NBER WORKING PAPER SERIES

STAGGERED PRICE INDEXATION

Martín Uribe

Working Paper 27657
http://www.nber.org/papers/w27657

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2020

I would like to thank Stephanie Schmitt-Grohe for invaluable comments and discussions and Ken Teoh for excellent research assistance The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

Staggered Price Indexation
Martín Uribe
NBER Working Paper No. 27657
August 2020
JEL No. E1,E3,E5

ABSTRACT

Empirical studies using micro data find that about two thirds of all product prices do not change in a given quarter. This evidence has been interpreted as indicating the absence of price indexation. Further, models of staggered price setting without indexation interpret all price changes as optimal. However, the empirical evidence is mute with regard to whether price changes are optimal or not. To reconcile the possibility of price indexation with the micro evidence on the frequency of price changes, I modify the Calvo sticky price model by allowing each period a fraction of randomly picked prices to change optimally, another fraction of randomly picked prices to change due to indexation, and the remaining prices to be constant. The paper presents five main findings: (1) with staggered price indexation the Phillips curve includes a state variable that carries information about all past inflation rates; (2) as the degree of staggered price indexation increases, the Phillips curve becomes flatter; (3) staggered indexation dampens the short-run effect of monetary policy on inflation and amplifies its effect on output; (4) fixing the probability of a price change to 33% per quarter (in accordance with the empirical evidence), a small-scale new-Keynesian model estimated on U.S. data yields a probability of indexation of 19\% per quarter (and therefore a probability of an optimal price change of 14% per quarter); and
(5) according to the estimated model, staggered indexation explains more than half of the observed persistence of inflation in the United States.

Martín Uribe
Department of Economics
Columbia University
International Affairs Building
New York, NY 10027
and NBER
martin.uribe@columbia.edu

1 Introduction

Empirical studies using micro data find that product prices change with a probability of about 33% per quarter (e.g., Nakamura and Steinsson, 2008). The fact that prices don't change every quarter is often interpreted as indicating the absence of price indexation. Further, models of staggered price setting without indexation assume that all price changes are optimal. However, the empirical evidence is mute with regard to whether price changes are optimal or not. In this paper, I modify the Calvo sticky price model by allowing a fraction of randomly picked prices to change optimally, another fraction of randomly picked prices to change due to indexation, and the remaining prices to be constant.

Staggered price indexation introduces a new dimension of payoff-relevant states in the pricing problem of the firm. The reason is that in any given period, say t, given the path of all aggregate variables, a firm that gets to choose its price optimally takes into account that it can arrive at any given future date, say $t+j$, with a multitude of different possible prices, depending upon whether and when it got the chance to index its price between periods t and $t+j$. Thus, its expected profit in period $t+j$ depends not just on the accumulated inflation between periods t and $t+j$, as is the case under the standard indexation scheme, but on the expected inflation rate at every individual date within that period. In other words, the firm's expected present discounted value of profits features a double summation: the standard one over time horizons, capturing the presence of price stickiness, and an additional nested one over the length of each time horizon, capturing the presence of staggered price indexation.

The paper has two main theoretical findings: First, with staggered price indexation the Phillips curve includes a state variable that carries information about all past inflation rates. This state variable is the inflation rate of the basket of goods whose prices get to be indexed in the current period. In turn, this state variable takes the form of a geometric distributed lag of all past inflation rates, with a weight coefficient equal to the degree of price stickiness (0.66 if one uses the evidence reported at the top of this introduction). The second theoretical finding is that the slope of the Phillips curve as well as its expected-inflation coefficient are decreasing in the degree of staggered price indexation. Intuitively, given the total number of goods whose prices can change each period, the larger is the number of goods whose prices are indexed to past inflation, the fewer will be the number of goods whose prices can be adjusted in response to current or future expected disturbances in marginal costs.

The staggered price indexation model nests as special cases the standard Calvo model without price indexation and the Calvo model with the standard form of price indexation (i.e., one in which each period all prices that are not adjusted optimally are indexed to past inflation). For simplicity the formulation considered in this paper assumes full indexation to past inflation. Allowing for partial staggered indexation is relatively straightforward.

The empirical contribution of the present investigation is an econometric estimation of the degree of staggered price indexation and an assessment of its contribution to explaining inflation persistence in the United States. Fixing the probability of a price change to 33%, (in accordance with the available micro-data evidence), a small-scale new-Keynesian model estimated on U.S. data yields a probability of indexation of 19% per quarter (and therefore a probability of an optimal price change of 14% per quarter). Thus, according to this estimate, more than half of all price changes observed each period are indexatory in nature. The estimated model does a good job at explaining the dynamics of output, inflation, and the nominal interest rate. Shutting off staggered price indexation causes a drop in the serial correlation of inflation from 0.71 (which coincides with its observed value) to 0.30, suggesting that staggered indexation explains more than half of the observed persistence of inflation in the United States. The intuition behind this result is straightforward; the larger the fraction of goods that are updated on the basis of past changes of the general price level is, the larger the correlation of current inflation with past inflation will be. Finally, an analysis of impulse responses shows that staggered indexation dampens the short-run effects of monetary policy shocks on inflation and amplifies their effects on output.

This paper is related to a large body of empirical and quantitative work on price stickiness, indexation, inflation persistence, and the Phillips curve, too large to allow for an exhaustive account. On the empirical front, the evidence on the frequency of price changes used in this paper comes from Nakamura and Steinsson (2008) who, using micro data underpinning the U.S. consumer and producer price indices, find that after accounting for sales, the frequency of price changes is about 11 percent per month. Earlier, Bils and Klenow (2004), without controlling for sales, had found a frequency of price changes almost twice as large, 21 percent per month. The present paper contributes to this literature by showing that knowing what fraction of prices change each period might not suffice to ascertain how much price stickiness there is in the data, for holding this fraction constant, the partition between optimal and indexatory price adjustments can be key for understanding the dynamics of inflation. An early study on the difficulties of models with nominal rigidity in replicating observed inflation persistence is Fuhrer and Moore (1995). Pioneer work on estimating medium-scale DSGE models with nominal rigidity and indexation are Christiano et al. (2005) and Smets and Wouters (2007). These papers estimate a standard form of price indexation under which all prices change every period. The present paper extends the standard model of price indexation by assuming that this type of price updates are staggered over time as opposed to taking place every period. In this way, the aggregate model is in line with the fact that individual prices do not change every period. Ascari and Branzoli (2010) show that partial indexation to past inflation à la Christiano et al. (2005) can be optimal in models with staggered price setting and trend inflation. A number of papers have attempted to disentangle the roles played by indexation and the monetary regime in inducing inflation persistence, see Woodford, 2007, for a conceptual argument and Benati, 2008; and Cogley and Sbordone, 2008, for empirical investigations. The recent subdued response of inflation to relatively large swings in aggregate activity, especially since the onset of the great contraction of 2007-2009, has revitalized a literature that aims to document and explain changes in the slope of the Phillips curve (Del Negro et al., 2020; Stock and Watson, 2019; Galí and Gambetti, 2018; and McLeay and Tenreyro, 2019, among others). This paper contributes to this literature by identifying a new channel through which the slope of the Phillips curve can be affected.

The paper proceeds in six sections. Section 2 presents the model with staggered price indexation. Section 3 derives the Phillips curve under staggered price indexation. Section 4 presents a numerical illustration to shed light on the role of staggered price indexation in producing inflation persistence and amplification. Section 5 estimates the degree of staggered price indexation in the context of a small-scale new-Keynesian model on U.S. data. Section 6 uses the estimated model to ascertain how much of the observed inflation persistence in the United States is accounted for by staggered price indexation. Section 7 concludes.

2 The Model

Consider an economy populated by a large number of identical households with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t),$$

where c_t denotes consumption in period t, h_t denotes hours worked, $U(\cdot,\cdot)$ is the period utility function, assumed to be increasing in its first argument, decreasing in its second argument, and concave, $\beta \in (0,1)$ denotes the subjective discount factor, and E_t denotes the expectations operator conditional on information available in period t.

Every period $t \geq 0$ households face the budget constraint

$$P_t c_t + B_t + T_t = (1 + i_{t-1}) B_{t-1} + W_t h_t + \Phi_t,$$

where P_t denotes the nominal price of consumption, W_t denotes the nominal wage rate, B_t denotes holdings of a nominally riskless, one-period bond issued by the government in period t, Φ_t denotes nominal profit income, and T_t denotes lump-sum taxes paid in period t. Bonds issued in period t pay the nominal interest rate i_t in period $t+1$.

The household chooses processes $\{c_t, h_t, B_t\}_{t=0}^{\infty}$ to maximize its lifetime utility function subject to the above sequential budget constraint and to some borrowing limit that prevents it from engaging in Ponzi-type games. Letting $\beta^t \lambda_t / P_t$ be the Lagrange multiplier on the period budget constraint, the first-order conditions of the household's optimization problem with respect to c_t, h_t, and B_t are, respectively,

$$U_c(c_t, h_t) = \lambda_t, \tag{1}$$

$$-U_h(c_t, h_t) = w_t \lambda_t, \tag{2}$$

and

$$\lambda_t = \beta(1 + i_t) E_t \frac{\lambda_{t+1}}{1 + \pi_{t+1}}, \tag{3}$$

where $\pi_t \equiv P_t / P_{t-1} - 1$ denotes the inflation rate, and $w_t \equiv W_t / P_t$ denotes the real wage rate.

