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PRODUCT RECALLS AND FIRM REPUTATION

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Abstract

Product-recall data and information on stock-price reactions to recalls are used to estimate the value of reputation in a model in which product quality is not contractible. A recall is the result of a product defect that signals low effort. The recall triggers a reduction in the firm's product price and value which then both rise steadily until its next defect occurs. We estimate that reputation accounts for 8.3 percent of firm value and that welfare is 26 percent of its first best level. A policy intervention that attains first best is a recall tax accompanied by a flow subsidy.

1 Introduction

This paper estimates the value of a reputation using product-recall data from the transportation equipment sector and information about stock price movements in response to product-recall announcements. Reputation causes the values of firms to rise above the reproduction value of their physical assets. While previous research has documented the large losses in firm value associated with recalls and while there is even more theoretical research on reputation concerns, the present paper appears to be the first linking the two.

The model is that of a market for a homogenous, non-durable good with a continuum of buyers and sellers. Firms experience occasional shocks to product quality or "defects." Defects are contractible and the law requires that a firm must at least fully compensate its customers for them. Otherwise the quality of a seller's output is not observed before purchase and is not contractible; payment is up front. Interactions between buyers and sellers are short-term – there are no repeat meetings or long-term contacts. Costly effort reduces the probability that a defect will occur but if it does, the firm's revenue drops and then recovers gradually until the episode repeats itself. A defect signals that effort

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was low, and the signal is public. A firm's "reputation" is the history of its public signals, i.e., of its past defects or recalls.

Reputation, by our estimate, accounted for about 8.3 percent of firms' values in the transportation-equipment sector over the 1978-2007 period. This estimate derives from how much a firm's value falls when it recalls one of its products. Stock prices of publicly traded firms show this mechanism at work. Product recalls are common, but still rare enough that a recall represents significant news that produces a negative stock-price impact, often far larger than the direct costs associated with a recall, and this excess we interpret as reputational loss. A declining hazard of product recalls reflects the build-up of reputation and a rising effort on the part of the firm to maintain its reputation. The model fits the recall hazard, under a constraint on the stock-price impact of recalls – specifically the stock-price drop relative to direct recall costs; Jarrell & Peltzman (1985) estimate that value falls by twelve times the direct costs surrounding the recall. Other papers that estimate the effect of recalls on firm values are Hoffer, Pruitt and Reilly (1988), Barber and Darrough (1996) and Rupp (2004).

The decreasing hazard seems compatible with facts documented by Cabral and Hortacsu (2010) for eBay sellers. Sellers' sales drop significantly after the first negative feedback, consistent with the prediction that bad news leads to a large drop in the firm value. Moreover, subsequent negative feedback arrives more rapidly after the first negative feedback—firms decrease their effort level after public bad news.

We analyze a market in a steady state in which the product price per unit of quality is constant while prices of physical units rise and fall reflecting their reputations. Free entry of firms means that all the welfare gains go to the consumers. Firm values depart from the reproduction cost of their capital arises for roughly the same reason as in Hopenhayn (1992), but the model has multiple steady states. Consumer welfare and average reputation are positively correlated across these states, and welfare is negatively related to the price of the product. In other words, equilibria that feature higher reward to good performance yield higher social welfare – not surprisingly. Welfare is estimated to be 26 percent of its first-best level. Although estimates indicate that the reputational loss is up to twelve times higher than their direct recall cost, it is not large enough to correct for the inefficiency. The welfare analysis indicates that recalls should be taxed by an even larger multiple of their direct cost: The recall tax that is accompanied by a production subsidy, and it attains first best. Recall costs are a noisy signal of firms' hidden effort, but since firms are risk neutral, incentives can be restored by a policy that raises the sensitivity of compensation to the signal. As a result, a tax of sixty percent of average firm values restores first best.

Fig. 1 plots the steady state distribution of estimated values of reputation among sellers relative to their capital stocks. In other words it is the excess of market value over

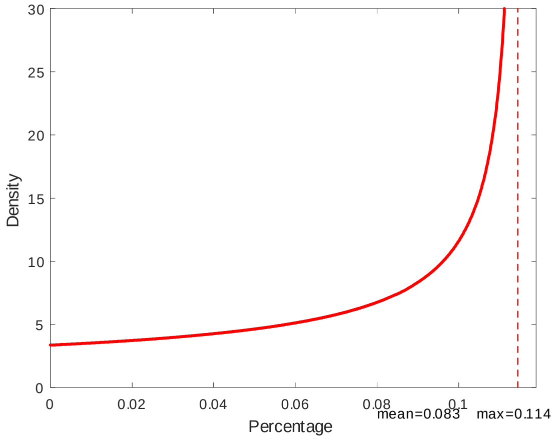

Figure 1: Market value of reputation relative to book value of capital

book value. The mean is 8.3 percent of book value.

This estimate is for the 1978-2007 period and for the transportation-equipment sector. Now, Hall (2001) reports an intangible capital value of 90 percent for that sector in 1998, but that was a year when market valuations in all sectors were unusually high.

Relation to the literature.—Reputation is but one of several intangibles. Bhandari and McGrattan (2018) report a value of intangibles of 65 percent of GDP – much larger than my estimate of the value of reputation. Gourio and Rudanko (2014) model the value of a customer base, Butters (1977) and Milgrom and Roberts (1986) model advertising, and Pakes (1985) studies how a firm's stock price reacts to changes in its patents.

There is a large literature in dynamic games in which a long lived agents with unknown types interact with short-term customers. Holmstrom (1999) has not only a hidden action but exogenous types. Board and Meyer-ter-Vehn (2013 henceforth BM13) model an agent who has a hidden investment in an evolving state and periodically generates a public signal. Cisternas (2018) models an evolving state with a produce-or-invest technology, with a privately chosen effort, and a publicly observed output. Tadelis (1999) and Mailath and Samuelson (2001) also feature models where types differ. Watson (1999, 2002) studies repeated interactions in which the level of trust starts at a low level and gradually rises as in the equilibria I focus on.

Related are models of governments that wish to borrow money or to refrain from confiscatory taxation, but have no ability to commit – Kydland and Prescott (1977) and Phelan and Stacchetti (2001). Organizational equilibrium studied by Basseto, Ho and Rios-Rull (2018) is similar in that value rises as a reward for good behavior except that government actions are observable and the equilibrium exhibits no on-path punishments. In Horner (2002) the threat of exit induces firms to choose high effort. And bandit models have sometimes assumed bad news events, "breakdowns" as in Keller and Rady (2015).

Closest to the model is Rob and Fishman (2005) who also have no types and who also focus on equilibria in which seller reputation consists of the time elapsed since the seller's last public bad-news signal. A technical difference is that the model is cast in continuous time and can be solved by hand. The solutions are simple and easy to estimate. The imperfect bad-news version of BM13 has types and has some similar implications but does not produce a monotone recall hazard or a monotone stock market penalty for recalls, as we shall explain later.

Azoulay, Bonatti and Krieger (2017, henceforth ABK) show that citations data can be explained by a model where learning about exogenously given types occurs via periodic bad-news signals. ABK is the only paper I am aware of where this general type of model has been estimated, and I discuss it in more detail at the end of the extensions section.

Section 2 lays out the model and its implications for the value of reputation and for optimal policy, Section 3 describes the data, the estimation, and the implications of the estimates for the magnitude of reputation capital and for policy. Section 4 discusses some extensions of the model and Section 5 concludes. Some mathematical proofs are in the Appendix.

2 Model

There is a continuum of buyers and sellers of a homogeneous good.

Buyers.—A buyer has a utility function $U(q, z)$ defined over consumption of quality units q of a "reputation good," and on the number of physical units z of an outside good. A buyer's income each period is m; he takes as given the price p per unit of the reputation good q, i.e., the price of quality. Both m and p are measured units of the numeraire good z. The buyer faces the period budget constraint $z + pq \leq m$. A single customer's demand for quality then is

$$D(p) = \arg \max_{q \in [0, (m-z)/p]} \{U(q, m - pq)\}. \tag{1}$$

There is a continuum of buyers of measure one so that market demand also is $D(p)$. We assume that $\lim_{\mathrm{q} \to 0} \partial U / \partial q = +\infty$.

Sellers.—Sellers are risk neutral with discount rate r. Each can sell up to one divisible physical unit per period[1] that is bought by many customers. Payment is in advance and between the continuum of customers and firms there are no repeat interactions. The quality of the output is equal to a seller's effort, x. Effort is unobserved, and the price per physical unit sold is px^*, where x^* is effort that buyers expect the seller to exert. The seller's cost of effort is $x^2/2$. All firms get the same price per unit of quality supplied,

[1]Proposition 4 shows that the results extend to the case where firms differ in their physical scale as long as returns to scale are constant.

and the firm's payoff (excluding any recall costs) is

$$px^* - \frac{1}{2}x^2.$$

Defects.—A customer can buy from various sellers. Periodically, the seller's output has a defect. A defective product reduces a customer's utility by an amount c per physical unit purchased. If defects were not compensated, a buyer i's utility would be

$$U\left(q, z - c\sum_{j=1}^{M_i} \kappa_{i,j} I_{\{\text{j was recalled}\}}\right) \tag{2}$$

where $\kappa_{i,j}$ is the number of physical units customer i bought from firm j, where $j \in \{1, 2, ..., M_i\}$, the latter being the set of sellers to customer i, and where c is a parameter.

Compensation for defects.—A seller must, by law, compensate each customer by the full amount of the loss; if seller j sells quantities $(\kappa_{i,j})_{i=1}^{N_j}$ to N_j customers and if he has a recall, his total recall cost is

$$c\sum_{i}^{N_j} \kappa_{i,j} = c, \tag{3}$$

because $\sum_i^{N_j} \kappa_{i,j} = 1$, i.e., his physical quantity is unity. This payment restores each of seller j's customers' utilities to their no-defect level of $U(q, z)$. There is, in other words, a full warranty; a seller must honor it if the product turns out to be defective, and the customer is fully insured[2].

Public histories.—Buyers cannot share their consumption experience. The only way they learn about a seller's performance is through public signals. A seller can try to avoid a recall by exerting effort x. Conditional on an effort path $(x_\tau)_{\tau=0}^{\infty}$, the waiting time τ until the next defect has CDF

$$\Pr(\tau \leq t) \equiv F(t) = 1 - \exp\left(-\int_0^t (\lambda - x_\tau)_+ \, d\tau\right). \tag{4}$$

Thus, the defect hazard is $\max(0, (\lambda - x))$, where $\lambda > 0$ is a parameter. There are no inherent differences among sellers, but their public histories will generally differ, and may influence buyers' expectations.

Public signals and reputation.—A defect is the only public signal about the seller's effort. Let t denote time elapsed since the last defect or, if the seller has had no defects yet, since the date of entry. We will focus on equilibria in which buyers do not distinguish new entrants from incumbents with recent recalls. Reputation matters if x_t depends on

[2]Strict products liability is now established law throughout the U. S. and the E.U. (Geisfeld 1988, 2012). Until Sec. 4.5 the full-insurance warranty is treated as exogenous.

t. When its next defect occurs, the price of the seller's output will fall and its market value will drop to k.[3]

The Hamilton Jacobi Bellman (HJB) equation.—Suppose that t is the only variable determining quality and not, for instance, the number of past recalls. Conditional on $(x_t^*)_0^\infty$, the Bellman equation for the lifetime value v is

$$rv_t = \max_{x \leq \lambda} \left(px_t^* - \frac{x^2}{2} - (\lambda - x)(v_t - k + c) + \frac{dv}{dt} \right). \tag{5}$$

The problem is concave in x and the seller's first-order condition is

$$x_t = v_t - k + c. \tag{6}$$

Free entry of sellers.—The supply of entrants is infinitely elastic at the value k. The initial condition in Eq. (5) reads

$$v_0 = k, \tag{7}$$

and it embodies two assumptions: A free entry condition at the cost k, and the assumption that the seller can sell the business to a new entrant for the price of k. If an incumbent's value ever dropped below k, it would be taken over by an entrant, and therefore[4]

$$v_t \geq k. \tag{8}$$

A seller cannot escape the cost of recall, however, regardless of whether or not it is taken over[5].

The stationary distribution $\mu(t)$.—Industry supply depends on the long-run distribution of x. In equilibrium, x_t depends on t which, in turn, follows a renewal process with inter-arrival distribution F. Let $\mu(t)$ be the pdf of sellers for whom the time elapsed since the last recall is t. The "age since last recall" distribution differs from $f(t)$ because unlike a recall (which is an event), age is a state which is reached only if no recall has taken place by then. Thus the pdf is proportional to $1 - F(t)$, and it is the long-run age

[3]If the firm can continue to operate under a new name without having to pay k again, its value cannot fall below k. Smaller punishments and more complicated history dependence are both possible equilibria. We shall assume, however, reputation works exclusively via the pair (t, k). The drop in value at recall is an on-path punishment, roughly as in Green and Porter (1984).

[4]The transaction involves the purchase of the capital along, implicitly, with the reputation of a new entrant. Tadelis (1999) discusses the market for reputation.

