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MONETARY POLICY AND ASSET PRICE OVERSHOOTING:
A RATIONALE FOR THE WALL/MAIN STREET DISCONNECT

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ABSTRACT

We analyze optimal monetary policy when asset prices influence aggregate demand with a lag. In this environment, when there is a current or anticipated output gap, the central bank optimally overshoots asset prices (i.e., significantly reduces discount rates to increase asset prices). Asset price overshooting leads to a temporary disconnect between the performance of financial markets and the real economy, but it also accelerates the recovery. A more intense overshooting policy weakens the relationship between inflation and the output gap (i.e., it flattens the Phillips curve). We quantify the policy-induced overshooting through risk-free rates in the Covid-19 recession and find that actual overshooting significantly exceeded the overshooting implied by a Taylor rule benchmark. Our calibrated model suggests this additional overshooting substantially accelerated the recovery. Finally, we show that policy-induced overshooting, along with monetary policy constraints, can shed light on the cyclical variation in the response of asset prices to macroeconomic news.

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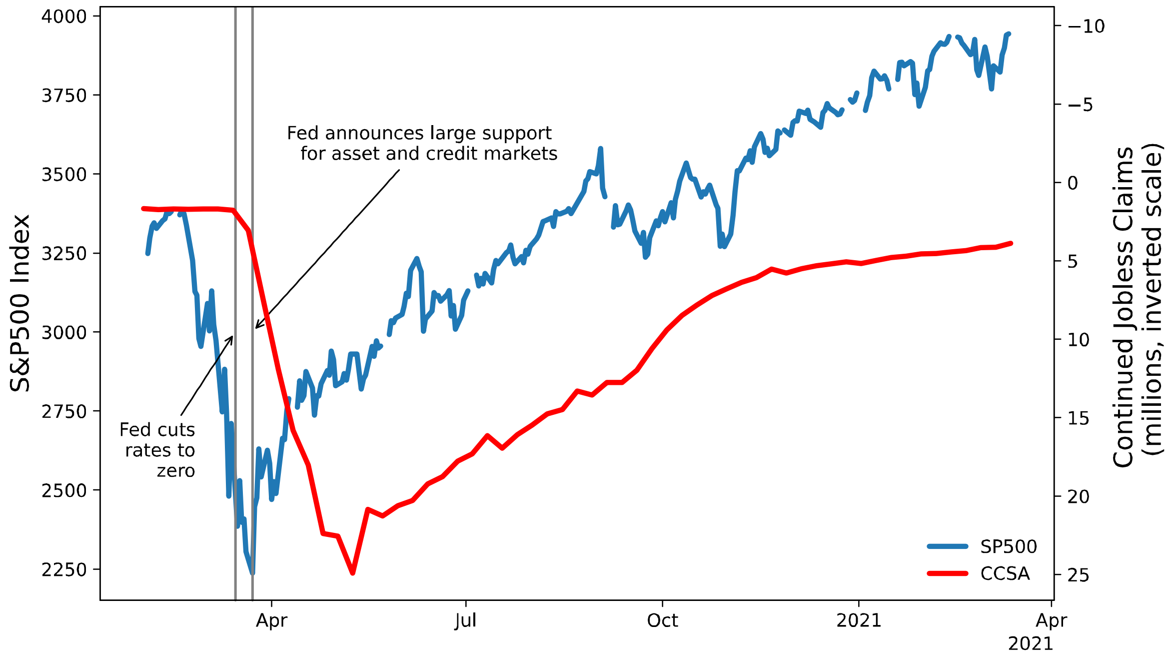

Figure 1: The disconnect between Wall Street and Main Street. Data: S&P500 index and continued jobless claims (seasonally adjusted). Source: FRED.

1. Introduction

The blue line in Figure 1 shows the acute drop in the S&P 500 once the economic impact of the Covid-19 shock became apparent. The figure also shows the equally dramatic recovery of the S&P 500 after the Fed announced its massive policy response (e.g., it cut the policy rate to zero and pledged close to 20% of GDP in asset purchases and credit support facilities). Other financial assets show a similar pattern. While the Fed was successful in reversing the financial meltdown, it did not prevent a collapse in the real economy. The red line in Figure 1 shows that the US continued jobless claims (inverted scale) declined substantially and recovered more slowly.[1] Beyond the early confusion brought about by the unique features of the Covid-19 shock, by 2021-Q1 the stock market vastly exceeded (it had "overshot") its pre-Covid-19 value, while the economy still exhibited a significant negative output gap. The policy support remained massive. In fact, by then a debate had started about whether macroeconomic policy support was excessive and creating frothy conditions in financial markets.

In this paper, we show that this type of disconnect and asset price overshooting is not

[1] This disconnect between the quick recovery of financial markets and the sluggish response of the real economy was the source of much debate, as highlighted by the cover page of The Economist, May 9th, 2020 ("A dangerous gap: The markets v the real economy").

necessarily anomalous: in fact, it is a general feature of well managed recessions. For this purpose, we develop a New-Keynesian model with the key feature that asset prices affect aggregate demand with a lag. In this environment, when there is a current or anticipated output gap, optimal monetary policy *overshoots* asset prices (i.e., significantly reduces discount rates to increase asset prices) and induces a *temporary* disconnect between markets and the economy.

Our model is set in continuous time. Output is determined by aggregate demand due to nominal rigidities. Aggregate demand depends on asset prices through a wealth effect on consumption. The key ingredient is that individual agents adjust their consumption infrequently, driven by a Poisson process that is independent across individuals. Therefore, aggregate asset prices (e.g., stocks, bonds, and houses) affect consumption with a transmission lag, consistent with empirical evidence (see, e.g., Chodorow-Reich et al. (2021)). Monetary policy affects aggregate demand through its impact on asset prices; thus, the policy also affects consumption and aggregate demand with a lag. The central bank sets discount rates to minimize output gaps, taking transmission lags into account.[2]

We focus on a "recovery" scenario in which aggregate demand, dragged down by a recent history of poor economic conditions, is initially below potential output. Over time, demand gradually catches up with potential output and closes the negative output gap. In this context, our main result shows that the equilibrium features *asset price overshooting*: aggregate asset prices are initially high (above their levels consistent with potential output) even though output is low. A central bank that dislikes output gaps boosts asset prices to close the output gap as fast as possible. This boost creates a large, temporary disconnect between financial markets and the real economy, but it also accelerates the recovery. In fact, the asset price boost (and its disconnect with the real economy) is largest at the beginning of the recovery phase, when the output gap is widest. In an appendix, we show that the policy also overshoots asset prices when the output gap is not currently negative but policymakers anticipate it will turn negative in the future (e.g., when the economy is experiencing a sharp but temporary decline in potential output as in the early stages of the Covid-19 recession).

Our baseline setting features fully sticky prices, but we also consider the case in which prices are partially flexible. In this case, inflation is endogenous and determined by a New-Keynesian Phillips curve. We find that the central bank overshoots real asset prices

[2]In a recent speech, Federal Reserve Chairman Powell emphasized the importance of transmission lags: "Finally, we continue to believe that monetary policy must be forward looking, taking into account... the lags in monetary policy's effect on the economy" (Powell (2020)).

by even more than in our baseline setting to fight the disinflationary pressure that results from negative output gaps. Since overshooting closes output gaps more quickly, it also weakens the relationship between inflation and the output gap. Thus, an increase in the central bank's willingness to overshoot asset prices provides one explanation for the weakening of the Phillips curve relation observed in recent decades (see, e.g., Stock and Watson (2019)). We also show that the optimal policy in our model is consistent with the monetary policy rules used in practice. For instance, a Taylor-type rule overshoots real asset prices as long as the interest rate is sufficiently sensitive to the output gap or inflation; and a greater sensitivity to either gap induces more overshooting.

Next, we quantify the policy-induced overshooting driven by risk-free rates, focusing on the Covid-19 recession.[3] To facilitate this exercise, we decompose the aggregate asset price in our setting into a "market-bond portfolio"—driven by forward interest rate changes, and a "residual"—driven by expected cash flows and other factors. The market-bond portfolio captures the policy support to asset prices through risk-free rates. The price of this portfolio increased substantially during the Covid-19 recession (close to the observed increase in stock and house prices). Moreover, while the boom of the market-bond portfolio in 2020 can be partly a response to the temporary rise in the aggregate risk premium, its boom later in the recovery more closely resembles the policy-induced overshooting in our model. We then calibrate the model to benchmark the observed asset price overshooting and to assess its impact on the path of recovery. The data and our calibration suggest that monetary policy overshot asset prices in the Covid-19 recession significantly more than implied by a Taylor-rule benchmark, and that this additional overshooting substantially accelerated the recovery.

In the last section of the paper we introduce a lower-bound constraint on the discount rate that restricts the central bank's ability to achieve a desired level of overshooting. While this constraint was less binding during the Covid-19 recession, where central banks were less (politically) constrained to implement LSAPs and other unconventional policies, it has been (and is likely to remain) a significant constraint for more typical recessions in a low interest rate environment. This lower-bound constraint not only slows down the recovery but it also makes overshooting non-monotonic over time. When the economy starts with a sufficiently negative output gap, the asset price boost starts low (due to the constraint), becomes greater as the economy recovers (and the constraint is relaxed),

[3] While central banks can also have a large impact on the risk premium through unconventional policies and commitments (and these actions were crucial during the Covid-19 recession), we focus on risk-free rates because: (i) they can be directly measured from the data and; (ii) we estimate that the wealth effect they generated accounted for most of the accumulated rise in households' wealth before our calibration target (2021-Q1).

and becomes smaller again as the economy approaches full recovery and the discount rate liftoff. This non-monotonicity provides one explanation for the fact that the impact of macroeconomic news on stock prices depends on the stage of the business cycle (e.g., McQueen and Roley (1993); Boyd et al. (2005); Andersen et al. (2007); Law et al. (2019)). In this context, good news about aggregate demand is good news for asset prices early in the recovery, but bad news once the output gap is small.

Literature review. The concept of an overshooting asset price in response to monetary policy actions was introduced by Dornbusch (1976). While our model shares the sticky prices and sluggish aggregate demand in his model, our overshooting result is a feature of optimal monetary policy rather than the implication of an arbitrage condition. The policy setup in Caballero and Lorenzoni (2014) is closer to our paper. Their small open economy model starts in an overvaluation phase (abnormally high demand for nontradables), and randomly transitions to a normal phase. The optimal policy in their setting is to reduce the overvaluation in the first phase and to overshoot the exchange rate at the transition, both with the goal of relaxing an export sector's financial constraint. The optimal policy also induces overshooting in our model, but for a different reason: to stimulate output in an environment with nominal rigidities and sluggish adjustment of aggregate demand.

Like in Caballero and Simsek (2020, 2021a,b), monetary policy in our model operates through financial markets. The central bank affects asset prices, which in turn affects aggregate demand. The distinctive feature of this paper is the delayed response of aggregate demand to asset prices. In the context of the Covid-19 recession, Caballero and Simsek (2021a) provides an explanation for the large decline in asset prices and highlights the key role of LSAPs in reversing that decline, and this paper provides a rationale for the subsequent Wall/Main Street disconnect (see Figure 1).

Our asset-price decomposition in the context of the Covid-19 recession is related to recent work by Van Binsbergen (2020); Knox and Vissing-Jorgensen (2021). Our market-bond portfolio is the same as the duration-matched fixed-income portfolio analyzed by Van Binsbergen (2020). We focus on the *price change* of this portfolio, which is central for our analysis, whereas Van Binsbergen (2020) focuses on the *total return.* Likewise, Knox and Vissing-Jorgensen (2021) focus on the total return of the stock market and provide a more general asset-price decomposition that incorporates the risk-premium and cash-flow news, in addition to the safe rate news that we analyze. For the Covid-19 recession, Knox and Vissing-Jorgensen (2021) find that the initial decline in stock prices is driven by an increase in the risk premium, but the subsequent recovery and the boom are heavily influenced by declining interest rates, consistent with our findings. More broadly, a growing empirical literature analyzes the stock price changes after the Covid-19 shock and finds a large role for monetary policy (see, e.g., Gormsen and Koijen (2020)).[4]

Our paper is also part of a theoretical New-Keynesian literature (see Woodford (2011); Galí (2015) for reviews). A strand of this literature analyzes optimal monetary policy in environments with transmission lags as in our model (see Svensson (2003); Svensson and Woodford (2007)). The literature typically uses these lags to compare the performance of different monetary policy rules (such as instrument and targeting rules), whereas we focus on the asset pricing implications of optimal policy with lags. We also relate the optimal policy in our environment to Taylor-type policy rules (e.g., Taylor (1993, 1999)).

In terms of the model's ingredients, this paper is related to and supported by an extensive empirical literature documenting that: (i) monetary policy affects asset prices (e.g., Jensen et al. (1996), Thorbecke (1997), Jensen and Mercer (2002), Rigobon and Sack (2004), Ehrmann and Fratzscher (2004), Bernanke and Kuttner (2005), Bauer and Swanson (2020))—these papers find that, on average, an unanticipated 100 bps increase in the policy rate or the one-year treasury yield is associated with a decrease in stock market returns in the range of 5% to 7%; (ii) asset prices affect aggregate demand and output (e.g., Davis and Palumbo (2001), Dynan and Maki (2001), Gilchrist and Zakrajšek (2012), Mian et al. (2013), Kyungmin et al. (2020), Di Maggio et al. (2020), Chodorow-Reich et al. (2021), Guren et al. (2021))—these papers find wealth and balance sheet effects in the range of 3–10 cents on the dollar depending on the sample and the specific asset price;[5] (iii) the effect of asset prices on aggregate demand and output is gradual (e.g., Davis and Palumbo (2001); Dynan and Maki (2001); Lettau and Ludvigson (2004); Carroll et al. (2011); Case and Shiller (2013); Chodorow-Reich et al. (2021))—these papers find that consumption typically takes about two years to fully adjust to stock price changes.

A central feature of our model is the infrequent adjustment of individual consumption (aggregate demand). An extensive literature on durables' consumption (and investment) uses fixed adjustment costs to explain this feature. This literature documents infrequent adjustment in microeconomic data, and explains the inertia it introduces for aggregate durables' consumption and investment (see Bertola and Caballero (1990) for an early survey). There is also a literature that emphasizes infrequent re-optimization for broader

[4]Several studies find that unconventional monetary policies (which we discuss in Section 4) also had a large positive impact on asset prices (e.g., Fed (2020); Cavallino and De Fiore (2020); Arslan et al. (2020); Haddad et al. (2021)). Other studies analyze the broader set of factors that drive asset prices during the Covid-19 recession (e.g., Ramelli and Wagner (2020); Landier and Thesmar (2020); Baker et al. (2020); Davis et al. (2021b,a)).

[5]In addition, Cieslak and Vissing-Jorgensen (2020) show that the Fed pays attention to the stock market, mainly because policymakers believe the stock market affects the economy through a wealth effect.

consumption categories—due to behavioral or informational frictions—and uses this feature to explain the inertial behavior of aggregate consumption (e.g., Caballero (1995); Reis (2006)) as well as asset pricing puzzles (e.g., Lynch (1996); Marshall and Parekh (1999); Gabaix and Laibson (2001)). We take the infrequent adjustment of individual consumption as given (driven by a Poisson process for simplicity) and study its implications for optimal monetary policy.

Finally, as we discuss in Section 5, our results with constrained monetary policy shed some light on the empirical literature documenting that the asset price impact of macroeconomic news depends on the stage of the business cycle.

The rest of the paper is organized as follows. Section 2 introduces our baseline model and establishes our main overshooting result. Section 3 analyzes asset price overshooting with endogenous inflation. Section 4 quantifies the extent and the impact of asset price overshooting in the Covid-19 recession driven by risk-free rates. Section 5 analyzes asset price overshooting with a discount rate lower-bound. Section 6 provides final remarks. Appendices A-C contain omitted derivations, extensions, and data sources, respectively.

2. A model with transmission lags

In this section, we describe our baseline model, define the equilibrium, and establish our main results. We focus on a *recovery* scenario following a recessionary shock to the economy, when aggregate demand is below potential output. We show that, when asset prices affect aggregate demand with transmission lags, optimal monetary policy during the recovery overshoots asset prices. We keep the baseline model simple and discuss various extensions at the end of the section.

2.1. Environment and equilibrium

Our model is a continuous-time variant of the textbook New-Keynesian model presented in Galí (2015). The key difference is the sluggish adjustment of demand to asset prices. For analytical tractability, we assume there are two types of agents denoted by superscript $i = s$ ("stockholders") and $i = h$ ("hand-to-mouth households"). Our focus is on the stockholders, whose sluggish response to asset prices drives aggregate demand. Stockholders own all financial assets (claims on firms' profits) but do not supply any labor. Hand-to-mouth households supply labor (endogenously) and spend all of their income in each period. We introduce these additional agents to decouple stockholders' consumption problem from the labor supply.

Supply side and nominal rigidities. Time $t \geq 0$ is continuous and there is no uncertainty. There is a single factor, labor, supplied elastically by hand-to-mouth households. They have the per-period utility function $\log C^h(t) - \chi \frac{L(t)^{1+\varphi}}{1+\varphi}$, which leads to a standard labor supply curve relegated to Appendix A.1.

A competitive final goods producer combines the intermediate goods according to the CES technology, $Y(t) = \left(\int_0^1 Y(t,\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}$ for some $\varepsilon > 1$. A continuum of monopolistically competitive firms, denoted by $\nu \in [0,1]$, produce the intermediate goods subject to the Cobb-Douglas technology

$$Y(t,\nu) = AL(t,\nu)^{1-\alpha},$$

where $1-\alpha$ denotes the share of labor. Notice that the intermediate good firms make pure profits due to their monopoly power. To simplify the distribution of output across factors, we assume the government taxes part of these profits (lump-sum) and redistributes to workers (lump-sum) so that workers earn and spend their production share of output,[6]

$$C^h(t) = (1-\alpha)Y(t). \tag{1}$$

With these assumptions, the equilibrium labor supply without nominal rigidities solves $\chi(L^*)^{1+\varphi} = \frac{\varepsilon-1}{\varepsilon}$ (see Appendix A.1). We refer to L^* as the *potential* labor supply and $Y^* = A(L^*)^{1-\alpha}$ as potential output.

Set against this flexible-prices benchmark, our intermediate good firms have, for now, fully sticky nominal prices (we endogenize inflation in Section 3). Since the intermediate goods producers operate with a markup, they find it optimal to meet the demand for their good, even when this demand deviates from potential output (for relatively small deviations, which we assume). Therefore, output is determined by aggregate demand, which depends on the consumption of stockholders and hand-to-mouth households:

$$Y(t) = C^s(t) + C^h(t). \tag{2}$$

Combining this with $C^h(t) = (1-\alpha)Y(t)$ from Eq. (1), we obtain

$$Y(t) = \frac{C^s(t)}{\alpha}. \tag{3}$$

[6]Formally, letting $T(t)$ denote the appropriate transfer, hand-to-mouth agents' income is $W(t)L(t) + T(t) = (1-\alpha)Q(t)Y(t)$, where $W(t)$ is the nominal wage and $Q(t) = \left(\int Q(t,\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}$ is the nominal price of the final good. $Q(t,\nu)$ denotes the nominal price of good ν.

Hand-to-mouth households create a multiplier effect, but output is ultimately determined by *stockholders'* spending, $C^s(t)$.

Demand side with sluggish adjustment. In financial markets, there are two assets. First, there is a "market portfolio" that is a claim on firms' profits, $\alpha Y(t)$ (the production share of capital). We let $P(t)$ and $R(t)$ denote the real price and the real discount rate of the market portfolio. Since there is no risk, by no arbitrage the discount rate satisfies

$$R(t) = \frac{\alpha Y(t) + \dot{P}(t)}{P(t)}. \tag{4}$$

Second, there is a risk-free asset in zero net supply with real interest rate $R^f(t)$. The central bank controls $R^f(t)$ by setting the nominal interest rate. Since there is no inflation (for now), the nominal and the real interest rates are the same. Since there is no risk, the interest rate and the discount rate are also the same, $R^f(t) = R(t)$. Going forward, we drop $R^f(t)$ from the notation and assume the central bank directly controls the discount rate. We discuss how the risk premium would affect our analysis in Section 2.3.4.

There is a continuum of identical stockholders that own the market portfolio and make a consumption-savings and a portfolio choice. These stockholders have time-separable log utility, with discount rate ρ. If there were no other frictions, stockholders would spend a constant fraction of their wealth, $C^s(t) = \rho P(t)$. Assuming the central bank sets the asset return to target an output level equal to its potential, and using Eq. (3), the economy would *immediately* reach a steady-state consistent with the flexible-prices benchmark:

$$\begin{aligned} Y(t) &= Y^*, \quad C^s(t) = C^{s,*} = \alpha Y^* \\ R(t) &= R^* = \rho \quad \text{and} \quad P^* = \frac{\alpha Y^*}{\rho}. \end{aligned} \tag{5}$$

We depart from this environment by assuming that *stockholders do not continuously adjust their allocations*. This is our key friction: At every instant, a random fraction of stockholders adjusts, with constant hazard θ. Their allocations remain unchanged until the next time they have a chance to adjust. Formally, there is a representative stockholder whose wealth equals aggregate wealth, $P(t)$. When adjusting, this stockholder chooses her new consumption level according to the following rule:

$$c^{s,adj}(t) = mp(t) + ny(t). \tag{6}$$

Here, $x(t) = \log\left(\frac{X(t)}{X^*}\right)$ is the log-deviation of the corresponding variable from its potential level in (5). The parameters, $m, n \in (0,1)$, capture the sensitivity of consumption to asset prices and output. Individual stockholders follow a similar rule scaled by their wealth.[7]

We adopt the consumption rule in (6) to simplify the exposition. When $m = 1$ and $n = 0$, we have the optimal rule of a continuously-adjusting stockholder, $c^{s,adj}(t) = p(t)$ (equivalently, $C^{s,adj}(t) = \rho P(t)$). This rule, however, is not optimal for a sluggish stockholder *who incorporates the fact that she might not get to adjust in future periods.* In Appendix B.1, we show that the optimal (log-linearized) consumption rule is given by (6) with appropriate weights m, n (see Section 2.3.2). The weights are endogenous to the dynamic equilibrium path but exogenous to monetary policy decisions (as long as the central bank sets discount rates without commitment, which we assume). We do not focus on this case and instead treat m, n as exogenous parameters to avoid fixed-point arguments that are orthogonal to our main contributions. In Appendix B.1, we show that our main result also holds with the optimal consumption rule.

Recall that stockholders adjust with constant hazard θ. Assuming that the adjusting stockholders are randomly selected, stockholders' total spending follows

$$\dot{C}^s(t) = \theta\left(C^{s,adj}(t) - C^s(t)\right).$$

Hence, total spending increases if and only if the adjusted spending exceeds the current level of spending, $C^s(t)$. Log-linearizing around the potential steady-state, we obtain

$$\dot{c}^s(t) = \theta\left(c^{s,adj}(t) - c^s(t)\right).$$

Using Eqs. (3) and (6), the output gap follows

$$\dot{y}(t) = \theta\left(mp(t) + ny(t) - y(t)\right). \tag{7}$$

The initial output gap, $y(0) = c^s(0)$, is exogenous (determined by an unmodeled history). The asset price gap, $p(t)$, affects the output gap, but with a lag.

