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# TRACKING BIASED WEIGHTS:
# ASSET PRICING IMPLICATIONS OF VALUE-WEIGHTED INDEXING

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**ABSTRACT**

We show theoretically and empirically that flows into index funds raise the prices of large stocks in the index disproportionately more than the prices of small stocks. Conversely, flows predict a high future return of the small-minus-large index portfolio. This finding runs counter to the CAPM, and arises when noise traders distort prices, biasing index weights. When funds tracking value-weighted indices experience inflows, they buy mainly stocks in high noise-trader demand, exacerbating the distortion. During our sample period 2000-2019, a small-minus-large portfolio of S&P500 stocks earns ten percent per year, while no size effect exists for non-index stocks.

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# 1 Introduction

The growth of passive investing over the recent decades has been explosive. Total assets in index mutual funds and exchange-traded funds (ETFs) rose from \$400 billion in 2000 to \$8.5 trillion in 2019, a larger than twentyfold increase. As of 2019, the assets managed by equity index mutual funds and ETFs accounted for 50 percent of assets managed by all US equity funds, and for 15 percent of the US stock market as a whole. The S&P500 index attracts the bulk of equity index investing: as of 2019, 42 percent of equity index mutual funds were tracking that index.[1]

Index funds provide households with a low-cost option to invest in financial markets. Their effects on equilibrium asset prices and market efficiency are less well-understood. Suppose, in the spirit of the CAPM, that index funds track the market portfolio, and that portfolio is held by the average of active funds. If households switch from active funds uniformly into index funds, then there should be no effect on asset prices. If instead passive investing grows because more households access financial markets, then the market risk premium should drop. Hence, asset prices should rise and expected returns should drop, and these effects should be more pronounced for high CAPM-beta assets.

In this paper we study theoretically and empirically how the growth of passive investing impacts stock prices. On the empirical side, we find that flows into equity index funds have sharply different effects than the CAPM-implied ones. Flows raise disproportionately the prices of large-capitalization stocks in the S&P500 index relative to the prices of the index's small stocks. Hence, flows are associated with a low return of a portfolio of small minus large index stocks. Conversely, flows predict a high future return of the small-minus-large index portfolio. These effects run counter to the CAPM because small stocks have higher CAPM beta than large stocks.[2] We find additionally a strong size effect, namely, small stocks earn higher average returns than large stocks even after adjusting for CAPM beta. Moreover, this effect is confined to stocks within the S&P500 index.

On the theoretical side, we show that in the absence of noise traders (or when these traders hold the index), flows into passive funds have the CAPM-implied effects. In the presence of noise traders,

[1]These data are from Page 39 and Table 42 of the 2020 Investment Company Institute Factbook, and from Page 9 of the 2012 Investment Company Institute Factbook.

[2]See, for example, Fama and French (1992).

however, the effects differ sharply and align with our empirical findings. Intuitively, stocks in high demand by noise traders are overvalued and enter with high weights into indices that weigh stocks proportionately to their market capitalization. Conversely, stocks in low demand are undervalued and enter with low weights. Hence, funds that track value-weighted indices overweight stocks in high noise-trader demand and underweight stocks in low demand, compared to the weights they would choose under portfolio optimization. When these funds experience inflows, they undertake investments that exacerbate the price distortions.

Our model is set up in continuous time and builds on Buffa, Vayanos, and Woolley (2020). Agents can trade one riskless asset, whose return is exogenous and constant over time, and multiple risky assets, whose prices are determined endogenously in equilibrium. Agents are of three types: experts, who can invest in the riskless asset and in the risky assets without any constraints; non-experts, whose risky-asset portfolio must track an index; and noise traders, who generate an exogenous demand for the risky assets that is constant over time. An increase in the measure of non-experts corresponds to more households accessing financial markets through index funds. An increase in the measure of non-experts accompanied by an equal decrease in the measure of experts corresponds to households switching from active into index funds.

In equilibrium, assets in high noise-trader demand trade at a high price. For these assets, volatility per share is also high, and so is the price impact of buying additional shares. Hence, an increase in the measure of non-experts, which triggers asset purchases, generates a larger percentage price increase for the assets in high noise-trader demand. An increase in the measure of non-experts that is accompanied by an equal decrease in the measure of experts generates an even stronger effect in the same direction. Indeed, assets in high noise-trader demand attract less investment by experts and are, therefore, less affected by a drop in the experts' demand.

The relationship between noise-trader demand, volatility and price impact is easiest to understand in the case where an asset is in such large demand that experts must short it in equilibrium. A positive shock to the asset's expected dividends causes the asset's price to rise. The experts' short position thus becomes larger and carries more risk. As a consequence, experts become more willing to unwind their position and to buy the asset. Their buying pressure amplifies the price rise, resulting in high volatility per share. The high volatility causes, in turn, high price impact.

Indeed, experts accommodate additional purchases of the asset by holding an even larger short position. This exposes them to even more volatility, causing the price to rise with demand in a convex manner.

We summarize our theoretical results into four hypotheses: (1) flows into index funds are associated with a low return of a portfolio of small- relative to large-capitalization index stocks, and predict a high future return of that portfolio; (2) flows into index funds raise the concentration of index weights, as measured by the cross-sectional standard deviation or the Herfindahl-Hirschman index, and conversely high concentration predicts a high future return of the small-minus-large index portfolio; (3) the rise in index investing in the recent decades is associated with a high average return of the small-minus-large index portfolio; and (4) this size effect is weaker for stocks not in the index. We test the four hypotheses by taking the index to be the S&P500, and the flows to be into index mutual funds and ETFs that track that index. We refer to these institutions collectively as index funds. Our sample period is 2000-2019.

To test the first hypothesis, we examine how the return of the small-minus-large index portfolio relates to contemporaneous and to lagged flows into index funds. Consistent with the model, we find a negative relationship between the return and contemporaneous flows, and a positive relationship between the return and lagged flows. The observed relationships are stronger when market volatility is high, which is also consistent with the model.

To test the second hypothesis, we examine whether changes in the concentration of index weights are positively related to flows into index funds. Consistent with the model, we find a positive contemporaneous relationship between flows and changes in concentration. We further examine whether changes in concentration relate to the subsequent return of the small-minus-large index portfolio. High concentration of index weights could be a manifestation of large flows into index funds or of high noise-trader demand for some index stocks, both of which are positively related to the future return of the small-minus-large index portfolio. Consistent with the model, we find a strong positive relationship between changes in concentration and the future return of the small-minus-large index portfolio.

To test the third hypothesis, we form portfolios based on index weights. The decile portfolio of lowest index weight stocks earns an average return of 10% per year above the decile portfolio of highest index weight stocks. This difference cannot be explained by differences in CAPM beta.

The "within S&P500" size effect that we find differs from the traditional size effect (Banz (1981), Fama and French (1992)) in important ways. First, the index weight-based portfolio spans (explains away the abnormal returns of) the Fama and French SMB size portfolio, but not vice-versa. Second, despite the previous findings of a strong January seasonality in returns of small stocks, we find no evidence that the within S&P500 size effect exhibits a January seasonality. Third, and consistent with the fourth hypothesis, while a stock's weight in the S&P500 index is a strong and negative predictor of its future return, such a relation is not statistically significant for non S&P500 index stocks. Interestingly, for the period 1964-2000 when indexing was less prevalent, the relationship between market capitalization and subsequent returns was similar for the two groups of stocks.

Our paper relates to various strands of the literature on mutual funds and indexing. One strand examines empirically the effects of index additions, deletions and rebalancings. Harris and Gurel (1986) and Shleifer (1986) find that when stocks are added to the S&P500 index, their prices rise, with the effect being partly temporary. Goetzmann and Garry (1986) likewise find a price drop for deleted stocks.[3] Barberis, Shleifer, and Wurgler (2005), Greenwood (2008) and Boyer (2011) find that inclusion in an index renders stocks more correlated with the index. Our work differs because we examine the effects of flows into index funds rather than of changes in index composition.

Another strand of related literature examines the effects of institutional flows. Most of these papers focus on institutions as a whole or on actively managed mutual funds. Badrinath, Kale, and Noe (1995) and Sias and Starks (1997) find that institutional trading can explain lead-lag patterns in stock returns. They attribute their findings to institutions reacting to information before other investors do, an explanation also supported by the findings in Chakravarty (2001). Nofsinger and Sias (1999) and Wermers (1999) find that institutional trading is positively related to contemporaneous stock returns and predicts positively future returns over a six-month to one-

[3]Subsequent papers on how index weight changes affect price levels include Beneish and Whaley (1996) and Lynch and Mendenhall (1997), who find that part of the effect occurs after weight changes are announced and before they are made; Kaul, Mehrotra, and Morck (2000) and Chang, Hong, and Liskovich (2015), who use mechanical index adjustments to rule out explanations other than price pressure; Wurgler and Zhuravskaya (2002) and Petajisto (2011), who find a larger effect for stocks with higher idiosyncratic risk; Chen, Noronha, and Singal (2004), who find a more lasting effect for additions than for deletions; Greenwood (2005), who finds that index rebalancings affect not only those stocks whose weight changes but also the stocks that covary highly with them; and Pandolfi and Williams (2019) who examine how rebalancings of sovereign bond indices affect bond yields and exchange rates.

year horizon. Griffin, Harris, and Topaloglu (2003) and Sias, Starks, and Titman (2006) find that the contemporaneous relationship remains positive in higher frequencies. Dasgupta, Prat, and Verardo (2011) find that the predictive relationship turns negative over horizons longer than two years. Coval and Stafford (2007) find that institutional trading in response to extreme flows is associated with strong price reversals even over shorter horizons. A key question in these papers is whether institutional trading causes price movements or whether it merely reflects them, either by leading them, if institutions are better informed, or by lagging them, if institutions are positive feedback traders. The evidence on price reversals is supportive of a causal relationship, i.e., price pressure. Price pressure lies at the core of our analysis as well. Our analysis differs because it concerns flows into index funds.

Relatively few papers study the effects of index fund flows. Goetzmann and Massa (2003) find that investors sell index mutual funds after market declines, and these flows are positively related to contemporaneous index returns but do not predict returns over the following week. More recent papers focus on ETF flows. Closest to our work is Ben-David, Franzoni, and Moussawi (2018), who find that trading by passive ETFs tend to destabilize the prices of the stocks they hold. Our analysis differs because we focus on how index fund flows affect price levels in the cross section.

A final strand of related literature is theoretical. In Vayanos and Woolley (2013), active funds exploit noise-trader induced price distortions. When investors move from active into index funds, assets in high noise-trader demand become more expensive, while assets in low demand become cheaper. Our model generates larger effects of index flows on assets in high noise-trader demand even when the flows come from outside the asset market. In Kapur and Timmermann (2005) and Cuoco and Kaniel (2011), asset managers receive a fee that depends on their performance relative to an index, and in Brennan (1993), Basak and Pavlova (2013), Buffa and Hodor (2018) and Kashyap, Kovrijnykh, Li, and Pavlova (2020), managers derive direct utility from their performance relative to an index. These papers show that managers' concerns with relative performance induce them to buy assets in the index, causing their prices to rise. In our model, flows into index funds also cause prices to rise, with the effect being stronger for assets with high index weights. In Chabakauri and Rytchkov (2020), flows into index funds cause asset return volatilities to decline and have ambiguous effects on return correlations.

## 2 Theory

Our model builds on Buffa, Vayanos, and Woolley (2020, BVW), who examine how limits on asset managers' deviations from market indices affect equilibrium prices. We focus on the special case of BVW where the limits are infinitely tight, i.e., managers must track indices perfectly. We extend BVW by allowing for a more general index and by examining how changes in the measure of index investors affect prices. We first present our version of the BVW model and solve for equilibrium prices. We then perform comparative statics on how changes in the measure of index investors affect prices and expected returns, and derive our empirical hypotheses.

### 2.1 Model

Time $t$ is continuous and goes from zero to infinity. The riskless rate is exogenous and equal to $r > 0$. There are $N$ risky assets. Asset $n = 1, .., N$ pays a dividend flow $D_{nt}$ per share and is in supply of $\eta_n > 0$ shares. The dividend flow $D_{nt}$ follows the square-root process

$$dD_{nt} = \kappa_n \left(\bar{D} - D_{nt}\right) dt + \sigma_n \sqrt{D_{nt}} dB_{nt}, \tag{2.1}$$

where $\bar{D}$ and $\{\kappa_n, \sigma_n\}_{n=1,..,N}$ are positive constants and $B_{nt}$ is a Brownian motion. Setting the long-run mean of the dividend flow to a value $\bar{D}$ common across assets is without loss of generality because we can redefine the number $\eta_n$ of shares of each asset. For simplicity, we take the Brownian motions $\{B_{nt}\}_{n=1,..,N}$ to be mutually independent, thus assuming that assets have independent cashflows.

Denoting by $S_{nt}$ the price of risky asset $n$, the asset's return per share in excess of the riskless rate is

$$dR^{sh}_{nt} \equiv D_{nt}dt + dS_{nt} - rS_{nt}dt, \tag{2.2}$$

and the asset's return per dollar in excess of the riskless rate is

$$dR_{nt} \equiv \frac{dR^{sh}_{nt}}{S_{nt}} = \frac{D_{nt}dt + dS_{nt}}{S_{nt}} - rdt. \tag{2.3}$$

We refer to $dR_t^{sh}$ as share return, omitting that it is in excess of the riskless rate. We refer to $dR_t$ as return, omitting that it is per dollar and in excess of the riskless rate.

Agents are competitive and form overlapping generations living over infinitesimal time intervals. Each generation includes agents of three types. *Experts* observe the dividend flow and the supply of all risky assets, and can invest in the riskless asset and in the risky assets without any constraints. These agents can be interpreted as investors who invest with active managers. *Non-experts* do not observe the dividend flow and the asset supply, and their risky-asset portfolio must track an index. These agents can be interpreted as investors who invest with passive managers.[4] *Noise traders* generate an exogenous asset demand, which is constant over time.

We denote by $W_{1t}$ and $W_{2t}$ the wealth of an expert and a non-expert, respectively, by $z_{1nt}$ and $z_{2nt}$ the number of shares of risky asset $n$ that these agents hold, and by $\mu_1$ and $\mu_2$ these agents' measure. We denote by $\eta'_n$ the number of shares of asset $n$ included in the index. A non-expert thus holds $z_{2nt} = \lambda\eta'_n$ shares of asset $n$, where $\lambda$ is a proportionality coefficient that the agent chooses optimally. We denote by $u_n$ the number of shares of asset $n$ held by noise traders, and assume that $u_n$ is smaller than the asset's supply $\eta_n$.

The index does not include some assets, possibly small-capitalization ones, and weighs the remaining assets proportionately to their capitalization. We refer to the included and non-included assets as index and non-index assets, respectively. Denoting by $\mathcal{I}$ the set of index assets, $\eta'_n = 0$ for $n \notin \mathcal{I}$. Since the weights of index assets $n \in \mathcal{I}$ are proportional to capitalization, included supply $\eta'_n$ and actual supply $\eta_n$ are proportional. Without loss of generality, we set them to be equal.