The consumption good c_t is assumed to be a composite of a continuum of varieties c_{it} indexed by $i \in [0, 1]$. The aggregation technology is assumed to be of the form

$$c_t = \left[\int_0^1 c_{it}^{1-1/\eta} di \right]^{\frac{1}{1-1/\eta}},$$

where the parameter $\eta > 0$ denotes the elasticity of substitution across varieties. Given c_t, the household chooses the consumption of varieties c_{it} to minimize total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the above aggregation technology, where P_{it} denotes the nominal price of variety i. This problem delivers a demand for individual varieties of the form

$$c_{it} = c_t \left(\frac{P_{it}}{P_t} \right)^{-\eta}, \tag{4}$$

where the price level P_t is given by

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}, \tag{5}$$

and represents the minimum cost of one unit of the composite consumption good.

The firm producing variety i operates in a monopolistically competitive market. The production technology is linear and uses labor and is buffeted by exogenous productivity shocks. Specifically, output of variety i is given by

$$y_{it} = e^{a_t} h_{it}, \tag{6}$$

where y_{it} denotes output of variety i in period t, h_{it} denotes labor input used in the production of variety i, and a_t denotes the productivity shock.

The main innovation of the model is in the formulation of nominal rigidity. The model builds on Calvo (1983) and Yun (1996). Each period, firm $i \in [0,1]$ gets to change its price with constant probability $1-\theta$. Thus, the probability of being stuck with the previous period's price is θ. The point of departure from the Calvo-Yun model is that in the present formulation having the right to change the price is not equal to having the right to choose the price optimally. Specifically, I assume that with probability χ the firm indexes its price to past inflation. Thus, each period the firm chooses its price optimally with probability $1-\theta-\chi$. I refer to this price-setting mechanism as staggered price indexation. The parameter χ resides in the interval $[0, 1-\theta]$. The special case $\chi = 0$ corresponds to the standard Calvo-Yun model. The special case $\chi \in (0,1)$ and $\theta = 0$ corresponds to the Calvo-Yun model with standard indexation—i.e., a model in which all prices that are not set optimally are indexed to past inflation.

Let $\widetilde{P}_{it}$ be the price charged in period t by a firm $i \in [0,1]$ that gets to set its price optimally in period t. In period $t+j$, $j \geq 0$, a firm that has not gotten the permission to set its price optimally since period t and has not gotten the permission to index since period $t+k$, $k \leq j$, charges the price $\widetilde{P}_{it} P_{t+k-1}/P_{t-1}$.[1] The probability of this event conditional on information available in period t is $[\chi(\theta+\chi)^{k-1}]^{I(k>0)}\theta^{j-k}$, where $I(k>0)$ is the indicator function taking the value 1 if $k>0$ and 0 otherwise.

At the posted price, firms commit to meet demand. Then, a firm i that gets to optimize its price in period t chooses $\widetilde{P}_{it}$ and employment to maximize the present discounted value of profits,

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \sum_{k=0}^{j} [\chi(\theta+\chi)^{k-1}]^{I(k>0)} \theta^{j-k} \left[c_{t+j} \left(\frac{\widetilde{P}_{it} P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{1-\eta} - w_{t+j} h_{it+j,k} \right],$$

subject to the participation constraint

$$e^{a_t} h_{it+j,k} \geq c_{t+j} \left(\frac{\widetilde{P}_{it} P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{-\eta},$$

where $h_{it+j,k}$ denotes the demand for labor in period $t+j$ by any firm $i \in [0,1]$ that chooses the price optimally in period t and gets to index its price for the last time in $t+k$, for $0 \leq k \leq j$ and $j \geq 0$. Unlike in the standard Calvo-Yun formulation, expected profits

[1] Note that this firm will charge this price regardless of how many times it got the right to index between periods t and $t+k-1$.

features a double summation. The summation over k reflects the fact that the firm can arrive at period $t+j$ with $k+1$ different possible prices, depending on the period in which it last was allowed to index since period t. The first-order optimality conditions with respect to $\widetilde{P}_{it}$ and $h_{it+j,k}$ are, respectively,

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \sum_{k=0}^{j} [\chi(\theta+\chi)^{k-1}]^{I(k>0)} \theta^{j-k} c_{t+j} \left(\frac{\widetilde{P}_{it} P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{-\eta} \left[\frac{\eta-1}{\eta} \frac{\widetilde{P}_{it} P_{t+k-1}}{P_{t+j} P_{t-1}} - \mu_{it+j,k} \right] = 0 \tag{7}$$

and

$$\mu_{it+j,k} = \frac{w_{t+j}}{e^{a_{t+j}}}. \tag{8}$$

where $\beta^j \frac{\lambda_{t+j}}{\lambda_t} [\chi(\theta+\chi)^{k-1}]^{I(k>0)} \theta^{j-k} \mu_{it+j,k}$ is the Lagrange multiplier associated with the firm's participation constraint. The interpretation of these optimality conditions is the same as in the standard sticky-price model: The second condition says that at the optimum, the Lagrange multiplier on the participation constraint equals the firm's marginal cost. The first condition states that the firm picks the price $\widetilde{P}_{it}$ in period t so that the demand-weighted present discounted value of discrepancies between marginal cost and marginal revenue is nil. The difference with the standard model is that here the firm faces a larger set of payoff-relevant future states, due to the presence of staggered price indexation.

2.1 The Government

Monetary policy takes the form of a Taylor-type interest-rate feedback rule of the form

$$\frac{1+i_t}{1+i} = \left(\frac{1+\pi_t}{1+\pi} \right)^{\alpha_\pi} \left(\frac{y_t}{y} \right)^{\alpha_y} e^{\psi_t}, \tag{9}$$

where y_t is aggregate output, to be defined later, π is the central bank's inflation target, i and y are the steady-state values of i_t and y_t, respectively, and ψ_t is an exogenous monetary shock.

The fiscal authority consumes no goods and maintains a passive fiscal stance, in the sense that it sets lump-sum taxes, T_t, to guarantees fiscal solvency independently of the paths of the price level or the nominal interest rate.

2.2 Equilibrium

The key step in characterizing the equilibrium conditions of the present model is to express the firm's optimality condition (7) in recursive form. To this end, we proceed as follows:

The firm's optimality condition (8) shows that the Lagrange multiplier $\mu_{it,k}$ is independent of i and k, so we will write it without these two subscripts. Then we have

$$\mu_t = \frac{w_t}{e^{a_t}}, \tag{10}$$

for all $t \geq 0$. Similarly, since the firm problem features no idiosyncratic state variables other than its own price, all firms choosing the price optimally in a given period pick the same price, which allows one to drop the subscript i from $\widetilde{P}_{it}$. Now let's split optimality condition (7) in two parts, the demand-weighted present discounted value of marginal revenue, denoted z_t^1, and the present discounted value of marginal costs, denoted z_t^2. Formally, let

$$z_t^1 = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \sum_{k=0}^{j} [\chi(\theta+\chi)^{k-1}]^{I(k>0)} \theta^{j-k} c_{t+j} \frac{\eta-1}{\eta} \left(\frac{\widetilde{P}_t P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{1-\eta}$$

and

$$z_t^2 = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \sum_{k=0}^{j} [\chi(\theta+\chi)^{k-1}]^{I(k>0)} \theta^{j-k} c_{t+j} \left(\frac{\widetilde{P}_t P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{-\eta} \mu_{t+j}$$

So that, by equation (7), we have that

$$z_t^1 = z_t^2 \tag{11}$$

To write the first of the above three expressions recursively, begin by rewriting it as

$$z_t^1 = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\theta^j c_{t+j} \frac{\eta-1}{\eta} \left(\frac{\widetilde{P}_t}{P_{t+j}} \right)^{1-\eta} + \chi \sum_{k=1}^{j} (\theta+\chi)^{k-1} \theta^{j-k} c_{t+j} \frac{\eta-1}{\eta} \left(\frac{\widetilde{P}_t P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{1-\eta} \right].$$

Now define

$$z_t^{11} = E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{\lambda_{t+j}}{\lambda_t} c_{t+j} \frac{\eta-1}{\eta} \left(\frac{\widetilde{P}_t}{P_{t+j}} \right)^{1-\eta}$$

and

$$z_t^{12} = E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{\lambda_{t+j}}{\lambda_t} c_{t+j} \frac{\eta-1}{\eta} \left[\sum_{k=1}^{j} \left(\frac{\theta}{\theta+\chi} \right)^{-k} \left(\frac{\widetilde{P}_t P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{1-\eta} \right],$$

so that

$$z_t^1 = z_t^{11} + \frac{\chi}{\theta+\chi} z_t^{12}. \tag{12}$$

In turn, letting $p_t \equiv \widetilde{P}_t / P_t$ denote the relative price of every variety whose price is optimized in period t in terms of the composite consumption good, one can write z_t^{11} and z_t^{12} recursively as

$$z_t^{11} = \frac{\eta - 1}{\eta} c_t p_t^{1-\eta} + \beta\theta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t}{p_{t+1}} \right)^{1-\eta} (1 + \pi_{t+1})^{\eta-1} z_{t+1}^{11} \tag{13}$$

and

$$z_t^{12} = \beta(\theta + \chi) E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t}{p_{t+1}} \frac{1 + \pi_t}{1 + \pi_{t+1}} \right)^{1-\eta} \left[z_{t+1}^{11} + z_{t+1}^{12} \right]. \tag{14}$$