For symmetry, we assume stockholders are also sluggish with respect to their portfolio choices, although this does not play any role in our analysis. Specifically, Eq. (4) also holds with sluggish stockholders. Given these returns, the stockholders that adjust are

[7] Formally, let $a^i(t)$ denote a stockholder's wealth and $\alpha^i(t) = \frac{a^i(t)}{P(t)}$ denote her wealth share. A stockholder with wealth share $\alpha^i(t)$ follows the rule $C^{i,adj}(t) = C^{s,*}\alpha^i(t)\exp(mp(t) + ny(t))$. Aggregating across all adjusting stockholders, we obtain (6) since wealth shares satisfy $\sum_i \alpha^i(t) = 1$. With this rule, there might be paths along which some stockholders' wealth becomes zero, e.g., if the stockholder does not readjust for a long time. If this happens, the budget constraint binds and consumption falls to zero. We ignore these paths since we focus on relatively small shocks and log-linearized dynamics.

indifferent to changing their portfolios. We assume all stockholders invest all of their wealth in the market portfolio, which ensures market clearing. We also log-linearize Eq. (4) around the potential levels in (5), to obtain

$$r(t) = \frac{\rho}{1+\rho}(y(t) - p(t)) + \frac{1}{1+\rho}\dot{p}(t). \tag{8}$$

Here, $r(t) \equiv \log \frac{1+R(t)}{1+\rho}$ is the log-deviation of the *gross* discount rate from its potential. Eq. (8) is a continuous time version of the standard Campbell-Shiller approximation.

Monetary policy. The central bank implements a path of output, asset price, and discount rate gaps, $[y(t), p(t), r(t)]_{t\in(0,\infty)}$, that satisfy Eqs. (7) and (8) given $y(0)$. We can think of the central bank as targeting a path of output and asset price gaps, $[y(t), p(t)]_{t\in(0,\infty)}$, that satisfy Eq. (7) given $y(0)$. Then Eq. (8) describes the equilibrium rate path, $[r(t)]_{t\in(0,\infty)}$, that the central bank needs to set to achieve its target.

We assume the central bank's objective function is

$$V(0, y(t)) = \int_0^\infty e^{-\rho t}\left(-\frac{1}{2}y(t)^2 - \frac{\psi}{2}p(t)^2\right)dt. \tag{9}$$

As usual, the central bank dislikes output gaps, $y(t)$. We assume a quadratic cost function, which leads to closed-form solutions. In addition, the central bank also dislikes asset price gaps, $p(t)$. In the limit $\psi \to 0$, we have the conventional setup in which the central bank does not (directly) pay attention to asset prices. Our main results hold in this conventional limit, but optimal policy overshoots asset prices by an extreme amount. In practice, a large asset price overshooting could lead to a number of concerns that range from financial stability (e.g., high asset prices can increase the risk of a collapse) to wealth redistribution (e.g., high asset prices can increase inequality). We capture these types of concerns with the parameter $\psi > 0$, which we refer to as "aversion-to-overshooting." The central bank's discount rate is the same as that of the stockholders, ρ.

Finally, we assume the central bank sets the current policy *without* commitment: that is, it sets the current asset price gap $p(t)$, taking the path of future gaps as given. In this case, the central bank's policy problem can be formulated recursively as[8]

$$\begin{aligned}
\rho V(y) &= \max_p -\frac{y^2}{2} - \psi\frac{p^2}{2} + V'(y)\,\dot{y}, \\
\dot{y} &= \theta\left(mp - (1-n)\,y\right), \\
V(y) &\leq 0 \text{ and } V(0) = 0.
\end{aligned} \tag{10}$$

The constraints in the last line follow from the objective function in (9) and ensure that we pick the correct solution to the recursive problem. We define the equilibrium as follows.

Definition 1. *A (log-linearized) equilibrium with optimal monetary policy,* $[y(t), p(t), r(t)]_{t=0}^{\infty}$, *is such that the path of output and asset price gaps,* $[y(t), p(t)]$, *solve the recursive problem* (10) *and the discount rate satisfies Eq.* (8)*.*

2.2. Optimal overshooting during the recovery

We next solve for the equilibrium and establish our main results about overshooting. To capture the recovery from a recessionary shock, we focus on the case with a negative initial output gap, $y(0) < 0$, where aggregate demand has yet to catch up with potential output. Our main result shows that the optimal policy features *asset-price overshooting:* the central bank optimally chooses a *positive asset price gap.* That is, even though output is below its potential level, the asset price is high and above its potential level.

Consider the planner's problem (10). In the appendix, we conjecture and verify that the solution is a quadratic function,

$$V(y) = -\frac{1}{2v}y^2, \tag{11}$$

$$\text{where } 0 = v^2 - (\rho + 2\theta(1-n))\,v - \frac{\theta^2 m^2}{\psi}. \tag{12}$$

Here, $v > 0$ ("value") denotes an endogenous coefficient. Since $V(y) < 0$, we also have $v > 0$: the solution corresponds to the positive root of (12).

Combining the value function in (11) with the optimality condition, we solve for the optimal asset price as

$$p = \frac{\theta m}{\psi}V'(y) \Longrightarrow p(t) = -\frac{\theta m}{\psi v}y(t). \tag{13}$$

[8] The lack of commitment does not restrict monetary policy in our baseline model. The Principle of Optimality implies that maximizing (9) subject to (7) is equivalent to solving problem (10). Lack of commitment is restrictive when we introduce inflation (see Section 3) or when we endogenize the consumption function in (6) (see Section 2.3).

This expression illustrates the overshooting of asset prices. Starting with a negative output gap, the optimal asset price is above its potential level. The asset price is the central bank's policy lever. Since the asset price increases the output gap, the central bank increases the price whenever the output gap is negative.

Next consider the change in output gaps along the optimal path. Combining Eqs. (13) and (7), we obtain

$$\dot{y}(t) = -\gamma y(t), \qquad \text{where } \gamma \equiv \theta\left(\frac{\theta m^2}{\psi v} + 1 - n\right) > 0. \tag{14}$$

The composite parameter, γ, captures the convergence rate. Starting with a negative output gap, and an associated positive asset price gap (in view of overshooting), both gaps monotonically converge to zero at rate γ.

Finally, consider the discount rate gap. Combining Eqs. (8), (13), and (14), we obtain

$$\begin{aligned} r(t) &= \frac{\rho}{1+\rho}\left(y(t) - p(t)\right) + \frac{1}{1+\rho}\dot{p}(t) \\ &= \frac{\rho}{1+\rho}\left(1 + \frac{\theta m}{\psi v}\right) y(t) + \frac{1}{1+\rho}\frac{\theta m}{\psi v}\gamma y(t) \\ &= \mathcal{R} y(t), \\ \text{where } \mathcal{R} &\equiv \frac{\rho}{1+\rho} + \frac{\theta m}{\psi v}\left(\frac{\gamma+\rho}{1+\rho}\right). \end{aligned} \tag{15}$$

Hence, the discount rate gap is (positively) proportional to the output gap. Starting with a negative output gap, the discount rate gap starts below zero and gradually increases, $r(0) < 0$ and $\dot{r}(t) > 0$. The following result summarizes this discussion.

Proposition 1. *The value function is given by* (11), *where* $v > 0$ *is the positive solution to* (12). *The equilibrium path of the output, asset price, and discount rate gaps,* $[y(t), p(t), r(t)]_{t=0}^{\infty}$, *is characterized by Eqs.* $(13), (14), (15)$. *Starting with a negative output gap,* $y(0) < 0$, *the equilibrium features* ***asset price overshooting****: the asset price is above its potential,* $p(0) > 0$, *and the discount rate is below its potential,* $r(0) < 0$. *Over time, all gaps converge to zero at the exponential rate* $\gamma = v - (\rho + \theta(1-n)) > 0$.

Figure 2 illustrates the equilibrium dynamics for a particular parameterization starting with a negative output gap. The equilibrium (the solid lines) features overshooting: the central bank sets a positive asset price gap and gradually closes the output gap. The central bank achieves this outcome by starting with a low discount rate, and then gradually increasing the discount rate and reducing the asset price gap. The figure also illustrates the case without overshooting ($\psi = \infty$), where the central bank sets the asset price equal

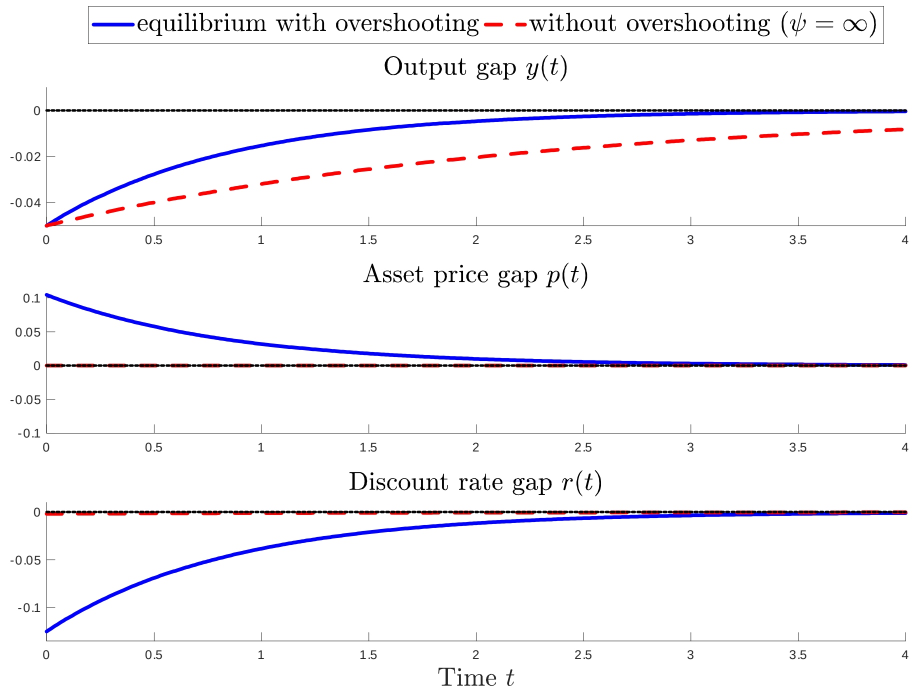

Figure 2: A simulation of the equilibrium with low ψ (solid lines) and high ψ (dashed lines). The dotted lines correspond to the potential levels for the corresponding variables.

to its potential. In this case, output gaps are closed more slowly and the economy operates below its potential for a longer time.

In Appendix A.2, we further establish the comparative statics of overshooting. We find that the central bank overshoots by more (greater $\left|\frac{p(t)}{y(t)}\right| = \frac{\theta m}{\psi v}$) and induces a faster recovery (greater γ) when the output-asset price relation is stronger—in the sense that the spending response to asset prices is either faster (with smaller lags) or larger. Strengthening the output-asset price relation generates two counteracting effects. On the one hand, it makes the asset price overshooting more effective—which induces more overshooting. On the other hand, it enables the central bank to achieve the same impact on the output gap with smaller (or shorter) overshooting—which induces less overshooting. With quadratic preferences (which are standard and commonly used in the literature), the first force dominates and the central bank induces more overshooting when it is more effective.

2.3. Overshooting in richer environments

While we keep our baseline model simple, our main overshooting result extends to richer environments.

2.3.1. Overshooting with growth

In our model, the asset price *level* also declines over time (after an initial jump). This feature is not essential to the argument. The same result would still apply for the asset price *gap* in a variant with productivity growth. In this case, the potential asset price would also be increasing over time (at the growth rate), so overshooting would not necessarily imply a declining asset price level. It would only imply a *frontloading* of some of the future gains on the market portfolio.

2.3.2. Overshooting with sophisticated stockholders

For simplicity, we assume that adjusting stockholders exogenously follow the rule in (6). In Appendix B.1, we consider the case with fully sophisticated stockholders who choose their consumption optimally *anticipating that they will readjust in the future with Poisson probability* θ*.* Proposition 6 shows that, as long as stockholders' adjustment is not too sluggish ($\theta > \rho$) and the planner sets monetary policy without commitment (which is the case we focus on), our main result holds in this setting. Along the equilibrium path, the optimal consumption rule takes the form in (6) with endogenous coefficients:

$$\begin{aligned} c^{s,adj}(t) &= m(\gamma)\, p(t) + n(\gamma)\, y(t), \\ \text{where } m(\gamma) &= \frac{\theta - \rho}{\theta + \gamma} \in (0,1) \text{ and } n(\gamma) = \frac{\rho}{\theta + \gamma}. \end{aligned}$$

Absent commitment, the planner takes these coefficients as given. Therefore, our earlier analysis also applies in this case. Recall that the convergence rate γ also depends on the coefficients of the consumption rule, m, n. Hence, with fully rational stockholders, the equilibrium corresponds to a fixed point that we characterize in the appendix.

2.3.3. Preemptive overshooting

In the main text, we focus on a recovery scenario in which the output gap is negative and the central bank's main concern is to close it as quickly as possible. In Appendix B.2, we extend our analysis to a situation in which the main concern is not the current output gap but the anticipation that it will widen in the near future. This situation may arise, for example, when the economy is experiencing a sharp but temporary decline in potential output, as in the Covid-19 recession. In the recession phase, potential output has experienced a deep contraction but is expected to increase according to a Poisson event—in which case the economy transitions to a recovery phase (as in the main text). We show that the recession features *preemptive overshooting*: the central bank boosts asset prices *even if output is at its (depressed) potential level.* The central bank anticipates that the recovery will start with a large negative output gap due to the inertial behavior of aggregate demand. Therefore, the central bank acts preemptively to boost asset prices and aggregate demand during the recession, to ensure that aggregate demand is not too depressed during the early stages of the recovery. We also find that the central bank overshoots asset prices more when it anticipates a faster transition to recovery. In fact, by frontloading much of the overshooting to the recession, the central bank might do less overshooting when the recovery arrives (compared to the early stages of the recession).

2.3.4. Overshooting with a time-varying risk premium

A more important omission from our analysis is the lack of a risk premium. In practice, aggregate wealth is associated with a time-varying risk premium (see, e.g., Cochrane (2011)). We could incorporate a risk premium without changing our main conclusions. Suppose the discount rate on the market portfolio is given by $r(t) = r^f(t) + \xi(t)$, where $r^f(t)$ is the risk-free rate and $\xi(t)$ is the risk premium. As long as the elasticity of intertemporal substitution is equal to one, our analysis would still apply, but the central bank would target *the discount rate on the market portfolio*, $r(t)$, as opposed to the risk-free rate, $r^f(t)$. For instance, Eq. (13) would apply: the optimal policy would still overshoot asset prices. Eq. (15) would also apply and imply that the central bank should set the risk-free rate according to, $r^f(t) = -\xi(t) + \mathcal{R}y(t)$. If the risk premium is countercyclical (as suggested by empirical evidence), $\xi(t) = \xi_0 - \xi_1 y(t)$, then the risk-free rate becomes more procyclical than in our model $r^f(t) = -\xi_0 + (\xi_1 + \mathcal{R})\, y(t)$. In a demand recession, the optimal policy cuts the risk-free rate aggressively to first "undo" the rise in the risk premium and then to overshoot asset prices as in our model.

The presence of a risk premium not only leaves our main result unchanged but it also expands the policies the central bank can use to affect the discount rate on the market portfolio, $r(t)$. Even if conventional monetary policy is constrained due to, e.g., an effective lower bound on the interest rate, the central bank can reduce the risk premium, $\xi(t)$, via unconventional policies. In Caballero and Simsek (2020), we formalize this argument in a model without transmission lags. In that model, large-scale asset purchases can reduce the risk premium by transferring risk to the government's balance sheet. These policies are especially powerful after a large surprise shock, such as Covid-19, that leads to an endogenous asset price spiral that increases the risk premium. We discuss the role of different types of monetary policies in the Covid-19 recession in Section 4.

3. Overshooting with endogenous inflation

So far we have assumed nominal prices are fully sticky. In this section, we allow some price flexibility and endogenize inflation. In this environment, negative output gaps induce disinflation, and *these disinflationary concerns induce the central bank to overshoot by more than in our baseline model.* We also find that overshooting weakens the relationship between inflation and the output gap. Hence, an increase in the central bank's willingness to overshoot asset prices can provide one explanation for the weakening of the Phillips curve relation observed in the data in recent decades. Finally, we show that the optimal policy in our model is consistent with the monetary policy rules used in practice.

Consider the same model as in Section 2 with the difference that intermediate firms' nominal prices are not fully sticky. We adopt the standard Calvo setup: at each instant a randomly selected fraction of firms reset their nominal price, with constant hazard. This price remains unchanged until the firm gets to adjust again. In Appendix A.1, we characterize the firm's optimal price setting decision. We log-linearize the equilibrium around a zero-inflation benchmark and obtain the New-Keynesian Phillips curve,

$$\rho\pi(t) = \kappa y(t) + \dot{\pi}(t) \quad \Longrightarrow \pi(t) = \kappa \int_t^\infty e^{-\rho(s-t)} y(s)\, ds. \tag{16}$$

Here, $\pi(t)$ denotes the log-deviation of the price level from its steady-state level. The parameter, κ, is a composite price flexibility parameter that depends on the rate of price adjustment along with other parameters. We also express inflation as a discounted sum of future output gaps (assuming inflation is bounded, which is the case in equilibrium).

Since the economy features inflation, the nominal and the real rates are no longer the same. We denote the log-linearized nominal and real discount rates with $r^n(t)$ and $r(t) = r^n(t) - \pi(t)$. The planner effectively "chooses" the real discount rate by setting the nominal rate appropriately, given its implemented inflation path. We now assume the planner maximizes a value function that incorporates the costs of inflation [see (9)],

$$V(0, y(0)) = \int_0^\infty e^{-\rho t} \left(-\frac{y(t)^2}{2} - \phi \frac{\pi(t)^2}{2} - \psi \frac{p(t)^2}{2} \right) dt. \tag{17}$$

The parameter, $\phi \geq 0$, captures the planner's weight on inflation relative to output gaps. We assume the planner sets the discount rate *without commitment.*[9]

[9] Unlike in Section 2, the no-comitment constraint binds in this section. The planner might want to promise to fight inflation more aggressively in the future in order to influence current inflation expectations. We abstract from these types of commitment policies since they are not our focus and their

In this setting, the equilibrium is a fixed point of an inflation *function*, $\boldsymbol{\pi}(y)$, and an asset price *policy function*, $\mathbf{p}(y)$. Given the policy function, and the implied path for the output gap, inflation satisfies (16) . Given the inflation function, the asset price policy is optimal for the planner. That is, it solves the following analogue of problem (10):

$$\begin{aligned}
\rho V(y; \boldsymbol{\pi}) &= \max_p -\frac{y^2}{2} - \phi \frac{\boldsymbol{\pi}(y)^2}{2} - \psi \frac{p^2}{2} + \frac{dV(y; \boldsymbol{\pi})}{dy} \dot{y}, \\
\dot{y} &= \theta (mp - (1-n) y), \\
V(y) &\leq 0 \text{ and } V(0) = 0.
\end{aligned}$$

The equilibrium has the same structure as in the previous section. In particular, the output gap converges to zero at a constant, endogenous rate, $y(s) = y(t) e^{-\gamma(s-t)}$. Substituting this into Eq. (16), the inflation function has a closed-form solution,

$$\boldsymbol{\pi}(y) = \frac{\kappa}{\rho + \gamma} y. \tag{18}$$

Output gaps translate into more inflation when prices are more flexible (greater κ) and when the gaps converge more slowly (smaller γ).

Likewise, the value function is still a quadratic, $V(y; \boldsymbol{\pi}) = -\frac{1}{2v(\gamma)} y^2$, which leads to a similar policy function as before [cf. (13)],

$$\mathbf{p}(y) = -\frac{\theta m}{\psi v(\gamma)} y. \tag{19}$$

The difference is that the value coefficient, $v(\gamma)$, is now endogenous to the convergence rate and solves a quadratic that we relegate to the appendix. The equilibrium is then a fixed-point of Eq. (14), which describes the convergence rate as a function of $v(\gamma)$,

$$\gamma \equiv \theta \left(\frac{\theta m^2}{\psi v(\gamma)} + 1 - n \right). \tag{20}$$

The following result establishes the existence and comparative statics of equilibrium.

Proposition 2. *Consider the model with endogenous inflation. In equilibrium, the output gap converges to zero at a constant rate,* $\gamma > 0$*. The convergence rate corresponds to the fixed point of Eq.* (20)*, where* $v(\gamma)$ *is the positive solution to Eq.* (A.30) *in the appendix. Inflation and output are linear functions of the output gap given by Eqs.* (18) *and* (19)*.*

The equilibrium features asset price overshooting, $\frac{d\mathbf{p}(y)}{dy} = -\frac{\theta m}{\psi v(\gamma)} < 0$*. Greater price*

benefits are well understood (see, e.g., Clarida et al. (1999)).

flexibility strengthens the relation between inflation and the output gap, $\frac{d}{d\kappa}\left|\frac{d\boldsymbol{\pi}}{dy}\right| > 0$*; it decreases the planner's value for a given output gap,* $\frac{dv(\gamma)}{d\kappa} < 0$*; it increases the extent of overshooting,* $\frac{d}{d\kappa}\left|\frac{d\mathbf{p}}{dy}\right| > 0$*; and it accelerates convergence to potential outcomes,* $\frac{d\gamma}{d\kappa} > 0$.

The result shows that the planner also overshoots asset prices in this case. In fact, *greater price flexibility induces greater overshooting.* Intuitively, price flexibility makes output gaps costlier due to the endogenous inflation response. The planner then induces greater asset price overshooting to close the output gaps more quickly.

Implications for the slope of the Phillips curve. In recent decades, the slope of the Phillips curve in the U.S. has declined substantially: specifically, the output gap does not seem to correlate with inflation as strongly as it did before the early 2000s (see, e.g., Ball and Mazumder (2011); Stock and Watson (2019)). Our model in this section offers one explanation based on an increase in the central bank's willingness to overshoot asset prices.

Corollary 1. *Consider the model with inflation. Reducing the planner's aversion to overshooting induces faster convergence of output gaps to zero,* $\frac{d\gamma}{-d\psi} > 0$*; and weakens the relation between inflation and the current output gap,* $\frac{d}{-d\psi}\left|\frac{d\boldsymbol{\pi}}{dy}\right| < 0$.

