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ABSTRACT

We evaluate the effects of COVID19 restrictions and fiscal policy in a model featuring economic slack. The restrictions can reduce current-period GDP by more than is directly associated with the restrictions themselves even if prices and wages are flexible, households can smooth consumption, and workers are mobile across sectors. The most effective fiscal policies depend on (a) the joint distribution of capital operating costs with respect to firm revenues, (b) the extent to which the price of capital adjusts, and (c) additional factors that determine whether the economy will enter a boom or a slump after the restrictions are lifted, such as the effect of the restrictions on inequality and on spending by high-income households.

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1. Introduction

What is the economic effect of the fiscal policy response to the COVID19 crisis? While this is a truly $2 trillion question, there is little clarity on how the stimulus works in the current conditions. Indeed, one may think that fiscal policy is more stimulative in recessions but e.g., Brunet (2018) documents evidence suggesting that fiscal multipliers were smaller during World War II because the government imposed restrictions on how households could spend their income.

To shed light on this key policy issue, we examine COVID19-related restrictions and spending multipliers in a model of economic slack.[1] We demonstrate that in principle output losses and spending multipliers can be *much larger* than those implied by models in the New Keynesian tradition. This difference is driven by the possibility that goods markets need not clear in the near future. In the slack framework, today's output depends on current spending and future output depends on future spending, whereas in New Keynesian-style models, future output is determined by supply factors and a goods-market-clearing condition (as price and wage rigidities dissipate in the steady state). Perhaps most strikingly, large spending multipliers can occur even in the *absence* of credit constraints and in the *presence* of flexible price and wage contracts. Furthermore, even if all households can smooth consumption and prices are flexible, current spending decisions today have strong effects on output today and in the future.[2]

Specifically, we extend the negligible-marginal-cost (NMC) framework of Murphy (2017) to examine heterogeneous (high-income and low-income) households that consume a variety of services. We model the economic restrictions associated with COVID19 as a temporary decrease in the share of varieties of goods/services that can be exchanged. Similar to Guerrieri et al (2020), a fraction of firms are restricted from selling, which causes a decrease in aggregate income. Our model features large declines in output even in the absence of credit constraints or strong intertemporal substitution and in the presence of income sharing across sectors (e.g., through worker mobility).

[1] Theories of economic slack posit that workers and capital experience periods of idleness that represent wasted resources (e.g., Michaillat and Saez 2015, Murphy 2017). For empirical evidence of the relevance of models of slack, see e.g. Auerbach et al. (2020a, 2020b), Demyanyk et al. (2019), Egger et al. (2020), Boehm and Pandali-Nayar (2017).

[2] This feature of the slack framework implies the effects on output of key macroeconomic factors are potentially very different (and under different conditions) than is implied by other frameworks. For example, inequality can have large and persistent effects in the slack framework, as higher inequality implies lower permanent income for low-income households. This lower income causes lower consumption over time, potentially causing prolonged slumps. In contrast, inequality tends to have a muted effect on output in models in which the supply side determines future output (Auclert and Rognlie 2020).

The NMC framework offers additional new insights into the effects of these restrictions. First, the model predicts large multiplier effects of the restrictions when firms face fixed costs of operation and hence firm entry/exit is an important margin of adjustment. In particular, if the fixed operating costs are sufficiently large (relative to firms' revenues) and do not adjust to changes in firms' demand for capital, then restrictions on a subset of firms' products causes some firms to exit and therefore cease production of other unrestricted goods and services. For example, restaurants are restricted from serving customers in the establishment but are able to provide carry-out and delivery services. If restrictions cause some restaurants' revenues to decline below their fixed costs, then these restaurants will cease producing carry-out services. This firm exit channel leads to large indirect (multiplier) effects of economic restrictions and provides a strong rationale for policies aimed at mitigating fixed capital operating costs. In the absence of these multiplier effects, it might be optimal to allow firms to temporarily exit and then re-enter once restrictions are lifted. But the large multipliers imply that such exit can be very costly. If fixed operating costs are flexible or negligible, then the output loss is proportional to the fraction of varieties that are directly subject to COVID19 restrictions. While this output loss is substantial, there are no implied multiplier effects.

Second, the model delivers conditions under which a rapid recovery and even a boom in economic activity occurs after the restrictions are lifted. Households smooth the marginal utility of consumption across varieties *and* across time. When fewer varieties are available in the current period, households may spend more in the future. Even though lifetime income decreases due to the COVID19-related restrictions, spending and income in the future may increase beyond the level that would have occurred in the absence of the restrictions. This is because households tilt their expenditure toward the future when there are more varieties. In this sense, the restrictions act as a reduction in households' discount factor.

We examine the effect of fiscal transfers in this environment. Government transfers to low-income households have multiplier effects, which can offset the adverse secondary (multiplier) effects of the COVID19 restrictions. However, the transfers can have smaller multiplier effects during the presence of COVID19 restrictions, since there are fewer products on which to spend. This result is similar to that in Guerrieri et al. (2020). Transfers also increase low-income households' total spending capacity, which leads to a larger boom after restrictions are lifted.

All fiscal policy is not equal, however. In our framework the government has various fiscal levers that it can pull: direct transfers to households, direct transfers to firms, as well as various targeted transfers. The preferred policy depends on the policy objective and the extent to which the government can target households and firms.

Targeted transfers to low-income households can increase spending on unrestricted items, thus supporting income during the restrictions. However, the transfers have stronger effects after the restrictions are lifted. Whether these future effects stabilize future output at its counterfactual level (or cause a boom that might not warrant the cost of the transfers) depends on the strength of forces that could cause a prolonged slump. Since the strength of these forces is unclear ex ante, transfers to households could be postponed until after the restrictions are lifted to determine whether they are necessary. Furthermore, the output effect of transfers is falling in inequality, as spending multipliers are increasing in the income share of the poor.

The strongest effect of fiscal stimulus is targeted transfers to multiproduct firms for which the restrictions push their revenues below their fixed operating costs. Such targeted transfers prevent firm exits that lead to large secondary (multiplier) output declines. In practice it may be difficult to identify and target such firms, although the model offers some guidance. The firms most at risk of exit are those with relatively low profitability and for which capital operating costs are the largest or most rigid. As documented by Gilje et al. (2020), rigid capital contracts can arise from asymmetric information regarding firms' ability to cover capital costs. In our context, the asymmetric information friction is perhaps the most severe for smaller businesses that are not subject to the same reporting requirements as public firms. Direct loans and transfers to small private businesses may therefore target the firms on the margin of exit and have large benefits per dollar spent.

While targeted transfers to firms has the larger potential benefit, untargeted transfers to firms have among the least benefit. Not only is some income spent on firms that are not in danger of exit, but a large share of the income received by the firms accrues to high-income households for whom spending is less sensitive to transfers.

This paper is broadly related to emerging work evaluating the indirect economic effects of COVID19. Most closely related is Guerrieri et al. (2020), who model COVID19 as a restriction on labor supplied to a subset of firms. They document that COVID19 restrictions can cause a fall in output in the presence of strong complementarities between restricted goods and other goods, large elasticities of intertemporal substitution, and large shares of credit-constrained households. If these conditions are sufficiently strong, the economy can exhibit a multiplier whereby output falls by more than the size of the direct supply restrictions. Our approach to modeling the COVID19 restrictions is similar in that a subset of firms cannot sell output to consumers. However, we find that output effects of the restrictions are large even if credit is unrestricted and even if wage contracts can be renegotiated. We furthermore show that large output effects are limited to the direct effects of restrictions on a subset of goods and services unless firms face fixed capital operating costs. Finally, we evaluate the benefits of alternative fiscal stimulus measures, including (targeted and untargeted) transfers to households and firms. The relative effectiveness of alternative fiscal stimulus measures depends on a number of conditions, including the joint distribution of firms' revenues and capital costs. In this sense our framework can guide empirical work examining the relative merits of alternative stimulus measures.

