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A THEORY OF THE NOMINAL CHARACTER OF STOCK SECURITIES

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ABSTRACT

We construct recursive solutions for, and study the properties of the dynamic equilibrium of an economy with three types of agents: (i) house- hold/investors who supply labor with a finite elasticity, consume a large variety of goods that are not perfect substitutes and trade government bonds; (ii) firms that produce those varieties of goods, receive productivity shocks and set prices in a Calvo manner; (iii) a government that collects an exogenous fiscal surplus and acts mechanically, buying and selling bonds in accordance with a Taylor policy rule based on expected inflation. In this setting we show that stock market returns are much less than one-for-one related to inflation over a one-year holding period, which means that stock securities have a strong nominal character. We also show that their nominal character diminishes as the length of the stock-holding period increases, in accordance with empirical evidence.

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> Fama and Schwert (1977) found that expected stock returns did not increase one-for-one with inflation. They interpreted this result to say that expected returns are higher in bad economic times, since people are less willing to hold risky assets, and are lower in good times. Inflation is lower in bad times and higher in good times, so lower expected returns in times of high inflation are not a result of inflation, but a coincidence.
>
> Cochrane (2005b)

Traders in the stock market take monetary policy very much into account. Yet, the literature in monetary economics by and large ignores the stock market and, given the complementary focus on fiscal policy, is only interested in pricing government bonds, a task which is accomplished by means of the private sector's Euler condition of portfolio choice (known in this context as the Fisher equation). There have been several exceptions (Marshall (1992), Challe and Giannitsarou (2014), Swanson (2014)),[1] and a few attempts to relate monetary economics to financial economics. Several papers going in that direction are: Svensson (1989), Benhabib, Schmitt-Grohe and Uribe (2001, 2002), Nakajima and Polemarchakis (2005) and Magill and Quinzii (2009, 2012). We draw inspiration from these papers, our purpose being to describe the key features of the relation between monetary and fiscal policy on the one hand and the stock market on the other.

The main and most useful result of the model will be the manner in which the stochastic process of equilibrium securities prices corresponds to a given monetary-policy process.[2] It will indicate how any multi-period government behavior is transmitted dynamically to financial markets. Investors are interested in that transmission because, among other things, they would very much like to know how shares of stock can serve as a hedge against inflation.[3]

The empirical evidence most cogently related to the present paper pertains to the relations between stock returns and inflation. Lintner (1975), Bodie (1976), Jaffee and Mandelker (1976), Nelson (1976), Fama and Schwert (1977), Fama (1981), Gultekin (1983), Boudoukh and Richardson (1993), Goto and Valkanov (2000) all document a negative correlation between nominal stock returns and inflation at monthly or annual frequency.

To explain the correlation, Fama (1981) suggested a "proxy hypothesis". When money demand is stable and money supply is fixed so that no monetary

[1] Some more recent theoretical contributions that deal with asset prices in New Keynesian settings include Nístico (2005), De Paoli, Scott and Weeken (2007), Milani (2008), Li and Palomino (2009), Wei (2009), Castelnuovo and Nístico (2010) and Challe and Giannitsarou (2014), who focus on the response of the stock market to a monetary policy shock, whereas we focus exclusively on a productivity shock. And Rudebusch and Swanson (2008) provide a calibration and apply it to bond prices.

[2] See Asness (2003).

[3] Conversely, the knowledge of that transmission can guide central bankers in their attempt to utilize information from financial markets to gauge anticipations of monetary policy (Bernanke and Gertler (1999, 2001), Bernanke and Kuttner (2004)).

effect is at play, a positive real shock both increases real stock returns and reduces inflation. The negative correlation is then just due to the existence of real shocks. The model to be outlined below does not satisfy Fama's money-supply assumptions; yet the negative correlation between real returns and inflation that we find will also be attributable to real productivity shocks, in the way mentioned in the epigraph of our article, the directions of the two contributing effects being, however, reversed. To the opposite of Fama, Geske and Roll (1983) suggested that the negative response is due to counter-cyclical monetary policy and the monetization of government debt.

Boudoukh and Richardson (BR), whose dataset covers close to two hundred years of annual data, introduce an important distinction between the ex ante and the ex post forms of the correlation of stock returns with inflation. To capture the ex post correlation, BR simply regress one-year holding-period realized nominal stock returns on one-year realized inflation. They do the same for five-year holding-period realized nominal stock returns and five-year realized inflation. In both cases the slope coefficient is found to be significantly positive but, more importantly, significantly less than 1: stocks are very much "nominal", as opposed to "real" assets. And the slope is many times larger for the five-year data: stocks are less nominal for a five-year holding period than for a one-year holding period.[4]

The ex ante relation, otherwise called the "Fisher" hypothesis (applied to stocks as opposed to bonds or Treasury Bills), relates conditionally expected nominal stock returns to conditionally expected inflation. Under the null hypothesis, the regression slope is expected to be equal to 1, reflecting a constant real rate of return. When anticipating inflation, agents have available an information set, which the econometrician treats as instrumental variables. BR use past inflation and past interest rates as instrumental variables. They do not reject the null hypothesis on five-year data but reject it on one-year data.

Katz and Lustig (2017) using a panel of countries confirm that stock markets are slow to incorporate news about future inflation, so that they do not qualify to be called "real" assets, whereas bond markets are not. Gorodnichenko and Weber (2016) show empirically that, after monetary policy announcements, the conditional volatility of stock market returns rises more for firms with stickier prices than for firms with more flexible prices and that sticky prices are, indeed, costly for firms.

The chief goal of this paper is to explain these stylized facts, especially those brought out by Boudoukh and Richardson concerning the slope of the relationship, using for the purpose well-recognized economic models, which have been shown in the past to be empirically relevant.

Before we begin, it is important to note that our purpose is analytical in nature. To that aim, we take the analytical derivations as far as they go, after which numerical solutions will take over. The use of numbers in this paper does not mean that we undertake the quantitative exercise of calibrating a

[4] Goto and Valkanov (2002) and Hagmann and Lenz (2005), using a vector autoregression, show an attenuation of the negative relationship following the Volcker reform of monetary policy.

DSGE model to a real-world economy. We want to illustrate a mechanism. For the same purpose, we make simplifying assumptions. For instance, we consider a bare-bone form of monetary policy (a Taylor rule; see Section 1), with no monetary shock. There is only one shock in our model, namely the productivity or supply shock.

With this single shock, the conditional correlation between all stochastic variables is equal to 1 and their unconditional correlation is also high. For that reason, the R^2 of the relation between stock return and inflation will be very high, which is not true in the data. When we succeed in obtaining a slope very much smaller than 1, it will follow mechanically that the volatility of stock returns will be smaller than the volatility of inflation, which is counterfactual. Our purpose is singlemindedly to explain the slope, not the R^2.

The article is organized as follows. In Section 1, we discuss our modelling choices in relation to the existing literature. In Section 2, we set up a purely financial economy in which income is given exogenously, thus obtaining the aggregate-demand schule. In Section 3, we turn to aggregate supply, adding to the financial side of the economy a productive sector in which oligopolistic firms can set prices in a fully flexible way. In Section 4, the productive sector functions along the lines of the New Keynesian model with Calvo pricing. Section 5 provides the main result of the paper as it derives the connection between stock returns and inflation; extensive simulations are performed and the degree to which the model matches the evidence is discussed. In Section 6, we add a demand for cash in the form of a Baumol-Tobin inventory demand and perform new simulations. Finally, in Section 7 we consider the pricing of long-term bonds and discuss the 'Fed model' of price comparison between bonds and stocks, as spelled out in Asness (2003).

1 Modelling choices and related literature

A financial economist, building on monetary and fiscal policy research, cannot treat the government as just any other trader that seeks to optimize his/her lifetime utility function under some budget constraint. Indeed, most of the existing work in that area attributes no explicit objective function to the government.[5] Issues of feasibility, stability and determinacy are discussed at length, but the objective functions of the government and the central bank are not stated explicitly. Instead, a behavior rule is postulated as a quasi-mechanical intervention formula. Today, the majority of central banks follows a policy called "inflation targeting" epitomized by the famous Taylor (1993) or Henderson-McKibbin (1993) rule.

[5] Three strands of monetary economics provide exceptions. First, ad hoc mean-variance objective functions are used to justify the linear Taylor rule (Woodford (2003), pages 535ff). Second, in the context of incomplete markets where nominal assets are traded, some researchers (e.g., Chari et al. (1993), Allen et al. (2012)) ask: can monetary policy maximize welfare by serving to render the market more complete. The optimal policy involves unrealistically volatile inflation rate and nominal interest rate. Third, Ramsey-optimal inflationary taxes have been derived by e.g., Persson et al. (1987).

Following Sargent and Wallace (1975), the literature has stressed the unavoidable financial linkage between monetary and fiscal policies when the central bank intervenes in the money market, including the Treasury-bill market, to set the nominal interest rate.[6] Indeed, another distinction between the government and regular investors arises in the specification of its income. In an exchange economy, regular investors receive an income, which is exogenous. Or, in a production economy, they may draw some income from their labor and that income is dictated by the production function, which is specified *ab initio*. The government is different in that it draws income from taxes. A major distinction must be drawn between a specification in which the budget surplus of the government is exogenous – a so-called "non Ricardian" fiscal policy – and one in which it will at some point or the other have to raise enough taxes to repay its debt – a "Ricardian" policy. The distinction between Ricardian and non-Ricardian fiscal regimes can be traced back to Aiyagari and Gertler (1985), Leeper (1991) and Canzoneri et al. (2011). In this paper, we assume that fiscal policy is non Ricardian (or "active" in the vocabulary of Leeper (1991)). Under a non Ricardian policy, it is conceivable for the government in some sense to default but we do not model that event. More importantly for our purposes, in a non Ricardian regime, some of the debt may be monetized.

When setting the nominal rate of interest, the principal aim of the central bank is to anchor inflationary expectations. In most models of monetary economics, the Taylor rule is backward looking in that it captures the central bank's reaction to *realized inflation*. Realized inflation is really a proxy for rationally expected inflation, a proxy that a central bank would rely on when it has access to incomplete information. In this paper, we make the assumption that the central bank has access to the same full information as the private sector. For that reason, we write the Taylor rule as a forward-looking formula relating the nominal rate of interest to the *rationally-expected rate of inflation*, as in Clarida et al. (2000), Bernanke and Bovin (2000) and Svensson and Woodford (2009) who refer to this implementation as "Inflation-Forecast Targeting."[7]

In most work in Monetary Economics, an initial price level of goods is picked and the model is left to run its way forward into the indefinite future; one then checks, often with the help of a linearized version of the model, whether the path of the price level is stable or explosive.[8] Instead, we postulate a finite terminal date for the economy although, stepping backward, we are able to postpone it indefinitely until we reach an unchanging solution. The private agents' utility functions do not extend beyond the terminal date and the prices of all securities, including the agents' financial wealth, after final payments have been made, are set equal to zero, both in real and in nominal terms. In this way, the enforcement

[6] A recent, elaborately argued exposition of that view is to be found in Leeper and Leith (2016).

[7] The literature has studied at length issues of stability of inflation over time, which arise from the lag that the proxy introduces in the backward-looking Taylor rule. References to that enormous literature and a convincing opinion on the matter can be found in Cochrane (2005a, 2011).

[8] Recently, Kollman (2019) has illustrated the fact that commonly used criteria for stability are not valid for non linear models. Linearizing them is not a proper way out.

of terminal conditions is ensured.[9] The price level of goods at the terminal date is endogenous, as it is at any other date, including the initial one; it is always finite. In our specification, the issue of stability over time does not arise. It is replaced by an issue of uniqueness of the equilibrium, which we examine. Provided one assumes as we do here, that the price level of goods is forward looking – as is done in the Fiscal Theory of the Price Level – as opposed to being predetermined, it makes no difference whether one solves by means of forward shooting or by backward induction.

Finally, we do not resort to the customary technique of approximation (by linearization or Taylor expansion) around the deterministic steady state.[10] We handle explicitly the non linearities of the model and obtain an exact solution. We even sometimes discover two viable equilibria. We are able to do that because we reach explicit solutions for the aggregate-demand curve (inclusive of the policy rule) and, in this way, can enumerate and locate solution points exactly.

The empirical estimation of the money-demand curve has become a harder and harder exercise to perform, so much so that, in recent years, many central bankers have stopped paying attention to monetary aggregates and focused exclusively on realized inflation and interest rates. The difficulty, of course, is that money demand and money supply shift simultaneously so that there is an identification problem. We adopt successively two specifications of household monetary behavior, which are standard.[11] The first is the "cashless economy" of Woodford (2003). Our second specification will be the "square-root" model of money demand developed some sixty years ago by Allais (1947, pages 238-241), Baumol (1952) and Tobin (1956). In that simple, inventory-theoretic model, households incur a fixed cost every time they go to the bank to turn securities into cash. They regulate their stock of money to minimize the average cost so incurred while making sure that they can always have enough money to meet a fixed, exogenous flow of consumption needs.[12]

As mentioned, the closest antecedents to the present paper are the articles by Benhabib, Schmitt-Grohe and Uribe (2001a, 2001b, 2002), Nakajima and Polemarchaskis (2005) and Magill and Quinzii (2009, 2012). They study issues of indeterminacy (unrelateded to issues of stability) of the price level and of the

[9] See Michel (1982) for the interpretation of "transversality conditions," which are used in infinite-horizon models.

[10] Recently, some authors have superimposed on the policy rule a zero lower bound on the nominal rate of interest. They were thus lead to worry about non linearities and multiple solutions of the resulting system of equations (which previously was linearized without a qualm). See Fernandez-Villaverde et al. (2012), Mertens and Ravn (2012), Aruoba and Schorfeide (2013), Christiano and Eichenbaum (2013) and Braun et al. (2013).

[11] See Tin (2000).

[12] While several attempts have been made at developing general-equilibrium versions of the Baumol-Tobin model (see, e.g., Romer (1986), Smith (1986), Heathcote (1998), Schwartz (2006), Leo (2006), Bai (2005), Silva (2011) etc..), most of them tend to simplify the model by postulating, e.g., an overlapping-generation model, for the sole purpose of cutting down to size the dynamic program to be solved. Danthine and Donaldson (1986) assume a money demand resulting from money in the utility function and an exogenous supply of money. In that context, they establish the conditions under which stock returns and inflation are negatively correlated.

rate inflation, and their potential solutions by means of Taylor-like intervention rules. The same issues arise here.

2 The aggregate-demand side of the model

We begin our investigation with an economy in which economic agents need no money to transact and in which prices of goods and services are fully flexible.

We consider a financial market populated with one (or a continuum of identical) household(s), for which we use a subscript 1, and one central bank, subscripted 2, and a set of exogenous time sequences of individual income (or output) received by households $\{y_t \in \mathbb{R}_{++}; t = 0, ...T\}$, which are placed on a tree or lattice. These are received by the households only. For simplicity, we consider a binomial tree so that a given node at time t is followed by two nodes $\{u, d\}$ at time $t+1$ at which the two values of income are denoted $\{y_{t+1,u}, y_{t+1,d}\}$. The transition probabilities are equal to 1/2. Notice that the tree accommodates the exogenous state variables only.

In the financial market, there are several securities with at least one nominally riskless security, viz. a one-period nominal bond. The household trades all securities to maximize some lifetime utility.

Assumption 1 *The central bank only trades the one-period nominal bond in a mechanical way described by a Taylor policy rule:*

$$1 + i_t = (1 + \bar{\imath}) \times \left(\frac{\frac{\frac{1}{2}P_{t+1,u} + \frac{1}{2}P_{t+1,d}}{P_t}}{1 + \bar{\pi}} \right)^{\phi}; \phi \geq 0; \phi \neq 1 \tag{1}$$

The Taylor rule aims to set *expected inflation*. It does not respond to realized inflation, so that it should have little effect on its conditional volatility. We would call *monetary shock* a deviation from the exact rule. Throughout this paper we assume zero monetary shock.

The government's primary surplus (taxes in excess of expenditures) is denoted s_t in real terms, S_t in nominal terms. The number of units (measured by the nominal amount of the future payoff) of the one-period bond with which the private sector *exits* time t is denoted $\theta_{1,t}$ and its exiting financial wealth $F_{1,t} \triangleq \theta_{1,t}/(1 + i_t)$ is the present value of the nominally riskless bond holdings. We handle the stock market separately as the central bank does not trade it anyway.

We assume that the utility function of the private sector is time-additive and isoelastic. Let the relative risk aversion of the household be $1-\gamma$ and their impatience factor be $\rho < 1$. The private sector (agent carrying a subscript 1) maximizes:

$$\sup_{\{c,\theta_1\}} \mathbb{E}_0 \sum_{t=0}^{T} \mathfrak{u}(c_t, t)$$

subject to:

- terminal conditions:
$$\theta_{1,T} = 0, \tag{2}$$

- a sequence of flow budget constraints:
$$P_t \times c_t + \frac{\theta_{1,t}}{1+i_t} + s_t \times P_t = \theta_{1,t-1} + P_t \times y_t \tag{3}$$

- and given initial holdings:
$$\theta_{1,-1} = \bar{\theta}_1 \tag{4}$$

The initial condition at $t = 0$ is given in terms of a *nominal* outstanding claim $\bar{\theta}_1 = -\bar{\theta}_2$ of the public on the government. Please, bear in mind that $\theta_{2,t}$ is a negative number, except in very unusual and temporary fiscal situations.

The government (agent carrying a subscript 2) acts mechanically according to the constraints:

$$\frac{\theta_{2,t}}{1+i_t} = \theta_{2,t-1} + s_t \times P_t; j = u, d \tag{5}$$

and to the Taylor rule (1) with initial holdings:

$$\theta_{2,-1} = \bar{\theta}_2$$

and terminal condition:

$$\theta_{2,T} = 0$$

Citing Nakajima and Polemarchakis (2005), "a fiscal policy is called 'Ricardian' if it guarantees that the public debt vanishes at each terminal node *for all possible, equilibrium or non-equilibrium, values of price levels and other endogenous variables*" [Emphasis added]. In that case, the fiscal surplus cannot be exogenous throughout. Nakajima and Polemarchakis (2005) demonstrates that, as long as fiscal policy is Ricardian in a cashless economy, the value of government debt is indeterminate.[13]

For that reason, in the balance of this paper, we maintain the following

Assumption 2 *The government pursues a non Ricardian fiscal policy.*

Therefore, let government surplus s_t be exogenously fixed in real terms. As explained earlier, the government's debt is managed mechanically according to a Taylor rule (1), which aims to anchor inflationary expectations.[14]

[13] This is in conformity with Woodford (2003, page 125) and Cochrane (2011). And this indeterminacy induces an indeterminacy of the entire future path of inflation. When later we introduce a production side of the economy, the indeterminacy would also be physical.