Experts and non-experts born at time $t$ are endowed with wealth $W$. Their budget constraint is

$$dW_{it} = \left(W - \sum_{n=1}^{N} z_{int}S_t\right) rdt + \sum_{n=1}^{N} z_{int}(D_t dt + dS_t) = Wrdt + \sum_{n=1}^{N} z_{int} dR_{nt}^{sh}, \tag{2.4}$$

where $dW_{it}$ is the infinitesimal change in wealth over their life, $i = 1$ for experts, and $i = 2$ for

[4] Agents' choice to invest with active or passive managers could result from trading off the superior returns of active managers with their higher fees, in the spirit of Grossman and Stiglitz (1980).

non-experts. They have mean-variance preferences

$$\mathbb{E}_t(dW_{it}) - \frac{\rho}{2}\mathbb{V}\mathrm{ar}_t(dW_{it}) \tag{2.5}$$

over $dW_{it}$, where $\rho$ is a risk-aversion coefficient. The objective (2.5) can be derived from any VNM utility $u$, as can be seen from the second-order Taylor expansion

$$u(W + dW_{it}) = u(W) + u'(W)dW_{it} + \frac{1}{2}u''(W)dW_{it}^2 + o(dW_{it}^2). \tag{2.6}$$

Maximizing the conditional expectation of (2.6) is equivalent to maximizing (2.5), with $\rho = -\frac{u''(W)}{u'(W)}$. Non-experts, who do not observe $\{D_{nt}\}_{n=1,..,N}$, maximize the unconditional expectation of (2.6), which is equivalent to maximizing that of (2.5). The latter expectation is

$$\mathbb{E}(dW_{it}) - \frac{\rho}{2}\mathbb{V}\mathrm{ar}(dW_{it}), \tag{2.7}$$

because with infinitesimal wealth changes, $\mathbb{E}\left[\mathbb{V}\mathrm{ar}_t(dR_{nt}^{sh})\right] = \mathbb{V}\mathrm{ar}(dR_{nt}^{sh})$.[5]

## 2.2 Equilibrium

We look for an equilibrium where the price $S_{nt}$ of risky asset $n$ is a function of the asset's dividend flow $D_{nt}$. Denoting that function by $S_n(D_{nt})$ and assuming that it is twice continuously differentiable, we can write the share return $dR_{nt}^{sh}$ as

$$dR_{nt}^{sh} = D_{nt}dt + dS_n(D_{nt}) - rS_n(D_{nt})dt$$

[5] We can write $\mathbb{E}\left[\mathbb{V}\mathrm{ar}_t(dR_{nt}^{sh})\right]$ as

$$\mathbb{E}\left[\mathbb{V}\mathrm{ar}_t(dR_{nt}^{sh})\right] = \mathbb{E}\left[\mathbb{E}_t\left[(dR_{nt}^{sh})^2\right] - \left[\mathbb{E}_t(dR_{nt}^{sh})\right]^2\right].$$

Since the first term in the square bracket is of order $dt$ and the second of order $dt^2$, we can keep only the first term and find

$$\mathbb{E}\left[\mathbb{V}\mathrm{ar}_t(dR_{nt}^{sh})\right] = \mathbb{E}\left[\mathbb{E}_t\left[(dR_{nt}^{sh})^2\right]\right] = \mathbb{E}\left[(dR_{nt}^{sh})^2\right].$$

We likewise find

$$\mathbb{V}\mathrm{ar}(dR_{nt}^{sh}) = \mathbb{E}\left[(dR_{nt}^{sh})^2\right] - \left[\mathbb{E}(dR_{nt}^{sh})\right]^2 = \mathbb{E}\left[(dR_{nt}^{sh})^2\right].$$

$$= \left[D_{nt} + \kappa_n(\bar{D} - D_{nt})S_n'(D_{nt}) + \frac{1}{2}\sigma_n^2 D_{nt} S''(D_{nt}) - rS_n(D_{nt})\right] dt + \sigma_n \sqrt{D_{nt}} S_n'(D_{nt}) dB_{nt}, \tag{2.8}$$

where the second step follows from (2.1) and Ito's lemma.

Using the budget constraint (2.4), we can write the objective (2.5) as

$$\sum_{n=1}^{N} z_{int}\mathbb{E}_t(dR_{nt}^{sh}) - \frac{\rho}{2}\sum_{n=1}^{N} z_{int}^2 \mathbb{V}\text{ar}_t(dR_{nt}^{sh}).$$

Experts maximize (2.5) over positions $\{z_{1nt}\}_{n=1,..,N}$. The first-order condition is

$$\mathbb{E}_t(dR_{nt}^{sh}) = \rho z_{1nt} \mathbb{V}\text{ar}_t(dR_{nt}^{sh}). \tag{2.9}$$

The expected share return $\mathbb{E}_t(dR_{nt}^{sh})$ is the drift term in (2.8), and the share return variance $\mathbb{V}\text{ar}_t(dR_{nt}^{sh})$ is the square of the diffusion term. Non-experts maximize (2.7) over positions $\{z_{2nt}\}_{n=1,..,N}$ that satisfy $z_{2nt} = \lambda\eta_n'$. This amounts to maximizing

$$\sum_{n=1}^{N} \lambda\eta_{nt}'\mathbb{E}\left(dR_{nt}^{sh}\right) - \frac{\rho}{2}\sum_{n=1}^{N} \lambda^2(\eta_{nt}')^2\mathbb{V}\text{ar}(dR_{nt}^{sh})$$

over $\lambda$. The first-order condition is

$$\sum_{n=1}^{N} \eta_n'\mathbb{E}(dR_{nt}^{sh}) = \rho\lambda\sum_{n=1}^{N}(\eta_n')^2\mathbb{V}\text{ar}(dR_{nt}^{sh}). \tag{2.10}$$

Market clearing requires that the demand of experts, non-experts and noise traders equals the supply coming from asset issuers:

$$\mu_1 z_{1nt} + \mu_2\lambda\eta_n' + u_n = \eta_n. \tag{2.11}$$

Solving for $z_{1nt} = \frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}$, and substituting into the first-order condition (2.9) of experts, we find the following ordinary differential equation (ODE) for the function $S_n(D_{nt})$:

$$D_{nt}+\kappa_n(\bar{D}-D_{nt})S_n'(D_{nt})+\frac{1}{2}\sigma_n^2 D_{nt}S_n''(D_{nt})-rS_n(D_{nt}) = \rho\frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}\sigma_n^2 D_{nt}S_n'(D_{nt})^2. \quad (2.12)$$

We look for an affine solution to the ODE (2.12):

$$S_n(D_{nt}) = a_{n0} + a_{n1}D_{nt}, \tag{2.13}$$

where $(a_{n0}, a_{n1})$ are constant coefficients. Substituting (2.13) into (2.12) and identifying terms, we compute $(a_{n0}, a_{n1})$. Substituting the affine solution into the first-order condition (2.10) of non-experts, we compute $\lambda$, completing our characterization of the equilibrium.

**Proposition 2.1.** *In equilibrium, the price of risky asset $n$ is given by $S_n(D_{nt}) = a_{n0} + a_{n1}D_{nt}$, with*

$$a_{n0} = \frac{\kappa_n}{r}a_{n1}\bar{D}, \tag{2.14}$$

$$a_{n1} = \frac{2}{r+\kappa_n+\sqrt{(r+\kappa_n)^2 + 4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}}, \tag{2.15}$$

*and where $\lambda > 0$ solves*

$$\sum_{n=1}^{N}\eta_n'(\eta_n - u_n)a_{n1}^2 = \lambda(\mu_1+\mu_2)\sum_{n=1}^{N}(\eta_n')^2 a_{n1}^2. \tag{2.16}$$

*The price depends on $(\eta_n, \sigma_n, \eta_n', u_n, \mu_1, \mu_2)$ only through $\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2$, and is decreasing and convex in that variable.*

The dependence of the price on $\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2$ is key for our analysis. The quantity $\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2$ is the risk-adjusted net supply (RANS) of asset $n$ that each expert holds in equilibrium. RANS is equal to the supply $\eta_n$ coming from the issuer, minus the demand $\mu_2\lambda\eta_n'$ and $u_n$ coming from non-experts and noise traders, respectively. It is expressed in per-capita terms by dividing by the measure $\mu_1$ of experts, and is adjusted for risk by multiplying by $\sigma_n^2$. An asset $n$ in small RANS trades at a high price. Moreover, its price is highly sensitive to changes in the dividend flow $D_{nt}$. Indeed, consider the extreme case in which RANS is negative, so experts are short the asset. An increase in $D_{nt}$ tends to raise the asset's price because it raises expected dividends. At the same time, dividends become riskier due to the square-root specification of $D_{nt}$. Since experts hold a short position, the increase in risk makes them more willing to unwind their position and to buy the asset. This amplifies the price rise. Thus, holding constant the volatility of dividends through the parameter $\sigma_n^2$, assets in small RANS have high volatility per share.

The negative relationship between RANS and price is more pronounced for smaller values of RANS, i.e., the price is a decreasing and convex function of RANS. Intuitively, convexity arises because of the negative relationship between RANS and volatility per share holding $\sigma_n^2$ constant. Since assets in small RANS have higher volatility per share than assets in large RANS, experts require a larger price rise to accommodate a decline in RANS when RANS is small than when it is large.

### 2.3 Comparative Statics

Our main comparative statics exercise is to increase the measure $\mu_2$ of non-experts holding the measure $\mu_1$ of experts constant. This exercise can be interpreted as a increase in asset-market participation by households through index funds. We also perform an alternative exercise to increase $\mu_2$ holding the measure $\mu_1 + \mu_2$ of experts and non-experts constant. This exercise can interpreted as a switch by households from active to index funds. We examine how these changes affect the size (market capitalization) of different risky assets, and the relationship between size and expected returns.

#### 2.3.1 Measuring Size and Expected Returns

We begin by constructing our measures of size and expected returns. We measure size by weight in a capitalization-weighted portfolio. The weight of an asset $n$ in a portfolio of assets in a set $\mathcal{S}$ is

$$w_{nt} = \frac{\eta_n S_n(D_{nt})}{\sum_{m \in \mathcal{S}} \eta_m S_m(D_{mt})}. \tag{2.17}$$

The weight $w_{nt}$ varies over time because dividend flows do. With a large number of assets $m$ in $\mathcal{S}$, independence of dividend flows $D_{mt}$ and linearity of $S_m(D_{mt})$ imply that the denominator of (2.17) is $\sum_{m\in\mathcal{S}} \eta_m S_m(\bar{D})$ plus smaller-order terms.[6] Hence, the unconditional expectation of $w_{nt}$ is approximately

$$\begin{aligned}
\mathbb{E}\left(w_{nt}\right) &\approx \frac{\eta_n S_n(\bar{D})}{\sum_{m\in\mathcal{S}} \eta_m S_m(\bar{D})} \\
&= \frac{\frac{\eta_n}{r+\kappa_n+\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}}}{\sum_{m\in\mathcal{S}} \frac{\eta_m}{r+\kappa_m+\sqrt{(r+\kappa_m)^2+4\rho\frac{\eta_m-\mu_2\lambda\eta_m'-u_m}{\mu_1}\sigma_m^2}}},
\end{aligned} \tag{2.18}$$

where the second equality follows by using (2.13) and the values of $(a_{n0}, a_{n1})$ in Proposition 2.1. We refer to $\mathbb{E}(w_{nt})$ as index weight when the portfolio consists of the index assets ($\mathcal{S} = \mathcal{I}$) and by non-index weight when the portfolio consists of the non-index assets.

The unconditional expected return of risky asset $n$ is

$$\begin{aligned}
\mathbb{E}\left(dR_{nt}\right) &= \mathbb{E}\left(\frac{dR_{nt}^{sh}}{S_n(D_{nt})}\right) \\
&= \mathbb{E}\left(\frac{\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2 D_{nt} S_n'(D_{nt})^2}{S_n(D_{nt})}\right) dt \\
&= \frac{2\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}{r+\kappa_n+\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}} \mathbb{E}\left(\frac{D_{nt}}{\frac{\kappa_n}{r}\bar{D}+D_{nt}}\right) dt.
\end{aligned} \tag{2.19}$$

where the second equality follows by keeping only the drift term in (2.8) and replacing it by its value in (2.12), and the third equality follows by using (2.13) and the values of $(a_{n0}, a_{n1})$ in Proposition 2.1. We use $\mathbb{E}(w_{nt})$ and $\mathbb{E}(dR_{nt})$ to measure portfolio weight and expected return in the propositions derived in Sections 2.3.2 and 2.3.3.

### 2.3.2 Increase in Market Participation

When the measure $\mu_2$ of non-experts increases, their aggregate investment $\mu_2\lambda$ in the index rises and so do the prices of all index assets. Prices of non-index assets do not change because non-experts

[6] Denoting the number of assets in $\mathcal{S}$ by $M$, $\sum_{m\in\mathcal{S}} \eta_m S_m(D_{mt})$ is equal to $\sum_{m\in\mathcal{S}} \eta_m S_m(\bar{D})$, which is of order $M$, plus a term which is of order $\sqrt{M}$, plus smaller-order terms.

do not invest in them.

**Proposition 2.2.** *Suppose that the measure $\mu_2$ of non-experts increases, holding the measure $\mu_1$ of experts constant.*

- *The prices and expected returns of non-index assets do not change.*
- *Non-experts' aggregate investment $\mu_2\lambda$ in the index rises.*
- *The prices of all index assets rise and their expected returns drop.*

To derive cross-sectional implications for index assets, we focus on two polar opposite cases. In the first case, all index assets have identical characteristics except for noise-trader demand. That case captures a market where noise-trader demand is the main driver of cross-sectional variation. We refer to it as the *noise-trader model*. In the second case, noise-trader demand for each index asset is proportional to the asset's supply. That case is equivalent to noise traders being absent from the market and to supply being reduced proportionately across all index assets (i.e., multiplied by a scalar smaller than one and equal across assets). For that reason, we refer to it as the *no noise-trader model*. The two models differ in some of their predictions for expected returns and their response to fund flows. Our empirical results are consistent with the noise-trader model, and hence with the notion that index weights are biased.

**Proposition 2.3.** *Suppose that all assets in the index have identical characteristics except for their noise-trader demand ($(\eta_n, \kappa_n, \sigma_n) = (\eta, \kappa, \sigma)$ for all $n \in \mathcal{I}$). Consider index assets $n, m \in \mathcal{I}$ with asset $n$ being in larger demand ($u_n > u_m$).*

- *Asset $n$ has higher index weight than asset $m$ ($\mathbb{E}(w_{nt}) > \mathbb{E}(w_{mt})$) and earns lower expected return ($\mathbb{E}(dR_{nt}) < \mathbb{E}(dR_{mt})$).*
- *When the measure $\mu_2$ of non-experts increases, holding the measure $\mu_1$ of experts constant:*
  - *The price of asset $n$ rises more in percentage terms than the price of asset $m$.*
  - *The expected return difference $\mathbb{E}(dR_{mt}) - \mathbb{E}(dR_{nt})$ between assets $m$ and $n$ increases.*

When all index assets have identical characteristics except for noise-trader demand, an asset $n$ in higher noise-trader demand than an asset $m$ is in smaller risk-adjusted net supply (RANS).

Asset $n$ must earn a lower expected return than asset $m$ so that experts are induced to hold its smaller RANS, and hence trades at a higher share price ($S_{nt} > S_{mt}$). It has higher capitalization because of its higher share price and because all assets are in the same number of shares. Hence, when cross-sectional variation is driven only by noise-trader demand, index weight and expected return are negatively related.

When there are flows into index funds, assets $n$ and $m$ experience a equal increase in demand (in terms of number of shares) because the index includes an equal number of shares of both. Because the price is convex in RANS (Proposition 2.1) and asset $n$ is in smaller RANS than asset $m$, its price rises more in percentage terms. Moreover, asset $n$'s expected return drops more, and hence the difference in expected returns between assets $m$ and $n$ becomes larger.

**Proposition 2.4.** *Suppose that for all assets in the index, noise-trader demand is proportional to asset supply ($u_n = U\eta_n$ with $U < 1$ for all $n \in \mathcal{I}$) and mean-reversion is the same ($\kappa_n = \kappa$ for all $n \in \mathcal{I}$). Consider index assets $n, m \in \mathcal{I}$, with asset $n$ being in smaller risk-adjusted supply ($\eta_n\sigma_n^2 < \eta_m\sigma_m^2$).*

- *Asset $n$ earns lower expected return than asset $m$ ($\mathbb{E}(dR_{nt}) < \mathbb{E}(dR_{mt})$). It has higher index weight than asset $m$ ($\mathbb{E}(w_{nt}) > \mathbb{E}(w_{mt})$) if*

$$\frac{\eta_n}{\eta_m} > \frac{r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{(1-U)\eta_n}{\mu_1+\mu_2}\sigma_n^2}}{r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{(1-U)\eta_m}{\mu_1+\mu_2}\sigma_m^2}}. \tag{2.20}$$

- *When the measure $\mu_2$ of non-experts increases, holding the measure $\mu_1$ of experts constant,*
  - *The price of asset $n$ rises less in percentage terms than the price of asset $m$.*
  - *The expected return difference $\mathbb{E}(dR_{mt}) - \mathbb{E}(dR_{nt})$ between assets $m$ and $n$ decreases.*

When noise-trader demand is proportional to asset supply, an asset $n$ in smaller risk-adjusted supply than an asset $m$ is also in smaller RANS. (Supply and net supply are proportional because non-experts invest in the index and absorb a fixed fraction of the difference between supply and noise-trader demand.) Asset $n$ must earn a lower expected return than asset $m$ so that experts are induced to hold its smaller RANS, and hence trades at a higher share price ($S_{nt} > S_{mt}$). It can have a higher or lower capitalization (number of shares times share price) depending on its supply. If the smaller risk-adjusted supply of asset $n$ is due to its smaller supply, then asset $n$ has a lower index weight. If instead it is due to lower risk, then asset $n$ can have a higher index weight.

