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RETROSPECTIVE CAPITAL GAINS TAXATION

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ABSTRACT

This paper presents a new approach to the taxation of capital gains that eliminates the deferral advantage present under current realization-based systems, along with the lock-in effect and tax arbitrage possibilities associated with this deferral advantage. The new approach also taxes capital gains only upon realization but, by effectively charging interest on past gains when realization finally occurs, eliminates the incentive to defer such realization. Unlike a similar scheme suggested previously by Vickrey, the present one does not require knowledge of the potentially unobservable pattern of gains over time. It thus is applicable to a very broad range of capital assets.

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I. Introduction

Virtually every country that taxes income imposes a capital gains tax only upon the realization of gains rather than on accrual. Though countries vary with respect to indexing for inflation and the relative tax rates on capital gains and ordinary income, the realization-based tax system sets capital gains taxation apart from other forms of taxation and is associated with a variety of economic distortions.

The most frequently discussed problem arising from taxing capital gains upon realization is the "lock-in" effect, the desire to hold appreciated assets in order to defer taxes on gains already accrued. This effect leads investors to accept a lower rate of return before-tax than they would for new investments without such accrued gains, resulting in a distorted allocation of capital and inefficient portfolio selection.

As an illustration of the lock-in effect, consider a simple two-period example in which an investor, having accrued a first-period gain, g, must decide whether to realize the gain and reinvest at the safe rate of return, i, or hold the asset for an additional, safe rate of return r. Assuming all capital income is taxed at the same rate, t, then the investor's terminal wealth under the first strategy is

$$(1) \qquad W_R = (1+g(1-t))(1+i(1-t)) = (1+g)(1+i) - t[g(1+i(1-t))+(1+g)i]$$

In second-period units, total taxes equal those paid in the first period, accumulated at the net-of-tax interest rate, plus those due in the second period.

If the investor chooses to hold rather than sell, the terminal wealth is:

(2) $$W_H = (1+g)(1+r)-t[(1+g)(1+r)-1] = (1+g)(1+i) - t[g+(1+g)r]$$

so that the tax on the first-period gain is deferred, <u>without interest</u>, to the second period. This makes the investor willing to hold even for a range of returns $r < i$. The larger is g, the larger the deferral advantage and hence the lower r must be to induce the investor to sell.

A convenient way to express this deferral advantage is in terms of the effective tax rates on alternative investments. Compared to the rate t imposed if the investor realizes and reinvests, the additional taxes, per unit of additional gain $(1+g)r$, are (comparing (1) and (2)) $t[1-\frac{gi(1-t)}{r(1+g)}] < t$. It is through this lower effective tax rate that an investor can achieve at least his alternative after-tax return $i(1-t)$ despite receiving a before-tax return $r < i$.

Closely related to the lock-in effect is the general problem of tax avoidance facilitated by the voluntary nature of realization. Because losses as well as gains have their tax burdens deferred until realization, investors have the incentive to realize losses immediately, to maximize the associated tax reductions. Aggressive application of the simple rule of holding winners and realizing losers potentially permits individuals to generate tax reductions without incurring major transaction costs (Constantinides 1983, Stiglitz 1983). This arbitrage possibility has led to a second major distortion arising from the present system of capital gains taxation. To prevent investors from generating capital losses to offset ordinary income, tax systems typically limit the allowable annual deduction for such losses. In the U.S., this limit is presently 3000 dollars. While perhaps representing an effective response to the problem of tax arbitrage, this loss-offset limitation also distorts the choice of investment away from the risky assets more likely to produce losses (e.g. Stiglitz 1969).

Given such problems, there is great appeal to the prospect of switching to a tax on accrued capital gains. Taxing gains on accrual would make the actual realization of gains irrelevant to individual tax liabilities, thereby eliminating both the lock-in effect and the ability to engage in tax arbitrage through selective realizations. It would also obviate the need for limiting loss-offsets and the associated discouragement of risky investments. Though proposals to adopt accrual taxation have received serious scholarly attention (e.g. Shakow 1986), there seems little chance that such a system will be adopted on a broad scale.[1] Beyond the criticism that accrual taxation would increase annual taxpayer compliance costs, perhaps the most significant arguments against it are that some assets are hard to value except when they are sold and that liquidity constraints could force the premature sale of indivisible assets simply to pay the accruing taxes. These two problems would often apply at the same time, as with the case of closely held family businesses, for example.