2. Baseline Model

Here we examine fiscal policy in the heterogeneous-household version of the negligible-marginal-cost (NMC) model in Murphy (2017). This version of the model features rich and poor households, denoted by $h \in \{\mathbb{R}, \mathbb{P}\}$, each of which receives different shares of income from the NMC sector and consumes services from the NMC sector. The model also features an endowment that is owned and consumed by the rich. The endowment represents land or other factors of production that are used to produce goods consumed primarily by the rich (e.g., beach homes and luxury items). The endowment pins down the interest rate and the consumption path of the rich household. Agents trade bonds to satisfy their desired time paths of consumption, subject to a no-Ponzi constraint that the present value of their asset position must be weakly greater than zero.

We evaluate policy responses to a one-time restriction in spending at date $t = 0$. Without loss of generality, we subsume all future periods into a single date $t = 1$. To facilitate derivation of analytical results, we assume that all uncertainty is resolved after the initial period.

2.1. Model

There is a unit mass of homogenous varieties in the NMC sector. Households inelastically supply labor to the NMC sector, and there are zero marginal costs of labor associated with increasing output.[3] In the initial period, a share $1 - \xi$ of the varieties is restricted from being sold.

Households. Household type h maximizes

[3] See Auerbach et al. (2020b) for an overview of the empirical relevance of negligible marginal labor costs.

$$U^h = \sum_{t=0}^{1} \beta^t \left(y_t^h + \int_0^{\psi_t} \int_0^{\xi_t} \left(\theta q_{jkt}^h - \frac{\gamma}{2} \left(q_{jkt}^h \right)^2 \right) dk\, dj \right), \qquad (1)$$

subject to the budget constraints

$$\int_0^{\psi_0} \int_0^{\xi_0} p_{j0} q_{j0}^h dkdj + y_0^h + QB = \Pi_0^h + e_0^h + T_0^h, \qquad (2)$$

$$\int_0^{\psi_1} \int_0^{\xi_1} p_{j1} q_{j1}^h dkdj + y_1^h = \Pi_1^h + e_1^h + T_1^h + B, \qquad (3)$$

where q_{jkt}^h is type h's consumption of variety $k \in [0,1]$ from firm $j \in [0,1]$ from the NMC sector in period t. The household's preferences are over each producer-commodity (jk) element. ξ_t is the fraction of goods that can be sold without restriction and $\psi_t \leq 1$ is the endogenously determined number of firms in the economy. We will assume that $\xi_t = 1$ and $\psi_t = 1$ in the absence of COVID-related restrictions, and that the restrictions imply $\xi_0 \equiv \xi < 1, \xi_1 = 1$ (and potentially $\psi_t < 1$).[4] Π_t^h is agent h's income from the NMC sector of the economy, e_t^h and $y_t^h$are h's endowment and consumption of the numeraire, where $e_t^{\mathbb{P}} = 0$. T_t^h is net transfers from the government. Q is the price of a bond B that pays a unit of the numeraire in period 1. Since agents can smooth consumption (and hence the effect of the present value of future net transfers is the same as the effect of present-period net transfers), we will write the present value of total net transfers as $T^h \equiv T_0^h + QT_1^h$.

A convenient feature of the quasilinear utility function is that agents consume only the good from the NMC sector when their income is sufficiently low (depending on θ and γ).[5] This feature, along with the assumption that poor agents are not endowed with the numeraire, $e_t^{\mathbb{P}} = 0 \; \forall \; t$, simplifies the analysis and maintains the focus on Keynesian-type multipliers in the NMC sector. We assume parameter values such that only the rich household consumes the numeraire endowment good. One implication of this assumption is that, similar to the Lucas-tree model, variation in endowments e pins down the interest rate Q to the discount factor β of the rich, that is

[4] Given the separability of preferences, shutting down access to any jkt element has symmetric effects on all other jkt elements and hence there are no changes in the composition of remaining commodities (and hence no *direct* demand spillover effects on unaffected producer-good commodities).

[5] While the rich households' marginal propensity to consume (MPC) on NMC goods is zero, their MPC that includes spending on the endowment e is equal to the poor households' MPC on NMC goods (the poor do not spend anything on endowment good e). Hence, the "total" MPC is the same for the poor and the rich.

$Q = 1/\beta$. This assumption is a reduced-form attempt to model the economy when interest rates are fixed at some level (for example, the effective lower bound).

Firms. Output in the NMC sector is produced by firms who hire workers as fixed costs and pay a fixed capital operating cost f_{jt}. Firm j faces demand for product jk from household type h

$$q_{jkt}^h = \frac{1}{\gamma}\left(\theta - \lambda_{ht} p_{jkt}^h\right),$$

where λ_{ht} is household h's budget multiplier at time t. Prices are flexible in each period. For analytic convenience, we assume that firms can price discriminate between the rich and the poor. The profit-maximizing price charged to household type h is

$$p_{jkt}^h = \frac{\theta}{2\lambda_{ht}}, \tag{4}$$

and resulting expenditure on the NMC sector is

$$c_{jkt}^h \equiv p_{jkt}^h q_{jkt}^h = \frac{\theta^2}{4\gamma\lambda_{ht}}.$$

The rich household's budget multiplier is pinned down by marginal utility of the numeraire, $\lambda_{rt} = 1$. Therefore its expenditure on any given firm is a function only of exogenous parameters and we treat $c_{jkt}^{\mathbb{R}} = \theta^2/4\gamma$ as exogenous for the remainder of the analysis.

A firm's revenues are equal to expenditure across households: $R_{jt} = \int_0^{\xi_t}\left(c_{jkt}^{\mathbb{R}} + c_{jkt}^{\mathbb{P}}\right)dk$. By symmetry of varieties, we can write $R_{jt} = \xi_t\left(c_{jkt}^{\mathbb{R}} + c_{jkt}^{\mathbb{P}}\right)$. Firm j pays a fixed capital operating cost f_{jt} in period t. We assume that households own capital in the same proportion to their share of firm profits and so we roll capital income into profits (i.e., Π includes profits and f_{jt}). A firm exits for period t if $R_{jt} < f_{jt}$. We assume that the distribution of fixed costs is such that the unit mass of firms all produce if $\xi = 1$ and that only a share $\psi_0(\xi) < 1$ continue to produce in the initial period if $\xi < 1$. If there are additional costs to re-entry once restrictions are lifted, then $\psi_1 < 1$. In the absence of such costs to re-entry, $\psi_1 = 1$.

The poor household receives a share κ of the revenues from the NMC sector in each period, while the rich household receives the remaining $1 - \kappa$ share. The poor household also owns a share κ of the capital stock (and therefore earns a share κ of the payments from firms for fixed capital operating costs). It can be shown that there exists a threshold value $\bar{\kappa}$ such that $\forall\ 0 < \kappa <$

$\bar{\kappa}$, the poor consume output only from the NMC sector. $\bar{\kappa}$ depends on model parameters and fiscal policy. We assume parameter values such that $\kappa < \bar{\kappa}$.