[14] It is asserted in Canzoneri et al. (2011) that the backward looking Taylor rule based on realized inflation is incompatible with non Ricardian fiscal policy. With a forward looking Taylor rule involving expected inflation, there is no incompatibility as we show now.

Definition 1 *An equilibrium is defined as a joint process for the allocation of consumption c_t, the price level P_t, the amount of government bonds outstanding $\theta_{2,t}$ and the nominal rate of interest i_t such that the supremum of the private sector's objective function (31) is reached for all t, the government abides by its period budget constraints (5) and follows the mechanical rule (1), and the market-clearing conditions:*

$$\theta_{1,t} + \theta_{2,t} = 0 \tag{6}$$

are also satisfied with probability 1 *at all times* $t = 0, ...T$.

2.1 Equation system

It is shown in Appendix A that, for a utility function $\mathfrak{u}(c,t) = \rho^t c^\gamma / \gamma; \gamma < 1$, a recursive (backward-induction) equilibrium can be obtained by solving, at each node (the exogenous state variable here being y_t),[15] the following system of equations:[16]

$$\begin{gathered}
\text{Flow budget constraints of private sector at time } t+1 \\
P_{t+1,u} \times c_{t+1,u} + F_{1,t+1,u} + s_{t+1,u} \times P_{t+1,u} = \theta_{1,t} + P_{t+1,u} \times y_{t+1,u}; F_{1,T,u} = 0 \\
P_{t+1,d} \times c_{t+1,d} + F_{1,t+1,d} + s_{t+1,d} \times P_{t+1,d} = \theta_{1,t} + P_{t+1,d} \times y_{t+1,d}; F_{1,T,d} = 0 \\
\text{Flow budget constraints of government at time } t+1 \\
F_{2,t+1,u} = \theta_{2,t} + s_{t+1,u} \times P_{t+1,u}; F_{2,T,u} = 0 \\
F_{2,t+1,d} = \theta_{2,t} + s_{t+1,d} \times P_{t+1,d}; F_{2,T,d} = 0 \\
\text{Portfolio-choice, or Euler, or Fisher condition at time } t \\
\frac{1}{1+i_t}\frac{1}{P_t} = \rho \frac{\frac{1}{2}(c_{t+1,u})^{\gamma-1}\frac{1}{P_{t+1,u}} + \frac{1}{2}(c_{t+1,d})^{\gamma-1}\frac{1}{P_{t+1,d}}}{(c_t)^{\gamma-1}} \\
\text{Taylor rule at time } t \\
1 + i_t = (1+\bar{\imath}) \times \left(\frac{\frac{\frac{1}{2}P_{t+1,u} + \frac{1}{2}P_{t+1,d}}{P_t}}{1+\bar{\pi}} \right)^{\phi} \\
\text{Market clearing at time } t \\
\theta_{1,t} + \theta_{2,t} = 0
\end{gathered} \tag{7}$$

The functions carried backward in the backward-induction procedure are $F_{1,t}$ $(= -F_{2,t})$:

$$F_{1,t} \triangleq \frac{\theta_{1,t}}{1+i_t}.$$

The unknowns are $\{i_t, c_{t+1,u}, c_{t+1,d}, \theta_{1,t}, \theta_{2,t}, P_{t+1,u}, P_{t+1,d}\}$. The only endogenous state variable is the current price level P_t, which is determined at time zero from the outstanding nominal amount of government debt, as in the Fiscal

[15] In later sections, y_t is endogenized and productivity is exogenous.

[16] In principle, there is also an endogenous state variable (here being P_t) but, in the absence of nominal illusion, it can be factored out on grounds of homogeneity.

Theory of the Price Level.[17] At time zero, the *initial condition* to be solved for the unknown initial price P_0 is

$$F_{2,0} = \theta_{2,-1} + s_0 \times P_0 \tag{8}$$

where $\theta_{2,-1}$ is a given (negative) amount of nominal claim outstanding and s_0 a given time-0 surplus.

Since the government does not trade the *equity* and private agents are homogeneous, it is not traded at all and its price is virtual. If the equity security is defined – *for the time being* – as paying the total income, its price x_t is equal to

$$\begin{aligned} x_t &= \rho \frac{\frac{1}{2}\left(c_{t+1,u}\right)^{\gamma-1} \times \left(y_{t+1,u} + x_{t+1,u}\right) + \frac{1}{2}\left(c_{t+1,d}\right)^{\gamma-1} \times \left(y_{t+1,d} + x_{t+1,d}\right)}{\left(y_t\right)^{\gamma-1}}; \\ x_T &= 0 \end{aligned} \tag{9}$$

Since the government consumes no goods, there exists an obvious analytical solution for which

$$c_{t+1,u} = y_{t+1,u}; c_{t+1,d} = y_{t+1,d}$$

and for which, for that reason, the market clears. Once consumption is known, and given that the government surplus is exogenous, the real value of government debt $f_{2,t} = F_{2,t}/P_t$ follows as a discounted present value. All the endogenous quantities (inflation, interest rate, government debt etc.) that prevail under that solution are derived in Appendix B. The formula for the real price x_t of the stockmarket (9) owes nothing to the price level but the real rate of return on it is conditionally correlated with inflation since $y_{t+1}, s_{t+1}, f_{2,t+1}$ are correlated with each other. We now turn to a special case, which will occupy us for the rest of the article. The derivations for it are identical to those contained in the appendix.

2.2 The case of IID growth and proportional taxes

In case growth is stochastic and identically and independently distributed (IID) over time,

$$\frac{y_{t+1,u}}{y_t} = 1 + u; \frac{y_{t+1,d}}{y_t} = 1 + d; u > d$$

the output shock is, by construction, permanent or infinitely persistent,[18] and there is scale invariance in the sense that the quantity $f_{2,t}$ does not depend on the level of income y_t at time t once the surplus process $\{s_t\}$ is given.

If, however, the exogenous surplus is specified to be at all times and in all states proportional to income, $s_t = \tau \times y_t$, where τ can interpreted as a constant

[17] See Cochrane (2005a) and Niepelt (2004). A recent, comprehensive survey is Leeper and Leith (2016).

[18] The IID growth example will suffice for our purposes. But a somewhat more realistic mean-reverting growth could just as easily be handled by means of a Markov chain.

tax rate, then the real discounted value of government debt $f_{2,t}$ is proportional to the level of income y_t at time t:

$$f_{2,t} = \hat{f}_{2,t} \times y_t \tag{10}$$

where $\hat{f}_{2,t}$ approaches zero *deterministically* as one approaches the terminal date. Indeed:

$$\begin{aligned} \frac{\hat{f}_{2,t}}{-\tau + \hat{f}_{2,t+1}} &= k; \hat{f}_{2,T} = 0 \\ \hat{f}_{2,t} &= -\tau \times \frac{k \times \left(-1 + k^{T-t}\right)}{-1 + k} \end{aligned} \tag{11}$$

where we define:

$$k \triangleq \rho \times \left[\frac{1}{2}(1+u)^{\gamma} + \frac{1}{2}(1+d)^{\gamma}\right] \tag{12}$$

$k < 1$ being assumed for now, so that for an infinite horizon, we get the Gordon formula and a constant real debt factor:

$$\hat{f}_{2,t} = \tau \times \frac{k}{-1 + k}$$

The realized inverse real gross rates of return on government debt are:

$$\begin{aligned} \frac{f_{2,t}}{-s_{t+1,u} + f_{2,t+1,u}} &= \frac{k}{1+u} \text{ in a } u \text{ state} \\ \frac{f_{2,t}}{-s_{t+1,d} + f_{2,t+1,d}} &= \frac{k}{1+d} \text{ in a } d \text{ state} \end{aligned}$$

The quantity:

$$k \times \left(\frac{1}{2}\frac{1}{1+u} + \frac{1}{2}\frac{1}{1+d}\right)$$

can be viewed as the expected inverse gross *real* rate of interest on *nominally* riskless claims, which is not equal to the inverse gross real rate on really riskless claims ($\rho \times \left[(1+u)^{\gamma-1} + (1+d)^{\gamma-1}\right]/2$).

Proposition 1 *Under the IID growth assumption, the realized rates of inflation are:*

$$\begin{aligned} \frac{P_{t+1,u}}{P_t} &= \frac{k}{1+u} \times (1+i_t) \\ \frac{P_{t+1,d}}{P_t} &= \frac{k}{1+d} \times (1+i_t) \end{aligned}$$

independent of the tax rate. Inflation is lower in the u state than in the d state.

The result goes the same way as in the traditional quantity theory of money but for completely different, in this case fiscal, reasons. A high output shock today implies higher real fiscal surpluses into the indefinite future. For a given outstanding nominal debt, the price level is lower. In the remarks that follow Proposition 6 below, we comment on the empirical validity of a generalized version of Proposition 1.

Proposition 2 *Under the IID growth assumption, the nominal rate of interest is constant:*

$$1 + i_t = \left(\frac{1+\bar{\imath}}{(1+\bar{\pi})^{\phi}}\right)^{\frac{1}{1-\phi}} \times \left[k \times \left(\frac{1}{2}\frac{1}{1+u} + \frac{1}{2}\frac{1}{1+d}\right)\right]^{\frac{\phi}{1-\phi}} \tag{13}$$

The price level at time 0, according to the Fiscal theory, solves the initial condition (8):

$$-\tau \times \frac{k \times \left(-1+k^T\right)}{-1+k} \times y_0 \times P_0 = \theta_{2,-1} + \tau \times y_0 \times P_0$$

so that it is equal to

$$P_0 = -\frac{\theta_{2,-1}}{\tau \times y_0 \times \left[1 + \frac{k \times (-1+k^T)}{-1+k}\right]}$$

The stock-market price (excluding current output) is proportionnal to output:

$$x_t = \hat{x}_t \times y_t; x_T = 0$$

where $\hat{x}_t$ (a form of dividend-price ratio) is *deterministic*:[19]

$$\begin{aligned} \hat{x}_t &= k \times (1 + \hat{x}_{t+1})\, ; \hat{x}_T = 0 \\ \hat{x}_t &= \frac{k \times \left(-1+k^{T-t}\right)}{-1+k} \end{aligned}$$

and, when the horizon is infinite:

$$\hat{x}_t = \frac{k}{1-k}$$

[19] Not surprisingly, because of the proportional tax, there exists a systematic relation between the real stock market price per unit of output $\hat{x}_t$ (the price-dividend ratio) and the real discounted value of government debt per unit of output $\hat{f}_{2,t}$:

$$-\frac{\hat{f}_{2,t}}{\tau} = \hat{x}_t$$

Over time, they both decline deterministically.

Proposition 3 *Under the IID growth assumption, the real gross rates of return on the stock market are:*

$$\frac{1+u}{k} \text{ in a } u \text{ state}$$
$$\frac{1+d}{k} \text{ in a } d \text{ state}$$

On a u node, inflation is lower than in the d node with the same predecessor while the real stock market return is higher but that is just a "proxy" result of a common cause, namely the output shock, which acts both on the stock market and on tax collection. The real rate of return on equity being low in a state in which inflation is high, it is conditonally negatively correlated with the rate of inflation and the stock market is not a one-for-one hedge against inflation. In fact, for their product, which is the realized gross *nominal* rate of return on stocks, we have:

Proposition 4 *Under the IID growth assumption, the realized gross nominal rate of return on stocks is equal to the gross nominal interest rate, which is constant.*

In that sense, stock market returns are 100% nominal. In later sections, we introduce additional features – including endogenous output – that will produce more realistic results. More importantly, these features will explain the fact that the link between inflation and nominal stock returns varies depending on the length of the holding period.

In the current section, the *output* growth rates u and d are given parameters, which cannot be varied. To prepare for the situation in which output is endogenous, however, we derive for the two states the *aggregate-demand schedules* (or IS curves) that relate the future prices P_u, P_d to the future outputs y_u, y_d. We draw Figure 1 which shows these relations in the two states at time $t+1$ for the two output shocks at time $t+1$ for a fixed level of output and a fixed price level at time t. Proposition 1 shows that, for a given rate of interest, inflation is decreasing in output. However, the relationships displayed in the figure incorporate the influence of future output on the current rate of interest as per Equation (13). In that way, the two schedules are interdependent (in a symmetric way, in the sense that $P_u(y_u, y_d)$ and $P_d(y_d, y_u)$ are the same function).

Proposition 5 *As long as $\phi < 1 + (1+d)/(1+u)$, the aggregate demand schedules (inclusive of policy rule) are increasing functions of income or output y when $\phi > 1$ and decreasing functions when $\phi < 1$.*

The proof is in Appendix C.

3 Flexible-price aggregate supply

The model we have built in Section 2 combines the policy rule with the aggregate-demand or "IS" side of the economy. The solution we calculated is complete

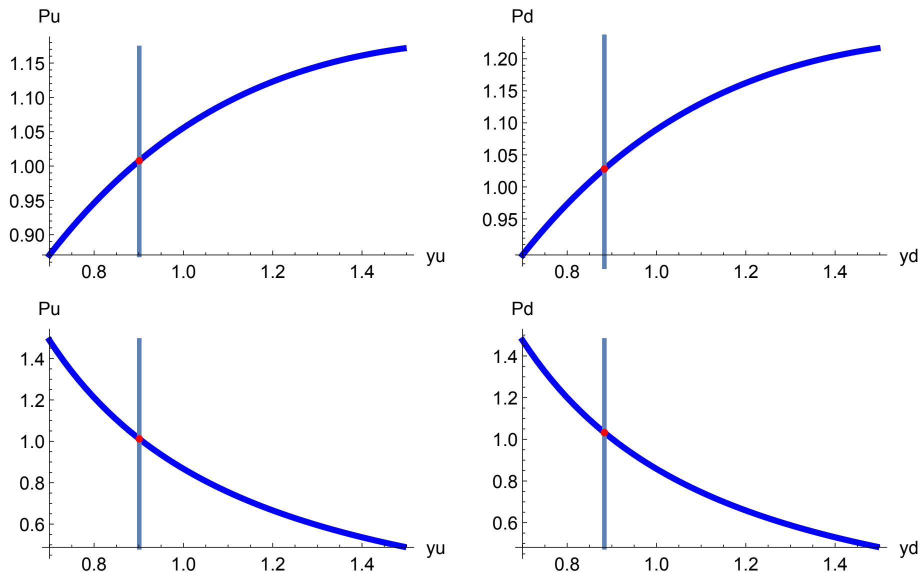


Figure 1: **Aggregate-demand curve (inclusive of policy rule) with fixed outputs, $\phi = 1.5$ (top panels) and $\phi = 0.5$ (bottom panels)**. In each row, the left-hand panel shows $P_{t+1,u}$ plotted against $y_{t+1,u}$ for the fixed value of $y_{t+1,d}$. The fixed value of $y_{t+1,u}$ is shown as a vertical line. Right-hand panel: $P_{t+1,d}$ plotted against $y_{t+1,d}$ for the fixed value of $y_{t+1,u}$. The fixed value of $y_{t+1,d}$ is shown as a vertical line. Parameter values are as in Table 1. $T = 6$. The current price level is set at 1. The current level of output y_t is set at 0.9011.

Parameter	**Value**
ρ	0.99
$\bar{\pi}$	$2\%/year$
$\bar{\imath}$ (in figures)	$1/\rho - 1 + \bar{\pi}$
or $\bar{\imath}$ (in simulations)	"neutral" as per Definition 2
$1 - \gamma$	1
tax rate τ	1/3
σ	4
η	2
volatility of z growth	$1\%/year$
expected value of z growth	0
probability u and d	1/2
ω (price stickiness)	0.6

Table 1: Parameter values for the numerical illustration; one-year periods

when income or output is exogenous. We now introduce firms and endogenize output. We use that part of the model to obtain future (time-$t+1$) output. We develop the aggregate-supply side, productivity z being now the exogenous state variable, which is assigned to the nodes of the tree. In this section, we assume that firms are free to adjust their prices, this being only a transition to the next section in which prices are sticky.

Households: There exists a continuum $\iota \in [0,1]$ of differentiated varieties of the good.[20] The argument c_t of the households' utility is a composite defined as

$$c_t \triangleq \left(\int_0^1 c_{\iota,t}^{\frac{\sigma-1}{\sigma}} d\iota \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution between the separate varieties. As a result, their demand for each separate variety ι is

$$c_{\iota,t} = \left(\frac{P_{\iota,t}}{P_t} \right)^{-\sigma} c_t$$

where $P_{\iota,t}$ is the nominal price of variety ι and P_t is the general price index, which is defined generally as

$$P_t \triangleq \left(\int_0^1 P_{\iota,t}^{1-\sigma} d\iota \right)^{\frac{1}{1-\sigma}} \tag{14}$$

but will be particularized below. In addition, the utility function of households now contains a separate, additive term for the dis-utility of labor. The full utility function that households optimize is

$$\sup_{\{c,l,\theta_1\}} \mathbb{E}_0 \sum_{t=0}^{T} \mathfrak{u}(c_t, t) - \mathfrak{v}(l_t, t)$$

subject to terminal conditions (2), a sequence of flow budget constraints:

$$P_t \times c_t + \frac{\theta_{1,t}}{1+i_t} + \theta_{X,t} \times P_t \times x_t + s_t \times P_t = \theta_{1,t-1} + \theta_{X,t-1} \times P_t \times (\delta_t + x_t) + W_t \times l_t$$

and given initial holdings:

$$\begin{aligned} \theta_{1,-1} &= \bar{\theta}_1 \\ \theta_{X,-1} &= 1 \end{aligned}$$

where W_t is the nominal wage rate, l_t the number of hours worked, $\theta_{X,t}$ equity holdings, x_t the real price of equity and δ_t real dividends distributed. Since households alone hold the stock, it will be the case at equilibrium that $\theta_{X,t} = \theta_{X,t-1} = 1$. We have in mind, however, that the first-order condition for equity holdings will serve to price the equity.

[20] Here, we follow Chapter 8 in Walsh (2010) and Challe (2005).

We assume an isoelastic dis-utility of work: $\mathfrak{v}(l,t) = \rho^t \times l^\eta/\eta; \eta > 1$. The households' first-order condition for hours worked is obviously

$$\frac{l_t^{\eta-1}}{c_t^{\gamma-1}} = \frac{W_t}{P_t} \tag{15}$$

Firms: The production function for variety ι of the good is

$$y_{\iota,t} = z_t \times l_{\iota,t} \tag{16}$$

where z_t is a productivity shock, the same for all firms and $l_{\iota,t}$ is the amount of labor utilized for the production of good ι.