A potential solution to the problems of both realization and accrual taxation is a realization-based tax that offsets the deferral advantage of holding gains by imposing a higher tax rate on gains held for longer periods of time. The effect is to simulate a system under which capital gains taxes are computed on an accrual basis but collected, with interest, only upon realization. From a comparison of (1) and (2), it is clear that charging tax-deductible interest on the taxes accruing on unrealized gains would eliminate the deferral advantage. Such an approach was originally conceived of by Vickrey (1939). By construction, it would eliminate the lock-in effect and the tax arbitrage possibilities generated by selective realization, because of its equivalence to an accrual tax. At the same time, it would also remedy the liquidity problem of accrual taxation by collecting the tax only when sales actually occurred.

Unfortunately, this "cumulative averaging" approach is plagued by the second problem of accrual taxation mentioned above, that of valuation. For assets that are hard for the government to value except when sold, it will be unclear upon sale what the time pattern of accrual of the realized gain was. This will make it impossible to compute retrospectively the tax liability equal in present value to an annual tax on the asset's accrued gains (Green and Sheshinski 1978). For example, if an asset has increased in value over a ten year period, the tax rate on the realized gain needed to simulate accrual taxation would by the ordinary tax rate if the gain occurred entirely in the tenth year, but this tax rate compounded by one plus the relevant interest rate to the ninth power if the entire gain occurred during the first year of ownership. Simply to assume, for tax purposes, that a realized gain accrued smoothly at a constant annual rate would not solve the problem. Assets achieving above-normal rates of return initially would still be subject to a lock-in effect, because an investor anticipating only normal returns from the asset in the future would be able to spread the accrual pattern retrospectively imputed for this gain over several years by holding on to the asset. Likewise, an asset that had declined in value would offer its owner the incentive to sell. Thus, basic arbitrage transactions involving the holding of winners and the sale of losers would still be attractive, though perhaps less so than under a pure realization-based tax.

Clearly, many capital assets such as common shares of large companies could be marked to market each year to avoid the valuation problem. But an effective method of dealing with hard-to-value assets would still be necessary to make a switch to accrual taxation or accrual-equivalent realization taxation practical. This paper presents such a method. The new approach does not require any information on the past pattern of accrued gains, and yet eliminates the lock-in effect and the benefits of deferral-based tax arbitrage. In place of the private information on the accrued gains of individual assets, the scheme uses public information, the market interest rate, combined with the assumption of optimal portfolio choice by investors. It does not impose the same effective tax rate on accrued gains, ex post, regardless of their time pattern, but it does impose the same tax rate, ex ante, after adjusting for risk.

This new approach to capital gains taxation can accommodate any asset currently subject to capital gains treatment, and its information requirements are small. Indeed, it does not require knowledge of the asset's purchase price, only its year of purchase.

In the next section, we formalize the criterion that a capital gains tax must satisfy in order not to distort the holding period decision or allow deferral-based arbitrage. To provide the basic intuition about the new scheme developed here and how it works, we introduce and analyze it first, in Section III, for a special class of assets (such as precious metals) that generate no cash flows or tax liabilities until they are sold. Section IV presents the solution for the general class of assets, and Section V offers some concluding remarks.

II. Holding-Period Neutrality

The present system of taxation upon realization distorts behavior because the rate at which it taxes the income arising from an asset depends on the size of the asset's previous unrealized gains. This induces both the lock-in effect and deferral-related tax arbitrage.

Suppose the risk-free interest rate is i,[2] and the investor's tax rate on all forms of income, including realized capital gains, is t. Then, as shown above, an investor holding an appreciated asset will require a before-tax return (adjusted for risk) less than i from that asset to achieve his after-tax opportunity cost of $i(1-t)$, because the tax rate t applied to new gains is offset by the continued deferral, without interest, of taxes payable on the gains already generated but not yet realized. This is the lock-in-effect. It encourages the holding of assets likely to generate a significant portion of their returns in the form of capital gains, because their income faces a tax rate below t if they are held for longer than one year, but it also imposes a tax-induced transaction cost on selling assets that increases over time and reflects no underlying social cost.

The problem of tax arbitrage relates to the lower effective tax rate generated by deferral rather than the distorted realization decision. Since assets turned over immediately face an effective tax rate t, investors can acquire two assets with offsetting risk characteristics and generate negative tax payments in present value by realizing positions that have declined in value sooner, and more frequently, than positions that have increased in value. To the extent that such offsetting positions can be maintained, the investor bears no risk, but there is a social cost nonetheless because social transaction costs are being incurred simply to transfer revenue from other taxpayers to the investor in question.

It is clear that neither of these distortions of the realization-based capital gains tax system would be present under an accrual tax. The latter would tax income at the same rate regardless of unrealized appreciation or holding period. The result would be a required rate of return independent of these other characteristics. It is this result that we refer to as "holding-period neutrality."

