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ABSTRACT

We study supply and demand shocks in a general disaggregated model with multiple sectors, multiple factors, input-output linkages, downward nominal wage rigidities, credit-constraints, and a zero lower bound. We use the model to understand how the Covid-19 crisis, an omnibus of supply and demand shocks, affects output, unemployment, and inflation, and leads to the coexistence of tight and slack labor markets. Negative sectoral supply shocks are stagflationary, whereas negative sectoral demand shocks are deflationary. Furthermore, complementarities in production amplify Keynesian spillovers from negative supply shocks but mitigate them for negative demand shocks. In a stylized quantitative model of the US, we find supply and demand shocks each explain about half the reduction in real GDP. Although there is as much as 7% Keynesian unemployment, this is concentrated in certain markets. Hence, aggregate demand stimulus is less than half as effective as in a typical recession where all labor markets are slack.

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1 Introduction

Covid-19 is an unusual macroeconomic shock. It cannot easily be categorized as an aggregate supply or demand shock. Rather, it is a messy combination of disaggregated supply and demand shocks. These shocks propagate through supply chains to create different cyclical conditions in different parts of the economy. Some sectors are tight, constrained by supply constraints, and struggling to keep up with demand. Other sectors are slack and shedding workers to reduce excess capacity because of lack of demand.

To analyze this situation of divergent outcomes and take advantage of the available disaggregated data, we use a general disaggregated model and aggregate up from the micro to the macro level. We allow for an arbitrary number of sectors and factors as well as unrestricted input-output linkages and elasticities of substitution. We incorporate downward nominal wage rigidities, credit-constraints, and a zero lower bound on nominal interest rates.

We model the outbreak of the pandemic as a combination of supply and demand shocks. We define demand shocks as changes in households' expenditures for fixed prices and income and supply shocks as changes in the economy's production possibilities. On the one hand, the epidemic set off demand shocks by changing final demand within and across periods. Even fixing incomes and prices, households rebalanced their current expenditures across sectors for fear of infection from consuming certain goods and disliking the steps needed to consume other goods safely. Households also reduced their current expenditures overall, deciding to postpone spending to the future when conditions for consumption are back to normal. On the other hand, the epidemic also triggered supply shocks that reduced the availability of some goods. For example, lockdowns, social distancing in the workplace, and insurance and liability concerns reduced the supply of labor that can safely be used and the productivity of different industries.

In this paper, we provide local and global comparative statics with respect to these shocks and study how the comparative statics depend on complementarities and credit-constraints. We then use a calibrated version of the model to study the Covid-19 crisis, decomposing the supply and demand sources of the recession and conducting counterfactuals. We describe these contributions in turn.

First, we provide general *local* comparative statics, characterizing the response of aggregates such as output, inflation, and unemployment, as well as of disaggregated variables. In particular, we show how the elasticities of substitution in production and in consumption interact with the input-output network to redirect demand away from some factors and towards others, causing Keynesian unemployment in labor markets where demand goes down more than supply.[1]

In some special cases, we also provide *global* comparative statics. These global comparative statics allow us to capture the nonlinearities of the model and in particular how the shocks interact with each other and get amplified or mitigated. Under some conditions, we show that as long as there are *complementarities*, the set of equilibria can be ranked so that there is a unique best equilibrium with the minimal number of slack labor markets and the minimal amount of Keynesian unemployment in each labor market.

We show that complementarities in production amplify negative supply shocks to one market by causing Keynesian spillovers in other markets. Intuitively, a negative supply shock raises the price of the shocked sectors, and because of complementarities, expenditures are redirected towards those sectors. This reduces demand in other sectors and causes Keynesian unemployment. In contrast, the same complementarities that amplify supply shocks also mitigate demand shocks. In response to a negative demand shock, flexible factor prices fall, and because of complementarities, expenditures are redirected away from flexibly-priced factors towards Keynesian factor markets, which stabilizes those markets.

In the best equilibrium, we show that output is monotone decreasing in negative supply and negative demand shocks, whereas inflation is monotone increasing in negative supply and monotone decreasing in negative demand shocks. Hence, although supply shocks can generate Keynesian unemployment, such shocks are generically inflationary.

