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OPTIMAL MANAGEMENT OF A PANDEMIC IN THE SHORT RUN AND THE LONG RUN

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ABSTRACT

Social policy to limit interactions can slow the spread of infection, but this benefit comes at the cost of reduced output. We solve an optimal control problem to choose the degree of interaction to maximize an objective function that rewards output and penalizes excess deaths. Optimal policy restricts the degree of interaction—permanently and perhaps substantially—but, surprisingly, not so much as to eradicate the disease. This finding holds regardless of how much weight the objective function places on excess deaths, provided the weight is finite. Complete eradication is optimal only if achieved by science or medicine.

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> "It's important to put this on the table: This virus may become just another endemic virus in our communities, and this virus may never go away," Dr. Michael Ryan, WHO, May 14, 2020

Efforts to fight the global COVID-19 pandemic comprise medical/scientific activities and social policies. Medical/scientific activities include pursuit of therapies to accelerate recovery and reduce the death rate among infected people, as well as development of a safe and effective vaccine. Social policies aim to reduce the spread of the disease through shutdowns of business and social activities, social distancing, wearing of face coverings, contact tracing and quarantine. Such policies have had massive impacts on output and employment that have led to bitter divisions about how aggressively to pursue these policies and when to relax them.

As in the burgeoning economics literature, we treat medical/scientific efforts as exogenous and focus on social policies. We use a version of the SIR model with vital dynamics and excess deaths to provide the constraints in a planner's optimal control problem. As in Alvarez et al. (2020), Piguillem and Shi (2020), and Acemoglu et al. (2020), the planner's objective function rewards output and penalizes excess deaths. The key tradeoff facing the planner is that increased interaction leads to more output, but also more deaths from the disease. We characterize the optimal solution of this problem and compare it to the outcome of a "laissez-faire" economy, an economy where the social interaction rate is chosen freely by individuals, as in the optimization frameworks of Eichenbaum et al. (2020) and Toxvaerd (2020), or informally as in Cochrane (2020).

A surprising finding of our analysis is that even though it is feasible for the planner to eradicate the disease by limiting interactions, it is not optimal to do so, even in the long run. That is, the planner's optimal policy leads to an endemic equilibrium. Remarkably, this finding holds no matter how large is the penalty on excess deaths. Nevertheless, it is optimal for the planner to limit interactions until the random arrival of a cure and vaccine. Moreover, the planner restricts interactions more than individuals would in a laissez-faire economy.

Two features distinguish our paper from existing economics literature on pandemics.

First, by using versions of the SIR model in which there is no entry in the pool of susceptible people, existing papers (including all of the work cited above, as well as Atkeson (2020), Berger et al. (2020), and Fernandez-Villaverde and Jones (2020)) preclude the possibility of endemic equilibria in which the infection share remains positive even in the long run. We overcome this problem by including vital dynamics (births and non-disease deaths) to allow replenishment of the pool of susceptible people, thus opening the possibility of endemic equilibria. (Alternatively, we could specify that recovered people lose their immunity after a period of time and become susceptible, which would also open the possibility of endemic equilibria.) Second, our analysis produces robust theoretical results that do not depend on specific parameter values, unlike the papers above that typically[1] rely on numerical simulations, which, of course, depend on specific parameter values.

1 SIR Model with Population Growth and Excess Deaths

The total population, N, is the sum of susceptible people, S, infected people, I, and recovered people, R, who are no longer susceptible to the disease. Let $\phi > 0$ be the birth rate per unit of population per unit of time, $\mu \geq 0$ be the baseline death rate per unit of population per unit of time, $\gamma > 0$ be the recovery rate per infected person per unit of time and $\delta > 0$ be the excess death rate of infected people per unit of time. As in conventional SIR epidemiological models, the flow of new infections per unit of time is $\beta S \frac{I}{N}$, where $\beta > 0$ is a contagion parameter reflecting the extent of social and professional interactions. In this section we treat β as a fixed parameter. From section 2 onward, we treat beta as a time-varying, choice variable.

The differential equations governing the evolution of S, I, and R are

$$\frac{dS}{dt} = \phi N - \mu S - \beta S \frac{I}{N} \tag{1}$$

[1] Toxvaerd (2020) provides analytic results in a laissez-faire context.

$$\frac{dI}{dt} = \beta S \frac{I}{N} - (\mu + \delta + \gamma) I \tag{2}$$

and

$$\frac{dR}{dt} = \gamma I - \mu R. \tag{3}$$

Because population can potentially grow without bound, we work with the population shares $s \equiv \frac{S}{N}$, $i \equiv \frac{I}{N}$, and $r \equiv \frac{R}{N}$, where $s + i + r = 1$. The change in population per unit time is $gN = \phi N - \mu N - \delta I$, which is births, ϕN, less baseline deaths, μN, and less excess deaths, δI, so

$$g = g(i) \equiv \phi - \mu - \delta i. \tag{4}$$

The change in the susceptible share $s \equiv \frac{S}{N}$ is $\frac{ds}{dt} = \frac{1}{N}\frac{dS}{dt} - gs$, so

$$\frac{ds}{dt} = \phi - \beta si - (\mu + g) s. \tag{5}$$

Similarly,

$$\frac{di}{dt} = [\beta s - (\delta + \gamma + \mu + g)] i \tag{6}$$

and

$$\frac{dr}{dt} = \gamma i - (\mu + g) r. \tag{7}$$

Since g always appears as $g + \mu$, define the "adjusted growth rate," Γ, which satisfies

$$\phi - \delta \leq \Gamma \equiv g + \mu = \phi - \delta i \leq \phi. \tag{8}$$

Define

$$R_0 \equiv \frac{\beta}{\delta + \gamma + \Gamma}, \tag{9}$$

which generalizes the basic reproduction rate in a conventional SIR model. R_0 depends on the endogenous growth rate g, so it is endogenous. R_0 plays a pivotal role in determining whether the long-run equilibrium of the economy is a disease-free equilibrium (DFE) with a zero measure of infected people or an endemic equilibrium (EE) with a positive measure of infected people.

1.1 Steady-State Equilibria

In a steady state, S, I, R, and N all grow at the rate g, so s, i, and r are constant and equal s^*, i^*, and r^*, respectively. Throughout, an asterisk (*) denotes steady-state values of variables. In a steady state, the rates of change of the population shares in (5) - (7) are zero. In a DFE steady state, $s^* = 1$ and $i^* = r^* = 0$, so (4) implies that the population growth rate is $g^* = \phi - \mu$.

1.1.1. An EE Steady State

An EE steady state has strictly positive values of s^*, i^*, and r^*.

Proposition 1 *If $R_0^* \geq 1$, then*

1. $s^* = R_0^{*-1}$
2. $i^* = \frac{\Gamma^*}{\gamma + \Gamma^*}\left(1 - R_0^{*-1}\right)$
3. $r^* = \frac{\gamma}{\gamma + \Gamma^*}\left(1 - R_0^{*-1}\right)$.

