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MONETARY POLICY AND ASSET PRICE OVERSHOOTING: A RATIONALE FOR THE WALL/MAIN STREET DISCONNECT

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ABSTRACT

We analyze optimal monetary policy and its implications for asset prices, when aggregate demand has inertia and responds to asset prices with a lag. If there is a negative output gap, the central bank optimally overshoots aggregate asset prices (asset prices are initially pushed above their steady-state levels consistent with current potential output). Overshooting leads to a temporary disconnect between the performance of financial markets and the real economy, but it accelerates the recovery. When there is a lower-bound constraint on the discount rate, overshooting becomes a concave and non-monotonic function of the output gap: the asset price boost is low for a deeply negative initial output gap, grows as the output gap improves over a range, and shrinks toward zero as the output gap improves further. This pattern also implies that good macroeconomic news is better news for asset prices when the output gap is more negative. Finally, we document that during the Covid-19 recovery, the policy-induced overshooting was large—sufficient to explain the high levels of stock and house prices in 2021.

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Dynamic link to most recent draft is available at
https://www.dropbox.com/s/e92ev3owfw286fy/overshooting_

1. Introduction

The initial recovery from the Covid-19 recession featured a large *disconnect* between the performance of the real economy and financial markets. The left panel of Figure 1 shows that, by the end of 2020, the U.S. output was still significantly below its long-run potential, whereas stock prices (as well as house and bond prices) vastly exceeded their pre-pandemic levels.[1] The robust recovery of asset markets was primarily due to the aggressive monetary (and fiscal) policy response to the Covid-19 shock. During the early stages of the recession, monetary policy stabilized asset prices by containing and then reversing the large spike in the risk-premium (see, e.g., Caballero and Simsek (2021a)). Subsequently, monetary policy supported asset prices by keeping short and long interest rates low. By the end of 2020, the excess valuation in asset prices (relative to pre-Covid) was mostly attributable to the sharp decline in *safe real rates*, rather than to a decline in the risk premium (see Section 4 and Knox and Vissing-Jorgensen (2021)). A debate started about whether monetary policy support was excessive and creating frothy financial market conditions.

Fast-forward to mid 2022 and the disconnect between the real economy and the markets disappeared. The economy recovered faster than most people expected, and the rapid recovery created inflationary pressures. The Fed responded to the rise in inflation by announcing the gradual withdrawal of monetary policy support. This announcement led to a sharp decline in asset prices and induced a *reconnect* between the markets and the economy. The decline in stock prices in 2022 can be largely explained by the increase in real interest rates (see Section 4).

In this paper, we present a model where this type of initial *disconnect* and subsequent *reconnect* between the markets and the economy, driven by policy induced fluctuations in real rates, is not an anomaly but a desirable feature. Our model is similar to the textbook New-Keynesian model, with the key difference that aggregate demand has *inertia* and responds to asset prices gradually (see, e.g., Chodorow-Reich et al. (2021) for empirical evidence supporting this property). Our main result shows that, when output is below its potential, monetary policy *optimally* induces *asset price overshooting*: aggregate asset prices are initially high (above their steady-state levels consistent with current potential output) even though output is low. A central bank that dislikes output gaps reduces the real rates and boosts asset prices to close the output gap as fast as possible. This boost

[1] The early disconnect between the quick recovery of financial markets and the sluggish response of the real economy was the source of much debate, as highlighted by the cover page of The Economist, May 9th, 2020 ("A dangerous gap: The markets v the real economy").

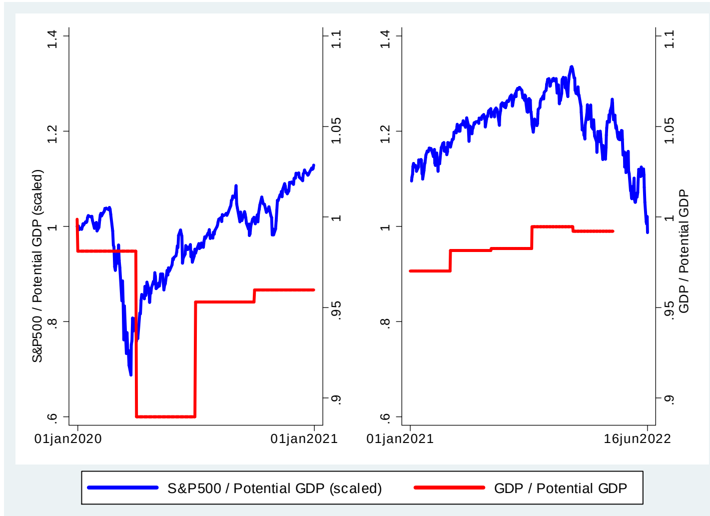

Figure 1: **The disconnect and the subsequent reconnect between Wall Street and Main Street.** The series for S&P500 over potential GDP is scaled to equal one on December 31, 2019. Potential GDP is the estimate by the Congressional Budget Office (CBO). See Appendix D for data sources.

creates a large, temporary disconnect between financial markets and the real economy, but it also accelerates the recovery. As output recovers, the central bank gradually raises the real rates and reverses the asset price overshooting, which reconnects the markets and the economy.

The specific reason for the overshooting result in our model is that individual agents adjust their consumption infrequently. Output is determined by aggregate demand, and aggregate demand depends on asset prices through a wealth effect on consumption. With continuous and full microeconomic adjustments, the central bank "sets" asset prices at the right level, to make aggregate demand equal to potential output at all times. In contrast, with infrequent microeconomic adjustments, a central bank facing a negative output gap due to a depressed aggregate demand needs to overshoot asset prices so that the agents that do adjust (partially) compensate for the depressed consumption of those that do not. At a more general level, the key ingredient that drives overshooting is the inertial response of aggregate demand to asset prices. Aggregate demand inertia also emerges from frictions outside of our model, such as habit formation (see the literature review for further discussion).[2]

[2] Aggregate demand inertia also provides a natural explanation for the "long and variable" monetary policy transmission lags observed in practice (see Woodford (2005), Chapter 5, for a formalization). In a recent speech, Federal Reserve Chairman Powell emphasized the importance of these transmission lags: "Finally, we continue to believe that monetary policy must be forward looking, taking into account... the lags in monetary policy's effect on the economy" (Powell (2020)).

Our second set of results follows from adding a lower-bound constraint on the discount rate. This constraint makes the overshooting a concave and non-monotonic function of the output gap. For a deeply negative initial output gap, the asset price boost is low since the constraint is severely binding. As the output gap improves up to a threshold, the asset price boost grows since the constraint is effectively relaxed. As the output gap improves beyond the threshold, the asset price boost shrinks toward zero. In this range, the central bank is nearly unconstrained and it optimally "tapers" the overshooting in response to an improvement in the output gap. This interaction between constraints and optimal overshooting also provides an explanation for the fact that the impact of macroeconomic news on stock prices depends on the stage of the business cycle (e.g., McQueen and Roley (1993); Boyd et al. (2005); Andersen et al. (2007); Law et al. (2019)). In our model, as in the data, good macroeconomic news is *better* news for asset prices when the output gap is lower (more negative) and the economy is farther from full recovery.

Our last set of results documents the size and impact of the policy-induced overshooting during the recovery from the Covid-19 recession. To facilitate this exercise, we decompose the aggregate asset price in our setting into a "market-bond portfolio"—driven by forward interest rate changes, and a "residual"—driven by expected cash flows and other factors. The market-bond portfolio captures the policy support to asset prices through risk-free rates. The price of this portfolio increased substantially during the Covid-19 recovery—sufficient to explain the high levels of stock and house prices in 2021. A back-of-the-envelope calculation suggests that this asset price overshooting in 2021 increased 2022 output by about 2.3%. We also compare the Covid-19 episode with the Great Recession and show that the market-bond portfolio response was much faster during the Covid-19 recovery.

Literature review. In our model, monetary policy operates through financial markets as in Caballero and Simsek (2020, 2021a,b). The central bank affects asset prices, which in turn affect aggregate demand. The distinctive feature of this paper is the delayed response of aggregate demand to asset prices. Also, in the context of the Covid-19 recession, Caballero and Simsek (2021a) provides an explanation for the large initial decline in asset prices and highlights the key role of LSAPs in reversing that decline; while this paper provides a rationale for the subsequent Wall/Main Street disconnect (see Figure 1).

Our paper is part of a large New-Keynesian literature (see Woodford (2005); Galí (2015) for textbook treatments). Our key ingredient, aggregate demand inertia, is routinely assumed in quantitative New-Keynesian models because it helps match the observed gradual response of spending to a variety of shocks (see Brayton et al. (2014)). However, the policy implications of aggregate demand inertia are less well understood. Fuhrer (2000); Amato and Laubach (2004) study the optimal monetary policy implications of habit formation—a specific source of aggregate demand inertia.[3] We also study optimal monetary policy with aggregate demand inertia, but we focus on the implications for asset prices and obtain an overshooting result. In follow-on work (Caballero and Simsek (2022)), we investigate the effects of aggregate demand inertia in an environment with a temporary supply shock. There, we focus on the implications for overheating and inflation, rather than on asset prices and overshooting.

We capture aggregate demand inertia by assuming infrequent adjustment of individual consumption. An extensive literature on durables' consumption (and investment) uses fixed adjustment costs to document this type of infrequent adjustment and its implications for aggregate durables' consumption and investment (see Bertola and Caballero (1990) for an early survey). There is also a literature that emphasizes infrequent re-optimization for broader consumption categories—due to behavioral or informational frictions—and uses this feature to explain the inertial behavior of aggregate consumption (e.g., Caballero (1995); Reis (2006)) as well as asset pricing puzzles (e.g., Lynch (1996); Marshall and Parekh (1999); Gabaix and Laibson (2001)). We take infrequent adjustment of individual consumption as given (driven by a Poisson process for simplicity) and study its implications for optimal monetary policy.

Our asset-price decomposition in the context of the Covid-19 recovery is related to recent work by Van Binsbergen (2020); Knox and Vissing-Jorgensen (2021). Our market-bond portfolio is the same as the duration-matched fixed-income portfolio analyzed by Van Binsbergen (2020). We focus on the *price change* of this portfolio, which is central for our analysis, whereas Van Binsbergen (2020) focuses on the *total return.* Likewise, Knox and Vissing-Jorgensen (2021) focus on the total return of the stock market and provide a more general asset-price decomposition that incorporates the risk-premium and cash-flow news, in addition to the safe rate news that we analyze. For the Covid-19 episode, Knox and Vissing-Jorgensen (2021) find that the initial decline in stock prices is driven by an increase in the risk premium, but the subsequent recovery and boom are heavily influenced by declining interest rates, consistent with our findings. More broadly, a growing empirical literature analyzes the stock price changes after the Covid-19 shock and finds that monetary policy plays a large role (see, e.g., Gormsen and Koijen (2020)).[4]

[3] A strand of the literature uses models of aggregate demand inertia to compare the performance of different monetary policy rules (see, e.g., Svensson (2003); Svensson and Woodford (2007)).

[4] Several studies find that unconventional monetary policies (which we discuss in Section 4) also had a large positive impact on asset prices (e.g., Fed (2020); Cavallino and De Fiore (2020); Arslan et al. (2020); Haddad et al. (2021)). Other studies analyze the broader set of factors that drive asset prices

Also, as we discuss in Section 3.2, our results with constrained monetary policy shed some light on the empirical literature documenting that the impact of macroeconomic news on asset prices depends on the stage of the business cycle.

In terms of the model's ingredients, this paper is related to and supported by an extensive empirical literature documenting that: (i) monetary policy affects asset prices (e.g., Jensen et al. (1996), Thorbecke (1997), Jensen and Mercer (2002), Rigobon and Sack (2004), Ehrmann and Fratzscher (2004), Bernanke and Kuttner (2005), Bauer and Swanson (2020))—these papers find that, on average, an unanticipated 100 bps increase in the policy rate or the one-year treasury yield is associated with a decrease in stock market returns in the range of 5% to 7%; (ii) asset prices affect aggregate demand and output (e.g., Davis and Palumbo (2001), Dynan and Maki (2001), Gilchrist and Zakrajšek (2012), Mian et al. (2013), Kyungmin et al. (2020), Di Maggio et al. (2020), Chodorow-Reich et al. (2021), Guren et al. (2021))—these papers find wealth and balance sheet effects in the range of 3–10 cents on the dollar depending on the sample and the specific asset price;[5] (iii) the effect of asset prices on aggregate demand and output is gradual (e.g., Davis and Palumbo (2001); Dynan and Maki (2001); Lettau and Ludvigson (2004); Carroll et al. (2011); Case and Shiller (2013); Chodorow-Reich et al. (2021))—these papers find that consumption typically takes about two years to fully adjust to stock price changes.

The rest of the paper is organized as follows. Section 2 introduces our baseline model, establishes our main overshooting result, and discusses several extensions. Section 3 analyzes asset price overshooting with a discount rate lower-bound. Section 4 quantifies the asset price overshooting driven by risk-free rates during the recovery from the Covid-19 recession. Section 5 provides final remarks. Appendices A-D contain omitted microfoundations, proofs, extensions, and data sources, respectively.

2. A model with aggregate demand inertia

In this section, we describe our baseline model, define the equilibrium, and establish our main results. We focus on a *recovery* scenario following a recessionary shock to the economy, when aggregate demand is below potential output. We show that, when aggregate demand has *inertia* and responds to asset prices gradually, optimal monetary

during the Covid-19 episode (e.g., Ramelli and Wagner (2020); Landier and Thesmar (2020); Baker et al. (2020); Davis et al. (2021b,a)).

[5] In addition, Cieslak and Vissing-Jorgensen (2020) show that the Fed pays attention to the stock market, mainly because policymakers believe the stock market affects the economy through a wealth effect.

policy overshoots asset prices. We keep the baseline model simple and discuss various extensions at the end of the section.

2.1. Environment and equilibrium

Our model is a variant of the textbook New-Keynesian model presented in Galí (2015). The key difference is *aggregate demand inertia.*

Agents. There are two types of agents, denoted by superscript $i = s$ ("stockholders") and $i = h$ ("hand-to-mouth households"). This separation is useful because it allows us to decouple stockholders' consumption problem from the labor supply decision. Our focus is on the stockholders, whose spending has inertia and responds to asset prices sluggishly. Stockholders own all financial assets (claims on firms' profits) but do not supply any labor. Hand-to-mouth households supply labor (endogenously) and spend all of their income in each period.

Supply side and nominal rigidities. Time $t \geq 0$ is continuous and there is no uncertainty. A competitive final goods producer combines the intermediate goods according to the CES technology, $Y(t) = \left(\int_0^1 Y(t,\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}$ for some $\varepsilon > 1$. A continuum of monopolistically competitive firms, denoted by $\nu \in [0,1]$, produce the intermediate goods. These firms have fully sticky nominal prices (we endogenize inflation in Appendix C.2). Since these firms operate with a markup, they find it optimal to meet the demand for their good (for relatively small demand shocks, which we assume). Therefore, output is determined by aggregate demand, which depends on the consumption of stockholders, $C^s(t)$, and hand-to-mouth households, $C^h(t)$:

$$Y(t) = C^s(t) + C^h(t). \tag{1}$$

Labor, L, is the only factor of production. The intermediate good firms produce according to the Cobb-Douglas technology

$$Y(t,\nu) = AL(t,\nu)^{1-\alpha},$$

where $1-\alpha$ denotes the share of labor. Labor is supplied by hand-to-mouth households. They have the per-period utility function

$$\log C^h(t) - \chi \frac{L(t)^{1+\varphi}}{1+\varphi},$$

which leads to a standard labor supply curve (see Appendix A).

With these production technologies, if the model was fully competitive, the labor's share of output would be constant and given by, $(1-\alpha) Y(t)$. However, since the intermediate good firms have monopoly power and make pure profits, the labor's share is smaller than $(1-\alpha) Y(t)$. To simplify the exposition, we assume the government taxes part of the firms' profits (lump-sum) and redistributes to workers (lump-sum) so that the labor's share is as in the fully competitive case. This implies the spending of hand-to-mouth households (who supply all labor) is[6]

$$C^h(t) = (1-\alpha) Y(t). \tag{2}$$

Combining Eqs. (1) and (2), yields

$$Y(t) = \frac{C^s(t)}{\alpha}. \tag{3}$$

Hand-to-mouth households create a Keynesian multiplier effect, but output is ultimately determined by *stockholders'* spending, $C^s(t)$.

In Appendix A, we characterize the equilibrium in a flexible-price benchmark economy without nominal rigidities (the same setup except the intermediate good firms have fully flexible prices). In this benchmark, the labor supply is the solution to $\chi (L^*)^{1+\varphi} = \frac{\varepsilon-1}{\varepsilon}$ and output is given by $Y^* = A(L^*)^{1-\alpha}$. We refer to Y^* as the *potential* output. In our model with sticky prices, output is determined by Eq. (3) and can deviate from potential output.

Financial markets. There are two assets. First, there is a "market portfolio" that is a claim on firms' profits, $\alpha Y(t)$ (the firms' share of output). We let $P(t)$ and $R(t)$ denote the real price and the real discount rate of the market portfolio. Since there is no risk, by no arbitrage the discount rate satisfies

$$R(t) = \frac{\alpha Y(t) + \dot{P}(t)}{P(t)}. \tag{4}$$

Second, there is a risk-free asset in zero net supply with real interest rate $R^f(t)$. The central bank controls $R^f(t)$ by setting the nominal interest rate. Since there is no inflation

[6] Formally, letting $T(t)$ denote the appropriate transfer, hand-to-mouth agents' income is $W(t) L(t) + T(t) = (1-\alpha) Q(t) Y(t)$, where $W(t)$ is the nominal wage and $Q(t) = \left(\int Q(t,\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}$ is the nominal price of the final good. $Q(t,\nu)$ denotes the nominal price of good ν. See Appendix A for details.

(except for Appendix C.2), the nominal and the real interest rates are the same. Since there is no risk, the interest rate and the discount rate are also the same, $R^f(t) = R(t)$. Going forward, we drop $R^f(t)$ from the notation and assume the central bank directly controls the discount rate. We discuss how a risk premium would affect our analysis in Section 2.3.5.

Steady-state with potential output. There is a continuum of identical stockholders that own the market portfolio and make consumption-savings and portfolio choices. These stockholders have time-separable log utility, with discount rate ρ. If there were no other frictions, stockholders would spend a constant fraction of their wealth, $C^s(t) = \rho P(t)$. Using Eq. (3), if the central bank adjusts the asset return to target an output level equal to its potential, the economy *immediately* reaches a steady-state consistent with the flexible-price benchmark:

$$\begin{aligned} Y(t) &= Y^*, \quad C^s(t) = C^{s,*} = \alpha Y^* \\ R(t) &= R^* = \rho \quad \text{and} \quad P^* = \frac{\alpha Y^*}{\rho}. \end{aligned} \tag{5}$$

Aggregate demand inertia. We depart from this environment by assuming that *stockholders adjust their spending (and portfolio allocations) infrequently.* At every instant, a random fraction of stockholders adjusts, with constant hazard θ. Their allocations remain unchanged until the next time they have a chance to adjust. Let $C^{s,adj}(t)$ denote the adjusting stockholders' total spending. Assuming that the adjusting stockholders are randomly selected, stockholders' total spending follows

$$\dot{C}^s(t) = \theta\left(C^{s,adj}(t) - C^s(t)\right).$$

Total spending increases if and only if the adjusting stockholders' spending exceeds the current level of spending, $C^s(t)$. Using Eq. (3), and log-linearizing around the potential steady-state, we further obtain

$$\dot{y}(t) = \theta\left(c^{s,adj}(t) - y(t)\right). \tag{6}$$

Here, $x(t) = \log\left(\frac{X(t)}{X^*}\right)$ is the log-deviation of the corresponding variable from its potential level in (5). We refer to the log-deviation of output, $y(t)$, as the *output gap.*

Eq. (6) captures our key friction: Aggregate demand (and the output gap) has *inertia* and responds to the spending decisions of adjusting stockholders sluggishly. The hazard parameter θ captures the degree of aggregate demand inertia.

Consumption wealth effect with demand inertia. We next specify the adjusting stockholders' total spending, $C^{adj}(t)$. Note that there is a representative stockholder whose wealth equals aggregate wealth, $P(t)$. When adjusting, this adjusting stockholder chooses her new consumption level according to the following log-linear rule:

$$c^{s,adj}(t) = mp(t) + ny(t). \tag{7}$$

The parameters, $m > 0, n \in [0, 1)$, capture the sensitivity of consumption to asset prices and output. Individual stockholders follow a similar rule scaled by their wealth.[7]

We adopt the consumption rule in (7) to simplify the exposition. When $m = 1$ and $n = 0$, we have the optimal rule of a continuously-adjusting stockholder, $c^{s,adj}(t) = p(t)$ (equivalently, $C^{s,adj}(t) = \rho P(t)$). This rule is not optimal for a sluggish stockholder *who incorporates the fact that she might not get to adjust in future periods.* In Appendix C.1, we show that the optimal (log-linearized) consumption rule is given by (7) with appropriate weights m, n (see Section 2.3.2). The weights are endogenous to the dynamic equilibrium path but exogenous to monetary policy decisions (as long as the central bank sets discount rates without commitment, which we assume). We do not focus on this fully rational case and instead treat m, n as exogenous parameters, to avoid fixed-point arguments that are orthogonal to our main contributions. Appendix C.1 shows that our main result also holds with the optimal (rational-expectations) consumption rule.