> Definition: A realization-based tax system is holding-period neutral if it leads each investor in an asset to require a before-tax return having a certainty-equivalent value that is not a function of the length of holding period or the asset's past pattern of returns.

We will confirm later that a tax system satisfying this criterion does, indeed, makes the realization decision irrelevant.

III. Retrospective Taxation

Suppose an investor holds an asset that generates no cash flows or tax liabilities until it is sold and is taxed only upon sale. We wish to design a tax at realization that satisfies the criterion of holding-period neutrality, as just defined.

One formulation that would satisfy this objective is the Vickrey (1939, 1947) cumulative averaging approach. With the problem formulated in continuous time, if T_s is the total tax payment required of an asset held for s years (with $T_0 = 0$), a Vickrey-type tax system would satisfy:

$$\dot{T}_s = i(1-t)T_s + tg_sA_s \qquad (3)$$

where g_s is the actual, *ex post*, rate of return on the asset at time (after purchase) s and A_s is the asset's value at date s. It is clear from (3) that such a tax system would be equivalent to actually taxing asset income on accrual but letting the tax liability accumulate at the investor's opportunity cost until the asset is sold.

As already indicated, though, the tax system described in (3) cannot be imposed retrospectively without knowledge of the time pattern of gains g_s. However, this expression is not a necessary condition for a holding-period-neutral tax. The fact that individual decisions are influenced by ex ante distributions of returns rather than ex post returns allows us to pursue a weaker condition.

Suppose that, at any date s, the investor knows the current value of his asset but not its current rate of return. Let V(•) be the valuation operator at each date that converts that date's distribution of uncertain returns into their certainty equivalents, from the investor's perspective. Then, intuition suggests that a holding-period-neutral tax system must satisfy, at each instant s, the following condition:

$$V(\dot{T}_s) = i(1-t)T_s + tiA_s \tag{4}$$

where, again, i is the risk-free interest rate (assumed to be constant only for the sake of exposition). Expression (4) says that the investor faces an increase in the realization tax liability associated with the asset equal to the interest on the unpaid liability plus the ordinary tax on the asset based on a rate of return equal to the risk-free rate.

The motivation for (4) is that, by definition, the certainty-equivalent return on risky assets should equal the risk-free rate if investors' portfolios are balanced and taxes do not distort behavior. We can formalize the relationship of expression (4) to the achievement of holding-period-neutrality.

Proposition 1: Condition (4) is necessary and sufficient for the achievement of holding period neutrality for the class of assets considered in this section.

Proof: At any date s, the net-of-tax value of an asset to the investor is the value of the asset A_s less the accumulated tax liability T_s. To continue to hold the asset for another instant, the investor requires a certainty-equivalent rate of return equal to the after-tax interest rate i(1-t). Thus, in portfolio equilibrium:[3]

$$V(\dot{A}_s - \dot{T}_s) = (A_s - T_s)i(1-t) \tag{5}$$

Combined with equation (4), (5) implies that $V(\dot{A}_s) = iA_s$, regardless of A_s or s. Hence, (4) implies holding-period neutrality. Combined with the requirement for holding-period neutrality that $V(\dot{A}_s) = iA_s$, (5) implies (4). QED

Since the certainty-equivalent value of the before-tax asset return g will equal i when an accrual-equivalent tax is imposed, it is clear that the Vickrey-type tax system described in (3) satisfies (4), and hence is holding-period neutral. However, the converse need not be true: the class of tax systems defined by (4) may be larger. The challenge is to find some other tax scheme also satisfying (4) that has weaker informational requirements. Fortunately, such a tax system exists.

Proposition 2: Suppose the realization tax liability at date s is

$$T_s = (1 - e^{-tis})A_s \tag{6}$$

Then the tax system satisfies (4) for all s and hence is holding-period neutral.

Proof: Taking the time derivative of (6), we obtain:

$$\dot{T}_s = (1 - e^{-tis})\dot{A}_s + tie^{-tis}A_s$$

$$= (1 - e^{-tis}) (\frac{\dot{A}}{A})_s A_s - (1-e^{-tis}) tiA_s + tiA_s$$

$$= (1 - e^{-tis}) \left[(\frac{\dot{A}}{A})_s - ti\right] A_s + tiA_s$$

By Proposition 1, $V(\frac{\dot{A}}{A}) = i$ if (4) is satisfied. Our strategy will be to assume $V(\frac{\dot{A}}{A}) = i$. Once proving that (4) is satisfied, our assumption will prove correct.[4]