Appendix A presents a detailed derivation of equation (14). The derivation of equation (13) is analogous. Similarly, one can write z_t^2 recursively as

$$z_t^2 = z_t^{21} + \frac{\chi}{\theta + \chi} z_t^{22}, \tag{15}$$

with

$$z_t^{21} = c_t p_t^{-\eta} \mu_t + \beta\theta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t}{p_{t+1}} \right)^{-\eta} (1 + \pi_{t+1})^{\eta} z_{t+1}^{21} \tag{16}$$

and

$$z_t^{22} = \beta(\theta + \chi) E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t}{p_{t+1}} \frac{1 + \pi_t}{1 + \pi_{t+1}} \right)^{-\eta} \left[z_{t+1}^{21} + z_{t+1}^{22} \right]. \tag{17}$$

This completes the recursive representation of optimality condition (7).

In equilibrium, a fraction θ of all firms is stuck with the previous period's price, a fraction $1 - \theta - \chi$ sets their optimal price $\widetilde{P}_t$, and the remaining χ firms index their price to past inflation. So the price level given in equation (5) can be written as

$$P_t^{1-\eta} = \theta P_{t-1}^{1-\eta} + (1 - \theta - \chi) \widetilde{P}_t^{1-\eta} + \int_{i \in I_x} P_{it}^{1-\eta} di, \tag{18}$$

where I_x denotes the set of goods whose prices are indexed in period t. By definition, I_x has measure χ. A firm that has not optimized since period $t - j$ and gets to index in period t charges the price $\widetilde{P}_{t-j} P_{t-1} / P_{t-j-1}$. The size of this cohort in period $t - j$ is $1 - \theta - \chi$. Of these firms, a fraction $(\theta + \chi)^{j-1}$ arrives in period $t - 1$ not having gotten the right to optimize again, and of these a fraction χ receives the permission to index in period t. So we have that

$$\int_{i \in I_x} P_{it}^{1-\eta} di = \chi(1 - \theta - \chi) \sum_{j=1}^{\infty} (\theta + \chi)^{j-1} \left(\frac{\widetilde{P}_{t-j} P_{t-1}}{P_{t-j-1}} \right)^{1-\eta}. \tag{19}$$

Define the price level of the basket of goods whose prices are indexed in period t as

$$P_t^x \equiv \left[\frac{1}{\chi} \int_{i \in I_x} P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}.$$

Then, we can write equations (18) and (19), respectively, as

$$P_t^{1-\eta} = \theta P_{t-1}^{1-\eta} + (1-\theta-\chi)\widetilde{P}_t^{1-\eta} + \chi P_t^{x1-\eta} \tag{20}$$

and

$$P_t^{x1-\eta} = (1-\theta-\chi)\sum_{j=1}^{\infty}(\theta+\chi)^{j-1}\left(\frac{\widetilde{P}_{t-j}P_{t-1}}{P_{t-j-1}}\right)^{1-\eta} \tag{21}$$

Equation (21) can be written recursively as (see section B of the appendix for a proof)

$$P_t^{x1-\eta} = (1+\pi_{t-1})^{1-\eta}\left[(1-\theta-\chi)\left(p_{t-1}P_{t-1}\right)^{1-\eta} + (\theta+\chi)P_{t-1}^{x1-\eta}\right]. \tag{22}$$

Let $1+\pi_t^x \equiv P_t^x/P_{t-1}$. The variable π_t^x is the rate of inflation of the basket of goods whose prices are indexed in period t. To see this, note that since the right to index is random across goods, the price level of this basket of goods in period $t-1$ is P_{t-1}. Then, after shifting the time subscript one period forward, we can write (22) as

$$(1+\pi_{t+1}^x)^{1-\eta} = (1-\theta-\chi)p_t^{1-\eta}(1+\pi_t)^{1-\eta} + (\theta+\chi)(1+\pi_t^x)^{1-\eta}. \tag{23}$$

The inflation rate of indexed goods, π_t^x, is a predetermined state variable, because both P_t^x and P_{t-1} are determined prior to period t. Similarly, we can write equation (20) as

$$1 = \theta(1+\pi_t)^{\eta-1} + (1-\theta-\chi)p_t^{1-\eta} + \chi\left(\frac{1+\pi_t^x}{1+\pi_t}\right)^{1-\eta}. \tag{24}$$

Let us now derive an expression for aggregate output, which, as mentioned earlier, we denote by y_t. Clearing of the labor market requires that

$$h_t = \int_0^1 h_{it}di.$$

In the goods market output of each variety must equal demand,

$$e^{a_t}h_{it} = c_t\left(\frac{P_{it}}{P_t}\right)^{-\eta}.$$

Integrating over varieties and using the above labor resource constraint yields

$$e^{a_t}h_t = c_t\int_0^1\left(\frac{P_{it}}{P_t}\right)^{-\eta}di. \tag{25}$$

The integral on the right-hand side of (25) is a measure of price dispersion that arises naturally in models of staggered price stickiness. As shown next, the presence of staggered price indexation adds more persistence of this distortion. Let

$$S_t^{-\eta} \equiv \int_0^1 P_{it}^{-\eta} di.$$

By the same argument used to derive equations (20) and (22), in equilibrium this equation can be written as

$$S_t^{-\eta} = \theta S_{t-1}^{-\eta} + (1-\theta-\chi)\widetilde{P}_t^{-\eta} + \chi S_t^{x-\eta} \tag{26}$$

with

$$S_t^{x-\eta} = (1+\pi_{t-1})^{-\eta}\left[(1-\theta-\chi)\left(p_{t-1}P_{t-1}\right)^{-\eta} + (\theta+\chi)S_{t-1}^{x-\eta}\right], \tag{27}$$

where

$$S_t^x \equiv \left[\frac{1}{\chi}\int_{i\in I_x} P_{it}^{-\eta} di\right]^{-\frac{1}{\eta}}$$

is a measure of price dispersion among goods whose prices are allowed to be indexed in period t. This variable is backward looking, adding persistence to the general measure of price dispersion S_t, relative to the standard Calvo-Yun model. Defining $s_t \equiv S_t/P_t$ and $s_t^x \equiv S_t^x/P_{t-1}$, we can write equations (26) and (27) as the following two expressions describing the equilibrium dynamics of price dispersion:

$$s_t^{-\eta} = \theta(1+\pi_t)^{\eta} s_{t-1}^{-\eta} + (1-\theta-\chi)p_t^{-\eta} + \chi(1+\pi_t)^{\eta} s_t^{x-\eta} \tag{28}$$

and

$$s_{t+1}^{x-\eta} = (1-\theta-\chi)(1+\pi_t)^{-\eta} p_t^{-\eta} + (\theta+\chi)s_t^{x-\eta}. \tag{29}$$

The variables s_{t-1} and s_t^x are predetermined states. Finally, using this definition, we can write equation (25) as

$$y_t = c_t, \tag{30}$$

with

$$y_t \equiv s_t^{\eta} e^{a_t} h_t. \tag{31}$$

We are now ready to define a competitive equilibrium in this new-Keynesian model with staggered price indexation.

Definition 1 (Competitive Equilibrium with Staggered Price Indexation) *A competitive equilibrium is a set of process* c_t, h_t, y_t λ_t, μ_t, w_t, π_t, i_t, p_t, z_t^1, z_t^2, z_t^{11}, z_t^{12}, z_t^{21}, z_t^{22}, π_t^x, s_t, *and* s_{t+1}^x *satisfying equations* (1)-(3), (9)-(17), (23), (24), (28)-(31), *given the initial conditions* π_0^x, s_0^x, s_{-1}, *and the laws of motion of the exogenous shocks.*

3 The Phillips Curve with Staggered Price Indexation

To understand the role of staggered price indexation in equilibrium, this section derives the Phillips curve that arises from a linearized version of the equilibrium conditions around a zero-inflation steady state. The main result of this section is that the Phillips curve under staggered price indexation includes a new term featuring the state variable $\widehat{\pi}^x_t$, the inflation rate of the basket of goods that get to be indexed in period t. Formally, the Phillips curve takes the form

$$\widehat{\pi}_t = \gamma_\pi E_t \widehat{\pi}_{t+1} + \gamma_\mu \widehat{\mu}_t + \gamma_x \widehat{\pi}^x_t, \tag{32}$$

with

$$\widehat{\pi}^x_{t+1} = \widehat{\pi}_t + \theta \widehat{\pi}^x_t. \tag{33}$$

where hatted variables correspond to the log deviations of μ_t, $1 + \pi_t$, and $1 + \pi^x_t$ from their deterministic steady-state values. Equation (33) says that the state variable π^x_t carries information about all past inflation rates. The higher the degree of price stickiness, the larger the effect of past inflations on $\widehat{\pi}^x_t$. The loading of $\widehat{\pi}^x_t$ on $\widehat{\pi}_t$ the Phillips curve, given by the parameter γ_x depends on χ, the fraction of prices that are indexed each period. In particular, γ_x is increasing in χ and vanishes at $\chi = 0$. The next two sections show that this property of the Phillips curve under staggered indexation has a significant role in determining the persistence of the rate of inflation in equilibrium.