Equilibrium. Equilibrium consists of prices and quantities such that households maximize (1) subject to (2) and (3), firms' prices are given by (4), and ψ_t is determined by the number of firms for which revenues exceed fixed capital operating costs. Below we examine equilibrium under the condition that poor households consume only goods from the NMC sector. We also examine scenarios in which the capital market clears (capital operating costs are flexible) and when it does not (capital operating costs are rigid).

Proposition 1: Household consumption smoothing motives imply that variety-level consumption is equal across periods

$$q_{jk0}^{h} = q_{jk1}^{h}, \tag{5}$$

and expenditure on a variety across periods is related by

$$c_{jk0}^{h} = Qc_{jk1}^{h}. \tag{6}$$

Proof: Appendix.

The interesting aspects of the equilibrium are based on the expenditure of poor households (since the rich household's expenditure is effectively exogenous). Total expenditure by household h in period t is the sum of expenditure on the varieties. Given the assumptions about ξ_t, we can write

$$c_0^h = \int_0^{\psi_0}\int_0^{\xi_0} c_{jk0}^{h} dk\, dj \ = \psi_0 \xi c_{jk0}^{h}, \quad c_1^h = \int_0^{\psi_1}\int_0^{\xi_1} c_{jk1}^{h} dk\, dj \ = \psi_1 c_{jk1}^{h}. \tag{7}$$

Let $C^{\mathbb{P}}$ be the present value of the poor household's total lifetime expenditure. Then substituting (6) and (7) into (2) and (3) and simplifying implies that the present value of the poor household's total lifetime expenditure is

$$C^{\mathbb{P}} = c_0^{\mathbb{P}} + Qc_1^{\mathbb{P}} = \ \psi_0 \xi c_{jk0}^{\mathbb{P}} + \psi_1 c_{jk0}^{\mathbb{P}}. \tag{8}$$

To be clear, c_{jkt} represents the equilibrium level of spending, which is the same for any jk. To save notation, from now on, jk denotes spending on any variety. The poor household's lifetime income $I^{\mathbb{P}}$ is

$$I^{\mathbb{P}} = \kappa\psi_0\xi\left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}}\right) + Q\left(\kappa\psi_1\left(c_{jk1}^{\mathbb{P}} + c_{jk1}^{\mathbb{R}}\right)\right) + T^{\mathbb{P}}, \tag{9}$$

which reflects the fact that the poor household earns a share κ of total expenditure. Since households own capital in the same proportion to their share of firm profits and households (as firm owners) are both liable for firms' capital operating costs and receive income from payments to capital, capital costs and income are netted out of household income. Simplifying and solving for $c_{jk0}^{\mathbb{P}}$ yields

$$c_{jk0}^{\mathbb{P}} = \frac{\kappa}{1-\kappa} c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)}, \tag{10}$$

where we have substituted β for Q based on the rich household's first-order conditions with respect to the numeraire and the bond. The poor household's consumption of NMC goods is proportional to their ownership share κ (in the absence of transfers). The higher is κ, the more the poor household receives of every dollar spent, and the more they recycle back into further spending on NMC goods.

To solve for the poor household's total expenditure, we can substitute $c_0^{\mathbb{P}}/(\psi_0 \xi)$ for $c_{jk0}^{\mathbb{P}}$:

$$c_0^{\mathbb{P}} = \psi_0 \xi \left[\frac{\kappa}{1-\kappa} c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)} \right]. \tag{11}$$

From this, it is straightforward to write nominal GDP for the NMC sector in periods 0 and 1 as:

$$Y_0 = \psi_0 \xi c_{jk0}^{\mathbb{P}} + \psi_0 \xi c_{jk0}^{\mathbb{R}}, \quad Y_1 = \frac{\psi_1}{\beta} \left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}} \right). \tag{12}$$

At this point it is helpful to observe some important aspects of the model. First, output in both periods is increasing in κ (and hence falling in inequality). This is because a larger income share for the poor leads to higher spending, which drives up aggregate income and output. Since there is slack (negligible marginal costs) in the economy, this higher spending translates directly into higher output. Second, higher consumption by the rich in the initial period leads to a multiplier effect on output in both periods. As the rich spend more (e.g., due to increases in θ), the income of poor households increases. This increases their spending and income in a multiplier feedback loop. We summarize these observations in the following results:

Result 1 (*GDP in each period falling in inequality*). *Let* $(1-\kappa)/\kappa$ *– the ratio of the income share of the rich to the income share of the poor in the NMC sector - be a measure of inequality. Higher inequality causes lower GDP in the current and future periods:*

$$\frac{dY_0}{d\left(\frac{1-\kappa}{\kappa}\right)}|_{\xi=1,T^{\mathbb{P}}=0} = -\left(\frac{\kappa}{1-\kappa}\right)^2 c_{jk0}^{\mathbb{R}} < 0,$$
$$\frac{dY_1}{d\left(\frac{1-\kappa}{\kappa}\right)} = -\frac{1}{\beta}\left(\frac{\kappa}{1-\kappa}\right)^2 c_{jk0}^{\mathbb{R}} < 0, \tag{13}$$

where $\xi = 1 \Rightarrow \psi_0 = 1, \psi_1 = 1.$

Result 2 *(GDP in each period is increasing in desired spending by the rich). The effect of rich-household spending on GDP is increasing in the income share of the poor (κ):*

$$\left.\frac{dY_0}{dc_{jk0}^{\mathbb{R}}}\right|_{\xi=1} = \frac{1}{1-\kappa}, \qquad \left.\frac{dY_1}{dc_{jk0}^{\mathbb{R}}}\right|_{\xi=1} = \frac{1}{\beta}\frac{1}{1-\kappa}, \tag{14}$$

Therefore, a lower propensity to spend on NMC-sector goods by the rich or a rise in inequality is associated with large output multipliers and can cause a permanent slump.

NK vs. NMC frameworks

To draw contrast between the NMC framework and the mainstream New Keynesian (NK) approach, note that a simple way of capturing the mechanics of a New Keynesian model is to assume

$$Y_0^{NK} = C_0, \qquad Y_1^{NK} = \bar{Y}, \tag{15}$$

where the superscript indicates the New Keynesian representation of the model. Here, future output is determined by the endowment, reflecting the supply-side dominance of the New Keynesian models at horizons after which price rigidities have dissipated. To solve the model, one must simply determine C_0, which in general will be based on consumption smoothing and and intertemporal budget constraint. A simple version of consumption smoothing can be written as

$$C_0 = C_1, \tag{16}$$

and the budget constraint can be written (assuming $\beta = 1$) as

$$C_0 + C_1 = Y_0^{NK} + Y_1^{NK}, \tag{17}$$

Substituting the equilibrium conditions from (15) and solving for C_0 yields

$$C_0 = \frac{\bar{Y}}{2} \Rightarrow Y_0 = \frac{\bar{Y}}{2}, \tag{18}$$

Therefore, in the presence of consumption smoothing (the absence of credit constraints), output in the demand-determined period depends on the future supply side of the economy. In short, in the absence of credit constraints, the supply side dominates. As a result, credit constraints (and associated high MPCs) and the strength of intertemporal substitution is a key consideration for policymakers in thinking about the macroeconomic effect of the restrictions (e.g., Guerrieri et al. 2020). If policymakers are persuaded by recent evidence that many low-income households are not credit-constrained but rather have low MPCs (see, e.g., Miranda-Pinto et al. (2020) for a survey), or if they are persuaded by evidence that the elasticity of intertemporal substitution is well below unity (e.g., Cashin and Unayama 2016; Schmidt and Toda 2019), then policymakers may conclude that output effects of the restrictions are not a concern.