If $V(\frac{\dot{A}}{A}) = i$, then $\frac{\dot{A}}{A} = i + \varepsilon$, where ε is a random return satisfying $V(\varepsilon) = 0$. (Note that, in general, $E(\varepsilon) \neq 0$). Hence,

$$\dot{T}_s = (1 - e^{-tis})(i(1-t) + \varepsilon_s)A_s + tiA_s$$

which, by (6), may be written:

$$\dot{T}_s = i(1 - t)T_s + tiA_s + (1 - e^{-tis})\varepsilon_s A_s \quad (7)$$

Since, by construction, $V(\varepsilon) = 0$, application of $V(\cdot)$ to both sides of (7) yields (4).QED

Clearly, the evolution of the tax liability T_s described by (7) differs from that of the Vickrey-type system based on ex post returns described by (3). Since the gain $g = i + \varepsilon$, (7) differs from (3) in taxing the excess return ε at rate $(1 - e^{-tis})$ rather than t. This is a tax rate that starts at 0 and approaches 1 as $s \to \infty$. But the tax rate on the excess return has no effect on the investor's welfare, because by construction the excess return has zero value to him (e.g. Gordon 1985, Sandmo 1985).[5]

A simple example is useful in demonstrating how this tax system works to eliminate the lock-in effect. Suppose an investor purchased an asset at some past date 0. At date s_1, he chooses between realizing at price p_1 and repurchasing the asset versus not realizing, in both cases realizing the asset again at some future date s_2. The asset's price at s_2, p_2, is uncertain at s_1 but not influenced by the investor's decision.

Under the realization strategy, the investor pays a tax of $p_1(1-e^{-its_1})$ at s_1 and $p_2(1-e^{-it(s_2-s_1)})$ at s_2. Under the alternative strategy, he pays $p_2(1-e^{-its_2})$ at s_2. A comparison of the two cases shows that the choice is between a tax payment of $e^{-its_2}(e^{its_1}-1)\cdot p_1 e^{it(s_2-s_1)}$ at s_1 versus $e^{-its_2}(e^{its_1}-1)\cdot p_2$ at s_2. But the certainty-equivalent value of p_2 at s_1 is just $p_1 e^{it(s_2-s_1)}$, so the investor is indifferent, _et ante_. The two cases differ only in the _ex post_ treatment of the asset's risk premium.

Proposition 2 offers a very simple system of capital gains taxation. Computation of the tax burden when an asset is sold requires knowledge of the risk-free interest rate, the investor's marginal tax rate, the holding period of the asset and the final sales price. (Nothing in the proof depends on either i or t being constant, so variations over time in rates of interest and marginal taxation present no difficulty.) The initial purchase price, the pattern of accrued gains and the asset's stochastic properties are irrelevant to the calculation. The tax itself is expressed as a time-dependent fraction of the asset's value at sale, with this fraction going from 0 at $s = 0$ to 1 as $s \to \infty$.[6]

To interpret the tax formula (6), consider again the Vickrey type tax system described in (3). For a terminal asset value of A_s, a holding period of s and a rate of capital gain always equal to the risk-free rate (implying an initial cost of $A_s e^{-is}$), that system would impose a realization tax liability of

$$(8) \qquad T_s = t\int_0^s e^{i(1-t)(s-z)}(A_s e^{-i(s-z)})dz = A_s(1-e^{-its})$$

Thus, the tax schedule (6) treats investors <u>as if</u> they had arrived at their current position by investing at risk-free rate. Since in terms of certainty-equivalents, this is precisely what they did, the tax system "works" in the same way that a Vickrey-type system would.[7]

It is natural to ask whether there could be other tax systems achieving holding-period neutrality based on the same information. Proposition 3 shows that this tax system is unique.

<u>Proposition 3</u>: The tax system described in (6) is the only one based on the information set (t,i,s,A_s) that satisfies the condition for holding-period neutrality, (4).

<u>Proof</u>: Consider a tax rule based on the admissible information set:

$$T_s = F(t,i,s,A_s) \tag{9}$$

Differentiating (9) with respect to s yields:

$$\dot{T}_s = F_s + F_A\dot{A}_s = F_s + F_A A(i+\varepsilon_s) = F_s + F_A iA_s + F_A\varepsilon_s \tag{10}$$

