AGGREGATE-DEMAND AMPLIFICATION OF SUPPLY DISRUPTIONS:
THE ENTRY-EXIT MULTIPLIER

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ABSTRACT

Due to its impact on nominal firm profits, price rigidity amplifies the response of entry and exit to adverse supply shocks, such as COVID-19. This "entry-exit multiplier" triggers substantial magnification of the welfare losses due to negative supply shocks—even in an efficient-entry benchmark. In addition to those second-order effects, price rigidity also induces first-order amplification under external returns, when entry is no longer efficient. Endogenous entry-exit thus radically changes the consequences of nominal rigidities: in addition to the aggregate-demand amplification of supply disruptions, it also reconciles the response of hours worked across the benchmark New Keynesian and RBC models.

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1 Introduction

The economic recession following the recent COVID-19 crisis has been associated with very sharp responses in business entry and exit. The increase in exit has reached hitherto unseen magnitudes: real-time data from Womply reported by Chetty et al (2020) shows that 40% of U.S. small businesses closed down in march 2020 and, despite a timid recovery in late 2020, almost 40% were still closed in mid-2021. Data from a different source (Homebase) reported in Crane et al (2021) paints a similar picture, as do model-based estimates by Kalemli-Ozcan, Gourinchas, Penciakova, and Sander (2020) for Europe.[1] These supply-side developments are mirrored on the demand-side: many of the consumer or intermediate input varieties customarily consumed by households and firms have simply become unavailable. At the same time, a deep recession happened: measures of real activity fell sharply and stayed under their long-run average for more than a year; most macroeconomists would agree that this reflects a fall of output below potential, i.e. that the "output gap" has been negative throughout. What does a business-cycle macroeconomic model consistent with these entry-exit features imply about the propagation of supply shocks to macroeconomic aggregates?

The negative supply impulse we consider as a metaphor for the COVID-19 crisis is a classic negative productivity shock: a downward shift in the production function associated with severe restrictions on the availability of inputs. The classic question we revisit is: can this supply disruption lead to a demand-recession, understood as a fall in activity, output and income, when output is demand-determined, e.g. with nominal rigidities as in New Keynesian (hereinafter NK) models? We find that it does and that the endogenous responses of entry-exit associated with those nominal rigidities plays a key role.

We first show that the response of entry-exit to supply shocks is amplified in a model with sticky prices: a supply-side phenomenon we dub *the entry-exit multiplier*. The response of the number of firms is amplified when firms cannot adjust prices in response to productivity shocks. Those distortions induce changes in profits that trigger entry-exit dynamics, setting off a feed-

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[1]Entry data that would parallel this for the production side is unavailable; existing real-time measures of broad "entry" such as the Census Bureau's Business Formation Statistics discussed in Haltiwanger (2020) are unlikely to reflect "real" entry translating fully into jobs and production. That measure of new business applications also fell by 40% at the onset of the crisis, to then recover to unprecedented levels throughout Q2 and Q3; however, this was not reflected in the Q2 and Q3 entry data from the Business Employment Dynamics that came out shortly before writing the current version.

back loop to (endogenous) aggregate productivity. Consider a negative shock. Firms wish to increase their price to reflect their increased marginal cost. With sticky prices they cannot, so they are "stuck" with their suboptimal price. This induces further losses and triggers further exit, engendering an additional (endogenous) aggregate productivity decrease that amplifies the initial impulse. The endogenous fall in aggregate productivity is due to the standard variety effect in models with entry and exit.

This mechanism captures an intuition that is more general than the inability to reset prices. It applies more generally to profitability shocks induced by monetary rigidities. Thus, this is a reduced form for frictions that impinge upon intensive-margin adjustments, with negative consequences for profitability. Kalemli-Ozcan, Gourinchas, Penciakova, and Sander (2020) present an example of such an alternative model. They use it to explain exit in the aftermath of COVID-19 but focus on direct, first-round effects—whereas our complementary focus is on general-equilibrium effects propagated through endogenous entry-exit.

In our benchmark economy where output is a constant elasticity of substitution aggregate of intermediate inputs, henceforth CES, entry responds proportionately with the supply shock when prices are flexible. This is the well-known market size effect on entry with constant markups going back to Krugman (1980). When prices are sticky, however, the same supply shock leads to a more than proportionate response of entry. This is the entry-exit "multiplier" under sticky prices. While this channel is present and operates in any model with entry and nominal rigidities studying monetary policies (see i.a. Bilbiie Ghironi Melitz 2007; Bergin Corsetti 2008; Bilbiie Fujiwara Ghironi 2014; Bilbiie 2019), it has not been identified, isolated, and analytically characterized before. This is our paper's first contribution.[2] This amplification is important not only in and of itself but especially due to its consequences for the further amplification of the output response leading to a demand-determined recessions. This transmission channel has not been previously analyzed.

We characterize the conditions under which *aggregate-demand amplification* of supply shocks (a magnified response of aggregate output under sticky prices) occurs—relative to the flexible-price benchmark. It is well-known that such a negative supply shock *cannot* drive a demand recession

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[2]Our results generalize to models of entry with sunk costs where the number of firms acts as a state variable providing propagation and matching profits' dynamics, such as Bilbiie Ghironi, and Melitz (2007, 2012) and Gutierrez, Jones, and Philippon (2021). We nevertheless focus on the free-entry, zero-profits model of entry with a fixed per-period cost for analytical tractability (as in e.g. Jaimovich and Floetotto (2008) for flexible prices and Bilbiie (2019) for sticky prices).

in a standard NK model, wherein a temporary negative productivity shock implies an *increase* in the output gap: a smaller fall in output under sticky prices than under flexible prices. Our second contribution is thus to analyze the entry-exit multiplier's ensuing impact on aggregate demand.

We first show that in our benchmark CES economy with sticky prices, amplification of *negative* shocks always occurs. Indeed, we identify an asymmetry in the effects of shocks on the "output gap": negative shocks make sticky-price output over-react and positive shocks make it under-react. In this benchmark case, the responses to shocks under either sticky or flexible prices are identical to the first order, thus isolating the contribution of higher-order nonlinear terms. This effect is driven by the curvature of output in intermediate input variety. It is increasing in the elasticity of substitution between goods under sticky prices. Throughout, we focus on a nonlinear solution of the model that captures higher-order effects. The latter are especially important with large shocks like the COVID-19 crisis, which is associated with unprecedented changes in aggregate variables rendering first-order perturbation methods insufficient and potentially misleading.[3]

We then show that this amplification of aggregate demand through our entry-exit multiplier can have a first-order impact when the equilibrium level of entry is inefficient. To highlight this, we explore deviations from CES aggregation that feature such an inefficiency, e.g. the presence of external returns to intermediate input variety. Our main takeaway is that the entry-exit multiplier then yields first-order aggregate-demand amplification when the aggregate productivity benefit of input variety is larger than the net markup (the profit incentive for entrants): supply-driven demand recessions occur when "demand" forces exceed "supply" forces for the creation and destruction of new input varieties.

With *no* (or *exogenous*) entry and exit—such as the standard NK model—the response of aggregate activity is proportional to the adverse supply shock when prices are flexible: if productivity falls by 1%, consumption and output fall by 1%. In this case, the response is at most proportional, and generally smaller than 1% under sticky prices. In other words, there is aggregate-demand *dampening* of supply shocks, an issue well-known in NK models.[4]

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[3]We use first-order approximations only for illustrative purposes, to elucidate some key mechanisms that survive to first-order (even when other mechanisms do not).

[4]The response ou output with sticky prices itself can be positive (if prices are not entirely fixed, etc.) – but the key point is that it is always less than one. That is, the output gap (the key summary statistic) is positive in response to negative supply shocks (the response under sticky is smaller than under flexible prices).

With *endogenous* entry and exit, there is amplification of the aggregate response relative to this no-entry model, even under *flexible* prices. This is due to the "increasing returns" inherent in any expanding-variety model magnifying the effect of productivity shocks, whereby entry variations act as endogenous aggregate productivity.[5] But there is *further* amplification under sticky prices. Thus, endogenous entry-exit radically changes the consequences of sticky prices for supply disruptions: price stickiness *dampens* the aggregate response without entry, but it *amplifies* that aggregate response with entry-exit. Furthermore, the sticky-price amplification of recessions under entry-exit is an increasing function of the *size* of the exogenous disruption: the amplification works in part through higher-order, nonlinear terms due to the concavity of welfare in the number of input varieties. This is particularly relevant for large shocks like those associated with the COVID-19 crisis, where such nonlinearities are likely to be especially important.

Nevertheless, aggregate-demand amplification can also occur though first-order, linear terms as soon as we depart from the efficient-entry CES benchmark. In a nutshell, supply-driven demand recessions can occur when the benefit of input variety exceeds the markup, making entry-exit inefficiently low in the market equilibrium relative to the planner's optimum. Under sticky prices the response of entry-exit is magnified through the multiplier effect that we identified. This immediately translates into first-order magnification of the output response too.

Our result can also be viewed through the lens of the intertemporal logic that is the backbone of business-cycle models, including NK models. With no entry and fixed prices, there is no change in the real interest rate, so consumption is fixed by the Euler equation. However, the real interest rate does change even when prices are fixed when entry is endogenous. Consumption falls if there is exit because this creates expected deflation in the welfare-relevant CPI, and an increase in the real interest rate, generating intertemporal substitution towards future consumption.

An important additional implication of our framework for business-cycle analysis is that entry-exit brings the response of hours worked in our sticky-price model in line with the response of its flexible-price counterpart (akin to the workhorse RBC model). This resolves a long-standing issue with NK models that has been the subject of a spirited debate. In particular, while RBC models

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[5]This amplification is studied in detail i.a. in Bilbiie, Ghironi, and Melitz (2012) in a model with sunk-cost dynamic entry, and earlier in Devereux, Head and Lapham (1996) and Chatterjee and Cooper (1993) in a "static entry" model. It is also related to the welfare gain of trade and market size in the "new trade theory" with monopolistic competition, e.g. Melitz (2003). Gopinath and Neiman (2014) provide trade-based empirical evidence for the negative efects of adverse shocks on endogenous productivity.

focus on—and embed at their core—procyclical hours worked, standard NK models customarily imply countercyclical hours in response to productivity shocks. Since this is driven by income effects on labor due to profit variations, the entry-exit channel endogenously eliminates those income effects and can thus generate procyclical hours when the flexible-price model does. Of course, the model can still imply arbitrary hours' responses to TFP shocks: their sign, however, will not be governed by price stickiness but by whether the shock is transitory of persistent, by labor supply elasticity and income effect—features that seem more inherently relevant for the dynamics of hours.

Finally, we extend our analysis to the case of arbitrary intertemporal substitution in consumption. The requirement for the entry-exit multiplier and for aggregate-demand amplification of *aggregate* supply shocks is then that the elasticity of substitution between goods be *higher* than the elasticity of intertemporal substitution in consumption. Virtually all empirical estimates for those two elasticities satisfy this ranking. We compare and contrast our analysis with a recent paper by Guerrieri et al (2002). In their benchmark model with *exogenous* variation in the number of good-sectors and *sector-specific shocks*, the condition for aggregate-demand amplification requires prima facie a *reversal* in the ranking of those two elasticities: higher intertemporal than across goods-sectors. We highlight these key differences in further detail below and in Section 4.

Although we initially focus on the non-linear implications of our basic NK model, we also develop a loglinearized framework of a substantially more general model (in terms of functional forms). We highlight the connections with the textbook treatments of the NK model such as Woodford (2003) and Gali (2008).

Related literature

A large and growing literature emphasizes the role of endogenous entry and variety with *flexible prices* for business cycles, studying macro fluctuations and normative properties i.a. Bilbiie, Ghironi, and Melitz (2012, 2019), Colciago and Etro (2010), Jaimovich and Floetotto (2008), Lewis and Poilly (2014), Hamano and Zanetti (2017), Cacciatore and Fiori (2016), Dixon and Savagar (2020), Edmond, Midrigan, and Xu (2020), Michelacci, Paciello, and Pozzi (2019). A few papers have analyzed these models with nominal rigidities, focusing on monetary policy (Bilbiie Ghironi Melitz,

2007; Bergin and Corsetti, 2008; Bilbiie, Fujiwara, and Ghironi, 2014).[6]

In our model, the direction of the response of hours worked to supply shocks is invariant to price stickiness. This has significant implications for the literature studying and contrasting the empirical properties of RBC and NK models, e.g. Gali (1999), Basu, Fernald and Kimball (2006), Christiano, Eichenbaum, and Vigfusson (2003), Chari, Kehoe, and McGrattan (2008), and Alexopoulos (2011). Cantore et al (2014) provide different mechanisms, complementary to ours, that can reduce the discrepancy between the hours' response in RBC vs NK models: the factor-augmenting nature of shocks and the capital-labor substitutability in CES production.[7]

The standard NK model's failure to produce demand-recessions in response to negative supply shocks is the starting point of a recent important contribution, Guerrieri et al (2020), that is complementary to ours. The authors call such occurrences "Keynesian supply shocks". They build a 2-sector model that predicts those responses to *sector-specific* exogenous-exit shocks. In a benchmark CES economy (across goods and over time), the necessary condition is that the elasticity of intertemporal substitution be larger than the elasticity of substitution between sectors/goods. In other words, the requirement is that goods-sectors be Edgeworth complements: that households be more willing to substitute intertemporally in the same good than they are to substitute between two goods (understood as sectors). The authors explore extensions that allow relaxing this restriction, most notably the interaction with liquidity constrained, high marginal propensity to consume households.[8] Our focus in the current paper is at the *disaggregated level*, *within* sectors. At this level, the above parameter restriction is at odds with most of the empirical literature measuring these key elasticities: the intertemporal elasticity is at most two (and most often estimated to be below one), while the elasticity of substitution between varieties is estimated to be

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[6]Earlier pioneering contributions on RBC-like models with entry include Chatterjee and Cooper (1993), Devereux, Head, and Lapham (1996), Campbell (1998), and Cook (2001); Chaterjee et al (1993) and Jaimovich (2007) focused on entry and strategic complementarities leading to multiple equilibria and endogenous fluctuations. More recently, Bilbiie, (2019), Cooke and Damianovic (2020), Colciago and Silvestrini (2020), Gutierrez et al (2021) and Hamano and Zanetti (2020) used models with entry and nominal rigidities to study departures from monetary neutrality, the effects of market concentration, the implications of the ZLB, and selection with firm heterogeneity, respectively.

[7]For more recent evidence supporting a positive response of hours worked to a positive *transitory* productivity shock, see e.g. Peersman and Straub (2009) and Foroni et al (2018). A different recent literature studies the response of the labor *share to demand* shocks under nominal rigidities, e.g. Kaplan and Zoch (2020).