[5]A product recall often results in a takeover of the firm in question. Some examples are listed in online Appendix B. Jovanovic and Rousseau (2002) introduce takeovers into a Hopenhayn (1992) type of model and treat the value of acquired capital as k net of a "salvage cost." The salvage cost represents the costs of transferring the capital to new owners, a process that often involves a private equity firm. See online Appendix and particularly Table A1 for examples where a takeover occurred following a product recall.

distribution of products, and μ which in turn depends on t.[6]

$$\mu(t) = \frac{1 - F(t)}{\int_0^\infty [1 - F(s)]\, ds}. \tag{9}$$

Then, the average quality per seller, $\bar{x}$, is:

$$\bar{x} = \int_0^\infty x_t \mu(t)\, dt. \tag{10}$$

Market clearing.—Individual sellers' behavior was solved conditional on p, and that also implies $\bar{x}$ in Eq. (10). Given the price p, the number of sellers, n, is determined by the market-clearing condition:

$$D(p) = \bar{x}n, \tag{11}$$

where $q = D(p)$ is the argmax in Eq. (1). When t does affect x, quality varies over sellers. The parameters are (r, m, c, k, λ, U). We consider a steady state in which the aggregates (q, p, n) are fixed.

Prices of physical units.—A firm sells its one physical unit at the price px_t^*. Variation of v_t over firms induces a cross-section price distribution, similar to the distributions depicted in Fig. 1. The total number of physical units sold is n, their average quality is $\bar{x}$ and the total quality supplied is $\bar{x}n$.

Market shares.—Market sales volume is constant at $pn\bar{x}$, and therefore t periods after the firm's last recall,

$$\text{market share} = \frac{px_t}{pn\bar{x}} = \frac{1}{n\bar{x}} x_t \tag{12}$$

and so if x_t rises with t, so does the firm's share of aggregate sales.

The distribution of customers over firms.—Buyers are homogeneous, and each buyer buys q units in total, generally not all from the same firm. A firm is indifferent as to how many customers buy from it because the recall compensation is prorated to the quantity sold, so that the total recall compensation is always c. Similarly, a customer is indifferent as to how many firms it buys from, the total cost of a number of quality units q is always pq.

2.1 Equilibrium

Definition.— Equilibrium consists of the triple (q, p, n) and the pair of real-valued functions $(x_t, v_t)_{t=0}^\infty$ such that for all $t \geq 0$, (i) $v_t \geq k$ satisfies (5), (ii) $x_t \geq 0$ satisfies (6),

[6]Eq. (9) is a restatement of Eq. (3) in Cox (1962), p. 61. The renewal theory term for this distribution is the *backward recurrence time* or the *age of the component in use*. online Appendix provides examples of (f, μ) pairs.

(*iii*) buyers' expectations are correct in that $x_t = x_t^*$, (*iv*) the market clears so that given p and the supply per firm, $\bar{x}$ defined in Eq. (10), determines the number of firms n.

The ODE for x_t.—We now use parts (*ii*) and (*iii*) of the definition of equilibrium to express (5) in terms of x_t. From (6), $dv/dt = dx/dt$, and so

$$\frac{dx}{dt} = r(k - c) + (r + \lambda - p)x - \frac{1}{2}x^2. \tag{13}$$

Since $v_0 = k$, the initial condition is $x_0 = c$. Let x_1 and x_2 be the two roots of x at which the RHS of (13) is zero:

$$x_1 = r + \lambda - p - \sqrt{(r + \lambda - p)^2 + 2r(k - c)} < 0, \quad \text{and} \tag{14}$$

$$x_2 = r + \lambda - p + \sqrt{(r + \lambda - p)^2 + 2r(k - c)} > 0. \tag{15}$$

To guarantee that the roots are real and that $x_1 < 0 < x_2$, we shall assume that the entry cost exceeds the recall cost,

$$k - c > 0. \tag{16}$$

Recalls.—Since $v \geq k$, for the solution for x_t in Eq. (6) to generate recalls, we need that

$$c < \lambda, \tag{17}$$

otherwise the optimum would be at $x_t = \lambda$, and a zero recall hazard. Moreover, for $x_t < \lambda$ for all t, we require that $x_2 < \lambda$, i.e., that

$$r(k - c) < \left(p - r - \frac{\lambda}{2}\right)\lambda. \tag{18}$$

Then Appendix proves

Proposition 1 *If and only if Eqs. (17) and (18) hold, the ODE (13) has the solution*

$$x_t = x_1 + \frac{x_2 - x_1}{1 + \frac{x_2 - c}{c - x_1}\exp\{-\frac{1}{2}(x_2 - x_1)t\}} < \lambda \tag{19}$$

for all $t \geq 0$, with

$$x_1 < 0 < x_2 < \lambda. \tag{20}$$

Thus x_t rises and the recall hazard declines monotonically as a function of time elapsed since the last recall. The right panel of Fig. 2 shows that $x_2 < \lambda$ is the maximal value of x_t reached as $t \to \infty$ if no defect occurs, and since $x_2 < \lambda$ the hazard remains positive and a recall must eventually occur.

Typical equilibrium play is illustrated in Fig. 2. Since $k > c$, we have $x_1 < 0 < x_2$,

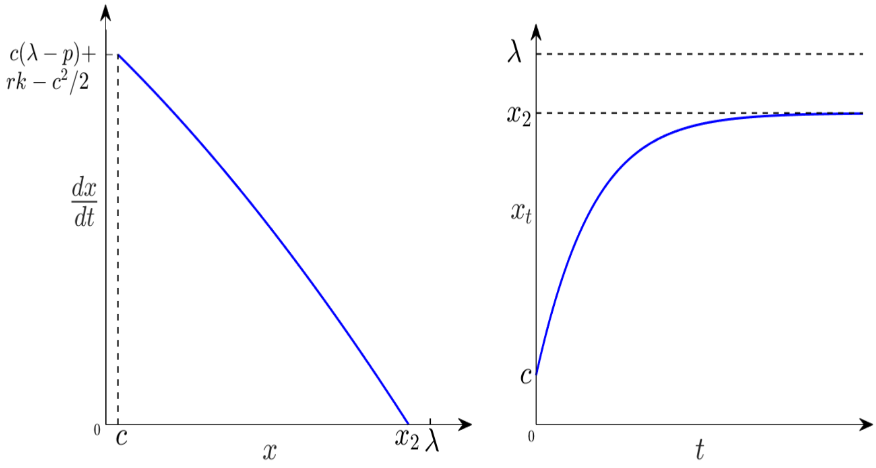

Figure 2: THE RHS OF (13) AS A FUNCTION OF x (LEFT PANEL) AND THE TIME PATH OF x_t (RIGHT PANEL)

so that the curve crosses the positive half line exactly once[7].

2.2 Equilibria and Welfare

The equilibrium x_t in (19) depends on p through its influence on x_1 and x_2. If $\lambda > c$, there is a continuum of equilibria indexed by $p \in [p_{\min}, p_{\max}]$; we shall solve for $p_{\min}$ and $p_{\max}$ presently. Since sellers get zero rents, all rents go to the buyers. Welfare therefore declines with p – the equilibria are Pareto ranked. Even in the most efficient equilibrium, $p_{\min}$, welfare is below its maximum level.

2.2.1 The no-reputation equilibrium at $p_{\max}$

The firm must compensate its customers for recall disutilities and this induces a minimal level of effort at which (x, v) are constant. It entails $x_t = c$ and $v_t = k$ for all t. Eqs. (5), (7), and the fact that $dv/dt = 0$ imply that $p_{\max}$ solves $rk = p_{\max}c - c^2/2 - (\lambda - c)\, c = p_{\max}c + c^2/2 - \lambda c$, so that the firm's discounted profits equal its entry cost plus discounted expected recall costs. This gives us the no-reputation equilibrium price

$$p_{\max} = \frac{rk}{c} + \lambda - \frac{c}{2}, \tag{21}$$

which is increasing in λ. And $p_{\max}$ is decreasing in c because c raises x, and therefore the firm's sales, px^*, rise by more than the production and recall costs do.

The no-reputation equilibrium entails the smallest welfare and the highest p. This equilibrium exists if $p_{\max} \geq 0$. If the RHS of (21) is negative, there is no positive price

[7] As x varies on the horizontal x axis, dx/dt in (13) is inverted-U-shaped and it peaks at $x = r + \lambda - p$. A larger view of Eq. (13) is shown in Panel 1 of Fig. 5.

at which the firm's rents can cover its entry cost and the market shuts down if there are no reputations.

2.2.2 The highest-welfare equilibrium at $p_{\min}$

Next, Eq. (15) implies that

$$\frac{dx_2}{dp} = -1 - \frac{r + \lambda - p}{\sqrt{(r + \lambda - p)^2 + 2r\,(k - c)}} < 0. \tag{22}$$

and since equilibrium requires that $x_2 \leq \lambda$, the lowest admissible p is one at which $x_2 = \lambda$.[8] This yields

$$p_{\min} = r\left(1 + \frac{k - c}{\lambda}\right) + \frac{\lambda}{2}. \tag{23}$$

The equilibrium set.—The size of the set of equilibria that Proposition 1 covers, $[p_{\min}, p_{\max}]$, depends mostly on the difference between λ and c. Simple algebra shows that

$$p_{\max} - p_{\min} = \left(\frac{1}{2} + \frac{r}{\lambda}\frac{k - c}{c}\right)(\lambda - c) \searrow 0 \quad \text{as } c \nearrow \lambda. \tag{24}$$

2.2.3 The first-best, contractible-x equilibrium at $\hat{p}$

The optimum could be decentralized if x was contractible; intervention would then be unnecessary. The market would then be complete with the risk-averse consumer fully insured against c by risk-neutral firms. The first welfare theorem would apply and equilibrium coincides with the Pareto optimal outcome.

The complete market equilibrium would be summarized by the triple $(\hat{n}, \hat{x}, \hat{p})$ that solves (25), (26), and (27) as follows. Price of quality, $\hat{p}$, then equals its marginal utility:

$$p = \frac{U_q\,(xn, m - pnx)}{U_z\,(xn, m - pnx)} \qquad \text{(consumer optimization).} \tag{25}$$

The social opportunity cost of an additional seller is k. Equating k to the discounted net benefit of the seller's output yields

$$rk = \max_x \left(px - \frac{x^2}{2} - (\lambda - x)\,c\right) \qquad \text{(free entry condition).} \tag{26}$$

In (26) x is chosen optimally:

$$x = p + c \qquad \text{(firms optimize over } x\text{)} \tag{27}$$

The unique solution for $(\hat{n}, \hat{x}, \hat{p})$.—Substituting for x into (26), $rk = (\hat{p} + c)^2/2 - \lambda c$,

[8] See Appendix for the argument why x_2 cannot exceed λ.

which together with (27) implies that

$$\hat{p} = \sqrt{2\,(rk + \lambda c)} - c \quad \text{and} \quad \hat{x} = \sqrt{2\,(rk + \lambda c)}. \tag{28}$$

2.3 Optimal policy intervention

A simple tax-subsidy scheme attains first best. It consists of

a) A tax T at each recall,

b) A flow subsidy S paid to each firm per period,

c) Full compensation by firms to consumers for defects.

The tax raises firms' effort to its first-best level $\hat{x}$, but at the first-best price $\hat{p}$ firms would then be making a loss, because in addition to compensating customers for recall they would be facing the additional expense T, adding up in expectation to $(\lambda - x)\,T$. We have the following characterization:

Proposition 2 *The tax-subsidy scheme that attains first best is*

$$T = \hat{p}, \tag{29}$$

$$S = (\lambda - c - \hat{p})\,T > 0. \tag{30}$$

Proof. Since $c < \lambda$, $\hat{p} > 0 \Rightarrow T > 0$. The HJB Eq. now is

$$rk = S + \max_{x \leq \lambda} \left(px_t^* - \frac{x^2}{2} - (\lambda - x)(c + T) \right) \tag{31}$$

and the FOC is

$$x_t = c + T \tag{32}$$

Eqs. (27) and (29) imply that $x = \hat{x}$. Substituting for x and for S into (31), the latter reads

$$rk = p\hat{x} - \frac{\hat{x}^2}{2} - (\lambda - \hat{x})c,$$

which is identical to (26) evaluated at $x = \hat{x}$. Finally, $x = \hat{p} + c$, and therefore $S = (\lambda - x)\,T$. Existence of equilibrium requires that $\lambda > x$, and so $S > 0$. ■

2.4 The private value of a reputation

The private value of the firm's reputation is $v_t - k$. If the firm is publicly traded and if k is the firm's book value[9], the firm's market to book value is v/k. Define the firm's value of reputation relative to its book value as

$$w_t \equiv \frac{v_t - k}{k} = \frac{x_t - c}{k} \leq \frac{x_2 - c}{k} \equiv w_{\max}, \tag{33}$$

[9] Hopenhayn (1992) also assumes that k is book value.

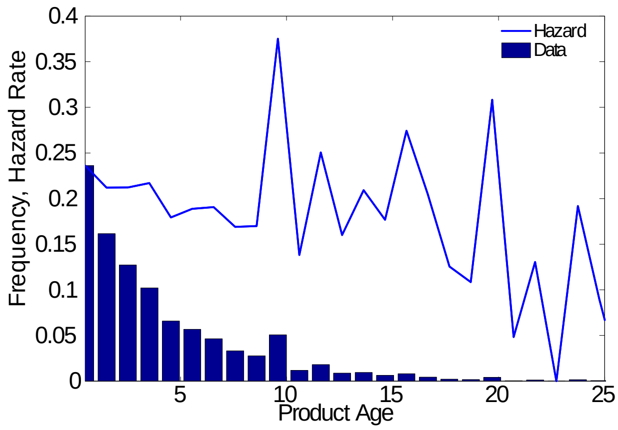

Figure 3: THE WAITING-TIME FREQUENCY DISTRIBUTION AND ITS HAZARD RATE.

with x_2 given in (15). The first equality in (33) follows from (6) and the inequality uses the fact that $x_t \leq x_2$.