The effects of both negative supply and demand shocks are stronger if unemployed households are unable to borrow against their future income. In this case, unemployed workers are forced to cut back their spending more aggressively than they would if they could borrow. Therefore, credit-constraints magnify spending reductions given income losses and act like endogenous negative demand shocks. As with exogenous demand shocks, these endogenous demand shocks are also mitigated by complementarities.

Although our model is disaggregated, under some conditions, it nevertheless admits an aggregate supply (AS) and aggregate demand (AD) representation. We use this to illustrate graphically how the equilibrium responds to shocks. A novelty of our model is that supply shocks do not simply shift the AS and AD curves, but they also change their shape, resulting in apparent instability of the AS-AD relationship. The unstable shape of these curves reflects the nonlinearities arising from the interaction of complementarities

[1] Keynesian unemployment measures the amount of slack in a given factor market. It captures underemployment due to lack of demand for the good that the factor is producing because of downwardly rigid wages. Measured unemployment in the data reflects not only Keynesian unemployment but other forms of supply-driven underemployment due to the pandemic. See Section 2.2 for a discussion.

and occasionally-binding downward nominal wage rigidities.

We use a parsimonious quantitative input-output model of the US economy to gauge the importance of the various theoretical forces that we identify. We calibrate the model to match the reduction in sectoral employment and nominal expenditures in May, 2020 compared to February, 2020. The benchmark model predicts that real GDP falls by 9%, inflation is −1% and there is up to 7% Keynesian unemployment. Negative supply shocks on their own reduce output by only 6%, cause mild Keynesian unemployment of around 1%, and imply inflation should be close to 7%. On the other hand, negative demand shocks on their own reduce output by 5%, cause 10% Keynesian unemployment and predict inflation of −4%. Hence, *both* supply and demand shocks are necessary to match the data, which features large reductions in real GDP but only mild deflation.

Furthermore, we use the model to classify sectoral labor markets as supply-constrained (tight) or demand-constrained (slack). Supply-constrained sectors experienced mild inflation, and demand-constrained sectors experienced mild deflation in both the model and the data over our sample period.

Using the model, we quantify the importance of complementarities. We find that although complementarities amplify negative supply shocks, they also mitigate demand shocks by roughly an equal amount, and therefore, do not have strong effects on the overall aggregate response of inflation or output. However, complementarities change the breakdown between the relative importance of supply versus demand.

Separating demand shortfalls from supply constraints is important because they have different implications for policy. Policies that boost demand, like lowering interest rates or increasing government spending, exacerbate problems of inadequate supply, leading to shortages and inflation. Similarly, policies that boost supply, like relaxing lock-downs or providing liability exemptions, are ineffective at restoring activity when applied to demand-constrained sectors.

In this vein, we consider policy counterfactuals for social insurance and monetary policy. The sectoral nature of the Covid-19 shock, which affects sectors in different ways, blunts the power of untargeted aggregate demand stimuli like monetary policy. Compared to a purely aggregate-demand-driven recession, monetary or untargeted fiscal stimulus is less than half as effective in the current crisis since around half the labor markets are supply-constrained. Realistic complementarities further sap the efficacy of aggregate demand stimulus by dissipating more of it as inflation.

We also study the importance of social insurance in stabilizing inflation, output, and employment. Our baseline model has complete markets, and we quantify how credit-constraints further depress output, inflation, and employment. For example, if 50% of unemployed workers become credit-constrained and receive no income support from the government, then output falls by an additional 1%, and Keynesian unemployment increases by an extra 2%. As with monetary policy, the importance of social insurance depends on the strength of complementarities, and in a Cobb-Douglas model with weaker complementarities, social insurance is three times more important.

The outline of the paper is as follows. In Section 2, we set up the model, define the equilibrium and notation, and discuss the shocks. In Section 3, we establish local comparative static results for a general model with arbitrary elasticities of substitution. In Section 4, we specialize the environment to Cobb-Douglas, give some examples, and conduct some global (rather than local) comparative statics. We also show how the disaggregated Cobb-Douglas model can be represented via an AS-AD diagram. In Section 5, we generalize the global comparative statics and discuss the way complementarities can amplify supply shocks and mitigate demand shocks. In Section 6, we conduct a quantitative exercise to understand the importance of the various mechanisms we have emphasized for the Covid-19 crisis and their implications for policy. We touch upon some extensions of the basic framework in Section 7 before concluding in Section 8.