[8]Guerrieri et al observe that the failure of NK models to generate keynesian supply shocks is robust to the introduction of incomplete markets; see also Bilbiie (2008) for an illustration of that point in a two-agent TANK model, and for an alternative mechanism generating aggregate-demand recessions in response to negative productivity shocks (labelled "inverted aggregate demand logic" therein), through high enough elasticity of income of constrained agents. See also Auerbach et al (2021) for a related "inverted Keynesian cross" mechanism.

substantially higher, somewhere in the range between four and eight. We also focus on a different, complementary channel driven by the *endogenous* entry-exit decisions of profit-maximizing producers. And we consider *aggregate* supply shocks affecting all firms and products symmetrically. We thus abstract from the important consequences of both sector-specific exogenous shocks and of household heterogeneity, both essential ingredients in Guerrieri et al's analysis.[9] Instead, we focus on the role of the *endogenous* entry-exit response in general equilibrium, at the disaggregated level; we view our mechanism as compatible and indeed complementary, whereby substitutability and endogenous entry-exit at the disaggregated level coexist with and reinforce complementarity and exogenous reallocations at the sectoral level. .

More recently, Auerbach, Gorodnichenko, and Murphy (2021) also emphasize exit as an amplification channel following the reduction in revenues due to restrictions on a subset of products with rigid capital operating costs. Other contributions emphasize related supply-side mechanisms, such as inter-sectoral linkages and complementarities (Woodford (2020)), unmployment and endogenous growth (Fornaro and Wolf (2020)), input-output network structures (Baqaee and Farhi (2020)), and investment (Basu et al, 2021). We abstract from such features to focus on endogenous entry-exit and its interaction with price stickiness.

2   The Entry-Exit Multiplier and Aggregate Demand

In this section, we outline the simplest model of endogenous entry-exit with nominal rigidities. In our benchmark economy, households maximize the expected present value of utility defined over a consumption good C and hours worked L. The utility function is logarithmic in consumption $\ln C_t - \chi \frac{L_t^{1+\varphi}}{1+\varphi}$, where total consumption is equal to the output of a final-good sector consisting of a CES aggregate of intermediates. This provides an important benchmark distilling our core mechanism. We then show how our key results hold more generally: with external effects, with a utility function with different income effects on labor, and with arbitrary elasticity of intertemporal substitution (relaxing log utility in consumption).

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[9]Furthermore, the sector-specific shocks in Guerrieri et al are isomorphic to good-specific "demand" shocks, i.e. disturbances to the utility function. Cesa-Bianchi and Ferrero (2021) quantify empirically the contribution of sectoral shocks to aggregate fluctuations.

2.1 A Simple New Keynesian Model with Endogenous Entry-Exit

At time t, the household consumes C_t, equal to final good production Y_t. The latter is produced using a continuum of intermediate inputs with measure N_t: $Y_t = \left(\int_0^{N_t} y_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the symmetric elasticity of substitution across intermediate goods.[10] Let $p_t(\omega)$ denote the nominal price of good ω and $P_t = \left(\int_0^{N_t} p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}$ the price of the final good. The demand for each intermediate ω is then $y_t(\omega) = (p_t(\omega)/P_t)^{-\theta} Y_t$.

There is a continuum of monopolistically competitive firms, each producing a different intermediate $\omega \in [0, N_t]$. Production requires only one factor, labor, whose productivity is scaled exogenously by a factor A_t. We model the COVID-19 economic impact as a large negative shock to this productivity term. (In our simple framework, this is identical to a downward shift in labor supply). Output supplied by firm ω is:

$$
y_t(\omega) = \begin{cases} A_t l_t(\omega) - f, & \text{if } A_t l_t(\omega) > f \\ 0, & \text{otherwise,} \end{cases}
$$

where $l_t(\omega)$ is the firm's labor demand and f a fixed per-period cost. Under free (endogenous) entry, this fixed cost determines the number of firms in equilibrium, whereas with no or exogenous entry, it determines the profit share. Cost minimization, taking the wage as given, implies that the real marginal cost is equal to the real wage deflated by productivity W_t/A_t, with $W_t \equiv \tilde{W}_t/P_t$ and $\tilde{W}_t$ the nominal wage.

We consider a symmetric equilibrium with N_t producing firms and drop the ω qualifier. The relative price of intermediates in units of the final good is a key object that captures the aggregate productivity benefit of input variety, also known as "increasing returns to specialization":

$$
\rho_t \equiv \frac{p_t}{P_t} = N_t^{\frac{1}{\theta-1}}. \tag{1}
$$

Variations in the number of intermediates induce changes to endogenous aggregate productivity, an insight that is at the core of all the expanding-variety endogenous growth literature.

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[10]This specification follows Ethier (1982) and Romer (1987)'s extension of the Spence-Dixit-Stiglitz aggregator. Our results carry through, albeit with some differences in interpretation, to a setup where the CES aggregate is defined over indvidual varieties in consumption instead.

Let μ_t denote the firms' markup (potentially time-varying):

$$\mu_t \equiv \frac{\rho_t}{W_t/A_t}. \tag{2}$$

Firm ω profit in period t can be written as:

$$d_t = \frac{p_t}{P_t} y_t - W_t l_t.$$

The household's budget constraint enforces the aggregate accounting identity equating expenditures (consumption plus the fixed cost "investment" for all firms) with income (labor income and profits for all firms). That is:

$$C_t + \frac{W_t}{A_t} f N_t = W_t L_t + \left(\frac{p_t}{P_t} - \frac{W_t}{A_t} \right) y_t N_t.$$

Combining the above equations and aggregating across goods, anticipating a symmetric equilibrium, we obtain:

$$Y_t = N_t^{\frac{\theta}{\theta-1}} \left(\frac{A_t L_t}{N_t} - f \right). \tag{3}$$

With **endogenous entry-exit**, the number of firms is determined by a zero-profit condition every period for aggregate profits, and hence individual firm profits $d_t = 0$ in our symmetric equilibrium. Replacing the firm production function in the expression for profits, equating to zero, and solving, we obtain firm-level labor demand: $l_t = \frac{\mu_t}{\mu_t-1} \frac{f}{A_t}$.[11]

A key equation is *aggregate labor demand*, obtained by aggregating l_t across producers:

$$L_t = \frac{\mu_t}{\mu_t - 1} \frac{f N_t}{A_t}. \tag{4}$$

Combined with the markup rule, this yields:

$$W_t = A_t^{\frac{\theta}{\theta-1}} \left(\frac{\mu_t - 1}{\mu_t} \frac{1}{f} L_t \right)^{\frac{1}{\theta-1}} \frac{1}{\mu_t}. \tag{5}$$

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[11] The free-entry, zero-profit condition with per-period fixed costs differs from previous work e.g. Ghironi and Melitz (2005) and Bilbiie et al (2007, 2012) which used dynamic entry subject to a sunk cost. The purpose of this is to distill the novel channel we focus on here in the simplest framework, and thus maximize the role of extensive margins in the sharpest setup; the main insight about the entry-exit multiplier and comovement of the output gap would transfer to a model with sunk costs (and heterogeneity), even though the diffusion pattern of entry-exit over time would change.

Three important observations are in order: first, endogenous entry implies that the aggregate labor demand is upward sloping. Its slope is the degree of increasing returns. Second, aggregate labor demand shifts as usual with changes in labor productivity, but that effect is amplified here by the increasing returns; and finally, aggregate labor demand shifts with endogenous changes in markups. The last effect is also present in sticky-price models with fixed entry, even though the endogenous change in markups depends on the equilibrium adjustment in the number of firms.

Using the zero-profit condition, aggregate accounting (3) can be written:

$$C_t = Y_t = W_t L_t. \tag{6}$$

The households' choice between consumption and hours yields the standard labor supply:

$$\chi L_t^\varphi = \frac{1}{C_t} W_t. \tag{7}$$

Logarithmic utility in consumption implies that income and substitution effects cancel out: (7) and the resource constraint (6) imply fixed equilibrium hours worked $L_t = \bar{L} = \chi^{-\frac{1}{1+\varphi}}$. This simplifies the algebra and allows us to focus on the core novel channel associated with endogenous entry. We relax this assumption later on.

An important distinction concerns input versus final-good prices and their corresponding inflation rates. We refer to the former as the producer price p and to the latter as the consumer price. Producer-price inflation $1 + \pi_t = p_t / p_{t-1}$ and consumer-price inflation $1 + \pi^C = P_t / P_{t-1}$ are related to the growth in the number of intermediate inputs through (1):

$$\frac{1 + \pi_t}{1 + \pi_t^C} = \left(\frac{N_t}{N_{t-1}} \right)^{\frac{1}{\theta - 1}}. \tag{8}$$

This distinction is particularly important with nominal rigidities because these apply at the *individual* firm-level price p_t. The relevant inflation rate for aggregate demand is *consumer* inflation π_t^C, insofar as it determines the ex-ante real interest rate that governs intertemporal substitution. Indeed, the solution to the household's intertemporal problem is the standard Euler equation for

consumption:[12]

$$\frac{1}{C_t} = \beta E_t \left(\frac{1 + I_t}{1 + \pi^C_{t+1}} \frac{1}{C_{t+1}} \right). \tag{9}$$

The model is closed by specifying the price-setting equation—delivering a Phillips curve for PPI inflation and a Taylor rule for the nominal interest rate in response to PPI inflation.

2.2 The Entry-Exit Multiplier: Closed-form Solution

In order to highlight the role of nominal rigidities as starkly as possible, we first consider an extreme form of sticky prices that are indefinitely fixed. Later on, we generalize this to a model with a Phillips curve and Taylor rule and show that our main qualitative results remain unchanged. Under **flexible prices**, the equilibrium (denoted by superscript F), is fully determined by combining (3), (4), (1), and (2). The markup is constant and given by $\mu^*_t = \theta / (\theta - 1)$, delivering the lower left corner of Table 1. In this equilibrium firm-size is also constant, $y^*_t(\omega) = (\theta - 1) f$; Relative to the no-entry model (also under flexible prices), this is the opposite extreme whereby all the adjustment is born at the extensive margin (and none at the intensive).[13]

Under **sticky prices** (denoted by superscript S), we assume momentarily that rather than a Taylor rule setting the nominal interest rate, the central bank sets the amount of nominal expenditure, e.g. money supply M_t. This yields the "quantity equation": $M_t = P_t Y_t$. We adopt this for simplicity, but show in Appendix A that this has exactly the same interpretation as a fixed real rate combined with the Euler equation.[14] Since individual prices are fixed, the relative-price equation is $P_t = \bar{p} N_t^{-\frac{1}{\theta-1}}$, and the markup μ_t is now endogenous and given by (4). The equilibrium is outlined in the lower right corner of Table 1.

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[12]The full solution also implies a standard transversality condition.

[13]This feature of the equilibrium is due to the combination of free entry (no sunk-cost delays) and fixed costs' being denominated in the ouptut of the respective intermediate. Deviating from either of these assumptions would generate some adjustment in the intensive margin too.

[14]We later on solve the dynamic version of the model with a Phillips curve and Taylor rule that does not entirely neutralize PPI inflation.

Table 1: Closed-Form Nonlinear Solution		
	Flexible Prices (**F**)	Sticky Prices (**S**)
No Entry-Exit (**N**)	$Y_t^{NF} = A_t \bar{L}$ $M_t = P_t^{NF} Y_t^{NF}$	demand: $Y_t^{NS} = \frac{M_t}{\bar{P}}$ supply: $Y_t^{NS} = A_t L_t^{NS}$
Endogenous Entry-Exit (**E**)	$N_t^{EF} = \frac{1}{\theta} \frac{A_t \bar{L}}{f}$ $Y_t^{EF} = \frac{\theta-1}{\theta(\theta f)^{\frac{1}{\theta-1}}} \left(A_t \bar{L} \right)^{\frac{\theta}{\theta-1}}$	$N_t^{ES} = \frac{A_t \bar{L}}{f} - \frac{M_t}{f \bar{p}}$ $Y_t^{ES} = \frac{M_t}{\bar{p}} \left(\frac{A_t \bar{L}}{f} - \frac{M_t}{f \bar{p}} \right)^{\frac{1}{\theta-1}}$

Comparing the equilibrium expressions of entry-exit leads to our first, core proposition:

Proposition 1 *The Entry-Exit Multiplier. The response of the number of firms (entry-exit) N_t to the supply shock A_t is proportionately higher under sticky prices (relative to flexible prices):*

$$\frac{d \log N_t^{ES}}{d \log A_t} > \frac{d \log N_t^{EF}}{d \log A_t}.$$

This is a powerful result that operates in models with entry-exit and nominal rigidities; for example, it is a feature of Bilbiie, Ghironi, and Melitz (2007) and Bergin and Corsetti (2008), although it has not been identified or discussed as such. The intuition is very simple and general. With sticky prices, the intensive margin cannot adjust in some key dimension and the extensive margin inefficiently bears all the adjustment. For a productivity decrease, the firm would like to increase its price to keep its scale constant, thus selling the same quantity at a higher price, but cannot (sticky prices). This generates a demand shortage and exit, with each remaining firm hiring more workers, producing more, and ending up "too large"; whereas with flexible prices, there would still be exit but each firm would keep its scale constant.[15] Firms are bigger than they would be absent price rigidities, and there are fewer of them. This is a distortion that increases with the demand elasticity θ. In other words, more intensive-margin adjustment would be desirable, and this is relatively more important when inputs are closer substitutes. This last argument is related to the impact on aggregate output, that we will study next.

We note that the equilibrium is determined by two key equations: 1. free (endogenous) entry-exit, implying zero aggregate profits; and 2. individual profit maximization, implying the pricing

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[15]Note that the effect of productivity on entry-exit is symmetric for positive and negative shocks; as we discuss momentarily, this is no longer true for the effect on aggregate output.

condition that marginal cost equal marginal revenue.[16] When a (say) negative exogenous productivity shock hits ($dA < 0$), there is a ceteris paribus decrease in profits for each firm (keeping relative prices ρ fixed). *Free entry-exit* implies the number of firms N goes down to restore the zero-profit condition (the first equation). Due to increasing returns to specialization this feeds back into a further—now *endogenous*—fall in aggregate productivity.

To find *how much* equilibrium entry-exit occurs, we need to consider the pricing condition. Notice that marginal revenue is given by ρ/μ. Keeping the wage fixed, a productivity fall implies an increase in marginal cost. With flexible prices (at given N), the markup μ is constant and individual prices increase. But the equilibrium response of the *relative* price ρ depends on the extent of entry-exit. Each individual firm contracts its labor demand, *and* there is a lower number of firms (one-to-one with the productivity decrease).

With sticky firm prices, marginal cost and revenue are still equalized. But now when firms' profits go down, they are stuck with prices too low, generating an incentive to exit. The markup goes down (it was constant under flex prices), which dampens the fall in individual labor demand. The number of firms however falls by more, generating exactly the same aggregate labor demand response. Thus, the relative price falls by more under sticky prices to compensate for the fall in markup and generates the same real marginal cost (and revenue) regardless of whether prices are flexible or sticky. In other words, the final-good price (CPI) P falls by more under sticky prices.

The above discussion hints that our mechanism is likely to be more relevant and realistic for negative productivity shocks than for positive ones, in terms of the relative timing of entry and pricing decisions. For a positive shock, an undesirable model feature is that entry itself happens before individual firms can adjust their price. This can be fixed by introducing a sunk cost, but this would need to be lower than the price adjustment cost (Bilbiie, Ghironi, and Melitz 2007 uses a sunk-cost model and investigates those issues).