Whenever $x_2 > c$, the function x_t is strictly increasing. Let $t = t(x)$ denote the inverse of x_t in Eq. (19). The CDF of w is $\int_0^{t(c+wk)} \mu(s)\, ds$. The pdf of w is

$$\zeta(w) \equiv kt'(c+wk)\,\mu(t(c+wk)), \tag{34}$$

which is solved explicitly in Appendix. Fig. 1 portrays the estimated density $\zeta(w)$.

3 Estimation

3.1 Data and Identification

Product-recall data.—We use the auto recall data from the Department of Transportation, obtaining 48,000 observations covering the period 1978-2007. The data cover only recalled products. We measure "age at recall" as the difference between the product's recall date and the "start of manufacture" of the product. Let n_t be the fraction of all recalled products that were recalled at age t. The average age at recall is 4.14 years.[10] The resulting data are portrayed in Fig. 3 which shows the frequency distribution n_t of the ages of the products at recall, and the annualized hazard rate $\hat{h}(t) = n_t / \left(1 - \sum_{\tau=1}^{t-1} n_\tau\right)$. Details are in online Appendix A.

Recall costs.—Recall costs are included in warranty expenses; they are not a separate line item. The 10Ks and 10Qs (the official financial reports filed with the SEC) sometimes include information on large recalls, but recall costs are not mandated disclosure items.

[10] Automobile manufacturers are required to correct a safety defect at no charge to the owner only for vehicles that are less than 10 years old – see https://www-odi.nhtsa.dot.gov/recalls/recallprocess.cfm Hence some of the observations on defects are less likely to show up in the recall data after 10 years.

We therefore rely on the estimates of several studies of the stock-price impact of recalls and other public bad news. The first to estimate this were Jarrell and Peltzman who report (1985, p. 521) that for publicly traded firms value loss is twelve times recall costs. Armour, Meyer and Polo (2009) estimate financial losses to be nine times as large as the fines imposed for financial misconduct. Hoffer, Pruitt and Reilly (1988), Barber and Darrough (1996) and Rupp (2004) estimate abnormal percentage returns associated with recalls, but the percentages understate the loss relative to revenues from the product in question because firms typically sell more than one product. A summary of the findings is in online Appendix C, Table A.2. Some recent examples are portrayed graphically in online Appendix B.

The model interprets a firm's reputation as the value of its t – the time since the last recall. The model explains the decreasing hazard as follows: Incentives after a recall are at their minimal level since a second recall (if it happened right away) would not reduce the firm's value any further. As time passes and if recalls do not happen, x_t keeps rising and along with it the firm's market share $x_t/\bar{x}$. The recall hazard steadily declines. And since v_t rises with t, price declines should rise as a function of time elapsed since the last recall. Thus

1. *Impact of recalls on market shares.*—Since x_t rises with t, the higher is a firm's reputation, larger should be its loss of market share when a recall occurs. Supporting evidence is in Rhee and Haunschild (2006) who combine the same recall data with measures of reputation measured as quality ratings and as fleet depreciation rates. They find that when its product is recalled, a firm with a high reputation suffers more market penalties in terms of subsequent sales of their products than would a firm with a poor reputation.

2. *Impact of recalls on firm value.*—Since v_t rises with t, the larger should be its absolute and its percentage stock price decline. When several recalls occur over a period of time, each should have a smaller impact on the firm's value. Table 1 of Barber and Darrough (1998) shows this to be the case.[11] In the above equilibrium incentives after a recall are zero since a second recall does not take the firm any further below $v = k$. This offers a contrast to the imperfect bad news version of BM13 Sec. 5.2 which has effort/investment taking on values of either 0 or 1, that are a non-monotonic step function of beliefs (0 for extreme beliefs and 1 for middle beliefs about quality) as described in Fig. 4 of their paper. When reputation is

[11] The variables in columns 1 and 4 of their Table 1 are negatively correlated. Comparing the small price impact of GM's frequent recalls when compared to Toyota's isolated recall Brauer (2014) writes: "Toyota's history of uninterrupted success in the U.S. market had established very high expectations for the brand. When the unintended acceleration recall hit in 2009 it hit Toyota hard in the image department..... By comparison, GM doesn't have the same infallible reputation Toyota possessed in 2008, meaning a recall (even a massive one) doesn't impact GM to the same degree."

high, one bad news shock may matter little, but then the firm starts investing a lot since another bad news shock would finish it off by pushing effort to zero. The drop in value due to a second recall (shortly after a first one) is small in my model, but large in BM13.

Equilibrium implies that reputation is related to the slope of the recall hazard; a recall is judged to be less likely to occur if it has not occurred for a long time.

Assuming that c is financed by debt or from future profits, we may interpret $v_t + c - k$ as the stock-price reduction at the time of the recall, and we now calculate its distribution in the population of all recalls. From (6), the loss is equal to x_t, and so we shall need the distribution of x_t conditional on recall.

A summary of what we shall do is as follows: We estimate the parameter k and the equilibrium price p, from recall data in the transportation equipment sector; the interest rate will be pre-set, and the recall cost c will be restricted so that the expected drop in firm value at a recall $E[v_t + c - k] = E^{\mathrm{G}}(x)$ equals 6c (or 12c), to fit previous findings by Jarrell and Peltzman (1985), relating direct recall costs to loss in stock market value. Furthermore, (k, λ, p, c) will be estimated alternatively by maximum likelihood and by and nonlinear OLS to fit the distribution of product age at recall.

Distribution of t conditional on recall occurring at age t.—Suppose we had a collection of N products on $[0, T]$ each with an initial date starting at zero, and if the first k products had a single recall $t_i < T$ while the remaining $N - k$ products had no recall. If the recall dates were identically distributed, the likelihood would be $\prod_{i=1}^{k} f(t_i)\,[1 - F(T)]^{N-k}$. This would be the likelihood if, upon failing, the product never again reappeared in the sample.[12]

Our sample consists only of recalled products, however, and the equilibrium involves a re-initialization, with the inter-arrival CDF F defined in (4). The long-run product-age density is $\mu(t)$ in Eq. (9), and $\mu'(t) < 0$ reflects the fact that young products are over-represented. Being an unconditional age distribution, it acts as the prior on t. Using Bayes rule, the likelihood of a recalled product having age t is

$$b(t) = \frac{f(t)\,\mu(t)}{\int_0^\infty f(s)\,\mu(s)\,ds}, \tag{35}$$

with $B(t) = \int_0^t b(s)\,ds$ being its CDF. Eq. (66) of Appendix proves

Proposition 3 *With $\mu(t)$ given in Eq. (9), if the two pdfs, $b(t)$ and $f(t)$ satisfy (35) then the CDFs $B(t)$ and $F(t)$, is*

$$B(t) = 1 - (1 - F(t))^2, \tag{36}$$

[12] See eq. (2.5) of Lancaster and Nickell (1980) for the same expresion in a different context.

which is equivalent to the hazard of b *being twice that of* f*:*

$$\frac{b(t)}{1-B(t)} = 2\frac{f(t)}{1-F(t)}. \tag{37}$$

The model thus predicts a hazard that declines from $2(\lambda - c)$ to $2(\lambda - x_2)$. The CDF of x conditional on recall then is

$$G(x) \equiv B(t(x)), \tag{38}$$

with density denoted by $g(x)$.

3.1.1 Identification

We have no data on prices of physical units. The parameter r is pre set at 0.05. We can identify the remaining four parameters (p, λ, k, c) as follows: Eq. (36) gives a one-to-one relation between B and F. We interpret B as the CDF of a product's age at time of recall, and hence $G(x)$ in Eq. (38) as the distribution of x at time of recall. To constrain the ML estimates , by the average stock-price drop at recall, we shall use the two alternatives:

$$E^{\mathrm{G}}(x) = \int x dG(x) \in \{6c, 12c\}. \tag{39}$$

Using (13) and the fact that $x(0) = c$, simple algebra shows that the level, slope and curvature of the hazard $\lambda - x$ at $t = 0$ are

$$\begin{aligned} \text{intercept} &= 2(\lambda - c), & (40)\\ \text{slope} &= -2\left.\frac{dx}{dt}\right|_{t=0} = -2\left(rk + (\lambda - p)c - \frac{1}{2}c^2\right) < 0, \quad \text{and} & (41)\\ \text{curvature} &= -2\left.\frac{d^2x}{dt^2}\right|_{t=0} = 2(c + p - r - \lambda)\left.\frac{dx}{dt}\right|_{t=0} > 0. & (42) \end{aligned}$$

These are three additional independent restrictions. Informally, one can think of Eqs. (39) and (40) identifying (c, λ), (42) then identifying p, and (41) identifying k.

Caveats.—Two maintained assumptions affect the implications: (i) unit-elastic product demand and (ii) quadratic cost of effort.

(i) *Unit-elastic* D.— We cannot identify n or the parameters of the demand curve $D(\cdot)$; the welfare conclusions hinge on the assumption that $D(p)$ is unit elastic. The smaller is the elasticity of substitution between z and q in U, the smaller is the first best welfare gain because under first best a lower p yields less additional q and less additional utility. Even in the the Leontief case, however, the welfare gain is positive because although the ratio q/z does not change, z and q are both higher.

(ii) *Quadratic effort cost.*—Other functional forms for costs could significantly change the welfare implications. For example, if $c(x) = \left(\frac{x}{\omega}\right)^{1+\delta}$, as $\delta \to \infty$, $c(x) \to 0$ for $x < \omega$ and $c(x) \to \infty$ for $x > \omega$; we would get $\lim_{\delta\to\infty} x_t \to \omega$ for all $t > 0$, the hazard would become flat at $2(\lambda - \omega)$ and equilibrium welfare would converge to 100%. Of course a flat hazard is not what the data show and it would be hard to accurately estimate two additional parameters using $B(\cdot)$ and and (39) alone. Appendix reports the ODE for x_t for the form $c(x) = x^{1+\delta}/(1+\delta)$, but we cannot obtain analytically the solution for x_t.

3.2 Estimates for one market

The first round of estimates presumes that all sellers share the same parameters (r, λ, p, k, c). The ML procedure is[13]

$$\max_{(p,c,k,\lambda)} \prod_{i=1} b(t_i) \qquad \text{s.t. (39).} \tag{43}$$

The parameter estimates are reported in Table 1 which also reports x_2, and $w_{\max}$, the upper bound on the value of reputation relative to the entry cost k. It also reports the percentage loss at recall, the fraction

$$l = \frac{v-(k-c)}{v} = \frac{x}{x+k-c}, \tag{44}$$

so that $x = (k-c)\frac{l}{1-l}$, and so that the CDF of l is

$$L(l) = B\left(t\left[(k-c)\frac{l}{1-l}\right]\right) \qquad \text{for } l \in \left[0, \frac{1}{1+\frac{k-c}{x_2}}\right]. \tag{45}$$

Table 1: Constrained ML estimates

λ	p	k	c	x_2	$w_{\max}$	$E^{\mathrm{L}}(l)$	$E^{\mathrm{G}}(x)/c$	Ln ML
0.172 [0.17,0.17]	0.595 [0.58,0.61]	0.234 [0.22,0.25]	0.003 [−0.01,0.01]	0.030	0.115	0.071	6	−2.243
0.203 [0.19,0.21]	0.584 [0.53,0.64]	0.278 [0.24,0.31]	0.002 [−0.01,0.01]	0.039	0.133	0.066	12	−2.273

There are two sets of estimates. Line 1 reports estimates constrained by a value-drop-recall ratio of 6, so as to reflect the estimate averaged over the various papers cited above. Line two of the table reports the estimates constrained by value-drop-to-recall-cost ratio 12 as reported by Jarrell and Peltzman (1985 p. 521). Visually, the plots in Fig. 4 favor the estimates in Line 1 which will be used in subsequent calculations.[14] Since $x_0 = c$, Eq.

[13] Appendix provides explicit solutions for $F(t), \mu(t), G(x), b(t)$ and $t(x)$ that were then used in the estimation.

[14] Jarrell and Peltzman focused on major recalls, this may explain their higher estimate of firms losing 12 times their recall costs. If smaller recalls get less publicity, it is not surprising that Figure 4 shows a

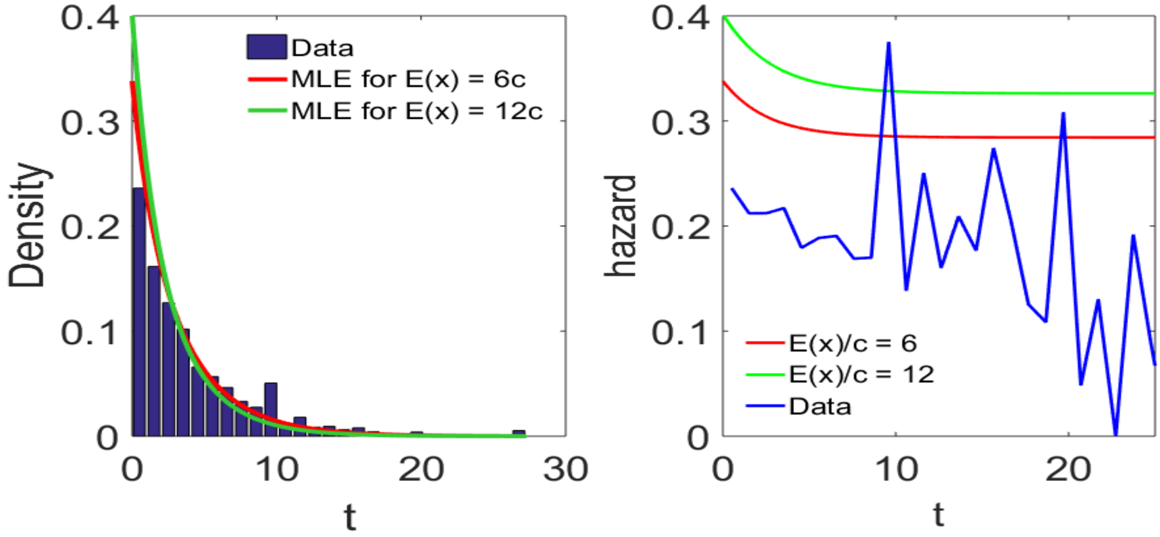

Figure 4: Maximum Likelihood estimate

(37) implies that the intercept of the hazard of B is $2(\lambda - c)$. The model overpredicts the intercept and underpredicts the decline in the hazard even for the case $E(x) = 6$.