Suppose that there are flows into index funds. If asset $n$ is in smaller supply than asset $m$, then it experiences a smaller increase in demand (in terms of number of shares) because the index includes fewer shares of asset $n$ than of asset $m$. Because of the lower demand for asset $n$, that asset's price rises less in percentage terms than the price of asset $m$. The same conclusion holds if asset $n$ is less risky than asset $m$ because a given increase in demand affects its price less. In both cases, asset $n$'s expected return drops less than asset $m$, and hence the difference in expected returns between assets $m$ and $n$ becomes smaller.

The effects of flows in Proposition 2.4 are in line with the CAPM. Indeed, since agents have mean-variance preferences and there are no noise traders, the expected returns of the assets in the index are given by the CAPM. Since asset $n$ earns a lower expected return than asset $m$, it has a lower CAPM beta. Hence, Proposition 2.4 implies that flows into index funds raise the price of the low CAPM-beta asset $n$ less than of the high CAPM-beta asset $m$.

### 2.3.3 Switch from Active to Passive

When the measure $\mu_2$ of non-experts increases, holding the measure $\mu_1 + \mu_2$ of experts and non-experts constant, all assets are affected, including non-index ones. Non-index assets drop in price because there are fewer experts to invest in them and non-experts do not pick up the slack. To characterize the cross-sectional effects, we focus on the same two special cases as in Section 2.3.2, generalizing their definitions to include non-index assets. We derive counterparts of Propositions 2.3 and 2.4 in the Appendix (Propositions A.1 and A.2), and summarize them below.

Under the noise-trader model, an increase in $\mu_2$ holding $\mu_1 + \mu_2$ constant has the same effects as in Proposition 2.3: index assets in high noise-trader demand rise in price more in percentage terms than index assets in low demand, and their relative expected return drops. Two mechanisms drive these effects. As in Proposition 2.3, index assets in high noise-trader demand are more affected by the rise in the demand of non-experts because the price is convex. Moreover, the same assets are less affected by the drop in the demand of experts, because being in higher noise-trader demand they attract less investment by experts. The difference in expected return between non-index assets in high and low noise-trader demand moves in the same direction as for index assets. The movement is smaller, however, under plausible sufficient conditions derived in Proposition A.1, because the effect through non-expert demand (and price convexity) is absent.

Under the no noise-trader model, an increase in $\mu_2$ holding $\mu_1 + \mu_2$ constant does not affect index assets. This is because, in line with the CAPM, experts and non-experts hold the index, and their total measure does not change. On the other hand, because demand for non-index assets drops, and more so for assets in larger supply, the effects on those assets are the reverse of those in Proposition 2.4. In particular, the difference in expected return between non-index assets in high and low RANS rises.

#### 2.3.4 Empirical Hypotheses

Our empirical hypotheses follow from the noise-trader model, analyzed in Propositions 2.3 and A.1. Hypothesis 1 concerns the relationship between flows into index funds and the return of small- relative to large-capitalization index assets. According to each of Propositions 2.3 and A.1, index assets in higher noise-trader demand (i) have higher index weight, (ii) experience a higher percentage price increase following flows into index funds, and (iii) experience a larger decline in their future expected return following flows into index funds. Combining (i) and (ii) yields the first statement in Hypothesis 1. Combining (i) and (iii) yields the second statement. The third statement follows because the effects of flows on prices in Propositions 2.3 and A.1 converge to zero when the volatility parameter $\sigma$ goes to zero.

**Hypothesis 1.** *Flows into index funds during Period $t$ are:*

- *Negatively related to the return of small- minus that of large-capitalization index assets during Period $t$.*
- *Positively related to the return of small- minus that of large-capitalization index assets during Periods $t' > t$.*

*These effects are stronger during times of high market volatility.*

Hypothesis 2 concerns the relationship between the concentration in index weights on the one hand, and flows and returns on the other. Concentration reflects the extent to which the index weight of large-capitalization index assets exceeds that of small-capitalization ones. In our empirical analysis, we use two measures of concentration: the cross-sectional standard deviation of index weights, and the Herfindahl-Hirschman Index of index weights.

Since flows into index funds cause the price of large-capitalization index assets to rise more in percentage terms than the price of small-capitalization ones, they raise concentration. This yields the first statement in Hypothesis 2. The second statement follows because high concentration can arise following flows into index funds or following changes to noise-trader demand that make it more heterogeneous across assets (higher for assets in high demand, and lower for assets in low demand). In both cases, the future expected return of small-capitalization index assets increases more relative to that of large-capitalization ones.

**Hypothesis 2.** *High concentration of index weights during Period $t$ is:*

- *Positively related to flows into index funds during Period $t$.*
- *Positively related to the return of small- minus that of large-capitalization index assets during Periods $t' > t$.*

Hypothesis 3 concerns the unconditional relationship between market capitalization and expected return for index assets. According to each of Propositions 2.3 and A.1, index assets in high noise-trader demand have higher index weight than assets in low demand. Moreover, the former assets earn lower expected return. Combining the two results yields Hypothesis 3.

**Hypothesis 3.** *Small-capitalization index assets earn higher average return than large-capitalization index assets.*

Hypothesis 4 compares the relationship between market capitalization and expected return for index and for non-index assets. This relationship is negative for index assets, and becomes more negative following flows into index funds. Hence the rise in indexing should generate a more negative relationship between market capitalization and expected return for index assets than for non-index assets.

**Hypothesis 4.** *The average return difference between small- and large-capitalization assets is higher for index assets than for non-index assets.*

Hypotheses 1-3 cannot hold simultaneously in the no noise-trader model. Suppose that assets in lower risk-adjusted supply have higher index weight (i.e., (2.20) holds) and hence are the large-capitalization assets. Proposition 2.4 implies that these assets experience a lower percentage price increase following flows into index funds, contradicting Hypothesis 1. Suppose instead that assets in lower risk-adjusted supply are the small-capitalization assets. Proposition 2.4 implies that they earn lower expected return, contradicting Hypothesis 3.

# 3 Empirics

We test Hypotheses 1–4 by taking the index to be the S&P500, and the flows to be into index mutual funds and ETFs tracking that index. The S&P500 index attracts the bulk of equity index investing. We refer to index mutual funds and ETFs tracking the S&P500 index as index funds.

By considering only flows into index funds, we exclude the broader groups of institutions whose performance is benchmarked against the S&P500 index. For example, many active managers are evaluated against the S&P500 index or face tracking-error constraints limiting their deviation from that index. We focus on index funds because it is easier to measure their assets.

## 3.1 Descriptive Statistics

Our data on stock returns and firm accounting variables come from the Center for Research in Security Prices (CRSP) and Compustat. Our data on assets and flows for index mutual funds tracking the S&P500 index come from the Investment Company Institute (ICI). ICI does not report data on S&P500 ETFs. We instead collect those data from CRSP. CRSP reports data on domestically listed ETFs. We include in our analysis only plain-vanilla ETFs, excluding alternative ETFs such as leveraged ETFs, inverse ETFs and buffered ETFs. Our ETF sample consists of the SPDR S&P 500 ETF Trust, the iShares Core S&P 500 ETF, and the Vanguard S&P 500 Index Fund ETF, which collectively account for almost all of the plain-vanilla S&P500 ETF market.

Table 1 reports descriptive statistics for our main variables, for the sample of S&P500 stocks and the period July 2000 to June 2019. The descriptive statistics in Panel A of Table 1 concern firm-level variables. The average stock earns an average monthly return of 0.91%, with a standard deviation of 9.76%. It has an average market capitalization of $27 billion and an average index weight of 0.18%. The distributions of market capitalization and index weight are skewed to the right, with high skewness and kurtosis. In the Fama-MacBeth regressions, we use the natural logarithm of index weight ($log(IndexWeight)$), which has skewness and kurtosis closer to zero. We estimate a stock's CAPM beta by regressing the stock's monthly excess return on the excess return of the S&P500 index on a rolling five-year basis. The average stock has CAPM beta close to one. For each stock, we also compute the industry-adjusted book-to-market (BM) ratio, using the procedure of Fama and French (1992) and Daniel, Grinblatt, Titman, and Wermers (1997); the return momentum, measured by the past one-year return skipping the most recent month ($Ret_{-12,-2}$), following Jegadeesh and Titman (1993); and the short-term return reversal, measured by the past one-month return ($Ret_{-1}$).

In our sample period, the growth of index funds was substantial. As shown in Figure 1, the assets of index funds more than tripled, growing from less than $500 billion in July 2000 to more than $1.5 trillion in July 2019. As a result, the funds' ownership of S&P500 stocks more than doubled, expanding from 2.5% to more than 6%. Because index fund holdings exhibit a secular trend during our sample period, we focus on fund flows in our empirical tests.

The descriptive statistics in Panel B of Table 1 concern aggregate variables, sourced at a quarterly frequency. The variables are: index fund holdings, fund flows, and the concentration of index weights. We define index fund holdings at the end of quarter $t$ as the ratio of the value of index fund net assets to the value of the S&P500 index (i.e., the combined value of the S&P500 firms):

$$IndexFund_t = \frac{\$IndexAssets_t}{\$SP500_t}.$$

We use two measures of index fund flows. The first is the change in index fund holdings between the end of quarters $t-1$ and $t$:

$$Flow_{1,t} = IndexFund_t - IndexFund_{t-1}.$$

The second is dollar flows in quarter $t$ divided by the index value at the end of that quarter:

$$Flow_{2,t} = \frac{\$Flow_t}{\$SP500_t}.$$

ICI reports dollar flows into index mutual funds, so we use that direct measure. ICI does not report dollar flows into ETFs, so we infer these indirectly from CRSP using the change in ETF net assets and the ETF return as:

$$\$ETFFlow_t = \$ETFAssets_t - \$ETFAssets_{t-1} \times (1 + ETFRet_t).$$

We measure concentration of index weights at the end of quarter $t$ by the cross-sectional standard deviation of index weights ($Dispersion_t$) or alternatively by the Herfindahl-Hirschman Index ($HHI_t$).

## 3.2 Time-Series Relationships

Hypotheses 1 and 2 concern the time-series relationships between index fund flows and concentration of index weights, respectively, with the return spread of the small-minus-large index portfolio. We construct that portfolio in each quarter by forming decile portfolios based on size, with Decile 1 containing the smallest stocks in the S&P500 index (i.e., the stocks with the smallest index weights), and Decile 10 containing the largest stocks (i.e., the stocks with the largest index weights). The small-minus-large index portfolio consists of a long position in the stocks in Decile 1 combined with an equal short position in the stocks in Decile 10. We construct that portfolio in both equally-weighted and value-weighted terms.

### 3.2.1 Index Fund Flows

We test Hypothesis 1 using the regression specification:

$$SMB_{SPi,t} = \alpha_{i,j} + \gamma_{i,j,contemp} \times Flow_{j,contemp,t} + \gamma_{i,j,past} \times Flow_{j,past,t} + \epsilon_{i,j,t},$$

where $SMB_{SPi,t}$ is the return of the small-minus-large S&P500 index portfolio in quarter $t$, with $i = ew$ if the return is computed in equal-weighted terms, and $i = vw$ if the return is computed in value-weighted terms; $Flow_{j,contemp,t}$ are contemporaneous index fund flows, with $j = 1, 2$ corresponding to the two measures of flows; and $Flow_{j,past,t}$ are past index fund flows. Hypothesis 1 implies $\gamma_{i,j,contemp} < 0$ and $\gamma_{i,j,past} > 0$.

We define contemporaneous flows in quarter $t$ to also include flows in the previous quarter:

$$Flow_{j,contemp,t} = Flow_{j,t} + Flow_{j,t-1}.$$

We define past flows to include flows in more distant quarters going back to one year and a half:

$$Flow_{j,past,t} = \sum_{i=2}^{6} Flowj_{j,t-i}.$$

We include the previous quarter into contemporaneous flows to account for lags between flows and trade. For example, when ETF sponsors accept cash from authorized participants (APs) to create new ETF shares, they may not purchase the constituent securities immediately. Similarly, when APs redeem ETF shares, ETF sponsors return the constituent securities to APs but APs may not sell the securities immediately.

Table 2 shows the regression results. For both equal- and value-weighted returns, and for both measures of flows, we find the pattern of regression coefficients consistent with Hypothesis 1.

The coefficient $\gamma_{i,j,contemp}$ on contemporaneous flows is negative across the four specifications in Columns 1–4. The effects are statistically significant and economically large. For example, a one standard deviation increase in $Flow_{1,contemp,t}$ is associated with a contemporaneous decline in the return of the equally weighted small-minus-large index portfolio by 3.56% per quarter.[7] Hence, flows into index funds tend to drive up the prices of large stocks in the S&P500 index by more than the prices of small stocks in that index.

It is, of course, possible that a negative $\gamma_{i,j,contemp}$ reflects reverse causality. Suppose that large stocks in the S&P500 index perform well. Since they are the main driver of the index, the index

[7] A one standard deviation increase in $Flow_{1,contemp,t}$ is 0.14% ($= 0.10\% \times \sqrt{2}$). Multiplied by the slope coefficient -25.47, it yields an effect of -3.56%.

performs well too. If investors are performance-chasers, then they invest more in the index. This gives rise to a negative relationship between index fund flows and the return of the small-minus-large index portfolio. To partly address this concern, we include the index return in the regressions. The regression results remain similar.

The coefficient $\gamma_{i,j,past}$ on past flows is positive across the four specifications in Columns 1–4. It is statistically significant, however, only for $Flow_{2,past,t}$ (Columns 3–4). That effect is economically large. For example, a one standard deviation increase in $Flow_{2,past,t}$ predicts an increase in the future return of the equally weighted small-minus-large index portfolio by 2.82% per quarter.[8]

Our model implies that the relationship between index fund flows and the return of the small-minus-large index portfolio should be stronger when volatility is high. In Columns 5–8 of Table 2, we perform the same regressions as in Columns 1–4 for quarters when VIX is above average. In all specifications, the results are consistent with Hypothesis 1 and are statistically significant despite a smaller sample. Moreover, their economic significance strengthens. In particular, the coefficient $\gamma_{i,j,past}$ on past flows is four times as large for $Flow_{1,contemp,t}$ and twice as large for $Flow_{2,past,t}$.

### 3.2.2 Concentration of Index Weights

We test the first part of Hypothesis 2 using the regression specification

$$\Delta Concentration_t = \alpha_j + \gamma_j \times Flow_{j.contemp,t} + \epsilon_{j,t},$$

where $Concentration_t$ is the concentration of index weights at the end of quarter $t$, measured by $Dispersion_t$ or $HHI_t$, and $Flow_{j,contemp,t}$ are contemporaneous index fund flows, with $j = 1, 2$. The regression also includes the lagged equal- or value-weighted return of the small-minus-large index portfolio, to control for momentum. The first part of Hypothesis 2 implies $\gamma_j > 0$.

Table 3 shows the regression results. The coefficient $\gamma_j$ is positive and statistically significant across all four specifications. This finding is consistent with the first part of the Hypothesis 2 that flows into index funds increase the concentration of index weights. It is also consistent with Hypothesis 1 and the findings in Table 2, which indicate that flows into index funds are associated

[8]A one standard deviation increase in $Flow_{2,past,t}$ is 0.20% (=0.09% $\times \sqrt{5}$). Multiplied by the slope coefficient 14.01, it yields and effect of 2.82%.

with a low return of the small-minus-large index portfolio. The lagged return of the small-minus-large index portfolio is not statistically significant in the regressions.

