NBER WORKING PAPER SERIES

MONETARY POLICY WITH OPINIONATED MARKETS

Ricardo J. Caballero
Alp Simsek

Working Paper 27313
http://www.nber.org/papers/w27313

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2020, Revised December 2020

We thank Chris Ackerman, Marios Angeletos, Ben Bernanke, Jonathan Brownrigg, Anthony Diercks, Tishara Garg, Ozer Karagedikli, Arvind Krishnamurthy, Yueran Ma, Guillermo Ordonez, Alexi Savov, Jaume Ventura, Annette Vissing-Jorgensen, Christian Wolf, and seminar participants at Bilkent, Bocconi, Chicago, Columbia, CREI, Dallas Fed, Duke, ECB, NBER SI, NYU Stern, Richmond Fed, Stanford GSB, and Yale SOM, for their comments. Caballero and Simsek acknowledge support from the National Science Foundation (NSF) under Grant Numbers SES-1848857 and SES-1455319, respectively. First draft: 10/07/2019. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Monetary Policy with Opinionated Markets
Ricardo J. Caballero and Alp Simsek
NBER Working Paper No. 27313
June 2020, Revised December 2020
JEL No. E00,E12,E21,E32,E43,E44,G11,G12

ABSTRACT

Central banks (the Fed) and markets (the market) often disagree about the path of interest rates. We develop a model where these different views stem from disagreements between the Fed and the market about future aggregate demand. We then study the implications of these disagreements for monetary policy, the term structure of interest rates, and economic activity. In our model, agents learn from the data but not from each other—they are opinionated. In this context, the market perceives monetary policy "mistakes" and the Fed partially accommodates the market's view to mitigate the impact of perceived "mistakes" on output and inflation. The Fed plans to implement its own view gradually, as it expects the market to receive more information and move closer to the Fed's belief. Disagreements about future demand, together with learning, translate into disagreements about future interest rates. Disagreements also provide a microfoundation for monetary policy shocks: after a surprise policy announcement, the market (partially) learns the Fed's belief and the extent of future "mistaken" interest rate changes. We categorize these shocks into three groups: Fed belief shocks, market reaction shocks, and tantrum shocks, and analyze their impact on forward interest rates and economic activity. Tantrum shocks are the most damaging, as they arise when the Fed fails to forecast the forward rates' reaction. These shocks motivate additional gradualism as well as communication policies that reveal the Fed's belief, not to persuade the market (which is opinionated) but to prevent a misinterpretation of the Fed's belief. Finally, we also find that disagreements affect inflation and create a policy trade-off between stabilizing output and inflation.

Ricardo J. Caballero
Department of Economics, E52-528
MIT
77 Massachusetts Avenue
Cambridge, MA 02139
and NBER
caball@mit.edu

Alp Simsek
Department of Economics, E52-552
MIT
77 Massachusetts Avenue
Cambridge, MA 02139
and NBER
asimsek@mit.edu

1. Introduction

Figure 1 plots the evolution of the Fed funds rate over time (thin black line), along with predicted paths. The dotted lines plot the Fed's predictions—either the FOMC members' median dot forecast (the right panel) or the Fed staff's assumption for the Greenbook (the left panel). The solid lines plot the forward interest rates that reflect the financial market's predictions. Each color-matched pair of lines plots data from the same FOMC meeting. The figure shows large disagreements between the Fed's predictions and the forward rates, especially around policy-inflection episodes. Ubide (2015) observes similar disagreements in other countries where central banks publish their expected interest rate paths (e.g., Sweden, Norway, and New Zealand).[1] These disagreements about interest rates are difficult to explain with conventional macroeconomic models. The literature typically focuses on the Fed's superior information about its policy rule or economic activity (and its willingness to signal this information). However, the right panel of Figure 1 shows that financial markets expect a different interest rate than the Fed *even after the FOMC members announce the interest rates they plan to set.* In fact, market participants often have their own opinions and do not necessarily consider the Fed to have superior information about the state of the economy.[2]

These disagreements are a source of concern for the Fed, as they suggest that the market might perceive the Fed's actions as "mistakes." How do interest rate announcements perceived as "mistakes" affect the term structure of interest rates and economic activity? How should the Fed manage monetary policy in this environment? To address these, and related issues, we build a model in which the market and the Fed have disagreements about *future aggregate demand.* Our model delivers several positive and normative results: First, we explain the differences in interest rate predictions between the Fed and financial markets. Second, we find that the Fed's *optimal* interest rate target partially reflects the market's view. Third, we provide a microfoundation for monetary policy shocks driven by the Fed's announcements, and analyze the role of communication policy in attenuating these shocks. Finally, we show that disagreements

[1] There are many adjustments one could make to Figure 1, while preserving its main qualitative features. The most significant one is to remove the embedded risk premium from the forward curve. We chose not to do so because there is a wide range of estimates for this premium. The most recent estimates by the Fed researchers suggest that this adjustment is of the order of one basis point per month (positive in the pre GFC period and negative after that), which is not nearly enough to eliminate the disagreements (see, e.g., Diercks et al. (2019)). Moreover, we also found large disagreements between the Fed's predictions and the professional forecaster's consensus (in both the Blue Chip Financial Forecasts and the Survey of Professional Forecasters).

[2] To illustrate how opinionated the market can be, consider the FOMC meeting in December 2007—the run-up to the financial crisis—in which the Fed cut interest rates by 25 basis points. The market was expecting a larger interest rate cut, so this was a "hawkish" policy surprise that led to a decline in stock prices. According to journal coverage, some market participants were quite pessimistic that deteriorating financial conditions would adversely affect the economy, and they thought the Fed did not realize the scope of the problem. For example, the day after the FOMC meeting the Wall Street Journal wrote: "Some on Wall Street yesterday criticized the Fed's actions so far as inadequate. 'From talking to clients and traders, there is in their view no question the Fed has fallen way behind the curve,' said David Greenlaw, economist at Morgan Stanley. 'There's a growing sense the Fed doesn't get it.' Markets believe a weakening economy will force the Fed to cut rates even more than they expected before yesterday, Mr. Greenlaw said."

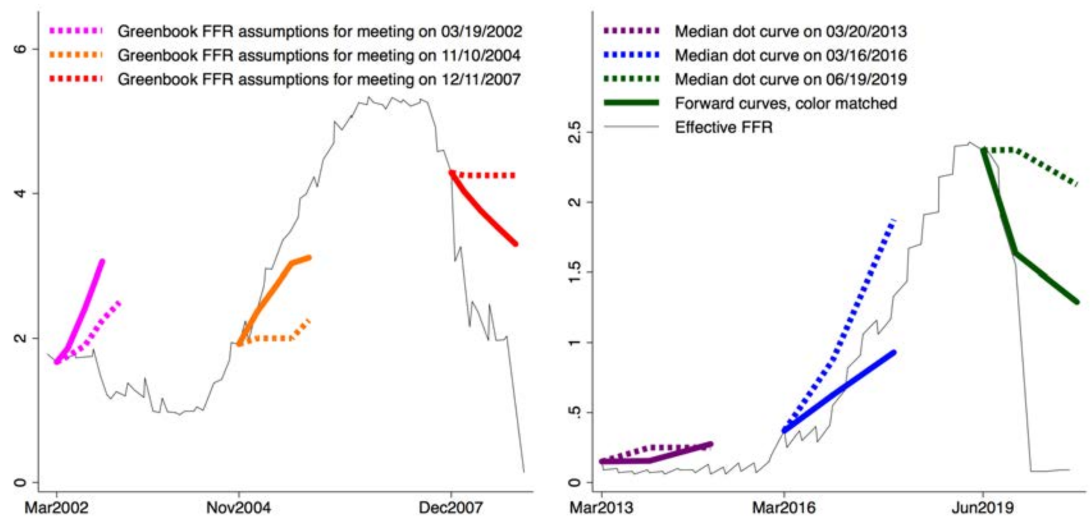

Figure 1: Dotted lines: the Fed prediction for Fed funds rates for select FOMC meetings—from either the Greenbook assumptions (the left panel) or the FOMC dots (the right panel). Solid lines: the forward Fed funds rates for the same meetings. Thin black line: the Fed funds rate.

affect inflation and create a policy trade-off between stabilizing output and inflation.

Our model is a variant of the canonical New Keynesian model (e.g., Clarida et al. (1999); Galí (2015)). Nominal prices are fully sticky in our baseline setup (and partially sticky in an extension). There is a representative household (the market) that makes consumption-saving and labor supply decisions. In each period, the economy is subject to a shock that affects current spending without changing current potential output—we refer to this as an *aggregate demand shock*. The Fed sets the risk-free interest rate in an attempt to insulate output from aggregate demand shocks. However, the Fed sets the interest rate *under uncertainty about aggregate demand* for the current period. This assumption implies that the Fed cannot fully stabilize the output gap (output relative to potential). Instead, the Fed ensures the output gap is zero "on average" *according to the Fed's belief.*

Our main assumption is that the market and the Fed can have *opinionated belief disagreements* about the evolution of aggregate demand. In our baseline setup, agents know each others' beliefs and *agree to disagree.* Agents also *learn over time*: they update their beliefs as they observe realizations of aggregate demand. These assumptions provide the central insight of our paper: the market considers the Fed's interest rate decisions *that do not match the market's belief* to be *"mistakes"* (we occasionally use quotes to remind the reader that these are mistakes under the market's belief, not under the Fed's belief or the objective belief). The anticipation of these "mistakes" affects economic activity and drives the Fed's optimal interest rate decision.

For example, suppose the Fed becomes more optimistic than the market about a persistent component of aggregate demand. Since the Fed is optimistic about aggregate demand, it raises the interest rate to stabilize the output gap. However, since the market doesn't share the Fed's optimism, it considers the interest rate hike to be a “mistake” and expects the output gap to be negative. Moreover, since disagreements are expected to disappear only gradually as agents learn over time, the market expects “mistakenly high” interest rates in future periods as well. The forward interest rates immediately increase and put downward pressure on *current* economic activity. Therefore, even though the Fed is optimistic about aggregate demand, it does not need to raise the current interest rate by much to stabilize the output gap under its belief. In equilibrium, the Fed raises the interest rate by a relatively small amount, which—together with the increase in the forward rates—reduces aggregate demand just enough to counteract the increase in the Fed’s optimism. The Fed also expects to continue raising rates over time, as it expects the data to support its view and persuade the market to move closer to its view. That is, when combined with learning, disagreements naturally imply (expected) *gradualism* in monetary policy implementation.

Our first result formalizes this logic and shows that the Fed’s optimal interest rate reflects a *weighted average of the Fed’s belief and the market’s belief.* Moreover, the relative weight on the market’s belief is greater when agents are more confident in their initial beliefs, since confidence translates into more persistent disagreements. The observation that the interest rate is a weighted average of agents’ beliefs, together with learning, also explains the observed differences between the market’s and the Fed’s expectation for future interest rates—“the forward curve” and “the dot curve” (see Figure 1). Specifically, for sufficiently distant horizons, the forward curve reflects the market’s *current* belief whereas the dot curve reflects the Fed’s *current* belief. Intuitively, the market thinks *the Fed* will learn from data and come to *the market’s* belief. Hence, the market thinks the Fed will set future interest rates that are more closely aligned with the market’s current belief. Conversely, the Fed thinks the market will learn from data and it will set future interest rates reflecting its current belief.

We then provide a microfoundation for textbook monetary policy shocks. These shocks are typically modeled as exogenous, random fluctuations around an interest rate rule. In contrast, we envision monetary policy shocks as times when the market updates its belief about the Fed’s belief about aggregate demand. To capture these shocks, we extend the model to make the market *initially* uncertain about the Fed’s belief. In this setup, a surprise interest rate announcement informs the market about the Fed’s belief. Since the market is opinionated, the announcement does not change the market’s belief about aggregate demand. Instead, the announcement affects the market’s expectation of “mistaken” interest rate changes. The induced market response affects economic activity in the same manner as textbook shocks—but with important differences and policy implications. We categorize our microfounded monetary policy shocks into three groups: Fed belief shocks, market reaction shocks, and tantrum shocks.

A *Fed belief shock* corresponds to a situation in which the Fed’s interest rate decision fully reveals the Fed’s belief to the market. In particular, a surprise interest rate hike implies the Fed has become more optimistic. Since beliefs adjust only gradually, the market anticipates further (“mistaken”) interest rate hikes. Therefore, the shock lifts the forward curve and reduces the market's expected output gap (and asset prices)—like textbook shocks. However, the shock's subsequent impact on the output gap is more subtle and depends on whether the Fed's or the market's belief is closer to the data generating process. In fact, if the Fed has the objective belief and sets the interest rate optimally—accommodating the disagreement as in our baseline model—then the shock has no impact on the output gap on average.

A *market reaction shock* arises when the Fed's belief has multiple dimensions and is not fully revealed by the interest rate. We focus on scenarios in which a surprise rate hike can result from either short or long-term optimism. This type of shock affects the equilibrium *according to the market's reaction*—whether the market interprets the signal as short or long-term optimism—*as opposed to the Fed's actual belief about its optimism's horizon.* For instance, a reactive market attributes the signal to long-term optimism and responds by raising the forward rates. Facing such a market, the Fed needs to act as if it has long-term optimism—raising the interest rate by a small amount as in our baseline setup—even when it only has short-term optimism.

A *tantrum shock* is similar to the market reaction shock with the difference that the Fed fails to forecast correctly how the market will react to its interest rate decision. This is a costlier shock because the interest rate set by the Fed is suboptimal *given the market's reaction.* For concreteness, suppose the Fed has become more optimistic about the short term and thinks the (unreactive) market will attribute an interest rate hike to short-term optimism. However, the market is actually reactive. In this setup, anticipating no change in the forward rates, the Fed raises the current interest rate substantially to address its short-term optimism. The market interprets this aggressive hike as a large increase in long-term optimism and the forward rates increase substantially. This unanticipated market reaction depresses current aggregate demand and makes the Fed miss its output target—even under its own belief.

Tantrum shocks have two other important implications. First, when the Fed anticipates these shocks, it acts *even more gradually* than in our baseline setting (where the Fed knows how the market will react to its policy). Intuitively, an optimistic Fed does not hike the interest rate as much as in the baseline, to mitigate the tantrum shock that would arise if the market were revealed to be reactive. Second, despite the Fed's more conservative policy stance, these shocks induce the Fed to miss its output gap target more often than in the baseline setting. This motivates *Fed communication* policies designed to mitigate tantrum shocks. With these shocks, Fed communication can be a highly effective policy tool, not for its persuasion power (since the market is opinionated), but because it reveals the Fed's actual belief to the market. For example, by announcing the future interest rates it expects to set ("the dot curve") in addition to the current rate, the Fed can reveal whether it has long or short-term optimism and reduce the chance of tantrum shocks in which the market misinterprets the Fed's belief.

In the final part of the paper we extend the model to allow for partial price flexibility, which gives rise to a standard New Keynesian Phillips curve. This extension strengthens our mechanism, in the sense that the Fed accommodates the market's belief even more than with fully sticky prices. The reason is that, for optimal policy purposes, disagreements closely resemble the cost-push shocks in the textbook New-Keynesian model. Consider the earlier example with an optimistic Fed in which the market expects the Fed to set high interest rates and induce negative output gaps. With partially flexible prices, the market also expects disinflation which, via the Phillips curve, reduces *current* inflation. The Fed is then forced to set an even lower interest rate than before—closer to the market's pessimistic belief—to create a positive output gap and fight the disinflationary pressure. In fact, the "divine coincidence" breaks down and the Fed faces a trade-off between stabilizing current inflation and the current output gap.

The rest of the paper is organized as follows. After discussing the related literature, we start in Section 2 by documenting facts about interest rate disagreements among forecasters that motivate our modeling ingredients. Section 3 introduces our general environment, describes the belief structure we focus on, and derives the equilibrium conditions. Section 4 shows how disagreements affect optimal interest rate policy and (together with learning) explain the gap between the forward curve and the dot curve. Section 5 introduces the market's uncertainty about the Fed's belief and derives our results about monetary policy shocks. Section 6 analyzes the extension with partial price flexibility. Section 7 provides final remarks, and is followed by several appendices.

Related literature. Our paper has normative and positive parts, both of which relate to multiple literatures about monetary policy. The distinctive feature of our model is *belief disagreements* between the Fed and the market. In particular, the market has its own belief and *does not* consider the Fed to have superior information about economic activity.

Our policy analysis contributes to a large literature that investigates *gradualism* in monetary policy: the idea that the Fed tends to adjust the interest rates in small steps in the same direction (see, e.g., Woodford (2003); Bernanke (2004); Stein and Sunderam (2018)). Our model features a novel form of *expected* gradualism. When the Fed becomes more optimistic than the market, it hikes the interest rate by a small amount—partially accommodating the market's view—but it also *expects* to continue to hike rates. Importantly, *the market does not expect the rate hikes to continue*, which might help explain why gradualism has been difficult to detect from the term structure of interest rates (e.g., Rudebusch (2002)). With tantrum shocks, our model features a second, more standard rationale for gradualism (similar to Brainard (1967); Sack (1998)): The Fed adjusts the policy rate conservatively in fear of a large market reaction. However, our model can also generate rapid policy responses with respect to other types of shocks, e.g., an increase in the market's optimism not matched by the Fed's optimism (see Remark 1 in Section 4).

Our policy analysis is also related to the growing literature on central bank communication (see Blinder et al. (2008) for a review). The literature documents that central bank transparency has increased in recent years, and that the common forms of communication has made monetary policy shocks more predictable. Our model is consistent with these findings and provides a theoretical rationale for Fed communication. As anticipated by Blinder (1998), the Fed in our setup communicates *to let the market know its own belief.* This improves the Fed's ability to predict how the market will react to its actions and to devise appropriate policies. In particular, the Fed can avoid the tantrum shocks in which the market misinterprets the Fed's belief.[3]

More broadly, our normative analysis is part of a large literature that investigates optimal macroeconomic policy without rational expectations (see Woodford (2013) for a review). This literature typically assumes the planner is rational, but agents are boundedly rational due to frictions such as learning (e.g., Evans and Honkapohja (2001); Eusepi and Preston (2011)), level-k thinking (e.g., García-Schmidt and Woodford (2019); Farhi and Werning (2019); Angeletos and Sastry (2018)), or cognitive discounting (Gabaix (forthcoming)).[4] The focus is on designing policies that address or are robust to agents' bounded rationality. Our approach has two key differences. First, we do not take a stand on who has rational beliefs: in fact, the market thinks it has correct beliefs and the Fed has incorrect beliefs—the opposite of the typical assumption. Second, our agents are not boundedly rational in the usual sense: both the market and the Fed have *dogmatic* beliefs about exogenous states and understand how those states map into endogenous outcomes. These assumptions lead to a different policy analysis and results. In our setting, the Fed's main non-standard concern is to mitigate the macroeconomic impact of the monetary policy "mistakes" perceived by the market.

Our positive analysis contributes to the large literature that empirically investigates the effects of monetary policy shocks on economic activity (see Ramey (2016) for a recent review). We introduce *Fed belief shocks (and variants)* as microfounded monetary policy shocks. Our shocks are related to *the Fed information effect* emphasized in the recent literature (see, e.g., Campbell et al. (2012); Nakamura and Steinsson (2018); Andrade et al. (2019))—the idea that the Fed's policy announcements might contain information about fundamentals. In our model, *the market does not think policy announcements have information about fundamentals.* Instead, the market updates its belief about the Fed's belief. Our monetary policy shocks generate *contingent* predictions for the impact of monetary policy shocks on subsequent economic activity that depend on the accuracy of the Fed's belief (or the market's understanding of it). As we discuss in the concluding section, these predictions can help shed light on the recent empirical findings that monetary policy shocks seem to have a smaller effect on activity in recent years.

Our analysis generates some of the asset price responses to monetary policy shocks identified by the empirical literature that uses high-frequency event study methods (e.g., Bernanke

[3]See Woodford (2005) for other arguments for Fed communication and Amato et al. (2002) for a model in which the Fed communication might be excessive. A parallel debate concerns the best practices for central bank communication; for instance, whether the central bank should speak with a single voice or with many voices—reflecting the differences of opinion among policymakers (see, e.g., Ehrmann and Fratzscher (2007)). In recent work, Vissing-Jorgensen (2019) analyzes "the quiet cacophony of voices": *informal* communication by multiple FOMC members. She argues that the market beliefs *do* influence actual monetary policy decisions (as in our model), and the FOMC members know this and selectively reveal information to influence the market's belief. In her model, informal communication resembles a Prisoner's dilemma and is welfare reducing.

[4]A closely related literature assumes agents are also rational but lack common knowledge of each other's beliefs, and illustrates how the resulting coordination problems can lead to aggregate behavior that resembles some forms of bounded rationality (e.g., Woodford (2001); Angeletos and La'O (2010); Morris and Shin (2014); Angeletos and Lian (2018); Angeletos and Huo (2018)).

and Kuttner (2005); Gürkaynak et al. (2005a,b); Hanson and Stein (2015); Goodhead and Kolb (2018)). For instance, Gürkaynak et al. (2005b) find the forward curve reaction can be summarized by two factors: a policy *target factor* and a *path factor*. Our monetary policy shocks can accommodate both factors with appropriate beliefs (see Remark 2 in Section 5).

A strand of the literature documents that the high-frequency "policy surprises" are predictable from information *publicly available* before the announcement (see, e.g., Miranda-Agrippino (2016); Miranda-Agrippino and Ricco (2018); Cieslak et al. (2018)). In recent work, Sastry (2019); Bauer and Swanson (2020) investigate this puzzle and find that the Fed has reacted to public data about the state of the economy more than the market had anticipated. The evidence further suggests that, at the time of the announcement, the market learns the Fed's belief (or reaction) and disagrees with it. Instead of adopting the Fed's belief, the market independently updates its own belief from the same public data—possibly at a different time. These findings are consistent with our key ingredients, disagreements and learning from data.

Our analysis with partial price flexibility is related to the New Keynesian literature on the limits of inflation stabilization policy. In the textbook model, stabilizing inflation also replicates the flexible-price outcomes. This *divine coincidence* applies for supply shocks as well as demand shocks and implies that the central bank does not face a policy trade-off (e.g., Goodfriend and King (1997); Blanchard and Galí (2007); Galí (2015)). This feature seems counterfactual, which has lead the literature to introduce "cost-push" shocks—often motivated by markup fluctuations or wage rigidities—that affect firms' price setting (the Phillips curve) and create a policy trade-off. We show that disagreements between the Fed and the market (the price setters) create a policy trade-off *even without cost-push shocks*. Intuitively, perceived policy "mistakes" shift agents' inflation expectations and affect their price setting *as-if* there is a cost-push shock.

Our analysis of the Blue Chip data is related to a literature that uses survey data to document belief distortions about macroeconomic outcomes. Much of the recent literature focuses on whether agents over-or-underreact to data (e.g., Coibion and Gorodnichenko (2015); Bordalo et al. (2018); Broer and Kohlhas (2018); Angeletos et al. (2020); Ma et al. (2020)). In contrast, we focus on agents' disagreements (see also Andrade et al. (2016)) and their reaction to each other's beliefs. We provide evidence for confident disagreement: forecasters' beliefs are persistent and largely insensitive to the consensus belief. We also show that, as in our model, beliefs about the interest rate correlate with beliefs about aggregate demand—proxied by inflation (see also Giacoletti et al. (forthcoming)).

Finally, this paper is related to a large literature that studies the macroeconomic implications of belief disagreements and speculation (see Simsek (2020) for a recent survey). We analyze the disagreements between the Fed and investors, whereas the literature mostly focuses on the disagreements among investors (see, e.g., our previous work, Caballero and Simsek (2020, 2019)).

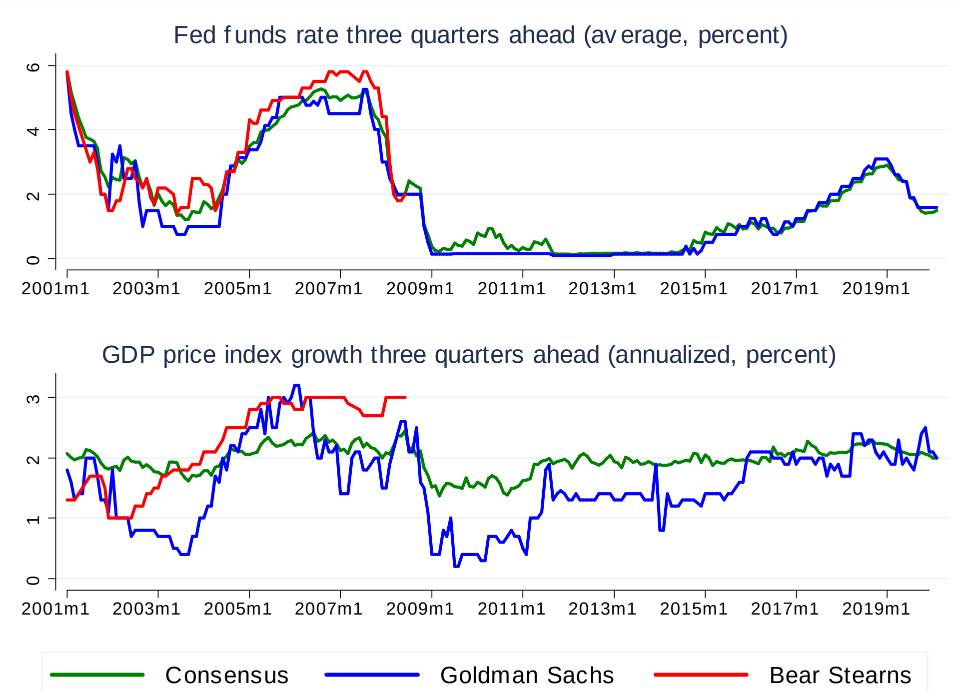

Figure 2: Select Blue Chip predictions for the Fed funds rate (top panel) and the GDP price index (bottom panel).