When the planner overshoots asset prices by more, it closes the output gaps faster. This keeps future inflation expectations close to the inflation target. When price setters expect future inflation to be close to the target, they change their prices by a smaller amount in response to the current output gap [see Eq. (16)]. Therefore, overshooting weakens the correlation between inflation and the *current* output gap, *even if the structural price flexibility coefficient,* κ*, remains unchanged.*[10]

Relationship to monetary policy rules. A strand of the New-Keynesian literature focuses on *interest rate rules* for monetary policy that are easy to implement and that approximate the fully optimal policy. For instance, *Taylor rules* prescribe the interest rate response to output and inflation gaps (see, e.g., Taylor (1993, 1999)). These rules describe the *actual* monetary policy in the U.S. reasonably well in recent decades, and they are used as benchmarks at the Fed's policy meetings. We next show that these rules

[10] In recent work, Hazell et al. (2020) use cross-regional data to argue that κ has indeed remained relatively stable over time. They offer a complementary interpretation of the weakening of the Phillips curve that relies on changes in *long-run* inflation expectations, whereas we emphasize the medium-run inflation expectations associated with changes in (anticipated) output-gap dynamics.

are related to the optimal policy in our setting. To see the relationship, consider the Taylor rule in our setting:

$$r^n(t) = \pi(t) + \tau_y y(t) + \tau_\pi \pi(t). \tag{21}$$

In particular, the policy sets the *real* discount rate relative to the steady state, $r(t) = r^n(t) - \pi(t)$, as a function of the output gap and inflation relative to the target (which we assume to be zero). The parameters τ_y, τ_π capture the rule's sensitivities to the output gap and inflation, respectively. In our model with optimal policy, inflation is proportional to the output gap, $\pi(t) = \frac{\kappa}{\rho+\gamma} y(t)$. Substituting this relation into Eq. (21) and comparing the resulting expression with Eq. (15), we establish an equivalence result.

Corollary 2. *Consider the model with inflation. Let $v(\psi), \gamma(\psi)$ denote the equilibrium coefficients as a function of the aversion to overshooting (characterized in Proposition 2). The optimal policy is equivalent to a Taylor rule with coefficients that satisfy*

$$\tau_y + \tau_\pi \frac{\kappa}{\rho + \gamma(\psi)} = \mathcal{R}(\psi), \tag{22}$$

where $\mathcal{R}(\psi) = \frac{\rho}{1+\rho} + \frac{\theta m}{\psi v(\psi)} \frac{\gamma(\psi)+\rho}{1+\rho}$. For any τ_y, τ_π that satisfy,

$$\tau_y + \tau_\pi \frac{\kappa}{\rho + \gamma(\infty)} > \frac{\rho}{1+\rho}, \tag{23}$$

there exists a unique $\psi \in (0, \infty)$ that ensures Eq. (22). In particular, a Taylor rule that is sufficiently sensitive to the output gap or inflation is equivalent to an optimal overshooting policy. All else equal, the implied aversion to overshooting is smaller when the Taylor rule is more sensitive to either gap, $\frac{\partial \psi}{\partial \tau_y} < 0$, $\frac{\partial \psi}{\partial \tau_\pi} < 0$.

When the policy follows a Taylor rule, the output gap has counteracting effects on the price of the market portfolio. On the one hand, a low output gap decreases cash flows, which tends to reduce the asset price. On the other hand, a low output gap decreases the real discount rate, which tends to increase the asset price. Condition (23) says that the second effect dominates—*the asset price overshoots*—as long as the discount rate is sufficiently sensitive to the output gap. The required condition is satisfied for the parameters proposed by Taylor (1993), $\tau_y = \tau_\pi = 0.5$, when we set the steady-state dividend yield to a standard level, $\rho = 0.03$. Hence, *asset price overshooting is implied by the Taylor rules used in practice.* Moreover, making the rule more sensitive to either the output gap or inflation is equivalent to decreasing the aversion to overshooting. As observed by Taylor (1999), the policy has become more sensitive to gaps in recent years, which suggests that central banks have become less averse to overshooting asset prices.

An important caveat is that Taylor rules apply to the risk-free rate, whereas the overshooting policy targets the discount rate on the market portfolio, which might also include a risk premium. In general, our model suggests the Taylor rule should be modified to include a direct response to the aggregate risk premium, $\xi(t)$ (see also Section 2.3.4):

$$r^{f,n}(t) - \pi(t) = -\xi(t) + \tau_y y(t) + \tau_\pi \pi(t). \tag{24}$$

If the policy does not respond to the risk premium, then it might fail to overshoot asset prices in recessions in which the risk premium increases substantially (such as financial crises). Viewed from this lens, our model suggests overshooting has been particularly salient in the Covid-19 recession because this recession was *not* driven by a financial shock, and the initial damage to financial markets was contained by aggressive unconventional monetary policy (as well as fiscal policy).

Finally, while we focus on Taylor-type interest rate rules for concreteness, it should be clear from our analysis that asset price overshooting is also implied by other monetary policy rules *as long as the rule prescribes a sufficiently aggressive response of the interest rate to output or inflation gaps*. In fact, *inflation-targeting* rules (as discussed in, e.g., Svensson and Woodford (2007); Svensson (2010)) might be more likely to induce asset price overshooting because they are more flexible. For instance, these rules would allow the interest rate to respond to risk-premium shocks as in (24), as long as this response helps achieve the inflation target in subsequent periods, which is the case in our model.

4. Quantifying the overshooting in the Covid-19 recession

Although our main results are qualitative, in this section we attempt to quantify the extent and impact of asset price overshooting. We focus on the recovery from the Covid-19 recession, but our approach is flexible and can be applied to other recessions. While there were many policy tools used during the Covid-19 shock (see Section 2.3.4), we focus on risk-free rates because: (i) they can be directly measured from the data and; (ii) we estimate that the wealth effect they generated accounts for most of the accumulated rise in households' wealth before our calibration target date (2021-Q1).

To quantify the overshooting through risk-free rates, we decompose the market portfolio into a component driven by forward interest rates ("market-bond portfolio") and a component driven by other factors ("residual/other"), along the lines of Van Binsbergen (2020); Knox and Vissing-Jorgensen (2021). The market-bond portfolio provides a measure of the policy support (on asset prices) that operates through risk-free rates. This component increased substantially in the Covid-19 recession (close to the observed increase in stock and house prices). While its increase in 2020 might be partly in response to a rise in the risk premium, its subsequent behavior (as of 2021-Q1) more closely resembles the policy-induced overshooting in our model—since the aggregate risk premium was no longer elevated. Our analysis also suggests that the Fed's LSAP programs for safe assets played an important role in driving the market-bond portfolio in this recession.

We conclude the section with a calibration exercise to benchmark the observed asset price overshooting and to assess its impact on the path of recovery. The data and our calibration suggest that monetary policy overshot asset prices in the Covid-19 recession substantially more than implied by a Taylor-rule benchmark, and that this additional overshooting significantly accelerated the recovery.

4.1. A price decomposition based on the market-bond portfolio

To state our decomposition result, we define a fixed-income portfolio that matches the *duration* of the market portfolio strip-by-strip. Formally, consider a portfolio of zero-coupon bonds with face values that match the steady-state payoffs of the dividend strips of the market portfolio (αY^*). We refer to this portfolio as the *market-bond portfolio* and denote its price with $P^{MB}(t)$. By no-arbitrage, this price satisfies

$$P^{MB}(t) = \int_0^\infty P^{MB}(t,\mu)\, d\mu, \qquad \text{where } P^{MB}(t,\mu) \equiv \alpha Y^* e^{-\int_t^{t+\mu} R(s)ds}. \tag{25}$$

$P^{MB}(t,\mu)$ is the time-t price of the μ-maturity strip of the market-bond portfolio. We also let $p^{MB}(t) \equiv \log \frac{P^{MB}(t)}{P^{MB,*}}$ denote the log-linearized price of the market-bond portfolio; $y(t,\mu) = \frac{\int_t^{t+\mu} R(s)ds}{\mu}$ denote the continuously compounded zero-coupon yield with maturity μ; and $f(t,\mu) = R(t+\mu)$ denote the μ-period-ahead instantaneous forward rate.

Proposition 3. *Consider the baseline model. Let* $[y(t), p(t), r(t)]_{t=0}^{\infty}$ *denote a feasible path that satisfies Eqs.* $(7-8)$ *along with the no-bubble condition,* $\lim_{t\to\infty} e^{-\rho t} p(t) = 0$. *The following identities hold up to a log-linear approximation:*

(i) ***Decomposition:*** *The log-price of the market portfolio satisfies*

$$\begin{aligned} p(t) &= p^{MB}(t) + p^{O}(t), \\ \text{where } p^{MB}(t) &= -\int_t^\infty e^{-\rho(s-t)}(1+\rho)\, r(s)\, ds \\ \text{and } p^{O}(t) &= \int_t^\infty e^{-\rho(s-t)} \rho y(s)\, ds. \end{aligned} \tag{26}$$

(ii) ***Yield-based measurement:*** *The log-price change of the market-bond portfolio satisfies*

$$\dot{p}^{MB}(t) = -\int_0^\infty w_\mu \mu \frac{\partial y(t,\mu)}{\partial t} d\mu, \quad \text{where } w_\mu = \frac{P^{MB*}(\mu)}{P^{MB*}} = e^{-\rho\mu}\rho. \tag{27}$$

(iii) ***Forward-rate-based measurement:*** *The log-price change of the market-bond portfolio also satisfies*

$$\dot{p}^{MB}(t) = -\int_0^\infty W_\mu \frac{\partial f(t,\mu)}{\partial t} d\mu, \quad \text{where } W_\mu = \int_\mu^\infty w_{\tilde{\mu}} d\tilde{\mu} = e^{-\rho\mu}. \tag{28}$$

The first part of the proposition decomposes the price of the market portfolio into the market-bond portfolio and a residual. The market-bond portfolio isolates asset price changes driven by the risk-free rates. In our model, monetary policy affects asset prices *primarily* through $p^{MB}(t)$: by changing the forward rates, the central bank has a *direct* effect on the valuation of cash flows. This change in asset prices also affects expected cash flows, creating indirect knock-on effects captured by $p^{O}(t)$. In general, monetary policy can affect asset prices through other channels, e.g., by changing the risk premium. We view the market-bond portfolio as capturing the asset price impact of monetary policy via risk-free rates, and the residual term as capturing other channels of monetary policy as well as other drivers of asset prices such as a time-varying risk premium.

The price of the market-bond portfolio can be measured from data on treasury yields or forwards. The last two parts of the proposition facilitate this measurement. Eq. (27) shows that the price change of the portfolio depends inversely on the yield changes of the individual strips multiplied by the *weighted* duration, $w_\mu \mu$. The weights are proportional to the (steady-state) value of the corresponding strip, $w_\mu = \frac{P^{MB*}(\mu)}{P^{MB*}}$. Eq. (28) expresses the price change in terms of forward rates. The price depends inversely on a cumulative-weighted-average of forward rates at all horizons μ. The cumulative weights capture the weights of bond strips with maturity beyond μ, that is, $W_\mu = \int_\mu^\infty w_{\tilde{\mu}} d\tilde{\mu}$. Intuitively, each forward rate affects the valuation of strips with maturities that exceed its horizon.

Since we do not observe yields or forward rates for distant horizons, we fix some $\overline{\mu}$ and bunch the values of all bond strips with maturities beyond $\overline{\mu}$ at the strip with maturity $\overline{\mu}$. This bunching yields the following approximation to Eqs. (27) and (28):

$$\dot{p}^{MB}(t) \simeq -\int_0^{\overline{\mu}} w_\mu \mu \frac{\partial y(t,\mu)}{\partial t} d\mu - W_{\overline{\mu}} \overline{\mu} \frac{\partial y(t,\mu)}{\partial t} \tag{29}$$

$$\simeq -\int_0^{\overline{\mu}} W_\mu \frac{\partial f(t,\mu)}{\partial t} d\mu, \tag{30}$$

where recall that $w_\mu = e^{-\rho\mu}\rho$ and $W_\mu = e^{-\rho\mu}$. Our bunching procedure and the resulting approximation are similar to the bond portfolio return analyzed by Van Binsbergen (2020).

4.2. Overshooting in the Covid-19 recession via risk-free rates

We next use Eq. $(29-30)$ to measure the policy support in the Covid-19 recession through risk-free rates. We adopt a yearly calibration for the bond maturity (μ) and let $\rho = 0.03 = \frac{\alpha Y^*}{P^*}$ to roughly match the annual dividend-price ratio (and the inverse duration) of the stock market index. This choice generates bond-strip weights that are consistent with the available data from dividend futures (see Van Binsbergen (2020)). We focus on real (inflation-adjusted) prices and obtain daily one-year-ahead TIPS forward rates up to a 30-year horizon ($\overline{\mu} = 30$) from the term structure data provided by the Federal Reserve, based on the approach by Gürkaynak et al. (2007). We use the forward-rate-based measure in Eq. (30) (see Appendix C for details).

The blue line in the top panel of Figure 3 illustrates the evolution of $p^{MB}(t)$ from January 2020 until June 2021. The market-bond portfolio increased substantially during the Covid-19 recession. It reached its peak in early August 2020 (about 19 log points) and it was still sizeable in 2021-Q1 (our calibration target). The figure also plots the S&P500 index and the house price index, as well as household net worth (from the Federal Reserve) which aggregates various sources of wealth. We view these assets as proxies for $p(t)$ in our model and normalize them by potential output (from the CBO) to adjust for inflation and growth. In the stock market, the residual component dragged prices down earlier in the recession, whereas the increase of the market-bond component helped stabilize the price drop. More recently, the residual has recovered (and this recovery was partly driven by aggressive monetary policy), contributing to the stock price boom. In the housing market, which adjusts more slowly to shocks, the increase of the market-bond component gradually increased prices. Accordingly, household net worth increased by an unprecedented amount: from about \$117.8 trillion in 2019-Q4 to about \$137 trillion in

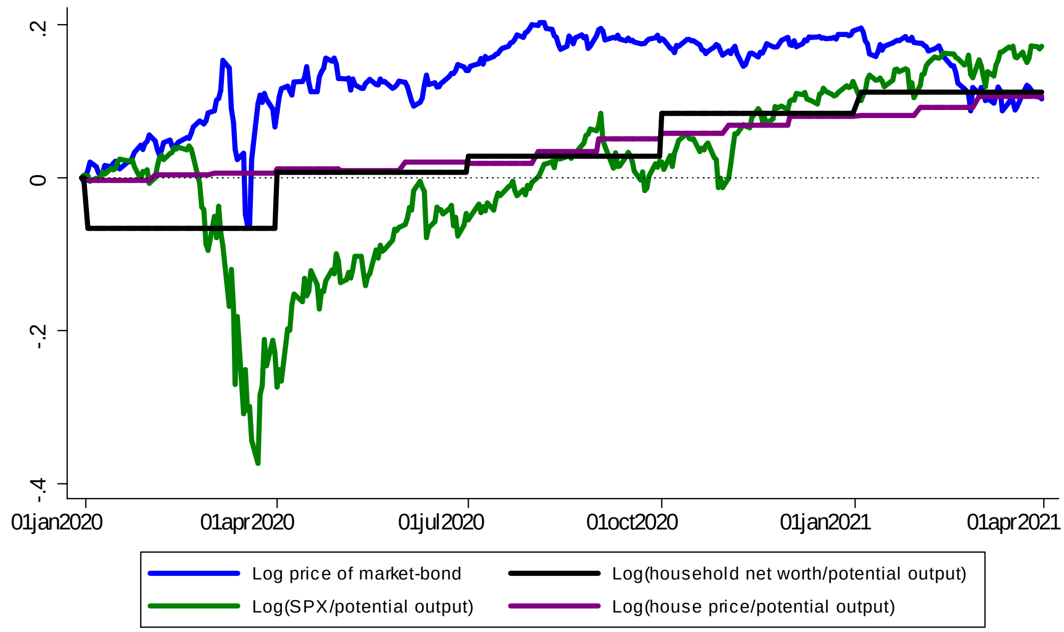

Figure 3: The evolution of the log price of the market-bond portfolio, $p^{MB}(t)$, along with the log S&P500 index, log house price index, and log household net worth normalized by potential output. All series are normalized to zero at the last trading date of 2019.

2021-Q1 (an 11 log points increase even after normalizing by potential output).

Figure 3 also illustrates that, while the increase in $p^{MB}(t)$ in 2020 was in all likelihood partly an endogenous response to the rise in the risk premium, this is unlikely to still be the case in early 2021. The recovery of the residual component in the stock market suggests the risk premium is no longer elevated. In fact, Knox and Vissing-Jorgensen (2021) provide a more detailed decomposition of the stock market returns and argue that the risk premium increased substantially earlier in the recession but had declined to close to its pre-shock level by the end of 2020. These observations suggest that the behavior of $p^{MB}(t)$ as of 2021-Q1 (our calibration target) more closely resembled the policy-induced overshooting of our model—rather than a passive response to a rise in the risk premium.

Figure 4 plots select TIPS forward rates to illustrate the drivers of $p^{MB}(t)$ over this period. Early in the recession, the shorter-term forward rates were compressed due to the zero lower bound (ZLB) on the nominal rates and expected disinflation. Nonetheless, $p^{MB}(t)$ increased because the longer-term forward rates also declined (except for March 2020). Over time, the policy support for $p^{MB}(t)$ gradually shifted from longer-term to shorter-term rates, which declined substantially due to an increase in expected inflation. The decline of the forward rates at distant horizons suggests that monetary policy partly

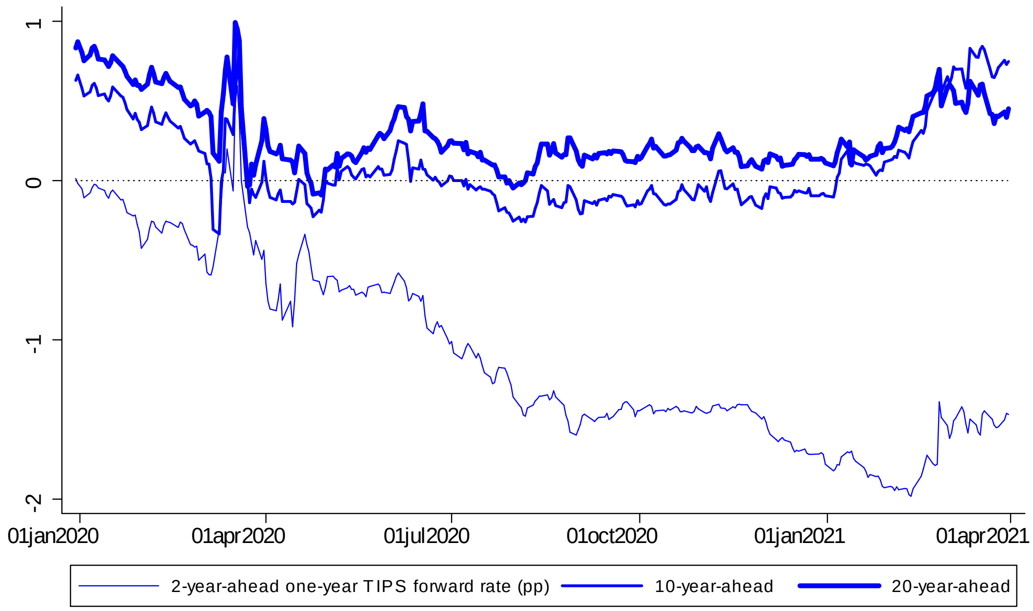

Figure 4: The drivers of the market-bond portfolio in the Covid-19 recession: One-year TIPS forward rates at select horizons.

operated through LSAPs. We return to the role of LSAPs at the end of the section.

Figure 5 compares the evolution of $p^{MB}(t)$ in the last two recessions. While monetary policy also supported asset prices through risk-free rates in the Great Recession, the response was much more gradual: it took about five years for $p^{MB}(t)$ to reach its peak. This is in sharp contrast with the Covid-19 recession, where the policy was swift and $p^{MB}(t)$ peaked within one year. One interpretation is that the Fed gradually discovered the power of LSAPs during the recovery from the Great Recession, and they deployed these tools immediately in the Covid-19 recession.[11]

4.3. The impact of overshooting on the recovery

Policy-induced overshooting in the Covid-19 recession was unprecedented not only in its speed but also in its magnitude relative to the size of the shock. We next calibrate our model to benchmark the overshooting of aggregate wealth, and to assess its likely impact on the path of recovery. We start the calibration in 2021-Q1 ($t = 0$). By that time, the U.S. economy was well on its way to recovery: people were getting vaccinated rapidly and the supply constraints were significantly relaxed. Another advantage of this quarter

[11] The same interpretation applies to the magnitude and speed of the fiscal response, which in all likelihood contributed to preventing deflationary expectations from undoing the monetary policy effort.

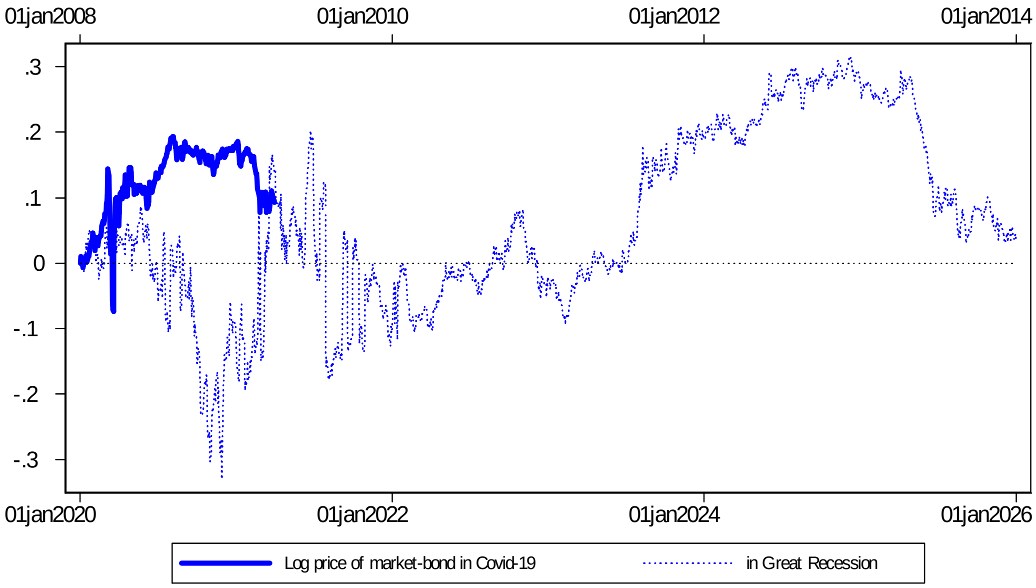

Figure 5: The evolution of the log price of the market-bond portfolio, $p^{MB}(t)$, in the Covid-19 recession (solid line) and in the Great Recession (dotted line).

is that, as illustrated by Figure 3, the increase in household net worth (normalized by potential output) is close to the increase in the market-bond portfolio. Hence, in this quarter the residual factors were on net small and the aggregate wealth increase was arguably driven by policy support through risk-free rates.

For parsimony, we focus on the baseline model with exogenous inflation ($\kappa = 0$). Recent empirical studies typically find that the Phillips curve is very flat, which implies endogenizing inflation would not significantly change our analysis.[12] As before, we set $\rho = 0.03$ to match the duration of the stock market index (see Section 4.2). That leaves us with the consumption parameters, m, n, θ, along with the planner's aversion to overshooting, ψ.[13]

We calibrate the consumption parameters, θ, m, based on the estimates from Chodorow-Reich et al. (2021). They find that the MPC out of a dollar of stock's wealth is

[12] For instance, the cross-regional analysis in Hazell et al. (2020) suggests a very low κ. Setting a positive but small κ would complicate the calibration by adding a parameter: the planner's perceived cost of inflation ϕ (see Section 3).