Yet for negative productivity shocks, the same criticism has less bind. If firms are stuck with a price too high and a scale too large, a greater proportion of them fail. In case of a big negative shock, if it were possible to redistribute the fall in individual sales (intensive margin), more firms would survive. But this is impossible, so disproportionately more firms fail. While price

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[16]A key observation is that the labor market equilibrium is identical under flexible and sticky prices: the real wage and marginal cost change by exactly the same amount.

stickiness is probably not the most micro-plausible mechanism for this failure of intensive-margin adjustment, the firms' inability to increase prices enough in a slump certainly seems realistic for crisis such as the current one (alternative mechanism would have to take into account coordination and wage stickiness). So we take price stickiness as a metaphor for firms' inability to contract even though a large negative profit results in exit.

A final remark is that the difficulty of increasing prices to stabilize individual production is likely to apply to *product* (as opposed to *firm*) level, so the exit emphasized here applies as well to multi-product firms dropping products as it does to the exit of firms.[17]

2.3 Aggregate-Demand Amplification Through Entry-Exit

When does this entry-exit multiplier of the supply, productivity shock lead to aggregate demand amplification—a higher response of aggregate output (and consumption)? A key point to note in this context is that N is *linear* in A, but Y is *nonlinear* in A, because $Y(N)$ is nonlinear. It is useful for comparison to review the standard NK model with no entry-exit, with a fixed number of varieties $N_t = \bar{N}$, which we normalize to 1. For reference, we denote throughout variables in the No-Entry-Exit model by the superscript N. Labor supply is still given by (7), but labor demand is simply: $W_t^N = A_t/\mu_t^N$, a special case of (5) with no aggregate productivity benefit to variety. Furthermore, there is now no distinction between producer and consumer prices. Since we normalize the mass of goods to 1, individual and aggregate variables coincide. The production function is $Y_t^N = A_t L_t^N$, where we normalize the fixed cost in the no-entry economy to zero (this is immaterial for our analysis).

Under *flexible prices*, optimal pricing implies a constant markup rule $\mu_t^{NF} = \frac{\theta}{\theta-1}$. Denoting this equilibrium with the superscript NF, the solution is $W_t^{NF} = \frac{\theta-1}{\theta} A_t$ with hours and consumption $L_t^{NF} = \bar{L} = \left(\frac{\theta-1}{\theta\chi}\right)^{\frac{1}{1+\varphi}}$ and $Y_t^{NF} = A_t \bar{L}$. Hours are constant in equilibrium, even though labor supply is endogenous, because income and substitution effects cancel out (a consequence of logarithmic utility in consumption). With *sticky prices* we now have $Y_t^{NS} = A_t L_t^{NS} = \frac{M_t}{P} \rightarrow L_t^{NS} = \frac{M_t}{P}\frac{1}{A_t}$: as long as they stay within the time constraint, hours go up when productivity goes down in order to keep consumption constant at the demand-determined level.[18] This core intuition of the NK

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[17]See Argente et al (2018) for recent evidence on the cyclical relevance of this margin.

[18]Evidently, there is rationing as soon as $\frac{M_t}{P}\frac{1}{A_t}$ becomes larger than the feasible time endowment.

model is at odds with the data and is at work in richer versions of this model. It is overturned in our model with endogenous entry-exit. This restores procyclical hours in response to productivity, just like under flexible prices. We return to this issue in the dynamic model later on.

The effect on hours is due to income effects. Note that wages and profits are: $W_t^{NS} = A_t^{-\varphi} \chi \left(\frac{M_t}{\bar{P}} \right)^{1+\varphi}$ and $D_t^{NS} = (M_t/\bar{P}) \left(1 - A_t^{-(1+\varphi)} \chi (M_t/\bar{P})^{1+\varphi} \right)$. Wages are countercyclical and profits procyclical conditional on supply shocks. In particular, wages go up and profits down in response to a bad shock. Agents work more because of the extra income effect of profits relative to the free-entry ($Y = wL$) case, whereby income and substitution effects cancel out. With entry-exit, this decrease in profits results in exit. In the top row of Table 1, we record the closed-form equilibrium solution of the no-entry model with flexible and sticky prices.

We plot final output Y as a function of the shock A for the two equilibria in the left panel of Figure 1. Since output is demand-determined under flexible prices, it is the upward sloping line with slope $\bar{L}$. Under sticky prices, it is the horizontal line $Y_t^{NS} = \frac{M_t}{\bar{P}}$. We choose the domain of A_t such that there is no rationing. That is, the equilibrium level of output is equal to demand and the adjustment is borne by hours worked. Those hours increase to compensate for the productivity-driven shortfall.[19]

The main takeaway is that in response to a bad supply shock (lower A), output goes down proportionally under flexible prices. But under sticky prices, it either stays unchanged (if labor is elastic enough) or at most falls by as much as under flexible prices: in other words, there is never a demand shortage in response to a negative supply shock, and there can even be excess demand. The "output gap" is positive in response to supply disruptions. This is a well-known property of the standard no-entry sticky-price model restated here as a benchmark.

Consider now the role of endogenous entry and exit. In the right panel of Figure 1, we plot the EF (red dash) and ES (blue solid) equilibria. The economies are again calibrated so that the steady-state equilibria ($A = 1$) coincide, and also coincide with the steady-state of the NF model (see Appendix A for details). The only remaining free parameter is θ, which we set to 6, in the

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[19]With a negative enough shock, demand can exceed supply (what can be produced), so the equilibrium amount produced and consumed would be represented by a kinked line (where to the left, the upward-sloping part would be supply-determined). The kink point itself is determined by labor supply elasticity: for instance with inelastic labor, any small negative A shock would lead to rationing. In particular, there is rationing as soon as $\frac{M_t}{\bar{P}} \frac{1}{A_t} > L^{tot}$ (the total time endowment), which calibrating real money balances to equate the two equilibria at $A = 1$ (in the absence of shocks) delivers $A_t < \frac{\bar{M}}{\bar{P}} \frac{1}{L^{tot}} = \left(\frac{\theta-1}{\theta\chi} \right)^{\frac{1}{1+\varphi}} \frac{1}{L^{tot}}$.

middle of the estimates' interval and in line with NK calibrations. As the figure makes clear, output under sticky prices is always lower than under flexible prices. In particular, output falls by more in response to a bad supply shock when prices are sticky. That is, the output gap is negative in response to supply disruptions. Moreover, the larger the disruption, the larger the demand recession, and the more negative the output gap.
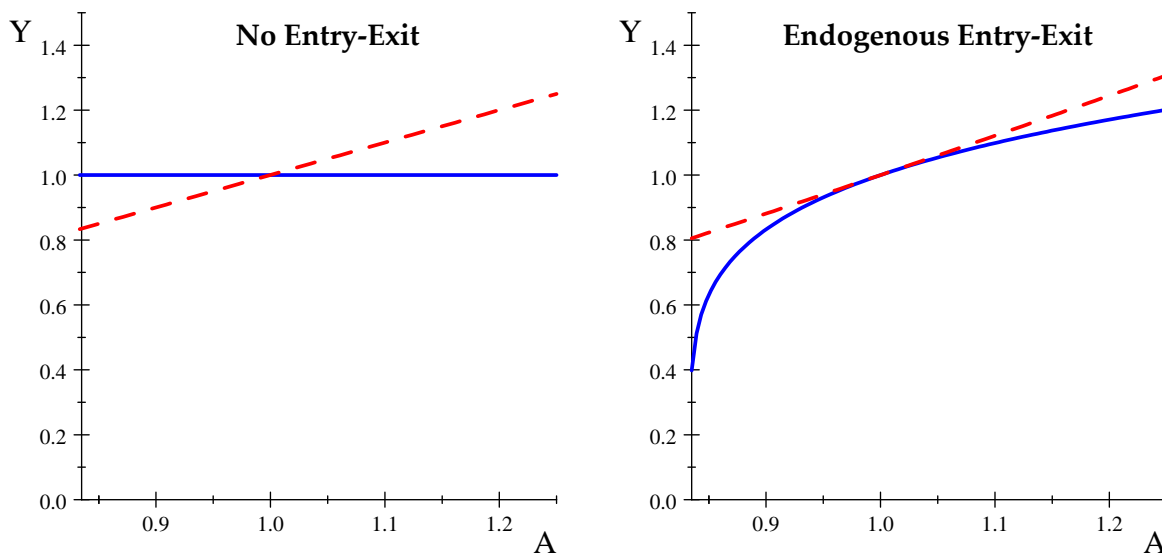
Figure 1: CES. Y^{xF} (flex. prices) red dash, Y^{xS} (sticky prices) solid blue

To understand what drives this key result that completely overturns the propagation of supply shocks in the no-entry New Keynesian model, we compare the free-entry equilibria formally. We take a second-order approximation around the point $Y^{ES} = Y^{EF}$ denoting the percentage deviation from steady-state by a small letter e.g. $a_t \equiv (A_t - A)/A$, to obtain our second Proposition.

Proposition 2 *To second order, output under flexible and sticky prices is, respectively:*

$$y_t^{EF} \simeq \frac{\theta}{\theta - 1} a_t + \frac{1}{2} \frac{\theta}{(\theta - 1)^2} a_t^2,$$

$$y_t^{ES} \simeq \frac{\theta}{\theta - 1} a_t + \frac{1}{2} \frac{\theta^2 (2 - \theta)}{(\theta - 1)^2} a_t^2.$$

Therefore, the "output gap" is:

$$y_t^{ES} - y_t^{EF} \simeq -\frac{1}{2} \theta a_t^2.$$

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We note that the first order elasticities are identical under flexible and sticky prices. The fundamental reason is that, under the CES aggregator, the market equilibrium is Pareto optimal: as in Dixit and Stiglitz, the number of input varieties is efficient. By an envelope argument, first-order deviations from that allocation are negligible (a consequence of the neutrality proposition in Bilbiie (2019)). But through the second-order term, the output gap response is always negative. That is, there is an asymmetry: output increases by less in response to positive shocks, but falls by more in response to negative shocks. For large negative shocks in particular, the response under sticky prices can be much larger.

Dissecting the Mechanism. The key to understanding these second-order (concavity) effects lies in the equilibrium dependence of aggregate output to the number of intermediate inputs $Y(N)$. As we already noted, N itself is a linear function of A. In other words, N is (linearly) amplified through our entry-exit multiplier, while Y is then amplified further through second-order effects. In particular, the key equation is the "aggregate production function":

$$Y_t = N_t^{\frac{\theta}{\theta-1}} \left(\frac{A_t \bar{L}}{N_t} - f \right).$$

A second-order approximation around the steady-state (common to F and S) equilibrium yields:

$$y_t \simeq -\frac{1}{2} \frac{\theta}{(\theta-1)^2} n_t^2. \tag{10}$$

The amplification of N to (negative) A generates the amplification of Y by this curvature. This effect, through the concavity of consumption in the number of intermediates, is decreasing with the benefit of variety $(\theta - 1)^{-1}$. It is thus determined by the same parameter governing the amplification of entry-exit itself. Naturally, when the benefit of variety (the degree of increasing returns to specialization) vanishes there is no curvature of output in the number of varieties. The degree of increasing returns to specialization is crucial for the balance between the extensive and intensive margin adjustment that becomes distorted under sticky prices. More intensive-margin adjustment would be desirable but is unfeasible, and this distortion is *less* important when goods are closer substitutes: θ larger, less returns to scale (less benefit of variety), less distortion. To summarize, θ determines both entry-exit amplification and the concavity distortion, but has opposite effects on

these two forces.

Overall, the net effect of θ is to amplify the difference between flexible and sticky-price allocations: the positive effect through the entry-exit multiplier is proportional to θ^2, while the negative effect through (10) is proportional to θ^{-1}, (i.e. $\theta(\theta-1)^{-2}$). We disentangle these two forces subsequently, using preferences that break this link between the degree of returns to scale and the elasticity of substitution.

A Numerical Nonlinear Model

Up to now, we relied on some strong functional form assumptions. This allowed us to analytically characterize the full nonlinear solution to this dynamic general-equilibrium model (which we used to produce Figure 1). However, our insights carry through under more general conditions. We assume elastic labor supply (unit elasticity), a Phillips curve (adjustment cost parameter to deliver a first-order slope of around 0.01), a Taylor rule responding to PPI inflation (which is justified on welfare grounds, see e.g. Bilbiie, Ghironi, Melitz (2007)) with response 1.5 and a persistent 10% decrease in productivity perfectly anticipated to survive next period with persistence 0.5.[20]

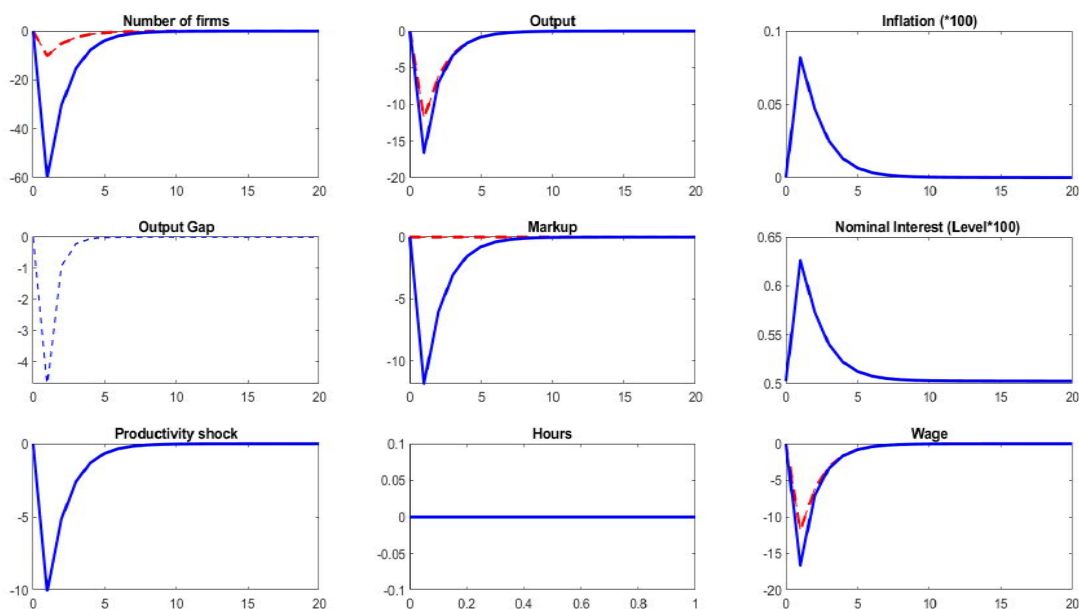Figure 2: Effect of a 10% fall in productivity with persistence .5. CES aggregator. Flexible (red dash) and Sticky (solid blue) prices.

[20]We still use logarithmic utility in consumption; we explore alternatives like CRRA and GHH analytically in the loglinearized model below. The conclusions for the nonlinear model are largely unaffected (beyond the insight we provide in the loglinearized model).

In Figure 2, we plot the dynamic equilibrium responses of key macroeconomic variables in our nonlinear model to a 20% fall in productivity with persistence 0.5 under flexible prices (red dash) and under sticky prices (solid blue). The output gap plots the difference between the two respective output values: the larger response of sticky-price output is driven exclusively by higher-order terms.[21] In the next section, we show that it can occur to first order too, when departing from the CES benchmark; but before, we emphasize an important implication for the business-cycle properties of the model.