Related Literature

The paper is part of the literature on economic effects of the Covid-19 crisis, as well as the literature on multi-sector models with nominal rigidities.

Guerrieri et al. (2020) show that negative supply shocks can have negative demand spillovers, under the condition that the intersectoral elasticity of substitution is less than the intertemporal one. They also show that this condition is weaker under incomplete markets. Our results about supply shocks build on and are related to theirs. We show that complementarities in the production network, rather than consumption, can also amplify negative supply shocks, even if the intersectoral and intertemporal elasticities of substitution in consumption are the same. Furthermore, we show that while complementarities amplify negative supply shocks, they also mitigate negative demand shocks. In our quantitative exercise, a Cobb-Douglas model, without complementarities, predicts almost the same reduction in output and employment as a model with stronger intersectoral complementarities because of these off-setting effects.

Bigio et al. (2020) study optimal policies in response to the Covid-19 crisis in a two-sector Keynesian model. We differ in both focus and framework, since we are not focused on optimal policy and instead try to understand the importance of the production structure.[2] Fornaro and Wolf (2020) study Covid-19 in a New-Keynesian model where the pandemic is assumed to have persistent effects on productive capacity in the future by lowering aggregate productivity growth. The expected loss in future income reduces aggregate demand. They show that a feedback loop can arise between aggregate supply and aggregate demand if productivity growth in turn depends on the level of economic activity.[3] We differ in that we focus on the effects of current disruptions. Caballero and Simsek (2020) study a different kind of spillover, between asset prices and demand shortages.

Our paper also relates to quantitative multi-sector models. Barrot et al. (2020) study the effect of Covid-19 using a quantitative production network with complementarities and detailed administrative data from France. Bonadio et al. (2020) study the effect of Covid-19 in a quantitive international trade model. Bodenstein et al. (2020) analyze optimal shutdown policies in a two-sector model with complementarities and minimum-scale requirements. Our approach differs from these papers due to our focus on nominal rigidities and Keynesian effects. Brinca et al. (2020) use a statistical model to decompose sectoral outcomes in the Covid-19 crisis into demand- and supply-side sources. Our classification of demand and supply drivers are conceptually different to theirs for reasons we discuss in Section 2. Kaplan et al. (2020) combine an SIR model with a multi-sector heterogeneous agent New Keynesian model to study the economic impact of the pandemic.

This paper is also related to other work by the authors, especially Baqaee and Farhi (2020b). Whereas in this paper, we study how exogenous shocks interact with nominal frictions and result in involuntary unemployment, Baqaee and Farhi (2020b) is a companion paper where we analyze the nonlinear mapping from changes in hours and household preferences to real GDP. In this companion paper, we find that the negative supply and demand shocks associated with Covid-19 are large enough that accounting for nonlinearities is quantitatively important.

Our analysis is also related to production network models with nominal rigidities, like Baqaee (2015), who studies the effect of targeted fiscal policy and shocks to the sectoral composition of demand in a production network with downward wage rigidity, Pasten et al. (2017) and Pasten et al. (2019) who study propagation of monetary and TFP shocks in models with sticky prices, Ozdagli and Weber (2017) who study the interaction of monetary policy, production networks, and asset prices, and Rubbo (2020) and La'O and Tahbaz-Salehi (2020) who study optimal monetary policy with sticky prices.

[2]Bigio et al. (2020) study a fully dynamic model specified in continuous time, which allows them to analyze how the effects unfold over time.

[3]This could be because of reduced investment in research and development due to a reduced size of the market à la Benigno and Fornaro (2018).

2 Setup

In this section, we set up the basic model. We break the description of the model in two. First, we discuss the intertemporal problem of how households choose to spend their income across periods. Second, we discuss the intratemporal problem of how a given amount of expenditures is spent across different goods within a period. We then define the equilibrium notion and discuss the shocks that we will be studying.