As shown later in this section, equation (32) nests the Phillips curves of two special economies, one with no indexation ($\chi = 0$), and one with full indexation ($\theta = 0$). Setting $\chi = 0$, equation (32) becomes

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \widehat{\mu}_t$$

which is the familiar expression implied by the standard Calvo-Yun model. Setting $\theta = 0$, the state variable $\widehat{\pi}^x_t$ collapses to $\widehat{\pi}_{t-1}$, and the Phillips curve becomes

$$\widehat{\pi}_t = \frac{\beta}{1+\beta} E_t \widehat{\pi}_{t+1} + \frac{(1-\beta\chi)(1-\chi)}{(1+\beta)\chi} \widehat{\mu}_t + \frac{1}{1+\beta} \widehat{\pi}_{t-1} \tag{34}$$

One might find it surprising that π^x_t does not depend on the probability of indexation,χ. After all, π^x_t is the inflation rate of the basket of goods whose prices got the permission to be indexed in period t. To build intuition for this property of π^x_t, let us deconstruct its law of motion, given in equation (33) (a formal derivation appears later in this section). Of the set of goods that are indexed in period $t + 1$, the fraction of prices that are increased by exactly $\widehat{\pi}_t$ is given by the fraction of goods whose prices are allowed to change in period t, namely, $1-\theta-\chi$ prices that are optimized in period t plus χ prices that are indexed in t, for a total of $1-\theta$ prices. The fraction of indexed prices that will increase by exactly $\widehat{\pi}_t+\widehat{\pi}_{t-1}$ in $t+1$ is given by the fraction of prices that were allowed to change in $t-1$ but not in t, that is, $\theta(1-\theta)$. Similarly, the fraction of indexed prices that will increase by exactly $\widehat{\pi}_t+\widehat{\pi}_{t-1}+\widehat{\pi}_{t-2}$ in $t+1$ is $\theta^2(1-\theta)$. In general, the fraction of indexed prices that will increase by exactly $\widehat{\pi}_t+\widehat{\pi}_{t-1}+\cdots+\widehat{\pi}_{t-j}$ in period $t+1$ is $(1-\theta)\theta^j$. So we have that $\widehat{\pi}^x_{t+1}=(1-\theta)[\widehat{\pi}_t+\theta(\widehat{\pi}_t+\widehat{\pi}_{t-1})+\theta^2(\widehat{\pi}_t+\widehat{\pi}_{t-1}+\widehat{\pi}_{t-2})+\cdots=\widehat{\pi}_t+\theta\widehat{\pi}^x_t$, which is precisely equation (33).

To derive the Phillips curve and the law of motion of π^x_t (equations (32) and (33)), begin by linearizing equilibrium conditions (11), (12), and (15). This operation yields

$$\widehat{z}^1_t=\frac{1-\beta(\theta+\chi)}{1-\beta\theta}\widehat{z}^{11}_t+\frac{\beta\chi}{1-\beta\theta}\widehat{z}^{12}_t,$$

$$\widehat{z}^2_t=\frac{1-\beta(\theta+\chi)}{1-\beta\theta}\widehat{z}^{21}_t+\frac{\beta\chi}{1-\beta\theta}\widehat{z}^{22}_t$$

and

$$\widehat{z}^1_t=\widehat{z}^2_t.$$

In deriving these linearizations and those of the other equilibrium conditions of the model, it is of use to first calculate the steady state of the model under zero inflation. This information appears in appendix C. Combining the above three linear expressions gives

$$\widehat{z}^{12}_t-\widehat{z}^{22}_t=-\frac{1-\beta(\theta+\chi)}{\beta\chi}(\widehat{z}^{11}_t-\widehat{z}^{21}_t) \tag{35}$$

Linearizing equations (13) and (16) we have

$$\widehat{z}^{11}_t=(1-\beta\theta)[\widehat{c}_t+(1-\eta)\widehat{p}_t]+\beta\theta[E_t\widehat{\lambda}_{t+1}-\widehat{\lambda}_t+(1-\eta)(\widehat{p}_t-E_t\widehat{p}_{t+1}-E_t\widehat{\pi}_{t+1})+E_t\widehat{z}^{11}_{t+1}]$$

and

$$\widehat{z}^{21}_t=(1-\beta\theta)[\widehat{c}_t-\eta\widehat{p}_t+\widehat{\mu}_t]+\beta\theta[E_t\widehat{\lambda}_{t+1}-\widehat{\lambda}_t-\eta(\widehat{p}_t-E_t\widehat{p}_{t+1}-E_t\widehat{\pi}_{t+1})+E_t\widehat{z}^{21}_{t+1}]$$

Subtracting the second of these two expressions from the first, one can write

$$\widehat{z}^{11}_t-\widehat{z}^{21}_t=(1-\beta\theta)(\widehat{p}_t-\widehat{\mu}_t)+\beta\theta[\widehat{p}_t-E_t\widehat{p}_{t+1}-E_t\widehat{\pi}_{t+1}+E_t\widehat{z}^{11}_{t+1}-E_t\widehat{z}^{21}_{t+1}] \tag{36}$$

Performing the same operations with equilibrium conditions (14) and (17) produces

$$\widehat{z}_t^{12} = E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + (1-\eta)[\widehat{p}_t - E_t\widehat{p}_{t+1} + \widehat{\pi}_t - E_t\widehat{\pi}_{t+1}] + [1-\beta(\theta+\chi)]E_t\widehat{z}_{t+1}^{11} + \beta(\theta+\chi)E_t\widehat{z}_{t+1}^{12},$$

$$\widehat{z}_t^{22} = E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \eta[\widehat{p}_t - E_t\widehat{p}_{t+1} + \widehat{\pi}_t - E_t\widehat{\pi}_{t+1}] + [1-\beta(\theta+\chi)]E_t\widehat{z}_{t+1}^{21} + \beta(\theta+\chi)E_t\widehat{z}_{t+1}^{22},$$

and

$$\widehat{z}_t^{12} - \widehat{z}_t^{22} = \widehat{p}_t - E_t\widehat{p}_{t+1} + \widehat{\pi}_t - E_t\widehat{\pi}_{t+1} + [1-\beta(\theta+\chi)](E_t\widehat{z}_{t+1}^{11} - E_t\widehat{z}_{t+1}^{21}) + \beta(\theta+\chi)(E_t\widehat{z}_{t+1}^{12} - E_t\widehat{z}_{t+1}^{22})$$

Using equation (35) to eliminate $\widehat{z}_t^{12} - \widehat{z}_t^{22}$ and $\widehat{z}_{t+1}^{12} - \widehat{z}_{t+1}^{22}$ and rearranging gives

$$\widehat{z}_t^{11} - \widehat{z}_t^{21} = -\frac{\beta\chi}{1-\beta(\theta+\chi)}[\widehat{p}_t - E_t\widehat{p}_{t+1} + \widehat{\pi}_t - E_t\widehat{\pi}_{t+1}] + \beta\theta(E_t\widehat{z}_{t+1}^{11} - E_t\widehat{z}_{t+1}^{21}) \qquad (37)$$

Comparing (36) and (37) yields the restriction

$$(1-\beta\theta)\widehat{p}_t - \beta(\theta+\chi)(1-\beta\theta)E_t\widehat{p}_{t+1} + \beta\chi\widehat{\pi}_t - \beta\theta(1-\beta(\theta+\chi))E_t\widehat{\pi}_{t+1} - (1-\beta(\theta+\chi))(1-\beta\theta)\widehat{\mu}_t - \beta\chi E_t\widehat{\pi}_{t+1} = 0$$

To eliminate $\widehat{p}_t$ and $\widehat{p}_{t+1}$ log-linearize equation (24). This yields

$$\widehat{p}_t = \frac{(\theta+\chi)\widehat{\pi}_t - \chi\widehat{\pi}_t^x}{1-\theta-\chi} \qquad (38)$$

Combining the last two equations yields

$$\begin{aligned}[\theta(1-\beta\theta) + \chi[1+\beta(1-\theta(1+\beta\chi+\beta\theta))]]\widehat{\pi}_t \quad = \quad & [\chi\beta(1-\beta\theta) + \beta\theta(1-\beta\theta)]E_t\widehat{\pi}_{t+1} \\ & +\chi(1-\beta\theta)(1-\beta\theta^2-\beta\chi\theta)\widehat{\pi}_t^x \\ & +[1-\beta(\chi+\theta)](1-\beta\theta)(1-\theta-\chi)\widehat{\mu}_t,\end{aligned}$$

Dividing both sides by the coefficient on $\widehat{\pi}_t$ yields the Phillips curve (32) as desired. It is straightforward to see that setting $\chi = 0$ the Phillips curve collapses to its standard form given by equation (34) and that setting $\theta = 0$ it becomes the Phillips curve implied by a model with the standard form of indexation, given in equation (34).