Firms are free to adjust their prices at will. Firms producing variety ι that choose price $P_{\iota,t}$ sell an amount of goods equal to $(P_{\iota,t}/P_t)^{-\sigma} c_t$, for which they will have to hire an amount of labor equal to $(P_{\iota,t}/P_t)^{-\sigma} c_t/z_t$. Their profits are:

$$P_{\iota,t}\left(\frac{P_{\iota,t}}{P_t}\right)^{-\sigma} \times c_t - W_t \times \left(\frac{P_{\iota,t}}{P_t}\right)^{-\sigma}\frac{c_t}{z_t}$$

Optimizing the selling price:

$$(1-\sigma)\left(\frac{P_{\iota,t}}{P_t}\right)^{-\sigma} \times c_t - \frac{W_t}{P_t} \times (-\sigma)\left(\frac{P_{\iota,t}}{P_t}\right)^{-\sigma-1}\frac{c_t}{z_t} = 0$$

so that:

$$\frac{P_{\iota,t}^*}{P_t} = \frac{\sigma}{\sigma-1}\varphi_t$$

where

$$\varphi_t \triangleq \frac{W_t}{z_t \times P_t} \tag{17}$$

We interpret φ_t as the real marginal cost of labor. Profits are maximized by setting a mark up and an optimal price P^* related to the price-elasticity of demand, *the same* P_t^* *for all varieties.*

In the aggregate, firms produce $(P_t^*/P_t)^{-\sigma} \times y_t$. Total labor employed is:

$$\left(\frac{P_t^*}{P_t}\right)^{-\sigma} \times \frac{y_t}{z_t}$$

Letting l_t stand for the labor supplied by households, the clearing of the labor market requires:

$$l_t = \left(\frac{P_t^*}{P_t}\right)^{-\sigma} \times \frac{y_t}{z_t} \tag{18}$$

Equilibrium: By Walras' law, the equilibrium in the financial market and the equilibrium in the labor market imply the equilibrium in the goods market: $c_t = y_t$. Furthermore, since all the firms behave the same way, (14) implies that

$P_t = P_t^*$. Equations (17), (15) and (18) imply that the flexible-price level of output is:

$$y_{f,t} = \left(\frac{\sigma - 1}{\sigma} z_t^{\eta} \right)^{\frac{1}{\eta - \gamma}} \tag{19}$$

and that the supply price is indeterminate. The determination of the price level is then left entirely to the aggregate demand side (inclusive of the policy rule) exactly as in Section 2. Since $\eta > 1$ and $\gamma < 1$, output is an increasing function of productivity.

The special IID-growth case described in Section 2.2 can be recast in terms of productivity shocks. From this point on, we assume that

Assumption 3 *The growth rate of productivity z is IID.*

Equation (19) shows that, since $\eta > 1$ and $\gamma < 1$, the resulting equilibrium diagrams remain identical to Figure 1, reinterpreted as showing *endogenous* values of the output y.[21]

Proposition 6 *Under Assumption 3, proportional taxes and flexible prices, the equilibrium is unique and Propositions 1 to 4 remain true with $1 + u$ replaced by $1 + \hat{u} \triangleq (1 + u)^{\frac{\eta}{\eta - \gamma}}$, $1 + d$ replaced by $1 + \hat{d} \triangleq (1 + d)^{\frac{\eta}{\eta - \gamma}}$ and k replaced by*

$$\hat{k} \triangleq \rho \times \left[\frac{1}{2} (1 + \hat{u})^{\gamma} + \frac{1}{2} \left(1 + \hat{d} \right)^{\gamma} \right] \tag{20}$$

($\hat{k} < 1$ being now assumed anew). In particular, the gross nominal rate of return on stocks remains equal to the gross nominal interest rate, which is constant.

The empirical evidence is very much in line with this generalization of Proposition 1.[22] Smets and Wouters (2011) fit both a DSGE model and a VAR specification, both producing very similar results, to seven US macroeconomic time series with a highly persistent total factor productivity (TFP) shock. Their Figure 7 indicates clearly that the DSGE model produces a negative response on impact of inflation to a positive TFP shock. Altig et al. (2011) after performing a VAR analysis on US data comment their Figure 2 saying that: "Finally notice that a neutral technology shock leads to an initial sharp fall in the inflation rate." Furthermore, the VAR fit performed by Alves (2004, Figure 2) on six OECD countries leads uniformly to the same conclusion.

Our productivity shock, being a (geometric) random walk, is highly persistent. For that reason the shock is also news about future productivity. The empirical literature confirms a negative impact of news shocks about TFP on

[21] In each row, the left-hand panel shows $P_{t+1,u}$ plotted against $y_{t+1,u}$ for the *flexible-price* value of $y_{t+1,d}$. The flexible-price value of $y_{t+1,u}$ is shown as a vertical line. Right-hand panel: $P_{t+1,d}$ plotted against $y_{t+1,d}$ for the *flexible-price* value of $y_{t+1,u}$. The flexible-price value of $y_{t+1,d}$ is shown as a vertical line. Additional parameter values are as in Table 1. The current level of output y_t is set at its flexible-price level equal to 0.9011.

[22] We are very grateful to Rafael Wouters for these references.

prices and inflation. See, for instance, Kurmann and Sims (2017), Barsky and Sims (2011) and Miranda-Agrippino et al. (2019).

Dividends and stock prices: As the firms enjoy market power, they generate positive profits. We now re-define the aggregate stock security as paying corporate profits (as opposed to paying output, which it was in Section 2). The real, future profits, assumed to be distributed as dividends, are:

$$\delta_t = \left[\left(\frac{P_t^*}{P_t} \right)^{1-\sigma} - \frac{W_t}{P_t} \times \left(\frac{P_t^*}{P_t} \right)^{-\sigma} \frac{1}{z_t} \right] \times y_t = \frac{1}{\sigma} y_t$$

The value of the stock market, in real terms, not including current profits, is:

$$\begin{aligned} x_t &= \rho \frac{1}{\left(\left(\frac{\sigma-1}{\sigma} z_t^{\eta} \right)^{\frac{1}{\eta-\gamma}} \right)^{\gamma-1}} \left[\frac{1}{2} \left(\left(\frac{\sigma-1}{\sigma} z_{t+1,u}^{\eta} \right)^{\frac{1}{\eta-\gamma}} \right)^{\gamma-1} \right. \\ &\quad \times \left(\frac{1}{\sigma} \left(\frac{\sigma-1}{\sigma} z_{t+1,u}^{\eta} \right)^{\frac{1}{\eta-\gamma}} + x_{t+1,u} \right) \\ &\quad \left. + \frac{1}{2} \left(\left(\frac{\sigma-1}{\sigma} z_{t+1,d}^{\eta} \right)^{\frac{1}{\eta-\gamma}} \right)^{\gamma-1} \times \left(\frac{1}{\sigma} \left(\frac{\sigma-1}{\sigma} z_{t+1,d}^{\eta} \right)^{\frac{1}{\eta-\gamma}} + x_{t+1,d} \right) \right]; \\ x_T &= 0 \end{aligned}$$

The stock-market price (not including current profits) is proportionnal to productivity to a power:

$$x_t = \hat{x}_t \times \left(\frac{\sigma-1}{\sigma} z_t^{\eta} \right)^{\frac{1}{\eta-\gamma}}; x_T = 0$$

where $\hat{x}_t$ is *deterministic*:

$$\begin{aligned} \hat{x}_t &= \frac{1}{\sigma} \times \hat{k} \times (1 + \hat{x}_{t+1}); \hat{x}_T = 0 \\ \hat{x}_t &= \frac{1}{\sigma} \times \frac{\hat{k} \times \left(-1 + \hat{k}^{T-t} \right)}{-1 + \hat{k}} \end{aligned}$$

and where $\hat{k}$ is as defined in (20). When the horizon is infinite:

$$\hat{x}_t = \frac{1}{\sigma} \times \frac{\hat{k}}{1 - \hat{k}}$$

In the next section, we introduce sticky prices. They will explain the main fact that we are trying to understand, i.e., that the link between inflation and nominal stock returns varies depending on the length of the holding period.

4 Sticky-price aggregate supply

We now develop in standard New Keynesian fashion (see, for instance, Galí (2008), Walsh (2010) or Challe (2005)), the case in which firms are not free to set their prices, thus generating the Phillips curve, which endogenizes total income. The Phillips curve relates the price level to output or total income contemporaneously. Taking a cue from Dumas and Lyasoff (2012), we later shift it to time $t+1$, so that, in our rendition, it will relate the *future* price level to *future* income. Here again, productivity growth is IID but income growth, inflation and stock returns are no longer IID as they depend on an endogenous state variable reflecting path dependence.

Firms: Firms are not free to adjust their prices at will. Instead, as in Calvo (1983), each firm at each point in time has a probability $1-\omega$ of being allowed to adjust its price to an optimal level P_t^* (which will be the same for all firms). By the Law of Large Numbers, a fraction $1-\omega$ do so, so that the price index, or general price level, P_t particularizes to:[23]

$$P_t \triangleq \left[(1-\omega)\times\left(P_t^*\right)^{1-\sigma}+\omega\times\left(P_{t-1}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{21}$$

Firms maximize their market value on the equity market. With regard to setting the current price $P_{\iota,t}$ of variety ι, the part of each firm's objective function that depends on it is:[24]

$$\sup_{P_{\iota,t}}\sum_{i=0}^{T-t}\mathbb{E}_t\left[(\rho\omega)^i\frac{\left(c_{t+i}\right)^{\gamma-1}}{\left(c_t\right)^{\gamma-1}}\left(\frac{P_{\iota,t}}{P_{t+i}}-\varphi_{t+i}\right)\left(\frac{P_{\iota,t}}{P_{t+i}}\right)^{-\sigma}y_{t+i}\right]$$

(where: $y_t \triangleq \left(\int_0^1 y_{\iota,t}^{\frac{\sigma-1}{\sigma}}d\iota\right)^{\frac{\sigma}{\sigma-1}}$), with a solution $P_{\iota,t}=P_t^*$ which is:[25]

$$\frac{P_t^*}{P_t}=\frac{\sigma}{\sigma-1}\frac{\mathbb{E}_t\sum_{i=0}^{T-t}(\rho\omega)^i\left(c_{t+i}\right)^{\gamma-1}y_{t+i}\varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\sigma}}{\mathbb{E}_t\sum_{i=0}^{T-t}(\rho\omega)^i\left(c_{t+i}\right)^{\gamma-1}y_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\sigma-1}} \tag{22}$$

a function of y_t for which the numerator and the denominator will be computed

[23] This equation should really be:

$$P_t \triangleq \left[(1-\omega)\times\int_0^1\left(P_{\iota,t}^*\right)^{1-\sigma}d\iota+\omega\times\int_0^1\left(P_{\iota,t-1}\right)^{1-\sigma}d\iota\right]^{\frac{1}{1-\sigma}}$$

We are going to find that $P_{\iota,t}^*$ is the same for all ι but that is not true for $P_{\iota,t-1}$. The index of price dispersion across firms should really be present in the derivations below. We ignore it, as does most of the literature. For more details on this, see the appendix of Challe and Giannitsarou (2014). We thank Edouard Challe for confirmation.

[24] The overall objective function is the maximization of equity value, which includes additional terms not dependent on $P_{\iota,t}$. See the value of the stock market (25) and (26) below.

[25] The proof is standard and is reproduced in Appendix D.

by backward induction. To that aim, we restate Equation (22) in recursive form:

$$\begin{aligned}
\frac{P_t^*}{P_t} &= \frac{\sigma}{\sigma-1}\frac{c_t^{\gamma-1}y_t\varphi_t + A(t,y_t)}{c_t^{\gamma-1}y_t + B(t,y_t)} \qquad (23)\\
A(t,y_t) &\triangleq \mathbb{E}_t\rho\omega\left(\frac{P_{t+1}}{P_t}\right)^{\sigma}\left[(c_{t+1})^{\gamma-1}y_{t+1}\varphi_{t+1} + A(t+1,y_{t+1})\right]\\
A(T,y_T) &= 0\\
B(t,y_t) &\triangleq \mathbb{E}_t\rho\omega\left(\frac{P_{t+1}}{P_t}\right)^{\sigma-1}\left[(c_{t+1})^{\gamma-1}y_{t+1} + B(t+1,y_{t+1})\right]\\
B(T,y_T) &= 0
\end{aligned}$$

Equilibrium: As a result of their choice of price, a proportion ω of firms produce $(P_{t-1}/P_t)^{-\sigma}\times y_t$ on an average and employ $(P_{t-1}/P_t)^{-\sigma}\times y_t/z_t$ units of labor and a proportion $1-\omega$ of firms produce $(P_t^*/P_t)^{-\sigma}\times y_t$ and employ $(P_t^*/P_t)^{-\sigma}\times y_t/z_t$ units of labor.[26] Total labor employed is:

$$\left[\omega\times\left(\frac{P_{t-1}}{P_t}\right)^{-\sigma} + (1-\omega)\times\left(\frac{P_t^*}{P_t}\right)^{-\sigma}\right]\times\frac{y_t}{z_t}$$

Letting l_t stand for the labor supplied by households, the clearing of the labor market requires:

$$l_t = \left[\omega\times\left(\frac{P_{t-1}}{P_t}\right)^{-\sigma} + (1-\omega)\times\left(\frac{P_t^*}{P_t}\right)^{-\sigma}\right]\times\frac{y_t}{z_t} \qquad (24)$$

Substitution of Equations (17), (15), (24) and (21) into (23) gives the equi-

[26] Because of (21):

$$y_t \equiv \left\{\omega\times\left[\left(\frac{P_{t-1}}{P_t}\right)^{-\sigma}\times y_t\right]^{\frac{\sigma-1}{\sigma}} + (1-\omega)\times\left[\left(\frac{P_t^*}{P_t}\right)^{-\sigma}\times y_t\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}$$

i.e., the amounts produced by the two categories of firms add up to y_t.

librium Phillips curve $P_t/P_{t-1} = Phill_t(y_t)$ in implicit form:[27,28]

$$\left(\frac{1-\omega\times\left(\frac{P_{t-1}}{P_t}\right)^{1-\sigma}}{1-\omega}\right)^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1}\frac{1}{y_t^{\gamma}+B(t,y_t)}$$

$$\times\left\{\left(\frac{y_t}{z_t}\right)^{\eta}\left[\omega\times\left(\frac{P_{t-1}}{P_t}\right)^{-\sigma}+(1-\omega)\times\left(\frac{1-\omega\times\left(\frac{P_{t-1}}{P_t}\right)^{1-\sigma}}{1-\omega}\right)^{-\frac{\sigma}{1-\sigma}}\right]^{\eta-1} + A(t,y_t)\right\}$$

where:

$$A(t,y_t) = \rho\omega\mathbb{E}_t\left(\frac{P_{t+1}}{P_t}\right)^{\sigma}$$

$$\times\left\{\left(\frac{y_{t+1}}{z_{t+1}}\right)^{\eta}\left[\omega\times\left(\frac{P_t}{P_{t+1}}\right)^{-\sigma}+(1-\omega)\times\left(\frac{P^*_{t+1}}{P_{t+1}}\right)^{-\sigma}\right]^{\eta-1}+A(t+1,y_{t+1})\right\}$$

and:

$$B(t,y_t) = \rho\omega\mathbb{E}_t\left(\frac{P_{t+1}}{P_t}\right)^{\sigma-1}\left[y_{t+1}^{\gamma}+B(t+1,y_{t+1})\right]$$

The shapes of the Phillips curves are illustrated in Figures 2 and 3, along with the accompanying aggregate-demand curves derived according to Section 3.

The time $t+1$ Phillips curves and the system to be solved: Because the time-t aggregate-demand relations established in Section 3 relate time-$t+1$ prices to time-$t+1$ output, it is convenient to shift the Phillips curves to time $t+1$, separately and independently for states u and d. In this way, we are left with a system of four equations in four unknowns: $\{P_{t+1,u}, P_{t+1,d}, y_{t+1,u}, y_{t+1,d}\}$ which must be solved numerically for each node of the tree (each capturing exogenous state variable z_t) and for each value of the endogenous state variable y_t, recursively for $t = T-1, ..., 0$. Equivalently, since productivity is exogenous, the *output gap* – defined as the ratio of the actual, sticky-price output y_t to the flexible-price output (19) minus 1 – can be recognized as the endogenous state

[27] As we saw in the previous section, in the case of full price flexibility ($\omega = 0$), the Phillips curve is vertical at the flexible-price level of output: $y_t = \left((\sigma-1)z_t^{\eta}/\sigma\right)^{1/(\eta-\gamma)}$ and, on its own, leaves the price indeterminate.

[28] There exists an explicit, approximate form for the Phillips function, as suggested in Galí (2008).

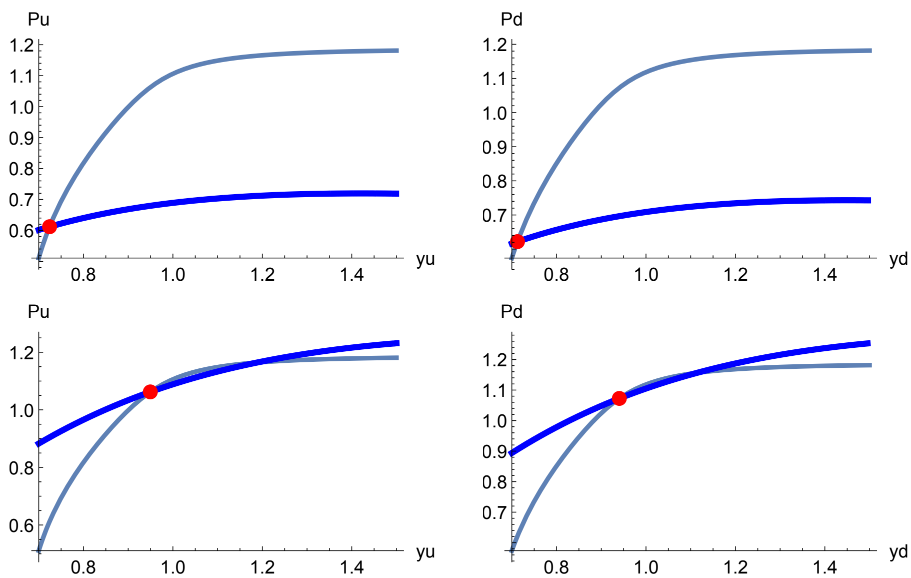


Figure 2: **Aggregate-demand (inclusive of policy rule) and aggregate-supply curves with sticky-price output and** $\phi = 1.5$. **Top panels: low-output equilibrium. Bottom panels: high-output equilibrium** (left-most intersection)**.** In each row, left-hand panel: $P_{t+1,u}$ plotted against $y_{t+1,u}$ for the equilibrium sticky-price value of $y_{t+1,d}$. Right-hand panel: $P_{t+1,d}$ plotted against $y_{t+1,d}$ for the equilibrium sticky-price value of $y_{t+1,u}$. The lighter solid line is the Phillips or aggregate-supply curve; the darker solid line is the aggregate-demand (inclusive of policy rule) curve. Parameter values are as in Table 1. $T = 6$. The time-t price level is set at 1. The time-t level of output y_t is set at 3.5% above the flexible-price level.

