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ABSTRACT

We develop new network measures of exposure to foreign productivity and trade cost shocks for more than 140 countries over more than 40 years from 1970-2012. We derive these exposure measures from a friend-enemy matrix representation of the first-order general equilibrium effects of productivity and trade cost shocks. This representation is exact for small shocks, permits an analytical characterization of the quality of the approximation for large shocks, and is almost exact for empirically-reasonable productivity shocks and trade elasticities. We recover the entire network of bilateral income and welfare exposure from a single matrix inversion, without requiring a separate counterfactual for each productivity or trade cost shock. Our measures are therefore well suited to empirical applications in which this entire network of income and welfare exposure is a key object of interest. We provide an external validation of our measures by showing that they predict country selection into future preferential trade agreements (PTAs) and successfully capture the subsequent effects of these agreements in increasing interdependence between countries. We show that as countries become greater economic friends in terms of welfare exposure predicted by geography, they become greater political friends in terms of United Nations voting and strategic rivalries.

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1 Introduction

A central question in international economics is the impact of foreign productivity and trade cost shocks on domestic income and welfare. A related key debate in political economy is the extent to which large-scale changes in the relative economic size of nations necessarily entail heightened political tension and realignments in the international balance of power. To address these questions, we develop new network measures of exposure to foreign productivity and trade cost shocks for more than 140 countries over more than 40 years from 1970-2012. Our measures correspond to closed-form solutions for the elasticity of income and welfare in each country with respect to these shocks with any foreign country in the class of models with a constant trade elasticity. We derive these closed-form solutions from a friend-enemy matrix representation of the general equilibrium effects of productivity and trade cost shocks. This matrix representation is exact for small shocks, permits an analytical characterization of the quality of the approximation for large shocks, and is almost exact for productivity shocks of the magnitude implied by the observed data. We use our closed-form solutions to provide evidence on the large-scale changes in the network of income and welfare exposure that occur over our sample period, which includes the emergence of China into the global economy. We provide an external validation of our exposure measures by showing that they predict country selection into future preferential trade agreements (PTAs) and successfully capture the subsequent effects of these agreements in increasing economic interdependence between countries. We use variation in the network of welfare exposure predicted by geography to provide new evidence that as as countries become greater economic friends, they become greater political friends in terms of United Nations voting and strategic rivalries.

Although the theoretical mechanisms through which productivity or trade cost shocks affect the income and welfare of countries have been understood for some time, obtaining analytical predictions for the impact of these shocks on individual trade partners in realistic settings with many countries and sectors has proved challenging for several reasons. First, standard models of international trade appear to be highly non-linear, which suggests that the effect on individual trade partners depends in a complicated way on the entire general equilibrium structure of consumption, production and trade. Second, although one can solve the full non-linear model numerically to obtain predictions for individual trade partners, these complex general equilibrium interactions make it hard to obtain intuition for the role of different economic mechanisms in explaining these predictions. Third, the fact that these numerical predictions depend on the full non-linear solution of the model makes it difficult to assess the robustness of results across alternative possible quantitative model structures.

Our first main contribution is to develop new network measures of exposure to productivity and trade cost shocks, which correspond to closed-form solutions for the elasticity of income and welfare in each country with respect to productivity of trade cost shocks with any country in the class of models with a constant trade elasticity. If one country's productivity growth raises (reduces) the income/welfare of another country, we refer to it being a "friend" ("enemy") for income (welfare). We show that these elasticities depend on only observed trade shares and the constant trade elasticity. We derive these closed-form solutions from a friend-enemy matrix representation of the first-order general equilibrium effect of productivity or trade cost shocks on income and welfare. A key advantage of this representation is that these closed-form solutions involve a single matrix inversion, which yields the complete network of bilateral exposure to productivity and trade cost shocks. As our approach is based on a linearization, we can use this closed-form solution to evaluate the impact of a productivity or trade cost shock of any magnitude, and to assess the effect of any combination of productivity or trade cost shocks. In contrast, computing numerical predictions using the non-linear model, requires undertaking a counterfactual for each value and combination of productivity or trade costs shocks. Our approach therefore lends itself to applications, such as ours, in which the complete network of bilateral exposure to productivity or trade cost shocks is the key empirical object of interest.

Our second main contribution is to show that this matrix representation permits an analytical characterization of the quality of the approximation of the first-order general equilibrium effect to the full non-linear model solution. First, we show that the Hessian matrix that controls the magnitude of the second-order terms can be written solely in terms of the observed trade shares and the constant trade elasticity. Second, we show that a weighted average of the second-order terms (weighted by initial country income) is equal to zero across countries. Third, we show that the absolute magnitude of the second-order term for each country is controlled by the largest eigenvalue of this Hessian matrix, which can be computed using the observed trade shares and the constant trade elasticity. Fourth, we use Lagrange's Remainder Theorem to bound the magnitude of all higher-order terms (second-order and above) using the Hessian matrix evaluated over the support of the distribution of productivity shocks. While the use of sufficient statistics that capture first-order effects has become more prevalent in international trade in recent years, a distinctive feature of our friend-enemy matrix representation is that it permits this analytical characterization of the quality of the approximation error to the full non-linear model solution.

Our third main contribution is to show that this friend-enemy matrix representation permits an additive decomposition of the the first-order general equilibrium effect of productivity and trade cost shocks into the contribution of different mechanisms in the model. The impact on domestic income operates through a *market-size effect* (as foreign productivity growth raises foreign income, it increases demand for domestic goods, which in turn raises domestic income) and a *cross-substitution effect* (as foreign productivity growth enhances the price competitiveness of foreign goods, it reduces the demand for domestic goods in all markets). The impact on domestic welfare occurs through these changes in *income* (from the market-size and cross-substitution effects) and a *cost of living effect* (as foreign productivity growth enhances the price competitiveness of foreign goods, it reduces the domestic cost of living). We are thus able to connect our closed-form solutions for each country's income and welfare exposure to productivity growth to directly interpretable economic mechanisms. As these closed-form solutions closely approximate the full non-linear model solution, it thus becomes possible to understand the numerical solutions of the full non-linear model in terms of these economic mechanisms.

Our fourth main contribution is to show that our matrix representation holds across a wide range of different models with a constant trade elasticity, including single-sector models, multi-sector models and multi-sector models with input-output linkages. For each of these different model structures, we derive the closed-form solutions for the elasticity of income and welfare in each country with respect to a productivity shock in any country. Comparing these closed-form solutions across the different models, it becomes straightforward to transparently evaluate the sensitivity of bilateral predictions for the income and welfare incidence of productivity growth to alternative quantitative model assumptions. As our decomposition of the first-order general equilibrium effect holds across all of these different models, one can connect the change in the income and welfare incidence of productivity growth to the impact of these different model assumptions on the market-size, cross-substitution and cost of living effects. Finally, we show that our friend-enemy matrix representation also can be used to derive bounds on income and welfare exposure for departures from a constant elasticity.

We internally validate our closed-form solutions using three different approaches. First, we compare them to the full non-linear model solution for the empirical distribution of productivity shocks. Using the observed trade and income data and a central value of the trade elasticity of 5, we invert the non-linear model and recover the changes in unobserved productivity and trade costs implied by the observed data (up to a normalization). Starting from the observed initial equilibrium in the data, we undertake counterfactuals using the full non-linear model and the empirical distribution of productivity shocks, and compare the resulting numerical predictions to our closed-form solutions. We show that the two sets of predictions are almost visibly indistinguishable from one another.

Second, we compare our closed-form solutions to the full non-linear model solution using simulated productivity shocks and for the range of empirically-plausible values for the trade elasticity ranging from 2 to 20. Across all of these simulations, we find that our closed-form solutions provide an almost exact approximation to the full non-linear model solutions with a coefficient of correlation of more than 0.99. Even when we consider extremely stylized trade networks, such as a circular network consisting of a small number of countries, in which each country i only trades with two countries $i + 1$ and $i + 2$, we continue to find that our approximation performs well. As we increase the number of countries in these stylized trade networks, the quality of the approximation improves further.

Third, we use our analytical characterization for the quality of the approximation for large shocks to understand why our closed-form solutions correspond closely to the full non-linear model solution. We first show that our approximation is exact in the limiting cases of autarky and free trade. We next use Lagrange's Remainder Theorem, to show that it is almost exact for observed trade matrices, because of the low eigenvalues of the Hessian of second derivatives, with the approximation error for all higher-order terms less than 0.62 percent of the variance of the productivity shocks. Intuitively, our approximation is exact in the limiting cases of autarky and free trade, and performs well for random trade matrices. As observed trade matrices are well approximated by a linear combination of autarky, free trade and random trade matrices, our approximation also performs well using these observed trade matrices.

We illustrate the wide range of applications for our network measures using four different empirical implementations: (i) global income and welfare exposure to productivity growth from 1970-2012; (ii) the emergence of China into the global economy; (iii) preferential trade agreements (PTAs), and (iv) the debate about whether countries with more similar economic interests also have more similar political interests. In each of these applications, we focus on our extension with multiple sectors and input-output linkages, because of its greater empirical realism. We use our first application to document an increase in both the average level and dispersion of global welfare exposure over time, which is consistent with the idea that increased globalization has enhanced countries' interdependence. We show that our exposure measures are not fully captured by simpler measures of trading relationships, such as the value of bilateral trade or the aggregate share of expenditure of each importer on each exporter.

In our second empirical application, we show that our closed-form solutions capture the large-scale changes in the network of income and welfare exposure following China's emergence into the global economy. We find increasingly large negative effects of Chinese productivity growth on the relative income of industrialized countries, such as the United States and most Western European countries. In contrast, we find increasingly large positive effects on relative income for resource-rich economies, including a number of African countries, and for a cluster of South-East Asian countries. Despite these increasingly negative effects of Chinese productivity growth on the relative income of industrialized countries, we find increasing positive effects on their aggregate welfare, highlighting the strength of the cost of living effect. We find that input-output linkages play an important role in shaping this pattern of results. While

Chinese productivity growth expands the Electrical, Medical and Office Equipment sectors in other Asian countries, it draws resources into the Mining, Agricultural and Basic Metals sectors in resource-rich countries in Africa and Latin America, consistent with a form of general equilibrium "Dutch Disease."

In our third empirical application, we provide further external validation for our closed-form solutions, by showing that they are successful in predicting future preferential trade agreements (PTAs) and detecting their subsequent impact on economic interdependence between countries. We find that our measure of welfare exposure to bilateral trade cost reductions predicts the formation of preferential trade agreements (PTAs) almost two decades into the future. Using a conventional event-study differences-in-differences research design to control for selection on time-invariant factors, we find no evidence of differences in pre-trends in welfare exposure to either productivity growth or trade cost reductions leading up to the formation of a PTA, but a sharp and statistically significant increase in the years after its formation. We find a similar but marginally weaker pattern of results for income exposure, which is consistent with the idea that political decision makers place some weight on the welfare of their constituents, and internalize that changes in the cost of living are part of the mechanism through which these welfare effects occur.

In our fourth empirical application, we revisit a classic political economy debate about the relationship between economic and political conflict. A number of scholars have drawn parallels between the current China-US tensions and earlier historical episodes, such as the confrontation between Germany and Great Britain around the turn of the twentieth century, and the rise of Athens that instilled fear in Sparta that itself made war more likely (the *Thucydides Trap*).[1] Our network exposure measures of welfare exposure are well suited for this application, because they directly capture the bilateral impact of productivity growth in each country on all countries. We consider a range of measures of countries' political attitudes towards one another, including the similarity of their voting patterns in the United Nations General Assembly (UNGA) and measures of strategic rivalry. We first show that China's emergence into the global economy has been followed by a realignment of bilateral political attitudes away from the United States and towards China, particularly in Africa and East Asia. We next provide regression evidence from a long differences specification, in which we instrument our bilateral welfare exposure measures using geographical determinants of trade, in the form of exporter and importer population and bilateral distance. We find that increases in welfare exposure predicted by our instruments lead to a statistically significant increase in the frequency with which countries vote similarly in the UNGA, and a statistically significant decrease in the propensity with which they are strategic rivals. We show that these results robust to controlling simpler measures of trading relationships between countries, such as bilateral trade flows or aggregate expenditure shares. Taken together, these results are consistent with the view that greater similarity of economic interests does indeed promote greater congruence of political interests.

Our research is related to several strands of existing work. First, our work is connected to a long line of research on the impact of foreign productivity and trade cost shocks on domestic welfare, including Hicks (1953), Johnson (1955), Bhagwati (1958), Eaton and Kortum (2002), Arkolakis et al. (2012), Costinot et al. (2012), di Giovanni et al. (2014), Caliendo and Parro (2015), Hsieh and Ossa (2016), Levchenko and Zhang (2016), Adão et al. (2017), Burstein and Vogel (2017), Caliendo et al. (2018), and Caliendo et al. (2019). Our key contribution relative to this research is to develop network exposure measures that correspond to closed-form solutions for the elasticity of income and welfare in each country with respect to productivity and trade cost shocks in any country. These exposure measures lend themselves

[1] See for example Brunnermeier et al. (2018) and "China-US rivalry and threats to globalization recall ominous past, " Martin Wolf, *Financial Times*, 26th May, 2020.

to empirical applications, such as ours, in which the entire network of bilateral impacts on welfare is a key object of interest. In referring to countries as "friends" and "enemies" depending on whether the impact of a shock is positive or negative, we use the same terminology as for the relationship between goods and factor prices in neoclassical theories of trade, following Jones and Scheinkman (1977).

Second, our work is related to the literature on sufficient statistics and networks in international trade, including Wilson (1980), Arkolakis et al. (2012), Caliendo et al. (2017), Galle et al. (2018), Baqaee and Farhi (2019a,b), Huo et al. (2019), Bartelme et al. (2019), Liu (2019), Kim and Vogel (2020) and Carvalho et al. (2021). In the closed economy, the celebrated theorem of Hulten (1978) shows that for efficient economies, the aggregate impact of a microeconomic productivity shock is summarized by the shocked producer's sales as a share of GDP. This result is exact for a Cobb-Douglas production technology and a first-order approximation more generally. In this general case, Baqaee and Farhi (2019a) derives a nonparametric formula for the higher-order terms, and shows that these non-linearities can be substantial for the nested constant elasticity substitution (CES) production technology. In the open economy, Baqaee and Farhi (2019b) shows that Hulten's formula no longer holds in general, and derives the corresponding open economy nonparametric generalizations. Again, for departures from Cobb-Douglas production technologies, such as nested CES, the non-linearities from the higher-order terms can be quantitively important. To incorporate these non-linearities, the paper shows how to integrate local comparative statics to arrive at exact global comparative statics. In contrast, we focus on the class of international trade models with a constant trade elasticity, in which the import demand system takes the CES form, but production technologies are either linear or Cobb-Douglas. In this influential class of trade models, we derive new friend-enemy closed-form solutions for each country's exposure to productivity and trade shocks in any country, which are exact to first-order, and can be used to provide an analytical characterization of the magnitude of the higher-order terms.

Within this class of models with a constant trade elasticity, Arkolakis et al. (2012) shows that the welfare gains from trade can be measured using only a country's domestic trade share and the constant trade elasticity. In a general class of gravity models, Allen et al. (2020) use the network structure of trade to prove existence and uniqueness, and show that counterfactual predictions in this class of models have a series expansion representation in terms of demand and supply matrices that are functions of trade data and demand and supply elasticities. In a model with general spatial links between local labor markets, Adão et al. (2019) characterize general equilibrium elasticities of employment, wages, and real wages in each market with respect to shift-share measures of exposure to foreign trade shocks using revenue and consumption shares. Our main contribution relative to this research on sufficient statistics is to derive our new friend-enemy measures of each country's exposure to a productivity or trade cost shock in any country. Our approach involves two key steps. We begin by stacking the first-order general equilibrium effects of productivity and trade cost shocks in any country in matrix form. We next invert this matrix system of equations to recover the full bilateral network of each country's exposure to productivity or trade cost shocks in any country. The central advantages of this friend-enemy matrix representation are that it yields closed-form solutions for the elasticity of income and welfare with respect to these productivity and trade shocks and that it permits an analytical characterization of the quality of the approximation to the full non-linear model solution.

Third, our research connects with the large reduced-form literature that has examined the domestic effects of trade shocks (such as the China shock), including Topalova (2010), Kovak (2013), Dix-Carneiro and Kovak (2015), Autor et al. (2013), Autor et al. (2014), Amiti et al. (2017), Pierce and Schott (2016), Feenstra et al. (2019), Borusyak and Jaravel (2019), and Sager and Jaravel (2019). A key contribution of this empirical research has been to provide compelling causal evidence on the effects of trade shocks using quasi-experimental variation. A continuing source of debate in implementing this empirical analysis is the appropriate measurement of trade shocks, including whether to focus on imports from one country, a group of countries or all countries; how to capture imports of final goods versus intermediate inputs; how to incorporate exports as well as imports; and how to measure third-market effects. Our research contributes to this debate by deriving a closed-form solution for the elasticity of income and welfare in each country with respect to a productivity shock in any country, which incorporates all of the above channels.

Fourth, our empirical investigation of preferential trade agreements (PTAs) is related to a large empirical literature in international trade, including Frankel (1997), Frankel and Wei (1998), Limao (2007), Romalis (2007) and Estevadeordal et al. (2008), as reviewed in Freund and Ornelas (2010). Our analysis of the relationship between economic interests and political attitudes connects with a large literature in economics, history and political science, including Scott (1955), Cohen (1960), Signorino and Ritter (1999), Alesina and Spolaore (2003), Martin et al. (2008), Kuziemko and Werker (2006), Guiso et al. (2009), Häge (2011), Dicaprio and Sokolova (2018), and Bao et al. (2019). Our network measures of welfare exposure are well suited to addressing this question, because they capture the full bilateral matrix of effects of economic growth in each country on welfare in all countries. We show that as countries become greater economic friends in terms of welfare exposure predicted by geographical determinants of trade, they become greater political friends in terms of United Nations voting and strategic rivalries.

The remainder of the paper is structured as follows. Section 2 provides a characterization of the effects of productivity shocks in each country on income and welfare in all countries in a neoclassical trade model with Armington differentiation of goods by origin and a general homothetic utility function. Section 3 derives our closed-form solutions for income and welfare exposure to small productivity shocks for the special case of this model with a constant trade elasticity. Section 4 provides an analytical characterization of the quality of the approximation of our linearization to the full non-linear model solution for large productivity shocks. Section 5 reports a number of extensions and generalizations, including trade imbalances, small departures from a constant trade elasticity, multiple sectors, and input-output linkages. Section 6 reports our empirical evidence on global income and welfare exposure and the emergence of China into the global economy. Section 7 provides empirical evidence on PTAs and the relationship between the similarity of countries' economic and political interests. Section 8 concludes. A separate online appendix contains the derivations of the results in each section of the paper and the proofs of the propositions.

2 Neoclassical Trade Model

We begin by introducing the mechanisms through which productivity and trade cost shocks affect income and welfare in a neoclassical trade model with Armington differentiation of goods by origin and a general homothetic utility function. We consider a world of many countries indexed by $n, i \in \{1, \ldots, N\}$. Each country has an exogenous supply of ℓ_n workers, who are each endowed with one unit of labor that is supplied inelastically.

2.1 Preferences

The representative consumer in country n has the following homothetic indirect utility function:

$$u_n = \frac{w_n}{\mathcal{P}\left(\boldsymbol{p}_n\right)}, \tag{1}$$

where $\boldsymbol{p}_n$ is the vector of prices in country n of the goods produced by each country i with elements p_{ni} (inclusive of trade costs); w_n is the wage; and $\mathcal{P}(\cdot)$ is a continuous and twice differentiable function that corresponds to the ideal price index for consumption. From Roy's Identity, country n's demand for the good produced by country i is:

$$c_{ni} = c_{ni}(\boldsymbol{p_n}) = -\frac{\partial \left(1/\mathcal{P}(\boldsymbol{p}_n)\right)}{\partial p_{ni}} w_n \mathcal{P}(\boldsymbol{p}_n). \tag{2}$$

2.2 Production

Each country's good is produced with labor according to a constant returns to scale production technology, with productivity z_i in country i. Markets are perfectly competitive. Goods can be traded between countries subject to iceberg trade costs, such that $\tau_{ni} \geq 1$ units of a good must be shipped from country i in order for one unit to arrive in country n (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the cost in country n of consuming one unit of the good produced by country i is:

$$p_{ni} = \frac{\tau_{ni} w_i}{z_i}. \tag{3}$$

2.3 Expenditure Shares and Market Clearing

Country n's expenditure share on the good produced by country i can be written as:

$$s_{ni} = \frac{p_{ni} c_{ni}(\boldsymbol{p}_n)}{\sum_{\ell=1}^{N} p_{n\ell} c_{n\ell}(\boldsymbol{p}_n)}. \tag{4}$$

Totally differentiating this expenditure share equation, the proportional change in expenditure shares in country n depends on the proportional change in the prices of the goods from each country i and the own and cross-price elasticities for each good:

$$\mathrm{d}\ln s_{ni} = \sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh}\right] \mathrm{d}\ln p_{nh}, \tag{5}$$

where

$$\theta_{nih} \equiv \left(\frac{\partial \left(p_{ni} c_{ni}(\boldsymbol{p}_n)\right)}{\partial p_{nh}} \frac{p_{nh}}{p_{ni} c_{ni}(\boldsymbol{p}_n)}\right),$$

is the elasticity in country n of the expenditure share for good i with respect to the price of good h. Totally differentiating prices, the proportional change in the price in country n of the good produced by country i depends on the proportional changes in the underlying trade costs, wages and productivities as follows:

$$\mathrm{d}\ln p_{ni} = \mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i. \tag{6}$$

Market clearing requires that income in country i equals the expenditure on goods produced by that country:

$$w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n, \tag{7}$$

where for simplicity we begin by considering the case of balanced trade and show how the analysis generalizes to imbalanced trade in Section 5 below.

2.4 Comparative Statics

Using preferences (1) and market clearing (7), we now characterize the general equilibrium effect of shocks to productivity and trade costs. First, totally differentiating the market clearing condition (7) holding constant country endowments, the change in income in each country i depends on the share of value-added that it derives from each market n (t_{in}), the own and cross-price elasticities (θ_{nih}), and the proportional changes in the price of the good from each country h as determined by (6):

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in} \left(\mathrm{d}\ln w_n + \left[\sum_{h=1}^{N} \left[\theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh} \right] \left[\mathrm{d}\ln \tau_{nh} + \mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] \right] \right), \tag{8}$$

where the share of value-added that country i derives from each market n is defined as:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}. \tag{9}$$

Second, totally differentiating the indirect utility function (1), the change in welfare in country n equals the change in income in that country minus the expenditure share weighted average of the proportional change in the price of each country's good, as determined by (6):

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni} \left[\mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right]. \tag{10}$$

The market clearing condition for each country (8) shapes how exogenous changes in productivities ($\mathrm{d}\ln z_i$) and trade costs ($\mathrm{d}\ln \tau_{ni}$) map into endogenous changes in wages ($\mathrm{d}\ln w_i$). The utility function (10) determines how these endogenous changes in wages ($\mathrm{d}\ln w_i$) and the exogenous changes in productivities ($\mathrm{d}\ln z_i$) and trade costs ($\mathrm{d}\ln \tau_{ni}$) translate into endogenous changes in welfare in each country ($\mathrm{d}\ln u_n$). In general, both the own and cross-price elasticities of expenditure with respect to prices (θ_{nih}) are variable and depend on the entire price vector ($\boldsymbol{p}_n$), complicating the mapping from exogenous to endogenous variables.

3 Constant Elasticity of Import Demand

We now show that a sharp friend-enemy matrix representation of the first-order general equilibrium effect of foreign productivity or trade cost shocks can be obtained under the assumption of a constant trade elasticity. In Subsections 3.1 through 3.4, we derive this new matrix representation for small changes in productivity or trade costs. In Section 4, we show that this friend-enemy representation permits an analytical characterization of the quality of our approximation for large changes in productivity or trade costs as a function of observed trade shares.