As a reality check on our estimate of c, note that that warranty expenses are an upper bound on recall costs. Cohen *et al.* (2011) report that warranty expenses are from 1.45 and 1.82 percent of sales revenue in the industries they studied. Our estimates say that relative to sales recall costs, c/p, are between 0.34 (using row 2) and 0.5 (row 1) percent of sales revenue, i.e., at most about one third of all warranty expenses.

The solutions are depicted in the 4-quadrant Fig. 5, evaluated at the parameter estimates in the top row of Table 1. Panel 1 is an expanded version of Fig. 2, where we see the RHS of (13) crossing the zero axis at $x_1 = -0.78$ and at $x_2 = 0.03$.

Panel 2 shows that x approaches x_2 and as x rises, v also rises. Additionally, v rises because the hazard rate $2(\lambda - x)$ declines, reducing the likelihood of a reversion to $v_0 = k$. Panel 4 shows the distribution of x conditional on a recall which, by Eq. (6), is equal to c plus the loss in value.

The next figure compares the planner's solution and the equilibrium outcome. Panel 1 of Fig. 6 assumes the utility function

$$U(q, z) = q^{\alpha} z^{\beta} \quad \text{and} \quad \Rightarrow D(p) = \frac{A}{p} \quad \text{where } A = \frac{\alpha}{\alpha + \beta} m \tag{46}$$

where m is income per head. Panel 1 and plots the value of n as a function of p. We use (9), (11) and the solution for x_t in (19). Once p is specified, equations (9), (11) and our x_t do not depend on the demand parameters. Because the entry cost is constant at k, n is proportional to A and so we set $A = 1$. The other parameters used in Fig 6 are listed

better fit for the MLE for 6c than for 12c.

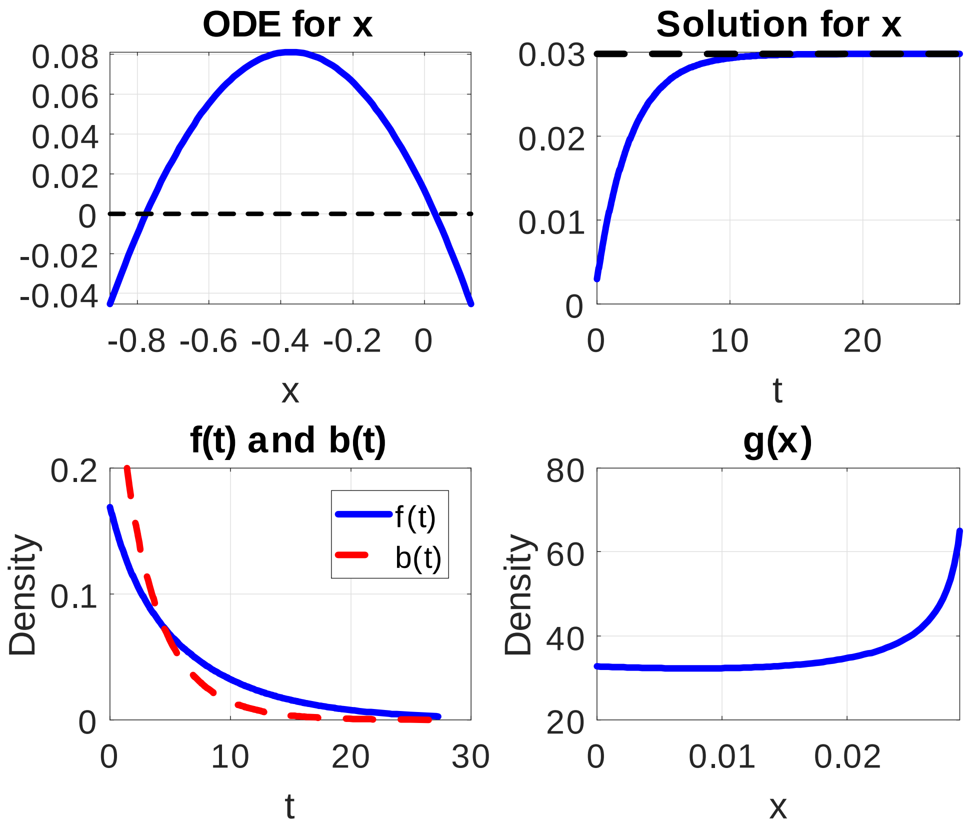

Figure 5: Plot of eqs (13) in Panel 1, (19) in Panel 2, (4) for $f = F'$ and (35) in Panel 3, and (38)for $g = G'$ in Panel 4.

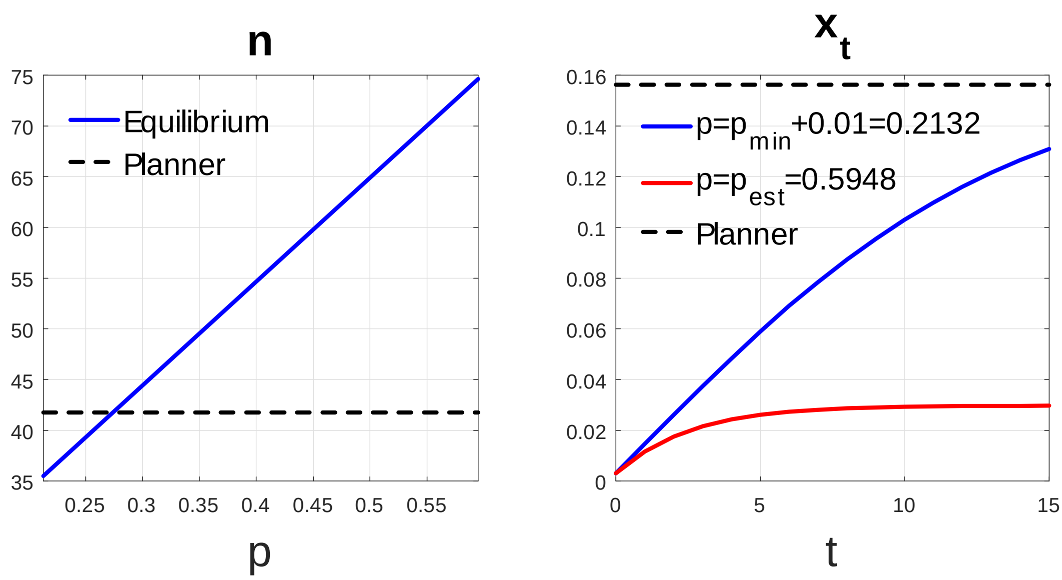

Figure 6: $A = 1$. PARAM. VALUES FROM THE TOP ROW OF TABLE 1

in Table 1.

Panel 1 shows that the planner wants fewer firms and more effort per firm.

3.2.1 Consistency of the estimates with the assumptions made on the parameter values

Several inequality restrictions were imposed and now we check that they are satisfied at the parameter estimates in Table 1.

Regarding Fig. 2.— Since $p > r + \lambda$, in the positive orthant of the figure dx/dt is positive and decreasing in x as shown in Fig. 2. A larger version of the equation system is in Fig. 5

On the existence of the equilibrium at $p_{\max}$.—The RHS of (21) is estimated to be positive which means that the $p_{\max}$ equilibrium exists.

Checking that equilibrium p exceeds first best p.—Eq. (28) implies that $p > \sqrt{2(rk + \lambda c)} - c$, and this is indeed so at the estimates in Table 1.

Checking condition (16).—and the condition (16) is met so that Proposition 1 and the solution for x in Eq. (6) is valid.

3.2.2 Fitting the hazard by OLS instead

The ML estimator targets the density, not the hazard, hence the left panel in Fig. 4 shows a much better fit than the right panel. To get better fit for the hazard rate itself, we estimate (λ, p, k, c) that fits the model to the empirical hazard by least squares, i.e.,

$$\min_{(p,c,k,\lambda)} \sum_{i=1} (h(t_i) - \hat{h}(t_i))^2, \qquad \text{s.t. (39).}$$

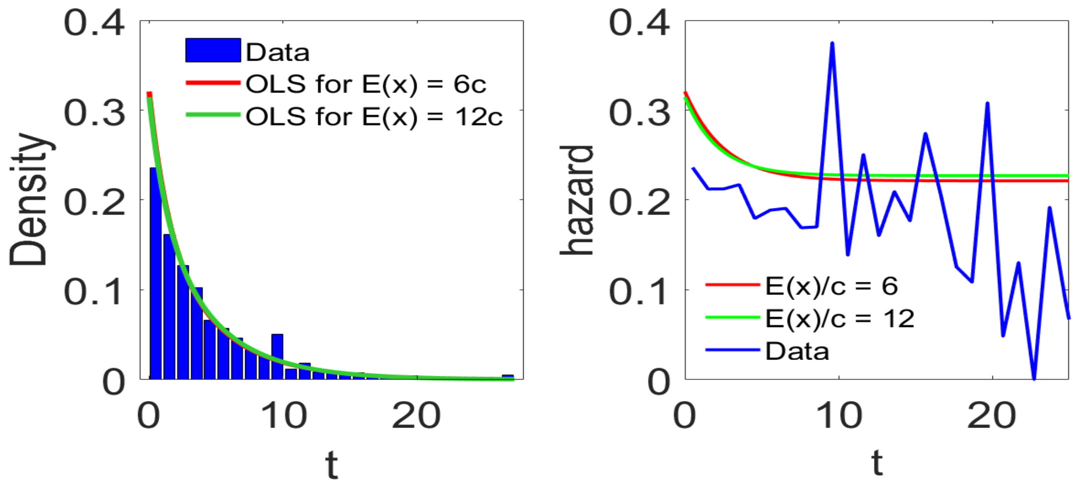

Figure 7: OLS ESTIMATES

where $h(t) = 2(\lambda - x_t)$, and $\hat{h}(t)$ is the empirical hazard, The parameter estimates are constrained by (39) and are reported in Table 2.

Table 2: OLS ESTIMATES CONSTRAINED BY (39)

λ	p	k	c	x_2	$w_{\max}$	$E^{\mathrm{G}}(l)$	$E^{\mathrm{G}}(x)/c$	MSE
0.166 [0.11,0.22]	0.577 [0.10,1.06]	0.438 [0.16,0.72]	0.006 [0.006,0.006]	0.056	0.116	0.074	6	0.062
0.160 [0.13,0.19]	0.649 [0.27,1.02]	0.429 [0.04,0.82]	0.003 [0.003,0.003]	0.046	0.100	0.064	12	0.062

The hazard, pictured in Fig. **??** is only slightly steeper compared to that generated by the ML estimate.

3.3 Unobserved heterogeneity: Two markets

Our estimated hazard rates tend to be too flat. Now, unobserved differences in hazard rates are known to create a steeper hazard for the group, as the movers leave and the stayers remain. Suppose then that there are two groups and estimate p, k, c, λ separately across groups, treating them as separate markets. We assume that the customers know the firm types but that the econometrician does not. We also estimate the fraction π_1 of the observations that fall in group 1, with the remaining fraction π_2 falling in group 2. With two λs the procedure is

$$\max_{(p_j, c_j, k_j,, \lambda_j, \pi_j)_{j=1}^2} \prod_{i=1}^{N} \sum_{j=1}^{2} \pi_j b(t_i \mid \lambda_j),$$

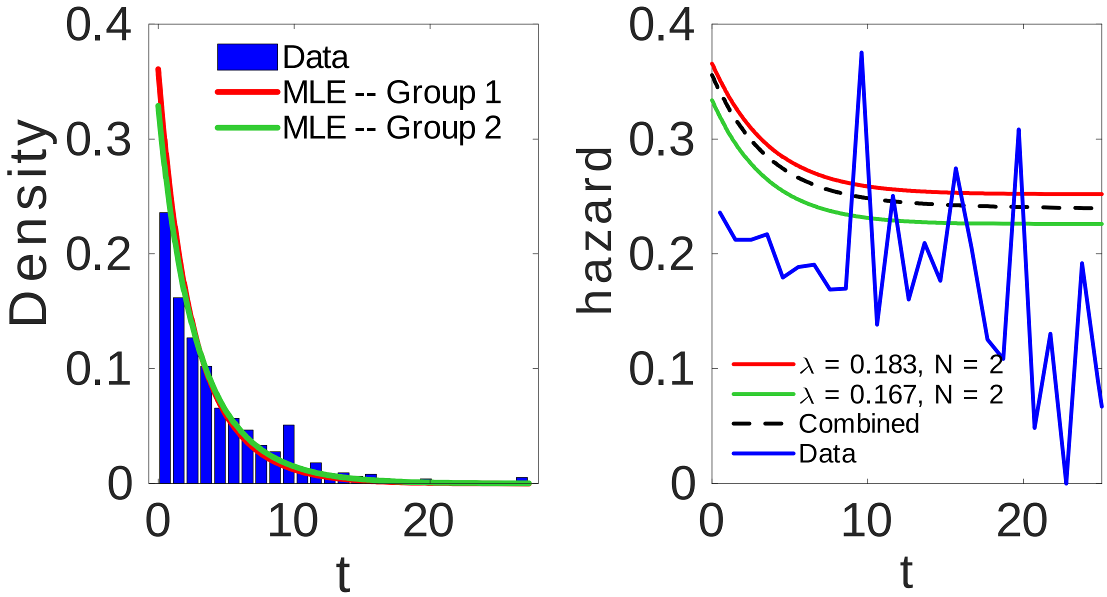

Figure 8: Two-market estimates

i.e., we estimate the mixing distribution (the πs) on the 2-vector (λ_1, λ_2).[15] Table 3 reports the estimates and Fig. 8 plots the model fit.