2.4 Entry-Exit Solves the NK vs RBC Hours Controversy

A well-known controversy between sticky-price (NK) and RBC models concerns the response of hours worked to productivity shocks. In the former framework, hours fall in response to positive labor productivity shocks (and increase with negative shocks). This stands in stark contrast with the implications for standard flexible-price, RBC models, whose transmission greatly relies upon (and that try hard to match) procyclical hours worked in response to productivity shocks. As our discussion above highlights, the different sticky-price response is driven by an income effect stemming from the response of profits.

It follows immediately that entry and exit, which operate precisely in response to these profit variations, can bring the responses of hours worked in line between the flexible- and sticky-price models. Indeed, it seems a desirable property of a model to deliver a response of hours worked to productivity that is invariant to the largely orthogonal model feature of whether firms can reset prices or not. Our model with entry and exit does that, more closely aligning the responses of hours between the flexible and sticky price versions.

The responses of hours can be aligned even more closely when we eliminate the income effects on labor supply driving their divergence. In Appendix D.1, we solve the model under preferences that eliminate those effects (called "GHH", from Greenwood, Hercowitz, and Huffmann) and show that the response of hours becomes in fact positive ceteris paribus under both flexible and sticky prices, as long as there is entry and exit.

The full solution is outlined in Table A1 in the Appendix. In the no-entry-exit model, the

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[21]The responses are produced by solving the full model globally using Dynare's nonlinear perfect-foresight solver (Adjemian et al, 2011). We plot PPI inflation and nominal interest only for the sticky-price model since their flex.price magnitudes are so large that they dwarf the sticky-price responses.

response of hours worked is $l_t^{NF} = \eta^{-1}a_t$ with flexible prices (η is the inverse labor elasticity), whereas it has the opposite sign $l_t^{NS} = -\theta a_t$ with sticky prices. This sharply illustrates the dichotomy we previously highlighted. With entry-exit, the responses are instead:

$$l_t^{EF} = l_t^{ES} = \frac{\theta}{\eta(\theta-1)-1}a_t. \tag{11}$$

This clearly illustrates that in both cases, hours are procyclical: $\eta(\theta-1) > 1$ is a restriction for productivity improvements to be expansionary (see Appendix). Indeed, the responses are *identical*, a property of the CES benchmark (we show in the Appendix that for a more general aggregator hours are still procyclical in both cases, but their responses are no longer identical).[22]

3 External Effects and First-order Amplification

In this section, we introduce a generalized CES aggregator reflecting the presence of external returns in intermediate input variety: we assume that the mass of input varieties contributes directly to aggregate productivity in addition to its indirect impact via the standard CES aggregator. The production function for the final good is then:

$$Y_t = N_t^\lambda \left(\int_0^{N_t} y_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}},$$

where $\lambda > 0$ parameterizes the degree of external returns to variety (we assume that this externality is positive). The returns to variety are therefore magnified by this externality relative to the standard CES aggregator. We use this functional form for tractability, but our results generalize easily to non-CES homothetic aggregators as we show in Appendix D.4.[23] The final good price is now $P_t = N_t^{-\lambda} \left(\int_0^{N_t} p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{\theta-1}}$, and the relative price (replacing equation (1) above) is: $\rho_t \equiv p_t/P_t = N_t^{\lambda + \frac{1}{\theta-1}}$. Crucially for our results, the benefit of an additional input is now $\lambda + \frac{1}{\theta-1}$ and thus no longer aligned with the producers' incentive to provide it—the net markup. The aggregate accounting equation (3) is now $Y_t = N_t^{\lambda + \frac{\theta}{\theta-1}} \left(\frac{A_t L_t}{N_t} - f \right)$, leading to similar changes to

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[22] As we show in the Appendix, the requirement for the output gap to go down with adverse supply shocks is the same as under separable preferences.

[23] For aggregates of individual varieties of consumption goods, this is akin to assuming an arbitrary benefit of variety $\lambda + 1/(\theta-1)$ as in the working paper version of Dixit and Stiglitz and further elaborated in i.a. Benassy (1996), Blanchard and Giavazzi (2007), Bilbiie, Ghironi, and Melitz (2019).

aggregate labor demand (5) and to the relationship between PPI and CPI inflation (8).[24]

The equilibrium number of firms is unchanged under both flexible and sticky prices, since it is determined by markups, which govern the incentive for entry. It follows that the entry-exit multiplier we previously uncovered remains unchanged. But the equilibrium values of output change respectively to:

$$Y_t^{EF} = \left(\frac{1}{\theta f}\right)^{\lambda + \frac{1}{\theta - 1}} (A_t \bar{L})^{\lambda + \frac{\theta}{\theta - 1}} \frac{\theta - 1}{\theta}; \; Y_t^{ES} = \frac{M_t}{\bar{p}} \left(\frac{A_t \bar{L}}{f} - \frac{M_t}{f \bar{p}}\right)^{\lambda + \frac{1}{\theta - 1}}.$$

A key property of models with endogenous entry under general input aggregation is that the equilibrium amount of entry may be inefficient. In our model, the wedge between the flexible-price market equilibrium and a Pareto optimal level chosen by a planner N_t^{opt} is given by $\frac{N_t^{opt}}{N_t^{EF}} = 1 + \lambda \frac{\theta - 1}{\lambda + \frac{\theta}{\theta - 1}}$. It follows that the market number of firms is inefficiently low whenever $\lambda > 0$, as we have assumed.[25]

Since the number of varieties is inefficiently low and its elasticity with respect to productivity shocks is also inefficiently low, a mechanism that provides a magnification of the response of entry to productivity shocks will yield first-order welfare improvements: this is indeed the case for our entry-exit multiplier.

To understand the amplification properties of the model under external returns, we take a second-order approximation of the equilibrium value of consumption, obtaining Proposition (3) (the derivation is in the Appendix):

Proposition 3 *To second order, output under flexible and sticky prices is, respectively:*

$$y_t^{EF} = \left(\lambda + \frac{\theta}{\theta - 1}\right) a_t + \frac{1}{2} \left(\lambda + \frac{1}{\theta - 1}\right) \left(\lambda + \frac{\theta}{\theta - 1}\right) a_t^2,$$

$$y_t^{ES} = \left(\lambda + \frac{1}{\theta - 1}\right) \theta a_t + \frac{1}{2} \left(\lambda + \frac{1}{\theta - 1}\right) \left(\left(\lambda + \frac{1}{\theta - 1}\right) - 1\right) \theta^2 a_t^2.$$

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[24]The insights from the nonlinear model also apply to the general CES aggregator with external effects, subject to some qualifications described in Appendix B.

[25]The planner solution is found by maximizing the number of goods subject to technology and resource constraints only; see Bilbiie, Ghironi, and Melitz (2019) for a detailed analysis of the welfare implications of entry, variety, and markups.

Therefore, the output gap is:

$$y_t^{ES} - y_t^{EF} = \lambda (\theta - 1) a_t + \frac{1}{2} \left(\lambda + \frac{1}{\theta - 1} \right) \left[\lambda (\theta^2 - 1) + \theta - \theta^2 \right] a_t^2.$$

To help intuition, it is useful again to take a second-order approximation of consumption as a function of the number of varieties, yielding:

$$y_t \simeq \lambda n_t + \frac{1}{2} \left(\lambda + \frac{1}{\theta - 1} \right) \left(\lambda - \frac{\theta}{\theta - 1} \right) n_t^2. \tag{12}$$

The *first-order* response of the output gap to a negative supply shock $d(-a_t)$ with endogenous entry is $-\lambda (\theta - 1)$ and is thus negative. The combination of an inefficiently low (and inelastic) number of firms with the entry-exit multiplier, which increases the responsiveness of the number of firms to productivity shocks under sticky prices, translates into first-order aggregate demand amplification. As (12) makes clear, there is a first-order welfare benefit of expanding the number of firms. This first-order amplification generalizes to the model with an Euler equation, Phillips curve, and Taylor rule; in Appendix C, we outline a loglinearized model that is isomorphic to the textbook NK no-entry model and is amenable to an aggregate demand-aggregate supply analysis.

The equilibrium responses are also different to second order and, importantly, we can now disentangle the effects of input variety from the demand elasticity, which were conflated under CES preferences. The second-order term in (12) illustrates that Y is concave in N whenever $\lambda < \frac{\theta}{\theta - 1}$. This disentangles the effect of the curvature of output in the degree of returns to scale from that of the elasticity of substitution, discussed after Proposition 2. Ceteris paribus, Y is more concave the smaller the degree of returns to specialization $\lambda + \frac{1}{\theta - 1}$ and the smaller the elasticity of substitution θ—but the former effect dominates. For the benchmark CES aggregator, the overall effect is that a higher θ makes Y more concave because, implicitly, it reduces the degree of increasing returns.

4 Substitution Across Goods and Over Time: CRRA Utility

We now study the role of the elasticity of intertemporal substitution in consumption, normalized to one until now by the assumption of logarithmic utility in consumption. Consider the more general CRRA utility in consumption: $U(C_t) = \left(C_t^{1 - \frac{1}{\sigma}} - 1 \right) / (1 - \frac{1}{\sigma})$, with $\ln C_t$ as a limit when

$\sigma \to 1$. We assume inelastic labor and CES preferences for simplicity but treat the case of elastic labor and arbitrary benefit of product variety for full generality in Appendix D.2.

With this change in preferences, the only change to our model (in the inelastic-labor case) is that the aggregate Euler equation becomes (in loglinear terms): $c_t = E_t c_{t+1} - \sigma \left(i_t - E_t \pi_{t+1}^C \right)$. Solving our model under flexible and sticky prices yields the generalization of the entry-exit multiplier (the solution is outlined in the Appendix for the more general case):

$$n_t^{ES} = \frac{\theta}{\sigma} n_t^{EF} = \frac{\theta}{\sigma} a_t. \tag{13}$$

This illustrates that entry's response under sticky prices is larger than the response under flexible prices if and only if:

$$\theta > \sigma. \tag{14}$$

Our next Proposition emphasizes that under this same condition needed for the entry-exit multiplier, a negative output gap also occurs under CES preferences—through second-order terms, as in our log-utility benchmark.

Proposition 4 *To second order, output under flexible and sticky prices is, respectively:*

$$y_t^{ES} \simeq \frac{\theta}{\theta - 1} a_t + \frac{1}{2} \frac{\theta^2 (1 + \sigma - \theta)}{\sigma (\theta - 1)^2} a_t^2,$$

$$y_t^{EF} \simeq \frac{\theta}{\theta - 1} a_t + \frac{1}{2} \frac{\theta}{(\theta - 1)^2} a_t^2.$$

Therefore, the output gap is:

$$y_t^{ES} - y_t^{EF} \simeq -\frac{1}{2} \left(\frac{\theta}{\sigma} - 1 \right) \frac{\theta}{\theta - 1} a_t^2.$$

A demand recession in response to negative supply shocks occurs again when output is more concave under sticky prices, that is if (14) holds ($\theta > 1$ is again a restriction). This is the same condition required for the entry-exit multiplier, and is very plausible empirically—since most estimates of the substitution between goods θ are between 4 and 8, while estimates of intertemporal substitution σ are smaller than 2, and in fact most often smaller than 1.

The intuition for these results is similar to the one we previously discussed. In response to a negative supply shock, aggregate activity can adjust through two margins: intensive and ex-

tensive. With endogenous entry and when the entry-exit multiplier is at work ($\theta > \sigma$), adjustment happens disproportionately at the extensive margin. This then translates into aggregate-demand amplification, even though the first-order responses are still identical in the CES benchmark, through the concavity of output in the number of goods.

The condition (14) is, however, prima facie the *opposite* of the requirement found by Guerrieri et al (2020) to generate aggregate-demand recessions in response to (sector-specific) supply shocks. The reason is precisely that we focus on the response of aggregate activity in general equilibrium, comprising both an intensive and an *endogenous* extensive margin: individual goods are produced or not depending on whether it is profitable to do so. On the other hand, Guerrieri et al focus on the endogenous response of the intensive margin (in the surviving goods) to an *exogenous* change in the extensive margin. Furthermore, our focus is on the endogenous variations in the set of goods at a highly *disaggregated* level, where substitutability is the more plausible assumption; while Guerrieri et al's mechanism pertains to a more *aggregated*, *sectoral* interpretation wherein complementarity is more plausible. We thus view the two respective mechanisms—complementarity at the aggregated, sectoral level, and substitutability at the disaggregated, good level—as mutually compatible and indeed complementary for explaining aggregate amplification.[26] We further elaborate on this connection by formalizing this distinction in our framework, and showing how exogenous variations in extensive margins can lead to aggregate-demand amplification under complementarity in Appendix D.3.

5   Conclusion

The responses of entry-exit to adverse supply shocks like the recent COVID-19 crisis are amplified by firms' inability to increase their prices, which leads to additional losses for individual firms. This in turn amplifies the response of exit relative to a flexible-price benchmark. We call this simple mechanism, which operates in any model with endogenous entry-exit and price stickiness, *the entry-exit multiplier.*

This "supply-side" amplification further induces an aggregate-demand recession; that is a fall in sticky-price output that is larger than the fall in flexible-price output: a negative output gap.

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[26]Guerrieri et al show how the $\sigma > \theta$ restriction is relaxed in an economy with liquidity constraints; we abstract from this and focus instead on endogenous entry-exit as a complementary amplification channel.

We analyze two separate mechanisms that generate this negative output gap. First, when final output is a standard CES aggregate of intermediate goods, aggregate-demand amplification occurs through second-order terms: the concavity of aggregate output in the number of varieties, combined with the entry-exit multiplier. In this case, the elasticity of the output gap to the shock is a function of the size of the shock—underscoring the importance of considering higher-order, nonlinear terms for large crises such as the Covid-19 recession.

Second, when there are external returns to variety, such when the benefit of an additional variety is larger than the net markup, a further source of aggregate-demand amplification operates, this time to a first-order. The entry-exit multiplier now magnifies the response of entry which is "too low" to start with, compared to what a planner would choose: the combination of the two directly implies an amplification of aggregate demand, through entry-exit.

We generalize our results to the case of an arbitrary elasticity of intertemporal substitution. The parameter condition for our amplification to occur is very plausible empirically: that the elasticity of substitution between goods be larger than the elasticity of intertemporal substitution.

Our a benchmark New Keynesian sticky-price model with endogenous entry-exit is solved in closed-form. Another potential empirical advantage of our model pertains to its business-cycle properties: the (sign of the) response of hours worked to supply shocks is largely invariant to price stickiness. This solves a well-known controversy between the standard RBC and NK models. In their baseline versions, they generate diametrically opposed employment responses: under sticky prices, hours worked fall after a positive supply shock. With endogenous entry-exit in addition to sticky prices, hours worked increase following a transitory productivity increase—just as in standard RBC models.

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A Derivations under CES Benchmark

A.1 Equivalence of quantity equation and Euler equation with fixed real rate

We show that the quantity, money-supply rule considered in text is equivalent to the more standard Euler equation with an interest rate rule fixing the real rate (or with fixed prices). Consider for generality the case of CRRA utility (used later in the paper) $U = \left(C^{1-\frac{1}{\sigma}} - 1 \right) / \left(1 - \frac{1}{\sigma} \right)$. Labor is inelastic for simplicity.