Throughout this section, we derive our results in a single-sector, constant elasticity Armington model, which is a special case of the framework developed in the previous section. In Section E of the online appendix, we show that these results hold in the entire class of international trade models considered in Arkolakis et al. (2012), henceforth ACR, which satisfy the four primitive assumptions of (i) Dixit-Stiglitz preferences; (ii) one factor of production; (iii) linear cost functions; and (iv) perfect or monopolistic competition; as well as the three macro restrictions of (i) a constant elasticity import demand system, (ii) a constant share of profits in income, and (iii) balanced trade. In addition to the Armington model considered here, this class includes models of perfect competition and constant returns to scale with Ricardian technology differences, as in Eaton and Kortum (2002), and those of monopolistic competition and increasing returns to scale, in which goods are differentiated by firm, as in Krugman (1980) and Melitz (2003) with an untruncated Pareto productivity distribution.

While at the beginning of this section we allow for both productivity and trade cost shocks, we focus from subsection 3.3 onwards on productivity shocks alone. In Section 5 below, we consider a variety of extensions, including trade imbalances, trade cost shocks, multiple sectors following Costinot et al. (2012), and input-output linkages following Caliendo and Parro (2015). We also derive sensitivity bounds for countries' income and welfare exposure to foreign productivity shocks for departures from a constant trade elasticity.

3.1 Trade Matrices

We begin by introducing the matrices of trade shares that play a key role in our closed-form solutions for the elasticity of income and welfare with respect to foreign productivity and trade cost shocks. We use boldface, lowercase letters for vectors, and boldface, uppercase letters for matrices. We use the corresponding non-bold, lowercase letters for elements of vectors and matrices. We use $\mathbf{I}$ to denote the $N \times N$ identity matrix.

Expenditure Share and Income Share Matrices Let $\mathbf{S}$ be the $N \times N$ matrix with the ni-th element equal to importer n's expenditure on exporter i. Let $\mathbf{T}$ be the $N \times N$ matrix with the in-th element equal to the fraction of income that exporter i derives from selling to importer n. We refer to $\mathbf{S}$ as the *expenditure share* matrix and to $\mathbf{T}$ as the *income share* matrix. Intuitively, s_{ni} captures the importance of i as a *supplier* to country n, and t_{in} captures the importance of n as a *buyer* for country i. Note the order of subscripts: in matrix $\mathbf{S}$, rows are buyers and columns are suppliers, whereas in matrix $\mathbf{T}$, rows are suppliers and columns are buyers. Both matrices have rows that sum to one.

These $\mathbf{S}$ and $\mathbf{T}$ matrices are equilibrium objects that can be obtained directly from observed trade data. We derive comparative statics results using these observed matrices. Using $\mathbf{S}^k$ to represent the matrix $\mathbf{S}$ raised to the k-th power, we impose the following technical assumption on the matrix $\mathbf{S}$, which is satisfied in the observed trade flow data.

Assumption 1. *(i) For any i, n, there exists k such that $\left[\mathbf{S}^k\right]_{in} > 0$. (ii) For all i, $s_{ii} > 0$.*

The first part of this assumption states that all countries trade with each other *directly* or *indirectly*. That is, in the language of graph theory, the global trade network is *strongly connected.* This assumption is important because shocks propagate in general equilibrium through changes in relative prices, which are only well-defined if countries are connected (potentially indirectly) to each other through trade. When the global trade network has disconnected components—for instance, if a subset of countries only trade among themselves but not with other nations, or if some countries are in autarky—our results can be applied to study the general equilibrium propagation of shocks within each of the connected components separately. In practice, we find that the global trade network is strongly connected throughout our sample period. The second part of this assumption ensures that every country consumes a positive amount of domestic goods, which again is satisfied in all years.

Using Assumption 1, we now establish the relationship between the $\mathbf{S}$ and $\mathbf{T}$ matrices, which shapes the general equilibrium impact of productivity shocks on income and welfare.

Lemma 1. *Assuming that trade is balanced,*

1. *$\mathbf{S}$ has a unique left-eigenvector $\mathbf{q}'$ with all positive entries summing to one; the corresponding eigenvalue is one.*

2. *The* i*-th element of this left-eigenvector* q_i *is the equilibrium income of country* i *relative to world nominal GDP,* $q_i = w_i \ell_i / \left(\sum_{n=1}^N w_n \ell_n\right)$.

3. $\mathbf{q}'$ *is also a left-eigenvector of* $\mathbf{T}$ *with eigenvalue one, and* $q_i t_{in} = q_n s_{ni}$.

4. *Under free-trade (i.e.* $\tau_{ni} = 1$ *for all* n, i*),* $\mathbf{q}'$ *is equal to every row of* $\mathbf{S}$ *and of* $\mathbf{T}$.

Proof. See Section B.1 of the online appendix. □

Going forward, we refer to the vector $\mathbf{q}'$ as simply the income vector, reflecting our normalization that world nominal GDP is equal to one. Lemma 1 shows that, under balanced trade, one could recover $\mathbf{q}$ and $\mathbf{T}$ from the expenditure share matrix $\mathbf{S}$. Therefore, $\mathbf{S}$ is a sufficient statistic for the general equilibrium effect of small productivity shocks on income and welfare under balanced trade.[2]

In the remainder of this section, we use these properties of the trade matrices to derive our analytical expressions for the elasticity of country income and welfare to productivity shocks in any country in the constant elasticity version of the Armington model developed in Section 2 above.

3.2 First-Order Comparative Statics

In the constant elasticity Armington specification, the preferences of the representative consumer in country n in equation (1) are characterized by the following functional form:

$$u_n = \frac{w_n}{\left[\sum_{i=1}^N p_{ni}^{-\theta}\right]^{-\frac{1}{\theta}}}, \qquad \theta = \sigma - 1, \qquad \sigma > 1, \tag{11}$$

where $\sigma > 1$ is the constant elasticity of substitution between country varieties and $\theta = \sigma - 1$ is the trade elasticity. Using Roy's Identity, country n's share of expenditure on the good produced by country i is:

$$s_{ni} = \frac{p_{ni}^{-\theta}}{\sum_{m=1}^N p_{nm}^{-\theta}}. \tag{12}$$

Using these functional forms in the market clearing condition (7) and totally differentiating holding constant country endowments, the system of equations for the change in income (8) now simplifies to:

$$\mathrm{d}\ln w_i = \sum_{n=1}^N t_{in} \left(\mathrm{d}\ln w_n + \theta \left(\sum_{h=1}^N \begin{array}{c} s_{nh} \left[\mathrm{d}\ln \tau_{nh} + \mathrm{d}\ln w_h - \mathrm{d}\ln z_h \right] \\ - \left[\mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right] \end{array} \right) \right). \tag{13}$$

The system of equations for the change in welfare again takes the same form as in equation (10):

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^N s_{ni} \left[\mathrm{d}\ln \tau_{ni} + \mathrm{d}\ln w_i - \mathrm{d}\ln z_i \right]. \tag{14}$$

Given exogenous changes in productivities ($\mathrm{d}\ln z_i$) and trade costs ($\mathrm{d}\ln \tau_{ni}$), the market clearing condition for each country (8) provides a system of N equations that can be used to determine the N endogenous changes in wages in each country ($\mathrm{d}\ln w_i$). Combining these endogenous changes in wages ($\mathrm{d}\ln w_i$) with the exogenous changes in productivities ($\mathrm{d}\ln z_i$) and trade costs ($\mathrm{d}\ln \tau_{ni}$), the utility function (10) determines the N endogenous changes in welfare in each country ($\mathrm{d}\ln u_n$).

[2] As the expenditure and income shares sum to one, both the matrices $\mathbf{S}$ and $\mathbf{T}$ represent row-stochastic Markov chains, and $\mathbf{q}'$ is their stationary distribution. Assumption 1 ensures that the matrix $\mathbf{S}$ is *primitive*. Since the elements of the matrix $\mathbf{T}$ satisfy $q_i t_{in} = q_n s_{ni}$, the Markov chain $\mathbf{S}$ is *reversible* if and only if $\mathbf{S} = \mathbf{T}$, which holds if and only if trade is balanced bilaterally between each country-partner-pair, a condition which is not satisfied in the data. Finally, the matrix $\mathbf{TS}$, which we show below determines the cross-price elasticity under a constant trade elasticity, is the *multiplicative reversiblization* of $\mathbf{S}$ (Fill 1991), with $q_i \left[\mathbf{TS}\right]_{in} = q_n \left[\mathbf{TS}\right]_{ni}$. Note that the income vector $\mathbf{q}'$ is a left-eigenvector of this matrix $\mathbf{TS}$ with eigenvalue one.

3.3 Friends-and-Enemies Matrix Representation

We now use these comparative statics results in equations (13) and (14) to derive our friend-enemy matrix representation of countries' income and welfare exposure to productivity shocks. To streamline the exposition and in light of our empirical applications, we focus from now onwards on productivity shocks ($\mathrm{d}\ln z_i \neq 0$), assuming that trade cost shocks are zero ($\mathrm{d}\ln \tau_{ni} = 0\, \forall n, i$). In Subsection 5.1 below, we show that our approach naturally also accommodates trade cost shocks ($\mathrm{d}\ln \tau_{ni} \neq 0$).

3.3.1 Friends-and-Enemies for Income

We begin by stacking the first-order general equilibrium effects of small productivity shocks on income in each country in equation (13) in a matrix representation, which has three key advantages. First, we use this representation to derive our closed-form solutions for the elasticity of one country's income with respect to a productivity shock in any other country. Second, this matrix representation permits an analytical characterization of the quality of the approximation of the first-order general equilibrium effect full non-linear model solution, as shown in Section 4 below. Third, this matrix representation yields an intuitive decomposition of income exposure to productivity growth into the contributions of market-size and cross-substitution effects.

Using $\mathrm{d}\ln \mathbf{z}$ and $\mathrm{d}\ln \mathbf{w}$ to denote column vectors of country-level productivity shocks and wage responses, and recalling $\mathrm{d}\ln \tau_{ni} = 0$, we stack the comparative statics in equation (13) in the following matrix form:

$$\underbrace{\mathrm{d}\ln \mathbf{w}}_{\text{income effect}} = \underbrace{\mathbf{T}\,\mathrm{d}\ln \mathbf{w}}_{\text{market-size effect}} + \underbrace{\theta \cdot \mathbf{M} \times (\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z})}_{\text{cross-substitution effect}}, \tag{15}$$

where $\mathbf{M} \equiv \mathbf{TS} - \mathbf{I}$ is an $N \times N$ matrix with in-th entry $m_{in} \equiv \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}$.

As in any general equilibrium model, in order to solve for nominal variables such as income, we need a choice of numeraire. We choose world GDP as our numeraire, which with unchanged country endowments (ℓ_i) implies the following normalization: $\sum_{i=1}^{N} q_i\, \mathrm{d}\ln w_i = 0$. Starting with equation (15), dividing both sides by $(\theta + 1)$, re-arranging terms, and using this normalization, we obtain:

$$(\mathbf{I} - \mathbf{V})\ \mathrm{d}\ln w = -\frac{\theta}{\theta+1}\mathbf{M}\,\mathrm{d}\ln \mathbf{z}, \qquad \mathbf{V} \equiv \frac{\mathbf{T} + \theta\mathbf{TS}}{\theta+1} - \mathbf{Q}, \tag{16}$$

where $\mathbf{Q}$ is an $N \times N$ matrix with the income row vector $\mathbf{q}'$ stacked N times and recall our assumption that $\theta > 0$. Under free-trade (i.e. $\tau_{ni} = 0$ for all n, i), $\mathbf{Q} = \mathbf{S} = \mathbf{T}$.

The presence of the term $\mathbf{Q}\,\mathrm{d}\ln \mathbf{w} = \mathbf{0}$ on the left-hand side in equation (16) reflects our choice of numeraire. In the absence of this term, the matrix $\left(\mathbf{I} - \frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}\right)$ is not invertible: the income shares and expenditure shares sum to one ($\sum_{n=1}^{N} t_{in} = 1$ and $\sum_{n=1}^{N} s_{ni} = 1$), thus the rows of $\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}$ also sum to one, and the columns of $\left(\mathbf{I} - \frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}\right)$ are not linearly independent. This non-invertibility reflects the fact that the trade share matrices $\boldsymbol{T}$, $\boldsymbol{S}$ and $\boldsymbol{M}$ are homogeneous of degree zero, which implies that income can only be recovered from these trade shares up to a normalization or choice of units. Although we choose world GDP as a convenient numeraire, all of our predictions for relative country incomes are invariant to whatever normalization is chosen.[3]

[3]Note that the matrix $\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}$ represents a row-stochastic Markov chain; its left eigenvector $\mathbf{q}'$ is also the stationary distribution of the Markov chain, and $\lim_{k\to\infty}\left(\frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1}\right)^k = \mathbf{Q}$.

Inverting the matrix system (16), we obtain our closed-form solution for the elasticity of each country's income with respect to a productivity shock in any country (including itself), in terms of the observed trade matrices ($\boldsymbol{S}$, $\boldsymbol{T}$ and $\boldsymbol{M}$) and the constant trade elasticity (θ):

Proposition 1. *In the class of international trade models characterized by a constant trade elasticity (θ), the elasticity of each country's income with respect to a productivity shock in any other country is given by:*

$$\mathbf{W} \equiv -\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M}, \tag{17}$$

$$\mathrm{d}\ln\mathbf{w} = \mathbf{W}\,\mathrm{d}\ln\mathbf{z}. \tag{18}$$

Proof. The Proposition follows from equations (15) and (16), using our choice of world GDP as numeraire ($\mathbf{Q}\,\mathrm{d}\ln\mathbf{w} = \mathbf{0}$), as shown in Section D of the online appendix. □

The elements of this matrix $\mathbf{W}$ capture countries' bilateral income exposure to productivity shocks. In particular, the in-th element of this matrix is the elasticity of income in country i (row) with respect to a small productivity shock in country n (column). We refer to country n as being a "friend" of country i for income when this elasticity is positive and an "enemy" of country i for income when this elasticity is negative. In general, $\mathbf{W}$ is not necessarily symmetric: i could view n as a friend, while n views i as an enemy. We now show that the friends-and-enemies matrix $\mathbf{W}$ in Proposition 1 exists, because the matrix $(\boldsymbol{I}-\boldsymbol{V})$ is invertible under Assumption 1.

Proposition 2. *Let $\mathbf{V} \equiv \frac{\mathbf{T}+\theta\mathbf{TS}}{\theta+1} - \mathbf{Q}$. Under Assumption 1 and $\theta > 0$, the matrix $(\mathbf{I}-\mathbf{V})$ is invertible, $(\mathbf{I}-\mathbf{V})^{-1} = \sum_{k=0}^{\infty}\mathbf{V}^k$, and the power series converge at rate $|\mu| < 1$, where $|\mu|$ is the spectral radius (absolute value of the largest eigenvalue) of $\mathbf{V}$ (i.e., $||\mathbf{V}^k|| \leq c \cdot |\mu|^k$ for some constant c).*

Proof. See Section B.2 of the online appendix. □

As the spectral radius of the matrix $\mathbf{V}$ is less than one, the matrix inversion in Proposition 1 has a power-series or Neumann-series representation. Therefore, we can use this representation to decompose the overall first-order impact of the productivity shock into a partial equilibrium effect, which captures the direct impact at initial goods and factor prices, and general equilibrium effects, which capture the endogenous adjustment of goods and factor prices:

$$\mathbf{W} = -\frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M} = -\frac{\theta}{\theta+1}\sum_{k=0}^{\infty}\mathbf{V}^k\mathbf{M} = -\underbrace{\frac{\theta}{\theta+1}\mathbf{M}}_{\text{partial equilibrium}} - \underbrace{\frac{\theta}{\theta+1}\left(\mathbf{V}+\mathbf{V}^2+\dots\right)\mathbf{M}}_{\text{general equilibrium}}. \tag{19}$$

Substituting our solution for income changes ($\mathrm{d}\ln\mathbf{w}$) in response to productivity shocks ($\mathrm{d}\ln\mathbf{z}$) from Proposition 1 in equation (15), we obtain a direct economic interpretation for these first-order general equilibrium effects in terms of different mechanisms in the model. The matrix $\mathbf{T}$ in the first term on the right-hand side of equation (15) captures a *market-size effect*: To the extent that the productivity shock vector $\mathrm{d}\ln\mathbf{z}$ increases incomes in countries n, this raises income in country i through increased demand for its goods. In particular, the elements of $\mathbf{T}$ are the share of income that country i earns through selling to each market n (t_{in}), and capture how dependent country i is on markets in each country n.

The matrix $\mathbf{M}$ in the second term on the right-hand side of equation (15) captures a *cross-substitution effect*. To understand this effect, consider the in-th element of this matrix: $m_{in} \equiv \sum_{h=1}^{N} t_{ih}s_{hn} - 1_{n=i}$. For $i \neq n$, the sum

$\sum_{h=1}^{N} t_{ih} s_{hn}$ captures the overall competitive exposure of country i to country n, through each of their common markets h, weighted by the importance of market h for country i's income (t_{ih}). As the competitiveness of country n increases, as measured by a decline in its wage relative to its productivity ($\mathrm{d}\ln w_n - \mathrm{d}\ln z_n$), consumers in all markets h substitute towards country n and away from other countries $i \neq n$, thereby reducing income in country i and raising it in country n. With a constant elasticity import demand system, the magnitude of this cross-substitution effect in market h depends on the trade elasticity (θ) and the share of expenditure in market h on the goods produced by country n (s_{hn}): consumers in market h increase expenditure on country n by $(s_{hn} - 1)$ and lower expenditure on country i by s_{hn}. Summing across all markets h, we obtain the overall impact of the shock to country n's production cost on country i's income, as captured in the in-th element of the matrix $\mathbf{M}$.

3.4 Friends-and-Enemies for Welfare

We now derive an analogous closed-form solution for the elasticity of welfare in each country with respect to a productivity shock in any other country. Using $\mathrm{d}\ln \mathbf{u}$ to denote the column vector of country-level welfare changes, and recalling $\mathrm{d}\ln \tau_{ni} = 0$, we stack the comparative statics in equation (14) in the following matrix form:

$$\underbrace{\mathrm{d}\ln \mathbf{u}}_{\text{welfare effect}} = \underbrace{\mathrm{d}\ln \mathbf{w}}_{\text{income effect}} - \underbrace{\mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right)}_{\text{cost-of-living effect}}. \tag{20}$$

Using our solution for income changes ($\mathrm{d}\ln \boldsymbol{w}$) from Proposition 1, we obtain an analogous result for welfare exposure to productivity shocks, in terms of the observed trade matrices ($\boldsymbol{S}$, $\boldsymbol{T}$ and $\boldsymbol{M}$) and the trade elasticity (θ):

Proposition 3. *In the class of international trade models characterized by a constant trade elasticity (θ), the elasticity of each country's welfare with respect to a productivity shock in any other country is given by:*

$$\mathbf{U} \equiv (\mathbf{I} - \mathbf{S})\,\mathbf{W} + \mathbf{S}. \tag{21}$$

$$\mathrm{d}\ln \mathbf{u} = \mathbf{U}\,\mathrm{d}\ln \mathbf{z}. \tag{22}$$

Proof. The proposition follows from Proposition 1 and equation (20), as shown in Section D of the online appendix. □

The elements of the matrix $\mathbf{U}$ capture countries' bilateral welfare exposure to productivity shocks. In particular, the ni-th element of this matrix is the elasticity of welfare in country n (row) with respect to a small productivity shock in country i (column). We refer to country i as being a "friend" of country n for welfare when this elasticity is positive and an "enemy" of country n for welfare when this elasticity is negative. As for income exposure, welfare exposure $\mathbf{U}$ is not necessarily symmetric: i could view n as a friend, while n views i as an enemy. Welfare exposure in Proposition 3 is invariant to our choice of numeraire, because the elements of the expenditure share matrix ($\mathbf{S}$) are homogeneous of degree zero in per capita income and sum to one for each importer. Therefore, adding any constant vector $\boldsymbol{k}$ to changes in log per capita incomes ($\mathrm{d}\ln \mathbf{w} = \mathrm{d}\ln \mathbf{w} + \mathbf{k}$) leaves the welfare effect in equation (20) unchanged (since $\mathbf{k} - \mathbf{S}\mathbf{k} = \mathbf{0}$).

From Proposition 3, welfare exposure ($\boldsymbol{U}$) depends on income exposure ($\boldsymbol{W}$) and a cost of living effect, which reflects the impact of the productivity shock in country i on the price index in country n. This cost of living effect depends on the expenditure share matrix ($\mathbf{S}$), which appears in the second term on the right-hand side of equation

(20). The elements of this matrix s_{ni} capture the relative importance of each country i in the consumer expenditure bundle of country n. A productivity shock in country i will have a large positive effect on welfare in country n if it has a large positive effect on wages in country n (through the income effect) and a large negative effect on wages and production costs in the countries from which country n sources most of its goods (through the cost of living effect).

In Propositions 1 and 3, the impact of a productivity shock in one country on income and welfare in another country can be either positive or negative, such that countries can be either friends or enemies of one another in this class of single-sector trade models. On the one hand, the direct effect of higher productivity in a given foreign country on domestic welfare is necessarily positive, because higher foreign productivity reduces the price of imported foreign goods. On the other hand, there are indirect effects through cross-substitution in each market around the world, which can cause higher productivity growth in a given foreign country to lower domestic welfare. As this foreign country becomes more productive, consumers in all markets substitute towards its goods and away from the domestic country's goods, which lowers domestic income and welfare.[4]

4 Quality of the Approximation

Although our friend-enemy matrix representation of income and welfare exposure to productivity shocks is only exact for small shocks, we now show that it permits an analytical characterization of the quality of the approximation for large shocks. In particular, a key contribution of our friend-enemy matrix representation is that we can derive explicit bounds for the magnitude of the second-order and higher terms in terms of the observed trade shares ($\boldsymbol{S}$, $\boldsymbol{T}$ and $\boldsymbol{M}$) and the constant trade elasticity (θ). In our empirical analysis below, we use these analytical results to show that our linearization is almost exact for empirically-realistic productivity shocks and values for the constant trade elasticity.

We begin by comparing our linearization to the full non-linear solution of the model for large changes using the exact-hat algebra approach of Dekle et al. (2007). We re-write the market clearing condition (7) and indirect utility function (11) in a counterfactual equilibrium (denoted by a prime) following a vector of productivity shocks in terms of the observed values of variables in an initial equilibrium (no prime) and the relative changes of variables between the counterfactual and initial equilibria (denoted by a hat such, that $\hat{x} = x'/x$):

$$\ln \hat{w}_i = \left(\frac{\theta}{\theta+1}\right) \ln \hat{z}_i + \frac{1}{\theta+1} \ln \left[\sum_{n=1}^{N} t_{in} \frac{\hat{w}_n}{\sum_{\ell=1}^{N} s_{n\ell} \hat{w}_\ell^{-\theta} \hat{z}_\ell^{\theta}}\right], \tag{23}$$

$$\ln \widehat{u}_i = \ln \hat{w}_i + \frac{1}{\theta} \ln \left[\sum_{n=1}^{N} s_{in} \hat{w}_n^{-\theta} \hat{z}_n^{\theta}\right].$$

Using equations (15) and (20), we can re-write our friends-and-enemies income and welfare exposure measures in the following similar but log linear form:

$$\mathrm{d} \ln w_i = \left(\frac{\theta}{\theta+1}\right) \mathrm{d} \ln z_i + \frac{1}{\theta+1} \sum_{n=1}^{N} t_{in} \left[\mathrm{d} \ln w_n + \theta \sum_{\ell=1}^{N} s_{n\ell} \left[\mathrm{d} \ln w_\ell - \mathrm{d} \ln z_\ell\right]\right], \tag{24}$$

$$\mathrm{d} \ln u_i = \mathrm{d} \ln w_i - \sum_{n=1}^{N} s_{in} \left[\mathrm{d} \ln w_n - \mathrm{d} \ln z_n\right].$$

[4] One simple example in which the cross-substitution effect is particularly powerful is when countries A and B both trade with country C, but do not directly trade with one another. In this case, an increase in country B's productivity reduces income and welfare in country A, as consumers in country C substitute away from country A and towards country B.

