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**ABSTRACT**

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# TFPR: Dispersion and Cyclicality*

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December 2, 2020

**Abstract**

This paper studies the determinants of TFPR, a revenue based measure of total factor productivity. Recent business cycle models are built upon the countercyclical dispersion of TFPR. But, the distribution of TFPR is endogenous, dependent upon other exogenous shocks and the endogenous determination of prices. This paper studies the determination the distribution of TFPR in an overlapping generations model with monopolistic competition and state dependent pricing. Changes in the mean and the dispersion of a quantity based measure of total factor productivity, TFPQ, and monetary shocks are analyzed as exogenous variations that influence the distribution of TFPR. None of these shocks alone can generate countercyclical dispersion in TFPR **and** match observed countercyclical dispersion in price changes and countercyclical movements in the frequency of price changes. Large enough shocks to the dispersion in TFPQ along with an appropriately responsive monetary policy can match these facts. But the required monetary feedback does not reproduce the positive correlation between money innovations and the dispersion in TFPR seen in the data. In this framework, uncertainty *per se* plays a very limited role.

## 1 Motivation

The dispersion of productivity has been shown to be countercyclical.[1] This finding plays a major role in recent quantitative analyzes of aggregate fluctuations. A prominent example is Bloom (2009) which studies the effects of uncertainty over the dispersion of productivity on investment activity. Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) go further to document the business cycle implications of countercyclical dispersion in, *inter alia*, firm level productivity.[2] Bachmann and Bayer (2014) provide complementary evidence from German data.[3] As another leading example, Vavra (2014) provides evidence that price changes are more dispersed in recessions and the frequency of price adjustment is higher. He argues that these patterns can be reproduced in a model with variations in the volatility of firm level productivity as these fluctuations induce some sellers to adjust prices upwards and others to adjust downwards.[4]

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1See the evidence and discussion in, for example, Kehrig (2011), Bachmann and Bayer (2014), and Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). The evidence is presented as changes in the distribution of total factor productivity and/or the correlation in the dispersion of total factor productivity with a measure of economic activity.

2Here there is an important but distinction between uncertainty and dispersion. Uncertainty refers to an *ex ante* situation of not knowing, say, some moment of the distribution of a random variable, such as not knowing the future variance. Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) contains both uncertainty and dispersion effects.

3They also add to the set of observations the procylicality in the dispersion of investment rates.

4His calibration relies upon the same measures of dispersion as Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). The connection between firm specific shocks and the distribution of price changes is highlighted in Golosov and Lucas (2007) as well.

But, there is a fundamental inconsistency in the above description of evidence and models. It has to do with the term "productivity". The measured productivity that underlies the evidence is revenue based total factor productivity, hereafter TFPR. But, the models are built upon dispersion (and uncertainty) effects from changes in the distribution of a quantity based measure of total factor productivity, hereafter TFPQ.

The problem is that these are different measures of productivity, both in the data and in theory. The distinction between these measures of productivity is central to the empirical analysis in Foster, Haltiwanger, and Syverson (2008).[5] The facts presented in Foster, Haltiwanger, and Syverson (2008) make clear that: (i) the distributions of TFPQ and TFPR differ and (ii) the distribution of TFPR is not degenerate. The first point implies that any model attempting to study both of these distributions needs to rationalize the difference between TFPR and TFPQ. Further, that model, following the discussion in Hsieh and Klenow (2009), must explain why the distribution of TFPR is not degenerate.

The key difference between the distribution of TFPR and the distribution of TFPQ is the distribution of prices. Thus, understanding the cyclicality of TFPR dispersion requires a model of price determination. With that in mind, the central question of this paper is: what factors, both in the process of price determination and shocks, generate the observed countercyclical dispersion in TFPR?

This question is addressed through a model of state dependent pricing, with heterogenous firms, to obtain a mapping from the distribution of TFPQ to the distribution of TFPR. In contrast to the flexible price case, state dependent pricing due to menu costs introduces both extensive and intensive margins of pricing decisions and thus allows for a variety of factors, both monetary and real, to influence the distribution of TFPR.

In our environment, following Hsieh and Klenow (2009), in the absence of frictions, the distribution of TFPR would be degenerate. Interestingly, price stickiness is sufficient to create a non-degenerate distribution of TFPR, other types of frictions or wedges are not needed.[6] Thus state dependent pricing is a key input into the economic mechanism determining the distribution of TFPR.

This paper does not contest the cyclicality of TFPR dispersion. Rather, it studies the determinants of this cyclicality through the effects of three types of exogenous shocks: (i) variations in the distribution of TFPQ, (ii) aggregate money shocks and (iii) (both aggregate and idiosyncratic) productivity shocks.[7]

The first source of fluctuations seems natural: the dispersion in TFPR is driven by the dispersion in TFPQ. The second and third types of shocks can also generate changes in the dispersion in TFPR since, in our framework, the distribution of prices is endogenous.

The framework for the analysis is an overlapping generations model with monopolistic competition and sticky prices specified in section 2. Young agents have market power, set prices *ex ante* and can, at a cost, change them *ex post*, once the various shocks specified above along with the menu cost are realized. Old agents take money earnings from youth as well as monetary policy induced transfers and spend them on a variety of goods. The analysis is conducted through a stationary rational expectations equilibrium for this environment.

One benefit of this model framework is the transparency of the equilibrium characterization as the *ex*

[5] As in Foster, Haltiwanger, and Syverson (2008), suppose the technology at the plant-level is $y = Af(l)$, where $y$ is output, $A$ is a technology shock, and $l$ is the labor input into a technology $f(\cdot)$. TFPQ is given by $A = \frac{y}{f(l)}$ while TFPR is $pA = \frac{pY}{f(l)}$ where $p$ is the relative output price.

[6] We are grateful to John Haltiwanger for emphasizing this point to us.

[7] These are leading sources of fluctuations but there are other candidates worth considering. Kehrig (2011) stresses the distributional implications of entry and exit as well as the distinction between durable and nondurable goods producers. Cooper and Schott (2013) emphasize the importance of variations in the cost of reallocation as generating aggregate fluctuations as well as variance in the dispersion of productivity.

*post* pricing decisions of sellers have no dynamic component. That is, the pricing problem is associated with young agents who set a price *ex ante* and have an option to pay a cost to adjust their price *ex post*. In old age, they are buyers not sellers.[8]

A second benefit is that money demand and monetary shocks are an integral part of the environment. The response of prices and quantities at both the individual and aggregate levels are fully determined in a stationary rational expectations equilibrium. There is no need to restrict the analysis to one-time unanticipated shocks.

Some of the results are quantitative and rely on a particular form of preferences and adjustment costs.[9] These are calibrated from the existing literature. Section 3 presents the quantitative model.

It is natural to think that variations in the dispersion of TFPR, hereafter denoted $disp_R$, is driven by changes in the dispersion of TFPQ, hereafter denoted $disp_Q$. This is indeed the case, despite the endogenous component of TFPR due to price setting. That is, variations in the distribution of TFPQ impacts TFPR dispersion directly and also indirectly through the frequency and magnitude of price adjustment. This includes the responsive of prices on both the extensive and intensive margins.

But this does not generate countercyclical $disp_R$. In the model, output is itself positively correlated with $disp_Q$. In a flexible price model, the increased dispersion of productivity allows for the reallocation of inputs towards more productive uses and output increases.[10] That effect exists also when prices are sticky. Thus one main finding is that a model economy driven by dispersion shocks to TFPQ fails to capture the countercyclicality in $disp_R$. To clear, this result is not immediate because of the endogeneity of price determination.

As suggested by Vavra (2014), shocks to the dispersion of TFPQ do succeed in matching two prominent features of pricing: (i) countercyclical dispersion in price changes and (ii) the countercyclical frequency of price adjustment. This qualitatively matches the data patterns Vavra (2014) uncovers but the model delivers the counterfactual prediction of procyclical $disp_R$.[11]

Note that this seems to contradict the findings of Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). That model relies on a "wait and see" aspect of investment with nonconvex adjustment costs that is not present in our setting. Thus we focus largely (though see below) on the effects of dispersion rather than volatility. The findings in Bachmann and Bayer (2014), Berger, Dew-Becker, and Giglio (2020) and Cooper and Schott (2013) and create considerable doubt that uncertainty dominates volatility effects. This further motivates our emphasis on pricing as the source of countercyclical $disp_R$.

Due to price setting behavior, $disp_R$ responds to other shocks. Holding fixed the distribution of TFPQ, money shocks alone can impact the distribution of TFPR through pricing decisions of sellers. In this case, the money shocks also generate countercyclical dispersion in price changes and in the frequency of price changes, but the dispersion of TFPR is again procyclical.

Finally, if fluctuations are driven by (aggregate) shocks to the mean of TFPQ, denoted $\mu_Q$, then extreme shocks to the mean of TFPQ will reduce the dispersion in TFPR while increasing output. Interestingly, the

[8]Other papers in the literature make assumptions that limit the power of state dependent pricing. For example, Christiano, Eichenbaum, and Evans (2005) invoke a Calvo pricing framework so that adjustment probabilities are exogenous and prices are indexed to the rate of lagged inflation. A similar approach is taken by Smets and Wouters (2007).

[9]Thus the overlapping generations framework is used here to provide a framework for conducting experiments in a stochastic equilibrium setting. The results are intended to be qualitative and suggestive for more detailed empirical analyses.

[10]This is sometimes called the Abel-Hartman effect and is discussed by Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) alongside the effects of uncertainty (see their section 5.2.4). These reallocation effects are emphasized in Cooper and Schott (2013).

[11]The distinction between the dispersion in TFPQ and TFPR is not discussed in Vavra (2014). The analysis does not evaluate the cyclicality of $disp_R$ and considers variation in $disp_Q$ along with the mean of TFPQ as sources of fluctuations

same goes for the employment response to the shock: for extreme shocks aggregate employment increases with aggregate productivity, else it falls.[12] But in this setting the frequency of price adjustment is procyclical and the correlation of output and the dispersion of price changes is very close to zero.

Throughout these exercises, one theme emerges: there are non-linearities in the response of $disp_R$ to various shocks. Regardless of the source of aggregate fluctuations, $disp_R$ is generally lowest for extremely low and high realizations of the money shock and highest for the average state.

This suggests that allowing the monetary authority to respond to shocks to either $disp_Q$ or $\mu_Q$ shocks might alter the cyclical patterns. To study this, Section 5.1 adds a monetary feedback rule. Depending on the feedback rule and the source of the shock, we are able to match all of these moments. In particular, if fluctuations are driven by $disp_Q$ shocks **and** the monetary authority tightens money growth when $disp_Q$ is high, then the equilibrium displays countercyclical dispersion in TFPR (despite having procyclical dispersion in TFPQ), countercyclical frequency and dispersion of price changes as well as countercyclical employment dispersion. A similar finding about the role of monetary policy applies when both $disp_Q$ and $\mu_Q$ shocks are present and are perfectly negatively correlated. However, the monetary feedback, making money shocks dependent on $disp_Q$, produces a negative correlation between monetary innovations and $disp_R$, contrary to the data.

Section 6 looks at two additional properties of the model economy. First, as discussed by Vavra (2014) as well, there is another link between dispersion and monetary policy. An increase in the dispersion of real productivity reduces the effectiveness of monetary policy.[13] This reflects the fact that following an increase in $disp_Q$, both the frequency of adjustment and the dispersion of price adjustment increase.

Second, the model also provides insights into the pricing and output effects of uncertainty, as distinguished from dispersion shocks. In general, dispersion shocks refer to variations in the *ex post* distribution of idiosyncratic or aggregate shocks. The uncertainty effect arises *ex ante* as agents are uncertain of the future distribution from which the shocks are drawn.[14] In our model, an increase in *ex ante* uncertainty over the future distribution of idiosyncratic productivity influences the *ex ante* price set by sellers. Quantitatively we find that these effects are tiny.[15]

Returning to our motivating question, in the end it does not appear that the endogenous evolution of the distribution of prices is enough to match the basic facts on pricing and to generate a countercyclical dispersion in TFPR. While we do succeed in finding setting in which all these moments are matched, this requires, as argued below, monetary interventions that are quite different from those observed in the data.

# 2 Model

We study these issues in an infinite horizon overlapping generations model with differentiated products and market power. Agents live for two periods, youth and old age. Generation $t$ young agents produce and, when old, these agents consume a basket of goods produced by the next generation of young producers. The sequence of choices is shown in Figure 1.

[12] This is clearly related to the findings in Galí (1999) and the literature that followed. Here we link these patterns to price setting at the plant-level.

[13] There is some additional empirical support for this proposition. Tenreyro and Thwaites (2016) argue that the response of the economy to monetary (federal funds rate) innovations is considerably stronger in expansions compared to recessions. The findings reported in Bachmann, Born, Elstner, and Grimme (2019) provide additional evidence linking price setting behavior to uncertainty.

[14] Under some assumptions, increases in uncertainty reduce economic activity. Bloom (2009) and related papers focus on the effects of uncertainty on spending on durables, such as firm capital.

[15] This is consistent with Vavra (2014) and Bachmann, Born, Elstner, and Grimme (2019).

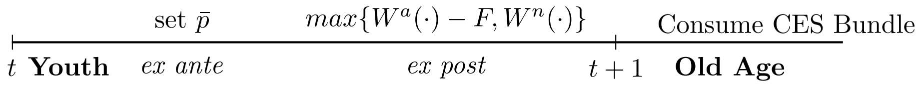



Figure 1: Time Line: Generation $t$

Saving occurs through the holding of fiat money. The quantity of fiat money is stochastic, representing monetary shocks.

Young producers are distinct in three dimensions. First, they produce a differentiated product. Second, their output is a stochastic function of their labor input. Finally, they have an idiosyncratic cost of price adjustment.

The focus of the analysis is on price setting by sellers. Each young agent freely sets a price *ex ante*. After the realization of their idiosyncratic and aggregate shocks, they decide to adjust their price or not.[16] Importantly, this *ex post* decision on readjustment depends on the realization of all shocks. In this way, the dispersion of the distribution of productivity shocks impacts the frequency of adjustment and thus the real effects of money shocks.

