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ABSTRACT

We propose a dynamic theory of banking where the role of deposits is akin to that of productive capital in the classical Q-theory of investment for non-financial firms. As a key source of leverage, deposits create value for well-capitalized banks. However, unlike productive capital of nonfinancial firms that typically has a positive marginal q, the deposit q can turn negative for undercapitalized banks. Demand deposit accounts commit banks to allow holders to withdraw or deposit funds at will, so banks cannot perfectly control leverage. Therefore, for banks with insufficient capital to buffer risk, deposit inflow destroys value through the uncertainty it brings in future leverage. This intertemporal channel complements the focus of static models on value destruction of deposit outflow and bank run. Our model predictions on bank valuation and dynamic asset-liability management are broadly consistent with the evidence. Moreover, our model lends itself to a re-evaluation of the costs and benefits of leverage regulation, offers alternative perspectives on banking in a low interest rate environment, and reveals new aspects of deposit market power that has unique implications on bank franchise value.

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1 Introduction

During the unfolding COVID-19 pandemic, the U.S. banks have undergone unprecedented balance-sheet expansions as a result of massive inflows into deposit accounts. Most dramatically, deposits of US banks increased by $865 billion just in April 2020 alone. From Q4 2019 to Q1 2020, JPMorgan Chase experienced an increase of 18% percent of its deposit base, and the deposit liabilities of Citigroup and Bank of America increased by 11% and 10%, respectively.[1] Contrary to the conventional wisdom, abundant funding liquidity did not benefit bank valuation. The banking sector is among the slowest sectors to recover from the pandemic lows of equity valuation.

Large deposit inflows are both an opportunity and a challenge for banks. As the literature has emphasized, the bank business model rests on deposit taking. Demand deposit account is a source of cheap funding that banks rely on to finance their lending and trading activities. Depositors accept relatively low rates for the convenience of using deposits as means of payment. But the consequence of allowing depositors to freely move funds in and out of their accounts is that banks cannot perfectly control the size of their deposit base and balance sheet. Therefore, deposit inflows present a challenge to banks' risk management because it is uncertain whether the new deposits will stay in the customers' accounts or be paid out in the near future.

To the extent that the banking literature is modeling the deposit-flow risk, it has done so only by assuming that loss of control of the balance sheet manifests itself by deposit outflows in a bank-run equilibrium. Such models assume that, in the no-run equilibrium, banks face no risk with respect to the size of deposit base so that deposits can be treated as fixed-maturity debts.

We depart from this narrow framing and treat banks' deposit base more generally as a sticky and random variable subject to shocks of both inflows and outflows in a fully dynamic setting. The stochastic law of motion of deposit stock captures the aggregate behavior of a large number of atomic depositors. As in Drechsler, Savov, and Schnabl (2020), deposits are effectively long-term debts, but different from their paper, the maturities of debts are random in our model. The key ingredient is that the law of motion of deposit stock can be only partially controlled through

[1] See "U.S. Banks are 'Swimming in Money' as deposits increase by 2 trillion dollars amid the coronavirus" by Hugh Son, CNBC June 21, 2020. Such deposit influx also happened in the financial crisis of 2007–2008.

deposit rate. Such uncertainty in the size and composition of balance sheet distinguishes a bank from a non-financial firm. We then add two important frictions to this model.

First, we assume in line with the evidence that it is costly for a bank to issue new equity. As in Brunnermeier and Sannikov (2014), the issuance costs cause the bank to be endogenously risk averse. The degree of risk aversion varies with the ratio of equity capital to deposit stock (the key state variable). When the ratio is high, the bank is well-capitalized and deposit inflow creates value by allowing the bank to cheaply finance risky lending. The bank's risk-taking behavior is summarized by a formula akin to that of Merton (1969). When the ratio is low, the bank has relatively less equity capital to buffer the uncertainty in its deposit stock and the risk from lending. As a result, deposit inflow becomes burdensome as it further reduces the equity capital-to-deposit ratio, making risk management more challenging and costly equity issuance more likely.

The bank adjusts deposit flows through deposit rate. When it is well-capitalized, it increases deposit rate to attract deposits. In our model and as documented by Drechsler, Savov, and Schnabl (2017), deposit base is sticky and flows are persistent. Setting a higher rate to attract deposits is just like investing in a customer base that brings cheap financing for future risk-taking. In contrast, an undercapitalized bank decreases deposit rate to stem inflows and avoid involuntary expansion of future leverage, as the risk-management incentive dominates. We derive the optimal deposit-rate policy in a similar fashion as the optimization of firm's investment policy in Hayashi (1982).

The second friction we add is a lower bound for the remuneration of deposits. A natural bound is zero, which can be motivated by the threat of withdrawal en masse and liquidation. While this friction is not required to generate endogenous risk aversion, it further limits the bank's ability to adjust the size of its deposit liabilities and balance sheet, strengthening the mechanism. Empirically, this deposit rate lower bound has become increasingly binding in the current low-rate environment. Indeed, banks are loath to impose negative rates on deposits in practice even if this could help stem an inflow of new deposits and the associated involuntary expansion of leverage.[2]

Finally, another distinguishing feature of banks is that to be able to continue operating, a bank must sustain a level of equity capital that is above a regulatory minimum. Due to the deposit-flow

[2]Heider, Saidi, and Schepens (2019) document the bunching of deposits rates at zero in the Euro area.

risk, a bank does not have full control over the balance-sheet size and composition. Unexpected deposit inflows increase leverage, so when the bank is undercapitalized, it has to raise costly equity to avoid violating the regulatory restriction. Regulation does not cause endogenous risk aversion (equity issuance costs do) but regulation amplifies it. Therefore, leverage regulation can make deposit-taking more costly and make banks more reluctant to lend. During the Covid-19 pandemic, the U.S. banking regulators relaxed the supplementary leverage ratio requirements. Our model provides a rationale for such response. We do not aim to comprehensively evaluate leverage regulation. In fact, by modelling diffusive risk (rather than jump risk), we allow the bank to avoid insolvency via continuous balance-sheet adjustments. Moreover, we do not model the externalities associated with bank risk-taking and insolvency that can justify tightening leverage regulation.

We also do not add any other frictions that may be relevant in practice. One important friction that is absent from our model is fire-sale pricing (or other forms of asset adjustment costs), which happens when a bank seeks to quickly shrink its asset base. In our model, the bank can costlessly change the size of its asset portfolio, so our focus is entirely on the lack of control of deposit liabilities rather than assets. This is a reasonable assumption for financial securities, which can be quickly adjusted at low costs. Admittedly, this is a stronger assumption as far as the bank's loan book is concerned. Adding an adjustment cost to the loan book, however, is a major complication as this would introduce a second state variable, rendering the mechanism much less transparent.

Next, we provide a more detailed summary of the bank's dynamic asset-liability management decisions including optimal deposit remuneration, short-term borrowing, lending, equity issuance, and dividend payout. We solve for the franchise value of the bank and show how it varies with the equity capital-to-deposit ratio, generating endogenous risk aversion. We also solve for the marginal value of deposits ("deposit q") and show that it can turn negative when the bank is undercapitalized. This key result explains why deposit influx during the Covid-19 pandemic can hurt bank valuation when bank capital was weakened by enormous loan loss provisions. After presenting the model solution, we summarize its implications on capital regulation, banking in a low interest rate environment, and the impact of deposit market power on bank valuation.

Because it is costly to issue equity, the marginal value of equity capital can be greater than one, and depends on the value of equity capital-to-deposit ratio, denoted by k, which is bounded by two endogenous reflecting boundaries. The marginal value of equity capital varies over a wide range: it is equal to one at the dividend payout boundary, when k is high and the bank is indifferent between retaining an extra unit of capital or paying it out (that is how the endogenous upper bound is defined), and when k is low, it can rise to above nine at the equity issuance boundary (the lower bound of k), even under conservative values for the issuance costs from the empirical literature. When the bank is close to hitting its regulatory leverage restriction, any additional unit of equity capital is very valuable as it reduces the likelihood of costly equity issuance.

The marginal value of equity capital does not decline with k linearly. The bank's franchise value is strictly concave in k, so that the bank is endogenously risk-averse even though shareholders are assumed to be risk neutral. We show that it is optimal for the bank to substantially reduce lending as k declines and the bank approaches the equity issuance boundary. This is consistent with empirical findings linking changes in bank equity capital to bank lending. When k increases and does approach the payout boundary, it is more likely to stay around than not. Indeed, at the peak of the stationary density of k the marginal value of equity is only slightly above one, so that, for the majority of time, the bank does not seem to be financially constrained. This nonlinearity captures a sharp contrast between the normal times and the crisis times when k is low, close to the equity issuance boundary, and the marginal value of equity capital shoots up dramatically.

In our model, deposits are valuable because depositors are willing to accept a deposit rate that is below the risk-free rate. This can be motivated by the convenience of payment services offered by deposit accounts, which at the same time subject the bank to deposit-flow uncertainty. When the bank has sufficient equity capital, deposits create value by allowing the bank to finance risky lending with cheap sources of funds. As a pool of cheap funding, the deposit stock in effect serves as a form of productive capital for the bank. However, when the bank's equity capital is depleted, the marginal value of deposits can be negative. The reason is that, near the (lower) boundary of costly equity issuance, uncontrollable deposit inflows destroy value for the bank's shareholders by increasing the bank's leverage and amplifying the likelihood of costly equity issuance. The bank then wants to deleverage and turn away deposits. However, the bank can only go as far as setting the deposit rate at the lower bound; it cannot turn down deposits by further lowering the rate.

Losing control of its deposit base is a problem when the bank faces equity issuance costs, for then, in effect, it also loses control of its leverage and runs the risk of paying the issuance costs. Indeed, we show that without the equity issuance costs the bank can costlessly offset any increase in deposits with a commensurate amount of newly raised equity capital. The bank's franchise value is then linear in k, so that the bank is no longer endogenously risk averse to loan risk and deposit risk management no longer matters. The emphasis on equity issuance costs sets out model apart from models that follows the tradition of Leland (1994) and assumes no cost of equity issuance.

In our model, a sharp distinction is drawn between deposits and short-term debt. With short-term debt, the bank can always choose to stop borrowing at maturity, and therefore, does not face the problem of unwanted debt. Deposits are long-term contracts and do not have a well-defined maturity. Deposits leave the bank only when depositors choose to withdraw funds. When the equity capital-to-deposit ratio, k, is high, the bank issues short-term debt to obtain leverage that is even higher than the leverage obtained through deposit-taking. If k declines, the bank deleverages by reducing short-term debt. If k declines further and approaches the lower boundary of costly equity issuance, the bank can even switch its position in short-term debt from borrowing to lending (i.e., holding safe assets) and thereby de-risk the asset side of its balance sheet. In contrast, the deposit stock is subject to shocks and cannot be perfectly controlled. Moreover, once the deposit rate hits the lower bound, the bank completely loses control of deposit stock. Managing the deposit risk is a key task that distinguishes banks from other financial intermediaries.

Capital regulation that imposes a cap on leverage amplifies the cost of deposit-taking because unexpected deposit inflow drives up leverage, pushing the bank closer to the violation of regulation and costly recapitalization. During the Covid-19 pandemic, the U.S. banking regulators relaxed leverage regulation. Our model shows that such responses stimulate lending and deposit-taking. Such measures can be particularly effective in a low interest rate environment where the deposit rate lower bound is likely to bind and the bank is very concerned about losing control of leverage. In contrast, tightening leverage regulation reduces deposit q and discourages deposit-taking.

Our model predicts a permanent negative impact of heightened leverage regulation on bank value. Kashyap, Stein, and Hanson (2010) argue that the impact should be temporary instead, because in a deterministic environment, the bank pays the equity issuance costs once and then settles on a lower leverage. Under shocks to deposit stock and loan returns, equity issuance is reoccurring in our model, and through shareholders' rational expectation, the issuance costs are reflected in bank value even when the bank is away from the equity issuance boundary. Therefore, tightening leverage regulation permanently reduces bank value by forcing the bank to raise equity more frequently. An unintended consequence is that to offset the increase in issuance costs, the bank has to generate more earnings for shareholders to break even in expectation, and to achieve that, the bank increases risk exposure per unit of equity capital. Leverage regulation, while successfully builds up bank capital, fails its original purpose of taming risk-taking.

Our model also sheds light on the critical role of interest rate level in bank valuation and balance-sheet management. As in Drechsler, Savov, and Schnabl (2017), the bank earns a deposit spread (the wedge between the risk-free rate and the lower deposit rate). The bank raises deposit rate when k is higher so that when k declines in the future (for example, following unexpected deposit inflows), the bank will have more room to reduce deposit rate before hitting the deposit rate lower bound. Therefore, when the risk-free rate is high, the bank has more flexibility in raising deposit rate in the high-k region without squeezing the deposit spread too much. The distance between the risk-free rate and deposit rate lower bound essentially determines the degree of flexibility to regulate deposit flows through deposit rate. In a low interest rate environment, the bank has less flexibility, so deposit q declines. Moreover, bank franchise value suffers in a low interest rate environment, which causes the bank to become more aggressive in shareholder payout. This helps explain the massive bank stock buybacks in the last decade of low interest rates.

Finally, our model introduces new aspects of deposit market power that have unique implications on bank franchise value. Strong deposit market power is typically associated with a low deposit demand elasticity (i.e., a more sticky deposit base) and a higher bank franchise value, because the bank does not need to pay high interest rate to attract depositors (Keeley, 1990; Drechsler, Savov, and Schnabl, 2017). Our model features a competing channel. A rate-insensitive deposit base weakens the bank's control of deposit flows through the adjustment of deposit rate. Therefore, a bank with stronger deposit market power faces a greater challenge of managing its deposit liabilities. This result speaks directly to the conundrum that large U.S. banks face during the Covid-19 pandemic. Another key aspect of deposit market power is the internalization of payments. A bank with a larger deposit market share is more likely to see its depositors sending payments to each other rather than sending payments to (or receiving payments from) other banks' depositors. Therefore, stronger deposit market power implies less uncertainty in the payment-driven deposit flows. The direct reduction of deposit risk enhances bank franchise value.

Literature. In the banking literature, the focus is on bank runs when it comes to banks' commitment to allow depositors to withdraw funds without prior notice (Diamond and Dybvig, 1983; Allen and Gale, 2004b; Goldstein and Pauzner, 2005). However, the deposit flow risk is more ubiquitous than the dramatic bank runs and influences banks' daily operation. Moreover, our paper is the first to show that the uncertainty in both deposit outflows and inflows poses a challenge to banks under equity issuance costs. Equity issuance costs have been incorporated in the models of bank capital structure (Bolton and Freixas, 2000; Allen, Carletti, and Marquez, 2015).

Depositors are willing to accept relatively low rates because of the convenience of using deposits as means of payment. What enables deposits as money is precisely banks' commitment to allow depositors to move funds in and out of their accounts. Banks are thus exposed to large payment flow shocks (Furfine, 2000; Bech and Garratt, 2003; Denbee, Julliard, Li, and Yuan, 2018). Deposit maturities are not chosen by the bank and depend on depositors' payment needs that are uncertain (Freixas, Parigi, and Rochet, 2000; Donaldson, Piacentino, and Thakor, 2018; Parlour, Rajan, and Walden, 2020). With a diversified depositor base, a bank essentially views deposits as debts that retire at a stochastic rate. Bianchi and Bigio (2014), De Nicolò, Gamba, and Lucchetta (2014), and Vandeweyer (2019) also recognize such payment shocks. However, in their models, deposits are one-period contracts (with intra-period shocks), so banks can freely adjust the deposit base every period. In contrast, deposit contract in reality (and in our model) is long-term – depositors can hold deposits as long as they want (Drechsler, Savov, and Schnabl, 2020). To adjust deposit base, banks can change deposit remuneration but completely loses control of deposit base when its deposit rate hits the lower bound. The key to our main results is precisely banks' lack of control of the deposit base, which exposes banks to the risk of paying equity issuance costs.