[13] There is also the parameter, α, which captures the share of output that accrues to stockholders. This parameter does not affect the dynamics of output. Setting a higher α strengthens the direct wealth effect from stocks but weakens the Keynesian multiplier effect (see Eq. (3)). We set $\alpha = 0.40$ to roughly match the share of capital. This implied partial-equilibrium Keynesian multiplier, $1/\alpha = 2.5$, is relatively high but not too far from the estimates of the aggregate multiplier when monetary policy remains passive (see, e.g., Chodorow-Reich (2019)).

around 3 cents per year, but the adjustment is sluggish and the effects stabilize in about two years after the wealth shock. In our model, the MPC out of a dollar stock wealth is approximately $m\frac{C^{s*}}{P^{*}} = m\rho$ (see Eq. (6)). Given $\rho = 0.03$, we set $m = 1$ to hit the MPC target. We also set $\theta = 0.5$ to match the adjustment target: in the calibrated model, the average time it takes a stockholder to adjust her consumption level is two years.

The consumption parameter, n, affects the speed of recovery in the model: all else equal, increasing n slows down the recovery (see Appendix A.2). We calibrate this parameter to match the CBO's February 2021 projections for the convergence rate of the output gap between 2021-Q1 and 2024-Q1. Note, however, that the aversion to overshooting, ψ, also affects the convergence rate. As a benchmark, we calibrate n together with ψ to match the CBO-implied convergence rate, γ, along with a standard Taylor rule for monetary policy.[14]

The first panel of Figure 6 shows that this benchmark calibration (black dotted line) is similar to the CBO's February 2021 projections for the output gap (purple solid line). On the other hand, the second panel shows that the calibration misses the observed asset price overshooting: The calibration predicts about two log points initial increase in the asset price gap, $p(t)$, whereas the actual increase in household net worth normalized by potential output was about 11 log points as of 2021-Q1 (see Figure 3). While the standard Taylor rule also implies overshooting, the magnitudes are small. With an initial output gap between two and three percent, and an expected recovery in about three years, the Taylor rule cuts the risk-free interest rate by about one percentage point for a few years (bottom panel). This response implies less overshooting than observed in the data.

We then turn to our main calibration, where we keep n unchanged but recalibrate ψ to match the observed response of $p(t)$ (as of 2021-Q1). The blue solid lines in Figure 6 show that overshooting substantially accelerates the recovery relative to the benchmark case with a Taylor rule (and even more so relative to the case without overshooting). With the observed amount of overshooting, our model predicts that the output gap recovers almost fully by 2022, whereas a Taylor rule predicts a similar outcome by 2024. The second and the third panels show that optimal policy tapers overshooting very quickly: in view of the transmission lags, the initial burst of overshooting gradually closes the output gaps, obviating the need for further overshooting. Why does the model predict a very fast recovery? As we mentioned earlier, household net worth increased by about \$20 trillion

[14] The CBO relies on a Taylor rule to construct its projections for interest rates (see Arnold (2018)). That said, the purpose of this calibration is not to replicate the CBO's projection exercise (which is much more detailed than in our analysis) but to obtain a reasonable benchmark with a Taylor rule against which we can measure the extent and the impact of asset price overshooting.

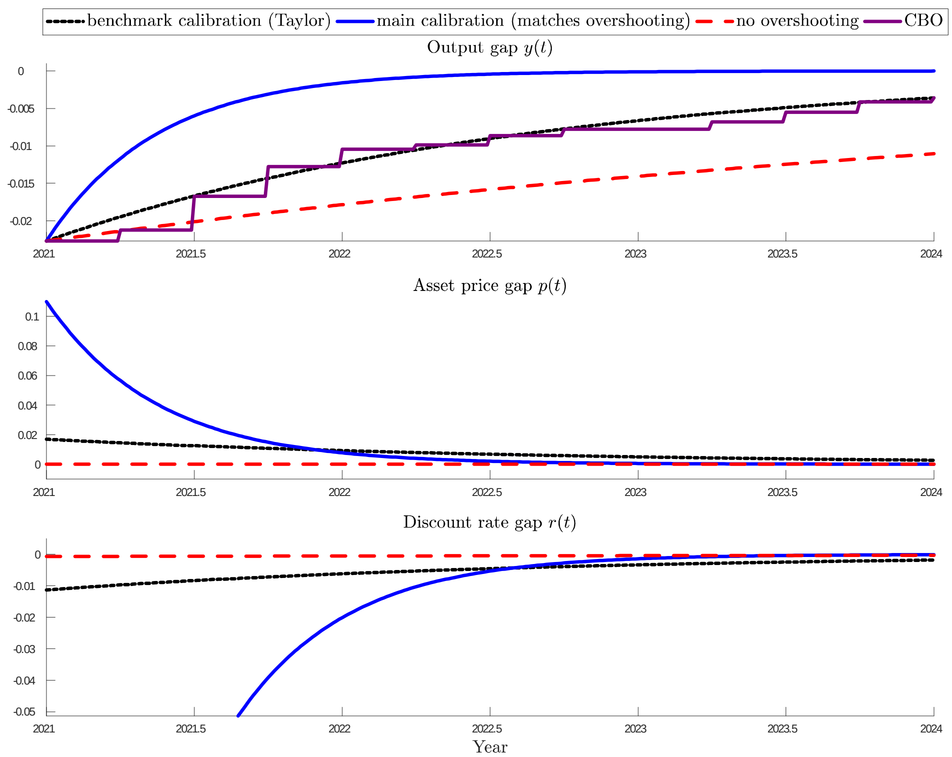

Figure 6: Dotted black line: Benchmark calibration with a Taylor rule that matches the CBO's February 2021 projections for future output gap (purple line). Blue solid line: Main calibration that matches the overshooting of $p\left(t\right)$ as of 2021-Q1. Red dashed line: Calibration without overshooting.

between 2019-Q4 and 2021-Q1—close to the size of the U.S. annual GDP (see Figure 3). This large increase in wealth translates into a large increase in spending, even with a conservative MPC calibration, which quickly fills a two or three percent output gap.

The role of LSAPs. The bottom panel of Figure 6 shows that in our model the optimal policy induces and then tapers overshooting by adjusting the short-term discount rates by a large amount and then quickly undoing this aggressive cut. This aspect of our model does not fully match the data: in the Covid-19 recession, monetary policy partly operated through distant-horizon forward rates (see Figure 3). These very long-term rate changes were in all likelihood driven by the large LSAP programs for safe assets the Fed implemented in this recession.[15] The Fed purchased trillions of dollars of treasuries and agency mortgage-backed securities between March and June 2020; and about \$120 billion a month between June 2020 and 2021-Q1 (and beyond).[16]

Our model is stylized and does not have the appropriate frictions, such as risk absorption by the government (e.g., Caballero and Simsek (2021a)) or segmented markets (e.g., Vayanos and Vila (2021); Ray (2019); Sims et al. (forthcoming)), that make LSAPs operational. Nonetheless, from the perspective of our model, we view LSAPs as a close substitute for conventional monetary policy, *conditional on them inducing the same impact on aggregate wealth, $p(t)$*. In particular, the price of the market-bond portfolio also captures the wealth effect driven by long-term *safe* asset purchases typical of quantitative easing policies. These purchases can substitute for short-term rate cuts by reducing the long-term rates, e.g., by absorbing the duration risk and reducing the term premium. By now, there is an extensive empirical literature documenting that the LSAPs in recent years have indeed been good substitutes for conventional monetary policy (see, e.g., Swanson and Williams (2014); Swanson (2018); Sims and Wu (2020, 2021)).

In sum, our analysis in this section quantifies the policy-induced asset price overshooting through risk-free rates in the Covid-19 recession and its likely impact on the recovery. The data and our calibration suggest asset-price overshooting in this recession was significantly greater than implied by a Taylor-rule benchmark, and this additional overshooting substantially accelerated the recovery.

[15] Although, see Hanson and Stein (2015) for the puzzling finding that conventional monetary policy shocks seem to affect *real long-term* interest rates. See also Bianchi et al. (forthcoming) for an explanation of these long lasting effects of monetary policy over real rates and asset prices based on a regime-switching model with sticky inflation expectations. Note, however, that the space for conventional monetary policy during the Covid-19 recession was very limited, which suggests that LSAPs also played a central role in driving $p^{MB}(t)$ in this episode.

[16] For the Fed's response, see `https://www.brookings.edu/research/fed-response-to-covid19/`

5. Overshooting with constrained monetary policy

So far, we have assumed that the planner faces no constraints in adjusting the discount rate to achieve its target level of overshooting. While such constraints appear to have been less binding during the Covid-19 recession, where central banks were able to quickly implement large LSAPs and other unconventional policies, they have become a central feature of recent recessions. In practice, central banks might find it difficult or costly to reduce the policy rate—due to, e.g., the zero lower-bound constraint, concerns about financial institutions' balance sheets, or a rapidly depreciating exchange rate. Central banks might also be unable to implement significant LSAPs—due to, e.g., political and institutional constraints.

In this section, we analyze the optimal policy when there is a limit on how much the policy can reduce discount rates. We focus on a lower-bound constraint on the discount rate but interpret it more broadly as a limit on both conventional monetary policy and LSAPs. We find that the lower-bound constraint makes overshooting non-monotonic over time: When the economy starts with a sufficiently negative output gap, the asset price boost starts low, grows as the economy recovers, and eventually shrinks as the economy approaches full recovery. This pattern also implies that good macroeconomic news about *aggregate demand* can raise or lower asset prices depending on the stage of the recovery.

Consider the baseline model in Section 2 with the only difference that the central bank sets the discount rate subject to a lower-bound constraint,

$$r(t) \geq \overline{r} \quad \text{for each } t. \tag{31}$$

The parameter, $\overline{r} < 0$, captures the severity of the constraint. Combining Eqs. (10) and (8), the central bank's recursive problem becomes

$$\begin{aligned} \rho V(y) &= \max_{p} -\frac{y^2}{2} - \psi \frac{p^2}{2} + V'(y)\,\dot{y}, \\ \dot{y} &= \theta\left(mp - (1-n)\,y\right) \\ r(t) &= \frac{\rho}{1+\rho}\left(y(t) - p(t)\right) + \frac{1}{1+\rho}\dot{p}(t) \geq \overline{r} \end{aligned} \tag{32}$$

As before, we assume the central bank sets the current policy *without* commitment. The planner takes future output and asset price gaps (as well as the price drift $\dot{p}(t)$) as given and sets the instantaneous asset price, $p(t)$, subject to the lower-bound constraint.[17]

[17] Unlike in Section 2, the no-comitment constraint binds in this section. The planner might want to promise low interest rates in the future so as to relax the current lower bound constraint. We abstract from

Consider the solution corresponding to a negative initial output gap, $y(0) < 0$. Recall that when there is no lower-bound constraint the discount rate is increasing over time [cf. (15)]. With a lower-bound constraint, there exists a cutoff output gap, $\overline{y} \leq 0$, such that the discount rate constraint binds for $y(t) < \overline{y}$ but not for $y(t) \in (\overline{y}, 0)$. When the constraint does not bind, the solution is exactly as in Section 2.2. In particular, the optimal asset price gap is given by Eq. (13) and the corresponding discount rate is given by (15). Setting $r(t) = \overline{r}$, we solve for the cutoff output and asset price gaps as follows,

$$\overline{y} = \frac{\overline{r}}{\frac{\rho}{1+\rho} + \frac{\theta m}{\psi v}\frac{\gamma+\rho}{1+\rho}} < 0 \quad \text{and} \quad \overline{p} = -\frac{\frac{\theta m}{\psi v}\overline{r}}{\frac{\rho}{1+\rho} + \frac{\theta m}{\psi v}\frac{\gamma+\rho}{1+\rho}} > 0. \tag{33}$$

When the discount rate constraint binds, the solution satisfies the following differential equation system over time in variables $y(t), p(t)$:

$$\begin{aligned} \dot{y} &= \theta\left(mp - (1-n)\,y\right) \\ \overline{r} &= \frac{\rho}{1+\rho}\left(y(t) - p(t)\right) + \frac{1}{1+\rho}\dot{p}(t). \end{aligned} \tag{34}$$

Starting with the point, $(\overline{y}, \overline{p})$, we can uniquely solve this system *backward* over time. The resulting path describes the optimal asset price gap p corresponding to each output gap $y \leq \overline{y}$. Over time, the gaps travel along the solution path until they reach the point $(\overline{y}, \overline{p})$. Subsequently, the gaps follow the unconstrained solution.

Our main result in this section characterizes the solution for the constrained range. We show that the asset price gap follows a *non-monotonic* path over time, increasing in time for sufficiently low levels of the output gap but decreasing in time for higher levels of the output gap. The proof (in the appendix) relies on the phase diagram corresponding to the differential equation system in (34). We assume the parameters satisfy a technical condition that ensures the system has two real eigenvalues and a unique steady-state.

Proposition 4. *Consider the model with a lower-bound on the discount rate $\overline{r} < 0$ for parameters that satisfy $(\theta(1-n) - \rho)^2 > 4\theta\rho(m - (1-n)) \neq 0$. Let $\overline{y} < 0, \overline{p} > 0$ denote the output and asset price gap cutoffs given by (33).*

When $y(0) \geq \overline{y}$, the constraint does not bind and the solution is the same as in Proposition 1. The asset price gap is a function of the output gap, $p = \mathbf{p}(y) = -\frac{\theta m}{\psi v}y$.

When $y(0) < \overline{y}$, the constraint binds and the path, $(y(t), p(t))$, solves the system in (34) and reaches $(\overline{y}, \overline{p})$ in finite time. The output gap is increasing over time ($\dot{y}(t) > 0$).

these types of *forward guidance* policies as they are not our focus and their benefits are well understood (see, e.g., Eggertsson and Woodford (2003)).

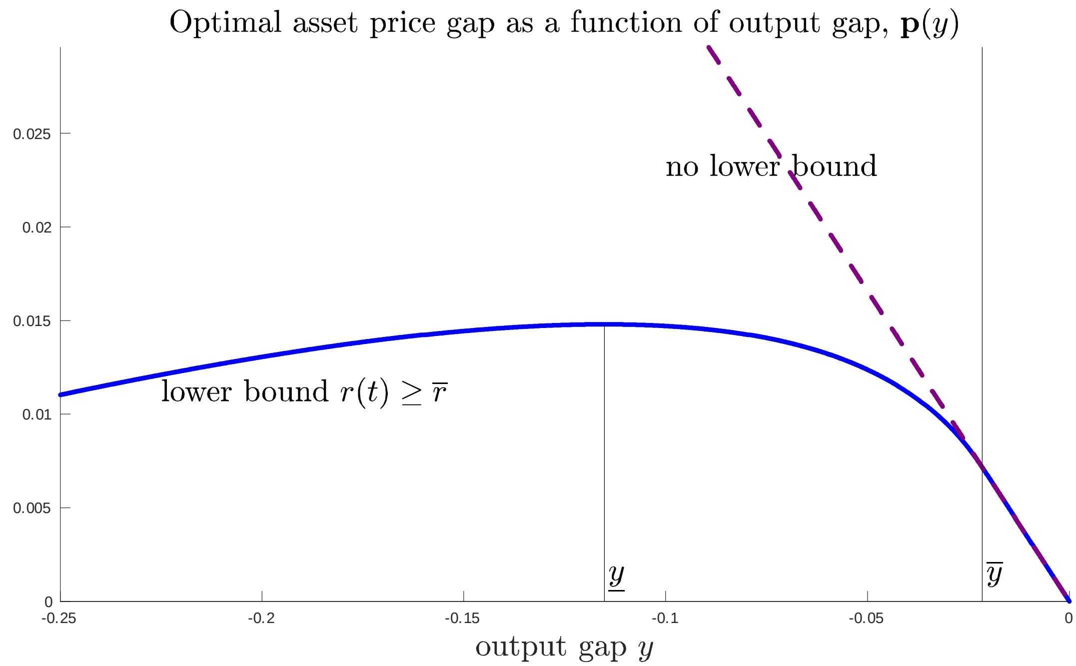

Figure 7: Equilibrium with an interest rate lower bound (the solid curve) and without a lower bound (the dashed line).

There exists another output gap cutoff below the first cutoff, $\underline{y} < \overline{y}$*, such that the asset price gap is increasing over time* ($\dot{p}(t) > 0$) *iff* $y < \underline{y}$*. The asset price gap is a function of the output gap,* $\mathbf{p}(y)$*, and it is increasing in the output gap* ($\frac{d\mathbf{p}(y)}{dy} > 0$) *iff* $y < \underline{y}$*.*

Below a cutoff output gap ($\underline{y}$), the asset price and the output gap both increase over time. Above the cutoff, the asset price gap decreases over time whereas the output gap continues to increase. Equivalently, the asset price gap is a non-monotonic function of the output gap, $\mathbf{p}(y)$: increasing below a cutoff level of the output gap ($\underline{y}$) and decreasing for higher levels of the output gap. Figure 7 illustrates this function for a numerical example.

For intuition, consider Figure 8, which illustrates the dynamics over time starting with a low level of output gap ($y(0) = -0.25$). In this example, the discount rate stays at the lower-bound until time $T \in [4, 5]$ at which point the gaps satisfy $y(T) = \overline{y}$ and $p(T) = \overline{p}$, and the discount rate "lifts off" above the lower-bound. Before time T the output gap shrinks over time (although more slowly than in the unconstrained case), whereas the asset price gap follows a non-monotonic path. To understand the non-monotonic behavior of asset prices, note that Eq. (26) implies the present discounted value formula:

$$p(t) = \int_t^{\infty} e^{-\rho(s-t)} \left[\rho y(s) - (1+\rho) r(s)\right] ds.$$

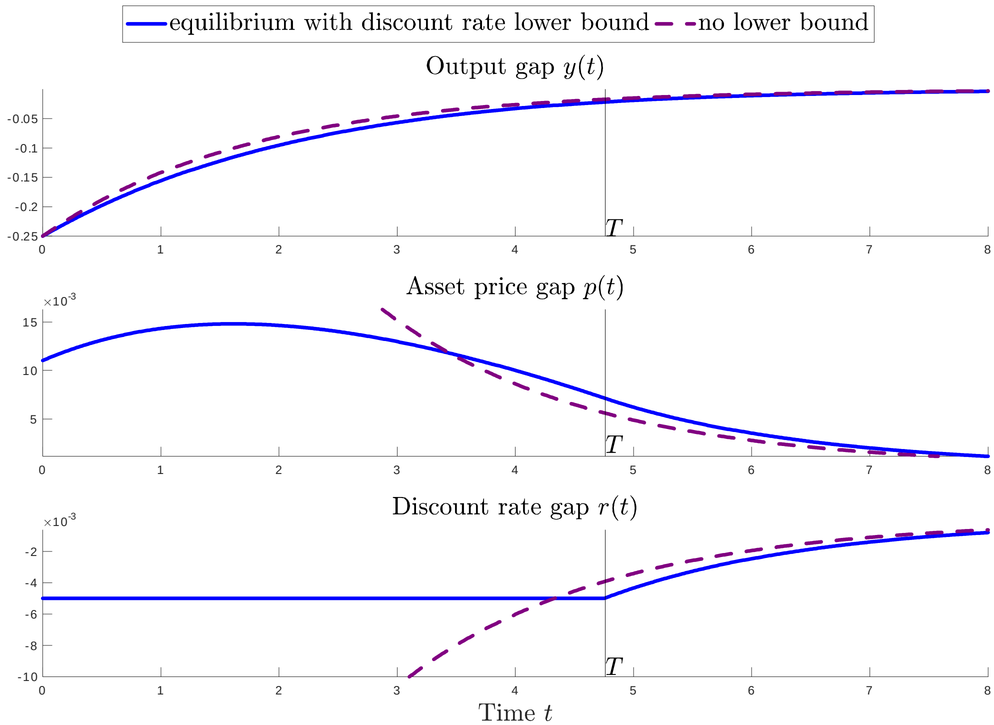

Figure 8: A simulation of the equilibrium over time with an interest rate lower bound (solid lines) and no lower bound (dotted lines).

Starting backwards, when t is sufficiently close to the liftoff time T, the planner is nearly unconstrained and she effectively "sets" the asset price by adjusting the discount rates beyond T. In this range, as the output gap recovers, the extent of overshooting declines as in the baseline model. While expected cash flows increase, expected discount rates increase by even more and the asset price gap decreases. In contrast, when t is sufficiently far from the liftoff time T, the planner is severely constrained: she would like a greater overshooting but cannot achieve it. In this range, as the output gap recovers over time, the asset price gap increases and the planner allows asset prices to stay high. Expected cash flows increase and expected discount rates mostly remain constant.

Implications for the stock price impact of aggregate demand news. An empirical literature finds that the impact of macroeconomic news on stock prices depends on the stage of the business cycle. For instance, Law et al. (2019) show that stock prices react to macroeconomic news announcements more strongly when the output gap is sufficiently negative, and the relationship becomes weaker (and it can have a negative sign) when the output gap is closer to zero. In earlier work, Boyd et al. (2005) observe a similar pattern and attribute the cyclicality of the response to changes in the relative strength of the interest-rate and the cash-flow effects of news (see also McQueen and Roley (1993); Andersen et al. (2007)). Our analysis in this section provides an explanation for these findings when good news reflects mostly *stronger aggregate demand*.

Corollary 3. *Consider the setup in Proposition 4. Suppose there is a one-time surprise change in the initial output gap, $y(0)$, after which the economy follows the equilibrium we have analyzed. Good news (a surprise improvement in the output gap) decreases the price of the market portfolio if and only if the output gap is sufficiently high,*

$$\frac{dp(0)}{dy(0)} < 0 \quad \textit{iff} \quad y(0) > \underline{y}.$$

As we discussed earlier, the intuition stems from competing effects of good news (about demand) on cash flows and discount rates. A greater initial output gap accelerates the recovery, which increases the expected future output and cash flows. This is good news for capital when the discount rate is mostly constrained (the economy is far from the discount rate liftoff) and the planner does not overturn the price impact of higher cash flows. Conversely, a larger output gap is bad news for capital when the discount rate is mostly unconstrained (the economy is close to the liftoff) and the planner *optimally* overturns the price impact of higher cash flows by accelerating interest-rate hikes. Importantly, this interest-rate response is strong enough to dominate the cash-flow effect of news, because the planner chooses to undo some of the induced *overshooting*.

An important caveat is that these results apply for good news that increases *aggregate demand* without changing aggregate supply. In our context, good news that raises *aggregate supply* (beyond aggregate demand) would increase asset prices. In practice, macroeconomic news is likely to reflect a mix of news about aggregate demand and supply. Our analysis suggests that the asset price impact is likely to be more procyclical (and good news is more likely to be bad news) for news that primarily concerns aggregate demand, e.g., announcements about inflation, unemployment, or fiscal policy.

6. Final Remarks

We proposed a model to illustrate that large gaps between the performance of financial markets and the real economy can result from optimal monetary policy when aggregate demand is below its potential and responds to asset prices with a lag. The central bank boosts asset prices to close the output gap as fast as possible. In fact, the central bank also boosts asset prices if aggregate demand is currently at its potential but is expected to fall below potential in the near future (see Appendix B.2). These optimal policy responses create large temporary gaps between financial markets and the real economy, but they also accelerate the recovery. We also showed that in this context endogenous inflation leads to more overshooting, and an aggressive overshooting policy weakens the relationship between inflation and the output gap (i.e., it flattens the Phillips curve).