Table 3: ML estimates; constrained by $E^{G}(x|\lambda_i) = 6c$ for both i

Parameter estimates and the implied $(x_2, w_{\max})$

i	λ	π	p	k	c	x_2	$w_{\max}$
1	0.183	0.686	0.467	0.160	0.003	0.057	0.340
	$[0.183, 0.183]$	$[0.685, 0.687]$	$[0.466, 0.468]$	$[0.160, 0.161]$	$[0.003, 0.003]$		
2	0.167	0.314	0.460	0.156	0.003	0.054	0.330
	$[0.166, 0.158]$	$[0.313, 0.315]$	$[0.459, 0.460]$	$[0.155, 0.157]$	$[0.002, 0.003]$		

The dashed line in Fig. 8 is the hazard of the combined population in which the fraction surviving to date t is

$$1 - F(t) = \sum_{i=1}^{2} \pi_i \exp\left(-\int_0^t (\lambda_i - x_{i,s})\, ds\right),$$

and the combined hazard, $f/(1 - F(t))$, that the dashed line portrays is only slightly steeper than the hazards of the two subpopulations.

[15] This is a simplified version of a method that Kiefer and Wolfowitz (1956) proposed and that Heckman and Singer (1984) have used. It does not require that the data be arranged into groups (in this case two), the groups are chosen so that the likelihood is maximized. It amounts to raising the number of parameters in the likelihood from one (i.e., λ) to three: $(\lambda_1, \lambda_2, \pi_1)$.

3.4 Implications of the estimates

We now refer back to the model and the issues it raises in light of these estimates. We shall focus on the estimates in Table 1 except for the welfare results where we also include the two-market results reported in Table 3.

3.4.1 Welfare

We shall use the estimates of the first row of Table 1, and Table 3. Thus in Table 4 for the two market case, $p_{\min}$ and $p_{\max}$ are calculated in each market using its estimated parameters (λ, k, c) reported in Table 3.

Table 4: WELFARE

	One Market		Two Markets	
	source of info	$1/p$	source of info	$\frac{\pi_1}{p_1} + \frac{\pi_2}{p_2}$
First best (contractible)	Eq. (28)	6.52	Eq. (28)	$\frac{.69}{.13} + \frac{.31}{.13} = 7.86$
highest-welfare equilib. $p_{\min}$	Eq. (23)	4.92	Eq. (23)	$\frac{.69}{.18} + \frac{.31}{.18} = 5.47$
estimated equilib.	Table 1	**1.68**	Table 2	$\frac{.69}{.47} + \frac{.31}{.46} = \mathbf{2.15}$
worst equilib. $p_{\max}$	Eq. (21)	0.24	Eq. (21)	$\frac{.53}{2.85} + \frac{.47}{2.77} = 0.35$

The two markets have higher welfare than the one-market estimates, and the estimated first-best level is also higher. In both cases welfare is estimated to be about 25 percent of its first-best level. With two λ values, the estimated reputation building is slightly higher because the individual hazards are somewhat steeper than the combined hazard.

3.4.2 Estimate of the optimal policy intervention

Evaluating (29) and (30) at the parameter estimates in Table 1, we derive the estimates of the optimal policy in Table 5:

Table 5: OPTIMAL POLICY

Assumed average price drop	$E(x)/c$	6	12
Recall tax	$T = \hat{p}$	0.153	0.167
Period subsidy	$S = (\lambda - c - \hat{p})\, T$	0.002	0.006
Tax relative to firm value	$T/E(v)$	0.605	0.550
Subsidy relative to firm value	$S/E(v)$	0.010	0.019
equilibrium recall rate (est.)	$\lambda - \bar{x}$	0.149	0.175
optimal recall rate	$\lambda - \hat{x}$	0.016	0.034
recall rate difference (equilib. minus opt.)		0.133	0.141
recall rate % reduction		0.895	0.808

where $E(x) = \int x_t \mu(t)\, dt$, $E(v) = k - c + E(x)$, and μ is given in (9).

Note first, that T is 55-60 percent of the average market value; it is high because recalls are relatively infrequent and the policy has to leverage them to induce higher effort in all periods. Second, compared to equilibrium, recalls are much smaller under the optimal policy, especially if the market punishment is only 6 times costs.

Since the proposed policy is quite simple, we would expect it to be already in place, and there is some evidence that this is so, at least qualitatively. Recall-related fines are imposed in several countries and jurisdictions. In the U.S., the FDA can exact civil fines when it can demonstrate that a Federal law was violated.[16] Additionally, in the U.S., GM was fined $900 million after it's ignition switch scandal.[17]

In the U.K., fines are imposed on firms that engaged in financial "misconduct" which we can think of as a defective financial service. Armour, Meyer & Polo (2017) and Karpoff (2012) summarize the stock-price impact which, relative to the fines imposed, have reputational effects similar in size to those that product recalls have relative to the direct recall costs – their results are listed in online Appendix C Table A2. Online Appendix D Table A3 lists more examples of fines and provides details. Online Appendix D also includes more examples of fines as well as examples of offsetting flow subsidies – counterparts of S.

3.5 The value of a reputation

Fig. 9 shows the stationary distribution of w in two equilibria, and Table 6 summarizes the findings.

From Eq. (15) we find that x_2 is decreasing in p, which means that welfare is positively related to the value of reputation. The larger is x_2, the greater the tendency for firms to be bunched near the maximum where the recall hazard is at its lowest, and we find in Appendix that $\lim_{w \to w_{max}} \zeta(w) = \infty$ if $2\lambda + x_1 < 3x_2$, which holds at the estimated parameter values.

Table 6: AVERAGE VALUE OF REPUTATION, $\mathrm{E}^{\zeta}(w)$

equilibrium	E(w)
MLE	**0.084**
OLS-estimator	0.074

The shape of the cross-firm value distributions depicted Fig. 1 and Fig. 9 are quite

[16]The FDA's website https://www.fda.gov/AnimalVeterinary/ResourcesforYou/ucm268127.htm lists the fine schedule under the Food Drug and Cosmetics Act. Discussion is in Urban (1992) and Olson (1996).

[17]The fine, however, was because GM knew about the problem for over a decade before issuing the recall.
https://www.washingtonpost.com/news/business/wp/2015/09/17/why-general-motors-900-million-fine-for-a-deadly-defect-is-just-a-slap-on-the-wrist/?utm_term=.0b65b339f126

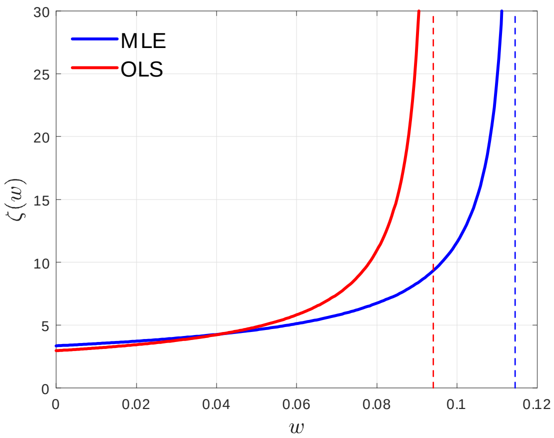

Figure 9: THE DENSITY $\zeta(w)$ OF THE VALUE OF REPUTATION, w, UNDER DIFFERENT ESTIMATES.

similar to those in Panel E and Panel F of BM18's Fig. 1, but once again their reputation density does not rise as steeply on the right because their investment is not monotone in reputation, it declines as reputation rises towards its highest value.

4 Discussion and extensions

Several issues are addressed in this section. First, in the model sellers are homogenous; but we will show within a market some heterogeneity in firms' scale can be handled with no change in the implications. Second, we estimated one particular equilibrium of the game, but there are other equilibria some of which we outline. Third, firms may not want to announce their defects and generate recalls right away – the recall decision is endogenous. Fourth, transportation equipment is durable and we show how the model extends to this case. Fifth, if firms can commit to warranty payments in excess of coverage of defects, they may be able to raise their values. Sixth, there may be other information channels that affect reputation. The seventh and last subsection discusses three mechanisms other than reputational concerns that motivate effort.

4.1 Heterogeneity of firm scale

The data include recalls of transportation equipment ranging from cars to windshield wipers. There are trivial recalls that customers do not respond to, perhaps because c is much smaller for such recalls. Then there are major recalls involving automobile safety. Firms also differ in their size, which in this model would amount to differences in the total amount of quality they supply. We now show that if the parameters are scaled in a particular way, the model can accommodate a specific form of one-dimensional heterogeneity with no change in the recall hazard.

Let us assume that the industry has an integer-valued number of firms n (instead of a continuum of measure n), but that n is large enough that firms take p as fixed and constant. This simplifies the exposition. Now the quality consumed, q, is an aggregate of individual firms' qualities X_i:

$$q_t = \sum_{i=1}^{n} X_{i,t}, \tag{47}$$

Eq. (1) holds at the market-clearing price p with the consumers' FOC for q still satisfying $U_q/U_z = p$. The unit price of quality is the same for all firms, and firm i's revenue is pX_i. Let η_i denote an exogenous characteristic of firm $i \in \{1, ..., n\}$ that we shall refer to as the "scale" of firm i. The solution of the previous section survives if heterogeneity is one dimensional as follows:

Supply of product i.—For product i, let $X_i^2/(2\eta_i)$ = effort cost, $\lambda - X_i/\eta_i$ = recall hazard, K_i = entry cost, and C_i = recall cost. Firm i's Bellman equation reads

$$rV_{i,t} = \max_X \left(pX_{i,t}^* - \frac{1}{2\eta_i} X^2 - \left(\lambda - \frac{X}{\eta_i} \right) (V_{i,t} - K_i + C_i) + \frac{dV_{i,t}}{dt} \right) \tag{48}$$

with the FOC

$$X_{i,t} = V_{i,t} - K_i + C_i \tag{49}$$

Then in Appendix we prove

Proposition 4 *If $K_i = \eta_i k$ and $C_i = \eta_i c$, then*

$$X_{i,t} = \eta_i x_t \quad \textit{and} \quad V_{i,t} = \eta_i v_t \tag{50}$$

will solve equations (47), (48), and (49), and each firm's recall hazard will be $\lambda - x_t$, where (x_t, v_t) satisfy Eqs. (5) and (19).

Under these assumptions we can fit a common hazard function to the data as plotted in Fig. 3

Other types of heterogeneity Some variation in types cannot, however, be handled in the way outlined above, and this is true both even when types are exogenous, as well as when they are endogenous:

(i) *Exogenous types.*—Let's start with ABK who assume that high types have lower but still a positive likelihood of retractions, and we may assume that high types have higher values for consumers. The conditional probability of a retraction falls in the absence of a retraction and jumps up following a retraction. In contrast to what happens in my model, these hazard dynamics are purely a function of selection – the probability, call it p, that the firm's type is high, rises the longer it has been since the last retraction – see ABK's Fig. 2.

Now the estimates in Sec. 3.3 are for two types and we found in Panel 2 of Fig. 8 that even conditional on market type the hazard rate declines. This suggests that if, contrary to my model, the hazard conditional on type was constant, the firm's type would need to change, so as to generate the decreasing type-specific hazards that Fig. 8 shows.

(*ii*) *Endogenous types.*—This is what BM13 adds; there the investment in quality is chosen continually but only matters at random dates. The perfect bad news version of BM13 does produce a monotone recall hazard rate and monotone valuation penalties for recalls: The firm exerts effort when its reputation lies above some cutoff and shirks below this cutoff. In some of their perfect bad news equilibria firms start out with low effort but, if they suffer no breakdowns, beliefs about the firm's type drift up into the region where high effort (which is even less likely or not-at-all likely to lead to a breakdown) becomes optimal and the hazard rate drops.[18]

4.2 Other equilibria

So far we have assumed that a firm's value at a recall coincides with new entrant's value k and that the evolution of value and of quality is thereafter deterministic. We shall now briefly discuss the consequences of dropping these two assumptions. First we define a class of equilibria in which behavior is random, and then equilibria in which upon recall v_t drops to a value other than k.

4.2.1 Equilibria indexed by reversion to $v_R \neq k$ at recall

Takeovers limit punishment values to no lower than k. Effectively, the firm erases its reputation by selling its hard assets to a newcomer. This assumes that the firm's capital is salvageable. If the punishment following a recall can force its value to some $v_{\mathrm{R}} \neq k$, and then behavior would be as described in the previous sections, but with the HJB equation

$$rv = \max_{x \leq \lambda} \left(px^* - \frac{x^2}{2} - (\lambda - x)(v - v_{\mathrm{R}} + c) + \frac{dv}{dt} \right),$$

An firm's initial condition is still $v_0 = k$ and the decline in the recall hazard would be larger the lower is v_{R}.