Applying V(•) to (10), and combining the result with (4) and (9) to eliminate $V(\dot{T}_s)$ and T_s, we obtain the partial differential equation:

$$\frac{1}{i(1-t)}F_s + \frac{A}{1-t}F_A = F + \frac{tA}{1-t} \tag{11}$$

Since the division of assets is arbitrary, it must be the case that F is homogeneous of degree one with respect to A_s. That is, dividing an asset into two pieces and realizing each half separately can have no effect on the capital gains tax liability. Thus, there must exist some function $F^1(\cdot)$ such that:

$$(12) \qquad F(i,t,s,A_s) = F^1(i,t,s)\cdot A_s$$

Substituting the expression for F_s and F_A obtained from (12) into (11), we obtain the ordinary differential equation:

$$(13) \qquad \frac{1}{i(1-t)} \cdot \frac{dF^1}{ds} + \frac{1}{1-t}F^1 = F^1 + \frac{t}{1-t}$$

which, combined with the initial condition $F^1(i,t,0)=0$, yields the unique solution $F^1(i,t,s) = (1-e^{-its})$ and hence $T_s = F(i,t,s,A_s) = F^1(i,t,s)A_s = (1-e^{-its})A_s$. QED

One may extend the tax system given in (6) to accommodate the more general situation in which marginal tax rates vary across assets. Suppose it is desired that income from the risk-free asset and the capital asset be taxed differentially, at rates t' and t, respectively.[8] (For $t < t'$, capital assets would be tax-favored.) In this case, the preceding analysis goes through for a required return before-tax equal to $i(1-t')/(1-t)$. That is, replacing (6) with

$$(6') \qquad T_s = (1 - e^{-ti\left(\frac{1-t'}{1-t}\right)s})A_s$$

results in a flow tax rule:

$$(7') \qquad \dot{T}_s = i(1-t')T_s + tiA_s + (1 - e^{ti\left(\frac{1-t'}{1-t}\right)s})\varepsilon_s A_s$$

Once again, the investor is charged the <u>relevant</u> after-tax interest rate

$i(1-t')$ on the outstanding tax liability and taxed on the certainty-equivalent accruals of income at the capital asset's tax rate t. The significance of this result is that it shows that holding-period-neutral retrospective taxation is perfectly compatible with the favorable tax treatment of capital assets. The tax benefit need not be provided via a distortionary deferral advantage.

If investors face different tax rates and, indeed, even if they receive different relative after-tax returns on different assets, the analysis applies to each investor individually, as long as he is in portfolio equilibrium, with his after-tax risk-adjusted return equal to his opportunity cost. That is, (6') and (7') always imply that the investor will require a certainty-equivalent before-tax return of $i(\frac{1-t'}{1-t})$, even if the ratio $(1-t')/(1-t)$ varies across the population. By construction, the risk premium ε equals the total return g less the required, risk-adjusted before-tax return $i(\frac{1-t'}{1-t})$, so differences in $(\frac{1-t'}{1-t})$ imply different risk premia on the same asset for different investors. But this is precisely what gives rise to portfolio sorting and clientele formation, with investors holding diversified portfolios but gravitating toward those assets in which they obtain a relatively favorable trade-off between risk and return (Auerbach and King 1983). In equilibrium, each investor will require the available risk premium to hold each risky asset, assuming there is an interior solution to the portfolio choice problem.[9]

Thus, for the class of assets considered in this section, a simple realization-based tax system exists that is holding-period neutral, has limited informational requirements, and can be applied under a tax system with marginal tax rates that vary over time, assets and investors. We next show how the tax system described by (6) can be generalized for the class of assets broader than those yielding returns only upon sale. The tax formula becomes more complicated than that described in (6), but follows the same approach.

IV. The General Tax System

Most assets presently subject to capital gains taxes generate cash flows and are subject to tax charges before disposition of the assets themselves. In the case of corporate equities, shareholders receive dividends and pay taxes on them. For other assets, taxes and cash flows may not be so closely tied. For real estate investments qualifying for accelerated depreciation allowances, for example, investors might in some years receive positive cash flows and tax refunds at the same time while in later years paying taxes equal to a substantial fraction of cash flows. In this section we treat the general class of assets with arbitrary patterns of cash flows and tax payments.

Let D_s be the cash distribution received at date s, and let τ_s be the tax payment made at date s. For some assets, we might impose a restriction relating τ_s to D_s, but this is unnecessary for the derivation of a holding-period neutral capital gains tax. To the extent that there are transaction costs associated with purchasing, selling or holding the asset, these can be treated as negative distributions.

We follow the same strategy as in Section III, first discussing the evolution of the tax liability T that is necessary to ensure holding-period neutrality. As before, we assume initially that the government wishes to tax all asset income at a single rate t.