We argued that the optimal overshooting policy is consistent with the monetary policy rules used in practice. When the interest rate rule is more sensitive to the output gap or inflation, the policy involves more asset price overshooting. Having said this, we gauged the extent of policy-induced overshooting in the Covid-19 recession driven by risk-free rates and found it to be exceptionally large. To facilitate this exercise, we decomposed the aggregate asset price in our setting into a "market-bond portfolio"—driven by expected interest rate changes, and a "residual"—driven by expected cash flows and other factors. The market-bond portfolio increased substantially in the Covid-19 recession, and its boom as of early 2021 resembles the policy-induced overshooting in our model. Our calibrated model suggests that this policy-induced overshooting is significantly greater than implied by a Taylor-rule benchmark, and that this additional overshooting substantially accelerated the recovery. Our quantitative analysis shows that this overshooting was driven partly by distant-horizon forward rates, suggesting a key role for the Fed's LSAP programs for safe assets.

We also showed that, when the central bank faces a lower-bound on the discount rate it can set, the overshooting result still holds but becomes non-monotonic: When the economy starts with a sufficiently negative output gap, the asset price boost starts low, grows as the economy recovers, and eventually shrinks as the economy approaches full recovery and the interest-rate liftoff. This result connects our analysis to a literature documenting that stock prices react to macroeconomic news announcements more strongly when the output gap is more negative (see, e.g., Law et al. (2019)).

While we do not explicitly model fiscal policy, our analysis of the price impact of news suggests that fiscal policy is likely to complement monetary policy when the output gap is significantly negative, and substitute for it when the output gap is small. When the output gap is large (and negative), fiscal policy increases asset prices and the extent of overshooting—an outcome that the central bank desires but cannot achieve due to the discount rate constraint. When the output gap is small, fiscal policy induces the central bank to accelerate the interest-rate hikes sufficiently to *decrease* asset prices and the extent of overshooting.

Throughout our analysis we assume that all firms are equally affected by the recession. Adding heterogeneity in productivity (a central feature of the Covid-19 recession) does not change our main results, but it introduces a large dispersion in asset prices across firms. In particular, firms whose relative productivity is positively affected by the recession see their shares' value rise by even more since they benefit from the central bank's attempt to boost asset prices without suffering from a decline in productivity. In the Covid-19 recession, this provides a rationale for the extraordinary performance of indices such as the Nasdaq 100, whose main components consist of "Covid-sheltered" firms.

Finally, while we demonstrated that the broad features of asset markets during the Covid-19 episode are consistent with a well managed monetary policy framework, we do not wish to imply that there are no anomalies or pockets of irrational exuberance in some markets. Having said this, the logic of the model suggests that experiencing an episode of irrational exuberance during a deep recession with a difficult recovery ahead has a positive dimension, since it reduces the burden on the central bank to engineer an overshooting.

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A. Appendix: Omitted derivations and proofs

This appendix presents the analytical derivations and proofs omitted from the main text. We first provide the microfoundations for the supply side with nominal rigidities that we describe in Sections 2 and 3. We then characterize the comparative statics of overshooting that we discuss in Section 2.2. We then present the proofs for the results in the main text.

A.1. Microfoundations for the supply side

There are two types of agents denoted by superscript $i = s$ ("stockholders") and $i = h$ ("hand-to-mouth"). There is a single factor, labor.

Hand-to-mouth households supply labor according to relatively standard intra-period preferences. They do not hold financial assets and spend all of their income. We write the hand-to-mouth agents' problem as,

$$\begin{aligned} &\max_{L(t)} \log C^h(t) - \chi \frac{L(t)^{1+\varphi}}{1+\varphi} \\ &Q(t)\, C^h(t) = W(t)\, L(t) + T(t)\,. \end{aligned} \tag{A.1}$$

Here, φ denotes the Frisch elasticity of labor supply, $Q(t)$ denotes the nominal price for the final good, $W(t)$ denotes the nominal wage, and $T(t)$ denotes lump-sum transfers to labor (described subsequently). Using the optimality condition for problem $(A.1)$, we obtain a standard labor supply curve,

$$\frac{W(t)}{Q(t)} = \chi L(t)^{\varphi} C^h(t)\,. \tag{A.2}$$

Stockholders own (and trade) the market portfolio and they supply no labor. We analyze these agents' consumption-savings problem in the main text as well as in Appendix B.1. They receive the profits from the production firms that we will describe subsequently.

Production is otherwise similar to the standard New Keynesian model. There is a continuum of monopolistically competitive firms, denoted by $\nu \in [0, 1]$. These firms produce differentiated intermediate goods, $Y(t,\nu)$, subject to the Cobb-Douglas technology,

$$Y(t,\nu) = AL(t,\nu)^{1-\alpha}. \tag{A.3}$$

Here, $1-\alpha$ denotes the share of labor in production and A is a productivity shifter.

A competitive final goods producer combines the intermediate goods according to the CES technology,

$$Y(t) = \left(\int_0^1 Y(t,\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}, \tag{A.4}$$

for some $\varepsilon > 1$. This implies the price of the final consumption good is determined by the ideal price index,

$$Q(t) = \left(\int_0^1 Q(t,\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}, \tag{A.5}$$

and the demand for intermediate good firms satisfies,

$$Y(t,\nu) \leq \left(\frac{Q(t,\nu)}{Q(t)}\right)^{-\varepsilon} Y(t). \tag{A.6}$$

Here, $Q(t,\nu)$ denotes the nominal price set by the intermediate good firm ν.

Labor market clearing condition is

$$\int_0^1 L(t,\nu)\, d\nu = L(t). \tag{A.7}$$

Goods market clearing condition is

$$Y(t) = C^s(t) + C^h(t). \tag{A.8}$$

Finally, to simplify the distribution of output across factors, we assume the government taxes part of the profits lump-sum and redistributes to workers to ensure they receive their production share of output. Specifically, each intermediate firm pays lump-sum taxes determined as follows:

$$T(t) = (1-\alpha) Q(t) Y(t) - W(t) L(t). \tag{A.9}$$

This ensures that in equilibrium hand-to-mouth households receive and spend their production share of output, $(1-\alpha) Q(t) Y(t)$, and consume [see $(A.1)$]

$$C^h(t) = (1-\alpha) Y(t). \tag{A.10}$$

Stockholders receive the total profits from the intermediate good firms, which amounts to the residual share of output, $\Pi(t) \equiv \int_0^1 \Pi(t,\nu)\, d\nu = \alpha Q(t) Y(t)$.

Benchmark equilibrium without nominal rigidities. To characterize the equilibrium, it is useful to start with a benchmark setting without nominal rigidities. In this benchmark, an intermediate good firm ν solves the following problem,

$$\begin{aligned} \Pi &= \max_{Q,L} QY - W(t) L - T(t) \\ & \quad \text{where } Y = AL^{1-\alpha} = \left(\frac{Q}{Q(t)}\right)^{-\varepsilon} Y(t) \end{aligned} \tag{A.11}$$

The firm takes as given the aggregate price, wage, and output, $Q(t), W(t), Y(t)$, and chooses its price, labor input, and output Q, L, Y.

The optimal price is given by

$$Q = \frac{\varepsilon}{\varepsilon - 1} W(t) \frac{1}{(1-\alpha) AL^{-\alpha}}. \tag{A.12}$$

Intuitively, the firm sets an optimal markup over marginal costs of output, where the marginal cost depends on the wage and (inversely) on the marginal product of labor.

In equilibrium, all firms choose the same prices and allocations, $Q(t) = Q$ and $L(t) = L$. Substituting this into $(A.12)$, and using $Y = AL^{1-\alpha}$, we obtain a labor demand equation,

$$\frac{W(t)}{Q(t)} = \frac{\varepsilon - 1}{\varepsilon} (1-\alpha) AL^{-\alpha}. \tag{A.13}$$

Combining this with the labor supply equation $(A.2)$, and substituting the hand-to-mouth consumption $(A.10)$, we obtain the equilibrium labor as the solution to,

$$\chi (L^*)^{\varphi} (1-\alpha) Y^* = \frac{\varepsilon - 1}{\varepsilon} (1-\alpha) A (L^*)^{-\alpha}.$$

In equilibrium, output is given by $Y^* = A(L^*)^{1-\alpha}$. Therefore, the equilibrium condition simplifies to,

$$\chi (L^*)^{1+\varphi} = \frac{\varepsilon - 1}{\varepsilon}.$$

Fully sticky prices. We next describe the equilibrium with nominal rigidities for the baseline case with full price stickiness. In particular, intermediate good firms have a preset nominal price that remains fixed over time, $Q(t,\nu) = Q^*$. This implies the nominal price for the final good is also fixed and given by $Q(t) = Q^*$ [cf. $(A.5)$]. Then, each intermediate good firm ν at time t solves the following version of problem $(A.11)$,

$$\begin{aligned} \Pi &= \max_{L} QY - W(t) L - T(t) \\ & \quad \text{where } Y = AL^{1-\alpha} \leq Y(t) . \end{aligned} \tag{A.14}$$

Here, we dropped the index, (t, ν), from firm-specific variables to simplify the notation. By symmetry, each firm chooses the same allocation. For small aggregate demand shocks (which we assume) each firm optimally chooses to meet the demand for its goods, $Y = AL^{1-\alpha} = Y(t)$. Therefore, each firm's output is determined by aggregate demand, which is equal to spending by stockholders and hand-to-mouth households [cf. $(A.8)$],

$$Y(t) = C^s(t) + C^h(t) .$$

This establishes Eq. (2) in the main text.

Finally, recall that hand-to-mouth agents' spending is given by $C^h(t) = (1-\alpha) Y(t)$ [see Eq. $(A.10)$]. Combining this with $Y(t) = C^s(t) + C^h(t)$, the aggregate demand for goods is determined by the stockholders' spending,

$$Y(t) = \frac{C^s(t)}{\alpha}.$$

This establishes Eq. (3) in the main text.

Partially flexible prices and the New-Keynesian Phillips curve. We next consider the case with partially flexible prices that we analyze in Section 3. Specifically, at each instant, a random fraction of intermediate good firms adjusts their price, with constant hazard ζ. Their prices remain unchanged until the next time they have a chance to adjust. We characterize the optimal price for an adjusting firm. We then characterize the dynamics of inflation and derive the New-Keynesian Phillips curve.

Consider the firms that adjust their price in period t. These firms' optimal price, $Q^{adj}(t)$, solves

$$\begin{aligned} &\max_{Q^{adj}(t)} \int_{T=t}^{\infty} e^{-\zeta(T-t)} e^{-\int_t^T R(s) ds} \left(Q^{adj}(t) Y(T|t) - W(T) L(T|t) \right) dT \\ &\text{where } Y(T|t) = AL(T|t)^{1-\alpha} = \left(\frac{Q^{adj}(t)}{Q(T)} \right)^{-\varepsilon} Y(T) . \end{aligned} \tag{A.15}$$

The integral captures the sum of expected profits over histories at which the firm's price remains unchanged. At time T, the price set at time t remains unchanged with probability $e^{-\zeta(T-t)}$. Since there is no aggregate risk, the firm discounts the profits at time T according to the discount rate between times t and T. The terms, $L(T|t), Y(T|t)$, denote the input and the output of the firm at time T (for a firm that reset its price at time t). We dropped the lump-sum taxes from the expression since they do not affect the firm's optimal pricing decision.

The optimality condition is given by,

$$\int_{T=t}^{\infty} e^{-\zeta(T-t)} e^{-\int_t^T R(s)ds} Q(T)^{\varepsilon} Y(T) \left(\begin{array}{c} Q^{adj}(t) \\ -\frac{\varepsilon}{\varepsilon-1} \frac{W(T)}{(1-\alpha)AL(T|t)^{-\alpha}} \end{array} \right) dT = 0 \quad \text{(A.16)}$$

$$L(T|t) = \left(\frac{Q^{adj}(t)}{Q(T)} \right)^{\frac{-\varepsilon}{1-\alpha}} \left(\frac{Y(T)}{A} \right)^{\frac{1}{1-\alpha}}.$$

Here, we have substituted $\frac{Y(T|t)}{Q^{adj}(t)} = Q^{adj}(t)^{-(1+\varepsilon)} Q(T)^{\varepsilon} Y(T)$ and dropped $Q^{adj}(t)^{-(1+\varepsilon)}$ since it is common across all of the terms inside the integral.

We next combine Eq. $(A.16)$ with the remaining equilibrium conditions to derive the New-Keynesian Phillips curve. Specifically, we log-linearize the equilibrium around the allocation that features potential real outcomes and zero nominal inflation, that is, $L_t = L^*, Y_t = Y^*$ and $Q_t = Q^*$ for each t. Recall that we use the notation $x(t) = \log \frac{X(t)}{X^*}$ to denote the log-deviation of the corresponding variable $X(t)$ from its potential.

We first log-linearize the labor supply Eq. $(A.2)$ (after substituting $(A.10)$) to obtain

$$w(t) - q(t) = \varphi l(t) + y(t). \quad \text{(A.17)}$$

Likewise, we log-linearize Eqs. $(A.4 - A.3)$ and $(A.7)$, to obtain

$$y(t) = (1-\alpha) l(t). \quad \text{(A.18)}$$

Finally, we log-linearize the optimality condition $(A.16)$ to obtain,

$$\int_{T=t}^{\infty} e^{-(\rho+\zeta)(T-t)} \left[\begin{array}{c} (\varepsilon q(T) + y(T)) (Q^*)^{\varepsilon} Y^* \left(Q^* - \frac{\varepsilon}{\varepsilon-1} \frac{W^*}{(1-\alpha)A(L^*)^{-\alpha}} \right) \\ + (Q^*)^{\varepsilon} Y^* \left(Q^* q^{adj}(t) - \frac{\varepsilon}{\varepsilon-1} \frac{W^*}{(1-\alpha)A(L^*)^{-\alpha}} (w(t) + \alpha l(T|t)) \right) \end{array} \right] dT$$

After substituting $Q^* = \frac{\varepsilon}{\varepsilon-1} \frac{W^*}{(1-\alpha)A(L^*)^{-\alpha}}$ [see Eq. $(A.12)$] and calculating $l(T|t)$ [see $(A.16)$]

we obtain

$$\int_{T=t}^{\infty} e^{-(\rho+\zeta)(T-t)} \left(q^{adj}(t) - (w(T) + \alpha l(T|t))\right) dT = 0 \tag{A.19}$$

$$\text{where } l(T|t) = \frac{-\varepsilon}{1-\alpha}\left(q^{adj}(t) - q(T)\right) + l(T).$$

Here, the second line uses $\frac{1}{1-\alpha} y(T) = l(T)$.

We next combine Eqs. $(A.17 - A.19)$ to express the optimality condition in terms of the firm's price, $q^{adj}(t)$, the aggregate price after each history, $q(T)$, and the aggregate labor after each history, $l(T)$:

$$\int_{T=t}^{\infty} e^{-(\rho+\zeta)(T-t)} \left(\left(q^{adj}(t) - q(T)\right)\frac{1-\alpha+\varepsilon\alpha}{1-\alpha} - \frac{\varphi+1}{1-\alpha} y(T)\right) dT = 0.$$

After rearranging terms, we obtain an expression for the optimal price of an adjusting firm,

$$\begin{aligned} q^{adj}(t) &= (\rho+\zeta)\int_{T=t}^{\infty} e^{-(\rho+\zeta)(T-t)} \left(\Theta y(T) + q(T)\right) dT \\ \text{where } \Theta &= \frac{1+\varphi}{1-\alpha+\varepsilon\alpha}. \end{aligned}$$

Since the expression is recursive, we can also write it as a differential equation

$$(\rho+\zeta)\, q^{adj}(t) = (\rho+\zeta)\left(\Theta y(t) + q(t)\right) + \dot{q}^{adj}(t). \tag{A.20}$$

Next, we consider the aggregate price index $(A.5)$. Log-linearizing the expression around Q^*, we obtain $q(t) = \int q(t,\nu)\, d\nu$. Differentiating over time and using the observation that firms adjust their prices with constant hazard ζ, we obtain,

$$\pi(t) \equiv \dot{q}(t) = \zeta\left(q^{adj}(t) - q(t)\right). \tag{A.21}$$

Differentiating this, we also have

$$\dot{\pi}(t) = \zeta\left(\dot{q}^{adj}(t) - \dot{q}(t)\right). \tag{A.22}$$

Substituting Eqs. $(A.21 - A.22)$ into $(A.20)$, we obtain the New-Keynesian Phillips curve

$$\begin{aligned} \rho\pi(t) &= \kappa y(t) + \dot{\pi}(t) \\ \text{where } \kappa &= \zeta(\rho+\zeta)\frac{1+\varphi}{1-\alpha+\varepsilon\alpha}. \end{aligned} \tag{A.23}$$

This completes the microfoundations for the supply side that we use in the main text.

A.2. Comparative statics of overshooting

In Section 2.2, we show that the central bank optimally induces asset price overshooting in response to an output gap. In this section, we discuss how different parameters in our model affect the overshooting result. In particular, the following result describes the extent of overshooting for a given output gap, $\left|\frac{p(t)}{y(t)}\right| = \frac{\theta m}{\psi v}$ (cf. (13)) as well as the convergence rate, γ (cf. (14)).

Proposition 5. *Consider the equilibrium path with optimal monetary policy. Then, the asset price overshoots by more per unit of output gap (greater $\left|\frac{p(t)}{y(t)}\right|$) and output gaps are closed faster (greater γ) in each of the following cases:*

(i) When the central bank has smaller aversion to overshooting (smaller ψ)

(ii) When a larger fraction of stockholders adjusts at any moment (larger θ)

(iii) When adjusting stockholders respond more to the asset price (larger m)

In addition:

(iv) The asset price overshoots by more (larger $\left|\frac{p(t)}{y(t)}\right|$) but output gaps are closed more slowly (smaller γ), when adjusting stockholders respond more to current income (larger n).

The first part of Proposition 5 formalizes the comparative statics illustrated in Figure 2. The second and the third parts say that the central bank overshoots more (and induces a faster recovery) when the output-asset price relation is stronger—in the sense that the spending response to asset prices is either faster (with smaller lags) or larger. Strengthening the output-asset price relation generates two counteracting effects. On the one hand, it makes the asset price overshooting more effective—which induces more overshooting. On the other hand, it enables the central bank to achieve the same impact on the output gap with smaller (or shorter) overshooting—which induces less overshooting. With quadratic preferences (which are standard and commonly used in the literature), the first force dominates and the central bank induces more overshooting when it is more effective.

The final part of Proposition 5 considers the effect of increasing the stockholders' response to current income. This strengthens the *amplification* force by which current output gaps persist: for instance, negative output gaps tend to reinforce further negative gaps (cf. (7)). This induces the central bank to overshoot more in order to mitigate the amplification loop and speed up the convergence. Despite the central bank's response, the amplification force prevails and implies that the output gaps are closed more slowly.

A.3. Omitted proofs

Proof of Proposition 1. Most of the analysis is presented in the main text. Here, we verify that the value function is a quadratic function as in (11) and we characterize the endogenous coefficient, $v > 0$.

We first consider the optimality condition for problem (10),

$$p = \frac{\theta m}{\psi} V'(y). \tag{A.24}$$

We then differentiate Eq. (10) with respect to y and use the Envelope Theorem to obtain

$$(\rho + \theta(1-n)) V'(y) = -y + V''(y)\theta(mp - (1-n)y). \tag{A.25}$$

Eqs. $(A.24)$ and $(A.25)$ correspond to a second order ODE that characterizes the value function. We conjecture that the solution is a quadratic function [see Eq. (11)]

$$V(y) = -\frac{1}{2v}y^2.$$

Substituting the conjecture together with $p = \frac{\theta m}{\psi} V'(y)$ into $(A.25)$, we obtain:

$$(\rho + 2\theta(1-n)) \frac{y}{v} = y - \frac{\theta^2 m^2}{\psi} \frac{y}{v^2}.$$

After canceling y's from both sides and rearranging terms, we obtain the quadratic Eq. (12):

$$P(v) \equiv v^2 - (\rho + 2\theta(1-n)) v - \frac{\theta^2 m^2}{\psi} = 0.$$

This quadratic equation has one positive root and one negative root. The solution corresponds to the positive root, $v > 0$, since we require $V(y) = -\frac{1}{2v}y^2 \leq 0$. This root has the following closed form solution:

$$v = \frac{\rho + 2\theta(1-n) + \sqrt{(\rho + 2\theta(1-n))^2 + 4\frac{\theta^2 m^2}{\psi}}}{2}. \tag{A.26}$$

The rest of the analysis is in the main text. Note that Eq. (12) implies the convergence rate in Eq. (14) also satisfies

$$\gamma \equiv \theta\left(\frac{\theta m^2}{\psi v} + 1 - n\right) = v - (\rho + \theta(1-n)) > 0. \tag{A.27}$$

Combining this with Eq. (A.26), we can also calculate the convergence rate in closed form as follows:

$$\gamma = \frac{\sqrt{(\rho + 2\theta(1-n))^2 + 4\frac{\theta^2 m^2}{\psi}} - \rho}{2}. \tag{A.28}$$

This completes the proof. □

We next prove Proposition 5 that describes the comparative statics of the equilibrium. The following lemma facilitates the proof.

Lemma 1. *Consider the normalized value v which corresponds to the positive root of the polynomial* (12) *given by* (A.26).

(i) Increasing ψ decreases v and increases $v\psi$.

(ii) Increasing θ increases v and decreases v/θ.

(iii) Increasing m increases v and decreases v/m.

(iv) Increasing n decreases v and v/m.

Proof of Lemma 1. Eq. (A.26) implies v is decreasing in ψ, but $v\psi$ is increasing in ψ because

$$v\psi = \frac{(\rho + 2\theta(1-n))\psi + \sqrt{\psi^2(\rho + 2\theta(1-n))^2 + 4\psi\theta^2 m^2}}{2}$$

This proves the first part.

The same equation also implies v is increasing in θ, but v/θ is decreasing in θ because:

$$\frac{v}{\theta} = \frac{\frac{\rho}{\theta} + 2(1-n) + \sqrt{\left(\frac{\rho}{\theta} + 2(1-n)\right)^2 + 4\frac{m^2}{\psi}}}{2}.$$

This proves the second part.

The same equation also implies v is increasing in m, but v/m is decreasing in m

because:

$$\frac{v}{m} = \frac{\frac{\rho+2\theta(1-n)}{m} + \sqrt{\left(\frac{\rho+2\theta(1-n)}{m}\right)^2 + 4\frac{\theta^2}{\psi}}}{2}.$$

This proves the third part. Finally, this expression together with Eq. $(A.26)$ imply that both v and v/m are decreasing in n. This proves the last part. □

Proof of Proposition 5. Note that the overshooting and the convergence rate satisfy, $\left|\frac{p(t)}{y(t)}\right| = \frac{\theta m}{\psi v}$ and $\gamma = v - (\rho + \theta(1-n))$ [cf. $(A.27)$]. Thus, parts (i) and (iii) follow directly from the corresponding parts in Lemma 1.