Proof of Proposition 1 . Use (8) and (9) to rewrite (6) as $\frac{di}{dt} = \left(s - R_0^{-1}\right)\beta i$. Consider two cases for R_0^*. Case I: $R_0^* = 1$, so $\frac{di}{dt} = (s^* - 1)\beta i^* = 0$ implies that $s^* = 1$ or $i^* = 0$. Thus, the steady state is DFE with $s^* = 1$, $i^* = r^* = 0$ and Statements 1 - 3 are satisfied. Case II: $R_0^* > 1$. Assume that the steady state is DFE. In the neighborhood of $i = 0$ and

$s = 1$, $\frac{di}{dt} = \left(1 - R_0^{-1}\right)\beta i > 0$ so i will not approach 0. Therefore, the steady state is EE with $i^* > 0$ and $\frac{di}{dt} = \left(s^* - R_0^{-1}\right)\beta i^* = 0$ so $s^* = R_0^{-1}$ (Statement 1). Set $\frac{ds}{dt} = 0$ in (5) to obtain $\phi - \Gamma s^* = \beta s^* i^*$ and use $s^* = R_0^{*-1}$ to obtain $i^* = \frac{\phi R_0^* - \Gamma^*}{\beta}$. Set the change in r in (7) equal to zero to obtain $\gamma \frac{i^*}{r^*} = \Gamma^*$, which implies $r^* = \frac{\gamma}{\Gamma^*} i^*$. Since $s^* + i^* + r^* = 1$, $R_0^{*-1} + \left(1 + \frac{\gamma}{\Gamma^*}\right) i^* = 1$, so $i^* = \frac{\Gamma^*}{\gamma + \Gamma^*}\left(1 - R_0^{*-1}\right)$ (Statement 2). Statement 3 follows from Statement 2 and $r^* = \frac{\gamma}{\Gamma^*} i^*$. ■

Corollary 1 *A steady state will be an EE if and only if $R_0^* > 1$.*

Proof of Corollary 1. From Proposition 1, if $R_0^* > 1$, then $i^* > 0$, so the steady state is EE. If $R_0^* = 1$, Proposition 1 implies that $s^* = 1$ and $i^* = r^* = 0$, so the steady state is DFE. If $R_0^* < 1$, then $\frac{di}{dt}$ in (6) equals zero if and only if $i^* = 0$, so the steady state is DFE. ■

Define the critical value of the contagion parameter β

$$\beta_c \equiv \delta + \gamma + \phi. \tag{10}$$

The definition in (8) implies $i^* = \frac{\phi - \Gamma^*}{\delta}$ and, if $R_0^* \geq 1$, Statement 2 in Proposition 1 implies $i^* \leq \frac{\Gamma^*}{\gamma + \Gamma^*}$, so

$$\frac{\phi - \Gamma^*}{\delta} \leq \frac{\Gamma^*}{\gamma + \Gamma^*}. \tag{11}$$

Proposition 2 *If $\beta > \beta_c$, then*

1. *$R_0^* > 1$, so the steady state is EE*
2. *the steady-state adjusted growth rate, $\Gamma^* \equiv g^* + \mu$, is the positive root of $q(\Gamma) \equiv (\beta - \delta)\Gamma^2 + [(\beta - \delta)(\delta + \gamma) - \phi\beta]\Gamma - \gamma\phi\beta = 0$.*

Sketch of Proof of Proposition 2. See Appendix B, which rewrites the inequality in (11) as a quadratic function of Γ^* and shows that one root of that function satisfies (8). ■

Remark 1 *If the birth rate, ϕ, equals 0, then the roots of $q(\Gamma) = 0$ are 0 and $-(\delta + \gamma)$, so $\Gamma^* = 0$. Therefore, Statement 2 of Proposition 1 implies that $i^* = 0$ and the steady state is DFE, even if $R_0^* > 1$. As stated in the introduction, if there is no replenishment of the pool of susceptible people, the steady state cannot be endemic.*

Corollary 2 *If $\beta \leq \beta_c$ then*

1. *the steady state adjusted growth rate is $\Gamma^* = \phi$*
2. *$R_0^* \leq 1$, so the steady state is DFE.*

Proof of Corollary 2. From the proof of Proposition 2, $q(\phi) = (\beta - \beta_c)\delta\phi$. If $\beta = \beta_c$, then $q(\phi) = 0$, and therefore $\Gamma^* = \phi$. If $\beta < \beta_c$, then $q(\phi) < 0$, and since $q(\Gamma^*)$ is convex, the positive root of $q(\Gamma^*) = 0$ is greater than ϕ, which violates the inequality in (8). In this case, $\Gamma^* = \phi$, which satisfies inequalities (B.1) in the proof of Proposition 2 and (8); therefore $i^* = -\frac{\Gamma^* - \phi}{\delta} = 0$. ■

Proposition 3 *If $\beta > \beta_c$, then $i^{*\prime}(\beta) > 0$.*

Proof of Proposition 3. Since the quadratic function $q(\Gamma)$ is linear in β, it can be written as $q(\Gamma) = -\delta\Gamma^2 - \delta(\delta + \gamma)\Gamma + \beta\frac{dq(\Gamma)}{d\beta}$. Thus $q(\Gamma^*) = 0$ implies $\frac{dq(\Gamma^*)}{d\beta} = (\Gamma^* + \delta + \gamma)\frac{\delta}{\beta}\Gamma^* = \delta\Gamma^* R_0^{-1} > 0$. Since Γ^* is the larger root of the convex function $q(\Gamma)$, $q'(\Gamma^*) > 0$. Therefore, $\frac{d\Gamma^*}{d\beta} = -\frac{dq(\Gamma^*)/d\beta}{q'(\Gamma^*)} < 0$. Since $\Gamma^* = \phi - \delta i^*$, $\frac{di^*}{d\beta} = -\frac{1}{\delta}\frac{d\Gamma^*}{d\beta} > 0$. ■

The following Proposition provides a simple upper bound on i^* that holds for both EE and DFE steady states.

Proposition 4 $0 \leq i^* < \frac{\phi}{\gamma + \phi} < 1$.

Proof of Proposition 4. If $\beta > \beta_c$, then $R_0^* > 1$ and Statement 2 of Proposition 1 implies $i^* < \frac{\Gamma^*}{\gamma + \Gamma^*} \leq \frac{\phi}{\gamma + \phi} < 1$. If $\beta \leq \beta_c$, then from Corollary 2, the steady state is DFE so $i^* = 0$. ■

The quantitative analysis in Section 5 uses $\phi = 0.015$ and $\gamma = 12$, so Proposition 4 implies $i^* < \frac{\phi}{\gamma + \phi} = 0.0012$. Remarkably, this upper bound holds for any finite value of the contagion parameter β, even arbitrarily large values.

2 Effect of Public Health Policy on Output

Public health policies aimed at reducing the contagion parameter β, such as those designed to restrict interaction, can reduce aggregate output. In this section, we specify output as a function of β.

Define z_s as an index of a susceptible person's social and productive interactions. A susceptible person's effective labor is strictly increasing in z_s for $z_s < \overline{z}$ and is constant for $z \geq \overline{z}$. The contagion parameter β is strictly increasing in z_s, so we can write effective labor per susceptible worker as $e(\beta)$ with $e'(\beta) > 0$ for $\beta < \overline{\beta}$ and $e'(\beta) \equiv 0$ for $\beta \geq \overline{\beta}$, where $\overline{\beta}$ is the value of β when $z_s = \overline{z}$.