<u>Proposition 4</u>: For the general class of assets just described, the following condition is necessary and sufficient for a tax to be holding-period neutral:

(14) $$V(\dot{T}_s) = i(1-t)T_s + tiA_s - \tau_s$$

Proof: Following the proof of Proposition 1, we note that the yield on the net of tax asset value A-T must equal i(1-t). This yield consists of the return on the asset D plus the net capital gain $\dot{A} - \dot{T}$ minus the tax payment τ; thus[10]

(15) $$V(\dot{A}_s - \dot{T}_s) + D_s - \tau_s = (A_s - T_s)i(1-t)$$

Combined with equation (14), (15) implies that $V(\dot{A}_s) + D_s = iA_s$, regardless of A_s or s. Hence (14) implies holding-period neutrality. Combined with the requirement for holding-period neutrality that $V(\dot{A}_s) + D_s = iA_s$, that the before-tax return required in the asset be independent of A_s or s, (15) implies (14). QED

Expression (14) says that, in computing their increase in tax liability $\dot{T}$, investors should be given credit for taxes paid currently. Again, such a provision is present in Vickrey's original scheme. As before, the rule described in (14) is less restrictive in that it applies to the valuation of returns *ex ante* rather than actual *ex post* returns in each state of nature. Once again, there is a tax system that will satisfy (14) without requiring information on the pattern of an asset's growth in value.

Proposition 5: Suppose the realization tax liability is:

(16) $$T_s = (1 - e^{-tis})A_s - e^{i(1-t)s}\left[\int_0^s (e^{-iz} - e^{-i(1-t)z})D_z dz + \int_0^s e^{-i(1-t)z}\tau_z dz\right]$$

Then the tax system satisfies (14) for all s and hence is holding-period neutral.

Proof: Taking the time derivative of (16), we obtain (substituting (16) into the result):

$$\dot{T}_s = (1-e^{-tis})\dot{A}_s + tie^{-tis}A_s + i(1-t)[T_s - (1-e^{-tis})A_s]$$
$$-e^{i(1-t)s}[(e^{-is}-e^{-i(1-t)s})D_s + e^{-i(1-t)s}\tau_s]$$

$$= (1-e^{-tis})((\frac{\dot{A}}{A})_s - i)A_s + tiA_s + i(1-t)T_s + (1-e^{-tis})D_s - \tau_s$$

$$= (1-e^{-tis})((\frac{\dot{A}}{A})_s + D_s - i)A_s + tiA_s + i(1-t)T_s - \tau_s$$

Again, without restriction (see the proof of Proposition 2) we may assume that the risk-adjusted, before-tax required return $V(\frac{\dot{A}}{A}) + D = i$, so that $\frac{\dot{A}}{A} + D = i + \varepsilon$ with $V(\varepsilon) = 0$. Thus,

$$\dot{T}_s = i(1-t)T_s + tiA_s - \tau_s + (1-e^{-tis})\varepsilon_s A_s \tag{17}$$

Since, by construction, $V(\varepsilon) = 0$, application of $V(\cdot)$ to both sides of (17) yields (14).QED

As in the previous case, the solution involves taxing the asset's risk premium at a rate $(1 - e^{-tis})$ rather than t. A way of interpreting (16) is to rewrite it as:

$$T_s = (1-e^{-tis})(A_s + \int_0^s e^{i(s-z)}D_z dz) - (\int_0^s e^{i(s-z)}D_z dz$$
$$- \int_0^s e^{i(1-t)(s-z)}D_z dz) - \int_0^s e^{i(1-t)(s-z)}\tau_z dz \tag{16'}$$

The term $(A_s + \int_0^s e^{-i(s-z)}D_z dz)$ is the present value, at date s, of the asset plus all previous distributions. Thus, the tax scheme begins by treating this entire value as subject to the tax rate $(1 - e^{-tis})$, as in Section III. Had all distributions been received tax free and reinvested in the asset itself,[11] this would be appropriate, for then the asset would be of the type analyzed there. However, because taxes have been paid in the past and the distributions invested elsewhere, two corrections are necessary for taxes already paid. The last term in (16′) is a credit for taxes already paid directly on the asset, while the middle term in (16′) is an imputation for taxes paid on the income generated by distributions invested in other assets facing an income tax rate t. That is, the treatment of distributions as having been reinvested in the same asset assumes that they continue to generate income at the before-tax rate of return i, adjusted for risk. Since they were actually invested in other assets, which we may assume to face an accrual-equivalent income tax rate t, we are therefore ignoring the subsequent income taxes attributable to such reinvested distributions. The present value of these imputed taxes at date s is $(\int_0^s e^{i(s-z)}D_z dz - \int_0^s e^{i(1-t)(s-z)}D_z dz)$. Thus, the tax system in (16) can be interpreted as treating all distributions as being reinvested and then applying the tax scheme described in Section III, but giving credit for taxes paid along the way.