Comparing equations (23) and (24), the difference between the full non-linear model solution and our linearization corresponds to the difference between the log of a weighted mean and a weighted mean of logs. These two expressions take the same value in the two limiting cases of autarky ($t_{nn} \to 1$ and $s_{nn} \to 1$ for all n) and free trade ($t_{in} \to \bar{t}_i$ and $s_{ni} \to \bar{s}_i$ for all n, i). More generally, these two expressions take different values, with the difference between them equal to the second and higher-order terms in a Taylor-series expansion. We now characterize the properties of the second-order term in this expansion, before bounding the magnitude of all higher-order terms. To simplify notation, we define $\tilde{z}_i$ as $\ln \hat{z}_i$. We use $f_i(\tilde{\mathbf{z}})$ to denote the implicit function that defines the log changes in wages $\tilde{w}_i$ in equation (23) as a function of the log productivity shocks $\{\tilde{\mathbf{z}}\}$, and we use $\epsilon_i(\tilde{\mathbf{z}})$ to denote the second-order term in the Taylor-series expansion of $f_i(\tilde{\mathbf{z}})$. Using this notation, we can rewrite equation (23) as:

$$\tilde{w}_i = \underbrace{-\theta\left(\tilde{w}_i - \tilde{z}_i\right) + \sum_n t_{in}\tilde{w}_n + \theta \sum_n m_{in}\left[\tilde{w}_n - \tilde{z}_n\right] +}_{\text{first-order}} \underbrace{\epsilon_i\left(\tilde{\mathbf{z}}\right)}_{\text{second-order}} + \underbrace{O\left(\|\tilde{\mathbf{z}}\|^3\right)}_{\text{higher-order}}.$$

The properties of the second-order term depend on the Hessian $\mathbf{H}_{f_i}$ of the function f_i evaluated at $\tilde{z}_\ell = 0 \;\forall\; \ell$:

$$\mathbf{H}_{f_i} \equiv \begin{bmatrix} \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_1^2} & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_1 \partial \tilde{z}_2} & \cdots & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_1 \partial \tilde{z}_N} \\ \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_2 \partial \tilde{z}_1} & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_2^2} & \cdots & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_2 \partial \tilde{z}_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_N \partial \tilde{z}_1} & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_N \partial \tilde{z}_2} & \cdots & \frac{\partial^2 f_i(\mathbf{0})}{\partial \tilde{z}_N^2} \end{bmatrix}, \tag{25}$$

where we can write this second-order term as $\epsilon_i(\tilde{\mathbf{z}}) = \tilde{\mathbf{z}}' \mathbf{H}_{f_i} \tilde{\mathbf{z}}$.

We now proceed as follows. First, we derive an expression for this Hessian in terms of matrices of observed trade data (Proposition 4). Second, we show that a cross-country average of the second-order terms is exactly zero (Proposition 5). Third, we show that the absolute magnitude of this second-order term for each country can be bounded by the largest eigenvalue (in absolute) value of this Hessian (Proposition 6). As this largest eigenvalue can be measured using observed trade data, we can use this result to bound the quality of the approximation for each country given the observed trade matrices. Fourth, we aggregate these results for the second-order terms across countries, and provide an upper bound on their sums of squares (Proposition 7). Again this bound can be computed using observed trade data and provides a summary measure of the overall performance of our linearization. Finally, Proposition 8 provides a bound on *all* higher order terms, including the second-order term and beyond.

In Proposition 4, we show that the Hessian ($\mathbf{H}_{f_i}$) depends solely on the trade elasticity (θ) and the three observed matrices that capture the market-size effects ($\mathbf{T}$), cross-substitution effects ($\mathbf{M}$), and expenditure shares ($\mathbf{S}$). In particular, the second-order term depends on expectations and variances taken across the elements of these matrices, as summarized in the following proposition.

Proposition 4. *The Hessian matrix can be explicitly written as*

$$\mathbf{H}_{f_i} = -\frac{1}{2}\left(\mathbf{A}'\left(diag\left([\mathbf{M}+\mathbf{I}]_i\right) - \mathbf{S}' diag\left(\mathbf{T}_i\right)\mathbf{S}\right)\mathbf{A} - \mathbf{B}'\left(diag\left(\mathbf{T}_i\right) - \mathbf{T}_i'\mathbf{T}_i\right)\mathbf{B}\right).$$

where $\mathbf{A} \equiv \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{I}-\mathbf{T}\right)$ *and* $\mathbf{B} \equiv \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\mathbf{M} + \mathbf{S}\mathbf{A}$, *and* $\mathbf{T}_i$, $\mathbf{M}_i$ *are the* i*-th rows of* $\mathbf{T}$ *and* $\mathbf{M}$, *respectively.*

The second-order term $\epsilon_i(\tilde{\mathbf{z}}) \equiv \tilde{\mathbf{z}}' \mathbf{H}_{f_i} \tilde{\mathbf{z}}$ *can be re-written more intuitively as*

$$\epsilon_i(\tilde{\mathbf{z}}) = -\frac{\theta^2 \mathbb{E}_{T_i} \mathbb{V}_{S_n}\left[\ln \hat{w}_k - \tilde{z}_k\right]}{2} + \frac{\mathbb{V}_{T_i}\left(\ln \hat{w}_i + \theta \mathbb{E}_{S_n}\left[\ln \hat{w}_k - \tilde{z}_k\right]\right)}{2},$$

where $\mathbb{E}_{T_i}$, $\mathbb{E}_{M_i}$, $\mathbb{E}_{S_n}$, $\mathbb{V}_{T_i}$, *and* $\mathbb{V}_{S_n}$ *are expectations and variances taken using* $\{\mathbf{T}_{in}\}_{n=1}^{N}$, $\{\mathbf{M}_{in}\}_{n=1}^{N}$, *and* $\{\mathbf{S}_{nk}\}_{k=1}^{N}$ *as measures (e.g.* $\mathbb{E}_{T_i}[x_n] \equiv \sum_{n=1}^{N} \mathbf{T}_{in} x_n$, $\mathbb{V}_{T_i}[x_n] \equiv \sum_{n=1}^{N} \mathbf{T}_{in} x_n^2 - \left(\sum_{n=1}^{N} \mathbf{T}_{in} x_n\right)^2$*).*

Proof. See Section B.3 of the online appendix. □

As a first step towards characterizing the magnitude of the second-order terms in this expression, we next show in Proposition 5 that the average across countries (weighted by country size in the initial equilibrium before the productivity shock) of these second-order terms is exactly zero: $\mathbf{q}'\epsilon(\tilde{\mathbf{z}}) = 0$. Therefore, these second-order terms raise or reduce the predicted change in the wage of individual countries in response to the productivity shock, but when weighted appropriately they average out across countries.

Proposition 5. *Weighted by each country's income, the second-order terms average to zero for any productivity shock vector:* $\mathbf{q}'\epsilon(\tilde{\mathbf{z}}) = 0$ *for all* $\tilde{\mathbf{z}}$*.*

Proof. See Section B.4 of the online appendix. □

We now bound the absolute value of the second-order term for the income response of each country, following any vector of productivity shocks. First, note that because the model features constant returns to scale, a uniform shock to the productivity of all countries across the globe does not affect relative income. It is therefore without loss of generality to focus on productivity shocks that average to zero. We now show in Proposition 6 that the absolute value of the second-order term for the log-change in income of each country i is bounded, relative to the variance of productivity shocks, by the largest eigenvalue $\mu^{\max,i}$ (by absolute value) of the Hessian matrix $\mathbf{H}_{f_i}$ ($|\epsilon_i(\tilde{\mathbf{z}})| \leq |\mu^{\max,i}| \cdot \tilde{\mathbf{z}}^T \tilde{\mathbf{z}}$). The corresponding eigenvector $\tilde{\mathbf{z}}^{\max,i}$ is the productivity shock vector that achieves the largest second-order term for country i. As these eigenvalues of the Hessian matrix for each country can be evaluated using the observed trade matrices, we thus obtain a bound on the size of the second-order term for each country that can be computed in practice using the observed trade data. In our empirical application below, we show that for each country, even the largest eigenvalue is close to zero, which in turn implies that the second-order term for each country is close to zero.

Proposition 6. $|\epsilon_i(\tilde{\mathbf{z}})| \leq |\mu^{\max,i}| \cdot \tilde{\mathbf{z}}'\tilde{\mathbf{z}}$ *for all* $\tilde{\mathbf{z}}$*, where* $\mu^{\max,i}$ *is the largest eigenvalue of* $\mathbf{H}_{f_i}$ *by absolute value. Let* $\tilde{\mathbf{z}}^{\max,i}$ *denote the corresponding eigenvector (such that* $\mathbf{H}_{f_i}\tilde{\mathbf{z}}^{\max,i} = \mu^{\max,i}\tilde{\mathbf{z}}^{\max,i}$*). The upper bound for* $|\epsilon_i(\tilde{\mathbf{z}})|$ *is achieved when productivity shocks are represented by* $\tilde{\mathbf{z}}^{\max,i}$*:* $\left|\epsilon_i\left(\tilde{\mathbf{z}}^{\max,i}\right)\right| = |\mu^{\max,i}| \cdot \left(\tilde{\mathbf{z}}^{\max,i}\right)^T \tilde{\mathbf{z}}^{\max,i}$*.*

Proof. See Section B.5 of the online appendix. □

We next aggregate the second-order terms across countries and provide an upper-bound on their sum-of-squares in Proposition 7, which enables us to assess the overall performance of our linear approximation. As we show in our empirical application later, the standard unit vector $\mathbf{e}^{\ell}$ comes close to achieving the upper-bound for the ℓ-th equation, i.e. $\mathbf{e}^{\ell} \approx \tilde{\mathbf{z}}^{\max,\ell}$ for all ℓ. Intuitively, because $\mathbf{e}^{i}$ is orthogonal to $\mathbf{e}^{j}$ for all $i \neq j$, this implies that the productivity shock vectors $\tilde{\mathbf{z}}^{\max,i}$ and $\tilde{\mathbf{z}}^{\max,j}$ that maximize second-order effects for different countries $i \neq j$ are almost orthogonal. Hence, given any productivity shock vector $\tilde{\mathbf{z}}$, at most one country $\ln \hat{w}_i = f_i(\tilde{\mathbf{z}})$ can have a second-order term close to the upper-bound $\mu^{\max,i}$, which is small, and the second-order terms for all other countries are close to zero. To formalize this intuition, Proposition 7 constructs a symmetric order-4-tensor $\mathcal{A}$ such that $\frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\epsilon_i^2(\tilde{\mathbf{z}})}}{\tilde{\mathbf{z}}^T\tilde{\mathbf{z}}}$ is bounded above by the square-root of the spectral norm of $\mathcal{A}$. Note that $\frac{1}{N}\sum_{i=1}^{N}\epsilon_i^2\left(\tilde{\mathbf{z}}\right)$ is exactly the mean-square-residuals from a linear regression of the second-order-approximation on our linearized solution.

Proposition 7. *Let $\mathcal{A}:\mathbb{R}^N\to\mathbb{R}_{\geq 0}$ denote the order-4 symmetric tensor defined by the polynomial*

$$g\left(\tilde{\mathbf{z}}\right)=\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{a,b,c,d=1}^{N}\left[\mathbf{H}_{f_i}\right]_{ab}\cdot\left[\mathbf{H}_{f_i}\right]_{cd}\cdot\tilde{z}_a\cdot\tilde{z}_b\cdot\tilde{z}_c\cdot\tilde{z}_d\right),$$

where $\left[\mathbf{H}_{f_i}\right]_{ab}$ is the ab-th entry of $\mathbf{H}_{f_i}$. By construction, $g\left(\tilde{\mathbf{z}}\right)=\langle\mathcal{A},\tilde{\mathbf{z}}\otimes\tilde{\mathbf{z}}\otimes\tilde{\mathbf{z}}\otimes\tilde{\mathbf{z}}\rangle$ represents the inner product and is equal to the cross-equation sum-of-square of the second-order terms ($g\left(\tilde{\mathbf{z}}\right)=\frac{1}{N}\sum_i\epsilon_i^2\left(\tilde{\mathbf{z}}\right)$) under productivity shock $\tilde{\mathbf{z}}$. Let $\mu^{\mathcal{A}}$ be the spectral norm of $\mathcal{A}$:

$$\mu^{\mathcal{A}}\equiv\sup_{\mathbf{z}}\frac{\langle\mathcal{A},\mathbf{z}\otimes\mathbf{z}\otimes\mathbf{z}\otimes\mathbf{z}\rangle}{\|\mathbf{z}\|_2^4},$$

where $\|\cdot\|_2$ is the ℓ_2 norm ($\|\mathbf{z}\|_2\equiv\sqrt{\mathbf{z}'\mathbf{z}}$). Then

$$\sqrt{\frac{1}{N}\sum_i\epsilon_i^2\left(\tilde{\mathbf{z}}\right)}\leq\sqrt{\mu^{\mathcal{A}}}\|\tilde{\mathbf{z}}\|_2^2=\sqrt{\mu^{\mathcal{A}}}\tilde{\mathbf{z}}'\tilde{\mathbf{z}}.$$

Proof. See Section B.6 of the online appendix. □

The spectral norm of $\mathcal{A}$ can be computed using the observed trade data, and the norm being close to zero implies that the second-order terms are close to zero. Furthermore, Lagrange's remainder theorem implies that if productivity shocks are bounded, we can obtain a bound on all the higher-order terms including second-order and above. Using $\mathbf{H}_{f_i}\left(\tilde{\mathbf{z}}\right)$ to denote the Hessian of $f_i\left(\tilde{\mathbf{z}}\right)$ evaluated at productivity shock $\tilde{\mathbf{z}}$ (not necessarily equal to the zero vector), we have the following result.

Proposition 8. *Suppose productivity shocks are bounded, $\tilde{\mathbf{z}}\in\mathcal{X}\equiv\prod_{i=1}^{N}\left[\underline{z},\bar{z}\right]$. For any $\tilde{\mathbf{z}}$, there exists $\mathbf{x}\in\mathcal{X}$ such that*

$$\ln\hat{w}_i=\underbrace{-\theta\left(\ln\hat{w}_i-\tilde{z}_i\right)+\sum_n t_{in}\ln\hat{w}_n+\theta\sum_n m_{in}\left[\ln\hat{w}_n-\tilde{z}_n\right]}_{\textit{first-order}}+\underbrace{\tilde{\mathbf{z}}'\mathbf{H}_{f_i}\left(\mathbf{x}\right)\tilde{\mathbf{z}}}_{\substack{\textit{second and}\\ \textit{higher-order}}}.$$

Proof. This is a direct application of Lagrange's remainder theorem. □

Proposition 8 demonstrates that the Hessian matrix, evaluated at some productivity shock vector $\mathbf{x}$, provides the *exact* error for our first-order approximation. A bound on the eigenvalue of the Hessian evaluated over the entire support $\mathcal{X}$ of productivity shocks therefore provides an upper-bound on the exact approximation error. We exploit this result in our empirical analysis below and show that approximation errors are close to zero for productivity shocks of the magnitude implied by the observed trade data. We thus conclude that our linearization provides an almost exact approximation to the full non-linear solution of the model given the observed trade matrices. Consistent with this, when we correlate the full non-linear solution from the exact-hat algebra on our linear approximation in our empirical analysis below, we find correlation coefficients close to one (>0.99) in all of our simulations.

5 Extensions

We now show that our friend-enemy matrix representation admits a large number of extensions and generalizations. In Section 5.1, we derive our exposure measures allowing for both productivity and trade cost shocks. In Section 5.2, we relax one of the ACR macro restrictions to allow for trade imbalance. In Section 5.3, we relax another of these restrictions to consider small deviations from a constant elasticity import demand system. In Section 5.4, we show that our results generalize to a multi-sector environment following Costinot et al. (2012). In Section 5.5, we extend this specification further to incorporate input-output linkages following Caliendo and Parro (2015).

5.1 Productivity and Trade Cost Shocks

Whereas productivity shocks are common across all trade partners, trade cost shocks are bilateral, which implies that our comparative static results in equations (13) and (14) now have a representation as a three tensor. To reduce this three tensor down to a matrix (two tensor) representation, we aggregate bilateral trade costs across partners using the appropriate weights implied by the model. In particular, we define two measures of outgoing and incoming trade costs, which are trade-share weighted averages of the bilateral trade costs across all export destination and import sources, respectively. We define *outgoing* trade costs for country i as $\mathrm{d}\ln\tau_i^{\mathbf{out}} \equiv \sum_n t_{in}\,\mathrm{d}\ln\tau_{ni}$, where the weights are the income share (t_{in}) that country i derives from selling to each export destination n. We define *incoming* trade costs for country n as $\mathrm{d}\ln\tau_n^{\mathbf{in}} \equiv \sum_i s_{ni}\,\mathrm{d}\ln\tau_{ni}$, where the weights are the expenditure share (s_{ni}) that country n devotes to each import source i. Using these definitions in equations (13) and (14), we obtain:

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln w_n + \theta\left(\begin{array}{c}\sum_{h=1}^{N}\sum_{n=1}^{N} t_{in}s_{nh}\left[\mathrm{d}\ln w_h - \mathrm{d}\ln z_h\right] - \left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right]\\ +\sum_{n=1}^{N} t_{in}\,\mathrm{d}\ln\tau_n^{\mathbf{in}} - \mathrm{d}\ln\tau_i^{\mathbf{out}}\end{array}\right), \tag{26}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln w_n - \sum_{i=1}^{N} s_{ni}\left[\mathrm{d}\ln w_i - \mathrm{d}\ln z_i\right] - \mathrm{d}\ln\tau^{\mathbf{in}}. \tag{27}$$

From this representation, we obtain the following closed-form solutions for the elasticities of country income and welfare with respect to trade cost shocks.

Proposition 9. *In the class of international trade models characterized by a constant trade elasticity (θ), the elasticities of each country's income and welfare with respect to productivity and trade cost shocks satisfy:*

$$\begin{aligned}\underbrace{\mathrm{d}\ln\mathbf{w}}_{\text{income effect}} &= \underbrace{\mathbf{T}\,\mathrm{d}\ln\mathbf{w}}_{\text{market-size effect}} + \underbrace{\theta\left[\mathbf{M}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right) + \mathbf{T}\,\mathrm{d}\ln\tau^{\mathbf{in}} - \mathrm{d}\ln\tau^{\mathbf{out}}\right]}_{\text{cross-substitution effect}} \\ &= \mathbf{W}\,\mathrm{d}\ln\mathbf{z} + \frac{\theta}{\theta+1}\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{T}\,\mathrm{d}\ln\tau^{\mathbf{in}} - \mathrm{d}\ln\tau^{\mathbf{out}}\right)\end{aligned} \tag{28}$$

$$\begin{aligned}\underbrace{\mathrm{d}\ln\mathbf{u}}_{\text{welfare effect}} &= \underbrace{\mathrm{d}\ln\mathbf{w}}_{\text{income effect}} \underbrace{-\,\mathbf{S}\left(\mathrm{d}\ln\mathbf{w} - \mathrm{d}\ln\mathbf{z}\right) - \mathrm{d}\ln\tau^{\mathbf{in}}}_{\text{cost-of-living effect}} \\ &= \boldsymbol{U}\,\mathrm{d}\ln\mathbf{z} + \frac{\theta}{\theta+1}\left(\boldsymbol{I}-\boldsymbol{S}\right)\left(\mathbf{I}-\mathbf{V}\right)^{-1}\left(\mathbf{T}\,\mathrm{d}\ln\tau^{\mathbf{in}} - \mathrm{d}\ln\tau^{\mathbf{out}}\right) - \mathrm{d}\ln\tau^{\mathbf{in}}\end{aligned} \tag{29}$$

Proof. The proposition follows from equations (26) and (27) and Propositions 1 and 3, as shown in Section F.1 of the online appendix. □

The closed-form solutions in Proposition 9 again have an intuitive interpretation. Holding productivity constant, country n's demand for the goods supplied by country i increases if the bilateral trade cost τ_{ni} between these countries falls relative to country n's trade costs with all other nations. These effects are aggregated into $\mathrm{d}\ln\tau_n^{\mathbf{in}}$ and $\mathrm{d}\ln\tau_i^{\mathbf{out}}$, which weight the bilateral changes in trade costs by their appropriate income and expenditure shares. From equation (28), country i's income increases if its outgoing trade cost ($\mathrm{d}\ln\tau_i^{\mathbf{out}}$) falls relative to the incoming trade cost of its export markets, weighted by the importance of each market for country i's income ($\mathbf{T}\,\mathrm{d}\ln\tau^{\mathbf{in}}$). In this equation, productivity shocks are pre-multiplied by the matrix $\mathbf{M}$. In contrast, incoming trade cost shocks are pre-multiplied by the matrix $\mathbf{T}$, because they already include the expenditure share weights (s_{ni}), and outgoing trade cost shocks already incorporate the income share weights (t_{in}). From equation (29), incoming trade cost shocks ($\mathrm{d}\ln\tau^{\mathbf{in}}$) also directly affect welfare through a higher cost of imports, which raises the cost of living. In addition to these direct effects, trade cost shocks like productivity shocks also have indirect general equilibrium effects, through the resulting endogenous changes in incomes.

5.2 Trade Imbalance

Our income and welfare exposure measures in Propositions 1 and 3 are derived under the ACR macro restrictions, including balanced trade. We now show that these results naturally generalize to the case of exogenous trade imbalances commonly considered in the quantitative international trade literature. We measure the flow welfare of the representative agent as per capita expenditure deflated by the consumption price index:

$$u_n = \frac{w_n\ell_n + \bar{d}_n}{\ell_n\left[\sum_{i=1}^{N} p_{ni}^{-\theta}\right]^{-\frac{1}{\theta}}} \tag{30}$$

where $\bar{d}_n$ is the nominal trade deficit. Market clearing requires that income in each country equals expenditure on goods produced in that country:

$$w_i\ell_i = \sum_{n=1}^{N} s_{ni}\left[w_n\ell_n + \bar{d}_n\right]. \tag{31}$$

Trade Matrices We begin by establishing some properties our trade matrices under trade imbalance. We continue to use $q_i \equiv w_i\ell_i/\left(\sum_n w_n\ell_n\right)$ to denote country i's share of world income. Let $e_i \equiv \left(w_i\ell_i + \bar{d}_n\right)/\left(\sum_n w_n\ell_n\right)$ denote country i's share of world expenditures, where we use the fact that the aggregate deficit for the world as a whole is equal to zero. Let $d_i \equiv q_i/e_i$ denote country i's income-to-expenditure ratio, which is equal to one divided by one plus its nominal trade deficit relative to income. Let $\mathbf{D} \equiv Diag\left(\mathbf{d}\right)$ be the diagonalization of the vector $\mathbf{d}$; note $\mathbf{q}' = \mathbf{e}'\mathbf{D}$. Under trade balance, $q_i = e_i$ for all i, and $\mathbf{D} = \mathbf{I}$.

We continue to use $\mathbf{S}$ to denote the expenditure share matrix and $\mathbf{T}$ to denote the income share matrix: s_{ni} captures the expenditure share of importer n on exporter i and t_{in} captures the share of exporter i's income derived from selling to importer n. Under trade balance, $q_i t_{in} = q_n s_{ni}$, but this is no longer the case under trade imbalance. Instead, we have the following results.

Lemma 2. *Under trade imbalance,* $\mathbf{q}' = \mathbf{e}'\mathbf{S}$, $\mathbf{e}' = \mathbf{q}'\mathbf{T}$. *Moreover,*

1. $\mathbf{q}'$ *is the unique left-eigenvector of* $\mathbf{D}^{-1}\mathbf{S}$ *with all positive entries summing to one; the corresponding eigenvalue is one.* $\mathbf{q}'$ *is also the unique left-eigenvector of* $\mathbf{TD}$ *and* $\mathbf{TS}$ *with eigenvalue equal to one.*

2. $\mathbf{e}'$ *is the unique left-eigenvector of* $\mathbf{SD}^{-1}$ *with all positive entries summing to one; the corresponding eigenvalue is one.* $\mathbf{e}'$ *is also the unique left-eigenvector of* $\mathbf{DT}$ *and* $\mathbf{ST}$ *with eigenvalue equal to one.*

Proof. See Section B.1 of the online appendix. □

Comparative Statics Using these properties of the trade matrices, we now derive countries' income and welfare exposure to productivity shocks under trade imbalance. As the model does not generate predictions for how trade imbalances respond to shocks, we follow the common approach in the quantitative international trade literature of treating them as exogenous. In particular, we assume that trade imbalances are constant as a share of world GDP, which given our choice of world GDP as the numeraire, corresponds to holding the nominal trade deficits $\bar{d}_n$ fixed for all countries n. Totally differentiating (30) and (31), we obtain the following generalizations of equations (13) and (14):

$$\mathrm{d}\ln w_i = \sum_{n=1}^{N} t_{ni} \left(\mathrm{d}\ln e_n + \theta \left(\sum_{h=1}^{N} s_{nh}\, \mathrm{d}\ln p_{nh} - \mathrm{d}\ln p_{ni} \right) \right), \tag{32}$$

$$\mathrm{d}\ln u_n = \mathrm{d}\ln e_n - \sum_{m=1}^{N} s_{nm}\, \mathrm{d}\ln p_{nm}. \tag{33}$$

The introduction of trade imbalances has three main implications for these comparative static relationships. First, trade imbalances complicate the relationship between the expenditure share ($\mathbf{S}$), income share ($\mathbf{T}$) and cross-substitution ($\mathbf{M}$) matrices, because with income no longer equal to expenditure for each country ($e_i \neq q_i$), we have $q_i t_{in} \neq q_n s_{ni}$. Second, the market-size effect in the income equation depends on changes in expenditure rather than changes in income (the first term in equation (32)). Third, the income effect in the welfare equation also depends on changes in expenditure rather than changes in income (the first term in equation (33)). Under our assumption that trade imbalances stay constant as a share of world GDP, we have the following generalization of our earlier results.