Deposits are inside money of the private sector – stores of value and means of payment issued by banks. This feature has been well recognized in the recent macro-finance literature. Deposits are modelled as short-term debts (Piazzesi and Schneider, 2016; Drechsler, Savov, and Schnabl, 2018) with interest rates below the risk-free rate by a money premium (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Greenwood, Hanson, and Stein, 2015; Li, 2019; Begenau, 2019). Brunnermeier and Sannikov (2016) and Drechsler, Savov, and Schnabl (2020) are notable exceptions. Brunnermeier and Sannikov (2016) model deposits as infinite-maturity nominal liabilities and study the general equilibrium implications. Our model is more related to Drechsler, Savov, and Schnabl (2020) who find that sticky deposit liabilities have long duration. Our contribution is to incorporate the random evolution of deposit liabilities and analyze the impact of deposit risk on bank valuation and asset-liability management. Dynamic banking models typically differentiate short-term debts and deposits in both interest rate and operation costs (Hugonnier and Morellec, 2017; Van den Heuvel, 2018; Begenau, 2019). In these models, banks do not face uncertainty in the size of deposit liabilities.

2 Model

We model a single bank's decisions under the risk-neutral measure, effectively assuming no arbitrage and taking as exogenous the pricing kernel (stochastic discount factor) that depends on the aggregate dynamics of the broader economy. Let r denote the risk-free rate, which is also the expected return of all financial assets under the risk-neutral measure.

Risky Assets. We use A_t to denote the value of the bank's holdings of loans and other investments at time t.[3] It has the following law of motion:

$$dA_t = A_t \left(r + \alpha_A\right) dt + A_t \sigma_A d\mathcal{W}_t^A, \tag{1}$$

[3] The bank's assets include not only loans but also other assets that generate revenues of trading and services such as cash management, trade credit, derivatives, structured products, and underwriting of securities (Bolton, 2017).

The parameter α_A reflects the return from the bank's expertise. Because we set up our model under the risk-neutral measure, α_A is the risk-adjusted value-added.[4] The second term in (1) describes the Brownian shock, where σ_A is the diffusion-volatility parameter and $\mathcal{W}^A$ is a standard Brownian motion. Examples of these shocks include unexpected charge-offs of delinquent loans. At any time t, the bank may adjust its risky assets and the liability structure (i.e., deposits, bonds, and equity).

Deposits. Deposits are at the core of our model. Let X_t denote the value of deposits at time t on the liability side of the bank's balance sheet. It has the following law of motion:

$$dX_t = -X_t \left(\delta_X dt - \sigma_X d\mathcal{W}_t^X\right) + X_t n\left(i_t\right) dt. \tag{2}$$

where $\mathcal{W}_t^X$ is a standard Brownian motion. Given a diversified depositor base, a $\left(\delta_X dt - \sigma_X d\mathcal{W}_t^X\right)$ fraction are withdrawn in dt because depositors may need cash or pay agents who hold accounts at other banks. If $\left(\delta_X dt - \sigma_X d\mathcal{W}_t^X\right) > 0$, the bank's own depositors receive payments into their accounts. Deposits thus have an average *effective duration* of $1/\delta_X$, and σ_X captures payment flow uncertainty.[5] The stochastic withdrawal is in line with the three-dates models (Diamond and Dybvig, 1983; Allen and Gale, 2004b), where agents' stochastic preferences over early and late consumption translate into uncertainty in the deposit outflow. The deposit flow shock, $d\mathcal{W}_t^X$, is likely to be positively correlated with the loan repayment shock, $d\mathcal{W}_t^A$, as a healthy asset portfolio can attract depositors. Let ϕdt denote the instantaneous covariance between $d\mathcal{W}_t^X$ and $d\mathcal{W}_t^A$.

In the presence of diffusive shocks (instead of jump shocks), the bank can avoid default by adjusting the balance sheet locally and thus preserve a positive continuation value for equityholders. Therefore, deposits are risk-free for depositors. The deposit rate is i_t, chosen by the bank. The spread, $r - i_t$, can be positive if agents value the convenience of deposits as means of payment (DeAngelo and Stulz, 2015; Nagel, 2016; Piazzesi and Schneider, 2016; Li, 2018, 2019). For

[4] The bank may have expertise in monitoring (Diamond, 1984), loan screening (Ramakrishnan and Thakor, 1984), relationship lending (Boot and Thakor, 2000), restructuring (Bolton and Freixas, 2000), and serving local markets (Gertler and Kiyotaki, 2010). More generally, in the macro-finance literature, banks are often modelled as agents with expertise in asset management (He and Krishnamurthy, 2012, 2013; Brunnermeier and Sannikov, 2014, 2016).

[5] The value of δ_X largely depends on where the bank sits in the payment network, and the payment flow uncertainty σ_X can be significant in data (Denbee, Julliard, Li, and Yuan, 2018).

deposits to function as means of payment, depositors must be able to move funds in and out of their accounts freely (Freixas, Parigi, and Rochet, 2000; Parlour, Rajan, and Walden, 2020). By allowing this, the bank exposes itself to the deposit-base risk in (2).

The bank can adjust the growth rate of deposits via i_t in $n(i_t)\,dt$, where the deposit demand elasticity depends on the bank's deposit market power (Drechsler, Savov, and Schnabl, 2017). Reducing the deposit rate causes deposit outflow, i.e., $n'(i_t) < 0$. Moreover, following Hugonnier and Morellec (2017) and Drechsler, Savov, and Schnabl (2020), we assume that to maintain the existing deposits and attract new deposits, the bank pays a flow cost $C(n(i_t), X_t)\,dt$, which captures the expenses of maintaining branches, marketing products, and servicing customers.

Deposits are essentially long-term debts with stochastic maturity and controllable increments. Not all depositors withdraw at the same time, and withdrawal depends on depositors' payment needs. Therefore, a diversified depositor base implies an effective duration of deposits that depends on the average rate of withdrawal.

Our treatment of deposits stands in contrast with the macro-finance literature and dynamic banking literature that generally treats deposits simply as short-term debts (motivated by depositors' right to withdraw at any time). We share with Drechsler, Savov, and Schnabl (2020) the view that the right to withdrawal does not necessarily translate into a low duration of deposits. We emphasize that the right to withdrawal imposes a lower bound on the feasible deposit rate – the bank cannot set a negative deposit rate because depositors will withdraw en masse and earn the zero return on dollar bills that also work as means of payment. We assume that in such a bank run, the shareholders' equity is wiped out, so the bank always avoids such scenario. In reality, depositors may tolerate an effective negative deposit rate (often in the form of fees) due to the inconvenience of carrying dollar bills, but as long as there exists a lower bound, our results hold.

Bonds. The bank issues short-term bonds (e.g., financial commercial papers), and it is costless to do so. Let B_t denote the value of bonds issued at t that will mature at $t + dt$. Without default risk, the contractual rate of return for short-term debt initiated at t is the risk-free rate r. The bank's bond interest payment over time interval dt is $B_t r dt$. The bank may choose not to issue bonds but instead invest in risk-free bonds issued by other entities in the economy (e.g., the government). In this case, we have $B_t < 0$. Whether the bank issues or holds risk-free bonds will depend on its risk-taking capacity, which in turns depends on the existing deposit liabilities and equity capital.

Equity, Dividend, and Costly Issuance. Let K_t denote the bank's equity (or "capital"), so the following identity summarizes all the balance-sheet items:

$$K_t = A_t - (B_t + X_t) \ . \tag{3}$$

The bank can pay out dividends that reduce K_t. We use U_t to denote the (undiscounted) cumulative dividends, so the amount of (non-negative) incremental payout is dU_t.

The bank may find it optimal to issue external equity. In reality, banks face significant external financing costs due to asymmetric information and incentive issues.[6] A large empirical literature has sought to measure these costs, in particular, the costs arising from the negative stock price reaction in response to the announcement of a new equity issue.[7] Let F_t denote the bank's (undiscounted) cumulative net external equity financing up to time t and H_t to denote the corresponding (undiscounted) cumulative costs of external equity financing up to time t.

The bank's equityholders are protected by limited liability. Let τ denote the stochastic stopping time when the bank defaults. Therefore, the bank maximizes the equityholders' value,

$$V_0 = \mathbb{E}\left[\int_{t=0}^{\tau} e^{-\rho t}\left(dU_t - dF_t - dH_t\right)\right].^{8} \tag{4}$$

[6]Explicitly modeling informational asymmetry would result in a substantially more involved analysis. Lucas and McDonald (1990) provides a tractable analysis by making the simplifying assumption that the informational asymmetry is short lived, i.e. it lasts one period.

[7]Lee, Lochhead, Ritter, and Zhao (1996) document that for initial public offerings (IPOs) of equity, the direct costs (underwriting, management, legal, auditing and registration fees) average 11.0% of the proceeds, and for seasoned equity offerings (SEOs), the direct costs average 7.1%. IPOs also incur a substantial indirect cost due to short-run underpricing. An early study by Asquith and Mullins (1986) found that the average stock price reaction to the announcement of a common stock issue was -3% and the loss in equity value as a percentage of the size of the new equity issue was as high as -31% (Eckbo et al., 2007).

[8]In our model, the bank manager's incentive is aligned with equityholders. To some extent, the issuance cost partially captures insiders' distinct incentives, as a high insider ownership is empirically associated with reluctance to issue equity (Goetz, Laeven, and Levine, 2020). Becht, Bolton, and Röell (2011) discuss the issues of corporate governance in the banking sector.

Because the bank only faces (locally continuous) diffusive shocks, it can avoid default as long as the continuation value is positive. In our numeric solution, this is indeed the case, so $\tau = +\infty$. We assume that the discount rate ρ, i.e., the equityholders' required return, is greater than r. This impatience can be microfounded by a Poisson death rate that is equal to $\rho - r$.

Capital Requirement. The bank must meet a capital requirement. For example, the Basel III accords stipulate that banks must back a specific percentage of risk-weighted assets with equity.[9] As in Nguyen (2015), Davydiuk (2017), Van den Heuvel (2018), and Begenau (2019), we introduce

$$\frac{A_t}{K_t} \leq \xi_K \,. \tag{5}$$

In accordance with Basel III capital standards, banks must maintain the Tier 1 capital ratio (Tier 1 capital divided by total risk-weighted assets) of 6% (increased to 7% from 2019 onward). We set ξ_K equal to $1/0.07 = 14.3$.[10]

Leverage Restriction. Since January 1, 2018, banks in the U.S. face a supplementary leverage ratio restriction (SLR). It supplements the capital requirement that can be vulnerable to manipulation (Plosser and Santos, 2014). Banks are required to maintain a ratio of tier 1 capital to total consolidated assets at a minimum level of 3%. The U.S. bank holding companies that have been identified as global systemically important banks ("U.S. G-SIBs") must maintain an SLR of greater than 5%, and if they fail to do so, they will be subject to increasingly stringent restrictions on its ability to make capital distributions and discretionary bonus payments.[11]

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[9] See Thakor (2014) for a review of the debate on bank capital and its regulations.

[10] Davydiuk (2017) and Begenau (2019) set ξ_K to be the sample average of the ratio of Tier 1 equity to risky assets in quantitative models for the reason that banks typically maintain a buffer over the regulatory thresholds in order to prevent regulatory corrective action. In our model, the buffer arises endogenously, driven by banks' precaution to avoid paying the equity issuance costs, so we set ξ_K to the regulatory threshold. In theoretical studies on banking regulations, De Nicolò, Gamba, and Lucchetta (2014) calibrates the capital requirements to 4% and 12%, Hugonnier and Morellec (2017) calibrates the thresholds to 4% , 7%, 9%, and 20% to investigate the effects of the proposal by Admati and Hellwig (2013), and Phelan (2016) calibrates the threshold to 7.7% and 10.6% in a macroeconomic model.

[11] During the Covid-19 crisis, the U.S. regulators excluded Treasury securities and reserve held at the Federal Reserve System from the denominator of SLR, as banks faced an influx of deposits on the liability side of their balance sheets and, on the asset side, acquired significant amounts of U.S. Treasury securities and news loans (especially due

The ratio of its total assets (or liabilities) to equity capital cannot exceed ξ_L. When the bank has short-term debts, i.e., $B > 0$, the leverage ratio requirement is given by

$$\frac{A}{K} = \frac{K + X + B}{K} \leq \xi_L \,, \tag{6}$$

and when $B < 0$, the leverage ratio requirement is given by

$$\frac{A - B}{K} = \frac{K + X}{K} \leq \xi_L \,. \tag{7}$$

SLR capital requirement of 5% translates into $\xi_L = 20$. When $B > 0$, both capital requirement and SRL restriction become a restriction of A/K.

the U.S. banking regulators relaxed the supplementary leverage ratio requirements.

3 Dynamic Banking without Equity Issuance Costs

A key friction in our model is the equity issuance cost. Next, we show that without such costs, the value function is linear in the deposit stock, X, and capital, K. As a result, the bank does not exhibit endogenous risk aversion and the marginal value of deposits is constant.

Without the issuance costs, the marginal value of capital is equal to one, i.e., $V_K(X, K) = 1$, because if $V_K(X, K) > 1$, the bank raises equity, and the bank pays out dividend if $V_K(X, K) < 1$. In Appendix A, we show that there exists a constant Q such that

$$V(X, K) = QX + K \,. \tag{8}$$

In Appendix A, we provide the analytical solution of Q.

A key result is that Q does not depend on any of the risk parameters, i.e., σ_A and σ_X. Without the equity issuance costs, the bank is not concerned about risks, because when it needs capital following adverse shocks, it can always raise capital. Intuitively, as long as $\alpha_A > 0$, i.e., lending

to customers drawing on credit lines).

generates excess return, the bank will borrow short-term debt, increasing leverage beyond what is already obtained through deposit-taking. As previously discussed, when $B > 0$, both capital requirement and SLR restriction become a restriction on A/K. Because $\xi_L > \xi_K$, capital requirement binds and the bank's optimal lending is proportional to equity capital, i.e., $A/K = \xi_K$. Moreover, the bank sets a constant deposit rate.

4 Dynamic Banking under Equity issuance Costs

4.1 Bank Optimization

We derive the optimality conditions for the bank's control variables and the Hamilton-Jacobi-Bellman (HJB) equation for the bank's value function. In the next subsections, we parameterize the deposit maintenance cost and provide intuitive characterizations of the bank's optimal policies.

State and Control Variables. The bank solves a dynamic optimization problem with two state variables, the deposit stock X_t and the equity capital K_t. We denote the shareholders' value at time t as V_t. This present value results from the bank's optimal control of the stochastic processes of loan portfolio size A_t, short-term borrowing B_t, the deposit rate i_t, the payout of dividends dU_t, and the value of newly issued equity shares dF_t:

$$V_t = V\left(X_t, K_t\right) = \max_{\{A,B,i,U,F\}} \mathbb{E}\left[\int_{t=0}^{\tau} e^{-\rho t}\left(dU_t - dF_t - dH_t\right)\right]. \tag{9}$$

The value function is a function of the state variables, i.e., $V_t = V\left(X_t, K_t\right)$. Every instant, given the state variables, X_t and K_t, the bank optimizes the control variables before the realization of diffusion shocks, taking into consideration the impact on the evolution of state variables (and through such impact, the continuation value). To solve the bank's optimal choices and value function, we need the laws of motion of state variables that show how the choice variables affect their evolution. The law of motion for X_t is given by (2). For the equity capital K_t, we have

$$dK_t = A_t \left[(r + \alpha_A)\, dt + \sigma_A d\mathcal{W}_t^A \right] - B_t r dt - X_t i_t dt - C\left(n\left(i_t\right), X_t\right) dt - dU_t + dF_t\,. \quad (10)$$

The first three terms on the right side record the return on loans, bond interest expenses, and deposit interest expenses. The fourth term is the operation cost associated with adjusting and maintaining the deposit stock. The last two terms are the dividend payout and capital raised via equity issuance.