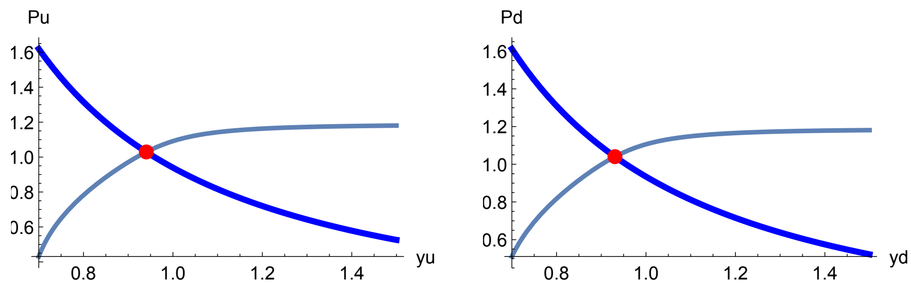


Figure 3: **Aggregate-demand (inclusive of policy rule) and aggregate-supply curves with sticky-price output and $\phi = 0.5$; single equilibrium.** Left-hand panel: $P_{t+1,u}$ plotted against $y_{t+1,u}$ for the equilibrium sticky-price value of $y_{t+1,d}$. Right-hand panel: $P_{t+1,d}$ plotted against $y_{t+1,d}$ for the equilibrium sticky-price value of $y_{t+1,u}$. The lighter solid line is the Phillips or aggregate-supply curve; the darker solid line is the aggregate-demand (inclusive of policy rule) curve. $T = 6$. Additonal parameter values are as in Table 1. The time-t price level is set at 1. The time-t level of output y_t is set at 3.5% above the flexible-price level.

variable.[29],[30]

By Walras' law, the equilibrium in the financial market and the equilibrium in the labor marlet imply the equilibrium in the goods market: $c_{t+1,u} = y_{t+1,u}$ and $c_{t+1,d} = y_{t+1,d}$.[31]

The shapes of the aggregate-demand and Phillips curves are such that there may not exist solutions, that there may be multiple solutions and that gradient-based solvers do not find them easily. We may find our way towards one of the solution by starting with the flexible-price solution ($\omega = 0$) and by gradually increasing ω in small increments, or by starting at no price adjustment ($\omega = 1$) and by gradually decreasing ω.

When $\phi > 1$, there can be two solutions,[32] which are shown in the two panels of Figure 2, for the point in time $t = T - 1$. In such a case, it is impossible to pursue the recursion to earlier points in time. This difficulty would not have even been spotted by the large number of researchers who work not with the exact system of equations but with a system that is linearized around the flexible-price

[29] The current general price level P_t is also an endogenous state variable but, in the absence of nominal illusion, it can be factored out on grounds of homogeneity, as noted before.

[30] In addition, when household utility is isoelastic and the production function satisfies the property of constant returns ot scale, a scale-invariance property can be exploited: we need not do the calculation for every node of each point in time t, which differ only in the level of productivity z_t. For the several nodes of time t, the functions that are carried backward (f_1 or f_2, A and B) can be deduced from a single one of them.

[31] In Appendix E, we verify that clearing of the financial market and of the labor market do imply clearing of the goods market.

[32] And, for low enough values of current output y_t there are no solutions.

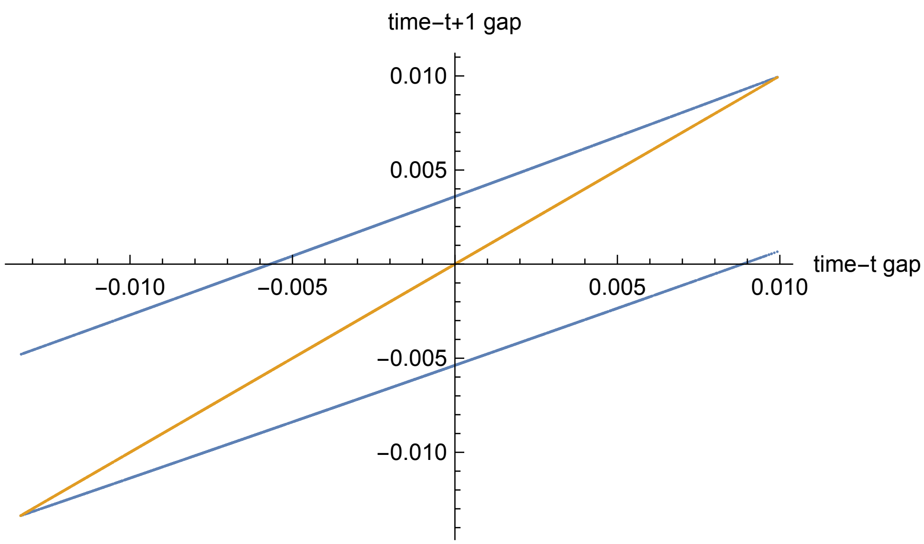
solution.

When $\phi < 1$, the equilibrium is surely unique as shown in Figure 3. The reason is that, in that case, as we have seen under Proposition 8, the aggregate-demand functions (inclusive of the policy rule) are decreasing, while the Phillips curve, of course, is increasing. In what follows, we assume that $\phi < 1$ so that we can obtain a unique solution at any point in time. In other words, we make the following assumption:

Assumption 4 *Monetary policy is "passive," in the terminology of Leeper (1991).*

In association with the active fiscal policy already assumed, we are considering Regime F in the vocabulary of Leeper and Leith (2018).

Figure 4: **Relation between output gap at time t (on the x axis) and at time $t+1$ conditional on a u productivity shock (lower line) and a d shock (upper line)**, across 10,000 paths at a fixed date. The 45° line is also shown. Parameters are as in Table 1 with $\bar{\tau}$ set at a neutral level (see Definition 2). The 45° line is also drawn.



Under the set of assumptions made so far, the behavior of the output gap is illustrated in Figure 4 and formulated in the following

Observation 1

1. *A u productivity shock at time $t+1$ decreases the output gap relative to its value at t. A d shock increases it.*

2. *The lower the time-t output gap, the smaller the decrease. A d shock increases the output gap. The higher the time-t output gap, the smaller the increase. In other words:*

$$\begin{aligned} \frac{y_{u,t+1}}{y_{f,u,t+1}} &< \frac{y_t}{y_{f,t}} < \frac{y_{d,t+1}}{y_{f,d,t+1}} \\ \frac{d\left(\frac{y_{u,t+1}}{y_{f,u,t+1}}\right)}{d\left(\frac{y_t}{y_{f,t}}\right)} &< 1; \frac{d\left(\frac{y_{d,t+1}}{y_{f,d,t+1}}\right)}{d\left(\frac{y_t}{y_{f,t}}\right)} < 1 \end{aligned}$$

3. *The gap is bounded above and below. When it is at its lower bound, a u productivity shock leaves it unchanged. When it is at its upper bound, a d productivity shock leaves it unchanged.*

When prices are flexible, a u productivity shock induces firms to increase equilibrium output. When prices are sticky they are not able to increase it as much as they would with flexible prices. Therefore, the output gap decreases. The upper and the lower bounds are temporary "anchor points" for d and u shocks, respectively.

Initial conditions: At time zero, given the initial productivity z_0, the initial conditions to be solved for the unknown initial price P_0 and the initial income y_0 are

$$\begin{cases} f_{2,0} \times P_0 = \theta_{2,-1} + s_0 \times P_0 \\ \frac{P_0}{P_{-1}} = Phill\,(0, y_0) \end{cases}$$

where the first condition is identical to the initial condition of Section 2 and the second one is just the Phillips curve at time 0, and where $\theta_{2,-1}$ is a given (negative) amount of nominal claim outstanding and s_0 a given time-0 surplus. The solution for P_0 is unique as long as the backward recursion provided a unique function $Phill$.

Stationary solution: As mentioned, we solve the system for each node of the tree (each node capturing exogenous state variable z_t) and for each value of the endogenous state variable, recursively for $t = T-1, ..., 0$. With the impatience parameter set at $\rho = 0.99$, the value $T = 270$ years is sufficiently large for functions carried backward to be unchanging by the time we get to time 0. The stationary functions capture the equilibrium of an economy with an horizon that has been increased indefinitely.

5 Stock returns and inflation

As the firms enjoy market power, they generate profits. We now *re-define the aggregate stock security* as paying corporate profits (as opposed to paying output, which it was in Section 2)). The real, future profits, assumed to be distributed as dividends, are:[33]

$$\delta_{t+1} \triangleq \left[\omega \times \left(\frac{P_t}{P_{t+1}} - \varphi_{t+1}\right)\left(\frac{P_t}{P_{t+1}}\right)^{-\sigma} + (1-\omega) \times \left(\frac{P^*_{t+1}}{P_{t+1}} - \varphi_{t+1}\right)\left(\frac{P^*_{t+1}}{P_{t+1}}\right)^{-\sigma}\right] \times y_{t+1} \tag{25}$$

The value of the stock market, in real terms, current profits not included, is:

$$x_t = \rho \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t}\right)^{\gamma-1} (\delta_{t+1} + x_{t+1})\right] \tag{26}$$

Numerical illustrations will indicate the correlation between the rate of return on the stock market and inflation.

After solving all the equations of all times in a backward sequence, we use the stationary functions to simulate the economy, drawing at random the event of a u or a d productivity shock z over 200 time steps of one year each. Ten thousand paths are drawn. For this and the next simulations, we assume again IID growth of productivity and we now set $\bar{\imath}$ to be equal to the neutral rate of interest, where we define "neutral" as follows:

Definition 2 *Under IID growth of productivity, the neutral rate of interest of an economy is the interest rate that would prevail in a flexible-price economy when $\bar{\imath}$ is equal to the equilibrium interest rate (13) (with $1+u$, $1+d$ and k replaced as in Proposition 6).*

The value of the neutral rate is[34]

$$1 + i = \frac{1+\bar{\pi}}{\hat{k} \times \left(\frac{1}{2}\frac{1}{1+\hat{u}} + \frac{1}{2}\frac{1}{1+\hat{d}}\right)}$$

Assumption 5 *The Taylor-rule parameter $\bar{\imath}$ is equal to the neutral rate of interest.*

5.1 Impulse responses to a productivity shock

To obtain impulse response functions, we segregate the paths that experience an a u productivity shock at $t = 45$ from those that experience a d productivity shock at that time. We then compute conditional average paths for each of the two subsets of paths. We call "impulse-response function" the difference

[33] Current profit δ_t differs from one firm to the other, depending on which firm is allowed currently to change its price. For future profits, we ignore current price dispersion, as explained in footnote 23.

[34] With the parameter values of Table 1, the neutral rate is equal to 3.02%. With that value of $\bar{\imath}$, the rate of interest that prevails in the sticky-price economy at an output gap equal to zero is equal to 3.109%.

between the two conditional averages, normalized (thereby detrended) by the unconditional average path. Figure 5 shows the responses of output, the real stock market return index and the price level to a +1% productivity shock compared to a −1% shock. Output and the price level take several years to reach new levels, increased by 2% and reduced by 3% respectively, while, because of isoelastic utility, the real stock market return index reacts exactly like output.[35] The output gap is reduced by more than 0.8% on impulse and returns to approximately zero in five to six years.

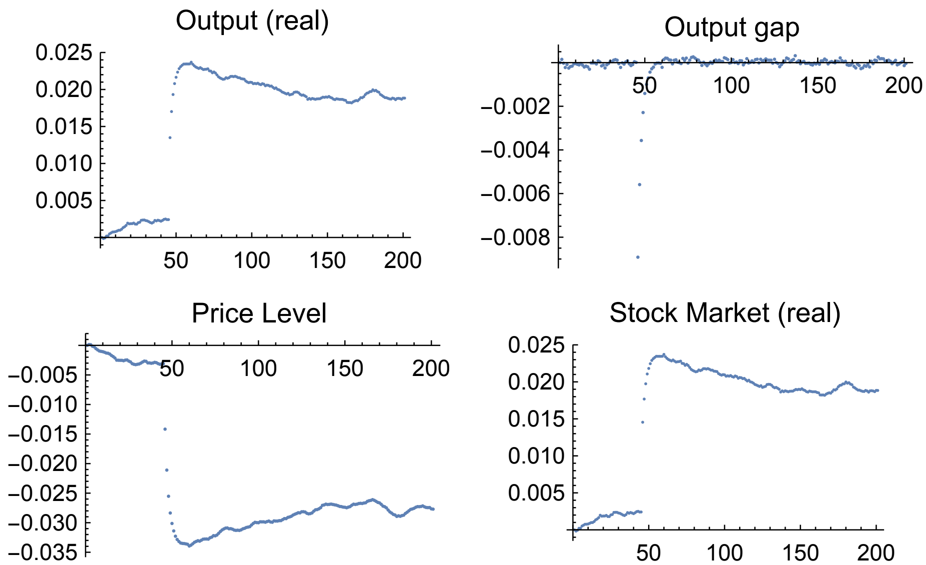


Figure 5: **Impulse responses: average path conditional on a u productivity shock occurring at $t = 45$ minus average path conditional on a d productivity shock occurring at that time.** Top left-hand panel: response of output (t in years on the x-axis). Top right-hand panel: response of the optput gap. Bottom left panel: response of the price level**.** Bottom right panel: response of the stock market (in real terms). Bottom panel:All responses are scaled by the corresponding unconditional average. Parameter values are as in Table 1 with $\bar{\imath}$ set at a neutral level (see Definition 2). The figure is obtained from 10,000 paths drawing at random the event of a u or a d productivity shock z over 200 time steps of one year each.

[35] The long-run reaction is equal to the one that prevails under flexible prices (with $\gamma = 0$, as in our numerical example, a 2% increase in productivity leads to a 2% increase in output when prices are flexible; see formula (19)).

5.2 The role of productivity shocks over one vs. several periods

As we saw in Sections 2 and 3, if prices set by firms were fully flexible, there would be zero relation whatever between the nominal return on stocks and the rate of inflation.

When the prices set by firms are sticky, however, the output depends on the previous-period output gap, thus generating richer dynamics for stock returns and inflation. There can occur many values for inflation and many values for nominal stock returns depending on the value of the preexisting output gap. Over a single time-step, productivity can be increased by a factor u or d. Figure 6, left-hand panel, displays the two variables in a cross-section of paths.[36] The cloud of simulated points is highly structured.

Observation 2

1. *For the same time-t value of the output gap, when productivity is increased by a factor u, inflation is lower and the nominal stock return is higher than when the factor is d.*

2. *Conditional upon productivity growth being u or d at time $t+1$, the time-$t+1$ realized inflation and realized nominal stock returns are near-linearly, positively related across different values of the time-t output gap. Upon a u move, the lowest values of inflation and nominal stock return are reached when the gap is at its upper bound. Upon a d move, the highest values of inflation and nominal stock return are reached when the gap is at its lower bound.*

In accordance with Observation 2, the two radii that appear are increasing near-straight lines. Of the two radii, the upper (lower) one portrays the relation conditional upon productivity growth being u (d). On a given radius, the points that plot farther from the origin correspond to higher values of the output gap prior to the shock, as the labelling of some points indicates. And, as we have seen, for any given value of the output gap at time t, a u productivity shock causes the output gap to be down at time $t+1$, the more so as the time-t gap is higher.[37] Observation 2 invites empirical tests that would be conducted conditional on observed productivity shocks.

Not conditioning on the productivity growth, i.e., across the entire cloud of points, the slope coefficient of a regression of the nominal stock return on inflation is equal to 0.0966.

When measuring returns over a longer holding period, the relationship is similar but more combinations of u and d productivity moves are possible. For

[36] When drawn along one path, the picture is nearly identical, as it should be under a stationary-growth equilibrium.

[37] These are effects opposite to those of the epigraph of this article, but they leave unchanged the conclusion about the sign of the correlation.

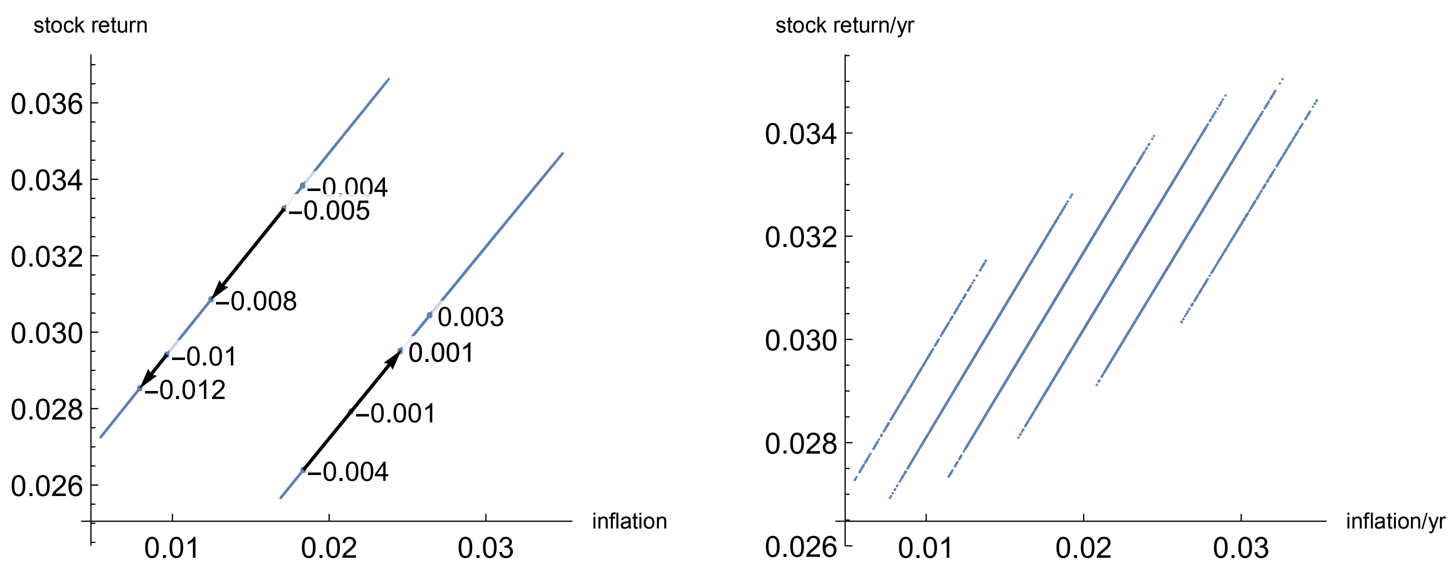


Figure 6: **Relation between one-period nominal stock return and one-period inflation (left-hand panel) and relation between the same two variables measured over five periods (right-hand panel)**, across 10,000 paths at a fixed date. Parameters are as in Table 1 with $\bar{\iota}$ set at a neutral level (see Definition 2). In the left-hand panel, the labelling of some of the points indicates the level of the output gap. The arrows show examples of transitions of the gap between two successive points in time.

instance, over five periods, six combinations are possible. The six corresponding radii are shown in Figure 6, right-hand panel: the highest-slope radius reflects realizations in which all five productivity moves are u; the line supporting that radius is identical to that of a single u move on the left-hand panel The radius second from the top contains realizations for which four moves where u and one d in any order etc.[38] The lowest-slope radius reflects realizations in which all five productivity moves are d; the line supporting that radius is identical to that of a single d move in the left-hand panel.