As in Lucas (1972), in the absence of price stickiness, there would be a stationary rational expectations equilibrium in which money was neutral. This is because money transfers are made to the old in proportion to money holding earned in youth. And, as in that paper, the analysis rests on the coexistence of real and nominal shocks. But, in our setting the friction of costly price adjustment replaces his assumption of imperfect information.[17]

## 2.1 Choice of Old Agents

Lifetime utility is represented by $u(c) - g(n) = \frac{c^{1-\sigma}}{1-\sigma} - g(n)$. Here $c$ is a CES aggregator given by $c = \left(\sum_i c^{i\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$.[18] The function $g(\cdot)$ is increasing and convex in hours worked, with $0 \leq n \leq 1$.

When old, agents take their money holdings from income earned in youth and allocate it across goods to maximize $u\left(\left[\sum_i (c^i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right)$, subject to a budget constraint of $\sum_i c^i p^i = M$ where $M$ is their nominal income and $p^i$ is the money price of good $i$.[19]

For these preferences, the demand for good $i$ is given by

$$c^i = d(p^i, P, M) = (\frac{p^i}{P})^{-\varepsilon} \frac{M}{P}. \tag{1}$$

Here $P$ is an aggregate price index defined as $P = \left(\sum_i (p^i)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$. Note that the only shock to demand is from variations in the stock of money, $M$: there are no product specific taste shocks common to all agents.

Let $V(\frac{M}{P})$ be the value of the solution to the optimization problem of an old agent with nominal income of $M$ with prices given by $P$. Given the definition of $c$,

[16] In this version of the model, these include idiosyncratic shocks to profitability and menu costs and aggregate shocks to money. Appendix section 8.2 extends the equilibrium to include shocks to the distribution of productivity.

[17] Of course, in his model the real shock was to the fraction of sellers in a particular market while we focus on productivity shocks.

[18] We normalize the number of young agents and thus products to 1. With the CES assumption, markups are constant.

[19] To simplify the notation, the time subscript is repressed. The money holdings come from income earned in youth as money is the store of value in this economy. Many other general equilibrium models, such as Dotsey, King, and Wolman (1999), impose money demand. In Golosov and Lucas (2007), money is in the utility function.

$$V(\frac{M}{P}) = u\left(\left[\sum_i \left((\frac{p^i}{P})^{-\varepsilon}\frac{M}{P}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right) = u\left(\left[\sum_i \left((\frac{p^i}{P})^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\frac{M}{P}\right) \tag{2}$$

with $P$ given above. From this, the marginal value of nominal income is given by $V_M = \frac{u'(c)}{P}$.

At this point, these are generic demands and values for an old age given nominal income and prices. We will take this structure and use it to study the choices of young agents in the OG framework, summarizing the utility they obtain when old through $V(\frac{M}{P})$.

## 2.2 Choice of Young Agents

We start with the pricing decisions of generation $t$ young agents. When young agents choose the price of their product *ex ante*, they take into account the option, at a fixed cost, of adjusting their price *ex post*. Since this is a model of a menu rather than a quadratic cost at the micro-level, the *ex ante* price will influence the frequency of adjustment but not the *ex post* price conditional on adjustment.

As is common in the literature, see for example Galí (2015), agents are assumed to meet the demand forthcoming at their price. Thus the prices they set will determine their nominal income in youth.

This nominal income is held over time in the form of money to purchase consumption goods when old. Holdings of money are altered through monetary policy. Thus in our framework, money holdings and monetary policy interventions are made explicit.

To study the pricing choice, consider the *ex post* decision of generation $t$ sellers. If they choose to adjust, these sellers choose a price $\tilde{p}$ to solve

$$W^a(z_t, M_{t-1}, x_t, P_t) = max_{\tilde{p}} E_{x_{t+1},P_{t+1}} V((R(\tilde{p}, P_t, M_t))x_{t+1}/P_{t+1}) - g(\frac{d(\tilde{p}, P_t, M_t)}{z_t}). \tag{3}$$

Here the demand, denoted $d(\tilde{p}, P_t, M_t)$ and specified in (1), is the spending of the old agents on the product of this seller. The function $V((R(\tilde{p}, P_t, M_t))x_{t+1}/P_{t+1})$ is given by (2) with, in that notation, $M = R(\tilde{p}, P_t, M_t)x_{t+1}$ being the nominal revenue earned as a seller in period $t$ supplemented by the period $t+1$ money shock and $P = P_{t+1}$, the period $t+1$ aggregate price level.

Since this decision is made *ex post*, the value and the price depend on the current state: $(z_t, M_{t-1}, x_t, P_t)$. Here $z_t$ is the current idiosyncratic productivity shock, $M_{t-1}$ is the inherited money supply, $x_t$ is the money shock and $P_t$ is the aggregate price level, determined in equilibrium as described below.

There is also a seller specific menu cost, denoted $F_t$, that influences whether adjustment occurs or not but not the price selected given adjustment. The adjustment cost is written as a utility loss. This specification has a convenient property that the optimal price is independent of the adjustment cost. So, the extensive margin of adjustment will depend on the realized menu cost and idiosyncratic productivity but the intensive margin does not so that the price dispersion of adjusters reflects only heterogeneity in $z_t$.

Notice that the price set by these sellers is independent of any price they may have set *ex ante* so that the *ex ante* choice does not appear in the state. Importantly, once the cost of adjustment is incurred, the price reflects both the monetary shock and seller specific productivity. In this sense, there is an underlying complementarity at work. If a seller pays an adjustment cost to respond to one type of shock, then the marginal cost of responding to another type of shock is zero. This is important for the analysis that follows as it explains why price dispersion and thus TFPR dispersion is influenced by monetary policy.

With the production function of $y = zn$, the labor input of the seller is given by $\frac{d(\tilde{p},P_t,M_t)}{z_t}$.[20] As the seller meets all demand, the labor input varies inversely with productivity, **given** demand.

The first-order condition is

$$E_{x_{t+1},P_{t+1}}\left(u'(c_{t+1})x_{t+1}\frac{d(p_t,P_t,M_t)(1-\varepsilon)}{P_{t+1}}\right) = g'(\frac{d(p_t,P_t,M_t)}{z_t})\left(-\varepsilon\frac{d(p_t,P_t,M_t)}{p_t z_t}\right). \tag{4}$$

Denote this *ex post* optimal price by $p_t = \tilde{p}(z_t, M_{t-1}, x_t, P_t)$ for all a seller with realized productivity $z_t$.

This is the standard condition for optimal price setting, equating marginal revenue with marginal cost.[21] But in this overlapping generations model, the value marginal revenue is determined by the marginal utility of the future consumption that can be acquired with the additional money income. And that income is itself impacted by future monetary policy, through the stochastic transfer $x_{t+1}$.

Alternatively, if the seller does not adjust, then expected lifetime utility is given by:

$$W^n(z_t, M_{t-1}, x_t, P_t, \bar{p}) = E_{x_{t+1},P_{t+1}}V((R(\bar{p},P_t,M_t))x_{t+1}/P_{t+1}) - g(\frac{d(\bar{p},P_t,M_t)}{z_t}). \tag{5}$$

Here, expected utility depends on the preset price, $\bar{p}$.

Given this, consider the *ex ante* choice. When this price is set, the young agent just knows the money supply from the past. Let $W^{xa}(M_{t-1})$ be the value to a young agent of setting the price *ex ante*. The value is given by:

$$\begin{aligned} W^{xa}(M_{t-1}) \quad &= \quad max_{\bar{p}}E_{(z_t,x_t,x_{t+1}P_t,P_{t+1})}[(1-\Omega(F^*(\Omega_t)))W^n(z_t,M_{t-1},x_t,P_t,\bar{p}) + \\ &\int_0^{F^*(\Omega_t)} W^a(M_{t-1},x_t,P_t) - F]d\Omega(F) \end{aligned} \tag{6}$$

where $F^*(z_t, M_{t-1}, x_t, P_t)$ is the critical menu cost in state $(z_t, M_{t-1}, x_t, P_t)$ such that price adjustment occurs iff $F \leq F^*(z_t, M_{t-1}, x_t, P_t)$. Here the menu cost $F$ has a cdf of $\Omega(\cdot)$. Let $\bar{p}(M_{t-1})$ denote the optimal *ex ante* choice.

## 2.3 SREE

The analysis is based on a stationary rational expectations equilibrium (SREE) with valued fiat money.[22] The current aggregate state is represented as $(M, x)$ where $M$ is the inherited money supply and $x$ is the current shock, so that the current money supply is $Mx$. At the individual supplier level, productivity and the cost of price adjustment are the two elements in the idiosyncratic state: $(z, F)$.

There are four state dependent functions to be determined. The *ex ante* price set knowing only $M$ is denoted $\bar{p}(M)$. The *ex post* price set by sellers who choose to adjust their price is given by $\tilde{p}(M, z, x)$, indicating the price depends on both the realized money shock and productivity. There is a critical level of the adjustment cost, $F^*(M, x, z)$, such that adjustment occurs iff $F \leq F^*(M, x, z)$. Finally, the *ex post* money price of goods, $P(M, x)$, clears the goods market.

[20] When we discuss below the case of an aggregate TFP shock, $\mu_Q$, then $y = \mu_Q zn$.

[21] To understand this condition in a static setting, let $d = (\frac{p}{P})^{-\varepsilon}y$ be the level of produce demand if the seller sets the price $p$ and the aggregate price is $P$ and the level of real spending is $y$. So $d_p = -\varepsilon\frac{d}{p}$. Further, revenue is given by $R = pd = p^{1-\varepsilon}(\frac{1}{P})^{-\varepsilon}y$. Hence $R_p = (1-\varepsilon)d$. The left side of (4) is the product of $R_p$ and $\frac{u'(c_{t+1})x_{t+1}}{P_{t+1}}$. The right side is the product of $d_p$ and the marginal disutility of work, $g'(\frac{d(p_t,P_t,M_t)}{z_t})\frac{1}{z_t}$.

[22] The more general SREE including shocks to the distribution of idiosyncratic productivity is presented in Appendix Section 8.2 for the linear-quadratic economy.

At this point of the analysis, the distribution of the idiosyncratic productivity shocks is not in the state vector. An equilibrium is defined and characterized given that distribution.

**Definition 1** *A SREE is a set of functions* $(\bar{p}(M), \tilde{p}(M,z,x), F^*(M,x,z), P(M,x), W^n(M,x,z), W^a(M,x,z))$ *such that:*

- $\bar{p}(M)$ *solves the ex ante pricing problem given the state dependent price index* $P(M,x)$*;*

$$\bar{p}(M) = argmax_p E_{x,z,x'} V((R(p, P(M,x), Mx)x')/P(Mx,x')) - g(\frac{d(p, P(M,x), Mx)}{z}). \tag{7}$$

*for all* $M$.

- $\tilde{p}(M,x,z)$ *solves the ex post pricing problem:*

$$\tilde{p}(M,x,z) = argmax_p E_{x'} V\left((R(p, P(M,x), Mx))x'/P(Mx,x')\right) - g(\frac{d(p, P(M,x), Mx)}{z}) \tag{8}$$

*given the state dependent price vector,* $P(M,x)$*, for all* $(M,x,z)$.

- *At the critical adjustment cost,* $F^*(M,x,z)$*, the seller is just indifferent between adjusting and not:*

$$F^*(M,x,z) \equiv W^n(M,x,z) - W^a(M,x,z)$$

*for all* $(M,x,z)$*, with* $W^a(M,x,z)$ *given by:*

$$W^a(M,x,z) = E_{x'} V\left((R(\tilde{p}(M,x,z), P(M,x), Mx))x'/P(Mx,x')\right) - g(\frac{d(\tilde{p}(M,x,z), P(M,x), Mx)}{z}) \tag{9}$$

*and* $W^n(M,x,z)$ *given by*

$$W^n(M,x,z) = E_{x'} V((R(\bar{p}(M), P(M,x), Mx))x'/P(Mx,x')) - g(\frac{d(\bar{p}(M), P(M,x), Mx)}{z}). \tag{10}$$

- $P(M,x)$ *is the aggregate price index in state* $(M,x)$ *given by:*

$$P(M,x) = [E_z(1 - \Omega(F^*(M,x,z)))\bar{p}(M)^{1-\varepsilon} + E_z(\Omega(F^*(M,x,z))\tilde{p}(M,x,z)^{1-\varepsilon})]^{\frac{1}{1-\varepsilon}} \tag{11}$$

*where* $d(\bar{p}(M), P(M,x), Mx) = (\frac{\bar{p}(M)}{P(M,x)})^{-\varepsilon} Y$ *and* $d(\tilde{p}(M,x,z), P(M,x), Mx) = (\frac{\tilde{p}(M,x,z)}{P(M,x)})^{-\varepsilon} Y$*. Here* $Y = \frac{Mx}{P(M,x)}$ *is the equilibrium determined real value of money holdings.*

There are two main properties of a SREE that are verified in the analysis that follows.

**Proposition 1** *There exists a SREE in which: (i) all prices are proportional to* $M$ *and real quantities are independent of* $M$ *and (ii) all prices are not proportional to* $x$ *and so real quantities are not independent of* $x$*.*

First, the inherited money supply is neutral: i.e. prices are proportional to $M$ and all real quantities are independent of $M$. Formally, this amounts to guessing and verifying that there is a SREE in which $\bar{p}(M) = QM$ where $Q$ is an unknown constant and $\tilde{p}(M,x,z) = M\tilde{\phi}(x,z)$. From this all relative prices and thus quantities demanded (and thus supplied) are independent of $M$.

The second property is money non-neutrality. If prices were not costly to adjust, i.e. the distribution of $F$ was degenerate at $F = 0$, then there would exist a SREE with prices proportional to $Mx$. In this case, real quantities would be independent of the current money supply, $Mx$. But, in the presence of non-degenerate menu costs, as long as some sellers choose not to adjustment their prices *ex post*, a SREE with prices proportional to $Mx$ cannot exist simply because the preset price, $\bar{p}$, must be independent of $x$.[23]

In equilibrium, aggregate real GDP is given by: $Y(x) = \frac{Mx}{P(M,x)} = \frac{x}{\phi(x)}$. In this model, the difference between TFPQ and TFPR is transparent. Here, $z$ corresponds to the TFPQ measure of productivity. It is exogenous to the seller. The variable $\frac{zp}{P}$ is TFPR, where $p \in \{\tilde{p}, \bar{p}\}$ is the seller's price and $P$ is the aggregate price.[24] It is endogenous as prices are set by sellers. The distribution of TFPR responds to shocks insofar as sellers adjust prices in response to those shocks.[25]

# 3 Quantitative Analysis

The quantitative analysis rests upon a linear-quadratic economy where $u(c) = c, g(n) = \frac{n^2}{2}$.[26] To be clear, the goal of the quantitative analysis is not to match data moments. This seems off the domain of the model as it is cast as an OG structure, with pricing decisions made once and thus apparently far from the high frequency decision problem that underlies plant and firm-level data.