We test the second part of Hypothesis 2 using the regression specification

$$SMB_{SPi,t} = \alpha_i + \gamma_{i,t-n} \times Concentration_{t-n} + \epsilon_{i,t},$$

where $SMB_{SPi,t}$ is the return of the small-minus-large S&P500 index portfolio in quarter $t$, with $i = ew$ if the return is computed in equal-weighted terms, and $i = vw$ if the return is computed in value-weighted terms; and $Concentration_{t-n}$ is the concentration of index weights in quarter $t-n$, measured by $Dispersion_{t-n}$ or $HHI_{t-n}$. The coefficient $\gamma_{i,t-n}$ represents the predictive effect of index concentration on the returns on the small-minus-large index portfolio $n$ quarters ahead. The second part of Hypothesis 2 implies $\gamma_{i,t-n} > 0$.

Table 4 shows the regression results. The coefficient $\gamma_{i,t-n}$ is positive and statistically significant across all four specifications. These results support the second part of the Hypothesis 2 that concentration of index weights predicts the future return of the small-minus-large index portfolio. The effects are economically large. For example, Column 1 shows that a one standard deviation increase in $Dispersion_t$ predicts an increase in the return of the equally weighted small-minus-large index portfolio two quarters ahead by 4.17% (= 0.03% × 139.2), with an adjusted $R^2$ of 19.1%. Column 2 shows that $HHI_t$ has a similar predictive power for the future return of the small-minus-large index portfolio. The predictive power of concentration carries through for six to seven quarters ahead.

### 3.3 Unconditional Averages

Hypotheses 3 and 4 concern the unconditional averages of returns. Hypothesis 3 compares the unconditional averages of returns across small and large stocks in the index. Hypothesis 4 examines how this comparison differs across index and non-index stocks.

### 3.3.1 Size Effect for Index Stocks

A simple way to test Hypothesis 3 is to form portfolios based on index weights for stocks in the S&P500 index. At the end of each June from 2000 to 2018, we form decile portfolios based on size, with Decile 1 containing the smallest stocks in the index (i.e., the stocks with the smallest index weights), and Decile 10 containing the largest stocks (i.e., the stocks with the largest index weights). We compute equal- and value-weighted returns on the ten portfolios and on the small-minus-large index portfolio that buys stocks in Decile 1 and shorts stocks in Decile 10.

Table 5 shows a sizable return spread between small and large index stocks. On an equal-weighted basis, stocks in Decile 1 earn average excess returns of 1.16% per month, while stocks in Decile 10 earn 0.37% per month. The resulting return spread of 0.79% per month is statistically significant at the 1% level. On a value-weighted basis, the return spread is even larger, 0.81% per month.

The return spread between small and large index stocks cannot be explained by differences in CAPM beta. After controlling for CAPM beta, the return spread (i.e., the spread in CAPM alpha) becomes 0.58% per month on an equally-weighted basis, and 0.61% per month on a value-weighted basis.

The "within S&P500" size effect that we find is reminiscent of the traditional size effect identified by Banz (1981) and Fama and French (1992), among others. We explore the relationship between the two effects through a series of tests.

A first test is to examine whether the payoff space of $SMB_{SP}$ spans the payoff space of the Fama and French $SMB_{FF}$ factor designed to capture the equity size effect, or vice versa. We do this by regressing the equal- or value-weighted return of $SMB_{SP}$ on the return of $SMB_{FF}$, and testing whether the intercept (alpha) is statistically different from zero. We then reverse the regression and perform the same test on the new intercept.

Columns 1–4 of Table 6 show the results. When regressing $SMB_{SP}$ on $SMB_{FF}$, the alpha is 0.57% and 0.59% per month for $SMB_{SPew}$ and $SMB_{SPvw}$, respectively. Both alphas are statistically significant. By contrast, when regressing $SMB_{FF}$ on $SMB_{SPew}$ and $SMB_{SPvw}$, the alpha is approximately -0.04% per month and statistically insignificant. These results indicate that the small-minus-large index portfolio spans the traditional size factor, but not the other way around.

A second test is to examine whether the "within S&P500" size effect exhibits a strong January seasonality, as previous papers document for the traditional size effect. Columns 5–8 of Table 6 show that the large average returns on the small-minus-large index portfolio are not associated with a January seasonality. The intercepts in the regressions of $SMB_{SPew}$ and $SMB_{SPvw}$ on the January indicator variable are 0.76% and 0.77% per month, respectively, both statistically significant; the slope coefficients for the January dummy are 0.42% and 0.54%, respectively, both statistically insignificant. For $SMB_{FF}$, the intercept is 0.17% and the slope coefficient for the January dummy 0.44%, both statistically insignificant.

To put the findings in Table 6 in historical perspective and connect them to previous papers, we perform the same regressions for an earlier sample period ranging from July 1964 to June 2000. The findings, reported in Table 7, show an entirely different picture. The returns on $SMB_{SPew}$, $SMB_{SPvw}$, and $SMB_{FF}$ are closely related, rendering the alphas from the spanning tests statistically insignificant. Moreover, there is a strong January seasonality that dominates the average returns on $SMB_{SPew}$, $SMB_{SPvw}$, and $SMB_{FF}$. In non-January months, the average returns are statistically insignificant, but in January, the returns are sizable. The historical evidence reinforces the notion that the "within S&P500" size effect that we identify is distinct from the traditional size effect.

#### 3.3.2 Size Effect for Index versus Non-Index Stocks

To test Hypothesis 4, we divide the universe of stocks available in CRSP and Compustat into two sets: S&P500 index stocks and non S&P500 index stocks. For an index stock, we compute its index weight by dividing the stock's market capitalization by the total capitalization of the stocks in the index. For a non-index stock, we compute an analogous portfolio weight, by dividing the stock's market capitalization by the total capitalization of non-index stocks. We then use the Fama and MacBeth (1973) cross-sectional regression to test if the relationship between portfolio weight and future returns is more negative within index stocks.

We start with index stocks. At the end of each June from 2000 to 2018, we compute the index weight of each stock and use it to predict the stock's monthly returns in the subsequent 12 months from July to next June. As additional predictors, we use the stock's CAPM beta, industry-adjusted book-to-market (BM) ratio, return momentum measured by the past one-year return skipping the most recent month ($Ret_{-12,-2}$), and short-term return reversal measured by the past one-month return ($Ret_{-1}$).

We report the results in Panel A of Table 8. Columns 1–5 shows univariate regressions of stock return on each of the five predictors. Index weight is strongly negatively related to future return, and is the only predictor that is statistically significant. Column 6 shows a multivariate regression of stock return on all five predictors. After controlling for CAPM beta, book-to-market ratio, return momentum and short-term return reversal, the negative relationship between index weight and future return strengthens, with the $t$-statistic increasing from $-2.95$ in the univariate regression to $-3.45$. It is noteworthy that the "within S&P500" size effect is distinct from the value effect, as proxied by BM. For S&P500 stocks, index weight is a strong predictor of future returns but BM is not.

We repeat the exercise with non-index stocks, and report the results in Panel B of Table 8. Column 1 shows that the relationship between portfolio weight and future return is negative but statistically insignificant, with a $t$-statistic of -0.80. In terms of economic significance, the slope coefficient on portfolio weight among stocks outside the S&P500 index is only one fifth of that among S&P500 stocks. Among other predictors, only short-term return reversal is statistically significant.

We next test whether the mean of the Fama-MacBeth regression coefficient for $Log(IndexWeight)$ in Column 6 of Panel A ($\gamma_{IndexWeight}$) equals that for $Log(PortfolioWeight)$ in Column 6 of Panel B ($\gamma_{PortfolioWeight}$). Consistent with Hypothesis 4, equality of the two coefficients is rejected, at the 5% significance level.

To put the findings in Table 8 in historical perspective, we perform the same regressions for an earlier sample period from July 1964 to June 2000. The findings, reported in Table 9, differ from those in Table 8 in two important ways. First, the relationship between portfolio weight and future return for non-index stocks is negative, statistically significant and close to that for index stocks. The multivariate regression coefficients for index and non-index stocks are -0.118 and -0.119, respectively, as shown in Column 6 of Panels A and B. This finding is consistent with the finding in Section 3.3.1 that $SMB_{SP}$ and $SMB_{FF}$ tend to be indistinguishable from each other in the spanning test in the earlier sample. Second, the "outside S&P500" size effect diminishes over time: the slope coefficient changes from -0.119 to 0.002, essentially becoming non-existent. On the other hand, the "within S&P500" size effect increases over time: the slope coefficient changes from -0.118 to -0.171, strengthening by almost one half.

#### 3.3.3 Anomalies in the S&P500 Index

Our finding of a strong negative relationship between index weight and future return among stocks in the S&P500 index is surprising for two reasons. First, S&P500 stocks are liquid and widely followed by security analysts and institutional investors. Hence, there are reasons to believe that they are efficiently priced. Consistent with this conjecture, the cross-sectional association between the "anomaly" variables and stock returns tends to be stronger in small and micro-cap stocks (see, e.g., Fama and French (2008)). Second, several studies have documented that the efficiency of the stock market has increased over time, in the sense that some return anomalies seem to have gone away (see, e.g., McLean and Pontiff (2016); Green, Hand, and Zhang (2017)). For example, Green, Hand, and Zhang (2017) argue that only 12 out of 94 stock characteristics identified in previous literature as predicting returns have predictive power for non-microcap stocks over the period 1980 to 2014. Against this backdrop, our post-millennium finding for a strong "within S&P500" size effect is surprising.

To provide more context for our finding, we examine the predictive power of the 12 stock characteristics identified by Green, Hand, and Zhang (2017) in our sample. We exclude two of the characteristics that they identify, book-to-market ratio and short-term return reversal, as they are examined in Table 8. This leaves us with 10 characteristics: cash holdings (cash), changes in 6-month momentum (chmom), changes in analyst coverage (chnanalyst), earnings announcement returns (aer), number of earnings increases (nincr), ratio of R&D expenditures to market value (rdve), return volatility (retvol), volatility of share turnover (std_turn), share turnover ratio (turn), and number of zero trading days (zerotrade). We include three additional characteristics, following Fama and French (2015): corporate investment and two proxies for profitability, gross profitability (gma) and operating profitability (operof).

We perform Fama-MacBeth regressions for the 13 characteristics, similar to the regressions in Table 8. The results, shown in Table 10, indicate that for stocks in the S&P500 index, none of the 13 characteristics predicts returns over the period 2000 to 2019.

## 4 Conclusion

We study theoretically and empirically how the growth of passive investing impacts stock returns. In a CAPM world, flows into equity index funds would not affect stock prices if the flows represent a uniform switch from active to index funds. If instead the flows represent new investment in the stock market, they would raise stock prices, with the impact being stronger for high CAPM-beta stocks. We instead find empirically that flows into funds tracking the S&P500 index raise disproportionately the prices of large-capitalization stocks in the index relative to the prices of the index's small stocks. Moreover, the flows predict a high future return of the small-minus-large index portfolio. We find additionally a strong "within S&P500" size effect: a small-minus-large portfolio of S&P500 stocks earns ten percent per year, while the return of the counterpart portfolio of non-S&P500 stocks is smaller and statistically insignificant.

Our theoretical model generates results in line with our empirical findings when noise traders do not hold the index, distorting prices away from the CAPM. When prices are distorted, weights of value-weighted indices are biased, and flows into index funds exacerbate the distortions. Intuitively, stocks in high demand by noise traders are overvalued and enter with high weights into value-weighted indices. Conversely, stocks in low demand are undervalued and enter with low weights. Hence, funds that track value-weighted indices overweight the former stocks and underweight the latter, compared to the weights they would choose under portfolio optimization. When these funds experience inflows, they undertake investments that exacerbate the distortions.

Our results suggest that passive investing and benchmarking can have important effects on market efficiency, and hence on the allocation of capital in the economy.[9] The strength of these effects can depend on the design of indices and on the decisions by passive funds on which indices to track. Examining these issues seems an interesting direction for future research.

[9] Kashyap, Kovrijnykh, Li, and Pavlova (2020) explore the links between benchmarking and investment decisions by firms.

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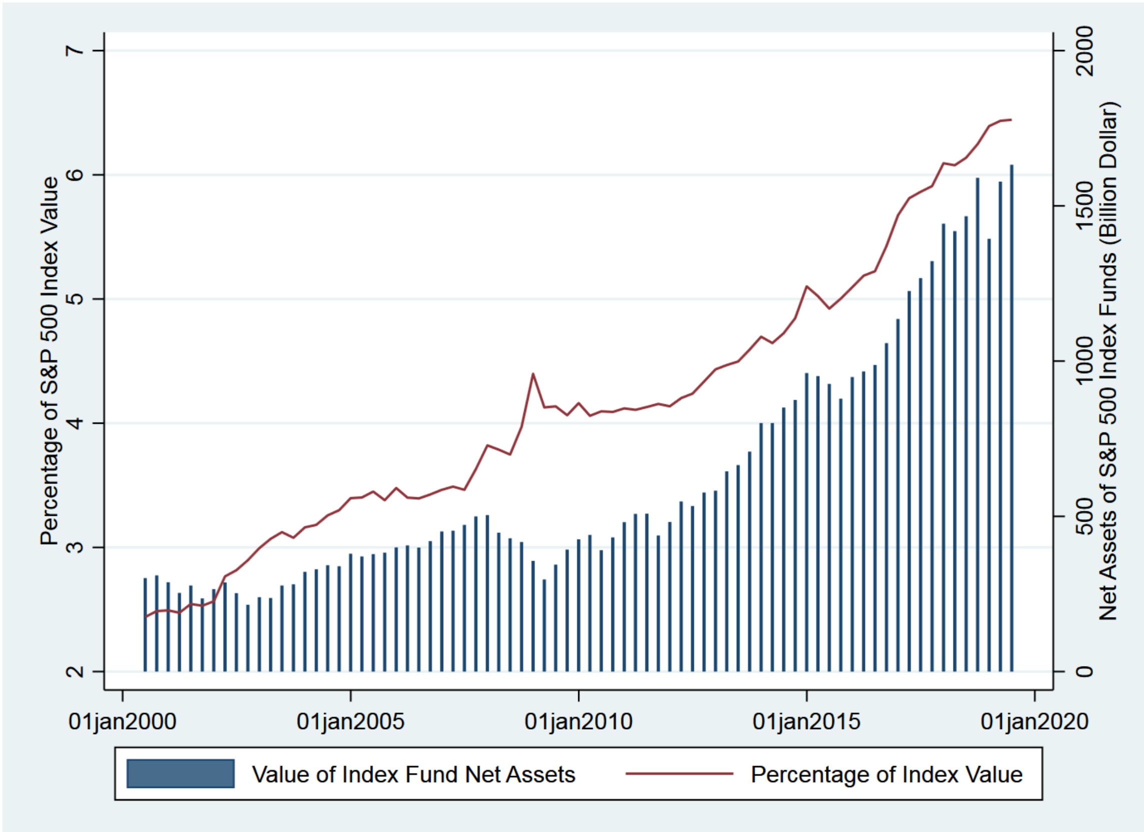


Figure 1: **Assets of S&P500 Index Funds**

This figure plots the value of the assets of mutual funds and ETFs tracking the S&P500 index over the period June 2000 to June 2019. The red line represents the ratio of index fund net assets to index value (left y-axis). The blue bars represent index fund net assets in millions of dollars (right y-axis). The data come from CRSP and ICI.