Combining Eqs. (6) and (7), the output gap follows

$$\dot{y}(t) = \theta \left(mp(t) + ny(t) - y(t) \right). \tag{8}$$

The initial output gap, $y(0) = c^s(0)$, is exogenous (determined by an unmodeled history). The output gap responds to the asset price gap, $p(t)$, due to the standard consumption wealth effect. However, the response is gradual due to aggregate demand inertia.

Portfolio choice and equilibrium in financial markets. For symmetry, we assume stockholders are also sluggish with respect to their portfolio choices, although this does

[7] Formally, let $a^i(t)$ denote a stockholder's wealth and $\alpha^i(t) = \frac{a^i(t)}{P(t)}$ denote her wealth share. A stockholder with wealth share $\alpha^i(t)$ follows the rule $C^{i,adj}(t) = C^{s,*}\alpha^i(t)\exp(mp(t) + ny(t))$. Aggregating across all adjusting stockholders, we obtain (7) since wealth shares satisfy $\sum_i \alpha^i(t) = 1$. With this rule, there might be paths along which some stockholders' wealth becomes zero, e.g., if the stockholder does not adjust for a long time. If this happens, the budget constraint binds and consumption falls to zero. We ignore these paths since we focus on relatively small shocks and log-linearized dynamics.

not play any role in our analysis. Specifically, Eq. (4) also holds with sluggish stockholders. Given these returns, the stockholders that adjust are indifferent to changing their portfolios. We assume all stockholders invest all of their wealth in the market portfolio, which ensures market clearing. We also log-linearize Eq. (4) around the potential levels in (5), to obtain

$$r(t) = \frac{\rho}{1+\rho}\left(y(t) - p(t)\right) + \frac{1}{1+\rho}\dot{p}(t). \tag{9}$$

Here, $r(t) \equiv \log \frac{1+R(t)}{1+\rho}$ is the log-deviation of the *gross* discount rate from its potential. Eq. (9) is a continuous time version of the standard Campbell-Shiller approximation.

Monetary policy. The central bank implements a path of output, asset price, and discount rate gaps, $[y(t), p(t), r(t)]_{t\in(0,\infty)}$, that satisfy Eqs. (8) and (9) given $y(0)$. We can think of the central bank as targeting a path of output and asset price gaps, $[y(t), p(t)]_{t\in(0,\infty)}$, that satisfy Eq. (8) given $y(0)$. Then Eq. (9) describes the equilibrium rate path, $[r(t)]_{t\in(0,\infty)}$, that the central bank needs to set to achieve its target.

We assume the central bank's objective function is

$$V(0, y(t)) = \int_0^\infty e^{-\rho t}\left(-\frac{1}{2}y(t)^2 - \frac{\psi}{2}p(t)^2\right)dt. \tag{10}$$

As usual, the central bank dislikes output gaps, $y(t)$. We assume a quadratic cost function, which leads to closed-form solutions. In addition, the central bank also dislikes asset price gaps, $p(t)$. In the limit $\psi \to 0$, we have the conventional setup in which the central bank does not (directly) pay attention to asset prices. Our main results hold in this conventional limit, but optimal policy overshoots asset prices by an extreme amount. In practice, a large asset price overshooting could lead to a number of concerns that range from financial stability (e.g., high asset prices can increase the risk of a collapse) to wealth redistribution (e.g., high asset prices can increase inequality). We capture these types of concerns with the parameter $\psi > 0$, which we refer to as "aversion-to-overshooting." The central bank's discount rate is the same as that of the stockholders, ρ.

Finally, we assume the central bank sets the current policy *without* commitment: it sets the current asset price gap $p(t)$, taking the path of future gaps as given. In this case, the central bank's policy problem can be formulated recursively as,[8]

$$\begin{aligned}\rho V(y) &= \max_p -\frac{y^2}{2} - \psi\frac{p^2}{2} + V'(y)\dot{y}, \qquad (11)\\ \dot{y} &= \theta\left(mp - (1-n)y\right),\\ V(y) &\leq 0 \text{ and } V(0) = 0.\end{aligned}$$

The constraints in the last line follow from the objective function in (10) and ensure that we pick the correct solution to the recursive problem. We define the equilibrium as follows.

Definition 1. *A (log-linearized) equilibrium with optimal monetary policy,* $[y(t), p(t), r(t)]_{t=0}^{\infty}$*, is such that the path of output and asset price gaps,* $[y(t), p(t)]$*, solve the recursive problem* (11) *and the discount rate satisfies Eq.* (9)*.*

2.2. Asset price overshooting in a recovery

We next solve for the equilibrium and establish our overshooting result. To capture the recovery from a recessionary shock, we focus on the case with a negative initial output gap, $y(0) < 0$, where aggregate demand has yet to catch up with potential output. Our main result shows that the optimal policy features *asset-price overshooting:* the central bank optimally chooses a *positive asset price gap*. That is, even though output is below its potential level, the asset price is high and above its potential level. We also show that the central bank overshoots asset prices by *more* when aggregate demand is more inertial and responds to asset prices more gradually.

Consider the planner's problem (11). In the appendix, we conjecture and verify that the solution is a quadratic function,

$$V(y) = -\frac{1}{2v}y^2, \qquad (12)$$

$$\text{where } 0 = v^2 - \left(\rho + 2\theta(1-n)\right)v - \frac{\theta^2 m^2}{\psi}. \qquad (13)$$

Here, $v > 0$ ("value") denotes an endogenous coefficient. Since $V(y) < 0$, we also have $v > 0$: the solution corresponds to the positive root of (13).

[8] The lack of commitment does not restrict monetary policy in our baseline model. The Principle of Optimality implies that maximizing (10) subject to (8) is equivalent to solving problem (11). Lack of commitment is restrictive when we introduce inflation (see Appendix C.2) or when we endogenize the consumption function in (7) (see Appendix C.1).

Combining the value function in (12) with the optimality condition, we solve for the optimal asset price as

$$p = \frac{\theta m}{\psi} V'(y) \Longrightarrow p(t) = -\frac{\theta m}{\psi v} y(t). \tag{14}$$

This expression illustrates the overshooting of asset prices. Starting with a negative output gap, the optimal asset price is above its potential level.

Next consider the change in output gaps along the optimal path. Combining Eqs. (14) and (8), we obtain

$$\dot{y}(t) = -\gamma y(t), \qquad \text{where } \gamma \equiv \theta\left(\frac{\theta m^2}{\psi v} + 1 - n\right) > 0. \tag{15}$$

The composite parameter, γ, captures the convergence rate. Starting with a negative output gap, and an associated positive asset price gap (in view of overshooting), both gaps monotonically converge to zero at rate γ.

Finally, consider the discount rate gap. Combining Eqs.(9), (14), and (15), we obtain

$$\begin{aligned} r(t) &= \frac{\rho}{1+\rho}(y(t) - p(t)) + \frac{1}{1+\rho}\dot{p}(t) \\ &= \frac{\rho}{1+\rho}\left(1 + \frac{\theta m}{\psi v}\right) y(t) + \frac{1}{1+\rho}\frac{\theta m}{\psi v}\gamma y(t) \\ &= \mathcal{R} y(t), \\ \text{where } \mathcal{R} &\equiv \frac{\rho}{1+\rho} + \frac{\theta m}{\psi v}\left(\frac{\gamma+\rho}{1+\rho}\right). \end{aligned} \tag{16}$$

Hence, the discount rate gap is (positively) proportional to the output gap. Starting with a negative output gap, the discount rate gap starts below zero and gradually increases, $r(0) < 0$ and $\dot{r}(t) > 0$. The following result summarizes this discussion.

Proposition 1. *The value function is given by* (12), *where* $v > 0$ *is the positive solution to* (13). *The equilibrium path of the output, asset price, and discount rate gaps,* $[y(t), p(t), r(t)]_{t=0}^{\infty}$, *is characterized by Eqs.* $(14), (15), (16)$. *Starting with a negative output gap,* $y(0) < 0$, *the equilibrium features* ***asset price overshooting****: the asset price is above its potential,* $p(0) > 0$, *and the discount rate is below its potential,* $r(0) < 0$. *Over time, all gaps converge to zero at the exponential rate* $\gamma = v - (\rho + \theta(1-n)) > 0$.

Figure 2 illustrates the equilibrium dynamics for a particular parameterization starting with a negative output gap. The equilibrium (the solid lines) features overshooting: the central bank sets a positive asset price gap and gradually closes the output gap. The

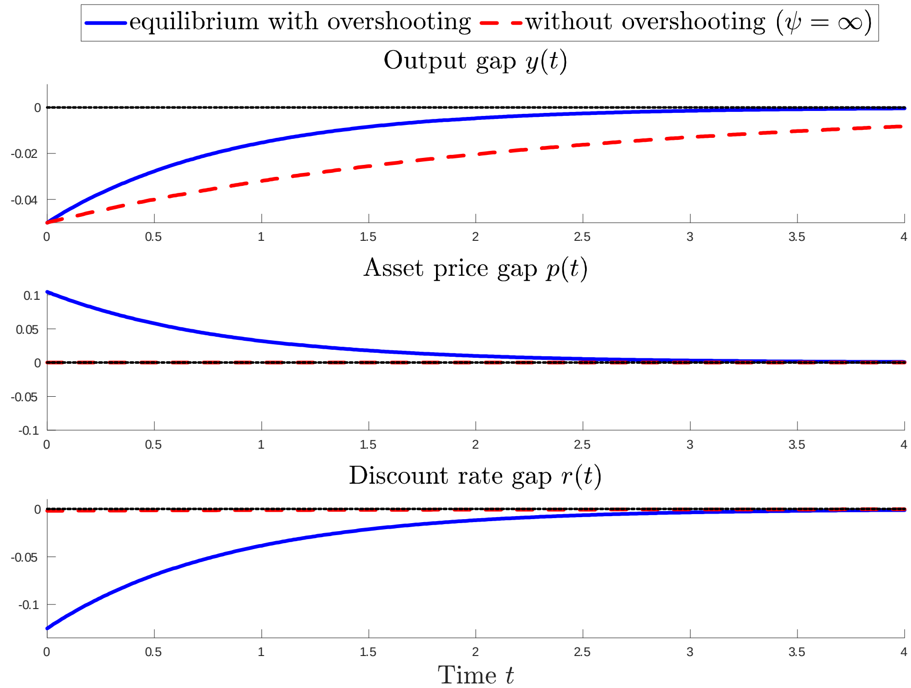

Figure 2: A simulation of the equilibrium with low ψ (solid lines) and high ψ (dashed lines). The dotted lines correspond to the potential levels for the corresponding variables.

central bank achieves this outcome by starting with a low discount rate, and then gradually increasing the discount rate and reducing the asset price gap. The figure also illustrates the case without overshooting ($\psi = \infty$), where the central bank sets the asset price equal to its potential. In this case, output gaps are closed more slowly and the economy operates below its potential for a longer time.

Why does the central bank overshoot asset prices? Intuitively, since only a fraction of stockholders adjust at any moment, the central bank sends a stronger "spend" signal to the adjusting stockholders to (partially) compensate for the depressed consumption of those that do not. This policy requires overshooting asset prices, which is costly, but it also accelerates the demand recovery and shrinks the negative output gaps. The following corollary reinforces this intuition by establishing the comparative statics of overshooting with respect to the adjustment hazard rate θ. We measure asset price overshooting with the *cumulative* sum of expected asset price gaps, $\int_0^\infty p(t)\,dt$—the area under the asset price panel of Figure 2.

Corollary 1. *The equilibrium features* greater *cumulative asset price overshooting per unit of negative output gap (greater* $\frac{\int_0^\infty p(t)dt}{-y(0)}$*), when a* smaller *fraction of stockholders adjust at any moment (smaller* θ*). In the limit with very frequent adjustment, the cumulative* *asset price overshooting is zero,* $\lim_{\theta\to\infty} \frac{\int_0^\infty p(t)dt}{-y(0)} = 0$.

When aggregate demand has more inertia, and therefore responds to asset prices more sluggishly, the central bank overshoots asset prices by more. This further highlights that the asset price overshooting illustrated in Figure 2 is driven by our key friction: aggregate demand inertia. In fact, in the limit of no demand inertia, there is effectively *no* asset price overshooting: demand recovers immediately and asset prices remain at their potential level throughout (except for a vanishingly small initial period).

2.3. Overshooting in richer environments

Our main overshooting result extends to richer environments.

2.3.1. Overshooting with growth

In our model, the asset price *level* declines over time (after an initial jump). This feature is not essential to the argument. The same result would still apply for the asset price *gap* in a variant with productivity growth. In this case, the potential asset price would also be increasing over time (at the growth rate), so overshooting would not necessarily imply a declining asset price level. It would only imply a *frontloading* of some of the future gains on the market portfolio.

2.3.2. Overshooting with sophisticated stockholders

For simplicity, we assume that adjusting stockholders exogenously follow the rule in (7). In Appendix C.1, we consider the case with fully sophisticated stockholders who choose their consumption optimally, *anticipating that they will readjust in the future with Poisson probability* θ. Proposition 5 in the appendix shows that, as long as stockholders' adjustment is not too sluggish ($\theta > \rho$), our main result still holds. Along the equilibrium path, the optimal consumption rule takes the form in (7) with endogenous coefficients:

$$\begin{aligned} c^{s,adj}(t) &= m(\gamma)\,p(t) + n(\gamma)\,y(t), \\ \text{where } m(\gamma) &= \frac{\theta-\rho}{\theta+\gamma} \in (0,1) \text{ and } n(\gamma) = \frac{\rho}{\theta+\gamma}. \end{aligned}$$

Absent commitment (which is the case we focus on), the planner takes these coefficients as given, and our earlier analysis still applies. Also recall that the convergence rate γ depends on the coefficients of the consumption rule, m, n. Hence, with fully rational stockholders, the equilibrium corresponds to a fixed point that we characterize in the appendix.

2.3.3. Overshooting with inflation

For simplicity, we assume fully sticky prices. In Appendix C.2, we extend our analysis to allow for partially flexible prices. In this case, inflation is endogenous and determined by a New-Keynesian Phillips curve (see (C.15)). Proposition 6 in the appendix shows that inflation reinforces our main result. Starting with a negative output gap, the central bank still overshoots real asset prices. Moreover, *when nominal prices are more flexible, the central bank overshoots real asset prices by more.* When nominal prices are flexible, negative output gaps create disinflationary pressures: they reduce inflation below its target. These negative inflation gaps are costly and create an additional reason for the central bank to fight negative output gaps. The central bank overshoots real asset prices by even more than in our baseline setting to close the output gaps more quickly.

2.3.4. Preemptive overshooting and overheating

In the main text, we focus on a recovery scenario in which the output gap is negative and the central bank's main concern is to close it as quickly as possible. In Appendix C.3, we extend our analysis to a situation in which the main concern is not the current output gap but the anticipation that the output gap will become negative in the near future. This situation may arise, for example, when the economy is experiencing a sharp but temporary decline in potential output, as in the Covid-19 recession. In the low-supply phase, potential output has experienced a deep contraction but is expected to recover according to a Poisson event—in which case the economy transitions to a high-supply phase (as in the main text). Proposition 7 in the appendix shows that the recession features *preemptive overshooting:* The central bank boosts asset prices *even if output is at its (depressed) potential level.*

By preemptively overshooting asset prices, the central bank temporarily *overheats* the economy: It induces *positive* output gaps until the potential output recovers. The central bank anticipates that the high-supply phase will start with a large negative output gap due to aggregate demand inertia. Therefore, the central bank acts preemptively to boost asset prices and aggregate demand during the low-supply phase, to ensure that aggregate demand is not too depressed during the early stages of transition to high supply. This boost temporarily induces positive asset price and output gaps, which are costly, but it also shrinks the expected negative output gaps after the potential output recovers. In Caballero and Simsek (2022), we investigate this trade-off further by focusing on the implications for overheating and inflation—rather than on overshooting and asset prices.

2.3.5. Overshooting with a time-varying risk premium

An important omission from our analysis is the lack of a risk premium. In practice, aggregate wealth is associated with a time-varying risk premium (see, e.g., Cochrane (2011)). We could incorporate a risk premium without changing our main conclusions. Suppose the discount rate on the market portfolio is given by $r(t) = r^f(t) + \xi(t)$, where $r^f(t)$ is the risk-free rate and $\xi(t)$ is the risk premium. As long as the elasticity of intertemporal substitution is equal to one, our analysis would still apply, but the central bank would target *the discount rate on the market portfolio*, $r(t)$, as opposed to the risk-free rate, $r^f(t)$. For instance, Eq. (14) would apply: the optimal policy would still overshoot asset prices. Eq. (16) would also apply and imply that the central bank should set the risk-free rate according to, $r^f(t) = -\xi(t) + \mathcal{R}y(t)$. If the risk premium is countercyclical (as suggested by empirical evidence), $\xi(t) = \xi_0 - \xi_1 y(t)$, then the risk-free rate becomes more procyclical than in our model $r^f(t) = -\xi_0 + (\xi_1 + \mathcal{R})\, y(t)$. *In a demand recession, the optimal policy cuts the risk-free rate aggressively to first "undo" the rise in the risk premium and then to overshoot asset prices as in our model.*

The presence of a risk premium not only leaves our main result unchanged but it also expands the policies the central bank can use to affect the discount rate on the market portfolio, $r(t)$. Even if conventional monetary policy is constrained due to, e.g., an effective lower bound on the interest rate, the central bank can reduce the risk premium, $\xi(t)$, via unconventional policies. In Caballero and Simsek (2021a), we formalize this argument in a model without transmission lags. In that model, large-scale asset purchases can reduce the risk premium by transferring risk to the government's balance sheet. These policies are especially powerful after a large surprise shock, such as Covid-19, that damages the risk-tolerant agents' balance sheets and increases the risk premium. We discuss the role of different types of monetary policies in the Covid-19 recession in Section 4.

3. Constrained overshooting and the news effect on asset prices

So far, we have assumed that the central bank can achieve any desired level of overshooting by appropriately adjusting the discount rate. In this section, we analyze the optimal policy when there is a limit on how much the policy can reduce the discount rate.[9] We find that, with a lower-bound constraint, overshooting is a concave and non-monotonic function of

[9]We focus on a lower-bound constraint on the discount rate but interpret it more broadly as a limit on both conventional monetary policy and LSAPs.

the output gap: the asset price boost is low for a deeply negative initial output gap, grows as the output gap improves over a range, and shrinks toward zero as the output gap improves further. This pattern also implies that good macroeconomic news is better news for asset prices when the output gap is lower (more negative) and the economy is farther from full recovery.

3.1. Overshooting with an interest rate constraint

Consider the baseline model in Section 2, with the only difference that the central bank sets the discount rate subject to a lower-bound constraint,

$$r(t) \geq \overline{r} \quad \text{for each } t. \tag{17}$$

The parameter, $\overline{r} < 0$, captures the severity of the constraint. Combining Eqs. (11) and (9), the central bank's recursive problem becomes

$$\begin{aligned} \rho V(y) &= \max_{p} -\frac{y^2}{2} - \psi \frac{p^2}{2} + V'(y)\dot{y}, \\ \dot{y} &= \theta(mp - (1-n)y) \\ r(t) &= \frac{\rho}{1+\rho}(y(t) - p(t)) + \frac{1}{1+\rho}\dot{p}(t) \geq \overline{r} \end{aligned} \tag{18}$$

As before, we assume the central bank sets the current policy *without* commitment. The planner takes future output and asset price gaps (as well as the price drift $\dot{p}(t)$) as given and sets the instantaneous asset price, $p(t)$, subject to the lower-bound constraint.[10]

Consider the solution corresponding to a negative initial output gap, $y(0) < 0$. Recall that when there is no lower-bound constraint the discount rate is increasing over time (see Figure 2). With a lower-bound constraint, there exists a cutoff output gap, $\overline{y} \leq 0$, such that the discount rate constraint binds for $y(t) < \overline{y}$ but not for $y(t) \in (\overline{y}, 0)$. When the constraint does not bind, the solution is exactly as in Section 2.2. In particular, the optimal asset price gap is given by (14) and the corresponding discount rate is given by (16). Setting $r(t) = \overline{r}$, we solve for the cutoff output and asset price gaps as follows,

$$\overline{y} = \frac{\overline{r}}{\frac{\rho}{1+\rho} + \frac{\theta m}{\psi v}\frac{\gamma+\rho}{1+\rho}} < 0 \quad \text{and} \quad \overline{p} = -\frac{\frac{\theta m}{\psi v}\overline{r}}{\frac{\rho}{1+\rho} + \frac{\theta m}{\psi v}\frac{\gamma+\rho}{1+\rho}} > 0. \tag{19}$$

[10] Unlike in Section 2, the no-comitment constraint binds in this section. The planner might want to promise low interest rates in the future so as to relax the current lower bound constraint. We abstract from these types of *forward guidance* policies as they are not our focus and their benefits are well understood (see, e.g., Eggertsson and Woodford (2003)).

When the discount rate constraint binds, the solution satisfies the following differential equation system over time in variables $y(t), p(t)$:

$$\begin{aligned} \dot{y} &= \theta\left(mp - (1-n)\,y\right) \\ \overline{r} &= \frac{\rho}{1+\rho}\left(y(t) - p(t)\right) + \frac{1}{1+\rho}\dot{p}(t). \end{aligned} \tag{20}$$

Starting with the point, $(\overline{y}, \overline{p})$, we can uniquely solve this system *backward* over time. The resulting path describes the optimal asset price gap p corresponding to each output gap $y \leq \overline{y}$. Over time, the gaps travel along the solution path until they reach the point $(\overline{y}, \overline{p})$. Subsequently, the gaps follow the unconstrained solution.