Further equilibria entail punishments that depend on recall order. Let v_{R}^j denote the value following the jth recall. Starting with $v_0 > k$, any decreasing sequence $\left(v_{\mathrm{R}}^j\right)_{j \geq 1}$is an equilibrium. And when the second punishment entails $v_{\mathrm{R}}^2 = 0$, this would resemble one of the the outcomes in the imperfect bad news version of BM13 pp. 2411-2 in which the second piece of bad news in a shirk-work-shirk equilibrium reduces reputation by so much as to eliminate investment altogether.

[18] Board and Meyer-ter-Vehn (2018) extends BM13 in some ways, but assumes only good news signals and thus does not apply to events such as recalls.

4.2.2 Equilibria in which firm quality evolves randomly

Assume that, following a recall v_t drops to k and stays there until a Poisson shock hits, upon which it rises to $v_{\mathrm{H}} > k$. Assume that during the punishment phase (also the immediate-post-entry phase) quality is assumed to be $x_{\mathrm{L}} = c$ for a period of random duration (independent over firms) distributed exponentially with hazard rate θ, following which action is assumed to revert to

$$x_{\mathrm{H}} = c + v_{\mathrm{H}} - k \tag{51}$$

with

$$rk = pc - \frac{c^2}{2} - (\lambda - c)\, c + \theta\, (v_{\mathrm{H}} - k) \tag{52}$$

$$rv_{\mathrm{H}} = px^*_{\mathrm{H}} - \frac{x_H^2}{2} - (\lambda - x_{\mathrm{H}})\, (v_{\mathrm{H}} - k + c) \tag{53}$$

where $v_0 = k$ is the firm's value in the punishment phase, and v_{H} is its value when in the normal phase. Each firm's recall hazard would then jump up to $\lambda - x_{\mathrm{L}}$ after a recall (as in the equilibrium discussed above and estimated), and remain constant until the punishment phase was over at which point it would jump down to $\lambda - x_{\mathrm{H}}$. The aggregate hazard would mix over the firms of the two types and over the random length of the punishment.

The aggregate hazard.—The waiting time distribution, F satisfies

$$1 - F(t) = e^{-(\lambda - x_{\mathrm{L}})t} e^{-\theta t} + \int_0^t e^{-(\lambda - x_{\mathrm{H}})(t-s)} \theta e^{-\theta s} ds. \tag{54}$$

The ML estimation procedure then uses (35)-(43), except there are only two values of x so that $x_{\mathrm{L}} = c$ and v_{H} and x_{H} solve (51) and (53).

This type of equilibrium seems to be unique in its class – Eq. (52) gives us p which seems uniquely determined. So, in total there are 3 unknowns $(x_{\mathrm{H}}, v_{\mathrm{H}}, p)$. This has implications similar to the two-action model of BM13 when, with perfect bad news signals, incentives increase in reputation and the bad-news hazard is lower the longer it has been since the previous one. The recall data do seem to favor a gradually rising x_t along the equilibrium path,

4.3 Endogeneity of recall announcements

Why would a firm disclose the defect and announce a costly recall? Perhaps to forestall lawsuits that would result in certain cases such as defects (e.g., death due to brake failure).[19] Thus, even if the car was not sold under warranty, the companies must recall the car or face legal action. Table A.3 reports recall related fines, but fines are also imposed for non-reporting defects. For example, GM was fined $900 million after its ignition switch scandal because GM knew about the problem for over a decade before issuing the recall.[20]

The question of when one would report bad news about oneself has been studied more generally. In the absence of any legal sanctions, a pure adverse selection argument suggests that defects will always be disclosed, as Grossman (1981) and Milgrom (1981) showed. Suppose that a defect was publicly known to have occurred, but that its magnitude was unknown so that the c that the firm would incur to fix it was uncertain. Buyers would then expect that any product information withheld by a firm is unfavorable to his product which causes unraveling and everyone discloses. That is, lowest-c type would always disclose and this would then induce the next-to-lowest-c type to disclose and so on until every type discloses.

4.4 Durable goods

We now interpret q as durable-good services. Assume as before that the manufacturer must replace a failed product at the cost c in units of z, but that recalls affect *only the latest generation of products.* If past generations of the product were also affected and customers hold a stock that was revealed to be in some way defective, the following would not apply.[21]

Consumers.—Then (1) still applies with the following modification: Investment by consumers in durables is I, measured in quality units and services of durables consumed in quality units are

$$q_t = \int_0^\infty e^{-\delta s} I_{t-s} ds.$$

Since bad products are replaced, recalls do not appear in the above equation. The budget constraint is $m = pI + z$. In steady state $q = \delta^{-1} I$ is a constant and assuming all income m is spent each period, the consumer's FOC is

$$p = \int_t^\infty e^{-(r+\delta)(s-t)} U_q \left(\frac{I}{\delta}, m - pI \right) ds = \frac{1}{r+\delta} U_q \left(\frac{I}{\delta}, m - pI \right). \quad (55)$$

Producers.—Conditional on p, (5) is unchanged and the main propositions and impli-

[19] In the recall literature, Rupp and Taylor (2002) find that auto manufactures initiate high potential liability defects. Sometimes, however, manufacturers bunch their recall announcements (Rupp 2004, p. 23), suggesting that they can at least postpone some of them.

[20] https://www.washingtonpost.com/news/business/wp/2015/09/17/why-general-motors-900-million-fine-for-a-deadly-defect-is-just-a-slap-on-the-wrist/?utm_term=.0b65b339f126

[21] A model where the quality of a durable asset remains uncertain is Hong and Stein (1999).

cations again go through subject to a reinterpretation of p as given in (55).

4.5 Warranties

So far we assumed that a firm is legally liable to cover the recall damage and pay c. If, however, a firm can offer a larger payment in the event of a recall, this becomes a substitute for contractible quality of output because both the quality and the probability of recall are governed by the same effort variable, x.[22] The firm can then raise its payoff and the only equilibrium then coincides with the first best described in Sec. 2.2.3.

If the warranty w exceeds c, the customer is effectively getting an expected premium for each physical unit purchased. Note first that since n is the number of firms and also the total number of physical units sold, and since the number of customers is normalized to one, $n = \#$ of physical units that each customer buys. We first establish:

Lemma 1 *In units of z the customer's expected premium for each physical unit purchased would equal*

$$\text{PHYSICAL UNIT PREMIUM} = (w - c)(\lambda - x). \tag{56}$$

Proof. Using Eq. (2), if M_i is large, customer i's expected benefit from defective products is, by the SLLN,

$$\sum_{j=1}^{M_i} (w_j - c)\, \kappa_{i,j} I_{\{\text{j was recalled}\}} \overset{\text{a.s.}}{\to} (\lambda - x)(w - c)\, n, \tag{57}$$

in which case (56) follows because the expression in (57) is deterministic. ■

The customer is buying n physical units but the firm sells only one unit and therefore each firm receives a premium of $(\lambda - x)(w - c)$ for the one unit that it has sold. If, in addition w was set so that

$$w = p + c, \tag{58}$$

the excess recall reimbursement, $w - c$, would play the same exact role as the recall tax T in Sec. 3.4.2 provided that $p = \hat{p}$ and the equilibrium would be first best. Formally, Appendix proves

Proposition 5 *If*

$$p = \hat{p} \quad \textit{and} \quad w - c = T, \tag{59}$$

then (i) *condition (56) is the same as (30), and (58) is the same as (29);* (ii) w *is the firms' optimal warranty under commitment.* (iii) *The equilibrium at* $\hat{p}$ *is unique.*

[22] Chu and Chintagunta (2009) find that better warranty coverage correlates positively with product prices.

In other words the warranty exceeds c by an amount equaling the recall tax and the planner's optimal subsidy equals the warranty premium customers are willing to pay. This happens because of the assumption that customers are diversified and therefore effectively risk neutral with respect to $w-c$. The warranty can then serve two functions: a) compensating customers and b) providing incentives to the firm. As MacLeod (2007) points out, when the buyer and seller are risk neutral, as long as the signal of performance is informative, one can achieve the first best. In our case the buyer draws utility from two outputs of an agent with effort affecting both, then it is enough to be able to condition on only one of them as long as the agent and principal are both risk neutral – the risk neutrality is what the diversification assumption delivers.

If firms' qualities differed and were privately known by the firms, warranties would naturally emerge as signals that would separate the types (Spence 1977). Warranties are costly to a seller and their costs increase with product liability. This would likely lead quality to be reflected in the firms' stock prices and a recall itself would then not be news about the firm's quality and not cause a firm's stock price to drop beyond the direct cost of fixing the recall.[23]

4.6 Other information channels that may affect reputation

There are signals of quality other than product recalls; word-of-mouth information diffusion, on-line-reviews and so on. A tractable way to incorporate them is to have a second bad news signal with hazard rate $\rho-\beta x$ that does not entail a recall cost but that still reflects x. If, moreover, the signal prompts a drop in v again to k, we will have an equilibrium with the only state being $t \equiv$ time since the last signal (of either type), the HJB equation

$$rv_t = \max_{x \leq \min\left(\lambda, \frac{\rho}{\beta}\right)} \left(px_t^* - \frac{x^2}{2} - (\lambda - x)(v_t - k + c) - (\rho - \beta x)(v_t - k) + \frac{dv}{dt} \right),$$

and the FOC

$$x_t = (1+\beta)(v_t - k) + c, \tag{60}$$

which reduces to (6) when $\beta = 0$. For the FOC (60) to hold for all t where both hazard rates are positive, instead of (18) we now need $x_2 \leq \min\left(\lambda, \frac{\rho}{\beta}\right)$. The initial condition is still $x_0 = c$ but the response of x to reputation as measured by $v-k$ is now higher which implies a higher reputational effect on effort x. The social optimum $\hat{x}$ in (28) remains the same and now x grow faster and further towards it absent any bad news signal. At $\rho = \beta = 0$ we have the original structure with x_2 given in Eq. (15) and Appendix derives x_2 and proves that a small additional signal raises it:

[23] Cohen, Darrough, Huang and Zach (2011) find that stock prices are indeed positively related to measures of warranty coverage measured relative to the industry median.

Proposition 6

$$\left.\frac{\partial x_2}{\partial \rho}\right|_{\beta=\rho=0} = \frac{x_2 - c}{x_2 - (r + \lambda - p)} > 0. \tag{61}$$

This result does not imply that welfare will rise because conditional on x the signals are now more frequent and this forces x to revert more often to c.[24]

4.7 Mechanisms other than reputation

Several other mechanisms could be at work to explain the declining recall hazard and the large stock price impact of recall. These would be complementary hypotheses. We list three here: (a) Prevention of possible defects as a product ages, (b) Random, autocorrelated recall costs, and (c) multi-product firms.

(a) The declining hazard may in part be caused by a gradual discovery process on the part of the manufacturer; the pattern in Fig. 3 could arise if obvious defects are manifest from early on, and the less obvious ones take time before the company can gather enough evidence. As time passes, the product is therefore improved and further defects are less likely, hence the declining recall hazard. To the ABK model, for instance, one could add a discovery mechanism whereby the probability of defects declines perhaps by checking the components of the product one at a time, so that their hazard declines even conditional on type. In that case more complex products would show a stronger decline in the hazard.

(b) To explain why stock prices fall much more than the direct recall costs alone one could add uncertainty over the magnitude of c itself (in addition to its uncertain timing). An autocorrelated stochastic process for c would lead investors to interpret a large recall as a signal that such recalls were more likely in the future. In other words, the market may be concerned that the firm may have systemic problems in quality control that will affect its future operating profits and its future stream of recalls, as in the standard bandit model with unknown payoffs. To maintain an effect of recall news on the value of the firm as the firm gets older (e.g., Toyota was founded in 1937), there would need to be periodic exogenous shifts in the underlying quality of the firm (perhaps as a result of new product introductions or changes in management) so that the market never fully learns the firm's quality.

(c) Another reason why stock prices fall much more than the direct recall costs alone could be that the firm produces more than one product. A recall of one product could signals poor management practices that may spill over to affect the quality of all of the firm's products.

[24] In repeated games with moral hazard, a shorter the delay with which actions are observed need not produce equilibria with higher welfare – see Abreu, Milgrom and Pearce (1991).

5 Conclusion

The paper has structurally estimated model of reputation building in a market in which firm reputations consist of the public histories of their product recalls. On-path punishments were periodic, arriving through a sequence of defects interpreted as product recalls. Product recalls – when made by publicly traded firms – are typically accompanied by stock price reductions. The recall data and information from the stock-prices were used to estimate the model. The model fits the recall data fairly well and we find that reputation accounts for about 8.3 percent of firm value. We then drew welfare and policy conclusions. These conclusions pertain to the transportation-equipment sector only.

Welfare was estimated at 25 percent of first best, but an easily implementable policy can attain first best. First best is attained by a recall tax that is substantially larger than the direct recall cost; to maintain the right incentives for firms to enter, the tax has to be accompanied by a subsidy that can be paid every period. Such a policy is currently used sporadically by the FDA in the U.S. and in other countries; if the fines were higher and more broadly applied, that would bring recall rates down by an order of magnitude and raise quality of goods produced to its first best level.

Several extensions and alternative mechanisms are listed at the end of the previous section, and it is clear that our welfare estimate may change considerably when they are taken into account, most notably other information channels and warranties, both of which are common in many industries, including the automobile industry, as well as different costs of effort. It is hoped that future work will assess how much they may contribute to explaining the phenomena that we have described in this paper and possibly modifying its welfare conclusions.