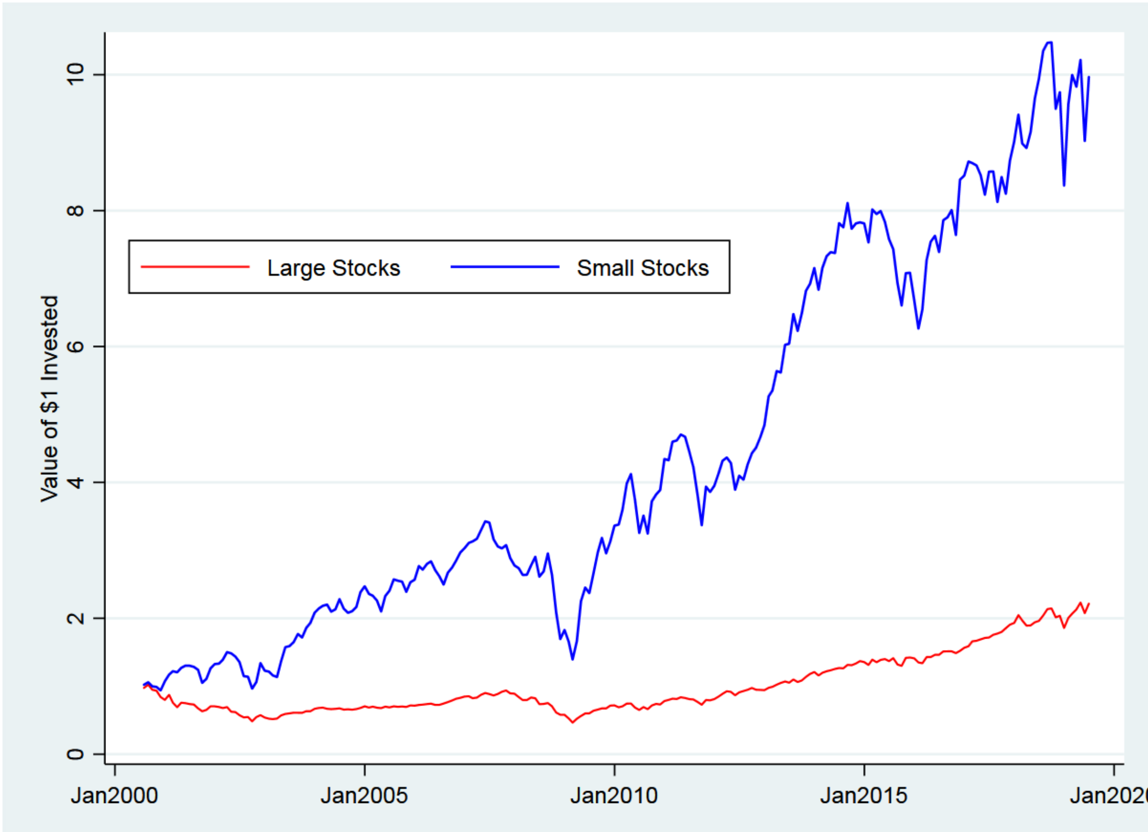


Figure 2: **Value of $1 Invested in Small and Large Stocks in the S&P500 Index**
This figure plots the value of $1 invested at the end of June 2000 in the bottom 10% of S&P500 stocks based on market capitalization (small stocks, blue line) and in the top 10% of S&P500 stocks (large stocks, red line). The portfolios are rebalanced annually at the end of each subsequent June and are liquidated in June 2019.

Table 1: **Descriptive Statistics**

This table shows descriptive statistics for the main variables in our sample from July 2000 to June 2019. Panel A includes the following firm-level variables: monthly stock return in percent, market capitalization in millions of dollars, weight of a stock in the S&P500 index in percent, CAPM beta, industry-adjusted book-to-market ratio, and return momentum (cumulative return from month $t-12$ to month $t-2$) for stocks in the S&P500 index. Panel B includes the following aggregate variables: index fund holdings (ratio of S&P500 index fund net assets to index value), changes in index fund holdings ($Flow_1$), dollar flows into index funds divided by index value ($Flow_2$), cross-sectional standard deviation of S&P500 index weights ($Dispersion$), and Herfindahl-Hirschman Index ($HHI$) of S&P500 index weights. The variables in Panel B are sourced at a quarterly frequency and are multiplied by 100.

Panel A: Firm-Level Variables

| | Mean | Std Dev | 25th Pctl | 50th Pctl | 75th Pctl | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| Monthly Return ($R_t \times 100$) | 0.91 | 9.76 | -3.75 | 1.06 | 5.62 | 0.46 | 10.93 |
| Market Cap ($millions) | 27,393 | 51,383 | 5,9021 | 11,827 | 25,305 | 5.71 | 50.56 |
| Index Weight ($\times$100) | 0.18 | 0.32 | 0.04 | 0.08 | 0.16 | 5.05 | 34.25 |
| Log(Index Weight) | -7.05 | 1.10 | -7.79 | -7.16 | -6.41 | 0.47 | 0.37 |
| Beta | 1.03 | 0.63 | 0.59 | 0.95 | 1.36 | 1.04 | 2.16 |
| Industry-Adjusted BM | -0.46 | 0.51 | -0.74 | -0.53 | -0.27 | 3.37 | 31.28 |
| Momentum ($R_{t-12,t-2}$) | 0.10 | 0.34 | -0.08 | 0.09 | 0.26 | 1.93 | 23.41 |

Panel B: Time-Series Variables (Quarterly, $\times$100)

| | Mean | Std Dev | 25th Pctl | 50th Pctl | 75th Pctl | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| Index Fund Holdings | 4.17 | 1.08 | 3.40 | 4.11 | 4.88 | 0.46 | -0.57 |
| $Flow_1$ | 0.05 | 0.10 | 0.00 | 0.04 | 0.10 | 0.51 | 3.44 |
| $Flow_2$ | 0.03 | 0.09 | -0.02 | 0.02 | 0.07 | 0.27 | 3.73 |
| $Dispersion$ | 0.35 | 0.03 | 0.32 | 0.34 | 0.37 | 0.38 | -0.79 |
| $HHI$ | 0.81 | 0.11 | 0.72 | 0.79 | 0.88 | 0.52 | -0.63 |

Table 2: **Index Fund Flows and the Return of the Small-Minus-Large S&P500 Index Portfolio**

This table shows the dynamic relationship between S&P500 index fund flows and the return of the small-minus-large index portfolio, computed in equal- and value-weighted terms. $Flow_{1,contemp}$ and $Flow_{1,past}$ are the contemporaneous and past index fund flows measured as changes in index fund holdings. $Flow_{2,contemp}$ and $Flow_{2,past}$ are the contemporaneous and past index fund flows measured as dollar flows divided by index value. $SMB_{SPew}$ and $SMB_{SPvw}$ are the return of the small-minus-large S&P500 index portfolio, computed in equal- and value-weighted terms, respectively. Columns 1–4 use the full sample, and Columns 5–8 use the sample when the VIX is above the sample mean.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| | $SMB_{SPew}$ | $SMB_{SPvw}$ | $SMB_{SPew}$ | $SMB_{SPvw}$ | $SMB_{SPew}$ | $SMB_{SPvw}$ | $SMB_{SPew}$ | $SMB_{SPvw}$ |
| $Flow_{1,contemp}$ | -25.47 | -23.13 | | | -39.78 | -36.52 | | |
| | (-3.13) | (-2.93) | | | (-2.63) | (-2.49) | | |
| $Flow_{1,past}$ | 5.77 | 4.44 | | | 22.71 | 21.36 | | |
| | (1.06) | (0.84) | | | (1.92) | (1.86) | | |
| $Flow_{2,contemp}$ | | | -17.95 | -14.91 | | | -31.00 | -27.38 |
| | | | (-2.04) | (-1.75) | | | (-1.95) | (-1.79) |
| $Flow_{2,past}$ | | | 14.01 | 13.32 | | | 35.08 | 34.40 |
| | | | (2.21) | (2.17) | | | (2.85) | (2.91) |
| $Intercept$ | 0.0358 | 0.0360 | 0.0118 | 0.0103 | 0.0189 | 0.0167 | -0.00586 | -0.00911 |
| | (1.80) | (1.87) | (0.73) | (0.65) | (0.48) | (0.43) | (-0.17) | (-0.28) |
| Observations | 76 | 76 | 76 | 76 | 28 | 28 | 28 | 28 |
| R-squared | 0.131 | 0.114 | 0.123 | 0.108 | 0.331 | 0.310 | 0.362 | 0.355 |
| Full Sample | Y | Y | Y | Y | | | | |
| High VIX | | | | | Y | Y | Y | Y |

Table 3: **Index Fund Flows and Concentration in Index Weights**

This table shows the relationship between S&P500 index fund flows and changes in the concentration of index weights. Panel A shows the results for $Flow_{1,contemp}$; Panel B for $Flow_{2,contemp}$. We use two measures of concentration of index weights: the cross-sectional standard deviation ($Dispersion$) and the Herfindahl-Hirschman Index ($HHI$). We use the lagged return of the small-minus-large index portfolio as a control variable.

Panel A: $Flow_{1,contemp}$

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | $\Delta Dispersion$ | $\Delta HHI$ | $\Delta Dispersion$ | $\Delta HHI$ | $\Delta Dispersion$ | $\Delta HHI$ |
| $Flow_{1,contemp}$ | 0.0282 | 0.104 | 0.0271 | 0.100 | 0.0268 | 0.0992 |
| | (2.97) | (2.97) | (2.89) | (2.96) | (2.84) | (2.91) |
| $L.SMB_{SPew}$ | | | -2.29e-05 | -2.19e-05 | | |
| | | | (-0.18) | (-0.05) | | |
| $L.SMB_{SPvw}$ | | | | | -4.55e-05 | -0.000108 |
| | | | | | (-0.35) | (-0.23) |
| $Intercept$ | -3.99e-05 | -0.000151 | -3.46e-05 | -0.000130 | -3.37e-05 | -0.000127 |
| | (-2.47) | (-2.53) | (-2.09) | (-2.18) | (-2.03) | (-2.12) |
| Observations | 76 | 76 | 75 | 75 | 75 | 75 |
| $R^2$ | 0.106 | 0.112 | 0.107 | 0.110 | 0.109 | 0.112 |

Panel B: $Flow_{2,contemp}$

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | $\Delta Dispersion$ | $\Delta HHI$ | $\Delta Dispersion$ | $\Delta HHI$ | $\Delta Dispersion$ | $\Delta HHI$ |
| $Flow_{2,contemp}$ | 0.0209 | 0.0758 | 0.0226 | 0.0836 | 0.0225 | 0.0831 |
| | (1.99) | (1.95) | (2.23) | (2.28) | (2.22) | (2.26) |
| $L.SMB_{SPew}$ | | | -7.75e-05 | -0.000224 | | |
| | | | (-0.61) | (-0.49) | | |
| $L.SMB_{SPvw}$ | | | | | -0.000105 | -0.000330 |
| | | | | | (-0.80) | (-0.69) |
| $Intercept$ | -2.25e-05 | -8.60e-05 | -1.77e-05 | -6.79e-05 | -1.70e-05 | -6.52e-05 |
| | (-1.55) | (-1.61) | (-1.23) | (-1.31) | (-1.18) | (-1.25) |
| Observations | 76 | 76 | 75 | 75 | 75 | 75 |
| $R^2$ | 0.051 | 0.049 | 0.070 | 0.071 | 0.073 | 0.074 |

Table 4: **Concentration in Index Weights and the Return of the Small-Minus-Large Index Portfolio**

This table shows the relationship between concentration of index weights and the future return of the small-minus-large index portfolio. In Columns 1–4 we measure concentration by *Dispersion*, and in Columns 5–8 we measure concentration by $HHI$. The regressions are univariate, and we use concentration lagged up to eight quarters.

| | | (1) | (2) | (3) | (4) | | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $SMB_{SPew}$ | $SMB_{SPvw}$ | $SMB_{SPew}$ | $SMB_{SPvw}$ | | $SMB_{SPew}$ | $SMB_{SPvw}$ | $SMB_{SPew}$ | $SMB_{SPvw}$ |
| Specification | *Dispersion* | Slope Coefficients | | Adj. $R^2$ | | $HHI$ | Slope Coefficients | | Adj. $R^2$ | |
| (1) | L1. | 133.3 | 133.7 | 0.169 | 0.185 | L1. | 38.46 | 38.63 | 0.174 | 0.191 |
| | | (3.85) | (4.07) | | | | (3.92) | (4.15) | | |
| (2) | L2. | 139.2 | 133.5 | 0.191 | 0.198 | L2. | 40.23 | 38.58 | 0.198 | 0.205 |
| | | (4.12) | (4.22) | | | | (4.21) | (4.31) | | |
| (3) | L3. | 101.8 | 98.00 | 0.105 | 0.110 | L3. | 28.80 | 27.69 | 0.104 | 0.109 |
| | | (2.89) | (2.96) | | | | (2.88) | (2.95) | | |
| (4) | L4. | 75.94 | 80.81 | 0.058 | 0.074 | L4. | 20.97 | 22.43 | 0.055 | 0.071 |
| | | (2.08) | (2.37) | | | | (2.02) | (2.31) | | |
| (5) | L5. | 95.98 | 100.6 | 0.094 | 0.116 | L5. | 27.19 | 28.53 | 0.093 | 0.115 |
| | | (2.67) | (3.00) | | | | (2.66) | (3.00) | | |
| (6) | L6. | 86.15 | 89.88 | 0.080 | 0.099 | L6. | 24.50 | 25.57 | 0.080 | 0.099 |
| | | (2.43) | (2.73) | | | | (2.43) | (2.73) | | |
| (7) | L7. | 67.45 | 69.19 | 0.050 | 0.060 | L7. | 19.06 | 19.54 | 0.050 | 0.059 |
| | | (1.88) | (2.07) | | | | (1.87) | (2.06) | | |
| (8) | L8. | 50.78 | 55.14 | 0.028 | 0.038 | L8. | 14.16 | 15.46 | 0.027 | 0.037 |
| | | (1.38) | (1.61) | | | | (1.36) | (1.59) | | |

Table 5: **Size Effect for S&P500 Index Stocks**

This table shows the average monthly return and CAPM alpha in percent for decile portfolios formed on the basis of stocks' weights in the S&P500 index. Specifically, at the end of each June from 2000 to 2018, we sort stocks in the index into ten portfolios according to their market capitalization-based index weights, with Decile 1 containing the stocks with smallest index weights, and Decile 10 containing the stocks with largest index weights. We compute equal- and value-weighted returns on the ten portfolios. We also compute $SMB_{SP}$, the difference in returns between stocks in deciles 10 and 1.