Our next result characterizes the solution for the constrained range. The proof (in the appendix) relies on the phase diagram corresponding to the differential equation system in (20). We assume the parameters satisfy a technical condition that ensures the system has two real eigenvalues and a unique steady-state.

Proposition 2. *Consider the model with a lower-bound on the discount rate* $\overline{r} < 0$ *for parameters that satisfy* $(\theta(1-n) - \rho)^2 > 4\theta\rho(m - (1-n)) \neq 0$. *Let* $\overline{y} < 0, \overline{p} > 0$ *denote the output and asset price gap cutoffs given by* (19).

When $y(0) \geq \overline{y}$*, the constraint does not bind and the solution is the same as in Proposition 1. The asset price gap is a function of the output gap,* $p = \mathbf{p}(y) = -\frac{\theta m}{\psi v}y$.

When $y(0) < \overline{y}$*, the constraint binds and the path,* $(y(t), p(t))$*, solves the system in* (20) *and reaches* $(\overline{y}, \overline{p})$ *in finite time. The output gap is increasing over time* ($\dot{y}(t) > 0$)*. There exists another cutoff* $\underline{y} < \overline{y}$ *such that the asset price gap is increasing over time* ($\dot{p}(t) > 0$) *iff* $y < \underline{y}$*. The asset price gap is a* strictly concave *function of the output gap,* $\mathbf{p}(y)$*, and it attains its maximum at the lower cutoff,* $\underline{y}$.

The asset price gap is a *concave and non-monotonic* function of the output gap, $\mathbf{p}(y)$: increasing below a cutoff level of the output gap ($\underline{y}$) and decreasing for higher levels of the output gap. Consequently, below a cutoff output gap ($\underline{y}$), both the asset price and the output gap increase over time. Above the cutoff, the asset price gap decreases over time whereas the output gap continues to increase.

Figure 3 illustrates the asset price function for a numerical example. Figure 4 illustrates the dynamics of equilibrium for the same example. In this example, the discount rate stays at the lower-bound until time $T \in [5, 6]$ at which point the gaps satisfy $y(T) = \overline{y}$ and $p(T) = \overline{p}$. After time T, the discount rate "lifts off" above the lower-bound. Before time T the output gap improves over time (although more slowly than in the unconstrained case), and the asset price gap follows a non-monotonic path.

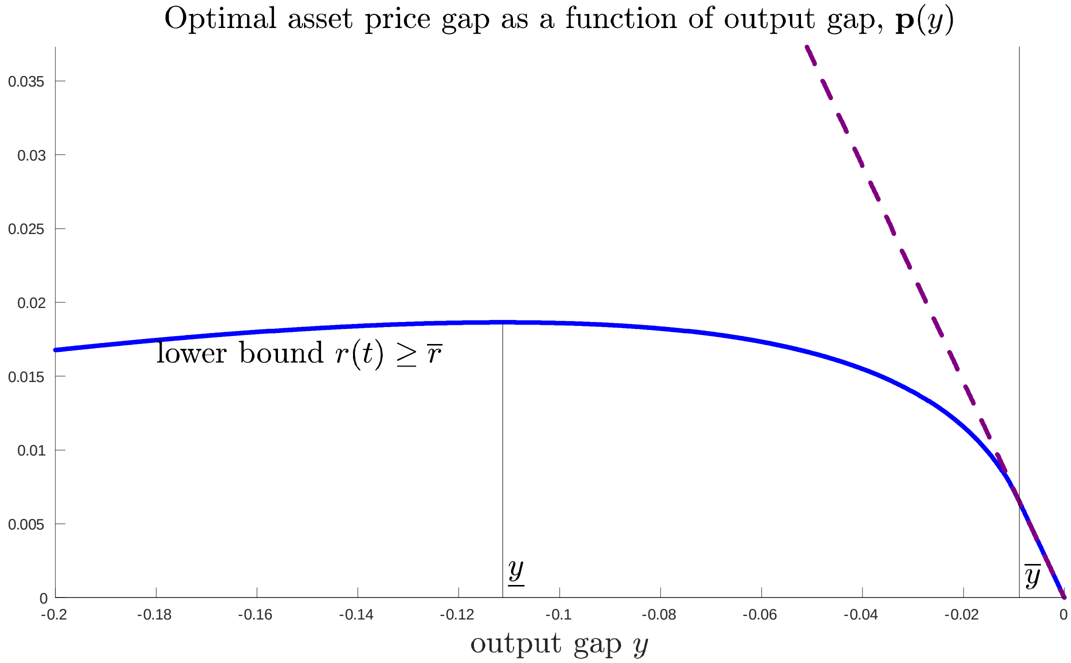

Figure 3: Equilibrium with an interest rate lower bound (the solid curve) and without a lower bound (the dashed line).

Why is the asset price gap a concave and non-monotonic function of the output gap? For intuition, note that Eq. (9) implies the present discounted value formula:

$$p(t) = \int_t^\infty e^{-\rho(s-t)} \left[\rho y(s) - (1+\rho) r(s)\right] ds.$$

Consider an initial output gap slightly below $\overline{y}$, which corresponds to an initial time t slightly before the lift-off time T. In this range, the central bank is nearly unconstrained and effectively "sets" asset prices by adjusting the discount rates beyond T. As the output gap improves, the central bank optimally "tapers" the overshooting, as in the baseline model. While a higher output gap increases expected cash flows, it increases expected discount rates by even more so it results in a smaller asset price gap.

Now consider a lower initial output gap, e.g., closer to $\underline{y}$, which corresponds to an initial time t farther from the lift-off time T. In this range, the central bank is more constrained: it would like a greater overshooting but cannot achieve it. A higher output gap still increases expected cash flows but it induces a smaller increase in expected discount rates than before. The cash flow and the discount rate effects roughly cancel each other, and the extent of overshooting is relatively insensitive to the output gap.

Finally consider a much lower initial output gap, e.g., to the left of $\underline{y}$, which corresponds

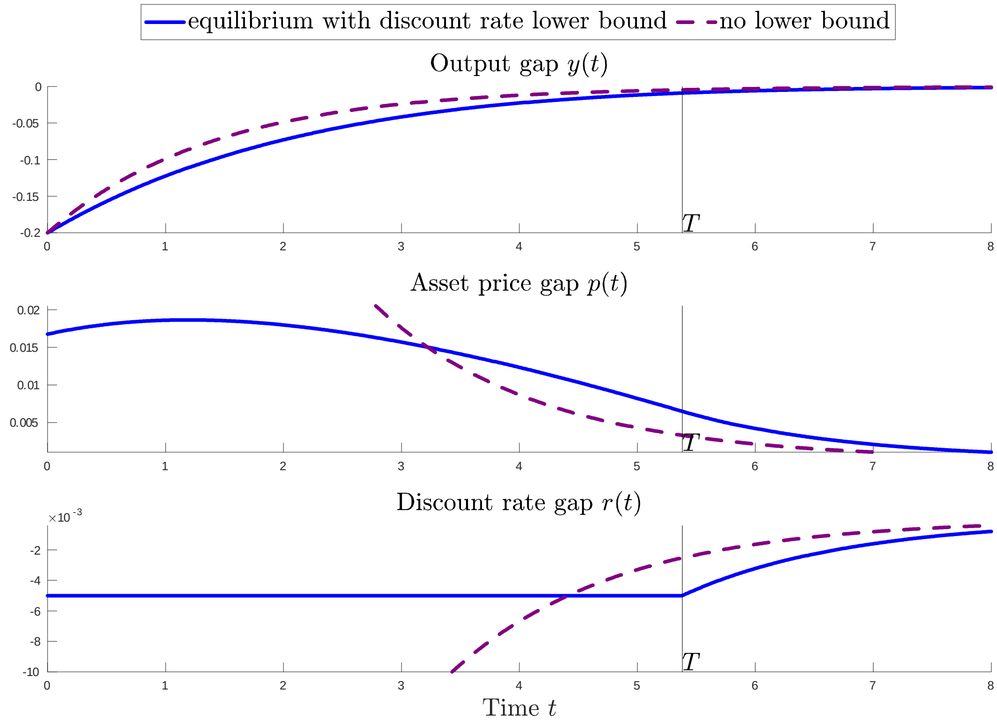

Figure 4: A simulation of the equilibrium over time with an interest rate lower bound (solid lines) and no lower bound (dotted lines).

to an initial time t very far from the lift-off time T. In this range, the planner is severely constrained. A higher output gap induces an even smaller increase in expected discount rates than before. The cash flow effects dominate the discount rate effects, and the extent of overshooting is increasing in the output gap.

3.2. Macroeconomic news and asset prices

We next show that in our model good macroeconomic news is better news for asset prices when the output gap is lower (more negative) and the economy is farther from full recovery. This pattern is consistent with existing empirical evidence. For instance, Law et al. (2019) show that stock prices react to macroeconomic news announcements more strongly when the output gap is sufficiently negative, and the relationship becomes weaker (and it can have a negative sign) when the output gap is closer to zero. In earlier work, Boyd et al. (2005) observe a similar pattern and attribute the cyclicality of the response to changes in the relative strength of the interest-rate and the cash-flow effects of news (see also McQueen and Roley (1993); Andersen et al. (2007)). Our model in this section provides an explanation for these findings.

Formally, let N denote a zero-mean random variable that captures macroeconomic news. For instance, N might correspond to the surprise component of a scheduled macroeconomic announcement such as nonfarm payrolls. Suppose initial and potential outputs are increasing functions of N:

$$Y(0) = \tilde{Y}(0) \exp(aN) \text{ and } Y^* = \tilde{Y}^* \exp(bN). \tag{21}$$

Here, $a, b > 0$ capture the impact of the news on output and potential output, respectively. A positive piece of news (e.g., nonfarm payrolls above expectations) implies stronger economic activity, which reflects both higher aggregate demand and aggregate supply. The news also affects the output gap, which is given by

$$y(0) = \tilde{y}(0) + (a - b) N. \tag{22}$$

Here, $\tilde{y}(0) = \log\left(\tilde{Y}(0)/\tilde{Y}^*\right)$ is the *expected* output gap. We assume $a - b > 0$ so that good news improves the output gap. This assumption is natural since good macroeconomic news typically increases bond yields and forward interest rates (see Andersen et al. (2007)). In our model, a higher output gap implies higher future interest rates (see Figure 4).

The news is realized at the beginning of the model. Once the news is realized, there is no further uncertainty and the setup is the same as in Section 3. Our next result shows that the asset price impact of news depends on the level of the output gap.

Proposition 3. *Consider the setup in Proposition 2 with a one-time macroeconomic news that affects both aggregate demand and aggregate supply according to* (21)*. Suppose $a > b$, so that good news improves the output gap. Then:*

(i) The impact of the news on asset prices is given by

$$\left.\frac{d \log P(0)}{dN}\right|_{N=0} = (a - b) \left.\frac{d\mathbf{p}(y)}{dy}\right|_{y=\tilde{y}(0)} + b, \tag{23}$$

where $\mathbf{p}(y)$ is the function characterized in Proposition 2.

(ii) Good news has a greater impact on asset prices when the expected output gap is lower (more negative):

$$\frac{d}{d\tilde{y}(0)} \left.\frac{d \log P(0)}{dN}\right|_{N=0} \leq 0. \tag{24}$$

The first part of the result characterizes the asset price impact of macroeconomic news. Good news affects the price of the market portfolio through two channels: by increasing potential output, captured by b, and by changing the output gap, captured by $(a-b)\left.\frac{d\mathbf{p}(y)}{dy}\right|_{y=\tilde{y}(0)}$. Since $\mathbf{p}(y)$ is a concave function, this second term becomes smaller as the output gap increases (see Figure 3). The second part of the result uses this observation to show that the asset price impact of good news becomes smaller as the output gap increases.

While good news always raises the price of the market portfolio through its impact on aggregate supply, it induces competing effects through its impact on aggregate demand and the output gap. A higher output gap raises the expected cash flows but it also raises the expected discount rates. Therefore, a higher output gap is good news for asset prices when the discount rate is mostly constrained (the economy is far from the discount rate lift-off) and the central bank does not overturn the price impact of higher cash flows. Conversely, a higher output gap is bad news for asset prices when the discount rate is mostly unconstrained (the economy is close to the lift-off) and the central bank *optimally* overturns the price impact of higher cash flows by accelerating interest-rate hikes. *Combining the supply and demand effects, good macroeconomic news is "better news" for the market when the output gap is lower (more negative).*

4. Real interest rates and overshooting during the Covid-19 recovery

In this section, we document the magnitude of the asset market overshooting generated by the decline in real interest rates during the Covid-19 episode. We find that it was large and happened much sooner than during the Great Recession. A back-of-the-envelope calculation suggests that this overshooting had a sizeable impact on output in the Covid-19 recovery. Our analysis also suggests that the Fed's LSAPs for safe assets played an important role in driving the asset price overshooting in this episode.

We use an asset price decomposition that applies in different variants of our model. As Section 3 illustrates, the magnitude and the dynamics of the optimal overshooting policy depends on the precise constraints faced by the central bank. In practice, these constraints are richer than in our stylized model (e.g., a rise in expected inflation can reduce the real interest rate and alleviate the lower-bound constraint). In addition, the central bank might deviate from the optimal overshooting policy for reasons outside our model. Therefore, we attempt to quantify the asset price overshooting by imposing a minimal theoretical structure.

4.1. A price decomposition based on the market-bond portfolio

To quantify the overshooting induced by the decline in risk-free rates, we decompose the market portfolio into a component driven by forward interest rates ("market-bond portfolio") and a component driven by other factors ("residual/other"), along the lines of Van Binsbergen (2020); Knox and Vissing-Jorgensen (2021). The market-bond portfolio provides a measure of the policy support (on asset prices) that operates through risk-free rates.

To state our decomposition result, we define a fixed-income portfolio that matches the *duration* of the market portfolio strip-by-strip. Formally, consider a portfolio of zero-coupon bonds with face values that match the steady-state payoffs of the dividend strips of the market portfolio (αY^*). We refer to this portfolio as the *market-bond portfolio* and denote its price with $P^{MB}(t)$. By no-arbitrage, this price satisfies

$$P^{MB}(t) = \int_0^\infty P^{MB}(t,\mu)\, d\mu, \quad \text{where } P^{MB}(t,\mu) \equiv \alpha Y^* e^{-\int_t^{t+\mu} R(s)ds}. \tag{25}$$

$P^{MB}(t,\mu)$ is the time-t price of the μ-maturity strip of the market-bond portfolio. We also let $p^{MB}(t) \equiv \log \frac{P^{MB}(t)}{P^{MB,*}}$ denote the log-linearized price of the market-bond portfolio; $y(t,\mu) = \frac{\int_t^{t+\mu} R(s)ds}{\mu}$ denote the continuously compounded zero-coupon yield with maturity μ; and $f(t,\mu) = R(t+\mu)$ denote the μ-period-ahead instantaneous forward rate.

Proposition 4. *Let $[y(t), p(t), r(t)]_{t=0}^\infty$ denote a feasible path that satisfies Eqs. $(8-9)$ along with the no-bubble condition, $\lim_{t\to\infty} e^{-\rho t} p(t) = 0$. The following identities hold up to a log-linear approximation:*

(i) ***Decomposition:*** *The log-price of the market portfolio satisfies*

$$\begin{aligned} p(t) &= p^{MB}(t) + p^O(t), \\ \textit{where } p^{MB}(t) &= -\int_t^\infty e^{-\rho(s-t)}(1+\rho)\, r(s)\, ds \\ \textit{and } p^O(t) &= \int_t^\infty e^{-\rho(s-t)} \rho y(s)\, ds. \end{aligned} \tag{26}$$

(ii) ***Yield-based measurement:*** *The log-price change of the market-bond portfolio satisfies*

$$\dot{p}^{MB}(t) = -\int_0^\infty w_\mu \mu \frac{\partial y(t,\mu)}{\partial t} d\mu, \quad \textit{where } w_\mu = \frac{P^{MB*}(\mu)}{P^{MB*}} = e^{-\rho\mu}\rho. \tag{27}$$

(iii) ***Forward-rate-based measurement:*** *The log-price change of the market-bond*

portfolio also satisfies

$$\dot{p}^{MB}(t) = -\int_0^\infty W_\mu \frac{\partial f(t,\mu)}{\partial t} d\mu, \qquad \textit{where } W_\mu = \int_\mu^\infty w_{\tilde{\mu}} d\tilde{\mu} = e^{-\rho\mu}. \tag{28}$$

The first part of the proposition decomposes the price of the market portfolio into the market-bond portfolio and a residual. The market-bond portfolio isolates asset price changes driven by the risk-free rates. In our model, monetary policy affects asset prices *primarily* through $p^{MB}(t)$: by changing the forward rates, the central bank has a *direct* effect on the valuation of cash flows. This change in asset prices also affects expected cash flows, creating indirect knock-on effects captured by $p^O(t)$. In general, monetary policy can affect asset prices through other channels, e.g., by changing the risk premium. We view the market-bond portfolio as capturing the asset price impact of monetary policy via risk-free rates, and the residual term as capturing other channels of monetary policy as well as other drivers of asset prices such as a time-varying risk premium.

The price of the market-bond portfolio can be measured from data on treasury yields or forwards. The last two parts of the proposition facilitate this measurement. Eq. (27) shows that the price change of the portfolio depends inversely on the yield changes of the individual strips multiplied by the *weighted* duration, $w_\mu \mu$. The weights are proportional to the (steady-state) value of the corresponding strip, $w_\mu = \frac{P^{MB*}(\mu)}{P^{MB*}}$. Eq. (28) expresses the price change in terms of forward rates. The price depends inversely on a cumulative-weighted-average of forward rates at all horizons μ. The cumulative weights capture the weights of bond strips with maturity beyond μ, that is, $W_\mu = \int_\mu^\infty w_{\tilde{\mu}} d\tilde{\mu}$. Intuitively, each forward rate affects the valuation of strips with maturities that exceed its horizon.

Since we do not observe yields or forward rates for distant horizons, we fix some $\overline{\mu}$ and bunch the values of all bond strips with maturities beyond $\overline{\mu}$ at the strip with maturity $\overline{\mu}$. This bunching procedure yields the following approximation to Eqs. (27) and (28):

$$\dot{p}^{MB}(t) \simeq -\int_0^{\overline{\mu}} w_\mu \mu \frac{\partial y(t,\mu)}{\partial t} d\mu - W_{\overline{\mu}} \overline{\mu} \frac{\partial y(t,\mu)}{\partial t} \tag{29}$$

$$\simeq -\int_0^{\overline{\mu}} W_\mu \frac{\partial f(t,\mu)}{\partial t} d\mu, \tag{30}$$

where recall that $w_\mu = e^{-\rho\mu}\rho$ and $W_\mu = e^{-\rho\mu}$. Our bunching procedure and the resulting approximation are similar to the bond portfolio return analyzed by Van Binsbergen (2020).

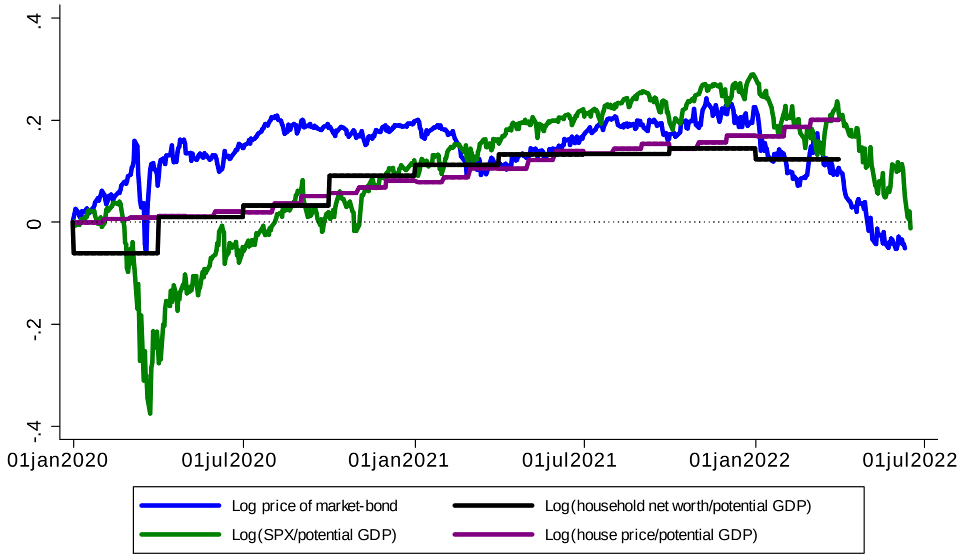

Figure 5: The evolution of the log price of the market-bond portfolio, $p^{MB}(t)$, along with the log S&P500 index, log house price index, and log household net worth normalized by potential output according to the CBO. All series are normalized to zero on December 31, 2019.

4.2. Overshooting in the Covid-19 recovery via risk-free rates

We next use Eq. $(29-30)$ to measure the policy support in the Covid-19 recovery through risk-free rates. We adopt a yearly calibration for the bond maturity (μ) and let $\rho = 0.03 = \frac{\alpha Y^*}{P^*}$ to roughly match the annual dividend-price ratio (and the inverse duration) of the stock market index. This choice generates bond-strip weights that are consistent with the available data from dividend futures (see Van Binsbergen (2020)). We focus on real (inflation-adjusted) prices and obtain daily one-year-ahead TIPS forward rates up to a 30-year horizon ($\overline{\mu} = 30$) from the term structure data provided by the Federal Reserve, based on the approach by Gürkaynak et al. (2007). We use the forward-rate-based measure in Eq. (30) (see Appendix D for details).