Consider part (ii). Lemma 1 implies increasing θ increases θ/v. Thus, it also increases the overshooting, $\left|\frac{p(t)}{y(t)}\right| = \frac{\theta m}{\psi v}$ as well as the convergence rate, $\gamma \equiv \theta\left(\frac{\theta m^2}{v\psi} + 1 - n\right)$ [cf. $(A.27)$].

Consider part (iv). Lemma 1 implies increasing n increases m/v. Therefore, it increases overshooting, $\left|\frac{p(t)}{y(t)}\right| = \frac{\theta m}{\psi v}$. Eq. $(A.28)$ illustrates that it also decreases the convergence rate, γ, completing the proof. □

Proof of Proposition 2. We first complete the characterization of equilibrium and establish the existence of a unique fixed point that solves Eq. (20). After substituting the inflation function, $\boldsymbol{\pi}(y) = \frac{\kappa}{\rho+\gamma} y$ [see Eq. (18)], the planner's recursive problem becomes

$$\begin{aligned} \rho V(y;\boldsymbol{\pi}) &= \max_p - \left(1 + \phi\left(\frac{\kappa}{\rho+\gamma}\right)^2\right)\frac{y^2}{2} - \psi\frac{p^2}{2} + \frac{dV(y;\boldsymbol{\pi})}{dy}\dot{y} \qquad \text{(A.29)} \\ \text{where } \dot{y} &= \theta(mp - (1-n)y) \end{aligned}$$

As before, we conjecture a quadratic value function

$$V(y;\boldsymbol{\pi}) = -\frac{1}{2v(\gamma)} y^2.$$

The coefficient, $v(\gamma)$, depends on the growth rate through the inflation function $\boldsymbol{\pi}(y)$. The optimality condition implies Eq. (19) in the main text,

$$\mathbf{p}(y) = \frac{\theta m}{\psi}\frac{dV(y;\boldsymbol{\pi})}{dy} = \frac{-\theta m}{\psi v(y)} y.$$

Differentiating Eq. $(A.29)$ with respect to y and using the Envelope Theorem, we obtain

$$(\rho+\theta(1-n))\frac{dV(y;\boldsymbol{\pi})}{dy}=-\left(1+\phi\left(\frac{\kappa}{\rho+\gamma}\right)^2\right)y+\frac{d^2V(y;\boldsymbol{\pi})}{dy^2}\theta(m\mathbf{p}(y)-(1-n)y).$$

After substituting the functional form for the value function and the optimal policy, $\mathbf{p}(y)$, we obtain the following analogue of the quadratic in (12),

$$0=v(y)^2\left(1+\phi\left(\frac{\kappa}{\rho+\gamma}\right)^2\right)-v(y)(\rho+2\theta(1-n))-\frac{\theta^2m^2}{\psi}. \tag{A.30}$$

As before the coefficient, $v(\gamma)$, corresponds to the positive root of this quadratic. Using the shape of the quadratic, it is easy to check that $v'(\gamma)>0$. Note also that $v(\infty)$ corresponds to the solution in the benchmark with fully sticky prices.

Next note that Eq. (7) implies Eq. (20),

$$\gamma=\frac{-\dot{y}(t)}{y(t)}=\theta\left(\frac{\theta m^2}{\psi v(y)}+1-n\right).$$

The left side of this expression is an increasing function of γ whereas the right side is a decreasing function. There exists a unique fixed point over the range $\gamma\in(0,\infty)$, which corresponds to the equilibrium convergence rate.

We next establish the comparative statics of the equilibrium with respect to the price flexibility parameter, κ. Eq. $(A.30)$ implies $\frac{\partial v(\gamma)}{\partial\kappa}<0$: all else equal, greater price flexibility induces smaller value coefficient. Intuitively, with more flexible prices, the planner obtains a lower value due to the endogenous response of inflation to output gaps. The observation, $\frac{\partial v(\gamma)}{\partial\kappa}<0$, implies that increasing κ shifts the function on the right side of Eq. (20) upward. This increases the fixed point, $\frac{d\gamma}{d\kappa}>0$, and decreases the value coefficient corresponding to the fixed point, $\frac{dv(\gamma)}{d\kappa}<0$. Intuitively, the endogenous increase in the convergence rate, γ, mitigates but does not overturn the decline in the planner's value induced by greater κ.

Combining $\frac{dv(\gamma)}{d\kappa}<0$ with the quadratic in $(A.30)$ also implies $\frac{d\left(1+\phi\left(\frac{\kappa}{\rho+\gamma}\right)^2\right)}{d\kappa}>0$ (since otherwise the positive root of the quadratic would increase). This implies $\frac{d}{d\kappa}\left|\frac{d\boldsymbol{\pi}}{dy}\right|=\frac{d}{d\kappa}\frac{\kappa}{\rho+\gamma}>0$. As before, the endogenous increase in the convergence rate, γ, mitigates but does not overturn the increase in the inflation response to the output gap induced by greater κ. This completes the proof of the proposition. $\square$

Proof of Corollary 1. Next consider the comparative statics of equilibrium with respect to the planner's aversion to overshooting, ψ. Adapting the proof for the first part of Lemma 1 to this case implies that smaller ψ increases $v(\gamma)$ but decreases the product, $v(\gamma)\psi$. This in turn implies smaller ψ shifts the function on the right side of Eq. (20) upward. This increases the fixed point, $\frac{d\gamma}{-d\psi} > 0$, and increases the extent over overshooting evaluated at the fixed point, $\frac{d}{-d\psi}\left|\frac{d\mathbf{p}}{dy}\right| = \frac{d}{-d\psi}\frac{\theta m}{\psi v(y)} > 0$. Using Eq. (18), we conclude that smaller ψ also weakens the relation between inflation and the output gap, $\frac{d}{-d\psi}\left|\frac{d\boldsymbol{\pi}}{dy}\right| = \frac{d}{-d\psi}\frac{\kappa}{\rho+\gamma} < 0$. □

Proof of Corollary 2. Consider a Taylor rule that satisfies the condition in (23). We first show that there is a unique $\psi \in (0,\infty)$ that solves Eq. (22). Consider the function

$$\begin{aligned} f(\psi) &= \mathcal{R}(\psi) - \tau_y - \tau_\pi \frac{\kappa}{\rho+\gamma(\psi)} \\ \text{where } \mathcal{R}(\psi) &= \frac{\rho}{1+\rho} + \frac{\theta m}{\psi v}\left(\frac{\gamma+\rho}{1+\rho}\right). \end{aligned}$$

From the proof of Corollary 1, we have $\frac{d}{d\psi}\frac{\theta m}{\psi v(\psi)} < 0$ and $\frac{d\gamma}{d\psi} < 0$. It is also easy to check that $\lim_{\psi\to 0}\frac{\theta m}{\psi v(\psi)} = \infty$ and $\lim_{\psi\to\infty}\frac{\theta m}{\psi v(\psi)} = 0$. Combining these observations, we obtain

$$\begin{aligned} \frac{df(\psi)}{d\psi} &< 0, \\ \lim_{\psi\to 0} f(\psi) &= \infty, \text{ and} \\ \lim_{\psi\to\infty} f(\psi) &= \frac{\rho}{1+\rho} - \tau_y - \tau_\pi\frac{\kappa}{\rho+\gamma(\infty)} < 0. \end{aligned}$$

Here, the last inequality follows from condition (23). It follows that there is a unique $\psi \in (0,\infty)$ such that $f(\psi) = 0$.

We next establish the comparative statics of the solution to Eq. (22). Consider an increase in either τ_y or τ_π. This shifts the function $f(\psi)$ downward and reduces $\psi \in (0,\infty)$ that solves Eq. (22). This implies $\frac{d\psi}{d\tau_y} < 0$ and $\frac{d\psi}{d\tau_\pi} < 0$. □

Proof of Proposition 3. Consider part (i). We first integrate Eq. (8) forward and use $\lim_{t\to\infty} e^{-\rho t}p(t) = 0$ to obtain a present discounted value formula:

$$p(t) = \int_t^\infty e^{-\rho(s-t)}\left[\rho y(s) - (1+\rho) r(s)\right] ds. \tag{A.31}$$

We then log-linearize Eq. (25) to obtain:

$$
\begin{aligned}
p^{MB}(t) &= \int_0^\infty \frac{P^{MB*}(\mu)}{P^{MB*}} \log \frac{P^{MB}(t,\mu)}{P^{MB*}(\mu)} d\mu \\
&= -\int_0^\infty e^{-\rho\mu}\rho \int_t^{t+\mu} (R(s)-\rho)\, d\mu \\
&= -\int_0^\infty \int_t^{t+\mu} e^{-\rho\mu}\rho(1+\rho) r(s)\, ds d\mu \\
&= -\int_t^\infty \left(\int_{s-t}^\infty \rho e^{-\rho\mu} d\mu\right)(1+\rho) r(s)\, ds \\
&= -\int_t^\infty e^{-\rho(s-t)} r(s)(1+\rho)\, ds.
\end{aligned}
\tag{A.32}
$$

Here, the second line uses $P^{MB*} = \frac{\alpha Y^*}{\rho}$ and $P^{MB*}(\mu) = \alpha Y^* e^{-\rho\mu}$ along with Eq. (25). The third line substitutes $r(s) \simeq \frac{R(s)-\rho}{1+\rho}$. The fourth line uses Fubini's Theorem to switch the order of integration; and the last step simplifies the expression. Combining the final expression with Eq. $(A.31)$, we establish Eq. (26).

Differentiating the second line in $(A.32)$ with respect to time, we obtain

$$
\dot{p}^{MB}(t) = \int_0^\infty e^{-\rho\mu}\rho \left(-\frac{d}{dt}\left(\int_t^{t+\mu} R(s)\, ds\right)\right) d\mu.
$$

After substituting the definition of the zero-coupon yield, $y(t,\mu) = \frac{\int_t^{t+\mu} R(s)ds}{\mu}$, we establish Eq. (27).

Finally consider part (iii). Note that the last line in $(A.32)$ implies

$$
p^{MB}(t) = -\int_0^\infty e^{-\rho\mu}(R(t+\mu)-\rho)\, d\mu.
$$

Here, we have substituted $r(s)(1+\rho) \simeq R(s)-\rho$ and applied the change-of-variables, $\mu = s-t$. After differentiating the expression with respect to time, and substituting the definition of the forward rate, $f(t,\mu) = R(t+\mu)$, we establish Eq. (28). This completes the proof of the proposition. □

Proof of Proposition 4. Consider the constrained region $y(0) < \overline{y}$. The equilibrium corresponds to the solution to the differential equation system in (34), which we write as

$$\begin{bmatrix} \dot{y} \\ \dot{p} \end{bmatrix} = A \begin{bmatrix} y \\ p \end{bmatrix} + b$$
$$\text{where } A = \begin{bmatrix} -\theta(1-n) & \theta m \\ -\rho & \rho \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ \overline{r}(1+\rho) \end{bmatrix}.$$

In view of the parametric condition, this system has a unique steady-state given by,

$$\begin{bmatrix} y^* \\ p^* \end{bmatrix} = \begin{bmatrix} \zeta m \\ \zeta(1-n) \end{bmatrix} \text{ where } \zeta = \frac{\overline{r}(1+\rho)}{\rho(m-(1-n))}. \tag{A.33}$$

Away from the steady-state, the qualitative behavior of the system is governed by the eigenvalues of the matrix, A, which correspond to the zeros of the polynomial,

$$P(\lambda) = \lambda^2 + \lambda(\theta(1-n)-\rho) + \theta\rho(m-(1-n)). \tag{A.34}$$

Under the parametric condition, there are two real eigenvalues, $\lambda_1 < \lambda_2$, with corresponding eigenvectors given by

$$e_1 = \begin{bmatrix} \lambda_1 - \rho \\ -\rho \end{bmatrix} \text{ and } e_2 = \begin{bmatrix} \lambda_2 - \rho \\ -\rho \end{bmatrix}. \tag{A.35}$$

We denote the eigenmatrix with $E = [e_1, e_2]$. Given a generic vector $\begin{bmatrix} y - y^* \\ p - p^* \end{bmatrix}$ (measured in deviation from the steady-state), we find the corresponding eigencoordinates by solving $\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = E^{-1} \begin{bmatrix} y - y^* \\ p - p^* \end{bmatrix}$. Given an initial vector with eigencoordinates $\begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix}$, the solution is given by

$$\begin{bmatrix} y(t) - y^* \\ p(t) - p^* \end{bmatrix} = E \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \text{ where } \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} z_1(0)e^{\lambda_1 t} \\ z_2(0)e^{\lambda_2 t} \end{bmatrix}. \tag{A.36}$$

To characterize the solution further, we consider two cases depending on the sign of the term, $m - (1-n)$.

Case (i) $m - (1-n) > 0$. In this case, $\zeta < 0$ so the steady-state lies in the lower-left quadrant in the $y - p$ space [see $(A.33)$]. Moreover, Eqs. $(A.34 - A.35)$ (together with

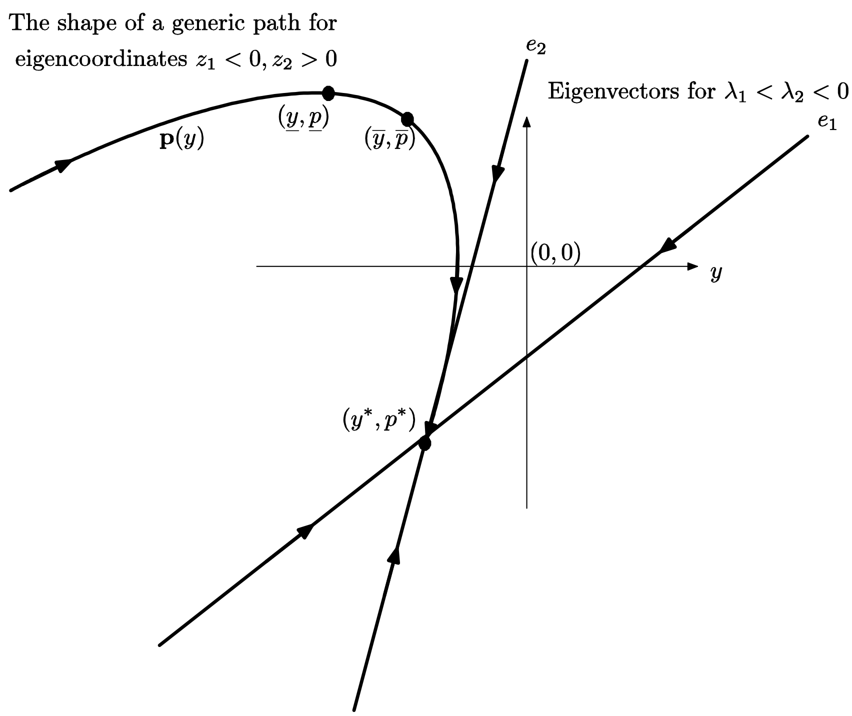

Figure 9: Phase diagram for the system in (34) when $m - (1-n) > 0$.

parametric condition) imply that both eigenvalues are negative $\lambda_1 < \lambda_2 < 0$ and that both eigenvectors, e_1, e_2, have a strictly positive slope in the $y - p$ space. Moreover, the eigenvector e_1 corresponding to the smaller (more negative) eigenvalue λ_1 has a smaller slope. Figure 9 illustrates the steady-state, the eigenvalues, and the eigenvectors for to this case.

Now consider a generic initial vector in the upper-left quadrant in the eigenspace: that is, with eigencoordinates $z_1(0) < 0 < z_2(0)$. For such vectors, Eq. $(A.36)$ together with the fact that $\lambda_1 < \lambda_2 < 0$ implies $\lim_{t \to \infty} \frac{z_1(t)}{z_2(t)} = 0$. Hence, as we solve the equation forward, the solution becomes qualitatively similar to the paths along the eigenvector e_2. Conversely, we have $\lim_{t \to -\infty} \frac{z_2(t)}{z_1(t)} = 0$. As we solve the equation backward, the solution becomes qualitatively similar to the paths along the eigenvector e_1. Combining the two cases, a generic solution path for the upper-left quadrant in the eigenspace has the shape illustrated in Figure 9. In particular, $\dot{y}(t) > 0, \dot{p}(t) > 0$ when t is sufficiently small and negative, whereas $\dot{y}(t) < 0, \dot{p}(t) < 0$ when t is sufficiently large and positive.

Now consider the vector $(\overline{y}, \overline{p})$ at which the discount rate constraint starts to bind. Since the discount rate is effectively unconstrained, our analysis in Section 2.2 implies $\dot{y}(t) > 0$ and $\dot{p}(t) < 0$ (this can also be seen from (34)). Combining this observation with Figure 9, the vector, $(\overline{y}, \overline{p})$, must be in the upper-left quadrant in the eigenspace.

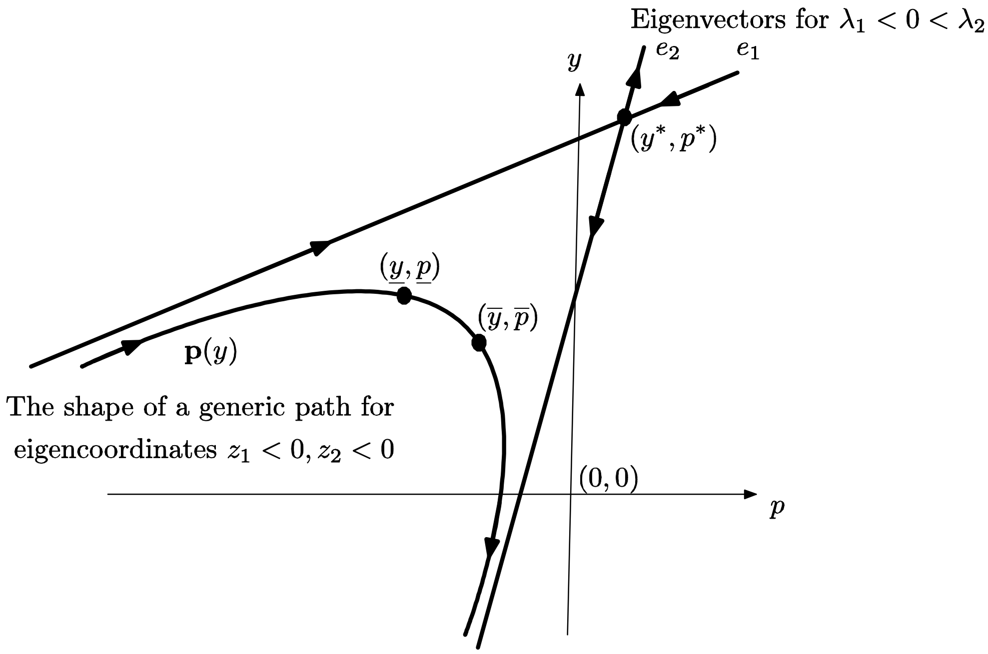

Figure 10: Phase diagram for the system in (34) when $m - (1 - n) < 0$.

Moreover, $(\overline{y}, \overline{p})$ must lie on the part of a solution path where $\dot{p}(t)$ has become strictly negative but $\dot{y}(t)$ has not yet become negative. These observations prove the proposition for case (i). As illustrated in Figure 9, the function $\mathbf{p}(y)$ corresponds to the solution path over the range, $y < \overline{y}$, and the cutoff $\underline{y}$ corresponds to the maximum of the function $\mathbf{p}(y)$.

Case (ii) $m - (1 - n) < 0$. In this case, $\zeta > 0$ so the steady-state lies in the upper-right quadrant in the $y - p$ space [see $(A.33)$]. Moreover, Eq. $(A.34)$ implies that one eigenvalue is strictly negative while the other one is strictly positive $\lambda_1 < 0 < \lambda_2$ and that the positive eigenvalue satisfies, $\lambda_2 < \rho$. Therefore, Eq. $(A.35)$ still implies that both eigenvectors have a positive slope and the eigenvector e_1 corresponding to the smaller (negative) eigenvalue λ_1 has a smaller slope. Figure 10 illustrates the steady-state, the eigenvalues, and the eigenvectors corresponding to this case.

Now consider a generic initial vector in the lower-left quadrant in the eigenspace: that is, with eigencoordinates $z_1(0) < 0, z_2(0) < 0$. For such vectors, Eq. $(A.36)$ together with the fact that $\lambda_1 < 0 < \lambda_2$ implies $\lim_{t\to\infty} z_1(t) = 0$ and $\lim_{t\to\infty} z_2(t) = -\infty$. Hence, as we solve the equation forward, the solution moves away from the steady-state at an exponential rate along the eigenvector e_2. Conversely, we also have $\lim_{t\to-\infty} z_1(t) = -\infty$ and $\lim_{t\to-\infty} z_2(t) = 0$. Hence, as we solve the equation backward, the solution moves away from the steady-state at an exponential rate along the eigenvector e_1. Combining the two cases, a generic solution path in the lower-left quadrant in the eigenspace has the shape illustrated in Figure 10. In particular, $\dot{y}(t) > 0, \dot{p}(t) > 0$ when t is sufficiently small and negative whereas $\dot{y}(t) < 0, \dot{p}(t) < 0$ when t is sufficiently large and positive.

Now consider the vector $(\overline{y}, \overline{p})$ at which the discount rate constraint starts to bind. As before, this vector satisfies $\dot{y}(t) > 0$ and $\dot{p}(t) < 0$. Combining this observation with Figure 10 implies that $(\overline{y}, \overline{p})$ must be in the lower-left quadrant in the eigenspace. Moreover, it must lie on the part of a solution path where $\dot{p}(t)$ has become strictly negative but $\dot{y}(t)$ has not yet become negative. These observations prove the proposition for case (ii). As illustrated in Figure 10, the function, $\mathbf{p}(y)$, corresponds to the solution path over the range, $y < \overline{y}$, and the cutoff $\underline{y}$ corresponds to the output gap where $\mathbf{p}(y)$ is maximized.

Finally, to facilitate the numerical solution, we also write the function, $\mathbf{p}(y)$, as a solution to a differential equation in y-domain. Observing that $\frac{dp}{dy} = \frac{dp}{dt} / \frac{dy}{dt}$, we combine the two differential equations in (34) to obtain,

$$\frac{d\mathbf{p}(y)}{dy} = \frac{(1+\rho)\overline{r} + \rho(\mathbf{p}(y) - y)}{\theta(m\mathbf{p}(y) - (1-n)y)} \text{ with } \mathbf{p}(\overline{y}) = \overline{p}. \tag{A.37}$$

We can obtain the functions $\mathbf{p}(y)$ illustrated in Figures $9-10$ by solving this differential equation backward in y-domain. □

Proof of Proposition 7, Part (i). First consider the value function that solves problem $(B.17)$. After changing the variables to log-deviations, and using (7), we can rewrite the problem as:

$$\rho V_1(\tilde{y}_1) = \max_{\tilde{p}_1} -\frac{\tilde{y}_1^2}{2} - \psi\frac{\tilde{p}_1^2}{2} + V_1'(\tilde{y}_1)\theta(m\tilde{p}_1 - (1-n)\tilde{y}_1) + \lambda(V_2(\tilde{y}_1 - k) - V_1(\tilde{y}_1)). \tag{A.38}$$

Here, with a slight abuse of notation, we continue to use $V_1(\cdot)$ to denote the value function defined over the output gap $\tilde{y}_1$ instead of the normalized output level y_1. After transition to recovery, the log output gap declines by k even though the normalized output remains unchanged (since log potential output increases by k).