Let's now derive the law of motion of the state variable $\widehat{\pi}_t^x$ given in equation (33). Log-linearizing equation (23) evaluated in period $t+1$ yields

$$\widehat{\pi}_{t+1}^x = (1-\theta-\chi)(\widehat{p}_t + \widehat{\pi}_t) + (\theta+\chi)\widehat{\pi}_t^x. \qquad (39)$$

Now using equation (38) to eliminate $\widehat{p}_t$ one obtains

$$\widehat{\pi}_{t+1}^{x} = \widehat{\pi}_t + \theta \widehat{\pi}_t^{x}$$

which is equation (33).

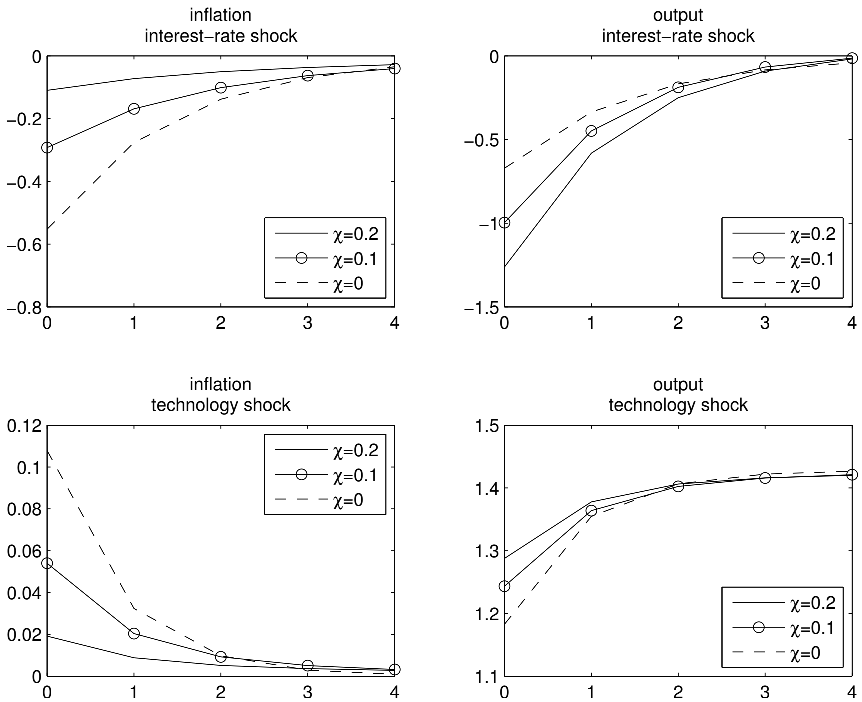
4 The Effects of Staggered Price Indexation

How does staggered indexation affect aggregate dynamics? It is natural to think that if, as in the standard Calvo-Yun model, all of the $1-\theta$ prices that change every period adjust optimally, inflation will be more responsive and aggregate activity less responsive to aggregate disturbances than if a nonzero fraction χ of these $1-\theta$ prices adjust autonomously to past inflation.

Figure 1 confirms this intuition. It plots the response of inflation and output to a contractionary interest-rate shock, $\psi_0 = 1$, with persistence equal to 0.5 (this shock appears in the interest-rate feedback rule (9)), and to a positive permanent technology shock, $z_0 \equiv a_0 - a_{-1} = 1$, with persistence equal to 0.3 (a_t appears for the first time in the production function, equation (6)). The impulse responses are computed for three values of χ, 0, 0.1, and 0.2. The fraction of prices that remain unchanged each period is kept constant and equal to 66% ($\theta = 0.66$), in accordance with the available micro evidence. The parameterization of the model is described in detail in the notes to the figure. The key message of this exercise is that, in line with the intuition given at the beginning of this section, an increase in the degree of staggered price indexation, χ, causes a smaller and flatter short-run response of inflation and a more pronounced response of output.

The result that the short-run response of inflation to aggregate shocks becomes more muted as the degree of staggered price indexation increases suggests that inflation persistence could be increasing in χ, the parameter measuring the fraction of prices that are indexed to past inflation each period. Figure 2 shows that this is indeed the case under the illustrative parameterization we are considering in this section. It displays the serial correlation of inflation as a function of χ. The economy continues to be driven by the monetary shock, ψ_t, and the permanent technology shock. $z_t = a_t - a_{t-1}$, which, as before, follow AR(1) processes with serial correlations of 0.5 and 0.3, respectively. The innovations to both processes are assumed to have standard deviations equal to 0.01. All other parameter values are as described in the notes to Figure 1. Figure 2 shows that staggered price indexation can have a significant effect on the persistence of inflation. In this illustration, the serial correlation of π_t almost doubles as χ increases from 0 to 0.33.

Figure 1: Impulse Responses of Inflation and Output to Monetary and Technology Shocks: A Numerical Illustration



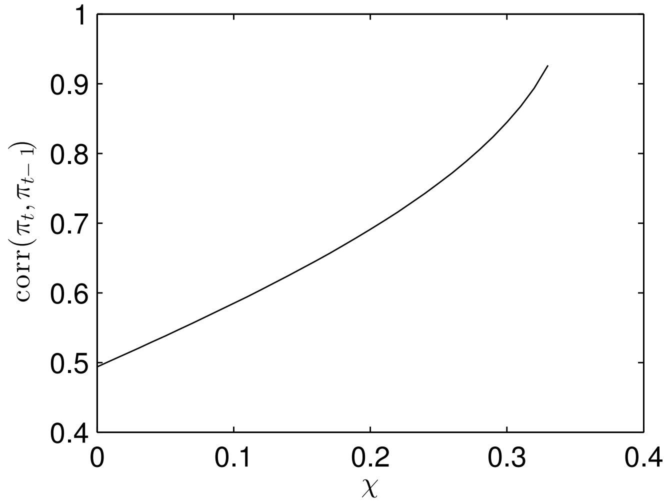
Notes. The top panels display impulse responses to a unit increase in ψ_t, and the bottom panels display impulse responses to a unit increase in $z_t \equiv a_t - a_{t-1}$. The underlying parameterization of the model is as follows: $U(c,h) = \ln c - h^2/2$; $\beta = 1.03^{-1/4}$; $\alpha_\pi = 1.5$; $\alpha_y = 0$; $\theta = 0.66$; $\eta = 6$; $\pi = 0$; ψ_t is AR(1) with mean 0 and serial correlation 0.5; z_t is AR(1) with mean 0.0041 and serial correlation 0.3.

Table 1: Staggered Price Indexation, the Phillips Curve, and the Policy Function of Inflation

	Phillips Curve			Inflation Policy Function		
χ	γ_π	γ_μ	γ_x	$\widetilde{\gamma}_x$	$\widetilde{\gamma}_z$	$\widetilde{\gamma}_\psi$
0	0.993	0.169	0	0	0.108	-0.552
0.1	0.835	0.061	0.054	0.128	0.054	-0.292
0.2	0.770	0.017	0.076	0.215	0.019	-0.110
0.3	0.747	0.001	0.084	0.280	0.002	-0.011

Notes. The Phillips curve is given by $\widehat{\pi}_t = \gamma_\pi E_t \widehat{\pi}_{t+1} + \gamma_\mu \widehat{\mu}_t + \gamma_x \widehat{\pi}_t^x$, and the inflation policy function by $\widehat{\pi}_t = \widetilde{\gamma}_x \widehat{\pi}_t^x + \widetilde{\gamma}_z \widehat{z}_t + \gamma_\psi \widehat{\psi}_t,$. The parameterization of the model is as described in the notes to Figures 1 and 2.

Figure 2: Inflation Persistence as a Function of the Degree of Staggered Price Indexation



Notes. The economy is driven by a monetary shock, ψ_t and a technology shock, $z_t \equiv a_t - a_{t-1}$, both of which follow AR(1) processes with serial correlations 0.5 and 0.3, respectively, and innovations with standard deviations of 0.01. The parameterization of the model is as shown in the notes to Figure 1.

The degree of staggered price indexation, χ, affects inflation persistence not only by increasing the loading of the state variable π_t^x on inflation in the Phillips curve, but also by affecting the slope of the Phillips curve (the loading of marginal cost, μ_t, on inflation), and the loading of expected inflation on inflation itself. To see this, Table (1) displays the coefficients of the Phillips curve, $\widehat{\pi}_t = \gamma_\pi E_t \widehat{\pi}_{t+1} + \gamma_\mu \widehat{\mu}_t + \gamma_x \widehat{\pi}_t^x$, for $\chi = 0$, 0.1, 0.2, and 0.3. As the degree of staggered price indexation increases, the Phillips curve becomes flatter (γ_μ falls) and less responsive to expected inflation (γ_π falls). Intuitively, as χ increases, holding constant the total number of firms that can change prices each period, $1-\theta$, fewer firms will update their prices in response to current or future expected disturbances and more will simply limit themselves to charging past inflation.