Now consider a situation in which future output is demand-determined as in the NMC framework. In this case, the equilibrium conditions can be written as

$$C_0 = \frac{1}{2}(Y_0 + Y_1), \tag{19}$$

where $Y_0 = C_0$ and $Y_1 = C_0$ (by consumption smoothing). Here, any level of desired consumption is a potential equilibrium. A unique equilibrium arises in setups featuring households with different income shares and a mix of sectors with demand-determined output (such as the model presented above) and supply-determined output. The unique equilibrium can support large spending multipliers (e.g., from government spending), since households' income is limited by their spending in both periods rather than by the supply side of the economy in a future period. Thus, with demand-driven output, the effects of social distancing and other constraints on the economy can be quite large under a less restrictive set of conditions.

Effects of COVID19 restrictions. The social distancing restrictions associated with COVID19 can be modeled as a decrease in ξ from an initial value of 1, reflecting the restrictions on the exchange of services such as restaurant meals, movie theaters, and sporting events. Let Y_t^U be NMC-sector output in the absence of COVID19 restrictions. Then

$$\frac{Y_0}{Y_0^U} = \psi_0 \xi \frac{\left(\frac{1}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)}}{\left(\frac{1}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{2(1-\kappa)}}, \tag{20}$$

with $\frac{Y_0}{Y_0^U} \to \psi_0 \xi$ as $T^{\mathbb{P}} \to 0$.

Consider first the effects of a decline in ξ in the absence of net transfers ($T^{\mathbb{P}} = 0$). If all firms survive (e.g., if $f_{j0} = 0 \; \forall \; j$), then the share of output lost is equal to the share of services that are restricted $(1-\xi)$. Because of demand-determined output, the decline in income by a share $1-\xi$ is balanced with an equal decline in spending. We refer to this as a restriction multiplier of unity.

If some firms exit due to revenues falling below fixed capital operating costs, then output falls by a multiple of the direct effect of the restriction (ξ) and the indirect effect of firm exits (ψ). Because $\psi \leq 1$ is (weakly) increasing in ξ, restricted output (as a fraction of counterfactual output) is $\psi\xi < \xi$ and the restriction multiplier is above 1. In particular, for any given distribution of the fixed capital operating cost f with probability distribution function $v(f)$, the effect of restrictions on the mass of firms is

$$\left.\frac{d\psi_0}{d\xi}\right|_{\xi=1} = \left(\frac{v(R_{jt})}{1+v(R_{jt})\mathrm{K}}\right)\left(\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{4(1-\kappa)}\right), \tag{21}$$

where $\mathrm{K} \equiv \frac{\xi T^{\mathbb{P}}}{(\psi_0\xi+\psi_1)^2(1-\kappa)}$, and R_{jt} is total spending on firm j's output; the derivation is provided in the Appendix. With $T^{\mathbb{P}} = 0$, $\left.\frac{d\psi_0}{d\xi}\right|_{\xi=1} = \left(\frac{v(R_{jt})}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} > 0$.

More generally, we can define the restriction multiplier as the ratio of the total effect of restrictions to the partial (direct) effect. We show in the Appendix that this ratio evaluated around $\psi_1 = 1, \xi = 1, T^{\mathbb{P}} = 0$ is

$$\left.\frac{dY_0/d\xi}{\partial Y_0/\partial\xi}\right|_{\xi=1,T^{\mathbb{P}}=0} = \frac{\psi_0 + \xi(d\psi_0/d\xi)}{\psi_0} = 1 + \frac{\xi}{\psi_0}\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} > 1. \tag{22}$$

The total effect exceeds the direct effect only if $d\psi_0/d\xi > 0$, that is, if some firms are forced to exit as a result of the lower revenues. Note that the relative size of the total multiplier increases in spending by the rich (e.g., $c_{jk0}^{\mathbb{R}}$ can increase because the rich have a stronger preference for NMC goods θ) and the share of income that goes to the poor κ (lower inequality). Specifically, using (21) we find

$$\begin{aligned}\left.\frac{d^2Y_0}{d\xi dc_{jk0}^{\mathbb{R}}}\right|_{T^{\mathbb{P}}=0} &= \frac{1}{1-\kappa}\left[\left(\psi_0 + \frac{\xi d\psi_0}{d\xi}\right) + c_{jk0}^{\mathbb{R}}\left(2\frac{d\psi_0}{dc_{jk0}^{\mathbb{R}}} + \frac{\xi d^2\psi_0}{d\xi dc_{jk0}^{\mathbb{R}}}\right)\right] > 0,\\ \left.\frac{d^2Y_0}{d\xi d\kappa}\right|_{T^{\mathbb{P}}=0} &= \frac{1}{1-\kappa}c_{jk0}^{\mathbb{R}}\left[\left(\psi_0 + \frac{\xi d\psi_0}{d\xi}\right)\frac{1}{1-\kappa} + \left(2\frac{d\psi_0}{d\kappa} + \frac{\xi d^2\psi_0}{d\xi d\kappa}\right)\right] > 0.\end{aligned} \tag{23}$$

In general ($T^{\mathbb{P}} \neq 0$), the size of the total effect will depend on the joint distribution of firm sales and operating capital costs. The firms most susceptible to exit are those with large operating costs as a share of revenues. These may include smaller businesses for which the leasing of space is particularly important. It may also include highly levered firms for which debt payments are large relative to revenues. We summarize this observation in Result 3:

Result 3 *(The output effects of COVID19 restrictions can be large). In the absence of a firm exit margin, the decline in output is proportional to the share of products that are restricted (the restriction multiplier is unity). Firm exit causes a larger fall in output – a restriction multiplier greater than unity.*

Firm exit is a result of rigid costs of operating capital. If the capital market is flexible, then the rate will adjust so that the rental rate equals the revenues of the marginal firm and (given an inelastic supply of capital) in equilibrium there would remain a unit mass of firms.[6]

The effect of restrictions on future output is

$$\frac{dY_1}{d\xi} = \frac{1}{\beta}\left(\frac{1}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} \frac{d\psi_1}{d\xi} - \frac{T^{\mathbb{P}}}{(1-\kappa)(\psi_0 \xi + \psi_1)^2}\left(\psi_0 + \xi \frac{d\psi_0}{d\xi} + \frac{d\psi_1}{d\xi}\right). \qquad (24)$$

Somewhat surprisingly, the restriction can increase future output if the poor household receives net transfers and if most firms produce in the future ($\psi_1 \approx 1$ and so $\frac{d\psi_1}{d\xi} \approx 0$). In that case, there is an economic expansion between periods 0 and 1 that exceeds the amount of output lost in period 0. This future-period expansion is due to the fact that the poor household smooths its government transfers across varieties *and* across time. In the initial period, there are fewer goods to buy, so the household spends less of the transfer wealth in the initial period and it spends relatively more in the future period when more goods are available.

Result 4 *(Future output expands in response to positive transfers): If low-income households receive positive net transfers (or have other forms of wealth), then the economy will expand in the future. Future output can exceed what it would have been in the absence of COVID19 restrictions (e.g., if costs to re-entry are not too large).*

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[6] There are plenty of reasons to expect that capital costs may not be flexible, at least in the short run. Asymmetric information between capital owners and the firms that rent the capital is among the reasons for rigid capital prices. If capital is imperfectly substitutable such that owners have pricing power, then capital owners may be reluctant to adjust if they cannot identify which firms can pay and which cannot. Indeed, recent empirical evidence documents a strong role for asymmetric information in preventing renegotiations between capital owners and firms even when such renegotiations would otherwise benefit both (Gilje et al 2020).