But there is a benefit to this abstraction through an opportunity to clarify various channels of influence through the stationary rational expectations equilibrium of a monetary economy. In this setting, the monetary shocks as well as those to productivity are part of the basic economic environment and thus the responses are part of the equilibrium outcome. The goal is to provide a framework upon which empirics can be generated. These points are brought out here and in the following sections on additional implications.

Further, the OG model is not that far from the more standard models of Calvo price adjustment. In those models, as in the OG structure, the probability of price adjustment and the price set conditional on adjustment are both independent of the previously set price. Further, in some specifications, such as Christiano, Eichenbaum, and Evans (2005), price setters who do not adjust get to freely reset prices based upon inflation. This added feature further reduces the role of history for price setting. Or, put differently, the fact that *ex post* price setters do not look beyond the current period is also present in these other formulations.

This point is reinforced by the quantitative analysis which generates familiar patterns of price adjustment. The analysis makes clear that the overlapping generations model with state dependent price adjustment retains the essential features of the more standard infinitely lived agent specifications.

[23] Formally, this requires that the support of menu costs be large enough so that even if all other sellers adjust their prices *ex post*, the remaining seller, for any $x$, will have a high enough adjustment cost so that adjustment will not occur. See Ball and Romer (1991) for a discussion of this related to multiplicity of equilibria.

[24] Since TFPQ is measured directly in simulated data, there is no need to infer TFPR from revenue and thus no discussion of output or revenue factor shares. See the discussion of these measurement issues in Decker, Haltiwanger, Jarmin, and Miranda (2019).

[25] Using a static, flexible price version of the model and returning to a point made earlier, TFPR= $pz = q^{-\eta}z = z^{1-\eta}n^{-\alpha\eta}$. From the first order condition with respect to $n$, if marginal cost of labor is $\omega$, we have

$$(1-\eta)\alpha n^{(-\alpha\eta+\alpha-1)}z^{1-\eta} = \omega.$$

At $\alpha = 1$, the FOC becomes $(1-\eta)n^{-\eta}z^{(1-\eta)} = \omega$ for this to hold for all $z$. This implies that TFPR is given by $\frac{\omega}{1-\eta}$ and hence is independent of $z$. In our model, both price stickiness and non-linear production costs will contribute to the non-degenerate distribution of TFPR.

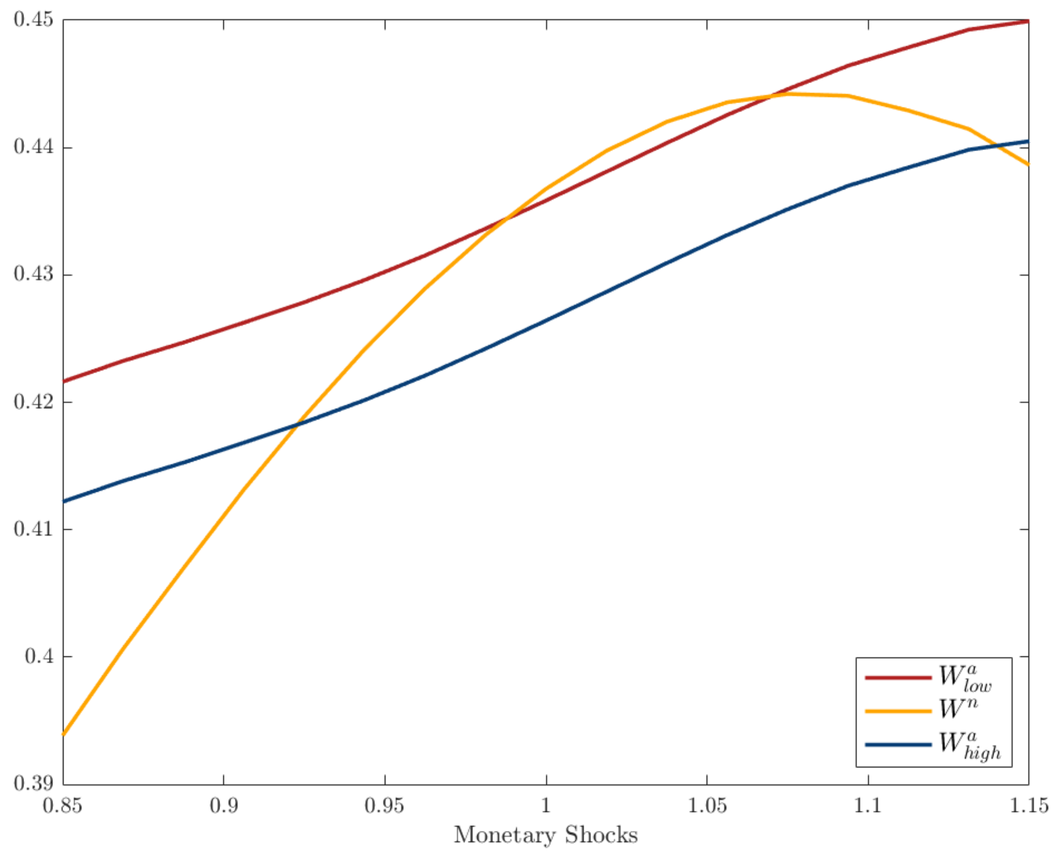
[26] Sub-section 8.1 of the Appendix characterizes the SREE for this special case. The methods and parameterization for the quantitative analysis is presented in Appendix sub-section 8.3.

For this analysis, the distribution of idiosyncratic productivity shocks is taken as given, so that $\sigma_z$ is not in the state vector. The effects of changes in this distribution are discussed in the next section.

## 3.1 Seller Choices

This section illustrates the quantitative properties of the seller's choices for the linear-quadratic economy.

As in the traditional state dependent pricing model, prices are adjusted only for sufficiently large monetary shocks and the region of adjustment depends on the adjustment costs. This property is illustrated for a given productivity shock to make clear that our model also contains the usual result. A second important property is the interaction of the real productivity shock and the monetary shock: adjustment can occur even for monetary shocks near their expected value if productivity shocks are sufficiently large.

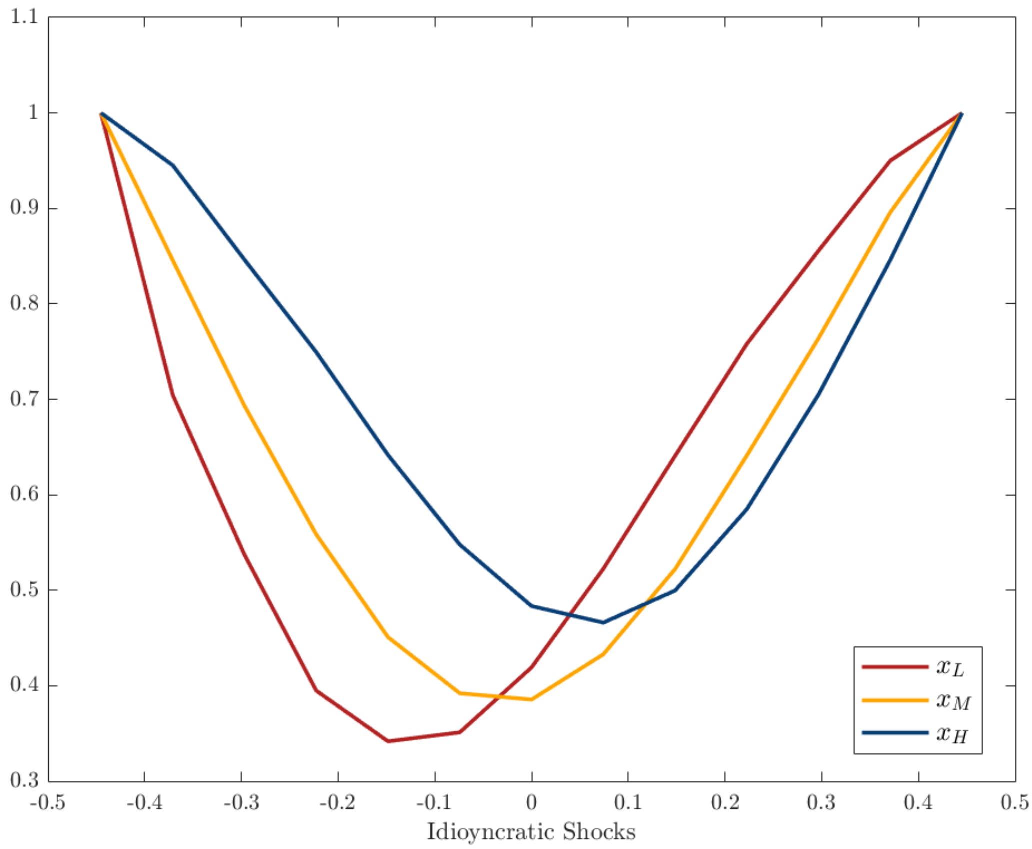



This figure shows the choice to adjust or not for different realizations of the money shock, $x$, for "high" and "low" values of the menu cost, for a given $z$.

Figure 2: Adjustment Values

Figure 2 illustrates the *ex post* choices of a seller. The figure shows the value of no-adjustment, $W^n$, along with the values of adjustment for two levels of the menu cost. The blue value is associated with a higher menu cost than the red value. As is clear, for sufficiently low (high) values of the money shock $x$, adjustment is preferred to no adjustment. The region of adjustment is larger for the low menu cost.

Figure 3 illustrates the dependence of the probability of price adjustment on the real productivity shock,




This figure shows the adjustment probability as a function of the idiosyncratic profitability shock, for 3 values of the monetary shock $x_l < x_m < x_h$.

Figure 3: Adjustment Probability as a function of Producitivity

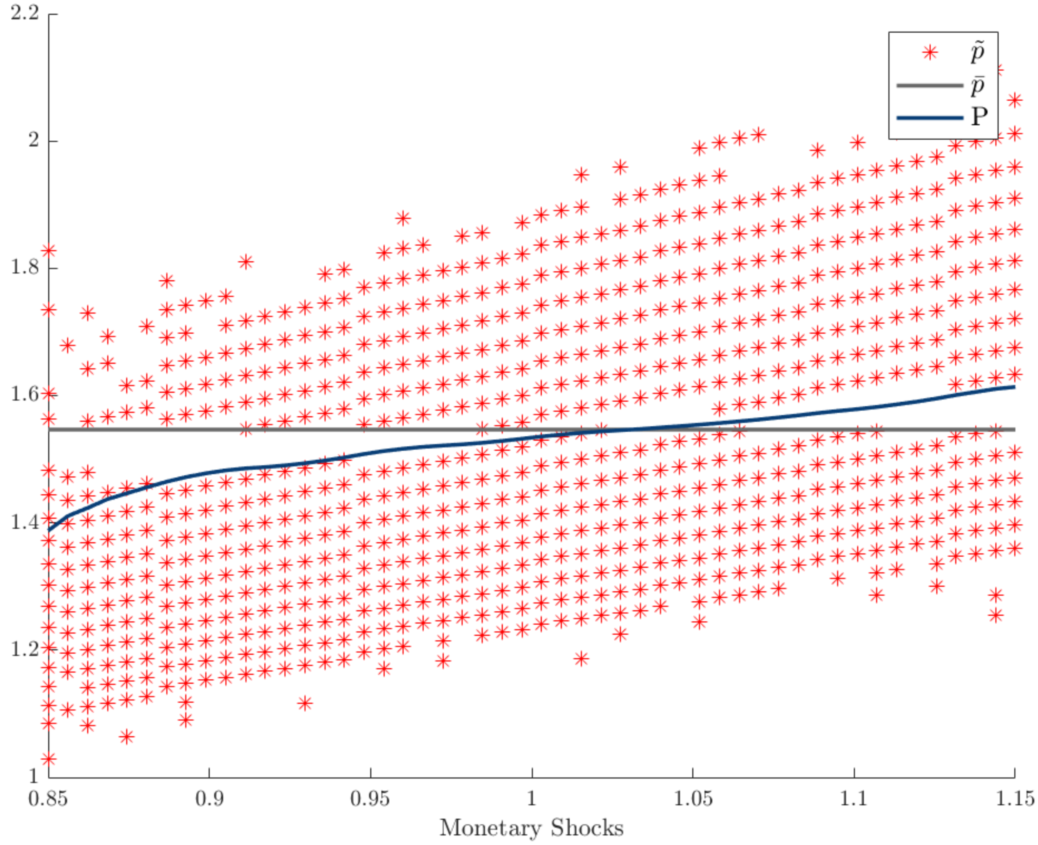
for three values of $x$. For each value of $x$, there is a realization of the productivity shock such that the likelihood firms adjustment their price in minimal. The frequency of adjustment is then U-shaped around this minimum. The higher the money shock, the higher is this critical productivity to offset the incentive to adjust.

The fact that the model economy produces this shape for the adjustment rate is important for two reasons. First, it confirms that state dependent pricing in the overlapping generations model produces patterns that are similar to other models. Second, as the analysis develops, the aggregate economy will display non-monotonic responses to various types of shocks. Those patterns can be traced back to the U-shaped adjustment rate.

## 3.2 Aggregate Effects

Given these responses at the firm level, we now turn to briefly describe the aggregate implications of the model in terms of the overall price level, the frequency of adjustment and output. We return to these effects when we look at other implications of the model.

Figure 4 shows the aggregate price, $P$, as well as the *ex ante* price, $\bar{p}$ and the distribution of *ex post* prices as a function of the monetary shock. The aggregate price, which is a CES index, is a combination of the state independent *ex ante* price and the state dependent *ex post* price. It is increasing in the money shock, reflecting responses on both the extensive and intensive margins. The frequency of adjustment, as established earlier, is U-shaped, while the *ex post* price is monotonically increasing in the shock.