Given X_t and K_t, the bank's choices of A_t and B_t resemble a portfolio problem (Merton, 1969).[12] Let π_t^A denote the portfolio weight on loans, i.e., $\pi_t^A\left(X_t + K_t\right) = A_t$, so the weight on bonds is $\left(\pi_t^A - 1\right)$ because $B_t = A_t - (X_t + K_t)$. Note that if $A_t > X_t + K_t$, the bank issues bonds, $B_t > 0$, paying the interest rate r; if $A_t < X_t + K_t$, the bank lends in the short-term debt market (i.e., $B_t < 0$) and earns the interest rate r. We can rewrite the law of motion for K_t as

$$dK_t = (X_t + K_t)\left[\left(r + \pi_t^A \alpha_A\right) dt + \pi_t^A \sigma_A d\mathcal{W}_t^A\right] - X_t i_t dt - C\left(n\left(i_t\right), X_t\right) dt - dU_t + dF_t\,. \quad (11)$$

Given the Markov nature of the bank's problem, we suppress the time subscript of X and K going forward to simplify the notations wherever it does not cause confusion.

Payout and Equity Issuance. The bank pays out dividends, i.e, $dU_t > 0$, only if the decrease of continuation value is equal to or less than the consumption value of dividends, $V(X, K) - V(X, K - dU_t) \leq dU_t$, i.e.,

$$V_K(X, K) \leq 1\,. \quad (12)$$

The optimality of payout also requires the following condition:

$$V_{KK}(X, K) = 0\,. \quad (13)$$

[12] The bank may adjust the loan amount A_t by selling loans in the secondary market. The technological progress on the reduction of information asymmetries between loan buyers and loan sellers facilitate the trading of loans, and and the design of contract between the loan buyers and originators can alleviate the moral hazard problem (reduced monitoring incentive) on the part of loan originators (Pennacchi, 1988; Gorton and Pennacchi, 1995).

Following Bolton, Chen, and Wang (2011), we assume that the bank incurs proportional and fixed costs of issuing equity. Let M_t denote the amount raised. $\psi_1 M_t$ is the proportional equity-issuance cost. We further assume that the fixed cost is linear in X_t, so that $\psi_0 X_t$ denotes the fixed equity-issuance cost. This makes the bank's problem homogeneous in X_t and, as will be shown shortly, significantly simplifies the solution. Moreover, as the bank grows geometrically with X_t, modeling the fixed cost as increasing in X_t avoids it becoming eventually negligible.

The bank raises equity, i.e., $dF_t > 0$, only if the increase of shareholders' value after issuance is equal to or greater than the cost,

$$V\left(X, K + dF_t\right) - V\left(X, K\right) \geq dF_t + dH_t = \psi_0 X + \left(1 + \psi_1\right) M_t \,. \tag{14}$$

where $dF_t = M_t$ is the capital raised and, as previously discussed, the issuance costs have a fixed and a proportional components, $dH_t = \psi_0 X + \psi_1 M_t$. The optimal amount of issuance is given by the following condition:

$$V_K\left(X, K + M_t\right) = 1 + \psi_1 \,. \tag{15}$$

HJB Equation. Given the laws of motion (2) for X and (11) for K, in the interior region where $dU_t = 0$ and $dF_t = 0$, the bank's HJB equation is

$$\begin{aligned} \rho V\left(X, K\right) = \max_{\{\pi^A, \pi^R, i\}} \; & V_X\left(X, K\right) X \left[-\delta_X + n\left(i\right)\right] + \frac{1}{2} V_{XX}\left(X, K\right) X^2 \sigma_X^2 \\ & + V_K\left(X, K\right)\left(X + K\right)\left(r + \pi^A \alpha_A\right) + \frac{1}{2} V_{KK}\left(X, K\right)\left(X + K\right)^2 \left(\pi^A \sigma_A\right)^2 \\ & - V_K\left(X, K\right)\left[X i + C\left(n\left(i\right), X\right)\right] + V_{XK}\left(X, K\right) X\left(X + K\right) \pi^A \sigma_A \sigma_X \phi \,. \end{aligned} \tag{16}$$

Lending. The first-order condition for π^A gives the following solution:

$$\pi^A = \frac{\alpha_A + \epsilon\left(X, K\right) \sigma_A \sigma_X \phi}{\gamma\left(X, K\right) \sigma_A^2 \left(\frac{X+K}{K}\right)} \,, \tag{17}$$

where we define the endogenous risk aversion parameter based the value function

$$\gamma\left(X,K\right)\equiv\frac{-V_{KK}\left(X,K\right)K}{V_{K}\left(X,K\right)}\,, \tag{18}$$

and the elasticity of marginal value of capital, $V_K\left(X,K\right)$, to the stock of deposit liabilities

$$\epsilon\left(X,K\right)\equiv\frac{V_{XK}\left(X,K\right)X}{V_{K}\left(X,K\right)}\,. \tag{19}$$

Even though the bank evaluates the equityholders' payoffs with a risk-neutral objective in (4), it can be effectively risk-averse, i.e., $\gamma\left(X,K\right)>0$, due to the equity issuance cost. When $\epsilon\left(X,K\right)>0$, deposits and capital are complementary in creating value for banks' shareholders.

While setting up $\pi^A = A/\left(X+K\right)$ as the control variable is convenient for solving the model, it is intuitive to express the solution in loan-to-capital ratio, i.e., $A/K=\pi^A\left(X+K\right)/K$:

$$\frac{A}{K}=\frac{\alpha_A+\epsilon\left(X,K\right)\sigma_A\sigma_X\phi}{\gamma\left(X,K\right)\sigma_A^2}\,, \tag{20}$$

This solution resembles Merton's portfolio choice. In the numerator, a higher excess return, α_A, increases lending. The bank's incentive to lend is also strengthened when deposits are natural hedge – the asset-side shock, $d\mathcal{W}^A$, and the liability-side (deposit) shock, $d\mathcal{W}^A$ are positively correlated ($\phi>0$) and more deposits make capital more valuable (i.e., $\epsilon\left(X,K\right)>0$). [13].

Deposit Rate. The bank sets the deposit rate, i, to equate the marginal value of new deposits, $V_X\left(X,K\right)n'\left(i\right)X$, and the marginal costs from reducing the shareholders' profits (i.e., return on equity capital) by paying interests on the existing deposits, $V_K\left(X,K\right)X$, and by paying the costs of maintaining a larger deposit franchise, $V_K\left(X,K\right)C_n\left(n\left(i\right),X\right)n'\left(i\right)$:

$$V_X\left(X,K\right)n'\left(i\right)X=V_K\left(X,K\right)\left[X+C_n\left(n\left(i\right),X\right)n'\left(i\right)\right]\,. \tag{21}$$

[13]While different in mechanism, this feature of our model echoes the literature on the synergy between lending and deposit-taking (Calomiris and Kahn, 1991; Berlin and Mester, 1999; Kashyap, Rajan, and Stein, 2002; Gatev and Strahan, 2006; Hanson, Shleifer, Stein, and Vishny, 2015). Our mechanism also echoes the recent finding of Drechsler et al. (2020) that deposit liabilities serve as a natural hedge (against interest-rate risk) for long-term loans.

4.2 Optimal Deposit Rate

We parameterize the deposit-flow function and the deposit cost function to obtain more intuitive solutions. First, we specify the deposit demand as a simple linear function of the deposit rate

$$n(i) = \omega i\,, \tag{22}$$

where, as shown in (2), ω is the semi-elasticity of deposits X_t with respect to i, which we will calibrate to the estimate from Drechsler, Savov, and Schnabl (2017) in our numeric solution.[14] Next, we specify the deposit maintenance/adjustment cost as follows,

$$C(n(i), X) = \frac{\theta}{2} n(i)^2 X\,. \tag{23}$$

The cost is increasing in the existing amount of deposits, X_t, and is increasing and convex in the flow of new deposits $n(i)$, reflecting the increasing marginal cost of expanding the depositor base.

This functional form leads to a Hayashi style optimal policy of deposit rate. In Hayashi (1982), firms make investments in productive capital, while, in our model, the bank attracts depositors by raising the deposit rate, building up its customer capital. Using (21), we obtain

$$i = \frac{\frac{V_X(X,K)}{V_K(X,K)} - \frac{1}{\omega}}{\theta\omega}\,. \tag{24}$$

Consistent with the evidence in Drechsler, Savov, and Schnabl (2017), the deposit rate is higher when the demand is more elastic, i.e., ω is high. The bank also sets a higher rate to attract more deposits when the marginal adjustment cost increases slowly, i.e., θ is low.

The bank sets a high deposit rate when the marginal value of deposits, $V_X(X, K)$, is high relative to the marginal value of equity capital, $V_K(X, K)$. Paying a higher deposit rate attracts more deposits but paying more interest expenses reduce earnings and equity. Section 3 presents the solution of the bank's problem without the equity issuance costs. In that case, the marginal value

[14] We also experiment with an alternative specification of quadratic $n(i)$ that allows the deposit flow to be increasingly sensitive to deposit rate as i approaches zero. The results are very similar and are available upon request.

of equity is always equal to one and the marginal value of deposits is a constant Q. Therefore, the optimal rate is a constant:

$$i = \frac{Q - \frac{1}{\omega}}{\theta\omega}\,. \tag{25}$$

In the presence of equity issuance cost, the optimal policy of deposit rate depends on X and K.[15]

An interesting feature of the optimal deposit rate is that it hits the zero lower bound when

$$\frac{V_X\left(X,K\right)}{V_K\left(X,K\right)} \leq \frac{1}{\omega}\,. \tag{26}$$

Once the deposit rate reaches zero, the bank cannot further decrease the deposit rate to reduce deposits. Later we show that this restriction makes deposits undesirable, especially when the bank is undercapitalized, and thus, is concerned of a high leverage from large deposits that amplifies the impact of negative shocks on equity, increasing the likelihood of costly equity issuance.

When the deposit demand is more elastic, i.e., ω is high, the bank has to pay a higher deposit rate, as shown in (24), which turns to decrease the shareholders' value. However, given the value function, it is less likely for the condition (26) to hold, because a high demand elasticity allows the bank to control the deposit flow more effectively and thereby to avoid hitting the zero lower bound. This result suggests that the deposit-rate lower bound is more acute a problem for larger banks with greater deposit market power or stickier deposit base (i.e., smaller ω). Smaller banks with less deposit market power are less concerned of the deposit-rate lower bound, but they have to pay higher interest rates to attract depositors.

4.3 Optimal Risk-Taking

Given the functional forms of deposit flows and costs, the bank's problem is homogeneous in X and its value function $V\left(X,K\right) = v\left(k\right)X$, where

$$k = \frac{K}{X}\,, \tag{27}$$

[15]The difference between (24) and (25) is akin to the difference in a firm's optimal investment in Hayashi (1982) and Bolton, Chen, and Wang (2011). In Bolton, Chen, and Wang (2011), the cost of raising equity induces a state-dependent value of liquidity, so the ratio of marginal value of capital to that of liquidity drives the firm's investment.

Therefore, instead of working with X and K as the state variables, we will work with X and k. The capital-to-deposit ratio, k, captures the composition of *long-term* funding on the liability side of the bank's balance sheet. We will show that the choice variables are functions of k only.

Next, we simplify the expression of loan-to-capital ratio, a measure of the bank's risk-taking. First, note that the expression of the effective risk aversion in (18) can be simplified to

$$\gamma\left(k\right)=\frac{-V_{KK}\left(X,K\right)K}{V_{K}\left(X,K\right)}=-\frac{v''\left(k\right)k}{v'\left(k\right)}\,. \tag{28}$$

And, the elasticity of marginal value of capital to deposits in is given by (19)

$$\epsilon\left(k\right)=\frac{V_{XK}\left(X,K\right)X}{V_{K}\left(X,K\right)}=-\frac{v''\left(k\right)k}{v'\left(k\right)}\,, \tag{29}$$

which happens to be equal to $\gamma\left(k\right)$.

Using $\epsilon\left(k\right)=\gamma\left(k\right)$, we simplify the optimal loan-to-capital ratio from (20):

$$\frac{A}{K}=\frac{\alpha_{A}}{\gamma\left(k\right)\sigma_{A}^{2}}+\frac{\sigma_{X}}{\sigma_{A}}\phi\,, \tag{30}$$

The bank's risk-taking is state-dependent and only depends on k through $\gamma\left(k\right)$. When the effective risk aversion is low, the bank chooses a high loan-to-capital ratio; when the effective risk aversion is high, the bank reduces its risk exposure. In our numeric solution, we show that $\gamma\left(k\right)$ decreases in k, so the loan-to-capital ratio increases when the bank has a high equity buffer relative to its deposit liabilities. The correlation between the loan return shock and the deposit flow shock, ϕ, induces a hedging demand. The risk of deposit flow is essentially the bank's background risk from the perspective of portfolio management and a natural hedge when $\phi>0$. The parameter ϕ captures the synergy between lending and deposit-taking that has been studied extensively in the banking literature (Kashyap, Rajan, and Stein, 2002; Gatev and Strahan, 2006).

4.4 Solving the Value Function

Value Function ODE. To solve the bank's value function, we simplify the HJB equation to obtain an ordinary differential equation for $v(k)$. First, given that $V(X,K) = v(k)X$, we obtain

$$V_K(X,K) = v'(k)\ ,\ V_X(X,K) = v(k) - v'(k)k$$
$$V_{KK}(X,K) = v''(k)\frac{1}{X}\ ,\ V_{XX}(X,K) = v''(k)\frac{k^2}{X}\ ,\ V_{XK}(X,K) = -v''(k)\frac{k}{X}. \tag{31}$$

Using these expressions, we can rewrite the HJB equation (16) as

$$\begin{aligned}\rho v(k) = &\max_{\pi^A,i}\ [v(k) - v'(k)k](-\delta_X + \omega i) + \frac{1}{2}v''(k)k^2\sigma_X^2 \\ &+ v'(k)(1+k)\left(r + \pi^A\alpha_A\right) + \frac{1}{2}v''(k)(1+k)^2\left(\pi^A\sigma_A\right)^2 \\ &- v'(k)\left[i + \frac{\theta}{2}(\omega i)^2\right] - v''(k)k(1+k)\pi^A\sigma_A\sigma_X\phi\,.\end{aligned} \tag{32}$$

To show that (32) is an ODE for $v(k)$, we need to show that the control variables only depend on k and the level and derivatives of $v(k)$. First, by definition, $\pi^A = A/(X+K)$, so we obtain the following simplified expression for π^A from (30):

$$\pi^A = \left(\frac{A}{K}\right)\left(\frac{K}{K+X}\right) = \left(\frac{\alpha_A}{\gamma(k)\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi\right)\left(\frac{k}{1+k}\right). \tag{33}$$

The optimal deposit rate given by (24) only depends on $V_X(X,K) = v(k) - v'(k)k$ and $V_K(X,K) = v'(k)$. Then we can substitute these optimal choices into the HJB equation to obtain an second-order ODE for $v(k)$ that contains only k and the level and derivatives of $v(k)$. Fully solving the model then takes two steps, first, solving the ODE to obtain $v(k)$, and second, using the solved $v(k)$ and its derivatives to solve the bank's optimal choices.