Across all the paths, i.e., not conditioning on the productivity growth combinations, the slope coefficient of an across-paths regression of the nominal stock return on inflation is equal to 0.2348.[39] This result about the slope is motivated by the following:

Observation 3

1. *For the same time-t value of the output gap, when productivity is increased five times by a factor u, inflation per period and the nominal stock return per period are lower than they would be with a single u productivity move over one period, and are closer to the anchor point of the lower bound ot the gap. In the limit, when the gap is at its lower bound, inflation per annum and the stock return per annum are equal to what they would be with a single u move.*[40]

2. *For the same time-t value of the output gap, when productivity is changed five times by a factor d, inflation per period and the nominal stock return per period are higher than they would be with a single d productivity move over one period, and are closer to the anchor point of the upper bound ot the gap. In the limit, when the gap is at its upper bound, inflation per annum and the stock return per annum are equal to what they would be with a single d move.*[41]

3. *Hybrid combinations of productivity moves have an intermediate effect, over a wider range of values for inflation and the nominal stock return.*

The observation implies that the unconditional regression slope is higher over five periods than it is over one. The observation follows from Observation 1 above, which says that increases in the gap decrease with the starting value

[38] Except for the highest-slope and lowest-slope radii, all other radii are actually a bundle of radii that are partially superimposed and overlapping, each depending on the order (or permutation) in which the productivity shocks occur. In the (inflation, stock return, starting output gap) space, they would be separate radii.

[39] For a ten-year holding period, the slope would be only slightly larger: 0.2639.

[40] Because of the anchor effect, the point of the highest-slope radius closest to the origin on the right-hand panel is identical to the point of the higher-slope radius closest to the origin on the left-hand panel of Figure 6.

[41] Because of the anchor effect, the point of the lowest-slope radius farthest from the origin on the right-hand panel is identical to the point of the lower-slope radius farthest from the origin on the left-hand panel of Figure 6.

of the gap; five moves in the same direction have less effect per period than a single move over one period. One can also imagine on the basis of the same observation and Figure 4 that hybrid combinations of moves impart the gap with more volatility.

We stress that the direction of the increase in the regression slope is not just the mechanical result of frictions slowly working themselves out over time. Indeed, the increase is a move *away* from the frictionless outcome, which, as the reader will recall, was a zero slope.

5.3 *Ex post* regression result

We ask whether the message conveyed by Figure 6 is confirmed by *time-series* regressions. Can the model fit the facts listed in the introduction? Having simulated 10,000, 200-period long paths of the economy of Section 4, we run on each an *ex post* regression in the manner of Boudoukh Richardson (1993) (BR).[42]

The *ex post* regression being run is quite simply:[43]

$$R_{t\to t+j} = \alpha_j + \beta_j \times \pi_{t\to t+j} + \varepsilon_{t,j} \tag{27}$$

where R is the nominal rate of return on the equity and π is the rate of inflation, with $j = 1$ for the one-year time interval and $j = 5$ for the five-year time interval. If the real rate of return on stock were constant, one would expect $\alpha_j = 0$ and $\beta_j = 1$. Because the five-year rates of return are calculated every year, there is overlap in the data and the Generalized Method of Moments is used to compute heteroskedasticity- (and autocorrelation-) consistent standard errors. The results are shown in Table 2.

The results are exactly in conformity with the intuition conveyed above, in that the five-year regression slope is higher than the one-year slope. The results are also in close conformity with the empirical results of BR. Recall that both their slope coefficients were positive, with the exact same disparity between them.[44]

Basically, therefore, we have discovered the reason for which BR found different slopes for different lengths of close holding-period.

5.4 *Ex ante* regression result

We test the moment conditions:

$$\mathbb{E}\left[\left(R_{t\to t+j} - \alpha_j - \beta_j \times \pi_{t\to t+j}\right) \otimes Z_t\right] = 0 \tag{28}$$

[42] We drop the first ten periods of the paths to ensure that statistical results do not depend on the initial condition, which is just the nominal amount $\bar{\theta}_2$ of government debt outstanding at $t = 0$.

[43] The exercise is descriptive. We are not testing a hypothesis and do not assume that inflation is an exogenous variable. Furthermore, if one wanted to hedge inflation risk using equities, one should calculate the hedge ratio (i.e., the number of units of stock to buy) by regressing inflation on stock returns.

[44] Had we included in our model a monetary shock, we could also have increased both our simulated slope coefficients at will.

Statistic	α_1	β_1	α_5	β_5
Median	0.029	0.091	0.14	0.233
Upper quintile	0.03	0.127	0.142	0.242
Lower quintile	0.028	0.048	0.139	0.22
Std error				
Median	0.00	0.013	0.001	0.012
Upper quintile	0.00	0.016	0.002	0.014
Lower quintile	0.00	0.011	0.001	0.01

Table 2: **Stock Returns and Contemporaneous Inflation:** the regressions are those of Equation (27). Parameters are as in Table 1 with $\bar{\imath}$ set at a neutral level (see Definition 2). The table is obtained from 10,000 paths drawn at random.

Statistic	α_1	β_1	α_5	β_5
Median	0.027	0.2	0.139	0.247
Upper quintile	0.051	1.327	0.214	0.972
Lower quintile	0.004	-1.011	0.064	-0.484
Std error				
Median	0.000	0.407	0.023	0.225
Upper quintile	0.646	32.673	1.641	15.714
Lower quintile	0.002	0.1	0.006	0.057

Table 3: **Stock Returns and Expected Inflation: the Instrumental Variable Approach as in Equation (28).** Past output is the instrument. Parameters are as in Table 1 with $\bar{\imath}$ set at a neutral level (see Definition 2). The table is obtained from 10,000 paths drawn at random.

where Z_t is some set of instrumental variables known to investors at time t.

The exact *ex ante* formula that would correspond to the model is the CAPM (26) that applies to the equity. That CAPM in no way implies that the conditionally expected real rate of return on equity is constant. It is obviously a function of the state variable y_t, the current level of output. For that reason, we try one of the ex ante specifications of BR that involves the current level of output as the instrumental variable.

The results are shown in Table 3.

Recall that BR found one-year slope coefficients that were markedly smaller than the five-year coefficients. To the degree that the moments are conditional on the previous year's level of output, one may understand that the relationship between stock returns and inflation has become more positive for the one-year holding period. However, the comparative magnitudes of our simulated results for β_1 and β_5 in the *ex ante* specifications are not in conformity with BR.

In our model, the previous year's level of output is a strong instrument. It is conceivable that, in the data, the instruments used by BR were not as strong.

6 Money demand and the zero lower bound

We now investigate the behavior of money demand and supply in the equilibrium of the last section. To do that, we build an equilibrium model of money demand along the lines of Baumol (1952) and Tobin (1956). We must observe at the outset that, when the nominal rate of interest approaches zero, money demand grows steadily thereby creating *a natural lower bound* on the rate of interest. Meanwhile, the demand of the private sector for the government bond drops steadily. Eventually the demands for money and bonds become indeterminate while their sum remains determinate and finite. The government cum central bank, as noted by J. M. Keynes, falls into a "liquidity-trap" regime that is akin to Quantitative Easing.[45]

Now there is cash explicitly in the economy, side by side with government bonds. Calling M monetary claims, money supply at time t is: $M_{2,t}$ (a negative number because, like $\theta_{2,t}$, it is a liability of the government cum central bank); money demand is $M_{1,t}$; the seignorage, an indirect tax, collected at time t and measured in nominal terms of that date is: $M_{1,t} \times (1 - 1/(1 + i_t))$. Households receive an income of a single good and no income in cash. At time t, the financial wealth available for consumption is:

$$P_t \times y_t + \theta_{1,t-1} + M_{1,t-1} - F_{1,t} - S_t$$

The proceeds $P_t \times y_t$ from the sale of the physical income are in the form of a deposit at a bank. Cash on hand $M_{1,t-1}$ and the other terms are assumed to be readily available in cash. Cash can be withdrawn by taking trips to the bank. Each trip costs a fixed real amount ν. The smaller the number of trips $N_{1,t}$ the household decides to take to the bank, the more cash the household holds on an average over the time period $[t, t + 1)$:[46]

$$M_{1,t} = \frac{P_t \times y_t}{2 \times N_{1,t}}$$

[45] On the zero lower bound, a very active topic of research during the Great Recession, see the following papers, which have implications for Finance: McCallum (2000), Krippner (2012), Wright (2012), Gavin et al. (2013), Priebsch (2013), Greenwood et al. (2014), Swanson and Williams (2014).

[46] We could have assumed that all the financial wealth except cash on hand is deposited with a bank. Then,

$$M_{1,t} = \frac{P_t \times y_t + \theta_{1,t-1} - F_{1,t} - S_t}{2 \times N_{1,t}}$$

so that the cost of the trips at current prices is:

$$k \times P_t \times N_{1,t} = \nu \times P_t \times \frac{P_t \times y_t + \theta_{1,t-1} - F_{1,t} - S_t}{2 \times M_{1,t}}$$

The derivation of the nodal system under that assumption is available in Appendix G.

so that the cost of the trips at current prices is:

$$\nu \times P_t \times N_{1,t} = \nu \times P_t \times \frac{P_t \times y_t}{2 \times M_{1,t}}$$

That cost is truly a deadweight loss; no one gets the benefit of it. But, for the sake of computational simplicity we imagine that it is refunded to the private sector in the form of a transfer $\zeta_{1,t} = P_t \times y_t \times \nu \times P_t / (2 \times M_{1,t})$, thus keeping in our equation system only the distortionary effect of the cost but not its wealth effect.[47,48] At the terminal point T, however, money is not "refunded." Even without a refund, the private sector holds it till the end because it has to. We set $1/(1+i_T) = 0$.

In Appendix F, we derive the set of equations (41) to be solved at each node of the tree. Eliminating the money terms from it and taking (40) into account:

Flow budget constraints of private sector

$$P_{t+1,j} \times c_{t+1,j} + F_{1,t+1,j} + \frac{P_{t+1,j} \times \sqrt{\frac{1}{2} y_{t+1,j} \times \frac{\nu}{1-\frac{1}{1+i_{t+1,j}}}}}{1+i_{t+1,j}} + S_{t+1,j}$$

$$= \theta_{1,t} + P_t \times \sqrt{\frac{1}{2} y_t \times \frac{\nu}{1-\frac{1}{1+i_t}}} + P_{t+1,j} \times y_{t+1,j};$$

$$F_{1,T,j} = 0; j = u, d$$

Flow budget constraints of government cum central bank

$$F_{2,t+1,j} - \frac{P_{t+1,j} \times \sqrt{\frac{1}{2} y_{t+1,j} \times \frac{\nu}{1-\frac{1}{1+i_{t+1,j}}}}}{1+i_{t+1,j}}$$

$$= \theta_{2,t} - P_t \times \sqrt{\frac{1}{2} y_t \times \frac{\nu}{1-\frac{1}{1+i_t}}} + S_{t+1,j}; F_{2,T,j} = 0; j = u, d$$

Portfolio-choice or Euler conditions

$$\frac{1}{1+i_t} \frac{1}{P_t} = \rho \frac{\frac{1}{2} (c_{t+1,u})^{\gamma-1} \frac{1}{P_{t+1,u}} + \frac{1}{2} (c_{t+1,d})^{\gamma-1} \frac{1}{P_{t+1,d}}}{(c_t)^{\gamma-1}}$$

Market clearing

$$\theta_{1,t} + \theta_{2,t} = 0$$

Initial conditions are:

$$\begin{aligned} \frac{P_0}{P_{t-1}} &= Phill_0 (y_0) \\ F_{2,0} (y_0) - \frac{P_0 \times \sqrt{\frac{1}{2} y_0 \times \frac{\nu}{1-\frac{1}{1+i_0}}}}{1+i_0} &= \theta_{2,-1} + M_{2,-1} + S_0 \end{aligned} \tag{29}$$

[47] Without that assumption, the trips to the bank being deadweight losses $c_t \neq y_t$.

[48] In addition, to preserve scale invariance (see footnote 30), we do not take ν to be a constant; we assume it proportional to output.

A change of unknown variables θ:

$$\hat{\theta}_{1,t} \triangleq \theta_{1,t} + P_t \times \sqrt{\frac{1}{2} y_t \times \frac{\nu}{1 - \frac{1}{1+i_t}}}; \quad \hat{\theta}_{2,t} \triangleq \theta_{2,t} - P_t \times \sqrt{\frac{1}{2} y_t \times \frac{\nu}{1 - \frac{1}{1+i_t}}}$$

along with a change of backward iterates:[49]

$$\hat{F}_{1,t} \triangleq F_{1,t} + \frac{P_t \times \sqrt{\frac{1}{2} y_t \times \frac{\nu}{1 - \frac{1}{1+i_t}}}}{1+i_t}; \quad \hat{F}_{2,t} \triangleq F_{2,t} - \frac{P_t \times \sqrt{\frac{1}{2} y_t \times \frac{\nu}{1 - \frac{1}{1+i_t}}}}{1+i_t}$$

transforms the system of equations into one that is identical to the system (7), which we solved in the absence of money. We thus demonstrate that, *for a given value of the endogenous variable y_t*, money is simply added to government bonds and is *otherwise irrelevant*. The government surplus being exogenous anyway,[50] seignorage being refunded and inflation targeting being an infinitely elastic central-bank reaction function, money demand only serves to determine money supply, as has been pointed out by many authors.

This is true with two caveats. Firstly, the change of variables is valid only for strictly positive nominal interest rates. If we implemented it blindly, the nominal rate of interest could become negative, despite the natural lower bound. To prevent that error in the computation, we superimpose on the Taylor rule an artificial zero lower bound on the nominal rate of interest:[51]

$$1 + i_t = \max \left[1, (1+\bar{\imath}) \times \left(\frac{\frac{\frac{1}{2} P_{t+1,u} + \frac{1}{2} P_{t+1,d}}{P_t}}{1+\bar{\pi}} \right)^{\phi} \right] \tag{30}$$

Secondly, since we have assumed that money is not refunded, the terminal conditions, which were originally $F_{1,T,j} = F_{2,T,j} = 0$ must be replaced by: $\hat{F}_{1,T,j} = -\hat{F}_{2,T,j} = P_T \times \sqrt{\frac{1}{2} y_T \times \nu}$. We intend to study the paths of the economy in a stationary situation. For that, the change of terminal condition is not very important except for the fact that it modifies the solution to the initial conditions (29), so that the initial price level P_0 and the initial output y_0 *are affected by the presence of money.* The initial point being modified, every path of the economy will also be modified but the dynamics of the system will not, unless the nominal rate of interest approaches the zero lower bound. We point out that, with $M_{2,-1} > 0$, the initial level of government debt $\theta_{2,-1}$ could be set equal to zero. In the cashless economy, the initial condition of the Fiscal theory with zero debt would leave the price level of goods indeterminate at all times. But, in the economy with money, the initial condition does determine the initial price level even then, the present value of future government surpluses being compared to the outstanding stock of money $M_{2,-1}$. This shows that our results are not predicated on the validity of the strict, debt-based Fiscal theory.

[49] Note: $\hat{\theta}_{1,t}/(1+i_t) = \hat{F}_{1,t}$

[50] But see below the caveat concerning the terminal condition.

[51] We actually implement a smooth variant of that relation.

We amend the "aggregate demand" subsystem of equations of Section 2 to reflect the modified policy rule (30), leaving intact the "aggregate supply" subsystem of Section 4 and we solve by backward induction exactly as we did before (with the additional parameter $\nu = 1\%$ of output). Under the parameter and state variable combinations considered so far, the result is identical to that of Figure 3, simply because the cashless economy itself never produced a negative value for the rate of interest. In order to make liquidity-trap episodes possible, we replace Assumption 5 with the following:

Assumption 6 *The Taylor-rule interest parameter* $\bar{\imath}$ *is set* 1.5% *below the neutral rate of the flexible-price economy as defined in Definition 2.*

We now discuss the outcome of that experiment.

The new version of Figure 6 is Figure 7, which shows that the lower bound on the rate of interest introduces a support from below for realized nominal stock returns. For that reason, the relation between inflation and stock returns described above in section 5.2 is no longer near linear but is still positive, contingent on a given sequence of productivity shocks. Not conditioning on the productivity growth, the coefficient of an across-paths regression of the nominal stock return on inflation between the two variables is equal to 0.0743 over one period while it is equal to 0.1950 over five periods. Once again the slope is quite a bit larger over five periods than it is over one.