Recovered people are not susceptible to the disease, so they do not restrain interactions below $\overline{z}$. Therefore, $z_r = \overline{z}$ and a recovered person's effective labor is $e\left(\overline{\beta}\right)$.

We assume that infected people are not engaged in production, so the total amount of effective labor in the economy is $L = Se(\beta) + Re\left(\overline{\beta}\right)$.

As an example,[2] we assume that aggregate output equals AL, where A is the productivity of an effective unit of labor. Therefore, output per capita is $A\frac{L}{N} = A\left(se(\beta) + re\left(\overline{\beta}\right)\right)$, which we write as

$$Y(\beta, s, r) = sy(\beta) + ry\left(\overline{\beta}\right), \tag{12}$$

where $y(\beta) \equiv Ae(\beta)$ is the output per susceptible person and $y\left(\overline{\beta}\right) \equiv Ae\left(\overline{\beta}\right)$ is output per recovered person. Differentiating $Y(\beta, s, r)$ yields

$$Y_\beta(\beta, s, r) = sy'(\beta) \geq 0 \tag{13}$$

and

$$Y_s(\beta, s, r) = y(\beta) \leq y\left(\overline{\beta}\right) = Y_r(\beta, s, r). \tag{14}$$

[2]More generally, if aggregate output $F(L, K)$ is linearly homogeneous in L and K, where K is the aggregate capital stock, then output per capita is $Y\left(\beta, s, r, \frac{K}{N}\right) = \frac{1}{N}F(L, K) = F\left(\frac{L}{N}, \frac{K}{N}\right) = F\left(se(\beta) + re\left(\overline{\beta}\right), \frac{K}{N}\right)$. Therefore, $Y_\beta = F_L se'(\beta) \geq 0$, and $Y_s = F_L e(\beta) \leq F_L e\left(\overline{\beta}\right) = Y_r$. The specification in (12) is a special case in which $F(K, L)$ is linearly homogeneous and $F_K \equiv 0$.

The inequalities in (13) and (14) hold with equality if and only if $\beta = \overline{\beta}$.

3 Optimal β

As in Alvarez et al. (2020), we assume that there is a constant hazard of a "breakthrough date," denoted T, when medical therapies lead to complete and instantaneous recovery of all infected people and a vaccine prevents any new infections, so that the disease is completely eradicated. The recovery rate, γ, and the excess death rate, δ, remain constant until T; from T onward, $i = 0$, $g(i) = \phi - \mu$, and optimal $\beta = \overline{\beta}$. Therefore,

$$N_T \widetilde{V} \equiv N_T \int_T^\infty y\left(\overline{\beta}\right) e^{-(\rho-(\phi-\mu))(t-T)} dt = N_T \frac{y\left(\overline{\beta}\right)}{\rho - (\phi - \mu)} \tag{15}$$

is the present value, discounted at rate $\rho > 0$, of aggregate output from date T onward. To ensure that $\widetilde{V}$ is finite, assume that

$$\rho > \phi - \mu. \tag{16}$$

Before T, the optimal time path of β balances the benefit of reducing β in terms of reducing excess deaths against the cost of reducing β in terms of lost output. This tradeoff is reflected in the objective function

$$\max_{\beta_u, t \le u \le T} E_t \left\{ \int_t^T N_u e^{-\rho u} \left[Y\left(\beta_u, s_u, r_u\right) - \omega \delta i_u \right] du + N_T e^{-\rho(T-t)} \widetilde{V} \right\}, \tag{17}$$

where $0 < \omega < \infty$ is the weight the planner places on an excess death relative to a unit of aggregate output per capita.

Using $N_u = N_t \exp\left(\int_t^u g_z dz\right)$, the objective function at time t in (17), per unit of population, N_t, at time t is

$$V\left(s_t, i_t, r_r\right) = \max E_t \left\{ \int_t^T e^{-\int_t^u (\rho - g_z) dz} \left[Y\left(\beta_u, s_u, r_u\right) - \omega \delta i_u \right] du + \widetilde{V} e^{-\int_t^T (\rho - g_z) dz} \right\}. \tag{18}$$

The value function in (18) satisfies

$$(\rho - g(i))\, V(s,i,r) = \max_{\beta} \left\{ \begin{array}{c} Y(\beta,s,r) - \omega\delta i \\ +V_s \frac{ds}{dt} + V_i \frac{di}{dt} + V_r \frac{dr}{dt} + p\left(\widetilde{V} - V(s,i,r)\right) \end{array} \right\}, \tag{19}$$

where p is the (assumed constant) hazard rate of the breakthrough date, T. The left side of (19) is the required return per unit time, which is the growth-adjusted discount rate, $\rho - g(i)$, multiplied by V. The right side of this equation is the expected return, which comprises the instantaneous flow of welfare, $Y(\beta,s,r) - \omega\delta i$, and the expected change in $V(i,s,r)$, which consists of the change resulting from changes in the state variables, s, i, and r, $V_s \frac{ds}{dt} + V_i \frac{di}{dt} + V_r \frac{dr}{dt}$, and the expected change associated with the breakthrough, $p\left(\widetilde{V} - V(s,i,r)\right)$.

To obtain expressions for the dynamic behavior of $V_j(s,i,r)$, $j \in \{s,i,r\}$, differentiate both sides of (18) with respect to j and use $\frac{dV_j}{dt} = V_{js}\frac{ds}{dt} + V_{ji}\frac{di}{dt} + V_{jr}\frac{dr}{dt}$. Appendix A shows that these calculations lead to

$$(\rho + \mu + p)\, V_s = Y_s + \frac{dV_s}{dt} + \beta i\,(V_i - V_s) \tag{20}$$

$$(\rho + \mu + p + \delta)\, V_i = -\omega\delta + \frac{dV_i}{dt} + \gamma\,(V_r - V_i) - \delta\,[V - (sV_s + iV_i + rV_r)] - (V_s - V_i)\,\beta s \tag{21}$$

and

$$(\rho + \mu + p)\, V_r = Y_r. \tag{22}$$

In (20), the effective discount rate on the left side is $\rho + \mu + p$. As in models of uncertain lifetimes going back to Yaari (1965), the effective discount rate includes the rate of pure time preference, ρ, and the instantaneous hazard rate of death. Here, the hazard rate of death is the baseline death rate, μ, plus the hazard rate that T will arrive, terminating the regime in which the disease is present. Thus, the left side of (20) is the required return associated with

increasing s by one unit. It is equated with the expected return on the right side, which consists of the output Y_s produced by an additional unit of s and the change in valuation reflecting: (1) the passage of time, $\frac{dV_s}{dt}$, and (2) the increased hazard of becoming infected, βi, multiplied by the change in valuation, $V_i - V_s$, as a person moves from susceptible to infected. The interpretation of (22) is similar except there is no term reflecting the change in health status because there are no transitions to susceptibility or infected status from the recovered status.