2.1 Environment and Equilibrium

There are two periods, the present denoted without stars, and the future denoted with stars, and there is no investment.[4] We take the price level in the future as given. As in Krugman (1998) and Eggertsson and Krugman (2012), this is isomorphic to an infinite-horizon model where after an initial unexpected shock in period 1, the economy returns to a long-run equilibrium with market clearing and full employment.[5] We denote the supply of the future composite final-consumption good by $\bar{Y}_*$, its price by $\bar{p}^Y_*$, and future final income and expenditure by $\bar{E}_* = \bar{p}^Y_* \bar{Y}_*$, which are all taken to be exogenous.

There are a set of producers $\mathcal{N}$ and a set of factors $\mathcal{G}$ with supply functions $L_f \in [0, 1]$, which exist in both the present and the future. Full employment occurs when $L_f = 1$ for every $f \in \mathcal{G}$. We denote by $\mathcal{N} + \mathcal{G}$ the union of these sets. We abuse notation and also denote the number of producers and factors by $\mathcal{N}$ and $\mathcal{G}$.

Consumers. Consumers own the primary factors. When the quantity of employed factor f falls, we assume this change comes about via the extensive margin. That is, some fraction $1 - L_f$ of the owners become unemployed while the remaining fraction L_f continue to receive payment. Of the households who are unemployed, some fraction ϕ_f can borrow against their income tomorrow. The rest, $1 - \phi_f$, derive their entire income from f, cannot borrow, and therefore cannot consume today.

All households have the same intertemporal utility function

$$(1-\beta)\frac{y^{1-1/\rho}-1}{1-1/\rho} + \beta\frac{y_*^{1-1/\rho}-1}{1-1/\rho},$$

[4] We abstract from investment in the main body of the paper in order to keep the exposition manageable. We show in Appendix E how our approach generalizes to environments with investment.

[5] Our analysis extends to situations where the crisis lasts for multiple periods without change, as long as we maintain the assumption that there is no investment and no credit constraints; see footnote 12 for more information.

where ρ is the intertemporal elasticity of substitution (IES), $\beta \in [0, 1]$ captures households' time-preferences, and y and y_* are current and future consumption.

Since employed consumers and unemployed consumers that can borrow have the same homothetic preferences, we can aggregate their demand and refer to them as the representative *Ricardian* household. The rest of the households, who are unemployed and cannot borrow, we call the *hand-to-mouth* (HtM) households. The intertemporal budget constraint for the representative Ricardian household is

$$p^Y y + \frac{p_*^Y y_*}{1+i} = \sum_{f \in \mathcal{G}} w_f L_f + \sum_{f \in \mathcal{G}} \frac{w_f^* L_f^*}{1+i} \Big(1 - (1 - L_f)(1 - \phi_f)\Big),$$

where $(1 + i)$ is the nominal interest rate, the wage and quantity of factor f are w_f, L_f, w_f^*, and L_f^* in the current period and future period. The right-hand side of this equation is the permanent income of the Ricardian household, and the second summand on the right-hand side is the income of the Ricardian household in the future. Since some of the income earned in the future goes to the HtM households, the term $(1 - L_f)(1 - \phi_f)$ subtracts the income claimed by HtM households tomorrow from total future income. We omit the HtM households' budget constraint since they simply spend their exogenous future income on the future good and cannot consume in the present.

Now, we turn to the within-period problem. The consumption bundle in the present period is given by

$$Y = \mathcal{C}(c_1, \ldots, c_N; \omega_{\mathcal{D}}),$$

a homothetic final-demand aggregator of the final consumptions c_i of the different goods i. The parameter $\omega_{\mathcal{D}}$ is a preference shifter capturing changes in the sectoral composition of final demand. The price p^Y of the consumption bundle Y is denoted by

$$p^Y = \mathcal{P}(p_1, \ldots, p_N; \omega_{\mathcal{D}}).$$

where $\mathcal{P}$ is the dual price index of the quantity index $\mathcal{D}$. We also denote by

$$E = p^Y Y$$

the present final expenditure. In the rest of the paper, we will refer to Y as *output*.[6]