2. Interest rate disagreements: Some facts from forecasters

Our model is built on the observation that disagreements on expected interest rates are driven by *disagreements about expected aggregate demand*. Moreover, we assume *confident disagreement:* that is, agents have dogmatic beliefs and do not consider the other agent to have superior information. In this section, we present evidence for these modeling ingredients from disagreements among professional forecasters. We focus on disagreements among forecasters to exploit the high quality and *forecaster*-level data on beliefs, which we do not have for the Fed. Our presumption is that the beliefs' traits observed across forecasters (major financial institutions) should also carry over to the Fed.

Specifically, we measure beliefs from Blue Chip Financial Forecasts (Blue Chip). Blue Chip is a monthly survey of several major financial institutions. Forecasters report predictions about interest rates and other outcomes for up to six quarters. We focus on the Fed funds rate (reported as the quarterly average)—which captures beliefs about the policy interest rate, and the GDP price index as well as the real GDP (reported as the annualized quarterly growth rate)—which proxy for beliefs about aggregate demand. We analyze predictions for the third quarter (beyond the current quarter) but the results are similar for other horizons. Our sample runs from January 2001 until February 2020.

Figure 2 illustrates the consensus (average) prediction together with the predictions from two major institutions: Goldman Sachs and Bear Stearns (until its failure in 2008). The panels show that higher interest rate predictions are typically associated with higher aggregate demand predictions (proxied by the GDP price index). In the early 2000s, Goldman Sachs and Bear Stearns were both more pessimistic about aggregate demand and predicted lower interest rates than the consensus. In the mid 2000s, both institutions turned more optimistic about demand and predicted higher interest rates. In the run-up to the financial crisis of 2008, Goldman Sachs became more pessimistic about demand and predicted lower interest rates; whereas Bear Stearns remained optimistic and predicted higher interest rates (until it eventually failed in early 2008). After the crisis, Goldman Sachs remained pessimistic and predicted low interest rates during the zero lower bound and the lift-off episodes (until recent years).

Importantly, Figure 2 also highlights that relative predictions are quite persistent. This persistence is difficult to reconcile with dispersed information: forecasters see each other's prediction as well as the consensus prediction (with a delay of one month) and yet they largely stay with their own prediction. Rather, the persistence of predictions suggests *confident disagreement*: as in our model, forecasters seem to have *dogmatic beliefs* that they change only gradually.

We use regression analysis to show these observations more systematically. We first establish that higher interest rate predictions are associated with higher aggregate demand predictions. Specifically, we regress the Fed funds rate prediction on the GDP price index or the real GDP prediction, controlling for month and forecaster fixed effects. The first three columns of Table 1 illustrate that the coefficients are positive and significant. The coefficient for the GDP price index is larger and more significant—this is expected since nominal prices provide a more accurate proxy for aggregate demand than real output (which might also be driven by aggregate supply).

We next establish that interest rate predictions are persistent in a way that suggests confident disagreement rather than dispersed private information. With confident disagreement, predictions are correlated with their own past values. With dispersed private information, predictions instead are correlated with the consensus prediction's past values—assuming the consensus aggregates the dispersed information. We therefore run a "horse race" where we regress the Fed funds rate prediction on its one-month lag and the consensus prediction's one-month lag, controlling for forecaster fixed effects.[5] The fourth column of Table 1 shows that confident disagreement wins this horse race: the lagged own prediction has a much larger impact than the lagged consensus prediction (and adding more lags does not change this result).

In the horse-race regression, the lagged consensus also has a significant effect. While this might be driven by dispersed private information, it might also reflect other public signals that correlate with the consensus. To investigate, we control for the one-month lag of the Fed funds futures rate for three quarters ahead (the same quarter as the predictions).[6] The fifth column shows the lagged futures rate has a larger effect than the lagged consensus. Moreover, once

[5] We exclude month fixed effects from this regression, since including them would absorb the effect of the consensus prediction.

[6] We obtain the futures rate for the quarter by averaging the implied futures rates for the months in the quarter.

Table 1: Correlates of interest rate predictions

	Fed funds rate (FFR) prediction						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GDP price index prediction	0.11**		0.11**				0.04**
	(0.02)		(0.02)				(0.01)
Real GDP prediction		0.03*	0.03+				0.01+
		(0.02)	(0.02)				(0.01)
FFR prediction last month				0.69**	0.69**	0.69**	0.68**
				(0.03)	(0.03)	(0.02)	(0.02)
FFR consensus last month				0.29**	-0.17**		
				(0.03)	(0.06)		
FFR futures last month					0.47**		
					(0.06)		
Time FE	Yes	Yes	Yes	No	No	Yes	Yes
Forecaster FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2 (adjusted, within)	0.03	0.00	0.03	0.96	0.97	0.48	0.49
Forecasters	110	111	110	108	108	108	107
Months	230	230	230	229	226	229	229
Observations	10,365	10,645	10,363	10,370	10,244	10,370	10,052

Note: The sample is an unbalanced panel of monthly Blue Chip forecasts between 2001-2020. Predictions and futures are for 3 quarters ahead. FFR is the quarterly average (percent) and the GDP price index and the real GDP are annualized quarterly growth rates (percent). Estimation is via OLS. Standard errors are in parentheses and clustered by forecaster and month. +, *, and ** indicate significance at 0.1, 0.05, and 0.01 levels, respectively.

we control for the futures, the coefficient on the consensus changes signs (most likely due to collinearity). In contrast, the coefficient on the lagged own prediction is robust to including lagged futures (fifth column), month fixed effects—that capture forecasters' common reaction to all signals (sixth column), and the current own prediction for the GDP price index and the real GDP (the last column).

In sum, Table 1 shows that the results illustrated in Figure 2 hold more systematically. Interest rate forecasts correlate with aggregate demand forecasts. Forecasts also feature confident disagreement: forecasters seem to have dogmatic beliefs that they change only gradually. We next turn to our theoretical analysis, where we equip the Fed and the market with beliefs that satisfy these features and investigate the implications for monetary policy and asset prices.

3. Environment, equilibrium, and beliefs

In this section we introduce the model, characterize the equilibrium conditions for our baseline setting, and describe the evolution of beliefs. We also solve for the equilibrium in a benchmark case with common beliefs.

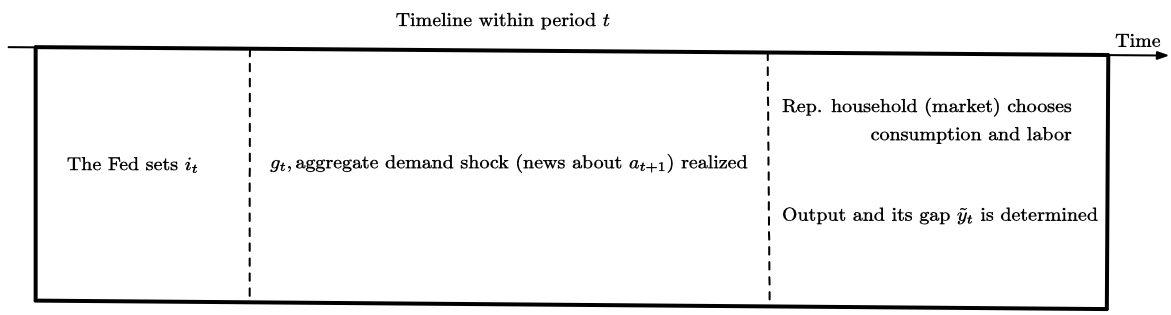

Figure 3: The timeline of events and the summary of the model.

3.1. The model

The model is similar to a textbook New Keynesian model with two distinct features. First, the economy is subject to *aggregate demand shocks* that move expenditure without changing potential output. Second, and more importantly, the Fed sets the interest rate *before* observing the aggregate demand shock for the current period. This assumption captures realistic lags in monetary policy transmission. It also makes *the Fed's belief about demand shocks* relevant for optimal monetary policy and allow us to study *disagreements* between the Fed and financial markets.

Figure 3 illustrates the timeline of events within a period. Each period has three phases. In the first phase, the Fed sets the risk-free interest rate. Then, a shock that determines aggregate demand within the period is realized. Finally, in the last phase, the market chooses optimal allocations, markets clear, and the equilibrium level of output is determined. Throughout, we denote the Fed and the market with the superscript $i \in \{F, M\}$. We use $E_t^i[\cdot]$ to denote agent i's expectation in period t before the realization of shocks (in the first phase), and we use $\overline{E}_t^i[\cdot]$ to denote the corresponding updated beliefs after the realization of shocks (in the last phase).

Preferences and technology. The economy is set in discrete time $t \in \{0, 1, ..\}$. The demand side features a representative household (*the market*) that maximizes utility in the last phase of each period,

$$\overline{E}_t^M \left[\sum_{\tilde{t}=t}^{\infty} e^{-\rho \tilde{t}} \left(\log C_{\tilde{t}} - \frac{N_{\tilde{t}}^{1+\varphi}}{1+\varphi} \right) \right].$$

The market observes the current aggregate demand shock (that we describe subsequently) and solves a standard problem that we relegate to the appendix.

The supply side features a competitive final goods sector and monopolistically competitive intermediate good firms that produce according to,

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } Y_t(\nu) = A_t N_t(\nu)^{1-\alpha}.$$

If nominal prices were fully flexible, the equilibrium labor and output would be equal to their potential levels denoted by, N^* and $Y_t^* = A_t (N^*)^{1-\alpha}$ (see Eq. $(A.12)$ in the appendix).

Nominal rigidities. We assume a fraction of the intermediate good firms have sticky nominal prices. We focus on the standard Calvo setup in which in each period a randomly selected fraction of firms reset their nominal prices, whereas the remaining fraction leaves their prices unchanged. For small aggregate demand shocks, this setup implies aggregate output is determined by aggregate demand, $Y_t = C_t$.

In the appendix, we log-linearize the equilibrium around allocations that feature potential (flexible-price) real outcomes and zero nominal inflation. We show that our price setting assumption implies the New-Keynesian Phillips Curve (NKPC),

$$\pi_t = \kappa \tilde{y}_t + \beta \overline{E}_t^M [\pi_{t+1}], \tag{1}$$

where $\tilde{y}_t = \log (Y_t / Y_t^*)$ denotes the output gap relative to potential and $\pi_t = \log (P_t / P_{t-1})$ denotes inflation. The coefficient, κ, is a price flexibility parameter (see Eq. $(A.21)$ in the appendix).

Aggregate demand shocks. We capture aggregate demand shocks via *news about potential growth.* Formally, log productivity, $a_t = \log A_t$, follows the process,

$$a_{t+1} = a_t + g_t,$$

where g_t denotes the growth rate of productivity between periods t and $t+1$, which is *realized in period* t. In particular, by the time the economy reaches period t, there is no uncertainty about the potential output of the economy in the current period: $a_t = a_{t-1} + g_{t-1}$ and g_{t-1} is already determined. However, there is uncertainty about potential growth between this and the next period, g_t.

In the appendix, we log-linearize the Euler equation for the market to obtain the IS equation,

$$\tilde{y}_t = -\left(i_t - \overline{E}_t^M [\pi_{t+1}] - \rho\right) + g_t + \overline{E}_t^M [\tilde{y}_{t+1}], \tag{2}$$

where $i_t - \overline{E}_t^M [\pi_{t+1}]$ corresponds to the (market-expected) real interest rate. Eq. (2) illustrates that for a given real interest rate, the equilibrium output gap increases one-to-one with the potential growth rate, g_t, as well as with the expected future output gap. This is why we refer to g_t as the *aggregate demand* shock in period t.

Monetary policy. The interest rate is set by the monetary authority (the Fed). Our key friction is that the Fed sets the interest rate *at the beginning of the period, before observing the aggregate demand shock for the period.* Otherwise, the Fed solves a standard gap-minimization problem. We assume the Fed sets policy without commitment. In each period and state, it takes the future allocations as given and implements the allocation that solves,

$$\min_{i_t,\tilde{y}_t,\pi_t} -\frac{1}{2}E_t^F\left[\phi\tilde{y}_t^2+\pi_t^2\right], \tag{3}$$

subject to (1) and (2).

3.2. Baseline equilibrium conditions

Except for Section 6, we focus on the special case with fully sticky prices, $\kappa = 0$. In this case, inflation is zero, $\pi_t = 0$, and the Fed focuses on stabilizing output. In particular, combining problem (3) with Eq. (2), the Fed's optimality condition implies,

$$E_t^F\left[\frac{d\tilde{y}_t}{di_t}\tilde{y}_t\right]=0 \text{ where } \frac{d\tilde{y}_t}{di_t}=-1+\frac{dE_{t+1}^M\left[\tilde{y}_{t+1}\right]}{di_t}. \tag{4}$$

Note that in this expression we substituted the market's end-of-period expectations in period t with its beginning-of-period expectations in period $t+1$, $\overline{E}_t^M\left[\tilde{y}_t\right]=E_{t+1}^M\left[\tilde{y}_{t+1}\right]$ (since no new information arrives between the last phase of period t and the first phase of period $t+1$—see Figure 3). For most of our analysis, we work with belief specifications under which the policy rate has a deterministic impact on the market's expected future output gap: that is, $\frac{dE_{t+1}^M[\tilde{y}_{t+1}]}{di_t}$ is deterministic. This is either because $E_{t+1}^M\left[\tilde{y}_{t+1}\right]$ does not depend on the current interest rate (Section 4); or because it has a (conditionally) constant slope with respect to the current interest rate (Section 5 except for Section 5.4). In these cases, the Fed's optimality condition simplifies to,

$$E_t^F\left[\tilde{y}_t\right]=0. \tag{5}$$

The Fed closes output gaps *in expectation* and *according to its own belief.*

We can then combine Eqs. (2) and (5) to solve for the equilibrium interest rate as

$$i_t=\rho+E_t^F\left[g_t\right]+E_t^F\left[E_{t+1}^M\left[\tilde{y}_{t+1}\right]\right]. \tag{6}$$

The Fed sets a higher interest rate when it expects greater aggregate demand, $E_t^F\left[g_t\right]$. More subtly, the Fed also sets a higher interest rate if it expects *the market* to be more optimistic about the subsequent output gap (higher $E_t^F\left[E_{t+1}^M\left[\tilde{y}_{t+1}\right]\right]$). As illustrated by Eq. (2), the market's optimism about future output increases current output, and the Fed increases the interest rate to offset this effect. This mechanism plays an important role for our results.

Substituting Eq. (6) into Eq. (2), we solve for the equilibrium output gap as,

$$\tilde{y}_t=g_t-E_t^F\left[g_t\right]+E_{t+1}^M\left[\tilde{y}_{t+1}\right]-E_t^F\left[E_{t+1}^M\left[\tilde{y}_{t+1}\right]\right]. \tag{7}$$

In equilibrium, the output gap depends on surprises relative to *the Fed's* expectations. The first two terms concern surprises to the aggregate demand shock, g_t. When aggregate demand is higher than the Fed expected when it set the interest rate, the output gap is higher. The last two terms concern the Fed's surprise about the market's expectation about the output gap in the next period. This second surprise will play no role until Section 5.3 (on tantrum shocks).

Finally, we also characterize risky asset prices along the equilibrium path. We focus on "the market portfolio" which we define as a financial asset (in zero net supply) whose payoff is equal to output in subsequent periods, $\{Y_{\tilde{t}}\}_{\tilde{t}\geq t+1}^{\infty}$. In the appendix, we show that the log price of this asset satisfies,

$$q_t = q^* + a_t + \tilde{y}_t, \tag{8}$$

where q^* is a constant. Under log-utility, the price of the market portfolio is proportional to output (see Eq. $(A.23)$). Therefore, the price moves either when productivity changes or when the output gap changes. In subsequent analysis, we focus on characterizing the output gap, $\tilde{y}_t$, and refer to Eq. (8) to describe the impact on asset prices.

Eqs. $(6-8)$ provide a generally applicable characterization of equilibrium (when prices are fully sticky and $\frac{dE_{t+1}^M[\tilde{y}_{t+1}]}{di_t}$ is deterministic). We next specify the agents' beliefs under which we apply these equations.

3.3. Uncertainty and beliefs

Motivated by our empirical analysis in Section 2, we equip agents with persistent belief disagreements. For simplicity, we start with a case in which agents disagree about a fully persistent component of aggregate demand. In Section 5, we analyze the case in which agents can (also) disagree about a transitory component.

Formally, we assume aggregate demand follows

$$\begin{aligned} g_t &= \mathbf{g} + v_t \text{ for each } t, \\ \text{where } v_t &\sim N(0, \Sigma) \text{ for each } t \text{ and independent across } t. \end{aligned} \tag{9}$$

The random variable v_t captures transitory shocks to aggregate demand. These shocks are i.i.d. across periods with a Normal distribution. The term $\mathbf{g}$ is an unknown parameter that captures the persistent component of aggregate demand. It is realized at the beginning of the model but it is not observed by the agents.

Agents have heterogeneous prior beliefs about $\mathbf{g}$. Specifically, at the beginning of period 0, each agent $j \in \{F, M\}$ believes the persistent component is drawn from a Normal distribution,

$$\mathbf{g} \sim N\left(\mathbf{g}_0^j, C_0^{-1}\Sigma\right), \tag{10}$$

where $\mathbf{g}_0^j$ denotes the perceived prior mean and $C_0^{-1}\Sigma$ denotes the perceived variance. The key assumption is that the beliefs $\mathbf{g}_0^F$ and $\mathbf{g}_0^M$ may differ. Throughout, agents have dogmatic disagreements in the sense that they do not think the other agent has any information they haven't already incorporated. Except for Section 5, we assume agents know each other's beliefs: "they agree to disagree." The parameter C_0 is a measure of the agent's *confidence* in its initial belief. In the main text, we focus on the case in which agents have common confidence. Confidence captures (inversely) how fast agents update their beliefs as they observe new data.

We are agnostic about the source of agents' dogmatic belief disagreements—our results do not require a specific interpretation. One possibility is that agents received news of an unusual event (e.g., a financial crisis) that will induce a persistent shock to aggregate demand. Since these events are rare, history does not provide sufficient guidance about how they affect the economy, so it is natural for agents to start with heterogeneous prior beliefs. An alternative possibility is that agents have heterogeneous and possibly misspecified learning models. For instance, suppose agents receive *public* signals that are informative about aggregate demand (e.g., macroeconomic data releases), but they interpret these signals differently, e.g., the Fed puts more weight on one signal whereas the market puts more weight on another signal. Our results would qualitatively apply under this alternative (behavioral) interpretation. For concreteness, however, we assume agents are "rational" *given their heterogeneous initial beliefs*.

Bayesian updating of beliefs. Specifically, the realization of aggregate demand in each period provides the agents with a noisy signal about the persistent component, $g_t = \mathbf{g} + v_t$. Agents combine this data with their prior beliefs in (10) to form their posterior beliefs according to Bayes' rule. In Appendix B.1, we show that agent j's conditional belief about the persistent component, $\mathbf{g}_t^j \equiv E_t^j[\mathbf{g}]$, evolves according to,

$$
\begin{aligned}
\mathbf{g}_t^j &= \mathbf{c}_{0,t}\mathbf{g}_0^j + (1-\mathbf{c}_{0,t})\,\overline{g}_{t-1} \text{ where } \overline{g}_{t-1} = \sum_{\tilde{t}=0}^{t-1} \frac{g_{\tilde{t}}}{t}, \\
&= \mathbf{c}_{t-1,t}\mathbf{g}_{t-1}^j + (1-\mathbf{c}_{t-1,t})\,g_{t-1} \text{ for each } t \geq 1,
\end{aligned} \tag{11}
$$

where $\mathbf{c}_{s,t}$ denotes the *relative confidence* in period s compared to a later period t,

$$
\mathbf{c}_{s,t} \equiv \frac{C_0 + s}{C_0 + t} \quad \text{for } s \leq t. \tag{12}
$$

Eq. (11) says that the conditional belief is a weighted average of the initial belief, $\mathbf{g}_0^j$, and the average realization, $\overline{g}_{t-1}$. The weights are determined by the relative confidence between periods 0 and t. The second line writes the belief as a weighted average of the most recent belief and the most recent realization.

Recall that the equilibrium depends on the agents' conditional belief of aggregate demand. This belief is the same as the conditional belief of the persistent component, $E_t^j[g_t] = \mathbf{g}_t^j$, since the transitory component has mean zero [cf. (9)]. We establish two additional properties of the conditional beliefs that facilitate the subsequent analysis. The first result describes the evolution of disagreements. The second result describes the higher order beliefs that matter for the equilibrium: in particular, the agents' expectations at time t about the conditional beliefs they will have at time $t+1$ [cf. Section 3.2].

Lemma 1. *Disagreements evolve according to,*

$$\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F = \mathbf{c}_{t,t+1}\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) = \mathbf{c}_{0,t+1}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right). \tag{13}$$

In particular, disagreements are deterministic and they decline over time.

Disagreements decline over time since agents update from the same data [cf. (11)]. Moreover, they decline *deterministically* because we have assumed agents have common confidence and therefore put the same weight on data (i.e., new realizations of g_t generate identical belief updates for both agents).

Lemma 2. *Consider the mean belief at the beginning of period s about the conditional mean belief (about aggregate demand) in a subsequent period $t \geq s$. For each agent $j \in \{F, M\}$ and $j' \neq j$, we have:*

$$E_s^j\left[\mathbf{g}_t^j\right] = \mathbf{g}_s^j, \tag{14}$$

$$E_s^j\left[\mathbf{g}_t^{j'}\right] = \mathbf{c}_{s,t}\mathbf{g}_s^{j'} + \left(1 - \mathbf{c}_{s,t}\right)\mathbf{g}_s^j. \tag{15}$$

Each agent expects its own conditional belief about aggregate demand in a future period to be the same as its current belief. In contrast, each agent expects *the other agent's* conditional belief in a future period to be a *weighted average* of the other agent's current belief and its own current belief. The weights depend on the relative confidence, $\mathbf{c}_{s,t}$. Intuitively, each agent expects the other agent to learn from the data and to come toward its own view. The expected speed of learning is decreasing in (the other agent's) confidence. This implication of learning will be important for our results.

3.4. Benchmark with common beliefs

We end this section by solving for the equilibrium in a benchmark scenario with no disagreement between the Fed and the market. Specifically, suppose $\mathbf{g}_0^F = \mathbf{g}_0^M \equiv \mathbf{g}_0$ so that agents have the same conditional belief, $\mathbf{g}_t$, in all periods.

Since agents share the same belief and the Fed sets output gaps to zero in expectation [cf. (5)], Eqs. (6) and (7) imply,

$$i_t = \rho + \mathbf{g}_t, \tag{16}$$

$$\tilde{y}_t = g_t - E_t\left[g_t\right] = g_t - \mathbf{g}_t. \tag{17}$$

With common beliefs, the market knows the Fed will, on average, stabilize future output gaps,

$E_{t+1}[\tilde{y}_{t+1}] = 0$. Therefore, there are no perceived "mistakes" and the Fed sets an interest rate that reflects its expected aggregate demand. Naturally, surprises relative to the Fed's belief shift the output gap.

Next suppose the economy is at the initial period 0, and consider the expected future interest rates according to the market's and the Fed's belief, respectively. With a slight abuse of terminology, we refer to the former as "the forward curve" and the latter as "the dot curve" (see Figure 1). Using Eq. (16) and Lemma 2, we obtain,

$$E_0^M[i_t] = E_0^F[i_t] = \rho + \mathbf{g}_0 \text{ for each } t. \tag{18}$$

With common beliefs, the forward and dot curves are the same and they reflect agents' *current* belief about aggregate demand.

4. Disagreements and interest rate policy

We next turn to disagreements. Our first result describes how disagreements affect the optimal interest rate as well as the expected interest rates.

Proposition 1. *Consider our setup with arbitrary initial beliefs,* $\mathbf{g}_0^M, \mathbf{g}_0^F$.

(i) The equilibrium interest rate and output gap are given by

$$i_t = \rho + (1 - \mathbf{c}_{t,t+1})\,\mathbf{g}_t^F + \mathbf{c}_{t,t+1}\mathbf{g}_t^M, \tag{19}$$

$$\tilde{y}_t = g_t - \mathbf{g}_t^F. \tag{20}$$

The optimal interest rate set by the Fed reflects in part the market's belief, with a weight that depends on relative confidence, $\mathbf{c}_{t,t+1}$*. In the limit of very high initial confidence, the interest rate reflects* ***only the market's*** *belief,* $\lim_{C_0 \to \infty} i_t = \rho + \mathbf{g}_t^M$.

(ii) The forward and dot curves in period 0 *are given by*

$$E_0^M[i_t] = \rho + \mathbf{g}_0^M + \mathbf{c}_{0,t}(1 - \mathbf{c}_{t,t+1})\left(\mathbf{g}_0^F - \mathbf{g}_0^M\right), \tag{21}$$

$$E_0^F[i_t] = \rho + \mathbf{g}_0^F + \mathbf{c}_{0,t+1}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right). \tag{22}$$

Each curve reflects the ***corresponding agent's current belief*** *for aggregate demand, with an adjustment toward the other agent's belief that declines with horizon. For sufficiently long horizons, each curve reflects* ***only*** *the corresponding agent's belief,* $\lim_{t \to \infty} E_0^j[i_t] = \rho + \mathbf{g}_0^j$*, and the difference reflects the level of current disagreement,* $\lim_{t \to \infty} E_0^M[i_t] - E_0^F[i_t] = \mathbf{g}_0^M - \mathbf{g}_0^F$.

The first part characterizes the equilibrium outcomes. The output is the same as in the benchmark with common beliefs [cf. (17)]. The interest rate is different and depends on a *weighted average of the Fed's and the market's beliefs about aggregate demand.* The Fed cannot set interest rates by focusing only on its own view of aggregate demand—it also needs to take into account the market's view and the extent of disagreement. Moreover, the more entrenched the disagreements, the more the Fed ignores *its own* view.

The second part shows that, unlike the benchmark case, *the forward and dot curves trace out different expected interest rate paths* [cf. (18)]. The forward curve reflects the market's belief for aggregate demand with an adjustment toward the Fed's belief—and vice versa for the dot curve. Therefore, belief disagreements about aggregate demand translate into differences between the forward and dot curves.