Panel A: Equal-Weighted Portfolio Returns

| | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | $SMB_{SP}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Average | 1.16 | 1.00 | 0.996 | 0.878 | 0.993 | 0.716 | 0.792 | 0.601 | 0.517 | 0.372 | 0.790 |
| | (2.38) | (2.68) | (2.63) | (2.57) | (3.12) | (2.34) | (2.63) | (1.94) | (1.64) | (1.27) | (2.48) |
| CAPM $\alpha$ | 0.487 | 0.459 | 0.437 | 0.371 | 0.521 | 0.255 | 0.323 | 0.119 | 0.0197 | -0.0886 | 0.576 |
| | (1.86) | (2.64) | (2.65) | (2.61) | (3.86) | (2.17) | (3.53) | (1.26) | (0.23) | (-1.06) | (1.97) |

Panel B: Value-Weighted Portfolio Returns

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Average | 1.13 | 0.987 | 0.987 | 0.876 | 0.981 | 0.717 | 0.788 | 0.602 | 0.504 | 0.32 | 0.813 |
| | (2.41) | (2.64) | (2.62) | (2.56) | (3.08) | (2.36) | (2.63) | (1.94) | (1.60) | (1.10) | (2.54) |
| CAPM $\alpha$ | 0.479 | 0.447 | 0.431 | 0.368 | 0.508 | 0.257 | 0.321 | 0.118 | 0.00805 | -0.13 | 0.608 |
| | (1.94) | (2.56) | (2.64) | (2.57) | (3.79) | (2.21) | (3.52) | (1.25) | (0.09) | (-1.36) | (2.05) |

Table 6: **S&P500 Size Factor and Fama-French Size Factor**

This table shows the relationship between the S&P500 size factor, defined as the return of the small-minus-large index portfolio in equal- and value-weighted terms ($SMB_{SPew}$ and $SMB_{SPvw}$, respectively), and the Fama and French SMB factor ($SMB_{FF}$). Columns 1–4 present the results of spanning tests. Columns 5–7 test for the January seasonality. We use monthly returns from July 2000 to June 2019.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| | Spanning Test | | | | January Seasonality | | |
| | $SMB_{SPew}$ | $SMB_{SPew}$ | $SMB_{FF}$ | $SMB_{FF}$ | $SMB_{SPew}$ | $SMB_{SPew}$ | $SMB_{FF}$ |
| $SMB_{FF}$ | 1.101 | 1.081 | | | | | |
| | (10.72) | (10.31) | | | | | |
| $SMB_{SPew}$ | | | 0.306 | | | | |
| | | | (10.72) | | | | |
| $SMB_{SPvw}$ | | | | 0.296 | | | |
| | | | | (10.31) | | | |
| January | | | | | 0.415 | 0.542 | 0.435 |
| | | | | | (0.36) | (0.47) | (0.72) |
| Intercept | 0.567 | 0.594 | -0.0389 | -0.0378 | 0.756 | 0.768 | 0.167 |
| | (2.18) | (2.23) | (-0.28) | (-0.27) | (2.27) | (2.29) | (0.95) |
| Observations | 228 | 228 | 228 | 228 | 228 | 228 | 228 |
| Adj. $R^2$ | 0.334 | 0.317 | 0.334 | 0.317 | -0.00385 | -0.00346 | -0.00215 |

Table 7: **S&P500 Size Factor and Fama-French Size Factor in the Earlier Period**
This table shows the same information as Table 6 for the period July 1964 to June 2000.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| | Spanning Test | | | | January Seasonality | | |
| | $SMB_{SPew}$ | $SMB_{SPew}$ | $SMB_{FF}$ | $SMB_{FF}$ | $SMB_{SPew}$ | $SMB_{SPew}$ | $SMB_{FF}$ |
| $SMB_{FF}$ | 1.027 | 0.986 | | | | | |
| | (20.01) | (19.20) | | | | | |
| $SMB_{SPew}$ | | | 0.470 | | | | |
| | | | (20.01) | | | | |
| $SMB_{SPvw}$ | | | | 0.468 | | | |
| | | | | (19.20) | | | |
| *January* | | | | | 5.770 | 5.100 | 2.260 |
| | | | | | (7.11) | (6.34) | (3.97) |
| *Intercept* | 0.138 | 0.197 | 0.0514 | 0.0289 | -0.112 | -0.00699 | 0.036 |
| | (0.81) | (1.15) | (0.44) | (0.24) | (-0.48) | (-0.03) | (0.22) |
| Observations | 432 | 432 | 432 | 432 | 432 | 432 | 432 |
| Adj. $R^2$ | 0.481 | 0.460 | 0.481 | 0.460 | 0.103 | 0.0832 | 0.0331 |

Table 8: **Size Effect for S&P500 Index Stocks Versus for Non-Index Stocks**

This table shows the relationship between stock characteristics and future stock returns. Panel A uses the stocks in the S&P500 index and Panel B uses non-S&P stocks. $IndexWeight$ is the weight of a stock in the S&P500 index; $PortfolioWeight$ is the weight of a stock in a hypothetical value-weighted portfolio of stocks not in the S&P500 index. The results are obtained using Fama-MacBeth cross-sectional regressions on monthly returns from July 2000 to June 2019.

Panel A: S&P500 Stocks

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $Log(IndexWeight)$ | -0.185 | | | | | -0.171 |
| | (-2.95) | | | | | (-3.45) |
| $Beta$ | | -0.152 | | | | -0.322 |
| | | (-0.55) | | | | (-1.40) |
| $BM$ | | | 0.100 | | | -0.067 |
| | | | (0.88) | | | (-0.67) |
| $Ret_{t-12,t-2}$ | | | | -0.334 | | -0.405 |
| | | | | (-0.60) | | (-0.83) |
| $Ret_{t-1}$ | | | | | -0.518 | -1.487 |
| | | | | | (-0.44) | (-1.62) |
| $Intercept$ | -0.426 | 1.044 | 0.948 | 0.621 | 0.822 | -0.338 |
| | (-1.15) | (5.00) | (2.86) | (2.11) | (2.79) | (-0.92) |
| Adj. $R^2$ | 0.011 | 0.058 | 0.008 | 0.044 | 0.026 | 0.114 |

Panel B: Non-S&P500 Stocks

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $Log(PortfolioWeight)$ | -0.037 | | | | | 0.002 |
| | (-0.80) | | | | | (0.04) |
| $Beta$ | | -0.132 | | | | -0.15 |
| | | (-0.66) | | | | (-0.87) |
| $BM$ | | | 0.085 | | | 0.035 |
| | | | (1.11) | | | (0.53) |
| $Ret_{t-12,t-2}$ | | | | -0.234 | | -0.266 |
| | | | | (-0.75) | | (-0.92) |
| $Ret_{t-1}$ | | | | | -2.224 | -2.437 |
| | | | | | (-3.06) | (-4.15) |
| $Intercept$ | 0.592 | 1.094 | 1.019 | 0.816 | 0.981 | 0.948 |
| | (0.83) | (4.97) | (2.92) | (2.48) | (2.97) | (1.87) |
| Adj. $R^2$ | 0.006 | 0.023 | 0.002 | 0.013 | 0.01 | 0.044 |
| $Pr(\gamma_{IndexWeight} = \gamma_{PortfolioWeight}) = 0.0131$ | | | | | | |

Table 9: **Size Effect for S&P500 Index Stocks Versus for Non-Index Stocks in the Earlier Period**

This table shows the same information as Table 8 for the period July 1964 to June 2000.

Panel A: S&P500 Stocks

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $Log(IndexWeight)$ | -0.096 | | | | | -0.118 |
| | (-2.11) | | | | | (-2.84) |
| $Beta$ | | 0.127 | | | | 0.029 |
| | | (0.67) | | | | (0.18) |
| $BM$ | | | 0.019 | | | 0.035 |
| | | | (0.62) | | | (1.25) |
| $Ret_{t-12,t-2}$ | | | | 0.883 | | 0.705 |
| | | | | (3.39) | | (3.08) |
| $Ret_{t-1}$ | | | | | -3.227 | -4.642 |
| | | | | | (-4.75) | (-7.87) |
| $Intercept$ | 0.501 | 1.080 | 1.203 | 1.086 | 1.265 | 0.339 |
| | (1.63) | (5.66) | (5.15) | (4.81) | (5.41) | (1.07) |
| Adj. $R^2$ | 0.018 | 0.03 | 0.001 | 0.027 | 0.016 | 0.076 |

Panel B: Non-S&P500 Stocks

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $Log(PortfolioWeight)$ | -0.118 | | | | | -0.119 |
| | (-2.68) | | | | | (-2.78) |
| $Beta$ | | 0.132 | | | | 0.082 |
| | | (0.86) | | | | (0.59) |
| $BM$ | | | 0.075 | | | 0.062 |
| | | | (4.56) | | | (4.31) |
| $Ret_{t-12,t-2}$ | | | | 1.046 | | 0.977 |
| | | | | (5.67) | | (6.48) |
| $Ret_{t-1}$ | | | | | -4.347 | -5.342 |
| | | | | | (-8.67) | (-12.65) |
| $Intercept$ | 0.212 | 1.158 | 1.290 | 1.106 | 1.287 | 0.058 |
| | (0.51) | (7.28) | (4.93) | (4.52) | (4.98) | (0.14) |
| Adj. $R^2$ | 0.012 | 0.023 | 0.001 | 0.017 | 0.01 | 0.054 |

Table 10: **Anomalies in the S&P500 Index Stocks**

This table shows the relationship between additional stock characteristics to those in Table 8 and future stock returns for the stocks in the S&P500 index. We use 10 characteristics from Green, Hand, and Zhang (2017): cash holdings (cash), changes in 6-month momentum (chmom), changes in analyst coverage (chnanalyst), earnings announcement returns (aer), the number of earnings increases (nincr), the ratio of R&D expenditures to market value (rdve), return volatility (retvol), volatility of share turnover (std_turn)share turnover ratio (turn), and the number of zero trading days (zerotrade). Following Fama and French (2015), we also include corporate investment and two proxies for profitability, the gross profitability (gma) and operating profitability (operof). The results are obtained using Fama-MacBeth cross-sectional regressions on monthly returns from July 2000 to June 2019.

| | cash | chmom | chnanalyst | ear | nincr | rd_mve | retvol | std_turn | turn | zerotrade | cinvest | gma | operprof |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Predictor* | -0.199 | -0.158 | -0.016 | 0.024 | -0.042 | 2.062 | -2.353 | 0.000 | -0.046 | -2800000 | -0.054 | -0.015 | -0.029 |
| | (-0.26) | (-0.48) | (-0.69) | (0.03) | (-0.93) | (1.00) | (-0.16) | (-0.00) | (-0.43) | (-0.14) | (-0.43) | (-0.06) | (-0.83) |
| *Intercept* | 1.010 | 0.885 | 0.995 | 0.991 | 1.046 | 0.952 | 0.882 | 0.945 | 0.981 | 0.968 | 0.948 | 0.957 | 0.976 |
| | (3.27) | (2.87) | (2.93) | (2.95) | (3.06) | (3.04) | (3.76) | (3.62) | (4.46) | (1.94) | (3.01) | (2.80) | (2.96) |
| Adj $R^2$ | 0.026 | 0.022 | 0.003 | 0.005 | 0.004 | 0.018 | 0.05 | 0.023 | 0.044 | 0.029 | 0.003 | 0.01 | 0.002 |

# Appendix

## A Proofs

**Proof of Proposition 2.1.** Substituting the affine price function (2.13) into the ODE (2.12), we find

$$D_{nt} + \kappa_n(\bar{D} - D_{nt})a_{n1} - r(a_{n0} + a_{n1}D_{nt}) = \rho\frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}\sigma_n^2 D_{nt}a_{n1}^2. \tag{A.1}$$

Equation (A.1) is affine in $D_{nt}$. Identifying the terms in (A.1) that are linear in $D_{nt}$ yields

$$\rho\frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}\sigma_n^2 a_{n1}^2 + (r + \kappa_n)a_{n1} - 1 = 0. \tag{A.2}$$

Equation (A.2) is quadratic in $a_{n1}$. When $\eta_n - \mu_2\lambda\eta_n' - u_n > 0$, the left-hand side is increasing for positive values of $a_{n1}$, and (A.2) has a unique positive solution, given by (2.15). When $\eta_n - \mu_2\lambda\eta_n' - u_n < 0$, the left-hand side is hump-shaped for positive values of $a_{n1}$, and (A.2) has either two positive solutions (including one double positive solution) or no solution. When two solutions exist, (2.15) gives the smaller of them, which is the continuous extension of the unique positive solution when $\eta_n - \mu_2\lambda\eta_n' - u_n > 0$. Identifying the constant terms in (A.1) yields

$$\kappa\bar{D}a_{n1} - ra_{n0} = 0,$$

whose solution is (2.14).

Equations (2.13) and (2.14) imply that the price is decreasing and convex in $z \equiv \frac{\eta_n - \mu_2\lambda\eta_n' + u_n}{\mu_1}\sigma_n^2$ if $a_{n1}$ is. Equation (2.15) implies that $a_{n1}$ is decreasing and convex in $z$ if the function

$$\Psi(z) \equiv \frac{1}{A + \sqrt{B + Cz}}$$

is, where $(A, B, C)$ are positive constants. The function $\Psi(z)$ is decreasing because its derivative

$$\Psi'(z) = -\frac{C}{2\sqrt{B + Cz}}\frac{1}{\left(A + \sqrt{B + Cz}\right)^2}$$

is negative. Since, in addition, $\Psi'(z)$ is increasing, $\Psi(z)$ is convex.

The unconditional expectation of the share return $dR^{sh}_{nt}$ is

$$\begin{aligned}
\mathbb{E}(dR^{sh}_{nt}) &= \mathbb{E}[\mathbb{E}_t(dR^{sh}_{nt})] \\
&= \mathbb{E}\left[D_{nt} + \kappa_n(\bar{D} - D_{nt})S'_n(D_{nt}) + \frac{1}{2}\sigma_n^2 D_{nt} S''(D_{nt}) - rS_n(D_{nt})\right] dt \\
&= \mathbb{E}\left[\rho \frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\sigma_n^2 D_{nt} S'_n(D_{nt})^2\right] dt \qquad \text{(A.3)} \\
&= \rho \frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\sigma_n^2 \bar{D} a_{n1}^2 dt, \qquad \text{(A.4)}
\end{aligned}$$

where the second inequality follows from (2.8), the third from (2.12), and the fourth from (2.13). The unconditional variance of the share return $dR^{sh}_{nt}$ is

$$\begin{aligned}
\mathbb{V}\mathrm{ar}(dR^{sh}_{nt}) &= \mathbb{E}[\mathbb{V}\mathrm{ar}_t(dR^{sh}_{nt})] \\
&= \mathbb{E}\left[\sigma_n^2 D_{nt} S'_n(D_{nt})^2\right] dt \\
&= \sigma_n^2 \bar{D} a_{n1}^2 dt, \qquad \text{(A.5)}
\end{aligned}$$

where the second inequality follows from (2.8), and the third from (2.13). Substituting (A.4) and (A.5) into (2.10), we find

$$\sum_{n=1}^{N} \eta'_n \frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1} a_{n1}^2 = \lambda \sum_{n=1}^{N} (\eta'_n)^2 a_{n1}^2, \qquad \text{(A.6)}$$

which we can rewrite as (2.16). Since $\eta_n > u_n$, (2.16) implies $\lambda > 0$.

An equilibrium exists if (2.16), in which $\{a_{n1}\}_{n=1,..,N}$ are implicit functions of $\lambda$ defined by (2.15), has a solution. For all non-positive values of $\lambda$, both sides of (2.16) are well-defined because the positivity of $\eta_n - \mu_2\lambda\eta'_n - u_n$ ensures that (A.2) has a solution for $a_{n1}$. Moreover, the right-hand side of (2.16) is positive, and exceeds the left-hand side which is non-positive. An equilibrium exists if both sides of (2.16) remain well-defined for a sufficiently large positive value of $\lambda$ that renders them equal. If there are multiple solutions $\lambda$ to (2.16), then we take the smallest. □

Lemma A.1 shows that an asset's unconditional expected return is increasing and concave in the asset's RANS.

**Lemma A.1.** *The unconditional expected return $\mathbb{E}(dR_{nt})$ that risky asset $n$ earns in equilibrium depends on $(\eta_n, \sigma_n, \eta'_n, u_n, \mu_1, \mu_2)$ only through $\frac{\eta_n - \mu_2 \lambda \eta'_n + u_n}{\mu_1}\sigma_n^2$, and is increasing and concave in that variable.*

**Proof of Lemma A.1.** Equation (2.19) implies that $\mathbb{E}(dR_{nt})$ is increasing and concave in $z \equiv \frac{\eta_n - \mu_2 \lambda \eta'_n + u_n}{\mu_1}\sigma_n^2$ if the function

$$\Phi(z) \equiv \frac{z}{A + \sqrt{B + Cz}}$$

is, where $(A, B, C)$ are positive constants. (The same constants as in the definition of $\Psi(z)$ in the proof of Proposition 2.1.) The function $\Phi(z)$ is increasing because its derivative

$$\Phi'(z) = \frac{A + \frac{B + \frac{Cz}{2}}{\sqrt{B+Cz}}}{\left(A + \sqrt{B + Cz}\right)^2}$$

is positive. Since, in addition,

$$\Phi'(z) = \frac{A + \frac{B + \frac{Cz}{2}}{\sqrt{B+Cz}}}{\left(A + \sqrt{B + Cz}\right)^2} = \frac{A + \frac{1}{2}\sqrt{B + Cz}}{\left(A + \sqrt{B + Cz}\right)^2} + \frac{\frac{B}{2\sqrt{B+Cz}}}{\left(A + \sqrt{B + Cz}\right)^2},$$

and both functions in the sum are decreasing, $\Phi'(z)$ is decreasing, and hence $\Phi(z)$ is concave. $\square$

**Proof of Proposition 2.2.** For a non-index asset $n'$, $\eta'_{n'} = 0$. Equations (2.13)-(2.15) imply that the asset's price is

$$S_{n't}(D_{n't}) = \frac{2}{r + \kappa_n + \sqrt{(r+\kappa_n)^2 + 4\rho\frac{\eta_{n'} - u_{n'}}{\mu_1}\sigma_n^2}}\left(\frac{\kappa_n}{r}\bar{D} + D_t\right). \tag{A.7}$$

Equation (2.19) implies that the asset's expected return is

$$\mathbb{E}\left(dR_{n't}\right) = \frac{2\rho\frac{\eta_{n'} - u_{n'}}{\mu_1}\sigma^2}{r + \kappa_n + \sqrt{(r+\kappa_n)^2 + 4\rho\frac{\eta_{n'} - u_{n'}}{\mu_1}\sigma_n^2}}\mathbb{E}\left(\frac{D_{nt}}{\frac{\kappa_n}{r}\bar{D} + D_{nt}}\right)dt. \tag{A.8}$$

Hence, the asset's price and expected return do not change when $\mu_2$ increases.