The more firms exit temporarily in period 0 (and hence the larger the indirect effect in period 0), the larger will be the future-period boom because fewer products are available in period 0 and so households tilt their spending more toward the future. However, various factors can mitigate or offset the future-period boom. First, an increase in inequality (a decline in κ) causes a decline in future-period output.[7] Second, high-income households might adjust their spending.[8] Finally, there may be permanent firm exit ($\psi_1 < 1$).[9] We do not explicitly model these forces but rather highlight the effects implied by the model.

2.2. Fiscal Policy

Government transfers to households and/or firms can mitigate the adverse effects of the restrictions. The effect of different transfers depends on how they are financed. From equations (11) and (12) it is clear that taxing low-income households (which pulls down $T^{\mathbb{P}}$) will reduce GDP (all else equal). An alternative source of funding is to exclusively tax the rich. As long as the rich maintain enough post-tax consumption of the numeraire, there will be no effect of this taxation on GDP in the NMC sector for either period.[10] There is also the possibility that the transfers could be money financed through the central bank (Gali 2019).[11] In our model this would have the same effect as taxing the rich. For the remainder of the analysis we assume that transfers

[7] This is because inequality is associated with lower permanent income of low-income households (and therefore lower spending in the future). Higher inequality could arise if COVID19 restrictions accelerate the substitution of technology for low-income workers, if there are scarring effects from unemployment that lead to long-term declines in income shares, or if small businesses are acquired by larger businesses in a way that alters the earnings distribution, for example.

[8] While the model does not predict such an adjustment, it could nonetheless result from a pessimistic view of the future or a change in consumption preferences arising from lifestyle adjustments in the initial period. A reduction in high-income households' spending would reduce low-income households' permanent income (through an effect similar to that of higher inequality), causing a permanent decline in spending and GDP.

[9] Large entry costs can prevent re-entry and decrease future output in response to initial-period restrictions.

[10] In general, the market for the endowment good clears even with taxes and transfers and no change in its price. For example, when the government taxes the endowment of the rich, the taxed portion eventually ends back in the hands of the rich as poor households spend the transfer on the NMC sector. If a poor household is given a dollar in transfers, it will spend the dollar on NMC goods. $1-\kappa$ share of the dollar will become income of the rich (who will spend it on the endowment good) while κ share will become income of the poor. This "second-round" income of the poor will be spent on the NMC goods again so that $(1-\kappa)\kappa$ will become income of the rich and κ^2 will become income of the poor. These rounds of spending will continue and, in the end, the rich will get their $1dollar in taxes back in income $(1-\kappa)+(1-\kappa)\kappa+(1-\kappa)\kappa^2+\cdots=1$ which they spend on the endowment good.

[11] The government could finance transfers with money if they had a technology to create the numeraire. Alternatively, one could interpret the numeraire as money (which the government can print).

are financed either through taxing the rich or through money, and we will examine the relative effectiveness of different types of spending.[12]

Transfers to Households. Consider first transfers to low-income households. The effect on GDP is

$$\begin{aligned}\frac{dY_0}{dT^{\mathbb{P}}} &= \frac{\psi_0\xi}{(\psi_0\xi+\psi_1)(1-\kappa)} - \frac{T^{\mathbb{P}}}{(\psi_0\xi+\psi_1)^2(1-\kappa)}\left(\xi\frac{d\psi_0}{dT^{\mathbb{P}}}+\frac{d\psi_1}{dT^{\mathbb{P}}}\right) \\ &\quad + \left[\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0\xi+\psi_1)(1-\kappa)}\right]\xi\frac{d\psi_0}{dT^{\mathbb{P}}}, \\ \frac{dY_1}{dT^{\mathbb{P}}} &= \frac{1}{\beta(1+\xi)(1-\kappa)} - \frac{T^{\mathbb{P}}}{(\psi_0\xi+\psi_1)^2(1-\kappa)}\left(\xi\frac{d\psi_0}{dT^{\mathbb{P}}}+\frac{d\psi_1}{dT^{\mathbb{P}}}\right) \\ &\quad + \left[\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0\xi+\psi_1)(1-\kappa)}\right]\xi\frac{d\psi_1}{dT^{\mathbb{P}}}.\end{aligned} \tag{25}$$

Transfers to low-income households of sufficient size can in principle fully offset secondary economic effects of the COVID-related restrictions. Transfers stimulate output through two channels. First, they increase spending on existing firms. Second, they induce firm entry, and this entry causes additional private-sector spending on the products of the entering firms. This firm entry margin is consistent with recent empirical evidence of the effects of fiscal stimulus (Auerbach et al. 2020b).

Result 5 *(Countercyclical effects of transfers): Transfers stimulate firm entry, and this firm entry margin leads to large total output effects.*

Under some circumstances, transfers can have larger effects on future output than they do on initial-period output. Fiscal policy is less stimulative in the initial period (when $\xi < 1$) *for a given mass of firms* (i.e., holding $\frac{d\psi_0}{dT^{\mathbb{P}}} = 0$), a reflection of the fact that there are fewer products on which to spend in the initial period. Endogenous firm entry has two counteracting effects on the transfer multiplier. It pushes up the multiplier, as higher transfers induce firm entry. But is also pushes down the multiplier, as the decline in ξ also is associated with a decline in ψ_0, and low ψ_0

[12] It might seem that an alternative policy is for the government to lend to poor households. However, since the households in this environment are already able to smooth their consumption, the lending has no effect. Therefore, one can think of our model as an environment in which monetary policy has extended credit to households to an extent that is sufficient for them to smooth consumption. The benefits of fiscal transfers are evaluated above and beyond the credit-enhancing benefits of monetary policy.

implies that there are even fewer products on which to spend in the current period. The net effect of transfers (accounting for endogenous firm entry) is

$$\left.\frac{dY_0}{dT^{\mathbb{P}}}\right|_{T^{\mathbb{P}}=0} = \frac{\xi}{(\psi_0\xi+\psi_1)(1-\kappa)}\left[\psi_0 + \frac{v(R_{j0})}{1-\kappa}c_{jk0}^{\mathbb{R}}\right]. \tag{26}$$

The effect of transfers is falling in inequality (derivation in Appendix): the smaller is the income share of low-income households, the less spending circulates back as income to low-income households (and hence the less they can spend).

Result 6 *(Fiscal multipliers and inequality): The fiscal transfer multiplier is falling in inequality (rising in the income share of the poor κ) under the sufficient condition of* $v'(R_{j0}) \geq 0$.

Transfers to firms. An alternative to household-level transfers is to provide transfers $T^{\mathbb{F}}$ to firms. In general, transfers to firms have the benefit of mitigating firm exit (if there are fixed operating costs). The downside of firm-level transfers is that low-income households (which drive spending multipliers) only end up with a share of the transfer κ. In the absence of a firm exit margin, household-level transfers would be more effective. But the possibility of firm exit implies potentially large benefits of firm-level transfers.