Boundary Conditions. Let $\overline{k}$ and $\underline{k}$ denote respectively the dividend payout and issuance boundaries, and let $m \equiv M/X$ denote the amount financing raised via issuance (scaled by X). The boundary conditions implied the optimality condition on payout (12) and (13) are

$$v'\left(\overline{k}\right) = 1\,, \tag{34}$$

and

$$v''\left(\overline{k}\right) = 0\,. \tag{35}$$

The boundary conditions implied by the optimality condition on issuance (14) and (15) are

$$v\left(\underline{k} + m\right) - v\left(\underline{k}\right) = \psi_0 + \left(1 + \psi_1\right) m\,, \tag{36}$$

and

$$v'\left(\underline{k} + m\right) = 1 + \psi_1\,. \tag{37}$$

Our numerical solution of $v(k)$ features global concavity, so (34) and (37) imply that $\overline{k} > \underline{k}$. Given $\underline{k}$, the four boundary conditions above solve the second-order ODE for $v(k)$ (i.e., the HJB equation), the upper boundary $\overline{k}$, and the endogenous issuance amount m. However, we still need one condition to pin down $\underline{k}$. In our numerical solution, $v(k)$ is globally concave, so $\underline{k} = 0$, i.e., the bank does not pay the issuance costs unless its capital drops to zero. However, in the presence of leverage ratio requirement, $\underline{k}$ must be positive. In our numerical solution, when k is small, $B < 0$ so the leverage ratio constraint implies that $(K + X)/K \leq \xi_L$, i.e.,

$$k \geq \underline{k} = \frac{1}{1 - \xi_L^{-1}} - 1\,. \tag{38}$$

Therefore, when k is small (and $B < 0$), SLR requirement becomes a lower bound on the state variable and triggers equity issuance. When k is large, the capital requirement binds before the SLR requirement (due to $\xi_L > \xi_K$) and restricts the control variable $A/K = \pi^A\left(\frac{1+k}{k}\right) \leq \xi_K$. We highlight that the newly introduced SLR requirement is more effective a tool to motivate bank recapitalization (that happens in bad times, i.e., when k is low), while the traditional capital requirement is a restriction on risk-taking (and binds in good times, i.e., when k is high).

In Appendix B, we provide a setup where, as in Drechsler, Savov, and Schnabl (2018), the bank has to hold assets that are more liquid than loans and is subject to a regulatory liquidity requirement, such as reserve and liquidity coverage ratio requirements. In this richer setup, our results on the value of deposits and the optimal strategies of payout, equity issuance, risk-taking, and deposit rate still hold.[16] The only difference is that the liquidity requirement generates another lower bound for k. Therefore, the bank raises equity to meet either the supplementary leverage ratio requirement binds or the liquidity requirement. Note that a binding regulatory constraint should not be interpreted literally because, once a bank is close to violating any regulatory constraint, regulators initiate various interventions that often restrict managerial compensation or payout to shareholders. Thus, empirically, equity issuance can happen before a constraint binds.

5 Quantitative Analysis

5.1 Parameter Choices

We set the unit of time to one year and r to 1% in line with the average Fed funds rate in the last decade. Shareholders' discount rate ρ is set to 4.5% in line with the commonly used discount rate in dynamic corporate finance models.[17]. We set α_A to 0.2% so that the model generates an average return on assets (ROA) equal to 1.05%, close to the average of US banks in the last decade (source: FRED). Note that when k is large, the bank only holds risky assets, but when k is small, the bank also holds risk-free assets ($B < 0$). Therefore, the ROA is state-dependent. To calculate the average ROA (and other averages later), we use the stationary distribution of k. We set asset volatility, σ_A, to 10% following Sundaresan and Wang (2014) and Hugonnier and Morellec (2017) who in turn refer to the calculation of Moody's KMV Investor Service.

For the deposit dynamics, we set δ_X to 0% and σ_X to 5% following Bianchi and Bigio (2014). We further set ω, the semi-elasticity of deposits to the deposit rate, to 5.3, an estimate from Drechsler, Savov, and Schnabl (2017). The correlation between asset-side and liability-side

[16] The solution of optimal liquidity holdings resembles the classic money demand (Baumol, 1952; Tobin, 1956).

[17] One example is Bolton, Chen, and Wang (2011). This is also consistent with the dynamic contracting literature (DeMarzo and Fishman, 2006; DeMarzo and Sannikov, 2006; Biais et al., 2007; DeMarzo and Fishman, 2007).

Table 1: PARAMETER VALUES

This table summarizes the parameter values for our baseline analysis. One unit of time is one year.

Parameters	Symbol	Value
risk-free rate	r	1%
discount rate	ρ	4.5%
bank excess return	α_A	0.2%
asset return volatility	σ_A	10%
deposit flow (mean)	δ_X	0
deposit flow (volatility)	σ_X	5%
deposit maintenance cost	θ	0.5
deposit demand semi-elasticity	ω	5.3
corr. between deposit and asset shocks	ϕ	0.8
equity issuance fixed cost	ψ_0	0.1%
equity issuance propositional cost	ψ_1	5.0%
SLR requirement parameter	ξ_L	20
capital requirement parameter	ξ_K	14.3

(deposit) shocks, ϕ, directly affects A/K in (30) and is set to 0.8 so that the (stationary) probability of a binding capital requirement is consistent with the evidence (Begenau, Bigio, Majerovitz, and Vieyra, 2019). As for the cost of maintaining deposit franchise, we set θ to 0.5. Under this value, the model generates an average deposit-to-total liabilities ratio equal to 96% in line with the evidence (Drechsler, Savov, and Schnabl, 2017). We set the proportional issuance cost to 5% (Boyson, Fahlenbrach, and Stulz, 2016). The fixed cost is set to 0.1%. Under this value, the model generates an issuance-to-equity ratio of 1% in line with the evidence (Baron, 2020). The regulatory parameters were discussed in Section 2. Table 5 summarizes our calibration.

5.2 Bank Franchise Value

In the frictionless world of Modigliani and Miller (1958) the bank shareholders' value V_t is equal to book equity, K_t. In our model, two forces creates a wedge, giving rise to a strictly positive bank franchise value $(V_t - K_t)$. First, as shown in Section 3, deposit-taking can create value for shareholders. Even if the bank is subject to a deposit maintenance cost, it can adjust the deposit

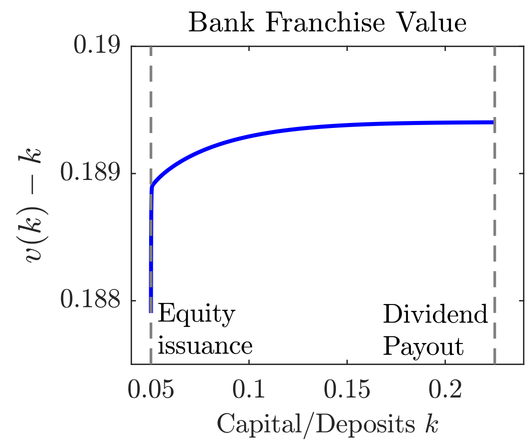

Figure 1: Bank Franchise Value

rate to exploit the downward-sloping demand for deposits and make a profit. Second, the equity issuance cost contributes to the wedge. The bank has to maintain a positive level of profits, the present value of which allows it to raise equity financing net of issuing costs. In Figure 1, we plot the franchise value, $(V_t - K_t)/X_t = v(k) - k$, against the key state variable k (the deposit-to-capital ratio). The franchise value increases when capital accumulates relative to deposits, which is consistent with the evidence from Minton, Stulz, and Taboada (2019).

At the lower boundary $\underline{k}$, the bank is required to raise new equity and then must pay an issuance cost. The further away k is from this issuance boundary, the lower the likelihood of hitting the boundary and paying an issuance cost. Therefore, the franchise value $(V_t - K_t)/X_t$ increases in k, which is consistent with the finding of Mehran and Thakor (2011) that bank value is positively associated with bank capital. The interior region ends at the upper (dividend payout) boundary $\overline{k}$. At that point the bank has sufficient retained earnings, so that it is safe to pay out dividends to impatient shareholders. Note that near $\overline{k}$ the franchise value is flat and relatively insensitive to variations in k because, at that point, the likelihood of a large loss of equity or a large deposit inflow that dramatically decrease k and force the bank to raise costly new equity is low.

These results have several implications on the empirical analysis of bank valuation (Atkeson, d'Avernas, Eisfeldt, and Weill, 2019; Minton, Stulz, and Taboada, 2019). First, the deposit stock

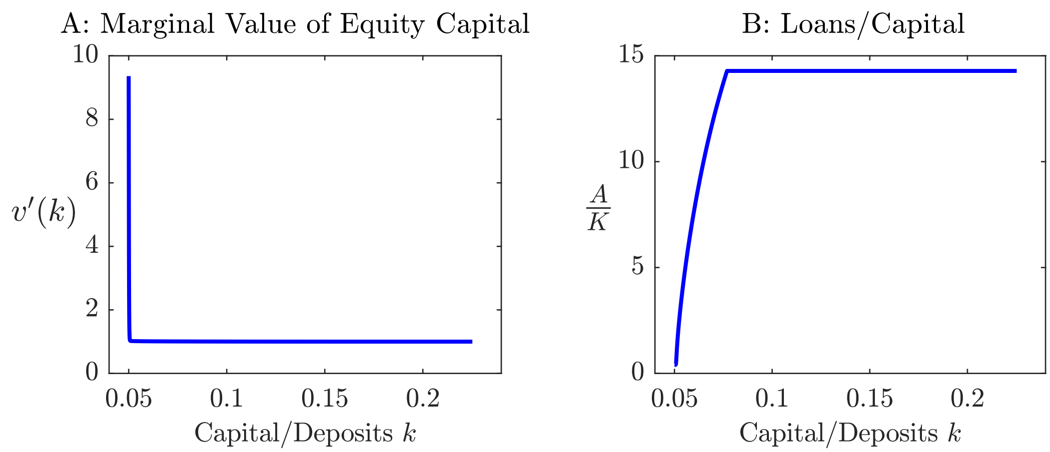

Figure 2: Marginal Value of Equity Capital and Bank Risk-Taking.

is a more natural normalization than book equity because of homogeneity property of the bank's problem. Second, shareholder value (scaled by deposits) increases in the capital-to-deposit ratio, reaching its highest level when the bank pays out dividends, and falling to its lowest level when the bank raises equity. This is consistent with the evidence that payout is procyclical while equity issuance is countercyclical (Adrian, Boyarchenko, and Shin, 2015; Baron, 2020).

5.3 Marginal Value of Equity Capital and Risk-Taking

Panel A of Figure 2 plots the marginal value of bank capital, $V_K(K, X) = v'(k)$. Without financial frictions (costly equity issuance) this variable should always be equal to one. When it is costly to issue equity, however, the marginal value of equity capital rises sharply above one near the issuance boundary. At $\underline{k}$, a value of $v'(k)$ close to eight means that one dollar of equity is worth nine dollars because of the imminence of costly equity issuance.[18]

In Panel B of Figure 2, we describe the bank's optimal risk-taking behavior. Panel A plots the target ratio of loan value to capital, A_t/K_t, given by (30). The bank obviously cannot exceed the regulatory capital requirement (i.e., $A/K \leq \xi_K = 14.3$), but it can expand its balance sheet up to

[18]Note that even though the proportional cost of issuance is only 5%, due to the fixed cost, the value of one dollar equity can be much higher than 1.05.

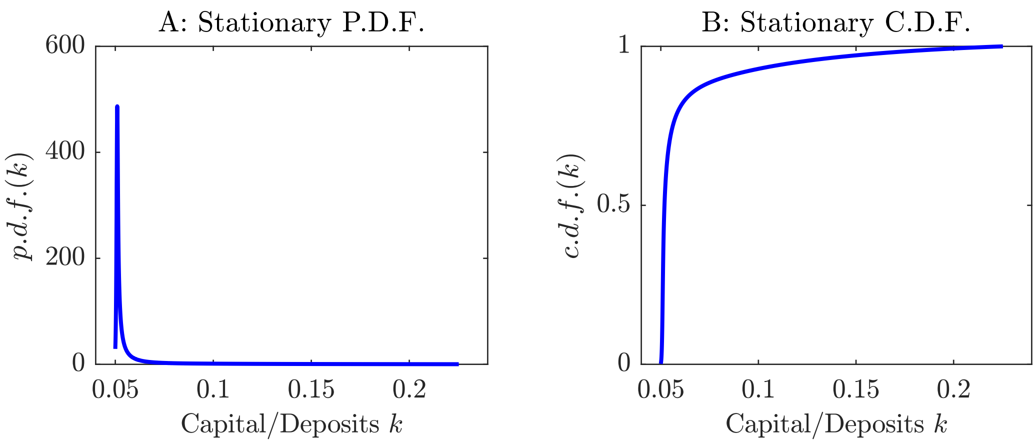

Figure 3: Stationary Probability Density and Cumulative Distribution Function.

that target. Risk-taking is procyclical. As capital accumulates relative to deposits (as k increases), the bank expands its balance sheet, financing the expansion through deposits and wholesale (short-term bond) funding. But when capital is depleted relative to deposits, the bank deleverages. This is consistent with the findings of Ben-David, Palvia, and Stulz (2020) that distressed banks deleverage and decrease observable measures of riskiness. A natural measure of risk-taking incentives in our model is $\gamma(k)$ the endogenous relative risk aversion coefficient of the bank defined in (18). $\gamma(k)$ decreases in k because, as shown in Panel A of Figure 2, the value function is extremely concave near the equity issuance (lower) boundary $\underline{k}$ but, as k increases, the concavity subdues quickly.

Panel A of Figure 3 plots the stationary probability density of the state variable k, and Panel B plots the corresponding cumulative probability function. It shows how much time the bank spends in various regions of k after the loan return and deposit flow shocks are realized over the long run. Note how the probability mass is highly concentrated in the area where k is close to the lower boundary $\underline{k}$, but still large enough that the marginal value of equity at that point is low. In fact, the marginal value of equity is 1.02 where the density function peaks. Importantly, even if for the majority of time the bank does not seem to be financially constrained, the shadow value of equity rises dramatically when equity is depleted to the point where the bank may be forced into a costly equity issuance, as shown in Panel A of Figure 2. These results illustrate the sharp contrast

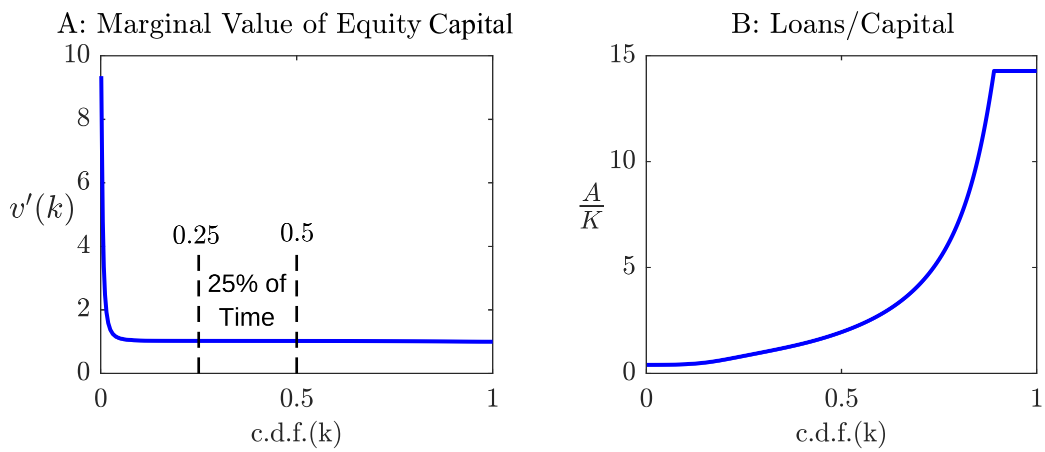

Figure 4: The Long Run Distribution of Marginal Value of Equity Capital and Bank Risk-Taking.

between normal times, when the bank is comfortably meeting its leverage requirements, and crisis times, when it is in danger of violating its leverage requirements and triggering equity issuance.