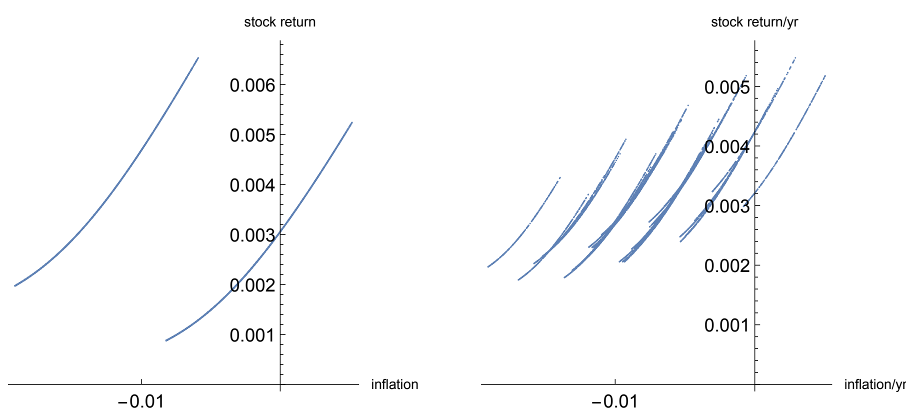


Figure 7: **Relation, in the presence of money, between one-period nominal stock return and one-period inflation (left-hand panel) and relation between the same two variables measured over five periods (right-hand panel)**, across 10,000 paths at a fixed date. Parameters are as in Table 1 with $\bar{\imath}$ set 1.5% below the neutral level (see Definition 2).

Statistic	α_1	β_1	α_5	β_5
Median	0.003	0.02	0.02	0.117
Upper quintile	0.004	0.045	0.02	0.13
Lower quintile	0.003	−0.006	0.019	0.104
Std error				
Median	0.00	0.009	0.00	0.008
Upper quintile	0.00	0.011	0.001	0.011
Lower quintile	0.00	0.008	0.00	0.006

Table 4: **Stock Returns and Contemporaneous Inflation in the presence of money:** the regressions are those of Equation (27). Parameters are as in Table 1 with $\bar{\imath}$ set at a neutral level (see Definition 2). The table is obtained from 10,000 paths drawn at random.

The new version of Table 2, which contained the results of *ex post* regressions across simulated paths, is Table 4.

The results are again in conformity with the empirical results of BR. We do not display the *ex ante* regression results, which are once again ambiguous.

7 Bonds and the 'Fed model'

7.1 Bond returns and inflation

As mentioned in the introduction, Katz and Lustig (2017) using a panel of countries have confirmed empirically that stock markets are slow to incorporate news about future inflation so that they do not qualify to be called "real" assets, but that the same is not true at all of bond markets. We now check whether our model can explain that fact. For that we draw figure 8, which relates the one-year and five-year nominal rates of return on a ten-year pure-discount nominal bond, across paths at a given point in time. The slope of an unconditional regression line of one-year returns on inflation is is equal to 0.7202 (as opposed to 0.0966 for stocks) while the slope for five-year returns is equal to 1.1028 (as opposed to 0.2348 for stocks). The difference between long-term bonds and stocks is in the payoff. In our model with sticky prices but flexible wages, dividends on stocks (Equation (25)) are adversely affected by inflation.

7.2 The 'Fed model'

Asness (2003) criticizes a heuristic approach of professional circles who compare yields on stock securities to yields on bonds, and expect the two to revert to each other, which, empirically speaking, they do, both yields being high when inflation is high.[52] He refers to this approach as the "Fed model". He points

[52] Maio (2013) shows empirically that the yield gap forecasts excess market returns, both at short and long forecasting horizons, and for both value- and equal-weighted stock indexes,

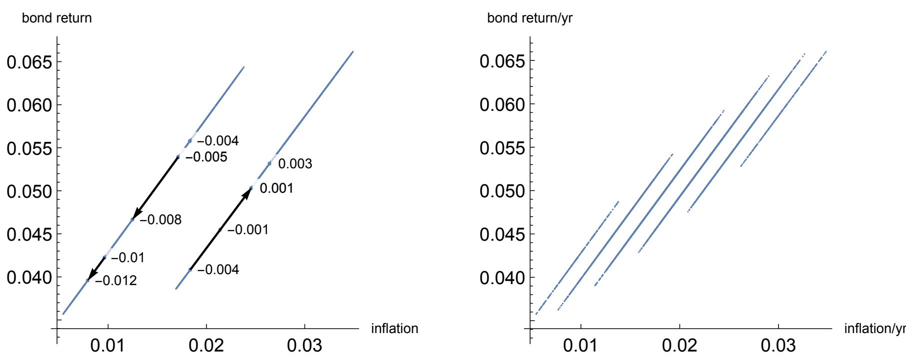


Figure 8: **Relation between one-period nominal 10-year bond return and one-period inflation (left-hand panel) and relation between the same two variables measured over five years (right-hand panel)**, across 10,000 paths at a fixed date. Parameters are as in Table 1 with $\bar{\imath}$ set at a neutral level (see Definition 2). In the left-hand panel, the labelling of the points indicates the level of the output gap. The arrows show examples of increments of the output gap between two successive points in time.

out correctly that the two yields are not comparable; the coupon payments on a bond are constant in current euros while the dividends on a share of stock will grow with inflation.

This attitude of professional circles may be a form of money illusion, reminiscent of Modigliani and Cohn (1979), unless one of two rational explanations of the fact that the "Fed model" works well empirically holds. One is the hypothesis that says that high current inflation is associated with *low* expected long-run nominal dividend growth in excess of the riskless rate, justifiably driving up the equity dividend yield. The other is that high inflation drives up the risk of the economy and thus the nominal equity risk premium. Campbell and Vuolteenaho (2004) (CV) ran an empirical investigation to determine which of the three possibilities transpires in the behavior of stock prices. They decompose the dividend yield on equity into three components: a constant, expected long-run future nominal equity excess returns and expected long-run nominal dividend growth in excess of the riskless rate. They find that high current inflation is associated with *high* expected long-run nominal dividend growth in excess of the riskless rate, and that inflation is not related to the anticipated nominal equity premium, thus leaving money illusion as the surviving hypothesis.

Indirect empirical evidence of the effect of monetary policy on the stock market is also provided by a recent paper of David and Veronesi (2013) relating stock returns to returns on bonds. The model allows for money illusion on the part of investors. The authors argue that realized inflation is interpreted very differently by investors depending on whether they fear stagflation (as in the 1980's) – a fear that leads to a high correlation – or deflation, which would lead to a lower correlation.

To examine these issues, we now revert to the cashless economy set up (with the parameter values of Table 1) and introduce a ten-year zero-coupon bond that pays one current monetary unit at maturity, just like the stock pays dividends forever. We find (Figure 9) that there exists a near-straight line *negatively sloped* relationship between dividend yield and bond yield. The labelling of a few of the simulated points tell the dynamic story: as negative productivity shocks accumulate, inflation becomes higher and higher, driving up the bond yield while the dividend yield is brought down by the d productivity shocks. Dividend yield and bond yield do not move in tango and should, therefore, not be compared.

The theoretical result is not consistent with the empirical evidence mentioned in the opening paragraphs of this section. This evidence, however, is entirely based on post-World War II data, which feature a long upward swing of yields to a peak in the early 80s followed by a long downward swing of both yields. The up and down swings are probably caused by the change of monetary policy regime that took place around 1980. If one extends the sample to the nineteenth century, one finds that the empirical relationship between yields no longer holds.

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and that it also outperforms competing predictors commonly used in the literature.

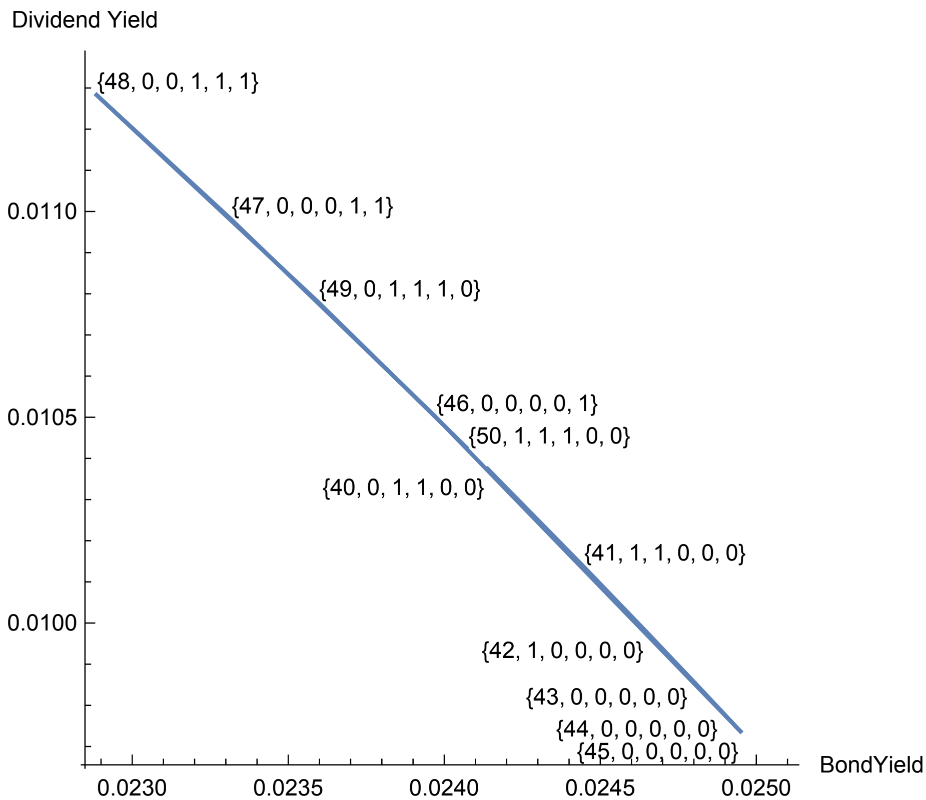

Figure 9: **Relation between nominal (or real) dividend yield on equity and nominal annualized yield on a ten-year nominal bond**, across 10,000 paths at a fixed date. The labeling of the points of dates 40 to 50 along one example path contains the following information: {date (year t), u or d productivity shock (coded 1 and 0) two years ago, one year ago and contemporaneously, and the contemporaneous rate of inflation}. Parameters are as in Table 1 with $\bar{\iota}$ set at a neutral level (see Definition 2).

8 Conclusion

Adopting a method that has been used to calculate dynamic financial-market equilibria, we have constructed the equilibrium of a cashless production economy with productivity shocks and with three types of agents: (i) household/investors who supply labor with a finite elasticity, consume a large variety of goods that are not perfect substitutes and trade government bonds; (ii) firms that produce those varieties of goods, setting prices in a Calvo manner; (iii) a government that collects an exogenous fiscal surplus and acts mechanically, buying and selling bonds in accordance with a Taylor policy rule based on expected inflation. Merging the consumption-financial behavior of households with the policy rule, under IID productivity growth and no monetary shock, we have derived explicitly at each point in time and in each state of nature, aggregate-demand schedules (inclusive of policy rule) relating, at the next point in time and in each successor state, the price level to the level of output. We have shown that these schedules are decreasing if and only if the exponent of the Taylor rule that falls on expected inflation is less than 1. The aggregate supply schedules (or Phillips curve) that also apply to the next point in time are always increasing. The equilibrium is unique if the exponent is less than 1. Otherwise, because of the non linearities of the two types of schedules, two equilibria can exist.

In this equilibrium, we have priced the stock market, defined as the present discounted value of firms' profits and demonstrated that, in a flexible price version of our economy, the equilibrium nominal return on stocks is just equal to the riskless interest rate, which is constant, whereas inflation is higher when productivity growth is low. That explains a zero-slope relation between these rates and gives stocks a nominal character.

Moving to a sticky-price version of the economy, we have simulated the joint behavior of stock returns and inflation. That has allowed us to discover the reason for which Boudoukh Richardson (1993) found different slopes for different holding-period lengths. The reason lies in the succession of productivity shocks that take place over several periods, which is a newfangled version of Fama (1981)'s "proxy hypothesis" explained in the introduction.

The equilibrium has then been expanded to incorporate an explicit money demand à la Baumol and Tobin. The only effect of the zero lower bound thus created as been to support stock returns when they are low.

Finally, we turned to long-term bonds to observe that their behavior vis-à-vis inflation is more "real" than that of stocks, which explains the surprising empirical findings of Katz and Lustig (2017). We examined the validity of the 'Fed model' in the context of our model. The model invalidates completely the suggestion that one might compare dividend yields to bond yields to assess the direction of the stock market. Under IID productivity-growth, the relationship between them is in fact negative: when inflation is high, the bond yield is high while the dividend yield is low.

All the results of this paper invite empirical tests that would be conducted conditional on observed productivity shocks, as opposed to being conducted across possible realizations of these shocks.

Appendixes

A The backward equation system for the non-Ricardian, real-surplus case of subsection 2.1

The dynamic programming formulation of the investor's problem is:

$$J_1\left(\theta_{1,t-1},\cdot,t\right)=\sup_{c_t,\{\theta_{1,t,i}\}}u\left(c_t,t\right)+\mathbb{E}_tJ_1\left(\theta_{1,t},\cdot,t+1\right) \tag{31}$$

subject to the flow budget constraint written at time t only.

The Lagrangian for problem (31) is:

$$\begin{aligned}\mathcal{L}_1\left(\theta_{1,t-1,i},\cdot,t\right)=&\sup_{c_t,\theta_{1,t}}\inf_{\phi_{1,t}}u\left(c_t,t\right)\\&+\frac{1}{2}\sum_{j=u,d}J_1\left(\theta_{1,t},\cdot,t+1\right)\\&+\phi_{1,t}\times\left[\theta_{1,t-1}+P_t\times y_t-P_t\times c_t-\frac{\theta_{1,t}}{1+i_t}-s_t\times P_t\right]\end{aligned}$$

where $\phi_{1,t}$ is the Lagrange multiplier attached to the flow budget constraint (3). The first-order conditions are:

$$\begin{aligned}u'\left(c_t,t\right)&=\phi_{1,t}\times P_t\\\theta_{1,t-1}+P_t\times y_t-P_t\times c_t-\frac{\theta_{1,t}}{1+i_t}-s_t\times P_t&=0\\\frac{1}{2}\sum_{j=u,d}\frac{\partial J_{1,t+1,j}}{\partial\theta_{1,t,i}}\left(\theta_{1,t},\cdot,t+1\right)&\\=\phi_{1,t}\times\frac{1}{1+i_t}&\end{aligned} \tag{32}$$

In order to eliminate the value function from the first-order conditions, we differentiate the Lagrangian with respect to $\theta_{1,t-1,i}$:

$$\frac{\partial J_1}{\partial\theta_{1,t-1,i}}=\frac{\partial\mathcal{L}_1}{\partial\theta_{1,t-1,i}}=\phi_{1,t}$$

so that the first-order conditions can also be written:

$$\begin{aligned}u'\left(c_t,t\right)&=\phi_{1,t}\times P_t\\\theta_{1,t-1}+P_t\times y_t-P_t\times c_t-\frac{\theta_{1,t}}{1+i_t}-s_t\times P_t&=0\\\frac{1}{2}\sum_{j=u,d}\phi_{1,t+1,j}&=\phi_{1,t}\times\frac{1}{1+i_t}\end{aligned} \tag{33}$$

As has been noted by Dumas and Lyasoff (2012) in a different context, the system made of (33) and (6) above has a drawback. It must be solved simultaneously (or globally) for all nodes of all times. As written, it cannot be solved recursively in the backward way because the unknowns at time t include consumptions at time t, c_t, whereas the third subset of equations in (33) if rewritten as:

$$\frac{1}{2}\sum_{j=u,d}\frac{u'\left(c_{t+1,j},t\right)}{P_{t+1,j}}=\phi_{1,t}\times\frac{1}{1+i_t}$$

can be seen to be a restriction on consumptions at time $t+1$, which at time t would already be solved for.

In order to "synchronize" the solution algorithm of the equations and allow recursivity, we first shift all first-order conditions, except the third one, forward in time and, second, we no longer make explicit use of the investor's position $\theta_{1,t-1}$ held when entering time t, focusing instead on the financial wealth: $F_{1,t}\triangleq\frac{\theta_{1,t}}{1+i_t}$ held when exiting time $t+1$, which are carried backward. Regrouping equations in that way leads to the equation system of subsection 2.1.

B Analytical solution

Assume homogeneity with respect to the price level (with notation: $\theta_{1,t}\equiv\vartheta_{1,t}\times P_t;\theta_{2,t}\equiv\vartheta_{2,t}\times P_t;F_{1,t+1,u}\equiv f_{1,t+1,u}\times P_{t+1,u};F_{2,t}\equiv f_{2,t}\times P_t;F_{2,t+1,u}\equiv f_{2,t+1,u}\times P_{t+1,u}$) and assume: $f_{1,t+1,u}=-f_{2,t+1,u},f_{1,t+1,d}=-f_{2,t+1,d}$. The system of equations simplifies to

Flow budget constraints of private sector

$$-f_{2,t+1,u}\times P_{t+1,u}+s_{t+1,u}\times P_{t+1,u}=\vartheta_{1,t}\times P_t$$

$$-f_{2,t+1,d}\times P_{t+1,d}+s_{t+1,d}\times P_{t+1,d}=\vartheta_{1,t}\times P_t$$

Flow budget constraints of government

$$f_{2,t+1,u}\times P_{t+1,u}=\vartheta_{2,t}\times P_t+s_{t+1,u}\times P_{t+1,u}\tag{34}$$

$$f_{2,t+1,d}\times P_{t+1,d}=\vartheta_{2,t}\times P_t+s_{t+1,d}\times P_{t+1,d}\tag{35}$$

Portfolio-choice, or Euler, or Fisher condition

$$\frac{1}{1+i_t}\frac{1}{P_t}=\rho\frac{\frac{1}{2}\left(y_{t+1,u}\right)^{\gamma-1}\frac{1}{P_{t+1,u}}+\frac{1}{2}\left(y_{t+1,d}\right)^{\gamma-1}\frac{1}{P_{t+1,d}}}{\left(y_t\right)^{\gamma-1}}$$

Taylor rule

$$1+i_t=(1+\bar{\imath})\times\left(\frac{\frac{\frac{1}{2}P_{t+1,u}+\frac{1}{2}P_{t+1,d}}{P_t}}{1+\bar{\pi}}\right)^{\phi}$$

Market clearing

$$\vartheta_{1,t}+\vartheta_{2,t}=0$$

Government debt: Government debt is nominal and can be priced by means of the Fisher equation, which means that the financial wealth of the government can be obtained by the following backward induction:

$$\frac{F_{2,t}}{P_t} \triangleq \frac{1}{P_t}\frac{\theta_{2,t}}{1+i_t} \tag{36}$$
$$= \rho\frac{\frac{1}{2}\left(c_{t+1,u}\right)^{\gamma-1}\left(-s_{t+1,u}+\frac{F_{2,t+1,u}}{P_{t+1,u}}\right)+\frac{1}{2}\left(c_{t+1,d}\right)^{\gamma-1}\left(-s_{t+1,d}+\frac{F_{2,t+1,d}}{P_{t+1,d}}\right)}{\left(y_t\right)^{\gamma-1}}$$

From (36), the backward dynamics of real government financial liabilities are provided by:

$$\begin{aligned} f_{2,t} &= \rho\frac{\frac{1}{2}\left(y_{t+1,u}\right)^{\gamma-1}\left(-s_{t+1,u}+f_{2,t+1,u}\right)+\frac{1}{2}\left(y_{t+1,d}\right)^{\gamma-1}\left(-s_{t+1,d}+f_{2,t+1,d}\right)}{\left(y_t\right)^{\gamma-1}}; \\ f_{2,T} &= 0 \end{aligned} \tag{37}$$

The current real discounted value $f_{2,t}$ of government debt depends only on future income and future surpluses. It does not depend on interest-rate policy.[53]

But the real *face value* $\vartheta_{2,t}$, which is the government's equilibrium portfolio choice or issuance decision, depends on the nominal rate of interest, which we now determine.