Magnitude of policy-induced overshooting. The blue line in the top panel of Figure 5 illustrates the evolution of $p^{MB}(t)$ from the end of 2019 until June 2022. The market-bond portfolio increased early in the Covid-19 recession and remained high through most of the recovery—the average value of $p^{MB}(t)$ between July 1, 2020 and the end of 2021 is about 19 log points. However, the market-bond portfolio declined substantially once the output recovered close to its pre-Covid level and the economy showed clear signs of overheating. By May 2022, $p^{MB}(t)$ was below its level at the end of 2019.

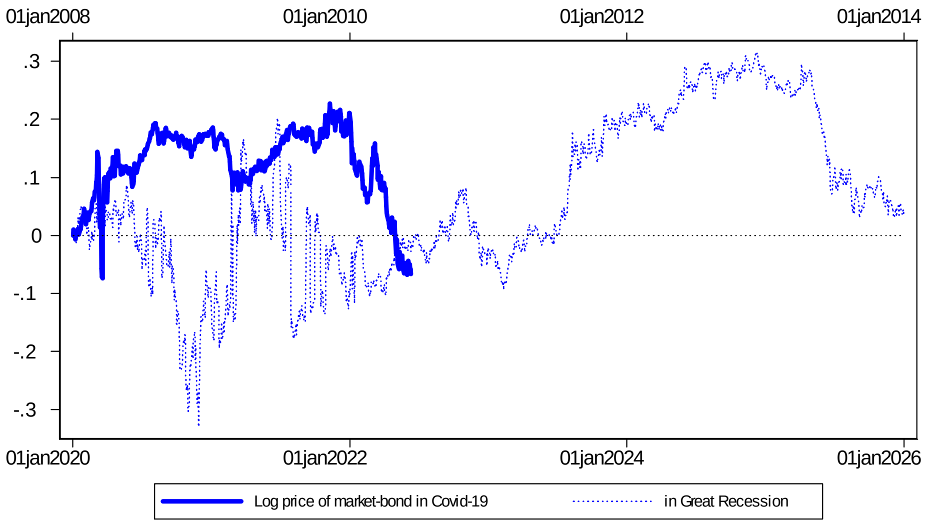

Figure 6: The evolution of the log price of the market-bond portfolio, $p^{MB}(t)$, in the recovery from the Covid-19 recession (solid line) and from the Great Recession (dotted line).

The figure also plots the S&P500 index and the house price index, as well as household net worth (from the Federal Reserve), which aggregates various sources of wealth. We view these assets as proxies for $p(t)$ in our model and normalize them by potential GDP (from the CBO) to adjust for inflation and growth. The figure illustrates that the stock and house prices boomed in the Covid-19 recovery along with the market-bond portfolio. Accordingly, household net worth increased by an unprecedented amount: from about \$116.7 trillion in 2019-Q4 to about \$149.8 trillion in 2021-Q4 (a 14 log points increase after normalizing by potential GDP). The figure also suggests that the high level of the stock and house prices throughout 2021 can be mostly attributed to the market-bond portfolio. Likewise, the decline in the stock prices in 2022 can be attributed to the decline in the market-bond portfolio. These observations suggest that, as in our model, monetary policy has been a key driver of the aggregate asset prices in the recovery from the Covid-19 recession.[11]

Comparison with the Great Recession. Figure 6 compares the evolution of $p^{MB}(t)$ in the last two recessions. While monetary policy also supported asset prices through risk-

[11] In the stock market, the residual component dragged prices down earlier in the recession, arguably due to a spike in the risk premium, but this residual effect disappeard (and might have flipped sign) by early 2021. In fact, Knox and Vissing-Jorgensen (2021) provide a more detailed decomposition of the stock market returns and argue that the risk premium increased substantially earlier in the recession but had declined to close to its pre-shock level by the end of 2020.

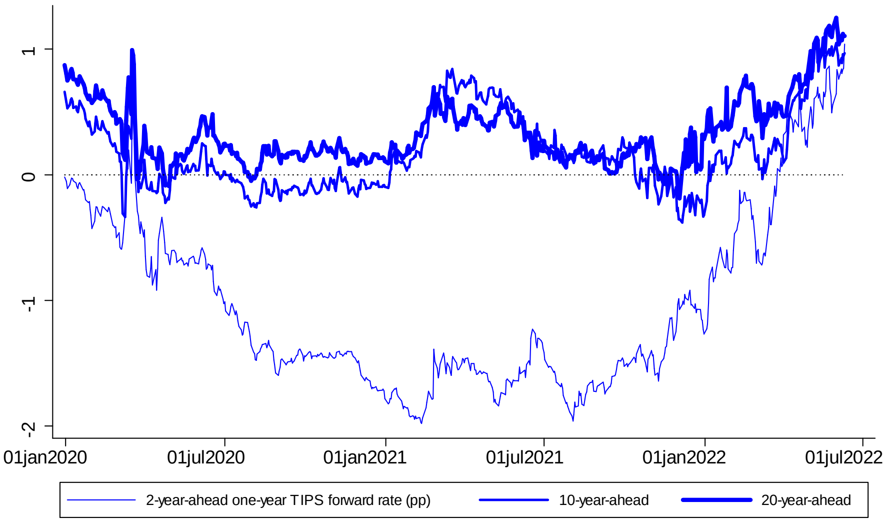

Figure 7: The drivers of the market-bond portfolio in the Covid-19 episode: One-year TIPS forward rates at select horizons.

free rates in the Great Recession, the response was much more gradual: it took about five years for $p^{MB}(t)$ to reach its peak. This is in sharp contrast with the Covid-19 recession, where the policy was swift and $p^{MB}(t)$ increased immediately and remained close to its peak throughout the recovery.

Decomposition. Figure 7 plots select TIPS forward rates to illustrate the drivers of $p^{MB}(t)$ in the Covid-19 episode. Early in the recession, the shorter-term forward rates were compressed due to the lower bound on the nominal rates and expected disinflation. Nonetheless, $p^{MB}(t)$ increased because the longer-term forward rates also declined (except for March 2020). During the recovery, the policy support for $p^{MB}(t)$ gradually shifted from longer-term to shorter-term rates, which declined substantially due to an increase in expected inflation. In early 2022, when the economy showed clear signs of overheating, $p^{MB}(t)$ declined because the short-term as well as the long-term rates recovered toward their pre-Covid levels.

The role of LSAPs. In our model, the optimal policy induces and then tapers overshooting by adjusting the short-term discount rates by a large amount and then quickly undoing this aggressive cut. This aspect of our model does not fully match the data: Figure 5 suggests that in the Covid-19 recovery monetary policy partly operated through distant-horizon forward rates. These very long-term rate changes were in all likelihood driven by the large LSAP programs for safe assets the Fed implemented in this episode.[12] The Fed purchased trillions of dollars of treasuries and agency mortgage-backed securities between March and June 2020; and about \$120 billion a month from mid-2020 until the end of 2021. The Fed started tapering its asset purchases in November 2021 and stopped expanding its balance sheet in March 2022.[13]

Our model is stylized and does not have the appropriate frictions that make LSAPs operational, such as risk absorption by the government (e.g., Caballero and Simsek (2021a)) or segmented markets (e.g., Vayanos and Vila (2021); Ray (2019); Sims et al. (forthcoming)). Nonetheless, from the perspective of our model, we view LSAPs as a close substitute for conventional monetary policy, *conditional on them inducing the same impact on aggregate wealth,* $p(t)$. In particular, the price of the market-bond portfolio also captures the wealth effect driven by long-term *safe* asset purchases typical of quantitative easing policies. These purchases can substitute for short-term rate cuts by reducing the long-term rates, e.g., by absorbing the duration risk and reducing the term premium.[14]

4.3. Effect of overshooting on the recovery

We end this section with a back-of-the-envelope calculation to assess the likely impact of the observed asset price overshooting on output's recovery. Consider the discretized version of Eq. (8) that describes output dynamics,

$$y(t+\Delta t) \simeq y(t) + \theta \left[mp(t) - (1-n)\, y(t) \right] \Delta t.$$

Setting $\Delta t = 1$ (interpreted as one year) and substituting $x(t) \simeq \frac{X(t)-X^*}{X^*}$, we write this as

$$\frac{Y(t+1)}{Y^*} \simeq \frac{Y(t)}{Y^*} + \theta m \frac{P(t)-P^*}{P^*} - \theta (1-n) \frac{Y(t)-Y^*}{Y^*}.$$

The equation describes the output in year $t+1$ in terms of the output in year t and the asset prices in year t. Note also that, if there were no asset price overshooting in year t, then output would be given by a similar expression with $P(t) = P^*$. Thus, the impact of

[12] See Hanson and Stein (2015) for the puzzling finding that conventional monetary policy shocks also seem to affect *real long-term* interest rates. See also Bianchi et al. (forthcoming) for an explanation of these long lasting effects of monetary policy over real rates and asset prices based on a regime-switching model with sticky inflation expectations. Note, however, that the space for conventional monetary policy during the Covid-19 recovery was very limited, which suggests that LSAPs also played a central role in driving $p^{MB}(t)$ in this episode.

[13] For the Fed's response, see `https://www.brookings.edu/research/fed-response-to-covid19/`

[14] There is an extensive empirical literature documenting that the LSAPs in recent years have been good substitutes for conventional monetary policy (see, e.g., d'Amico et al. (2012); Swanson and Williams (2014); Swanson (2018); Sims and Wu (2020, 2021)).

asset price overshooting (relative to a no-overshooting benchmark) is:

$$\frac{\Delta Y(t+1)}{Y^*} = \theta m \frac{P(t) - P^*}{P^*} = \frac{1}{\alpha}\theta M \frac{P(t) - P^*}{Y^*} \text{ where } M = m\rho. \tag{31}$$

The second equality substitutes $\frac{Y^*}{P^*} = \frac{\rho}{\alpha}$ from (5). The parameter $M = m\rho = m\frac{C^{s*}}{P^*}$ is the MPC out of stock wealth for the stockholders (see Eq. (7)). The impact of asset price overshooting depends on the MPC, M; the fraction of the stockholders that adjust their spending, θ; and the Keynesian multiplier, $\frac{1}{\alpha}$ (see (3)).

For a back-of-the-envelope calculation, we set M to target the (yearly) MPC out of wealth based on recent empirical estimates. Chodorow-Reich et al. (2021) estimate an MPC out of *stock* wealth equal to 3 cents, and Mian et al. (2013) estimate an MPC out of *housing* wealth equal to 5-7 cents. We set $M = 0.04$. Chodorow-Reich et al. (2021) also find that the response of spending to stock wealth is sluggish and stabilizes in about two years after the wealth shock. Based on this result, we set $\theta = 0.5$: half of the stockholders adjust their spending in a year. Finally, we set the Keynesian multiplier to a relatively conservative level, $\frac{1}{\alpha} = 1.5$.[15] Substituting these expressions, we obtain

$$\frac{\Delta Y(t+1)}{Y^*(t)} = \underbrace{1.5 \times 0.5 \times 0.04}_{0.03} \frac{P(t) - P^*(t)}{Y^*(t)}. \tag{32}$$

Each dollar of overshooting in a year, increases aggregate spending in the next year by about 3 cents. We index potential output and potential asset price with time, $Y^*(t), P^*(t)$, since these terms grow over time due to inflation and technological progress.

Consider the year $t = 2021$ in which asset price overshooting was substantial and mostly driven by the market bond portfolio (see Figure 5). Let $P(2021)$ denote the average household net worth in 2021. Let $P^*(2021) = P(2019Q4)\frac{Y^*(2021)}{Y^*(2019Q4)}$ denote the projected potential household net worth in 2021, based on the pre-Covid household net worth and the growth of potential output (we take the pre-Covid household net worth equal to potential). Substituting the data counterparts, we calculate:

$$\frac{P(2021) - P^*(2021)}{Y^*(2021)} = \frac{\$143.4T - \$116.7T \times \frac{\$23.4T}{\$21.7T}}{\$23.4T} \simeq 0.75.$$

[15] The analysis in Chodorow-Reich (2019) suggests that the aggregate zero lower bound multiplier is at least 1.7 (it could be considerably greater than this level since the empirical estimates often identify a cross-sectional multiplier, and the aggregate zero lower bound multiplier exceeds the cross-sectional multiplier in standard models). Note also that our calibration of the multiplier implies a share of capital that is larger than the empirical estimates, $\alpha = 0.66$. This discrepancy is due to the stark assumptions of our model (e.g., stockholders earn no labor income).

In 2021, the household net worth exceeded its pre-Covid level by about \$26.7 trillion. After adjusting for the projected increase due to inflation and technological progress, this amounts to an asset price overshooting of about 75% of potential output. Combining this with (32), we find that the asset price overshooting in 2021 increased output in 2022 by about 2.3% (0.03×0.75).

5. Final Remarks

Summary. We proposed a model to illustrate that when aggregate demand is below its potential and responds to asset prices with a lag, *optimal* monetary policy naturally generates large temporary gaps between the performance of financial markets and the real economy. The central bank boosts asset prices to close the output gap as fast as possible. We also showed that, when the central bank faces a lower-bound on the discount rate it can set, the overshooting becomes a concave and non-monotonic function of the output gap. Due to competing cash flow and interest rate effects, the asset price boost is low for a deeply negative initial output gap, grows as the output gap improves over a range, and shrinks toward zero as the output gap improves further. This result also implies that good macroeconomic news is better news for asset prices when the output gap is lower (more negative), which is consistent with the empirical literature on the news effect on asset prices (see, e.g., Law et al. (2019)).

While we do not explicitly model fiscal policy, our analysis of the price impact of news suggests that fiscal policy is likely to complement monetary policy when the output gap is significantly negative, and substitute for it when the output gap is closer to zero. When the output is significantly below its potential, fiscal policy increases asset prices—an outcome that the central bank desires but cannot achieve due to the discount rate constraint. When the output is close to its potential, fiscal policy induces the central bank to accelerate the interest-rate hikes sufficiently to *decrease* asset prices.

We estimate a large policy-induced overshooting in the Covid-19 recovery driven by risk-free rates. To facilitate this exercise, we decomposed the aggregate asset price in our setting into a "market-bond portfolio"—driven by expected interest rate changes, and a "residual"—driven by expected cash flows and other factors. The market-bond portfolio increased substantially in the Covid-19 recovery—a rise sufficient to explain the high levels of stock and house prices in 2021. A back-of-the-envelope calculation suggests that this asset price overshooting in 2021 increased output in 2022 by about 2.3%.

Observations. While we demonstrated that the broad features of asset markets during the Covid-19 episode are consistent with a well managed monetary policy framework, we do not wish to imply that there were no anomalies or pockets of irrational exuberance in some markets. Having said this, the logic of the model suggests that experiencing an episode of irrational exuberance during the recovery from a deep recession has a positive dimension, since it reduces the burden on the central bank to engineer an overshooting.

Finally, we note that adding heterogeneity in productivity (a central feature of the Covid-19 episode, not present in our model) does not change our main results, but it introduces a large dispersion in asset prices across firms. In particular, firms whose relative productivity is positively affected by the recession shock see their shares' value rise by even more since they benefit from the central bank's attempt to boost asset prices without suffering from a decline in productivity. In the Covid-19 episode, this provides a rationale for the extraordinary performance of indices such as the Nasdaq 100, whose main components consist of "Covid-sheltered" firms.

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Online Appendices: Not for Publication

The appendices present the analytical derivations and proofs omitted from the main text. Appendix A provides the microfoundations for the nominal rigidities. Appendix B contains the proofs of the results presented in the main text. Appendix C contains extensions of the baseline model. Appendix D contains the data details and sources.

A. Appendix: Microfoundations for the supply side

This appendix provides the microfoundations for the supply side with nominal rigidities that we describe in Section 2 and Appendix C.2.

There are two types of agents denoted by superscript $i = s$ ("stockholders") and $i = h$ ("hand-to-mouth"). There is a single factor, labor.

Hand-to-mouth households supply labor according to relatively standard intra-period preferences. They do not hold financial assets and spend all of their income. We write the hand-to-mouth agents' problem as,

$$\begin{aligned} &\max_{L(t)} \log C^h(t) - \chi \frac{L(t)^{1+\varphi}}{1+\varphi} \\ &Q(t) C^h(t) = W(t) L(t) + T(t). \end{aligned} \tag{A.1}$$

Here, φ denotes the Frisch elasticity of labor supply, $Q(t)$ denotes the nominal price for the final good, $W(t)$ denotes the nominal wage, and $T(t)$ denotes lump-sum transfers to labor (described subsequently). Using the optimality condition for problem $(A.1)$, we obtain a standard labor supply curve,

$$\frac{W(t)}{Q(t)} = \chi L(t)^{\varphi} C^h(t). \tag{A.2}$$

Stockholders own (and trade) the market portfolio and they supply no labor. We analyze these agents' consumption-savings problem in the main text as well as in Appendix C.1. They receive the profits from the production firms that we will describe subsequently.

Production is otherwise similar to the standard New Keynesian model. There is a continuum of monopolistically competitive firms, denoted by $\nu \in [0,1]$. These firms produce differentiated intermediate goods, $Y(t,\nu)$, subject to the Cobb-Douglas technology,

$$Y(t,\nu) = AL(t,\nu)^{1-\alpha}. \tag{A.3}$$

Here, $1-\alpha$ denotes the share of labor in production and A is a productivity shifter.

A competitive final goods producer combines the intermediate goods according to the CES technology,

$$Y(t) = \left(\int_0^1 Y(t,\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}, \tag{A.4}$$

for some $\varepsilon > 1$. This implies the price of the final consumption good is determined by the ideal price index,

$$Q(t) = \left(\int_0^1 Q(t,\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}, \tag{A.5}$$

and the demand for intermediate good firms satisfies,

$$Y(t,\nu) \leq \left(\frac{Q(t,\nu)}{Q(t)}\right)^{-\varepsilon} Y(t). \tag{A.6}$$

Here, $Q(t,\nu)$ denotes the nominal price set by the intermediate good firm ν.

Labor market clearing condition is

$$\int_0^1 L(t,\nu)\, d\nu = L(t). \tag{A.7}$$

Goods market clearing condition is

$$Y(t) = C^s(t) + C^h(t). \tag{A.8}$$

Finally, to simplify the distribution of output across factors, we assume the government taxes part of the profits lump-sum and redistributes to workers to ensure they receive their production share of output. Specifically, each intermediate firm pays lump-sum taxes determined as follows:

$$T(t) = (1-\alpha) Q(t) Y(t) - W(t) L(t). \tag{A.9}$$

This ensures that in equilibrium hand-to-mouth households receive and spend their production share of output, $(1-\alpha) Q(t) Y(t)$, and consume [see $(A.1)$]

$$C^h(t) = (1-\alpha) Y(t). \tag{A.10}$$

Stockholders receive the total profits from the intermediate good firms, which amounts to the residual share of output, $\Pi(t) \equiv \int_0^1 \Pi(t,\nu)\, d\nu = \alpha Q(t) Y(t)$.

Flexible-price benchmark and the potential output. To characterize the equilibrium, it is useful to start with a benchmark setting without nominal rigidities. In this benchmark, an intermediate good firm ν solves the following problem,

$$\begin{aligned} \Pi &= \max_{Q,L} QY - W(t) L - T(t) \\ & \quad \text{where } Y = AL^{1-\alpha} = \left(\frac{Q}{Q(t)}\right)^{-\varepsilon} Y(t) \end{aligned} \tag{A.11}$$

The firm takes as given the aggregate price, wage, and output, $Q(t), W(t), Y(t)$, and chooses its price, labor input, and output Q, L, Y.

The optimal price is given by

$$Q = \frac{\varepsilon}{\varepsilon - 1} W(t) \frac{1}{(1-\alpha) AL^{-\alpha}}. \tag{A.12}$$

Intuitively, the firm sets an optimal markup over marginal costs of output, where the marginal cost depends on the wage and (inversely) on the marginal product of labor.

In equilibrium, all firms choose the same prices and allocations, $Q(t) = Q$ and $L(t) = L$. Substituting this into $(A.12)$, and using $Y = AL^{1-\alpha}$, we obtain a labor demand equation,

$$\frac{W(t)}{Q(t)} = \frac{\varepsilon - 1}{\varepsilon} (1-\alpha) AL^{-\alpha}. \tag{A.13}$$

Combining this with the labor supply equation $(A.2)$, and substituting the hand-to-mouth consumption $(A.10)$, we obtain the equilibrium labor as the solution to,

$$\chi (L^*)^{\varphi} (1-\alpha) Y^* = \frac{\varepsilon - 1}{\varepsilon} (1-\alpha) A (L^*)^{-\alpha}.$$

In equilibrium, output is given by $Y^* = A(L^*)^{1-\alpha}$. Therefore, the equilibrium condition simplifies to,

$$\chi (L^*)^{1+\varphi} = \frac{\varepsilon - 1}{\varepsilon}.$$

We refer to L^* as the potential labor supply and $Y^* = A(L^*)^{1-\alpha}$ as the potential output.

Fully sticky prices. We next describe the equilibrium with nominal rigidities for the baseline case with full price stickiness. In particular, intermediate good firms have a preset nominal price that remains fixed over time, $Q(t,\nu) = Q^*$. This implies the nominal price for the final good is also fixed and given by $Q(t) = Q^*$ [see $(A.5)$]. Then, each intermediate good firm ν at time t solves the following version of problem $(A.11)$,

$$\Pi = \max_L QY - W(t) L - T(t) \quad \text{where } Y = AL^{1-\alpha} \leq Y(t). \tag{A.14}$$

Here, we dropped the index, (t,ν), from firm-specific variables to simplify the notation. By symmetry, each firm chooses the same allocation. For small aggregate demand shocks (which we assume) each firm optimally chooses to meet the demand for its goods, $Y = AL^{1-\alpha} = Y(t)$. Therefore, each firm's output is determined by aggregate demand, which is equal to spending by stockholders and hand-to-mouth households [see $(A.8)$],

$$Y(t) = C^s(t) + C^h(t).$$

This establishes Eq. (1) in the main text.