Now the "quantity equation" becomes, assuming that M enters utility logarithmically,

$$C_t^{\frac{1}{\sigma}} = \frac{M_t}{P_t}$$

Note: a policy whereby the central bank fixes M is equivalent to a policy whereby it fixes the (relative-to-PPI) real interest rate. Our previous work (Bilbiie Ghironi Melitz 2007) shows that PPI is the right object to target with price stickiness. Recall $1 + \pi_t = p_t/p_{t-1}$ and $1 + \pi^C = P_t/P_{t-1}$:

$$\frac{1 + \pi_t}{1 + \pi_t^C} = \left(\frac{N_t}{N_{t-1}} \right)^{\frac{1}{\theta-1}}.$$

The aggregate-demand relevant object, however, is the CPI π_t^C which matters for intertemporal substitution. Households' standard Euler equation is (take perfect foresight, no expectation):

$$C_t^{-\frac{1}{\sigma}} = \beta \frac{1 + I_t}{1 + \pi_{t+1}^C} C_{t+1}^{-\frac{1}{\sigma}}.$$

Replace CPI inflation definition

$$C_t^{-\frac{1}{\sigma}} = \beta \left(\frac{N_{t+1}}{N_t} \right)^{\frac{1}{\theta-1}} \frac{1 + I_t}{1 + \pi_{t+1}} C_{t+1}^{-\frac{1}{\sigma}}.$$

Now assume that the Taylor rule is such that it neutralizes expected PPI inflation entirely (it fixes the real rate with respect to it), i.e. $\frac{1+I_t}{1+\pi_{t+1}} = \beta^{-1}$. The same holds if individual prices p are fixed. Then we have:

$$C_t^{-\frac{1}{\sigma}} N_t^{\frac{1}{\theta-1}} = C_{t+1}^{-\frac{1}{\sigma}} N_{t+1}^{\frac{1}{\theta-1}} = \text{constant}$$

This is clearly identical to a model with fixed M and fixed prices. In the model with fixed variety,

$C_t^{-\frac{1}{\sigma}} = C_{t+1}^{-\frac{1}{\sigma}}$ =constant is the same as $C_t^{\frac{1}{\sigma}} = \frac{M_t}{P_t}$ =constant.

In the model with variety and fixed individual $\bar{p}$, we have $P_t = \bar{p} N_t^{-\frac{1}{\theta-1}}$. So fixed-money rule delivers $C_t^{\frac{1}{\sigma}} N_t^{-\frac{1}{\theta-1}} = \frac{M_t}{\bar{p}}$ =constant, which is exactly the same as the fixed-real-rate rule.

We work with the former for simplicity, but the reader can bear in mind throughout that this has exactly the same interpretation as a fixed real rate. We then solve the dynamic version of the model with a Phillips curve and Taylor rule that does not entirely neutralize PPI inflation.

A.2 Calibration equalizing steady states across models

In Figure 1, we choose the fixed cost f in order to make models consistent in the steady state, when the shock is absent $A = 1$; i.e. we pick f that equalizes Y^{EF} to Y^{NF}, $\theta f = \bar{L} \left(\frac{\theta-1}{\theta}\right)^{\theta-1}$. Then, we choose money supply M to equalize the SP equilibrium Y^{ES} with FE to this same $Y^{EF} = Y^{NF}$. This requires, using f:

$$Y = \frac{\theta-1}{\theta} \left(\frac{\bar{L}}{f} - \frac{M_t}{f\bar{p}}\right)^{\frac{1}{\theta-1}} = \bar{L} \rightarrow \frac{M}{\bar{p}} = \bar{L}\left(1 - \frac{1}{\theta}\right)$$

So for the free entry-exit (E) model we plot, for the sticky-price S case, replacing $f = \frac{\bar{L}}{\theta} \left(\frac{\theta-1}{\theta}\right)^{\theta-1}$ and $\frac{M}{\bar{p}} = \bar{L}\left(1 - \frac{1}{\theta}\right)$

$$Y_t^{ES} = \bar{L}\left(\theta A_t - \theta + 1\right)^{\frac{1}{\theta-1}}$$

and for flex-price F:

$$Y_t^{EF} = A_t^{\frac{\theta}{\theta-1}} \bar{L}.$$

While with no entry N (left panel) we plot $Y_t^{NF} = A_t \bar{L}$ and $Y_t^{NS} = \frac{M}{P}$.

B External Returns to Variety: General CES

Solving the benchmark model with the general CES aggregator with external returns introduced in text under flexible and fixed prices, respectively, delivers:

$$Y_t^{EF} = \left(\frac{1}{\theta f}\right)^{\lambda + \frac{1}{\theta-1}} (A_t \bar{L})^{\lambda + \frac{\theta}{\theta-1}} \frac{\theta-1}{\theta}$$

$$Y_t^{ES} = \frac{M_t}{\bar{p}} \left(\frac{A_t \bar{L}}{f} - \frac{M_t}{f\bar{p}}\right)^{\lambda + \frac{1}{\theta-1}}.$$

Consider a steady state equilibrium with $\frac{M}{f\bar{p}} = \frac{\theta-1}{\theta f} A\bar{L} \rightarrow \frac{A\bar{L}}{f} - \frac{M}{f\bar{p}} = \frac{A\bar{L}}{\theta f}$

$$Y^{EF} = \left(\frac{1}{\theta f}\right)^{\lambda + \frac{1}{\theta-1}} (A\bar{L})^{\lambda + \frac{\theta}{\theta-1}} \frac{\theta-1}{\theta}$$

$$Y^{ES} = \frac{M}{\bar{p}} \left(\frac{A\bar{L}}{\theta f}\right)^{\lambda + \frac{1}{\theta-1}} = \frac{\theta-1}{\theta} A\bar{L} \left(\frac{A\bar{L}}{\theta f}\right)^{\lambda + \frac{1}{\theta-1}} = Y^{EF}$$

Taking a Taylor approximation around $Y^{EF} = \left(\frac{1}{\theta f}\right)^{\lambda + \frac{1}{\theta-1}} (A\bar{L})^{\lambda + \frac{\theta}{\theta-1}} \frac{\theta-1}{\theta}$

$$Y_t^{EF} - Y^{EF} = \left(\lambda + \frac{\theta}{\theta-1}\right) \frac{\theta-1}{\theta} \left(\frac{1}{\theta f}\right)^{\lambda + \frac{1}{\theta-1}} (A\bar{L})^{\lambda + \frac{\theta}{\theta-1}} \left(\frac{A_t - A}{A}\right)$$

$$+ \frac{1}{2} \left(\lambda + \frac{1}{\theta-1}\right) \left(\lambda + \frac{\theta}{\theta-1}\right) \frac{\theta-1}{\theta} \left(\frac{1}{\theta f}\right)^{\lambda + \frac{1}{\theta-1}} (A\bar{L})^{\lambda + \frac{\theta}{\theta-1}} \left(\frac{A_t - A}{A}\right)^2 \rightarrow$$

$$\frac{Y_t^{EF} - Y^{EF}}{Y^{EF}} = \left(\lambda + \frac{\theta}{\theta-1}\right) \left(\frac{A_t - A}{A}\right) + \frac{1}{2} \left(\lambda + \frac{1}{\theta-1}\right) \left(\lambda + \frac{\theta}{\theta-1}\right) \left(\frac{A_t - A}{A}\right)^2$$

$$Y_t^{ES} - Y^{EF} = \left(\lambda + \frac{1}{\theta-1}\right) \frac{A\bar{L}}{f} \frac{M}{\bar{p}} \left(\frac{A\bar{L}}{f} - \frac{M}{f\bar{p}}\right)^{\lambda + \frac{1}{\theta-1} - 1} \left(\frac{A_t - A}{A}\right)$$

$$+ \frac{1}{2} \left(\lambda + \frac{1}{\theta-1}\right) \left(\lambda + \frac{1}{\theta-1} - 1\right) \left(\frac{A\bar{L}}{f}\right)^2 \frac{M}{\bar{p}} \left(\frac{A\bar{L}}{f} - \frac{M}{f\bar{p}}\right)^{\lambda + \frac{1}{\theta-1} - 2} \left(\frac{A_t - A}{A}\right)^2$$

Recall $Y = \frac{M}{\bar{p}} \left(\frac{A\bar{L}}{f} - \frac{M_t}{f\bar{p}}\right)^{\lambda + \frac{1}{\theta-1}}$ and $\frac{M}{f\bar{p}} = \frac{\theta-1}{\theta f} A\bar{L} \rightarrow \frac{A\bar{L}}{f} - \frac{M}{f\bar{p}} = \frac{A\bar{L}}{\theta f}$

$$\frac{Y_t^{ES} - Y^{EF}}{Y^{EF}} = \left(\lambda + \frac{1}{\theta-1}\right) \theta \left(\frac{A_t - A}{A}\right) + \frac{1}{2} \left(\lambda + \frac{1}{\theta-1}\right) \left(\lambda + \frac{1}{\theta-1} - 1\right) \theta^2 \left(\frac{A_t - A}{A}\right)^2$$

The *ES* response is larger to first-order iff $\lambda > 0$, as discussed in text. Here, we focus on the second-order difference. The *ES* response is larger second-order iff $\lambda > \frac{\theta}{\theta+1}$. (But now even with *negative* externality there can be over-reaction to negative shocks driven by higher-order effects. If

the shock is negative enough, the higher-order term eventually kicks in.)

In the figure, we plot the case $\lambda = 0.2$ for the two respective cases: blue solid for sticky prices, red dash for flexible prices. We use again the normalization with f that equalizes Y^{EF} to Y^{NF}, $\bar{L}\frac{\theta-1}{\theta} = (\theta f)^{\lambda+\frac{1}{\theta-1}}$. Then, we choose money supply M to equalize the SP equilibrium Y^{ES} with FE to this same $Y^{EF} = Y^{NF}$. This requires, using f:

$$Y = \bar{L} = \frac{M}{\bar{p}}\left(\bar{L} - \frac{M}{\bar{p}}\right)^{\lambda+\frac{1}{\theta-1}}\frac{\theta^{\lambda+\frac{1}{\theta-1}}}{\bar{L}^{\lambda+\frac{1}{\theta-1}}(\theta-1)} \rightarrow 1 = \frac{M}{\bar{p}\bar{L}}\left(1 - \frac{M}{\bar{L}\bar{p}}\right)^{\lambda+\frac{1}{\theta-1}}\frac{\theta^{\lambda+\frac{1}{\theta-1}}}{\frac{\theta-1}{\theta}},$$

again delivering $\frac{M}{\bar{p}} = \bar{L}\left(1 - \frac{1}{\theta}\right)$. Replacing these, we thus plot

$$Y_t^{EF} = A_t^{\lambda+\frac{\theta}{\theta-1}}\bar{L} \text{ and } Y_t^{ES} = \bar{L}\left(\theta A_t - (\theta-1)\right)^{\lambda+\frac{1}{\theta-1}}.$$
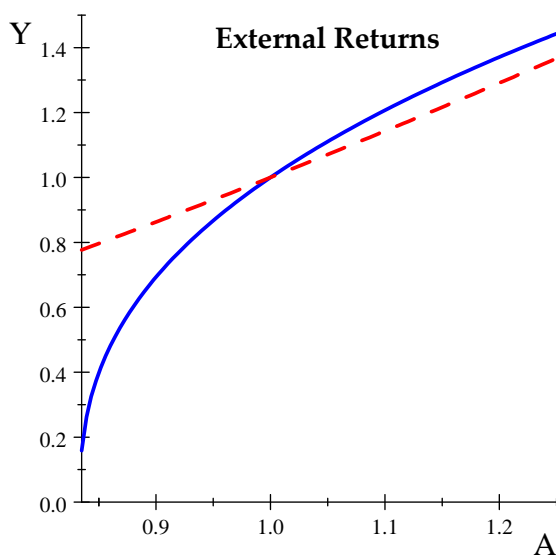

Figure A1: Y^{EF} (flex. prices) red dash, Y^{ES}

(sticky prices) solid blue. External-returns CES

$\lambda = 0.2$.

The response of the output gap to a negative shock can be found as follows to second order.

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$$c_t^{EF} = \left(\lambda + \frac{\theta}{\theta - 1}\right) a_t + \frac{1}{2}\left(\lambda + \frac{1}{\theta - 1}\right)\left(\lambda + \frac{\theta}{\theta - 1}\right) a_t^2$$

$$c_t^{ES} = \left(\lambda + \frac{1}{\theta - 1}\right) \theta a_t + \frac{1}{2}\left(\lambda + \frac{1}{\theta - 1}\right)\left(\lambda + \frac{1}{\theta - 1} - 1\right) \theta^2 a_t^2$$

The effect of supply shocks is thus:

$$\frac{dc_t^{EF}}{da_t} = \left(\lambda + \frac{\theta}{\theta - 1}\right) + \left(\lambda + \frac{1}{\theta - 1}\right)\left(\lambda + \frac{\theta}{\theta - 1}\right) da_t$$

$$\frac{dc_t^{ES}}{da_t} = \left(\lambda + \frac{1}{\theta - 1}\right) \theta + \left(\lambda + \frac{1}{\theta - 1}\right)\left(\lambda + \frac{1}{\theta - 1} - 1\right) \theta^2 da_t$$

The effect on the output gap is

$$\frac{d\left(c_t^{ES} - c_t^{EF}\right)}{da_t} = \lambda\left(\theta - 1\right) + \left(\lambda + \frac{1}{\theta - 1}\right)\left[\lambda\left(\theta^2 - 1\right) + \theta - \theta^2\right] da_t$$

This is larger than zero (so falls more to negative shocks) if:

$$\lambda\left(\theta - 1\right) + \left(\lambda + \frac{1}{\theta - 1}\right)\left[\lambda\left(\theta^2 - 1\right) + \theta - \theta^2\right] da_t > 0$$

Even with negative externalities $\lambda < 0$, this can still hold for negative enough shock, i.e.:

$$\left(\lambda + \frac{1}{\theta - 1}\right)\left[\lambda\left(\theta^2 - 1\right) + \theta - \theta^2\right] da_t > -\lambda\left(\theta - 1\right) > 0,$$

we need

$$\lambda < \frac{\theta}{\theta + 1}$$

which is always satisfied when $\lambda < 0$ since the right side is positive.

So the condition is:

$$d\left(-a_t\right) > \frac{\lambda}{\left(\lambda + \frac{1}{\theta-1}\right)\left[\lambda\left(\theta + 1\right) - \theta\right]}.$$

For a calibration with the overall benefit of variety $(\lambda + \frac{1}{\theta-1})$ equal to half the markup, $\lambda\left(\theta - 1\right) = -.5$ (a property of the translog preferences used in Bilbiie et at 2012 to match the cyclicality of markups and profits), the threshold is 0.215.