Proposition 10. *Assume constant trade deficits* $\bar{d}_n$ *for all countries* n*. The elasticities of each country's income and welfare with respect to productivity shocks are given by:*

$$\mathrm{d}\ln \mathbf{w} = \mathbf{W}\,\mathrm{d}\ln \mathbf{z}, \qquad \mathbf{W} \equiv -\frac{\theta}{\theta+1}\left(\mathbf{I} - \frac{\mathbf{TD}+\theta\mathbf{TS}}{\theta+1} + \mathbf{Q}\right)^{-1}\mathbf{M}, \tag{34}$$

$$\mathrm{d}\ln \mathbf{u} = \mathbf{U}\,\mathrm{d}\ln \mathbf{z}, \qquad \mathbf{U} \equiv (\mathbf{D} - \mathbf{S})\,\mathbf{W} + \mathbf{S}, \tag{35}$$

where recall that $\mathbf{D}$ *is the diagonalization of the vector of the ratio of income-to-expenditure* d_i*.*

Proof. The Proposition follows from equations (32) and (33), noting that for all n, $\mathrm{d}\ln \bar{d}_n = 0 \implies \mathrm{d}\ln e_n = \frac{q_n}{e_n}\,\mathrm{d}\ln w_n$, as shown in Section F.2 of the online appendix. □

5.3 Deviations from Constant Elasticity Import Demand

Another advantage of our friend-enemy representation is that it allows us to use results from matrix perturbation theory to provide bounds for the sensitivity of country income and welfare exposure to productivity shocks to departures from a constant trade elasticity. We now use our characterization of the neoclassical model of trade in Section 2 to derive these bounds. We begin by noting that a constant elasticity import demand system implies that the cross-price elasticities (θ_{nih}) in the market clearing condition (8) are:

$$\theta_{nih} = \begin{cases} (s_{nh} - 1)\,\theta & \text{if } i = h \\ s_{nh}\theta & \text{otherwise} \end{cases}. \tag{36}$$

Without loss of generality, we can represent the cross-price elasticity of any homothetic demand system as:

$$\theta_{nih} = \begin{cases} (s_{nh} - 1)\,\theta + o_{nih} & \text{if } i = h \\ s_{nh}\theta + o_{nih} & \text{otherwise,} \end{cases} \tag{37}$$

where o_{nih} captures the deviation from the predictions of the constant elasticity specification (36). Noting that homotheticity implies $\sum_{k=1}^{N} s_{nk} o_{nkh} = 0$, we obtain the following generalizations of our bilateral friend-enemy matrix representations of the income and welfare effects of productivity shocks:

$$\mathrm{d}\ln \mathbf{w} = \mathbf{T}\,\mathrm{d}\ln \mathbf{w} + (\theta \mathbf{M} + \mathbf{O}) \times (\,\mathrm{d}\ln \mathbf{w} - \,\mathrm{d}\ln \mathbf{z}\,)\,, \tag{38}$$

$$\mathrm{d}\ln \mathbf{U} = \,\mathrm{d}\ln \mathbf{w} - \mathbf{S}\,(\,\mathrm{d}\ln \mathbf{w} - \,\mathrm{d}\ln \mathbf{z}\,)\,, \tag{39}$$

where $\mathbf{O}$ is a matrix with entries $\mathbf{O}_{in} \equiv \sum_{h=1}^{N} t_{in} o_{nih}$ capturing the average across markets n of these deviations weighted by the share of country i's income derived from each market, as shown in Section F.3 of the online appendix. Using homotheticity, we can write $\mathbf{O} \equiv \epsilon \cdot \bar{\mathbf{O}}$ as the product between a scalar $\epsilon > 0$ and a matrix $\bar{\mathbf{O}}$ with an induced 2-norm equal to one ($\|\bar{\mathbf{O}}\| = 1$). By construction, $\|\mathbf{O}\| = \epsilon$. Using this representation, we can use results from matrix perturbation to obtain an upper bound on the sensitivity of income exposure to departures from the constant elasticity model, as a function of the observed trade matrices and the trade elasticity.

Proposition 11. *Let $\widetilde{\mathrm{d}\ln \mathbf{w}}$ be the solution to the general Armington model in equation (8) and let $\mathrm{d}\ln \mathbf{w}$ be the solution to the constant elasticity of substitution (CES) Armington model in equation (15). Then*

$$\lim_{\epsilon \to 0} \frac{\|\,\widetilde{\mathrm{d}\ln \mathbf{w}} - \,\mathrm{d}\ln \mathbf{w}\|}{\epsilon \cdot \|\,\mathrm{d}\ln \mathbf{w}\|} \leq \frac{\theta}{\theta + 1} \|\,(\mathbf{I} - \mathbf{V})^{-1}\,\| \|\mathbf{I} - (\mathbf{W} + \mathbf{Q})^{-1}\|. \tag{40}$$

Proof. See Section B.7 of the online appendix. □

Given this upper bound on the sensitivity of income exposure from Proposition 11, we can use equation (39) to compute the corresponding upper bound on the sensitivity of welfare exposure. All terms on the right-hand side of equation (40) can be computed using the observed trade matrices and the trade elasticity. Therefore, we can can compute these upper bounds for alternative assumed values of the trade elasticity. An immediate corollary of Proposition 11 is that as the departures from the constant elasticity model become small ($\epsilon \to 0$), income and welfare exposure under a variable trade elasticity converge towards their values in our constant elasticity specification.

Corollary 1. *As the deviations from a constant elasticity import demand system become small ($\lim \epsilon \to 0$), the elasticities of country income and welfare exposure to productivity shocks in the general Armington model converge to those in the constant elasticity of substitution (CES) Armington model in Propositions 1 and 3.*

Proof. This corollary follows immediately from Proposition 11. □

From Corollary 1, we can interpret the constant elasticity model as a limiting case of the variable elasticity model. In the neighborhood of this limiting case, our analytical expressions for the country income and welfare exposure to productivity shocks approximate those for the variable elasticity model. More generally, from Proposition 11, we can provide an upper bound for the sensitivity of income and welfare exposure to departures from the constant elasticity model that be computed using the observed trade matrices and assumed values for the trade elasticity. As such, our characterization of income and welfare exposure for a constant trade elasticity provides a useful benchmark for interpreting the results of quantitative trade models outside of this class.

5.4 Multiple Sectors

Our closed-form solutions for income and welfare exposure to productivity shocks in Propositions 1 and 3 also extend naturally to multi-sector environments with a constant trade elasticity. For continuity of exposition, we focus on a multi-sector version of the constant elasticity Armington model from Section 3 above, but the same results hold in the multi-sector version of the Eaton and Kortum (2002) model following Costinot et al. (2012), as shown in Section F.4 of the online appendix. The preferences of the representative consumer in country n are now defined across the consumption of a number of sectors k according to a Cobb-Douglas functional form:

$$u_n = \frac{w_n}{\prod_{k=1}^{K}\left[\sum_{i=1}^{N}\left(p_{ni}^k\right)^{-\theta}\right]^{-\alpha_n^k/\theta}}, \qquad \sum_{k=1}^{K}\alpha^k = 1, \qquad \theta = \sigma - 1, \qquad \sigma > 1. \tag{41}$$

where $\sigma > 1$ is the elasticity of substitution between country varieties and $\theta = \sigma - 1$ is the trade elasticity. For simplicity, we assume the same trade elasticity (θ) for all sectors, but in Section F.5 of the online appendix, we further generalize the analysis to allow for heterogeneous trade elasticities (θ^k) across sectors k.

Using expenditure minimization, the share of country n's expenditure in industry k on varieties from country i takes the standard constant elasticity form:

$$s_{ni}^k \equiv \frac{\left(p_{ni}^k\right)^{-\theta}}{\sum_{j=1}^{N}\left(p_{nj}^k\right)^{-\theta}}, \tag{42}$$

and we let $t_{in}^k \equiv s_{ni}^k \alpha_n^k \frac{w_n \ell_n}{w_i \ell_i}$ be the fraction of exporter i's income derived from selling to importer n in industry k.

Using the market clearing condition that country income equals expenditure on goods produced by that country together with the indirect utility function, we obtain the following generalization of our single-sector results.

Proposition 12. *In multi-sector international trade models, with constant elasticity of substitution (CES) preferences across sectors and a constant trade elasticity (θ), the elasticities of each country's income and welfare with respect to common productivity shocks across industries ($\mathrm{d}\ln z_\ell^k = \mathrm{d}\ln z_\ell$ for all k) in any country are given by:*

$$\underbrace{\mathrm{d}\ln \mathbf{w}}_{\textit{income effect}} = \underbrace{\mathbf{T}\,\mathrm{d}\ln \mathbf{w}}_{\textit{market-size effect}} + \underbrace{\theta \mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right)}_{\textit{cross-substitution effect}} = \boldsymbol{W}\,\mathrm{d}\ln \mathbf{z}, \tag{43}$$

$$\underbrace{\mathrm{d}\ln \mathbf{u}}_{\textit{welfare effect}} = \underbrace{\mathrm{d}\ln \mathbf{w}}_{\textit{income effect}} - \underbrace{\mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right)}_{\textit{cost-of-living effect}} = \boldsymbol{U}\,\mathrm{d}\ln \mathbf{z}, \tag{44}$$

where the expenditure share matrix ($\mathbf{S}$), income share matrix ($\mathbf{T}$) and cross-substitution matrix ($\mathbf{M}$) are now:

$$\mathbf{S}_{ni} \equiv \sum_{k=1}^{K}\alpha_n^k s_{ni}^k, \qquad \mathbf{T}_{in} \equiv \sum_{k=1}^{K} t_{ni}^k = \sum_{k=1}^{K}\frac{\alpha_n^k s_{ni}^k w_n \ell_n}{w_i \ell_i}, \qquad \mathbf{M}_{in} \equiv \sum_{h=1}^{N}\sum_{k=1}^{K} t_{ih}^k s_{hn}^k - 1_{n=i}, \tag{45}$$

$$\boldsymbol{W} = -\frac{\theta}{\theta+1}\left(\boldsymbol{I} - \boldsymbol{V}\right)^{-1}\boldsymbol{M}, \qquad \boldsymbol{U} = \left(\boldsymbol{I} - \boldsymbol{S}\right)\boldsymbol{W} + \boldsymbol{S}, \qquad \boldsymbol{V} \equiv \frac{\boldsymbol{T} + \theta \boldsymbol{T}\boldsymbol{S}}{\theta+1} - \boldsymbol{Q},$$

and $\boldsymbol{Q}$ denotes our choice of numeraire.

Proof. See Section F.4 of the online appendix. □

Although our closed-form solutions for income and welfare exposure take the same form in Proposition 12 as for the single-sector model in Propositions 1 and 3, the key difference is the way in which the expenditure share ($\boldsymbol{S}$), income share ($\boldsymbol{T}$) and cross-substitution ($\boldsymbol{M}$) matrices are constructed. In the multi-sector model, the elements of the $\mathbf{S}$ and $\mathbf{T}$ matrices in equation (45) now depend on the product of the share of country n's overall expenditure on sector k (α_n^k) times its share of expenditure on country i within that sector (s_{ni}^k). If sectors differ in size and vary in importance in the trade between different bilateral pairs of countries because of comparative advantage, the resulting elements of these matrices differ from those in the single-sector model. These differences in the elements of the $\mathbf{S}$ and $\mathbf{T}$ matrices in turn induce corresponding differences in the elements of the $\mathbf{M}$ matrix, which depend on the products of $s_{nh}^k t_{hi}^k$ for all markets h.

Therefore, in the multi-sector model, comparative advantage provides an additional source of terms of trade effects between countries. Even common changes in productivity across all sectors have heterogeneous bilateral effects on income and welfare depending on the extent to which pairs of countries share similar patterns of comparative advantage. Other things equal, the more similar are two countries' patterns of comparative advantage, the greater the extent to which higher productivity in one country will lead to cross-substitution away the other country in each market around the world. As for the single-sector model in the previous section, overall income and welfare exposure to productivity shocks again have a direct economic interpretation in terms of the underlying market-size, cross-substitution and cost of living effects in the model.

Additionally, the multi-sector model yields further disaggregated predictions for the impact of productivity shocks at the level of individual sectors. We derive these disaggregated predictions from the market clearing condition that equates industry value added in each country to expenditure on the goods produced by that industry. Using a matrix representation of this industry market clearing condition, we can derive an analogous closed-form expression for the elasticity of industry value added in each country with respect to a productivity shock in any country.

Proposition 13. *In multi-sector international trade models, with constant elasticity of substitution (CES) preferences across sectors and a constant trade elasticity (θ), the elasticity of industry income in each country with respect to common productivity shocks across industries ($\mathrm{d}\ln z_\ell^k = \mathrm{d}\ln z_\ell$ for all k) in any country are given by:*

$$\mathrm{d}\ln \mathbf{Y}^k = \mathbf{W}^k \,\mathrm{d}\ln \mathbf{z}, \qquad \mathbf{W}^k \equiv \mathbf{T}^k\mathbf{W} + \theta\mathbf{M}^k\left(\mathbf{W}-\mathbf{I}\right), \tag{46}$$

$$\mathbf{T}_{in}^k \equiv t_{ni}^k, \qquad \mathbf{M}_{in}^k \equiv \sum_{h=1}^{N} t_{ih}^k s_{hn}^k - 1_{n=i},$$

where $\mathbf{Y}^k$ is the vector of value-added in sector k across countries.

Proof. See Section F.4 of the online appendix. □

Aggregating across sectors, overall income exposure measure ($\mathbf{W}$) in Proposition 12 is the weighted average of sector value-added exposure measure ($\mathbf{W^k}$) in Proposition 13, with weights equal to sector value-added shares:

$$\mathbf{W}_i = \sum_k r_i^k \mathbf{W}_i^k, \qquad r_i^k \equiv \frac{w_i \ell_i^k}{\sum_{h=1}^{K} w_i \ell_i^h}, \tag{47}$$

where $\mathbf{W}_i$ is the income exposure vector for country i with respect to productivity shocks in its trade partners n and $\mathbf{W}_i^k$ is the analogous sector value-added exposure vector for country i and sector k.

5.5 Multiple Sectors and Input-Output Linkages

We now further extend our results for a multi-sector environment in the previous subsection to incorporate input-output linkages, following Caliendo and Parro (2015). Again for continuity of exposition, we focus on a multi-sector version of the constant elasticity Armington model, but the same results hold in a multi-sector version of the Eaton and Kortum (2002) model, as in Caliendo and Parro (2015).

The representative consumer's preferences are again defined across the consumption of a number of sectors, as in equation (41) in the previous subsection. Within each sector, each country's good is produced with labor and composite intermediate inputs according to a constant returns to scale production technology. These goods are subject to iceberg trade costs, such that $\tau_{ni}^k \geq 1$ units must be shipped from country i to country n in sector k in order for one unit to arrive (where $\tau_{ni}^k > 1$ for $n \neq i$ and $\tau_{nn}^k = 1$). Therefore, the cost to a consumer in country n of purchasing a good from country i within sector k is:

$$p_{ni}^k = \tau_{ni}^k c_i^k, \qquad c_i^k = \left(\frac{w_i}{z_i^k}\right)^{\gamma_i^k} \prod_{j=1}^{K} \left(P_i^j\right)^{\gamma_i^{k,j}}, \qquad \sum_{k=1}^{K} \gamma_i^{k,j} = 1 - \gamma_i^k, \tag{48}$$

where c_i^k denotes the unit cost function for sector k and country i; γ_i^k is the share of labor in production costs in sector k in country i; $\gamma_i^{k,j}$ is the share of materials from sector j used in sector k in country i; and z_i^k captures value-added productivity in sector k in country i.

Using the market clearing condition that country income equals expenditure on goods produced by that country together with indirect utility, we obtain the following further generalization of our single-sector results.

Proposition 14. *In multi-sector international trade models, with constant elasticity of substitution (CES) preferences across sectors, Cobb-Douglas input-output linkages between sectors and a constant trade elasticity (θ), the elasticities of each country's income and welfare with respect to common productivity shocks across industries ($\mathrm{d}\ln z_\ell^k = \mathrm{d}\ln z_\ell$ for all k) in any country are given by:*

$$\underbrace{\mathrm{d}\ln \mathbf{w}}_{\textit{income effect}} = \underbrace{\mathbf{T}\,\mathrm{d}\ln \mathbf{w}}_{\textit{market-size effect}} + \underbrace{\theta \mathbf{M}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right)}_{\textit{cross-substitution effect}} = \boldsymbol{W}\,\mathrm{d}\ln \mathbf{z}, \tag{49}$$

$$\underbrace{\mathrm{d}\ln \mathbf{u}}_{\textit{welfare effect}} = \underbrace{\mathrm{d}\ln \mathbf{w}}_{\textit{income effect}} - \underbrace{\mathbf{S}\left(\mathrm{d}\ln \mathbf{w} - \mathrm{d}\ln \mathbf{z}\right)}_{\textit{cost-of-living effect}} = \boldsymbol{U}\,\mathrm{d}\ln \mathbf{z}, \tag{50}$$

where the expenditure share matrix ($\mathbf{S}$), income share matrix ($\mathbf{T}$) and cross-substitution matrix ($\mathbf{M}$) are now:

$$\mathbf{S}_{ni} \equiv \sum_{h=1}^{N}\sum_{k=1}^{K} \alpha_n^k s_{nh}^k \Lambda_{hi}^k, \qquad \mathbf{T}_{in} \equiv \sum_{h=1}^{N}\sum_{k=1}^{K} \Pi_{ih}^k \vartheta_{hn}^k, \qquad \mathbf{M}_{in} \equiv \sum_{h=1}^{N}\sum_{k=1}^{K}\sum_{o=1}^{N} \Pi_{io}^k \left(\vartheta_{oh}^k + \sum_{j=1}^{N} \Theta_{oh}^{kj}\right) \Upsilon_{hon}^k,$$

where Λ_{hi}^k captures the share of revenue in industry k in country h that is spent on value-added in country i; Π_{ih}^k is the network-adjusted income share that country i derives from selling to industry k in country h; ϑ_{hn}^k is the share of revenue that industry k in country h derives from selling to country n; Θ_{oh}^{kj} captures the fraction of revenue in industry k in country o derived from selling to producers in industry j in country h; and Υ_{noh}^k captures the responsiveness of country h's expenditure on industry k in country o with respect to a shock to costs in country n.

Proof. See Section F.7 of the online appendix. □

Again our analytical expressions for income and welfare exposure take the same form as in the single-sector model, but the expenditure share ($\boldsymbol{S}$), income share ($\boldsymbol{T}$) and cross-substitution ($\boldsymbol{M}$) matrices are constructed differently. In particular, in the presence of input-output linkages, the elements of all three matrices must be further adjusted to take into account the network structure of production, using the observed industry-to-industry flows in the input-output matrix. For the $\mathbf{S}$ and $\mathbf{T}$ matrices that capture the share of an importer's expenditure on each exporter and the share of an exporter's income derived from each importer, respectively, this is largely a matter of accounting. We take into account that the gross value of trade from exporter i to importer n in industry k includes not only the direct value-added created in this exporter and industry but also indirect value-added created in previous stages of production. For the $\mathbf{M}$ matrix, this adjustment also takes into account that the effect of a foreign productivity shock now differs depending on whether it reduces intermediate input costs or competitors' output prices.

In Section F.7.12 of the online appendix, we report the analogous closed-form elasticities of each country's income and welfare with respect to bilateral trade cost shocks with input-output linkages, which take a similar form as in Section 5.1 above, and are used in our empirical analysis below.

6 Economic Friends and Enemies

In this section, we implement our network exposure measures empirically. In Subsection 6.1, we introduce our international trade data. In Subsection 6.2, we examine the quality of the approximation of our linearization to the non-linear solution of the model for empirically-reasonable productivity shocks and trade elasticities. In Subsection 6.3, we provide evidence on the evolution of the global network of welfare exposure our our sample period. In Subsection 6.4, we use our approach to quantify the impact of China's emergence into the global economy.

6.1 Data

Our data on international trade are from the NBER World Trade Database, which reports values of bilateral trade between countries for around 1,500 4-digit Standard International Trade Classification (SITC) codes, as discussed further in Section H of the online appendix. The ultimate source for these data is the United Nations COMTRADE database and we use an updated version of the dataset from Feenstra et al. (2005) for the time period 1970-2012.[5] We augment these trade data with information on countries' gross domestic product (GDP), population and bilateral distances from the GEODIST and GRAVITY datasets from CEPII.[6] We construct expenditure on domestic goods (X_{nnt}) using information on gross output, exports and imports, as discussed further in Section H of the online appendix. In our multi-sector models, we distinguish 20 tradeable and 20 non-tradeable sectors according to the International Standard Industrial Classification (ISIC). In our input-output specification, we use a common input-output matrix for all countries, based on the median input-output coefficients across the country sample in Caliendo and Parro (2015).[7] We use these datasets to construct the $\mathbf{S}$, $\mathbf{T}$ and $\mathbf{M}$ matrices for our three specifications of the single-sector constant elasticity Armington model (Section 3), our multi-sector extension (Section 5.4) and our input-output extension (Section 5.5).

[5]See https://cid.econ.ucdavis.edu/wix.html.

[6]See http://www.cepii.fr/cepii/en/bdd_modele/bdd.asp.

[7]In Section H of the online appendix, we report a robustness test, in which we construct domestic expenditure shares and country-specific input-output tables using the EORA Global Supply Chain Database (https://www.worldmrio.com/), for the shorter time period (1990-2015) and more aggregated industry classification for which these data are available. We find a strong correlation between our baseline measures using the NBER World Trade Database data and those using the EORA database for years where both data are available, for our input-output measures of income ($\boldsymbol{W}^{IO}$) and welfare ($\boldsymbol{U}^{IO}$) exposure, and for their components of expenditure ($\boldsymbol{S}^{IO}$) and income shares ($\boldsymbol{T}^{IO}$).

Our baseline sample consists of a balanced panel of 143 countries over the 43 years from 1970-2012.

6.2 Quality of the Approximation

In this section, we validate our closed-form solutions for income and welfare exposure, by showing that our linearization provides a close approximation to the full non-linear model solution for empirically-reasonable productivity shocks and values for the trade elasticity. For simplicity, and given its prominence in the existing literature, we focus in this section on our baseline single-sector Armington model from Section 3 above. In Section G.1.1 of the online appendix, we show that our linearization also closely approximates the full non-linear model solution for our extension to multiple sectors and input-output linkages from Section 5.5 above.

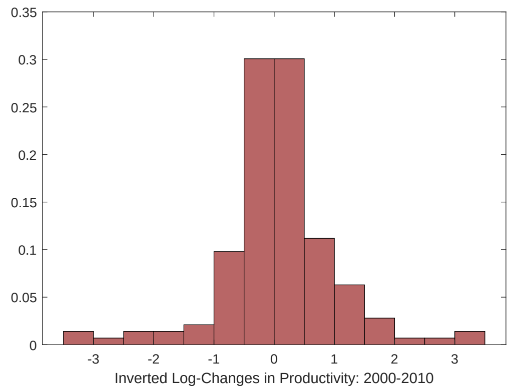
We report two sets of comparisons. First, we use the full non-linear model to recover the unobserved productivity and trade costs shocks that rationalize the observed data as an equilibrium. Using the empirical distribution of productivity shocks, we compare exact-hat algebra counterfactuals for the full non-linear model solution to the predictions of our linearization. Second, we use our analytical bounds for the quality of the approximation from Section 4 above to understand the reasons why our exposure measures provide such a good approximation, using the properties of the observed trade matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{M}$).