Sketch of proof. We sketch the proof of the proposition, which is useful to develop intuition. Recall from Lemma 1 that belief disagreements evolve deterministically. Therefore, we conjecture (and verify) an equilibrium in which the market's conditional belief about the subsequent output gap, $E_{t+1}^{M}\left[\tilde{y}_{t+1}\right]$, is also deterministic and independent of the current policy rate, $\frac{dE_{t+1}^{M}[\tilde{y}_{t+1}]}{di_t} = 0$. In particular, the baseline characterization in Section 3.2 applies. Eq. (7) then immediately implies (20). Since disagreements evolve deterministically, the Fed can still adjust the interest rate appropriately to hit its output target on average, according to its own belief. Disagreements manifest themselves in the *interest rate* that the Fed must set to achieve this outcome.

Next consider the IS curve (2) for the case without inflation,

$$\tilde{y}_t = -\left(i_t - \rho\right) + g_t + E_{t+1}^{M}\left[\tilde{y}_{t+1}\right].$$

All terms except for g_t are deterministic. Taking the expectations according to each agent and using $E_t^F\left[\tilde{y}_t\right] = 0$, we obtain,

$$E_t^M\left[\tilde{y}_t\right] - E_t^F\left[\tilde{y}_t\right] = E_t^M\left[\tilde{y}_t\right] = \mathbf{g}_t^M - \mathbf{g}_t^F. \tag{23}$$

In general, the market does not expect the output gap to be zero since it thinks the Fed will make "mistakes." The extent of "mistakes" depends on *disagreements.* For instance, when $\mathbf{g}_t^F > \mathbf{g}_t^M$, the market thinks the Fed is too optimistic about demand and therefore sets an interest rate too high, which will on average induce a negative output gap, $E_t^M\left[\tilde{y}_t\right] < 0$.

Eq. (23) together with Lemma 1 verifies that $E_{t+1}^{M}\left[\tilde{y}_{t+1}\right]$ is deterministic and satisfies $\frac{dE_{t+1}^{M}[\tilde{y}_{t+1}]}{di_t} = 0$. Using (6), we also solve for the the equilibrium interest rate,

$$\begin{aligned} i_t &= \rho + \mathbf{g}_t^F + E_{t+1}^M\left[\tilde{y}_{t+1}\right] \\ &= \rho + \mathbf{g}_t^F + \mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F \\ &= \rho + \mathbf{g}_t^F + \mathbf{c}_{t,t+1}\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \end{aligned} \tag{24}$$

This proves Eq. (19). The anticipation of future "mistakes" affects current activity and induces the Fed to adjust the interest rate in the direction of the market's belief. For instance, when $\mathbf{g}_t^F > \mathbf{g}_t^M$, the market thinks the Fed will remain optimistic also in the next period, $\mathbf{g}_{t+1}^F > \mathbf{g}_{t+1}^M$, and will set a negative output gap, $E_{t+1}^M\left[\tilde{y}_{t+1}\right] < 0$. This exerts downward pressure on the current output gap. Consequently, the Fed sets a lower interest rate than implied by its own (more optimistic) belief. The extent to which the Fed moves toward the market depends on the relative confidence, $\mathbf{c}_{t,t+1}$, because this determines the extent to which the market expects current disagreements to persist into the future.

Finally, we establish the second part of the proposition. Consider the forward curve. Taking the expectation of Eq. (19) according to the market's belief, we obtain

$$E_0^M [i_t] = \rho + (1 - \mathbf{c}_{t,t+1}) E_0^M \left[\mathbf{g}_t^F\right] + \mathbf{c}_{t,t+1} E_0^M \left[\mathbf{g}_t^M\right]. \tag{25}$$

Substituting the higher order belief from Lemma 2 proves Eq. (21). For intuition, recall that the higher order belief, $E_0^M \left[\mathbf{g}_t^F\right]$, monotonically converges to $\mathbf{g}_0^M$ as the horizon t increases. The market expects the Fed to learn over time and to converge to the market's belief. Therefore, the market expects future interest rates to be determined by its current belief, $\mathbf{g}_0^M$. A symmetric argument proves Eq. (22).

Illustration. Figure 4 illustrates the result and provides further intuition. In each panel, the thin dashed line corresponds to the (overlapping) expected interest rates with a common baseline belief. The thin solid line shows the expected rates when the common belief becomes more optimistic. The thicker purple and blue lines show the dot and the forward curves, respectively, when one agent becomes more optimistic and the other agent remains with the more pessimistic baseline belief.

First consider the case in which the Fed becomes more optimistic. The top panels of Figure 4 illustrate that this shifts both dot and forward curves upward, but with greater effects on the dot curve [cf. $(22 - 21)$]. This gap arises because the Fed expects the market to learn. Hence, over longer horizons, the Fed expects to set interest rates that reflect its optimism (whereas the market expects the *Fed* will learn instead).

These panels also illustrate that the Fed raises the interest rate less than the increase in its optimism [cf. (19)]. For a complementary intuition, note that the market's expected interest rates beyond the initial period also increase—illustrated by the shaded area in the figure. Moreover, the market considers these increases a "mistake." These "mistakenly high" future interest rates exert downward pressure on *current* output. Hence, even though the Fed becomes more optimistic, it only needs to increase the current interest rate slightly to achieve its target output gap. In fact, the Fed can be thought of as targeting an overall increase in the forward curve—the current rate hike plus the shaded area—that is just enough to counteract the increase in its optimism. Consistent with this intuition, the Fed increases the interest rate by more when agents are less confident in their initial beliefs. In that case, disagreements are less persistent and the market expects the interest rate hike to decline more quickly (see the top right panel of Figure 4).

Next consider the case in which the market is more optimistic. The bottom panels of Figure

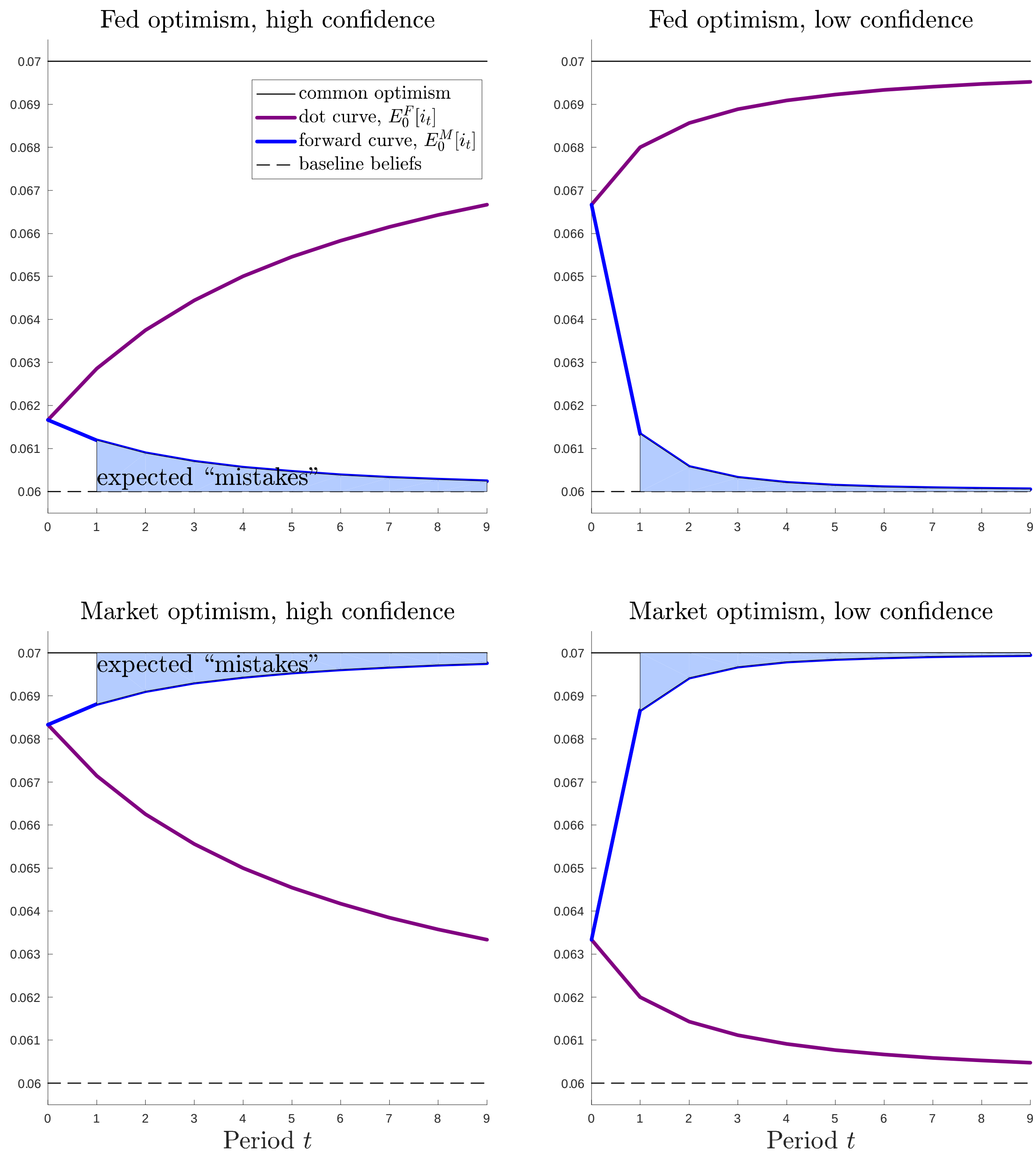

Figure 4: Top (resp. bottom) panels illustrate expected interest rates when the Fed (resp. the market) becomes more optimistic while the other agent remains with the baseline belief. Left (resp. right) panels correspond to higher (resp. lower) levels of initial confidence, C_0.

4 show that this also shifts both forward and dot curves upward, but with greater effects on the forward curve. As before, this gap arises because agents expect each other to learn. Note also that the Fed raises the initial interest rate even though its own belief did not change. In this case, the market expects the Fed to be too pessimistic and to set interest rates too low in future periods—illustrated by the shaded area in the figure. These "mistakenly low" future interest rates (together with the market's optimism) exert upward pressure on *current* output. Therefore, the Fed is forced to increase the interest rate to achieve its target output gap. In fact, the Fed can be thought of as hiking the current rate just enough to counteract the expected "shortfall" in future interest rates—the shaded area. Consistent with this intuition, the Fed increases the interest rate by less when agents are less confident in their initial beliefs. In that case, disagreements are less persistent and the market expects the interest rate to catch up with its optimism more quickly (see the bottom right panel of Figure 4).

Remark 1 (Expected Gradualism)**.** *In our baseline model, the optimal interest rate policy has a novel form of gradualism in the implementation of the Fed's view. When the Fed is more optimistic than the market, it chooses not to increase the policy rate by the full amount of its optimism (it partially accommodates the market's view), but it also expects to continue raising rates as it expects the data to sway the market toward the Fed's view over time (as reflected in the dot curve). Conversely, the Fed is very reactive to the market's optimism, as it raises rates immediately with the expectation to undo those changes over time as the market learns from data.*

5. Disagreements and monetary policy shocks

So far, we have assumed the Fed and the market know each others' belief. While the Fed might observe changes in the market's belief through asset prices (albeit with noise), it is harder for the market to observe changes in the Fed's belief. In fact, much of the Fed's communication policy can be viewed as an attempt to convey the Fed's belief to the public. In this section, we analyze the role of the Fed's announcements in revealing the Fed's belief to the market.

We derive *microfounded* monetary policy shocks and categorize them into three groups. "Fed belief shocks" occur when the Fed's interest rate decision optimally and fully reveals a change in its belief to the public. "Market reaction shocks" occur when the Fed's belief is multidimensional. In this case, the interest rate decision signals the Fed's belief only partially and its impact on the economy depends on the market's reaction to the signal, rather than on the Fed's actual belief. Finally, "tantrum shocks" occur when "market reaction shocks" are possible *and* the Fed does not fully know how the market will react to its interest rate decision. We also show that the fear of tantrum shocks induces the Fed to act *even more gradually* than in our baseline setting. Despite the Fed's more conservative policy stance, these (tantrum) shocks are welfare reducing and can be mitigated by appropriate Fed *communication* policies.

5.1. Fed belief shocks

Consider the baseline setup with the difference that the market does not know the Fed's initial belief, $\mathbf{g}_0^F$. Specifically, the market believes the Fed's belief is drawn from a distribution with mean, $\overline{\mathbf{g}}_0^F \equiv E_0^M\left[\mathbf{g}_0^F\right]$. The Fed still knows the market's belief, $\mathbf{g}_0^M$. The rest of the model is unchanged.

We conjecture an equilibrium that is *exactly the same* as in Section 4. That is, the Fed sets the interest rate

$$i_0 = \rho + \left(1 - \mathbf{c}_{0,1}\right)\mathbf{g}_0^F + \mathbf{c}_{0,1}\mathbf{g}_0^M.$$

Note that this rate is a one-to-one function of the Fed's belief, $\mathbf{g}_0^F$. Therefore, after observing the interest rate, the market infers the Fed's belief as, $\mathbf{G}_0^F\left(i_0\right) \equiv \frac{i_0-\rho-\mathbf{c}_{0,1}\mathbf{g}_0^M}{1-\mathbf{c}_{0,1}}$. Along the equilibrium path, the market's inference is correct, $\mathbf{G}_0^F\left(i\right) = \mathbf{g}_0^F$. Once the market learns $\mathbf{g}_0^F$, the analysis is the same as in Section 4.

We also need to check that the Fed does not have an incentive to deviate from the equilibrium interest rate policy. In the appendix, we show that the policy has a constant impact on the market's expected output gap,

$$\frac{dE_1^M\left[\tilde{y}_1|i_0\right]}{di_0} = \mathbf{c}_{0,1}\frac{d\mathbf{G}_0^F\left(i_0\right)}{di_0} = -\frac{\mathbf{c}_{0,1}}{1-\mathbf{c}_{0,1}}. \tag{26}$$

A greater interest rate makes the market infer greater Fed optimism and expect a smaller output gap. Consequently, the interest rate has a larger impact on current output than in the previous section, $\frac{d\tilde{y}_0}{di_0} = -\left(1 + \frac{\mathbf{c}_{0,1}}{1-\mathbf{c}_{0,1}}\right)$. However, the additional impact is constant across states and does not distort the Fed's optimal interest rate decision [cf. Eq. (4)]. In particular, the baseline characterization in Section 3.2 still applies and the optimal interest rate is the same as before, verifying the equilibrium.

In this equilibrium, the *Fed belief shock*—the revelation of the Fed's belief via the interest rate—affects *the market's* expected equilibrium outcomes. To characterize the impact, first consider the expected outcomes *after* the interest rate decision. Proposition 1 implies the market's expected interest rates are,

$$E_0^M\left[i_t|i_0\right] = \rho + \mathbf{g}_0^F\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right) + \mathbf{g}_0^M\left(1-\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right)\right) \quad \text{for } t \geq 1. \tag{27}$$

Likewise, Eq. (23) and Lemma 1 imply the market's expected output gaps are,

$$E_0^M\left[\tilde{y}_t|i_0\right] = \mathbf{c}_{0,t}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right) \quad \text{for } t \geq 1. \tag{28}$$

Next note that the market's expected outcomes *before* the interest rate decision corresponds to the same expressions with $\mathbf{g}_0^F$ replaced by $\overline{\mathbf{g}}_0^F$. This leads to the following result.

Proposition 2. *Suppose the market initially does not know the Fed's belief. Let Δx denote the* *equilibrium change of a variable in period* 0 *relative to its* ex-ante *expectation by the market. (For instance,* $\Delta \mathbf{g}_0^F = \mathbf{g}_0^F - \overline{\mathbf{g}}_0^F$ *denotes the surprise change in the Fed's belief and* $\Delta E_0^M [i_t] \equiv E_0^M [i_t|i_0] - E_0^M [i_t]$ *denotes the change in the market's expected interest rates.) The equilibrium is the same as in Proposition 1. The Fed's interest rate announcement in period* 0 *fully reveals its belief. A Fed optimism shock increases the current as well as the forward interest rates,*

$$\frac{\Delta i_0}{\Delta \mathbf{g}_0^F} = 1 - \mathbf{c}_{0,1} \text{ and } \frac{\Delta E_0^M [i_t]}{\Delta \mathbf{g}_0^F} = \mathbf{c}_{0,t} (1 - \mathbf{c}_{t,t+1}) \text{ for } t \geq 1. \tag{29}$$

The shock reduces the market's expectation for the output gap and the price of the market portfolio,

$$\frac{\Delta E_0^M [\tilde{y}_t]}{\Delta \mathbf{g}_0^F} = \frac{\Delta E_0^M [q_t]}{\Delta \mathbf{g}_0^F} = -\mathbf{c}_{0,t}. \tag{30}$$

Eq. (29) says that a Fed optimism shock affects the expected rates as we described previously [cf. Figure 4]. Eq. (30) shows that the shock reduces the market's expected output gap. After an interest hike, the market perceives greater monetary policy "mistakes" in the direction of high interest rates. This also reduces the expected price of the market portfolio, which is a one-to-one function of the output gap [see (8)].

These results highlight that Fed belief shocks affect interest rates and (the market's) expected economic activity like the textbook monetary policy shocks—typically modeled as random fluctuations around an interest rate rule (see, e.g., Galí (2015)). Finding an empirical counterpart to these shocks is challenging and requires a structural interpretation.[7] Proposition 2 describes a microfounded monetary policy shock, and clarifies the conditions under which it induces the classical effects of monetary policy. A Fed belief shock unambiguously generates conventional effects on *financial market outcomes* (e.g., forward interest rates and asset prices) that depend on *the market's* expectation.[8] However, the predictions for subsequent *real* outcomes are more subtle and depend on whether the market's or the Fed's belief is closer to the data generating process. The following result formalizes this point.

Corollary 1. *Consider the setup in Proposition 2. Suppose that at the beginning of date* 0 *the market's belief is fixed at some* $\mathbf{g}_0^M$*; whereas the Fed's belief,* $\mathbf{g}_0^F$*, and the* actual *persistent component of demand,* $\mathbf{g}$*, are jointly drawn from a distribution with some mean* $\overline{\mathbf{g}}, \overline{\mathbf{g}}_0^F$*. The market knows the Fed's belief is drawn from a distribution with mean* $\overline{\mathbf{g}}_0^F$ *but thinks it is uncorrelated*

[7] For instance, Ramey (2016) notes: "Because monetary policy is typically guided by a rule, most movements in monetary policy instruments are due to the systematic component of monetary policy rather than to deviations from that rule. We do not have many good economic theories for what a structural monetary policy shock should be."

[8] The financial market reaction also helps to differentiate our Fed belief shocks from the Fed information shocks emphasized in the recent literature. Unlike a Fed belief shock, an interest rate hike driven by a Fed information shock would typically *increase* stock prices—as it would make the market more optimistic about subsequent economic activity. In fact, Cieslak and Schrimpf (2019); Jarocinski and Karadi (2020) use the stock price response at the time of the policy announcement to disentangle conventional monetary policy shocks and Fed information shocks.

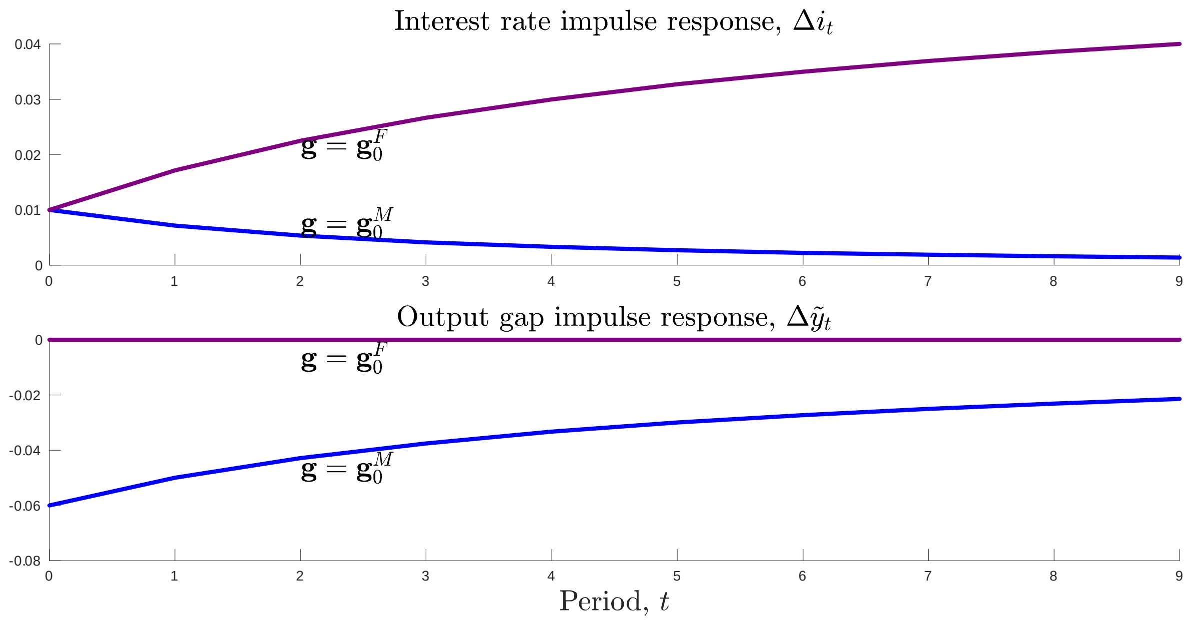

Figure 5: Impulse responses to a Fed optimism shock, $\Delta \mathbf{g}_0^F > 0$, that raises the interest rate by 1%. Blue lines (resp. purple lines) correspond to the case in which the actual persistent component demand is equal to the market's initial belief (resp. the Fed's initial belief). All transitory demand shocks are set to zero [cf. (9)].

with $\mathbf{g}$*. Let* $\beta^{DGP}(y,x) = \frac{cov^{DGP}(y,x)}{var^{DGP}(x)}$ *denote the beta coefficient between two variables under the data generating process. Then, we have,*

$$\beta^{DGP}(\tilde{y}_t, i_0) = \frac{\mathbf{c}_{0,t}}{1-\mathbf{c}_{0,1}} \left(\beta^{DGP}\left(\mathbf{g}, \mathbf{g}_0^F\right) - 1\right).$$

The result envisions a scenario in which a Fed belief shock—not matched by a market belief change—*might be* correlated with an actual (persistent) demand shock. In this scenario, regressing the output gap $\tilde{y}_t$ on the interest rate i_0 will produce the conventional (negative) coefficient, $\beta^{DGP}(\tilde{y}_t, i_0) < 0$, if and only if the data on average changes less than one-to-one with the Fed's belief, $\beta^{DGP}\left(\mathbf{g}, \mathbf{g}_0^F\right) < 1$. Intuitively, while the market thinks the Fed's interest rate change is a mistake, the Fed thinks it is appropriate and stabilizes the output gap according to its belief, $E_0^F[\tilde{y}_t] = 0$. If the Fed is right on average, $\beta^{DGP}\left(\mathbf{g}, \mathbf{g}_0^F\right) = 1$, then the regression coefficient is zero. If the market is right on average, $\beta^{DGP}\left(\mathbf{g}, \mathbf{g}_0^F\right) = 0$, then the regression coefficient is strictly negative [cf. Proposition 2]. If the truth is somewhere in between, $\beta^{DGP}\left(\mathbf{g}, \mathbf{g}_0^F\right) \in (0,1)$—arguably a reasonable assumption in a disagreement context—then the regression coefficient is still negative but smaller in magnitude.

Figure 5 illustrates the result by plotting the impulse responses to a Fed optimism shock under different realizations of the persistent component of demand, $\mathbf{g}$. We construct the impulse responses by setting all transitory demand shocks to zero: the realized aggregate demand is equal to the persistent component, $g_t = \mathbf{g}$ [cf. (9)]. If the market has the correct belief, $\mathbf{g}_0^M = \mathbf{g}$, the responses resemble a classical monetary policy shock: the interest rate increases and gradually declines, and the output gap declines and gradually recovers. If instead the Fed has the correct belief, $\mathbf{g}_0^F = \mathbf{g}$, the output gap remains at zero. Moreover, the interest rate keeps increasing—as the market learns from data and converges toward the Fed's more optimistic belief.

5.2. Market reaction shocks

In the previous subsection, the Fed's interest rate decision fully reveals its belief. In practice, beliefs are more complex and the interest rate might reveal the Fed's belief only partially. This leads to a second type of monetary policy shock that we refer to as "market reaction shocks."

To capture these shocks, we extend the setup to allow for short-term as well as long-term disagreement. In particular, suppose in (only) period 0 the Fed and the market can also disagree about the transitory component of demand, v_0. Specifically, the Fed believes $v_0 \sim N\left(\mathbf{v}_0^F, \Sigma\right)$, whereas the market believes $v_0 \sim N(0, \Sigma)$ as before. Suppose also that the market is uncertain about *both* dimensions of the Fed's belief: the market thinks $\mathbf{g}_0^F \sim N\left(\overline{\mathbf{g}}_0^F, \sigma_{\mathbf{g}}^2\right)$ and $\mathbf{v}_0^F \sim N\left(\overline{\mathbf{v}}_0^F, \sigma_{\mathbf{v}}^2\right)$, which are independent of each other. The parameters, $\sigma_{\mathbf{g}}^2, \sigma_{\mathbf{v}}^2$, denote the market's uncertainty about the Fed's long-term and short-term belief, respectively. As before, we use the notation Δx to denote the change of a variable in period 0 relative to its ex-ante expectation. In particular, $\Delta \mathbf{g}_0^F \equiv \mathbf{g}_0^F - \overline{\mathbf{g}}_0^F$ and $\Delta \mathbf{v}_0^F \equiv \mathbf{v}_0^F - \overline{\mathbf{v}}_0^F$ denote the change in the Fed's long-term and short-term optimism, respectively.