B.1 Insights from approximating $Y(N)$

Taking a second-order approximation of the aggregate production function/resource constraint:

$$Y_t = N_t^{\lambda + \frac{1}{\theta - 1}} \left(A_t \bar{L} - N_t f \right)$$

around the steady-state of the FP equilibrium (same as for SP equilibrium) with $N^{EF} = \frac{A\bar{L}}{f\theta}$, $Y^{EF} = \left(N^{EF} \right)^{\lambda + \frac{1}{\theta - 1}} \left(A\bar{L} - N^{EF} f \right) = \left(N^{EF} \right)^{\lambda + \frac{1}{\theta - 1}} \frac{\theta - 1}{\theta} A\bar{L}$

$$Y_t \simeq Y + \left(\left(\lambda + \frac{1}{\theta - 1} \right) N^{\lambda + \frac{1}{\theta - 1} - 1} \left(A\bar{L} - Nf \right) - N^{\lambda + \frac{1}{\theta - 1}} f \right) (N_t - N)$$

$$+ \frac{1}{2} \left(\begin{array}{c} \left(\lambda + \frac{1}{\theta - 1} \right) \left(\lambda + \frac{1}{\theta - 1} - 1 \right) N^{\lambda + \frac{1}{\theta - 1} - 2} \left(A\bar{L} - Nf \right) \\ - \left(\lambda + \frac{1}{\theta - 1} \right) N^{\lambda + \frac{1}{\theta - 1} - 1} f - \left(\lambda + \frac{1}{\theta - 1} \right) N^{\lambda + \frac{1}{\theta - 1} - 1} f \end{array} \right) (N_t - N)^2$$

and writing with percentage deviations:

$$y_t \simeq \lambda n_t + \frac{1}{2} \left(\lambda + \frac{1}{\theta - 1} \right) \left(\lambda - \frac{\theta}{\theta - 1} \right) n_t^2.$$

C Loglinearized general-CES NK model

This Appendix presents the loglinearized NK model with *arbitrary* benefit of input variety and first-order welfare effects, directly comparable with the plain-vanilla textbook version of the no-entry NK model.

In Table 2, we outline the key equilibrium responses of the loglinearized model, around a steady state with no supply shock mirroring the same structure as for Table 1, but for the loglinearized model. Letting a small letter denote the log-deviation from the respective steady-state, the loglinearized Euler equation (9) and Taylor rule are, respectively:

$$c_t = E_t c_{t+1} - \left(i_t - E_t \pi_{t+1}^C \right); \text{ and} \tag{15}$$

$$i_t = \phi \pi_t. \tag{16}$$

C.1 No-Entry Loglinearized Model

In the *No-Entry* model (superscript N) in the first row, under price flexibility all real variables are determined independently of any nominal forces (neutrality): in our case, exclusively by the supply shock $y_t^{NF} = c_t^{NF} = a_t$.[27] Given this optimal solution for consumption and output, the Euler equation serves to merely pin down the natural (Wicksellian), flexible-price interest rate: since we already solved for c_t^{NF} this implicitly defines the intertemporal price that confirms to agents that they are right to set that consumption path over time, $r_t^{NF} = E_t a_{t+1} - a_t$. Recalling that without entry $\pi_t = \pi_t^C$, the Taylor rule then determines uniquely the path of inflation through $\pi_t^{NF} = \phi^{-1} \left(E_t \pi_{t+1}^{NF} + r_t^{NF} \right)$ iff the Taylor principle is satisfied—but this is of course of no consequence for the real allocation.

With sticky (*fixed) prices*, the aggregate demand side, Euler equation (9) with no entry ($\pi_t = \pi_t^C$) is $\left(C_t^N \right)^{-1} = \beta E_t \left[\left(1 + r_t^N \right) \left(C_{t+1}^N \right)^{-1} \right]$, where the real interest rate is $1 + r_t^N \equiv \left(1 + I_t^N \right) / \left(1 + \pi_{t+1}^N \right)$. This illustrates most clearly that with either fixed prices and a Taylor rule ($P_t = \bar{P}$, $\pi_{t+1}^N = 0$, I_t^N fixed) or with a fixed real rate r_t^N, aggregate activity is invariant to supply shocks: it is fully pinned down by (9), where supply shocks do not appear. What bears the adjustment instead is the real wage, and with it hours worked, markups, and profits.[28]

In the upper right quadrant of Table 2 we outline the full equilibrium under *sticky* (fixed) prices. Hours worked increase proportionally with the negative supply shock $-a_t$ (as long as there is no rationing, which we implicitly assume); real wages increase, and markups and profits fall. Hours increase, even though the real wage goes up, because there is a negative income effect that dominates. This negative income effect arises because as wages go up, marginal cost goes up and profits go down—thus decreasing the income of households. This foreshadows the intuition for our model with entry, in which such profits variations cannot occur in equilibrium because they entail entry and exit, with different aggregate implications.

The key summary statistic describing whether the model generates or not a "demand recession" following a bad supply shock is whether output under sticky prices falls by more than output under flexible prices—that is, whether the output gap responds negatively to negative supply

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[27]Profits in the no-entry model are expressed as a share of steady-state Y: $d_t \equiv \frac{D_t - D}{Y} \simeq \frac{1}{\theta} y_t - mc_t$.

[28]Note that $\frac{\bar{p}_t}{\bar{P}}$ is no longer the profit-maximizing price. The aggregate production function and resource constraint is still $C_t = A_t \bar{L}_t$; this implicitly assumes that all markets clear, although prices are fixed. The adjustment necessary for equilibrium to obtain is borne by the nominal (and real) wage.

shocks. As it should be clear by now, in the no-entry-exit NK model the answer is no, the output gap being:

$$\frac{\partial \left(y_t^{NS} - y_t^{NF}\right)}{\partial \left(-a_t\right)} = 1,$$

and thus in fact *increasing* with bad supply shocks.

Table 2: Full loglinearized solution conditional on supply shock a_t		
	Flexible Prices F	Sticky Prices S
No-Entry (N)	$y_t^{NF} = c_t^{NF} = a_t$	$y_t^{NS} = c_t^{NS} = 0$
	$l_t^{NF} = 0$	$l_t^{NS} = -a_t$
	$w_t^{NF} = a_t$	$w_t^{NS} = -\varphi a_t$
	$\mu_t^{NF} = -mc_t^{NF} = 0$	$\mu_t^{NS} = -mc_t^{NS} = (1+\varphi)\, a_t$
	$d_t^{NF} = \frac{1}{\theta} a_t$	$d_t^{NS} = (1+\varphi)\, a_t$
	$r_t^{NF} = E_t a_{t+1} - a_t$	$r_t^{NS} = 0$
	$\pi_t^{NF} = \phi^{-1}\left(E_t \pi_{t+1}^{NF} + r_t^{NF}\right)$	$\pi_t^{NS} = 0$
Entry-Exit (E)	$y_t^{EF} = c_t^{EF} = \left(\lambda + \frac{\theta}{\theta-1}\right) a_t$	$y_t^{ES} = c_t^{ES} = \left(\lambda + \frac{1}{\theta-1}\right)\theta a_t$
	$n_t^{EF} = a_t$	$n_t^{ES} = \theta a_t$
	$l_t^{EF} = 0$	$l_t^{ES} = 0$
	$w_t^{EF} = \left(\lambda + \frac{\theta}{\theta-1}\right) a_t$	$w_t^{ES} = \left(\lambda + \frac{1}{\theta-1}\right)\theta a_t$
	$\mu_t^{EF} = -mc_t^{EF} = 0$	$\mu_t^{ES} = -mc_t^{ES} = a_t$
	$d_t^{EF} = 0$	$d_t^{ES} = 0$
	$r_t^{EF} = \left(\lambda + \frac{\theta}{\theta-1}\right)\left(E_t a_{t+1} - a_t\right)$	$r_t^{ES} = \left(\lambda + \frac{1}{\theta-1}\right)\theta\left(E_t a_{t+1} - a_t\right)$
	$\pi_t^{EF} = \phi^{-1}\left(E_t \pi_{t+1}^{EF} + r_t^*\right)$	$\pi_t^{ES} = 0$
	$\left(\pi_t^C\right)^{EF} = \pi_t^{EF} - \left(\lambda + \frac{1}{\theta-1}\right)\left(a_t - a_{t-1}\right)$	$\left(\pi_t^C\right)^{EF} = -\left(\lambda + \frac{1}{\theta-1}\right)\theta\left(a_t - a_{t-1}\right)$

Labor-Market Intuition with No Entry: The supply disruption (a_t falls) shifts labor demand downwards. This triggers an income effect on labor supply as the wage falls, so labor supply shifts rightward. By virtue of the log-utility assumption, income and substitution effects cancel out and hours stay unchanged: the wage falls one-to-one, and markup and profits stay unchanged. There is inflation as firms increase prices to keep real marginal cost (markup) constant at the desired level; how much inflation there is depends on the Taylor rule response. If the shock is transitory,

the natural interest rate goes up to give agents the right intertemporal incentives to consume less today.

With sticky prices, labor demand still moves down initially, as the marginal cost goes up; but now firms cannot increase prices, so the markup goes down, and profits go down too. Consumption and output do not change because there is no intertemporal substitution: with fixed prices (or with a fixed real rate) the Euler equation implies that consumption stays unchanged. In terms of labor market equilibrium, labor supply does not shift: we move *along* it. Markup and profits go down by enough to make it optimal to work more and keep consumption unchanged (income effect), while the real wage goes up by φa_t (substitution effect).

C.2   Endogenous Entry-Exit Loglinearized Model

Under endogenous entry-exit and flexible prices, the solution is readily obtained by noticing that $\mu_t^{EF} = \frac{\theta}{\theta-1}$. By virtue of logarithmic utility in consumption, hours worked stay constant (income and substitution effects on labor cancel out). Through (5), the real wage responds to labor productivity with elasticity $\lambda + \frac{\theta}{\theta-1}$; the effect is amplified relative to the no-entry model by the standard variety effect that acts like a form of increasing returns, making output and consumption also move with the shock in the same manner. The number of firms changes proportionally to the shock: a decrease in productivity triggers exit because it induces losses. The lower left quadrant of Table 2 outlines the full solution of the EF (endogenous entry-exit, flexible-price) model. Other than substantiating the above, notice that the natural interest rate responds with the same sign as under no entry-exit but with a larger elasticity, driven by the increasing returns. Since the natural rate increases with bad shocks, there is inflation in producer prices. And since there is exit, there is even higher inflation in consumer prices through the benefit of input variety. These inflation dynamics are nevertheless *still irrelevant* for the real allocation since prices are flexible.

Matters are different with sticky prices. Hours worked are again fixed in equilibrium, because income and substitution effects of the real wage cancel out (log utility in consumption), and in addition there are no extra income effects due to profits, which are zero by virtue of free entry. This can be seen by combining equations (7) and (6), and recalling the discussion after the latter, which implies that in loglinearized terms we have $w_t = c_t$.

Combining the loglinearized Euler equation (15) with the loglinearized (8) relating CPI, PPI inflation and variety growth:

$$\pi_t = \pi_t^C + \left(\lambda + \frac{1}{\theta - 1}\right)(n_t - n_{t-1}),\tag{17}$$

and imposing fixed producer prices $\pi_t = 0$, we obtain:

$$c_t = E_t c_{t+1} - \left(\lambda + \frac{1}{\theta - 1}\right)(E_t n_{t+1} - n_t).\tag{18}$$

Loglinearization of the markup-pricing rule (2) combined with the relative price (1) delivers:

$$w_t - a_t = \left(\lambda + \frac{1}{\theta - 1}\right)n_t - \mu_t,\tag{19}$$

while the free-entry condition (4) is:

$$n_t = a_t + l_t + (\theta - 1)\,\mu_t.\tag{20}$$

Combining the last two while imposing that hours are constant in this equilibrium $l_t = 0$ and replacing in $w_t = c_t$, we obtain:

$$c_t = \frac{\theta}{\theta - 1} a_t + \lambda n_t$$

Together with the Euler equation under fixed prices (18), this delivers

$$n_t^{ES} = \theta a_t;\ c_t^{ES} = \left(\lambda + \frac{1}{\theta - 1}\right)\theta a_t,$$

and the rest of the solution reported in the lower right quadrant of Table 2. Direct comparison with the solution under flexible prices delivers our condition for (first-order) negative output gap following a negative supply shock, $\lambda > 0$.

To help intuition, consider again the *labor market equilibrium*. With free entry-exit and flexible prices (EF), there is a larger recession than with no entry (NF) because of the variety effect which generates aggregate returns to scale: aggregate LD is upward sloping (with slope $\lambda + \frac{1}{\theta - 1}$) and shifts by $\lambda + \frac{\theta}{\theta - 1}$ with supply disturbances. Individual labor demand is as before, but now

an increase in marginal cost and fall in markup triggers *exit* (product destruction); since prices can be freely set, the amount of product destruction is dictated by the benefit of variety. This is represented with blue dashes in Figure 4.

Consider next sticky (fixed) prices ES. Since prices cannot increase now, the *markup goes down*. The crucial questions is: does LD shift up, or down? This depends on the benefit of input variety $\lambda + \frac{1}{\theta-1}$ versus the net markup $\frac{1}{\theta-1}$. When external returns are positive $\lambda > 0$, the benefit of input variety is higher and LD shifts further down: instead of a fall in profits, as under no entry), there is now exit. As a result, LS shifts further right due to the further negative income effect and, as we will see, consistent with intertemporal substitution. In other words, there is a *negative output gap*: consumption and income fall more than under flexible prices.

A complementary intuition starts from recalling that since prices cannot increase, the *markup goes down*. When the benefit of variety is higher than the markup, labor demand shifts further down: instead of a fall in profits (as under no entry-exit), there is now exit. The loglinear approximation of aggregate labor demand is:

$$w_t = \left(\lambda + \frac{\theta}{\theta-1}\right) a_t + \left(\lambda + \frac{1}{\theta-1}\right) l_t + \lambda \mu_t;$$

when the markup falls and real marginal cost increases there is a shift downwards in labor demand when $\lambda > 0$: demand forces dominate, labor demand plunges, and this demand shortage is met by dropping products. As a result, labor supply shifts further right due to the further negative income effect and, as we discuss in the dynamic model, consistent with intertemporal substitution.

Aggregate Demand and Variety: Intertemporal Interpretation

A key element of the model is the aggregate Euler equation governing aggregate demand (15), which written in gaps from the flexible-price equilibrium is:

$$c_t^{ES} - c_t^{EF} = E_t c_{t+1}^{ES} - E_t c_{t+1}^{EF} - \left(i_t - E_t \left(\pi_{t+1}^C\right)^{ES} - r_t^{EF}\right)$$

where $r_t^{EF} = \left(\lambda + \frac{\theta}{\theta-1}\right)\left(E_t a_{t+1} - a_t\right)$ is the natural interest rate. In this Euler equation, the relevant real rate is defined relative to CPI inflation. Spelling out CPI inflation using (17) we have:

$$c_t^{ES} - c_t^{EF} = E_t c_{t+1}^{ES} - E_t c_{t+1}^{EF} - \left[i_t - E_t \pi_{t+1}^{ES} + \left(\lambda + \frac{1}{\theta-1}\right)\left(E_t n_{t+1}^{ES} - n_t^{ES}\right) - r_t^{EF}\right], \quad (21)$$

which generalizes the aggregate-Euler IS curve with entry derived in Bilbiie, Ghironi, and Melitz (2007, equation 12).

