We calculate the ratio of the shaded area (approximated using trapezoids) to the area under the 45-degree line to measure the Gini coefficient.

Figure K.2 plots the estimated Gini coefficients from 1978-2018 for all firms, as well as within sectors provided by the BDS. The trends by sector are consistent with the trends described in Figure A.1 of Autor et al. (2020), who measure HHI across sectors: we find increasing concentration in retail, wholesale trade, utilities, and finance, and flat or decreasing concentration in manufacturing. We use the beginning and end of the time series for all firms and for the retail sector for calibrations in the main text.

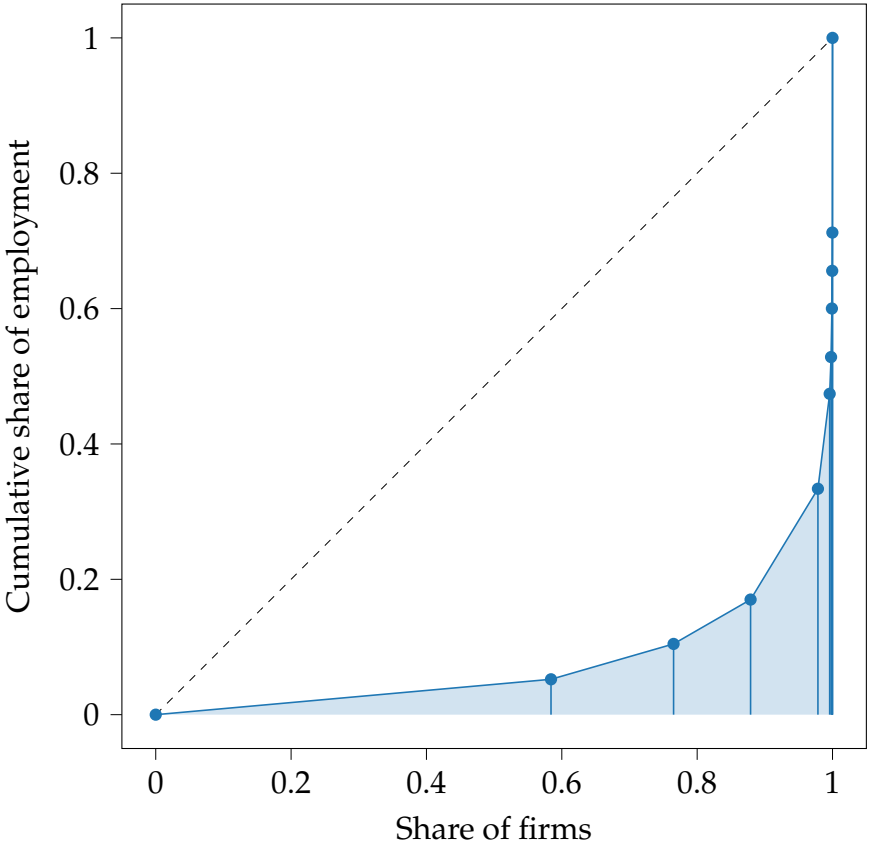Figure K.1: Lorenz curve of cumulative firm employment by share of firms in 2018.

Figure K.2: Estimated Gini coefficients in Census BDS data from 1978-2018.