Empirical Distribution of Productivity Shocks We begin by recovering the empirical distribution of productivity and trade cost shocks that rationalize the observed trade data in our baseline single-sector constant elasticity Armington model. We begin by assuming a central value for the trade elasticity of $\theta = 5$, but report results below for trade elasticities from 2 to 20, which spans the range of typical empirical estimates.[8] Changes in productivity and trade costs are only separately identified up to a normalization or choice of units, because an increase in a country's productivity is isomorphic to a reduction in its trade costs with all partners (including itself). Therefore, to separate these two variables, we use the normalization that there are no changes in own trade costs over time ($\hat{\tau}_{nn} = 1$), which absorbs common unobserved changes in trade costs across all partners into changes in productivity. But our findings for the quality of our approximation are not sensitive to the way in which we recover productivity shocks, as explored in the Monte Carlo simulations below.

We recover changes in trade costs and productivities ($\hat{\tau}_{ni}$, $\hat{z}_i$) from the model's gravity equation for bilateral trade flows and its market clearing condition that equates a country's income with expenditure on the goods produced by that country, as shown in Section G.1.1 of the online appendix. In Figure 1, we display the empirical distribution of log changes in productivities ($\ln \hat{z}_i$) implied by the observed data from 2000-2010 for our central value of the trade elasticity of $\theta = 5$. As apparent from the figure, we find that these log changes in productivities are clustered relatively closely around their mean of zero, although some individual countries can experience large changes in log productivities, in part because any common trade cost shocks across all partners are absorbed into these changes in log productivities.

[8] Eaton and Kortum (2002) reports estimates of the trade elasticity ranging from 2 to 12; Costinot and Rodríguez-Clare (2014) assumes a central value of 5; and Simonovska and Waugh (2014) estimates a value of 4.

Figure 1: Distribution Across Countries of Log Productivity Shocks ($\ln \hat{z}_{it}$) from 2000-2010 (Trade Elasticity $\theta = 5$)

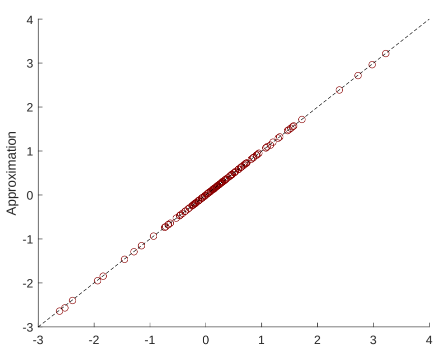


Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

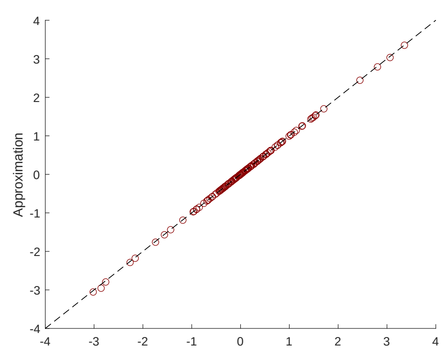
Actual Productivity Shocks in Actual Trade Networks Using this empirical distribution of changes in productivities, we compute exact-hat algebra counterfactuals for changes in country income ($\widehat{w}_i$) and welfare ($\widehat{u}_i$) in response to these productivity shocks ($\hat{z}_i$). In particular, we start from the observed equilibrium in the data in 2000, and shock productivity in each country by this empirical distribution of productivity shocks from 2000-2010, holding trade costs constant ($\hat{\tau}_{ni} = 1$). We compare the resulting counterfactual predictions for changes in income and welfare from the full non-linear model solution to those of our linearization, in which we pre-multiply the empirical distribution of productivity shocks by our income exposure ($\boldsymbol{W}$) and welfare exposure ($\boldsymbol{U}$) matrices: $\ln \hat{\mathbf{w}} = \mathbf{W} \ln \hat{\mathbf{z}}$. and $\ln \hat{\mathbf{u}} = \mathbf{U} \ln \hat{\mathbf{z}}$. In Figures 2a and 2b, we display the two sets of predictions against one another. Although they are not exactly the same as one another, they are visibly indistinguishable, with a regression slope coefficient close to one and a coefficient of correlation of more than 0.999.

Figure 2: Counterfactual Predictions for Empirical Distribution of Productivity Shocks from 2000-2010 (Trade Elasticity $\theta = 5$)

(a) Our Linearization Versus Non-linear Model Solution for Predicted Changes in Income ($\ln \hat{w}_{it}$)



(b) Our Linearization Versus Non-Linear Model Solution for Predicted Changes in Welfare ($\ln \hat{u}_{it}$)



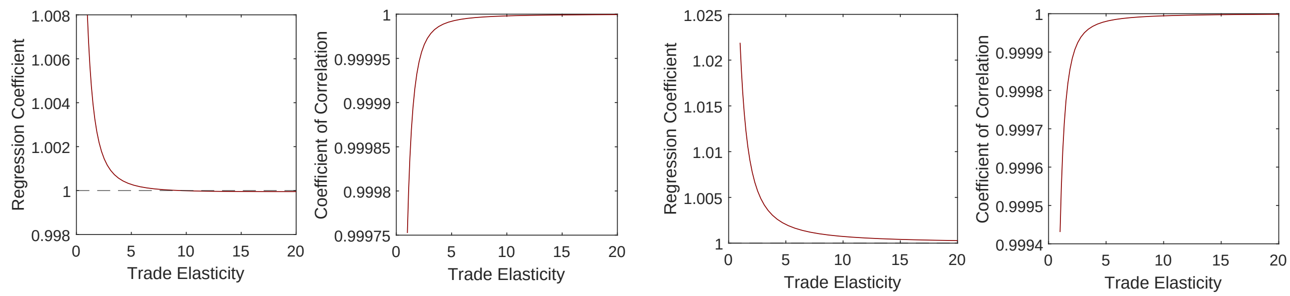
Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

We find similar results for the entire range of empirically-plausible values for the trade elasticity from 2 to 20. For each parameter value, we recover productivity and trade cost shocks from our model inversion, and compare counterfactual predictions for log changes in per capita income and welfare from our linearization and the full non-linear model solution. In Figures 3a and 3b, we show the regression slope coefficient and coefficient of correlation between the two sets of counterfactual predictions for log changes in income per capita and welfare. Across all values for the trade elasticity, we find a regression slope and coefficient of correlation close to one. As we increase the trade elasticity, our linearization converges even closer to the non-linear model solution, because the relative changes in productivity, income and welfare become smaller in absolute magnitude for larger trade elasticities.

Figure 3: Counterfactual Predictions for Empirical Distribution of Productivity Shocks 2000-2010 (Alternative Trade Elasticities from $\theta = 2$ to $\theta = 20$)

(a) Our Linearization Versus Non-linear Model Solution for Predicted Changes in Income ($\ln \hat{w}_{it}$)

(b) Our Linearization Versus Non-linear Model Solution for Predicted Changes in Welfare ($\ln \hat{u}_{it}$)



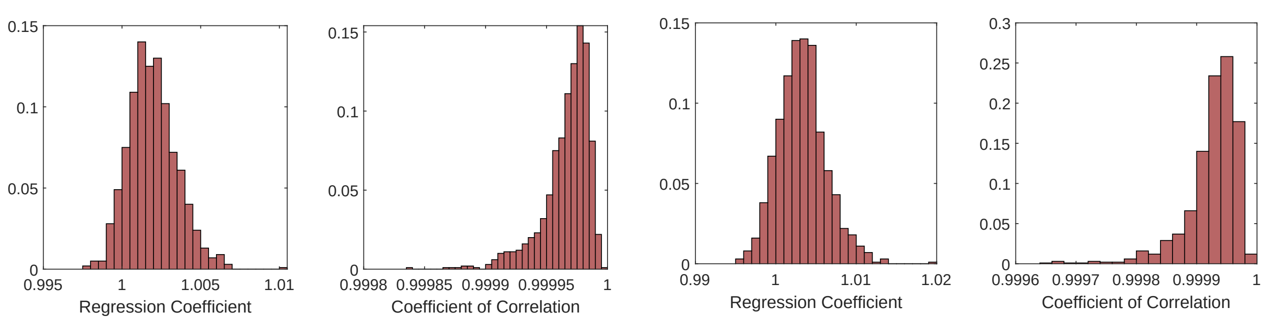
Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3.

Simulated Productivity Shocks in Actual Trade Networks We next demonstrate the robustness of our findings across alternative patterns of productivity shocks. In particular, we undertake 1,000 Monte Carlo simulations in which we draw (with replacement) productivity shocks for each country from the empirical distribution of productivity shocks from 2000-2010. Using these simulated shocks, we compare the predictions of our linearization and the full non-linear model solution. In Figures 4a and 4b, we show the distribution of regression slope coefficients and correlation coefficients between the two sets of predictions for log changes in income and welfare, respectively. Across all of our simulations, we find slope coefficients from 0.99-1.02 and correlation coefficients of more than 0.99.

Figure 4: Comparing our Linearization and the Non-linear Model Solution using Simulated Productivity Shocks from Monte Carlos (Trade Elasticity $\theta = 5$)

(a) Regression Slope Coefficients and Coefficients of Correlation for Predicted Changes in Income ($\ln \hat{w}_{it}$)

(b) Regression Slope Coefficients and Coefficients of Correlation for Predicted Changes in Welfare ($\ln \hat{u}_{it}$)



Source: NBER World Trade Database and authors' calculations authors' calculations using our baseline constant elasticity Armington model from Section 3. Monte Carlo simulations using 1,000 replications. Simulated productivity shocks drawn (with replacement) from the empirical distribution of productivity shocks from 2000-10.

Simulated Productivity Shocks in Simulated Trade Networks As discussed in Section 4 above, our linearization and the full non-linear model solution are identical in the two limiting cases of autarky and free trade. Furthermore, we find that our linearization also performs well for random trade matrices. In Section G.1.1 of the online appendix, we demonstrate that the observed trade matrices are well approximated by a weighted average of autarky, free trade, and random noise (Figure G.1), which provides an intuition why our approximation works well for actual trade networks. In Section G.1.1 of the online appendix, we report the results of an extensive search across simulated trade networks to try to find cases where our approximation is less successful (see in particular Figures G.2–G.5). The only examples that we have found in which our linearization performs less well are networks with a small number of countries and extreme trade patterns that differ substantially from actual trade patterns, such as a circular network, in which each country i only consumes goods $i+1$ and $i+2$ (and country N consumes goods 1 and 2). Even with this extreme trade pattern, we find that as we increase the number of countries, the quality of the approximation improves.

Actual Trade Cost Shocks We also perform an analogous exercise in which we undertake counterfactuals for changes in bilateral trade costs ($\hat{\tau}_{ni}^{-\theta}$), holding productivities constant ($\hat{z}_i = 1$). We again compare the counterfactual predictions of our linearization and the non-linear model solution, as discussed in Section G.1.1 of the online appendix. Although we again find a strong relationship between the predictions of our linearization and the exact-hat algebra counterfactuals, it is less strong on average than for productivity shocks. Since trade cost shocks are bilateral (as opposed to multilateral productivity shocks), the first-order and nonlinear response to trade cost shocks no longer coincide even if the economy starts in autarky or free-trade, which explains why the approximation performs less well. Nonetheless, for many shocks to trade costs with individual trade partners (e.g. for China-U.S. bilateral trade), our linearization continues to provide a close approximation to the full non-linear model solution.

Bounds on the Approximation Error for Productivity Shocks We now use our analytical results from Propositions 4-8 to show that the fact that our linearization provides a close to exact approximation to the non-linear model solution can be explained by the properties of the observed trade matrices. In Table 1, we report the distribution of the eigenvalues of the Hessian matrix that controls the magnitude of the second-order terms across the years of our sample period. We find that even the largest eigenvalue of the Hessian matrix ($\mathbf{H}_{f_i}$) is close to zero for each country. Therefore, as we approximate the log-income change for each country i separately, the second-order term ϵ_i, when maximized by a country-specific vector of TFP shocks $\left\{\tilde{\mathbf{z}}^{\max,i}\right\}$, accounts for at most a tiny fraction of the variation in $\ln \hat{w}_i$. For example, for the year 2000 and on average over time, we find that the second-order approximation error for the income exposure of each country is bounded by 0.26 percent and 0.36 percent of the variance of productivity shocks respectively.

Furthermore, for all countries, we find that the second-largest eigenvalues μ_i^{2nd} are substantially closer to zero, which implies that any productivity shock vector that is orthogonal to $\tilde{\mathbf{z}}^{\max,i}$ generates approximately zero second-order effects. We further find that the standard unit vector $\mathbf{e}^{\ell}$ comes close to achieving the upper bound for the ℓ-th equation, i.e. $\mathbf{e}^{\ell} \approx \tilde{\mathbf{z}}^{\max,\ell}$ for all ℓ. Hence, the second-order term for evaluating the effect of a productivity shock in country ℓ on income in country $i \neq \ell$ is small (approximately bounded by $\left|\mu^{2nd,i}\right|$) even relative to the own-effect on country ℓ itself, which is already small (approximately bounded by $\left|\mu^{\max,i}\right|$).

Table 1: Eigenvalues of the Hessian Matrix

	Eigenvalues of Hessians, ordered by absolute value, averaged across all countries									
	1	2	3	4	5	6	7	8	9	10
Year 2000										
	0.0026	0.0016	0.0010	0.0008	0.0006	0.0005	0.0004	0.0003	0.0003	0.0002
Average across years 1970–2012										
	0.0036	0.0020	0.0014	0.0010	0.0008	0.0006	0.0005	0.0004	0.0004	0.0003
Max across years 1970–2012										
	0.0062	0.0033	0.0023	0.0016	0.0013	0.0010	0.0008	0.0007	0.0006	0.0005

Source: NBER World Trade Database and authors' calculations using our baseline constant elasticity Armington model from Section 3 above.

Even when we consider all higher-order terms (second-order and above) in Proposition 8, using the assumption that the Hessian eigenvalues evaluated over the support of the distribution of productivity shocks are bounded by the Hessian eigenvalues observed during our sample period, we continue to find that the approximation error remains small. In particular, we find that the global approximation errors for income exposure to own productivity shocks are less than 0.62 percent of the variance of productivity shocks, and that these global approximation errors for welfare exposure to other countries' productivity shocks are 0.33 percent of the variance of productivity shocks.

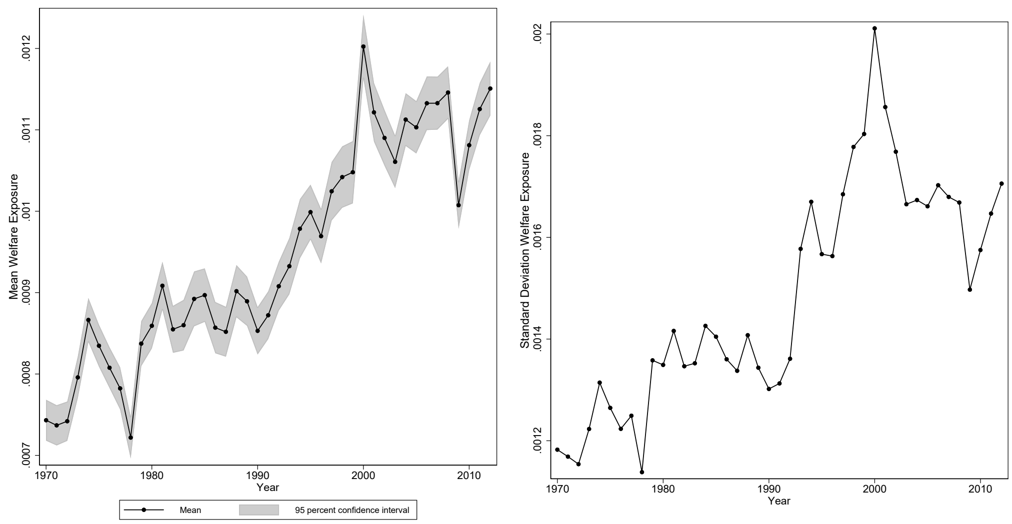
Based on the empirical results of this section as a whole, we conclude that our bilateral friend-enemy exposure measures for income and welfare are not only exact for small shocks but are close to exact for observed trade networks and empirically-reasonable productivity shocks and trade elasticities. A key advantage of our linearization is that it yields closed-form solutions for the elasticities of income and welfare with respect to any combination or value of productivity shocks, without having to solve a separate counterfactual for each combination and value of productivity shocks. Our approach is therefore well suited to applications in which the entire network of bilateral income and welfare exposure to productivity shocks is of interest, as explored in our empirical analysis below.

6.3 Global Income and Welfare Exposure 1970-2012

In this section, we use our approach to provide evidence on the evolution of the global network of income and welfare exposure to productivity shocks. We compute our exposure measures for our balanced panel of 143 countries over the 43 years from 1970-2012 ($143 \times 143 \times 43 = 879,307$ bilateral predictions for each variable). We focus on our extension to incorporate multiple sectors and input-output linkages from Section 5.5, because of its greater empirical realism, but we find a similar pattern of results using our single-sector model from Section 3 and our extension with multiple sectors from Section 5.4.

In Figure 5, we show mean and standard deviation of welfare exposure to foreign productivity shocks (excluding own productivity shocks) over time for our input-output model, as well as the associated 95 percent confidence intervals. Three features are apparent. First, we find that on average foreign productivity shocks raise domestic welfare, because the net effect of the market-size, cross-substitution and cost of living effects is typically positive. Second, there is considerable heterogeneity in welfare exposure across individual pairs of trading partners, with the standard deviation larger than the mean. Third, we find an increase in both the mean and standard deviation of welfare exposure to foreign productivity growth over time, which is consistent with the increased globalization that occurred over our sample period enhancing countries interdependence.

Figure 5: Mean Welfare Exposure to Productivity Shocks in Other Countries over Time

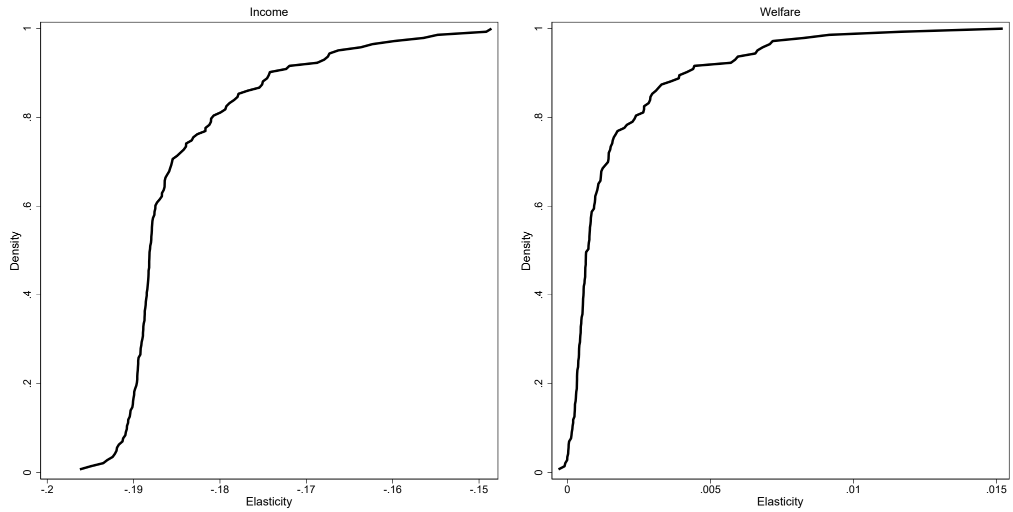


Note: Left panel shows mean welfare exposure (black line) and the 95 percent confidence interval (gray shading); right panel shows the standard deviation of welfare exposure (black line); both panels exclude own productivity shocks; NBER World Trade Database and authors' calculations using our input-output model from Section 5.5.

Recall that our measure of welfare exposure corresponds to the elasticity of welfare in one country to a productivity shock in another country. The mean elasticity in Figure 5 is naturally small for several reasons. First, foreign trade is typically a relatively small share of each country's overall expenditure. Second, although the world trade network is connected, in that sense that all countries trade with one another directly or indirectly (in accordance with Assumption 1), more than one quarter of exporter-importer pairs have zero bilateral trade, even at the end of our sample period in 2000, which contributes towards low mean welfare exposure, because these exporter-importer pairs are only linked indirectly. Third, even among exporter-importer pairs with positive bilateral trade, the distribution of trade flows is highly skewed, with countries typically having a small number of influential trade partners.

In Figure 6, we show the cumulative distributions of income and welfare exposure to U.S. productivity growth in the year 2000 (again excluding own productivity shocks). Even for a large exporter such as the United States, which trades with many importers, the distribution of importer exposure to its productivity growth is highly skewed. Income exposure is typically negative, given our choice of world GDP as the numeraire, because an increase in productivity that raises a country's own income trends to reduce the income of other countries (in order to hold world GDP constant). Welfare exposure is invariant to our choice of numeraire, and is typically positive, again because of the strength of the cost of living effect (higher productivity in a foreign country directly lower final output prices from that country) and input-output linkages (higher productivity in a foreign country directly lowers the cost of inputs sourced from that country). Nevertheless, welfare exposure to U.S. productivity growth is negative for some importers, in part because of the cross-substitution effect, whereby higher foreign productivity leads to substitution away from domestic goods in all markets around the globe.

Figure 6: Cumulative Distributions of Income and Welfare Exposure to U.S. Productivity Growth Across Importers in 2000



Note: Left panel shows the cumulative distribution of income exposure to U.S. productivity growth across importers in the year 2000; right panel shows the cumulative distribution of welfare exposure to U.S. productivity growth across importers in the year 2000; both panels exclude own productivity shocks; NBER World Trade Database and authors' calculations using our input-output model from Section 5.5.