The most effective form of firm-level transfers are those that are targeted to marginal firms, i.e., the firms for which fixed costs are a large share of their revenues and these firms are on the margin to exit. If the government can target such firms, the extra multiplier from targeted transfers $T^{\mathbb{F}:Target}$ (relative to untargeted firm-level transfers $T^{\mathbb{F}:All}$) is (see the Appendix for derivations)

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} - \frac{dY_0}{dT^{\mathbb{F}:All}} = \left(\kappa\frac{dY_0}{dT^{\mathbb{P}}} + \frac{\partial Y_0}{\partial\psi_0}\frac{1}{v(R_{jt})}\right) - \left(\kappa\frac{dY_0}{dT^{\mathbb{P}}}\right) = \frac{\partial Y_0}{\partial\psi_0}\frac{1}{v(R_{jt})}, \tag{27}$$

where

$$\left.\frac{\partial Y_0}{\partial\psi_0}\right|_{T^{\mathbb{F}}=0} = \left.R_{jt}\right|_{T^{\mathbb{F}}=0} = \frac{\xi}{1-\kappa}. \tag{28}$$

For example, if fixed costs are uniformly distributed, the marginal targeted tax dollar creates R_{jt} additional units of GDP compared to the marginal untargeted tax dollar![13] Small businesses are

[13] An alternative approach to evaluating the net benefit of targeted transfers is to derive the relative amount of each type of spending such that the restriction multiplier does not exceed unity. In the case of a uniform distribution of fixed costs, the amount saved by targeting is $T^{\mathbb{F}:All} - T^{\mathbb{F}:Target} = c_{j0}^{\mathbb{R}}\left[\frac{(\xi+1)(\xi-1)}{\kappa} - \frac{c_{j0}^{\mathbb{R}}}{2(1-\kappa)}(1-\xi^2)^2\right]$.

likely to be particularly prone to exit, therefore implying an important role of fiscal transfers to firms. Furthermore, higher inequality is associated with a larger net benefit from targeted transfers, as spending multiplier are increasing in inequality.

The relative benefit (in terms of GDP per dollar spent) of transfers to low-income households versus targeted transfers to firms depends on how many firms are kept afloat with each dollar spent. In this sense, the benefits of targeted transfers to firms are proportional to the indirect costs of the COVID19 restrictions. If there are large restriction multipliers (based on the joint distribution of fixed capital costs and firm revenues), then the relative benefits of targeted transfers are large and these benefits could be even larger if there are costs of reentry.

Result 7 *(The optimal composition of transfers): Targeted transfers to firms are the most cost-effective means of mitigating a restriction multiplier above unity. The relative benefit of targeted transfers depends on the joint distribution of firm revenues and capital operating costs. The relative benefit is also higher the greater is the income share of the poor (as equation (28) is increasing in* κ*;* $\left.\frac{\partial^2 Y_0}{\partial \psi_0 \partial \kappa}\right|_{T^{\mathbb{F}}=0} = \frac{v(R_{jt})\xi}{(1-\kappa)^2}$*)* .

An alternative policy to firm-level transfers is government loans to firms. But firms still need to cover their future-period fixed costs. Firms for which the present value revenues in both periods falls below the present value of fixed costs will not be helped by loans (specifically, ψ_0 and ψ_1 can fall below 1 even if the government offers loans). Loans are only effective for the firms that cannot cover their fixed costs in the initial period but nonetheless earn profits in present value.

3. Conclusion

The fiscal policy response to the COVID19 crisis to date has been a patchwork of transfers to households and transfers to firms. Ex ante it might seem that there are unnecessary or costly redundancies in this mix of policies. Our paper offers a rationale for some degree of combining transfers to households with transfers to firms. When the extent of fixed operating costs and the extent of low-income-household exposure to fixed operating costs are unclear, a mix of transfers to households and firms can address both channels through which COVID19 restrictions generate large indirect multiplier effects.

Our framework indicates a number of metrics that will be useful to monitor as the COVID19 crisis evolves. In the absence of rising inequality or reductions in spending by high-income households, GDP will rebound to a level beyond what it would have been in the absence of COVID. Rising inequality or reductions in spending by high-income households can mitigate this boom or cause a prolonged slump. Fiscal stimulus will be especially useful in the event of a slump, although its effect per dollar spent is decreasing in inequality.

Other important metrics include the prices of firms' operating capital, especially for firms that have large fixed operating costs relative to revenues and for multiproduct firms. Downward adjustment of capital prices can mitigate large restriction multipliers. Perhaps most surprisingly, monitoring wage and product price adjustments may be less relevant for understanding multiplier effects than monitoring capital price adjustments, since aggregate demand externalities can be present even if wages and output prices are flexible.

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Appendix

Proof of Proposition 1:

A household's first-order condition with respect to variety j can be written (omitting household superscripts) as

$$\theta - \gamma q_{jkt}^{h} = \lambda_t^h p_{jht},$$

which implies the following relationship across periods:

$$\frac{\theta - \gamma q_{jk0}^{h}}{\theta - \gamma q_{jk1}^{h}} = \frac{\lambda_0^h p_{jk0}^h}{\lambda_1^h p_{jk1}^h}.$$

Substituting in $p_{jkt}^h = \theta/2\lambda_t^h$, we have

$$\frac{\theta - \gamma q_{jk0}^{h}}{\theta - \gamma q_{jk1}^{h}} = 1,$$

which implies $q_{jk0}^h = q_{jk1}^h$. We can then write $\frac{c_{jk0}^h}{c_{jk1}^h} = \frac{p_{jk0}^h q_{jk0}^h}{p_{jk1}^h q_{jk1}^h} = \frac{\lambda_1^h}{\lambda_0^h}$. The household's first-order condition with respect to the bond implies that $\lambda_0^h Q = \lambda_1^h$, from which it follows that $c_{jk0}^h = Q c_{jk1}^h$. ■

Derivation of Equations (11) through (20):

Setting $C^{\mathbb{P}} = I^{\mathbb{P}}$ implies

$$\psi_0 \xi c_{jk0}^{\mathbb{P}} + \psi_1 c_{jk0}^{\mathbb{P}} = \kappa \psi_0 \xi \left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}}\right) + Q\left(\kappa \psi_1\left(c_{jk1}^{\mathbb{P}} + c_{jk1}^{\mathbb{R}}\right)\right) + T^{\mathbb{P}}$$

Substitute in $c_{jk1}^{\mathbb{P}} = \frac{c_{jk0}^{\mathbb{P}}}{Q}$, $c_{j1}^{\mathbb{R}} = \frac{c_{jk0}^{\mathbb{R}}}{Q}$ and simplify:

$$c_{jk0}^{\mathbb{P}}(\psi_0 \xi + \psi_1) = \kappa(\psi_0 \xi + \psi_1)\left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}}\right) + T^{\mathbb{P}}$$

$$c_{jk0}^{\mathbb{P}}(\psi_0 \xi + \psi_1)(1-\kappa) = \kappa(\psi_0 \xi + \psi_1) c_{jk0}^{\mathbb{R}} + T^{\mathbb{P}}$$

$$c_{jk0}^{\mathbb{P}} = \frac{\kappa}{1-\kappa} c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)},$$

Now substitute in $c_0^{\mathbb{P}} = \psi_0 \xi c_{jk0}^{\mathbb{P}}$ to arrive at (11):

$$c_0^{\mathbb{P}} = \psi_0 \xi \left[\frac{\kappa}{1-\kappa} c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)}\right].$$

Equation (12) follows from $Y_0 = c_0^{\mathbb{P}} + c_0^{\mathbb{R}} = \psi_0 \xi\left(c_{jk0}^{\mathbb{R}} + c_{jk0}^{\mathbb{P}}\right)$ and $Y_1 = \psi_1\left(c_{jk1}^{\mathbb{R}} + c_{jk1}^{\mathbb{P}}\right)$. In particular,

$$Y_0 = \psi_0 \xi \left[\left(1 + \frac{\kappa}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)}\right]$$

$$Y_1 = \frac{\psi_1}{\beta}\left(1 + \frac{\kappa}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)}$$

The unrestricted level of output in the initial period is based on Y_0 from above but with ψ_0, ψ_1, and ξ set equal to 1.

$$Y_0^U = \left[\left(\frac{1}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{2(1-\kappa)}\right].$$

Equation (20) follows directly.