As a benchmark, consider what happens when either $\sigma_{\mathbf{g}} = 0$ (and $\overline{\mathbf{g}}_0^F = \mathbf{g}_0^F$) or $\sigma_{\mathbf{v}} = 0$ ($\overline{\mathbf{v}}_0^F = \mathbf{v}_0^F$) so that the market knows one dimension of the Fed's belief but is uncertain about the other dimension. In this case, adapting our analysis from the previous sections implies the equilibrium interest rate is given by,

$$i_0 = \rho + \mathbf{g}_0^F + \mathbf{v}_0^F + \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right). \tag{31}$$

In these corner cases, the equilibrium rate fully reveals the remaining dimension of the Fed's belief. For instance, if the market knows $\mathbf{g}_0^F$ it can back out $\mathbf{v}_0^F$ from the interest rate (and vice versa). Therefore, the equilibrium is as if there is no belief uncertainty. The Fed accommodates for its long-term disagreement with the market but not for short-term disagreement—since this type of disagreement does not persist. Therefore, a short-term Fed optimism shock has a sizeable impact on the current rate, but no impact on the forward curve,

$$\frac{\Delta i_0}{\Delta \mathbf{v}_0^F} = 1 \text{ and } \frac{\Delta E_0^M\left[i_t\right]}{\Delta \mathbf{v}_0^F} = 0 \text{ for } t \geq 1. \tag{32}$$

In contrast, a long-term Fed optimism shock has an impact on both the current rate and the forward curve as before [cf. (29)].

Set against these benchmarks, consider arbitrary $\sigma_{\mathbf{g}}^2, \sigma_{\mathbf{v}}^2$. Since the market is uncertain about both dimensions of the Fed's belief, the interest rate cannot fully reveal the Fed's belief. In this case, the equilibrium consists of two functions that depend on each other: one describes the Fed's optimal interest rate policy, i_0, and the other describes the market's Bayesian posterior belief, $E_0^M \left[\mathbf{g}_0^F | i_0 \right]$. Our next result characterizes these functions as follows,

$$\begin{aligned} i_0 &= \rho + \mathbf{g}_0^F + \mathbf{v}_0^F + \mathbf{c}_{0,1} \left(\mathbf{g}_0^M - E_0^M \left[\mathbf{g}_0^F | i_0 \right] \right) \\ &= E_0^M \left[i_0 \right] + \left(1 - \mathbf{c}_{0,1} \tau \right) \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right); \end{aligned} \tag{33}$$

$$\begin{aligned} E_0^M \left[\mathbf{g}_0^F | i_0 \right] &= \overline{\mathbf{g}}_0^F + \tau \frac{i_0 - E_0^M \left[i_0 \right]}{1 - \mathbf{c}_{0,1} \tau} \\ &= \overline{\mathbf{g}}_0^F + \tau \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right) \text{ where } \tau = \frac{\sigma_{\mathbf{g}}^2}{\sigma_{\mathbf{v}}^2 + \sigma_{\mathbf{g}}^2}. \end{aligned} \tag{34}$$

The first row in Eq. (33) describes the Fed's *optimal* interest rate policy. The Fed still accommodates the long-term disagreement with the market. However, the extent of accommodation depends on *the market's posterior belief for the Fed's long-term belief*, $E_0^M \left[\mathbf{g}_0^F | i_0 \right]$, rather than on the Fed's actual long-term belief, $\mathbf{g}_0^F$ [cf. (31)]. The next row describes the equilibrium interest rate—obtained by substituting the market's posterior belief along the equilibrium path.

The first row in Eq. (34) describes the market's *Bayesian* posterior belief given the interest rate it observes. The next row describes the Bayesian posterior along the equilibrium path, obtained by substituting the equilibrium interest rate. In equilibrium, the posterior is centered around the market's prior with an adjustment toward the change in *total* Fed optimism, $\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F$, *regardless of where that optimism comes from.* Higher-than-expected interest rates reveal a bundled signal of Fed optimism, but not whether the optimism is short term or long term. The market interprets the signal according to its relative uncertainty about the long-term belief, $\tau = \frac{\sigma_{\mathbf{g}}^2}{\sigma_{\mathbf{v}}^2 + \sigma_{\mathbf{g}}^2}$. When τ is high, the market is more uncertain about the long-term belief and attributes high interest rates to long-term optimism. Going forward, we refer to τ as *the market's reaction type.*

The following result verifies the equilibrium and describes how a change in Fed optimism affects the forward rates as well as the current interest rate in equilibrium.

Proposition 3. *Consider the setup with both long-term and short-term disagreement. Suppose the market believes* $\mathbf{g}_0^F \sim N\left(\overline{\mathbf{g}}_0^F, \sigma_{\mathbf{g}}^2 \right), \mathbf{v}_0^F \sim N\left(\overline{\mathbf{v}}_0^F, \sigma_{\mathbf{v}}^2 \right)$*, independent of each other. In equilibrium, the interest rate and the market's posterior belief are given by Eqs.* (33) *and* (34)*. An increase in Fed optimism*—regardless of whether it is long-term or short-term—*increases the current interest rate and the forward rates according to the market's reaction type* $\tau = \frac{\sigma_{\mathbf{g}}^2}{\sigma_{\mathbf{v}}^2 + \sigma_{\mathbf{g}}^2}$*:*

$$\frac{\Delta i_0}{\Delta \mathbf{v}_0^F + \Delta \mathbf{g}_0^F} = 1 - \tau \mathbf{c}_{0,1} \tag{35}$$

$$\frac{\Delta E_0^M \left[i_t \right]}{\Delta \mathbf{v}_0^F + \Delta \mathbf{g}_0^F} = \tau \mathbf{c}_{0,t} \left(1 - \mathbf{c}_{t,t+1} \right) \text{ for } t \geq 1. \tag{36}$$

This results says that *both the current and the forward interest rates are determined by the* *market's reaction type—rather than by the Fed's actual belief type.* Figure 6 illustrates the result for the case in which the market's reaction type is relatively high. The solid lines plot the current and forward rates' response to a change in (long-term or short-term) Fed optimism. For comparison, the dotted and the dashed lines plot the forward rates' response to an equivalent change in long-term and short-term Fed optimism, respectively, when the market knows the Fed's belief. With a reactive market, the equilibrium is similar to the case with long-term Fed optimism, *regardless of the Fed's actual belief type.* In particular, *even when the Fed has short-term optimism,* it hikes the current interest rate by a small amount (smaller than the increase in its optimism).[9]

Why does the market's reaction drive the equilibrium? Forward rates are naturally determined by the market's reaction, as these rates reflect the market's belief. The current rate is also determined by the market's reaction, because the Fed *optimally* responds to the market's reaction. As before, the Fed targets an overall increase in the forward curve—the current rate plus the shaded area in the figure—that counteracts its initial optimism. Since the forward rates increase substantially, a Fed with short-term optimism raises the current interest rate by a small amount—closer to the baseline case in which it has long-term optimism [cf. Figure 4].

Finally, note that the analogues of Eq. (30) and Corollary 1 also apply in this setting. Market reaction shocks (driven by either long-term or short-term Fed beliefs) generate the conventional effects of monetary policy shocks on financial market outcomes but not necessarily on subsequent real outcomes.

Remark 2 (Interpreting the evidence on monetary policy shocks and forward rates)**.** *Empirical studies with high-frequency identification typically find that the forward interest rates' reaction to monetary policy shocks can be captured with a small number of factors. For instance, Gürkaynak et al. (2005b) emphasize a (Fed funds rate)* target factor *that captures the surprise changes to near-term interest rates, and an orthogonalized* path factor *that captures surprises to longer-term interest rates (see also Hamilton (2008); Barakchian and Crowe (2013)). These factors can be mapped into our analysis. A target factor can be thought of as a market reaction shock with an average $\overline{\tau}$—that is, the market attributes the Fed's belief change to an average horizon. In contrast, a path factor can be thought of as the market reaction relative to the average, $\tau - \overline{\tau}$—the market attributes the belief change to a longer or shorter horizon than usual.*[10] *In line with*

[9] In a memorable 2005 lecture, then BOE's Governor Mervyn King, wrote: "Maradona ran 60 yards from inside his own half beating five players before placing the ball in the English goal. The truly remarkable thing, however, is that, Maradona ran virtually in a straight line. How can you beat five players by running in a straight line? The answer is that the English defenders reacted to what they expected Maradona to do. Because they expected Maradona to move either left or right, he was able to go straight on.

Monetary policy works in a similar way. Market interest rates react to what the central bank is expected to do. In recent years the Bank of England and other central banks have experienced periods in which they have been able to influence the path of the economy without making large moves in official interest rates. They headed in a straight line for their goals. How was that possible? Because financial markets did not expect interest rates to remain constant. They expected that rates would move either up or down. Those expectations were sufficient—at times—to stabilise private spending while official interest rates in fact moved very little" (King (2005)).

[10] Equivalently, assuming the market knows the Fed's belief, we could modify the model slightly to interpret

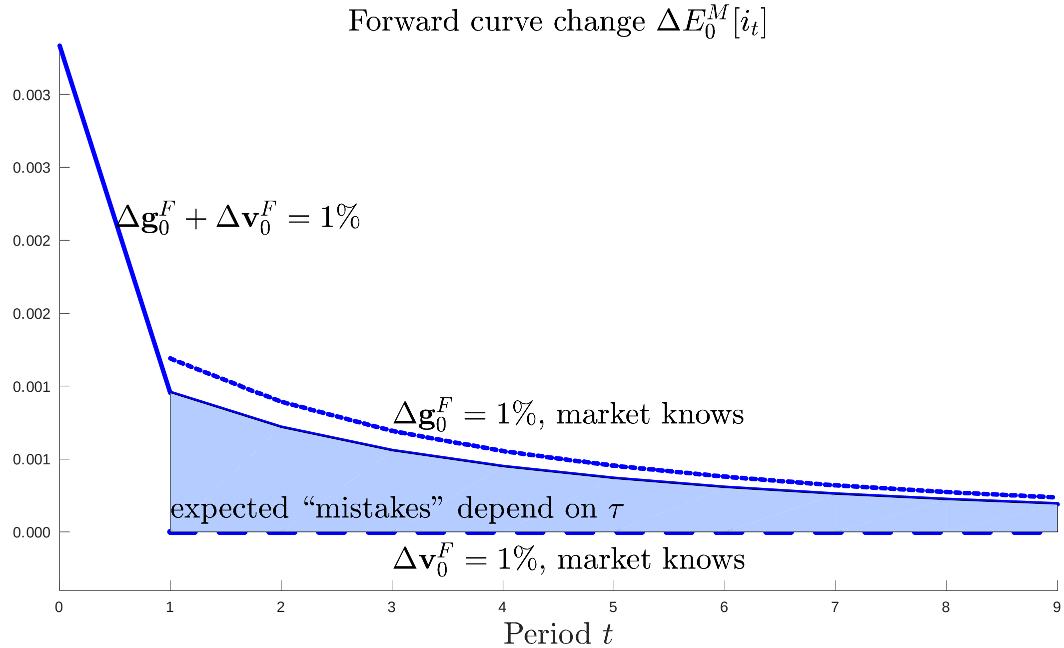

Figure 6: **Market reaction shock** from an increase in (long-term or short-term) Fed optimism when the market is relatively reactive (high τ). The solid line plots the current and the forward rates' response. The dotted (resp. the dashed) line plots the forward rates' response to an equivalent long-term (resp. short-term) optimism shock that the market knows.

this interpretation, empirical studies find that the path factor that drives the long-term interest rates is mainly driven by the content of Fed statements (see, e.g., Lucca and Trebbi (2011)).

5.3. Tantrum shocks

In the previous subsection, the Fed knows the market's reaction type τ and understands how the forward rates will change in response to its interest rate decisions. In practice, the Fed might be confused or uncertain about the market's reaction. This leads to a third type of monetary policy shock that we refer to as "tantrum shocks." These shocks are costlier (under the Fed's belief) than the previous shocks. Their anticipation induces the Fed to move more gradually and creates a role for Fed communication.[11]

these factors as appropriate Fed belief shocks—rather than market reaction shocks. Suppose the two dimensions of the Fed's belief change according to, $\mathbf{g}_0^F = \overline{\mathbf{g}}_0^F + \tau \Delta b^F$ and $\mathbf{v}_0^F = \overline{\mathbf{v}}_0^F + (1-\tau)\,\Delta b^F$, where Δb^F captures the change in Fed optimism and $\tau \in (0,1)$ captures the extent to which optimism concerns the long term rather than the short term. Suppose Δb^F and τ are independent random variables (realized at the beginning of the period and observed by the market) with mean 0 and $\overline{\tau}$, respectively. Then, the target factor would reflect the forward-rate impact of Δb^F given $\overline{\tau}$, and the path factor would reflect the impact of $(\tau - \overline{\tau})$ given Δb^F.

[11] On May 23, 2013, the day after Fed's Chairman Bernanke's testimony to Congress that touched off the "Taper Tantrum" episode, the WSJ wrote: "...The next step by the Fed could be especially tricky. One worry at the central bank is that a single small step to shrink the size of the program could be interpreted by investors as the first in a larger move to end it altogether. Mr. Bernanke sought to dispel that view, part of a broader effort by Fed officials to manage market expectations. If the Fed takes one step to reduce the bond buying, it won't mean the Fed is 'automatically aiming towards a complete wind-down,' Mr. Bernanke said. 'Rather we would be

We start with an extreme case in which the Fed does not anticipate tantrum shocks, which is useful to illustrate how the shocks affect the equilibrium. Formally, suppose the market is either very unreactive or very reactive, $\tau \in \{0,1\}$. The Fed thinks the market is unreactive ($\tau = 0$), whereas the market is actually reactive ($\tau = 1$). We also assume that both the unreactive and the reactive types think the Fed knows their type. The rest of the model is unchanged.

In this case, the Fed sets the interest rate according to (33) with $\tau = 0$, which implies,

$$i_0 = E_0^M [i_0] + \Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \tag{37}$$

An optimistic Fed hikes the interest rate by the full amount of its optimism. Since the Fed thinks the market is unreactive, it expects the market to receive the optimism signal, $\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F$, and to attribute it to short-term optimism [cf. Eq. (34) with $\tau = 0$].

However, the market reacts to the Fed's interest rate decision very differently than what the Fed anticipates. Since the market is reactive (and thinks the Fed knows this), it expects the Fed to set the interest rate according to (33) with $\tau = 1$, which implies,

$$i_0|_{\tau=1} = E_0^M [i_0] + (1 - \mathbf{c}_{0,1}) \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right).$$

In particular, the market expects an optimistic Fed to change the interest rate by a small amount. Therefore, after the market observes the interest rate in (37), it extracts a *larger* optimism signal, $(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F)/(1 - \mathbf{c}_{0,1})$. Moreover, the market attributes this signal to *long-term* optimism, so its posterior belief is given by [cf. (34)],

$$E_0^M \left[\mathbf{g}_0^F | i_0 \right] = \overline{\mathbf{g}}_0^F + \frac{\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F}{1 - \mathbf{c}_{0,1}}. \tag{38}$$

The forward curve and the market's expected output gap in equilibrium are then given by Eqs. (27) and (28) after substituting $\mathbf{g}_0^F$ with the posterior belief, $E_0^M \left[\mathbf{g}_0^F | i_0 \right]$.

In particular, Eqs. (27) and (38) imply,

$$\Delta E_0^M [i_t] = \frac{\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F}{1 - \mathbf{c}_{0,1}} \mathbf{c}_{0,t} \left(1 - \mathbf{c}_{t,t+1}\right) \text{ for } t \geq 1, \tag{39}$$

where $\Delta E_0^M [i_t] \equiv E_0^M [i_t | i_0] - E_0^M [i_t]$, as before. In equilibrium, an increase in Fed optimism raises the forward curve substantially—more than the case in which the market knows the Fed's belief [cf. (29)]. The top panel of Figure 7 illustrates this result. The solid line shows the actual change in the forward curve in period 0, whereas the dashed line is the change the Fed had anticipated when it announced the interest rate policy in period 0.

Likewise, Eqs. (28) and (38) imply that an increase in Fed optimism reduces the market's

looking beyond that to seeing how the economy evolves and we could either raise or lower our pace of purchases going forward. Again that is dependent on the data,' he said."

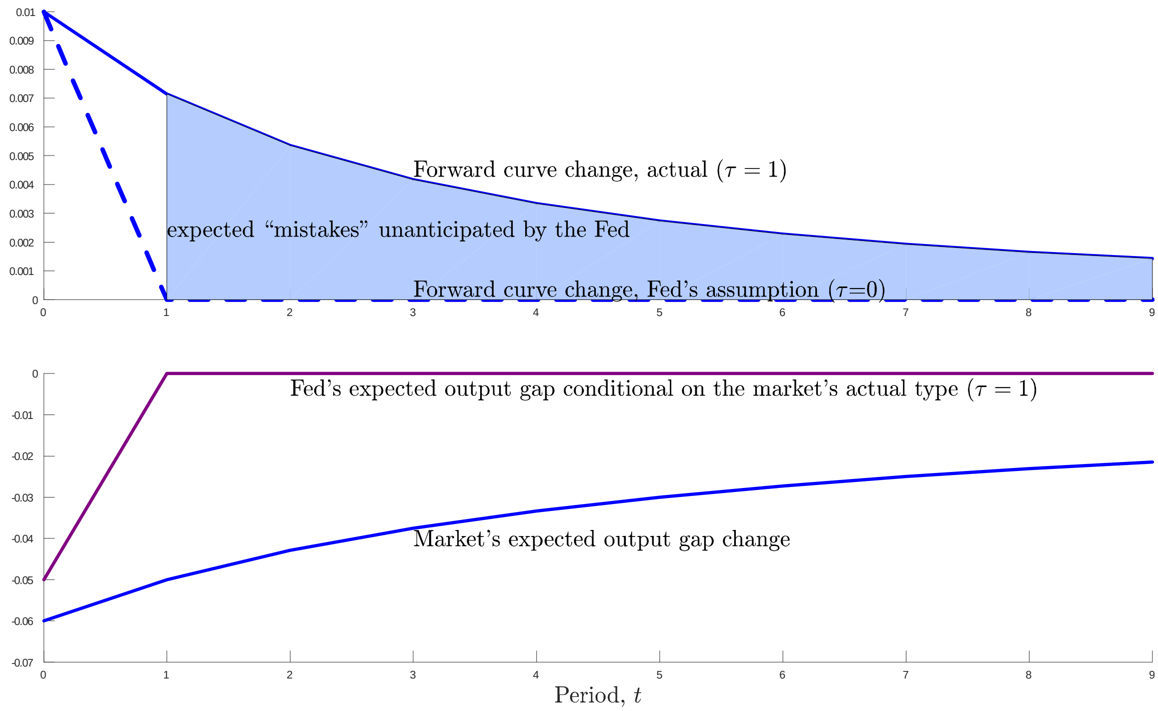

Figure 7: **Tantrum shock** from a 1% increase in (short-term or long-term) Fed optimism. The Fed thinks the market is not reactive, $\tau = 0$, when the market is actually reactive, $\tau = 1$. The solid (resp. the dashed) blue lines plot the actual realization (resp. the Fed's assumption) for the change in the market's expectations in period 0. The purple line in the bottom panel plots the Fed's expected output gap in period 0 conditional on the market's actual reaction type.

expected output gap substantially,

$$\Delta E_0^M [\tilde{y}_t] = -\mathbf{c}_{0,t} \frac{\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F}{1 - \mathbf{c}_{0,1}}. \tag{40}$$

This decline is greater than the perceived decline that would arise if the market knew the Fed's belief, $\Delta E_0^M [\tilde{y}_t] = -\mathbf{c}_{0,t} \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right)$. In fact, since the Fed sets i_0 under the wrong assumption about the market's reaction type, τ, it misses its expected output gap in period 0 *even under its own belief*,

$$E_0^F [\tilde{y}_0 | \tau = 1] = \Delta E_0^M [\tilde{y}_0] + \Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F = -\frac{\mathbf{c}_{0,1} \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right)}{1 - \mathbf{c}_{0,1}} < 0. \tag{41}$$

Here, $E_0^F [\tilde{y}_0 | \tau = 1]$ denotes the Fed's expected output gap given the market's actual reaction type. For subsequent periods $t \geq 1$, the Fed expects to hit its target, $E_0^F [\tilde{y}_t | \tau = 1] = 0$, because it has learned that the market is reactive and will adjust its future policy appropriately. The bottom panel of Figure 7 illustrates these results.

In this extreme case, the Fed operates under the assumption that the market is unreactive and will interpret its interest rate change as temporary. Thus, the Fed is surprised when the market is revealed to be reactive. This large reaction increases the forward curve substantially more than what the Fed anticipated when it set the policy rate. This makes the Fed miss its expected output gap target—even according to its own belief [cf. Figure 7]. Note also that the market's reaction is stronger—and the output gap is more negative—when the relative confidence, $\mathbf{c}_{0,1}$, is higher [cf. (38 – 41)]. In this case, the optimal policy with respect to a reactive market requires only a small adjustment of the interest rate. Therefore, adjusting the interest rate as-if the market is unreactive can be very costly.

5.4. Gradualism

In the previous subsection, we assumed the Fed sets the interest rate under incorrect beliefs about the market's reaction type, τ. We also assumed the market (incorrectly) assumes the Fed knows its τ. These assumptions simplified the analysis and are arguably relevant for episodes of extreme market reaction (such as the 2013 "Taper Tantrum" episode). However, tantrum shocks are costly even without these extreme assumptions. We next consider the case in which the Fed sets the interest rate policy under uncertainty about the market's type, and the market knows that the Fed is uncertain (so neither agent has incorrect views). In this case, the equilibrium features milder tantrum shocks that still reduce the Fed's welfare (under its own belief). Moreover, the fear of these shocks induces the Fed to act *more gradually* than in our baseline setting (see Remark 1). As emphasized by Brainard (1967), the Fed faces uncertainty about how a change in the policy rate will affect the economy, which induces it to act more conservatively.

Formally, suppose the Fed believes the market has the reactive type, $\tau = 1$, with probability $\delta \in (0, 1)$, and the unreactive type, $\tau = 0$, with probability $1 - \delta$. The market knows δ. The rest of the model is unchanged.

In this case, our next result shows the optimal interest rate and the market's posterior belief are given by the following analogues of Eqs. (33 – 34),

$$\begin{aligned} i_0 &= \rho + \mathbf{g}_0^F + \mathbf{v}_0^F + \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - \left\{\begin{array}{c} \tilde{\delta} E_0^M\left[\mathbf{g}_0^F | i_0, \tau = 1\right] \\ + \left(1 - \tilde{\delta}\right) E_0^M\left[\mathbf{g}_0^F | i_0, \tau = 0\right] \end{array}\right\}\right) \quad (42) \\ &= E_0^M\left[i_0\right] + \left(1 - \mathbf{c}_{0,1}\tilde{\delta}\right)\left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F\right) \text{ where } \tilde{\delta} > \delta \text{ solves } (B.5)\,; \\ E_0^M\left[\mathbf{g}_0^F | i_0, \tau\right] &= \overline{\mathbf{g}}_0^F + \tau \frac{i_0 - E_0^M\left[i_0\right]}{1 - \mathbf{c}_{0,1}\tilde{\delta}} = \overline{\mathbf{g}}_0^F + \tau\left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F\right) \text{ where } \tau = \frac{\sigma_{\mathbf{g}}^2}{\sigma_{\mathbf{v}}^2 + \sigma_{\mathbf{g}}^2}. \quad (43) \end{aligned}$$

Here, the parameter, $\tilde{\delta}$, is the solution to a quadratic equation that we relegate to the appendix.

Eq. (43) is the same as in the setup in which the Fed knows the market's reaction type. Along the equilibrium path, the market extracts a bundled optimism signal and forms a posterior belief that depends on its reaction type. Eq. (42) is different and says that the Fed accommodates the long-term disagreement with the market according to a *weighted-average* of the market's posterior beliefs over the cases in which the market is reactive and unreactive. Importantly, the

Fed *overweights* the case in which the market is reactive relative to its perceived prior probability of this case, $\tilde{\delta} > \delta$. Consequently, we also have $1 - \mathbf{c}_{0,1}\tilde{\delta} < 1 - \mathbf{c}_{0,1}\delta$ (recall that $\delta = E_0^F[\tau]$). That is, *the Fed acts more gradually than in a "certainty-equivalent" benchmark in which the market's reaction type is certain and equal to the Fed's ex-ante mean for the type* [cf. (33)].

Relatedly, and unlike any of the cases we have analyzed so far, Eq. (7) does not apply: the Fed *does not hit its output target on average.* Instead, we show in the appendix that the Fed's *ex-ante* expected output gap satisfies,

$$E_0^F[\tilde{y}_0] = \left(\tilde{\delta} - \delta\right)\mathbf{c}_{0,1}\left(\Delta\mathbf{g}_0^F + \Delta\mathbf{v}_0^F\right). \tag{44}$$

Since $\tilde{\delta} > \delta$, this expression implies that the Fed *does not fully stabilize the expected output gap changes that result from its belief changes.* For instance, when the Fed becomes more optimistic, $\Delta\mathbf{g}_0^F + \Delta\mathbf{v}_0^F > 0$, the Fed hikes the interest rate less than in the previous sections and allows for a positive output gap on average. We also characterize the Fed's output gap *conditional on the market's type* as,

$$E_0^F[\tilde{y}_0|\tau = 1] = -\left(1 - \tilde{\delta}\right)\mathbf{c}_{0,1}\left(\Delta\mathbf{g}_0^F + \Delta\mathbf{v}_0^F\right) \text{ and } E_0^F[\tilde{y}_0|\tau = 0] = \tilde{\delta}\mathbf{c}_{0,1}\left(\Delta\mathbf{g}_0^F + \Delta\mathbf{v}_0^F\right). \tag{45}$$

An optimistic Fed expects a negative output gap when the market is revealed to be reactive—a milder version of the tantrum shocks from the previous section—and a positive output gap when the market is unreactive. The following result verifies the equilibrium.