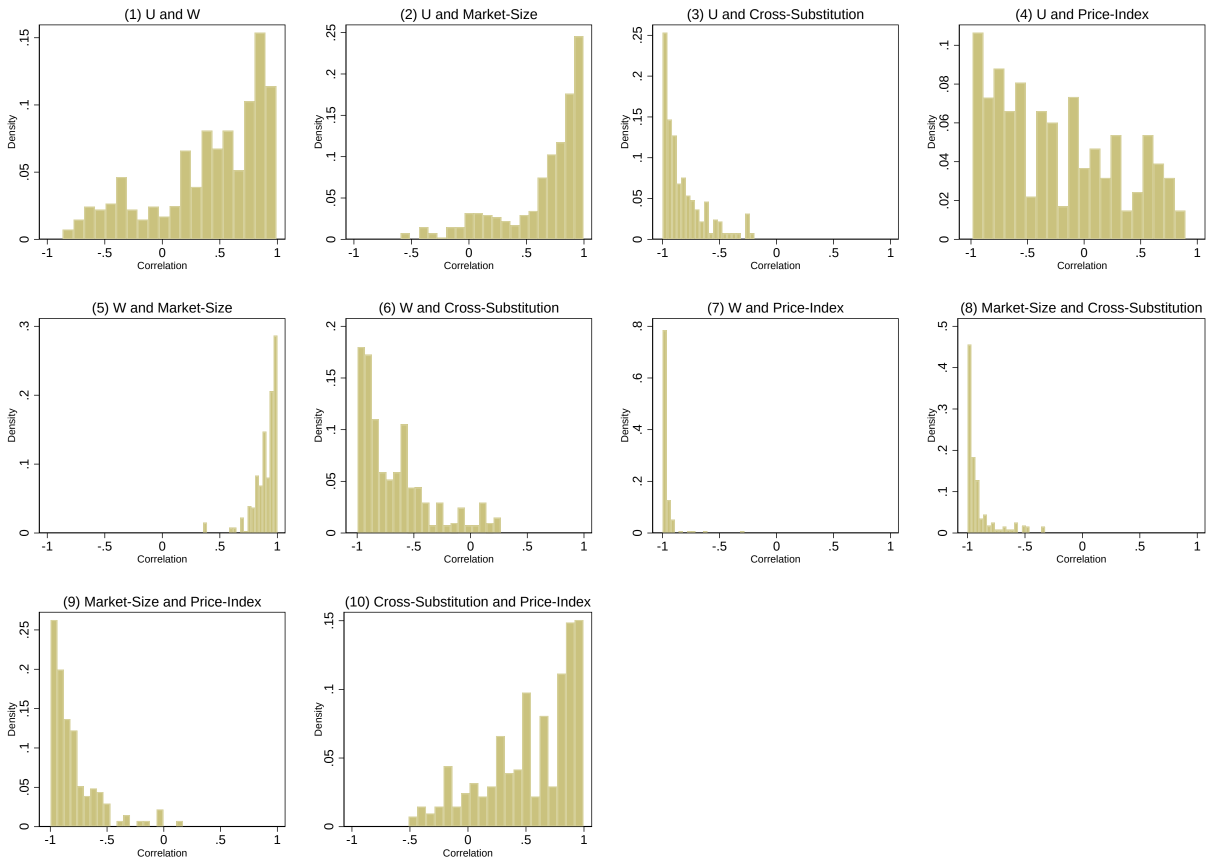
To obtain the percentage change in welfare in response to productivity shocks, one needs to multiply the elasticities in Figures 5 and 6 by the size of the productivity shock. When we do so, we obtain predicted changes in welfare in line with existing estimates in the quantitative international trade literature, as shown by the comparison of our linearization to the full non-linear model solution in the previous section. For example, our elasticity of US welfare to Chinese productivity growth ranges between 0.0003 and 0.0008 from 2000-2010. Over this time period, our measure of China's cumulative increase in log productivity relative to the world average from our model inversion is 110 percent. Multiplying these two numbers together, our linearization predicts that Chinese productivity growth over this period raised aggregate U.S. welfare by around 0.03-0.08 percent, which is the same to two decimal places as the prediction from exact-hat algebra counterfactuals using the full non-linear model solution.[9]

In Figure 7, we display correlations between welfare and income exposure and their components for the year 2000. For each exporter, we first compute the correlation across importers of the exposure measures in that year, excluding own productivity shocks. We next display the distribution of these correlations across exporters. Several features stand out. First, there is substantial heterogeneity in these correlations for different exporters, highlighting that they depend on the particular bilateral structure of the trade network. Second, welfare and income exposure can differ substantially from one another (Panel (1)), again indicating the importance of the price index effect. Third, both the market-size and cross-substitution effect matter for income exposure, although income exposure is more strongly correlated with the market-size effect than with the cross-substitution effect (Panels (5) and (6)). Fourth, the income effect and the price index effect are strongly negatively correlated, because increases in income in surrounding countries raise the cost of sourcing goods from those countries (Panel (7)). Fifth, the market-size and cross-substitution effects are typically strongly negatively correlated, because both are influenced by the gravity structure of trade (Panel (8)). When a home country is strongly exposed to positive market-size effects (because it derives a large share of its

[9]As other points of comparison, Caliendo et al. (2019) estimate that the China shock raised aggregate U.S. welfare by 0.2 percent, while Caliendo and Parro (2015) estimate the US welfare gains from NAFTA at around 0.08 percent. Therefore, the quantitative magnitude of our predictions is in line with the magnitudes typically found in the quantitative international trade literature. While we focus on these aggregate effects of productivity and trade cost shocks, there also can be distributional effects within countries.

income from a market), it is also highly exposed to negative cross-substitution effects (because substitution towards countries experiencing productivity growth leads to a large fall in its income from that market).

Figure 7: Histograms Across Exporters of the Correlations Between Importer Welfare and Income Exposure and Their Components in 2000



Note: for each exporter, we first compute the correlation between exposure measures across importers in 2000; the histograms then show the distributions of these correlations across exporters in that year; own productivity shocks are excluded; NBER World Trade Database and authors' calculations using our input-output model from Section 5.5.

In the previous section, we demonstrated that our income and welfare exposure measures provide a close approximation to the full non-linear solution of the model. We now show that these exposure measures cannot be fully captured by simpler measures of trading relationships between countries. In Table 2, we regress the income ($\boldsymbol{W}^{IO}$) and welfare ($\boldsymbol{U}^{IO}$) exposure measures from our input-model on the log value of trade between countries; aggregate import shares (the expenditure share matrix from our single-sector model ($\boldsymbol{S}^{SSM}$)); the expenditure share matrix from the input-output model ($\boldsymbol{S}^{IO}$); the income share matrix from the input-output model ($\boldsymbol{T}^{IO}$) ; and the cross-substitution matrix from the input-output model ($\boldsymbol{M}^{IO}$). While, in the interests of brevity, we report results for the year 2000, we find the same pattern across all years of our sample period.

Table 2: Correlations of Income ($\boldsymbol{W}^{IO}$) and Welfare ($\boldsymbol{U}^{IO}$) Exposure with Other Measures of Trading Relationships Between Countries

	(1) log value	(2) $\boldsymbol{S}^{SSM}$	(3) $\boldsymbol{S}^{IO}$	(4) $\boldsymbol{T}^{IO}$	(5) $\boldsymbol{M}^{IO}$
$\boldsymbol{W}^{IO}$	-0.000339***	-0.286***	-2.533**	-2.207***	-2.695***
	(0.0000104)	(0.0250)	(0.390)	(0.291)	(0.241)
Observations	20592	20592	20592	20592	20592
R-squared	0.0435	0.165	0.152	0.128	0.254
Year	2000	2000	2000	2000	2000
	(1) log value	(2) $\boldsymbol{S}^{SSM}$	(3) $\boldsymbol{S}^{IO}$	(4) $\boldsymbol{T}^{IO}$	(5) $\boldsymbol{M}^{IO}$
$\boldsymbol{U}^{IO}$	0.00000571***	0.0120***	0.123***	0.113***	0.0976***
	(0.000000231)	(0.000708)	(0.00357)	(0.00621)	(0.00448)
Observations	20592	20592	20592	20592	20592
R-squared	0.0290	0.688	0.843	0.781	0.783
Year	2000	2000	2000	2000	2000

Note: Observations are a cross-section of exporting and importing countries in the year 2000; $\boldsymbol{W}^{IO}$ is our income exposure measure for the input-output model from equation (49); $\boldsymbol{U}^{IO}$ is our welfare exposure measure for the input-output model from equation (50); log value is the log of one plus the value of bilateral trade; $\boldsymbol{S}^{SSM}$ is the share of each exporter in the aggregate expenditure of each importer (the expenditure share matrix in the single-sector model); $\boldsymbol{S}^{IO}$ is the expenditure share matrix in the input-output model; $\boldsymbol{T}^{IO}$ is the income share matrix in the input-output model; $\boldsymbol{M}^{IO}$ is the cross-substitution matrix in the input-output model; table reports the regressions of $\boldsymbol{W}^{IO}$ and $\boldsymbol{U}^{IO}$ (rows) on alternative measures of trading relationships between countries (columns); standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

As shown in the table, we find that our exposure measures have statistically significant correlations with all of these simpler measures of trading relationships, but the regression R-squared for our income exposure measure is always less than than 0.26. For our welfare exposure measure, we find that expenditure share matrix from the single-sector model ($\boldsymbol{S}^{SSM}$) has substantial explanatory power, although around one third of the variation in welfare exposure remains unexplained. Furthermore, the estimated coefficients in these reduced-form regressions do not have a structural interpretation, which implies that it would be hard to infer income and welfare exposure from these simpler measures of trading relationships without our closed-form solutions. When we regress our welfare exposure on the expenditure share matrix from the input-output model ($\boldsymbol{S}^{IO}$), the R-squared rises further to more than 0.80, highlighting the additional information from constructing the expenditure share matrix in the way implied by the structure of the input-output model. We find similar high R-squared for the income share ($\boldsymbol{T}^{IO}$) and cross-substitution ($\boldsymbol{M}^{IO}$) matrices from the input-output model, which is consistent with the close relationship between each of these other matrices and the expenditure share matrix. In Section G.1.2 of the online appendix, we show that the market-size, cross-substitution and price index components of income and welfare exposure are also imperfectly captured by these simpler measures of trading relationships between countries.

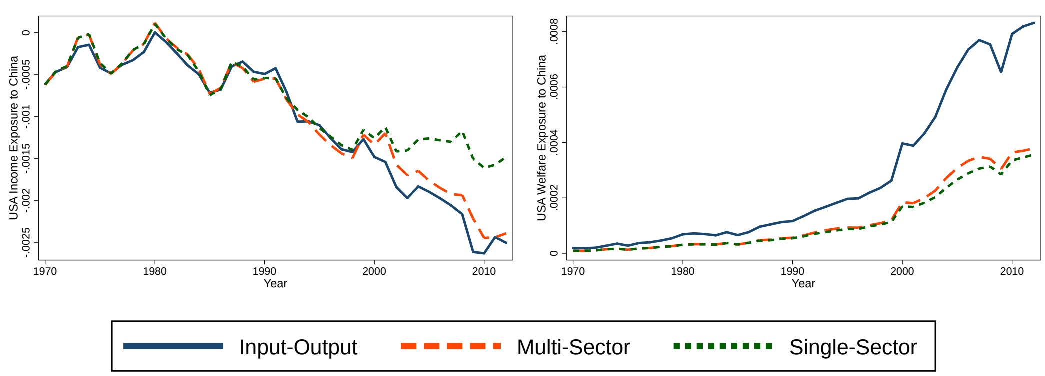
In Section G.1.2 of the online appendix, we report the results of a further validation exercise. We show that our welfare exposure measure captures the large-scale changes in the network of trade relationships that occurred over our sample period, including regional integration in North America following the North American Free Trade Agreement (NAFTA), the reorientation of European trade relationships following the fall of the Iron Curtain, and the reorganization of trading patterns in East Asia following the emergence of China into the global economy. Therefore, we find substantial changes over our sample period, not only in the mean and dispersion of welfare exposure, but also in the network of bilateral interdependencies between countries.

6.4 China's Emergence into the Global Economy

In the previous two sections, we validated our exposure measures by showing that they provide a close approximation to the full non-linear model solution, they differ from simpler measures of trading relationships between countries, and that they capture the large-scale changes in the global trading network that occurred over our sample period. In this section, we use our exposure measures to provide further evidence on the impact of China's emergence into the global economy following its market-orientated reforms of 1978.

U.S. Income and Welfare Exposure In Figure 8, we display income and welfare exposure in the United States to productivity growth in China for each year of our sample period from 1970-2012. Recall that our welfare exposure measure is invariant to the choice of numeraire, and to ensure that our income results are also unaffected by this choice of numeraire, we display income exposure for the United States relative to its income-weighted average for OECD countries. We display results for our input-output model (solid line), multi-sector model (long-dashed line) and single-sector model (short-dashed line). Across all three models, we find that China's emergence into the global economy implies that its productivity growth has an increasingly negative effect on U.S. income over time, but an increasingly positive impact on US welfare over time. The increase in the absolute magnitude of the negative income effect is larger in the models with multiple sectors than in the model with a single-sector, which is consistent with China's comparative advantage shifting closer to that of the United States over time (as captured in the models with multiple sectors but not incorporated in the model with a single-sector). Additionally, the increase in the absolute magnitude of the welfare effect is larger in the model with input-output linkages than in either of the other models, which is consistent with input-output linkages magnifying the welfare effects of productivity growth through reductions in the cost of imported intermediate inputs.

Figure 8: U.S. Relative Income and Welfare Exposure to Chinese Productivity Growth



Note: Left panel shows U.S. income exposure to Chinese productivity growth expressed relative to the income-weighted average for OECD countries to ensure that results are not sensitive to our choice of numeraire; Right panel shows U.S. welfare exposure to Chinese productivity growth; Welfare exposure is invariant to our choice of numeraire; NBER World Trade Database and authors' calculations using the single-sector model from Section 3, the multi-sector model from Section 5.4, and the input-output model from Section 5.5.

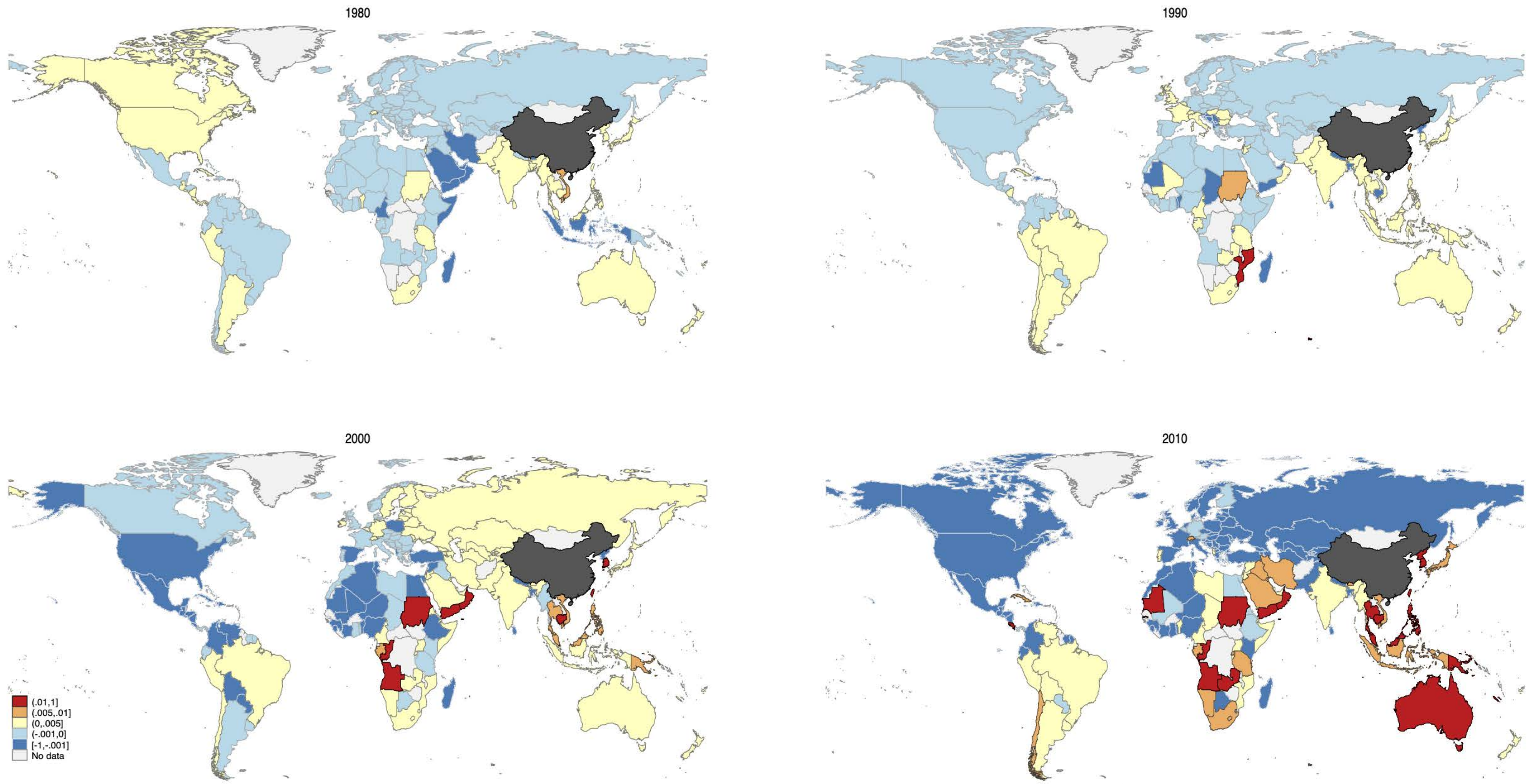
Global Income and Welfare Exposure In Figure 9, we show a map of country income exposure to Chinese productivity growth every decade from from 1980 (shortly after its market-orientated reforms) until 2010 (close to the end of our sample period). To ensure that results for income exposure are invariant to our choice of numeraire, we again normalize income exposure relative to the income-weighted average for OECD countries. Therefore, positive values represent an increase in income relative to the OECD average (shown in shades of red), and negative values correspond to a decrease in income relative to the OECD average (shown in shades of blue).

At the beginning of our sample period in 1980, Chinese productivity growth has modest effects on the relative income of countries around the globe, with small positive effects on Australia, India, South Africa and the United States, and small negative effects on most of the rest of Africa, Brazil, Western Europe and Russia. With China's rapid economic growth over the course of our sample, we observe an increase in the absolute magnitude of these income effects and a change in their spatial distribution. In particular, we find increasingly large negative effects on relative income for industrialized countries, such as the United States and most Western European countries. In contrast, we find increasingly large positive effects on relative income for resource-rich economies, including a number of African countries, as well as Australia and Chile. We also observe these increasingly large positive effects on relative income for a cluster of East Asian countries, consistent with the formation and growth of geographic production chains in East Asia over time.

In Figure 10 , we show a map of country welfare exposure to Chinese productivity growth for the same years. Recall that our welfare exposure measure is invariant to our choice of numeraire. We use darker shares of red to denote more positive welfare effects from Chinese productivity growth. For almost all countries, we observe positive welfare effects from Chinese productivity growth, highlighting the strength of the cost of living effect in the model, and the importance of distinguishing between income and welfare. Despite the increasingly large negative effects of Chinese productivity growth on relative income in the United States and most Western European countries in the previous figure, we find increasingly large positive effects on welfare in these countries. For resource-rich African countries, Australia and Chile, we find a similar pattern of results for welfare exposure as for income exposure, with increasingly large positive effects over time. Similarly, for East Asian countries, the cost of living effects reinforces our earlier results for relative income exposure, with Chinese productivity growth having increasing large positive effects on the welfare of these countries through geographic production chains.

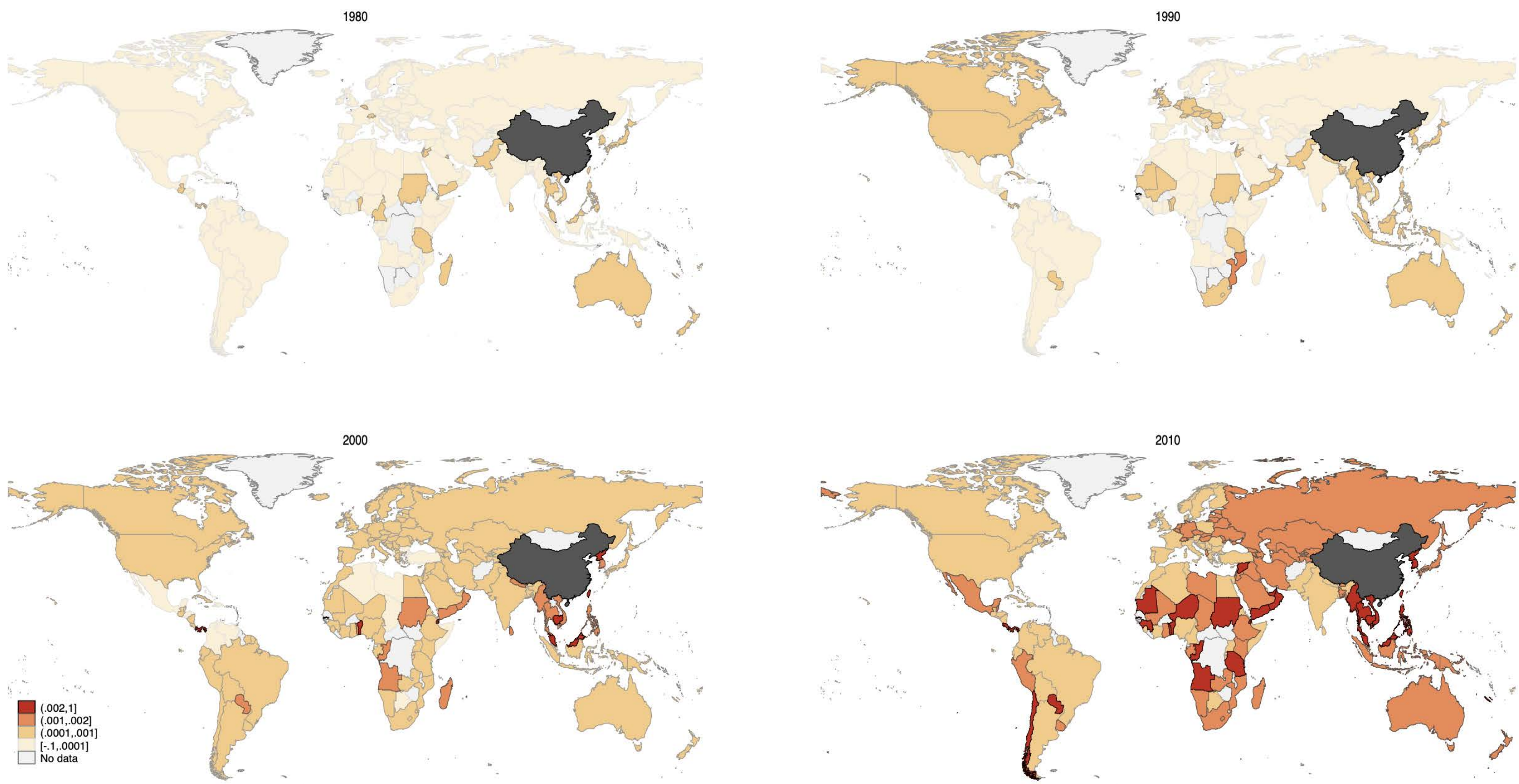
Cross-substitution Effects Throughout the class of theoretical models considered in Sections 3 and 5 of the paper, the key mechanism for negative effects of foreign productivity growth on domestic income (and potentially welfare) is the cross-substitution effect: as a foreign country becomes more productive, consumers in all markets around the world substitute away from the home country and towards the foreign country.

Figure 9: Country Income Exposure to Chinese Productivity Growth over Time



Note: income exposure is normalized relative to the income-weighted average of OECD countries; negative values (indicated using shades of blue, with darker shading denoting more negative effects) show decreases in income relative to the OECD average; positive values (indicated using shades of red, with darker shading denoting more positive effects) show increases in income relative to the OECD average; NBER World Trade Database and authors' calculations using the input-output model from Section 5.5.

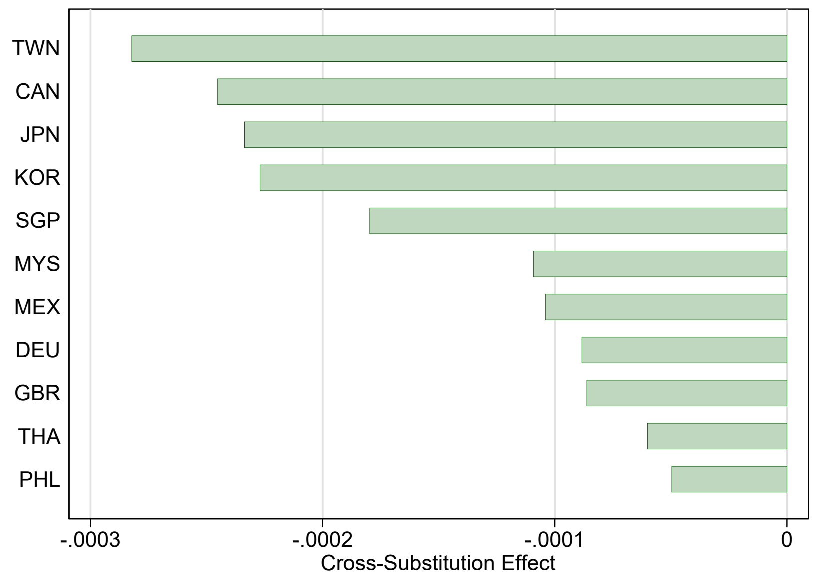
Figure 10: Country Welfare Exposure to Chinese Productivity Growth over Time



Source: welfare exposure is invariant to our choice of numeraire; darker shades of red denote more positive values; NBER World Trade Database and authors' calculations using the input-output model from Section 5.5.