In Panel A of Figure 4, we plot the marginal value of equity against the cumulative distribution function (c.d.f.) of the stationary distribution of k (with $c.d.f.\,(\underline{k}) = 0$ and $c.d.f.\,(\overline{k}) = 1$). In the graph, the horizontal span represents the amount of time the bank spends in a region on the long run. We illustrate that the bank spends 25% of the time with its marginal value of equity between 1.019 and 1.022. The bank spends less than 5% of the time in the region where it is in danger of violating the leverage requirement with a marginal value of equity above 1.08. In other words, crisis states are rare but they cast a long shadow over the bank's management of its balance sheet. As the bank becomes better capitalized relative to its deposit liabilities (i.e., k increases), the marginal value of equity declines dramatically, so that the bank's value $v(k)$ is concave and the bank is effectively risk averse. In Panel B of Figure 4, we plot the loan-to-capital ratio, A_t/K_t, against the c.d.f. of k, and show that around 11% of the time, the capital requirement binds. The capital requirement is relevant when the bank is well-capitalized and the risk-taking incentive is strong. Such procyclicality suggests that capital requirement can act as a macroprudential tool as also suggested by Gersbach and Rochet (2017). In contrast, the supplementary leverage ratio motivates the bank to replenish equity capital in bad times when it is undercapitalized.

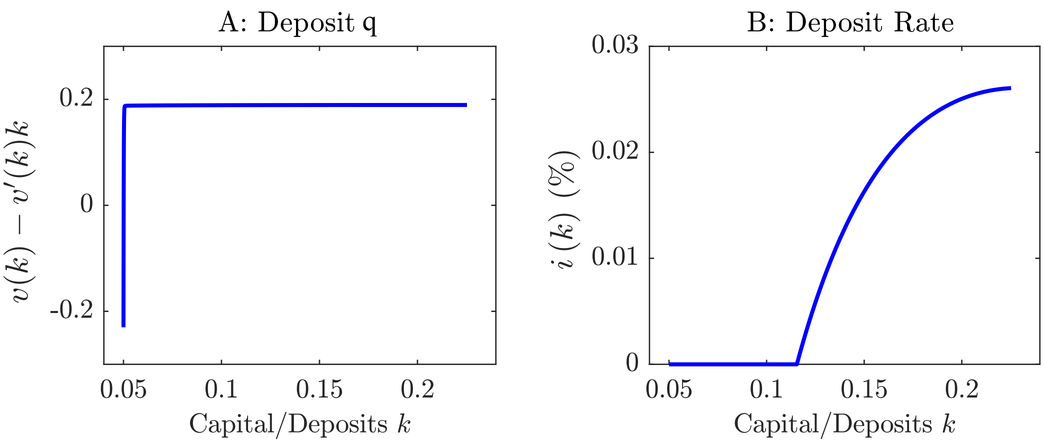

Figure 5: Marginal Value of Deposits and Deposit Rate.

5.4 The Value of Deposits

Panel A of Figure 5 plots the marginal value of deposits, i.e., $V_X(X, K) = v(k) - v'(k)k$, which we call the Deposit Q. When the bank has ample capital relative to deposits, i.e., k is large, the deposit Q is positive. However, it turns sharply negative when k nears the lower boundary of costly equity issuance. In Panel B of Figure 5, we plot the optimal deposit rate (given by (24)). When the bank has sufficient capital to buffer risk, it is willing to set a deposit rate that is above $r = 1\%$, i.e., the interest rate it pays to borrow in short-term debt. The bank sacrifices return on equity but, through a high deposit rate, gains a greater depositor base, which then can help the bank borrow at a deposit rate below $r = 1\%$ when its capital becomes lower (i.e., k decreases). This is due to the slow-moving (sticky) nature of deposits.

Fundamentally, deposits create value by allowing the bank to finance risky lending with relative cheap sources of funds. The deposit stock serves as a form of productive capital for the bank. The comovement of loan growth (Panel B of Figure 2) and deposit rate increase (Panel B of Figure 5) is consistent with the finding of Ben-David, Palvia, and Spatt (2017).

A key finding is that the deposit Q can turn negative when a bank's capital is low relative to its deposit liabilities. The reason is that when k is near the equity issuance boundary $\underline{k}$, deposits destroy value for the bank's shareholders by forcing the bank to sustain a high level of leverage

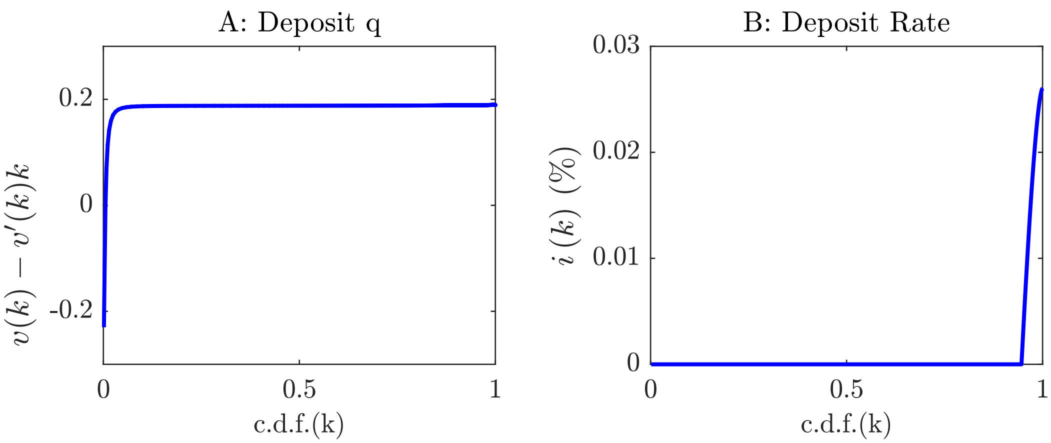

Figure 6: The Long Run Distribution of Marginal Value of Deposits and Deposit Rate.

that amplifies the impact of shocks on bank capital and makes it more likely to incur costly equity issuance. The bank may want to delever, turning away deposits by lowering the deposit rate. However, as shown by Panel B of Figure 5, doing so has a limit, that is the zero lower bound of deposit rate. Setting a deposit rate below zero causes depositors to withdraw deposits en masse and hoard dollar bills (which has a zero return). While we do not explicitly model the consequence of a run, the value destroyed through liquidation of loans and fire sale is likely to make the zero lower bound a binding constraint for the bank.

In Figure 6, we plot the deposit Q and deposit rate against the stationary c.d.f. of k. The deposit Q is positive, above 0.185 in 81% of the time, but near the issuance boundary (i.e., $c.d.f.(\underline{k}) = 0$), it can drop to -0.23. The deposit rate hovers around the lower bound at zero, showing that the bank is very conservative in deposit-taking. The deposits attracted by high rate today is helpful in financing lending (i.e., earning α_A) but can become burdensome when negative shocks deplete bank equity capital and k declines.

Deposits are very different from short-term debts. For short-term debts, the bank can always choose to stop borrowing at maturity, and therefore, does not face the problem of unwanted debts. However, deposit contracts do not have maturity. Deposits leave the bank only when depositors withdraw dollar bills or make payments to those who do not hold accounts at the bank. As long

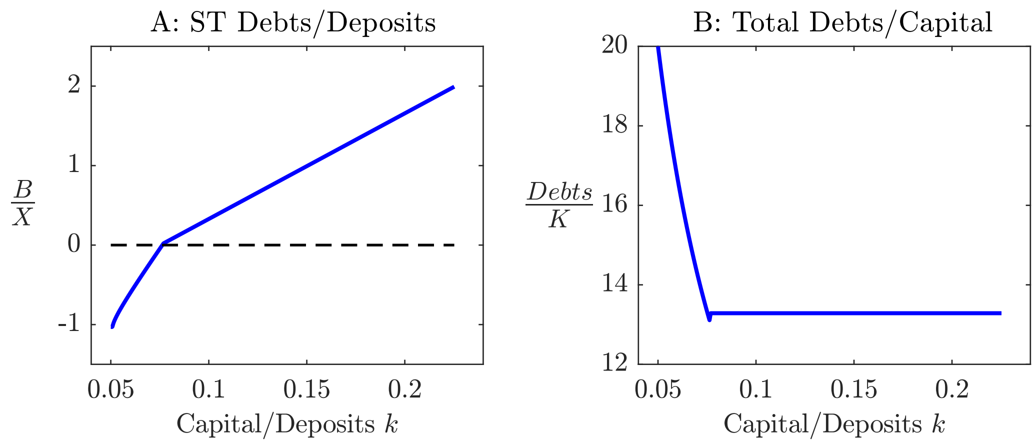

Figure 7: Short-Term Debt and Total Leverage

as depositors are willing to hold deposits, the bank cannot turn away the existing depositors. After hitting the zero lower bound, the bank loses control of its leverage, and when the bank is sufficiently close to incur costly equity issuance, the marginal value of deposits is negative.

5.5 Short-Term Borrowing vs. Safe Asset Demand

Figure 7 analyzes the the bank's debt structure. Panel A plots the ratio of short-term debts to deposits. When capital is abundant relative to deposits, the bank raises funds from short-term debts for risky lending, i.e., $B_t > 0$. In Panel B, we plot the total leverage. The capital requirement limits the loan-to-capital ratio, so, as the bank issues more short-term debts when k increases, the total leverage increases initially and quickly reaches the regulatory limit. Once the capital requirement binds, a further increase of k induces a substitution from deposits to short-term debts. The substitution from deposits to short-term debts reflects the bank's concern over the lack of control over deposit liabilities and the bank's preference for more controllable short-term debts in spite of higher debt costs. It captures deposit risk management that is distinct from loan risk management (dictated by the Merton-style formula (20)).

When the bank's equity capital is scarce relative to its deposit liabilities, the bank switches its short-term debt position, holding risk-free debts to reduce the overall riskiness of its asset portfolio.

When k is small and the deposit rate reaches the lower bound, the bank loses control of its leverage, and therefore, has to work on the asset-side of its balance sheet to de-risk (and to reduce the likelihood of costly equity issuance) by holding risk-free assets, i.e., $B_t < 0$.

As shown in Panel B of Figure 7, once the bank has stopped borrowing short-term debts and deposits become the only type of debts, a further decline of $k = K/X$ induces a lock-step increase of leverage X/K that resembles what the large U.S. banks have gone through during the Covid-19 crisis. In fact, Federal Reserve Board temporarily relaxes the supplementary leverage ratio requirement in April 2020 to alleviate banks' stress, precisely in response to the influx of deposits. Deposit inflows push banks towards costly equity issuance by reducing k, and close to the issuance boundary, $\underline{k}$, the deposits become burdensome and have a negative marginal value.

Capital requirement and leverage regulation play distinct roles in our model. The former is defined on the ratio of risky lending to equity capital, which translates into a constraint on the bank's control variable, A_t/K_t (see (30)). Instead of mandating a loss buffer (as the model does not feature bankruptcy), capital requirement acts as a macro-prudential regulation that limits risk-taking when the bank becomes better capitalized relative to its deposit liabilities, i.e., k increases. In contrast, the supplementary leverage ratio requirement (SLR) is a constraint on the bank's state variable, k, because when k is small, deposits are the bank's only liabilities and k is the total debt-to-equity ratio. As shown in (38), it is the SLR that triggers equity issuance.

6 Comparative Statics

6.1 Leverage Regulation

During the Covid-19 pandemic, U.S. banking regulators relaxed the supplementary leverage ratio (SLR) requirements. Jerome Powell, the Federal Reserve Chairman, emphasized that SLR provision is straining banks' ability to handle deposit influx. "Many, many bank regulators around the world have given leverage ratio relief," Powell said at a news conference following an FOMC meeting. "What it's doing is allowing [banks] to grow their balance sheet in a way that serves their

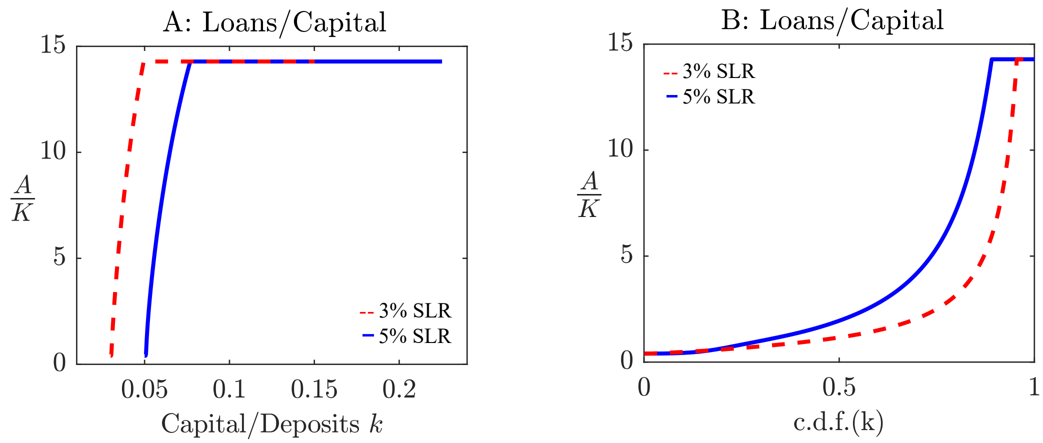

Figure 8: The Impact of Relaxing the SLR Requirement on Bank Lending

customers."[19] Next, we examine the impact of relaxing SLR requirement.

In our model, the bank raises equity and incurs the issuance costs to stay in compliance with the leverage requirement, as shown in (38). The equity issuance costs reflect a form of financial distress costs. Because the bank only faces small diffusive shocks, it can avoid insolvency (and the typical bankruptcy costs) by adjusting its balance sheet continuously, but once $k = K/X$ falls to $\underline{k}$, for example, after unexpected deposit inflow (positive shocks to X), the bank has to raise equity K. Therefore, relaxing the leverage constraint allows the bank to delay paying the distress costs and thereby stimulates lending and deposit-taking by making the bank less risk-averse. Note that it is not leverage regulation that causes risk aversion. Regulation only has an amplifying effect. Even without leverage regulation, the bank still has to raise equity when k falls to zero. Yet it is costly but optimal to do so since the continuation value is positive.

Next, we examine whether relaxing SLR requirement can achieve the intended effect of stimulating lending. To mimic the policy response to the Covid-19 pandemic, we consider a reduction of SLR requirement from 5% to 3%. In Panel A of Figure 8, we compare the risk-taking behavior of the bank (i.e., the ratio A/K) over different values of k. Both before and after the relaxation of

[19]See "Fed's Powell makes case why Congress should relax bank capital rule" by Hannah Lang, American Banker July 29, 2020.

leverage regulation, A/K peaks at the level given by the risk-based capital requirement (5). Before the capital requirement binds, the bank lends more at all values of k after the SLR requirement is relaxed. Relaxing SLR requirement achieves its goal of stimulating lending.