Finally, recall that hand-to-mouth agents' spending is given by $C^h(t) = (1-\alpha) Y(t)$ [see Eq. $(A.10)$]. Combining this with $Y(t) = C^s(t) + C^h(t)$, the aggregate demand for goods is determined by the stockholders' spending,

$$Y(t) = \frac{C^s(t)}{\alpha}.$$

This establishes Eq. (3) in the main text.

Partially flexible prices and the New-Keynesian Phillips curve. We next consider the case with partially flexible prices that we describe in Section 2.3.3 and analyze in Appendix C.2. Specifically, at each instant, a random fraction of intermediate good firms adjusts their price, with constant hazard ζ. Their prices remain unchanged until the next time they have a chance to adjust. We characterize the optimal price for an adjusting firm. We then characterize the dynamics of inflation and derive the New-Keynesian Phillips curve.

Consider the firms that adjust their price in period t. These firms' optimal price,

$Q^{adj}(t)$, solves

$$\max_{Q^{adj}(t)} \int_{T=t}^{\infty} e^{-\zeta(T-t)} e^{-\int_t^T R(s)ds} \left(Q^{adj}(t) Y(T|t) - W(T) L(T|t) \right) dT \qquad \text{(A.15)}$$

$$\text{where } Y(T|t) = AL(T|t)^{1-\alpha} = \left(\frac{Q^{adj}(t)}{Q(T)} \right)^{-\varepsilon} Y(T).$$

The integral captures the sum of expected profits over histories at which the firm's price remains unchanged. At time T, the price set at time t remains unchanged with probability $e^{-\zeta(T-t)}$. Since there is no aggregate risk, the firm discounts the profits at time T according to the discount rate between times t and T. The terms, $L(T|t), Y(T|t)$, denote the input and the output of the firm at time T (for a firm that reset its price at time t). We dropped the lump-sum taxes from the expression since they do not affect the firm's optimal pricing decision.

The optimality condition is given by,

$$\int_{T=t}^{\infty} e^{-\zeta(T-t)} e^{-\int_t^T R(s)ds} Q(T)^{\varepsilon} Y(T) \left(\begin{array}{c} Q^{adj}(t) \\ -\frac{\varepsilon}{\varepsilon-1} \frac{W(T)}{(1-\alpha)AL(T|t)^{-\alpha}} \end{array} \right) dT = 0 \qquad \text{(A.16)}$$

$$L(T|t) = \left(\frac{Q^{adj}(t)}{Q(T)} \right)^{\frac{-\varepsilon}{1-\alpha}} \left(\frac{Y(T)}{A} \right)^{\frac{1}{1-\alpha}}.$$

Here, we have substituted $\frac{Y(T|t)}{Q^{adj}(t)} = Q^{adj}(t)^{-(1+\varepsilon)} Q(T)^{\varepsilon} Y(T)$ and dropped $Q^{adj}(t)^{-(1+\varepsilon)}$ since it is common across all of the terms inside the integral.

We next combine Eq. $(A.16)$ with the remaining equilibrium conditions to derive the New-Keynesian Phillips curve. Specifically, we log-linearize the equilibrium around the allocation that features potential real outcomes and zero nominal inflation, that is, $L_t = L^*, Y_t = Y^*$ and $Q_t = Q^*$ for each t. Recall that we use the notation $x(t) = \log \frac{X(t)}{X^*}$ to denote the log-deviation of the corresponding variable $X(t)$ from its potential.

We first log-linearize the labor supply Eq. $(A.2)$ (after substituting $(A.10)$) to obtain

$$w(t) - q(t) = \varphi l(t) + y(t). \qquad \text{(A.17)}$$

Likewise, we log-linearize Eqs. $(A.4 - A.3)$ and $(A.7)$, to obtain

$$y(t) = (1-\alpha) l(t). \qquad \text{(A.18)}$$

Finally, we log-linearize the optimality condition $(A.16)$ to obtain,

$$\int_{T=t}^{\infty} e^{-(\rho+\zeta)(T-t)} \left[\begin{array}{c} (\varepsilon q(T) + y(T)) (Q^*)^{\varepsilon} Y^* \left(Q^* - \frac{\varepsilon}{\varepsilon-1} \frac{W^*}{(1-\alpha)A(L^*)^{-\alpha}} \right) \\ + (Q^*)^{\varepsilon} Y^* \left(Q^* q^{adj}(t) - \frac{\varepsilon}{\varepsilon-1} \frac{W^*}{(1-\alpha)A(L^*)^{-\alpha}} (w(t) + \alpha l(T|t)) \right) \end{array} \right] dT$$

After substituting $Q^* = \frac{\varepsilon}{\varepsilon-1} \frac{W^*}{(1-\alpha)A(L^*)^{-\alpha}}$ [see Eq. $(A.12)$] and calculating $l(T|t)$ [see $(A.16)$] we obtain

$$\int_{T=t}^{\infty} e^{-(\rho+\zeta)(T-t)} \left(q^{adj}(t) - (w(T) + \alpha l(T|t)) \right) dT = 0 \tag{A.19}$$
$$\text{where } l(T|t) = \frac{-\varepsilon}{1-\alpha} \left(q^{adj}(t) - q(T) \right) + l(T).$$

Here, the second line uses $\frac{1}{1-\alpha} y(T) = l(T)$.

We next combine Eqs. $(A.17 - A.19)$ to express the optimality condition in terms of the firm's price, $q^{adj}(t)$, the aggregate price after each history, $q(T)$, and the aggregate labor after each history, $l(T)$:

$$\int_{T=t}^{\infty} e^{-(\rho+\zeta)(T-t)} \left(\left(q^{adj}(t) - q(T) \right) \frac{1-\alpha+\varepsilon\alpha}{1-\alpha} - \frac{\varphi+1}{1-\alpha} y(T) \right) dT = 0.$$

After rearranging terms, we obtain an expression for the optimal price of an adjusting firm,

$$\begin{aligned} q^{adj}(t) &= (\rho+\zeta) \int_{T=t}^{\infty} e^{-(\rho+\zeta)(T-t)} (\Theta y(T) + q(T))\, dT \\ \text{where } \Theta &= \frac{1+\varphi}{1-\alpha+\varepsilon\alpha}. \end{aligned}$$

Since the expression is recursive, we can also write it as a differential equation

$$(\rho+\zeta) q^{adj}(t) = (\rho+\zeta)(\Theta y(t) + q(t)) + \dot{q}^{adj}(t). \tag{A.20}$$

Next, we consider the aggregate price index $(A.5)$. Log-linearizing the expression around Q^*, we obtain $q(t) = \int q(t,\nu)\, d\nu$. Differentiating over time and using the observation that firms adjust their prices with constant hazard ζ, we obtain,

$$\pi(t) \equiv \dot{q}(t) = \zeta \left(q^{adj}(t) - q(t) \right). \tag{A.21}$$

Differentiating this, we also have

$$\dot{\pi}(t) = \zeta\left(\dot{q}^{adj}(t) - \dot{q}(t)\right). \tag{A.22}$$

Substituting Eqs. $(A.21 - A.22)$ into $(A.20)$, we obtain the New-Keynesian Phillips curve

$$\begin{aligned} \rho\pi(t) &= \kappa y(t) + \dot{\pi}(t) \\ \text{where } \kappa &= \zeta(\rho+\zeta)\frac{1+\varphi}{1-\alpha+\varepsilon\alpha}. \end{aligned} \tag{A.23}$$

This completes the microfoundations for the supply side.

B. Appendix: Omitted proofs

This appendix contains the proofs omitted from the main text.

Proof of Proposition 1. Most of the analysis is presented in the main text. Here, we verify that the value function is a quadratic function as in (12) and we characterize the endogenous coefficient, $v > 0$.

We first consider the optimality condition for problem (11),

$$p = \frac{\theta m}{\psi} V'(y). \tag{B.1}$$

We then differentiate Eq. (11) with respect to y and use the Envelope Theorem to obtain

$$(\rho + \theta (1-n)) V'(y) = -y + V''(y) \theta (mp - (1-n) y). \tag{B.2}$$

Eqs. $(B.1)$ and $(B.2)$ correspond to a second order ODE that characterizes the value function. We conjecture that the solution is a quadratic function [see Eq. (12)]

$$V(y) = -\frac{1}{2v} y^2.$$

Substituting the conjecture together with $p = \frac{\theta m}{\psi} V'(y)$ into $(B.2)$, we obtain:

$$(\rho + 2\theta (1-n)) \frac{y}{v} = y - \frac{\theta^2 m^2}{\psi} \frac{y}{v^2}.$$

After canceling y's from both sides and rearranging terms, we obtain the quadratic Eq. (13):

$$P(v) \equiv v^2 - (\rho + 2\theta (1-n)) v - \frac{\theta^2 m^2}{\psi} = 0.$$

This quadratic equation has one positive root and one negative root. The solution corresponds to the positive root, $v > 0$, since we require $V(y) = -\frac{1}{2v} y^2 \leq 0$. This root has the following closed form solution:

$$v = \frac{\rho + 2\theta (1-n) + \sqrt{(\rho + 2\theta (1-n))^2 + 4\frac{\theta^2 m^2}{\psi}}}{2}. \tag{B.3}$$

The rest of the analysis is in the main text. Note that Eq. (13) implies the convergence rate in Eq. (15) also satisfies

$$\gamma \equiv \theta \left(\frac{\theta m^2}{\psi v} + 1 - n \right) = v - (\rho + \theta (1 - n)) > 0. \tag{B.4}$$

Combining this with Eq. $(B.3)$, we can also calculate the convergence rate in closed form as follows:

$$\gamma = \frac{\sqrt{(\rho + 2\theta (1 - n))^2 + 4\frac{\theta^2 m^2}{\psi}} - \rho}{2}. \tag{B.5}$$

This completes the proof. □

Proof of Corollary 1. Using the characterization in Section 2.2, we calculate the cumulative overshooting as:

$$\int_0^\infty p(t)\, dt = \int_0^\infty -\frac{\theta m}{\psi v} y(t)\, dt = -\frac{\theta m}{\psi v} y(0) \int_0^\infty \exp(-\gamma t)\, dt = -\frac{\theta m}{\psi v \gamma} y(0).$$

Here, the first equality substitutes $p(t) = y(t) \frac{\theta m}{\psi v}$ from (14) and the second equality substitutes the definition of the convergence rate, γ (see (15)). After substituting $\gamma = \theta \left(\frac{\theta m^2}{\psi v} + 1 - n \right)$ from (15), we further obtain

$$\frac{\int_0^\infty p(t)\, dt}{-y(0)} = \frac{\theta m}{\psi v \gamma} = \frac{m}{\psi v \left(\frac{\theta m^2}{\psi v} + 1 - n \right)} = \frac{1}{\theta m + \frac{1-n}{m} \psi v}.$$

Note from Eq. $(B.3)$ that increasing θ increases v. It follows that increasing θ increases $\frac{\int_0^\infty p(t)dt}{-y(0)}$. We also obtain $\lim_{\theta \to \infty} \frac{\int_0^\infty p(t)dt}{-y(0)} = 0$, completing the proof. □

Proof of Proposition 2. The characterization for the unconstrained region is presented in the main text. Consider the constrained region $y(0) < \overline{y}$. In this region, the equilibrium corresponds to the solution to the differential equation system in (20), which we write as

$$\begin{aligned} \begin{bmatrix} \dot{y} \\ \dot{p} \end{bmatrix} &= A \begin{bmatrix} y \\ p \end{bmatrix} + b \\ \text{where } A &= \begin{bmatrix} -\theta (1 - n) & \theta m \\ -\rho & \rho \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ \overline{r} (1 + \rho) \end{bmatrix}. \end{aligned}$$

In view of the parametric condition, this system has a unique steady-state given by,

$$\begin{bmatrix} y^* \\ p^* \end{bmatrix} = \begin{bmatrix} \zeta m \\ \zeta (1-n) \end{bmatrix} \text{ where } \zeta = \frac{\overline{r}(1+\rho)}{\rho(m-(1-n))}. \tag{B.6}$$

Away from the steady-state, the qualitative behavior of the system is governed by the eigenvalues of the matrix, A, which correspond to the zeros of the polynomial,

$$P(\lambda) = \lambda^2 + \lambda(\theta(1-n)-\rho) + \theta\rho(m-(1-n)). \tag{B.7}$$

Under the parametric condition, there are two real eigenvalues, $\lambda_1 < \lambda_2$, with corresponding eigenvectors given by

$$e_1 = \begin{bmatrix} \lambda_1 - \rho \\ -\rho \end{bmatrix} \text{ and } e_2 = \begin{bmatrix} \lambda_2 - \rho \\ -\rho \end{bmatrix}. \tag{B.8}$$

We denote the eigenmatrix with $E = [e_1, e_2]$. Given a generic vector $\begin{bmatrix} y - y^* \\ p - p^* \end{bmatrix}$ (measured in deviation from the steady-state), we find the corresponding eigencoordinates by solving $\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = E^{-1} \begin{bmatrix} y - y^* \\ p - p^* \end{bmatrix}$. Given an initial vector with eigencoordinates $\begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix}$, the solution is given by

$$\begin{bmatrix} y(t) - y^* \\ p(t) - p^* \end{bmatrix} = E \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \text{ where } \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} z_1(0) e^{\lambda_1 t} \\ z_2(0) e^{\lambda_2 t} \end{bmatrix}. \tag{B.9}$$

To characterize the solution further, we consider two cases depending on the sign of the term, $m - (1-n)$.

Case (i) $m - (1-n) > 0$. In this case, $\zeta < 0$ so the steady-state lies in the lower-left quadrant in the $y - p$ space [see $(B.6)$]. Moreover, Eqs. $(B.7 - B.8)$ (together with parametric condition) imply that both eigenvalues are negative $\lambda_1 < \lambda_2 < 0$ and that both eigenvectors, e_1, e_2, have a strictly positive slope in the $y - p$ space. Moreover, the eigenvector e_1 corresponding to the smaller (more negative) eigenvalue λ_1 has a smaller slope. Figure 8 illustrates the steady-state, the eigenvalues, and the eigenvectors for to this case.

Now consider a generic initial vector in the upper-left quadrant in the eigenspace: that is, with eigencoordinates $z_1(0) < 0 < z_2(0)$. For such vectors, Eq. $(B.9)$ together with the fact that $\lambda_1 < \lambda_2 < 0$ implies $\lim_{t\to\infty} \frac{z_1(t)}{z_2(t)} = 0$. Hence, as we solve the equation

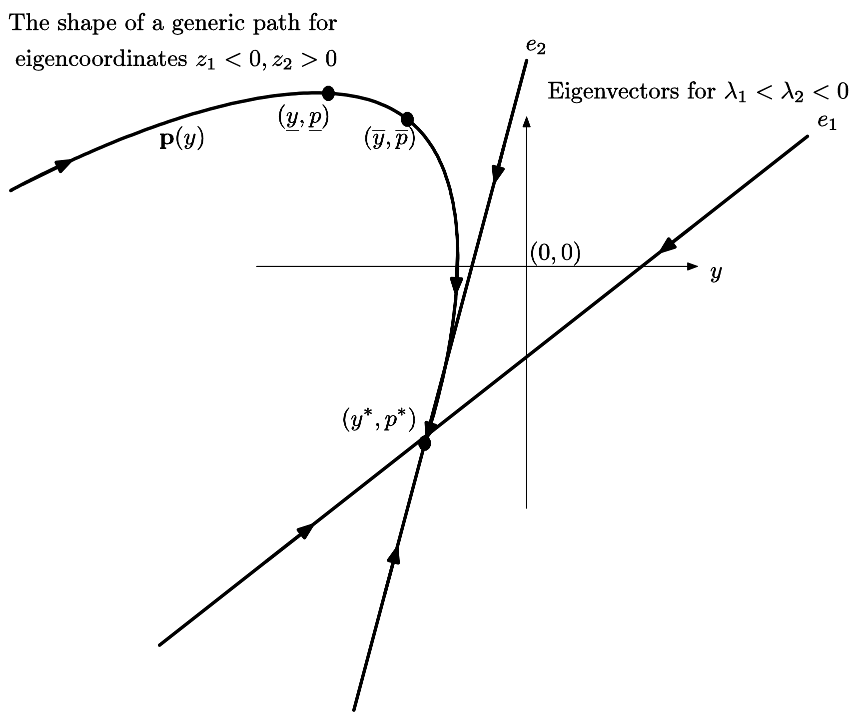

Figure 8: Phase diagram for the system in (20) when $m - (1 - n) > 0$.

forward, the solution becomes qualitatively similar to the paths along the eigenvector e_2. Conversely, we have $\lim_{t \to -\infty} \frac{z_2(t)}{z_1(t)} = 0$. As we solve the equation backward, the solution becomes qualitatively similar to the paths along the eigenvector e_1. Combining the two cases, a generic solution path for the upper-left quadrant in the eigenspace has the shape illustrated in Figure 8. In particular, $\dot{y}(t) > 0, \dot{p}(t) > 0$ when t is sufficiently small and negative, whereas $\dot{y}(t) < 0, \dot{p}(t) < 0$ when t is sufficiently large and positive.

Now consider the vector $(\overline{y}, \overline{p})$ at which the discount rate constraint starts to bind. Since the discount rate is effectively unconstrained, our analysis in Section 2.2 implies $\dot{y}(t) > 0$ and $\dot{p}(t) < 0$ (this can also be seen from (20)). Combining this observation with Figure 8, the vector, $(\overline{y}, \overline{p})$, must be in the upper-left quadrant in the eigenspace. Moreover, $(\overline{y}, \overline{p})$ must lie on the part of a solution path where $\dot{p}(t)$ has become strictly negative but $\dot{y}(t)$ has not yet become negative. These observations establish most of the claims in the proposition for this case (except for the concavity of $\mathbf{p}(y)$). As illustrated in Figure 8, the function $\mathbf{p}(y)$ corresponds to the solution path over the range, $y < \overline{y}$, and the cutoff $\underline{y}$ corresponds to the maximum of the function $\mathbf{p}(y)$. It remains to verify the concavity of $\mathbf{p}(y)$.

Case (ii) $m - (1 - n) < 0$. In this case, $\zeta > 0$ so the steady-state lies in the upper-right quadrant in the $y - p$ space [see $(B.6)$]. Moreover, Eq. $(B.7)$ implies that one eigenvalue is

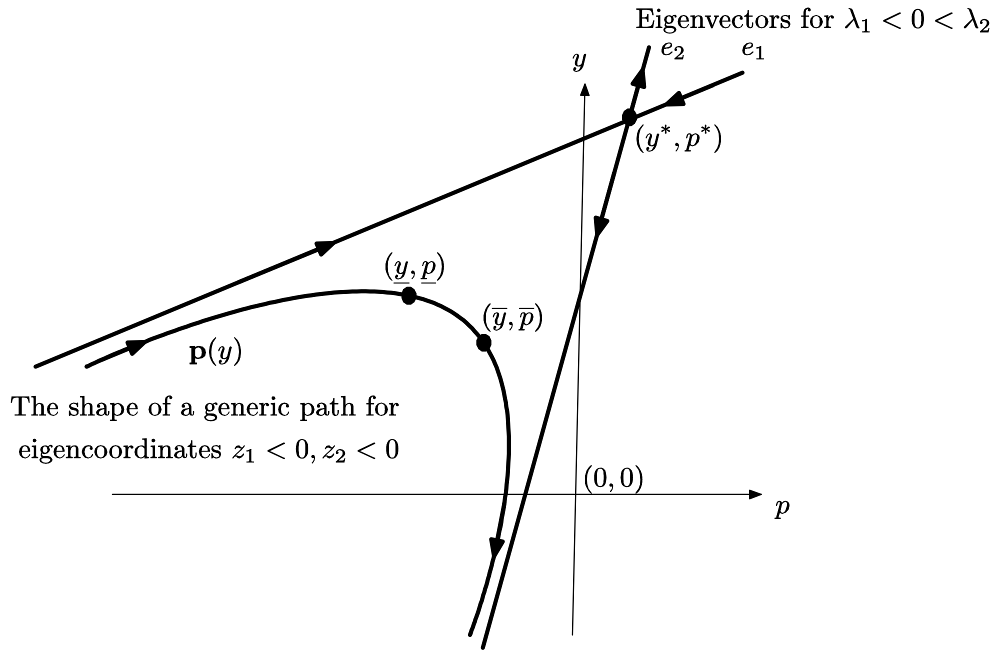

Figure 9: Phase diagram for the system in (20) when $m - (1 - n) < 0$.

strictly negative while the other one is strictly positive $\lambda_1 < 0 < \lambda_2$ and that the positive eigenvalue satisfies, $\lambda_2 < \rho$. Therefore, Eq. $(B.8)$ still implies that both eigenvectors have a positive slope and the eigenvector e_1 corresponding to the smaller (negative) eigenvalue λ_1 has a smaller slope. Figure 9 illustrates the steady-state, the eigenvalues, and the eigenvectors corresponding to this case.