Proposition 4. *Consider the setup in Proposition 3 with the differences that the market has one of two types, $\tau \in (0,1)$; the Fed believes $\tau = 1$ with probability $\delta \in (0,1)$; and the market knows δ. In equilibrium, the interest rate and the market's posterior belief are given by Eqs.* $(42-43)$, *where $\tilde{\delta} \in (\delta, 1)$ is the solution to Eq. $(B.5)$ in the appendix. The Fed acts as if the market is more reactive than implied by its ex-ante mean, $\tilde{\delta} > \delta = E_0^F[\tau]$. The Fed's ex-ante expected output gap is (typically) non-zero and given by* (44). *The Fed's output gap conditional on the market's type is also (typically) non-zero and given by,* (45).

Why does the Fed act more gradually than before and miss its expected output gap on average? Unlike in the previous sections, the Fed is uncertain about how a *change* in its policy interest rate i_0 will affect the output gap. As illustrated by Eq. (43), if the market is reactive, an interest rate hike increases the market's perception for the Fed's long-term optimism. Consequently, the interest rate hike also reduces the market's expected future output gap, $\frac{dE_1^M[\tilde{y}_1|i_0,\tau=1]}{di_0} < 0$. In view of the IS curve (2), this creates a large impact on the current output gap, $\frac{d\tilde{y}_0[\tau=1]}{di_0} < -1$. In contrast, if the market is unreactive, an interest rate hike does not change the market's expected future output gap, $\frac{dE_1^M[\tilde{y}_1|i_0,\tau=0]}{di_0} = 0$, and it has a smaller impact on the current output gap, $\frac{d\tilde{y}_0[\tau=0]}{di_0} = -1$. Since the economy is more sensitive to its interest rate decision when the market is reactive, the Fed overweights that case in its decision, $\tilde{\delta} > \delta$ [cf. Eq. (4)]. Therefore, the Fed acts as if the market is more reactive than implied by its prior mean, and adjusts the interest rate by a small amount. By acting conservatively, the Fed misses its output gap on average but it mitigates the tantrum shock that exacerbates its miss when the market is revealed to be reactive [cf. Eqs. $(44-45)$].

Note also that, despite acting conservatively, the Fed misses its output gap conditional on the market's type. Therefore, the possibility of tantrum shocks lowers the Fed's ex-ante objective in (3). When the market is uncertain about the Fed's belief, its reaction type τ becomes a key parameter for policy. If the Fed is confused about τ, there can be extreme tantrum shocks as in the previous subsection. If the Fed is uncertain about τ, there are still (milder) tantrum shocks that make the Fed miss its output target more often than without these shocks.

5.5. Fed communication

The welfare losses induced by tantrum shocks create a natural role for communication between the Fed and the market. First, the Fed can try to figure out the market's reaction type τ. Second, and perhaps more simply, the Fed can try to reveal its own belief to the market—mitigating the market reaction shocks and therefore also the tantrum shocks. In an early and insightful analysis, Blinder (1998) emphasized this mechanism as the key benefit of central bank communication:

> Greater openness might actually improve the efficiency of monetary policy... [because] expectations about future central bank behavior provide the essential link between short rates and long rates. A more open central bank... naturally conditions expectations by providing the markets with more information about its own view of the fundamental factors guiding monetary policy..., thereby creating a virtuous circle. By making itself more predictable to the markets, the central bank makes market reactions to monetary policy more predictable to itself. And that makes it possible to do a better job of managing the economy.

Our next result formalizes Blinder's insight. In our model with two belief types, the Fed can reveal its belief by announcing the average interest rate it plans to set in the next period in addition to the current rate.

Proposition 5. *Consider the setup in Propositions 3 and 4, with both long-term and short-term disagreement and the market uncertainty about the Fed's beliefs. Suppose in period* 0 *the Fed announces both* i_0 *and* $E_0^F[i_1]$*. In equilibrium, the Fed's interest announcements are given by*

$$\begin{aligned} i_0 &= \rho + \mathbf{g}_0^F + \mathbf{v}_0^F + \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right), \\ E_0^F[i_1] &= \rho + \mathbf{g}_0^F + \mathbf{c}_{0,2}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right). \end{aligned}$$

These announcements fully reveal both dimensions of the Fed's belief, $\mathbf{v}_0^F, \mathbf{g}_0^F$*. Therefore, the Fed belief shocks affect the equilibrium according to* the Fed's actual belief *[cf.* $(29-30)$ *and*

(32)]—*rather than the market's reaction type* $\tau = \frac{\sigma_{\mathbf{g}}^2}{\sigma_{\mathbf{v}}^2+\sigma_{\mathbf{g}}^2}$. *The Fed hits the output gap on average (according to its belief) regardless of its belief and the market's type,* $E_0^F[\tilde{y}_0] = E_0^F[\tilde{y}_0|\tau] = 0$.

This result provides a rationale for the enhanced Fed communication that we have seen in recent years (e.g., the dot curves). In our model, the role of these policies is *not* to persuade the market—the market is opinionated. Rather, communication is useful because it helps to reveal the Fed's belief to the market, reducing the chance of tantrum shocks in which the market misinterprets the Fed's belief.

6. Disagreements and inflation

So far, we have assumed nominal prices are fully sticky, $\kappa = 0$. In this section, we consider the case with partial price flexibility. We show that disagreements create a policy trade-off between stabilizing output and inflation that reinforces our earlier findings. In particular, the Fed accommodates the market's belief more than in the earlier sections with fully sticky prices. We further show that, for optimal policy purposes, disagreements closely resemble the cost push shocks in a textbook New Keynesian model. For simplicity, we focus on the baseline setup in which agents know each other's beliefs and "agree-to-disagree."[12]

As in Section 4, we conjecture an equilibrium in which each agent's expected output gap and inflation, $E_t^j[\tilde{y}_t]$ and $E_t^j[\pi_t]$, evolve deterministically. The difference is that inflation is not necessarily zero and is determined by the NKPC [cf. (1)],

$$\pi_t = \kappa \tilde{y}_t + \beta E_{t+1}^M[\pi_{t+1}].$$

Considering the equation (for period $t+1$) under each agent's belief and taking the difference, we obtain the key equation of this section,

$$\begin{aligned} E_{t+1}^M[\pi_{t+1}] &= E_{t+1}^F[\pi_{t+1}] + \kappa\left(E_{t+1}^M[\tilde{y}_{t+1}] - E_{t+1}^F[\tilde{y}_{t+1}]\right) \\ &= E_{t+1}^F[\pi_{t+1}] + \kappa\left(\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right). \end{aligned} \quad (46)$$

Here, the second line uses Eq. (23) which applies also in this context. Eq. (46) shows that the market can expect inflation or disinflation, $E_{t+1}^M[\pi_{t+1}] \neq 0$, even if the Fed were to set expected inflation to zero according to its own belief. Intuitively, with disagreements, the market thinks the Fed will make "mistakes" and won't be able to stabilize inflation. Since price setters are forward looking, expected inflation creates inflationary pressure in the current period as well.

[12]There is an important caveat for this section. Up to now we have referred to the private sector as *the market*, because we think in practice financial market participants' expectations, rather than the households' or the firms' expectations, determine the interest rates and asset prices (and households' consumption decisions are affected indirectly through asset prices). Instead, the new results in this section depend on the expectations of the price setters, which in practice would correspond to a different set of agents, e.g., firms, workers, or labor unions. We will ignore this distinction and leave this dimension of heterogeneity for future work.

Consequently, the divine coincidence breaks down: the Fed cannot simultaneously set average inflation and output to zero.

Next consider how the Fed trades off inflation and output. Problem (3) and the NKPC (1) imply the Fed chooses a particular split between expected output and inflation, $E_t^F[\tilde{y}_t] = -\frac{\kappa}{\phi}E_t^F[\pi_t]$. Combining this with the NKPC (under the Fed's belief), we obtain,

$$E_t^F[\pi_t] = \Theta\beta E_{t+1}^M[\pi_{t+1}] \tag{47}$$

$$E_t^F[\tilde{y}_t] = -\frac{1-\Theta}{\kappa}\beta E_{t+1}^M[\pi_{t+1}] \text{ where } \Theta = \frac{\phi}{\phi+\kappa^2}. \tag{48}$$

Here, $\beta E_{t+1}^M[\pi_{t+1}]$ captures the current inflationary pressure that results from the market's expected inflation. The composite parameter, $\Theta \in [0,1]$, captures the extent to which the Fed responds to the pressure by stabilizing output relative to inflation. As expected, the Fed focuses on output relatively more when it puts more weight on the output gap (greater ϕ) and when the nominal prices are more sticky (smaller κ).

Eqs. (46) and (47) characterizes inflation expectations as the solution to a recursive equation. Our main result in this section describes how the solution depends on disagreements between the Fed and the market.

Proposition 6. *Suppose prices are partially flexible, $\kappa > 0$. In equilibrium, the market's expected inflation is deterministic and given by,*

$$\begin{aligned} E_t^M[\pi_t] &= \kappa\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) + \Theta\beta E_{t+1}^M[\pi_{t+1}] \\ &= \sum_{n=0}^{\infty}(\Theta\beta)^n\kappa\left(\mathbf{g}_{t+n}^M - \mathbf{g}_{t+n}^F\right), \end{aligned} \tag{49}$$

where $\Theta = \frac{\phi}{\phi+\kappa^2}$ and $\mathbf{g}_t^M - \mathbf{g}_t^F = \mathbf{c}_{0,t}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right)$. The Fed's expected inflation and output gap are given by Eqs. (47) and (48). When the market is more optimistic, $\mathbf{g}_0^M > \mathbf{g}_0^F$, the market expects inflation, $E_{t+1}^M[\pi_{t+1}] > 0$, and the Fed induces (on average) inflation and negative output gaps, $E_t^F[\pi_t] > 0, E_t^F[\tilde{y}_t] < 0$. Conversely, when the market is more pessimistic, $\mathbf{g}_0^M < \mathbf{g}_0^F$, the market expects disinflation, $E_{t+1}^M[\pi_{t+1}] < 0$, and the Fed induces (on average) disinflation and positive output gaps, $E_t^F[\pi_t] < 0, E_t^F[\tilde{y}_t] > 0$.

When the market is more optimistic than the Fed, it expects positive output gaps ("too low interest rates") as in our earlier analysis. Expectations of positive output gaps translate into expected inflation. In turn, expected inflation exerts upward pressure on current inflation. When the market is more pessimistic than the Fed, the situation is the opposite.

We next characterize the equilibrium interest rate and generalize our main result from Section 4 [cf. Proposition 1]. Using Eqs. (2) and (48), we obtain,

$$r_t \equiv i_t - E_{t+1}^M[\pi_{t+1}] = \rho + \mathbf{g}_t^F + E_{t+1}^M[\tilde{y}_{t+1}] + \frac{1-\Theta}{\kappa}\beta E_{t+1}^M[\pi_{t+1}].$$

Here, r_t denotes the *real* interest rate the Fed sets to target its desired output gap. The first three terms on the right side are similar to their counterparts with fully sticky prices [cf. (24)]. The last term is new and captures the Fed's concern with stabilizing inflation. As before, output gap expectations satisfy Eq. (23). These observations imply the following.

Corollary 2. *The* real *interest rate corresponding to the equilibrium in Proposition 6 is,*

$$r_t = r_t^{sticky} + \frac{1-\Theta}{\kappa}\beta\left(E_{t+1}^M\left[\pi_{t+1}\right] - E_{t+2}^M\left[\pi_{t+2}\right]\right),$$

where $r_t^{sticky} = \rho + \left(1-\mathbf{c}_{t,t+1}\right)\mathbf{g}_t^F + \mathbf{c}_{t,t+1}\mathbf{g}_t^M$ *is the interest rate with fully sticky prices [cf. Proposition 1]. The interest rate reflects the market's belief relatively more than the case with fully sticky prices,*

$$\begin{cases} r_t > r_t^{sticky} & \text{when } \mathbf{g}_0^M > \mathbf{g}_0^F \\ r_t < r_t^{sticky} & \text{when } \mathbf{g}_0^M < \mathbf{g}_0^F \end{cases}.$$

For intuition, consider the case in which the market is more optimistic than the Fed. In this case, Proposition 1 says the market expects positive inflation, $E_{t+1}^M\left[\pi_{t+1}\right] > 0$ (and more so in earlier periods, $E_{t+1}^M\left[\pi_{t+1}\right] > E_{t+2}^M\left[\pi_{t+2}\right]$). Since the Fed is concerned with stabilizing inflation, it sets a higher rate than before. Effectively, this brings the interest rate closer to the level implied by the market's optimistic belief. Conversely, when the market is pessimistic and expects disinflation, the Fed sets a lower interest rate that reflects relatively more the market's pessimistic belief.

This result reinforces our earlier analysis and provides a complementary reason for why the Fed accommodates the market's belief. With fully sticky prices, the market's perceived monetary policy "mistakes" translate into future output gaps. This exerts pressure on the current output gap (via the IS curve) and forces the Fed's hand. With partially flexible prices, perceived "mistakes" translate into future inflation. This exerts pressure on the current inflation (via the NKPC) and forces the Fed's hand through a second channel.

Relationship to cost-push shocks. In the textbook New Keynesian model, the NKPC is usually augmented with cost-push shocks: a catchall for factors other than output gaps and inflation expectations that might affect firms' price setting. In our model, disagreements closely resemble cost-push shocks *from the Fed's perspective.* Therefore, our model inherits the optimal policy implications of cost-push shocks.

To illustrate this connection, we rewrite the NKPC (1) and substitute Eq. (46) to obtain,

$$\begin{aligned} \pi_t &= \kappa\tilde{y}_t + \beta E_{t+1}^F\left[\pi_{t+1}\right] + u_{t+1} \\ \text{where } u_{t+1} &\equiv \beta\left(E_{t+1}^M\left[\pi_{t+1}\right] - E_{t+1}^F\left[\pi_{t+1}\right]\right) = \beta\kappa\left(\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right). \end{aligned}$$

Therefore, the NKPC *under the Fed's belief* features an "as-if" cost-push shock—even though the actual NKPC (under the market's belief) features no such shock. Moreover, the as-if cost-push shock is positive, $u_{t+1} > 0$ (resp. negative $u_{t+1} < 0$) when the market is more optimistic, $\mathbf{g}_0^M > \mathbf{g}_0^F$ (resp. more pessimistic, $\mathbf{g}_0^M < \mathbf{g}_0^F$). This provides a complementary intuition for Proposition 6.

For the limit with high confidence, $C_0 \to \infty$, disagreements remain constant over time, $\mathbf{g}_t^M - \mathbf{g}_t^F = \mathbf{g}_0^M - \mathbf{g}_0^F$. In this case, there is a tighter relationship between our model and the textbook model. In particular, the equilibrium is *identical* to a corresponding equilibrium analyzed by Clarida et al. (1999) with an appropriate as-if cost-push shock.

Corollary 3. *Consider the equilibrium characterized in Proposition 6. When confidence is high, $C_0 \to \infty$, the Fed's expected output gap and inflation are constant over time and given by,*

$$\begin{aligned} E_t^F[\tilde{y}_t] &= -\frac{\kappa}{\kappa^2 + \phi(1-\beta)} u. \\ E_t^F[\pi_t] &= \frac{\phi}{\kappa^2 + \phi(1-\beta)} u \text{ where } u = \beta\kappa\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right) \end{aligned}$$

These expressions are the same as Eqs. (3.4) and (3.5) in Clarida et al. (1999) for the case with a fully persistent exogenous cost-push shock (after appropriately adjusting the notation).

This result implies the optimal policy in our model shares some of the properties in Clarida et al. (1999). In particular, the Fed can benefit from committing to put a higher relative weight on inflation than implied by its own preferences in (3). Intuitively, since inflation is forward looking, committing to stabilize future inflation aggressively also helps to stabilize current inflation. We leave a more complete analysis of the benefits of commitment in our setting for future work.

7. Final remarks

After illustrating the occasional large differences between FOMC interest rate predictions and the forward curve, we proposed a macroeconomic model where these differences emerge from disagreements about expected aggregate demand. We then studied the implications of these disagreements for monetary policy, the term structure of interest rates, and economic activity. The key feature of the environment is that the market constantly expects the Fed to make "mistakes" (under the market's belief). The anticipation of future "mistakes" affects current output and forces the Fed to partially accommodate the market's belief to stabilize the output gap. In particular, "rstar" (the natural interest rate) reflects the extent of disagreement as well as how entrenched each agent's beliefs are. The more entrenched the beliefs, the more the Fed needs to weight the market's belief. Partial price flexibility strengthens this result since perceived "mistakes" create inflationary or disinflationary pressures that induce the Fed to accommodate the market's belief even more. The Fed plans to implement its own belief gradually, as it expects the market to learn over time and move closer to the Fed's belief. The

Fed and the market disagree about future interest rates, as in the data, because agents expect the other agent to come to their own belief.

The model generates microfounded monetary policy shocks because the market learns the Fed's belief—and therefore the extent of future "mistaken" interest rate changes—from surprise policy announcements. These shocks come in different flavors that depend on the nature of the market's uncertainty about the Fed's belief. Fed belief shocks and market reaction shocks are benign (under the Fed's belief) as long as the Fed anticipates the market's reaction and embeds it into its interest rate policy. Tantrum shocks are more damaging, as they arise when the Fed fails to forecast the market's reaction. These shocks motivate additional gradualism as well as communication policies that reveal the Fed's belief. The reason for communication in our environment is *not* to persuade the market about the state of the economy—the market is opinionated. Rather, communication is useful because it helps to reveal the Fed's belief to the market, reducing the chance of tantrum shocks in which the market misinterprets the Fed's belief.

Our monetary policy shocks generate *contingent* effects on subsequent economic activity depending on whether the Fed's or the market's belief is closer to the objective belief. Fed belief shocks generate the textbook impulse responses for the output gap and inflation according to the market's belief, but not according to the Fed's belief. Tantrum shocks can generate the textbook impulse responses under both beliefs. While we do not test these empirical predictions, our results are in line with the empirical findings that monetary policy shocks seem to have a smaller effect on economic activity—and sometimes with flipped signs—after mid-1980s compared to earlier years (see, e.g., Boivin and Giannoni (2006); Barakchian and Crowe (2013); Ramey (2016)). One interpretation is that greater central bank transparency in recent years has made tantrum shocks rarer. It is also possible that the Fed's belief has become more accurate over time.

For simplicity, we assumed that both the Fed and the market were equally confident in their beliefs. In Appendix B.1, we analyze the case when they are not. Heterogeneous confidence leads to heterogeneous updating of beliefs as new macroeconomic shocks arrive. This implies that every demand shock now comes bundled with a market anticipation of a Fed belief shock. This bundling changes the impact of demand shocks on economic activity. Specifically, if the Fed is more data sensitive (less confident) than the market, then a positive demand shock has a dampened effect on output and asset prices—as the market anticipates interest rates to overreact due to the embedded Fed belief shock. Conversely, if the Fed is less data sensitive (more confident), then the market expects the interest rates to underreact, and the demand shock has an amplified effect on output and asset prices. We leave a fuller analysis of bundled shocks for future research.

Finally, it is important to clarify that the optimal policy we have characterized does *not* mean that the Fed should "surrender" to the market and avoid surprises at all costs. Instead, the optimal policy says that disagreements and surprises are normal, as long as the policy itself considers the effect of disagreement and surprises on output and inflation stabilization. Concretely, suppose that the Fed (but not the market) receives divine information that the long run "rstar" has risen by 100 basis points. If the market had received the same information or fully trusted the infinite wisdom (or divine connections) of the Fed, the optimal policy would be to hike the target rate immediately by 100bps. Instead, in our environment the market is opinionated, so the Fed knows that if it raises the rate by 100bps in one shot, it will trigger a much larger contraction in aggregate demand than it seeks. Thus, the Fed optimally raises the target rate by only 25bps today and it anticipates that it will continue raising rates by 25bps for three more meetings. This expected gradualism arises because the Fed expects the future data to confirm its belief. The Fed thinks that, by the next meeting, the market will update toward the Fed's belief and will expect smaller "mistakes" than it did after the previous hike, which will create more room for the Fed to raise rates in subsequent meetings. Rather than "surrendering" to the market, the Fed plans to implement its own belief more gradually.

References

Amato, J. D., Morris, S., Shin, H. S., 2002. Communication and monetary policy. Oxford Review of Economic Policy 18 (4), 495–503.

Andrade, P., Crump, R. K., Eusepi, S., Moench, E., 2016. Fundamental disagreement. Journal of Monetary Economics 83, 106–128.

Andrade, P., Gaballo, G., Mengus, E., Mojon, B., 2019. Forward guidance and heterogeneous beliefs. American Economic Journal: Macroeconomics 11 (3), 1–29.

Angeletos, G.-M., Huo, Z., 2018. Myopia and anchoring. NBER working paper No. 24545.

Angeletos, G.-M., Huo, Z., Sastry, K., 2020. Imperfect expectations: Theory and evidence. In: NBER Macroeconomics Annual 2020, volume 35. University of Chicago Press.

Angeletos, G.-M., La'O, J., 2010. Noisy business cycles. NBER Macroeconomics Annual 24 (1), 319–378.

Angeletos, G.-M., Lian, C., 2018. Forward guidance without common knowledge. American Economic Review 108 (9), 2477–2512.

Angeletos, G.-M., Sastry, K. A., 2018. Managing expectations without rational expectations. NBER working paper No. 25404.

Barakchian, S. M., Crowe, C., 2013. Monetary policy matters: Evidence from new shocks data. Journal of Monetary Economics 60 (8), 950–966.

Bauer, M. D., Swanson, E. T., 2020. The fed's response to economic news explains the 'fed information effect'. NBER working paper No. 27013.

Bernanke, B., 2004. Gradualism. Remarks at an economics luncheon co-sponsored by the Federal Reserve Bank of San Francisco (Seattle Branch) and the University of Washington, Seattle, Washington, May 20, 2004.

Bernanke, B. S., Kuttner, K. N., 2005. What explains the stock market's reaction to federal reserve policy? The Journal of finance 60 (3), 1221–1257.

Blanchard, O., Galí, J., 2007. Real wage rigidities and the new keynesian model. Journal of money, credit and banking 39, 35–65.

Blinder, A. S., 1998. Central banking in theory and practice. Mit press.

Blinder, A. S., Ehrmann, M., Fratzscher, M., De Haan, J., Jansen, D.-J., 2008. Central bank communication and monetary policy: A survey of theory and evidence. Journal of Economic Literature 46 (4), 910–45.

Boivin, J., Giannoni, M. P., 2006. Has monetary policy become more effective? The Review of Economics and Statistics 88 (3), 445–462.

Bordalo, P., Gennaioli, N., Ma, Y., Shleifer, A., 2018. Over-reaction in macroeconomic expectations. NBER working paper No. 24932.

Brainard, W. C., 1967. Uncertainty and the effectiveness of policy. The American Economic Review 57 (2), 411–425.

Broer, T., Kohlhas, A., 2018. Forecaster (mis-) behavior. CEPR Discussion Paper No. DP12898.

Caballero, R. J., Simsek, A., 2019. Prudential monetary policy. NBER working paper no. 25977.

Caballero, R. J., Simsek, A., 2020. A risk-centric model of demand recessions and speculation. The Quarterly Journal of Economics 135 (3), 1493–1566.

Campbell, J. R., Evans, C. L., Fisher, J. D., Justiniano, A., Calomiris, C. W., Woodford, M., 2012. Macroeconomic effects of federal reserve forward guidance [with comments and discussion]. Brookings Papers on Economic Activity, 1–80.

Cieslak, A., Morse, A., Vissing-Jorgensen, A., 2018. Stock returns over the fomc cycle. The Journal of Finance.

Cieslak, A., Schrimpf, A., 2019. Non-monetary news in central bank communication. Journal of International Economics 118, 293–315.

Clarida, R., Gali, J., Gertler, M., 1999. The science of monetary policy: a new keynesian perspective. Journal of economic literature 37 (4), 1661–1707.

Coibion, O., Gorodnichenko, Y., 2015. Information rigidity and the expectations formation process: A simple framework and new facts. American Economic Review 105 (8), 2644–78.

Diercks, A. M., Carl, U., et al., 2019. A simple macro-finance measure of risk premia in fed funds futures. FEDS Notes No. 2019-01-08.

Ehrmann, M., Fratzscher, M., 2007. Communication by central bank committee members: different strategies, same effectiveness? Journal of Money, Credit and Banking 39 (2-3), 509–541.

Eusepi, S., Preston, B., 2011. Expectations, learning, and business cycle fluctuations. American Economic Review 101 (6), 2844–72.

Evans, G. W., Honkapohja, S., 2001. Learning and expectations in macroeconomics.

Farhi, E., Werning, I., 2019. Monetary policy, bounded rationality, and incomplete markets. American Economic Review 109 (11), 3887–3928.

Gabaix, X., forthcoming. A behavioral new keynesian model. American Economic Review.

Galí, J., 2015. Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. Princeton University Press.

García-Schmidt, M., Woodford, M., 2019. Are low interest rates deflationary? a paradox of perfect-foresight analysis. American Economic Review 109 (1), 86–120.

Giacoletti, M., Laursen, K., Singleton, K. J., forthcoming. Learning from disagreement in the U.S. treasury bond market. The Journal of Finance.

Goodfriend, M., King, R. G., 1997. The new neoclassical synthesis and the role of monetary policy. NBER macroeconomics annual 12, 231–283.

Goodhead, R., Kolb, B., 2018. Monetary policy communication shocks and the macroeconomy.

Gürkaynak, R. S., Sack, B., Swanson, E., 2005a. The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models. American economic review 95 (1), 425–436.

Gürkaynak, R. S., Sack, B., Swanson, E. T., 2005b. Do actions speak louder than words? the response of asset prices to monetary policy actions and statements. International Journal of Central Banking.

Hamilton, J. D., 2008. Daily monetary policy shocks and new home sales. Journal of Monetary Economics 55 (7), 1171–1190.

Hanson, S. G., Stein, J. C., 2015. Monetary policy and long-term real rates. Journal of Financial Economics 115 (3), 429–448.

Jarocinski, M., Karadi, P., 2020. Deconstructing monetary policy surprises: the role of information shocks. American Economic Journal: Macroeconomics 12 (2), 1âĂŤ43.

King, M., 2005. Monetary policy: Practice ahead of theory. Mais Lecture, Cass Business School, London, May 17, 2005.