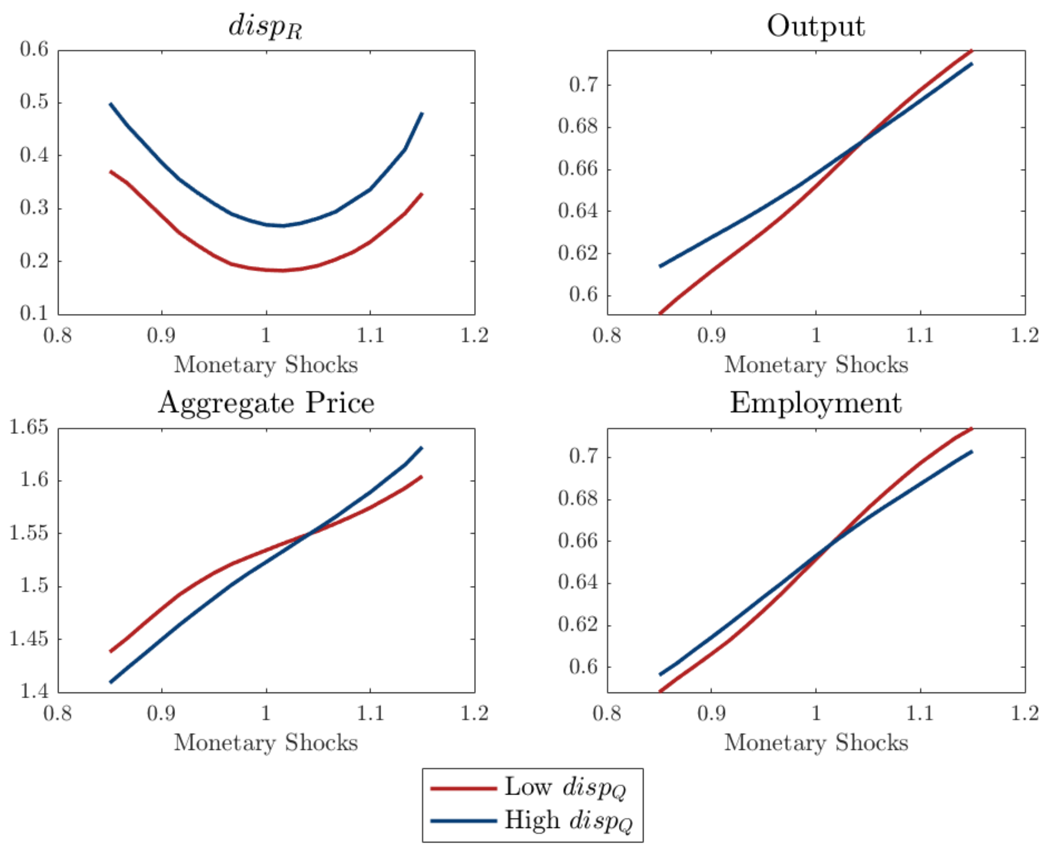

This figure shows patterns of prices for different realizations of the money shock, $x$.

Figure 4: Prices

The distribution of prices given $x$ illustrates the two forms of heterogeneity at the seller level. First, there are different productivity realizations that generate differences in prices, conditional on adjustment. Second, there are differences in adjustment costs that impact the extensive margin choice of whether to change prices. Different realizations of $x$ impact the firm choices on both of these margins.

Figure 5 illustrates the dependence of the adjustment frequency on the monetary shock and the real effects of the shock. Focusing on the blue line in the left panel, for low dispersion, the frequency of price adjustment is a U-shaped function of the money shock. Note that the adjustment frequency does not have a minimum at 0, reflecting the presence of the real shocks which create an independent value of adjustment.

The response of the aggregate price (index) to the money shock is shown in the top right panel. Here, reflecting both the extensive and intensive margins, the price level increases when the money supply expands. But, as indicated in the bottom left panel, the level of real economic activity output is clearly increasing in the monetary shock as well. This reflects both the stickiness of some prices and the choice of adjusted prices




This figure shows the effects of money shocks on the price adjustment rate, output, employment and the aggregate price.

Figure 5: Aggregate Implications

that are not proportional to the money supply.

This is summarized by the first row of Table 1. For the benchmark parameterization, about 25% of the sellers adjust their prices *ex post*. The average dispersion in the price change is about 0.10, reflecting the underlying distribution in productivity. The correlation of output and the money shock is nearly 1.

# 4 Cyclicality of TFPR Dispersion

The model of state dependent prices provides a basis to study the cyclicality of TFPR dispersion. As noted earlier, many theories are about the dispersion in TFPQ while, as emphasized by Foster, Haltiwanger, and Syverson (2008), the measurement commonly taken from plant-level studies is TFPR not TFPQ. Output and revenue measures of productivity are not same and their distributions may covary in different ways over the business cycle.

The question is whether the model of price setting can reproduce the countercyclical dispersion in TFPR seen in the data, as well as other pricing facts. This depends both on price setting behavior and exogenous variations. Here the exogenous variations include changes in $disp_Q$, money shocks and changes in the mean

| Case | Mean ($Freq_{\Delta P}$) | Mean ($disp_{\Delta P}$) | $corr(y, x)$ |
|---|---|---|---|
| Benchmark | 0.2556 | 0.0975 | 0.9959 |
| Higher Product Substitutability | 0.4799 | 0.0737 | 0.7035 |
| High Labor Supply Elasticity | 0.2166 | 0.1081 | 0.9956 |

This table shows basic moments in response to money shocks, for the baseline model and other parameterizations discussed in sub-section 5.2.

Table 1: Pricing Moments

of TFPQ, hereafter denoted $\mu_Q$.[27]

Table 2 summarizes our findings. It displays for the three sources of variation, the cyclical patterns of dispersion in TFPR, the dispersion of price changes, employment and the frequency of price adjustment.[28] The table is constructed using our baseline parameters. Variations on these parameters are reported in sub-section 5.2.

The table is discussed in detail in this section, first by looking at each shock independently. This allows us to focus on the cyclical effects of each shock independently. We then allow the monetary authority to respond to variations in the mean and dispersion of TFPQ and study the implications for the dispersion of TFPR.

The results are best evaluated relative to moments from the data, which are provided as well. From various studies, the dispersion of TFPR is countercyclical, the dispersion of price changes and frequency of price changes are countercyclical as is the dispersion of employment growth. In the data row, The correlation of output (growth) and $disp_R$ comes from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). Kehrig (2011) finds that the correlation of (detrended) output and the dispersion of productivity is -0.293 for non-durables and -0.502 for durables, in Table 2. His Table 4 makes clear that the countercyclicality is robust to various output measures. The moments on the dispersion and frequency of price changes comes from Vavra (2014), Table 4, and calculated at the business cycle frequency. The correlation of employment (growth) and dispersion is from Table I.3 in Ilut, Kehrig, and Schneider (2018).

## 4.1 Effects of Variation in TFPQ Dispersion

The analysis of countercyclical variation in TFPR dispersion starts with an obvious hypothesis: variations in $disp_Q$ drive the cyclicality of $disp_R$. To order for this explanation to be consistent with data patterns, it must be that: (i) increased dispersion in TFPQ creates increased dispersion in TFPR and (ii) increased dispersion in TFPQ causes economic downturns. We demonstrate that the model does not produce these patterns: **variations in the dispersion of TFPQ do not generate countercyclical fluctuations in the dispersion of TFPR.**

Specifically, here we study the effects on $disp_R$ of an increase in $disp_Q$, modeled as a mean preserving spread in the distribution of $z$.[29] To be clear, the effects highlighted here come from realized changes in the

[27] The SREE characterized above is for the case of money shocks alone. Exogenous variations in the dispersion of the idiosyncratic productivity shocks, $disp_Q$, is introduced as a mean preserving spread of $z$. Variations in the mean of TFPQ, denoted $\mu_Q$, infuence the production function at the individual level which becomes $y = \mu_Q zn$. For both cases the SREE is redefined and computed with the additional state variable in Appendix sub-section 8.2.

[28] Here we report correlations with the level of output as there is nothing to detrend. The same properties emerge from splitting the sample into "expansions" and "contractions" and computing conditional moments as displayed in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Ilut, Kehrig, and Schneider (2018).

[29] This is parameterized to match the cyclical change in TFPR dispersion as reported in Bloom, Floetotto, Jaimovich,

| Case | $corr(y, disp_R)$ | $corr(y, disp_{\Delta P})$ | $corr(y, disp_n)$ | $corr(y, freq)$ |
|---|---|---|---|---|
| Data | -0.45 | -0.41 | -0.50 | -0.27 |
| | | $disp_Q$ | | |
| Model | 0.4292 | -0.1348 | -0.3413 | -0.1695 |
| Flex. P | 0.9977 | 0.8626 | 0.9974 | *na* |
| | | $x$ | | |
| Model | 0.4574 | -0.2397 | -0.4863 | -0.1311 |
| | | $\mu_Q$ | | |
| Model | -0.0183 | -0.0010 | -0.3863 | 0.1878 |
| Flex. P | 0.9402 | -0.5259 | -0.3264 | *na* |

This table shows the correlation between output and the dispersion of TFPR, the dispersion and frequency of price changes and the dispersion of employment for three different types of shocks.

Table 2: Cyclical Variations

distribution of TFPQ. Another channel, studied below, is on the uncertainty caused by future changes in this distribution. Throughout, $disp_R$ is always less than $disp_Q$ , as it is in the data summarized by Foster, Haltiwanger, and Syverson (2008). Again, the magnitude of this difference depends on the state of the economy.

Variations in $disp_Q$ will impact $disp_R$ in two ways. First, of course, there is the direct effect: given prices, an increase in $disp_Q$ will translate into an increase in TFPR dispersion. Second, pricing behavior will adjust, potentially magnifying (reducing) the effects of the increase in $disp_Q$. The sign and size of this latter effect will depend on the properties of the revenue function and, as emphasized by our model, the pattern of price adjustment.

Figure 6 illustrates the effects of variations in $disp_Q$ on the $disp_R$, for different values of the money shock. Clearly an increase in $disp_Q$ leads to an increase in $disp_R$. The magnitude of this effect though does depend on the money shock, as discussed further below.

What are the effects of an increase in $disp_Q$ on output? If the increased dispersion causes a reduction in output, then shocks to $disp_Q$ will create countercyclical dispersion in TFPR, as in the data.

In an economy with flexible prices, a mean preserving spread in plant-level productivity will typically increase output as factors are reallocated to take advantage of high productivity plants. This property holds in our overlapping generations model with monopolistic competition as well if there are no costs of price adjustment.

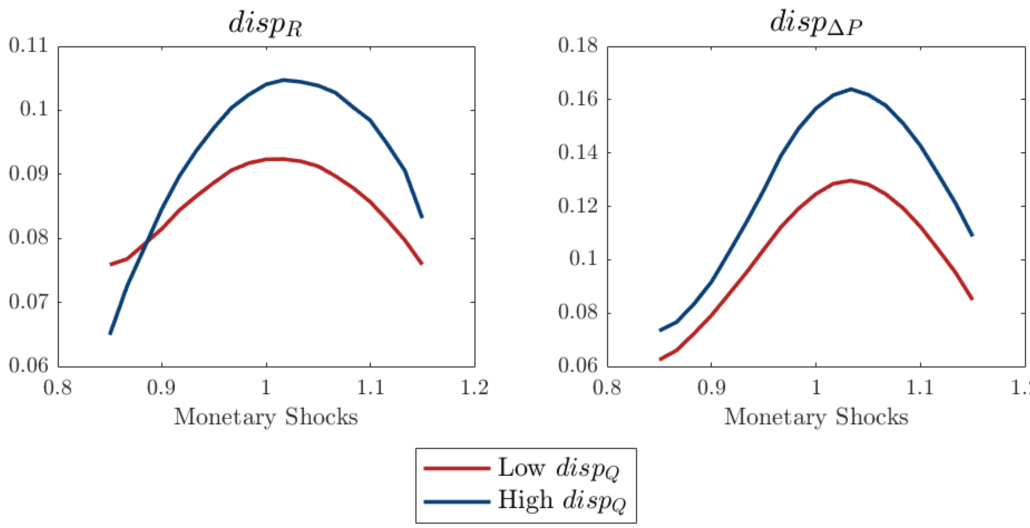
In a setting with price rigidities this reallocation will be weaker and may not even occur. To see why, consider a seller with a high productivity realization and a high adjustment cost. Without price adjustment, the high productivity means that the seller will supply a relatively low level of labor to meet demand.[30] But if the realized adjustment cost was low, the seller would choose a lower price and expand production and employment. Thus the output response to increased dispersion is hampered by price inflexibility.[31]

Returning to Figure 5, at $x = 1$, the response of the economy to an increase in $disp_Q$ is a slightly lower aggregate price level and thus slightly higher aggregate output. In this case, without a response by

Eksten, and Terry (2018). This determines the magnitude not the cyclicality of the change in the dispersion of TFPR. This parameterization applies when all the shocks, $(disp_Q, x, \mu_Q)$, are present to match the unconditional movements in the dispersion of TFPR. More details on the parameterization are provided in Appendix sub-section 8.3.3.

[30]This reflects the assumption that the seller meets demand at the posted price.

[31]As discussed in sub-section 5.2, the magnitude of the effect also depends on the marginal cost of employment.




This figure shows the effects of variations in $disp_Q$ on $disp_R$ and the dispersion of price changes.

Figure 6: Response of Dispersion in TFPR and Price Changes

the monetary authority, variations in $disp_Q$ are procyclical and lead to procyclical variations in $disp_R$ as indicated in Table 2.

Also from Table 2, the model predicts that the dispersion in price changes is countercyclical, consistent the evidence provided in Vavra (2014). The dispersion in employment is positively correlated with $disp_Q$. This makes sense, as the dispersion in productivity increases, so will the dispersion in employment. This effect is particularly strong for sellers who choose not to adjustment their price. Finally, the frequency of price adjustment is countercyclical in the model driven by $disp_Q$ shocks.

The findings about the cyclicality of the dispersion in price changes and frequency are consistent with the findings of Vavra (2014). **But the model is inconsistent with the data in terms of the motivating observation of countercyclical dispersion in TFPR.**

## 4.2 Money Shocks

A second shock comes from monetary innovations, $x$. Due to price rigidities, monetary shocks impact real output. From Figure 5, output and aggregate prices both respond positively to monetary shocks. Here we focus on the effects of money shocks on the dispersion of TFPR holding fixed the distribution of TFPQ.

Can they produce countercyclical TFPR dispersion? Figure 6 illustrates the effects of money shocks on the standard deviation of TFPR. As indicated in the top left panel, for either extremely low or high monetary shocks (around ± 10%), the dispersion in TFPR is actually lower than it is at the mean value of the money shock.