Many are concerned that relaxing the leverage regulation will cause the bank to take on more risks over the long run.[20] Consistent with this intuition, Panel A of Figure 8 shows that the payout and equity issuance boundaries both shift leftward after the regulatory change. Relaxing the SLR requirement makes the bank less risk-averse and hold less equity (relative to deposit liabilities). However, this does not necessarily imply a higher risk exposure per unit of equity capital over the long run as shown in Panel B. Drawing the distinction between Panel A and B is important for understanding the result. Panel A shows the impact of relaxing the SLR requirement given k, which summarizes the current state of balance-sheet conditions of the bank. The move from the solid line to the dashed line mimics the real-world scenario of regulatory change and its immediate effect. In contrast, Panel B shows the long-run effect. As previously discussed, the plot of A/K against the stationary c.d.f. of the state variable k shows how much time the bank spends (horizontal axis) at different values of A/K (vertical axis). Quite contrary to the conventional wisdom, relaxing the SLR actually leads to a smaller risk exposure per unit of equity capital over the long run.

Every time the bank raises equity, it pays the issuance costs. Therefore, over the long run, the bank must break even by generating a sufficient amount of earnings for shareholders that offset such costs. Relaxing the SLR requirement reduces the frequency of costly equity issuance, so the amount of earnings that the bank needs to generate declines. Therefore, the bank becomes less aggressive in earning the loan spread, α_A, through risk-taking. By the same logic, tightening the leverage regulation can actually lead to more aggressive risk-taking over the long run, as it means more frequent equity issuance. As a result, the bank has to engage in more risk-taking per unit of equity to generates earnings that offset the issuance costs in shareholders' expectation.

Tightening leverage regulation achieves the purpose of incentivizing the bank to raise more equity but fails to tame risk-taking. In fact, even though the bank maintains a higher level of equity

[20] When discussing the relaxation of SLR requirement, Fed chairman Powell emphasized that "This will not be a permanent change in capital standards." (see "Fed's Powell makes case why Congress should relax bank capital rule" by Hannah Lang, American Banker July 29, 2020).

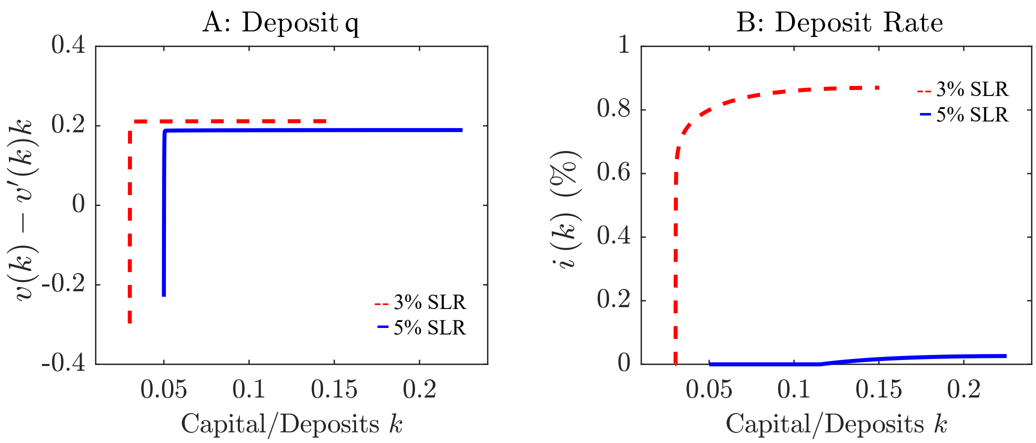

Figure 9: The Impact of Relaxing the SLR Requirement on Deposit Taking

capital, it takes on more risks per unit of capital. Equity issuance costs play a key role in our model by generating a reach-for-yield incentive. The mechanism captures the real-world bankers' focus on return on equity and is similar to the channel of financial instability in Li (2019).[21]

The risk-based capital requirement in (5) is effective in limiting risk-taking as it directly caps the loan-to-capital ratio, A/K. This seems to suggest that risk-based capital requirement is a superior tool relative to regulation on total leverage. However, this conclusion relies on an important assumption that the riskiness of loans is time-invariant, given by the parameter σ_A. When loan risk is countercyclical, risk-based capital requirement amplifies the procyclicality of bank risk-taking (Repullo and Suarez, 2012). Moreover, risk model is vulnerable to manipulation (Plosser and Santos, 2014). Because the model is designed to focus on deposit risk and the bank's imperfect control of balance-sheet size and composition, we do not model bank failure and the associated externalities that motivate both leverage regulation and risk-based capital requirement. Therefore, the paper does not aim to provide a comprehensive evaluation of banking regulations.

As mentioned at the beginning of this subsection, one key motivation for relaxing the SLR requirement during the Covid-19 pandemic is allowing banks to accommodate the unprecedented

[21] In Li (2019) presents a model of financial instability induced by government debt where the supply of government-issued money-like securities (e.g., Treasury bills) squeezes banks' profits from issuing money-like securities, so banks become more aggressive in risk-taking to sustain earnings that can offset the costs of issuing equity over the long run.

deposit inflows without concerns over violating the regulatory constraint. Next, we analyze the impact of relaxing the SLR requirement on deposit-taking. Figure 9 reports the results. In Panel A, we plot the marginal value of deposits, $V_X(X, K) = v(k) - v'(k)k$, before (solid line) and after (dashed line) the SLR requirement is reduced. As previously discussed, the region of k shifts leftward. To understand the model predictions, we pick any value of k on the solid line and consider the vertical movement to the dashed line. This mimics the response of a bank to the regulatory change given its balance-sheet condition (i.e., k). The regulatory change achieves its purpose of stimulating deposit-taking as the marginal value of deposits jumps up. The jump in deposit q is most significant at k where the deposit q was negative before the regulatory change.

If deposit influx continues after the regulatory change (for example, due to new rounds of stimulus payments to households) and outpaces the growth of bank equity via retained earnings, the bank moves along the dashed line in Panel A of Figure 10 to the left and the deposit q declines. Once the deposit q falls into the negative territory, further relaxing the SLR requirement becomes necessary. In fact, after the first regulatory change, deposit q becomes more negative near the new equity issuance boundary, which is due to the fact that the equity capital is lower relative to deposits (i.e., k is lower around the new lower $\underline{k}$) so the effect of deposit inflow on k is greater.

In Panel B of Figure 9, we plot the deposit rate. After the SLR requirement is reduced, the bank sets a higher rate to attract deposits because deposit q is higher. As a result, after the regulatory change, the region of k where the deposit rate lower bound binds shrinks significantly. Therefore, tightening leverage regulation has the unintended consequence of making the deposit rate lower bound a more binding constraint for the bank. The bank controls the stock of deposit liabilities through the deposit rate. When the deposit rate lower bound is more binding, the bank has less control over the size and composition of its balance sheet. This unintended consequence of leverage regulation is a unique implication of our model.

Finally, we examine the impact of SLR requirement on bank shareholders' value. Panel A of Figure 10 shows a clear increase of bank franchise value (divided by deposit stock), $(V(X, K) - K)/X = v(k) - k$, when the SLR requirement is reduced. A higher shareholders' value implies that the bank is more eager to protect its continuation value so the marginal value of equity is

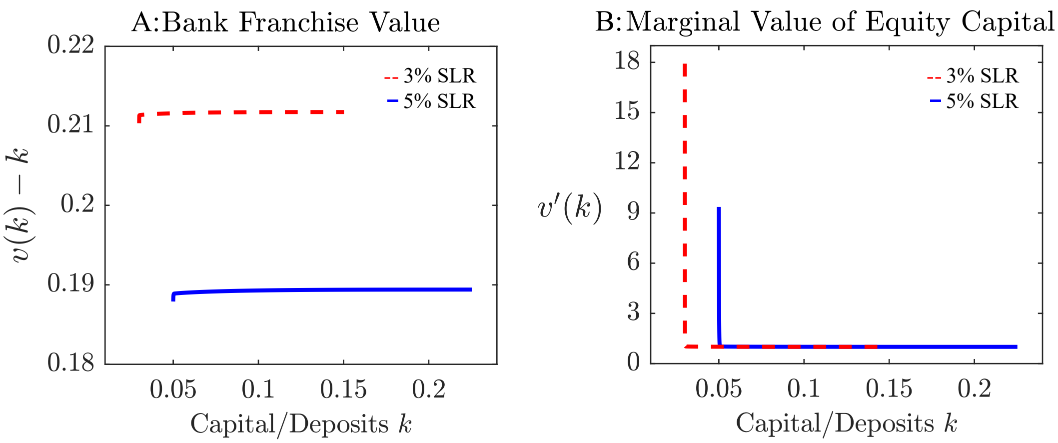

Figure 10: The Impact of Relaxing the SLR Requirement on Bank Valuation

higher near the equity issuance boundary, as shown in Panel B.

Tightening leverage regulation results in a sizeable loss of bank shareholders' value across all states of bank balance-sheet conditions (i.e., different values of k). Kashyap, Stein, and Hanson (2010) point out that the impact of tightening leverage regulation on bank shareholders' value is temporary because shareholders pay the equity issuance (dilution) costs once and then the bank will settle on a higher level of equity capital. This argument holds in a deterministic environment. In our model, uncertainty is the key. Either negative shocks to earnings due to loan losses ($d\mathcal{W}^A < 0$) or positive shocks to the stock of deposit liabilities ($d\mathcal{W}^X > 0$) can reduce $k = K/X$ and trigger costly equity issuance when k falls to $\underline{k}$. Therefore, in an uncertain environment, the impact of leverage regulation on bank shareholders' value is no longer a one-time cost of raising equity. The cost is now reoccurring, and through shareholders' rational expectation, is reflected in bank value even when k is away from $\underline{k}$. Moreover, to reduce the likelihood of incurring the equity issuance cost, the bank has to retain a higher level of equity capital when the leverage regulation tightens, which is also costly to shareholders because dividend payouts are delayed. Overall, our result contributes to the ongoing debt on cost of leverage regulation (Admati et al., 2013).

6.2 Deposit Risk and Interbank Market

Incomplete market is a key ingredient of our model. Because the shocks to deposit stock cannot be hedged, deposit inflow can be problematic because it adds to uncertainty in the future size and composition of balance sheet. Interbank market is often regarded as the place where banks hedge deposit shocks (Bhattacharya and Gale, 1987). Consider deposit flows that result from payment activities. When a depositor sends payment to another depositor at a different bank, the payer's bank loses deposits while the payee's bank gains deposits. The payee's bank can then lend to the payer's bank. As a result, a shock to deposit stock is offset by a simultaneous shock to *net interbank liabilities*. The payer's bank experiences a negative deposit shock but, through the interbank loan, its interbank liabilities increase. The payee's bank experiences a positive deposit shock and increases its interbank assets, so its position in net interbank liabilities decreases.

Given the idiosyncratic nature of such deposit shocks, banks can commit to each other a hedging mechanism that is implemented through interbank loans and automatically offsets deposit shocks with commensurate changes in net interbank liabilities. In fact, we can reinterpret X as the sum of net interbank liabilities and deposits, which is subject to smaller shocks thanks to interbank hedging. This hedging mechanism is implemented automatically in deferred net settlement (DNS) systems (e.g., the Canadian payment system). In real-time gross settlement (RTGS) systems, the implementation relies on an overnight interbank market after payments are settled intraday (Furfine, 2000; Bech and Garratt, 2003; Ashcraft et al., 2011).

In reality, trading frictions in the over-the-counter interbank market make implementing this hedging mechanism costly (Afonso and Lagos, 2015; Bianchi and Bigio, 2014). Moreover, the commitment of banks to lend to each other can break down in crises. Finally, a significant component of deposit shocks is systematic. For example, during the Covid-19 pandemic, the deposit influx in the U.S. is system-wide. Nevertheless, it is meaningful to examine the response of our model to the reduction of deposit risk possibly as a result of interbank hedging.

In Figure 11, we compare the bank franchise value (Panel A) and deposit rate (Panel B) under two values of σ_X. The solid line represents the baseline solution with $\sigma_X = 0.05$, and the dashed line represents the case with $\sigma_X = 0.04$. The reduction of deposit risk captures the effect of

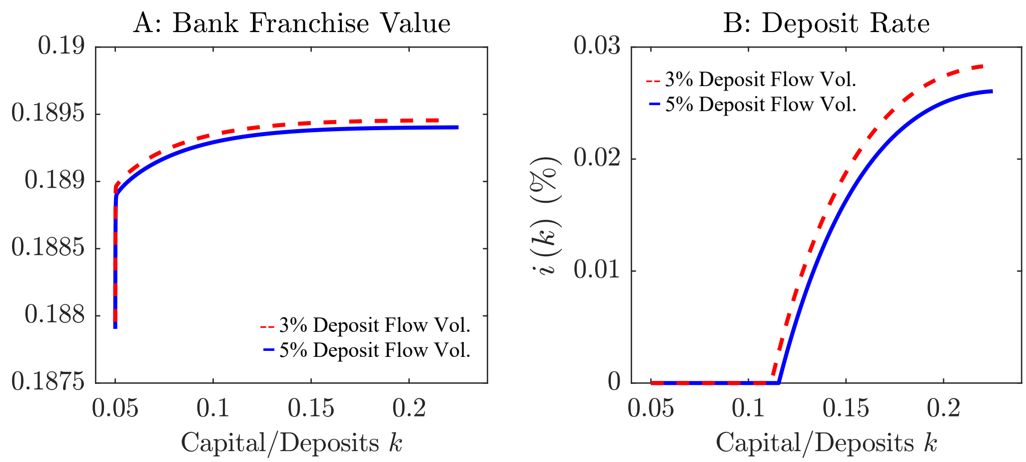

Figure 11: Deposit Risk and Bank Valuation

interbank hedging against the idiosyncratic component of deposit shocks. Less deposit risk leads to a higher bank franchise value, and because the bank is more in control of its deposit liabilities, it is willing to set a higher deposit rate and accept more deposit inflows. Due to the limited space, we omit the graph of deposit q, which increases when σ_X decreases.[22] Our results suggest that the interbank market as a hedging mechanism is important for depository institutions.

During the Covid-19 pandemic, banks' investment in high-quality liquid assets (HQLAs) increased alongside with their deposit liabilities. However, the deposit risk is still a concern because the hedging benefits of interbank net assets cannot be replicated by banks holding safe assets. In our model, the bank can hold risk-free assets by setting $B < 0$. As previously discussed, this helps the bank to reduce its exposure to loan risk, $d\mathcal{W}_t^A$ since the allocation of long-term funds $X + K$ (equity and deposits) tilts towards safe assets. However, holding safe assets does not decrease the bank's exposure to deposit shocks, $d\mathcal{W}_t^X$, which directly hit the amount of long-term liabilities. The interbank hedging mechanism works by netting off deposit shocks with commensurate changes in net interbank liabilities. Therefore, the interbank market plays an essential role in ameliorating the impact of deposit risk even when banks hold a large amount of HQLAs.

Finally, our results shed light on the impact of new payment technologies on banks. Alterna-

[22] The impact of a reduction in σ_X on bank risk-taking, A/K, is limited in our model.

tive payment service providers, such as PayPal and Square, are actively reshaping the topology of payment flows. Our model predicts that the resultant uncertainty in deposit flows leads to a decline of both bank franchise value and deposit rate. This channel is distinct from the standard narrative of bank losing customer base, which implies a decline of bank franchise value but, in contrast to our results, an increase of deposit rate that is necessary for retaining depositors.