Lucca, D. O., Trebbi, F., 2011. Measuring central bank communication: an automated approach with application to fomc statements. NBER working paper No. 15367.

Ma, Y., Ropele, T., Sraer, D., Thesmar, D., 2020. A quantitative analysis of distortions in managerial forecasts. NBER Working Paper No. 26830.

Miranda-Agrippino, S., 2016. Unsurprising shocks: information, premia, and the monetary transmission.

Miranda-Agrippino, S., Ricco, G., 2018. The transmission of monetary policy shocks.

Morris, S., Shin, H. S., 2014. Risk-taking channel of monetary policy: A global game approach. Unpublished working paper, Princeton University.

Nakamura, E., Steinsson, J., 2018. High-frequency identification of monetary non-neutrality: the information effect. The Quarterly Journal of Economics 133 (3), 1283–1330.

Ramey, V. A., 2016. Macroeconomic shocks and their propagation. In: Handbook of macroeconomics. Vol. 2. Elsevier, pp. 71–162.

Rudebusch, G. D., 2002. Term structure evidence on interest rate smoothing and monetary policy inertia. Journal of monetary economics 49 (6), 1161–1187.

Sack, B., 1998. Uncertainty, learning, and gradual monetary policy. FEDS Working Paper Series No. 98-34.

Sastry, K., 2019. Public sentiment and monetary surprises. Available at SSRN 3421723.

Simsek, A., 2020. The macroeconomics of financial speculation. working paper.

Stein, J. C., Sunderam, A., 2018. The fed, the bond market, and gradualism in monetary policy. The Journal of Finance 73 (3), 1015–1060.

Ubide, A., 2015. How the fed and markets are not in sync on interest rates. Peterson Institute for International Economics, www.piie.com/realtime.

Vissing-Jorgensen, A., 2019. Central banking with many voices: The communications arms race.

Woodford, M., 2001. Imperfect common knowledge and the effects of monetary policy. NBER working paper No. 8673.

Woodford, M., 2003. Optimal interest-rate smoothing. The Review of Economic Studies 70 (4), 861–886.

Woodford, M., 2005. Central bank communication and policy effectiveness. NBER working paper No. 11898.

Woodford, M., 2013. Macroeconomic analysis without the rational expectations hypothesis. Annu. Rev. Econ. 5 (1), 303–346.

A. Appendix: Model and the log-linearized equilibrium

In this appendix, we describe the details of the model and derive the log-linearized equilibrium conditions that we use in our analysis. The model and the analysis closely follows the textbook treatment in Galí (2015). The main difference is that the central bank sets the interest rate *before* observing the aggregate demand shock within the period.

Representative household (the market). The economy is set in discrete time with periods $t \in \{0, 1, ...\}$. In each period, a representative household that we refer to as "the market" makes consumption-savings and labor supply decisions. Formally, the market solves,

$$\max_{\{C_{\tilde{t}}, N_{\tilde{t}}\}_{\tilde{t}=t}^{\infty}} \overline{E}_t^M \left[\sum_{\tilde{t}=t}^{\infty} e^{-\rho \tilde{t}} \left(\log C_{\tilde{t}} - \frac{N_{\tilde{t}}^{1+\varphi}}{1+\varphi} \right) \right] \tag{A.1}$$

$$\text{s.t. } P_{\tilde{t}} C_{\tilde{t}} + \frac{B_{\tilde{t}}}{R_{\tilde{t}-1}^f} = B_{\tilde{t}-1} + W_{\tilde{t}} N_{\tilde{t}} + \int_0^1 \Pi_{\tilde{t}}(\nu)\, d\nu.$$

Here, C_t denotes consumption, N_t denotes the labor supply, and φ is the inverse labor supply elasticity. The market has log utility—we describe the role of this assumption subsequently. The expectations operator $E_t^M[\cdot]$ corresponds to the market's belief after the realization of uncertainty in period t (see Figure 3).

In the budget constraint, R_t^f denotes the gross risk-free nominal interest rate between periods t and $t+1$. The term B_t denotes the one-period risk-free bond holdings. In equilibrium, the risk-free asset is in zero net supply, $B_t = 0$. The term $\Pi_t(\nu)$ denotes the profits from intermediate good firms (that we describe subsequently). For simplicity, we do not allow households to trade the firms (in equilibrium, there would be no trade since this is a representative household).

The optimality conditions for problem $(A.1)$ are standard and given by,

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{C_t^{-1}} \tag{A.2}$$

$$C_t^{-1} = e^{-\rho} R_t^f \overline{E}_t^M \left[\frac{P_t}{P_{t+1}} C_{t+1}^{-1} \right]. \tag{A.3}$$

Final good firms. There is a competitive final good sector that combines intermediate inputs from a continuum of monopolistically competitive firms indexed by $\nu \in [0, 1]$. The final good sector produces according to the technology,

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.4}$$

This firm's optimality conditions imply a demand function for the intermediate good firms,

$$Y_t(\nu) = \left(\frac{P_t(\nu)}{P_t} \right)^{-\varepsilon} Y_t \tag{A.5}$$

$$\text{where } P_t = \left(\int_0^1 P_t(\nu)^{1-\varepsilon} d\nu \right)^{1/(1-\varepsilon)}. \tag{A.6}$$

Here, P_t denotes the ideal price index.

Intermediate good firms. Each intermediate good firm produces according to the technology,

$$Y_t(\nu) = A_t N_t(\nu)^{1-\alpha}. \tag{A.7}$$

Firms take the demand for their goods as given and set price to $P_t(\nu)$ to maximize the current market value of their profits, as we describe subsequently.

Market clearing conditions. The aggregate goods and labor market clearing conditions are given by,

$$\begin{aligned} Y_t &= C_t && \text{(A.8)} \\ N_t &= \int_0^1 N_t(\nu)\, d\nu. && \text{(A.9)} \end{aligned}$$

Potential (flexible-price) outcomes. We start by characterizing a potential (flexible-price) benchmark around which we log-linearize the equilibrium conditions. In this benchmark, firms are symmetric and set the same price that we denote with P_t^*. The optimal price solves [cf. $(A.5)$ and $(A.7)$]:

$$\begin{aligned} &\max_{P_t^*, Y_t^*, N_t^*} P_t^* Y_t^* - W_t N_t^* && \text{(A.10)} \\ \text{s.t. } Y_t^* &= A_t (N_t^*)^{1-\alpha} = \left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} Y_t. \end{aligned}$$

Here, Y_t denotes the aggregate output that firms take as given. The solution is given by,

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{(1-\alpha) A_t (N_t^*)^{-\alpha}}. \tag{A.11}$$

Hence, firms operate with a constant markup over their marginal cost. By symmetry, aggregate output is given by $Y_t = Y_t^* = A_t (N_t^*)^{1-\alpha}$. Combining these observations with Eqs. $(A.2)$ and $(A.8)$, we solve for the potential labor supply

$$N^* = \left(\frac{\varepsilon - 1}{\varepsilon}(1-\alpha)\right)^{1/(1+\varphi)}. \tag{A.12}$$

Likewise, potential output is given by

$$Y_t^* = A_t (N^*)^{1-\alpha}. \tag{A.13}$$

Note that potential output is determined by current productivity and is independent of expectations about the future.

Nominal rigidities. We next describe the nominal rigidities. In each period, a randomly selected fraction, $1-\theta$, of firms reset their nominal prices. The firms that do not adjust their price in period t, set their labor input to meet the demand for their goods (since firms operate with a markup and we focus on small shocks). Consider the firms that adjust their price in period t. These firms' optimal price, P_t^{adj}, solves

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \overline{E}_t^M \left\{ M_{t,t+k} \left(Y_{t+k|t} P_t^{adj} - W_t N_{t+k|t} \right) \right\} \tag{A.14}$$

$$\text{where } Y_{t+k|t} = A_{t+k} N_{t+k|t}^{1-\alpha} = \left(\frac{P_t^{adj}}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}$$

$$\text{and } M_{t,t+k} = e^{-\rho k} \frac{1/C_{t+k}}{1/C_t} \frac{P_t^{adj}}{P_{t+k}}.$$

The terms, $N_{t+k|t}, Y_{t+k|t}$, denote the input and the output of the firm (that resets its price in period t) in a future period $t+k$. The term, $M_{t,t+k}$, is the stochastic discount factor between periods t and $t+k$ (determined by the firm owners' preferences). Note that firms share the same belief as the representative household. The optimality condition gives,

$$\sum_{k=0}^{\infty} \theta^k \overline{E}_t^M \left\{ M_{t,t+k} \left(P_t^{adj} - \frac{\varepsilon}{\varepsilon - 1} \frac{W_{t+k}}{(1-\alpha) A_{t+k} N_{t|t+k}^{-\alpha}} \right) \right\} = 0 \tag{A.15}$$

$$\text{where } N_{t|t+k} = \left(\frac{P_t^{adj}}{P_{t+k}} \right)^{\frac{-\varepsilon}{1-\alpha}} \left(\frac{Y_{t+k}}{A_{t+k}} \right)^{\frac{1}{1-\alpha}}.$$

The New-Keynesian Phillips curve. We next combine Eq. $(A.15)$ with the remaining equilibrium conditions to derive the New-Keynesian Phillips curve. Specifically, we log-linearize the equilibrium around the allocation that features real potential outcomes and zero inflation, that is, $N_t = N^*, Y_t = Y_t^*$ and $P_t = P^*$ for each t. Throughout, we use the notation $\tilde{x}_t = \log(X_t/X_t^*)$ to denote the log-linearized version of the corresponding variable X_t. We also let $Z_t = \frac{W_t}{A_t P_t}$ denote the normalized (productivity-adjusted) real wage.

We first log-linearize Eq. $(A.2)$ (and use $Y_t = C_t$) to obtain,

$$\tilde{z}_t = \varphi \tilde{n}_t + \tilde{y}_t. \tag{A.16}$$

Log-linearizing Eqs. $(A.4 - A.7)$ and $(A.9)$, we also obtain

$$\tilde{y}_t = (1-\alpha) \tilde{n}_t. \tag{A.17}$$

Finally, we log-linearize Eq. $(A.15)$ to obtain,

$$\sum_{k=0}^{\infty} (\theta\beta)^k \overline{E}_t^M \left\{ \tilde{p}_t^{adj} - \left(\tilde{z}_{t+k} + \alpha \tilde{n}_{t+k|t} + \tilde{p}_{t+k} \right) \right\} = 0 \tag{A.18}$$

$$\text{where } \tilde{n}_{t|t+k} = \frac{-\varepsilon \left(\tilde{p}_t^{adj} - \tilde{p}_{t+k} \right)}{1-\alpha} + \tilde{n}_{t+k}.$$

Here, the second line uses $\tilde{y}_t = (1-\alpha) \tilde{n}_t$.

We next combine Eqs. $(A.16 - A.18)$ and rearrange terms to obtain a closed-form solution for the price set by adjusting firms,

$$\begin{aligned}\tilde{p}_t^{adj} &= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\,\overline{E}_t^M\left[\Theta\tilde{y}_{t+k}+\tilde{p}_{t+k}\right]\\ \text{where }\Theta &= \frac{1+\varphi}{1-\alpha+\alpha\varepsilon}\end{aligned}$$

Since the expression is recursive, we can also write it as a difference equation

$$\tilde{p}_t^{adj} = (1-\theta\beta)\left(\Theta\tilde{y}_t+\tilde{p}_t\right)+\theta\beta\overline{E}_t^M\left[\tilde{p}_{t+1}^{adj}\right]. \tag{A.19}$$

Here, we have used the law of iterated expectations, $\overline{E}_t^M[\cdot]=\overline{E}_t^M\left[\overline{E}_{t+1}^M[\cdot]\right]$.

Next, we consider the aggregate price index $(A.6)$,

$$\begin{aligned}P_t &= \left((1-\theta)\left(P_t^{adj}\right)^{1-\varepsilon}+\int_{S_t}\left(P_{t-1}(\nu)\right)^{1-\varepsilon}d\nu\right)^{1/(1-\varepsilon)}\\ &= \left((1-\theta)\left(P_t^{adj}\right)^{1-\varepsilon}+\theta P_{t-1}^{1-\varepsilon}\right)^{1/(1-\varepsilon)}\end{aligned}$$

Here, we have used the observation that a fraction θ of prices are the same as in the last period. The term, S_t, denotes the set of sticky firms in period t, and the second line follows from the assumption that adjusting terms are randomly selected. Log-linearizing the equation, we further obtain $\tilde{p}_t=(1-\theta)\,\tilde{p}_t^{adj}+\theta\tilde{p}_{t-1}$. After substituting inflation, $\pi_t=\tilde{p}_t-\tilde{p}_{t-1}$, this implies,

$$\pi_t=(1-\theta)\left(\tilde{p}_t^{adj}-\tilde{p}_{t-1}\right). \tag{A.20}$$

Hence, inflation is proportional to the price change by adjusting firms.

Finally, note that Eq. $(A.19)$ can be written in terms of the price change of adjusting firms as

$$\tilde{p}_t^{adj}-\tilde{p}_{t-1}=(1-\theta\beta)\,\Theta\tilde{y}_t+\tilde{p}_t-\tilde{p}_{t-1}+\theta\beta\overline{E}_t^M\left[\tilde{p}_{t+1}^{adj}-\tilde{p}_t\right].$$

Substituting $\pi_t=\tilde{p}_t-\tilde{p}_{t-1}$ and combining with Eq. $(A.20)$, we obtain the New-Keynesian Phillips curve (1) that we use in the main text,

$$\begin{aligned}\pi_t &= \kappa\tilde{y}_t+\beta\overline{E}_t^M\left[\pi_{t+1}\right]\\ \text{where }\kappa &= \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1+\varphi}{1-\alpha+\alpha\varepsilon}.\end{aligned} \tag{A.21}$$

Aggregate demand shocks. We focus on aggregate demand shocks, which we capture by assuming log productivity, a_{t+1}, follows the process

$$a_{t+1}=a_t+g_t. \tag{A.22}$$

Here, g_t denotes the growth rate of productivity between periods t and $t+1$, which is realized in period t.

The IS curve. Finally, we log-linearize the Euler equation $(A.3)$ to obtain Eq. (2) in the main text,

$$\tilde{y}_t = -\left(i_t - \overline{E}_t^M[\pi_{t+1}] - \rho\right) + g_t + \overline{E}_t^M[\tilde{y}_{t+1}].$$

Here, $i_t = \log R_t^f$ denotes the nominal risk-free interest rate. We have used the market clearing condition, $Y_t = C_t$ [cf. Eq. $(A.8)$], the definition of the potential output, $Y_t^* = A_t (N^*)^{1-\alpha}$ [cf. $(A.13)$], and the evolution of productivity, $A_{t+1} = A_t e^{g_t}$ [cf. $(A.22)$]. The equation illustrates g_t has a one-to-one effect on aggregate spending and output in period t. Hence, we refer to g_t as the *aggregate demand shock* in period t.

Monetary policy and equilibrium. Recall that we assume the Fed sets the interest rate to solve problem (3),

$$\min_{i_t, \tilde{y}_t, \pi_t} -\frac{1}{2} E_t^F \left[\phi \tilde{y}_t^2 + \pi_t^2\right] \text{ s.t. } (A.21) \text{ and } (2).$$

Here, ϕ denotes the relative weight on the output gap. This completes the equilibrium conditions.

Price of the market portfolio. For future reference, we also derive the equilibrium price of "the market portfolio." Specifically, in every period t, agents can also invest in a security in zero net supply whose payoff is proportional to output in subsequent periods, $\{Y_{\tilde{t}}\}_{\tilde{t} \geq t+1}^{\infty}$. We let $\omega_{\tilde{t}}$ denote the market's holding of this security and modify the budget constraint as follows,

$$P_{\tilde{t}} C_{\tilde{t}} + \frac{B_{\tilde{t}+1}}{R_{\tilde{t}}^f} + Q_{\tilde{t}} \omega_{\tilde{t}} = B_{\tilde{t}} + \omega_{\tilde{t}-1} (Y_{\tilde{t}} + Q_{\tilde{t}}) + W_{\tilde{t}} N_{\tilde{t}} + \int_0^1 \Pi_{\tilde{t}}(\nu)\, d\nu.$$

Here, $Q_{\tilde{t}}$ denotes the ex-dividend price of this security (excluding the current dividends). Using the optimality condition for ω_t, we obtain,

$$Q_t = \overline{E}_t^M \left[e^{-\rho} \frac{C_{t+1}^{-1}}{C_t^{-1}} (Q_{t+1} + Y_{t+1}) \right].$$

Solving the equation forward, and using the transversality condition, we further obtain

$$Q_t = \sum_{k \geq 1} e^{-\rho k} \frac{(C_{t+k})^{-1}}{(C_t)^{-1}} Y_{t+k}.$$

After substituting $Y_t = C_t$ and simplifying, we find

$$Q_t = \frac{e^{-\rho}}{1 - e^{-\rho}} Y_t. \tag{A.23}$$

Hence, in view of log utility, the equilibrium price of the market portfolio is proportional to output. Substituting $Y_t = \exp(\tilde{y}_t) Y_t^*$ and $Y_t^* = A_t (N^*)^{1-\alpha}$ and taking logs, we obtain Eq. (8) in the main text,

$$q_t = q^* + a_t + \tilde{y}_t \text{ where } q^* = \log\left(\frac{e^{-\rho}}{1 - e^{-\rho}} (N^*)^{1-\alpha}\right).$$

In equilibrium, asset prices are proportional to output. Therefore, asset prices change either when productivity (a_t) changes or when the output gap $(\tilde{y}_t)$ changes.

B. Appendix: Omitted derivations

This appendix presents the derivations omitted from the main text.

B.1. Omitted derivations in Section 3.3

Bayesian updating. The realization of aggregate demand provides each agent with an independent and noisy signal about the persistent component, $g_t = \mathbf{g} + v_t \sim N(0, \Sigma)$. Agents combine this data with their prior beliefs, $\mathbf{g} \sim N\left(\mathbf{g}_0^j, C_0^{-1}\Sigma\right)$ [cf. (10)], to form their posterior beliefs. Applying the Bayes' rule, at the beginning of each period t, agent j believes the persistent shock is Normally distributed with mean and variance given by,

$$\begin{aligned}
\mathbf{g}_t^j = E_t^j[\mathbf{g}] &= \frac{C_0\Sigma^{-1}\mathbf{g}_0^j + \sum_{\tilde{t}=0}^{t-1}\Sigma^{-1}g_{\tilde{t}}}{C_0\Sigma^{-1} + \sum_{\tilde{t}=0}^{t-1}\Sigma^{-1}} = \frac{C_0\mathbf{g}_0^j + \sum_{\tilde{t}=0}^{t-1} g_{\tilde{t}}}{C_0 + t} \\
\text{and } var_t^j(\mathbf{g}) &= \left(C_0\Sigma^{-1} + \sum_{\tilde{t}=0}^{t-1}\Sigma^{-1}\right)^{-1} = (C_0 + t)^{-1}\Sigma.
\end{aligned}$$

Here, $C_0\Sigma^{-1}$ denotes the precision of the initial belief and Σ^{-1} denotes the precision of each signal. The posterior belief is a precision-weighted average of the prior belief and the signals. The posterior precision (the inverse of the variance) is the sum of the precision of the initial belief and the precision of the signals. The posterior precision increases (or the posterior variance declines) over time.

Using the definition of relative confidence, $\mathbf{c}_{0,t} = \frac{C_0}{C_0+t}$ [cf. (12)], we write the posterior mean as

$$\mathbf{g}_t^j = \mathbf{c}_{0,t}\mathbf{g}_0^j + (1 - \mathbf{c}_{0,t})\,\overline{g}_{t-1} \text{ and } \overline{g}_{t-1} = \sum_{\tilde{t}=0}^{t-1}\frac{g_{\tilde{t}}}{t}.$$

The term, $\overline{g}_{t-1}$, denotes the average realization of aggregate demand up to (and including) the realization in period $t-1$. The agent's posterior (conditional) mean belief is a weighted average of her initial mean belief and the realized data. This proves the first line in (11).

To prove the second line in (11), note that

$$\begin{aligned}
\mathbf{g}_t^j &= \frac{C_0}{C_0+t}\mathbf{g}_0^j + \frac{t}{C_0+t}\frac{\sum_{\tilde{t}=0}^{t-2} g_{\tilde{t}-1} + g_{t-1}}{t} \\
&= \frac{C_0+t-1}{C_0+t}\left\{\frac{C_0}{C_0+t-1}\mathbf{g}_0^j + \frac{t-1}{C_0+t-1}\left(\sum_{\tilde{t}=0}^{t-2}\frac{g_{\tilde{t}-1}}{t-1}\right)\right\} + \frac{1}{C_0+t}g_{t-1} \\
&= \mathbf{c}_{t-1,t}\mathbf{g}_{t-1}^j + (1 - \mathbf{c}_{t-1,t})\,g_{t-1}.
\end{aligned}$$

Here, the last line substitutes the definitions of $\mathbf{g}_{t-1}^j$ and $\mathbf{c}_{t-1,t}$. □

Proof of Lemma 1. Follows from Eq. (11). □

Proof of Lemma 2. We next characterize the higher order beliefs. Note that the identities trivially hold when $t = s$. Suppose $t > s$. Consider $j, j' \in \{F, M\}$ where we allow j' to be the same as j. Consider the second line of Eq. (11) for agent j'. By repeatedly applying the equation and using Eq. (12), we obtain:

$$\mathbf{g}_t^{j'} = \mathbf{c}_{s,t}\mathbf{g}_s^{j'} + (1-\mathbf{c}_{s,t})\,\overline{g}_{s,t-1},$$

where $\overline{g}_{s,t-1} = \sum_{n=0}^{t-s-1} g_{s+n}/(t-s)$ denotes the average realization between periods s and $t-1$. Considering agent j's expectation of this expression in period s, we obtain:

$$\begin{aligned} E_s^j\left[\mathbf{g}_t^{j'}\right] &= \mathbf{c}_{s,t}\mathbf{g}_s^{j'} + (1-\mathbf{c}_{s,t})\,E_s^j\left[\overline{g}_{s,t-1}\right] \\ &= \mathbf{c}_{s,t}\mathbf{g}_s^{j'} + (1-\mathbf{c}_{s,t})\,\mathbf{g}_s^j. \end{aligned} \tag{B.1}$$

Here, the first line uses the observation that $\mathbf{g}_s^{j'}$ is known in period s. The second line observes that for each $n \geq 0$, we have $E_s^j[g_{s+n}] = \mathbf{g}_s^j$ by definition of the conditional belief $\mathbf{g}_s^j$. Applying Eq. $(B.1)$ for $j' = j$ implies Eq. (14). Applying it for $j' \neq j$ implies Eq. (15) and completes the proof. □

B.2. Omitted derivations in Section 4

Recall that the relative confidence is given by $\mathbf{c}_{s,t} = \frac{C_0+s}{C_0+t}$ [cf. (12)], Relative confidence satisfies the following identity, which we use in subsequent analysis,

$$\mathbf{c}_{s,t}\mathbf{c}_{t,t'} = \mathbf{c}_{s,t'} \text{ for } s \leq t \leq t'. \tag{B.2}$$

Proof of Proposition 1, part (i). Provided in the main text. □

Proof of Proposition 1, part (ii). The derivation of the forward curve is presented in the main text. Here, we derive the dot curve, and we establish the limit results as $t \to \infty$. Taking the expectation of Eq. (19) according to the Fed's belief, we obtain,

$$\begin{aligned} E_0^F[i_t] &= \rho + (1-\mathbf{c}_{t,t+1})\,E_0^F\left[\mathbf{g}_t^F\right] + \mathbf{c}_{t,t+1}E_0^F\left[\mathbf{g}_t^M\right] \\ &= \rho + (1-\mathbf{c}_{t,t+1})\,\mathbf{g}_0^F + \mathbf{c}_{t,t+1}\left(\mathbf{c}_{0,t}\mathbf{g}_0^M + (1-\mathbf{c}_{0,t})\,\mathbf{g}_0^F\right) \\ &= \rho + \mathbf{g}_0^F + \mathbf{c}_{1,t+1}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right). \end{aligned}$$

Here, the second line uses Lemma 2 and the third line uses Eq. $(B.2)$. This proves Eq. (22).

To derive the limit results, note that $\mathbf{c}_{0,t+1} = \frac{C_0}{C_0+t+1}$. Thus, $\mathbf{c}_{0,t+1}$ is decreasing in horizon t with $\lim_{t\to\infty}\mathbf{c}_{0,t+1} = 0$. This implies $\lim\limits_{t\to\infty} E_0^F[i_t] = \rho + \mathbf{g}_0^F$. Likewise, $(1-\mathbf{c}_{t,t+1})\,\mathbf{c}_{0,t}$ is decreasing in horizon t with $\lim_{t\to\infty}(1-\mathbf{c}_{t,t+1})\,\mathbf{c}_{0,t} = 0$. This implies $\lim\limits_{t\to\infty} E_0^M[i_t] = \rho + \mathbf{g}_0^M$, completing the proof. □

B.3. Omitted derivations in Section 5

Proof of Proposition 2. Most of the proof is provided in the main text. We verify that the conjectured actions correspond to an equilibrium. Specifically, we check that the Fed does not have an incentive to deviate from the equilibrium interest rate in (19),

$$i_0 = \rho + (1-\mathbf{c}_{0,1})\,\mathbf{g}_0^F + \mathbf{c}_{0,1}\mathbf{g}_0^M.$$

Recall that, after seeing this interest rate, the market thinks the Fed's belief is given by,

$$\mathbf{G}_0^F(i_0) \equiv \frac{i_0 - \rho - \mathbf{c}_{0,1}\mathbf{g}_0^M}{1 - \mathbf{c}_{0,1}}.$$

Along the equilibrium path, the market's belief is correct, $\mathbf{G}_0^F(i_0) = \mathbf{g}_0^F$.