The total effect of the restrictions can be obtained from $dY_0/d\xi$:

$$\frac{dY_0}{d\xi} = \left[\left(\frac{1}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)}\right]\left(\frac{\psi_0 d\xi}{d\xi} + \frac{\xi d\psi_0}{d\xi}\right) - \frac{\xi \psi_0 T^{\mathbb{P}}}{(1-\kappa)\left(\psi_0 \xi + \psi_1\right)^2}\left(\frac{\psi_0 d\xi}{d\xi} + \frac{\xi d\psi_0}{d\xi} + \frac{d\psi_1}{d\xi}\right)$$

The partial (direct) effect of the restrictions (i.e., holding ψ_0 constant) can be written as

$$\frac{\partial Y_0}{\partial \xi} = \psi_0 \left[\left(\frac{1}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0 \xi + \psi_1)(1-\kappa)}\right] - \frac{\xi \psi_0^2 T^{\mathbb{P}}}{(1-\kappa)\left(\psi_0 \xi + \psi_1\right)^2}$$

Evaluating these effects around $\psi = 1, \xi = 1, T^{\mathbb{P}} = 0$ implies that

$$\left.\frac{\frac{dY_0}{d\xi}}{\frac{\partial Y_0}{\partial \xi}}\right|_{T^{\mathbb{P}}=0} = \frac{\left(\psi_0 + \xi \frac{d\psi_0}{d\xi}\right)}{\psi_0}.$$

The effect of restriction on future output is

$$\frac{dY_1}{d\xi} = \frac{1}{\beta}\left(\frac{1}{1-\kappa}\right) c_{jk0}^{\mathbb{R}} \frac{d\psi_1}{d\xi} - \frac{T^{\mathbb{P}}}{(1-\kappa)\left(\psi_0 \xi + \psi_1\right)^2}\left(\frac{\psi_0}{d\xi} + \xi \frac{d\psi_0}{d\xi} + \frac{d\psi_1}{d\xi}\right).$$

The response of firm entry:

Let the PDF of the distribution of f be v and the CDF be V

$$\psi_0 = \int_0^{R_{jt}} v(f) df = V\left(R_{jt}\right)$$

Then

$$d\psi_0 = v\left(R_{jt}\right) dR_{jt}$$

and

$$dR_{jt} = \left(\frac{c_{jkt}^{\mathbb{R}}}{1-\kappa} + \frac{T^{\mathbb{P}}}{(\psi_0\xi + \psi_1)(1-\kappa)}\right)d\xi - \frac{\xi T^{\mathbb{P}}}{(\psi_0\xi + \psi_1)^2(1-\kappa)}(\xi d\psi_0 + \psi_0 d\xi + d\psi_1) + \frac{\xi}{(\psi_0\xi + \psi_1)(1-\kappa)}dT^{P}$$

imply

$$d\psi_0 = \frac{v(R_{jt})}{1 + v(R_{jt})\mathrm{K}}\left[\frac{\xi}{(\psi_0\xi + \psi_1)(1-\kappa)}dT^{\mathbb{P}} + (R_{jk0} - \mathrm{K}\psi_0)d\xi - \mathrm{K}d\psi_1\right]$$

where $\mathrm{K} \equiv \frac{\xi T^{\mathbb{P}}}{(\psi_0\xi+\psi_1)^2(1-\kappa)}$.

Hence, holding fixed $T^{\mathbb{P}}$ and for simplicity assuming that $d\psi_1 = 0$, we find:

$$\frac{d\psi_0}{d\xi} = \left(\frac{v(R_{jt})}{1 + v(R_{jt})\mathrm{K}}\right)(R_{jk0} - \mathrm{K}\xi)d\xi$$

$$\frac{d\psi_0}{d\xi} = \left(\frac{v(R_{jt})}{1 + v(R_{jt})\mathrm{K}}\right)\left(\left[\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0\xi + \psi_1)(1-\kappa)}\right] - \frac{\xi T^{\mathbb{P}}}{(\psi_0\xi + \psi_1)^2(1-\kappa)}\psi_0\right)d\xi$$

$$\frac{d\psi_0}{d\xi} = \left(\frac{v(R_{jt})}{1 + v(R_{jt})\mathrm{K}}\right)\left(\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0\xi + \psi_1)(1-\kappa)}\left(\frac{\psi_0\xi + \psi_1 - \psi_0\xi}{(\psi_0\xi + \psi_1)}\right)\right)$$

$$\frac{d\psi_0}{d\xi} = \left(\frac{v(R_{jt})}{1 + v(R_{jt})\mathrm{K}}\right)\left(\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{(\psi_0\xi + \psi_1)(1-\kappa)}\left(\frac{\psi_1}{(\psi_0\xi + \psi_1)}\right)\right) > 0$$

If we evaluate around $\xi = 1, \psi = 1$, then

$$\left.\frac{d\psi_0}{d\xi}\right|_{\xi=1} = \left(\frac{v(R_{jt})}{1 + v(R_{jt})\mathrm{K}}\right)\left(\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}} + \frac{T^{\mathbb{P}}}{4(1-\kappa)}\right)$$

Effect of transfers on entry.

The effect of transfers on entry is (assuming $d\xi = 0$ and for simplicity also assuming $d\psi_1 = 0$):

$$\frac{d\psi_0}{dT^{\mathbb{P}}} = \left(\frac{v(R_{jt})}{1 + v(R_{jt})\mathrm{K}}\right)\frac{\xi}{(\psi_0\xi + \psi_1)(1-\kappa)}$$

Therefore, the total effect of transfers on GDP is

$$\left.\frac{dY_0}{dT^{\mathbb{P}}}\right|_{T^{\mathbb{P}}=0} = \frac{\psi_0\xi}{(\psi_0\xi + \psi_1)(1-\kappa)} + \left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}}\xi\frac{\partial\psi_0}{\partial T_0}$$

$$\left.\frac{dY_0}{dT^{\mathbb{P}}}\right|_{T^{\mathbb{P}}=0} = \frac{\psi_0\xi}{(\psi_0\xi + \psi_1)(1-\kappa)} + \left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}}\left(\frac{v(R_{jt})}{1 + v(R_{jt})\mathrm{K}}\right)\frac{\xi^2}{(\psi_0\xi + \psi_1)(1-\kappa)}$$

$$\frac{dY}{dT^{\mathbb{P}}}|_{T^{\mathbb{P}}=0} = \frac{\xi}{(\psi_0\xi+\psi_1)(1-\kappa)}\left[\psi_0 + \xi\left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}}\left(\frac{v(R_{jt})}{1+v(R_{jt})\mathrm{K}}\right)\right]$$

$$= \frac{\xi}{(\psi_0\xi+\psi_1)(1-\kappa)}\left[\psi_0 + \xi\left(\frac{v(R_{jt})}{1-\kappa}\right)c_{jk0}^{\mathbb{R}}\right]$$

because $K = 0$ at $T^{\mathbb{P}} = 0$.