Inflation: Solving for inflation from the government flow budget constraints (34), (35):

$$\begin{aligned} \frac{P_{t+1,u}}{P_t} &= \frac{\vartheta_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}} \\ \frac{P_{t+1,d}}{P_t} &= \frac{\vartheta_{2,t}}{-s_{t+1,d}+f_{2,t+1,d}} \end{aligned}$$

so that the realized rates of inflation are:

$$\begin{aligned} \frac{P_{t+1,u}}{P_t} &= \frac{f_{2,t}\times(1+i_t)}{-s_{t+1,u}+f_{2,t+1,u}} \\ \frac{P_{t+1,d}}{P_t} &= \frac{f_{2,t}\times(1+i_t)}{-s_{t+1,d}+f_{2,t+1,d}} \end{aligned} \tag{38}$$

These relate the two levels of future inflation $(P_{t+1,u}/P_t, P_{t+1,d}/P_t)$ to calendar time t, to the two levels of future real government debt $(-s_{t+1,u}+f_{2,t+1,u}, -s_{t+1,d}+f_{2,t+1,d})$ and to the current level of real government debt $f_{2,t}$. We call $f_{2,t}/\left(-s_{t+1}+f_{2,t+1}\right)$ the "ex post *inverse* real gross rates of return on government debt". It is also the ex post inverse real gross rates of return on *any* nominally riskless debt.

Proposition 7 *The ex post levels of inflation in the two states of nature are separately*

[53] This result is, of course, a statement of the Fiscal Theory of the Price Level. In an overlapping-generations model, the real value of government debt may be larger than the present value of future government surpluses; for an extension of the Fiscal theory to overlapping generations, see Farmer and Zabczyk (2019).

- *increasing functions of the ex post* inverse *real gross rates of return on nominally riskless debt*
- *increasing functions of the (ex ante) nominal gross rate of interest.*

To illustrate, assuming that debt returns more in a u state than in a d state, which is

$$\frac{-s_{t+1,u}+f_{2,t+1,u}}{f_{2,t}} > \frac{-s_{t+1,d}+f_{2,t+1,d}}{f_{2,t}}$$

then

$$\frac{P_{t+1,u}}{P_t} < \frac{P_{t+1,d}}{P_t}$$

Inflation is lower in the u state than in the d state.

Relations between the rate of inflation and the nominal rate of interest are commonly known as the "aggregate-demand" schedules.[54] We now merge them with the policy rule.

Interest rate: Substituting into the Taylor rule:

$$1+i_t = (1+\bar{\imath}) \times \left(\frac{\frac{1}{2}\frac{\vartheta_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}} + \frac{1}{2}\frac{\vartheta_{2,t}}{-s_{t+1,d}+f_{2,t+1,d}}}{1+\bar{\pi}} \right)^{\phi}$$

so that (using $\vartheta_{2,t}/\left(1+i_t\right) = f_{2,t}$):

$$1+i_t = (1+\bar{\imath})^{\frac{1}{1-\phi}} \times \left(\frac{\frac{1}{2}\frac{f_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}} + \frac{1}{2}\frac{f_{2,t}}{-s_{t+1,d}+f_{2,t+1,d}}}{1+\bar{\pi}} \right)^{\frac{\phi}{1-\phi}} \tag{39}$$

The nominal rate of interest depends on future fiscal surpluses and output, as well as on the parameters of the Taylor rule. It is not generally equal to $\bar{\imath}$.

Proposition 8 *The (ex ante) nominal gross rate of interest is an increasing (decreasing) function of the expected* inverse *real gross rate of return on nominally riskless debt if $\phi < 1$ ($\phi > 1$).*

In total, a higher ex post inverse real gross rates of return on government debt has a double effect, one direct and increasing (Proposition 7), and one indirect because it affects the *expected* value of the inverse real gross rates of return. The sign of the second effect depends on whether ϕ is smaller or greater then 1. When $\phi < 1$, the direction of the effect is clear: a higher expected real rate of return on government debt implies a lower rate of inflation in both future states.

Finally, since (37) provides a unique value for the time-0 present value of the government debt, and since $\theta_{2,-1}$ is a given (negative) amount of nominal claim outstanding and s_0 a given time-0 surplus, the solution of the initial condition (8) for P_0 is unique. Cochrane (2011, page 579) says that we have determinacy in this case and, indeed, we do, irrespective of the value of the Taylor parameter so long as $\phi \neq 1$.

[54] The next two sections derive the "aggregate-supply" schedule. Here aggregate supply is exogenous and completely inelastic.

C Proof of Proposition 5

Combining (38) with (39), we get

$$\frac{P_{t+1,u}}{P_t} = \frac{f_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}} \times (1+\bar{\imath})^{\frac{1}{1-\phi}} \times \left(\frac{\frac{1}{2}\frac{f_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}} + \frac{1}{2}\frac{f_{2,t}}{-s_{t+1,d}+f_{2,t+1,d}}}{1+\bar{\pi}} \right)^{\frac{\phi}{1-\phi}}$$

and, therefore

$$\frac{\partial \frac{P_{t+1,u}}{P_t}}{\partial \frac{f_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}}} = (1+\bar{\imath})^{\frac{1}{1-\phi}} \times \left(\frac{\frac{1}{2}\frac{f_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}} + \frac{1}{2}\frac{f_{2,t}}{-s_{t+1,d}+f_{2,t+1,d}}}{1+\bar{\pi}} \right)^{\frac{\phi}{1-\phi}} \left[1 + \frac{\frac{1}{-s_{t+1,u}+f_{2,t+1,u}}}{\frac{1}{-s_{t+1,u}+f_{2,t+1,u}} + \frac{1}{-s_{t+1,d}+f_{2,t+1,d}}} \times \frac{\phi}{1-\phi}\right]$$

In view of (10),

$$\frac{\partial \frac{P_{t+1,u}}{P_t}}{\partial \frac{f_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}}} = (1+\bar{\imath})^{\frac{1}{1-\phi}} \times \left(\frac{\frac{1}{2}\frac{f_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}} + \frac{1}{2}\frac{f_{2,t}}{-s_{t+1,d}+f_{2,t+1,d}}}{1+\bar{\pi}} \right)^{\frac{\phi}{1-\phi}} \left[1 + \frac{\frac{1}{1+u}}{\frac{1}{1+u} + \frac{1}{1+d}} \times \frac{\phi}{1-\phi}\right]$$

And, from (37), (11) and (12)

$$\frac{\partial \frac{f_{2,t}}{-s_{t+1,u}+f_{2,t+1,u}}}{\partial y_{t+1,u}} = \frac{1}{y_{t+1,u}} \times \left[-\frac{\rho \times \frac{1}{2}(1+d)^{\gamma}}{1+u} + \rho(\gamma-1)\frac{1}{2}(1+u)^{\gamma-1} \right] < 0$$

This shows that $\partial \frac{P_{t+1,u}}{P_t} / \partial y_{t+1,u}$ is positive if and only if

$$1 + \frac{\frac{1}{1+u}}{\frac{1}{1+u} + \frac{1}{1+d}} \times \frac{\phi}{1-\phi} > 0$$

The proposition follows.

D Proof of equation (22)

The first-order condition is:

$$\sum_{i=0}^{T-t} \mathbb{E}_t \left[(\rho\omega)^i \frac{(c_{t+i})^{\gamma-1}}{(c_t)^{\gamma-1}} \left((1-\sigma)\frac{P_{\iota,t}^{-\sigma}}{P_{t+i}^{1-\sigma}} + \sigma\varphi_{t+i}\frac{P_{\iota,t}^{-\sigma-1}}{P_{t+i}^{-\sigma}} \right) y_{t+i} \right] = 0$$

Divide by $P_{\iota,t}^{-\sigma}$:

$$\sum_{i=0}^{T-t} \mathbb{E}_t \left[(\rho\omega)^i \frac{(c_{t+i})^{\gamma-1}}{(c_t)^{\gamma-1}} \left((1-\sigma) \frac{1}{P_{t+i}^{1-\sigma}} + \sigma \varphi_{t+i} \frac{P_{\iota,t}^{-1}}{P_{t+i}^{-\sigma}} \right) y_{t+i} \right] = 0$$

Split:

$$(1-\sigma) \sum_{i=0}^{T-t} \mathbb{E}_t \left[(\rho\omega)^i \frac{(c_{t+i})^{\gamma-1}}{(c_t)^{\gamma-1}} \frac{1}{P_{t+i}^{1-\sigma}} y_{t+i} \right] + \sigma P_{\iota,t}^{-1} \sum_{i=0}^{T-t} \mathbb{E}_t \left[(\rho\omega)^i \frac{(c_{t+i})^{\gamma-1}}{(c_t)^{\gamma-1}} \varphi_{t+i} \frac{1}{P_{t+i}^{-\sigma}} y_{t+i} \right] = 0$$

Multiply by $P_t^{1-\sigma}$:

$$(1-\sigma) \sum_{i=0}^{T-t} \mathbb{E}_t \left[(\rho\omega)^i \frac{(c_{t+i})^{\gamma-1}}{(c_t)^{\gamma-1}} \frac{P_t^{1-\sigma}}{P_{t+i}^{1-\sigma}} y_{t+i} \right] + \sigma P_t P_{\iota,t}^{-1} \sum_{i=0}^{T-t} \mathbb{E}_t \left[(\rho\omega)^i \frac{(c_{t+i})^{\gamma-1}}{(c_t)^{\gamma-1}} \varphi_{t+i} \frac{P_t^{-\sigma}}{P_{t+i}^{-\sigma}} y_{t+i} \right] = 0$$

Solving for $P_t P_{\iota,t}^{-1}$ gives (22).

E Walras' law in the sticky-price system

Aggregating the budget constraints in each state ($j = u, d$):

$$\begin{gathered} P_{t+1,j} \times c_{t+1,j} + F_{1,t+1,j} + F_{2,t+1,j} + s_{t+1,j} \times P_{t+1,j} \\ = \theta_{1,t} + \theta_{2,t} + s_{t+1,j} \times P_{t+1,j} + P_{t+1,j} \times \delta_{t+1,j} + W_{t+1,j} \times l_{t+1,j};\ F_{1,T,j} = 0 \\ P_{t+1,j} \times c_{t+1,j} = P_{t+1,j} \times \delta_{t+1,j} + W_{t+1,j} \times l_{t+1,j} \end{gathered}$$

$$P_{t+1,j} \times c_{t+1,j} = P_{t+1,j} \times \left[\omega \times \left(\frac{P_t}{P_{t+1,j}} - \varphi_{t+1,j} \right) \left(\frac{P_t}{P_{t+1,j}} \right)^{-\sigma} + (1-\omega) \times \left(\frac{P_{t+1}^*}{P_{t+1,j}} - \varphi_{t+1,j} \right) \left(\frac{P_{t+1,j}^*}{P_{t+1,j}} \right)^{-\sigma} \right] \times y_{t+1,j} + W_{t+1,j} \times l_{t+1,j}$$

$$P_{t+1,j} \times c_{t+1,j} = P_{t+1,j} \times \left[\omega \times \left(\frac{P_t}{P_{t+1,j}} - \frac{W_{t+1,j}}{z_{t+1,j} \times P_{t+1,j}} \right) \left(\frac{P_t}{P_{t+1,j}} \right)^{-\sigma} + (1-\omega) \times \left(\frac{P_{t+1}^*}{P_{t+1,j}} - \frac{W_{t+1,j}}{z_{t+1,j} \times P_{t+1,j}} \right) \left(\frac{P_{t+1,j}^*}{P_{t+1,j}} \right)^{-\sigma} \right] \times y_{t+1,j} + W_{t+1,j} \times l_{t+1,j}$$

$$P_{t+1,j} \times c_{t+1,j} = P_{t+1,j} \times \left[\omega \times \frac{P_t}{P_{t+1,j}} \left(\frac{P_t}{P_{t+1,j}} \right)^{-\sigma} + (1-\omega) \times \frac{P_{t+1}^*}{P_{t+1,j}} \left(\frac{P_{t+1,j}^*}{P_{t+1,j}} \right)^{-\sigma} \right] \times y_{t+1,j}$$
$$+ \left(-\frac{W_{t+1,j}}{z_{t+1,j}} \right) \times \left[\omega \times \left(\frac{P_t}{P_{t+1,j}} \right)^{-\sigma} + (1-\omega) \times \left(\frac{P_{t+1,j}^*}{P_{t+1,j}} \right)^{-\sigma} \right] \times y_{t+1,j} + W_{t+1,j} \times l_{t+1,j}$$

Cancellation produced by (24) gives:

$$P_{t+1,j} \times c_{t+1,j} = P_{t+1,j} \times \left[\omega \times \frac{P_t}{P_{t+1,j}} \left(\frac{P_t}{P_{t+1,j}} \right)^{-\sigma} + (1-\omega) \times \frac{P^*_{t+1}}{P_{t+1,j}} \left(\frac{P^*_{t+1,j}}{P_{t+1,j}} \right)^{-\sigma} \right] \times y_{t+1,j}$$

$$c_{t+1,j} = \left[\omega \times \left(\frac{P_t}{P_{t+1,j}} \right)^{1-\sigma} + (1-\omega) \times \left(\frac{P^*_{t+1,j}}{P_{t+1,j}} \right)^{1-\sigma} \right] \times y_{t+1,j}$$

$$c_{t+1,j} = y_{t+1,j}$$

Therefore, all accounts are straight: clearing of the financial market and of the labor market do imply clearing of the goods market.

F Backward equation system for the Baumol-Tobin case

In the entire paper, W_t is the nominal wage rate. In this appendix only, the symbol W stands for entering (or pre-trade) wealth.

$$\mathcal{L}_1(W_{1,t}, \cdot, t) = \sup_{c_t, \theta_{1,t}} \inf_{\phi_{1,t}} u_1(c_t, t) + \frac{1}{2} \sum_{j=u,d} J_1(\theta_{1,t}, M_{1,t}, \cdot, t+1)$$
$$+ \phi_{1,t} \times \left[W_{1,t} - \frac{\theta_{1,t}}{1+i_t} - S_t + P_t \times y_t \times \left(1 - \frac{\nu \times P_t}{2 \times M_{1,t}} \right) - P_t \times c_t - M_{1,t} + \zeta_{1,t} \right]$$

where: $W_{1,t} \triangleq M_{1,t-1} + \theta_{1,t-1}$. First-order condition with respect to $\theta_{1,t}$:

$$\frac{1}{2} \sum_{j=u,d} \frac{\partial}{\partial W_{1,t}} J_1(\theta_{1,t}, M_{1,t}, \cdot, t+1) - \frac{\phi_{1,t}}{1+i_t} = 0$$

First-order condition with respect to $M_{1,t}$:

$$\frac{1}{2} \sum_{j=u,d} \frac{\partial}{\partial W_{1,t}} J_1(\theta_{1,t}, M_{1,t}, \cdot, t+1) + \phi_{1,t} \times \left[P_t \times y_t \times \frac{\nu \times P_t}{2 \times (M_{1,t})^2} - 1 \right] = 0$$

Envelope condition:

$$\frac{\partial}{\partial W_{1,t-1}} J_1(\theta_{1,t-1}, M_{1,t-1}, \cdot, t) = \phi_{1,t}$$

The Euler conditions are:

$$\frac{1}{2} \sum_{j=u,d} \phi_{1,t+1,j} - \frac{\phi_{1,t}}{1+i_t} = 0; t = 0, ..., T-1$$

$$\frac{1}{2} \sum_{j=u,d} \phi_{1,t+1,j} = \phi_{1,t} \times \left[1 - P_t \times y_t \times \frac{\nu \times P_t}{2 \times (M_{1,t})^2} \right]; t = 0, ..., T$$

The latter is simply:

$$
\begin{aligned}
\frac{1}{1+i_t} &= 1-P_t\times y_t\times\frac{\nu\times P_t}{2\times\left(M_{1,t}\right)^2} \\
P_t\times y_t\times\frac{\nu\times P_t}{2\times\left(M_{1,t}\right)^2} &= 1-\frac{1}{1+i_t} \\
P_t\times y_t\times\frac{\nu\times P_t}{1-\frac{1}{1+i_t}} &= 2\times\left(M_{1,t}\right)^2 \\
M_{1,t} &= P_t\times\sqrt{\frac{1}{2}y_t\times\frac{\nu}{1-\frac{1}{1+i_t}}}
\end{aligned}
\tag{40}
$$

except at time $t=T$ where:

$$
\begin{aligned}
1-P_T\times y_T\times\frac{\nu\times P_T}{2\times\left(M_{1,T}\right)^2} &= 0 \\
M_{1,T} &= P_T\times\sqrt{\frac{1}{2}y_T\times\nu}
\end{aligned}
$$