As indicated by the right panel, the price changes for the adjusters are less dispersed in the tails of the money shock distribution since sellers with less dispersed values of the productivity shock are induced to adjust their price when $x$ is either very high or low. In doing so, they respond largely to the common shock to the money stock, thus reducing the dispersion in TFPR.[32]

[32] So, in contrast to Vavra (2014), it seems that dispersion and frequency can move in the same direction for some monetary

Since real output increases with the money shock, the model implies that the standard deviation of TFPR is not a monotone function of economic activity when fluctuations are induced by money shocks. It can be lower in recessions and also lower in expansions when the money shocks take relatively extreme values. **Thus, the model can produce countercyclical dispersion in TFPR, for a given distribution of TFPQ, but only when money shocks are surprisingly large**.

This suggests an empirical exercise that goes beyond the traditional focus on correlations between output and the dispersion of TFPR. From this model, the effects of the money shock on output and $disp_R$ depend on whether the money shock is above or below its average value, here $x = 1$. To the extent fluctuations are driven by money shocks correlation of output and $disp_R$ ought to be positive conditioning on below average innovations and negative for above average.

Leaving aside the nonlinearities, the basic correlations are indicated in Table 2. Here the overall correlation of output and $disp_R$ is positive, contrary to data. The model does reproduce a reduction in the dispersion of price changes during expansions. The dispersion in employment is lower in the expansion. This is because the higher frequency of price changes reduces the response of employment to productivity. Finally, the frequency of adjustment is countercyclical, but again this is a nonlinear relationship, partly masked by looked at this correlation.

## 4.3 Shocks to the Mean of TFPQ

The final source of variation is the more standard shock to the average productivity, i.e. the mean of TFPQ, denoted $\mu_Q$. As before, the interest is in the cyclicality of the dispersion in TFPR, as reported in Table 2 for this case.

Figure 7 summarizes the findings. The horizontal axis measures the *ex post* realization of $\mu_Q$. The left panel shows the dispersion in TFPR and the right indicates the dispersion in price changes. These are shown for two different levels of $disp_Q$.

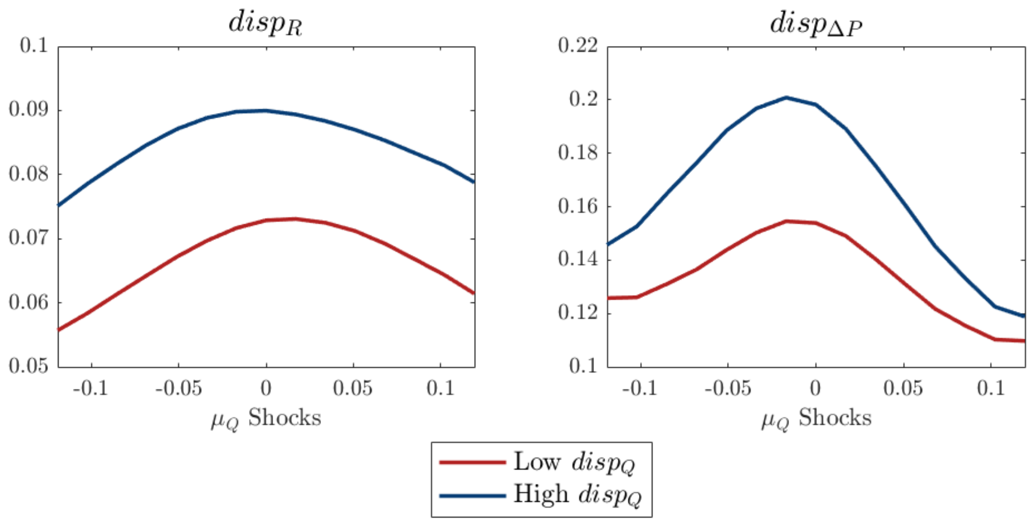
From the left panel, the relationship between $\mu_Q$ and $disp_R$ is not monotone. When the mean of TFPQ is either very large or very small, then $disp_R$ is lower. This again reflects the response of price setters. In the tails, the mean of TFPQ becomes a dominant force so that the dispersion of price changes falls. Note the asymmetry: the response of $disp_R$ is much larger for large realizations of $\mu_Q$ than for small ones. These patterns hold for both values of $disp_Q$.

As for the cyclical properties of a $\mu_Q$ shock, Figure 8 illustrates the effects of TFPQ shocks on output and employment, for two levels of TFPQ dispersion. Shocks to $\mu_Q$ are always procyclical. This is the case for both low and high dispersion in TFPQ.

Combining this with Figure 7, the relationship between output and the dispersion in TFPR is not monotone. If an economy with a relatively high aggregate TFPQ shock is compared with one with an average shock, then $disp_R$ appears to be procyclical. But if the comparison is between an average economy and one in a low $\mu_Q$ state, both output and dispersion are below average.

The right side of Figure 8 returns to the point about employment and productivity in modelprice models made in Galí (1999). Except for the lowest and highest productivity states, employment is falling productivity. This arises from the assumption that producers meet demand at the posted price. Hence sellers that do not adjust their price will decrease employment in the face of a productivity shock, be it aggregate or idiosyncratic. From Figure 8, this effect is dominating for most of the aggregate productivity states. But, in

shocks. Though clearly a reduction in $x$, increases the frequency but reduces the dispersion of change.




This figure shows the effects of shocks to $\mu_Q$, the mean of TFPQ, on the standard deviations of TFPR and price changes.

Figure 7: Effects of $\mu_Q$ shocks

the highest set of $\mu_Q$ realizations, there are enough sellers adjusting their prices in response to the aggregate shock, that employment increases with productivity.

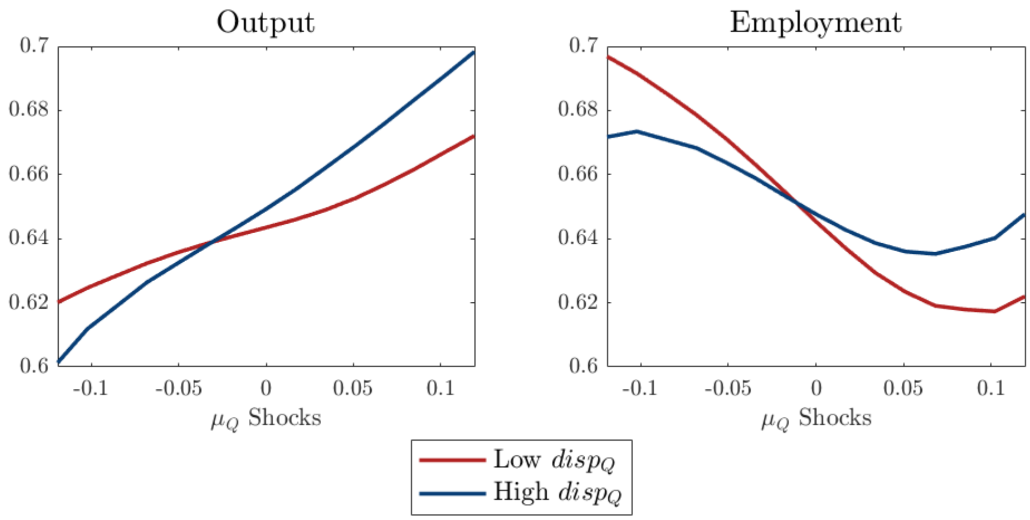
This interpretation is supported by behavior at the producer level. Table 3 reports regression results estimated from simulated data for three experiments characterized by the type of shocks: (i) idiosyncratic productivity shocks alone, (ii) idiosyncratic and aggregate productivity shocks and (iii) idiosyncratic and aggregate productivity and monetary shocks. The dependent variable is either the (log of) producer employment or output. The independent variation is the product of the idiosyncratic and the aggregate productivity shock, as in the revenue function.

For the employment column, the negative coefficients for those sellers choosing not to adjust their price indicate the role of these rigidities on the employment response. The negative effective is present, though weaker, even when monetary shocks are in the model. For the adjusters, the effect of productivity on employment is always positive.

The same is true for the output of adjusters: output expands with either productivity or money shocks. For non-adjusters, idiosyncratic productivity shocks have no output effects since demand is given. But, if there is a positive aggregate productivity shock then the sales of non-adjusters decline. Though nominal spending is held fixed, aggregate prices are lower so that a seller not adjusting its price has a high relative price and thus lower sales. When there is a money shock as well, the overall impact is to create a positive correlation of output and productivity, even for the non-adjusters.

Returning to the evidence, there are two issues. First, the aggregate response shown in Figure 7 clearly is dependent on the fraction of each type of producer, i.e. on the adjustment rate. This will determine the size of the two regions and the slope within each. At the aggregate level, the analysis points to a very non-linear response of employment to productivity, where the magnitude of the response depends on the underlying adjustment rate which itself is state dependent.

Second, at the plant level, there is ample evidence, for example in Decker, Haltiwanger, Jarmin, and Miranda (2019) that plant-level employment is increasing in profitability. To what extent this response




This figure shows the effects of shocks to $\mu_Q$ on output and employment.

Figure 8: Response to Aggregate Productivity Shocks

interacts with price setting at the plant-level remains an open question, made difficult to address by the lack of evidence on prices and employment at the micro-level.

| Shock | Employment | | Output | |
|---|---|---|---|---|
| | Adj. | No Adj. | Adj. | No Adj. |
| $z$ | 0.331 | -0.656 | 1.007 | 0 |
| $(z, A)$ | 0.472 | -0.847 | 1.164 | -0.172 |
| $(z, A, x)$ | 0.482 | -0.444 | 1.192 | 0.180 |

This table shows the effects of idiosyncratic,($z$), aggregate productivity, ($A$), and monetary shocks, $x$, on producer-level employment and output conditioning on price adjustment status.

Table 3: Dependence of Employment and Output on Productivity

Overall, our findings for the case of shocks to the mean of TFPQ are summarized in Table 2. The dispersion in TFPR is indeed countercyclical, as in the data, though the magnitude is very small. This is driven by the asymmetric response of the $disp_R$ to aggregate TFPQ shocks. The dispersion in price changes in also countercyclical, but again only slightly. Finally, the frequency of price adjustment is procyclical, counter to the data. **Thus shocks to the mean of TFPQ cannot match the data patterns.**

# 5 Extensions and Robustness

Here we undertake some extensions of the analysis. Given that the shocks, taken alone, are not able to match data patterns, a key extension looks at active monetary policy. A discussion of robustness is included as well.

## 5.1 Monetary Feedback Rules

From Figures 5 and 6, the effects of changes in the dispersion of profitability shocks, $disp_Q$, on the dispersion of TFPR, $disp_R$, were clearly dependent on the money shock. For example, from Figure 5, output actually falls slightly when there is an increase in $disp_Q$ along with a large positive money shock. And, from Figure 6, the effects of an increase in $disp_Q$ on $disp_R$ were quite small for extreme values of the money shock, particularly when money growth is low.

To study these interactions further, this sub-section allows some response by the monetary authority to the shocks to the mean and dispersion of TFPQ.[33] As we see, allowing the monetary authority to link the distribution of $x$ to the aggregate state can alter the cyclicality of $disp_R$. In this way, the implications of the model can be brought closer to some features of the data.

In particular, we highlight two cases which produce countercyclical $disp_R$ as well as countercyclical price changes. In the first, the economy is driven by fluctuations in $disp_Q$. When this dispersion is above average, the monetary authority intervenes so that the mean value of $x$ is below average, i.e. $\zeta < 0$. In the second, the economy is driven by fluctuations in $\mu_Q$. When aggregate productivity is above average, the monetary authority intervenes and selects a mean value of $x$ that is above average as well, i.e. $\zeta > 0$. In addition, following the evidence in Vavra (2014), only when there are $disp_Q$ shocks do we also produce countercyclical frequency of adjustment.

| Case | $corr(y, disp_R)$ | $corr(y, disp_{\Delta P})$ | $corr(y, disp_n)$ | $corr(y, freq)$ |
|---|---|---|---|---|
| Data | -0.45 | -0.41 | -0.50 | -0.27 |
| | | $disp_Q$ | | |
| $\zeta = 2.0$ | 0.2066 | 0.0089 | -0.3909 | 0.1470 |
| $\zeta = 1.3$ | 0.1296 | -0.0334 | -0.4602 | 0.1522 |
| $\zeta = 0$ | 0.4292 | -0.1348 | -0.3413 | -0.1695 |
| $\zeta = -1.3$ | -0.0093 | -0.0146 | -0.2517 | 0.0099 |
| $\zeta = -2.0$ | -0.0254 | 0.0065 | -0.1257 | -0.0812 |
| | | $\mu_Q$ | | |
| $\zeta = 2.0$ | -0.3044 | 0.0682 | -0.2829 | 0.4294 |
| $\zeta = 1.3$ | -0.0596 | -0.0868 | -0.3630 | 0.3130 |
| $\zeta = 0.0$ | -0.0183 | -0.0010 | -0.3863 | 0.1878 |
| $\zeta = -1.3$ | 0.2459 | -0.0865 | -0.3665 | 0.0320 |
| $\zeta = -2.0$ | 0.2340 | -0.0621 | -0.3505 | 0.1814 |

This table shows the correlation between output and the dispersion of TFPR, the dispersion and frequency of price changes and the dispersion of employment for different monetary feedback rules.

Table 4: Cyclical Variations: Monetary Feedback Rules

Specifically, suppose that the evolution of the money supply is given by:

$$M_{t+1} = M_t x_{t+1} = M_t[\phi(s_{t+1}) + \tilde{x}_{t+1}]. \tag{12}$$

In this specification, the money stock follows the same stochastic process as above, with $x_{t+1}$ representing the period $t+1$ money shock that is not predictable given period $t$ information. But here, the growth of the money supply, $[\phi(s_{t+1}) + \tilde{x}_{t+1}]$ has two components. The first is the feedback rule where $\phi(s_{t+1})$ allows

[33] In doing so, it makes clear the advantage of using an economy with valued money to study monetary policy.

money growth to depend on the period $t+1$ state of the economy. The second is the money shock, as above denoted $\tilde{x}_{t+1}$.

Following Table 2, we focus on two specific cases, distinguished by the source of fluctuations in the aggregate economy. In the first, the monetary authority responds to changes in the dispersion of TFPQ. Let $\mu_{disp_Q}$ be the average value of $disp_Q$ and consider

$$\phi(disp_Q) = \zeta(disp_Q - \mu_{disp_Q}). \tag{13}$$

In a similar fashion, let $\mu_{\mu_Q}$ be the average value of the mean of TFPQ and consider

$$\phi(\mu_Q) = \zeta(\mu_{\mu_Q} - \mu_Q). \tag{14}$$

In both formulations, the feedback is characterized by $\zeta$.