The last three columns of table 1 show that this effect is reflected in the policy function of inflation. The present model features one endogenous state, π_t^x, and two exogenous states, namely, the permanent technology shock, z_t, and the monetary shock, ψ_t.[2] Thus, the equilibrium dynamics of inflation are a function of these three variables and up to first order can be written as $\widehat{\pi}_t = \widetilde{\gamma}_x \widehat{\pi}_t^x + \widetilde{\gamma}_z \widehat{z}_t + \gamma_\psi \widehat{\psi}_t$,. The table displays the parameters of this policy function for the three values of χ considered. In line with the intuition given above, as the degree of staggered price indexation increases, inflation becomes more sensitive to changes in $\widehat{\pi}_t^x$, the inflation rate of the basket of goods whose prices are indexed in t, and less sensitive

[2]The model also includes the state variables s_t and s_t^x, measuring price dispersion. However, It can be shown that with zero steady-state inflation, these state variables are irrelevant up to first order.

to exogenous shocks to technology or monetary policy.

The effects of staggered price indexation on the slope of the Phillips curve (γ_μ) and on the persistence coefficient (γ_x) characterized in this section are relevant in light of the fact that existing empirical studies of price dynamics based on micro data do not provide much information on which fraction of observed price changes are optimal and which indexatory. In other words, based on the information provided by existing empirical studies, the four economies shown in table 1 have the same degree of price stickiness. This observation motivates the need to gauge the value of χ. The next section uses aggregate data to infer the value of this parameter.

5 Estimating the Degree of Staggered Price Indexation

This section estimates a version of the new-Keynesian model of section 2 on U.S. data, with the aim to extract information about χ, the new parameter introduced in the proposed model of staggered indexation. To make the model amenable to estimation, it is augmented with three additional sources of uncertainty, a real friction taking the form of habit formation, and a more realistic monetary policy rule, which incorporates interest-rate smoothing,

Specifically, the model is now driven by five shocks: a preference shock, denoted ξ_t, a labor-supply shock, denoted ϕ_t, a government purchases shock, denoted g_t, a technology shock, denoted a_t, and a monetary shock, denoted ψ_t. The technology and monetary shocks were introduced already in section 2. The technology shock is assumed to have a nonstationary component. Specifically, the law of motion of a_t is assumed to be

$$a_t = a_{t-1} + z_t,$$

where z_t, the growth rate of technology is assumed to be a stationary random variable with mean z.

With habit formation, the period utility function takes the form

$$e^{\xi_t}\left[\ln(C_t - \delta\widetilde{C}_{t-1}) - \Phi e^{\phi_t}\frac{h_t^{1+\nu}}{1+\nu}\right],$$

where C_t denotes nondetrended consumption, $\widetilde{C}_t$ denotes the cross-sectional average of C_t, which individual households take as exogenous, δ is a parameter measuring the degree of (external) habit formation, $1/\nu$ is the Frisch labor supply elasticity, and Φ is a scaling parameter. Upper case letters are used to indicate variables that have a stochastic trend inherited from a_t. In equilibrium, all households consume identical quantities of goods, so

$\widetilde{C}_t$ equals C_t.

Government spending is assumed to be exogenous and given by

$$G_t = \bar{g}e^{a_t+g_t},$$

where g_t is a stationary random variable with mean 0, and $\bar{g}$ is a parameter. Scaling G_t by the nonstationary productivity factor, e^{a_t}, is necessary to prevent public consumption from vanishing over time.

The interest-rate feedback rule is augmented to allow for interest-rate smoothing. This feature has become a standard component of monetary policy in estimated new-Keynesian models. Specifically, the Taylor rule (9) now takes the form

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\gamma_I} \left[\left(\frac{1+\pi_t}{1+\pi}\right)^{\alpha_\pi} \left(\frac{y_t}{y}\right)^{\alpha_y}\right]^{1-\gamma_I} e^{\psi_t},$$

where $\gamma_I \in [0, 1)$ is the interest-rate smoothing parameter. As explained next, the estimated model allows for persistence in all shocks, including the monetary shock, ψ_t. Since this shock is indistinguishable from trend inflation (i.e., from replacing the constant π that divides π_t in the Taylor rule by a random variable), the econometric estimation makes staggered indexation and trend inflation, among other shocks and frictions, compete for the data. This race is motivated by the work of Cogley and Sbordone (2008).

All five shocks are assumed to follow exogenous AR(1) processes. Formally, for $v = \xi, \phi, \psi, z, g$ it is assumed that

$$v_{t+1} = \rho_v v_t + \sigma_v \epsilon^v_{t+1},$$

where ρ_v and σ_v are parameters and ϵ^v_t is an i.i.d. disturbance with standard normal distribution.

The equilibrium conditions of the model are identical to those listed in Definition 1 in section 2, except for the household's optimality condition (1), which in stationary form becomes $(c_t - \delta e^{-z_t} c_{t-1})^{-1} = \lambda_t$, and the resource constraint (equations (30) and (31)) which becomes $y_t = e^{s_t} h_t = c_t + \bar{g}e^{g_t}$. To render the equilibrium conditions stationary, all endogenous variables other than h_t, π_t, and i_t are detrended by the nonstationary productivity shock, e^{a_t}. So, for example, $c_t \equiv C_t e^{-a_t}$.

As in much of the DSGE literature, the estimation strategy consists of estimating a subset of the parameters of the model and calibrating the remaining ones using standard values in business-cycle analysis. The set of estimated parameters includes the degree of staggered price indexation, χ, which is the focus of the present section, along with other parameters that play a central role in determining the model's implied short-run dynamics, such as those

Table 2: Calibrated Parameters

Parameter	Value	Description
β	0.9926	subjective discount factor (3%/yr.)
θ	0.67	fraction of unchanged prices per quarter
η	6	intratemporal elast. of subst.
z	0.0041	mean output growth rate (1.65%/yr.)
π	0.005	inflation target (2%/yr.)
Φ	1.11	preference parameter

Note. The time unit is one quarter.

governing habit formation, monetary policy, and the stochastic properties of the underlying sources of uncertainty.

Table 2 displays the values assigned to the calibrated parameters. The probability of not being able to change prices, θ, is set to 0.67, following the evidence from micro data (Nakamura and Steinsson, 2008). The subjective discount factor, β, is set equal to 0.9926, or 3% per year. The intratemporal elasticity of substitution across varieties of intermediate goods, η, is set to 6 (Galí, 2008). The unconditional mean of per capita output growth, z, takes the value 0.0041 (or 1.65 percent per year), which matches the average growth rate of real GDP per capita in the United States since the beginning of the great moderation. The parameter ν is fixed at unity, to ensure a unit Frisch elasticity of labor supply (Galí, 2008), and the scaling parameter Φ at 1.11 to normalize hours work to 1. Unlike in the numerical example of section 3, the inflation target, π, is now set at the more realistic value of 0.005, or 2% per year.

The remaining parameters of the model are estimated on three observables: the logarithm of real output, proxied by the logarithm of real GDP per capita; inflation, proxied by the growth rate of the GDP deflator; and the nominal interest rate, proxied by the federal funds rate. The sample is quarterly U.S. data from 1984:1 to 2008:1. Thus, the data ranges from the beginning of the great moderation to the onset of the global financial crisis. This is a period during which arguably monetary policy was relatively homogeneous, providing some degree of control for the possibility that inflation persistence be affected by changes in the policy regime (Woodford, 2007; Benati, 2008, Cogley and Sbordone, 2008). The data is detrended by first differencing. Appendix D provides data sources and a more detailed description of the construction of each series.