Consider the allocations in periods $t \geq 1$. The market *thinks* the equilibrium will be the same as in Section 4 given the Fed's initial belief $\mathbf{G}_0^F(i_0)$. Therefore, the market's expected output gap in period 1 is given Eq.(23),

$$E_1^M[\tilde{y}_1|i_0] = \mathbf{g}_1^M - \mathbf{g}_1^F = \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - \mathbf{G}_0^F(i_0)\right) \text{ where } \mathbf{G}_0^F(i_0) = \frac{i_0 - \rho - \mathbf{c}_{0,1}\mathbf{g}_0^M}{1 - \mathbf{c}_{0,1}}. \tag{B.3}$$

Next consider the equilibrium in period 0. Note that Eq. $(B.3)$ implies Eq. (26) in the main text, $\frac{dE_1^M[\tilde{y}_1|i_0]}{di_0} = -\frac{\mathbf{c}_{0,1}}{1-\mathbf{c}_{0,1}}$. Substituting this into the Fed's optimality condition (4), we obtain $E_t^F\left[\left(1 + \frac{\mathbf{c}_{0,1}}{1-\mathbf{c}_{0,1}}\right)\tilde{y}_0\right] = 0$. Since $\frac{\mathbf{c}_{0,1}}{1-\mathbf{c}_{0,1}}$ is constant, this implies Eq. (5) as before, $E_0^F[\tilde{y}_0] = 0$. Thus, the Fed's optimal interest rate is still given by Eq. (6),

$$\begin{aligned} i_0 &= \rho + E_0^F[g_0] + E_0^F\left[E_1^M[\tilde{y}_1|i_0]\right] \\ &= \rho + \mathbf{g}_0^F + \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - \mathbf{G}_0^F(i_0)\right). \end{aligned}$$

The second line substitutes $E_1^M[\tilde{y}_1|i_0]$ as well as the Fed's belief, $E_0^F[g_0] = \mathbf{g}_0^F$. Substituting the equilibrium condition, $\mathbf{G}_0^F(i_0) = \mathbf{g}_0^F$, we verify Eq. (19). □

Proof of Corollary 1. First consider the output gap in period t. Eq. (20) implies,

$$\tilde{y}_t = g_t - \mathbf{g}_t^F = \mathbf{g} - \mathbf{g}_t^F + v_t,$$

where we substituted $g_t = \mathbf{g} + v_t$. Next recall that the Fed's conditional belief is given by [cf. (11)],

$$\begin{aligned} \mathbf{g}_t^F &= \mathbf{c}_{0,t}\mathbf{g}_0^F + (1 - \mathbf{c}_{0,t})\sum_{\tilde{t}=0}^{t-1}\frac{g_{\tilde{t}}}{t} \\ &= \mathbf{c}_{0,t}\mathbf{g}_0^F + (1 - \mathbf{c}_{0,t})\left(\mathbf{g} + \sum_{\tilde{t}=0}^{t-1}\frac{v_{\tilde{t}}}{t}\right) \\ &= \mathbf{g} + \mathbf{c}_{0,t}\left(\mathbf{g}_0^F - \mathbf{g}\right) + (1 - \mathbf{c}_{0,t})\sum_{\tilde{t}=0}^{t-1}\frac{v_{\tilde{t}}}{t} \end{aligned}$$

Substituting this into the expression for the output gap, we obtain

$$\tilde{y}_t = \mathbf{c}_{0,t}\left(\mathbf{g} - \mathbf{g}_0^F\right) + \left\{v_t - (1 - \mathbf{c}_{0,t})\sum_{\tilde{t}=0}^{t-1}\frac{v_{\tilde{t}}}{t}\right\}.$$

The terms in set-parentheses are zero-mean random variables (transitory shocks) independent from both $\mathbf{g}$ and $\mathbf{g}_0^F$.

Next consider the interest rate in period 0. Eq. (19) implies

$$i_0 = \rho + (1 - \mathbf{c}_{0,1})\, \mathbf{g}_0^F + \mathbf{c}_{0,1}\mathbf{g}_0^M.$$

All of the terms except for $(1 - \mathbf{c}_{0,1})\, \mathbf{g}_0^F$ are deterministic.

Finally, we combine the expressions for the output gap and the interest rate to obtain the desired result,

$$\begin{aligned}
\beta^{DGP}(\tilde{y}_t, i_0) &= \frac{cov^{DGP}\left(\mathbf{c}_{0,t}\left(\mathbf{g}-\mathbf{g}_0^F\right), (1-\mathbf{c}_{0,1})\,\mathbf{g}_0^F\right)}{var^{DGP}\left((1-\mathbf{c}_{0,1})\,\mathbf{g}_0^F\right)} \\
&= \frac{\mathbf{c}_{0,t}(1-\mathbf{c}_{0,1})}{(1-\mathbf{c}_{0,1})^2}\frac{cov^{DGP}\left(\mathbf{g}-\mathbf{g}_0^F, \mathbf{g}_0^F\right)}{var^{DGP}\left(\mathbf{g}_0^F\right)} \\
&= \frac{\mathbf{c}_{0,t}}{1-\mathbf{c}_{0,1}}\left(\frac{cov\left(\mathbf{g}, \mathbf{g}_0^F\right)}{var\left(\mathbf{g}_0^F\right)} - 1\right) \\
&= \frac{\mathbf{c}_{0,t}}{1-\mathbf{c}_{0,1}}\left(\beta\left(\mathbf{g}, \mathbf{g}_0^F\right) - 1\right),
\end{aligned}$$

where the last line substitutes the definition of $\beta\left(\mathbf{g}, \mathbf{g}_0^F\right)$. □

Proof of Proposition 3. We conjecture an equilibrium that satisfies Eqs. $(33-34)$,

$$\begin{aligned}
i_0 &= \rho + \mathbf{g}_0^F + \mathbf{v}_0^F + \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - E_0^M\left[\mathbf{g}_0^F | i_0\right]\right) \\
&= E_0^M[i_0] + (1-\mathbf{c}_{0,1}\tau)\left(\mathbf{g}_0^F - \overline{\mathbf{g}}_0^F + \mathbf{v}_0^F - \overline{\mathbf{v}}_0^F\right) \\
E_0^M\left[\mathbf{g}_0^F | i_0\right] &= \frac{i_0 - E_0^M[i_0]}{1-\mathbf{c}_{0,1}\tau} \\
&= \overline{\mathbf{g}}_0^F + \tau\left(\mathbf{g}_0^F - \overline{\mathbf{g}}_0^F + \mathbf{v}_0^F - \overline{\mathbf{v}}_0^F\right).
\end{aligned}$$

The first row describes the optimal interest rate policy, i_0, given the market's posterior belief, $E_0^M\left[\mathbf{g}_0^F | i_0\right]$. The third row describes the market's Bayesian posterior belief, $E_0^M\left[\mathbf{g}_0^F | i_0\right]$, given the interest rate it observes, i_0. The second and the last rows describe the *equilibrium* levels of i_0 and $E_0^M\left[\mathbf{g}_0^F | i_0\right]$, respectively, obtained by substituting the equilibrium level of the *other* variable. The term, $E_0^M[i_0] = \rho + \overline{\mathbf{g}}_0^F + \overline{\mathbf{v}}_0^F + \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - \overline{\mathbf{g}}_0^F\right)$, describes the market's ex-ante expected interest rate.

To verify the conjecture, first consider the equilibrium starting period 1 onward given the market's posterior belief, $E_0^M\left[\mathbf{g}_0^F | i_0\right]$. The analysis is the same as in Section 5.1. Since there is no short-run disagreement (by assumption), the Fed's interest rate decision in period 1 fully reveals its conditional belief $\mathbf{g}_1^F$. Given $\mathbf{g}_1^F$, the equilibrium is the same as before. In particular, *after* the interest rate decision, the market's expected output gap is given by the following analogue of Eq.(28),

$$E_1^M[\tilde{y}_1 | i_1] = \mathbf{g}_1^M - \mathbf{g}_1^F = \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right).$$

Before the interest rate decision in period 1, we instead have,

$$E_1^M[\tilde{y}_1 | i_0] = \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - E_0^M\left[\mathbf{g}_0^F | i_0\right]\right) \text{ where } E_0^M\left[\mathbf{g}_0^F | i_0\right] = \frac{i_0 - E_0^M[i_0]}{1-\mathbf{c}_{0,1}\tau}. \quad \text{(B.4)}$$

The market expects an average mistake that reflects its posterior belief (in period 0) about the Fed's long-term belief.

Next consider the Fed's optimal interest rate policy in period 0. Note that Eq. $(B.4)$ implies $\frac{dE_1^M[\tilde{y}_1|i_0]}{di_0} = -\frac{\mathbf{c}_{0,1}}{1-\mathbf{c}_{0,1}\tau}$. Substituting this into the Fed's optimality condition (4), we obtain $E_0^F\left[\left(1+\frac{\mathbf{c}_{0,1}}{1-\mathbf{c}_{0,1}\tau}\right)\tilde{y}_0\right] = 0$. Since $\frac{\mathbf{c}_{0,1}}{1-\mathbf{c}_{0,1}\tau}$ is constant, this implies Eq. (5) as before, $E_0^F[\tilde{y}_0] = 0$. Thus, the Fed's optimal interest rate is still given by Eq. (6),

$$\begin{aligned} i_0 &= \rho + E_0^F[g_0] + E_0^F\left[E_1^M[\tilde{y}_1|i_0]\right] \\ &= \rho + \mathbf{g}_0^F + \mathbf{v}_0^F + \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - E_0^M\left[\mathbf{g}_0^F|i_0\right]\right), \end{aligned}$$

where the second line substitutes $E_1^M[\tilde{y}_1|i_0]$ as well as the Fed's belief, $E_0^F[g_0] = \mathbf{g}_0^F + \mathbf{v}_0^F$. This proves Eq. (33).

Next consider the market's Bayesian posterior belief in period 0. In equilibrium, the interest rate in (33) provides the market with an imperfect signal of the Fed's long-term belief (relative to its ex-ante mean),

$$x_0^F \equiv \frac{i_0 - E^M[i_0]}{1-\mathbf{c}_{0,1}\tau} = \mathbf{g}_0^F - \overline{\mathbf{g}}_0^F + \mathbf{v}_0^F - \overline{\mathbf{v}}_0^F \sim N\left(\mathbf{g}_0^F - \overline{\mathbf{g}}_0^F, \sigma_{\mathbf{v}}^2\right).$$

The market combines the signal with its prior belief, $\mathbf{g}_0^F - \overline{\mathbf{g}}_0^F \sim N\left(0, \sigma_{\mathbf{g}}^2\right)$, to form the Bayesian posterior,

$$E_0^M\left[\mathbf{g}_0^F - \overline{\mathbf{g}}_0^F|x_0^F\right] = x_0^F \frac{\frac{1}{\sigma_{\mathbf{v}}^2}}{\frac{1}{\sigma_{\mathbf{g}}^2}+\frac{1}{\sigma_{\mathbf{v}}^2}} = \left(\mathbf{g}_0^F - \overline{\mathbf{g}}_0^F + \mathbf{v}_0^F - \overline{\mathbf{v}}_0^F\right)\frac{\sigma_{\mathbf{g}}^2}{\sigma_{\mathbf{g}}^2+\sigma_{\mathbf{v}}^2}.$$

This proves Eq. (34).

Finally, consider the comparative statics of the equilibrium described in the proposition. Eq. (35) follows directly from Eq. (33). Consider Eq. (36) that describes the change in the forward interest rates for $t \geq 1$. Recall that starting period 1 onward, the Fed's belief is revealed and the equilibrium is the same as before. Therefore, *conditional on* $\mathbf{g}_0^F$, the forward rates are still given by Eq. (27),

$$E_0^M\left[i_t|\mathbf{g}_0^F\right] = \rho + \mathbf{g}_0^F\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right) + \mathbf{g}_0^M\left(1-\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right)\right).$$

Taking the expectation, we obtain,

$$\begin{aligned} E_0^M[i_t|i_0] &= \rho + E_0^M\left[\mathbf{g}_0^F|i_0\right]\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right) + \mathbf{g}_0^M\left(1-\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right)\right) \\ &= \rho + \left(\overline{\mathbf{g}}_0^F + \tau\left(\mathbf{g}_0^F - \overline{\mathbf{g}}_0^F + \mathbf{v}_0^F - \overline{\mathbf{v}}_0^F\right)\right)\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right) + \mathbf{g}_0^M\left(1-\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right)\right) \\ &= E_0^M[i_t] + \tau\mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right)\left(\mathbf{g}_0^F - \overline{\mathbf{g}}_0^F + \mathbf{v}_0^F - \overline{\mathbf{v}}_0^F\right). \end{aligned}$$

Here, the second line substitutes Eq. (34) and the last line substitutes the market's ex-ante expectation. Eq. (36) then follows after substituting $\Delta E_0^M[i_t|i_0] = E_0^M[i_t|i_0] - E_0^M[i_t]$. □

Proof of Proposition 4. We first describe the endogenous weight the Fed assigns to the case in which the market is reactive, $\tilde{\delta}$. Consider the quadratic,

$$P(x) \equiv x^2\mathbf{c}_{0,1} - x\left(1+2\delta\mathbf{c}_{0,1}\right) + \delta\left(\mathbf{c}_{0,1}+1\right).$$

Note that $P(\delta) = \delta(1-\delta)\mathbf{c}_{0,1} > 0$ and $P(1) = (1-\delta)(\mathbf{c}_{0,1}-1) < 0$. Since $P(\cdot)$ is an upward sloping parabola, these conditions imply that $P(.)$ has exactly one zero that falls in the interval $(\delta, 1)$. Let $\tilde{\delta}$

denote this zero: that is, $\tilde{\delta}$ is the unique solution to,

$$P\left(\tilde{\delta}\right) = 0 \text{ over } \tilde{\delta} \in (\delta, 1). \tag{B.5}$$

Given $\tilde{\delta}$, we conjecture an equilibrium that satisfies Eqs. (42) and (43), that is,

$$\begin{aligned}
i_0 &= \rho + \mathbf{g}_0^F + \mathbf{v}_0^F + \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - \left\{\begin{array}{c} \tilde{\delta} E_0^M\left[\mathbf{g}_0^F | i_0, \tau = 1\right] \\ +\left(1-\tilde{\delta}\right) E_0^M\left[\mathbf{g}_0^F | i_0, \tau = 0\right] \end{array}\right\}\right) \\
&= E_0^M[i_0] + \left(1 - \mathbf{c}_{0,1}\tilde{\delta}\right)\left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F\right); \\
E_0^M\left[\mathbf{g}_0^F | i_0, \tau\right] &= \overline{\mathbf{g}}_0^F + \tau \frac{i_0 - E_0^M[i_0]}{1 - \mathbf{c}_{0,1}\tilde{\delta}} \\
&= \overline{\mathbf{g}}_0^F + \tau\left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F\right) \text{ where } \tau = \frac{\sigma_{\mathbf{g}}^2}{\sigma_{\mathbf{v}}^2 + \sigma_{\mathbf{g}}^2}.
\end{aligned}$$

We also conjecture that the forward curve change in period 0 is given by the same expression as in Proposition 3, *conditional on the market's type* [cf. (36)]. Therefore, after observing the forward curve reaction to its interest rate decision in period 0, the Fed learns the market's reaction type τ.

To verify the conjecture, first consider the equilibrium in period 1 given the market's posterior belief, $E_0^M\left[\mathbf{g}_0^F | i_0, \tau\right]$. Since the Fed has learned the market's type τ, the analysis is the same as in Section 5.1. Following the steps in the proof of Proposition 3, we obtain the following analogue of $(B.4)$:

$$E_1^M\left[\tilde{y}_1 | i_0, \tau\right] = \mathbf{c}_{0,1}\left(\mathbf{g}_0^M - E_0^M\left[\mathbf{g}_0^F | i_0, \tau\right]\right) \text{ where } E_0^M\left[\mathbf{g}_0^F | i_0\right] = \overline{\mathbf{g}}_0^F + \tau \frac{i_0 - E_0^M[i_0]}{1 - \mathbf{c}_{0,1}\tilde{\delta}}. \tag{B.6}$$

Next consider the Fed's optimal interest rate policy in period 0. In this case, Eq. $(B.6)$ implies $\frac{dE_1^M[\tilde{y}_1 | i_0, \tau]}{di_0} = -\frac{\tau \mathbf{c}_{0,1}}{1 - \mathbf{c}_{0,1}\tilde{\delta}}$. Substituting this into the Fed's optimality condition (4), we obtain,

$$E_0^F\left[\frac{d\tilde{y}_0}{di_0}\tilde{y}_0\right] = 0 \text{ where } \frac{d\tilde{y}_0}{di_0} = -\left(1 + \frac{\tau \mathbf{c}_{0,1}}{1 - \mathbf{c}_{0,1}\tilde{\delta}}\right). \tag{B.7}$$

The marginal policy impact, $\frac{d\tilde{y}_0}{di_0}$, depends on the market's reaction type, τ. Since the Fed is uncertain about the market's type, this term does *not* drop out of the expectation. Therefore, unlike the equilibria we analyzed so far, the Fed's expected output gap, $E_0^F[\tilde{y}_0]$, is not necessarily zero.

To characterize the optimal policy, we rewrite Eq. $(B.7)$ in terms of conditional expectations,

$$0 = -E_0^F\left[\frac{d\tilde{y}_0}{di_0}\tilde{y}_0\right] = \delta\left(1 + \frac{\mathbf{c}_{0,1}}{1 - \mathbf{c}_{0,1}\tilde{\delta}}\right) E_0^F\left[\tilde{y}_0 | \tau = 1\right] + (1 - \delta) E_0^F\left[\tilde{y}_0 | \tau = 0\right].$$

Note that the quadratic in Eq. $(B.5)$ implies,

$$\tilde{\delta} = \frac{\delta\left(1 + \frac{\mathbf{c}_{0,1}}{1 - \mathbf{c}_{0,1}\tilde{\delta}}\right)}{\delta\left(1 + \frac{\mathbf{c}_{0,1}}{1 - \mathbf{c}_{0,1}\tilde{\delta}}\right) + 1 - \delta}.$$

Therefore, the Fed's optimality condition can be equivalently written as,

$$0 = \tilde{\delta} E_0^F [\tilde{y}_0 | \tau = 1] + \left(1 - \tilde{\delta}\right) E_0^F [\tilde{y}_0 | \tau = 0]. \tag{B.8}$$

Hence, the Fed targets a *weighted average* of the output gap over the cases in which the market is reactive and unreactive. The weight for the reactive case is given by the endogenous parameter, $\tilde{\delta}$, which exceeds the prior probability of this case, $\tilde{\delta} > \delta$.

We next solve the policy interest rate. Substituting the IS curve (2) into $(B.8)$, we obtain,

$$\begin{aligned} i_0 &= \rho + E_0^F [g_0] + \tilde{\delta} E_1^M [\tilde{y}_1 | i_0, \tau = 1] + \left(1 - \tilde{\delta}\right) E_1^M [\tilde{y}_1 | i_0, \tau = 0] \qquad \text{(B.9)} \\ &= \rho + \mathbf{g}_0^F + \mathbf{v}_0^F + \mathbf{c}_{0,1} \left(\mathbf{g}_0^M - \left\{ \begin{array}{c} \tilde{\delta} E_0^M \left[\mathbf{g}_0^F | i_0, \tau = 1\right] \\ + \left(1 - \tilde{\delta}\right) E_0^M \left[\mathbf{g}_0^F | i_0, \tau = 0\right] \end{array} \right\} \right). \end{aligned}$$

The second line substitutes $E_1^M [\tilde{y}_1 | i_0, \tau]$ from $(B.6)$ as well as the Fed's belief, $E_0^F [g_0] = \mathbf{g}_0^F + \mathbf{v}_0^F$. This proves Eq. (42).

Next consider the market's Bayesian posterior belief in period 0. In equilibrium, the interest rate in (42) provides the market with an imperfect signal of the Fed's long-term belief (relative to its ex-ante mean), $x_0^F = \frac{i_0 - E^M[i_0]}{1 - \mathbf{c}_{0,1}\tilde{\delta}} = \Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F$. Following the same steps as in the proof of Proposition 3, we prove Eq. (43) and verify the conjectured equilibrium.

We next establish Eqs. $(44 - 45)$. To this end, we substitute the interest rate from $(B.9)$ into the IS curve (2) to obtain

$$\tilde{y}_0 = g_0 - E_0^F [g_0] + E_1^M [\tilde{y}_1 | i_0, \tau] - \left\{ \tilde{\delta} E_1^M [\tilde{y}_1 | i_0, \tau = 1] + \left(1 - \tilde{\delta}\right) E_1^M [\tilde{y}_1 | i_0, \tau = 0] \right\}.$$

Taking the Fed's expectation conditional on $\tau = 1$, we obtain

$$\begin{aligned} E_0^F [\tilde{y}_0 | \tau = 1] &= \left(1 - \tilde{\delta}\right) \left(E_1^M [\tilde{y}_1 | i_0, \tau = 1] - E_1^M [\tilde{y}_1 | i_0, \tau = 0] \right) \\ &= -\left(1 - \tilde{\delta}\right) \mathbf{c}_{0,1} \left(E_0^M \left[\mathbf{g}_0^F | i_0, \tau = 1\right] - E_0^M \left[\mathbf{g}_0^F | i_0, \tau = 0\right] \right) \\ &= -\left(1 - \tilde{\delta}\right) \mathbf{c}_{0,1} \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right). \end{aligned}$$

The second and the third lines use Eqs. $(B.6)$ and (43), respectively. Likewise, taking the Fed's expectation conditional on $\tau = 0$, we obtain

$$\begin{aligned} E_0^F [\tilde{y}_0 | \tau = 0] &= \tilde{\delta} \left(E_1^M [\tilde{y}_1 | i_0, \tau = 0] - E_1^M [\tilde{y}_1 | i_0, \tau = 1] \right) \\ &= -\tilde{\delta} \mathbf{c}_{0,1} \left(E_0^M \left[\mathbf{g}_0^F | i_0, \tau = 0\right] - E_0^M \left[\mathbf{g}_0^F | i_0, \tau = 1\right] \right) \\ &= \tilde{\delta} \mathbf{c}_{0,1} \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right). \end{aligned}$$

This proves Eq. (45). Finally, note that the unconditional expectation is given by

$$\begin{aligned} E_0^F [\tilde{y}_0] &= \delta E_0^F [\tilde{y}_0 | \tau = 1] + (1 - \delta) E_0^F [\tilde{y}_0 | \tau = 0] \\ &= \left(-\delta \left(1 - \tilde{\delta}\right) + (1 - \delta) \tilde{\delta} \right) \mathbf{c}_{0,1} \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right) \\ &= \left(\tilde{\delta} - \delta \right) \mathbf{c}_{0,1} \left(\Delta \mathbf{g}_0^F + \Delta \mathbf{v}_0^F \right). \end{aligned}$$

This establishes Eq. (44) and completes the proof of the proposition. □

Proof of Proposition 5. We verify that the conjectured allocation is an equilibrium. The Fed's expected interest rate for the next period, $E_0^F[i_1]$, is a one-to-one function of the Fed's long-term belief, $\mathbf{g}_0^F$. Therefore, the announcement of the expected rate reveals $\mathbf{g}_0^F$. Given $\mathbf{g}_0^F$, the current interest rate is a one-to-one function of the Fed's short-term belief, $\mathbf{v}_0^F$. Thus, the announcement of the current rate reveals $\mathbf{v}_0^F$. Note also that the Fed does not benefit from deviating from the interest rate policy, since the policy enables it to hit its target (on average), $E_0^F[\tilde{y}_0] = 0$. This verifies the equilibrium. The rest of the result follows from the analysis preceding the proposition. □

B.4. Omitted derivations in Section 6

Proof of Proposition 6. Most of the proof is provided in the main text. Here, we complete the remaining steps. Eq. (49) follows from solving the difference equation forward and assuming that $\lim_{t\to\infty} E_t^M[\pi_t]$ remains bounded. Then, Eqs. $(47-49)$ provide a closed form and deterministic solution for $E_t^M[\pi_t], E_t^F[\pi_t], E_t^F[\tilde{y}_t]$. Note also that the IS curve implies $E_t^M[\tilde{y}_t] = E_t^F[\tilde{y}_t] + \mathbf{g}_t^M - \mathbf{g}_t^F$ [cf. (23)]. This verifies the conjecture that for each j, $E_t^j[\pi_t], E_t^j[\tilde{y}_t]$ evolve deterministically and completes the proof. □

Proof of Corollary 2. The analysis that precedes the corollary implies

$$r_t = \rho + \mathbf{g}_t^F + \mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F + E_{t+1}^F[\tilde{y}_{t+1}] + \frac{1-\Theta}{\kappa}\beta E_{t+1}^M[\pi_{t+1}].$$

Lemma 1 implies $\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F = \mathbf{c}_{t,t+1}\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)$, and Proposition 6 implies $E_{t+1}^F[\tilde{y}_{t+1}] = -\frac{1-\Theta}{\kappa}\beta E_{t+2}^M[\pi_{t+2}]$. Combining these observations, we obtain the expression for the interest rate,

$$r_t = r_t^{sticky} + \frac{1-\Theta}{\kappa}\beta\left(E_{t+1}^M[\pi_{t+1}] - E_{t+2}^M[\pi_{t+2}]\right).$$

To prove the inequalities in the second part of the result, let $\Delta x_t = x_t - x_{t+1}$ denote the first forward difference of a variable x_t. Note that Eq. (49) implies,

$$\Delta E_t^M[\pi_t] = \kappa\Delta\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) + \Theta\beta\Delta E_{t+1}^M[\pi_{t+1}], \tag{B.10}$$

Note also that Lemma 1 implies,

$$\begin{aligned}\Delta\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) &= \mathbf{g}_t^M - \mathbf{g}_t^F - \mathbf{c}_{t,t+1}\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)\\ &= \left(1-\mathbf{c}_{t,t+1}\right)\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)\\ &= \mathbf{c}_{0,t}\left(1-\mathbf{c}_{t,t+1}\right)\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right).\end{aligned}$$

Combining this with $(B.10)$, we obtain a closed-form solution for the forward difference of inflation,

$$\Delta E_t^M[\pi_t] = \kappa\left(\mathbf{g}_0^M - \mathbf{g}_0^F\right)\sum_{n=0}^{\infty}\mathbf{c}_{0,t+n}\left(1-\mathbf{c}_{t+n,t+n+1}\right).$$

Note that $\Delta E_t^M[\pi_t] > 0$ iff $\mathbf{g}_0^M > \mathbf{g}_0^F$. This implies $r_t > r_t^{sticky}$ iff $\mathbf{g}_0^M > \mathbf{g}_0^F$ and completes the proof of the corollary. □

Proof of Corollary 3. Substituting $\mathbf{g}_t^M - \mathbf{g}_t^F = \mathbf{g}_0^M - \mathbf{g}_0^F$, Eq. (49) implies

$$E_t^M [\pi_t] = \frac{\kappa \left(\mathbf{g}_0^M - \mathbf{g}_0^F\right)}{1 - \Theta\beta} = \frac{u}{\beta (1 - \Theta\beta)}.$$

Then, Eq. (48) implies

$$E_t^F [\tilde{y}_t] = -\frac{1 - \Theta}{\kappa (1 - \Theta\beta)} u = -\frac{\kappa}{\kappa^2 + \phi (1 - \beta)} u.$$

Here, the second substitutes $\Theta = \frac{\phi}{\phi + \kappa^2}$. Likewise, Eq. (47) implies

$$E_t^F [\pi_t] = \frac{\Theta}{1 - \Theta\beta} u = \frac{\phi}{\kappa^2 + \phi (1 - \beta)} u.$$

These equations are the same as Eqs. (3.4) and (3.5) in Clarida et al. (1999); Galí (2015) for the special case with a persistent cost-push shock ($\rho = 1$) after appropriately adjusting the notation (specifically, by setting $E_t^F [\tilde{y}_t] = x_t, E_t^F [\pi_t] = \pi_t, \kappa = \lambda$ and $\phi = \alpha$). □

B. Appendix: Omitted extensions

This appendix presents the extensions of the baseline model, omitted from the main text.