Yet another interpretation of expression (16) follows from the following logic.[12] As is well known, share repurchases and dividends are equivalent except for their tax treatment and, in this case, even the tax treatment is the same. Thus, one should be able to view each distribution as a share repurchase. Since each such repurchase amounts to the investor's realization of part of his assets, consistent treatment based on Proposition 1 ought to suffice. If each "partial" asset sale receives such treatment, there ought to be no deviation needed when the remainder of the asset is sold. Indeed, this conjecture is correct. Collecting terms in (16), we obtain:

$$(16'') \qquad T_s = (1-e^{-tis})A_s + \int_o^s e^{i(1-t)(s-z)}\{(1-e)^{-tiz})D_z-\tau_z\}dz$$

which says that the household's tax liability at date s equals the normal one due on asset without previous distributions or tax payments plus the accumulated deficit in tax payments on previous "realizations", i.e. distributions.[13]

Thus, one very simple approach to the achievement of holding period neutrality is to tax every distribution from a capital asset at the rate $(1-e^{-tis})$, where s is the time since the asset's purchase. In this event, the informational requirements are no worse than in the previous case without distributions.

More generally, expression (16) is more complex than expression (6), but its informational requirements are still minimal. In addition to what was needed in the previous case, the government now must also know the flows of previous taxes and distributions on the asset.

A record of previous taxes can be obtained from past tax returns. In many instances, as with common stock, the taxes are directly based on the distributions, so records of the distributions themselves are just as easily available. Even in cases where the taxes τ and distributions D are not so simply related (real estate investments, for example), the law requires taxpayers to supply enough information so that the distributions can be recovered. For example, a real estate investor would add interest payments and depreciation deductions back to reported profits in order to calculate the distribution from a property in a given year.

As before, the tax rule can be extended to the case of different tax rates on capital assets (t) and other income (t') by replacing the interest rate i with the required before-tax return i(1-t')/(1-t). In cases where t is known, this is a simple change. There are more complicated cases, though, where tax preferences are given not via a reduction in t but through tax credits or accelerated depreciation, each of which affects the present value of τ. In this case, it is necessary to determine what effective tax rate t is desired, and base the calculation in (16) on this value. Once this has been done, the continued presence or absence of tax credits or accelerated depreciation becomes irrelevant, for variations in these are simply offset by changes in the last term of (16).

For example, suppose the government wishes to lower an asset's effective tax rate from t' = .4 to t = .2, and might use an investment tax credit to do so. Once t = .2 is used to compute T in (16), the investment tax credit may be kept; but since it appears as a reduction in taxes paid by the investor in the last term in (16), it will simply increase T by an amount equal in present value. Put another way, the formula ensures that the specified effective tax rate will be achieved, regardless of the specific pattern of tax payments chosen by the government (or, for that matter, the investor who might choose or be required to make contributions toward his accumulating tax liability).

IV. Conclusion

This paper has presented a scheme that taxes capital gains upon realization without inducing a lock-in effect or providing the opportunity for tax arbitrage. The scheme requires information that is either publicly available (such as interest rates) or present on previous tax returns (such as past tax payments) but not the private (or potentially even unavailable) information on the time pattern of an asset's accrued gains. The scheme's simplicity may obscure its quite general applicability. It may be used for any assets subject to capital gains or losses, essentially all classes of assets. This includes not only common stock and real estate, for which capital gains treatment has historically been considered significant, but also, for example, depreciable assets, which currently are subject to capital gains taxes but also receive fixed, _ex ante_ depreciation allowances in lieu of deductions for accrued economic depreciation.[14]

Nothing about the tax system described here requires that all asset income be taxed at the same rate for a particular investor. Purchases of certain assets can still be encouraged through a lower overall tax burden, without the need to resort to _ad hoc_ measures such as accelerated depreciation or distortionary ones such as low rates of realization-based capital gains taxes that exacerbate the lock-in effect and the problem of tax arbitrage.

In achieving the economic benefits of accrual taxation without its associated liquidity or information problems, the new approach makes a move toward a less distortionary capital gains tax feasible and eliminates the need for the additional distortions induced by such anti-arbitrage provisions as limited loss offsets.

Footnotes

1. The tax system already has elements that effect accrual taxation, such as the mark-to-market requirements instituted in the 1981 Economic Recovery Tax Act to reduce tax arbitrage activity involving commodity straddles.

2. If the tax system is not indexed for inflation, then this rate should be viewed as a nominal interest rate. Moreover, in the absence of a risk-free asset, one may reinterpret the paper's results in terms of a "zero-beta" asset that carries no risk premium.

3. It might be argued that the investor may not achieve an interior solution to the portfolio choice problem in the case of assets subject to capital gains taxes. For example, one cannot freely buy and sell assets that are indexed by having already been held for a specified time period. However, our focus here is on the case in which the holding period becomes irrelevant to the portfolio choice problem. _A fortiori_, the assumption of portfolio balance is justified.