Given a monetary feedback rule, it is straightforward to extend the analysis of a SREE from sub-section 8.1 to include (12). Note that the monetary feedback rule impacts agents both as young price setters and as old agents, both in terms of the distribution of the stochastic transfer and the equilibrium prices they face as buyers. As in the previous analysis, all of the newly created money is distributed as a proportional transfer. But in this specification, it is feasible for the monetary authority to link these transfers to the current state of the economy. If prices were perfectly flexible, there would be no real effects of this monetary policy. Further, since private agents share the information of the monetary authority, there is no information transmitted to the private sector by this policy.

The SREE was characterized for both shocks to $\mu_Q$ and $disp_Q$, allowing both negative and positive responses by the monetary authority. The results are reported in Table 4 for a couple of values of $\zeta$. In addition, the moments calculated under the assumption of flexible prices is included for comparison. There are two cases that generate countercyclical dispersion in both TFPR and prices changes and thus match data patterns. Only one of these also creates countercyclical adjustment frequency.

Consider first the results when the economy is driven by variations in $dispQ$, along with money shocks. In this case, the only feedback rules that generate countercyclical dispersion in $dispR$ arise when the monetary authority sets $\zeta = -1.3$ and $\zeta = -2.0$. With this policy, the monetary authority responds to higher than average dispersion in idiosyncratic profitability shocks by reducing the average growth of the money supply. As output is positively correlated with $disp_Q$, the monetary authority appears to be leaning against the wind.

The mechanics are, in part, made clear by the panel in Figure 6 relating $disp_R$ to $x$. Since $disp_R$ is asymmetrically related to $x$, if the realized money shock (through the feedback rule) falls when the dispersion in TFPQ rises, then the dispersion in TFPR can fall. And, from Figure 5, the lower than average stock of money will imply higher output when $disp_Q$ is higher. Putting the pieces together, a high value of $disp_Q$ triggers, through the feedback rule, a lower realization for the money shock so that: (i) output on average is higher, (ii) $disp_R$ is lower and (iii) the dispersion in price changes in lower as well.[34] That is, while $disp_Q$ is procyclical, $\zeta < 0$ induces, through optimal pricing behavior a countercyclical $disp_R$.

This case matches other features of the data. For $\zeta = -1.3$ the model generates countercyclical dispersion in price changes. And at $\zeta = -2.0$, the frequency of price adjustment is countercyclical as well. But none of these correlations are as large as in the data moments taken from Vavra (2014).

[34]Here the statements are on average since the feedback influences the mean of the stochastic transfer, leaving some randomness.

Note that this result does not occur without monetary feedback. As noted earlier, with $\zeta = 0$ the model does match both the countercyclical dispersion in price changes and generates countercyclical frequency of price changes but it does not create procyclical dispersion in TFPR. Further, the result clearly requires state dependent pricing. If prices are flexible, then again the dispersion in TFPR is procyclical.

The lower block studies variations in aggregate TFPQ, denoted $\mu_Q$. If aggregate fluctuations are driven by a combination of shocks to the mean of TFPQ and monetary injections, the model is able to generate countercyclical $disp_R$ for $\zeta \geq 0$. And in some cases, the model also generates countercyclical variations in $disp_{\Delta p}$. But, in no cases is the frequency of adjustment countercyclical.

## 5.2 Alternative Parameters

| Case | $corr(y, disp_R)$ | $corr(y, disp_{\Delta p})$ | $corr(y, disp_n)$ | $corr(y, freq)$ |
|---|---|---|---|---|
| Data | -0.45 | -0.41 | -0.50 | -0.27 |
| | | Baseline | | |
| $disp_Q, \zeta = 0$ | 0.4292 | -0.1348 | -0.3413 | -0.1695 |
| $disp_Q, \zeta = -1.3$ | -0.0093 | -0.0146 | -0.2517 | 0.0099 |
| $mu_Q, \zeta = 0.0$ | -0.0183 | -0.0010 | -0.3863 | 0.1878 |
| $mu_Q, \zeta = 2.0$ | -0.3044 | 0.0682 | -0.2829 | 0.4294 |
| | | High Product Substitutability | | |
| $disp_Q, \zeta = 0$ | 0.2020 | 0.0412 | -0.2066 | 0.0822 |
| $disp_Q, \zeta = -1.3$ | 0.0842 | 0.0812 | -0.1555 | 0.1272 |
| $mu_Q, \zeta = 0$ | 0.1656 | -0.2432 | -0.4949 | 0.2213 |
| $mu_Q, \zeta = 2.0$ | -0.1669 | -0.3750 | -0.5912 | 0.5640 |
| | | High Labor Supply Elasticity | | |
| $disp_Q, \zeta = 0$ | 0.0348 | 0.0889 | -0.2521 | 0.1785 |
| $disp_Q, \zeta = -1.3$ | -0.1156 | 0.0258 | -0.2493 | 0.0161 |
| $mu_Q, \zeta = 0$ | 0.0770 | -0.0124 | -0.2388 | 0.0667 |
| $mu_Q, \zeta = 2.0$ | -0.2111 | -0.1221 | -0.4239 | 0.3641 |
| | | Larger $disp_Q$ Variation | | |
| $disp_Q, \zeta = 1.3$ | 0.3871 | 0.3855 | 0.4300 | 0.4417 |
| $disp_Q, \zeta = 0$ | -0.0065 | 0.0274 | 0.0521 | 0.1142 |
| $disp_Q, \zeta = -1.3$ | -0.4568 | -0.4300 | -0.4232 | -0.3693 |

This table shows moments for different monetary policy rules at the baseline and alternative parameter values.

Table 5: Cyclical Variations: Robustness

This sub-section looks at the robustness of our findings with respect to the parameterization of the problem. There are two key parameters: (i) the elasticity of substitution between products, $\varepsilon$ and (ii) the convexity in the disutility of work, $g(n) = \frac{n^{1+\phi}}{1+\phi}$. The point is to understand how the shapes of these functions impact the results in Table 4.[35]

The baseline model has a quadratic disutility of work so that $\phi = 1.0$ and $\varepsilon = 3$. In the high elasticity of substitution case, $\varepsilon = 4$. With products more substitutable, a sellers whose price is, for example, very high compared to competitors will lose a lot of sales and this will create an incentive for price adjustment. So all else the same, monetary shocks will have smaller effects on output. Further, in the case of an increase in the dispersion of productivity, there will be a larger output gain since demand and therefore production is more easily reallocated to high productivity production sites.

[35] Vavra (2014) has linear disutility of work and an elasticity of substitution of 6.8. Golosov and Lucas (2007) also have linear disutility with an elasticity of substitution of between 6 and 10. They also include money in the utility function.

The high labor supply elasticity sets $\phi = 0.4$. The reduction in the curvature of the disutility, towards being more linear, makes the marginal cost of production less variable. This reduces the probability of adjustment as the cost of not adjusting the price is lower. It also impacts the price chosen by adjusters.

The last two rows of Table 1 present the pricing moments for these two alternative parameterizations. Indeed, with higher product substitutability, price adjustment is more frequent and the correlation of output and the money shock is lower. The dispersion of price changes is also lower since sellers respond more to common than idiosyncratic shocks.

From Table 1, equilibrium outcomes with a lower $\phi$ looks more like the baseline. It is important to keep in mind that $\phi$ has no direct impact on the output (employment) response of sellers who do not adjust their price. These sellers, by assumption, meet demand. The lower $\phi$ reduces the adjustment rate since the marginal cost of meeting fluctuations demand is lower, as shown in Table 1.

A final robustness check looks at an alternative parameterization in the $disp_Q$ case. The baseline specification set the process for $disp_Q$ to match the magnitude of changes in the dispersion of TFPR reported in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) in an economy with multiple shocks, including those to the mean of TFPQ.[36] Here we allow only shocks to the standard deviation in $z$ as well as monetary shocks and set the distribution of the standard deviation of the idiosyncratic shocks to match the changes in the dispersion in TFPR. The result leads to more dispersion in the standard deviation of $z$ relative to the baseline.

| Case | $corr(x, disp_R)$ |
|---|---|
| Data | 0.045 |
| $disp_Q$ Shocks | |
| Baseline | |
| $\zeta = 1.3$ | 0.8534 |
| $\zeta = 0$ | 0.1083 |
| $\zeta = -1.3$ | -0.7367 |
| Larger $disp_Q$ Variation | |
| $\zeta = 1.3$ | 0.3857 |
| $\zeta = 0$ | -0.0257 |
| $\zeta = -1.3$ | -0.4790 |

This table shows the correlations between $disp_R$ and monetary innovations for different monetary policy rules at the baseline and alternative parameter values for $disp_Q$ shocks.

Table 6: Correlation between x and $disp_R$

Table 5 shows the resulting patterns of correlations for these cases. The top panel reproduces the patterns from the baseline parameters and the others are the three experiments.

In terms of fitting the moments, the increased competitiveness through higher product substitutability never produces countercyclical adjustment frequencies. Only when there are shocks to the mean of TFPQ and $\zeta = 2$ is the dispersion of TFPR countercyclical. In this case, so is the dispersion of price changes.

With a high labor supply elasticity, the model produces countercyclical dispersion in TFPR for both $disp_Q$. In some of these cases, the dispersion of price changes is also countercyclical but the frequency remains procyclical throughout.

[36] For further detail see Appendix sub-section 8.3.3.

For the final specification with higher dispersion in the standard deviation of $z$, the model seems to match the moments. For $\zeta = -1.3$, all of the key variables are countercyclical, as is the dispersion of employment.

Given this "success" we explore the issue of how well this monetary feedback matches the data. Table 6 displays the correlation between the money shock, $x$, and the dispersion of TFPR, $disp_R$. Here the focus in on $disp_R$ rather than $disp_Q$ as the former is observed.

For Table 6, the data row is calculated as follows. First, $x$ refers to the monetary policy shocks proxied with the high frequency/narrative shock series of Miranda-Agrippino and Ricco (2018). This series proxies for the changes in monetary policy which are not captured by the endogenous component reacting to the output or inflation gap. Second, following Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), Decker, Haltiwanger, Jarmin, and Miranda (2019) we identify TFPR shocks using a cost-share approach at the firm level.[37] We calculate $disp_R$ at the sectoral level by year and obtain annual $disp_R$ by averaging over the sectoral dispersion each year. Finally, the correlation between $x$ and $disp_R$ is calculated.

For the baseline parameterization, if the feedback rule entails $\zeta < 0$, then the correlation of monetary innovations and $disp_R$ is negative. That is, the negative correlation between $x$ and $disp_Q$ created by the feedback rule carries over to the correlation of the monetary innovation and $disp_R$. As indicated in the two bottom panels of the table, this pattern is sustained in parameterizations where the variation in $disp_Q$ is larger. Notably, in the case of the larger variation in $disp_Q$, the correlation is strongly negative and thus further from the data. In this sense, the monetary policy that generates moments qualitatively matching the data in Table 5, are far from the monetary innovations seen in the data.

This finding is strengthened by computing the $corr(x, disp_R)$ for a range of values of $\zeta \in \{-0.1, -2.5\}$. The results are shown in Figure 9. Clearly, for none of these values was the $corr(x, disp_R)$ computed from the simulated data positive.

## 5.3 Correlated Shocks

In many studies, such as Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Vavra (2014) the shock to dispersion and to the mean of TFPQ are studied jointly. Given the prominence of this case in the literature, it is important to study this case in detail. Here we follow the baseline model in Vavra (2014) and assume the shocks are perfectly negatively correlated: $corr(disp_Q, \mu_Q) = -1$.

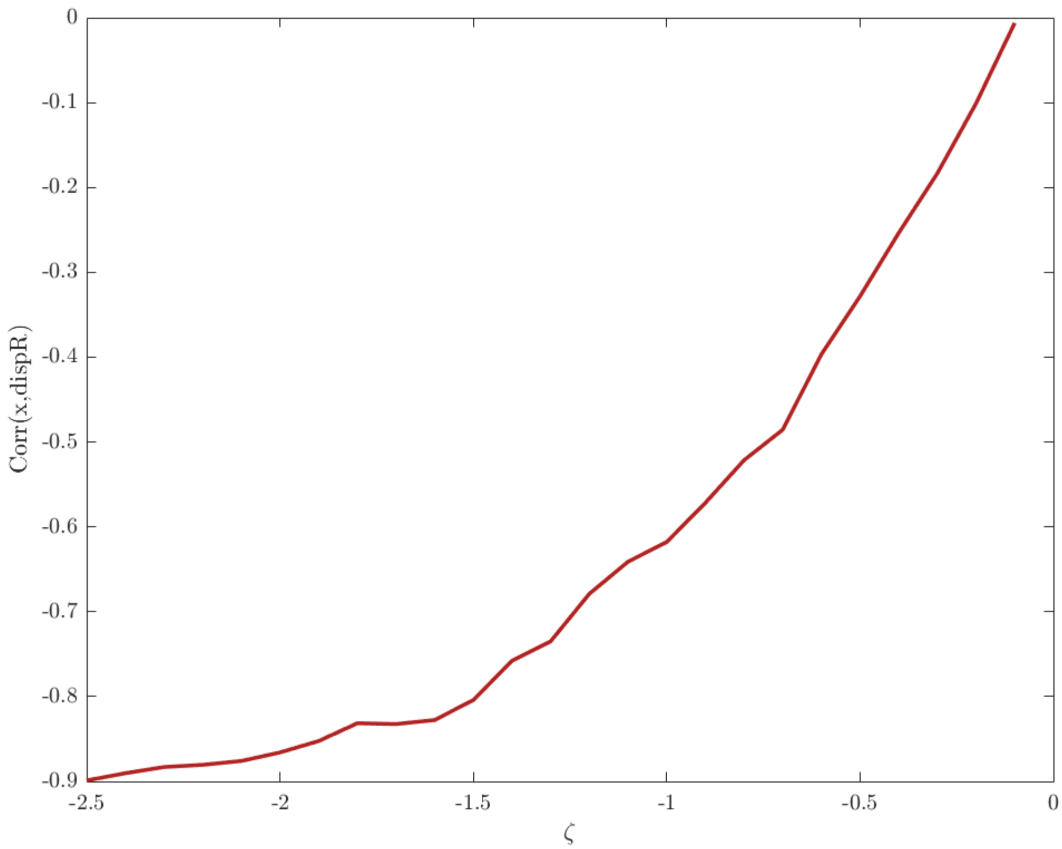
The moments from this exercise are shown in the bottom panel of Table 7. There are two cases. One in which both $disp_Q$ and $\mu_Q$ took on the same three values as in the baseline model and using the values for $disp_Q$ from the "Larger $disp_Q$ variation" parameterization. We also vary the size of $\mu_Q$ variation relative to the baseline in the last two blocks.[38] For each, we assume two values of $\zeta$.