A Mathematical Appendix

A.1 Proof of Proposition 1

We solve (13) using separation of variables. We can write (13) as $dx/dt = \rho(x-x_1)(x-x_2)$, where $\rho = -\frac{1}{2}$. This is equivalent to

$$\frac{1}{\rho(x_2 - x_1)}\left(\frac{1}{x - x_2} - \frac{1}{x - x_1}\right)dx = dt.$$

Integrating both sides, and using $x_1 < x < x_2$, we have

$$\frac{1}{\rho(x_2 - x_1)}\ln\frac{x_2 - x}{x - x_1} = t + C$$

Using the initial condition $x(0) = c$, we get $\exp\{\rho(x_2 - x_1)C\} = (x_2 - c) / (c - x_1)$. This implies that

$$x_t = \frac{x_2 + x_1 \frac{x_2 - c}{c - x_1} \exp\{\rho(x_2 - x_1)t\}}{1 + \frac{x_2 - c}{c - x_1} \exp\{\rho(x_2 - x_1)t\}} \tag{62}$$

or, alternatively

$$x_t = x_1 + \frac{x_2 - x_1}{1 + \frac{x_2 - c}{c - x_1} \exp\{\rho(x_2 - x_1)t\}}$$

which implies (19) since $\rho = -1/2$. Notice that x_t is strictly increasing in t, because the denominator is strictly decreasing (using the first definition for x_t and remembering $x_1 < 0 < x_2$). Also, $x_t \to x_2$ implies $v_t \to k - c + x_2$.

The necessity of $x_2 < \lambda$, i.e., of Eq. (18) – Assume, on the contrary, that an equilibrium entails $x_2 > \lambda$. We now show that this implies a discontinuity in v_t at the point $t = t_\lambda$ where x reaches λ. Suppose, on the contrary, that v_t is continuous at t_λ.

(i) For $t \geq t_\lambda$, we have a zero probability of a defect, and the flow profit at $x = \lambda$ is $p\lambda - \lambda^2/2$. That implies

$$v_{t_\lambda} = \frac{1}{r}\left(p\lambda - \frac{\lambda^2}{2}\right). \tag{63}$$

(ii) Evaluating (6) at $t = t_\lambda$ and $x_{t_\lambda} = \lambda$,

$$v_{t_\lambda} = k - c + \lambda. \tag{64}$$

(iii) $x_2 > \lambda$ is equivalent to $\sqrt{(r + \lambda - p)^2 + 2r(k - c)} > (p - r)$. Squaring both sides, this is equivalent to $(r - p)^2 + 2\lambda(r - p) + \lambda^2 + 2r(k - c) > (p - r)^2$. Canceling, this leaves

$$\begin{aligned} 2r(k - c) > 2\lambda(p - r) - \lambda^2 &\Leftrightarrow (k - c) > \frac{\lambda p}{r} - \lambda - \frac{\lambda^2}{2r} \\ &\Leftrightarrow (k - c + \lambda) > \frac{1}{r}\left(p\lambda - \frac{\lambda^2}{2}\right). \end{aligned} \tag{65}$$

But at (64) and (63) imply that Eq. (65) should hold as an equality, a contradiction. This completes the proof of Proposition 1.

A.2 Proof of Proposition 3

Using Eq. (35), the LHS of Eq. (37) reads

$$\frac{1}{\int_0^\infty f(s)\,\mu(s)\,ds} \frac{f(t)\,\mu(t)}{1 - \frac{\int_0^t f(s)\mu(s)ds}{\int_0^\infty f(s)\mu(s)ds}} = \frac{f(t)\,\mu(t)}{\int_t^\infty f(s)\,\mu(s)\,ds} = \frac{f(t)\,[1 - F(t)]}{\int_t^\infty f(s)\,[1 - F(s)]\,ds}$$

Integration by parts in the denominator yields

$$\int_t^{\infty} f(s)[1-F(s)]\,ds = -\frac{1}{2}[1-F(s)]^2\Big|_t^{\infty} = \frac{1}{2}[1-F(t)]^2$$

Canceling $1-F(t)$ from top and bottom of the ratio yields (37). We then have

$$B(t) = 1-\exp\left(-\int_0^t h_b(s)\,ds\right) = 1-\exp\left(-2\int_0^t \frac{f(s)}{1-F(s)}ds\right). \tag{66}$$

A.3 The ODE for $x^{1+\delta}/(1+\delta)$

Replacing $x^2/2$ by $x^{1+\delta}/(1+\delta)$ in Eq. (5), we get the FOC $x = (v-k+c)^{1/\delta}$ and $v = x^{\delta}-c+k$, so that $\frac{dv}{dt} = \delta x^{\delta-1}\frac{dx}{dt}$. Substituting into Eq. (5) and rearranging yields the ODE

$$\frac{dx}{dt} = \frac{r}{\delta}(k-c)\,x^{1-\delta} - \frac{p}{\delta}x^{2-\delta} - \frac{1}{1+\delta}x^2 + \frac{(r+\lambda)}{\delta}x \tag{67}$$

with initial condition $x_0 = c^{1/\delta}$. The overall decline in the hazard is $2\left(\hat{x}_2 - c^{1/\delta}\right)$., where $\hat{x}_2$ is the analog of x_2 that we would obtain by setting the RHS of (67) equal to zero. After dividing by x/δ and rearranging we find that

$$r(k-c)\,x^{-\delta} - px^{1-\delta} - \frac{\delta}{1+\delta}x + (r+\lambda) = 0$$

which when $\delta \neq 1$ we cannot solve for $\hat{x}_2$ analytically.

A.4 Derivation of $F(t)$, $\mu(t)$, $t(x)$, $\zeta(w)$, $b(t)$, $G(x)$ and $g(x)$

This Appendix derives explicit forms used in the estimation.

Derivation of F.—Note that by definition we have that $F(t) = 1-exp\left[-\int_0^t(\lambda - x_\tau)d\tau\right]$ Then, since (19) can be written as

$$x_t = x_2 - \frac{(x_2-x_1)\frac{x_2-c}{c-x_1}\exp\{-\frac{1}{2}(x_2-x_1)t\}}{1+\frac{x_2-c}{c-x_1}\exp\{-\frac{1}{2}(x_2-x_1)t\}}, \tag{68}$$

we have

$$\begin{aligned}\int_0^t(\lambda - x_\tau)d\tau &= (\lambda - x_2)t + \int_0^t \frac{(x_2-x_1)\frac{x_2-c}{c-x_1}\exp\{-\frac{1}{2}(x_2-x_1)s\}}{1+\frac{x_2-c}{c-x_1}\exp\{-\frac{1}{2}(x_2-x_1)s\}}ds \\ &= (\lambda - x_2)t - 2\ln\frac{1+\frac{x_2-c}{c-x_1}\exp\{-\frac{1}{2}(x_2-x_1)t\}}{1+\frac{x_2-c}{c-x_1}}\end{aligned}$$

Therefore,

$$1 - F(t) = \left[\frac{c - x_1 + (x_2 - c)\exp\{-\frac{1}{2}(x_2 - x_1)t\}}{x_2 - x_1} \right]^2 \exp(-(\lambda - x_2)t), \qquad (69)$$

Its derivative then yields the density $f(t)$ which is used in (35)

$$f(t) = \left(\frac{c - x_1}{x_2 - x_1}\right)^2 \exp\left(-(\lambda - x_2)t\right) \left(1 + \frac{x_2 - c}{c - x_1} \exp\left(-\frac{1}{2}(x_2 - x_1)t\right)\right) \\ \times \left[(\lambda - x_2) + (\lambda - x_1)\frac{x_2 - c}{c - x_1} \exp\left(-\frac{1}{2}(x_2 - x_1)t\right)\right]. \qquad (70)$$

Derivation of μ.—The numerator of (9) is (69). If $\lambda > x_2$, substitution from (69) into the denominator of (9) gives

$$\int_0^\infty [1 - F(s)]\, ds = \frac{1}{(x_2 - x_1)^2} \left[\frac{(c - x_1)^2}{(\lambda - x_2)} + \frac{4(c - x_1)(x_2 - c)}{(\lambda - x_1) + (\lambda - x_2)} + \frac{(x_2 - c)^2}{\lambda - x_1} \right]$$

Substituting into (9) and if $x_2 < \lambda$, then

$$\mu(t) = \frac{\left[c - x_1 + (x_2 - c)\exp\{-\frac{1}{2}(x_2 - x_1)t\}\right]^2 \exp(-(\lambda - x_2)t)}{\frac{(c - x_1)^2}{(\lambda - x_2)} + \frac{4(c - x_1)(x_2 - c)}{(\lambda - x_1) + (\lambda - x_2)} + \frac{(x_2 - c)^2}{\lambda - x_1}}. \qquad (71)$$

Derivation of $t(x)$.—Inverting the function in Eq. (19) yields

$$t(x) = -\frac{2}{x_2 - x_1} \ln\left(\frac{c - x_1}{x - x_1}\frac{x_2 - x}{x_2 - c}\right) \geq 0, \qquad (72)$$

Solution for $\zeta(w)$.—Differentiating in Eq. (72) we have

$$t'(x) = -\frac{2}{x_2 - x_1}\left(\frac{c - x_1}{x - x_1}\frac{x_2 - x}{x_2 - c}\right)^{-1}\left(\frac{c - x_1}{x_2 - c}\right)\left\{\frac{x_1 - x_2}{(x - x_1)^2}\right\} = \frac{2}{(x_2 - x)(x - x_1)},$$

and since $dx/dw = k$, we have

$$\zeta(w) \equiv kt'(c + wk)\,\mu(t(c + wk)) = \frac{2k}{(c + wk - x_1)(x_2 - c - wk)}\mu(t(c + wk)) \qquad (73)$$

Showing that at the parameter estimates $\lim_{w \to w_{\max}} \zeta(w) = +\infty$.—Let $L = \frac{1}{\int[1 - F(t)]dt} 2k(c-$

$x_1)^{\frac{2\lambda-2x_1}{x_2-x_1}}(x_2-c)^{\frac{2x_2-2\lambda}{x_2-x_1}}$. Using (73) and (9).

$$\lim_{w\to w_{\max}}\zeta(w)=\lim_{x\to x_2}\frac{2k\mu(t(x))}{(x-x_1)(x_2-x)}=\lim_{x\to x_2}\frac{L(x-x_1)^{\frac{2x_1-2\lambda}{x_2-x_1}}(x_2-x)^{\frac{2\lambda-2x_2}{x_2-x_1}}}{(x-x_1)(x_2-x)}$$

$$=\lim_{x\to x_2}\frac{L(\frac{2x_1-2\lambda}{x_2-x_1})(x-x_1)^{\frac{3x_1-2\lambda-x_2}{x_2-x_1}}(x_2-x)^{\frac{2\lambda-2x_2}{x_2-x_1}}-L(\frac{2\lambda-2x_2}{x_2-x_1})(x-x_1)^{\frac{2x_1-2\lambda}{x_2-x_1}}(x_2-x)^{\frac{2\lambda+x_1-3x_2}{x_2-x_1}}}{x_2-2x+x_1}$$

where the second line follows from substitution and the third line follows from L'Hôpital's rule. Hence,

$$\lim_{w\to w_{max}}\zeta(w)=\begin{cases}0 & 2\lambda+x_1>3x_2\\ \infty & 2\lambda+x_1<3x_2\end{cases}. \tag{74}$$

Derivation of $b(t)$ − −Differentiate (36) gives

$$\begin{aligned}b(t)&=2f(t)(1-F(t))\\&=2\left(\frac{c-x_1}{x_2-x_1}\right)^4\exp\left(-2(\lambda-x_2)t\right)\left(1+\frac{x_2-c}{c-x_1}\exp\left(-\frac{1}{2}(x_2-x_1)t\right)\right)^3\\&\quad\times\left[(\lambda-x_2)+(\lambda-x_1)\frac{x_2-c}{c-x_1}\exp\left(-\frac{1}{2}(x_2-x_1)t\right)\right]\end{aligned} \tag{75}$$

Derivation of $G(x)$.—Using (36) and (38), $G(x)=1-(1-F(t(x)))^2$ with $t(x)$ given in (72). Finally,

$$F(t(x))=1-\left(\frac{1-\frac{x_2-c}{x_1-c}\frac{x_1-c}{x_2-c}\frac{x-x_2}{x-x_1}}{1-\frac{x_2-c}{x_1-c}}\right)^2\exp\left(-(\lambda-x_2)t(x)\right)=1-\left(\frac{c-x_1}{x-x_1}\right)^{\frac{2(\lambda-x_1)}{(x_2-x_1)}}\left(\frac{x_2-x}{x_2-c}\right)^{\frac{2(\lambda-x_2)}{(x_2-x_1)}}.$$

Substituting into $G(x)$, we have

$$G(x)=1-\left(\frac{c-x_1}{x-x_1}\right)^{\frac{4(\lambda-x_1)}{(x_2-x_1)}}\left(\frac{x_2-x}{x_2-c}\right)^{\frac{4(\lambda-x_2)}{(x_2-x_1)}} \tag{76}$$

Differentiate the above yields

$$g(x)=\frac{4}{x_2-x_1}\left(\frac{c-x_1}{x-x_1}\right)^{\frac{4(\lambda-x_1)}{(x_2-x_1)}}\left(\frac{x_2-x}{x_2-c}\right)^{\frac{4(\lambda-x_2)}{(x_2-x_1)}}\left(\frac{\lambda-x_1}{x-x_1}+\frac{\lambda-x_2}{x_2-x}\right) \tag{77}$$

A.5 Proof of Proposition 4

Substitute into (48) to get

$$r\eta_i v=\max_{X_i}\left(p\eta_i x^*-\frac{\eta_i}{2}x^2-(\lambda-x)\eta_i(v-k+c)+\eta_i\frac{dv}{dt}\right). \tag{78}$$

and after canceling η, we get (5). Substituting into the (49) we have (6). Eq. (50) states that firms with larger η_i will have higher V_i, but they also a correspondingly higher entry cost so that $V_{i,0} = K_i$.