The interpretation of (21) is more complicated. On the left side, the effective discount rate, $\rho + \mu + p + \delta$, includes δ because a unit increase in i increases excess deaths by δ. The first three terms on the right side are similar to the terms on the right side of (20): The first term reflects that a unit increase in i increases deaths by δ, which reduces the flow of welfare by $\omega\delta$; the second term, $\frac{dV_i}{dt}$, captures the change in V_i with the passage of time; and the third term, $\gamma(V_r - V_i)$, is the hazard rate γ of switching from status i to status r, multiplied by the change in valuation, $V_r - V_i$, associated with that change. The fourth term reflects that a unit increase in i reduces the population change by δN, reducing the aggregate flow of utility by $\delta \frac{\partial}{\partial N} NV\left(\frac{S}{N}, \frac{I}{N}, \frac{R}{N}\right)$, which equals[3] $\delta[V - (sV_s + iV_i + rV_r)]$. The fifth term, $-(V_s - V_i)\beta s$, reflects an important externality, namely, that an increase in the infection share i increases by βs the hazard rate that a susceptible person will become infected. The planner takes account of this externality by including the change in welfare associated with this new infection $-(V_s - V_i)$ multiplied by βs.

The first-order condition for optimal β is

$$Y_\beta(\beta, s, r) = (V_s - V_i)\, si. \tag{23}$$

The left side of (23), $Y_\beta(\beta, s, r)$, is the marginal benefit of increasing β, which is the increase in per-capita output facilitated by an increase in β. The right side of (23) is the marginal cost of increasing β. A unit increase in β increases the infection rate by si, which reduces s by si units and increases i by si units, causing $V(s, i, r)$ to fall by $(V_s - V_i)\, si$.

[3] $\frac{\partial}{\partial N} NV\left(\frac{S}{N}, \frac{I}{N}, \frac{R}{N}\right) = V(s, i, r) - sV_s(s, i, r) - iV_i(s, i, r) - rV_r(s, i, r)$.

3.1 Steady State Under Optimal Policy

Let β^* denote the steady-state value of β under the optimal policy.

Lemma 1 *If $\beta^* \geq \beta_c \equiv \delta + \gamma + \phi$, then $V_s^* - V_i^*$ is positive and finite.*

Sketch of Proof of Lemma 1. The full proof of Lemma 1 in Appendix B uses the steady-state versions of (20), (21), and (22) and shows that if $V_s^* - V_i^* \leq 0$, then $\beta^* = \overline{\beta}$. That proof shows that if $\beta^* = \overline{\beta}$, then $V_s^* - V_i^* > 0$, thereby contradicting $V_s^* - V_i^* \leq 0$. ■

Lemma 1 helps prove the following proposition.

Proposition 5 *If $\overline{\beta} \geq \beta_c \equiv \delta + \gamma + \phi$, and $\omega > 0$ is finite, then under the optimal policy, the steady state is EE.*

Proof of Proposition 5. Suppose that, contrary to what is to be proved, the steady state under optimal policy is DFE, so that $s^* = 1$ and $i^* = r^* = 0$. Therefore, since $V_s^* - V_i^* > 0$ is finite (Lemma 1), the marginal cost of β, $(V_s^* - V_i^*)\, s^* i^*$, equals zero. Since $Y_\beta > 0$ for $\beta < \overline{\beta}$, the first-order condition for β in (23) implies that $\beta^* \geq \overline{\beta} > \beta_c$, which implies that $i^* > 0$. Therefore, the steady state under optimal policy cannot be DFE and hence is EE. ■

The first-order condition in (23) along with $Y_\beta\left(\overline{\beta}, s^*, r^*\right) = 0$ and $(V_s - V_i)\, s^* i^* > 0$ in an EE steady state imply

Corollary 3 *If $\overline{\beta} \geq \beta_c \equiv \delta + \gamma + \phi$ and $\omega > 0$ is finite, then $\beta^* < \beta$.*

4 Laissez Faire

In the absence of centralized policy to control β, individual susceptible people may choose to limit their interactions to reduce their own risks of becoming infected. Consider the decision of a susceptible person, who knows that in the future she may become infected and subsequently may recover from the disease. Using backward induction, first consider the recovered stage of life. The expected present value, discounted at rate ρ, of a recovered

person's earnings until the time of her death, or the arrival of T, whichever comes first, is v^R and satisfies

$$(\rho + \mu + p)\, v^r = y\left(\overline{\beta}\right). \tag{24}$$

For an infected person, the effective discount rate, $\rho + \mu + \delta + p$, includes the hazard rate, δ, that the person dies from the disease. Let v^I be the value of being in the infected state. The instantaneous flow of welfare, $-\omega\delta$, and the hazard-weighted change in value when recovering from the disease, $\gamma\left(v^R - v^I\right)$, satisfy

$$(\rho + \mu + \delta + p)\, v^I = -\omega\delta + \gamma\left(v^R - v^I\right). \tag{25}$$

A susceptible person chooses how much to expose herself to infection according to the Bellman equation

$$(\rho + \mu + p)\, v^S = \max_{\beta}\left\{y\left(\beta\right) + \frac{dv^S}{dt} + \beta i\left(v^I - v^S\right)\right\}, \tag{26}$$

where, unlike v^I and v^R, v^S is not constant. It depends on the aggregate infection share, i, which evolves over time. The first-order condition for the maximization in (26) is

$$y'\left(\beta\right) = i\left(v^S - v^I\right). \tag{27}$$

Now compare an individual's v^R, v^I, and v^S with the derivatives of the planner's value function V_r, V_i, and V_s, respectively. Comparing (24) with (22) and noting that $Y_r = y\left(\overline{\beta}\right)$, implies that $v^R = V_r$. Similarly, noting that $Y_s = y\left(\beta\right)$ and $Y_\beta = sy'\left(\beta\right)$ shows that (20) has the same form as (26) and the first-order condition (23) has the same form as (27), where V_s corresponds to v^S and V_i corresponds to v^I.

The optimal values of β in the two problems differ because the expression for v^I in (25) has a different form than the expression for V_i in (21). The last two terms on the right side of (21) have no counterpart in (25). In addition, the solution V_i of the ODE (21) is a function

of time, while the solution of (25) is a time-invariant, constant v^I. Because v^I differs from V_i, the laissez-faire choice of β differs from the planner's choice so welfare can be improved by mandating β rather than relying on individual precaution.

The two terms on the right side of (21) that are responsible for the discrepancy between V_i and v^I are $\delta\left[V-\left(sV_s+iV_i+rV_r\right)\right]$, and $\left(V_s-V_i\right)\beta s$. The first term is related to the fact that the planner takes into account the impact of i on population growth. The second term, $\left(V_s-V_i\right)\beta s$, reflects an important externality, as discussed earlier. An infected individual does not internalize the contagion of her infection, while the planner does. This externality is the fundamental reason that public health policy is useful.

5 Quantitative Behavior Along the Transition Path

This section presents a quantitative illustration of transition paths. We set $\phi = 0.015$, which is the sum of the annual birth and net immigration rates, and $\mu = 0.01$, the annual death rate in the United States.[4] The values of disease-related parameters are based on the US experience with Covid-19. We set $\gamma = 12$ to reflect that the average person who recovers was infected for about one month. We set $\delta = 0.01\gamma = 0.12$ to reflect that infected people are about 1% as likely to die from the disease as to recover from it. Therefore, the critical value $\beta_c \equiv \delta + \gamma + \phi = 12.135$. To calibrate $\overline{\beta}$, we use (9) and the fact that δi is so much smaller than β_c to obtain $\overline{\beta} = \left(\delta+\gamma+\phi-\delta i\right)\overline{R}_0 \approx \beta_c \overline{R}_0$, where $\overline{R}_0$ is the maximal value of R_0 observed at the beginning of the pandemic before any individual actions or any public health policies to reduce contagion. We use the high end of estimates for R_0 across US states in the last week of February 2020 and set $\overline{R}_0 = 3.5$, which implies $\overline{\beta} = 42.473$.[5] The value of the discount rate used by the planner and by individuals, ρ, is set to 0.03.