To show that $\mu_2\lambda$ increases in $\mu_2$, we write (2.16) as

$$\sum_{n=1}^{N} \eta_n' \left[\eta_n - u_n - \lambda(\mu_1+\mu_2)\eta_n'\right] a_{n1}^2 = 0. \tag{A.9}$$

Setting $M \equiv \mu_2\lambda$, we write (A.9) as

$$\sum_{n=1}^{N} \eta_n' \left[\eta_n - u_n - M\left(\frac{\mu_1}{\mu_2}+1\right)\eta_n'\right] a_{n1}^2 = 0, \tag{A.10}$$

and view the left-hand side of (A.10) as a function of $M$ rather than $\lambda$. At the smallest solution $\lambda$ of (2.16), the derivative of the left-hand side of (A.9) with respect to $\lambda$ is negative (since the smallest solution is positive and the left-hand side of (A.9) is positive for $\lambda = 0$). Hence, the derivative of the left-hand side of (A.10) with respect to $M$ is also negative at that solution. Since $a_{n1}$ depends on $(\mu_2, \lambda)$ only through $M$, the derivative of the left-hand side of (A.10) with respect to $\mu_2$ (holding $M$ constant) is

$$\sum_{n=1}^{N} \frac{M\mu_1}{\mu_2^2}(\eta_n')^2 a_{n1}^2 > 0.$$

Hence, the derivative of $M$ with respect to $\mu_2$ is positive, which means that $\mu_2\lambda$ increases in $\mu_2$. Since $\frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}\sigma_n^2$ decreases in $\mu_2$ for index assets ($\eta_n' > 0$), Proposition 2.1 implies that the price of these assets increases in $\mu_2$, and Lemma A.1 implies that these assets' expected return decreases in $\mu_2$. $\square$

**Proof of Proposition 2.3.** When $(\eta_n, \sigma_n) = (\eta, \sigma)$ for all $n \in \mathcal{I}$, RANS for an index asset $n$ is

$$\frac{\eta - \mu_2\lambda\eta - u_n}{\mu_1}\sigma^2 = \frac{(1-\mu_2\lambda)\eta - u_n}{\mu_1}\sigma^2.$$

Since RANS decreases in $u_n$, Lemma A.1 and $\kappa_n = \kappa$ for all $n \in \mathcal{I}$ imply $\mathbb{E}(dR_{nt}) < \mathbb{E}(dR_{mt})$. Moreover, (2.18) and $(\eta_n, \kappa_n, \sigma_n) = (\eta, \kappa, \sigma)$ for all $n \in \mathcal{I}$ imply $\mathbb{E}(w_{nt}) > \mathbb{E}(w_{mt})$.

When $\mu_2$ increases, (2.13)-(2.15) imply (through the same calculations as when differentiating the function $\Psi(z)$ defined in the proof of Proposition 2.1) that the price of asset $n$ changes by

$$\frac{\partial S_{nt}(D_{nt})}{\partial \mu_2} = \frac{\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\frac{4\rho\eta_n\sigma_n^2}{\mu_1}}{\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}\left[r+\kappa_n+\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}\right]^2}\left(\frac{\kappa_n}{r}\bar{D}+D_t\right),$$

and the percentage change is

$$\frac{1}{S_{nt}(D_{nt})}\frac{\partial S_{nt}(D_{nt})}{\partial \mu_2} = \frac{\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\frac{2\rho\eta_n\sigma_n^2}{\mu_1}}{\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}\left[r+\kappa_n+\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}\right]}. \tag{A.11}$$

Moreover, (2.19) implies (through the same calculations as when differentiating the function $\Phi(z)$ defined in the proof of Lemma A.1) that the expected return of asset $n$ changes by

$$\frac{\partial \mathbb{E}(dR_{nt})}{\partial \mu_2} = -\frac{\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\frac{2\rho\eta_n\sigma_n^2}{\mu_1}\left[r+\kappa_n+\frac{(r+\kappa_n)^2+2\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}{\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}}\right]}{\left[r+\kappa_n+\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}\right]^2}\mathbb{E}\left(\frac{D_{nt}}{\frac{\kappa_n}{r}\bar{D}+D_{nt}}\right)dt. \tag{A.12}$$

Using $(\eta_n,\kappa_n,\sigma_n)=(\eta,\kappa,\sigma)$ for all $n\in\mathcal{I}$ to simplify (A.11) and (A.12), we find that when $\mu_2$ increases, the percentage change in the price of asset $n$ and the change in that asset's expected return are

$$\frac{1}{S_{nt}(D_{nt})}\frac{\partial S_{nt}(D_{nt})}{\partial \mu_2} = \frac{\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\frac{2\rho\eta\sigma^2}{\mu_1}}{\sqrt{(r+\kappa)^2+4\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}\left[r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}\right]}, \tag{A.13}$$

$$\frac{\partial \mathbb{E}(dR_{nt})}{\partial \mu_2} = -\frac{\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\frac{8\rho^2\eta\sigma^4}{\mu_1^2}\left[r+\kappa+\frac{(r+\kappa)^2+2\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}{\sqrt{(r+\kappa)^2+4\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}}\right]}{\left[r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}\right]^2}\mathbb{E}\left(\frac{D_{nt}}{\frac{\kappa}{r}\bar{D}+D_{nt}}\right)dt, \tag{A.14}$$

respectively. Equation (A.13) implies that the price of asset $n$ rises more in percentage terms than the price of asset $m$ if the function

$$\Phi_{Su}(z) \equiv \frac{1}{\sqrt{B+Cz}\left(A+\sqrt{B+Cz}\right)}$$

is decreasing, where $(A, B, C)$ are positive constants. Since the denominator is increasing, $\Phi_{Su}(z)$ is decreasing. Equation (A.14) implies that the expected return difference $\mathbb{E}(dR_{mt}) - \mathbb{E}(dR_{nt})$ between assets $m$ and $n$ increases if the function

$$\Phi_{Ru}(z) \equiv \frac{A + \frac{B+\frac{Cz}{2}}{\sqrt{B+Cz}}}{\left(A+\sqrt{B+Cz}\right)^2}$$

is decreasing, where $(A, B, C)$ are positive constants. Since $\Phi_{Ru}(z) = \Phi'(z)$ for the concave function $\Phi(z)$ defined in the proof of Proposition 2.2, $\Phi_{Ru}(z)$ is decreasing. □

**Proof of Proposition 2.4.** When $u_n = U\eta_n$ for all $n \in \mathcal{I}$, (2.16) implies

$$\lambda = \frac{1-U}{\mu_1+\mu_2}. \tag{A.15}$$

Using $u_n = U\eta_n$ and (A.15), we can write RANS for an index asset $n$ as

$$\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\sigma_n^2 = \frac{(1-U)\eta_n}{\mu_1+\mu_2}\sigma_n^2. \tag{A.16}$$

Since $\eta_n\sigma_n^2 < \eta_m\sigma_m^2$, Lemma A.1 and $\kappa_n = \kappa$ for all $n \in \mathcal{I}$ imply $\mathbb{E}(dR_{nt}) < \mathbb{E}(dR_{mt})$. Moreover, (2.18), (A.16) and $\kappa_n = \kappa$ imply $\mathbb{E}(w_{nt}) > \mathbb{E}(w_{mt})$ if (2.20) holds.

Using $u_n = U\eta_n$ and $\kappa_n = \kappa$ for all $n \in \mathcal{I}$ to simplify (A.11) and (A.12), we find

$$\frac{1}{S_{nt}(D_{nt})}\frac{\partial S_{nt}(D_{nt})}{\partial \mu_2} = \frac{\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\frac{2\rho\eta_n\sigma_n^2}{\mu_1}}{\sqrt{(r+\kappa)^2 + 4\rho\frac{(1-U)\eta_n}{\mu_1+\mu_2}\sigma_n^2}\left[r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{(1-U)\eta_n}{\mu_1+\mu_2}\sigma_n^2}\right]}, \tag{A.17}$$

$$\frac{\partial \mathbb{E}(dR_{nt})}{\partial \mu_2} = -\frac{\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\frac{2\rho\eta_n\sigma_n^2}{\mu_1}\left[r+\kappa+\frac{(r+\kappa)^2+2\rho\frac{(1-U)\eta_n}{\mu_1+\mu_2}\sigma_n^2}{\sqrt{(r+\kappa)^2+4\rho\frac{(1-U)\eta_n}{\mu_1+\mu_2}\sigma_n^2}}\right]}{\left[r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{(1-U)\eta_n}{\mu_1+\mu_2}\sigma_n^2}\right]^2}\mathbb{E}\left(\frac{D_{nt}}{\frac{\kappa}{r}\bar{D}+D_{nt}}\right)dt, \tag{A.18}$$

respectively. Equation (A.17) implies that the price of asset $n$ rises less in percentage terms than the price of asset $m$ if the function

$$\Phi_{S\eta}(z) \equiv \frac{z}{\sqrt{B+Cz}\left(A+\sqrt{B+Cz}\right)}$$

is increasing, where $(A, B, C)$ are positive constants. Since

$$\Phi_{S\eta}(z) = \frac{z}{B+Cz} \times \frac{\sqrt{B+Cz}}{A+\sqrt{B+Cz}}$$

and both functions in the product are increasing, $\Phi_{S\eta}(z)$ is increasing. Equation (A.18) implies that the expected return difference $\mathbb{E}(dR_{mt}) - \mathbb{E}(dR_{nt})$ between assets $m$ and $n$ decreases if the function

$$\Phi_{R\eta}(z) \equiv \frac{z\left(A+\frac{B+\frac{Cz}{2}}{\sqrt{B+Cz}}\right)}{\left(A+\sqrt{B+Cz}\right)^2}$$

is increasing, where $(A, B, C)$ are positive constants. Since

$$\Phi_{R\eta}(z) = \frac{z}{B+Cz} \times \left[\frac{\sqrt{B+Cz}}{A+\sqrt{B+Cz}}\right]^2 \times \left(A+\frac{B+\frac{Cz}{2}}{\sqrt{B+Cz}}\right)$$

and all three functions in the product are increasing, $\Phi_{R\eta}(z)$ is increasing. $\square$

**Proposition A.1.** *Suppose that all assets in the index are in the same supply ($\eta_n = \eta$ for all $n \in \mathcal{I}$), all assets not in the index are in the same supply, which can differ from that of index assets ($\eta_{n'} = \bar{\eta}$ for all $n' \notin \mathcal{I}$), and $(\kappa_n, \sigma_n) = (\kappa, \sigma)$ for all $n$. Consider assets $n, m \in \mathcal{I}$, with asset $n$ being in larger noise-trader demand ($u_n > u_m$), and assets $n', m' \notin \mathcal{I}$, with asset $n'$ being in larger noise-trader demand ($u_{n'} > u_{m'}$).*

- *Asset $n'$ has higher non-index weight than asset $m'$ ($\mathbb{E}(w_{n't}) > \mathbb{E}(w_{m't})$) and earns lower expected return ($\mathbb{E}(dR_{n't}) < \mathbb{E}(dR_{m't})$).*
- *When $\mu_2$ increases, holding $\mu_1 + \mu_2$ constant:*
  - *The price of asset $n$ rises more, or drops less, in percentage terms than the price of asset $m$.*
  - *The expected return difference $\mathbb{E}(dR_{mt}) - \mathbb{E}(dR_{nt})$ between assets $m$ and $n$ increases.*
  - *The price of asset $n'$ drops less in percentage terms than the price of asset $m'$.*
  - *The expected return difference $\mathbb{E}(dR_{m't}) - \mathbb{E}(dR_{n't})$ between assets $m'$ and $n'$ increases.*
  - *The expected return difference $\mathbb{E}(dR_{mt}) - \mathbb{E}(dR_{nt})$ increases more than $\mathbb{E}(dR_{m't}) - \mathbb{E}(dR_{n't})$ under the sufficient conditions (i) $\bar{\eta} - u_{n'} \geq \eta(1-\mu_2\lambda) - u_n$ and (ii) $u_n - u_m \geq u_{n'} - u_{m'}$.*

The conditions ensuring that $\mathbb{E}(dR_m) - \mathbb{E}(dR_{nt})$ decreases more than $\mathbb{E}(dR_{m't}) - \mathbb{E}(dR_{n't})$ are (i) net supply for the index assets in high noise-trader demand is smaller than for their non-index counterparts, and (ii) the spread in noise-trader demand is larger for index than for non-index assets. Condition (ii) is plausible if index assets are mostly high-capitalization ones, with large numbers of shares. Condition (i) is plausible even if index assets are high-capitalization ones because the demand by non-experts reduces their net supply.

**Proof of Proposition A.1.** When $(\eta_{n'}, \sigma_{n'}) = (\bar{\eta}, \sigma)$ for all $n' \notin \mathcal{I}$, RANS for a non-index asset $n'$ is $\frac{\bar{\eta} - u_{n'}}{\mu_1}\sigma^2$. Since RANS decreases in $u_{n'}$, Lemma A.1 and $\kappa_{n'} = \kappa$ for all $n' \notin \mathcal{I}$ imply $\mathbb{E}(dR_{n't}) < \mathbb{E}(dR_{m't})$. Moreover, (2.18) and $(\eta_{n'}, \kappa_{n'}, \sigma_{n'}) = (\bar{\eta}, \kappa, \sigma)$ for all $n' \notin \mathcal{I}$ imply $\mathbb{E}(w_{n't}) > \mathbb{E}(w_{m't})$.