Using the optimality conditions, we obtain analogues of Eqs. $(A.24)$ and $(A.25)$:

$$\tilde{p}_1 = \frac{\theta m}{\psi} V_1'(\tilde{y}_1) \tag{A.39}$$

$$\begin{pmatrix} \rho + \\ \theta(1-n) + \lambda \end{pmatrix} V_1'(\tilde{y}_1) = -\tilde{y}_1 - \frac{\lambda}{v}(\tilde{y}_1 - k) + V_1''(y_1)\theta\begin{pmatrix} m\tilde{p}_1 - \\ (1-n)y_1 \end{pmatrix}. \tag{A.40}$$

Here, the second line uses (11) to substitute for $V_2'(\tilde{y}_1 - k)$.

We conjecture that the solution has the form in $(B.19)$:

$$V_1(\tilde{y}_1) = a + b\tilde{y}_1 - \frac{1}{2v}\tilde{y}_1^2,$$

Combining this conjecture with Eq. $(A.40)$, and using $y = \tilde{y}_1$ to denote gaps, we obtain:

$$(\rho + \theta(1-n) + \lambda)\left(b - \frac{1}{v}y\right) = -y - \lambda\frac{1}{v}(y-k) - \frac{1}{v}\frac{\theta^2 m^2}{\psi}\left(b - \frac{1}{v}y\right) + \frac{1}{v}\theta(1-n)y.$$

Collecting the terms with y, we obtain:

$$-\frac{1}{v}(\rho + \theta(1-n) + \lambda)y = -y - \frac{1}{v}\lambda y + \frac{1}{v^2}\frac{\theta^2 m^2}{\psi}y + \frac{1}{v}\theta(1-n)y.$$

After canceling $-\frac{1}{v}\lambda y$ from both sides, and dropping y, we obtain the same quadratic (12). Hence, v is the same as before.

Likewise, collecting the terms without y, we obtain:

$$(\rho + \theta(1-n) + \lambda)b = \frac{\lambda k}{v} - \frac{1}{v}\frac{\theta^2 m^2}{\psi}b.$$

After rearranging terms, we solve:

$$b = \frac{\frac{\lambda k}{v}}{\rho + \theta(1-n) + \lambda + \frac{1}{v}\frac{\theta^2 m^2}{\psi}} = \frac{\lambda}{\rho + \lambda + \gamma}\frac{k}{v}$$

Here, the second equality substitutes for γ from $(A.27)$. This proves the value function takes the form in $(B.19)$ with the coefficient $b > 0$ given by $(B.20)$.

Next consider the solution. Substituting the value function into Eq. $(A.39)$, we obtain:

$$\tilde{p}_1(t) = \frac{\theta m}{\psi}\left(b - \frac{1}{v}\tilde{y}_1(t)\right). \tag{A.41}$$

Combining this with $(B.18)$, and using $(A.27)$ to substitute γ, we obtain:

$$\frac{d\tilde{y}_1(t)}{dt} = \frac{\theta^2 m^2}{\psi}b - \gamma\tilde{y}_1(t). \tag{A.42}$$

This equation implies that the output gap converges to a steady-state given by:

$$\tilde{y}_1(\infty) = \frac{\theta^2 m^2}{\psi}\frac{b}{\gamma} = \frac{\gamma - \theta(1-n)}{\gamma}\frac{\lambda}{\rho + \lambda + \gamma}k.$$

Here, we have substituted for b [cf. $(B.20)$] as well as $\frac{\theta^2 m^2}{\psi v} = \gamma - \theta(1-n)$ [cf. $(A.27)$]. Note that this also implies $\tilde{y}_1(\infty) \in (0,k)$. Then, Eq. $(B.18)$ implies the asset price gap converges to a steady-state given by:

$$\tilde{p}_1(\infty) = \frac{1-n}{m}\tilde{y}_1(\infty) > 0.$$

This establishes Eq. $(B.21)$ completes the proof of the first part.

Part (ii). Suppose $y_1(0) < y_1(\infty)$, which implies the output gap starts below its steady-state, $\tilde{y}_1(0) < \tilde{y}_1(\infty)$. Absent transition, Eq. $(A.42)$ implies that the output gap monotonically increases toward its steady-state. Combining this with Eq. $(A.41)$, we also obtain that the asset price gap starts above its steady-state, $\tilde{p}_1(0) > \tilde{p}_1(\infty)$, and monotonically declines toward its steady-state. This also implies the normalized output $y_1(0)$ monotonically increases towards its steady-state $y_1(\infty)$; and the normalized asset price monotonically decreases toward its steady-state.

Next consider the time t' at which the economy transitions to the recovery (starting with $y_1(0) < y_1(\infty)$). Normalized output remains unchanged. It is below its new potential because, $y_2(t') = y_1(t') < y_1(\infty) < 0$. The asset price gap after transition is given by Eq. (13) whereas the asset price gap before transition is given by Eq. $(A.41)$. That is:

$$\begin{aligned}
\tilde{p}_2(t') = p_2(t') &= -\frac{\theta m}{\psi v} y_1(t') \\
\tilde{p}_1(t') &= \frac{\theta m}{\psi}\left(b - \frac{1}{v}\left(y_1(t') - y_1^*\right)\right)
\end{aligned}$$

After taking the difference, and substituting $y_1^* = -k$, we obtain:

$$\tilde{p}_2(t') - \tilde{p}_1(t') = \frac{\theta m}{\psi}\left(\frac{k}{v} - b\right) = \frac{\theta m}{\psi}\frac{k}{v}\frac{\rho+\gamma}{\rho+\lambda+\gamma} > 0.$$

Here, the second equality substitutes b from $(B.20)$. Since $p_1^* < 0$, we also have $\tilde{p}_1(t') > p_1(t')$. Thus, we obtain $\tilde{p}_2(t') = p_2(t') > \tilde{p}_1(t') > p_1(t')$. At the time of transition, the normalized asset price as well as its gap increases. This completes the proof. □

B. Appendix: Omitted extensions

This appendix presents the extensions of the model omitted from the main text. We first generalize the analysis to the case in which adjusting stockholders are fully rational and incorporate the fact that they will infrequently readjust in future periods (see Section 2.3.2). We then consider a scenario in which the output gap is not negative at the moment but anticipated to turn negative in the near future (see Section 2.3.3).

B.1. Overshooting with sophisticated stockholders

Consider the baseline model analyzed in Sections 2 and 2.2. In the main text, we assume adjusting stockholders exogenously follow the rule in (6) with coefficients $m > 0, n \in [0, 1)$. In this appendix, we generalize the analysis to the case with fully sophisticated adjusting stockholders that incorporate the fact that they will get to readjust in the future according to a Poisson process with intensity θ. Specifically, our main result in this section shows that there exists an equilibrium in which the sophisticated consumers follow the rule in (6) with *endogenous* coefficients, that is:

$$\begin{aligned} c^{s,adj}(t) &= m(\gamma)\, p(t) + n(\gamma)\, y(t) \\ \text{where } m(\gamma) &= \frac{\theta - \rho}{\theta + \gamma} \text{ and } n(\gamma) = \frac{\rho}{\theta + \gamma} \end{aligned} \tag{B.1}$$

Proposition 6. *Consider the baseline model with fully rational stockholders. When $\theta > \rho$, there exists an equilibrium in which the linearized optimal consumption rule for adjusting stockholders satisfies*

$$c^{s,adj}(t) = (1 - \eta)\, p(t) + \eta \frac{\rho y(t)}{\rho + \gamma} \quad \textit{where} \quad \eta = \frac{\rho + \gamma}{\theta + \gamma}. \tag{B.2}$$

Equivalently, the equilibrium consumption rule is given by Eq. (6) with the endogenous coefficients in $(B.1)$ that satisfy $m(\gamma), n(\gamma) \in (0, 1)$. The equilibrium path, $[y(t), p(t), r(t)]_{t=0}^{\infty}$, is characterized by Proposition 1 given the coefficients $m(\gamma), n(\gamma)$. In particular, ***the equilibrium with fully rational stockholders also features asset price overshooting.*** *The equilibrium convergence rate, $\gamma > 0$, solves:*

$$\gamma = F(\gamma) \equiv \frac{\sqrt{(\rho + 2\theta(1 - n(\gamma)))^2 + 4\frac{\theta^2 m(\gamma)^2}{\psi}} - \rho}{2}. \tag{B.3}$$

Eq. $(B.2)$ characterizes the optimal linearized consumption rule along a path in which the linearized equilibrium variables converge to zero at a constant rate $\gamma > 0$. The remaining equilibrium variables are then characterized by Proposition 1 given the endogenous coefficients $m(\gamma), n(\gamma)$. Recall from our earlier analysis that the convergence rate, γ, depends on the coefficients, m, n. Since the endogenous coefficients also depend on γ, the equilibrium convergence rate corresponds to a fixed point.

Before we prove Proposition 6, we discuss the intuition behind the consumption rule in $(B.2)$. Note that $\frac{\rho y(t)}{\rho+\gamma}$ captures the log wealth change driven by the consumer's *fixed-rate wealth*: the present value of her lifetime income discounted at the steady-state discount rate ρ (along the equilibrium path with convergence rate γ). Hence, Eq. $(B.2)$ says that log consumption reacts to a weighted average of the log actual wealth change, $p(t)$, and the log fixed-rate wealth change, $\frac{\rho y(t)}{\rho+\gamma}$. When asset prices increase because current income increases, the two wealth measures increase by the same amount and log consumption reacts one-to-one to log wealth changes. However, when asset prices increase because of a discount rate cut, log consumption reacts less than one-to-one to log wealth change ($m < 1$). Hence, the consumer *does* react to wealth changes induced by discount rate changes, but *less so* than when she can adjust continuously. As expected, $\theta \to \infty$ implies $\eta \to 0$: when consumption adjustment is sufficiently rapid, we recover the usual consumption rule.

Why does a sophisticated consumer react to the interest-rate driven wealth changes relatively less? As we show below, this is because interest-rate-driven wealth changes are associated with net income effects that mitigate the wealth effect. Intuitively, since the consumer cannot reoptimize in the future (in some states), the substitution effect becomes relatively weak. Therefore, despite log preferences, the substitution and income effects do not net out. Instead, the income effect dominates and implies that—keeping wealth constant—decreasing the discount rate decreases spending. Equivalently, lower discount rates make it costlier to finance a fixed consumption stream until the next adjustment opportunity.

We next prove Proposition 6 in three steps. First, we characterize the (log-linearized) optimal consumption rule for an arbitrary path of (log-linearized) future income and discount rate gaps, $[y(t), r(t)]_{t=0}^{\infty}$. Second, we show that, along the equilibrium path in which $y(t)$ and $r(t)$ converge to zero at an exponential rate $\gamma > 0$, the consumption rule is given by $(B.2)$. Third, we show there exists a convergence rate that solves the fixed point equation $(B.3)$ and prove Proposition 6.

Optimal consumption with Poisson adjustment. We establish optimal consumption rule for $t = 0$. Since the model is stationary, the results also apply for other times $t > 0$. Consider a stockholder with assets $A(0)$ that gets to adjust her consumption. For now, we take the initial asset level as exogenous (from the discount rate) and endogenize it subsequently. The stockholder's recursive problem is given by:

$$\begin{aligned}
V\left(A\left(0\right)\right) &= \max_{C}\int_{0}^{\infty}\theta e^{-\theta T}\left[\int_{0}^{T}e^{-\rho t}\log C dt+e^{-\rho T}V\left(A\left(T\right)\right)\right]dT \qquad \text{(B.4)}\\
&= \max_{C}\int_{0}^{\infty}\theta e^{-\theta T}\left[\frac{1-e^{-\rho T}}{\rho}\log C+e^{-\rho T}V\left(A\left(T\right)\right)\right]dT\\
\text{where } \dot{A}\left(t\right) &= R\left(t\right)A\left(t\right)-C
\end{aligned}$$

Here, we write the stockholder's problem as an integral over adjustment times T. The probability that adjustment takes place at time T is $\theta e^{-\theta T}$. Conditional on adjustment at time T, the stockholder receives the utility in the brackets.

Suppose consumption is strictly positive along the optimal path (this will be the case since we focus on small deviations from the steady-state). Then, the value function has the homogeneity property:

$$V\left(A\left(t\right)\right)=\frac{\log A\left(t\right)}{\rho}+V\left(1\right).$$

Note also that we can integrate the budget constraint forward to solve for the value of assets at time T. In particular, the present value of $A\left(T\right)$ is equal to the initial assets net of the present value of the fixed consumption path until time T:

$$\frac{A\left(T\right)}{\exp\left(\int_{0}^{T}R\left(s\right)ds\right)}=A\left(0\right)-C\int_{0}^{T}\exp\left(-\int_{0}^{t}R\left(s\right)ds\right)dt$$

Substituting this into $(B.4)$, we obtain the following optimality condition:

$$\int_{0}^{\infty}\theta e^{-\theta T}\left[\frac{1-e^{-\rho T}}{C\rho}-\frac{\exp\left(-\rho T\right)}{\rho}\frac{\int_{0}^{T}\exp\left(-\int_{0}^{t}R\left(s\right)ds\right)dt}{A\left(0\right)-C\int_{0}^{T}\exp\left(-\int_{0}^{t}R\left(s\right)ds\right)dt}\right]dT=0.$$

After rearranging terms, we have:

$$\frac{1}{C}\frac{1}{\theta+\rho}=\int_{0}^{\infty}\frac{\theta}{\rho}e^{-(\theta+\rho)T}\frac{\int_{0}^{T}\exp\left(-\int_{0}^{t}R\left(s\right)ds\right)dt}{A\left(0\right)-C\int_{0}^{T}\exp\left(-\int_{0}^{t}R\left(s\right)ds\right)dt.}dT.$$

The solution scales with initial wealth. Thus it can be written as

$$C=\rho X\left(0\right)A\left(0\right) \qquad \text{(B.5)}$$

for some $X(0) > 0$. After substituting, we further obtain

$$\frac{1}{X(0)} = \int_0^\infty \theta(\theta+\rho) e^{-(\theta+\rho)T} \frac{\int_0^T \exp\left(-\int_0^t R(s)\,ds\right) dt}{1 - \rho X(0) \int_0^T \exp\left(-\int_0^t R(s)\,ds\right) dt.} dT. \tag{B.6}$$

Consider the potential steady-state, $R^*(t) = \rho$ for each t. In this case, it is easy to check $X^*(0) = 1$ and thus $C = \rho A(0)$. Consider also the special case $\theta \to \infty$. It can be checked that this also gives the same solution $X(0) = 1$ and $C = \rho A(0)$.

Beyond these special cases, the solution is complicated. In particular, $X(0)$ depends on the whole future path of discount rates. To make progress, we log-linearize Eq. $(B.6)$ around the potential steady-state. Specifically, we define the log-deviation terms $x(0) = \log \frac{X(0)}{X^*(0)}$ and $r(t) = \log \frac{1+R(t)}{1+\rho}$. Substituting these variables into $(B.6)$ and using the approximation $r(s) \simeq \frac{R(s)-\rho}{1+\rho}$ along with $X^*(0) = 1$, we obtain:

$$\frac{1}{\exp(x(0))} = \int_0^\infty \theta(\theta+\rho) e^{-(\theta+\rho)T} \frac{\int_0^T \exp(-\rho t) \exp\left(-\int_0^t (1+\rho) r(s)\right) dsdt}{1 - \rho \exp(x(0)) \int_0^T \exp(-\rho t) \exp\left(-\int_0^t (1+\rho) r(s)\right) dt} dT.$$

Linearizing this expression around $x(0) = r(t) = 0$, we further obtain:

$$\begin{aligned} x(0) &= \int_0^\infty \theta(\theta+\rho) e^{-(\theta+\rho)T} \left\{ \begin{array}{c} -\rho x(0) \int_0^T e^{-\rho t} dt \frac{\int_0^T e^{-\rho t} dt}{\left(1-\rho \int_0^T e^{-\rho t} dt\right)^2} + \\ -\int_0^T e^{-\rho t} \left(-\int_0^t (1+\rho) r(s)\,ds\right) dt \frac{1}{1-\rho \int_0^T e^{-\rho t} dt} \\ -\rho \int_0^T e^{-\rho t} \left(-\int_0^t (1+\rho) r(s)\,ds\right) dt \frac{\int_0^T e^{-\rho t} dt}{\left(1-\rho \int_0^T e^{-\rho t} dt\right)^2} \end{array} \right\} dT \\ &= \int_0^\infty \theta(\theta+\rho) e^{-(\theta+\rho)T} \left\{ -x(0) \frac{\left(1-e^{-\rho T}\right)^2}{\rho \left(e^{-\rho T}\right)^2} + \frac{\int_0^T e^{-\rho t} \int_0^t (1+\rho) r(s)\,dsdt}{\left(e^{-\rho T}\right)^2} \right\} dT. \end{aligned}$$

Here, the first line uses the chain rule to evaluate the derivatives and cancels the constant terms from both sides (since the equation holds for $x(0) = r(t) = 0$). The second line calculates the integrals. After rearranging terms, we further obtain

$$\begin{aligned} & x(0) \left[1 + \int_0^\infty \frac{\theta(\theta+\rho)}{\rho} e^{-(\theta-\rho)T} \left(1-e^{-\rho T}\right)^2 dT \right] \\ = & \int_0^\infty \theta(\theta+\rho) e^{-(\theta-\rho)T} \int_0^T e^{-\rho t} \int_0^t (1+\rho) r(s)\,dsdtdT \end{aligned}$$

Calculating the integrals and simplifying further, we obtain:

$$x(0)\left(\frac{\theta+\rho}{\theta-\rho}\right)=\int_0^{\infty}\theta(\theta+\rho)e^{-(\theta-\rho)T}\int_0^T e^{-\rho t}\int_0^t(1+\rho)r(s)\,dsdtdT. \tag{B.7}$$

Recall that $C = X(0)\rho A(0)$ where $X(0) \simeq X^* \exp(x(0))$. Hence, Eq. $(B.7)$ illustrates that the slope of the consumption function deviates from the usual slope iff the discount rates are different from their steady-state levels. As long as $\theta > \rho$ (which we assume), keeping current wealth constant, greater discount rate increases (resp. decreases) spending. Intuitively, since stockholder cannot reoptimize in the future (in some states), the substitution effect becomes weaker. Therefore, despite log preferences, the substitution and income effects do not net out. Instead, the income effect dominates and implies—keeping the wealth constant—increasing the discount rate increases spending. A higher discount rate makes it cheaper to finance a steady consumption stream (C), which induces the stockholder to spend more.

As before, there is also a wealth effect once we endogenize the initial wealth $A(0)$. In particular, note that the wealth of the representative adjusting stockholder is equal to the price of capital, $A(0) = P(0)$. This implies:

$$\begin{aligned} C^{s,adj}(0) &= X(0)\rho P(0) \\ \text{where } P(0) &= \int_0^{\infty}\alpha Y(t)e^{-\int_0^t R(s)ds}dt. \end{aligned} \tag{B.8}$$

Log-linearizing the price of capital around the potential steady-state and using $r(s) \simeq \frac{R(s)-\rho}{1+\rho}$, we obtain

$$\begin{aligned} p(0) &= \int_0^{\infty}e^{-\rho t}\rho y(t)\,dt-\int_0^{\infty}\int_0^t e^{-\rho t}\rho r(s)(1+\rho)\,dsdt \\ &= \int_0^{\infty}e^{-\rho t}\rho y(t)\,dt-\int_0^{\infty}\left(\int_s^{\infty}e^{-\rho t}\rho dt\right)r(s)(1+\rho)\,ds \\ &= \int_0^{\infty}e^{-\rho t}\left(\rho y(t)-(1+\rho)r(t)\right)dt. \end{aligned} \tag{B.9}$$

Here, the second line switches the order of integration and the last line collects the terms (see Eq. $(A.31)$ in the main text for an alternative derivation). As usual, increasing the discount rate reduces the value of capital. All else equal, this reduces spending.

Finally, log-linearizing Eq. $(B.8)$ around the potential steady-state, we conclude that the optimal consumption rule is approximately,

$$c^{s,adj}(0) = x(0) + p(0), \tag{B.10}$$

where $x(0)$ is given by Eq. $(B.7)$ and $p(0)$ is given by $(B.9)$.

Optimal consumption along the equilibrium path. We next calculate consumption along the equilibrium path and prove Eq. (6). In equilibrium, we have:

$$y(t) = y(0)e^{-\gamma t} \text{ and } r(t) = r(0)e^{-\gamma t}.$$

Substituting these expressions in $(B.9)$, we solve for the asset price along the equilibrium path:

$$p(0) = \frac{\rho y(0) - (1+\rho) r(0)}{\rho + \gamma}. \tag{B.11}$$

As expected a greater initial output increases the price and a greater initial discount rate decreases the price.

Next, we define η such that:

$$x(0) = \frac{(1+\rho) r(0)}{\rho + \gamma}\eta. \tag{B.12}$$

This normalization is useful to simplify consumption function. Specifically, combining this with Eqs. $(B.10)$ and $(B.11)$, we obtain:

$$\begin{aligned} c^{s,adj}(0) &= \frac{(1+\rho) r(0)}{\rho+\gamma}\eta + p(0) \\ &= \left(\frac{\rho y(0)}{\rho+\gamma} - p(0)\right)\eta + p(0) \\ &= (1-\eta) p(0) + \eta \frac{\rho}{\rho+\gamma} y(0) \end{aligned} \tag{B.13}$$

Here, the second line substitutes $\frac{(1+\rho)r(0)}{\rho+\gamma} = \frac{\rho y(0)}{\rho+\gamma} - p(0)$ from Eq. $(B.11)$ to express the consumption function in terms of $y(0), p(0)$ (instead of $r(0), p(0)$).

It remains to solve for η along the equilibrium path. Combining Eqs. $(B.7)$ and

$(B.12)$, and substituting $r(s) = r(0)e^{-\gamma s}$, we calculate:

$$\begin{aligned}
\eta &= \frac{(\rho+\gamma)(\theta-\rho)}{\theta+\rho}\int_0^\infty \theta(\theta+\rho)e^{-(\theta-\rho)T}\int_0^T e^{-\rho t}\frac{1-e^{-\gamma t}}{\gamma}dtdT \\
&= \frac{(\rho+\gamma)(\theta-\rho)\theta}{\gamma}\int_0^\infty e^{-(\theta-\rho)T}\left[\frac{1-e^{-\rho T}}{\rho}-\frac{1-e^{-(\rho+\gamma)T}}{\rho+\gamma}\right]dT \\
&= \frac{(\rho+\gamma)(\theta-\rho)\theta}{\gamma}\left\{\frac{1}{\rho}\left(\frac{1}{\theta-\rho}-\frac{1}{\theta}\right)-\frac{1}{\rho+\gamma}\left(\frac{1}{\theta-\rho}-\frac{1}{\theta+\gamma}\right)\right\} \\
&= \frac{(\rho+\gamma)(\theta-\rho)\theta}{\gamma}\left\{\frac{1}{(\theta-\rho)\theta}-\frac{1}{(\theta-\rho)(\theta+\gamma)}\right\} \\
&= \frac{\rho+\gamma}{\theta+\gamma}. \qquad \text{(B.14)}
\end{aligned}$$

The final expression has intuitive comparative statics. For instance, as $\theta \to \infty$, we have $\eta \to 0$. That is, when the stockholder adjusts very rapidly, we recover the standard rule.