Then $X_{i,t} = \eta_i x_t$, and since x_t is common to all firms, instead of $D(p) = \bar{x}n$ in (11), we have

$$D(p) = \bar{x} \sum^{n} \eta_i$$

Next, $p_{\max}$ and $p_{\min}$ remain the same. About $p_{\max}$; Evaluating (78) at $v = k$ and $dv/dt = 0$ to get the no-reputation equilibrium at $p_{\max}$, once again η_i cancels and we have $rk = p_{\max}c - \frac{c^2}{2} - (\lambda - c)c$, i.e., (21). With $\varphi_i = \eta_i$, equilibrium exists as long as $\frac{1}{\eta_i}X_i < \lambda$ which (50) implies is the same as Eq. (18).

A.6 Proof of Proposition 5

(i) follows by inspection. (ii) Conditional on w and assuming that $v_t = k$ is a constant, the firm chooses its *ex-post* optimal value of x; the FOC w.r.t. x is $x = w$, and w then maximize the firm's constant expected flow payoff in units of z:

$$\begin{aligned} & pw + (w - c)(\lambda - w) - \frac{1}{2}w^2 - (\lambda - w)w \\ = \; & pw + cw - c\lambda - \frac{1}{2}w^2, \end{aligned} \tag{79}$$

with the FOC $w = p + c$, i.e., (58), which proves (ii).

(iii) The proof consists of showing that if $p \neq \hat{p}$, the firm has a profitable deviation. Substituting the flow profit from (79) into firm's HJB (5) gives:

$$rk = p(p+c) + c(p+c) - c\lambda - \frac{1}{2}(p+c)^2 = \frac{1}{2}(p+c)^2 - c\lambda.$$

Evidently, only $p = \hat{p}$ satisfies the above equation as the RHS is monotone in p.

A.7 Derivation of Eq. (61)

From (60), $\frac{dv}{dt} = \frac{1}{1+\beta}\frac{dx}{dt}$. Substituting from (60) into the proceeding HJB equation gives

$$r\left(\frac{x-c}{1+\beta} + k\right) = px - \frac{x^2}{2} - (\lambda - x)\left(\frac{x-c}{1+\beta} + c\right) - (\rho - \beta x)\frac{x-c}{1+\beta} + \frac{1}{1+\beta}\frac{dx}{dt}$$

and rearranging,

$$\begin{aligned} \frac{dx}{dt} &= r(x - c + (1+\beta)k) - (1+\beta)px + (1+\beta)\frac{x^2}{2} + (\lambda - x)(x + \beta c) + (\rho - \beta x)(x - c) \\ &= (1+\beta)rk + c(\lambda\beta - \rho - r) + (r + \lambda + \rho - (1+\beta)p)x - \frac{1}{2}(1+\beta)x^2. \end{aligned}$$

Since $v_0 = k$, the initial condition is $x_0 = c$. Let x_1 and x_2 be the two roots of x at which the RHS of dx/dt is zero:

$$x_1 = r + \lambda + \rho - (1+\beta)\,p - \sqrt{(r+\lambda+\rho-(1+\beta)\,p)^2 + 2\,((1+\beta)\,rk + c\,(\lambda\beta - \rho - r))\,(1+\beta)}$$
$$x_2 = r + \lambda + \rho - (1+\beta)\,p + \sqrt{(r+\lambda+\rho-(1+\beta)\,p)^2 + 2\,((1+\beta)\,rk + c\,(\lambda\beta - \rho - r))\,(1+\beta)},$$

Then

$$\frac{\partial x_2}{\partial \rho} = 1 + \frac{(r+\lambda+\rho-(1+\beta)\,p) - c\,(1+\beta)}{\sqrt{(r+\lambda+\rho-(1+\beta)\,p)^2 + 2\,((1+\beta)\,rk + c\,(\lambda\beta-\rho-r))\,(1+\beta)}}, \text{ and}$$
$$\left.\frac{\partial x_2}{\partial \rho}\right|_{\beta=\rho=0} = 1 + \frac{r+\lambda-p-c}{\sqrt{(r+\lambda-p)^2 + 2r\,(k-c)}} = 1 + \frac{r+\lambda-p-c}{x_2-(r+\lambda-p)} = \frac{x_2-c}{x_2-(r+\lambda-p)},$$

i.e., Eq. (61). If $r+\lambda-p-c>0$, then $\left.\frac{\partial x_2}{\partial \rho}\right|_{\beta=\rho=0} > 1$, which holds if c is small enough.

References

Abreu, Dilip, Paul Milgrom, and David Pearce. 1991. "Information and timing in repeated partnerships." *Econometrica: Journal of the Econometric Society*, 1713–1733.

Armour, John, Colin Mayer, and Andrea Polo. 2017. "Regulatory sanctions and reputational damage in financial markets." *Journal of Financial and Quantitative Analysis*, 52(4): 1429–1448.

Asquith, Paul. 1983. "Merger bids, uncertainty, and stockholder returns." *journal of Financial Economics*, 11(1-4): 51–83.

Azoulay, Pierre, Alessandro Bonatti, and Joshua L Krieger. 2017. "The career effects of scandal: Evidence from scientific retractions." *Research Policy*, 46(9): 1552–1569.

Barber, Brad M, and Masako N Darrough. 1996. "Product reliability and firm value: The experience of American and Japanese automakers, 1973-1992." *Journal of political economy*, 104(5): 1084–1099.

Bassetto, Marco, Zhen Huo, and JosÃľ-VÃŋctor RÃŋos-Rull. 2018. "Organizational Equilibrium with Capital." National Bureau of Economic Research Working Paper 25376.

Bhandari, Anmol, and Ellen R McGrattan. 2018. "Sweat Equity in U.S. Private Business." National Bureau of Economic Research Working Paper 24520.

Board, Simon, and Moritz Meyer-ter Vehn. 2013. "Reputation for quality." *Econometrica*, 81(6): 2381–2462.

Board, Simon, and Moritz Meyer-ter Vehn. 2014. "A reputational theory of firm dynamics." *Unpublished paper, UCLA.[430]*, 2381–2462.

Brauer, Karl. 2014. "Why A Massive Safety Recall Hurt Toyota More Than GM." *Forbes*, July 1.

Butters, Gerard R. 1978. "Equilibrium distributions of sales and advertising prices." In *Uncertainty in Economics*. 493–513. Elsevier.

Cabral, Luis, and Ali Hortacsu. 2010. "The dynamics of seller reputation: Evidence from eBay." *The Journal of Industrial Economics*, 58(1): 54–78.

Chu, Junhong, and Pradeep K Chintagunta. 2009. "Quantifying the economic value of warranties in the US server market." *Marketing Science*, 28(1): 99–121.

Cisternas, Gonzalo. 2018. "Career Concerns and the Nature of Skills." *American Economic Journal: Microeconomics*, 10(2): 152–89.

Cohen, Daniel, Masako N Darrough, Rong Huang, and Tzachi Zach. 2011. "Warranty reserve: Contingent liability, information signal, or earnings management tool?" *The Accounting Review*, 86(2): 569–604.

Cox, David Roxbee. 1962. "Renewal theory."

Food and Drug Administration (FDA). 2007-2017. "Archive for Recalls, Market Withdrawals & Safety Alerts." https://www.fda.gov/safety/recalls-market-withdrawals-safety-alerts/archive-recalls-market-withdrawals-safety-alerts.

Geistfeld, Mark. 1988. "Imperfect Information, the Pricing Mechanism, and Products Liability." *Columbia Law Review*, 88(5): 1057–1072.

Geistfeld, Mark. 2014. *Products liability law.* Wolters Kluwer Law & Business.

Gourio, Francois, and Leena Rudanko. 2014. "Customer capital." *Review of Economic Studies*, 81(3): 1102–1136.

Green, Edward J, and Robert H Porter. 1984. "Noncooperative collusion under imperfect price information." *Econometrica: Journal of the Econometric Society*, 87–100.

Grossman, Sanford J. 1981. "The informational role of warranties and private disclosure about product quality." *The Journal of Law and Economics*, 24(3): 461–483.

Hall, Robert E. 2001. "Struggling to understand the stock market." *American Economic Review*, 91(2): 1–11.

Heckman, James, and Burton Singer. 1984. "A method for minimizing the impact of distributional assumptions in econometric models for duration data." *Econometrica: Journal of the Econometric Society*, 271–320.

Hoffer, George E, Stephen W Pruitt, and Robert J Reilly. 1988. "The impact of product recalls on the wealth of sellers: A reexamination." *Journal of Political Economy*, 96(3): 663–670.

Holmström, Bengt. 1999. "Managerial incentive problems: A dynamic perspective." *The review of Economic studies*, 66(1): 169–182.

Hong, Harrison, and Jeremy C Stein. 1999. "A unified theory of underreaction, momentum trading, and overreaction in asset markets." *The Journal of finance*, 54(6): 2143–2184.

Hopenhayn, Hugo A. 1992. "Exit, selection, and the value of firms." *Journal of Economic Dynamics and Control*, 16(3-4): 621–653.

Hörner, Johannes. 2002. "Reputation and competition." *American economic review*, 92(3): 644–663.

Jarrell, Gregg, and Sam Peltzman. 1985. "The impact of product recalls on the wealth of sellers." *Journal of Political Economy*, 93(3): 512–536.

Jovanovic, Boyan, and Peter L Rousseau. 2002. "The Q-theory of mergers." *American Economic Review*, 92(2): 198–204.

Karpoff, Jonathan M. 2012. "Does reputation work to discipline corporate misconduct." *The Oxford handbook of corporate reputation*, 361–382.

Keller, Godfrey, and Sven Rady. 2015. "Breakdowns." *Theoretical Economics*, 10(1): 175–202.

Kiefer, Jack, and Jacob Wolfowitz. 1956. "Consistency of the maximum likelihood estimator in the presence of infinitely many incidental parameters." *The Annals of Mathematical Statistics*, 887–906.

Kydland, Finn E, and Edward C Prescott. 1977. "Rules rather than discretion: The inconsistency of optimal plans." *Journal of political economy*, 85(3): 473–491.

Lancaster, Tony, and Stephen Nickell. 1980. "The analysis of re-employment probabilities for the unemployed." *Journal of the Royal Statistical Society: Series A (General)*, 143(2): 141–152.

MacLeod, W Bentley. 2007. "Reputations, relationships, and contract enforcement." *Journal of economic literature*, 45(3): 595–628.

Mailath, George J, and Larry Samuelson. 2001. "Who wants a good reputation?" *The Review of Economic Studies*, 68(2): 415–441.

Milgrom, Paul, and John Roberts. 1986. "Price and advertising signals of product quality." *Journal of political economy*, 94(4): 796–821.

Milgrom, Paul R. 1981. "Good news and bad news: Representation theorems and applications." *The Bell Journal of Economics*, 380–391.

National Highway Traffic Safety Administration (NHTSA). 1966-2008. "Office of Defects Investigation (ODI) Recalls data." Department of Transportation. https://www-odi.nhtsa.dot.gov/downloads/(accessed May 11, 2016).

Olson, Mary. 1996. "Substitution in regulatory agencies: FDA enforcement alternatives." *The Journal of Law, Economics, and Organization*, 12(2): 376–407.

Pakes, Ariel. 1985. "On patents, R & D, and the stock market rate of return." *Journal of political economy*, 93(2): 390–409.

Phelan, Christopher, and Ennio Stacchetti. 2001. "Sequential equilibria in a Ramsey tax model." *Econometrica*, 69(6): 1491–1518.

Rhee, Mooweon, and Pamela R Haunschild. 2006. "The liability of good reputation: A study of product recalls in the US automobile industry." *Organization Science*, 17(1): 101–117.

Rob, Rafael, and Arthur Fishman. 2005. "Is bigger better? Customer base expansion through word-of-mouth reputation." *Journal of political Economy*, 113(5): 1146–1162.

Rupp, Nicholas G. 2004. "The attributes of a costly recall: Evidence from the automotive industry." *Review of Industrial Organization*, 25(1): 21–44.

Rupp, Nicholas G, and Curtis R Taylor. 2002. "Who initiates recalls and who cares? Evidence from the automobile industry." *The Journal of Industrial Economics*, 50(2): 123–149.

Rupp, Nicholas G, et al. 2001. "Newspaper coverage of automotive safety recalls." *East Carolina University Department of Economics Working Paper*.

Spence, Michael. 1977. "Consumer misperceptions, product failure and producer liability." *The Review of Economic Studies*, 44(3): 561–572.

Tadelis, Steven. 1999. "What's in a Name? Reputation as a Tradeable Asset." *American Economic Review*, 89(3): 548–563.

Urban, Marie A. 1992. "The FDA's policy on seizures, injunctions, civil fines, and recalls." *Food & Drug LJ*, 47: 411.

Watson, Joel. 1999. "Starting small and renegotiation." *Journal of economic Theory*, 85(1): 52–90.

Watson, Joel. 2002. "Starting small and commitment." *Games and Economic Behavior*, 38(1): 176–199.