The counterparts of (A.11) and (A.12) when $\mu_2$ increases holding $\mu_1 + \mu_2$ constant are

$$\frac{1}{S_{nt}(D_{nt})}\frac{\partial S_{nt}(D_{nt})}{\partial \mu_2} = \frac{\frac{2\rho\sigma_n^2}{\mu_1}\left(\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\eta_n - \frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\right)}{\sqrt{(r+\kappa_n)^2 + 4\rho\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\sigma_n^2}\left[r + \kappa_n + \sqrt{(r+\kappa_n)^2 + 4\rho\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\sigma_n^2}\right]}, \tag{A.19}$$

$$\frac{\partial \mathbb{E}(dR_{nt})}{\partial \mu_2} = -\frac{\frac{2\rho\sigma_n^2}{\mu_1}\left(\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\eta_n - \frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\right)\left[r+\kappa_n+\frac{(r+\kappa_n)^2+2\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}{\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}}\right]}{\left[r+\kappa_n+\sqrt{(r+\kappa_n)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\sigma_n^2}\right]^2}\mathbb{E}\left(\frac{D_{nt}}{\frac{\kappa_n}{r}\bar{D}+D_{nt}}\right)dt, \tag{A.20}$$

respectively. Moreover, the derivative of the left-hand side of (A.10) with respect to $\mu_2$ holding $\mu_1+\mu_2$ and $M$ constant is

$$\sum_{n=1}^{N}\frac{M(\mu_1+\mu_2)}{\mu_2^2}(\eta_n')^2 a_{n1}^2 > 0.$$

Hence, $\mu_2\lambda$ increases when $\mu_2$ increases holding $\mu_1+\mu_2$ constant. Using $(\eta_n,\kappa_n,\sigma_n)=(\eta,\kappa,\sigma)$ for all $n\in\mathcal{I}$ to simplify (A.19) and (A.20), we find

$$\frac{1}{S_{nt}(D_{nt})}\frac{\partial S_{nt}(D_{nt})}{\partial\mu_2} = \frac{\frac{2\rho\sigma^2}{\mu_1}\left(\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\eta - \frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\right)}{\sqrt{(r+\kappa)^2+4\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}\left[r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}\right]}, \tag{A.21}$$

$$\frac{\partial \mathbb{E}(dR_{nt})}{\partial \mu_2} = -\frac{\frac{2\rho\sigma^2}{\mu_1}\left(\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\eta - \frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\right)\left[r+\kappa+\frac{(r+\kappa)^2+2\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}{\sqrt{(r+\kappa)^2+4\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}}\right]}{\left[r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\sigma^2}\right]^2}\mathbb{E}\left(\frac{D_{nt}}{\frac{\kappa}{r}\bar{D}+D_{nt}}\right)dt, \tag{A.22}$$

respectively. Equation (A.21) implies that the price of asset $n$ rises more, or drops less, in percentage terms than the price of asset $m$ if the function $\Phi_{Su}(z)$ defined in the proof of Proposition 2.3 is decreasing and the function $\Phi_{S\eta}(z)$ defined in the proof of Proposition 2.4 is increasing. Equation (A.22) implies that the expected return difference $\mathbb{E}(dR_{mt})-\mathbb{E}(dR_{nt})$ between assets $m$ and $n$ increases if the function $\Phi_{Ru}(z)$ defined in the proof of Proposition 2.3 is decreasing and the function $\Phi_{R\eta}(z)$ defined in the proof of Proposition 2.4 is increasing. Both properties are shown in the proof of Propositions 2.3 and 2.4.

The counterparts of (A.19) and (A.20) for non-index assets are

$$\frac{1}{S_{n't}(D_{n't})}\frac{\partial S_{n't}(D_{n't})}{\partial \mu_2} = -\frac{\frac{2\rho(\eta_{n'}-u_{n'})\sigma_{n'}^2}{\mu_1^2}}{\sqrt{(r+\kappa_{n'})^2+4\rho\frac{\eta_{n'}-u_{n'}}{\mu_1}\sigma_{n'}^2}\left[r+\kappa_{n'}+\sqrt{(r+\kappa_{n'})^2+4\rho\frac{\eta_{n'}-u_{n'}}{\mu_1}\sigma_{n'}^2}\right]}, \tag{A.23}$$

$$\frac{\partial \mathbb{E}(dR_{n't})}{\partial \mu_2} = -\frac{\frac{2\rho(\eta_{n'}-u_{n'})\sigma_{n'}^2}{\mu_1^2}\left[r+\kappa_{n'}+\frac{(r+\kappa_{n'})^2+2\rho\frac{\eta_{n'}-u_{n'}}{\mu_1}\sigma_{n'}^2}{\sqrt{(r+\kappa_{n'})^2+4\rho\frac{\eta_{n'}-u_{n'}}{\mu_1}\sigma_{n'}^2}}\right]}{\left[r+\kappa_{n'}+\sqrt{(r+\kappa_{n'})^2+4\rho\frac{\eta_{n'}-u_{n'}}{\mu_1}\sigma_{n'}^2}\right]^2}\mathbb{E}\left(\frac{D_{n't}}{\frac{\kappa_{n'}}{r}\bar{D}+D_{n't}}\right)dt, \tag{A.24}$$

respectively. Using $(\eta_{n'},\kappa_{n'},\sigma_{n'}) = (\bar{\eta},\kappa,\sigma)$ for all $n' \notin \mathcal{I}$ to simplify (A.23) and (A.24), we find that when $\mu_2$ increases holding $\mu_1+\mu_2$ constant, the percentage change in the price of asset $n'$ and the change in that asset's expected return are

$$\frac{1}{S_{n't}(D_{n't})}\frac{\partial S_{n't}(D_{n't})}{\partial \mu_2} = -\frac{\frac{2\rho(\bar{\eta}-u_{n'})\sigma^2}{\mu_1^2}}{\sqrt{(r+\kappa)^2+4\rho\frac{\bar{\eta}-u_{n'}}{\mu_1}\sigma^2}\left[r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{\bar{\eta}-u_{n'}}{\mu_1}\sigma^2}\right]}, \tag{A.25}$$

$$\frac{\partial \mathbb{E}(dR_{n't})}{\partial \mu_2} = \frac{\frac{2\rho(\bar{\eta}-u_{n'})\sigma^2}{\mu_1^2}\left[r+\kappa+\frac{(r+\kappa)^2+2\rho\frac{\bar{\eta}-u_{n'}}{\mu_1}\sigma^2}{\sqrt{(r+\kappa)^2+4\rho\frac{\bar{\eta}-u_{n'}}{\mu_1}\sigma^2}}\right]}{\left[r+\kappa+\sqrt{(r+\kappa)^2+4\rho\frac{\bar{\eta}-u_{n'}}{\mu_1}\sigma^2}\right]^2}\mathbb{E}\left(\frac{D_{n't}}{\frac{\kappa}{r}\bar{D}+D_{n't}}\right)dt, \tag{A.26}$$

respectively. Equation (A.33) implies that the price of asset $n'$ drops less in percentage terms than the price of asset $m'$ if the function $\Phi_{S\eta}(z)$ defined in the proof of Proposition 2.4 is increasing. Equation (A.34) implies that the expected return difference $\mathbb{E}(dR_{m't})-\mathbb{E}(dR_{n't})$ between assets $m'$ and $n'$ increases if the function $\Phi_{R\eta}(z)$ defined in the proof of Proposition 2.4 is increasing. Both properties are shown in the proof of Proposition 2.4.

Equations (A.26) and (A.22) imply

$$\begin{aligned}
&\frac{\partial \mathbb{E}(dR_{mt})}{\partial \mu_2} - \frac{\partial \mathbb{E}(dR_{nt})}{\partial \mu_2} > \frac{\partial \mathbb{E}(dR_{m't})}{\partial \mu_2} - \frac{\partial \mathbb{E}(dR_{n't})}{\partial \mu_2} \\
&\Leftrightarrow \Phi_{R\eta}\left(\frac{(1-\mu_2\lambda)\eta-u_m}{\mu_1}\right) - \Phi_{R\eta}\left(\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\right)
\end{aligned}$$

$$
\begin{aligned}
&-\frac{\partial(\mu_2\lambda)}{\partial\mu_2}\eta\left[\Phi_{Ru}\left(\frac{(1-\mu_2\lambda)\eta-u_m}{\mu_1}\right)-\Phi_{Ru}\left(\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\right)\right]\\
&>\Phi_{R\eta}\left(\frac{\bar\eta-u_{m'}}{\mu_1}\right)-\Phi_{R\eta}\left(\frac{\bar\eta-u_{n'}}{\mu_1}\right).
\end{aligned}
\tag{A.27}
$$

Since $\Phi_{Ru}(z)$ is decreasing, $u_n > u_m$ and $\frac{\partial(\mu_2\lambda)}{\partial\mu_2} > 0$, (A.27) holds under the sufficient condition

$$
\Phi_{R\eta}\left(\frac{(1-\mu_2\lambda)\eta-u_m}{\mu_1}\right)-\Phi_{R\eta}\left(\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}\right)\geq\Phi_{R\eta}\left(\frac{\bar\eta-u_{m'}}{\mu_1}\right)-\Phi_{R\eta}\left(\frac{\bar\eta-u_{n'}}{\mu_1}\right),
$$

which we can write as

$$
\begin{aligned}
&\int_{\frac{(1-\mu_2\lambda)\eta-u_n}{\mu_1}}^{\frac{(1-\mu_2\lambda)\eta-u_m}{\mu_1}}\Phi'_{R\eta}(z)dz\geq\int_{\frac{\bar\eta-u_{n'}}{\mu_1}}^{\frac{\bar\eta-u_{m'}}{\mu_1}}\Phi'_{R\eta}(z)dz\\
&\Leftrightarrow\int_{\frac{\bar\eta-u_{n'}}{\mu_1}}^{\frac{\bar\eta-u_{n'}+u_n-u_m}{\mu_1}}\Phi'_{R\eta}\left(z+\frac{(1-\mu_2\lambda)\eta-u_n-(\bar\eta-u_{n'})}{\mu_1}\right)dz\geq\int_{\frac{\bar\eta-u_{n'}}{\mu_1}}^{\frac{\bar\eta-u_{m'}}{\mu_1}}\Phi'_{R\eta}(z)dz.
\end{aligned}
\tag{A.28}
$$

Equation (A.28) holds under the sufficient conditions in the proposition, provided that $\Phi_{R\eta}(z)$ is increasing and concave. Indeed, $\Phi_{R\eta}(z)$ increasing and $u_n-u_m \geq u_{n'}-u_{m'}$ imply that the left-hand side of (A.28) is not smaller than

$$
\int_{\frac{\bar\eta-u_{n'}}{\mu_1}}^{\frac{\bar\eta-u_{m'}}{\mu_1}}\Phi'_{R\eta}\left(z+\frac{(1-\mu_2\lambda)\eta-u_n-(\bar\eta-u_{n'})}{\mu_1}\right)dz.
\tag{A.29}
$$

Moreover, $\Phi_{R\eta}(z)$ concave and $\bar\eta-u_{n'}\geq\eta(1-\mu_2\lambda)-u_n$ implies that (A.29) is not smaller than the right-hand side of (A.28).

In the proof of Proposition 2.4 we show that $\Phi_{R\eta}(z)$ is increasing. To show that $\Phi_{R\eta}(z)$ is concave, we write it as

$$
\begin{aligned}
\Phi_{R\eta}(z)&=\frac{z\left(A+\frac{B+\frac{Cz}{2}}{\sqrt{B+Cz}}\right)}{\left(A+\sqrt{B+Cz}\right)^2}\\
&=\frac{z\left(A+\frac{B+Cz}{\sqrt{B+Cz}}\right)}{2\left(A+\sqrt{B+Cz}\right)^2}+\frac{z\left(A+\frac{B}{\sqrt{B+Cz}}\right)}{2\left(A+\sqrt{B+Cz}\right)^2}\\
&=\frac{z}{2\left(A+\sqrt{B+Cz}\right)}+\frac{zA}{2\sqrt{B+Cz}\left(A+\sqrt{B+Cz}\right)}
\end{aligned}
\tag{A.30}
$$

$$= \Phi(z) + \frac{A}{2}\Phi_{S\eta}(z) \tag{A.31}$$

where the third equality follows if $B = A^2$, a property that holds in the instances where we define $\Phi_{R\eta}(z)$, and the functions $\Phi(z)$ and $\Phi_{S\eta}(z)$ are defined in the proofs of Lemma A.1 and Proposition 2.4, respectively. In the proof of Lemma A.1 we show that $\Phi(z)$ is concave. Therefore, $\Phi_{R\eta}(z)$ is concave if $\Phi_{S\eta}(z)$ is concave. The derivative of $\Phi_{S\eta}(z)$ is

$$\begin{aligned}
\Phi'_{S\eta}(z) &= \frac{B + A\frac{B+\frac{Cz}{2}}{\sqrt{B+Cz}}}{(B+Cz)\left(A+\sqrt{B+Cz}\right)^2} \\
&= \frac{B + A\frac{B+Cz}{\sqrt{B+Cz}}}{2(B+Cz)\left(A+\sqrt{B+Cz}\right)^2} + \frac{B + A\frac{B}{\sqrt{B+Cz}}}{2(B+Cz)\left(A+\sqrt{B+Cz}\right)^2} \\
&= \frac{A}{2(B+Cz)\left(A+\sqrt{B+Cz}\right)} + \frac{B}{2(B+Cz)^{\frac{3}{2}}\left(A+\sqrt{B+Cz}\right)},
\end{aligned}$$

where the third equality follows if $B = A^2$. Since both functions in the sum are decreasing, $\Phi'_{S\eta}(z)$ is decreasing, and hence $\Phi_{S\eta}(z)$ is concave. □

**Proposition A.2.** *Suppose that noise-trader demand is proportional to asset supply ($u_n = U\eta_n$ with $U < 1$ for all $n$). Consider assets $n', m' \notin \mathcal{I}$, with asset $n'$ being in smaller risk-adjusted supply ($\eta_{n'}\sigma^2_{n'} < \eta_{m'}\sigma^2_{m'}$).*

- *Asset $n'$ earns lower expected return than asset $m'$ ($\mathbb{E}(dR_{n't}) < \mathbb{E}(dR_{m't})$).*
- *When $\mu_2$ increases, holding $\mu_1 + \mu_2$ constant:*
  - *The prices and expected returns of index assets do not change.*
  - *The price of asset $n'$ drops less in percentage terms than the price of asset $m'$.*
  - *The expected return difference $\mathbb{E}(dR_{m't}) - \mathbb{E}(dR_{n't})$ between assets $m'$ and $n'$ increases.*

**Proof of Proposition A.2.** When $u_n = U\eta_n$, RANS of a non-index asset $n'$ is

$$\frac{\eta_{n'} - u_{n'}}{\mu_1}\sigma^2_{n'} = \frac{(1-U)\eta_{n'}}{\mu_1}\sigma^2_{n'}. \tag{A.32}$$

Since $\eta_{n'}\sigma^2_{n'} < \eta_{m'}\sigma^2_{m'}$, Lemma A.1 and $\kappa_{n'} = \kappa$ for all $n' \notin \mathcal{I}$ imply $\mathbb{E}(dR_{n't}) < \mathbb{E}(dR_{m't})$.

When $\mu_2$ increases holding $\mu_1 + \mu_2$ constant, (A.16) implies that the net supply of an index asset $n$ does not change. Hence, (2.13)-(2.15) imply that the asset's price does not change, and (2.19) implies that the asset's expected return does not change.

Using $u_{n'} = U\eta_{n'}$ and $\kappa_{n'} = \kappa$ for all $n' \notin \mathcal{I}$ to simplify (A.23) and (A.24), we find

$$\frac{1}{S_{n't}(D_{n't})}\frac{\partial S_{n't}(D_{n't})}{\partial \mu_2} = -\frac{\frac{2\rho(1-U)\eta_{n'}\sigma_{n'}^2}{\mu_1}}{\sqrt{(r+\kappa)^2 + 4\rho\frac{(1-U)\eta_{n'}}{\mu_1}\sigma_{n'}^2}\left[r+\kappa+\sqrt{(r+\kappa)^2 + 4\rho\frac{(1-U)\eta_{n'}}{\mu_1}\sigma_{n'}^2}\right]}, \tag{A.33}$$

$$\frac{\partial \mathbb{E}(dR_{n't})}{\partial \mu_2} = \frac{\frac{2\rho(1-U)\eta_{n'}\sigma_{n'}^2}{\mu_1}\left[r+\kappa+\frac{(r+\kappa)^2+2\rho\frac{(1-U)\eta_{n'}}{\mu_1}\sigma_{n'}^2}{\sqrt{(r+\kappa)^2+4\rho\frac{(1-U)\eta_{n'}}{\mu_1}\sigma_{n'}^2}}\right]}{\left[r+\kappa+\sqrt{(r+\kappa)^2 + 4\rho\frac{(1-U)\eta_{n'}}{\mu_1}\sigma_{n'}^2}\right]^2}\mathbb{E}\left(\frac{D_{nt}}{\frac{\kappa}{r}\bar{D}+D_{nt}}\right)dt, \tag{A.34}$$

respectively. Equation (A.33) implies that the price of asset $n'$ drops less in percentage terms than the price of asset $m'$ if the function $\Phi_{S\eta}(z)$ defined in the proof of Proposition 2.4 is increasing. Equation (A.34) implies that the expected return difference $\mathbb{E}(dR_{m't}) - \mathbb{E}(dR_{n't})$ between assets $m'$ and $n'$ increases if the function $\Phi_{R\eta}(z)$ defined in the proof of Proposition 2.4 is increasing. Both properties are shown in the proof of Proposition 2.4. □