Combining Eqs. $(B.13)$ and $(B.14)$, we prove the consumption rule in Eq. $(B.2)$. This rule also implies the consumption rule in Eq. (6) with the endogenous coefficients in $(B.1)$ since

$$m(\gamma) = 1-\eta = \frac{\theta-\rho}{\theta+\gamma} \text{ and } n(\gamma) = \eta\frac{\rho}{\rho+\gamma} = \frac{\rho}{\theta+\gamma}.$$

Note that $m, n \in (0,1)$ since we assume $\theta > \rho$.

The equilibrium convergence rate. Note that the slope coefficients depend on the equilibrium convergence rate, that is, $m = m(\gamma)$ and $n = n(\gamma)$. Conversely, Proposition 1 shows that the convergence rate, γ, depends on m and n. Specifically, Eq. $(A.28)$ shows the convergence rate solves the fixed point equation $(B.3)$. We next complete the proof of Proposition 6.

Proof of Proposition 6. It remains to show that when $\theta > \rho$ there exists a solution to the fixed point equation $(B.3)$, which we replicate here:

$$\begin{aligned}
\gamma &= F(\gamma) \equiv \frac{\sqrt{(\rho+2\theta(1-n(\gamma)))^2+4\frac{\theta^2 m(\gamma)^2}{\psi}}-\rho}{2}, \\
&\text{where } m(\gamma) = \frac{\theta-\rho}{\theta+\gamma} \text{ and } n(\gamma) = \frac{\rho}{\theta+\gamma}.
\end{aligned}$$

We claim there is a solution that satisfies $\gamma \in (0, \overline{\gamma})$ where $\overline{\gamma} = \theta\left(1+\frac{1}{\sqrt{\psi}}\right)$.

First note that

$$F(\gamma = 0) \geq \frac{\sqrt{(\rho + 2\theta(1 - n(0)))^2} - \rho}{2} = \theta(1 - n(0)) = \theta - \rho > 0.$$

Here, the inequality follows since $\theta > \rho$. Next note that

$$\begin{aligned} F(\gamma) &\leq \frac{\sqrt{\left(\rho + 2\theta(1 - n(\gamma)) + 2\frac{\theta m(\gamma)}{\sqrt{\psi}}\right)^2} - \rho}{2} \\ &= \theta\left(1 - n(\gamma) + \frac{m(\gamma)}{\sqrt{\psi}}\right) \\ &< \theta\left(1 + \frac{1}{\sqrt{\psi}}\right) = \overline{\gamma} \end{aligned}$$

Here, the last equality uses $n(\gamma), m(\gamma) \in (0,1)$. In particular, $F(\overline{\gamma}) < \overline{\gamma}$. It follows that Eq. $(B.3)$ has a solution that satisfies $\gamma \in (0, \overline{\gamma})$. This establishes the existence of a fixed point and completes the proof. □

B.2. Preemptive asset price overshooting

In the main text, we focus on a recovery scenario in which the output gap is negative and the central bank's concern is to close the output gap as quickly as possible. We next extend the analysis to a case when the output gap is not (necessarily) negative at the moment but the central bank anticipates that it will turn negative in the near future. This situation may arise, for example, when the economy is experiencing a sharp but temporary decline in potential output that drags aggregate demand down with it, as in the Covid-19 recession. Our main result in this section shows that the central bank may find it optimal to *preemptively* overshoot asset prices also in this scenario.

Specifically, there are now two aggregate states denoted by subscript $s \in \{1, 2\}$. State $s = 2$ (the "recovery") corresponds to our analysis up to now. In particular, potential output is given by $Y_2^*(t) = Y^*$. State $s = 1$ (the "recession") corresponds to an earlier period in which potential output is lower, $Y_1^*(t) = \exp(-k) Y^*$ for some $k > 0$. The economy starts in the recession state $s = 1$ and transitions to the recovery state $s = 2$ according to a Poisson process with intensity $\lambda > 0$. Once the economy transitions, it remains in state $s = 2$ forever.[18]

[18] Our analysis is robust to the exact parametric change that induces the decline in potential output. For concreteness, suppose we only change the total factor productivity (TFP): that is, $A_1(t) = A\exp(-k)$ and $A_2(t) = A$, and the remaining parameters are the same across states (see Appendix A.1).

In this section, we use the notation $y_s = \log \frac{Y_s}{Y^*}, p_s = \log \frac{P_s}{P^*}, r_s = \log \frac{1+R_s}{1+\rho}$ to denote *normalized log variables*: their difference from potential levels *in the recovery state* [cf. (5)]. We use $\tilde{y}_s = \log \frac{Y_s}{Y_s^*}, \tilde{p}_s = \log \frac{P_s}{P_s^*}$ to denote the log output and the log asset price *gaps:* their difference from potential levels *within the corresponding state*. For the recovery state, the normalized and the gap variables are the same, $y_2 = \tilde{y}_2, p_2 = \tilde{p}_2$; but for the recession state they are different, $y_1 \leq \tilde{y}_1, p_1 \leq \tilde{p}_1$ (since the potential levels are lower).

We make three additional simplifying assumptions. First, stockholders have Epstein-Zin preferences with discount rate ρ, EIS equal to one, and RRA equal to 0. Hence, stockholders effectively have log utility with respect to consumption-saving decisions as before, but they are risk neutral with respect to portfolio decisions. This ensures that the asset pricing side of the model is the same as before, and the discount rate is the same as the expected return on capital. Thus, the log-linearized return is given by (8) in state $s = 2$ and by the following expression in state $s = 1$:[19]

$$r_1(t) = \frac{\rho}{1+\rho}(y_1(t) - p_1(t)) + \frac{\dot{p}_1(t) + \lambda(p_2(t) - p_1(t))}{1+\rho}. \tag{B.15}$$

The expected return on the market portfolio accounts for the change in the asset price when there is a transition.

Second, adjusting stockholders follow the rule in (6) in both the recovery and the recession states.[20] Thus, normalized output follows the same dynamics in the recession state [cf. (7)]:

$$\dot{y}_1 = \theta(mp_1 - (1-n)y_1). \tag{B.16}$$

Finally, we let the central bank solve the following version of problem (10) in state 1:

$$\begin{aligned} \rho V_1(y_1) &= \max_{p_1} -\frac{\tilde{y}_1^2}{2} - \psi\frac{\tilde{p}_1^2}{2} + V_1'(y_1)\dot{y}_1 + \lambda(V_2(y_1) - V_1(y_1)), \quad \text{(B.17)} \\ \text{where } \tilde{y}_1 &= y_1 - y_1^* \text{ with } y_1^* = -k < 0, \\ \text{and } \tilde{p}_1 &= p_1 - p_1^* \text{ with } p_1^* = -k(1-n)/m < 0. \end{aligned}$$

[19] We obtain $(B.15)$ by log-linearizing the following equation that describes the interest rate (around the potential levels in the recovery state $\left(Y^*, P^* = \frac{\alpha Y^*}{\rho}, R^* = \rho\right)$:

$$1 + R_1(t) = 1 + \frac{\alpha Y_1(t) + \dot{P}_1(t) + \lambda(P_2(t) - P_1(t))}{P_1(t)}.$$

[20] When consumers are fully rational, the optimal consumption rule would take this form with possibly state-dependent coefficients m_s, n_s. We assume the same rule for both states to keep the notation simple, although our results would qualitatively apply in the more general case (as long as $m_s, n_s \in (0,1)$ for both states).

Given current normalized output y_1, the central bank solves a gap-minimizing problem as before. However, the gaps in state $s = 1$ are no longer equal to the normalized variables. The potential output, y_1^*, is lower (recall that $Y_1^* = \exp(-k) Y^*$). The potential asset price, p_1^*, is also lower and reflects the lower potential output. If stockholders adjusted instantaneously ($\theta = \infty$), then keeping the asset price at its potential would ensure that output is also at its potential [cf. $(B.16)$]. The value function also accounts for the expected transition to the recovery state. In particular, $V_2(\cdot)$ denotes the recovery value function that we characterized in Section 2.2.

We define the equilibrium with optimal monetary policy as before. In the recession state, output and asset prices solve problem $(B.17)$ given $(B.16)$, and the discount rate is given by $(B.15)$. After transition to the recovery state, the equilibrium is given by Definition 1.

To analyze the equilibrium, note that Eq. $(B.16)$ also holds in terms of the gaps:

$$\frac{d\tilde{y}_1}{dt} = \theta\left(m\tilde{p}_1 - (1-n)\tilde{y}_1\right). \tag{B.18}$$

Thus, problem $(B.17)$ has a similar structure to the earlier problem (10). In the appendix, we show that the solution also has a similar form [cf. (11)]:

$$\begin{aligned} V_1 &= a + b\tilde{y}_1 - \frac{1}{2v}\tilde{y}_1^2, & \text{(B.19)} \\ \text{where} \quad b &= \frac{\lambda}{\lambda+\rho+\gamma}\frac{k}{v} > 0. & \text{(B.20)} \end{aligned}$$

Here, $v, \gamma > 0$ are the same as before. In this case, the value function features an additional linear component, $a + b\tilde{y}_1$, with $b > 0$. In particular, a positive output gap, $\tilde{y}_1 > 0$, *increases* the value function relative to the recovery state since $b\tilde{y}_1 > 0$.

A positive gap is now less costly since it will reduce the size of the output gap once potential output recovers. This suggests that the central bank might *preemptively* induce positive gaps in the recession state. We next state our main result in this section, which completes the characterization and verifies that the equilibrium features *preemptive* overshooting.

Proposition 7. *(i) In the recession state $s = 1$, the value function is given by $(B.19)$, where a is a constant and $b > 0$ is given by $(B.20)$. Absent a transition, the optimal asset* *price and output gaps follow the dynamics*

$$\begin{aligned}
\tilde{p}_1(t) &= \frac{\theta m}{\psi}\left(b - \frac{1}{v}\tilde{y}_1(t)\right), \\
\frac{d\tilde{y}_1(t)}{dt} &= \frac{\theta^2 m^2}{\psi} b - \gamma \tilde{y}_1(t).
\end{aligned}$$

These gaps monotonically converge to ***strictly positive steady-state levels*** *given by*

$$\tilde{y}_1(\infty) \equiv \frac{\gamma - \theta}{\gamma}\frac{\lambda}{\lambda + \rho + \gamma} k > 0 \quad \text{and} \quad \tilde{p}_1(\infty) = \frac{1-n}{m}\tilde{y}_1(\infty) > 0. \tag{B.21}$$

The normalized output and asset price $y_1(t), p_1(t)$ converge to corresponding steady-states $y_1(\infty) = \tilde{y}_1(\infty) + y_1^*$ and $p_1(\infty) = \tilde{p}_1(\infty) + p_1^*$. The steady-state output exceeds its potential in the recession state but remains below its potential in the recovery state, $y_1(\infty) \in (-k, 0)$ (equivalently, $\tilde{y}_1(\infty) \in (0, k)$).

(ii) Suppose the normalized output is initially not too high, $y_1(0) < y_1(\infty)$ (e.g., it is at potential $y_1(0) = y_1^* = -k$). Before transition, the equilibrium features ***preemptive asset price overshooting****: The normalized asset price starts above its steady-state level as well as its potential, $p_1(0) > p_1(\infty) > p_1^*$. Absent a transition, the normalized output monotonically increases toward $y_1(\infty)$, and the normalized asset price monotonically decreases toward $p_1(\infty)$. At the moment of the transition to recovery (time t'),* ***asset price overshooting increases****: the normalized output remains unchanged but the output gap becomes negative, $\tilde{y}_2(t') = y_2(t') = y_1(t') < 0$, and the normalized asset price level as well as its gap jump upward, $\tilde{p}_2(t') = p_2(t') > \tilde{p}_1(t') > p_1(t')$.*

Figure 11 illustrates the equilibrium (the solid lines) for a particular parameterization in which the economy starts with a zero output gap and the transition takes place at its expected time, $t' = 1/\lambda$. We set $k = 0.05$ so that the recession corresponds to a 5% decline in potential output. The dotted lines illustrate the within-state potential levels (the gaps are the distance from these lines). The equilibrium features preemptive overshooting: the central bank sets a positive asset price gap, which gradually induces a positive output gap. After transition, the central bank overshoots the asset price even more, which helps close the negative output gap. For comparison, the figure also illustrates what happens when the aversion to overshooting is extreme ($\psi = \infty$) so that the central bank always keeps the asset price at its potential. In this case, output does not start to recover until productivity actually recovers. As a result, the economy enters the recovery with a greater aggregate demand gap and closes this gap more slowly [cf. Figure 2].

Figure 12 illustrates the effect of increasing λ—the expected transition rate to recovery.

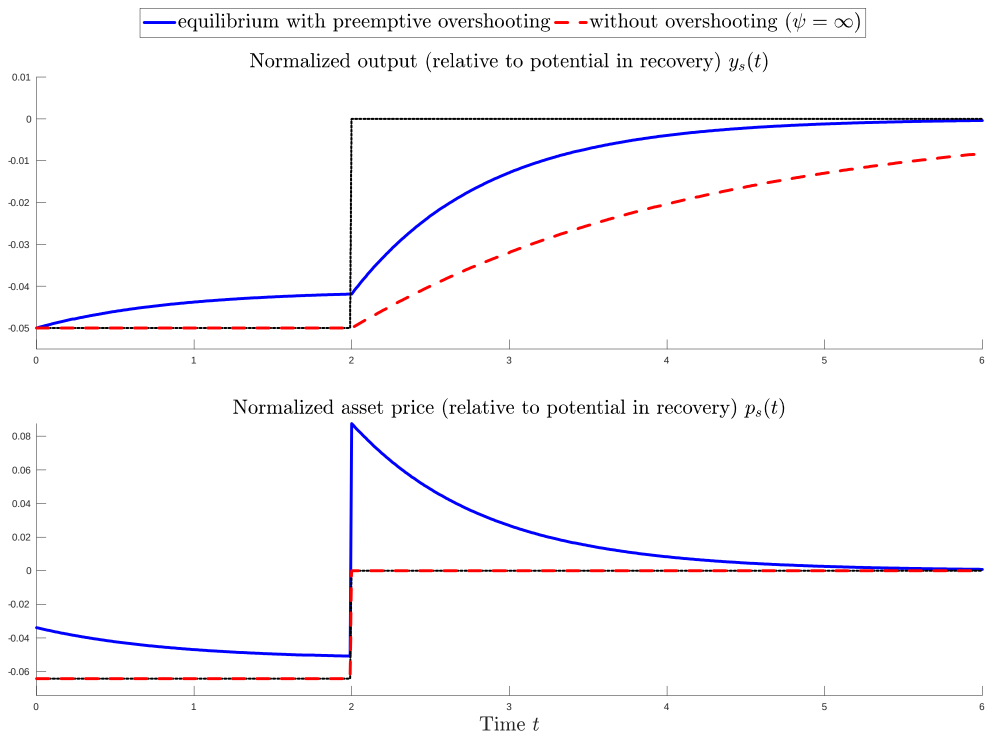

Figure 11: A simulation of the equilibrium variables starting in the shock state $s = 1$ with overshooting (solid lines) and with extreme aversion to overshooting ($\psi = \infty$, dashed lines). The dotted lines correspond to the within-state potential output and asset price.

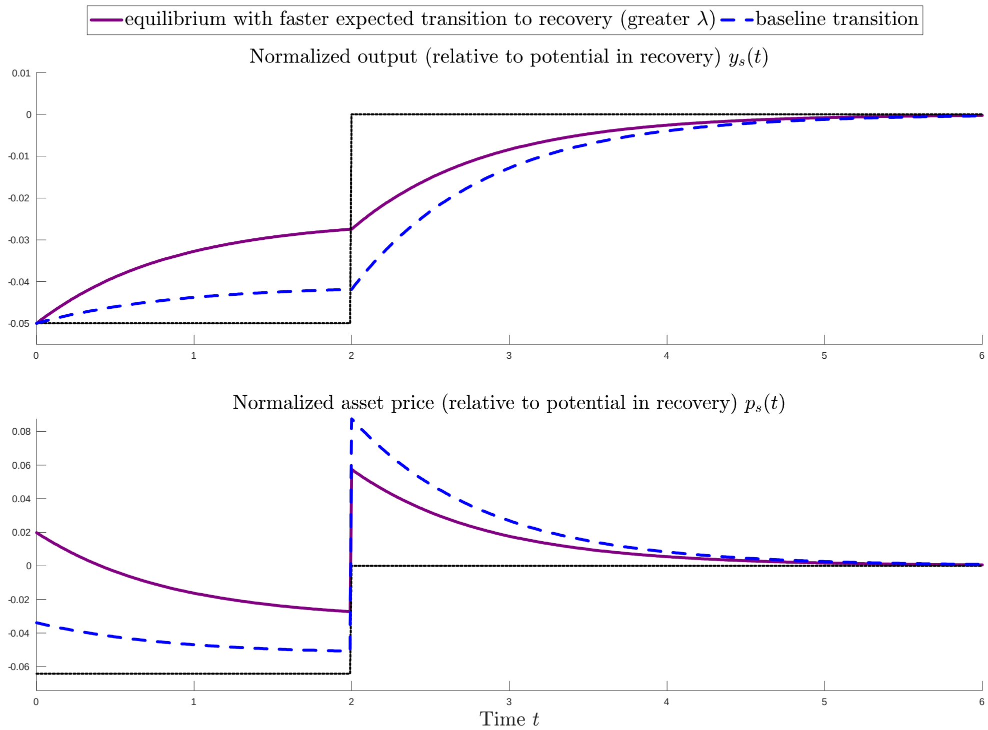

Figure 12: Equilibrium with a greater expected transition rate ($\tilde{\lambda} > \lambda$, solid lines) and with the baseline rate (λ, dashed lines).

The solid lines plot the equilibrium when the expected transition rate is higher ($\tilde{\lambda} > \lambda$) but the actual transition takes place at the same time as in the previous figure ($t' = 1/\lambda$). In this case, preemptive overshooting is stronger: asset prices increase by more and the output in the recession converges to a level closer to its potential in the recovery [cf. ($B.21$)]. Intuitively, since the central bank expects the recovery to start soon, it also engages in considerable overshooting in the recession. In fact, the figure highlights that, by frontloading much of the overshooting to the recession, the central bank might do less overshooting when the recovery finally arrives (compared to the early stages of the recession).

C. Appendix: Data sources

This appendix presents the data sources and variable construction used in the main text.

S&P 500 Index. Data for the S&P 500 index used in Figures 1 and 3 is taken from the St. Louis Fed's online database of Federal Reserve Economic Data (FRED) (available at https://fred.stlouisfed.org/series/SP500).

Continued Jobless Claims. Data for continued claims (seasonally adjusted) used in Figure 1 is also taken from FRED (available at https://fred.stlouisfed.org/series/CCSA).

Real Yields and One-Year-Ahead Forward Rates. To construct the forward rates in 4 and the market-bond portfolio in Figures 3 and 5, we use the estimated TIPS term structure data provided by the Federal Reserve, based on the approach by Gürkaynak et al. (2007) (available at https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm).

Specifically, Gürkaynak et al. (2007) estimate the TIPS term structure by properly tuning the parameters of the Nelson-Siegel-Svensson yield curve to approximate actual TIPS yield data. In order to estimate real yields of maturity beyond what is already included in the data set, we use the Nelson-Siegel-Svensson yield curve formula:

$$y(t,\mu) = \begin{array}{c} \beta_0 + \beta_1 \left(\frac{1-\exp(-\frac{\mu}{\tau_1})}{\frac{\mu}{\tau_1}} \right) + \beta_2 \left(\frac{1-\exp(-\frac{\mu}{\tau_1})}{\frac{\mu}{\tau_1}} - \exp\left(-\frac{\mu}{\tau_1}\right) \right) \\ +\beta_3 \left(\frac{1-\exp(-\frac{\mu}{\tau_2})}{\frac{\mu}{\tau_2}} - \exp\left(-\frac{\mu}{\tau_2}\right) \right) \end{array}.$$

Here, $y(t,\mu)$ is the (continuously-compounded) yield of a zero-coupon bond of maturity μ (in years) at date t. The parameters, $\beta_0, \beta_1, \beta_2, \tau_1, \tau_2$ are the Nelson-Siegel-Svensson yield curve parameters at date t (we suppress the dependence on t in the notation). More details can be found in Gürkaynak et al. (2007). From these estimated real yields, we then obtain the real one-year-ahead forward rates as follows:

$$f(t,\mu,1) = (\mu+1)y(t,\mu+1) - \mu y(t,\mu).$$

Here, $f(t,\mu,1)$ is the (continuously-compounded) one-year-ahead forward rate at date t beginning at horizon μ. It is the (continuously-compounded) rate one can obtain at date t for making a risk-free investment μ years later (at date $t+\mu$) for payment one year later (at date $t+\mu+1$).

Market-Bond Portfolio. We construct the price change of the market-bond portfolio by using discrete versions of Eqs. $(29-30)$. We adopt a yearly calibration for the bond maturity (μ) and set $\rho = 0.03$ and $\overline{\mu} = 30$ as described in the main text. We approximate the weights with their discrete-time counterparts:

$$w_\mu = \rho e^{-\rho} \text{ and } W_\mu = \sum_{\tilde{\mu}=\mu}^{\infty} w_{\tilde{\mu}} = \frac{\rho}{1-e^{-\rho}} e^{-\rho}.$$

For each date t, we then construct a yield-based and a forward-based measure:

$$\begin{aligned} p^{MB,yld}(t) &= -\sum_{\mu=1}^{\overline{\mu}-1} w_\mu \mu y(t,\mu) - W_{\overline{\mu}} y(t,\overline{\mu}) \\ p^{MB,fw}(t) &= -\sum_{\mu=1}^{\overline{\mu}} W_\mu f(t,\mu,1). \end{aligned}$$

The time derivative of these measures provides a discrete-time approximation for Eqs. (29) and (30), respectively. The two measures are approximately the same. In Figures 3 and 5, we plot the change of the forward-based measure over time.

Potential Output. Data for potential output used in Figures 3 and 6 is taken from the Congressional Budget Office (CBO) (available at https://www.cbo.gov/data/budget-economic-data#11). We use the projections made in February 2021.

Household Net Worth. Data for household net worth used in Figure 3 is taken from the Federal Reserve (available at https://www.federalreserve.gov/releases/z1/dataviz/z1/balance_sheet/table/).

House Price Index. Data for house price index used in Figure 3 is taken from the Federal Housing Finance Agency (FHFA) (available at https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index-Datasets.aspx#mpo).