With entry-exit, even when producer prices are fixed (or the real rate defined with respect to PPI inflation $i_t - E_t \pi_{t+1}^{ES}$ is fixed), the output gap is no longer proportional to the natural interest rate, as in a no-entry model. Indeed, the output gap then falls with bad supply shocks $\frac{\partial\left(c_t^{ES} - c_t^{EF}\right)}{\partial(-a_t)} < 0$ if:

$$\left(\lambda + \frac{1}{\theta-1}\right)\frac{\partial\left(E_t n_{t+1}^{ES} - n_t^{ES}\right)}{\partial(-a_t)} > \frac{\partial r_t^{EF}}{\partial(-a_t)},$$

that is if the increase in "expected inflation" that is purely due to the variety effect exceeds the increase in the natural rate. Replacing the responses of n_t^{ES} and r_t^{EF} we recover $\lambda > 0$.

Thus, with $i_t - E_t \pi_{t+1}^{ES}$ fixed, the real rate that is relevant for aggregate demand—i.e. real relative to CPI inflation—goes up since there is exit today, thus triggering intertemporal substitution towards the future. The labor supply then shifts right because of intertemporal substitution. This is a general mechanism that translates to our setup where producer prices are arbitrarily sticky, not fixed, outlined next.

The AD representation (21) also suggests a possible way out of a supply-driven, exit-amplified crisis: subsidize entry or sales temporarily so as to break the exit loop and generate future expected CPI inflation, and a boost in aggregate demand today by intertemporal substitution. This policy works even when interest rates are constrained against the lower bound.

C.3 The 3-Equation NK model with Free Entry-Exit

Like the textbook NK model (Woodford 2003, Gali 2008) our model can be summarized by an Aggregate Demand (IS curve) and Aggregate Supply (Phillips curve), with arbitrary degree of price stickiness. The former is given by (21), where we replace the number of firms using aggregate accounting $c_t = \frac{\theta}{\theta-1}a_t + \lambda n_t$ to obtain, after substitutions and using the flex-price equilibrium

$c_t^{EF} = \left(\lambda + \frac{\theta}{\theta-1}\right) a_t$ and $r_t^{EF} = E_t c_{t+1}^{EF} - c_t^{EF} = \left(\lambda + \frac{\theta}{\theta-1}\right)(E_t a_{t+1} - a_t)$:

$$c_t - c_t^{EF} = E_t \left(c_{t+1} - c_{t+1}^{EF}\right) + \lambda(\theta-1)\left(i_t - E_t \pi_{t+1} - \frac{1}{\lambda + \frac{\theta}{\theta-1}} r_t^{EF}\right) \tag{22}$$

or in levels (instead of gaps):

$$c_t = E_t c_{t+1} - \left(\lambda + \frac{1}{\theta-1}\right)\theta(E_t a_{t+1} - a_t) + \lambda(\theta-1)(i_t - E_t \pi_{t+1}) \tag{23}$$

We derive Aggregate Supply starting from the Phillips curve for PPI inflation obtained by assuming that it is costly for individual producers to change their prices, as in Bilbiie, Ghironi and Melitz (2007):

$$\pi_t = \beta E_t \pi_{t+1} - \psi \mu_t, \tag{24}$$

where $\psi = (\theta-1)/\kappa$ and κ is the Rotemberg adjustment-cost coefficient ranging from 0 (flexible prices) to infinity (fixed prices). The loglinearized free entry condition, using that hours worked are fixed in equilibrium, implies that $\mu_t = (\theta-1)^{-1}(n_t - a_t)$ and using aggregate accounting $c_t = \frac{\theta}{\theta-1} a_t + \lambda n_t$ to replace the number of goods we obtain:

$$\mu_t = \frac{1}{\lambda(\theta-1)}\left[c_t - \left(\lambda + \frac{\theta}{\theta-1}\right)a_t\right].$$

Replacing in the pricing equation and using $c_t^{EF} = \left(\lambda + \frac{\theta}{\theta-1}\right)a_t$ we obtain:

$$\pi_t = \beta E_t \pi_{t+1} - \psi \frac{1}{\lambda(\theta-1)}\left(c_t - c_t^{EF}\right) \tag{25}$$

Equations (22) and (25), together with a standard Taylor rule

$$i_t = \phi \pi_t, \tag{26}$$

constitute a full description of the model.

Notice that when prices are flexible, the equilibrium is fully determined by the supply side AS (25), $c_t = c_t^{EF}$. While when prices are completely rigid, it is determined exclusively by the demand side, AD (22) or (23) $c_t^{ES} = \left(\lambda + \frac{1}{\theta-1}\right)\theta a_t$. In between these two extremes, we need to solve the

model.

To do so, we first notice that the requirement for equilibrium determinacy in the model with entry-exit is exactly the same as in the no-entry model: the Taylor principle $\phi > 1$. To prove this, replace (25) and (26) into (22) to eliminate the output gap and interest rate, obtaining (let):

$$\pi_t - \beta E_t \pi_{t+1} = E_t \pi_{t+1} - \beta E_t \pi_{t+2} - \psi \left(\phi \pi_t - E_t \pi_{t+1} - \frac{1}{\lambda + \frac{\theta}{\theta-1}} r_t^{EF} \right), \tag{27}$$

Solving under AR1 shock with persistence ρ_a, $E_t a_{t+1} = \rho_a a_t$ and letting $\tilde{\psi} \equiv \psi / (1 - \beta \rho_a)$, we obtain:

$$\pi_t = -\tilde{\psi} \frac{1 - \rho_a}{1 - \rho_a + (\phi - \rho_a) \tilde{\psi}} a_t$$

$$c_t - c_t^{EF} = \lambda (\theta - 1) \frac{1 - \rho_a}{1 - \rho_a + (\phi - \rho_a) \tilde{\psi}} a_t$$

The result generalizes the previous one, derived with fixed prices: when the condition making demand, variety forces dominate supply, entry-exit forces holds ($\lambda > 0$), a bad supply shock causes a *negative output gap* and *PPI inflation*. Whereas in the opposite case, it causes a *positive output gap* that is still accompanied by PPI inflation. As a side note, this points to the possibility of deriving an implicit empirical test, based on macro comovements, of the mysterious micro parameter λ. Since there is exit regardless of whether $\lambda \gtrless 0$, CPI inflation is also going up.

An important point, which is related to determinacy results staying unchanged relative to the no-entry model, is that the crossing of the threshold $\lambda = 0$ triggers a swiveling of *both* AD and AS: in the $\lambda > 0$ region, AD slopes upwards and AS slopes downwards. A shift upwards of AD (as happens when the natural interest rate goes up, in response to an adverse supply shock) moves us leftward along the downward sloping AS, thus triggering a fall in output gap and inflation. Whereas for $\lambda < 0$ AS and AD have regular slopes and a shock shifting AD up causes an increase in the output gap and inflation, moving along an upward sloping AS curve.[29]

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[29]In the CES-DS case, AD is vertical and price stickiness is irrelevant, the neutrality result in Bilbiie (2019).

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C.4 No-entry NK model recap

It is important to understand that the effects we emphasize are altogether absent in the standard, no-entry, fixed-variety NK model. A recapitulation of that model's core equations illustrates that point. Recalling that we use as a benchmark a logarithmic utility function in consumption, the IS curve is:

$$c_t^N = E_t c_{t+1}^N - \left(i_t^N - E_t \pi_{t+1}^N \right), \text{ or in gaps} \tag{28}$$

$$c_t^N - c_t^{NF} = E_t \left(c_{t+1}^N - c_{t+1}^{NF} \right) - \left(i_t^N - E_t \pi_{t+1}^N - r_t^{NF} \right)$$

while the Phillips curve is $\pi_t = \beta E_t \pi_{t+1} - \psi \mu_t$ or, replacing the markup:

$$\pi_t^N = \beta E_t \pi_{t+1}^N + \psi \left(1 + \varphi \right) \left(c_t^N - a_t \right)$$

If we now allow for inflation to move in response to adverse supply shocks, it will increase and, through the active Taylor rule, trigger an increase in real interest rates and a fall in consumption. The output gap, however, is still always positive (in a determinate equilibrium). Take shock with persistence ρ_a and using the same notation for $\tilde{\psi} \equiv \psi / \left(1 - \beta \rho_a \right)$

$$\frac{dr_t^{NF}}{d \left(-a_t \right)} = 1 - \rho_a$$

$$\frac{d \left(c_t^N - c_t^{NF} \right)}{d \left(-a_t \right)} = \left(1 - \rho_a + \left(\phi - \rho_a \right) \tilde{\psi} \left(1 + \varphi \right) \right)^{-1} \frac{dr_t^{NF}}{d \left(-a_t \right)}$$

$$= \frac{1 - \rho_a}{1 - \rho_a + \left(\phi - \rho_a \right) \tilde{\psi} \left(1 + \varphi \right)} \geq 0$$

with the limit $dc_t^N = dc_t^{NF}$ reached when shocks are permanent, prices flexible (trivially), or labor inelastic. The response of the consumption (output) level is:

$$\frac{dc_t^N}{d \left(-a_t \right)} = -\frac{\left(\phi - \rho_a \right) \tilde{\psi} \left(1 + \varphi \right)}{1 - \rho_a + \left(\phi - \rho_a \right) \tilde{\psi} \left(1 + \varphi \right)}$$

D Extensions: Alternative Utility Functional Forms

We extend our results to GHH utility without income effects on labor; CRRA utility of C; and a general homothetic input aggregator instead of CES.

D.1 GHH Utility: no income effects on labor

To analyze the response of hours worked, we solve all our models with GHH utility function:

$$U = \frac{1}{1-\nu}\left(C - \frac{L^{1+\eta}}{1+\eta}\right)^{1-\nu} \text{ and } = \ln\left(C - \frac{L^{1+\eta}}{1+\eta}\right) \text{ if } \nu = 1$$

The key property of this is that it eliminates income effects on labor altogether, because the optimality condition for labor choice (the labor supply) is simply:

$$W = L^{\eta},$$

which does not shift when income changes—we always move along it. The equilibrium is dictated by shifts in labor demand. The full solution is outlined in Table A1.

Table A1: Full loglinearized solution. labor productivity shock a_t

Table A1: Full loglinearized solution conditional on supply shock a_t		
	Flexible Prices F	Sticky Prices S
No-Entry (N)	$y_t^{NF} = c_t^{NF} = \frac{1+\eta}{\eta} a_t$ $l_t^{NF} = \frac{1}{\eta} a_t$ $w_t^{NF} = a_t$ $\mu_t^{NF} = -mc_t^{NF} = 0$ $d_t^{NF} = \frac{1}{\theta} \frac{1+\eta}{\eta} a_t$ $r_t^{NF} = E_t a_{t+1} - a_t$ $\pi_t^{NF} = \phi^{-1} \left(E_t \pi_{t+1}^{NF} + r_t^{NF} \right)$	$y_t^{NS} = c_t^{NS} = -(\theta-1) a_t$ $l_t^{NS} = -\theta a_t$ $w_t^{NS} = -\eta\theta a_t$ $\mu_t^{NS} = -mc_t^{NS} = (1+\eta\theta) a_t$ $d_t^{NS} = \left(\frac{1}{\theta} + \eta\theta \right) a_t$ $r_t^{NS} = 0$ $\pi_t^{NS} = 0$
Entry-Exit (E)	$y_t^{EF} = c_t^{EF} = \frac{\left(\lambda+\frac{1}{\theta-1}\right)(1+\eta)}{\eta-\left(\lambda+\frac{1}{\theta-1}\right)} a_t$ $n_t^{EF} = \frac{1+\eta}{\eta-\left(\lambda+\frac{1}{\theta-1}\right)} a_t$ $l_t^{EF} = \frac{\lambda+\frac{\theta}{\theta-1}}{\eta-\left(\lambda+\frac{1}{\theta-1}\right)} \hat{a}_t$ $w_t^{EF} = \eta \frac{\lambda+\frac{\theta}{\theta-1}}{\eta-\left(\lambda+\frac{1}{\theta-1}\right)} a_t$ $\mu_t^{EF} = -mc_t^{EF} = 0$ $d_t^{EF} = 0$ $r_t^{EF} = \frac{\left(\lambda+\frac{1}{\theta-1}\right)(1+\eta)}{\eta-\left(\lambda+\frac{1}{\theta-1}\right)} \left(E_t a_{t+1} - a_t \right)$ $\pi_t^{EF} = \phi^{-1} \left(E_t \pi_{t+1}^{EF} + r_t^* \right)$ $\left(\pi_t^C\right)^{EF} = \pi_t^{EF} - \frac{\left(\lambda+\frac{1}{\theta-1}\right)(1+\eta)}{\eta-\left(\lambda+\frac{1}{\theta-1}\right)} \left(a_t - a_{t-1} \right)$	$y_t^{ES} = c_t^{ES} = \theta \frac{\left(\lambda+\frac{1}{\theta-1}\right)(1+\eta)}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} a_t$ $n_t^{ES} = \theta \frac{1+\eta}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} a_t$ $l_t^{ES} = \theta \frac{\lambda+\frac{1}{\theta-1}}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} \hat{a}_t$ $w_t^{ES} = \theta\eta \frac{\left(\lambda+\frac{1}{\theta-1}\right)}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} \hat{a}_t$ $\mu_t^{ES} = -mc_t^{ES} = \frac{1+\eta}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} a_t$ $d_t^{ES} = 0$ $r_t^{ES} = \theta \frac{\left(\lambda+\frac{1}{\theta-1}\right)(1+\eta)}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} \left(E_t a_{t+1} - a_t \right)$ $\pi_t^{ES} = 0$ $\left(\pi_t^C\right)^{EF} = -\theta \frac{\left(\lambda+\frac{1}{\theta-1}\right)(1+\eta)}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} \left(a_t - a_{t-1} \right)$

The output gap response is

$$\frac{\partial \left(y_t^{ES} - y_t^{EF} \right)}{\partial (-a_t)} = -\left(\lambda + \frac{1}{\theta-1} \right) \theta \frac{1+\eta}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} + \left(\lambda + \frac{\theta}{\theta-1} \right) \frac{1+\eta}{\eta-\left(\lambda+\frac{1}{\theta-1}\right)}$$

and it is negative whenever

$$\left(\lambda + \frac{1}{\theta-1} \right) \theta \frac{1+\eta}{1+\eta-\left(\lambda+\frac{1}{\theta-1}\right)\theta} > \left(\lambda + \frac{\theta}{\theta-1} \right) \frac{1+\eta}{\eta-\left(\lambda+\frac{1}{\theta-1}\right)}$$

Restricting attention to equilibria with standard responses (expansionary productivity improve-

ments in both equilibria) we obtain the same condition as before:

$$\lambda > 0.$$

D.2 CRRA utility of C: Income effects and intertemporal substitution

Assume that utility takes the CRRA form, allowing for arbitrary income effects on labor supply and intertemporal substitution, both parameterized by the curvature σ^{-1}

$$U(C, L) = \frac{C^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - \chi \frac{L^{1+\varphi}}{1 + \varphi}$$

This changes (only) the labor supply and the Euler equation for aggregate consumption in a standard way, i.e.:

$$\chi L_t^{\varphi} = C_t^{-\frac{1}{\sigma}} W_t; \tag{29}$$

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \left(\frac{1 + I_t}{1 + \pi_{t+1}^C} C_{t+1}^{-\frac{1}{\sigma}} \right). \tag{30}$$

To find the condition for **Edgeworth complementarity**, take the cross-derivative of utility with respect to demand of two goods ω and $\tilde{\omega}$; direct differentiation of the CRRA function having replaced the CES aggregate $\frac{1}{1-\frac{1}{\sigma}} \left[\left(\int_0^{N_t} c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}\left(1-\frac{1}{\sigma}\right)} - 1 \right]$ delivers:

$$U_{c_\omega c_{\tilde{\omega}}} = \left(\frac{1}{\theta} - \frac{1}{\sigma} \right) c_{\omega t}^{-\frac{1}{\theta}} c_{\tilde{\omega}t}^{-\frac{1}{\theta}} c_t^{\frac{2}{\theta} - \frac{1}{\sigma} - 1},$$

implying immediately that goods are Edgeworth complements $U_{c_\omega c_{\tilde{\omega}}} > 0$ when $\sigma > \theta$.