Discussion: Contingent Convertible Bond. An alternative to hedging deposit risk is to add contingent convertible bonds into the liability structure. When a recapitalization event is triggered, instead of paying the equity issuance costs to raise new capital, the bank enhances its equity position through the conversion of CoCos. A caveat is that CoCo issuance may also incur costs by the same logic of Myers and Majluf (1984) because CoCos are also information-sensitive securities.

6.3 Low Interest Rate Environment

The bank increases deposit rate when it is well-capitalized (i.e., k is high). Given the deposit rate lower bound, the higher the bank can set its deposit rate in the high-k region, the more flexibility it has to reduce deposit rate when k declines. However, raising deposit rate increases interest expenses and hurts earnings. Therefore, the bank faces a trade-off. It can sacrifices its earnings in the high-k region to gain flexibility of adjusting deposit rate in the low-k region.

As previously discussed, depositors are willing to accept a deposit rate below the risk-free rate r because depositors value the convenience of using deposit accounts to send and receive payments, and as documented by Drechsler, Savov, and Schnabl (2017), banks have market power in geographically segmented deposit markets. When the risk-free rate r is high, the bank can set a high deposit rate and still earn a positive deposit spread $r - i$. When the risk-free rate r is low, the bank has less room to manipulate deposit rate without squeezing the deposit spread too much.

Therefore, the flexibility to adjust deposit rate and to regulate deposit flows depends on the distance between the risk-free rate r and the deposit rate lower bound. When r is high, the bank has more flexibility in setting deposit rate and therefore is more in control of the size and composition of its balance sheet. In contrast, the bank in a low rate environment faces a greater challenge of

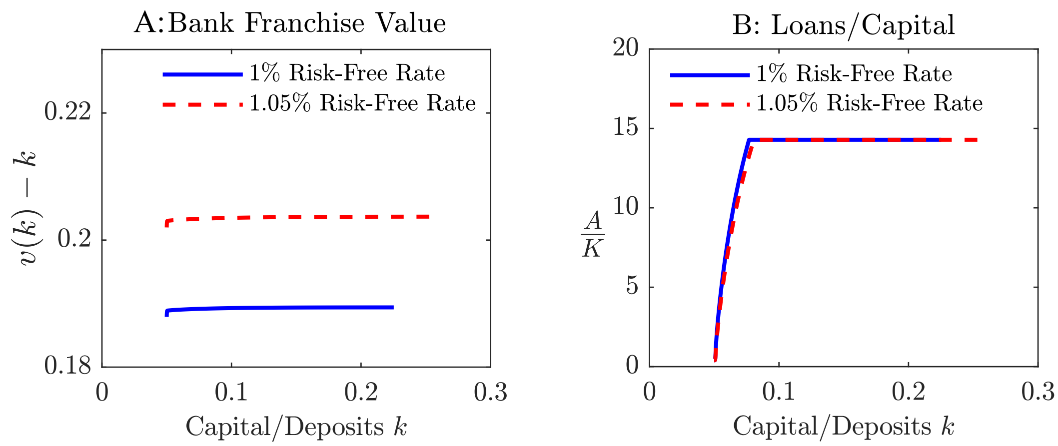

Figure 12: Interest Rate Level, Bank Valuation, and Risk-Taking

managing its deposit liabilities. Our model thus predicts a lower bank franchise value when r is lower. This mechanism is consistent with the empirical findings. For example, Heider, Saidi, and Schepens (2019) find that the distribution of deposit rates of euro-area banks is truncated at zero and more deposit rates bunch at zero once the ECB lowers the policy rate.

Panel A of Figure 12 compares the bank franchise value under different risk-free rates and shows that a higher r leads to a higher bank franchise value. Note that when r increases, the expected return from risky lending, $r + \alpha_A$ in (1), also increases. In other words, when we adjust the risk-free rate, we keep the loan spread constant in line with the evidence in Drechsler, Savov, and Schnabl (2020). In Panel B, we show that when r increases, the bank reduces its risk exposure per unit of equity capital. The increase of franchise value under a higher r results from more flexibility to adjust deposit rate rather than more aggressive risk-taking to earn the loan spread, α_A. Moreover, as shown in both Panel A and B, a higher franchise value incentivizes the bank to retain more equity capital as a risk buffer (i.e., to set the optimal payout boundary at a higher value of k).

Panel A of Figure 13 shows that when r is higher, the deposit q is higher at all levels of k. Deposits become more valuable when the bank can better control the deposit flows through deposit rate. In Panel B of Figure 13, we plot the deposit rate. When r is higher, the bank is more aggressive in raising deposit rate in the high-k region to preserve more flexibility for rate reduction

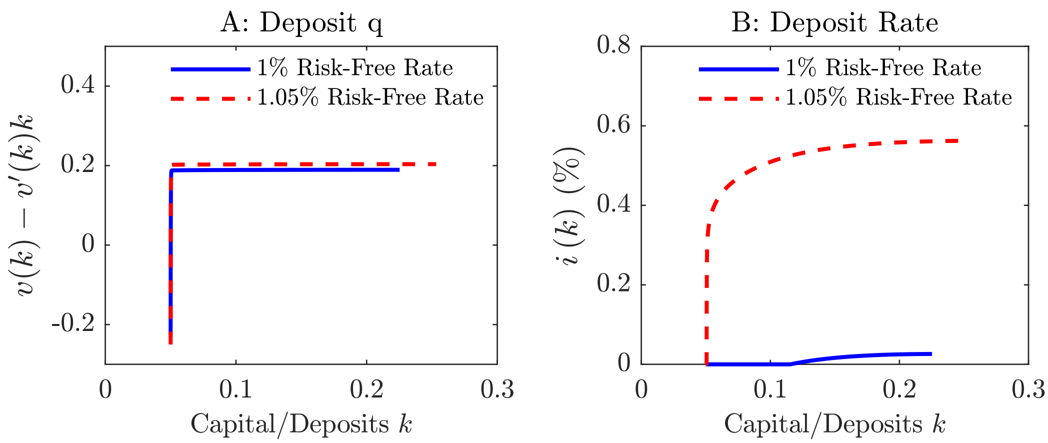

Figure 13: Interest Rate Level, Deposit q, and Deposit-Taking

when k declines in response to negative earning shocks ($d\mathcal{W}_t^A < 0$) or positive deposit shocks ($d\mathcal{W}_t^X > 0$). Under a higher r, the deposit rate lower bound becomes less binding.

Our model provides a rationale that links bank profitability and franchise value to the level of interest rate. The mechanism is related to the channel of deposit market power in Drechsler, Savov, and Schnabl (2017). In their paper, a higher risk-free rate makes cash becomes more expensive to hold, and this allows banks to raise deposit spreads, $r - i$, without losing much deposits to cash. Our specification of deposit flow (2) captures the bank's deposit market power through the stickiness of deposit stock. When the bank adjusts deposit rate, the outflow happens by the order of dt. Different from Drechsler, Savov, and Schnabl (2017), we highlight the risk in deposit flow and the fact that a higher risk-free rate offers the bank more flexibility to manage such risk.

As shown in Panel B of Figure 13, a lower r implies less flexibility to set deposit rate, and more importantly, a greater region of the state variable k where the deposit rate lower bound binds and the bank completely loses control of its deposit stock. The banking literature has largely focused on the positive effect of low interest rate on risk-taking, which we revisits in our setting (Panel B of Figure 12). Our paper puts more emphasis on the management of deposit risk. Moreover, our model predicts that in a low interest rate environment, the bank is more eager to pay out to shareholders (i.e., set a lower $\overline{k}$). This is consistent with the massive share repurchases done by

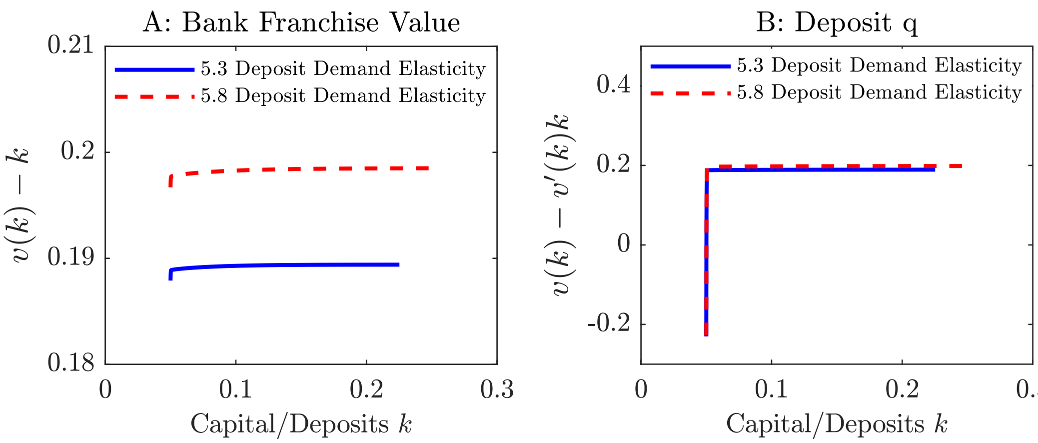

Figure 14: Deposit Demand Elasticity, Bank Valuation, and Deposit q

banks in the last decade of a low interest rate environment.

6.4 Deposit Demand Elasticity and Market Power

The deposit demand elasticity, ω, determines how responsive the deposit flow is to the variation of deposit rate (see $n(i) = \omega i$ in (2)). Therefore, under a higher value of demand elasticity, the bank can better control its deposit liabilities. Panel A of Figure 14 shows that bank franchise value increases in ω. In Panel B, we plot the deposit q, which also increases in ω.

In Panel A of Figure 15, we show that deposit rate is much higher under a higher value of ω. This is consistent with the mechanism that a higher deposit q incentivizes the bank to attract more deposits via a higher deposit rate. In Panel B of Figure 15, we plot the loan-to-equity capital ratio, A/K. Under a higher deposit demand elasticity, the bank reduces risky lending because the higher deposit rate drives up the cost of financing. In spite of earning less from the loan spread, bank value still increases because deposit risk management is more effective when the deposit flow is more responsive to changes in deposit rate.

A higher deposit demand elasticity is often associated with a more competitive deposit market. Consistent with the findings of Drechsler, Savov, and Schnabl (2017), our model generates a higher deposit rate when ω is higher. The typical narrative in the banking literature emphasizes

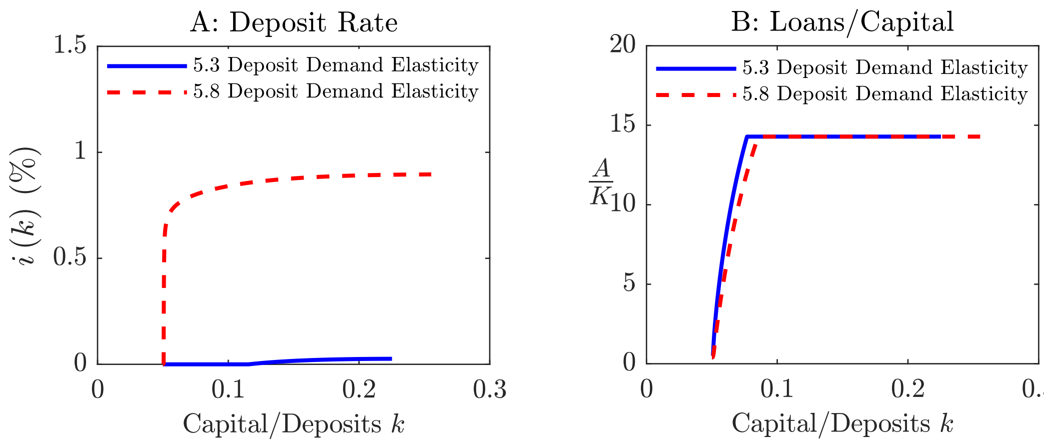

Figure 15: Deposit Demand Elasticity, Deposit-Taking, and Lending

the deposit demand side – when depositors are more price-sensitive, the bank has to set a higher interest rate to attract depositors. This narrative leads to the conclusion that competition erodes bank franchise value (Keeley, 1990).[23] Our model predicts the opposite. When deposit demand elasticity increases, bank franchise value increases. In our model, the increase in financing cost that results from a more elastic demand does have a negative impact on bank value, but such impact is dominated by the positive impact of the bank having more control over its deposit liabilities. Our focus is on the deposit supply side – when depositors are more price-sensitive, the bank can regulate deposit flows more effectively through deposit rate, so deposit q increases and the bank is more willing to pay a higher interest rate to depositors.

So far, our analysis seems to suggest that stronger deposit market power, represented by a more elastic deposit demand, amplifies the challenge of deposit management and hurts bank shareholders because the deposit base becomes less responsive to deposit rate. However, there is another key aspect of deposit market power. Depositors at a bank with a large deposit market share are more likely to send payments to and receive payments from depositors within the same bank. Therefore, the bank is less concerned about the uncertainty in deposit flow that results

[23]We refer the readers to the vast literature on how competition affects bank value (Petersen and Rajan, 1995; Jayaratne and Strahan, 1996; Allen and Gale, 2004a; Boyd and De Nicoló, 2005; Bertrand, Schoar, and Thesmar, 2007; Erel, 2011; Scharfstein and Sunderam, 2016; Drechsler, Savov, and Schnabl, 2017; Liebersohn, 2017).

from depositors' payment activities. In other words, a larger deposit market share translates into a smaller value of σ_X. In Section 6.2, we show that a smaller σ_X leads a higher bank value.

Our paper contributes to the literature on deposit market power (Drechsler, Savov, and Schnabl, 2017) by decomposing market power into two parameters, the deposit demand elasticity ω, and the size of deposit-flow uncertainty, σ_X, that have distinct implications on bank value. The level of risk-free rate r is also key to the impact of deposit market power on bank value. To the extent the bank can exploit its deposit market power, it does so by earning the deposit spread $r - i$. In a low interest rate environment (i.e., under low r), the bank has limited freedom in adjusting the spread given that i has a lower bound typically at zero. In contrast, a high r allows the bank to exploit its deposit market power more by earning a larger deposit spread, $r - i$, and having more flexibility in adjusting the deposit flow through the deposit rate i.

7 Conclusion

Deposits allow banks to cheaply finance lending. Depositors accept a low rate for the convenience of freely moving funds in and out of deposit accounts. The wedge between deposit rate and the prevailing risk-free rate is often termed as the money premium, because the freedom to transfer funds is essential for deposits to serve as means of payment. However, such commitment exposes banks to the uncertainty in deposit flow, so the value of deposits can be drastically different for well-capitalized banks and undercapitalized (risk-sensitive) banks. When a sequence of negative shocks deplete bank capital, the marginal q of deposits can turn negative, meaning that deposit inflows hurt bank shareholders. Our result stands in contrast with the existing literature that is mainly concerned about deposit outflows and bank runs. Our model shows that the key challenge of depository institutions is the lack of perfect control over the balance-sheet size and composition.

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A Solving the Model without Equity Issuance Costs

Without equity issuance cost, $V_X(X,K) = 1$ and the value function is linear in K. By inspecting the HJB equation (16), we know that the bank maximizes π^A (and borrow short-term debts) as long as $\alpha_A > 0$. Therefore, A is the bank's total assets and the capital requirement and SLR restriction are both a restriction on A/K. Because $\xi_L > \xi_K$, capital requirement binds, i.e., $A/K = \xi_K$ (or $\pi_A \left(1 + \frac{1}{k}\right) = \xi_K$). The optimal deposit rate is given by (24): $i = \frac{Q - \frac{1}{\omega}}{\theta}$.