Summing up, the set of equations to be solved at each node of the tree is:

$$
\begin{gathered}
\text{Flow budget constraints of private sector} \\
P_{t+1,j}\times c_{t+1,j}+F_{1,t+1,j}+M_{1,t+1,j}+S_{t+1,j} \\
=\theta_{1,t}+M_{1,t}+P_{t+1,j}\times y_{t+1,j}+\zeta_{1,t+1,j}; \\
F_{1,T,j}=0; j=u,d \\
\text{Flow budget constraints of government cum central bank} \\
F_{2,t+1,j}\ +M_{2,t+1,j}=\theta_{2,t}+M_{2,t}+S_{t+1,j}-\zeta_{1,t+1,j}; F_{2,T,j}=0; j=u,d \\
\text{Portfolio-choice or Euler conditions} \\
\frac{1}{1+i_t}\frac{1}{P_t}=\rho\frac{\frac{1}{2}\left(c_{t+1,u}\right)^{\gamma-1}\frac{1}{P_{t+1,u}}+\frac{1}{2}\left(c_{t+1,d}\right)^{\gamma-1}\frac{1}{P_{t+1,d}}}{\left(c_t\right)^{\gamma-1}}; t=0,...,T-1 \\
M_{1,t}=P_t\times\sqrt{\frac{1}{2}y_t\times\frac{\nu}{1-\frac{1}{1+i_t}}}; t=0,...,T-1; M_{1,T}=P_T\times\sqrt{\frac{1}{2}y_T\times\nu} \\
\text{Market clearing} \\
\theta_{1,t}+\theta_{2,t}=0; M_{1,t+1,u}+M_{2,t+1,u}=0; M_{1,t+1,d}+M_{2,t+1,d}=0; M_{1,t}+M_{2,t}=0
\end{gathered}
\tag{41}
$$

G Backward equation system for the Baumol-Tobin case under alternative specification

$$\mathcal{L}_1\left(\theta_{1,t-1,i}, M_{1,t-1}, \cdot, t\right) = \sup_{c_t,\theta_{1,t}} \inf_{\phi_{1,t}} u_1\left(c_t, t\right)$$
$$+\frac{1}{2}\sum_{j=u,d} J_1\left(\theta_{1,t}, M_{1,t}, \cdot, t+1\right)$$
$$+\phi_{1,t} \times \left\{ M_{1,t-1} + \left[P_t \times y_t + \theta_{1,t-1} - \frac{\theta_{1,t}}{1+i_t} - S_t\right] \times \left(1 - \frac{\nu \times P_t}{2 \times M_{1,t}}\right) - P_t \times c_t - M_{1,t} + \zeta_{1,t} \right\}$$

First-order condition with respect to $\theta_{1,t}$:

$$\frac{1}{2}\sum_{j=u,d} \frac{\partial}{\partial \theta_{1,t}} J_1\left(\theta_{1,t}, M_{1,t}, \cdot, t+1\right) - \frac{\phi_{1,t}}{1+i_t} \times \left(1 - \frac{\nu \times P_t}{2 \times M_{1,t}}\right) = 0$$

First-order condition with respect to $M_{1,t}$:

$$\frac{1}{2}\sum_{j=u,d} \frac{\partial}{\partial M_{1,t}} J_1\left(\theta_{1,t}, M_{1,t}, \cdot, t+1\right) + \phi_{1,t} \times \left\{ \left[P_t \times y_t + \theta_{1,t-1} - \frac{\theta_{1,t}}{1+i_t} - S_t\right] \times \frac{\nu \times P_t}{2 \times \left(M_{1,t}\right)^2} - 1 \right\} = 0$$

Envelope conditions:

$$\begin{aligned} \frac{\partial}{\partial \theta_{1,t-1}} J_1\left(\theta_{1,t-1}, M_{1,t-1}, \cdot, t\right) &= \phi_{1,t} \times \left(1 - \frac{\nu \times P_t}{2 \times M_{1,t}}\right) \\ \frac{\partial}{\partial M_{1,t-1}} J_1\left(\theta_{1,t-1}, M_{1,t-1}, \cdot, t\right) &= \phi_{1,t} \end{aligned}$$

so that the Euler conditions are:

$$\begin{aligned} \frac{1}{2}\sum_{j=u,d} \phi_{1,t+1,j} \times \left(1 - \frac{\nu \times P_{t+1,j}}{2 \times M_{1,t+1,j}}\right) - \frac{\phi_{1,t}}{1+i_t} \times \left(1 - \frac{\nu \times P_t}{2 \times M_{1,t}}\right) &= 0 \\ \frac{1}{2}\sum_{j=u,d} \phi_{1,t+1,j} + \phi_{1,t} \times \left\{ \left[P_t \times y_t + \theta_{1,t-1} - \frac{\theta_{1,t}}{1+i_t} - S_t\right] \times \frac{\nu \times P_t}{2 \times \left(M_{1,t}\right)^2} - 1 \right\} &= 0 \end{aligned}$$

The nodal system follows.

References:

Allais, M., 1947, *Economie et Intérêt*, Paris: Imprimerie Nationale.

Allen, F., E. Carletti and D. Gale, 2012, "Money, Financial Stability and Efficiency," working paper, Wharton School.

Alves, N., 2004, "A Flexible View on Prices," working paper, Banco de Portugal.

Aiyagari, S. R. and M. Gertler, 1985, "The Backing of Government Bonds and Monetarism," *Journal of Monetary Economics*, 16, 19-44.

Altig D., L. J. Christiano, M. Eichenbaum and J. Lindé, 2011, "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14, 225-247.

Aruoba, S. B. and F. Schorfheide, 2013, "Macroeconomic Dynamics near the ZLB: a Tale of Two Equilibria," NBER Working Paper 19248.

Asness, C., 2003, "Fight the Fed Model," *The Journal of Portfolio Management*, Fall, 11-24.

Bai, J. H., 2005, "Stationary Monetary Equilibrium in a Baumol-Tobin Exchange Economy: Theory and Computation," working paper, Georgetown University.

Barsky, R. and E. Sims, 2011, "News shocks and business cycles," *Journal of Monetary Economics*, 58, 273–289.

Basak, S. and H. Yan, 2010, "Equilibrium Asset Prices and Investor Behavior in the Presence of Money Illusion," *Review of Economic Studies*, 77, 914-936.

Baumol, W. J., 1952, "The Transactions Demand for Cash: an Inventory-theoretic Approach," *Quarterly Journal of Economics, 66, 545-556.*

Baumol, W. J. and J. Tobin, 1989, "The Optimal Cash Balance Proposition: Maurice Allais´ Priority," *Journal of Economic Literature*, September.

Bekaert, G. and E. Engstrom, 2010, "Inflation and the Stock Market: Understanding the 'Fed Model'," *Journal of Monetary Economics*, 57, 278-294.

Benhabib, J., S. Schmitt-Grohe, and M. Uribe, 2001a, "The Perils of Taylor Rules," *Journal of Economic Theory*, 96, 40-69.

Benhabib, J, S. Schmitt-Grohe, and M. Uribe, 2001b, "MonetaryP olicy and Multiple Equilibria," *American Economic Review*, 91, 167-186.

Benhabib, J, S. Schmitt-Grohe, and M. Uribe, 2002, "Avoiding Liquidity Traps," *Journal of Political Economy*, 110, 535-563.

Bernanke, B. and M. Gertler, 1999, "Monetary Policy and Asset Price Volatility," Federal Reserve Bank of Kansas City Economic Review, Fourth Quarter, 17-51.

Bernanke, B. and M. Gertler, 2001, "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review* Papers and Proceedings, 91, 253-257.

Bernanke, B. and K. N. Kuttner, 2004. "What Explains the Stock Market's Reaction to Federal Reserve Policy?," NBER Working Papers 10402, National Bureau of Economic Research, Inc.

Boudoukh, J. and M. Richardson, 1993, "Stock Returns and Inflation: a Long-Horizon Perspective," *The American Economic Review*, 83, 1346-1355.

Bodie, Z., 1976, "Common Stocks as a Hedge Against Inflation," *The Journal of Finance*, 31, 459-70.

Braun, R. A., M. Körber and Y. Waki, 2013, "Small and Orthodox Fiscal Multipliers at the Zero Lower Bound," Working Paper 2013-13, Federal Reserve Bank of Atlanta.

Calvo, G. A., 1983, "Staggered Prices in a Utility-Maximizing Model," *Journal of Monetary Economics*, 12, 983-998.

Campbell, J. Y. and T. Vuolteenaho, 2004, "Inflation Illusion and Stock Prices," *The American Economic Review*, 94, 19-23.

Canzoneri, M., R. Cumby and B. Diba, 2011, "The Interaction between Monetary and Fiscal Policy," in *Handbook of Monetary Economics*, 3B, B. Friedman and M. Woodford eds., 935-999.

Castelnuovo, E. and S. Nístico, 2010, "Stock Market Conditions and Monetary Policy in a DSGE Model for the U.S.," *Journal of Economic Dynamics and Control*, 34, 1700-1731.

Challe, E., 2005, "New Keynesian Macroeconomics," course notes at Cambridge University, https://sites.google.com/site/edouardchalle/lecture-notes

Challe, E. and C. Giannitsarou, 2014, "Stock Prices and Monetary Policy Shocks: a General Equilibrium Approach," *Journal of Economic Dynamics and Control*, 40, 46–66.

Chari, V., Christiano, L., Kehoe, P., 1991, "Optimal fiscal and monetary policy: Some recent results," *Journal of Money, Credit and Banking*, 23, 519–539.

Christiano, L. J. and M. Eichenbaum, 2012, "Notes on Linear Approximations, Equilibrium Multiplicity and E-learnability in the Analysis of the Zero Lower Bound," working paper Northwestern University.

Clarida, R., J. Gali and M. Gertler, 2000, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *The Quarterly Journal of Economics*, 115, No. 1, 147-180.

Cochrane, J. H., 2005a, "Money as stock," *Journal of Monetary Economics*, 52, 501–528.

Cochrane, J. H., 2005b, , "Financial Markets and the Real Economy," *Foundations and Trends in Finance*, 1, 1-101.

Cochrane, J. H., 2011, "Determinacy and Identification with Taylor Rules," *Journal of Political Economy*, 119, 565-615.

Danthine, J. P. and J. Donaldson, 1986, "Inflation and Asset Prices in an Exchange Economy," *Econometrica*, 54, 585-605.

David, A. and P. Veronesi, 2013, "What Ties Returns Volatilities to Price Valuations and Fundamentals?" *Journal of Political Economy*, 121, 682 – 746.

De Paoli, B., A. Scott and O. Weeken, 2009, "Asset pricing implications of a New Keynesian model," Bank of England Working Paper No 326.

Dumas, B. and A. Lyasoff, 2012, "Incomplete-Market Equilibria Solved Recursively on a Binomial Tree or a Lattice," *The Journal of Finance*, LXVII, 1887-1931.

Fama, E., 1981, "Stock Returns, Real Activity, Inflation and Money," *American Economic Review*, 71, 545–65.

Fama, E. F., and G. W. Schwert, 1977, "Asset Returns and Inflation," *Journal of Financial Economics*, 5: 115–146.

Farmer, R. E. A. and P. Zabczyk, 2019, "The Fiscal Theory of the Price Level in Overlapping Generations Models," working paper, University of Warwick.

Fernández-Villaverde, J., G. Gordon and P. A. Guerrón-Quintana, 2012, "Nonlinear Adventures at the Zero Lower Bound," NBER Working Paper 18058.

Galí, J., 2008, *Monetary Policy, Inflation, and the Business Cycle: an Introduction to the New Keynesian Framework*, Princeton University Press.

Gavin, W., B. Keen, A. Richter, and N. Throckmorton, 2013, "Global dynamics at the zero lower bound," Working Paper.

Geske, R., and Roll, R., 1983, "The Fiscal and Monetary Linkage Between Stock Returns and Inflation," *The Journal of Finance*, 38, 1–33.

Gorodnichenko, Y. and M. Weber, 2016, "Are Sticky Prices Costly? Evidence From The Stock Market," *American Economic Review*, 106, 165-199.

Goto, S., and R. Valkanov, 2000, "The Fed's Effect on Excess Returns and Inflation is Much Bigger than you Think," mimeo, University of California Los Angeles.

Greenwood, R., S. G. Hanson, J. S. Rudolph, and L. Summers, 2014, "Government Debt Management at the Zero Lower Bound," Hutchins Center Working Paper.

Gultekin, N. B., 1983, "Stock market returns and inflation: evidence from other countries," *The Journal of Finance*, 38, 49–65.

Hagmann, M. and C. Lenz, 2005. "Real Asset Returns and Components of Inflation: A Structural VAR Analysis," Working papers, Faculty of Business and Economics - University of Basel.

Heaton, J. and D. Lucas, 1996, "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy*, 104, 443-487.

Heathcote, J., 1998, "Interest Rates in a General Equilibrium Baumol-Tobin Model," working paper, University of Pennsylvania.

Henderson D. W. and W. McKibbin, 1993, "A Comparison of Some Basic Monetary Policy Regimes for Open Economies: Implications of Different Degrees of Instrument Adjustment and Wage Persistence," *Carnegie-Rochester Conference Series on Public Policy*, 39, 221-318.

Jaffe, J. and G. Mandelker, G., 1976, "The 'Fisher effect' for risky assets: an empirical investigation," *The Journal of Finance*, 31, 447–458.

Katz, M., H. Lustig and L. Nielsen, 2017, "Are Stocks Real Assets? Sticky Discount Rates in Stock Markets," *Review of Financial Studies*, 30, 539-587.

Kollmann, R., 2019, "Stationary Rational Bubbles in Non-Linear Business Cycle Models," working paper, Université Libre de Bruxelles.

Krippner, L., 2012, "Modifying Gaussian term structure models when interest rates are near the zero lower bound," Working Paper.

Kurmann, A. and E. Sims, 2017, "Revisions in Utilization-Adjjusted TFP and Robust Identificaiton of News Shocks," National Bureau of Economic Research, working paper #23142.

Leo, C. L., 2006, "A General Equilibrium Model of Baumol-Tobin Money Demand," working paper, The University of British Columbia.

Leeper, E. M., 1991, "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 27, 129-147.

Leeper, E. M. and C. Leith, 2016, "Understanding Inflation as a Joint Monetary-Fiscal Phenomenon," in *Handbook of Macroeconomics* vol. 2, J. B. Taylor, H. Uhlig eds, North Hoalland.

Li, E. X. N. and F. Palomino, 2009, "Monetary policy risk and the cross-section of stock returns," Mimeograph.

Lintner, J., 1975, "Inflation and Security Returns," *The Journal of Finance*, 1975, 30, 259-80.

Lucas, R. E. and N. L. Stokey, 1987, "Money and Interest Rates in a Cash-in-Advance Economy," *Econometrica*, 55, 491-513.

Magill, M. and M. Quinzii, 2009, "Anchoring Expectations of Inflation," working paper, University of Southern California and U.C. Davis.

Magill, M. and M. Quinzii, 2012, "Interest Policy and Expectations of Inflation," working paper, University of Southern California and U.C. Davis.

Maio, P., 2013, "The 'Fed Model' and the Predictability of Stock Returns," *Review of Finance*, 17, 1489–1533.

McCallum, B. T., 1981, "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations," *Journal of Monetary Economics*, 8, 319–29.

McCallum, B., 2000, "Theoretical analysis regarding a zero lower bound on nominal interest rates," *Journal of Money, Credit and Banking*, 32, 870–904.

Mertens, K. and M. O. Ravn, 2012, "Fiscal Policy in an Expectations Driven Liquidity Trap," CEPR Discussion Paper.

Michel, P., 1982, "On the Transversality Condition in Infinite Horizon Optimal Problems," *Econometrica*, 50, 975-985

Milani, F., 2008, "Learning About the Interdependence Between the Macroeconomy and the Stock Market," Mimeograph.

Miranda-Agrippino, S., Hacio¨glu Hoke S. and K. Bluwstein, 2019, "When Creativity Strikes: News Shocks and Business Cycle Fluctuations," working paper, Northwestern University.

Nakajima, T., and H. M. Polemarchakis (2005), "Money and Prices under Uncertainty," *Review of Economic Studies*, 72, 223-246.

Nelson, C. R., 1976, "Inflation and rates of return on common stocks," *The Journal of Finance*, 31, 471–483.

Niepelt, D., 2004, "The Fiscal Myth of the Price Level," *Quarterly Journal of Economics*, 119, 277-300.

Nístico, S., 2007, "Monetary Policy and Stock-Price Dynamics in a DSGE Framework," Mimeograph.

Persson, M., Persson, T., Svensson, L. E. O., 1987, "Time consistency of fiscal and monetary policy," *Econometrica,* 55, 1419–1431.

Priebsch, M., 2013, "Computing Arbitrage-Free Yields in Multi-Factor Gaussian Shadow-Rate Term Structure Models," Working Paper.

Romer, D., 1986, “A Simple General Equilibrium Version of the Baumol-Tobin Model,” *Quarterly Journal of Economics*, 101, 663-686.

Rudebusch, G. D. and E. T. Swanson, 2008, “The Bond Premium in a DSGE model with long-run real and nominal risks,” working paper, Federal Reserve Bank of San Francisco.

Sargent, T. J. and N. Wallace (1975), “Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule,” *Journal of Political Economy*, 83, 241-254.

Schwartz, I., 2006, “Monetary Equilibria in a Baumol-Tobin Economy,” Max Planck Institute of Research on Collective Goods.

Silva, A. C., 2011, “Individual and Aggregate Money Demands,” working paper, Nova School of Business and Economics.

Smets, F. and R. Wouters, 2007, “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 97, 586-606.

Smith, G. W., 1986, “A Dynamic Baumol-Tobin Model of Money Demand,” *Review of Economic Studies*, 53, 3, 465-69.

Svensson, L. E. O., 1989, “Trade in nominal assets: Monetary policy, and price level and exchange rate risk,” *Journal of International Economics*, 26, 1–28.

Svensson, L. E. O. and M. Woodford, 2005, “Implementing Optimal Policy through Inflation-Forecast Targeting,” in Bernanke, Ben S., and Michael Woodford, eds. (2005), *The Inflation-Targeting Debate*, University of Chicago Press, Chicago, 19-83.

Swanson, E. and J. Williams, 2014, “Measuring the effect of the zero lower bound on medium and longer-term interest rates,” *American Economic Review*, 104, 3154-3185.

Tin, J. S., 2000, “Transactions demand for money: The micro evidence,” *Quarterly Journal of Business and Economics*, Summer.

Taylor, J. B., 1993, “Discretion versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.

Tobin, J., 1956, “The Interest Elasticity of the Transactions Demand for Cash,” *Review of Economics and Statistics,* 88, 241-247.

Walsh, C. E., 2010, *Monetary Theory and Policy*, third edition, MIT Press.

Wei, C., 2009, “A Quartet of Asset Pricing Models in Nominal and Real Economies,” *Journal of Economic Dynamics and Control*, 33, 154-165.

Woodford, M., 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton, NJ.

Wright, J., 2012, “What does monetary policy do to long-term interest rates at the zero lower bound?,” *Economic Journal*, 122, F447–F466.