We specify the production function $y\left(\beta\right)$ to be quadratic with maximal value $y\left(\overline{\beta}\right)$ normalized to one. Therefore, $y\left(\beta\right) = 1-\alpha\left(\overline{\beta}-\beta\right)^2$, $y'\left(\beta\right) = 2\alpha\left(\overline{\beta}-\beta\right)$ for $\beta \leq \overline{\beta}$, and the "output gap" is $y\left(\overline{\beta}\right) - y\left(\beta\right) = \alpha\left(\overline{\beta}-\beta\right)^2$. Let $\Delta \equiv y\left(\overline{\beta}\right) - y\left(\beta_c\right)$ be the reduction in y

[4]Source: United Nations Population Division, (2015-2020).

[5]Source: Estimates of the reproductive rate provided by the websites http://rt.live and http://epiforecasts.io. For instance, rt.live estimates that on February 27, 2020, the effective reproduction rate of Covid 19 was 3.98 in New Jersey and approximately 3.6 in New York and Illinois.

when β is reduced from its pre-pandemic level, $\overline{\beta}$, to β_c, the level of β at which $R_0 = 1$, which was approximately the value of the reproductive rate for most states during the second quarter of 2020. We set $\Delta = 0.09$ to match the 9% drop in output in that quarter. Therefore, $\alpha\left(\overline{\beta} - \beta_c\right)^2 = \Delta$, which implies $\alpha = \frac{\Delta}{\left(\overline{\beta}-\beta_c\right)^2}$. Setting $\Delta = 0.09$ yields $\alpha = 9.779 \times 10^{-5}$.

To calibrate ω we use the concept of "Quality Adjusted Life Year" (QALY), defined as the value of extending quality life by an extra year. The World Health Organization consensus is that QALY is 1 - 3 annual GDP per capita.[6] Using a discount rate of $\rho + \mu = 0.03 + 0.01$ to discount the foregone stream of 1 QALY per year over the lost years of life implies that the present value of the losses from an excess death is 25 QALY. Assigning a value of 2 times GDP per capita to each QALY implies $\omega = 50$ times GDP per capita. With $y\left(\overline{\beta}\right) = 1$, we set ω equal to 50.[7]

Finally, i_0, the infection share of the population on the initial day of our simulation (March 1), is chosen so that the daily excess death count implied by the model matches the daily Covid-related deaths observed three weeks later (March 22). This calibration implies $\frac{\delta i_0}{365} = \frac{\text{Daily Deaths}}{\text{population}} = \frac{270}{330\times 10^6}$. With $\delta = 0.12, i_0 = 0.0025$.

To interpret data on the reproduction rate, we distinguish the *basic* reproduction rate at time t, $R_{0,t} \equiv \frac{\beta_t}{\gamma+\phi+\Gamma_t}$, from the *effective* reproduction rate at time t, $R_{t,t} \equiv \frac{\beta_t s_t}{\gamma+\phi+\Gamma_t}$, which is $s_t R_{0,t}$, the product of the susceptible share of the population and the basic reproduction rate. Using $R_{t,t}$, (6) can be written as

$$\frac{di}{dt} = \left(R_{t,t} - 1\right)\left(\delta + \gamma + \mu + g\right) i. \tag{28}$$

Therefore, the sign of $R_{t,t} - 1$ determines whether the infection share, i_t, is increasing or decreasing; in contrast, the sign of $R_0^* - 1$ determines whether the steady state is EE or DFE, where R_0^* is the steady-state value of $R_{0,t}$. Since available data typically refer to the effective reproduction rate, $R_{t,t}$, we will focus on that measure in Figure 1. However, near

[6]Source: "Overview of the ICER value assessment framework and update for 2017-2019", p. 15, available at: https://icer-review.org/wp-content/uploads/2018/03/ICER-value-assessment-framework-update-FINAL-062217.pdf

[7]Alvarez et al. (2020), citing Hall et al. (2020), set $\omega = 20$, but note that this value "is on the low range of the estimates in the literature." Our fundamental result that optimal policy leads to an endemic equilibrium holds for any positive value of ω.

the beginning of an epidemic of a new disease, s_t is very close to one, so the distinction between $R_{0,t}$ and $R_{t,t}$ is virtually immaterial.

In Figure 1, the optimal policy scenario (OP) shows the value of $R_{t,t}$ when the contagion parameter β_t is the socially optimal value determined by the first-order condition in (23) at each point of time. The laissez-faire scenario (LF) shows the value of $R_{t,t}$ when the contagion parameter β_t is the laissez-faire value determined by the first-order condition in (27) at each point of time. Figure 1 also shows data that are estimates of $R_{t,t}$ for the 50 individual US states.[8] The most notable feature of Figure 1 is that scenario OP exhibits more aggressive policy to fight the disease than scenario LF. Specifically, the values of $R_{t,t}$, which reflect values of β_t, are lower in scenario OP than in scenario LF. In particular, during the first 4-6 weeks, $R_{t,t}$ is substantially smaller than one in scenario OP and is substantially higher than one in scenario LF.

Figure 2 shows that under scenario OP, daily excess deaths initially decline and continue to decline throughout the 5 months shown. In contrast, under scenario LF, daily excess deaths spike upward abruptly, and after 2-3 months begin to decline very slowly. To illustrate the quantitative difference under the two scenarios, we find that on May 1, daily excess deaths are 114 under scenario OP and are 1501 under scenario LF. The data for nationwide daily excess deaths in Figure 2 resemble scenario LF for the first 6 weeks. Thereafter, daily excess deaths decline rapidly reflecting the effect of policies instituted by various states.

6 Concluding Remarks

Proposition 5 states that under the socially optimal policy, the steady state is an endemic equilibrium with a strictly positive measure infection share i^*. To avoid misunderstanding this finding, it is important to understand what the proposition does not say.

First, the proposition does not imply that public health policies should be abandoned in the steady state. On the contrary, Corollary 3 states that in the steady state, the optimal value of β is less than $\overline{\beta}$, which requires restraint on interaction.

[8]Source: http://rt.live. Data from http://epiforecasts.io imply quantitatively similar values.

Second, since the steady state under optimal policy is EE, $R_0^* > 1$ in the steady state. However, consistent with the comment regarding Corollary 3, R_0^* is less than its unfettered value, and may, for high values of ω be close to one.

Third, though R_0 is greater than one in the steady state, its optimal value can be smaller than one for a period of time along the transition path to the steady state. In the quantitative example in Section 5, socially optimal policy immediately decreases $R_{0,t}$ to 0.22. The steady-state value of i^* is small, implying $\delta i^* = 0.006\%$, which — for a population of 330 million — amounts to $19,800$ excess deaths annually.