Next we solve Q using the HJB equation (16) under the conjecture of value function $(k + Q)\, X$:

$$\rho k + \rho Q = Q\left(-\delta_X + \omega i\right) + r + rk + \xi_K \alpha_A k - \left[i + \frac{\theta}{2}\left(\omega i\right)^2\right].$$

For this equation to hold, we need the following quadratic equation to hold

$$\frac{\omega}{\theta}\left(1 - \frac{\omega}{2}\right) Q^2 - \left(\frac{2}{\theta} + \delta_X + \rho\right) Q + r + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta} = 0\,, \tag{A.1}$$

which solves Q, and the coefficient on k is equal to zero, i.e.,

$$\rho = r + \xi_K \alpha_A\,. \tag{A.2}$$

Equation (A.2) requires that the bank is indifferent between paying out dividend and retaining equity. If $\rho > r + \xi_K \alpha$, the bank prefers paying out dividends because the expected return on equity capital is below the shareholders' required rate of return. If $\rho < r + \xi_K \alpha$, the bank never pays dividend and prefers to raise an infinite amount of equity because the expected return on equity is greater than the shareholders' required return.

Under the condition,

$$\left(\frac{2}{\theta} + \delta_X + \rho\right)^2 \geq \frac{4\omega}{\theta}\left(1 - \frac{\omega}{2}\right)\left[r + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta}\right], \tag{A.3}$$

the roots of Equation (A.1) exist and are given by

$$Q = \frac{\sqrt{\left(\frac{2}{\theta} + \delta_X + \rho\right)^2 + \frac{4\omega}{\theta}\left(\frac{\omega}{2} - 1\right)\left[r + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta}\right]} - \left(\frac{2}{\theta} + \delta_X + \rho\right)}{\frac{2\omega}{\theta}\left(\frac{\omega}{2} - 1\right)}.\text{[24]} \tag{A.4}$$

[24] When $\omega > 2$ (as in our calibration), the other root is negative. We focus on the case where deposit value, $Q > 0$.

B Alternative Setup with Liquidity Requirement

In this appendix, we enrich the decision environment of the bank. On the asset side of balance sheet, the bank must hold assets that are more liquid than loans (Drechsler, Savov, and Schnabl, 2018). These assets can be reserves or other high-quality liquid assets (HQLA).

At time t, the bank chooses the value of liquidity holdings, denoted by R_t. Liquidity holdings pay an interest rate ι that is below the risk-free rate r. The bank is willing to pay the carry cost for the benefits of having a more liquid asset portfolio, as shown in the law of motion of equity capital

$$\begin{aligned} dK_t =& A_t \left[(r + \alpha_A) \, dt + \sigma_A d\mathcal{W}_t^A \right] - B_t r dt - X_t i_t dt - C \left(n \left(i_t \right), X_t \right) dt \\ & - dU_t + dF_t + R_t \iota dt - S \left(R_t, X_t, A_t \right) dt \, . \end{aligned} \tag{B.1}$$

In comparison to (10), the last two terms are new. The interest income from liquidity holdings is given by $R_t \iota dt$. The last term, $S(R_t, X_t, A_t)$, captures loss due to illiquidity of asset portfolio. This specification is isomorphic to the following microfounded setup: a Poisson-arriving withdrawal of a large amount of deposits can only be met by liquidity holdings and selling a large amount of loans in exchange for liquidity incurs a fire-sale cost (Moreira and Savov, 2017; Drechsler, Savov, and Schnabl, 2018). Accordingly, we assume $S_R(R_t, X_t, A_t) < 0$, $S_X(R_t, X_t, A_t) > 0$, and $S_A(R_t, X_t, A_t) > 0$. Note that in the main text, we only consider small (diffusive) deposit shocks.

The bank has to meet the regulatory requirement of liquidity holdings:

$$R_t \geq \xi_R X_t \, . \tag{B.2}$$

This regulatory constraint can be motivated by the traditional reserve requirement or more recent requirement on liquidity coverage ratio (Basel Committee on Banking Supervision, 2013). When $B < 0$, the bank holds risk-free assets that pay interest rate r. Note that these assets are not part of the liquidity holdings. Here we draw the distinction between liquid and illiquid safe assets in line with the evidence that these assets offer different yields (Krishnamurthy, 2002; Nagel, 2016).

The bank has long-term funding equal to $X_t + K_t$. As in the main text, let π_t^A denote the portfolio weight on loans, i.e., $\pi_t^A (X_t + K_t) = A_t$, and π_t^R denote the portfolio weight on liquid assets, i.e., $\pi_t^R (X_t + K_t) = R_t$, so the weight on bonds is $\left(\pi_t^A + \pi_t^R - 1 \right)$ because $B_t =$

$A_t + R_t - (X_t + K_t)$. We can rewrite the law of motion for K_t in (B.1) as

$$dK_t = (X_t + K_t)\left[r + \pi_t^A \alpha_A - \pi_t^R (r - \iota)\right] dt + (X_t + K_t)\, \pi_t^A \sigma_A d\mathcal{W}_t^A - X_t i_t dt \\ - C\left(n\left(i_t\right), X_t\right) dt - S\left(\pi_t^R \left(X_t + K_t\right), X_t, \pi_t^A \left(X_t + K_t\right)\right) - dU_t + dF_t\,. \tag{B.3}$$

Accordingly, the HJB equation in the interior region where $dU_t = 0$ and $dF_t = 0$ is

$$\begin{aligned}\rho V(X,K) = \max_{\{\pi^A, \pi^R, i\}}\ & V_X(X,K)\, X\left[-\delta_X + n(i)\right] + \frac{1}{2} V_{XX}(X,K)\, X^2 \sigma_X^2 \\ & + V_K(X,K)(X+K)\left[r + \pi^A \alpha_A - \pi^R (r-\iota)\right] + \frac{1}{2} V_{KK}(X,K)(X+K)^2 \left(\pi^A \sigma_A\right)^2 \\ & - V_K(X,K)\left[S\left(\pi^R (X+K), X, \pi^A (X+K)\right) + Xi + C\left(n(i), X\right)\right] \\ & + V_{XK}(X,K)\, X(X+K)\, \pi^A \sigma_A \sigma_X \phi\,.\end{aligned} \tag{B.4}$$

Risk-taking. The first-order condition for π^A gives the following solution:

$$\pi^A = \min\left\{\frac{\alpha_A + \epsilon(X,K)\,\sigma_A \sigma_X \phi - S_A(R,X,A)}{\gamma(X,K)\,\sigma_A^2\left(\frac{X+K}{K}\right)}\,,\ \frac{K}{\xi_K (X+K)}\right\}. \tag{B.5}$$

While setting up $\pi^A = A/(X+K)$ as the control variable is convenient for solving the model, it is intuitive to express the solution in loan-to-capital ratio, i.e., $A/K = \pi^A (X+K)/K$:

$$\frac{A}{K} = \min\left\{\frac{\alpha_A + \epsilon(X,K)\,\sigma_A \sigma_X \phi - S_A(R,X,A)}{\gamma(X,K)\,\sigma_A^2}\,,\ \frac{1}{\xi_K}\right\}. \tag{B.6}$$

In comparison with (20), the only difference is that the numerator is deducted by $S_A(R,X,A)$.

Liquidity Holdings. When the liquidity requirement (B.2) does not bind, the optimality condition for π^R equates the marginal cost of holding reserves, i.e., accepting the below-r rate of return ι, and the marginal benefit of holding reserves to reduce the payment settlement cost:

$$r - \iota = -S_R\left(\pi^R (X+K), X, \pi^A (X+K)\right)\,. \tag{B.7}$$

The reserve requirement can be rewritten as the following restriction on π^R:

$$\pi^R \geq \frac{\xi_R X}{(X+K)}\,. \tag{B.8}$$

Next, we specify the functional form of $S(R,X,A)$ that satisfies the properties that $S(R,X,A)$ decreases in R and increases in X and A:

$$S(R,X,A) = \frac{1}{2}\frac{(\chi_1 X + \chi_2 A)^2}{R}\,. \tag{B.9}$$

The numerator is convex in X and A while the denominator is linear in R. Therefore, to maintain the same level of $S(R,X,A)$, the bank will have to hold increasingly more liquidity as it expands its balance sheet (i.e., increases X and A). This captures the decreasing marginal return to liquidity holdings that have been microfounded in various ways (Moreira and Savov, 2017).

Under this functional form of $S(R,X,A)$, we obtain

$$S_R(R,X,A) = -\frac{1}{2}\left(\frac{\chi_1 X + \chi_2 A}{R}\right)^2\,. \tag{B.10}$$

Therefore, the optimality condition (**??**) for π_t^R implies that $r - \iota = \frac{1}{2}\left(\frac{\chi_1 X + \chi_2 A}{R}\right)^2$, so rearranging the equation we obtain the following reserve holding policy

$$R = \frac{\chi_1 X + \chi_2 A}{\sqrt{2(r-\iota)}} \tag{B.11}$$

This liquidity holding policy is in the spirit of Baumol (1952) and Tobin (1956) who show that the demand for liquidity is equal to the product of transaction costs (mapping to χ_1 and χ_2) and transaction needs (mapping to X and A) divided by the square root of two times the carry cost. As previously discussed, a microfoundation can be built for $S(R,X,A)$ where the transaction or liquidity needs of the bank arises from deposit withdrawal and depends the amount of relatively illiquid assets (loans) in the portfolio that are subject to fire-sale losses.

Given the functional forms of $S(R,X,A)$ and deposit maintenance costs in the main text, the bank's problem is homogeneous in X and its value function $V(X,K) = v(k)\,X$, where

$$k = \frac{K}{X}\,. \tag{B.12}$$

And, as in the main text, we simplify the expressions of the effective risk aversion in (18)

$$\gamma\left(k\right)=\frac{-V_{KK}\left(X,K\right)K}{V_{K}\left(X,K\right)}=-\frac{v''\left(k\right)k}{v'\left(k\right)}\,, \tag{B.13}$$

and the elasticity of marginal value of capital to deposits in (19)

$$\epsilon\left(k\right)=\frac{V_{XK}\left(X,K\right)X}{V_{K}\left(X,K\right)}=-\frac{v''\left(k\right)k}{v'\left(k\right)}\,, \tag{B.14}$$

which happens to be equal to $\gamma\left(k\right)$.

Next, we simplify the expression of loan-to-capital ratio, a measure of the bank's risk-taking. First, note that from (B.11), we obtain the marginal illiquidity cost of loans:

$$S_{A}\left(R,X,A\right)=\chi_{2}\left(\frac{\chi_{1}X+\chi_{2}A}{R}\right)=\chi_{2}\sqrt{2\left(r-\iota\right)}\,, \tag{B.15}$$

Using (B.15) and $\epsilon\left(k\right)=\gamma\left(k\right)$, we simplify the optimal loan-to-capital ratio from (20):

$$\frac{A}{K}=\min\left\{\frac{\alpha_{A}-\chi_{2}\sqrt{2\left(r-\iota\right)}}{\gamma\left(k\right)\sigma_{A}^{2}}+\frac{\sigma_{X}}{\sigma_{A}}\phi\,,\ \ \frac{1}{\xi_{K}}\right\}\,, \tag{B.16}$$

The only difference from (30) is that in the numerator, we subtract α_{A} by the marginal illiquidity cost $\chi_{2}\sqrt{2\left(r-\iota\right)}$. To make lending profitable, we impose the parameter restriction

$$\alpha_{A}>\chi_{2}\sqrt{2\left(r-\iota\right)}\,. \tag{B.17}$$

Using these expressions, we can rewrite the HJB equation (B.4) as

$$\begin{aligned}\rho v\left(k\right)=\max_{\pi^{A},\pi^{R},i}\ &\left[v\left(k\right)-v'\left(k\right)k\right]\left(-\delta_{X}+\omega i\right)+\frac{1}{2}v''\left(k\right)k^{2}\sigma_{X}^{2}\\&+v'\left(k\right)\left(1+k\right)\left[r+\pi^{A}\alpha_{A}-\pi^{R}\left(r-\iota\right)\right]+\frac{1}{2}v''\left(k\right)\left(1+k\right)^{2}\left(\pi^{A}\sigma_{A}\right)^{2}\\&-v'\left(k\right)\left[\frac{1}{2}\left(\frac{\frac{\chi_{1}}{1+k}+\chi_{2}\pi^{A}}{\pi^{R}}\right)^{2}\pi^{R}\left(1+k\right)+i+\theta_{0}+\frac{\theta_{1}}{2}\left(\omega i\right)^{2}\right]\\&-v''\left(k\right)k\left(1+k\right)\pi^{A}\sigma_{A}\sigma_{X}\phi\,.\end{aligned} \tag{B.18}$$

To show that (B.18) is an ODE for $v(k)$, we need to show that the control variables only depend on k and the level and derivatives of $v(k)$. First, by definition, $\pi^A = A/(X+K)$, so we obtain the following simplified expression for π^A from (B.16):

$$\pi^A = \left(\frac{A}{K}\right)\left(\frac{K}{K+X}\right) = \min\left\{\frac{\alpha_A - \chi_2\sqrt{2(r-\iota)}}{\gamma(k)\,\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi\,,\ \ \frac{1}{\xi_K}\right\}\left(\frac{k}{1+k}\right). \tag{B.19}$$

Rearranging (B.11), we can solve π^R as a linear function of π^A and the state variable k:

$$\pi^R = \frac{\chi_2}{\sqrt{2(r-\iota)}}\pi^A + \frac{\chi_1}{(1+k)\sqrt{2(r-\iota)}}\,, \tag{B.20}$$

so it also only depends on k and the level and derivatives of $v(k)$. The deposit rate, still given by (24) in the main text, only depends on $V_X(X,K) = v(k) - v'(k)\,k$ and $V_K(X,K) = v'(k)$.

After substituting the optimal control variables into the HJB equation, we obtained an ordinary equation with the same boundary conditions discussed in the main text. The determination of endogenous upper bound of k also follows the main text. The only difference is in the determination of endogenous lower bound of k, i.e., the equity issuance boundary.

Let $\underline{k}_S$ denote the lower bound in (38) implied by the supplementary leverage ratio (SLR) requirement. The liquidity requirement implies another lower bound $\underline{k}_L$. Substituting (B.20) into the reserve requirement (B.8), we have

$$\frac{\chi_2}{\sqrt{2(r-\iota)}}\pi^A + \frac{\chi_1}{(1+k)\sqrt{2(r-\iota)}} \geq \frac{\xi_R}{(1+k)}\,, \tag{B.21}$$

Using (33) to substitute out π^A and rearranging the equation, we have

$$\min\left\{\frac{\alpha_A - \chi_2\sqrt{2(r-\iota)}}{\gamma(k)\,\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi\,,\ \ \frac{1}{\xi_K}\right\} k \geq \frac{\xi_R\sqrt{2(r-\iota)} - \chi_1}{\chi_2}\,. \tag{B.22}$$

In our numeric solution, the right side increases in k (as $\gamma(k)$ increases in k). Therefore, (B.22) imposes a lower bound of k, denoted by $\underline{k}_L$. Therefore, we have

$$\underline{k} = \max\{0,\, \underline{k}_S,\, \underline{k}_L\}\,. \tag{B.23}$$

To sum up, introducing the bank's needs to hold reserves or HQLA leads to three changes in the solution. First, the new control variable, optimal liquidity-holding policy, is given by the Baumol-Tobin style money demand (B.11). Second, in the optimal risk-taking policy (B.16), α_A is subtracted by the marginal illiquidity cost of loans. Third, the equity issuance boundary is defined by (B.23) nesting considerations of liquidation, SLR requirement, and liquidity requirement.