Solving the model under this utility function delivers, for flexible prices:

$$n_t^{EF} = \frac{1}{1 - \left(\lambda + \frac{\theta}{\theta-1}\right)\frac{1-\sigma^{-1}}{1+\varphi}} \hat{a}_t$$

$$y_t^{EF} = c_t^{EF} = \frac{\lambda + \frac{\theta}{\theta-1}}{1 - \left(\lambda + \frac{\theta}{\theta-1}\right)\frac{1-\sigma^{-1}}{1+\varphi}} \hat{a}_t$$

and for fixed sticky prices SP:

$$n_t^{ES} = \frac{\theta}{\sigma} \frac{1}{1 - (1 - \sigma^{-1})\left[\frac{(\lambda + \frac{1}{\theta-1})\theta}{1+\varphi} - \lambda(\theta-1)\right]} \hat{a}_t$$

$$y_t^{ES} = c_t^{ES} = \frac{(\lambda + \frac{1}{\theta-1})\theta}{1 - (1 - \sigma^{-1})\left[\frac{(\lambda + \frac{1}{\theta-1})\theta}{1+\varphi} - \lambda(\theta-1)\right]} \hat{a}_t$$

The first observation is that the "entry multiplier" (ratio of ES and EF responses of n_t) is now:

$$\frac{\theta}{\sigma} \frac{1 - (\lambda + \frac{\theta}{\theta-1})\frac{1-\sigma^{-1}}{1+\varphi}}{1 - (1 - \sigma^{-1})\left[\frac{(\lambda + \frac{1}{\theta-1})\theta}{1+\varphi} - \lambda(\theta-1)\right]}$$

When is there amplification of the entry response? Under log utility, we recover $\theta > 1$; under inelastic labor, the solution in text θ/σ with the multiplier requirement $\theta > \sigma$. Under CES, likewise θ/σ. The interaction of non-log utility, elastic labor, and external effects generates richer dynamics. In particular, the responses of entry in both FP and SP can even change sign (positive to negative shock) when:

$$EF : \frac{\lambda + \frac{1}{\theta-1} - \varphi}{\lambda + \frac{\theta}{\theta-1}} > \sigma^{-1}$$

$$ES : 1 < (1 - \sigma^{-1})\left[\frac{(\lambda + \frac{1}{\theta-1})\theta}{1+\varphi} - \lambda(\theta-1)\right]$$

For CES, the condition is the same for EF and ES:

$$\theta < \sigma(1 - (\theta-1)\varphi)$$

Thus, under this condition—which under infinitely elastic labor $\varphi = 0$ in fact coincides with the amplification condition in Guerrieri et al—the endogenous response of entry to supply shocks flips sign under both flexible and sticky prices.

The response of the output gap to a supply disruption in the free entry-exit model is thus:

$$\frac{\partial (y_t^{ES} - y_t^{EF})}{\partial (-a_t)} = -\lambda(\theta-1)\frac{[1 + (\lambda + \frac{\theta}{\theta-1})(\sigma^{-1}-1)]}{\Omega}$$

where $\Omega \equiv \left(1 - (1-\sigma^{-1})\left[\frac{\left(\lambda+\frac{1}{\theta-1}\right)\theta}{1+\varphi} - \lambda(\theta-1)\right]\right)\left(1 - \left(\lambda+\frac{\theta}{\theta-1}\right)\frac{1-\sigma^{-1}}{1+\varphi}\right) > 0$ by the restriction that y_t^{ES} and y_t^{EF} both individually still fall with supply shocks. The question is, as before, when does the former fall by more than the latter?

Proposition 5 *Supply-driven demand recessions $\frac{\partial\left(y_t^{ES} - y_t^{EF}\right)}{\partial(-a_t)} < 0$ can occur in two cases:*

    1. If $\lambda > 0$, when $\sigma < 1 + \left(\lambda + \frac{1}{\theta-1}\right)^{-1} < \theta$

    2. If $\lambda < 0$, when $\sigma > 1 + \left(\lambda + \frac{1}{\theta-1}\right)^{-1} > \theta$

Case 1 is a generalization of our Proposition (3), in particular the condition for first-order effects $\lambda > 0$, to the case with larger income effect. Case 2 is different, and the parameter condition is the equivalent of Guerrieri et al in our different model. Thus, in this case there is a dampening of the entry-exit response under ES, and a magnification of the intensive-margin response. This translates into amplification of the aggregate response when the benefit of variety is smaller than the markup, the reason mirroring our benchmark case: entry is now inefficiently *high* and too elastic, so anything that reduces its response to supply shocks generates a welfare improvement; sticky prices, in this case, play precisely that dampening role.

As entry-exit becomes inefficiently low and to little responsive in the market equilibrium, i.e. $\lambda > 0$—a mechanism such as sticky prices that raises its responsiveness when $1 + \left(\lambda + \frac{1}{\theta-1}\right)^{-1} > \sigma$ then generates a first-order effect on consumption too, while engendering an increase in the intensive margin of surviving goods, as goods are substitutes. Conversely, with too high entry-exit $\lambda < 0$, we need the entry response to be dampened and the intensive margin to contract (by complementarity) in order to obtain a demand contraction.

D.3 Complementarity or substitutability

To illustrate the difference with Guerrieri et al's benchmark model more sharply, we perform an analysis similar to theirs but in our different framework. Namely, consider the good-specific Euler equation, linking the marginal rate of intertemporal substitution in one good and the "real" interest rate (with respect to inflation in the price of that good). In log-deviations from steady state, with $c_{\omega t}$ the log deviation of *individual* consumption of good ω a measure of the intensive margin, this is:

$$\frac{1}{\theta}c_{\omega t} + \left(\frac{1}{\sigma} - \frac{1}{\theta}\right)c_t = \frac{1}{\theta}E_t c_{\omega t+1} + \left(\frac{1}{\sigma} - \frac{1}{\theta}\right)E_t c_{t+1} - (i_t - E_t\pi_{t+1}). \tag{31}$$

The aggregation of individual into total consumption is: $c_t = c_{\omega t} + \frac{\theta}{\theta-1} n_t$;[30] replacing this above, we obtain an Euler equation for the *intensive* margin for an *exogenously given* extensive margin:

$$c_{\omega t} = E_t c_{\omega t+1} - \left(1 - \frac{\sigma}{\theta}\right) \frac{\theta}{\theta-1} \left(n_t - E_t n_{t+1}\right) - \sigma \left(i_t - E_t \pi_{t+1}\right). \tag{32}$$

An exogenous fall in the number of varieties $dn_t < 0$ (at fixed PPI-real interest rate $i_t - E_t \pi_{t+1}$) induces a fall in the demand for continuing goods if $\sigma > \theta$: this is exactly the condition in Guerrieri et al, the opposite of our requirement (14).[31] The intuition is that when $\sigma > \theta$, from the viewpoint of aggregate utility, any two individual goods are *Edgeworth complements*: a fall in the demand for one, or a fall in the number of goods, can only trigger a fall in the demand of surviving goods at constant real interest rates (and thus constant marginal utility *of those goods*) if the cross-derivative of utility with respect to any two goods is positive, i.e. complementarity. As it can be easily seen by direct differentiation of the CRRA function of the CES aggregator with respect to any two individual goods, the cross-derivative is proportional to $\left(\frac{1}{\theta} - \frac{1}{\sigma}\right)$ and thus positive when $\sigma > \theta$, a condition that seems plausible at the aggregated, sectoral level. Our amplification condition instead pertains to the *disaggregated* level and requires individual goods to be Edgeworth substitutes.

Furthermore, the above characterizes a *partial-equilibrium* response, whereas our focus is on the *general-equilibrium*, *endogenous* entry-exit response. Consider thus instead the same Euler equation rewritten in terms of aggregate consumption (having replaced the expression for CPI inflation):

$$c_t = E_t c_{t+1} + \frac{\sigma}{\theta-1} \left(n_t - E_t n_{t+1}\right) - \sigma \left(i_t - E_t \pi_{t+1}\right). \tag{33}$$

For aggregate activity to go down more than under flexible prices, the number of firms needs fall enough—since exit increases the aggregate-demand relevant real (with respect to CPI inflation) interest rate and triggers intertemporal substitution towards future consumption. Endoge-

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[30]This is the loglinear version of $C_t = \rho_t N_t c_{\omega t}$.

[31]An alternative illustration uses the solution under flexible prices to obtain the natural real (with respect to inflation in the individual good) interest rate $r_{\omega t}^{EF} \equiv (i_t - E_t \pi_{t+1})^{EF}$:

$$r_{\omega t}^{EF} = \left(\frac{\theta}{\sigma} - 1\right) \frac{1}{\theta-1} \left(E_t a_{t+1} - a_t\right).$$

This "natural" interest rate can fall with bad supply shocks when, again, $\sigma > \theta$.

nous changes in the extensive margin are thus key determinants of the aggregate response.

Under our benchmark of CES preferences, however, the equilibrium responses of consumption under flexible and sticky prices coincide *to a first order* as illustrated in Proposition (4). In this reference case, intensive and extensive margins move in exactly compensating ways: when goods are complements $\sigma > \theta$, the intensive margin still falls with negative supply shocks, following the logic described above. But the response of entry-exit itself is scaled down by $\theta/\sigma < 1$, which exactly compensates the former. When the opposite condition holds, $\theta/\sigma > 1$, the extensive margin response is magnified, but the intensive margin moves, again, in a compensating way.[32] Taking a first-order approximation of consumption given the number of products we have under sticky prices $c_t^{ES} = \frac{\sigma}{\theta-1} n_t^{ES}$, while under flexible prices $c_t^{EF} = \frac{\theta}{\theta-1} n_t^{EF}$. The sticky-price response of entry-exit to the shock is scaled by θ/σ, but here we see that the (partial-equilibrium) response of consumption to entry-exit is scaled by the inverse σ/θ, neutralizing the former.[33] As we saw above, deviating from CES preferences opens up an output gap to first order, by mechanisms similar to the ones emphasized in our benchmark case.

Lastly, when our condition (14) fails and $\sigma > \theta$, the entry-exit dampening generates aggregate-demand dampening too through second-order terms, as clear from Proposition (4): the output gap response to supply disruptions is in fact positive, just like in the no-entry NK model.

D.4 General Homothetic Input Aggregator

Assuming that the aggregator of intermediate gods takes the general homothetic form outlined in detail—in the context of preferences over individual varieties—in Bilbiie, Ghironi, and Melitz (2012, 2019), the model changes as follows. The relative price capturing the benefit of input variety is now an arbitrary function $\rho(N_t)$ and so is the elasticity of substitution between goods—and thus the markup. The pricing condition becomes

$$\mu(N_t)\frac{w_t}{A_t} = \rho(N_t)$$

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[32]The intuition for the exact compensation is the envelope argument stemming from the efficiency of entry with CES preferences.

[33]With elastic labor, the aggregate-demand amplification properties now depend in subtler ways on the balance of these parameters and labor elasticity; in particular, the economy may even exhibit perverse effects whereby consumption in both EF and ES goes up with negative shocks, but the output gap goes *down*, making it inappropriate as a sufficient statistic.

Loglinearization of the markup rule delivers model delivers

$$w_t - a_t = \epsilon n_t - \mu_t,$$

where the elasticity of the relative price to the number of goods capture the benefit of input variety and we denote it by $\epsilon \equiv \rho_N N / \rho$.

The free entry condition is (with μ the steady-state markup).

$$n_t = a_t + l_t + \frac{1}{\mu - 1} \mu_t.$$

Finally, letting ζ be the markup elasticity to N we have, under **flexible prices**:

$$\mu_t = \zeta n_t.$$

The other equations remain unchanged. Because of log utility in consumption, hours worked are fixed and solving the above we obtain:

$$n_t^{EF} = \frac{1}{1 - \frac{\zeta}{\mu - 1}} a_t$$

Intuitively, countercyclical markups $\zeta < 0$ imply less entry in response to a supply shock.

Substituting in the economy resource constraint we obtain

$$c_t^{EF} = \frac{(\mu - 1)(1 + \epsilon) - \zeta \mu}{\mu - 1 - \zeta} a_t$$

Under fixed prices, we have instead from the Euler equation with fixed real (relative to PPI) rate $c_t - \epsilon n_t = 0$ and replacing in the aggregate resource constraint $c_t = \mu a_t + [\epsilon - (\mu - 1)] n_t$:

$$n_t^{ES} = \frac{\mu}{\mu - 1} a_t \text{ and } c_t^{ES} = \epsilon \frac{\mu}{\mu - 1} a_t$$

This illustrates clearly that the "entry-exit multiplier" survives as long as $\frac{\mu}{\mu - 1} > \frac{1}{1 - \frac{\zeta}{\mu - 1}}$, which is always true when desired markups are countercyclical $\zeta < 0$ and generically true for $\zeta < \frac{\mu - 1}{\mu}$.

Note that we still have identical EF and ES elasticities in the knife-edge case $\epsilon = \mu - 1$, $c_t^{ES} =$

$c_t^{EF} = \mu a_t.$

AD amplification instead occurs when:

$$\epsilon \frac{\mu}{\mu - 1} > \frac{(\mu - 1)(1 + \epsilon) - \zeta\mu}{\mu - 1 - \zeta},$$

which (with countercyclical desired markups $\zeta < 0$ or $\zeta < \mu - 1$) implies:

$$\left(\frac{\epsilon}{\mu - 1} - 1 \right)(\mu - 1 - \zeta\mu) > 0 :$$

This yields the equivalent of our previous ($\lambda > 0$) condition:

$$\epsilon > \mu - 1.$$

The same condition also holds for procyclical desired markups $\zeta > \mu - 1 > 0$ since the requirement becomes:

$$\left(\zeta \frac{\mu}{\mu - 1} - 1 \right)(\epsilon - (\mu - 1)) > 0,$$

which also holds for $\epsilon > \mu - 1$. Therefore, our amplification condition applies to the wide class of general (non-CES) homothetic input aggregators.