Now consider a generic initial vector in the lower-left quadrant in the eigenspace: that is, with eigencoordinates $z_1(0) < 0, z_2(0) < 0$. For such vectors, Eq. $(B.9)$ together with the fact that $\lambda_1 < 0 < \lambda_2$ implies $\lim_{t \to \infty} z_1(t) = 0$ and $\lim_{t \to \infty} z_2(t) = -\infty$. Hence, as we solve the equation forward, the solution moves away from the steady-state at an exponential rate along the eigenvector e_2. Conversely, we also have $\lim_{t \to -\infty} z_1(t) = -\infty$ and $\lim_{t \to -\infty} z_2(t) = 0$. Hence, as we solve the equation backward, the solution moves away from the steady-state at an exponential rate along the eigenvector e_1. Combining the two cases, a generic solution path in the lower-left quadrant in the eigenspace has the shape illustrated in Figure 9. In particular, $\dot{y}(t) > 0, \dot{p}(t) > 0$ when t is sufficiently small and negative whereas $\dot{y}(t) < 0, \dot{p}(t) < 0$ when t is sufficiently large and positive.

Now consider the vector $(\overline{y}, \overline{p})$ at which the discount rate constraint starts to bind. As before, this vector satisfies $\dot{y}(t) > 0$ and $\dot{p}(t) < 0$. Combining this observation with Figure 9 implies that $(\overline{y}, \overline{p})$ must be in the lower-left quadrant in the eigenspace. Moreover, it must lie on the part of a solution path where $\dot{p}(t)$ has become strictly negative but

$\dot{y}(t)$ has not yet become negative. These observations establish most of the claims in the proposition for this case (except for the concavity of $\mathbf{p}(y)$). As illustrated in Figure 9, the function, $\mathbf{p}(y)$, corresponds to the solution path over the range, $y < \overline{y}$, and the cutoff $\underline{y}$ corresponds to the output gap where $\mathbf{p}(y)$ is maximized.

Numerical solution for the asset price function. To facilitate the numerical solution, we also write the function, $\mathbf{p}(y)$, as a solution to a differential equation in y-domain. Observing that $\frac{dp}{dy} = \frac{dp}{dt} / \frac{dy}{dt}$, we combine the two differential equations in (20) to obtain,

$$\frac{d\mathbf{p}(y)}{dy} = \frac{(1+\rho)\overline{r} + \rho(\mathbf{p}(y) - y)}{\theta(m\mathbf{p}(y) - (1-n)y)} \text{ with } \mathbf{p}(\overline{y}) = \overline{p}. \tag{B.10}$$

We can obtain the functions $\mathbf{p}(y)$ illustrated in Figures $8-9$ by solving this differential equation backward in y-domain.

Strict concavity of the asset price function. To complete the proof, it remains to verify that $\frac{d^2\mathbf{p}(y)}{d^2y} < 0$ over the constrained range $y < \overline{y}$. Note that

$$\frac{d^2\mathbf{p}(y)}{d^2y} = \frac{dy}{dt}\frac{d}{dt}\left(\frac{dp/dt}{dy/dt}\right).$$

Since $dy/dt > 0$ (over the relevant range), it suffices to show

$$\frac{d}{dt}\left(\frac{dp/dt}{dy/dt}\right) < 0 \text{ over } y < \overline{y}. \tag{B.11}$$

To prove this claim, recall from $(B.8)$ and $(B.9)$ that we have a closed-form solution for the output gap and the asset price gap (when $y < \overline{y}$):

$$\begin{aligned} y(t) &= y^* + (\lambda_1 - \rho) z_1(0) e^{\lambda_1 t} + (\lambda_2 - \rho) z_2(0) e^{\lambda_2 t} \\ p(t) &= p^* - \rho z_1(0) e^{\lambda_1 t} - \rho z_2(0) e^{\lambda_2 t}. \end{aligned}$$

Recall that $\lambda_1 < \lambda_2$ are the eigenvalues of the polynomial $(B.7)$. Using this solution, we calculate

$$\begin{aligned}
\frac{\dot{p}}{\dot{y}} &= \frac{Ae^{(\lambda_1-\lambda_2)t}+B}{Ce^{(\lambda_1-\lambda_2)t}+D} = \frac{A}{C} + \frac{B-\frac{AD}{C}}{Ce^{(\lambda_1-\lambda_2)t}+D} \\
&\text{where} \\
A &= -\rho\lambda_1 z_1(0) \\
B &= -\rho\lambda_2 z_2(0) \\
C &= (\lambda_1-\rho)\lambda_1 z_1(0) \\
D &= (\lambda_2-\rho)\lambda_2 z_2(0).
\end{aligned}$$

This in turn implies

$$\frac{d}{dt}\left(\frac{dp/dt}{dy/dt}\right) = \frac{(BC-AD)(\lambda_2-\lambda_1)}{\left(Ce^{(\lambda_1-\lambda_2)t}+D\right)^2}. \tag{B.12}$$

Note that $\lambda_2 - \lambda_1 > 0$. Note also that

$$BC - AD = (\lambda_2-\lambda_1)\rho\lambda_1\lambda_2 z_1(0) z_2(0).$$

It is easy to check that $BC - AD < 0$ in both cases analyzed earlier. In case (i), we have $\lambda_1 < \lambda_2 < 0$ and $z_1(0) < 0, z_2(0) > 0$. In case (ii), we have $\lambda_1 < 0 < \lambda_2$ and $z_1(0) < 0, z_2(0) < 0$. In either case, $BC - AD < 0$. Combining this observation with $(B.12)$, we establish $(B.11)$ and complete the proof of Proposition 2. □

Proof of Proposition 3. The log price of the market portfolio is given by

$$\begin{aligned}
\log P(0) &= p(0) + \log P^* \\
&= p(0) + \log\frac{\alpha Y^*}{\rho} \\
&= p(0) + bN + \log\frac{\alpha\tilde{Y}^*}{\rho} \\
&= \mathbf{p}(y(0)) + bN + \log\frac{\alpha\tilde{Y}^*}{\rho}
\end{aligned}$$

The first line uses the definition of the asset price gap, the second line substitutes P^* from (5), the third line substitutes Y^* from (21), and the last line substitutes the function $\mathbf{p}(y)$ from Proposition 2. Differentiating with respect to N (and evaluating at $N = 0$), we obtain,

$$\left.\frac{d\log P(0)}{dN}\right|_{N=0} = \left.\frac{dy(0)}{dN}\frac{d\mathbf{p}(y(0))}{dy(0)}\right|_{N=0} + b$$
$$= (a-b)\left.\frac{d\mathbf{p}(y)}{dy}\right|_{y=\tilde{y}(0)} + b.$$

This proves (23).

For the second part, note that $\frac{d^2\mathbf{p}(y)}{d^2y} = 0$ over the unconstrained range $y > \overline{y}$ and $\frac{d^2\mathbf{p}(y)}{d^2y} < 0$ for the constrained range $y < \overline{y}$. Combining this with the first part, we obtain $\frac{d}{d\tilde{y}(0)}\left.\frac{d\log P(0)}{dN}\right|_{N=0} \leq 0$, completing the proof. □

Proof of Proposition 4. Consider part (i). We first integrate Eq. (9) forward and use $\lim_{t\to\infty} e^{-\rho t}p(t) = 0$ to obtain a present discounted value formula:

$$p(t) = \int_t^\infty e^{-\rho(s-t)}\left[\rho y(s) - (1+\rho) r(s)\right] ds. \tag{B.13}$$

We then log-linearize Eq. (25) to obtain:

$$\begin{aligned} p^{MB}(t) &= \int_0^\infty \frac{P^{MB*}(\mu)}{P^{MB*}} \log \frac{P^{MB}(t,\mu)}{P^{MB*}(\mu)} d\mu \\ &= -\int_0^\infty e^{-\rho\mu}\rho \int_t^{t+\mu} (R(s) - \rho)\, d\mu \\ &= -\int_0^\infty \int_t^{t+\mu} e^{-\rho\mu}\rho(1+\rho) r(s)\, ds d\mu \\ &= -\int_t^\infty \left(\int_{s-t}^\infty \rho e^{-\rho\mu} d\mu\right)(1+\rho) r(s)\, ds \\ &= -\int_t^\infty e^{-\rho(s-t)} r(s)(1+\rho)\, ds. \end{aligned} \tag{B.14}$$

Here, the second line uses $P^{MB*} = \frac{\alpha Y^*}{\rho}$ and $P^{MB*}(\mu) = \alpha Y^* e^{-\rho\mu}$ along with Eq. (25). The third line substitutes $r(s) \simeq \frac{R(s)-\rho}{1+\rho}$. The fourth line uses Fubini's Theorem to switch the order of integration; and the last step simplifies the expression. Combining the final expression with Eq. $(B.13)$, we establish Eq. (26).

Differentiating the second line in $(B.14)$ with respect to time, we obtain

$$\dot{p}^{MB}(t) = \int_0^\infty e^{-\rho\mu}\rho\left(-\frac{d}{dt}\left(\int_t^{t+\mu} R(s)\, ds\right)\right) d\mu.$$

After substituting the definition of the zero-coupon yield, $y(t,\mu) = \frac{\int_t^{t+\mu} R(s)ds}{\mu}$, we establish Eq. (27).

Finally consider part (iii). Note that the last line in $(B.14)$ implies

$$p^{MB}(t) = -\int_0^\infty e^{-\rho\mu} \left(R(t+\mu) - \rho\right) d\mu.$$

Here, we have substituted $r(s)(1+\rho) \simeq R(s) - \rho$ and applied the change-of-variables, $\mu = s - t$. After differentiating the expression with respect to time, and substituting the definition of the forward rate, $f(t,\mu) = R(t+\mu)$, we establish Eq. (28). This completes the proof of the proposition. □

C. Appendix: Omitted extensions

This appendix presents the extensions of the baseline model omitted from the main text. We first generalize the analysis to the case in which adjusting stockholders are fully rational and incorporate the fact that they will infrequently readjust in future periods (see Section 2.3.2). We then consider the case in which prices are partially flexible and inflation is endogenous (see Section 2.3.3). Finally, we consider a scenario in which the output gap is not negative at the moment but anticipated to turn negative in the near future (see Section 2.3.4).

C.1. Overshooting with sophisticated stockholders

Consider the baseline model analyzed in Section 2. In the main text, we assume adjusting stockholders exogenously follow the rule in (7) with coefficients $m > 0, n \in [0, 1)$. In this appendix, we generalize the analysis to the case with fully sophisticated adjusting stockholders that incorporate the fact that they will get to readjust in the future according to a Poisson process with intensity θ. Specifically, our main result in this section shows that there exists an equilibrium in which the sophisticated consumers follow the rule in (7) with *endogenous* coefficients, that is:

$$\begin{aligned} c^{s,adj}(t) &= m(\gamma)\, p(t) + n(\gamma)\, y(t) \\ \text{where } m(\gamma) &= \frac{\theta - \rho}{\theta + \gamma} \text{ and } n(\gamma) = \frac{\rho}{\theta + \gamma} \end{aligned} \tag{C.1}$$

Proposition 5. *Consider the baseline model with fully rational stockholders. When $\theta > \rho$, there exists an equilibrium in which the linearized optimal consumption rule for adjusting stockholders satisfies*

$$c^{s,adj}(t) = (1 - \eta)\, p(t) + \eta \frac{\rho y(t)}{\rho + \gamma} \quad \text{where} \quad \eta = \frac{\rho + \gamma}{\theta + \gamma}. \tag{C.2}$$

Equivalently, the equilibrium consumption rule is given by Eq. (7) with the endogenous coefficients in (C.1) that satisfy $m(\gamma), n(\gamma) \in (0, 1)$. The equilibrium path, $[y(t), p(t), r(t)]_{t=0}^{\infty}$, is characterized by Proposition 1 given the coefficients $m(\gamma), n(\gamma)$. In particular, ***the equilibrium with fully rational stockholders also features asset price overshooting.*** *The equilibrium convergence rate, $\gamma > 0$, solves:*

$$\gamma = F(\gamma) \equiv \frac{\sqrt{(\rho + 2\theta(1 - n(\gamma)))^2 + 4\frac{\theta^2 m(\gamma)^2}{\psi}} - \rho}{2}. \tag{C.3}$$

Eq. $(C.2)$ characterizes the optimal linearized consumption rule along a path in which the linearized equilibrium variables converge to zero at a constant rate $\gamma > 0$. The remaining equilibrium variables are then characterized by Proposition 1 given the endogenous coefficients $m(\gamma), n(\gamma)$. Recall from our earlier analysis that the convergence rate, γ, depends on the coefficients, m, n. Since the endogenous coefficients also depend on γ, the equilibrium convergence rate corresponds to a fixed point.

Before we prove Proposition 5, we discuss the intuition behind the consumption rule in $(C.2)$. Note that $\frac{\rho y(t)}{\rho+\gamma}$ captures the log wealth change driven by the consumer's *fixed-rate wealth*: the present value of her lifetime income discounted at the steady-state discount rate ρ (along the equilibrium path with convergence rate γ). Hence, Eq. $(C.2)$ says that log consumption reacts to a weighted average of the log actual wealth change, $p(t)$, and the log fixed-rate wealth change, $\frac{\rho y(t)}{\rho+\gamma}$. When asset prices increase because current income increases, the two wealth measures increase by the same amount and log consumption reacts one-to-one to log wealth changes. However, when asset prices increase because of a discount rate cut, log consumption reacts less than one-to-one to log wealth change ($m < 1$). Hence, the consumer *does* react to wealth changes induced by discount rate changes, but *less so* than when she can adjust continuously. As expected, $\theta \to \infty$ implies $\eta \to 0$: when consumption adjustment is sufficiently rapid, we recover the usual consumption rule.

Why does a sophisticated consumer react to the interest-rate driven wealth changes relatively less? As we show below, this is because interest-rate-driven wealth changes are associated with net income effects that mitigate the wealth effect. Intuitively, since the consumer cannot reoptimize in the future (in some states), the substitution effect becomes relatively weak. Therefore, despite log preferences, the substitution and income effects do not net out. Instead, the income effect dominates and implies that—keeping wealth constant—decreasing the discount rate decreases spending. Equivalently, lower discount rates make it costlier to finance a fixed consumption stream until the next adjustment opportunity.

We next prove Proposition 5 in three steps. First, we characterize the (log-linearized) optimal consumption rule for an arbitrary path of (log-linearized) future income and discount rate gaps, $[y(t), r(t)]_{t=0}^{\infty}$. Second, we show that, along the equilibrium path in which $y(t)$ and $r(t)$ converge to zero at an exponential rate $\gamma > 0$, the consumption rule is given by $(C.2)$. Third, we show there exists a convergence rate that solves the fixed point equation $(C.3)$ and prove Proposition 5.

Optimal consumption with Poisson adjustment. We establish optimal consumption rule for $t = 0$. Since the model is stationary, the results also apply for other times

$t > 0$. Consider a stockholder with assets $A(0)$ that gets to adjust her consumption. For now, we take the initial asset level as exogenous (from the discount rate) and endogenize it subsequently. The stockholder's recursive problem is given by:

$$\begin{aligned} V(A(0)) &= \max_C \int_0^\infty \theta e^{-\theta T}\left[\int_0^T e^{-\rho t}\log C dt + e^{-\rho T} V(A(T))\right] dT \qquad \text{(C.4)} \\ &= \max_C \int_0^\infty \theta e^{-\theta T}\left[\frac{1-e^{-\rho T}}{\rho}\log C + e^{-\rho T} V(A(T))\right] dT \\ \text{where } \dot{A}(t) &= R(t)A(t) - C \end{aligned}$$

Here, we write the stockholder's problem as an integral over adjustment times T. The probability that adjustment takes place at time T is $\theta e^{-\theta T}$. Conditional on adjustment at time T, the stockholder receives the utility in the brackets.

Suppose consumption is strictly positive along the optimal path (this will be the case since we focus on small deviations from the steady-state). Then, the value function has the homogeneity property:

$$V(A(t)) = \frac{\log A(t)}{\rho} + V(1).$$

Note also that we can integrate the budget constraint forward to solve for the value of assets at time T. In particular, the present value of $A(T)$ is equal to the initial assets net of the present value of the fixed consumption path until time T:

$$\frac{A(T)}{\exp\left(\int_0^T R(s)\,ds\right)} = A(0) - C\int_0^T \exp\left(-\int_0^t R(s)\,ds\right) dt$$

Substituting this into $(C.4)$, we obtain the following optimality condition:

$$\int_0^\infty \theta e^{-\theta T}\left[\frac{1-e^{-\rho T}}{C\rho} - \frac{\exp(-\rho T)}{\rho}\frac{\int_0^T \exp\left(-\int_0^t R(s)\,ds\right)dt}{A(0) - C\int_0^T \exp\left(-\int_0^t R(s)\,ds\right)dt}\right] dT = 0.$$

After rearranging terms, we have:

$$\frac{1}{C}\frac{1}{\theta+\rho} = \int_0^\infty \frac{\theta}{\rho} e^{-(\theta+\rho)T}\frac{\int_0^T \exp\left(-\int_0^t R(s)\,ds\right)dt}{A(0) - C\int_0^T \exp\left(-\int_0^t R(s)\,ds\right)dt.} dT.$$

The solution scales with initial wealth. Thus it can be written as

$$C = \rho X(0) A(0) \tag{C.5}$$

for some $X(0) > 0$. After substituting, we further obtain

$$\frac{1}{X(0)} = \int_0^\infty \theta(\theta+\rho) e^{-(\theta+\rho)T} \frac{\int_0^T \exp\left(-\int_0^t R(s)\, ds\right) dt}{1 - \rho X(0) \int_0^T \exp\left(-\int_0^t R(s)\, ds\right) dt.} dT. \tag{C.6}$$

Consider the potential steady-state, $R^*(t) = \rho$ for each t. In this case, it is easy to check $X^*(0) = 1$ and thus $C = \rho A(0)$. Consider also the special case $\theta \to \infty$. It can be checked that this also gives the same solution $X(0) = 1$ and $C = \rho A(0)$.

Beyond these special cases, the solution is complicated. In particular, $X(0)$ depends on the whole future path of discount rates. To make progress, we log-linearize Eq. $(C.6)$ around the potential steady-state. Specifically, we define the log-deviation terms $x(0) = \log \frac{X(0)}{X^*(0)}$ and $r(t) = \log \frac{1+R(t)}{1+\rho}$. Substituting these variables into $(C.6)$ and using the approximation $r(s) \simeq \frac{R(s)-\rho}{1+\rho}$ along with $X^*(0) = 1$, we obtain:

$$\frac{1}{\exp(x(0))} = \int_0^\infty \theta(\theta+\rho) e^{-(\theta+\rho)T} \frac{\int_0^T \exp(-\rho t) \exp\left(-\int_0^t (1+\rho) r(s)\right) dsdt}{1 - \rho \exp(x(0)) \int_0^T \exp(-\rho t) \exp\left(-\int_0^t (1+\rho) r(s)\right) dt} dT.$$

Linearizing this expression around $x(0) = r(t) = 0$, we further obtain:

$$\begin{aligned} x(0) &= \int_0^\infty \theta(\theta+\rho) e^{-(\theta+\rho)T} \left\{ \begin{array}{c} -\rho x(0) \int_0^T e^{-\rho t} dt \frac{\int_0^T e^{-\rho t} dt}{\left(1-\rho \int_0^T e^{-\rho t} dt\right)^2} + \\ -\int_0^T e^{-\rho t} \left(-\int_0^t (1+\rho) r(s)\, ds\right) dt \frac{1}{1-\rho \int_0^T e^{-\rho t} dt} \\ -\rho \int_0^T e^{-\rho t} \left(-\int_0^t (1+\rho) r(s)\, ds\right) dt \frac{\int_0^T e^{-\rho t} dt}{\left(1-\rho \int_0^T e^{-\rho t} dt\right)^2} \end{array} \right\} dT \\ &= \int_0^\infty \theta(\theta+\rho) e^{-(\theta+\rho)T} \left\{ -x(0) \frac{\left(1-e^{-\rho T}\right)^2}{\rho \left(e^{-\rho T}\right)^2} + \frac{\int_0^T e^{-\rho t} \int_0^t (1+\rho) r(s)\, dsdt}{\left(e^{-\rho T}\right)^2} \right\} dT. \end{aligned}$$

Here, the first line uses the chain rule to evaluate the derivatives and cancels the constant terms from both sides (since the equation holds for $x(0) = r(t) = 0$). The second line calculates the integrals. After rearranging terms, we further obtain

$$\begin{aligned} & x(0)\left[1+\int_0^\infty \frac{\theta(\theta+\rho)}{\rho} e^{-(\theta-\rho)T}\left(1-e^{-\rho T}\right)^2 dT\right] \\ = & \int_0^\infty \theta(\theta+\rho) e^{-(\theta-\rho)T} \int_0^T e^{-\rho t} \int_0^t (1+\rho) r(s)\, ds dt dT \end{aligned}$$

Calculating the integrals and simplifying further, we obtain:

$$x(0)\left(\frac{\theta+\rho}{\theta-\rho}\right) = \int_0^\infty \theta(\theta+\rho) e^{-(\theta-\rho)T} \int_0^T e^{-\rho t} \int_0^t (1+\rho) r(s)\, ds dt dT. \tag{C.7}$$

Recall that $C = X(0)\rho A(0)$ where $X(0) \simeq X^* \exp(x(0))$. Hence, Eq. $(C.7)$ illustrates that the slope of the consumption function deviates from the usual slope iff the discount rates are different from their steady-state levels. As long as $\theta > \rho$ (which we assume), keeping current wealth constant, greater discount rate increases (resp. decreases) spending. Intuitively, since the stockholder cannot reoptimize in the future (in some states), the substitution effect becomes weaker. Therefore, despite log preferences, the substitution and income effects do not net out. Instead, the income effect dominates and implies—keeping the wealth constant—increasing the discount rate increases spending. A higher discount rate makes it cheaper to finance a steady consumption stream (C), which induces the stockholder to spend more.