B.1. Heterogeneous data sensitivity and bundled Fed belief shocks

In the main text, we focus on the case in which the Fed and the market have the same level of confidence in initial beliefs, $C_0^j = C_0$. In this appendix, we consider the general case in which agents have heterogeneous confidence levels, C_0^F and C_0^M, as well as heterogeneous initial beliefs, $\mathbf{g}_0^F$ and $\mathbf{g}_0^M$. We establish the results that we describe in the concluding section. For simplicity, we focus on the baseline setup in which agents know each other's initial beliefs and "agree-to-disagree."

In this case, the Fed and the market have *heterogeneous data sensitivity*. Specifically, Bayesian updating implies the following analogue of Eq. (11),

$$\begin{aligned} \mathbf{g}_t^j &= \mathbf{c}_{t-1,t}^j \mathbf{g}_{t-1}^j + \left(1 - \mathbf{c}_{t-1,t}^j\right) g_{t-1} \qquad \text{(B.1)} \\ \text{where } \mathbf{c}_{s,t}^j &= \frac{C_0^j + s}{C_0^j + t} \quad \text{for } s \leq t. \end{aligned}$$

Note that $C_0^j > C_0^{j'}$ implies $\mathbf{c}_{s,t}^j > \mathbf{c}_{s,t}^{j'}$ for each $t > s$: higher *initial* confidence implies a higher *relative* confidence at all times. Therefore, agents put heterogeneous weights on new realizations of aggregate demand. In particular, the agent with smaller confidence puts greater weight on new data. This feature changes how output and asset prices react to demand shocks. The demand shocks are *bundled* with *endogenous* Fed belief shocks similar to the ones analyzed in Section 5.1. We start by presenting our main result that characterizes the equilibrium. We then illustrate the comparative statics of demand shocks and contrast the effects with the case with common data sensitivity.

Proposition 7. *Consider arbitrary $C_0^F, C_0^M, \mathbf{g}_0^F, \mathbf{g}_0^M$ so that the Fed and the market can have heterogeneous data sensitivity as well as heterogeneous initial beliefs about aggregate demand.*

The equilibrium output gap and risky asset price are given by

$$\tilde{y}_t = \mathbf{D_t}\left(g_t - \mathbf{g}_t^F\right) \tag{B.2}$$
$$q_t = q^* + a_t + \mathbf{D_t}\left(g_t - \mathbf{g}_t^F\right) \tag{B.3}$$

Here, $\{\mathbf{D_t}\}_t$ *is a deterministic sequence that captures* ***the output impact of demand shocks.*** *It is the unique solution to the difference equation:*

$$\mathbf{D_t} = 1 + \left(\mathbf{c}_{t,t+1}^F - \mathbf{c}_{t,t+1}^M\right)\mathbf{D}_{t+1} \text{ with } \lim_{t\to\infty}\mathbf{D_t} = 1. \tag{B.4}$$

The solution satisfies $\mathbf{D_t} > 0$*: above-average shocks have a positive impact. If the Fed is more data sensitive than the market,* $C_0^F < C_0^M$ *(resp. less data sensitive than the market* $C_0^F > C_0^M$*), then the impact is dampened* $\mathbf{D_t} < 1$ *(resp. amplified* $\mathbf{D_t} > 1$*) relative to the case with common data sensitivity.*

The equilibrium interest rate is given by

$$i_t = \rho + \mathbf{g}_t^F + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \tag{B.5}$$

The forward and dot curves in a period s *(for interest rates in a subsequent period* $t \geq s$*) are given by*

$$E_s^M[i_t] = \rho + \mathbf{g}_s^M + \mathbf{c}_{s,t}^F\left(1 - \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right)\left(\mathbf{g}_s^F - \mathbf{g}_s^M\right) \tag{B.6}$$
$$E_s^F[i_t] = \rho + \mathbf{g}_s^F + \mathbf{D}_{t+1}\mathbf{c}_{s,t+1}^M\left(\mathbf{g}_s^M - \mathbf{g}_s^F\right). \tag{B.7}$$

This result generalizes our main result, Proposition 1, to the case with heterogeneous data sensitivity. Common data sensitivity corresponds to the special case with $\mathbf{D_t} = 1$. Eqs. $(B.2)$ and $(B.3)$ show that *heterogeneous data sensitivity changes the impact of demand shocks on output and asset prices.* If the Fed is more (less) data sensitive than the market, then demand shocks have a dampened (amplified) impact compared to the case with common sensitivity. Figure 8 illustrates the equilibrium sequence, $\{\mathbf{D_t}\}$, for a particular parameterization and heterogeneous data sensitivity.

Eqs. $(B.6 - B.7)$ characterize the equilibrium interest rate as well as the forward and the dot curves. These expressions are similar to their counterparts with common data sensitivity [cf. Proposition 1]. We describe the forward and the dot curves for each period s (as opposed to only the initial period 0). This enables us to analyze how a demand shock in the initial period 0, g_0, affects the forward and the dot curves. The initial demand shock affects the expected interest rates in period 1 (but not in period 0).

Sketch of proof of Proposition 1. We next provide a sketch of the proof for the result, which is useful to understand the intuition. We complete the proof at the end of the section. The limit condition holds, $\lim_{t\to\infty}\mathbf{D_t} = 1$, because in the long run disagreements disappear (due to learning) and the equilibrium approximates the benchmark case with common beliefs. We conjecture that the output gap satisfies Eq. $(B.2)$ for period $t+1$. We then establish the same condition for period t (along with the other equilibrium conditions) proving the result by backward induction.

The key observation is that Eq. $(B.2)$ and Eq. $(B.1)$ together imply:

$$\begin{aligned} E_{t+1}^M[\tilde{y}_{t+1}] &= \mathbf{D}_{t+1}\left(\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right) \\ &= \mathbf{D}_{t+1}\left(\mathbf{c}_{t,t+1}^M\mathbf{g}_t^M - \mathbf{c}_{t,t+1}^F\mathbf{g}_t^F + \begin{bmatrix} \left(1 - \mathbf{c}_{t,t+1}^M\right) \\ -\left(1 - \mathbf{c}_{t,t+1}^F\right) \end{bmatrix} g_t\right). \end{aligned} \tag{B.8}$$

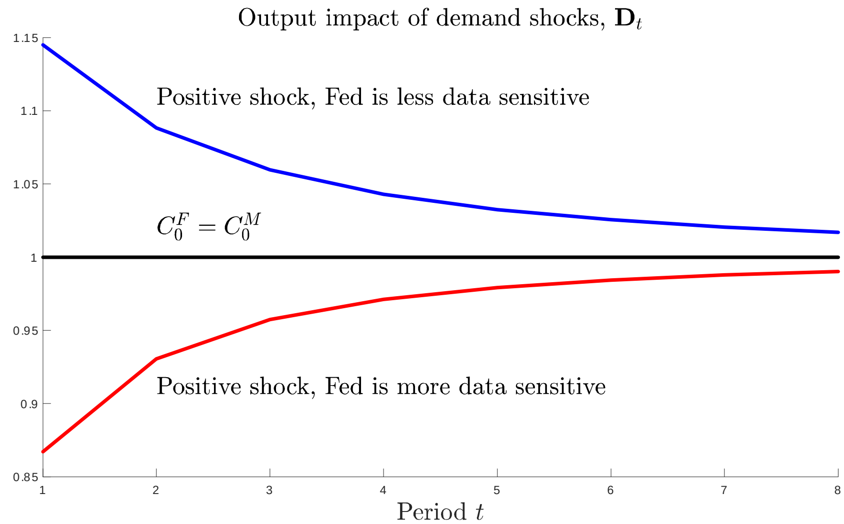

Figure 8: The equilibrium path for the output impact of demand shocks, $\{\mathbf{D}_t\}$, with heterogeneous data sensitivity.

Unlike the case with common data sensitivity, disagreement is stochastic and depends on the most recent realization of aggregate demand, g_t. Therefore, the market's expected output gap, $E_{t+1}^M[\tilde{y}_{t+1}]$, is no longer deterministic. For instance, when the Fed is more data sensitive, $1-\mathbf{c}_{t,t+1}^F > 1-\mathbf{c}_{t,t+1}^M$, an increase in aggregate demand, g_t, leads to a *decrease* in the expected output gap in the next period. How does this happen? An increase in demand affects the extent of disagreement: it increases the Fed's optimism compared to the market, because the Fed reacts to the data more than the market. As this happens, the market anticipates a Fed belief shock similar to the one in Section 5.1, which leads to a lower expected output gap.

Next recall from Eq. (7) that the equilibrium output gap depends on the surprises in the market's expectation for the future output gap, $E_{t+1}^M[\tilde{y}_{t+1}] - E_t^F\left[E_{t+1}^M[\tilde{y}_{t+1}]\right]$. Using Eq. $(B.8)$, we obtain,

$$E_{t+1}^M[\tilde{y}_{t+1}] - E_t^F\left[E_{t+1}^M[\tilde{y}_{t+1}]\right] = \mathbf{D}_{t+1}\left[\begin{array}{c}\left(1-\mathbf{c}_{t,t+1}^M\right)\\ -\left(1-\mathbf{c}_{t,t+1}^F\right)\end{array}\right]\left(g_t-\mathbf{g}_t^F\right). \tag{B.9}$$

The surprise in the expected output gap depends on the surprise in the demand shock, $g_t - \mathbf{g}_t^F$, because this determines the surprise in the extent of disagreement and the implied Fed belief shock.

Substituting Eq. $(B.9)$ into Eq. (7) from Section 3.2, we obtain

$$\tilde{y}_t = \left(1+\mathbf{D}_{t+1}\left[\begin{array}{c}\left(1-\mathbf{c}_{t,t+1}^M\right)\\ -\left(1-\mathbf{c}_{t,t+1}^F\right)\end{array}\right]\right)\left(g_t-\mathbf{g}_t^F\right).$$

Combining this with the difference equation $(B.4)$, the output gap satisfies Eq. $(B.2)$ also in period t.

The proof at the end of the section completes the remaining steps. Among other things, we show that the difference equation $(B.4)$ has a unique solution $\{\mathbf{D_t}\}$, that satisfies the properties in the proposition. In particular, $\mathbf{D_t} < 1$ if and only if $C_0^F < C_0^M$. For intuition, suppose the Fed is more data sensitive than the market, $1-\mathbf{c}_{t,t+1}^F > 1-\mathbf{c}_{t,t+1}^M$. In this case, the market expects the Fed to react to shocks more than (what the market thinks) is appropriate. Therefore, the bundled Fed belief shock dampens the direct impact of the demand shock [cf. $(B.9)$]. Conversely, when the Fed is less data sensitive, $1 - \mathbf{c}^F_{t,t+1} < 1 - \mathbf{c}^M_{t,t+1}$, the bundled Fed belief shock amplifies the direct impact of the shock.

Comparative statics of demand shocks. To shed further light on the mechanism, we next describe how a demand shock in period 0 affects the forward and the dot curves in the next period 1. Recall that agents' conditional belief in period 1 reflects a combination of their prior beliefs and the shock in period 0 [cf. $(B.1)$]:

$$\mathbf{g}_1^j = \mathbf{c}^j_{0,1}\left(g + u_0^j\right) + \left(1 - \mathbf{c}^j_{0,1}\right) g_0 \text{ for } j \in \{F, M\}. \tag{B.10}$$

Suppose agents' prior beliefs are the same, $\mathbf{g}_0^F = \mathbf{g}_0^M = \mathbf{g}$, so that agents initially have no disagreements. As a benchmark, suppose the initial shock in period 0 is the same as agents' prior, $g_0 = \mathbf{g}$. In this benchmark, agents' conditional belief is also the same as their prior, $\mathbf{g}_1^F = \mathbf{g}_1^M = \mathbf{g}$. Therefore, the forward and the dot curves in period 1 are also the same [cf. $(B.6 - B.7)$].

Now consider what happens when the economy experiences a positive demand shock, $g_0 = g + \Delta g_0$ with $\Delta g_0 > 0$. First consider the case in which the Fed is more data sensitive, $1 - \mathbf{c}^F_{0,1} > 1 - \mathbf{c}^M_{0,1}$. In this case, Eq. $(B.10)$ illustrates that a greater shock induces both agents to become more optimistic, but with a greater effect on the Fed, $\Delta \mathbf{g}_1^F > \Delta \mathbf{g}_1^M > 0$. Therefore, the curves shift as if there is a common-optimism shift, as in Section 3.4, combined with a Fed belief shift, as in Section 4. In particular, the dot curve increases more than the forward curve—especially for more distant horizons. The left panel of Figure 9 illustrates this case. Conversely, when the Fed is less data sensitive, $1 - \mathbf{c}^F_{0,1} < 1 - \mathbf{c}^M_{0,1}$, a greater demand shock has a larger effect on the market's optimism than the Fed's optimism, $\Delta \mathbf{g}_1^M > \Delta \mathbf{g}_1^F > 0$. Consequently, the forward curve increases more than the dot curve—especially for more distant horizons—as illustrated by the right panel of Figure 9.

This analysis also provides a complementary intuition for why heterogeneous data sensitivity changes the output impact of demand shocks [cf. Figure 8]. When the Fed is more data sensitive than the market, a positive demand shock increases the dot curve more than the forward curve [cf. the left panel of Figure 9]. This generates a dampened increase in output, because the market considers the Fed's additional reaction a "mistake"—driven by a Fed belief shock as in Section 5.1. Conversely, when the Fed is less data sensitive than the market, the shock increases the dot curve less than the forward curve [cf. the right panel of Figure 9]; and this induces an amplified effect on output since the market considers the Fed's muted reaction a "mistake."

We next complete the sketch proof of Proposition 7. The proof relies on the following generalization of Lemma 2.

Lemma 3. *Consider the setup with arbitrary C_0^F, C_0^M. Consider the mean belief at the beginning of period s about the conditional mean belief (about aggregate demand) in a subsequent period $t \geq s$. For each agent $j \in \{F, M\}$ and $j' \neq j$, we have:*

$$\begin{aligned} E_s^j\left[\mathbf{g}_t^j\right] &= \mathbf{g}_s^j. \\ E_s^j\left[\mathbf{g}_t^{j'}\right] &= \mathbf{c}^{j'}_{s,t}\mathbf{g}_s^{j'} + \left(1 - \mathbf{c}^{j'}_{s,t}\right)\mathbf{g}_s^j. \end{aligned}$$

As before, each agent expects *the other agent's* conditional belief about aggregate demand in a future period to be a weighted average of the other agent's current belief and its own belief. The weights depend

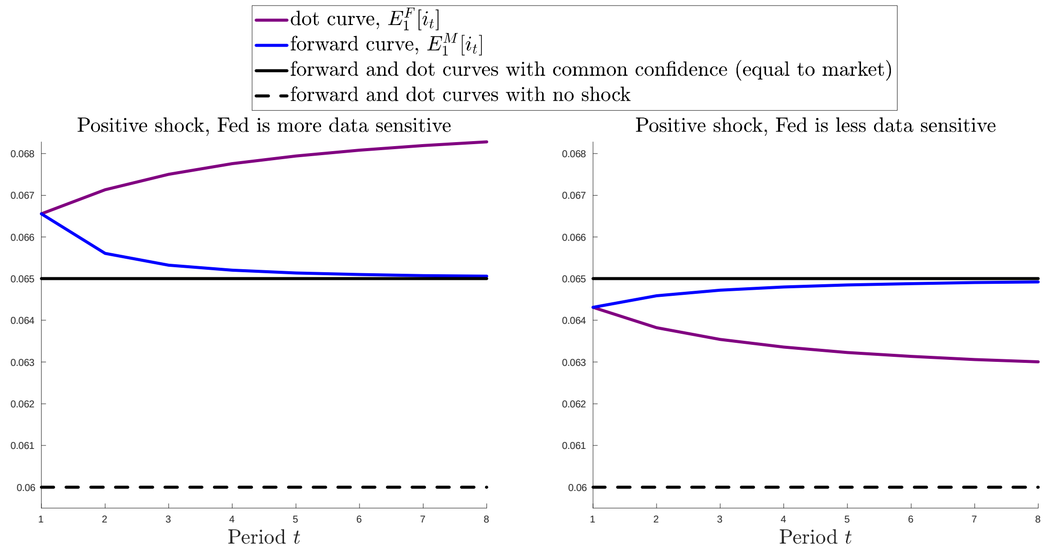

Figure 9: The comparative statics of an increase in the initial demand shock, $g_0 = g + \Delta g_0$ with $\Delta g_0 = 0.01$. The black solid lines illustrate the effects for the case analyzed in the main text in which the Fed and the market have common data sensitivity. The left panel (resp. the right panel) illustrates the effects when the Fed has greater (resp. smaller) data sensitivity than in the baseline, while the market has the same data sensitivity as in the baseline.

on *the other agent's* relative confidence, $\mathbf{c}_{s,t}^{j'}$. Intuitively, the agent expects the other agent to learn from the data with a speed that depends on *the other agent's* data sensitivity. The proof is omitted (it is a straightforward extension of the proof of Lemma 2 presented in Section 3.3).

Proof of Proposition 7. We first characterize the solution to the difference equation $(B.4)$, $\{\mathbf{D_t}\}$, and establish its properties. We then characterize the equilibrium interest rate as well as the forward curve and the dot curve.

To solve the difference equation, we first rewrite it as:

$$\mathbf{D_t} = 1 + \mathbf{C}_t \mathbf{D}_{t+1} \text{ with } \lim_{t\to\infty} \mathbf{D}_t = 1, \tag{B.11}$$

where we define

$$\mathbf{C}_t = \mathbf{c}_{t,t+1}^F - \mathbf{c}_{t,t+1}^M. \tag{B.12}$$

As a first step, we claim:

$$|\mathbf{C}_t| \le |\mathbf{C}_0| < 1 \text{ for each } t. \tag{B.13}$$

To prove this claim, we observe that $\mathbf{c}_{t,t+1}^j = \frac{C_0^j + t}{C_0^j + t + 1}$ implies:

$$\mathbf{C}_t = \frac{C_0^F - C_0^M}{\left(C_0^F + t + 1\right)\left(C_0^M + t + 1\right)}.$$

This in turn proves the claim since,

$$|\mathbf{C}_t| \le \frac{\left|C_0^F - C_0^M\right|}{\left(C_0^F + 1\right)\left(C_0^M + 1\right)} = |\mathbf{C}_0| = \left|\mathbf{c}_{0,1}^F - \mathbf{c}_{0,1}^M\right| < 1.$$

Here, the last inequality follows since $\mathbf{c}_{0,1}^F, \mathbf{c}_{0,1}^M \in (0,1)$.

We next iterate Eq. $(B.11)$ forward to obtain:

$$\mathbf{D_t} = 1 + \mathbf{C}_t + \mathbf{C}_t\mathbf{C}_{t+1} + ... + \mathbf{C}_t\mathbf{C}_{t+1}...\mathbf{C}_{t+n} + \mathbf{C}_t\mathbf{C}_{t+1}...\mathbf{C}_{t+n}\mathbf{D}_{t+n}.$$

Taking the limit of the expression as $n \to \infty$ and using $\lim_{n\to\infty} \mathbf{D}_{t+n} = 1$ and $|\mathbf{C}_t| \leq |\mathbf{C}_0| < 1$ [cf. $(B.13)$], we obtain:

$$\mathbf{D_t} = 1 + \sum_{n=0}^{\infty} \mathbf{C}_t\mathbf{C}_{t+1}...\mathbf{C}_{t+n}. \tag{B.14}$$

Note also that $|\mathbf{C}_t| \leq |\mathbf{C}_0| < 1$ implies $|\mathbf{D_t}| \leq \frac{1}{1-|\mathbf{C}_0|}$. Thus, there exists a unique and finite solution characterized by $(B.14)$.

Next consider the properties of the solution. First consider the case in which $C_0^F > C_0^M$. This implies $\mathbf{c}_{t,t+1}^F > \mathbf{c}_{t,t+1}^M$ for each t, which in turn implies $\mathbf{C}_t > 0$ for each t. Then, the closed-form solution in $(B.14)$ implies $\mathbf{D}_t > 1$ for each t.

Next consider the case in which $C_0^F < C_0^M$. This implies $\mathbf{c}_{t,t+1}^F < \mathbf{c}_{t,t+1}^M$ for each t, which in turn implies $\mathbf{C}_t < 0$ for each t. For this case, we next show $\mathbf{D}_t \in (0,1)$ for each t. This proves $\mathbf{D}_t$ satisfies the properties in the proposition.

To show the claim, first note that for an arbitrary $\varepsilon > 0$ we have $\mathbf{D}_T \in (0, 1+\varepsilon)$ for sufficiently large T (since $\lim_{t\to\infty} \mathbf{D}_t = 1$). Let $\varepsilon = 1/|\mathbf{C}_0| - 1$ and note that it is strictly positive [cf. $(B.13)$]. Suppose $\mathbf{D}_{t+1} \in (0, 1+\varepsilon)$ for some $t+1$. Then, Eq. $(B.11)$ implies:

$$\mathbf{D_t} = 1 + \mathbf{C}_t\mathbf{D}_{t+1} > 1 - |\mathbf{C}_t|(1+\varepsilon) \geq 1 - |\mathbf{C}_0|(1+\varepsilon) = 0.$$

Here, the second inequality uses $|\mathbf{C}_t| \leq |\mathbf{C}_0|$ [cf. $(B.13)$] and the last equality substitutes the definition of ε. Thus, we have $\mathbf{D_t} > 0$. Likewise, we have

$$\mathbf{D_t} = 1 + \mathbf{C}_t\mathbf{D}_{t+1} < 1,$$

where the inequality follows since $\mathbf{C}_t < 0$ and $\mathbf{D_{t+1}} > 0$. It follows that, for any $\mathbf{D}_{t+1} \in (0, 1+\varepsilon)$, we have $\mathbf{D_t} \in (0,1) \subset (0, 1+\varepsilon)$. By induction, this proves $\mathbf{D_t} \in (0,1)$ for each t.

Next consider the equilibrium interest rate. Using Eq. $(B.8)$, we obtain

$$E_t^F\left[E_{t+1}^M\left[\tilde{y}_{t+1}\right]\right] = \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right).$$

Substituting this into Eq. (6) from Section 3.2, we solve for the equilibrium interest rate as:

$$i_t = \rho + \mathbf{g}_t^F + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right).$$

This proves Eq. $(B.5)$.

Next consider the forward and the dot curves. Fix a period s and a subsequent period $t \geq s$. Consider the interest rate in period t. Taking the expectation of Eq. $(B.6)$ according to the market's belief in period s, we obtain,

$$\begin{aligned} E_s^M[i_t] &= \rho + \left(1 - \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right) E_s^M\left[\mathbf{g}_t^F\right] + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M E_s^M\left[\mathbf{g}_t^M\right] \\ &= \rho + \left(1 - \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right)\left(\mathbf{c}_{s,t}^F\mathbf{g}_s^F + \left(1 - \mathbf{c}_{s,t}^F\right)\mathbf{g}_s^M\right) + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\mathbf{g}_s^M \\ &= \rho + \mathbf{g}_s^M + \mathbf{c}_{s,t}^F\left(1 - \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right)\left(\mathbf{g}_s^F - \mathbf{g}_s^M\right). \end{aligned}$$

Here, the second line uses Lemma 3 to evaluate the higher order belief. This proves Eq. $(B.6)$.

Likewise, taking the expectation of Eq. $(B.6)$ according to the Fed's belief, we obtain,

$$\begin{aligned} E_s^F[i_t] &= \rho + \left(1 - \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right) E_s^F\left[\mathbf{g}_t^F\right] + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M E_s^F\left[\mathbf{g}_t^M\right] \\ &= \rho + \left(1 - \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right)\mathbf{g}_s^F + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\left(\mathbf{c}_{s,t}^M\mathbf{g}_s^M + \left(1 - \mathbf{c}_{s,t}^M\right)\mathbf{g}_s^F\right) \\ &= \rho + \mathbf{g}_s^F + \mathbf{D}_{t+1}\mathbf{c}_{1,t+1}^M\left(\mathbf{g}_s^M - \mathbf{g}_s^F\right). \end{aligned}$$

This establishes Eq. $(B.7)$ and completes the proof of the proposition. □