[6]As long there are no changes in the composition of final demand, $\omega_{\mathcal{D}}$ is constant, then changes in Y also coincide with changes in real GDP. To define real GDP, we mimic the chain-weighted procedures used by national income accountants, local changes in real GDP (output) are defined by the Divisia quantity index $d \log Y^{GDP} = \sum_{i \in \mathcal{N}} (p_i c_i)/E d \log c_i$, and changes in the GDP deflator are given by the Divisia price index

Producers. Producer i maximizes profits

$$\pi_i = \max_{\{y_i\},\{x_{ij}\},\{L_{if}\}} p_i y_i - \sum_{j \in \mathcal{N}} p_j x_{ij} - \sum_{f \in \mathcal{G}} w_f L_{if}$$

subject to production function

$$y_i = A_i F_i \left(\left\{ x_{ij} \right\}_{j \in \mathcal{N}}, \left\{ L_{if} \right\}_{f \in \mathcal{G}} \right),$$

where A_i is a Hicks-neutral productivity shifter, y_i is total output, and x_{ij} and L_{if} are intermediate and factor inputs used by i. Without loss of generality, we assume that F_i has constant returns to scale.[7]

Market equilibrium. Market equilibrium for goods is standard. The market for i is in equilibrium if

$$c_i + \sum_{j \in \mathcal{N}} x_{ji} = y_i.$$

Market equilibrium for factors is non-standard, the wages of factors cannot fall below some exogenous lower bound.[8] We say that factor market f is in equilibrium if the following there conditions hold:

$$(w_f - \bar{w}_f)(L_f - \bar{L}_f) = 0, \quad \bar{w}_f \leq w_{f,} \quad L_f \leq \bar{L}_f,$$

where

$$L_f = \sum_{i \in \mathcal{N}} L_{if}$$

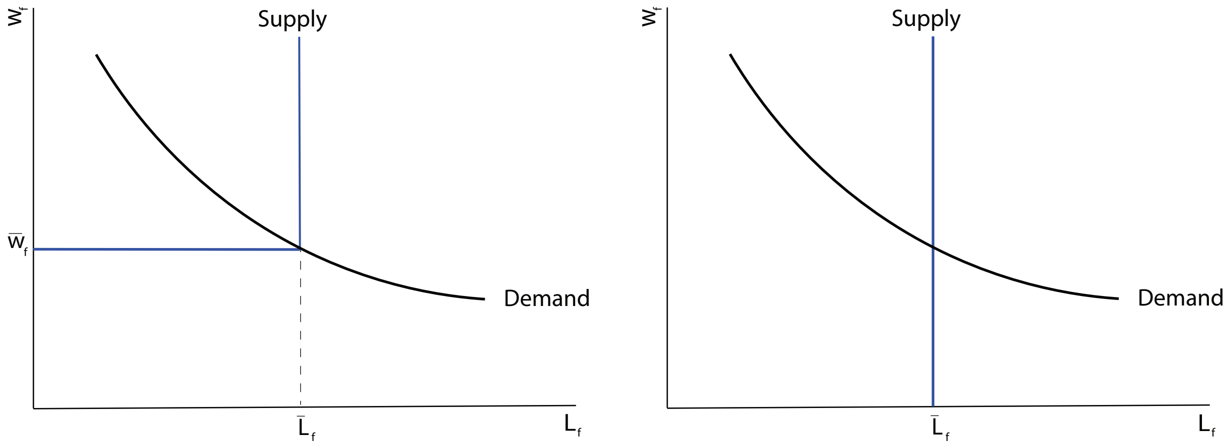
is the total demand for factor f. The parameters $\bar{w}_f$ and $\bar{L}_f$ are exogenous minimum nominal wage and endowment of the factor.

In words, there are two possibilities. One possibility is $w_f \geq \bar{w}_f$ and employment of the factor is equal to potential with $L_f = \bar{L}_f$. In this case, we say that the market is tight, that it clears, and that it is *supply-constrained*. The other possibility is that $w_f = \bar{w}_f$ and

$d \log p^{GDP} = \sum_{i \in \mathcal{N}} (p_i c_i)/(E) d \log p_i$. Therefore, nominal GDP can be decomposed into changes in real GDP and changes in the price level $d \log E = d \log Y^{GDP} + d \log p^{GDP}$. Discrete changes in real GDP and the price level are defined by integrating the Divisia indices. If the composition of final demand $\omega_{\mathcal{D}}$ changes, then real GDP $\Delta \log Y^{GDP}$ and the consumption bundle $\Delta \log Y$ are only equal up to a first order approximation. See Baqaee and Farhi (2020b) for more details. We return to these issues in the quantitative exercise in Section 6.