As before, there is also a wealth effect once we endogenize the initial wealth $A(0)$. In particular, note that the wealth of the representative adjusting stockholder is equal to the price of the market portfolio, $A(0) = P(0)$. This implies:

$$\begin{aligned} C^{s,adj}(0) &= X(0)\rho P(0) \\ \text{where } P(0) &= \int_0^\infty \alpha Y(t) e^{-\int_0^t R(s)ds} dt. \end{aligned} \tag{C.8}$$

Log-linearizing the price of the market portfolio around the potential steady-state and using $r(s) \simeq \frac{R(s)-\rho}{1+\rho}$, we obtain

$$\begin{aligned} p(0) &= \int_0^\infty e^{-\rho t}\rho y(t)\, dt - \int_0^\infty \int_0^t e^{-\rho t}\rho r(s)(1+\rho)\, ds dt \\ &= \int_0^\infty e^{-\rho t}\rho y(t)\, dt - \int_0^\infty \left(\int_s^\infty e^{-\rho t}\rho dt\right) r(s)(1+\rho)\, ds \\ &= \int_0^\infty e^{-\rho t}\left(\rho y(t) - (1+\rho) r(t)\right) dt. \end{aligned} \tag{C.9}$$

Here, the second line switches the order of integration and the last line collects the terms (see Eq. $(B.13)$ in the main text for an alternative derivation). As usual, increasing the discount rate reduces the value of the market portfolio. All else equal, this reduces spending.

Finally, log-linearizing Eq. $(C.8)$ around the potential steady-state, we conclude that the optimal consumption rule is approximately,

$$c^{s,adj}(0) = x(0) + p(0), \tag{C.10}$$

where $x(0)$ is given by Eq. $(C.7)$ and $p(0)$ is given by $(C.9)$.

Optimal consumption along the equilibrium path. We next calculate consumption along the equilibrium path and prove Eq. (7). In equilibrium, we have:

$$y(t) = y(0)e^{-\gamma t} \text{ and } r(t) = r(0)e^{-\gamma t}.$$

Substituting these expressions in $(C.9)$, we solve for the asset price along the equilibrium path:

$$p(0) = \frac{\rho y(0) - (1+\rho)r(0)}{\rho+\gamma}. \tag{C.11}$$

As expected a greater initial output increases the price and a greater initial discount rate decreases the price.

Next, we define η such that:

$$x(0) = \frac{(1+\rho)r(0)}{\rho+\gamma}\eta. \tag{C.12}$$

This normalization is useful to simplify consumption function. Specifically, combining this with Eqs. $(C.10)$ and $(C.11)$, we obtain:

$$\begin{aligned} c^{s,adj}(0) &= \frac{(1+\rho)r(0)}{\rho+\gamma}\eta + p(0) \\ &= \left(\frac{\rho y(0)}{\rho+\gamma} - p(0)\right)\eta + p(0) \\ &= (1-\eta)p(0) + \eta\frac{\rho}{\rho+\gamma}y(0) \end{aligned} \tag{C.13}$$

Here, the second line substitutes $\frac{(1+\rho)r(0)}{\rho+\gamma} = \frac{\rho y(0)}{\rho+\gamma} - p(0)$ from Eq. $(C.11)$ to express the consumption function in terms of $y(0), p(0)$ (instead of $r(0), p(0)$).

It remains to solve for η along the equilibrium path. Combining Eqs. $(C.7)$ and $(C.12)$, and substituting $r(s) = r(0)e^{-\gamma s}$, we calculate:

$$\begin{aligned}
\eta &= \frac{(\rho+\gamma)(\theta-\rho)}{\theta+\rho}\int_0^\infty \theta(\theta+\rho)e^{-(\theta-\rho)T}\int_0^T e^{-\rho t}\frac{1-e^{-\gamma t}}{\gamma}dtdT \\
&= \frac{(\rho+\gamma)(\theta-\rho)\theta}{\gamma}\int_0^\infty e^{-(\theta-\rho)T}\left[\frac{1-e^{-\rho T}}{\rho}-\frac{1-e^{-(\rho+\gamma)T}}{\rho+\gamma}\right]dT \\
&= \frac{(\rho+\gamma)(\theta-\rho)\theta}{\gamma}\left\{\frac{1}{\rho}\left(\frac{1}{\theta-\rho}-\frac{1}{\theta}\right)-\frac{1}{\rho+\gamma}\left(\frac{1}{\theta-\rho}-\frac{1}{\theta+\gamma}\right)\right\} \\
&= \frac{(\rho+\gamma)(\theta-\rho)\theta}{\gamma}\left\{\frac{1}{(\theta-\rho)\theta}-\frac{1}{(\theta-\rho)(\theta+\gamma)}\right\} \\
&= \frac{\rho+\gamma}{\theta+\gamma}. \qquad \text{(C.14)}
\end{aligned}$$

The final expression has intuitive comparative statics. For instance, as $\theta \to \infty$, we have $\eta \to 0$. That is, when the stockholder adjusts very rapidly, we recover the standard rule.

Combining Eqs. $(C.13)$ and $(C.14)$, we prove the consumption rule in Eq. $(C.2)$. This rule also implies the consumption rule in Eq. (7) with the endogenous coefficients in $(C.1)$ since

$$m(\gamma) = 1-\eta = \frac{\theta-\rho}{\theta+\gamma} \text{ and } n(\gamma) = \eta\frac{\rho}{\rho+\gamma} = \frac{\rho}{\theta+\gamma}.$$

Note that $m, n \in (0,1)$ since we assume $\theta > \rho$.

The equilibrium convergence rate. Note that the slope coefficients depend on the equilibrium convergence rate, that is, $m = m(\gamma)$ and $n = n(\gamma)$. Conversely, Proposition 1 shows that the convergence rate, γ, depends on m and n. Specifically, Eq. $(B.5)$ shows the convergence rate solves the fixed point equation $(C.3)$. We next complete the proof of Proposition 5.

Proof of Proposition 5. It remains to show that when $\theta > \rho$ there exists a solution to the fixed point equation $(C.3)$, which we replicate here:

$$\begin{aligned}
\gamma &= F(\gamma) \equiv \frac{\sqrt{(\rho+2\theta(1-n(\gamma)))^2+4\frac{\theta^2 m(\gamma)^2}{\psi}}-\rho}{2}, \\
&\text{where } m(\gamma) = \frac{\theta-\rho}{\theta+\gamma} \text{ and } n(\gamma) = \frac{\rho}{\theta+\gamma}.
\end{aligned}$$

We claim there is a solution that satisfies $\gamma \in (0, \overline{\gamma})$ where $\overline{\gamma} = \theta\left(1+\frac{1}{\sqrt{\psi}}\right)$.

First note that

$$F(\gamma = 0) \geq \frac{\sqrt{(\rho + 2\theta(1 - n(0)))^2} - \rho}{2} = \theta(1 - n(0)) = \theta - \rho > 0.$$

Here, the inequality follows since $\theta > \rho$. Next note that

$$\begin{aligned} F(\gamma) &\leq \frac{\sqrt{\left(\rho + 2\theta(1 - n(\gamma)) + 2\frac{\theta m(\gamma)}{\sqrt{\psi}}\right)^2} - \rho}{2} \\ &= \theta\left(1 - n(\gamma) + \frac{m(\gamma)}{\sqrt{\psi}}\right) \\ &< \theta\left(1 + \frac{1}{\sqrt{\psi}}\right) = \overline{\gamma} \end{aligned}$$

Here, the last equality uses $n(\gamma), m(\gamma) \in (0,1)$. In particular, $F(\overline{\gamma}) < \overline{\gamma}$. It follows that Eq. $(C.3)$ has a solution that satisfies $\gamma \in (0, \overline{\gamma})$. This establishes the existence of a fixed point and completes the proof. □

C.2. Overshooting with endogenous inflation

In the main text, we focus on a setup with fully sticky prices. We next extend the analysis to partially flexible prices and endogenous inflation. We show that *the central bank overshoots by more than in our baseline model.* This happens because in this case the central bank also needs to fight the disinflationary forces induced by a negative output gap.

Consider the same model as in Section 2 with the difference that intermediate firms' nominal prices are partially flexible. We adopt the standard Calvo setup: at each instant a randomly selected fraction of firms reset their nominal price, with constant hazard. This price remains unchanged until the firm gets to adjust again. In Appendix A, we characterized the firm's optimal price setting decision. We log-linearize the equilibrium around a zero-inflation benchmark and obtain the New-Keynesian Phillips curve,

$$\rho\pi(t) = \kappa y(t) + \dot{\pi}(t) \quad \Longrightarrow \pi(t) = \kappa \int_t^\infty e^{-\rho(s-t)} y(s)\, ds. \tag{C.15}$$

Here, $\pi(t) = \dot{q}(t)$ denotes inflation—the growth rate of nominal prices (the variable, $q(t)$, is the log-deviation of the nominal price level from its steady-state level). The parameter, κ, is a composite price flexibility parameter that depends on the rate of price adjustment along with other parameters. We also express inflation as a discounted sum of future output gaps (assuming inflation is bounded, which is the case in equilibrium).

Since the economy features inflation, the nominal and the real rates are no longer the same. We denote the log-linearized nominal and real discount rates with $r^n(t)$ and $r(t) = r^n(t) - \pi(t)$. The planner effectively "chooses" the real discount rate by setting the nominal rate appropriately, given its implemented inflation path. We now assume the planner maximizes a value function that incorporates the costs of inflation [see (10)],

$$V(0, y(0)) = \int_0^\infty e^{-\rho t} \left(-\frac{y(t)^2}{2} - \phi \frac{\pi(t)^2}{2} - \psi \frac{p(t)^2}{2} \right) dt. \tag{C.16}$$

The parameter, $\phi \geq 0$, captures the planner's weight on inflation relative to output gaps. We assume the planner sets the discount rate *without commitment*.[16]

In this setting, the equilibrium is a fixed point of an inflation *function*, $\boldsymbol{\pi}(y)$, and an asset price *policy function*, $\mathbf{p}(y)$. Given the policy function, and the implied path for the output gap, inflation satisfies $(C.15)$. Given the inflation function, the asset price policy is optimal for the planner. That is, it solves the following analogue of problem (11):

$$\begin{aligned} \rho V(y; \boldsymbol{\pi}) &= \max_p -\frac{y^2}{2} - \phi \frac{\boldsymbol{\pi}(y)^2}{2} - \psi \frac{p^2}{2} + \frac{dV(y; \boldsymbol{\pi})}{dy} \dot{y}, \\ \dot{y} &= \theta(mp - (1-n)y), \\ V(y) &\leq 0 \text{ and } V(0) = 0. \end{aligned}$$

The equilibrium has the same structure as in the previous section. In particular, the output gap converges to zero at a constant, endogenous rate, $y(s) = y(t) e^{-\gamma(s-t)}$. Substituting this into Eq. $(C.15)$, the inflation function has a closed-form solution,

$$\boldsymbol{\pi}(y) = \frac{\kappa}{\rho + \gamma} y. \tag{C.17}$$

Output gaps translate into more inflation when prices are more flexible (greater κ) and when the gaps converge more slowly (smaller γ).

[16] Unlike in Section 2, the no-comitment constraint binds in this section. The planner might want to promise to fight inflation more aggressively in the future in order to influence current inflation expectations. We abstract from these types of commitment policies since they are not our focus and their benefits are well understood (see, e.g., Clarida et al. (1999)).

After substituting the inflation function, the planner's recursive problem becomes

$$\begin{aligned}\rho V(y;\boldsymbol{\pi}) &= \max_p -\left(1+\phi\left(\frac{\kappa}{\rho+\gamma}\right)^2\right)\frac{y^2}{2}-\psi\frac{p^2}{2}+\frac{dV(y;\boldsymbol{\pi})}{dy}\dot{y} \qquad (C.18)\\ \text{where } \dot{y} &= \theta\left(mp-(1-n)y\right)\end{aligned}$$

As before, we conjecture a quadratic value function $V(y;\boldsymbol{\pi}) = -\frac{1}{2v(\gamma)}y^2$. The coefficient, $v(\gamma)$, depends on the growth rate γ through the inflation function. The optimality condition implies a similar policy function as before [see (14)],

$$\mathbf{p}(y) = \frac{\theta m}{\psi}\frac{dV(y;\boldsymbol{\pi})}{dy} = \frac{-\theta m}{\psi v(\gamma)}y. \qquad (C.19)$$

Differentiating Eq. $(C.18)$ with respect to y and using the Envelope Theorem, we obtain

$$(\rho+\theta(1-n))\frac{dV(y;\boldsymbol{\pi})}{dy} = -\left(1+\phi\left(\frac{\kappa}{\rho+\gamma}\right)^2\right)y+\frac{d^2V(y;\boldsymbol{\pi})}{dy^2}\theta\left(m\mathbf{p}(y)-(1-n)y\right).$$

After substituting the functional form for the value function and the optimal policy, $\mathbf{p}(y)$, we obtain the following analogue of the quadratic in (13),

$$0 = v(\gamma)^2\left(1+\phi\left(\frac{\kappa}{\rho+\gamma}\right)^2\right)-v(\gamma)(\rho+2\theta(1-n))-\frac{\theta^2m^2}{\psi}. \qquad (C.20)$$

As before the coefficient, $v(\gamma)$, corresponds to the positive root of this quadratic. Using the shape of the quadratic, it is easy to check that $v'(\gamma) > 0$. Note also that $v(\infty)$ corresponds to the solution in the benchmark with fully sticky prices.

Next note that Eq. (8) implies

$$\gamma \equiv \frac{-\dot{y}(t)}{y(t)} = \theta\left(\frac{\theta m^2}{\psi v(\gamma)}+1-n\right). \qquad (C.21)$$

The left side of this expression is an increasing function of γ whereas the right side is a decreasing function. There exists a unique fixed point over the range $\gamma \in (0,\infty)$, which corresponds to the equilibrium convergence rate. The following result summarizes this discussion and establishes the comparative statics of this equilibrium.

Proposition 6. *Consider the model with endogenous inflation. In equilibrium, the output gap converges to zero at a constant rate, $\gamma > 0$. The convergence rate corresponds to the fixed point of Eq. $(C.21)$, where $v(\gamma)$ is the positive solution to Eq. $(C.20)$. Inflation and* *output are linear functions of the output gap given by Eqs.* $(C.17)$ *and* $(C.19)$*.*

Starting with a negative output gap, $y(0) < 0$*, the equilibrium features* ***asset price overshooting****: the asset price is above its potential,* $p(0) > 0$*. Moreover, when prices are* more *flexible (greater* κ*), the equilibrium features* greater *cumulative asset price overshooting per unit of negative output gap (greater* $\frac{\int_0^\infty p(t)dt}{-y(0)}$*), along with smaller value for a given output gap (smaller* $v(\gamma)$*) and faster convergence to potential outcomes (greater* γ*).*

The planner also overshoots asset prices in this case. In fact, *greater price flexibility induces greater (cumulative) overshooting.* Intuitively, greater price flexibility makes output gaps costlier due to the endogenous inflation response. This induces the planner to overshoot asset prices by more to close the output gaps more quickly.

Proof of Proposition 6. We establish the comparative statics of the equilibrium with respect to the price flexibility parameter, κ. Eq. $(C.20)$ implies $\frac{\partial v(\gamma)}{\partial \kappa} < 0$: all else equal, greater price flexibility induces smaller value coefficient. Intuitively, with more flexible prices, the planner obtains a lower value due to the endogenous response of inflation to output gaps. The observation, $\frac{\partial v(\gamma)}{\partial \kappa} < 0$, implies that increasing κ shifts the function on the right side of Eq. $(C.21)$ upward. This increases the fixed point, $\frac{d\gamma}{d\kappa} > 0$, and decreases the value coefficient corresponding to the fixed point, $\frac{dv(\gamma)}{d\kappa} < 0$. Intuitively, the endogenous increase in the convergence rate, γ, mitigates but does not overturn the decline in the planner's value induced by greater κ.

Finally, following the same steps as in the proof of Corollary 1, we calculate the cumulative overshooting per unit of the output gap as:

$$\frac{\int_0^\infty p(t)\,dt}{-y(0)} = \frac{\theta m}{\psi v(\gamma)\gamma} = \frac{m}{\psi v(\gamma)\left(\frac{\theta m^2}{\psi v(\gamma)} + 1 - n\right)} = \frac{1}{\theta m + \frac{1-n}{m}\psi v(\gamma)}.$$

Since $\frac{dv(\gamma)}{d\kappa} < 0$, we also have $\frac{d}{d\kappa}\left(\frac{\int_0^\infty p(t)dt}{-y(0)}\right) > 0$. This completes the proof. □

C.3. Preemptive asset price overshooting

In the main text, we focus on a recovery scenario in which the output gap is negative and the central bank's concern is to close the output gap as quickly as possible. We next extend the analysis to a case when the output gap is not currently negative but the central bank anticipates that it will turn negative in the near future. This situation may arise, for example, when the economy is experiencing a sharp but temporary decline in potential output that drags aggregate demand down with it, as in the Covid-19 recession. Our main result in this section shows that the central bank may find it optimal to *preemptively* overshoot asset prices also in this scenario.

Specifically, now there are two aggregate states denoted by subscript $s \in \{1,2\}$. State $s = 2$ (the "high-supply state") corresponds to our analysis up to now. In particular, potential output is given by $Y_2^*(t) = Y^*$. State $s = 1$ (the "low-supply state") corresponds to an earlier period in which potential output is lower, $Y_1^*(t) = \exp(-k) Y^*$ for some $k > 0$. The economy starts in the low-supply state $s = 1$ and transitions to the high-supply state $s = 2$ according to a Poisson process with intensity $\lambda > 0$. Once the economy transitions, it remains in state $s = 2$ forever.[17]

We use the notation $y_s = \log \frac{Y_s}{Y^*}, p_s = \log \frac{P_s}{P^*}, r_s = \log \frac{1+R_s}{1+\rho}$ to denote *normalized log variables*: their difference from potential levels *in the high-supply state* [see (5)]. We use $\tilde{y}_s = \log \frac{Y_s}{Y_s^*}, \tilde{p}_s = \log \frac{P_s}{P_s^*}$ to denote the log output and the log asset price *gaps:* their difference from potential levels *within the corresponding state*. For the high-supply state, the normalized and the gap variables are the same, $y_2 = \tilde{y}_2, p_2 = \tilde{p}_2$; but for the low-supply state they are different, $y_1 \leq \tilde{y}_1, p_1 \leq \tilde{p}_1$ (since the potential levels are lower).

We make three additional simplifying assumptions. First, stockholders have Epstein-Zin preferences with discount rate ρ, EIS equal to one, and RRA equal to 0. Hence, stockholders effectively have log utility with respect to consumption-saving decisions as before, but they are risk neutral with respect to portfolio decisions. This ensures that the asset pricing side of the model is the same as before, and the discount rate is the same as the expected return on the market portfolio. Thus, the log-linearized return is given by (9) in state $s = 2$ and by the following expression in state $s = 1$:[18]

$$r_1(t) = \frac{\rho}{1+\rho}(y_1(t) - p_1(t)) + \frac{\dot{p}_1(t) + \lambda(p_2(t) - p_1(t))}{1+\rho}. \tag{C.22}$$

The expected return on the market portfolio accounts for the change in the asset price when there is a transition.

Second, adjusting stockholders follow the rule in (7) in both the high-supply and the

[17] Our analysis is robust to the exact parametric change that induces the decline in potential output. For concreteness, suppose we only change the total factor productivity (TFP): that is, $A_1(t) = A\exp(-k)$ and $A_2(t) = A$, and the remaining parameters are the same across states (see Appendix A).

[18] We obtain $(C.22)$ by log-linearizing the following equation that describes the interest rate (around the potential levels in the high-supply state $\left(Y^*, P^* = \frac{\alpha Y^*}{\rho}, R^* = \rho\right)$:

$$1 + R_1(t) = 1 + \frac{\alpha Y_1(t) + \dot{P}_1(t) + \lambda(P_2(t) - P_1(t))}{P_1(t)}.$$

low-supply states.[19] Thus, normalized output follows the same dynamics in the low-supply state [see (8)]:

$$\dot{y}_1 = \theta \left(m p_1 - (1-n) y_1 \right). \tag{C.23}$$

Finally, we let the central bank solve the following version of problem (11) in state 1:

$$\begin{aligned} \rho V_1 (y_1) &= \max_{p_1} -\frac{\tilde{y}_1^2}{2} - \psi \frac{\tilde{p}_1^2}{2} + V_1' (y_1) \dot{y}_1 + \lambda \left(V_2 (y_1) - V_1 (y_1) \right), \qquad \text{(C.24)} \\ \text{where } \tilde{y}_1 &= y_1 - y_1^* \text{ with } y_1^* = -k < 0, \\ \text{and } \tilde{p}_1 &= p_1 - p_1^* \text{ with } p_1^* = -k (1-n) / m < 0. \end{aligned}$$

Given current normalized output y_1, the central bank solves a gap-minimizing problem as before. However, the gaps in state $s = 1$ are no longer equal to the normalized variables. The potential output, y_1^*, is lower (recall that $Y_1^* = \exp(-k) Y^*$). The potential asset price, p_1^*, is also lower and reflects the lower potential output. If stockholders adjusted instantaneously ($\theta = \infty$), then keeping the asset price at its potential would ensure that output is also at its potential [see $(C.23)$]. The value function also accounts for the expected transition to the high-supply state. In particular, $V_2(\cdot)$ denotes the high-supply value function that we characterized in Section 2.2.