4. It is straightforward to show that this solution for required holding-period yields is unique. That is, there exists no other rate of return $j \neq i$ for which the implied tax rule corresponding to (7) is in fact consistent with the portfolio balance condition (5) and the assumed rate of return j.

5. In fact, as Gordon shows, the same general equilibrium outcome results from tax systems differing only in their treatment of excess returns, if private risk-pooling is efficient. Otherwise, taxes on excess returns that have no value to investors may be pooled by the government, creating value and reducing aggregate risk. In this event, the tax rate on risk premia influences the equilibrium outcome, even though the

investor's holding-period decision is not distorted.

6. Since the tax liability is bounded by the asset's value, the liquidity problem disappears under this tax system. It is important to stress that such an accumulating tax liability over time works to remove the lock-in effect only if the tax is eventually imposed. A provision that eliminates capital gains tax liability at death, for example, might cause the lock-in effect to be exacerbated by a move to such a tax system, since investors would have an even greater incentive to hold "to the end".

7. This utilization of _ex ante_ equivalence does suggest a potential political problem in implementing the retrospective tax scheme. It taxes investors on what, in a sense, their gains _should_ have been. _Ex post_, this means taxing winners' and losers' wealth at the same rate, treating them all as if their current wealth had been accumulated at the safe rate of return.

8. One could conceive of a variety of optimal tax or second-best arguments leading to such an objective. For example, see Auerbach (1981) or the related discussion in Sandmo (1985).

9. Such a solution will not exist, for example, if assets with different tax characteristics have the same return distributions, as in the case of perfect certainty. In such cases, constraints on investors' positions, on borrowing or short sales, perhaps, are required for any equilibrium to exist and corner solutions for individual portfolios will arise. Here, the equivalence among after-tax returns holds only if shadow prices on the binding constraints are taken into account. See Auerbach and King (1983). If, for example, an investor held no taxable debt, only tax exempt municipal bonds, the appropriate after-tax

opportunity cost would be the interest rate on municipal bonds.

10. We assume for the sake of exposition that D_s and r_s are known at date s, but this has no effect on the validity of the derivation.

11. The asset "itself" here refers to the account established for an asset, not a specific asset. If the unit of account were a business, for example, a corporation reinvesting all its profits would be such an asset.

12. I am grateful to Doug Bernheim for this suggestion.

13. It is particularly clear from (16'') why the initial purchase price does not appear in the tax calculation. One could view this initial cost as a negative distribution at date zero, but the appropriate tax on this negative distribution would be zero.

14. The economic effects of fixed depreciation allowances in the case of risky depreciation is discussed by Auerbach (1983) and Bulow and Summers (1984).

References

Auerbach, Alan J., 1981, "Evaluating the Taxation of Risky Assets," Harvard Institute of Economic Research, Discussion Paper no. 857.

------, 1983, "Corporate Taxation in the United States," Brookings Papers 14, pp. 451-505.

------ and Mervyn A. King, 1983, "Taxation, Portfolio Choice and Debt-Equity Ratios: A General Equilibrium Model," Quarterly Journal of Economics 98, November, pp. 587-609.

Bulow, Jeremy and Lawrence Summers, 1984, "The Taxation of Risky Assets," Journal of Political Economy 92, pp. 20-39.

Constantinides, George M., 1983, "Capital Market Equilibrium with Personal Tax," Econometrica 51, pp. 611-36.

Gordon, Roger H., 1985, "Taxation of Corporate Capital Income: Tax Revenue versus Tax Distortions," Quarterly Journal of Economics 100, pp. 1-27.

Green, Jerry R. and Eytan Sheshinski, 1978, "Optimal Capital-Gains Taxation under Limited Information," Journal of Political Economy 86, pp. 1143-58.

Sandmo, Agnar, 1985, "The Effects of Taxation on Savings and Risk-Taking," in A. J. Auerbach and M. Feldstein, eds., Handbook of Public Economics, vol 1.

Shakow, David, 1986, "Taxation without Realization: A Proposal for Accrual Taxation," University of Pennsylvania Law Review 134, pp. 1111-1205.

Stiglitz, Joseph E., 1969, "The Effects of Income, Wealth and Capital Gains Taxation on Risk-Taking," Quarterly Journal of Economics 83, pp. 203-83.

------, 1983, "Some Aspects of the Taxation of Capital Gains," Journal of Public Economics 21, pp. 257-294.

Vickrey, William, 1939, "Averaging Income for Income Tax Purposes," Journal of Political Economy 47, pp 379-97.

------, 1947, Agenda for Progressive Taxation, (New York: Ronald Press).