Fourth, while Proposition 5 implies that it is not optimal to eradicate the disease solely by reducing β, it leaves open the possibility that it is optimal to eradicate the disease by developing an effective vaccine that is widely used by the population.

Fifth, the finding that optimal policy does not reduce β enough to eradicate the disease does not depend on the possibility of a medical breakthrough that eliminates the disease. This finding prevails even if $p = 0$.

Sixth, the objective function in (17) treats deaths as the only harmful effect of the disease, though many survivors of the disease may have serious health problems that linger indefinitely. These non-fatal harmful effects can be incorporated into the objection function by increasing the value of ω. Since Proposition 5 holds for arbitrarily large ω, increasing ω will not change the result that under optimal policy, the steady state is EE.

Appendix

A System of ODEs

Use $\mu + g = \phi - \delta i$ to write equations (5) - (7) as

$$\frac{ds}{dt} = \phi\left(1 - s\right) - \left(\beta - \delta\right) si \tag{A.1}$$

$$\frac{di}{dt} = \left[\beta s - \left(\delta + \gamma + \phi - \delta i\right)\right] i \tag{A.2}$$

$$\frac{dr}{dt} = \gamma i - \left(\phi - \delta i\right) r. \tag{A.3}$$

Differentiate both sides of (19) with respect to s and use $\frac{dV_s}{dt} = V_{ss}\frac{ds}{dt} + V_{si}\frac{di}{dt} + V_{sr}\frac{dr}{dt}$ and (A.1) - (A.3) to obtain

$$\left(\rho - g\left(i\right)\right) V_s = Y_s + \frac{dV_s}{dt} - \left(\phi + \left(\beta - \delta\right) i\right) V_s + \beta i V_i - pV_s. \tag{A.4}$$

Use $g\left(i\right) = \phi - \mu - \delta i$ to obtain

$$\left(\rho + \mu + p\right) V_s = Y_s + \frac{dV_s}{dt} + \beta i\left(V_i - V_s\right). \tag{A.5}$$

Differentiate both sides of (19) with respect to i and use $\frac{dV_i}{dt} = V_{is}\frac{ds}{dt} + V_{ii}\frac{di}{dt} + V_{ir}\frac{dr}{dt}$, $g\left(i\right) = \phi - \mu - \delta i$ and (A.1) - (A.3) to obtain

$$\left(\rho - \phi + \mu + \delta i\right) V_i + \delta V = -\omega\delta + \frac{dV_i}{dt} - \left(\beta - \delta\right) sV_s + \left(\beta s - \left(\delta + \gamma + \phi - \delta i\right) + \delta i\right) V_i + \left(\gamma + \delta r\right) V_r - pV_i, \tag{A.6}$$

which can be rearranged to obtain

$$(\rho+\mu+p+\delta)\,V_i = -\omega\delta + \frac{dV_i}{dt} + \gamma\,(V_r - V_i) - \delta\,[V - (sV_s + iV_i + rV_r)] - \beta s\,(V_s - V_i)\,. \quad \text{(A.7)}$$

Differentiate both sides of (19) with respect to r and use $\frac{dV_r}{dt} = V_{rs}\frac{ds}{dt} + V_{ri}\frac{di}{dt} + V_{rr}\frac{dr}{dt}$, $g\,(i) = \phi - \mu - \delta i$, and equations (A.1) - (A.3) and rearrange to obtain $(\rho+\mu+p)\,V_r = Y_r + \frac{dV_r}{dt}$. The only solution of this differential equation consistent with the transversality condition $\lim_{t\to\infty} e^{-(\rho+\mu+p)t}V_r = 0$ is

$$(\rho+\mu+p)\,V_r = Y_r. \quad \text{(A.8)}$$

Equations (A.5), (A.7), and (A.8) can be written as a first-order system of nonhomogeneous linear ordinary differential equations with nonconstant coefficients

$$\begin{bmatrix} \frac{dV_s}{dt} \\ \frac{dV_i}{dt} \\ \frac{dV_r}{dt} \end{bmatrix} = A \begin{bmatrix} V_s \\ V_i \\ V_r \end{bmatrix} - b \quad \text{(A.9)}$$

where

$$A \equiv \begin{bmatrix} \rho+\mu+p+\beta i & -\beta i & 0 \\ (\beta-\delta)\,s & \rho+\mu+p+\delta+\gamma-\delta i-\beta s & -(\gamma+\delta r) \\ 0 & 0 & \rho+\mu+p \end{bmatrix} \quad \text{(A.10)}$$

and

$$b \equiv \begin{bmatrix} Y_s \\ -\delta\,(V+\omega) \\ Y_r \end{bmatrix}, \quad \text{(A.11)}$$

A.1 Steady State

In the steady state, $\frac{ds}{dt} = \frac{di}{dt} = \frac{dr}{dt} = \frac{dV_s}{dt} = \frac{dV_i}{dt} = \frac{dV_r}{dt} = 0$. Inspection of (A.8) reveals

$$V_r^* = \frac{Y_r^*}{\rho + \mu + p}. \tag{A.12}$$

Equation (19) along with $\widetilde{V} \equiv \frac{y(\overline{\beta})}{\rho+\mu-\phi}$ implies

$$V^* = \frac{Y^* - \omega\delta i^* + p\frac{y(\overline{\beta})}{\rho+\mu-\phi}}{\rho + p - g\left(i^*\right)}. \tag{A.13}$$

Using $g\left(i^*\right) = \phi - \mu - \delta i^*$, (A.13) implies

$$V^* + \omega = \frac{Y^* + p\frac{y(\overline{\beta})}{\rho+\mu-\phi}}{\rho + p - g\left(i^*\right)} + \left(1 - \frac{\delta i^*}{\rho + p - (\phi - \mu) + \delta i^*}\right)\omega > 0. \tag{A.14}$$

Notably, the coefficient on ω in (A.14) is positive, since $\rho + p + \delta i^* > \rho > \phi - \mu$, where the second inequality is (16).

Since the off-diagonal elements of the third row of A are zero, the expression for V_r^* in (A.12) implies

$$M_{33}^* \begin{bmatrix} V_s^* \\ V_i^* \end{bmatrix} = \begin{bmatrix} Y_s^* \\ -\delta\left(V^* + \omega\right) + \left(\gamma + \delta r^*\right)V_r^* \end{bmatrix}. \tag{A.15}$$

where

$$M_{33}^* \equiv \begin{bmatrix} \rho + \mu + p + \beta^* i^* & -\beta^* i^* \\ \left(\beta^* - \delta\right)s^* & \rho + \mu + p + \delta + \gamma - \delta i^* - \beta^* s^* \end{bmatrix} \tag{A.16}$$

is the matrix obtained by deleting the third row and the third column of A and

$$M_{33}^{*-1} = \frac{1}{\det M_{33}^*} \begin{bmatrix} \rho + \mu + p + \delta + \gamma - \delta i^* - \beta^* s^* & \beta^* i^* \\ -\left(\beta^* - \delta\right)s^* & \rho + \mu + p + \beta^* i^* \end{bmatrix}. \tag{A.17}$$