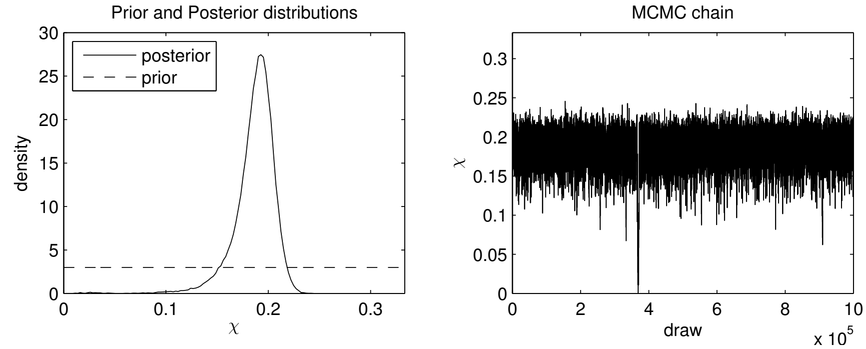
The econometric estimation employs Bayesian techniques. Table 3 displays means and standard deviations of the prior distributions of the estimated parameters. All estimated

Table 3: Prior and Posterior Parameter Distributions:

	Prior Distribution			Posterior Distribution			
Parameter	Distribution	Mean	Std	Mean	Std	5%	95%
χ	Uniform	0.167	0.0962	0.187	0.0205	0.151	0.212
δ	Beta	0.5	0.2	0.763	0.0994	0.577	0.895
α_π	Gamma	1.5	0.25	1.48	0.228	1.179	1.902
α_y	Gamma	0.125	0.1	0.394	0.159	0.152	0.675
γ_I	Uniform	0.5	0.289	0.744	0.0489	0.656	0.817
ρ_x	Beta	0.3	0.2	0.309	0.191	0.0409	0.655
ρ_g	Beta	0.5	0.2	0.759	0.175	0.363	0.926
ρ_ξ	Beta	0.5	0.2	0.759	0.0769	0.625	0.869
ρ_ϕ	Beta	0.5	0.2	0.324	0.145	0.107	0.58
ρ_ψ	Beta	0.5	0.2	0.567	0.139	0.324	0.781
σ_x	Uniform	0.025	0.0144	0.00366	0.00127	0.00177	0.00577
σ_ξ	Uniform	0.025	0.0144	0.0195	0.00518	0.0125	0.029
σ_ϕ	Uniform	0.025	0.0144	0.0388	0.00954	0.0198	0.0493
σ_ψ	Uniform	0.025	0.0144	0.000881	0.000126	0.000687	0.00108
σ_g	Uniform	0.025	0.0144	0.0175	0.00292	0.0124	0.0221
R_{11}	Uniform	1.41e-06	8.16e-07	1.49e-06	8.16e-07	1.62e-07	2.71e-06
R_{22}	Uniform	1.38e-07	7.98e-08	1.73e-07	7.21e-08	3.74e-08	2.68e-07
R_{33}	Uniform	8.33e-08	4.81e-08	7.57e-08	4.79e-08	6.3e-09	1.56e-07

Note. The time unit is one quarter. The parameters R_{ii}, for $i = 1, 2, 3$, are the diagonal elements of the diagonal matrix R, denoting the variance-covariance matrix of measurement errors.

Figure 3: Posterior Properties of the Degree of Staggered Price Indexation, χ



parameters have relatively loose priors. In particular, the novel parameter χ, measuring the degree of staggered price indexation is given a uniform distribution with the maximum possible support, namely, $[0, 1-\theta]$, to allow the data to freely pick the feature that constitutes the focal point of the present analysis. As is standard in the related literature, the estimation allows for i.i.d. measurement error in the observables. The prior distribution of the variances of measurement errors are assumed to be uniform with an upper bound of 10% of the variance of the data.

The last four columns of Table 3 displays means, standard deviations, and 5-95 percent intervals of the estimated posterior distributions, based on a Random Walk Metropolis Hastings MCMC chain of length one million after discarding (burning in) another one million draws. A number of parameters are estimated with significant uncertainty, a feature that is common in estimates of small-scale New Keynesian models. Nonetheless, the data speaks with a clear voice on the parameters χ, δ, and γ_I, governing nominal and real frictions in the model.

Of particular interest is the estimate of the staggered indexation parameter χ. It's posterior mean is 0.187, which means that each period about 19% of all prices change due to indexation and 14% change optimally. The degree of staggered price indexation is estimated with relative precision, with a posterior standard deviation of χ is 0.02. Figure 3 provides additional posterior information on χ. The left panel displays its prior and posterior distributions. The latter is significantly more concentrated than the former, suggesting that the data contains relevant information about this parameter. The right panel displays the MCMC chain of χ. Its relatively flat aspect indicates convergence.

In sum, the estimation of the model suggests that there is substantial staggered indexation in the United States, with more than half of all price changes being indexatory in nature.

To gauge the fit of the model, Table 4 displays selected actual and predicted second moments. The model does well in matching the standard deviations and serial correlations

Table 4: Actual and Predicted Unconditional Second Moments

	dy_t	π_t	i_t
Std. Dev.			
data	0.005	0.002	0.006
model	0.006	0.002	0.004
Autocorr.			
data	0.14	0.71	0.97
model	0.12	0.72	0.94
Corr. w. π_t			
data	-0.13	1.00	0.42
model	0.15	1.00	0.46

Note. Theoretical second moments are computed at the posterior means of the estimated parameters.

Table 5: Serial Correlations: Predicted, Actual, and Counterfactual

	dy_t	π_t	i_t
data	0.14	0.71	0.97
model	0.12	0.72	0.94
model, $\chi = 0$	0.09	0.30	0.86

Note. Theoretical serial correlations are computed at the posterior means of the estimated parameters (Table 3), except for the third row, in which all parameters other than χ take their posterior mean values and χ is set to zero.

of output growth, inflation, and the nominal interest rate. It also replicates well the correlation of inflation with the interest rate, and the low correlation between output growth and inflation, but it misses its sign.

6 Staggered Price Indexation and Inflation Persistence

The illustrative numerical exercise of section 4 (see in particular Figure 2), suggests that staggered price indexation has the potential to induce substantial inflation persistence. We can now address this question in the context of the estimated model.

To this end, Table 5 presents a counterfactual exercise consisting in computing serial correlations when χ is restricted to be 0 and all other estimated parameters are set at their posterior means shown in Table 3. For convenience, the table reproduces from Table 4 the actual and predicted serial correlations.

When χ is set to zero, the predicted serial correlation of inflation falls by more than half, from 0.71 to 0.30. The intuition behind this result is clear. When $\chi = 0$, all price changes (33 percent of all prices, the fraction $1-\theta$) are optimal. Because firms are forward looking, in this case all price changes are linked to current and future expected changes in marginal costs, aggregate demand, and the price level. By contrast, when χ takes its posterior mean value of 0.187, only 14 percent of prices are changed in an optimal, forward-looking fashion, and almost 19 percent in a backward-looking fashion, by incorporating the cumulative inflation rate since the last price change. Thus, in the latter case current inflation has a tighter link to past inflation, which makes it more serially correlated.

7 Conclusion

This paper introduces staggered price indexation in a model of staggered price setting. This theoretical contribution is motivated by the observation, based on micro evidence, that only about one third of all prices change each quarter, and by the fact that the same evidence is not informative as to whether the price changes that do occur are optimal or not. Staggered price indexation is an environment in which each period a random fraction of prices is allowed to incorporate the cumulative inflation since the last price change.

The main theoretical result derived from the present analysis is that under staggered price indexation the Phillips curve features an additional term, given by the inflation rate of the basket of goods that are indexed to past inflation. This is a state variable that carries information about all past rates of inflation. Its persistence is dictated by the degree of price stickiness, that is, by the fraction of goods whose prices do not change each period. The loading of this variable on the Phillips curve depends on a new parameter, χ, measuring the random fraction of goods that are indexed each period. Staggered price indexation is also shown to flatten the slope of the Phillips curve and to lower the loading of expected inflation on current inflation.

Given the degree of price stickiness (that is, the fraction of prices that are not allowed to change each period), staggered price indexation has the potential to affect inflation persistence in a significant way. Fixing the fraction of prices that are sticky each quarter at two thirds, in accordance with the available micro data, a small-scale new-Keynesian model estimated on U.S. data yields an estimate of the staggered indexation parameter χ of about 0.19, which, interpreted through the lens of the model means that each quarter more than half of all price changes are indexations to past inflation and less than half are optimal updates. The estimated model predicts that staggered price indexation is responsible for more than half of the inflation persistence observed in the United States.

8 Appendix

A Derivation of equation (14)

The variable z_t^{12} is defined as

$$z_t^{12} = E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{\lambda_{t+j}}{\lambda_t} c_{t+j} \frac{\eta - 1}{\eta} \left[\sum_{k=1}^{j} \left(\frac{\theta}{\theta + \chi} \right)^{-k} \left(\frac{\widetilde{P}_t P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{1-\eta} \right],$$

Noting that the j counter ranges from 0 to ∞ whereas the k counter ranges from 1 to j, we have that the term corresponding to $j = 0$ is nil. So we can write

$$z_t^{12} = E_t \sum_{j=1}^{\infty} (\theta\beta)^j \frac{\lambda_{t+j}}{\lambda_t} c_{t+j} \frac{\eta - 1}{\eta} \left[\sum_{k=1}^{j} \left(\frac{\theta}{\theta + \chi} \right)^{-k} \left(\frac{\widetilde{P}_t P_{t+k-1}}{P_{t+j} P_{t-1}} \right)^{1-\eta} \right]$$

Defining $j' = j - 1$ one can write

$$z_t^{12} = \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t+1}} \frac{P_t}{P_{t-1}} \right)^{1-\eta} \sum_{j'=0}^{\infty} (\theta\beta)^{j'} \frac{\lambda_{t+1+j'}}{\lambda_{t+1}} c_{t+1+j'} \frac{\eta - 1}{\eta} \left[\sum_{k=1}^{j'+1} \left(\frac{\theta}{\theta + \chi} \right)^{-k} \left(\frac{\widetilde{P}_{t+1} P_{t+k-1}}{P_{t+1+j'} P_t} \right)^{1-\eta} \right]$$