Of the cases explored, the ones that most closely matches the four data moments occur when the variation in $disp_Q$ is large relative to $\mu_Q$ and, once again, $\zeta < 0$. In these two cases the correlations are more strongly negative compared to the model with large $disp_Q$ variations alone. When $\zeta = 0$, the models do not generate countercyclical variation in the frequency of price adjustment.

But again, looking at the correlations in simulated data between the monetary innovations and $disp_R$, reported in Table 8, the monetary policy needed to match these moments is again counter to the data. That is, with $\zeta < 0$, the correlation of the monetary innovation and $disp_R$ is negative.

[37] This is based upon a Compustat annual sample from 1971 to 2018.

[38] The exact values and relationship to the literature are reported in Appendix sub-section 8.3.3.




This figure shows the correlation between $x$ and $disp_R$ for $\zeta \in \{-0.1, -2.5\}$ from the baseline parameterization.

Figure 9: $x$ and $disp_R$ Correlation

# 6 Other Implications

This section looks at other implications of the model. The first is the interaction between the dispersion of productivity shocks and the impact of monetary policy. The second is the effects of uncertainty rather than dispersion on pricing.

## 6.1 Real Dispersion and the Effects of Monetary Policy

This section continues to study the interaction between money shocks and TFPR dispersion. But instead of asking whether money shocks can create TFPR dispersion, here we study how the real effects of money depends on TFPQ dispersion.

There are two important empirical findings that guide this discussion. First, Vavra (2014) argues that the dispersion of price changes is countercyclical as is the frequency of price adjustment. Second, Tenreyro and Thwaites (2016), output is less responsive to monetary policy during recessions.

Putting these pieces together, if TFPR dispersion is countercyclical than recessions are associated with more frequent price adjustment and thus a smaller impact of monetary policy. This leads Vavra (2014) to argue that shocks to nominal spending will have a smaller effect on output when the dispersion of firm level

| Case | $corr(y, disp_R)$ | $corr(y, disp_{\Delta p})$ | $corr(y, disp_n)$ | $corr(y, freq)$ |
|---|---|---|---|---|
| Data | -0.45 | -0.41 | -0.50 | -0.27 |
| | | Baseline | | |
| $disp_Q, \zeta = 0$ | -0.1945 | -0.2621 | -0.4789 | 0.1378 |
| $disp_Q, \zeta = -1.3$ | 0.0181 | -0.4690 | -0.5616 | -0.1323 |
| | | Larger $disp_Q$ | | |
| $disp_Q, \zeta = 0$ | -0.6740 | -0.6255 | -0.8448 | 0.8450 |
| $disp_Q, \zeta = -1.3$ | -0.7807 | -0.6856 | -0.6101 | -0.5894 |
| | | Smaller $mu_Q$ | | |
| $disp_Q, \zeta = 0$ | -0.2222 | 0.0389 | -0.0553 | 0.1655 |
| $disp_Q, \zeta = -1.3$ | -0.5453 | -0.4618 | -0.5225 | -0.3950 |
| | | Larger $mu_Q$ | | |
| $disp_Q, \zeta = 0$ | -0.8533 | -0.3893 | -0.6577 | 0.5959 |
| $disp_Q, \zeta = -1.3$ | -0.8574 | -0.3119 | -0.6622 | 0.3344 |

This table shows moments for different mixtures of $disp_Q$ and $\mu_Q$ shocks as well as monetary policy rules at the baseline and alternative parameter values.

Table 7: Cyclical Variations: Combining Shocks

productivity is higher.

We use our model, with its explicit distinction between TFPQ and TFPR, to study the effects of monetary shocks. The question is whether the real impact of these shocks is lower when $disp_R$ is higher, given that this dispersion is endogenous.[39]

As we have already seen, changes in the distribution of $z$ will influence price setting and thus will the impact of monetary policy in a SREE. Intuitively, more variability in the distribution of $z$ implies that price adjustment, given a monetary shock, is more likely and thus the real effects of the monetary shock will be reduced. This is a variant of the point made by Vavra (2014).

This is illustrated in Figure 6 which compares low and high dispersion cases. Here the focus is on the response of real output and prices to $x$, rather than on the shifts in these curves due to variations in TFPQ dispersion. Clearly the frequency of adjustment is higher when the dispersion of $z$ is higher. From the right diagram, prices are more responsive to monetary shocks in the high uncertainty case so that output, left bottom, is less responsive.

Table 9 quantifies the effects of changes in dispersion on the response of output to a monetary innovation. It does so by regressing the log of real GDP on the (log of the) monetary shock. From that table, the response of output to a monetary innovation is 9% points higher in the low $disp_Q$ case. The bottom part of Table 9 shows the complementary effects of money shocks on prices.

Does this analysis support the finding of Tenreyro and Thwaites (2016) on the cyclical effectiveness of monetary policy? It would iff recessions were associated with large dispersion in TFPR. But, as discussed above, the sources of aggregate fluctuations studied here, particularly variations in $disp_Q$, do not generate countercyclical dispersion in TPPR.

[39] In contrast to Vavra (2014), we do so in a model with exogeneous variations in money rather than nominal spending "shocks".

| Case | $corr(x, disp_R)$ |
|---|---|
| Data | 0.045 |
| $disp_Q, \mu_Q$ | |
| Baseline | |
| $\zeta = 1.3$ | 0.8361 |
| $\zeta = 0$ | 0.0765 |
| $\zeta = -1.3$ | -0.6732 |
| Larger $disp_Q$ Variation | |
| $\zeta = 1.3$ | 0.0796 |
| $\zeta = 0$ | -0.3834 |
| $\zeta = -1.3$ | -0.5993 |
| Smaller $mu_Q$ Magnitudes | |
| $\zeta = 1.3$ | -0.0252 |
| $\zeta = 0$ | -0.1963 |
| $\zeta = -1.3$ | -0.5351 |
| Larger $mu_Q$ Magnitudes | |
| $\zeta = 1.3$ | 0.2017 |
| $\zeta = 0$ | -0.1058 |
| $\zeta = -1.3$ | -0.5733 |

This table shows the correlations between $disp_R$ and monetary innovations for different monetary policy rules at the baseline and alternative parameter values when both $disp_Q$ and $\mu_Q$ shocks are present.

Table 8: Correlation between x and $disp_R$

| | Constant | Slope |
|---|---|---|
| Output Response | | |
| Low $disp_Q$ | -0.4298 | 0.8866 |
| High $disp_Q$ | -0.4381 | 0.7993 |
| Price Response | | |
| Low $disp_Q$ | 0.4303 | 0.1167 |
| High $disp_Q$ | 0.4392 | 0.2087 |

Table 9: Regression of output and prices on log(x)

## 6.2 Effects of Uncertainty

The distinction between uncertainty and dispersion is often blurred. The main effect of uncertainty, again expressed in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), is to create an incentive to wait and allow the uncertainty to be resolved. To the extent this leads to a decrease in spending, largely on durables, the uncertainty can be recessionary. This is often quite different from the positive effects of dispersion which can lead to an expansion in output, as discussed above.

The previous discussion highlighted the effects of dispersion on the frequency of price adjustment and thus the real effects of monetary shocks. Here we focus on how *ex ante* price and *ex post* respond to uncertainty over a distribution, not the realization of that change.

Our analysis includes distributions over three dimensions: (i) idiosyncratic productivity, (ii) money transfers, and (iii) aggregate productivity. Thus in principle one can study the effects of uncertainty with respect to each of these three distributions.

To do so, it is natural to create a Markov switching process for the dispersion of, say, idiosyncratic productivity. Price setters in period $t$ would know the distribution of these shocks last period but in setting their *ex ante* price, the period $t$ distribution, as well as that for period $t+1$ would not be known. Further, for those who adjust *ex post*, the uncertainty would remain over the distribution in the following period when they are consumers.[40] This is the nature of the uncertainty.

One extreme version of this Markow switching process is for the dispersion to be permanently high (low). It turns out that for the price setting problem of young agents, the *ex ante* price is essentially the same with high dispersion of the idiosyncratic productivity shock as it is for the low dispersion case. In fact, this is true when the uncertainty is over the money transfer or the aggregate productivity distributions.

Given this, it is unlikely that *ex ante* uncertainty matters for the price setting problem. This is verified explicitly for the case of uncertainty over idiosyncratic productivity. Even if there is a positive probability of a regime shift in the distribution of $z$, the *ex ante* price is essentially unchanged.

This is an important finding. It makes clear that the effects come from dispersion not uncertainty. This is consistent with Berger, Dew-Becker, and Giglio (2020) who argue, at least for aggregate shocks, that uncertainty *per se*, had a negligible effect on real activity.

# 7 Conclusion

The analysis characterizes the properties of TFPR in a stationary rational expectations equilibrium in a monetary economy with state dependent pricing. A quantitative version of the model is used to determine the cyclicality of the dispersion in TFPR as well as other key pricing moments, the cyclicality of both the frequency of price changes and their dispersion. This is studied by determining pricing decisions and thus the distribution of TFPR in the face of aggregate shocks to: (i) the dispersion of TFPQ, (ii) the money supply, (iii) the mean of TFPQ. These are very conventional shocks for an aggregate economy, with recent attention given to variations in the dispersion of TFPQ.

The findings are not supportive of the view that variations in the dispersion of TFPQ drive countercyclical variations in TFPR. As indicated in the analysis, this can only arise from a particular form of monetary intervention: the money supply innovation must be negatively correlated with variations in the dispersion of TFPQ. This is the case even when the dispersion shock is accompanied with an offsetting change in the mean of TFPQ. Absent such monetary interventions, dispersion in TFPQ and TFPR are procyclical, reflecting the gains to reallocation associated with increased productivity dispersion.

Focusing on the case in which the model can reproduce both countercyclical TFPR and match pricing patterns, the paper provides evidence that the resulting monetary policy is not consistent with the data. In particular, the correlation between monetary innovations and the dispersion in TFPR is slightly positive. But, in order to match moments, the model requires this correlation to be quite negative.

Admittedly these results are suggestive rather than definitive. The OG model, with only one period of price setting, misses some of the forward looking aspect of price adjustment. But, as argued in the text, the pricing behavior in the model is similar to that produced by other state dependent pricing models. On the data side, it would be desirable to have higher frequency observations on both prices and quantities upon which to base a structural estimation exercise.

Throughout these exercises, one theme emerges: non-linearities in the response of the economy to monetary and dispersion shocks. Regardless of the source of aggregate fluctuations, the dispersion of TFPR is

[40] Thus the expectation on the left side of (16) is extended to include the conditional expectation over the future dispersion.

generally lowest for extremely low and high realizations and highest for the average state. This property of the model, driven by the U-shaped response of the frequency of price changes to money surprises, makes it useful to study the impact of monetary and productivity shocks using non-linear statistical models.

Finally, the model is used to study the effects of uncertainty on pricing. It seems clear that the effects highlighted in our analysis stem from dispersion not uncertainty. One interesting extension of our model would be to include some of the adjustment cost structure that creates a real options effect, as in Bloom (2009), **coupled with** state dependent pricing.

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# 8 Appendix

## 8.1 SREE: Linear Quadratic

For the case of linear quadratic preferences, the SREE defined in section 2.3 becomes a set of functions $(\bar{p}(M), \tilde{p}(M,z,x), F^*(M,x,z), P(M,x))$ such that:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M,x)$;

$$\hat{\varepsilon}\bar{p}(M)E_{x,x'}\left[\frac{x'}{P(Mx,x')}d(\bar{p}(M),M,x))\right] = \frac{(E_{z,x}d(\bar{p}(M),M,x))^2}{z^2}. \tag{15}$$

- $\tilde{p}(M,z,x)$ solves the *ex post* pricing problem given the state dependent price index $P(M,x)$

$$\hat{\varepsilon}\tilde{p}(M,z,x)E_{x'}(\frac{x'}{P(Mx,x')}) = \frac{d(\tilde{p}(M,z,x),M,x)}{z^2}. \tag{16}$$

- At the critical adjustment cost $F^*(M,x,z)$, the seller is just indifferent between adjusting and not: $F^*(M,x,z) = W^a(M,z,x) - W^n(M,z,x)$
- $P(M,x)$ is the aggregate price function in state $(M,x)$ given by:

$$P(M,x) = \Big[E_z(1-\Omega(F^*(M,x,z))\bar{p}(M)^{(1-\epsilon)} + E_z\Omega(F^*(M,x,z))\tilde{p}(M,z,x)^{(1-\epsilon)}\Big]^{\frac{1}{1-\epsilon}}. \quad (17)$$

Throughout, again, $d(\bar{p}(M),M,x) = (\frac{\bar{p}(M)}{P(M,x)})^{-\varepsilon}Y$ and $d(\tilde{p}(M,z,x),M,x) = (\frac{\tilde{p}(M,z,x)}{P(M,x)})^{-\varepsilon}Y$ and $Y = \frac{Mx}{P(M,x)}$.

The analysis builds on this case to add in both $disp_Q$ and $\mu_Q$ variations. The SREE for that more general model is defined in sub-section 8.2. The linear-quadratic economy is constructed from that definition using the linear-quadratic functional form.

## 8.2 Generalized Definition of SREE

Here the definition of a stationary rational expectations equilibrium is generalized to include shocks to the distribution of plant-level productivity through $\mu_Q$ and $disp_Q$. Let $S = (x, \mu_Q, disp_Q)$ be the aggregate state and $s = (z, F)$ be the idiosyncratic state.[41] As earlier, $M$ is the previous money stock and thus is known at the time prices are chosen *ex ante*. This definition is for the linear-quadratic economy.