Multiply both sides of (A.15) by M_{33}^{*-1} to obtain

$$\begin{bmatrix} V_s^* \\ V_i^* \end{bmatrix} = \frac{1}{\det M_{33}^*} \begin{bmatrix} (\rho+\mu+p+\delta+\gamma-\delta i^*-\beta^* s^*) Y_s^* + \beta^* i^* \left[-\delta\left(V^*+\omega\right)+(\gamma+\delta r^*) V_r^*\right] \\ -(\beta^*-\delta) s^* Y_s^* + (\rho+\mu+p+\beta^* i^*)\left[-\delta\left(V^*+\omega\right)+(\gamma+\delta r^*) V_r^*\right] \end{bmatrix}, \tag{A.18}$$

which implies

$$V_s^* - V_i^* = \frac{1}{\det M_{33}^*} \begin{pmatrix} (\rho+\mu+p+\delta+\gamma-\delta i^*-\delta s^*) Y_s^* \\ -(\rho+\mu+p)\left[-\delta\left(V^*+\omega\right)+(\gamma+\delta r^*) V_r^*\right] \end{pmatrix}. \tag{A.19}$$

Use the fact that $\delta - \delta s^* - \delta i^* = \delta r^*$ and (A.12) to simplify (A.19) to obtain

$$V_s^* - V_i^* = \frac{1}{\det M_{33}^*} \begin{pmatrix} (\rho+\mu+p)\left[Y_s^* + \delta\left(V^*+\omega\right)\right] \\ -(\gamma+\delta r^*)\left(Y_r^* - Y_s^*\right) \end{pmatrix}. \tag{A.20}$$

B Selected Proofs

Proof of Proposition 2. First, prove $\Gamma^* > 0$. Inequality (11) can be written as

$$f\left(\Gamma^*\right) \geq 0, \tag{B.1}$$

where $f(z) \equiv z^2 + (\gamma+\delta-\phi) z - \gamma\phi$. Since $f''(z) > 0$ and $f(0) = -\phi\gamma < 0$, the quadratic equation $f(z) = 0$ has two real roots, $z_1 < 0 < z_2$. Therefore, Γ^* satisfies (B.1) if and only if $\Gamma^* < z_1$ or $\Gamma^* > z_2$. Observe that $f(\phi-\delta) = (\phi-\delta)^2 + (\gamma+\delta-\phi)(\phi-\delta) - \phi\gamma = -\gamma\delta < 0$ so $z_1 < \phi - \delta$ and hence $\Gamma^* < z_1$ violates (11). Observe that $f(\phi) = \phi^2 + (\gamma+\delta-\phi)\phi - \phi\gamma = \delta\phi > 0$ so that $z_2 < \phi$. Therefore, $\Gamma^* \in [z_2, \phi]$ satisfies inequalities (B.1) and (11) so $\Gamma^* > 0$.

(Statement 1) The definition of R_0^* in (9) and the steady-state growth rate $g^* = \phi - \mu - \delta i^*$ imply that $R_0^* = \frac{\beta}{\delta+\gamma+\mu+g^*} = \frac{\beta}{\delta+\gamma+\phi-\delta i^*} = \frac{\beta}{\beta_c - \delta i^*} \geq \frac{\beta}{\beta_c^*} > 1$, where the final inequality follows from the assumption that $\beta > \beta_c$. (Statement 2) To calculate Γ^* when $R_0^* \geq 1$, set $\frac{ds}{dt}$ in (5) equal to zero to obtain $\phi - \Gamma s^* = \beta s^* i^*$ and use $s^* = R_0^{*-1}$ from Statement 1 of

Proposition 1 to obtain $i^* = \frac{\phi R_0^* - \Gamma^*}{\beta}$. Then subtract both sides of this equation from the corresponding sides of Statement 2 in Proposition 1 and use the definitions of Γ^* and R_0 to obtain the quadratic equation[9] $q(\Gamma^*) \equiv (\beta - \delta)\Gamma^{*2} + [(\beta - \delta)(\delta + \gamma) - \phi\beta]\Gamma^* - \gamma\phi\beta = 0$. Since $\beta > \beta_c > \delta$, $q(\beta)$ is convex and since $q(0) = -\gamma\phi\beta < 0$, $q(\Gamma) = 0$ has two distinct real roots $\Gamma_1 < 0 < \Gamma_2$. Since $\Gamma^* > 0$, ignore $\Gamma_1 < 0$. To prove that $\Gamma_2 < \phi$, it suffices to prove $q(\phi) > 0$ since $q(0) < 0$. Evaluate $q(\phi) = [(\beta - \delta)(\phi + \delta + \gamma) - \phi\beta - \gamma\beta]\phi = [-\delta(\phi + \delta + \gamma) + \beta\delta]\phi = (\beta - \beta_c)\delta\phi > 0$. ■

Proof of Lemma 1. First prove that $\det M_{33}^* > 0$ is finite. Since all four elements of M_{33}^* are finite and since $\rho + \mu + p + \beta^* i^* > 0$ and $-\beta^* i^* \leq 0$, it suffices to prove (a) $(\beta^* - \delta)s^* \geq 0$ and (b) $\rho + \mu + p + \delta + \gamma - \delta i^* - \beta^* s^* > 0$. The assumption $\beta^* \geq \beta_c \equiv \delta + \gamma + \phi$ implies $\beta^* - \delta \geq \gamma + \phi > 0$ which proves (a). To prove (b), consider two separate cases: (i) $i^* = 0$ and (ii) $i^* > 0$. In case (i), $i^* = 0$ implies $\beta^* \leq \beta_c$, which together with the assumption $\beta^* \geq \beta_c$ implies that $\beta^* = \beta_c$. Therefore, since $s^* = 1$, $\rho + \mu + p + \delta + \gamma - \delta i^* - \beta^* s^* \geq \rho + \mu + p + \delta + \gamma - \beta_c = \rho + p - (\phi - \mu) > 0$. In case (ii), setting $\frac{di}{dt}$ in equation (6) equal to zero implies $\beta^* s^* = \delta + \gamma + \mu + g^* = \delta + \gamma + \phi - \delta i^*$, so $\rho + \mu + p + \delta + \gamma - \delta i^* - \beta^* s^* = \rho + p - (\phi - \mu) > 0$. Therefore, $\det M_{33}^* > 0$ is finite.

Suppose, contrary to what is to be proved, that $V_s^* - V_i^* \leq 0$. Then the first-order condition for optimal β in (23) implies that $\beta^* = \overline{\beta}$. Therefore, using (A.14), which implies that $V^* + \omega > 0$ is finite, along with $\det M_{33}^* > 0$ finite and (14), which states that $Y_s\left(\overline{\beta}, s^*, r^*\right) = Y_r\left(\overline{\beta}, s^*, r^*\right)$, together in (A.20) implies that

$$V_s^* - V_i^* = \frac{1}{\det M_{33}^*}\left((\rho + \mu + p)\left[Y_s\left(\overline{\beta}, s^*, r^*\right) + \delta(V^* + \omega)\right]\right) > 0,$$

which contradicts the supposition that $V_s^* - V_i^* \leq 0$. Therefore, $V_s^* - V_i^*$ must be positive. ■

[9] $0 = \frac{\Gamma^*}{\gamma + \Gamma^*}\left(1 - R_0^{*-1}\right) - \frac{\phi R_0^* - \Gamma^*}{\beta} = \Gamma^*(\beta - (\gamma + \delta + \Gamma^*)) - \phi(\gamma + \Gamma^*)R_0^* + (\gamma + \Gamma^*)\Gamma^* = (\beta - \delta)\Gamma^* - \phi(\gamma + \Gamma^*)R_0^* = (\beta - \delta)(\gamma + \delta + \Gamma^*)\Gamma^* - \phi(\gamma + \Gamma^*)\beta = (\beta - \delta)\Gamma^{*2} + [(\beta - \delta)(\gamma + \delta) - \phi\beta]\Gamma^* - \phi\beta\gamma \equiv q(\Gamma^*)$.