[7]Following the replication argument of McKenzie (1959), we can treat every production function as though it has constant returns by adding producer-specific fixed factors to the model.

[8]In Appendix F, we extend the model to allow for some downward wage flexibility.



(a) Factors in $\mathcal{L}$ (b) Factors in $\mathcal{K}$

Figure 2.1: Equilibrium in the factor markets.

employment of the factor is less than potential $L_f \leq \bar{L}_f$. We then say that the market is slack, that it does not clear, and that it is *demand-constrained*. In this case, we call the underemployment $\bar{L}_f - L_f$ of the factor *Keynesian unemployment* since it is caused by a lack of demand for the good that the factor is producing given the rigid wage.

We only consider two cases: the case where $\bar{w}_f$ is equal to its pre-shock market-clearing value, denoting the set of such factors by $\mathcal{L} \subseteq \mathcal{G}$; and the case where $\bar{w}_f = -\infty$, making the wage of f flexible and ensuring the market for f always clears, denoting the set of such factors by $\mathcal{K} \subseteq \mathcal{G}$. For concreteness, we call $\mathcal{K}$ the capital factors and $\mathcal{L}$ the labor factors.

Of course, these are just names, in practice, one may easily imagine that certain capital markets could also be subject to nominal rigidities. This can be a way to model firm failures: imagine firms take out within-period loans to pay for their variable expenses, secured against their capital income. If the firm's capital income declines in nominal terms, then the firm defaults on the loan, exits the market, and its capital becomes unemployed for the rest of the period. We build on this observation further in Appendix D, where we formally introduce an extensive margin of firm exit. For the body of the paper, we treat capital markets as being frictionless.

We denote the endogenous set of supply-constrained factor markets by $\mathcal{S} \subseteq \mathcal{G}$. In other words, $f \in \mathcal{S}$ if, and only if, $L_f = \bar{L}_f$. We denote the endogenous set of demand-constrained factor markets by $\mathcal{D} \subseteq \mathcal{G}$. Hence, $f \in \mathcal{D}$ if, and only if, $w_f = \bar{w}_f$. Of course, capital markets are always supply-constrained $\mathcal{K} \subseteq \mathcal{S}$, and demand-constrained sectors are necessarily a subset of labor markets $\mathcal{D} \subseteq \mathcal{L}$. Figure 2.1 illustrates the supply and demand curves in the factor markets.

Equilibrium. Given a nominal interest rate $(1+i)$, factor supplies $\bar{L}_f$, productivities A_i, and demand shifters $\omega_{\mathcal{D}}$, an equilibrium is a set of prices p_i, factor wages w_f, intermediate input choices x_{ij}, factor input choices L_{if}, outputs y_i, and final demands c_i, such that: each producer maximizes its profits subject to its technological constraint; consumers maximize their utility; and the markets for all goods and factors are in equilibrium. Without loss of generality, we normalize $\bar{Y}_* = \bar{Y} = 1$ and $p_*^Y = 1$.

2.2 Supply and Demand Shocks

We provide comparative statics with respect to shocks, starting at an initial equilibrium with full employment of all factors. A natural disaster, like the Covid epidemic, can be captured as a combination of negative supply and demand shocks. We define a demand shock to be a shock that changes the household's expenditure shares on the different goods (across sectors and over time) *at given prices and incomes*. We define supply shocks to be shocks that change the possibilities to produce the different goods.