A SREE is a set of price functions $(\bar{p}(M), \tilde{p}(M,S,s), P(M,S))$, value functions $(W^n(M,S,s), W^a(M,S,s))$, and a critical value of the price adjustment cost, $F^*(M,S,s)$ satisfying: (i) individual optimization by young price setters and old consumers, (ii) market clearing and (iii) consistency of beliefs and expectations for all states. These conditions can be written:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M,S)$;

$$\bar{p}(M) = argmax_p E_{S,z,S'} V((R(p,P(M,S),Mx)x')/P(Mx,S')) - g(\frac{d(p,P(M,S),Mx)}{z}) \quad (18)$$

  for all $M$.

- $\tilde{p}(M,S,s)$ solves the *ex post* pricing problem:

$$\tilde{p}(M,S,s) = argmax_p E_{S'} V\left((R(p,P(M,S),Mx))x'/P(Mx,S')\right) - g(\frac{d(p,P(M,S),Mx)}{z}). \quad (19)$$

  given $P(M,S)$, for all $(M,S,s)$;

- At the critical adjustment cost, $F^*(M,S,s)$, the seller is just indifferent between adjusting and not:

$$F^*(M,S,s) \equiv W^n(M,S,s) - W^a(M,S,s)$$

  for all $(M,S,s)$, with $W^a(M,S,s)$ given by:

$$W^a(M,S,s) = E_{S'} V\left((R(\tilde{p}(M,S,s),P(M,S),Mx))x'/P(Mx,S')\right) - g(\frac{d(\tilde{p}(M,S,s),P(M,S),Mx)}{z}). \quad (20)$$

[41] So here the notation is different from that in the text to be more explicit about aggregate and idiosyncratic variables.

and $W^n(M,S,s)$ given by

$$W^n(M,S,s) = E_{S'}V((R(\bar{p}(M),P(M,S),Mx))x'/P(Mx,S')) - g(\frac{d(\bar{p}(M),P(M,S),Mx)}{z}). \quad (21)$$

- $P(M,S)$ is the aggregate price index in state $(M,S)$ given by:

$$P(M,S) = [E_z(1-\Omega(F^*(M,S,s)))\bar{p}(M)^{1-\varepsilon} + E_z(\Omega(F^*(M,S,s))\tilde{p}(M,S,s)^{1-\varepsilon})]^{\frac{1}{1-\varepsilon}} \quad (22)$$

where $d(\bar{p}(M),P(M,S),Mx) = (\frac{\bar{p}(M)}{P(M,S)})^{-\varepsilon}Y$ and $d(\tilde{p}(M,S,z),P(M,S),Mx) = (\frac{\tilde{p}(M,S,z)}{P(M,S)})^{-\varepsilon}Y$. Here $Y = \frac{Mx}{P(M,S)}$ is the equilibrium determined real value of money holdings.

## 8.3 Quantitative Approach

We first discuss how a SREE is computed and then present the various shocks and parameterization used in the analysis. For the price setting component of the SREE, all of the state variables are exogenous except for the aggregate price level, $P(M,S)$.[42] In contrast, the aggregate price level is an equilibrium object, and is therefore calculated from the choices of the sellers, as in (17). Thus the focus of the solution approach is to find the equilibrium price function, $P(M,S)$.

### 8.3.1 Computational Algorithm

**Step 1** Start with an initial guess of the aggregate price function, $P^{(0)}(M,S)$. This is a 3-dimensional matrix, since aggregate state variable set includes 3 elements $S = (x, \mu_Q, disp_Q)$ .

**Step 2** Calculate the new implied aggregate price function, $P^{(1)}(M,S)$, by solving the system. Specifically,

1. Solve the nonlinear system governed by (20) and (21). Note that (21) is not an independent equation *per se*, but a set of equations for each point in the state space. Solution to the system yields *ex ante* price, $\bar{p}(M)$ and *ex post* prices, $\tilde{p}(M,S,s)$ .

2. Using the *ex ante* $\bar{p}(M)$ and *ex post* prices $\tilde{p}(M,S,s)$, calculate the values of adjustment $W^a(M,S,s)$ and non-adjustment $W^n(M,S,s)$, given by (20) and (21) respectively, for each point in the state space.

3. Compare the values of adjustment $W^a(M,S,s)$ and non-adjustment $W^n(M,S,s)$ for each point in the state space, and store the maximum value of each case and also store whether adjustment or non-adjustment yield this maximum value.

4. Given the decisions about adjustment, pick the corresponding price (*ex ante* or *ex post*) and construct the realized price matrix for each point in the state space.

5. Given the probability of occurrences of each idiosyncratic state, calculate the new aggregate price matrix, $P^{(1)}(M,S)$ for each point in the aggregate state space.

[42] See Section 8.2 for the components of state variable set, along with the full definition of SREE.

**Step 3** If the distance between $P^{(0)}(M,S)$ and $P^{(1)}(M,S)$ is within the error tolerance level, the aggregate price function converges, which yields the aggregate price function and price policy functions. If not return to Step 1, setting the initial guess to the updated aggregate price function, *i.e.* $P^{(0)}(M,S) = P^{(1)}(M,S)$. Keep iterating until the aggregate price function converges.

Note that there is no approximation involved here. The approach simply solves a system of equations to find a SREE. So unlike an approach based upon Krusell and Smith (1998), there are no moments *per se* used to characterize an equilibrium.

### 8.3.2 Shocks

**Idiosyncratic productivity shock** In the model, idiosyncratic productivity shocks (TFPQ shocks) are one of the main sources of heterogeneity. In the baseline scenario, firms perfectly know the distribution which they are going to draw their productivities. Particularly, idiosyncratic productivity shocks have a mean of 1 and a standard deviation denoted by $\sigma_z$.[43] We employ Rouwenhorst algorithm to produce a Markov matrix for a *given* value of standard deviation of $z$ and mean of $z$.

Furthermore, as we discuss below, in the later steps, we relax the assumption of agents knowing the distribution which they are going to draw. In this setup, standard deviation $z$ ($disp_Q$ shocks) and mean of $z$ ( $\mu_Q$ shocks) are now stochastic processes, as well. Therefore, when $disp_Q$ and $\mu_Q$ shocks are imposed, firms do not know not only the value they will draw as their idiosyncratic productivity, but also the dispersion and the mean of the distribution.

$disp_Q$ **Shock** When imposed, the spread of idiosyncratic productivity distribution itself becomes a stochastic process. $disp_Q$ shocks change the dispersion of idiosyncratic productivity shocks for everyone, therefore is an aggregate shock. When $disp_Q$ shocks applied, agents *ex ante* do not know whether they are going to draw their idiosyncratic productivity from a wider or a narrower distribution.

$\mu_Q$ **Shock** $\mu_Q$ shocks move the mean value of TFPQ, and similar to $disp_Q$ shock, when imposed *ex ante* agents do not know the mean value TFPQ distribution. This shock is included to capture aggregate productivity shocks.

**Combined** $\mu_Q$ **and** $disp_Q$ **Shock** In Table 7, to impose perfectly negatively correlated combined shocks of $disp_Q$ and $\mu_Q$, first we decrease the number of states in $\mu_Q$ to be equal the number of $disp_Q$ states, 3. Therefore, as in Vavra (2014) we associate the highest state of $disp_Q$ to the lowest state of $\mu_Q$, and *vice versa.*

**Menu Cost** Firms are heterogeneous due to the realization of firm-specific price adjustment costs. Furthermore, firm heterogeneity stems not only from the realizations of menu costs, but we also impose another form of heterogeneity in the distribution of menu costs. A small fraction $\psi$ of firms face zero price adjustment cost and thus have perfectly flexible prices. The remaining fraction $1-\psi$ draws from a nondegenerate distribution of adjustment cost. Figure 10 exhibits the shape of menu cost distribution.

The menu cost distribution follows Dotsey and Wolman (2019), using a tangent function given by:

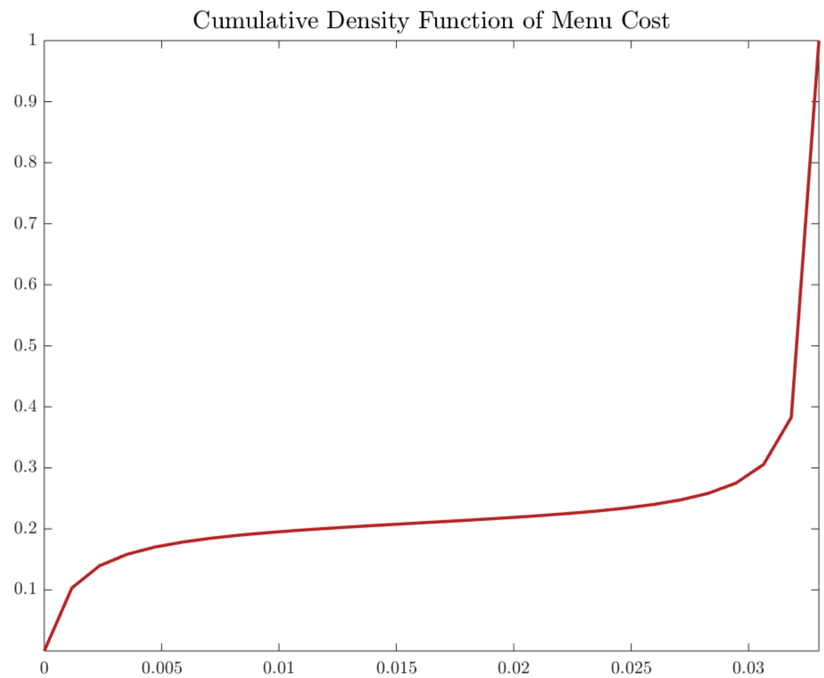
$$G(F) = \frac{1}{\omega}\left\{\tan\left(\frac{F-\kappa_2}{\kappa_1}\right) + \nu\cdot\pi\right\} \tag{23}$$

[43] Given the one period nature of price setting, there is no gain to specifying the AR(1) for these shocks.

with

$$\kappa_1 = \frac{\bar{F}}{[\tan^{-1}(\omega - \nu \cdot \pi) + \tan^{-1}(\nu \cdot \pi)]}; \qquad \kappa_2 = \arctan(\nu \cdot \pi) \cdot \kappa_1. \tag{24}$$

The upper bound on the fixed cost, $\bar{F}$, controls the extent of price stickiness. As $\bar{F}$ increases, higher values for menu cost is now available, making the adjustment harder. The curvature parameters $(\omega, \nu)$, are chosen so that $G(F)$ is monotonically increasing. As noted above, $\psi$ governs the fraction of flexible-price firms, and thus increasing this value leads to a larger number of small price changes and a higher overall frequency of price adjustment. Corresponding values can be found in Table 10.




This figure shows the non-degenerate distribution of price adjustment costs.

Figure 10: Menu Cost Distribution

### 8.3.3 Parameters

Standard deviations of idiosyncratic productivity, $disp_Q$ are parameterized to match some related the unconditional movements in the dispersion of TFPR moments reported in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Foster, Haltiwanger, and Syverson (2008). Our calibration strategy applies when all the shocks, $(disp_Q, x, \mu_Q)$ are imposed, however results do not significantly change, when we calibrate the model to separately include shocks to $(disp_Q, x)$ or $(\mu_Q, x)$.

More elaborately, we tried to match: **(i)** the mean value of $disp_R$: 0.08 taken from Foster, Haltiwanger, and Syverson (2008), and **(ii)** the standard deviation of idiosyncratic productivity shocks in the low dispersion state of 0.051 taken from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and **(iii)** the ratio of high volatility to low volatility of 4.1 taken from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). We choose our three $disp_Q$ values to match these moments. However, given the limitations of the model it is

| Parameter | Value | Description | Source |
|---|---|---|---|
| Utility Parameters | | | |
| $\epsilon$ | 3 | Elasticity of substitution between products | see sub-section 5.2 |
| $\phi$ | 1 | Elasticity of labor supply | see sub-section 5.2 |
| Dispersion | | | |
| Low $disp_Q$ | 0.051 | Volatility in the low idiosyncratic state | Bloom, Floetotto, Jaimovich, Eksten, ar |
| $\frac{\text{High } disp_Q}{\text{Low } disp_Q}$ | 4.1 | High $disp_Q$ to Low $disp_Q$ ratio | Bloom, Floetotto, Jaimovich, Eksten, ar |
| Mean $disp_R$ | 0.08 | Mean value of dispersion in TFPR | Foster, Haltiwanger, and Syverson (2008 |
| Low dispersion of $\mu_Q$ | 0.0067 | Volatility in the low aggregate state | Bloom, Floetotto, Jaimovich, Eksten, ar |
| Dispersion Ratio, $\mu_Q$ | 1.6 | The ratio of high to low volatility in the aggregate state | Bloom, Floetotto, Jaimovich, Eksten, ar |
| Menu Cost Distribution | | | |
| $\psi$ | 0.053 | Probability of zero menu cost | Dotsey and Wolman (2019) |
| $\bar{F}$ | 0.033 | Upper bound on menu cost | Dotsey and Wolman (2019) |
| $\omega$ | 41.9 | Curvature parameter | Dotsey and Wolman (2019) |
| $\nu$ | 2.8 | Curvature parameter | Dotsey and Wolman (2019) |

Table 10: Parameterization

not possible to closely match all these moments simultaneously. Under the baseline scenario, the calibrated values of $disp_Q = \{0.0571, 0.1284, 0.2111\}$ can match 0.0917 as the mean value of $disp_R$, and 0.0447 as the low volatility state of $disp_R$, and 3.3 as the ratio of high to low volatility.

In order to match the data moments, we relax first and second targets, and keep the third moment as our only calibration target. When we follow this strategy, calibrated values are $disp_Q = \{0.004, 0.015, 0.024\}$. Under this new calibration strategy, model yields the following moments: **(i)** 0.0470, **(ii)** 0.0156, and **(iii)** 4.08. As Table 5.2 presents, when $\zeta = -1.3$ model performs much better than other calibration strategy to match the data moments.

When calibrating $mu_Q$ shocks, we follow a similar approach as in $disp_Q$ shocks. This time our calibration target is the standard deviation of aggregate productivity shock distribution presented in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). Specifically, the baseline calibration sets the mean value of high and low volatility as stated in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), which is 0.0087. Related, when we combine $disp_Q$ and $\mu_Q$ shocks, we also imposed low and high variations. In Table 7 and 8, in smaller $mu_Q$ panel, the volatility is 0.0067 and in larger $mu_Q$ panel, it is 0.0174.