Define $k' = k - 1$ to write

$$\begin{aligned}
z_t^{12} &= \beta(\theta + \chi) E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t+1}} \frac{P_t}{P_{t-1}} \right)^{1-\eta} \sum_{j'=0}^{\infty} (\theta\beta)^{j'} \frac{\lambda_{t+1+j'}}{\lambda_{t+1}} c_{t+1+j'} \frac{\eta - 1}{\eta} \\
&\quad \times \left[\sum_{k'=0}^{j'} \left(\frac{\theta}{\theta + \chi} \right)^{-k'} \left(\frac{\widetilde{P}_{t+1} P_{t+1+k'-1}}{P_{t+1+j'} P_t} \right)^{1-\eta} \right] \\
&= \beta(\theta + \chi) E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t+1}} \frac{P_t}{P_{t-1}} \right)^{1-\eta} \sum_{j'=0}^{\infty} (\theta\beta)^{j'} \frac{\lambda_{t+1+j'}}{\lambda_{t+1}} c_{t+1+j'} \frac{\eta - 1}{\eta} \left[\left(\frac{\widetilde{P}_{t+1}}{P_{t+1+j'}} \right)^{1-\eta} \right. \\
&\quad \left. + \sum_{k'=1}^{j'} \left(\frac{\theta}{\theta + \chi} \right)^{-k'} \left(\frac{\widetilde{P}_{t+1} P_{t+1+k'-1}}{P_{t+1+j'} P_t} \right)^{1-\eta} \right] \\
&= \beta(\theta + \chi) E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t+1}} \frac{P_t}{P_{t-1}} \right)^{1-\eta} \left[\sum_{j'=0}^{\infty} (\theta\beta)^{j'} \frac{\lambda_{t+1+j'}}{\lambda_{t+1}} c_{t+1+j'} \frac{\eta - 1}{\eta} \left(\frac{\widetilde{P}_{t+1}}{P_{t+1+j'}} \right)^{1-\eta} \right. \\
&\quad \left. + \sum_{j'=0}^{\infty} (\theta\beta)^{j'} \frac{\lambda_{t+1+j'}}{\lambda_{t+1}} c_{t+1+j'} \frac{\eta - 1}{\eta} \sum_{k'=1}^{j'} \left(\frac{\theta}{\theta + \chi} \right)^{-k'} \left(\frac{\widetilde{P}_{t+1} P_{t+1+k'-1}}{P_{t+1+j'} P_t} \right)^{1-\eta} \right],
\end{aligned}$$

which can be written as

$$z_t^{12} = \beta(\theta+\chi)E_t\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{p_t}{p_{t+1}}\frac{1+\pi_t}{1+\pi_{t+1}}\right)^{1-\eta}\left[z_{t+1}^{11}+z_{t+1}^{12}\right]$$

B Derivation of Equation (22)

Beginning with equation (21) we have

$$\begin{aligned}
P_t^{x1-\eta} &= (1-\theta-\chi)\sum_{j=1}^{\infty}(\theta+\chi)^{j-1}\left(\frac{\widetilde{P}_{t-j}P_{t-1}}{P_{t-j-1}}\right)^{1-\eta} \\
&= (1-\theta-\chi)\left[\left(\frac{\widetilde{P}_{t-1}P_{t-1}}{P_{t-2}}\right)^{1-\eta}+\sum_{j=2}^{\infty}(\theta+\chi)^{j-1}\left(\frac{\widetilde{P}_{t-j}P_{t-1}}{P_{t-j-1}}\right)^{1-\eta}\right] \\
&= (1-\theta-\chi)\left[\left(\frac{\widetilde{P}_{t-1}P_{t-1}}{P_{t-2}}\right)^{1-\eta}+\sum_{j'=1}^{\infty}(\theta+\chi)^{j'}\left(\frac{\widetilde{P}_{t-1-j'}P_{t-1}}{P_{t-2-j'}}\right)^{1-\eta}\right] \\
&= (1-\theta-\chi)\left(\frac{P_{t-1}}{P_{t-2}}\right)^{1-\eta}\left[\widetilde{P}_{t-1}^{1-\eta}+(\theta+\chi)\sum_{j'=1}^{\infty}(\theta+\chi)^{j'-1}\left(\frac{\widetilde{P}_{t-1-j'}P_{t-2}}{P_{t-2-j'}}\right)^{1-\eta}\right] \\
&= (1+\pi_{t-1})^{1-\eta}\left[(1-\theta-\chi)\left(p_{t-1}P_{t-1}\right)^{1-\eta}+(\theta+\chi)P_{t-1}^{x1-\eta}\right],
\end{aligned}$$

where $j' \equiv j-1$.

C Zero-Inflation Steady State

For any variable, say, x_t, let x denote its deterministic steady-state value. Consider a steady state in which inflation is zero, $\pi = 0$. Then, by equations (23) and (24) we have that

$$p = 1+\pi^x = 1.$$

Then, from equation (13) we get

$$z^{11} = \frac{(\eta-1)/\eta}{1-\beta\theta}c,$$

and from equation (14)

$$z^{12} = \frac{\beta(\theta+\chi)}{1-\beta(\theta+\chi)}z^{11}$$

Similarly, from equations (16) and (17), respectively, we obtain

$$z^{21} = \frac{\mu}{1-\beta\theta} c$$

$$z^{22} = \frac{\beta(\theta+\chi)}{1-\beta(\theta+\chi)} z^{21}$$

The above steady-state conditions and equations (11), (12), and (15) then yield

$$z^1 = z^2 = \frac{1-\beta\theta}{1-\beta(\theta+\chi)} z^{11} = \frac{(\eta-1)/\eta}{1-\beta(\theta+\chi)} c,$$

$$\mu = \frac{\eta-1}{\eta},$$

$$z^{21} = z^{11},$$

and

$$z^{22} = z^{12}$$

From (28) and (29) we have

$$s = s^x = 1.$$

From (30) we have

$$c = h.$$

This expression together with (1), (2), and (10) implies that h is given by

$$-\frac{U_h(h,h)}{U_c(h,h)} = \mu.$$

Finally, the Euler equation (3) implies that

$$i = \frac{1}{\beta} - 1.$$

D The Data

The proxy for output is the logarithm of real GDP seasonally adjusted in chained dollars of 2012 minus the logarithm of the civilian noninstitutional population 16 years old or older. The proxy for the inflation rate is the growth rate of the implicit GDP deflator expressed in percent per year. In turn, the implicit GDP deflator is constructed as the ratio of GDP in current dollars and real GDP both seasonally adjusted. The proxy for i_t is the monthly Federal Funds Effective rate converted to quarterly frequency by averaging and expressed in percent per year. The source for nominal and real GDP is the Bureau of Economic Analysis (bea.gov), the source for population is the Bureau of Labor Statistics (bls.gov), and the source for the Federal Funds rate is the Board of Governors of the Federal Reserve System (federalreserve.gov).

References

Ascari, Guido and Nicola Branzoli, "The long-run optimal degree of indexation in new Keynesian models with price staggering à la Calvo," *Economics Bulletin 30,* February 2010, 482-493.

Benati, Luca, "Investigating Inflation Persistence Across Monetary Regimes," *The Quarterly Journal of Economics 123,* August 2008, 1005-1060.

Bils, Mark, and Peter J. Klenow, "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy 112,* 2004, 947-985.

Calvo, Guillermo, "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics 12,* September 1983, 383-398.

Christiano, Lawrence, Martin Eichenbaum, and Charles Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy 113,* 2005, 1-45.

Cogley, Timothy, and Argia M. Sbordone, "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review 98,* December 2008, 2101-26.

Del Negro, Marco Michele Lenza, Giorgio E. Primiceri, and Andrea Tambalotti, "Whats Up with the Phillips Curve?," BPEA Conference Drafts, March 2020.

Fuhrer, Jeff, and George Moore, "Inflation Persistence," *The Quarterly Journal of Economics 110,* February 1995, 127-159.

Galí, Jordi, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework,* Princeton University Press: Princeton, New Jersey, 2008.

Galí, Jordi. and Luca Gambetti, "Has the U.S. Wage Phillips Curve Flattened? A Semi-Structural Exploration," Mimeo, CREI, 2018.

McLeay, Michael and Silvana Tenreyro, "Optimal Inflation and the Identification of the Phillips Curve," in *NBER Macroeconomics Annual 2019, volume 34*, National Bureau of Economic Research, 2019.

Nakamura, Emi and Jon Steinsson , "Five Facts about Prices: A Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics 123,* November 2008, 1415-1464.

Smets, Frank R., and Raf Wouters, "Shocks and frictions in U.S. business cycles: a Bayesian DSGE approach," *American Economic Review 97,* 2007, 586-606.

Stock, James H. and Mark W. Watson, "Slack and Cyclically Sensitive Inflation," NBER working paper 25987, 2019.

Woodford, Michael, "Interpreting Inflation Persistence: Comments on the Conference on Quantitative Evidence on Price Determination," *Journal of Money, Credit, and Banking*

39, 2007, 203-211.

Yun, Tack, "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics 37,* April 1996, 345-370.