Supply shocks. We define supply shocks to be changes in the economy's production possibility frontier, which could come in the form of either reduced factors or reduced productivity. We call reductions in the available productive endowment of labor $\bar{L}_f$ shocks to *potential labor*. These are reductions that would take place absent any nominal frictions. These reductions could have different drivers. They could be driven directly by government action, like mandated shutdowns and stay at home orders. They could also be due to a reduced willingness to work by employees due to health concerns or policy disincentives such as overly generous unemployment insurance. Finally, reductions in potential labor could also be the result of a reorganization of production. For example, firms could be forced to operate at lower capacity to reduce legal liability and implement social distancing, such as a restaurant that can only safely serve a fraction of the customers it used to serve. In this case, workers would be involuntarily unemployed due to a reduced physical capacity to employ them and not because there is not enough demand for the good that they produce. This type of supply-driven underemployment would occur even in the absence of downward nominal wage rigidities. For this reason, we do not include this form of underemployment in our definition of Keynesian unemployment.[9]

[9] To model capacity constraints formally, imagine that $\bar{L}_f = \min\{\tilde{L}_f, S_f\}$, where $\tilde{L}_f$ is the physical endowment of labor and S_f is a "safety" input which, in the initial equilibrium, is not scarce. Since it is not scarce, it commands a price of zero initially. However, the pandemic reduces the supply of S_f so that it binds. At this point, the supply of potential labor $\bar{L}_f$ falls one-for-one with S_f. In this case, employers would refuse to hire any additional workers since their marginal product is zero. A formal capacity constraint like this

Similarly, the epidemic might have reduced the productivity A_i of the different producers by changing the way firms can operate, for instance by reducing person-to-person interactions.

Demand shocks. Whereas supply shocks change household's choices by changing prices and incomes, demand shocks change household choices for fixed prices and income.[10] Accordingly, the pandemic can change the current sectoral composition of final demand, since at given prices and income, households may shift expenditure away from some goods like cruises and air transportation, and towards other goods like groceries and online retail. We model this as a change in the preference shifter $\omega_{\mathcal{D}}$.

Similarly, the pandemic can reduce households' willingness to consume in the present relative to the future: at given prices and income, households may choose to consume less during the epidemic and more afterwards. We model this as an increase in the discount factor $\beta/(1-\beta)$. In Section 4, we provide a simple microfoundation for these demand shocks using a health-related disutility function.

2.3 Input-Output Definitions

To analyze the model, we define some input-output objects such as input-output matrices, Leontief inverse matrices, and Domar weights associated with any equilibrium. To make the exposition more intuitive, we slightly abuse notation by treating factors with the same notation as goods. For each factor f, we interchangeably use the notation L_{if} or $x_{i(\mathcal{N}+f)}$ to denote its use by producer i, the notation L_f or y_f to denote total factor supply, and p_f or w_f to refer to its price or wage. Furthermore, we denote final demand as an additional good produced by producer 0 using the final demand aggregator. We interchangeably use the notation c_i or x_{0i} to denote final consumption of good i. We write $1+\mathcal{N}$ for the union of the sets $\{0\}$ and $\mathcal{N}$, and $1+\mathcal{N}+\mathcal{G}$ for the union of the sets $\{0\}$, $\mathcal{N}$, and $\mathcal{G}$. With this abuse of notation, we can stack every market in the economy into a single input-output matrix that includes the household, the producers, and the factors.

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is isomorphic to our formulation where we directly shock $\bar{L}_f$ in terms of real GDP, inflation, and hours worked. The only difference is that the increase in the wage w_f would not take place and would instead be captured as a Ricardian rent by the firm.

[10]Our notion of supply and demand shocks are defined in the context of a general equilibrium, and are not the same as the one used by Brinca et al. (2020). They separate shocks based on whether they shift labor supply or labor demand, but for us, a "supply" shock can shift either labor supply or labor demand. For example, a capacity constraint placed on firms due to social distancing, described in the previous footnote, would manifest as a reduction in labor demand, but be classified as a supply shock under our definition since it reduces the production possibilities of the economy.

Input-output matrix. We define the input-output matrix to be the $(1+\mathcal{N}+\mathcal{G})\times(1+\mathcal{N}+\mathcal{G})$ matrix Ω whose ijth element is equal to i's expenditures on inputs from j as a share of its total income/revenues

$$\Omega_{ij} \equiv \frac{p_j x_{ij}}{p_i y_i} = \frac{p_j x_{ij}}{\sum_{k\in\mathcal{N}+\mathcal{G}} p_k x_{ik}}.$$

The input-output matrix Ω records the *direct* exposures of one producer to another.