In Figure 11, we provide evidence on these cross-substitution effects for the impact of Chinese productivity growth on the United States. For simplicity, we focus on the direct cross-substitution effect from higher Chinese productivity in each market ($-\theta \mathbf{M} \, \mathrm{d} \ln \mathbf{z}$ in equation (49)). While much of the direct effect of higher Chinese productivity growth occurs within the U.S. (importer's) market or the Chinese (exporter's) market, we also find that a substantial component also occurs through third markets, with the largest third-market effects occurring in Taiwan, Canada and Japan. This pattern of third market effects is intuitive, as this cross-substitution effect for the U.S. depends on the product of the share of U.S. income derived from a market (t_{ih}) and the share of that market's expenditure on China (s_{hn}). Of these three markets, Canada is one of the largest markets for the U.S. (high t_{ih}). Although Taiwan and Japan are smaller markets for the U.S. (lower t_{ih}), they have relative high shares of expenditure on China (high s_{nh}), and hence increased Chinese competitiveness in these markets has a large impact on US sales.

Figure 11: USA Exposure to Partial Equilibrium Cross-Substitution Effect from China, 2000



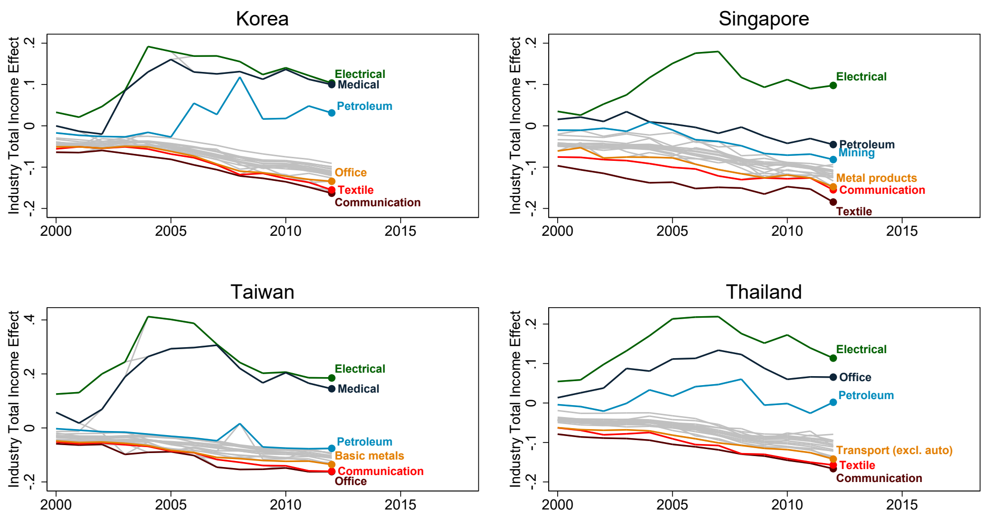
Source: NBER World Trade Database and authors' calculations using the input-output model from Section 5.5.

Sector Income Exposure In our multi-sector model with input-output linkages, even foreign productivity growth that is common across industries can have heterogeneous effects on industry income in other countries, depending on the extent to which those countries have similar patterns of industry comparative advantage in output markets, and the extent to which they source intermediate inputs from one another.

In Figures 12 and 13, we provide evidence on these heterogeneous industry effects of Chinese productivity growth for South-East Asian and resource-rich emerging economies, respectively. As for the aggregate income effect, our choice of world GDP as numeraire implies that a productivity shock that raises a country's own income tends to reduce income in other countries (in order to hold world GDP constant). For both the nearby South-East Asian countries (Figure 12) and the resource-rich emerging economies (Figure 13), we find some of the most negative effects for the Textiles sector.

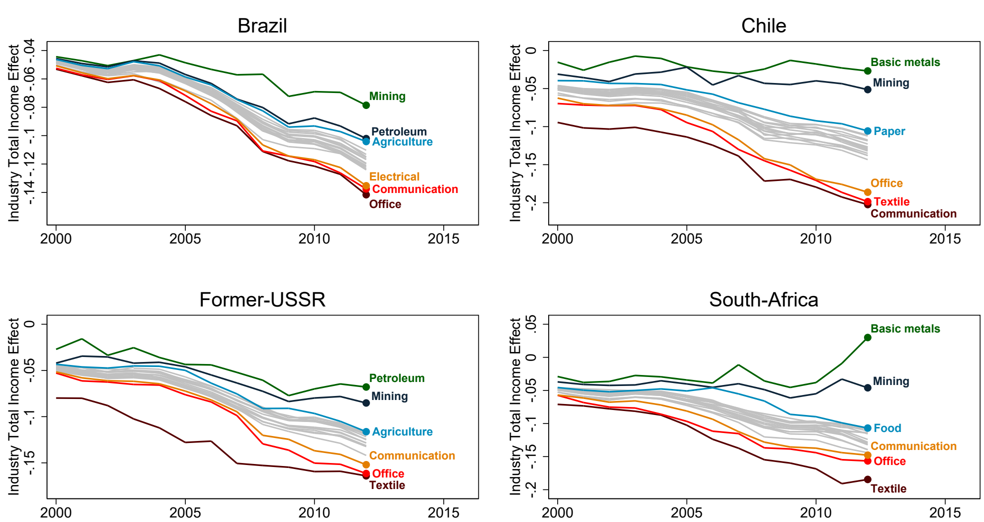
In contrast, we find striking differences between the two groups of countries in the sectors with the most positive or least negative income effects. For the nearby South-East Asian countries, the sectors that benefit most from Chinese productivity growth include the Electrical, Medical and Office Equipment sectors, which is consistent with input-output linkages between related sectors through global value chains in Factory Asia. However, for the resource-rich emerging economies, the sectors that benefit most include the Mining, Agricultural and Basic Metals sectors, which is in line with a form of "Dutch Disease," where the growth of resource-intensive sectors propelled by Chinese demand competes away factors of production from less resource-intensive sectors.

Figure 12: Industry Income Exposure in South-East Asia to Chinese Productivity Growth



Source: NBER World Trade Database and authors' calculations using the input-output model from Section 5.5.

Figure 13: Industry Income Exposure in Resource-Rich Countries to Chinese Productivity Growth



Source: NBER World Trade Database and authors' calculations using the input-output model from Section 5.5.

7 Economic and Political Friends and Enemies

We now use the property of our exposure measures that they capture the entire network of bilateral income and welfare effects between countries to provide new evidence on the connection between international relations and international trade. In Subsection 7.1, we examine two central questions about preferential trade agreements (PTAs). First, is the formation of PTAs endogenous to economic considerations, such that countries self select into PTAs that *ex ante* are expected to raise their welfare? Second, do observed PTAs in fact raise the *ex post* welfare of their members, taking into account general equilibrium effects? In Subsection 7.2, we provide econometric evidence on a classic debate in international relations and political science about the extent to which shared economic interests promote common political interests between countries.

7.1 Preferential Trade Agreements (PTAs)

One of the most striking features of international trade policy over the last few decades has been the proliferation of PTAs. Between 1965 and 2010, the share of world trade between countries that were members of a PTA rose from 22 percent in 1965 to 60 percent in 2010. Over the more recent period from 1990-2010, the number of PTAs rose by a factor of 4 form around 50 to just under 300 (Limao 2016). A large literature has developed examining the effect of these PTAs on the volume and pattern of international trade, including Frankel (1997), Frankel and Wei (1998), Limao (2007), Romalis (2007) and Estevadeordal et al. (2008), as reviewed in Freund and Ornelas (2010).

A central advantage of our income and welfare exposure measures is that they provide closed-form solutions for the impact of productivity and trade cost shocks on country income and welfare. As a further validation of these measures, we now examine whether they have predictive power for which countries self select into PTAs, and whether they detect increased interdependence between countries following the formation of a PTA. As tariff barriers for most countries have been low for several decades, much of the debate about PTAs has focused on non-tariff barriers, regulatory standards and deep integration, as emphasized in Grossman et al. (2021). Therefore, we interpret PTAs as a reduction in bilateral trade costs between member countries, abstracting from changes in tariff revenue.

Selection into PTAs We begin by examining selection into PTAs. First, we measure each importer's exposure to reductions in bilateral trade costs from each exporter at the beginning of our sample period in 1970 using our input-output model, as derived in Section F.7.12 of the online appendix. We use the subscript τ to distinguish this measure of welfare exposure to bilateral trade cost reductions ($\boldsymbol{U}_{\tau}^{IO}$) from our measure of welfare exposure to productivity growth ($\boldsymbol{U}^{\boldsymbol{IO}}$). We thus obtain a cross-section measure of welfare exposure to bilateral trade cost reductions at the beginning of our sample period for each exporter-importer pair. Second, we create a dummy variable that is equal to one if an exporter-importer pair is subsequently a member of a PTA in any year from 1971-2012 and zero otherwise. We thus obtain a cross-section measure of a future PTA for each exporter-importer pair. Third, we regress this cross-section measure of a future PTA from 1971-2012 on past welfare exposure to bilateral trade cost reductions in 1970. We estimate a linear probability model to allow the inclusion of exporter and importer fixed effects without introducing incidental parameter bias. These exporter and importer fixed effects control for unobserved heterogeneity in the average welfare gains from bilateral trade cost reductions and the average propensity to participate in trade agreements. We also control for past trade agreements by including a dummy variable that takes the value one for exporter-importer pairs that were members of PTAs in 1970 or earlier.

Table 3: Selection into Future Preferential Trade Agreements (PTAs) and Past Welfare Exposure to Bilateral Trade Cost Reductions ($\boldsymbol{U}_{\tau}^{IO}$)

	(1) PTA_{1971}^{2012}	(2) PTA_{1971}^{2012}	(3) PTA_{1971}^{2012}	(4) PTA_{1981}^{2012}	(5) PTA_{1991}^{2012}
$\boldsymbol{U}_{\tau}^{IO}$ 1970	4.874***	3.021**	8.150***	2.754**	2.716**
	(0.995)	(0.879)	(1.709)	(0.837)	(0.833)
PTA 1970	0.555***	0.534***	0.554***	-0.103***	-0.136***
	(0.0112)	(0.0108)	(0.0112)	(0.0115)	(0.0126)
Log value 1970		0.0203***		0.0187***	0.0179***
		(0.00127)		(0.00126)	(0.00125)
$\boldsymbol{S}^{SSM}$ 1970			-3.055*		
			(1.452)		
PTA_{1971}^{1980}				0.682***	
				(0.0120)	
PTA_{1971}^{1990}					0.717***
					(0.0125)
Exporter fixed effects	Yes	Yes	Yes	Yes	Yes
Importer fixed effects	Yes	Yes	Yes	Yes	Yes
Observations	17,292	17,292	17,292	17,292	17,292
R-squared	0.319	0.333	0.319	0.363	0.371

Note: Observations are a cross-section of exporter-importer pairs; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; PTA_{1971}^{2012} is a dummy variable that is equal to one if an exporter-importer pair is a member of a preferential trade agreement (PTA) from 1971-2012; PTA_{1981}^{2012}, PTA_{1991}^{2012}, PTA_{1971}^{1980} and PTA_{1971}^{1990} are defined analogously; $\boldsymbol{U}_{\tau}^{IO}$ 1970 is welfare exposure to bilateral trade cost reductions in 1970 in the input-output model from equation (50); PTA 1970 is a dummy variable that is equal to one if an exporter-importer pair is a member of a PTA in 1970 or earlier; log value 1970 is the log of one plus the value of bilateral trade flows in 1970; $\boldsymbol{S}^{SSM}$ 1970 is the share of each exporter in aggregate importer expenditure in 1970 (the expenditure share matrix in the single-sector model); standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

In Column (1) of Table 3, we report the estimation results. We find that exporter-importer pairs with larger welfare gains from bilateral trade cost reductions in 1970 are more likely to form future PTAs from 1971-2012. In Column (2), we show that this result is robust to controlling for the log value of bilateral trade in 1970 for each exporter-importer pair. In Column (3), we also find the same pattern of results controlling for each importer's aggregate share of expenditure on each exporter in 1970 (the expenditure share matrix from the single-sector model ($\boldsymbol{S}^{SSM}$)). Therefore, we find that our measure of past welfare exposure has predictive power for future trade agreements, even after controlling for the past values of these simpler measures of trading relationships between countries. In Columns (5) and (6), we show that welfare exposure to bilateral trade cost reductions in 1970 has predictive power for whether countries form PTAs more than a decade later from 1981-2012 and more than two decades later from 1991-2012, respectively. This predictive power holds even controlling for the log value of bilateral trade in 1970, past trade agreements in 1970 or earlier, and trade agreements in the intervening years of 1971-80 and 1971-90, respectively.

In Section G.2.1 of the online appendix, we show that we find a similar pattern of results for income exposure to bilateral trade cost reductions ($\boldsymbol{W}_{\boldsymbol{\tau}}^{\boldsymbol{IO}}$). Countries with larger income gains from bilateral trade cost reductions in 1970 are more likely to form future PTAs from 1971-2012. These income exposure results are marginally weaker than those for welfare exposure above, which is consistent with a role for cost of living effects in influencing selection into trade agreements. More broadly, these findings are consistent with the view that political actors place at least some weight on the welfare of their constituents when negotiating PTAs.

Taken together, these results provide further validation for our exposure measures, by showing that our closed-form solutions for the welfare gains from trade integration are successful in predicting future policy measures aimed at achieving this increased trade integration.

Impact of Trade Agreements on Economic Integration We next provide evidence on this subsequent impact of PTAs on economic integration between countries, including controls for the non-random selection established above. If PTAs are successful in enhancing deep integration between countries, we should expect to observe a subsequent increase in welfare exposure ($\boldsymbol{U}_{nit}^{IO}$) to productivity growth in member countries after the formation of these trade agreements. As our approach yields closed-form solutions for income and welfare exposure to productivity growth for each exporter-importer pair and time period, we can use these exposure measures to provide direct evidence on the magnitude of these effects and the time period over which they occur. To do so, we consider the following conventional event-study "difference-in-differences" specification:

$$\boldsymbol{U}_{nit}^{IO} = \sum_{s \in \{S_{-}, S_{+}\}} \beta_s (\mathbb{I}_{ni}^{PTA} \times \mathbb{I}_s) + \xi_{ni} + d_{ct} + h_{nit}, \tag{51}$$

where recall that n indexes importers, i denotes exporters and t corresponds to calendar year; $\mathbb{I}_{ni}^{PTA}$ is a dummy variable that equals one if an exporter-importer pair ever signs a trade agreement during our sample period; s is a treatment year index, which equals zero in the year an exporter-importer pair joins a PTA; therefore, negative values of s indicate years before joining a PTA and zero or positive values represent years after joining a PTA; $\mathbb{I}_s$ is a dummy variable that equals one in treatment year s and zero otherwise; we choose treatment year minus one as the excluded category; ξ_{ni} are exporter-importer pair fixed effects, which control for time-invariant factors that affect both bilateral welfare exposure and whether an exporter-importer joins a PTA; d_{ct} are continent-year dummies, which control for secular changes over time in welfare exposure and the propensity to join PTAs, where we allow these secular changes to vary by continent (results are similar with just year dummies); and h_{nit} is a stochastic error. We report standard errors clustered by exporter-importer pair to allow for serial correlation in the error term over time.

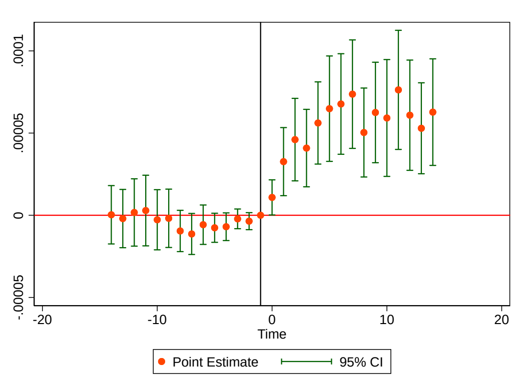
The key coefficients of interest are β_s on the treatment-year interactions, which capture the impact of the PTA on welfare exposure in treatment year s, relative to the excluded category of treatment year minus one. Our inclusion of an exporter-importer fixed effects controls for selection into PTAs based on time-invariant factors. Therefore, if exporter-importer pairs with high levels of welfare exposure are more likely to form PTAs in all years, this is controlled for in the exporter-importer fixed effect. The key identifying assumption in equation (51) is parallel trends between the treatment and control group within continents. As a check on this identifying assumption, we include the treatment-year interactions for years both before and after joining a PTA, which allows us to provide evidence on whether treated exporter-importer pairs exhibit different trends from control pairs even before joining a PTA.

In Figure 14a, we display the estimated treatment-year interactions and 95 percent confidence intervals for welfare exposure to productivity growth in our input-output model ($\boldsymbol{U}_{nit}^{IO}$) from equation (50). In Figure 14b, we show the corresponding estimates for welfare exposure to reductions in bilateral trade costs in our input-output model ($\boldsymbol{U}_{\tau,nit}^{IO}$). In both cases, we find no evidence of statistically significant differences in trends between the treatment and control group in the years leading up to the formation of a PTA, which is consistent with the idea that the inclusion of the exporter-importer fixed effect largely controls for the non-random selection into PTAs established above. In contrast, we observe a substantial and statistically significant increase in welfare exposure to both productivity shocks ($\boldsymbol{U}^{IO}_{nit}$) and reductions in bilateral trade costs ($\boldsymbol{U}^{IO}_{\tau,nit}$) immediately after the formation of a PTA. This pattern of results is consistent with that the idea that PTAs are successful in promoting economic interdependence between member countries, which thereby raises member countries' exposure to productivity growth or reductions in bilateral trade costs in other member countries.

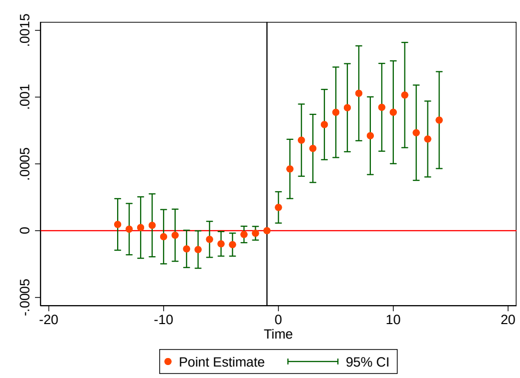
In Section G.2.1 of the online appendix, we report a number of further robustness checks. In Figure G.13, we show that we find a similar pattern of results for income exposure as for welfare exposure above. Again these income exposure results are marginally weaker than those for welfare exposure above, which is consistent with the idea that regional integration also increases the absolute magnitude of the cost of living effect between member countries. In Figure G.14, we show that find a similar pattern of welfare exposure even if we control for the log value of bilateral trade between each exporter-importer pair, which again confirms that our exposure measures cannot be fully captured by simpler measures of trading relationships between countries. Finally, the two-way fixed effects estimator uses variation over time within already-treated units, which can be hard to interpret in the presence of treatment heterogeneity and variable timing in the treatment, as recently pointed out in Chaisemartin and D'Haultfloeuille (2020), Borusyak et al. (2021) and Goodman-Bacon (2021). In Section G.2.1 of the online appendix, we show that we find the same pattern of results using the alternative difference-in-differences event-study estimator of Chaisemartin and D'Haultfloeuille (2020), which only exploits variation from transitions from untreated to treated status.

Figure 14: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Welfare Exposure

(a) Importer Welfare Exposure to Exporter Productivity Growth ($\boldsymbol{U}^{IO}_{nit}$)



(b) Importer Welfare Exposure to Exporter Bilateral Trade Cost Reductions ($\boldsymbol{U}^{IO}_{\tau,nit}$)



Note: Estimated treatment-year interactions (β_s) from the event-study specification in equation (51); Figure 14b shows results for the welfare exposure of importer n to productivity growth in exporter i at time t ($\boldsymbol{U}^{IO}_{nit}$) in our input-output model from equation (50); Figure 14b displays results for the welfare exposure of importer n to reductions in bilateral trade costs with exporter i at time t ($\boldsymbol{U}^{IO}_{\tau,nit}$) in our input-output model from equation (50); all specifications include exporter-importer fixed effects and continent-year fixed effects; standard errors clustered by exporter-importer pair to allow for serial correlation in the error term over time.

7.2 Bilateral Political Attitudes

We next use our network measures of bilateral welfare exposure to productivity growth to provide new evidence on the classic debate in international relations and political science about the extent to which increased conflict of economic interests between countries necessarily involves heightened political tension between them.

First, we introduce the data that we use to measure the similarity of countries' political attitudes. Second, we provide some descriptive evidence that changes in bilateral political attitudes over our sample period are systematically related to changes in welfare exposure. Third, we provide instrumental variables evidence that increases in the extent to which economic growth in a foreign country is welfare improving for a domestic country leads to an improvement in domestic political attitudes towards that foreign country.

7.2.1 Measuring Bilateral Political Attitudes

We consider two main measures of countries' bilateral political attitudes from the political science and international relations literature. First, we use data on observed voting behavior in the United Nations General Assembly (UNGA) to reveal the bilateral similarity of countries' foreign policies. Second, we use measures of strategic rivalries, as classified by political scientists, based on contemporary perceptions by political decision makers. Further details about the data sources and definitions are provided in Section G.2.2 of the online appendix.

United Nations Voting Country votes in the UNGA are recorded as "no" (coded 1), "abstain" (coded 2) or "yes" (coded 3). Our first measure of the similarity of countries' bilateral political attitudes is the S-score of Signorino and Ritter (1999), which equals one minus the sum of the squared actual deviation between a pair of countries' votes scaled by the sum of the squared maximum possible deviations between their votes. By construction, this S-score measure is bounded between minus one (maximum disagreement) and one (maximum agreement).

A limitation of this S-score measure is that is does not control for properties of the empirical distribution function of country votes. In particular, country votes may align by chance, such that the frequency with which any two countries agree on a "yes" depends on the frequency with which each country individually votes "yes." Therefore, we also consider two alternative measures of bilateral voting similarity that control in different ways for properties of the empirical distribution of votes. First, the π-score of Scott (1955) adjusts the observed variability of the countries' voting similarity using the variability of each country's own votes around the average vote for the two countries taken together. Second, the κ-score of Cohen (1960) adjusts this observed variability of the countries' voting similarity with the variability of each country's own votes around its own average vote.

Finally, a potential limitation of these three measures of the bilateral similarity of voting patterns is that they do not control for heterogeneity in the resolutions being voted on. To resolve this issue, Bailey et al. (2017) use the observed UN votes to estimate a time-varying measure of each country's political preferences or "ideal points." They show that these ideal points consistently capture the position of states vis-à-vis the US-led liberal order. We use this approach to derive a measure of bilateral distance between countries political attitudes by taking the absolute difference between the ideal points of countries i and j in each year t.

Strategic Rivalries Our second set of measures of countries' bilateral political attitudes are indicator variables that pick up whether country i is a strategic rival of j in year t, as classified by Thompson (2001) and Colaresi et al. (2010), and recently used in the economics literature in Aghion et al. (2018). These rivalry measures capture the risk of conflict with a country of significant relative size and military strength, based on contemporary perceptions by political decision makers, gathered from historical sources on foreign policy and diplomacy. Specifically, rivalries are identified by whether two countries regard each other as competitors, a source of actual or latent threats that pose some possibility of becoming militarized, or enemies. These rivalries are also further disaggregated into the following different types: (i) positional, where rivals contest relative shares of influence over activities and prestige within a system or subsystem; (ii) spatial, where rivals contest the exclusive control of a territory; and (iii) ideological, where rivals contest the relative virtues of different belief systems relating to political, economic or religious activities.

7.2.2 Descriptive Evidence on Bilateral Political Attitudes and Welfare Exposure

We next provide descriptive evidence of changes in bilateral political attitudes over our sample period that are systematically related to changes in bilateral welfare exposure. In particular, the most striking feature of our sample period is the large-scale change in the relative economic exposure of countries to productivity growth in China and the United States. We now show that the resulting change in other countries' relative welfare exposure to productivity growth in China and the United States is reflected in systematic changes in their relative political attitudes towards them. In the interests of brevity, we focus on our κ-score measure of the bilateral similarity of countries' voting patterns in the United Nations, which controls for the average frequency with which each country votes yes, no or abstain.

We begin by constructing a measure of relative political attitudes towards China and the United States. First, for all other countries n and years t, we compute the difference between each country's political attitudes to China and its attitudes to the United States ($\boldsymbol{A}^{\kappa}_{n,China,t} - \boldsymbol{A}^{\kappa}_{n,USA,t}$). Second, for each year t, we take the average of these relative political attitudes across countries within each of the following geographical areas of Africa, Asia/Oceania, Europe and North and South America. In Figure 15a, we display the evolution of these mean relative political attitudes over time. Following China's liberalization in 1978, we observe that other countries' political attitudes become more aligned towards China relative to the United States. We find that this realignment is stronger for Africa and Asia/Oceania, and weaker for Europe and North and South America.