We define the equilibrium with optimal monetary policy as before. In the low-supply state, output and asset prices solve problem $(C.24)$ given $(C.23)$, and the discount rate is given by $(C.22)$. After transition to the high-supply state, the equilibrium is given by Definition 1.

To analyze the equilibrium, note that Eq. $(C.23)$ also holds in terms of the gaps:

$$\frac{d\tilde{y}_1}{dt} = \theta \left(m \tilde{p}_1 - (1-n) \tilde{y}_1 \right). \tag{C.25}$$

Thus, problem $(C.24)$ has a similar structure to the earlier problem (11). The solution also has a similar form [see (12) in the proof]:

$$V_1 = a + b\tilde{y}_1 - \frac{1}{2v} \tilde{y}_1^2, \tag{C.26}$$

$$\text{where} \quad b = \frac{\lambda}{\lambda + \rho + \gamma} \frac{k}{v} > 0. \tag{C.27}$$

[19] When consumers are fully rational, the optimal consumption rule would take this form with possibly state-dependent coefficients m_s, n_s. We assume the same rule for both states to keep the notation simple, although our results would qualitatively apply in the more general case (as long as $m_s, n_s \in (0,1)$ for both states).

Here, $v, \gamma > 0$ are the same as before. In the low-supply state the value function features an additional linear component, $a + b\tilde{y}_1$, with $b > 0$. In particular, a positive output gap, $\tilde{y}_1 > 0$, *increases* the value function relative to the high-supply state since $b\tilde{y}_1 > 0$.

A positive gap is now less costly since it will reduce the size of the output gap once potential output recovers. This suggests that the central bank might *preemptively* induce positive gaps in the low-supply state. We next state our main result in this section, which completes the characterization and verifies that the equilibrium features *preemptive* overshooting.

Proposition 7. *(i) In the low-supply state $s = 1$, the value function is given by $(C.26)$, where a is a constant and $b > 0$ is given by $(C.27)$. Absent a transition, the optimal asset price and output gaps follow the dynamics*

$$\begin{aligned} \tilde{p}_1(t) &= \frac{\theta m}{\psi}\left(b - \frac{1}{v}\tilde{y}_1(t)\right), \\ \frac{d\tilde{y}_1(t)}{dt} &= \frac{\theta^2 m^2}{\psi} b - \gamma \tilde{y}_1(t). \end{aligned}$$

These gaps monotonically converge to ***strictly positive steady-state levels*** *given by*

$$\tilde{y}_1(\infty) \equiv \frac{\gamma - \theta}{\gamma}\frac{\lambda}{\lambda + \rho + \gamma}k > 0 \quad \text{and} \quad \tilde{p}_1(\infty) = \frac{1-n}{m}\tilde{y}_1(\infty) > 0. \tag{C.28}$$

The normalized output and asset price $y_1(t), p_1(t)$ converge to corresponding steady-states $y_1(\infty) = \tilde{y}_1(\infty) + y_1^*$ and $p_1(\infty) = \tilde{p}_1(\infty) + p_1^*$. The steady-state output exceeds its potential in the low-supply state but remains below its potential in the high-supply state, $y_1(\infty) \in (-k, 0)$ (equivalently, $\tilde{y}_1(\infty) \in (0, k)$).

(ii) Suppose the normalized output is initially not too high, $y_1(0) < y_1(\infty)$ (e.g., it is at potential $y_1(0) = y_1^* = -k$). Before transition, the equilibrium features ***preemptive asset price overshooting****: The normalized asset price starts above its steady-state level as well as its potential, $p_1(0) > p_1(\infty) > p_1^*$. Absent a transition, the normalized output monotonically increases toward $y_1(\infty)$, and the normalized asset price monotonically decreases toward $p_1(\infty)$. At the moment of the transition to high-supply (time t'),* ***asset price overshooting increases****: the normalized output remains unchanged but the output gap becomes negative, $\tilde{y}_2(t') = y_2(t') = y_1(t') < 0$, and the normalized asset price level as well as its gap jump upward, $\tilde{p}_2(t') = p_2(t') > \tilde{p}_1(t') > p_1(t')$.*

Figure 10 simulates the equilibrium (the solid lines) for a particular parameterization in which the economy starts with a zero output gap. We set $k = 0.05$ so that the low-supply

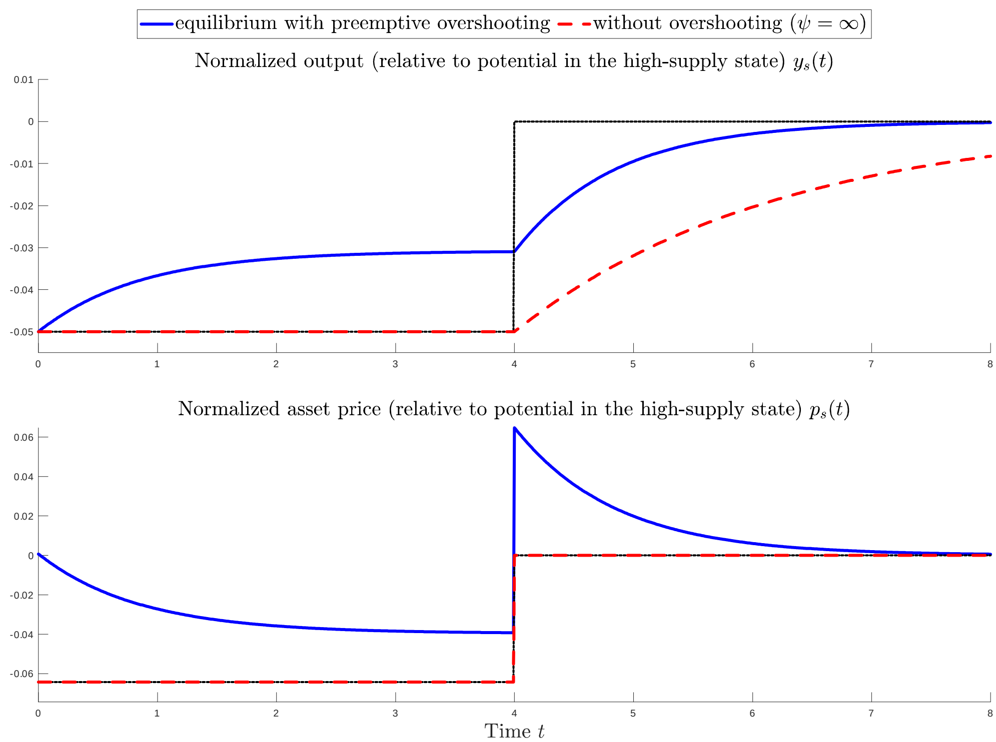

Figure 10: A simulation of the equilibrium variables starting in the low-supply state $s = 1$ with overshooting (solid lines) and with extreme aversion to overshooting ($\psi = \infty$, dashed lines). The dotted lines correspond to the within-state potential output and asset price.

state corresponds to a 5% decline in potential output. The dotted lines illustrate the within-state potential levels (the gaps are the distance from these lines). The equilibrium features preemptive overshooting: the central bank sets a positive asset price gap, which gradually induces a positive output gap. After transition, the central bank overshoots the asset price even more, which helps close the negative output gap. For comparison, the figure also illustrates what happens when the aversion to overshooting is extreme ($\psi = \infty$) so that the central bank always keeps the asset price at its potential. In this case, output does not start to recover until productivity actually recovers. As a result, the economy enters the high-supply state with a smaller aggregate demand gap and closes this gap more slowly [cf. Figure 2].

Figure 11 illustrates the effect of increasing λ—the expected transition rate to high supply. The solid lines plot the equilibrium when the expected transition rate is higher ($\tilde{\lambda} > \lambda$). In this case, preemptive overshooting is stronger: asset prices increase by more and the output in the low-supply state converges to a level closer to its potential in the high-supply state [see (C.28)]. Intuitively, since the central bank expects the supply to recover soon, it engages in considerable overshooting in the low-supply state. In fact, the figure highlights that, by frontloading much of the overshooting to the low-supply state, the central bank might do less overshooting when the supply finally recovers (compared to the early stages of the low-supply state).

Proof of Proposition 7, Part (i). First consider the value function that solves problem (C.24). After changing the variables to log-deviations, and using (8), we can rewrite the problem as:

$$\rho V_1(\tilde{y}_1) = \max_{\tilde{p}_1} -\frac{\tilde{y}_1^2}{2} - \psi\frac{\tilde{p}_1^2}{2} + V_1'(\tilde{y}_1)\,\theta\,(m\tilde{p}_1 - (1-n)\,\tilde{y}_1) + \lambda\,(V_2(\tilde{y}_1 - k) - V_1(\tilde{y}_1))\,. \tag{C.29}$$

Here, with a slight abuse of notation, we continue to use $V_1(\cdot)$ to denote the value function defined over the output gap $\tilde{y}_1$ instead of the normalized output level y_1. After transition to high supply, the log output gap declines by k even though the normalized output remains unchanged (since log potential output increases by k).

Using the optimality conditions, we obtain analogues of Eqs. (B.1) and (B.2):

$$\tilde{p}_1 = \frac{\theta m}{\psi} V_1'(\tilde{y}_1) \tag{C.30}$$

$$\begin{pmatrix} \rho + \\ \theta(1-n) + \lambda \end{pmatrix} V_1'(\tilde{y}_1) = -\tilde{y}_1 - \frac{\lambda}{v}(\tilde{y}_1 - k) + V_1''(y_1)\,\theta \begin{pmatrix} m\tilde{p}_1 - \\ (1-n)\,y_1 \end{pmatrix}. \tag{C.31}$$

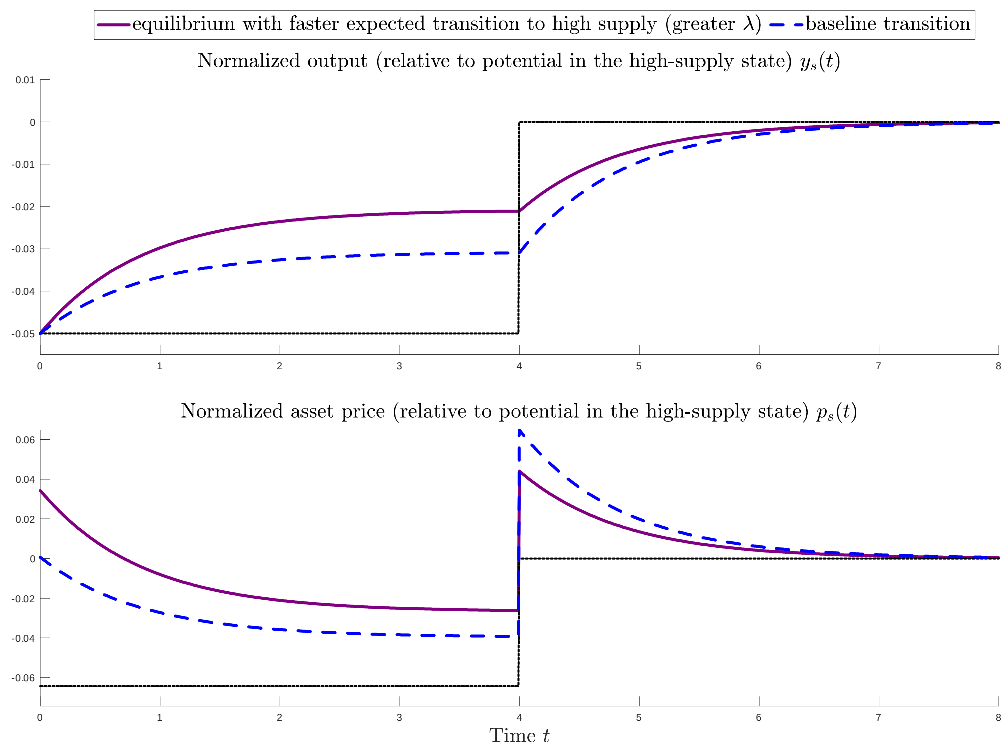

Figure 11: Equilibrium with a greater expected transition rate to the high-supply state ($\tilde{\lambda} > \lambda$, solid lines) and with the baseline rate (λ, dashed lines).

Here, the second line uses (12) to substitute for $V_2'(\tilde{y}_1 - k)$.

We conjecture that the solution has the form in $(C.26)$:

$$V_1(\tilde{y}_1) = a + b\tilde{y}_1 - \frac{1}{2v}\tilde{y}_1^2,$$

Combining this conjecture with Eq. $(C.31)$, and using $y = \tilde{y}_1$ to denote gaps, we obtain:

$$(\rho + \theta(1-n) + \lambda)\left(b - \frac{1}{v}y\right) = -y - \lambda\frac{1}{v}(y-k) - \frac{1}{v}\frac{\theta^2 m^2}{\psi}\left(b - \frac{1}{v}y\right) + \frac{1}{v}\theta(1-n)y.$$

Collecting the terms with y, we obtain:

$$-\frac{1}{v}(\rho + \theta(1-n) + \lambda)y = -y - \frac{1}{v}\lambda y + \frac{1}{v^2}\frac{\theta^2 m^2}{\psi}y + \frac{1}{v}\theta(1-n)y.$$

After canceling $-\frac{1}{v}\lambda y$ from both sides, and dropping y, we obtain the same quadratic (13). Hence, v is the same as before.

Likewise, collecting the terms without y, we obtain:

$$(\rho + \theta(1-n) + \lambda)b = \frac{\lambda k}{v} - \frac{1}{v}\frac{\theta^2 m^2}{\psi}b.$$

After rearranging terms, we solve:

$$b = \frac{\frac{\lambda k}{v}}{\rho + \theta(1-n) + \lambda + \frac{1}{v}\frac{\theta^2 m^2}{\psi}} = \frac{\lambda}{\rho + \lambda + \gamma}\frac{k}{v}$$

Here, the second equality substitutes for γ from $(B.4)$. This proves the value function takes the form in $(C.26)$ with the coefficient $b > 0$ given by $(C.27)$.

Next consider the solution. Substituting the value function into Eq. $(C.30)$, we obtain:

$$\tilde{p}_1(t) = \frac{\theta m}{\psi}\left(b - \frac{1}{v}\tilde{y}_1(t)\right). \tag{C.32}$$

Combining this with $(C.25)$, and using $(B.4)$ to substitute γ, we obtain:

$$\frac{d\tilde{y}_1(t)}{dt} = \frac{\theta^2 m^2}{\psi}b - \gamma\tilde{y}_1(t). \tag{C.33}$$

This equation implies that the output gap converges to a steady-state given by:

$$\tilde{y}_1(\infty) = \frac{\theta^2 m^2}{\psi}\frac{b}{\gamma} = \frac{\gamma - \theta(1-n)}{\gamma}\frac{\lambda}{\rho + \lambda + \gamma}k.$$

Here, we have substituted for b [see $(C.27)$] as well as $\frac{\theta^2 m^2}{\psi v} = \gamma - \theta(1-n)$ [see $(B.4)$]. Note that this also implies $\tilde{y}_1(\infty) \in (0, k)$. Then, Eq. $(C.25)$ implies the asset price gap converges to a steady-state given by:

$$\tilde{p}_1(\infty) = \frac{1-n}{m}\tilde{y}_1(\infty) > 0.$$

This establishes Eq. $(C.28)$ completes the proof of the first part.

Part (ii). Suppose $y_1(0) < y_1(\infty)$, which implies the output gap starts below its steady-state, $\tilde{y}_1(0) < \tilde{y}_1(\infty)$. Absent transition, Eq. $(C.33)$ implies that the output gap monotonically increases toward its steady-state. Combining this with Eq. $(C.32)$, we also obtain that the asset price gap starts above its steady-state, $\tilde{p}_1(0) > \tilde{p}_1(\infty)$, and monotonically declines toward its steady-state. This also implies the normalized output $y_1(0)$ monotonically increases towards its steady-state $y_1(\infty)$; and the normalized asset price monotonically decreases toward its steady-state.

Next consider the time t' at which the economy transitions to the high-supply state (starting with $y_1(0) < y_1(\infty)$). Normalized output remains unchanged. It is below its new potential because, $y_2(t') = y_1(t') < y_1(\infty) < 0$. The asset price gap after transition is given by Eq. (14) whereas the asset price gap before transition is given by Eq. $(C.32)$. That is:

$$\begin{aligned} \tilde{p}_2(t') = p_2(t') &= -\frac{\theta m}{\psi v}y_1(t') \\ \tilde{p}_1(t') &= \frac{\theta m}{\psi}\left(b - \frac{1}{v}\left(y_1(t') - y_1^*\right)\right) \end{aligned}$$

After taking the difference, and substituting $y_1^* = -k$, we obtain:

$$\tilde{p}_2(t') - \tilde{p}_1(t') = \frac{\theta m}{\psi}\left(\frac{k}{v} - b\right) = \frac{\theta m}{\psi}\frac{k}{v}\frac{\rho + \gamma}{\rho + \lambda + \gamma} > 0.$$

Here, the second equality substitutes b from $(C.27)$. Since $p_1^* < 0$, we also have $\tilde{p}_1(t') > p_1(t')$. Thus, we obtain $\tilde{p}_2(t') = p_2(t') > \tilde{p}_1(t') > p_1(t')$. At the time of transition, the normalized asset price as well as its gap increases. This completes the proof. □

D. Appendix: Data sources

This appendix presents the data sources and variable construction used in the main text.

S&P 500 Index and GDP. Data for the S&P 500 index and the GDP used in Figures 1 and 5 is taken from the St. Louis Fed's online database of Federal Reserve Economic Data (FRED) (available at https://fred.stlouisfed.org/series/SP500). The GDP series is nominal, quarterly, annualized, and seasonally adjusted.

Potential GDP. Data for potential GDP used in Figures 1 and 5 is taken from the Congressional Budget Office (CBO) (available at https://www.cbo.gov/data/budget-economic-data#11). We use the projections made in May 2022. The series is nominal, quarterly, and annualized.

Household Net Worth. Data for household net worth used in Figure 5 is taken from the Federal Reserve (available at https://www.federalreserve.gov/releases/z1/dataviz/z1/balance_sheet/table/). The series is nominal and quarterly.

House Price Index. Data for house price index used in Figure 5 is taken from the Federal Housing Finance Agency (FHFA) (available at https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index-Datasets.aspx#mpo). The series is nominal and monthly.

Real Yields and One-Year-Ahead Forward Rates. To construct the forward rates in 7 and the market-bond portfolio in Figures 5 and 6, we use the estimated TIPS term structure data provided by the Federal Reserve, based on the approach by Gürkaynak et al. (2007) (available at https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm).

Specifically, Gürkaynak et al. (2007) estimate the TIPS term structure by properly tuning the parameters of the Nelson-Siegel-Svensson yield curve to approximate actual TIPS yield data. In order to estimate real yields of maturity beyond what is already included in the data set, we use the Nelson-Siegel-Svensson yield curve formula:

$$y(t,\mu) = \begin{array}{c} \beta_0 + \beta_1 \left(\frac{1-\exp(-\frac{\mu}{\tau_1})}{\frac{\mu}{\tau_1}} \right) + \beta_2 \left(\frac{1-\exp(-\frac{\mu}{\tau_1})}{\frac{\mu}{\tau_1}} - \exp\left(-\frac{\mu}{\tau_1}\right) \right) \\ +\beta_3 \left(\frac{1-\exp(-\frac{\mu}{\tau_2})}{\frac{\mu}{\tau_2}} - \exp\left(-\frac{\mu}{\tau_2}\right) \right) \end{array}.$$

Here, $y(t,\mu)$ is the (continuously-compounded) yield of a zero-coupon bond of maturity μ (in years) at date t. The parameters, $\beta_0, \beta_1, \beta_2, \tau_1, \tau_2$ are the Nelson-Siegel-Svensson yield curve parameters at date t (we suppress the dependence on t in the notation). More details can be found in Gürkaynak et al. (2007). From these estimated real yields, we then obtain the real one-year-ahead forward rates as follows:

$$f(t,\mu,1) = (\mu+1)y(t,\mu+1) - \mu y(t,\mu).$$

Here, $f(t,\mu,1)$ is the (continuously-compounded) one-year-ahead forward rate at date t beginning at horizon μ. It is the (continuously-compounded) rate one can obtain at date t for making a risk-free investment μ years later (at date $t+\mu$) for payment one year later (at date $t+\mu+1$).

Market-Bond Portfolio. We construct the price change of the market-bond portfolio by using discrete versions of Eqs. $(29-30)$. We adopt a yearly calibration for the bond maturity (μ) and set $\rho = 0.03$ and $\overline{\mu} = 30$ as described in the main text. We approximate the weights with their discrete-time counterparts:

$$w_\mu = \rho e^{-\rho} \text{ and } W_\mu = \sum_{\tilde{\mu}=\mu}^{\infty} w_{\tilde{\mu}} = \frac{\rho}{1-e^{-\rho}} e^{-\rho}.$$

For each date t, we then construct a yield-based and a forward-based measure:

$$\begin{aligned} p^{MB,yld}(t) &= -\sum_{\mu=1}^{\overline{\mu}-1} w_\mu \mu y(t,\mu) - W_{\overline{\mu}} y(t,\overline{\mu}) \\ p^{MB,fw}(t) &= -\sum_{\mu=1}^{\overline{\mu}} W_\mu f(t,\mu,1). \end{aligned}$$

The time derivative of these measures provides a discrete-time approximation for Eqs. (29) and (30), respectively. The two measures are approximately the same. In Figures 5 and 6, we plot the change of the forward-based measure over time.