This effect of transfers is increasing in κ (falling in inequality):

$$\left.\frac{d^2Y_0}{dT^{\mathbb{P}}}\right|_{T^{\mathbb{P}}=0} = \frac{\xi}{(\psi_0\xi+\psi_1)(1-\kappa)}\left[d\psi_0 + c_{jk0}^{\mathbb{R}}\xi\left(\frac{v(R_{jt})}{(1-\kappa)^2}d\kappa + \frac{v'(R_{jt})}{1-\kappa}dR_{jt}\right)\right]$$
$$+\,\xi\left[\psi_0 + \frac{v(R_{jt})\xi}{1-\kappa}c_{jk0}^{\mathbb{R}}\right]\left(\frac{1}{(\psi_0\xi+\psi_1)(1-\kappa)^2}d\kappa + \frac{\xi d\psi_0 + d\psi_1}{(\psi_0\xi+\psi_1)^2(1-\kappa)}\right)$$

$$\left.\frac{d^2Y_0}{dT^{\mathbb{P}}d\kappa}\right|_{T^{\mathbb{P}}=0} = \frac{\xi}{(\psi_0\xi+\psi_1)(1-\kappa)}\left[\frac{d\psi_0}{d\kappa} + c_{jk0}^{\mathbb{R}}\frac{\xi}{(1-\kappa)^2} + \frac{v'(R_{jt})}{1-\kappa}\frac{dR_{jt}}{d\kappa}\right]$$
$$+\,\xi\left[\psi_0 + \frac{v(R_{jt})\xi}{1-\kappa}c_{jk0}^{\mathbb{R}}\right]\left(\frac{1}{(\psi_0\xi+\psi_1)(1-\kappa)^2} + \frac{\frac{\xi d\psi_0}{d\kappa} + \frac{d\psi_1}{d\kappa}}{(\psi_0\xi+\psi_1)^2(1-\kappa)}\right)$$

$\left.\frac{d^2Y_0}{dT^{\mathbb{P}}d\kappa}\right|_{T^{\mathbb{P}}=0}$ is guaranteed to be positive if $v'(R_{jt}) \geq 0$. For example, $\left.\frac{d^2Y_0}{dT^{\mathbb{P}}d\kappa}\right|_{T^{\mathbb{P}}=0} > 0$ if f_j is uniformly distributed.

Effect of Targeted Firm-Level Transfers

For each dollar targeted to marginal firms, the government would create $d\psi_0 = \frac{1}{v(R_{jt})}$ firms. Equivalently, if fixed costs for the marginal firm is $v(R_{jt})$, the government must spend that amount to keep them alive. So $\frac{d\psi_0}{dT^{Target}} = \frac{1}{v(R_{jt})}$.

$\kappa dT^{\mathbb{F}:Target}$ would also be transferred to households (as they own a share κ of capital).

Therefore

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} = \frac{\partial Y_0}{\partial T^{\mathbb{P}}}\frac{dT^{\mathbb{P}}}{dT^{\mathbb{F}:Target}} + \frac{\partial Y_0}{\partial \psi_0}\frac{d\psi_0}{dT^{\mathbb{F}:Target}}$$

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} = \kappa\frac{dY_0}{dT^{\mathbb{P}}} + \frac{\partial Y_0}{\partial \psi_0}\frac{1}{v(R_{jt})}$$

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} = \kappa\frac{dY_0}{dT^{\mathbb{P}}} + \frac{\partial Y_0}{\partial \psi_0}\frac{1}{v(R_{jt})}$$

If the government could not target firms – but rather spent across all firms, it would need create only

$$\frac{dY_0}{dT^{\mathbb{F}:Firms}} = \frac{\partial Y_0}{\partial T^{\mathbb{P}}}\frac{dT^{\mathbb{P}}}{dT^{\mathbb{F}:Firms}} = \kappa\frac{dY_0}{dT^{\mathbb{P}}}.$$

Targeted firm transfers have an additional multiplier effect given by

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} - \frac{dY_0}{dT^{\mathbb{F}:Firms}} = \frac{\partial Y_0}{\partial \psi_0}\frac{1}{v(R_{jt})},$$

where

$$\left.\frac{\partial Y_0}{\partial \psi_0}\right|_{T^{\mathbb{P}}=0} = R_{jt}|_{T^{\mathbb{P}}=0} = \frac{\xi}{1-\kappa}c_{j0}^{\mathbb{R}}.$$

So if fixed costs are uniformly distributed, the marginal targeted tax dollar creates R_{jt} additional units of GDP compared to the marginal untargeted tax dollar.

Another way to compare measure the relative benefit of targeted dollars is by determining how much of targeted versus untargeted dollars would need to be spent to prevent a spending multiplier greater than unity. The government must spend $T^{\mathbb{F}:All}$ such that

$$Y_0|_{\psi_0=1} = Y_0|_{T=0}$$

$$\xi\left[\left(\frac{1}{1-\kappa}\right)c_{j0}^{\mathbb{R}} + \frac{\kappa T^{\mathbb{F}:All}}{(\xi+1)(1-\kappa)}\right] = \left(\frac{1}{1-\kappa}\right)c_{jk0}^{\mathbb{R}}\psi_0\xi$$

$$T^{\mathbb{F}:All} = \left(\frac{\xi+1}{\kappa}\right)c_{jk0}^{\mathbb{R}}(1-\psi_0)$$

With respect to targeted dollars, the government would spend

$$T^{\mathbb{F}:Target} = \int_{R_{jt}}^{\bar{f}}(f - R_{jt})v(f)df$$

If f is uniformly distributed, then

$$T^{\mathbb{F}:Target} = \frac{1}{2}\left(\bar{f} - R_{jt}\right)^2$$

Let $\bar{f}$ be $R_{jt}|_{T=0,\xi=1}$. Then

$$T^{\mathbb{F}:Target} = \frac{1}{2}\left(\frac{1}{1-\kappa}c_{j0}^{\mathbb{R}}(1-\xi\psi_0)\right)^2$$

This transfer would be paid to capital, a fraction κ of which would be remitted to low-income households. The government could then tax low-income households by this amount to prevent additional multiplier effects (and hence prevent additional spending on unrestricted items).

Therefore, to ensure that output falls by only a share $1-\xi$, the government would spend

$$T^{\mathbb{F}:Target} = \frac{1}{2}\left(\frac{1}{1-\kappa} c_{j0}^{\mathbb{R}} (1-\xi\psi_0)\right)^2 (1-\kappa)$$

The amount saved by targeting is

$$\begin{aligned} T^{\mathbb{F}:All} - T^{\mathbb{F}:Target} &= \left(\frac{\xi+1}{\kappa}\right) c_{j0}^{\mathbb{R}} (1-\xi) - \frac{1}{2}\left(\frac{1}{1-\kappa} c_{j0}^{\mathbb{R}} (1-\xi\psi_0)\right)^2 (1-\kappa) \\ &= c_{j0}^{\mathbb{R}} \left[\frac{(\xi+1)(\xi-1)}{\kappa} - \frac{c_{j0}^{\mathbb{R}}}{2(1-\kappa)} (1-\xi^2)^2\right] \end{aligned}$$

The stronger is inequality, the larger is this difference.