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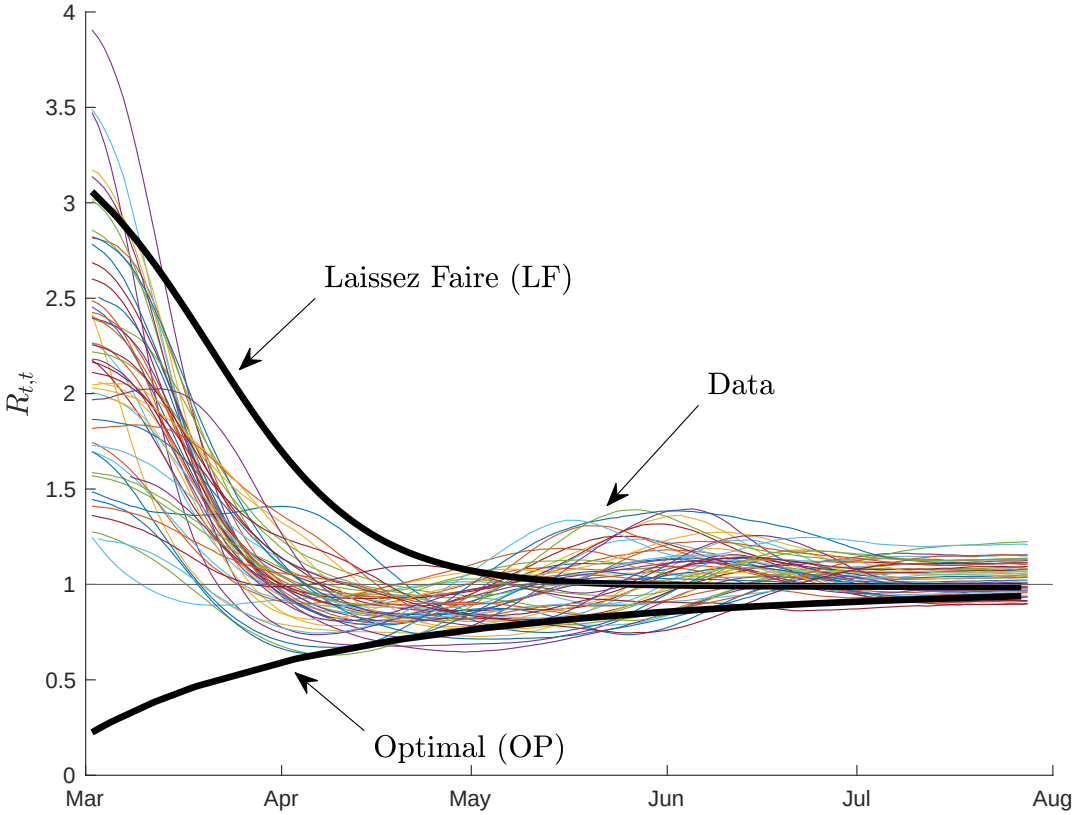

Figure 1: The thick lines depict the effective reproduction rate ($R_{t,t}$) according to the model under the optimal policy (OP) and laissez faire (LF) scenarios from beginning of March to end of July, 2020. The thin lines depict estimated $R_{t,t}$ for each US state.

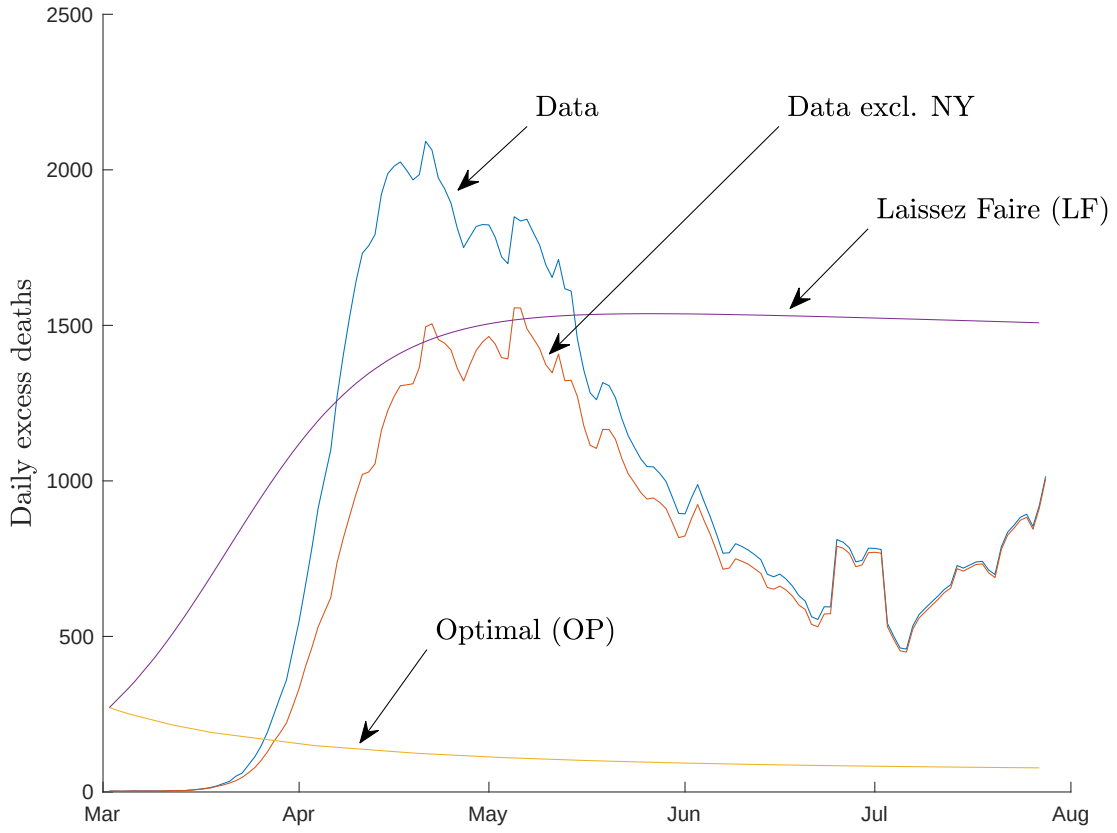

Figure 2: Model-implied daily excess deaths under the optimal policy (OP) and laissez faire (LF) scenarios. The line "Data" corresponds to the daily excess deaths observed in the US over this period. The line "Data Excl NY" excludes New York from the computation of daily excess deaths.