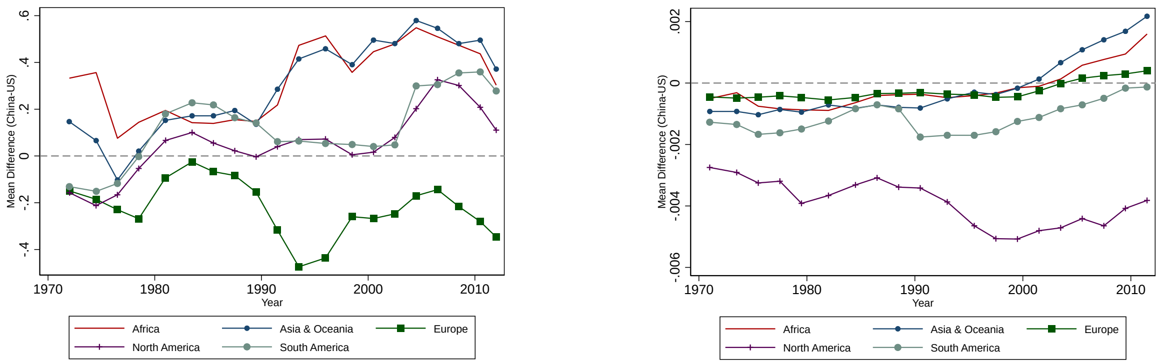
We next construct a measure of relative economic exposure towards China and the United States. First, for all other countries n and years t, we compute the difference between each country's welfare exposure to China and its welfare exposure to the United States ($\boldsymbol{U}^{IO}_{n,China,t} - \boldsymbol{U}^{IO}_{n,USA,t}$). Second, for each year t, we take the average of this relative economic exposure across countries within each of the same geographical areas. In Figure 15b, we display the evolution of these mean relative welfare exposures over time. Following China's liberalization in 1978, we also observe that its productivity growth has a more positive effect on other countries' welfare relative to that of productivity growth in the United States. We again find that this change in the pattern of relative welfare exposure is stronger for Africa and Asia/Oceania, and weaker for Europe and North and South America.

In Figure 16, we show that the countries whose relative political attitudes change towards China and away from United States are the same countries that experience an increase in positive welfare exposure from China relative to the United States. In particular, we display ventiles from a binscatter of the change in relative political attitudes against the change in relative welfare exposure, after conditioning on country and year fixed effects and each importer's aggregate share of expenditure on each exporter in 1970 (the expenditure share matrix from the single-sector model ($\boldsymbol{S}^{SSM}$)). The inclusion of these fixed effects implies that this relationship is identified from differential changes in relative political attitudes and welfare exposure within countries. We also show the corresponding linear fit between the two variables. We find a positive and statistically significant relationship between the two variables, with an estimated coefficient of 18.833 (standard error 2.998).

Figure 15: Average Relative Political Attitudes and Average Relative Welfare Exposure by Continent Over Time (Average Towards China Minus Average Towards the United States)

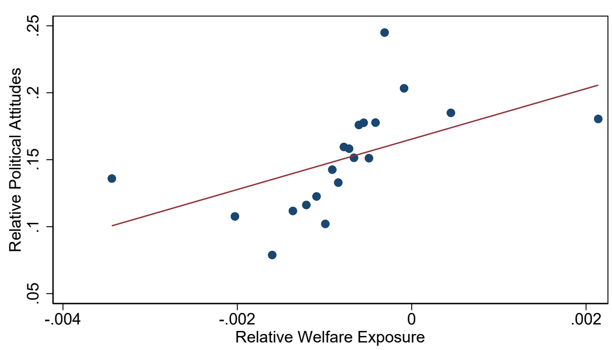
(a) Political Similarity ($\boldsymbol{\kappa}_{n,China,t} - \boldsymbol{\kappa}_{n,USA,t}$)

(b) Welfare Exposure ($\boldsymbol{U}^{IO}_{n,China,t} - \boldsymbol{U}^{IO}_{n,USA,t}$)



Notes: In the left panel, we first measure the bilateral attitudes of each importer n to each exporter i in each year t using the kappa measure (κ_{nit}) of the similarity of country votes in the United Nations General Assembly (UNGA); we next compute each importer's political attitudes to China minus its political attitudes to the United States in each year ($\boldsymbol{A}^{\kappa}_{n,China,t} - \boldsymbol{A}^{\kappa}_{n,USA,t}$); finally, we take averages of attitudes to China relative to attitudes to the United States in each year across all importers within each continent (excluding China and the United States); in the right panel, we first measure the economic exposure of each importer n to each exporter i in each year t using the welfare exposure measure ($\boldsymbol{U}^{IO}_{nit}$) in our input-output model from equation (50); we next compute each importer's welfare exposure to China minus its welfare exposure to the United States in each year ($\boldsymbol{U}^{IO}_{n,China,t} - \boldsymbol{U}^{IO}_{n,USA,t}$); finally, we take averages of welfare exposure to China relative to welfare exposure to the United States in each year across all importers within each continent (excluding China and the United States).

Figure 16: Changes in Country Relative Political Attitudes Against Changes in Country Relative Welfare Exposure (Relative Attitudes and Exposure to China Minus the United States)



Notes: Figure shows a binscatter of country relative political attitudes against country relative welfare exposure, after conditioning on country and year fixed effects and each importer's aggregate share of expenditure on each exporter (the expenditure share matrix from the single-sector model ($\boldsymbol{S}^{SSM}$)); relative political attitudes equals each other country's κ-score for China minus its κ-score for the United States in each year ($\boldsymbol{A}^{\kappa}_{n,China,t} - \boldsymbol{A}^{\kappa}_{n,USA,t}$); relative welfare exposure equals each other country's welfare exposure to China minus its welfare exposure to the United States in each year ($\boldsymbol{U}^{IO}_{n,China,t} - \boldsymbol{U}^{IO}_{n,USA,t}$); the inclusion of country and year fixed effects implies that the figure shows the relationship between changes in relative political attitudes and changes in relative welfare exposure; the red line shows the linear fit with coefficient 18.833 (standard error 2.998); each blue dot corresponds to a ventile (twenty quantile) of the country-year distribution.

Although this descriptive evidence by itself is not definitive, this pattern of results is consistent with the idea that changes in countries' economic influences on one another affect their bilateral political attitudes. The fact that both these realignments are particularly strong for Africa and Asia/Oceania is in line with a number of Asian countries having close input-output linkages to China, and with several resource-rich African countries exporting primary products to China. These findings are also in line with the broader literature that has emphasized the political implications of Chinese economic initiatives, including for example foreign direct investment (FDI) in Africa and the Belt and Road

Initiative (BRI), as discussed for example in Alden (2007) and Hillman (2020).

7.2.3 Regression Evidence on Bilateral Political Attitudes and Welfare Exposure

Motivated by the above empirical findings, we now provide econometric evidence in support of a causal interpretation of this relationship between changes in welfare exposure and changes in bilateral political attitudes. We instrument changes in our welfare exposure measure using exogenous changes in the geographical determinants of bilateral trade. In particular, we consider the following long-differenced instrumental variables (IV) regression specification, using five-year long differences from 1970 through 2010:

$$\begin{aligned} \Delta \boldsymbol{A}_{nit} &= \beta \Delta \boldsymbol{U}_{nit}^{IO} + \eta_{nt}^{A} + \mu_{it}^{A} + \epsilon_{nit}^{A}, \\ \Delta \boldsymbol{U}_{nit}^{IO} &= \gamma \Delta \boldsymbol{S}_{nit}^{IOG} + \eta_{nt}^{U} + \mu_{it}^{U} + \epsilon_{nit}^{U}, \end{aligned} \tag{52}$$

where $\Delta \boldsymbol{A}_{nit}$ is the five-year change in one of our measures of bilateral political attitudes; $\Delta \boldsymbol{U}_{nit}^{IO}$ is the five-year change in bilateral welfare exposure in our input-output model from equation (50); $\Delta \boldsymbol{S}_{nit}^{IOG}$ is the five-year change in the expenditure share matrix in our input-output model predicted by the five-year change in the geographical determinants of bilateral trade; η_{nt}^{A} and η_{nt}^{U} are importer-year fixed effects; μ_{it}^{A} and μ_{it}^{U} are exporter-year fixed effects; and ϵ_{nit}^{A} and ϵ_{nit}^{U} are stochastic errors. We report standard errors that are clustered by exporter-importer pair to allow for serial correlation in these error terms over time.

We focus on five-year changes to capture longer-run changes in relative welfare exposure to growth in trade partners that are relevant for countries' political attitudes towards one another. By focusing on these five-year changes, we difference out any fixed effect in the level of bilateral political attitudes and welfare exposure. Therefore, we allow for unobserved heterogeneity in the time-invariant determinants of these variables that is specific to each exporter-importer pair, which is differenced out when we take long differences. We include exporter-year fixed effects to control for changes in bilateral political attitudes and welfare exposure that are common across importers for each exporter. Recall that the change in our welfare exposure measure captures the change in the elasticity of importer welfare with respect to exporter productivity. In general, the magnitude and sign of exporter productivity growth could also matter for changes in bilateral political attitudes, which is captured through the inclusion of these exporter-year fixed effects, since exporter productivity growth is common across all partner importing countries. We also include importer-year fixed effects to control for changes in bilateral political attitudes and welfare exposure that are common across exporters for each importer.

Our key coefficient of interest β is identified from the relationship between five-year changes in political attitudes and welfare exposure that are specific to individual exporter-importer pairs. We instrument the five-year changes in bilateral welfare exposure ($\Delta \boldsymbol{U}_{nit}^{IO}$) using five-year changes in geographical determinants of bilateral trade ($\Delta \boldsymbol{S}_{nit}^{IOG}$) to address the simultaneity concern that bilateral shocks to political attitudes could induce changes in bilateral trade, and hence result in endogenous changes in bilateral welfare exposure. We construct these instruments using the close relationship between welfare exposure ($\boldsymbol{U}_{nit}^{IO}$) and the expenditure share matrix in our input-output model ($\boldsymbol{S}_{nit}^{IO}$) established in Section 6.3 above. Following Frankel and Romer (1999), we model the exogenous geographic determinants of this expenditure share matrix ($\boldsymbol{S}_{nit}^{IO}$) using a gravity equation in importer population (ℓ_{nt}), exporter population (ℓ_{it}), and bilateral distance (dist_{ni}):

$$\boldsymbol{S}_{nit}^{IO} = \ell_{nt}^{\xi_t} \ell_{it}^{\vartheta_t} \text{dist}_{ni}^{\delta_t} \varpi_{nit}, \tag{53}$$

where we allow the exponent on each variable to change over time; and ϖ_{nit} is a stochastic error.

We estimate this gravity equation (53) separately for each year using the Poisson Pseudo Maximum Likelihood estimator of Santos Silva and Tenreyro (2006). To abstract from shocks to bilateral political attitudes, we use the fitted values from this gravity equation to measure the component of the expenditure share matrix in our input-output model ($\boldsymbol{S}_{nit}^{IO}$) that is solely driven by the geographical determinants of trade: $\boldsymbol{S}_{nit}^{IOG} = \ell_{nt}^{\xi_t} \ell_{it}^{\vartheta_t} \text{dist}_{ni}^{\delta_t}$. We use changes in these geographical determinants of trade ($\Delta \boldsymbol{S}_{nit}^{IOG}$) as our instrument for changes in welfare exposure ($\Delta \boldsymbol{U}_{nit}^{IO}$). These changes in the geographical determinants of trade ($\Delta \boldsymbol{S}_{nit}^{IOG}$) reflect changes in the estimated coefficients on importer and exporter population and bilateral distance (ξ_t, ϑ_t and δ_t^d) and changes in the products of importer and exporter populations weighted by their estimated coefficients ($\ell_{nt}^{\xi_t} \ell_{it}^{\vartheta_t}$), since importer and exporter populations enter the gravity equation (53) multiplicatively, whereas the exporter-year and importer-year fixed effects enter our political attitudes and exposure equations (52) additively.

In Table 4, we report results of estimating our instrumental variables specification (52) using our measures of bilateral political attitudes based on UNGA voting. Column (1) uses the S-score measure of Signorino and Ritter (1999); Column (2) considers the π-score measure of Scott (1955); and Column (3) examines the κ-score measure of Cohen (1960). Across all three specifications, we find that increases in welfare exposure to productivity growth predicted by our instruments lead to a positive and statistically significant increase in the propensity to vote similarly in the UNGA. We find that our instrument has power in the first-stage regression, with first-stage F-statistics above the conventional threshold of 10. We use the κ-score as our preferred measure of bilateral political attitudes, because it controls for the empirical frequency with which each country individually votes yes, no or abstain.[10]

In Columns (4) and (5), we show that our results are robust to controlling for simpler measures of trading relationships between countries, including the change in the log value of bilateral trade and the change in aggregate import shares (the expenditure share in our single-sector model ($\Delta \boldsymbol{S}_{nit}^{SSM}$)).[11] Both these specifications should be interpreted with caution, because these simpler measures of trading relationships are endogenous. Additionally, the change in aggregate expenditure shares ($\Delta \boldsymbol{S}_{nit}^{SSM}$) is naturally correlated with the change in the geographical determinants of expenditure shares in our input-output model ($\Delta \boldsymbol{S}_{nit}^{IOG}$). Nevertheless, there remains independent variation from the pattern of input-output linkages within each industry between countries and from the instrumentation of changes in expenditure shares in our input model by changes in geographical determinants. Even when we control for these simpler measures of trading relationships, we continue to find that increases in welfare exposure to productivity growth predicted by our instruments lead to a positive and statistically significant increase in the frequency with which countries vote in more similar ways in the UNGA.

[10] As for our other specifications above, we find stronger results for welfare exposure ($\boldsymbol{U}_{\boldsymbol{nit}}^{\boldsymbol{IO}}$) than for income exposure ($\boldsymbol{W}_{\boldsymbol{nit}}^{\boldsymbol{IO}}$), with for example an estimated coefficient (standard error) for income exposure of 78.692 (26.136) in the specification in Column (3) using the κ-score. This pattern of results is again consistent with political decision makers placing some weight on the welfare of their constituents, and internalizing that changes in the cost of living are part of the mechanism through which these welfare effects occur.

[11] Our results using the S-score and π-score in Columns (1) and (2) are also robust to controlling for the log value of bilateral trade and aggregate import shares, with estimated coefficients (standard errors) of 39.573 (12.309) and 73.328 (29.245) for the S-score and 160.821 (36.866) and 305.879 (100.377) for the π-score, respectively.

Table 4: Changes in Political and Economic Friends (Five-Year Long Differences)

	(1)	(2)	(3)	(4)	(5)
	$\Delta \boldsymbol{A}^{S}_{nit}$	$\Delta \boldsymbol{A}^{\pi}_{nit}$	$\Delta \boldsymbol{A}^{\kappa}_{nit}$	$\Delta \boldsymbol{A}^{\kappa}_{nit}$	$\Delta \boldsymbol{A}^{\kappa}_{nit}$
$\Delta \boldsymbol{U}^{IO}_{nit}$	36.82***	151.1***	202.5***	215.0***	408.2***
	(11.56)	(34.27)	(38.90)	(42.24)	(124.9)
Δ log value$_{nit}$				-0.00276***	
				(0.000520)	
$\Delta \boldsymbol{S}^{SSM}_{nit}$					-16.92**
					(6.954)
Estimation	IV	IV	IV	IV	IV
Exporter-year fixed effects	Yes	Yes	Yes	Yes	Yes
Importer-year fixed effects	Yes	Yes	Yes	Yes	Yes
First-stage F-statistic	51.36	51.36	51.36	48.43	26.61
Observations	114,426	114,426	114,426	114,426	114,426

Note: Observations are pooled five-year long differences from 1970 through 2010 for exporter-importer pairs; the first subscript n denotes the importer, the second subscript i corresponds to the exporter, and the third subscript t indexes the five-year difference; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; $\Delta \boldsymbol{A}^{S}_{nit}$ is the five-year change in the S-score measure of the political similarity in voting patterns in the United Nations General Assembly (UNGA); $\Delta \boldsymbol{A}^{\pi}_{nit}$ is the five-year change for the π measure of political similarity; $\Delta \boldsymbol{A}^{\kappa}_{nit}$ is the five-year change for the κ measure of political similarity; $\Delta \boldsymbol{U}^{IO}_{nit}$ is the five-year change in welfare exposure in the input-output model from equation (50); Δ log value$_{nit}$ is the five-year change in the log of one plus the value of bilateral trade; $\Delta \boldsymbol{S}^{SSM}_{nit}$ is the five-year change in the aggregate share of each importer's expenditure on each exporter (the expenditure share matrix in the single-sector model); the five-year change in welfare exposure ($\Delta \boldsymbol{U}^{IO}_{nit}$) is instrumented using the five-year change in the expenditure share matrix in the input-output model predicted by geographic variables ($\Delta \boldsymbol{S}^{IOG}_{nit}$) from a gravity equation (exporter and importer population and bilateral distance with time-varying coefficients); first-stage F-statistic is a test of the statistical significance of the instrument in the first-stage regression; the second-stage R-squared is not reported, because it does not have a meaningful interpretation; standard errors in parentheses are clustered by exporter-importer pair; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

In Table 5, we show that this pattern of results is robust across alternative measures of bilateral political attitudes. Column (1) uses the bilateral distance in countries' ideal points based on UNGA voting patterns from Bailey et al. (2017); Column (2) uses the measure of any strategic rivalry from Thompson (2001) and Colaresi et al. (2010); Column (3) focuses on positional strategic rivalries from Colaresi et al. (2010); Column (4) considers spatial strategic rivalries from Colaresi et al. (2010); and Column (5) examines ideological strategic rivalries from Colaresi et al. (2010). We find that increases in welfare exposure to productivity growth predicted by our instruments decrease the distance between countries' ideal points (Column (1)) and reduce the propensity with which countries are strategic rivals (Column (2)). We find the same negative relationship for each of the different types of strategic rivalries (Columns (3)-(5)), although the estimated coefficient for ideological rivalries is not significant at conventional critical values, which is consistent with the idea that ideological rivalries could be less sensitive to economic factors. In Table G.3 in the appendix, we show that these results for alternative measures of bilateral political attitudes are also robust to controlling for simpler measures of trading relationships between countries, such as the log value of trade. While we use five-year differences to capture longer-run changes, we find similar results using ten-year differences, or using a yearly panel data specification that exploits deviations from time means within exporter-importer pairs.

Taken together, the empirical results of this subsection are consistent with the view that increasing alignment of countries' economic interests does indeed lead to increasing alignment of their political attitudes.

Table 5: Robustness of Changes in Political and Economic Friends (Five-year Long Differences)

	(1) $\Delta \boldsymbol{A}_{nit}^{D}$	(2) $\Delta \boldsymbol{A}_{nit}^{R}$	(3) $\Delta \boldsymbol{A}_{nit}^{RP}$	(4) $\Delta \boldsymbol{A}_{nit}^{RS}$	(5) $\Delta \boldsymbol{A}_{nit}^{RI}$
$\Delta \boldsymbol{U}_{nit}^{IO}$	-1760.2***	-40.89**	-25.90**	-35.60***	-5.336
	(304.9)	(16.30)	(10.78)	(13.43)	(4.102)
Estimation	IV	IV	IV	IV	IV
Exporter-year fixed effects	Yes	Yes	Yes	Yes	Yes
Importer-year fixed effects	Yes	Yes	Yes	Yes	Yes
First-stage F-statistic	51.36	51.36	51.36	51.36	51.36
Observations	113,694	133,748	133,748	133,748	133,748

Note: Observations are pooled five-year long differences from 1970 through 2010 for exporter-importer pairs; the first subscript n denotes the importer, the second subscript i corresponds to the exporter, and the third subscript t indexes the five-year difference; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; $\Delta \boldsymbol{A}_{nit}^{D}$ is the five-year change in the ideal-distance measure of political attitude; $\Delta \boldsymbol{A}_{nit}^{R}$ is the five-year change in the measure of any strategic rivalries; $\Delta \boldsymbol{A}_{nit}^{RP}$ is the five-year change in the measure of positional strategic rivalries; $\Delta \boldsymbol{A}_{nit}^{RS}$ is the five-year change in the measure of spatial strategic rivalries; $\Delta \boldsymbol{A}_{nit}^{RI}$ is the five-year change in the measure of ideological strategic rivalries; $\Delta \boldsymbol{U}_{nit}^{IO}$ is the five-year change in welfare exposure in the input-output model from equation (50); the five-year change in welfare exposure ($\Delta \boldsymbol{U}_{nit}^{IO}$) is instrumented by the five-year change in the expenditure share matrix ($\Delta \boldsymbol{S}_{nit}^{IO}$) in the input-output model predicted by geographic variables ($\Delta \boldsymbol{S}_{nit}^{IOG}$) from a gravity equation (exporter and importer population and bilateral distance with time-varying coefficients); first-stage F-statistic is a test of the statistical significance of the instrument in the first-stage regression; the second-stage R-squared is not reported, because it does not have a meaningful interpretation; standard errors in parentheses are clustered by exporter-importer pair; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

8 Conclusions

We develop new network measures of exposure to foreign productivity and trade cost shocks for more than 140 countries over more than 40 years from 1970-2012. Our measures correspond to closed-form solutions for the elasticity of income and welfare in each country with respect to these shocks with any foreign country in the class of models with a constant trade elasticity. We derive these closed-form solutions from a friend-enemy matrix representation of the general equilibrium effects of productivity and trade cost shocks. This matrix representation is exact for small shocks, permits an analytical characterization of the quality of the approximation for large shocks, and is almost exact for productivity shocks of the magnitude implied by the observed data. A key advantage of our measures is that they recover the entire network of bilateral income and welfare exposure from a single matrix inversion, without requiring a separate counterfactual for each productivity of trade cost shock. Our measures are therefore well suited to empirical applications in which the entire network of income and welfare exposure is a key object of interest.

We provide an internal validation for our closed-form solutions by comparing them to counterfactual predictions from the full non-linear model solution. We show that our closed-form solutions provide an almost exact approximation to the full non-linear model solution for empirically-reasonable productivity shocks and trade elasticities. We use our analytical results to show that our linearization performs so well, because of the low eigenvalues of the matrix of second derivatives implied by observed trade matrices. Intuitively, our linearization is exact in the limiting cases of autarky and free trade, and performs well for random trade matrices. As observed trade matrices are well approximated by a linear combination of autarky, free trade and random trade matrices, our closed-form solutions also perform well using these observed trade matrices.

We illustrate the wide range of applications for our network measures using four different empirical implementations: (i) global income and welfare exposure to productivity growth from 1970-2012; (ii) the emergence of China into the global economy; (iii) preferential trade agreements (PTAs), and (iv) the debate about whether countries with more similar economic interests also have more similar political interests. We show that our exposure measures are not fully captured by simpler measures of trading relationships, such as the value of bilateral trade or the aggregate share of expenditure of each importer on each exporter. We find increasingly large negative effects of Chinese productivity growth on the relative income of industrialized countries, such as the United States and most Western European countries. In contrast, we find increasingly large positive effects on relative income for resource-rich economies, including a number of African countries, and for a cluster of South-East Asian countries. Despite these increasingly negative effects of Chinese productivity growth on the relative income of industrialized countries, we find increasing positive effects on their aggregate welfare, highlighting the strength of cost of living effects.

We provide further external validation for our closed-form solutions, by showing that they are successful in predicting future preferential trade agreements (PTAs) and detecting their subsequent impact on economic interdependence between countries. Finally, we use our network measure of exposure to provide new evidence on the classic political economy debate about the relationship between economic and political interests. We show that China's emergence into the global economy since its liberalization in 1978 has been followed by a realignment of the political attitudes of other countries away from the United States and towards China, particularly in Africa and East Asia. We provide regression evidence that increases in welfare exposure predicted by our geographical instruments lead to a statistically significant increase in the frequency with which countries vote similarly in the United Nations General Assembly (UNGA), and a statistically significant decrease in the propensity with which they are strategic rivals. Taken together, these results are consistent with the view that greater similarity of economic interests does indeed promote greater congruence of political interests.

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