Leontief inverse matrix. We define the Leontief inverse matrix as

$$\Psi \equiv (I-\Omega)^{-1} = I + \Omega + \Omega^2 + \ldots.$$

The Leontief inverse matrix Ψ records instead the *direct and indirect* exposures through the supply chains in the production network. This can be seen from the fact that $(\Omega^n)_{ij}$ measures the weighted sums of all paths of length n from producer i to producer j.

Nominal expenditure and Domar weights. Recall that nominal expenditure is the total sum of all final expenditures

$$E = \sum_{i\in N} p_i c_i = \sum_{i\in N} p_i x_{0i}.$$

We define the Domar weight λ_i of producer i to be its sales share as a fraction of GDP

$$\lambda_i \equiv \frac{p_i y_i}{E}.$$

Note that $\sum_{i\in N} \lambda_i > 1$ in general since some sales are not final sales but intermediate sales. The Domar weight λ_f of factor f is simply its total income share and factor income shares sum to one $\sum_{f\in\mathcal{G}} \lambda_f = 1$.

The accounting identity $p_i y_i = p_i x_{0i} + \sum_{j\in\mathcal{N}} p_i x_{ji} = \Omega_{0i}E + \sum_{j\in\mathcal{N}} \Omega_{ji}\lambda_j E$ links the Domar weights to the Leontief inverse via

$$\lambda_i = \Psi_{0i} = \sum_{j\in\mathcal{N}} \Omega_{0j}\Psi_{ji},$$

where $\Omega_{0j} = (p_j x_{0j})/(\sum_{k\in\mathcal{N}+\mathcal{G}} p_k x_{0k}) = (p_j c_j)/E$ is the share of good j in final expenditure.

2.4 Nested-CES Economies

For simplicity, we restrict attention to nested-CES economies. That is, we assume every production function and the final demand function can be written as nested-CES functions (albeit with an arbitrary set of nests).

More precisely, any nested-CES economy can be written in *standard form*, defined by a tuple $(\bar{\omega}, \theta)$, where $\bar{\omega}$ is a $(1 + \mathcal{N} + \mathcal{G}) \times (1 + \mathcal{N} + \mathcal{G})$ matrix of input-output parameters and θ is a $(1 + \mathcal{N}) \times 1$ vector of microeconomic elasticities of substitution. Each good $i \in \mathcal{N}$ is produced with the production function

$$\frac{y_i}{\bar{y}_i} = \frac{A_i}{\bar{A}_i} \left(\sum_{j \in \mathcal{N}+\mathcal{G}} \bar{\omega}_{ij} \left(\frac{x_{ij}}{\bar{x}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}},$$

where x_{ij} are intermediate inputs from j used by i. Final demand is produced by producer 0 using

$$\frac{y_0}{\bar{y}_0} = \left(\sum_{j \in \mathcal{N}+\mathcal{G}} \bar{\omega}_{0j} \frac{\omega_{0j}}{\bar{\omega}_{0j}} \left(\frac{x_{0j}}{\bar{x}_{0j}} \right)^{\frac{\theta_0 - 1}{\theta_0}} \right)^{\frac{\theta_0}{\theta_0 - 1}},$$

where ω_{0j} are sectoral demand shocks with $\sum_j \omega_{0j} = 1$. In these equations, variables with over-lines are normalizing constants.To simplify the notation below, we think of ω_0 as a $1 \times (1 + \mathcal{N} + \mathcal{G})$ vector with k-th element ω_{0k}.

Through a relabelling, this structure can represent any nested-CES economy with an arbitrary pattern of nests and elasticities. Intuitively, by relabelling each CES aggregator to be a new producer, we can have as many nests as desired.[11]

3 Local Comparative Statics

In this section, we describe the comparative statics of the basic model and provide some examples. Our results here are local (first-order) comparative statics. In Sections 4 and 5, we provide global comparative statics in important special cases.

Because of downward wage-rigidity, variables like aggregate output and inflation are not differentiable everywhere. Therefore, our local comparative statics should be understood as holding almost-everywhere. Furthermore, there are potentially multiple equilibria, in which case, local comparative statics should be understood as perturbations of a given locally-isolated equilibrium.

We write $d \log X$ for the differential of an endogenous variable X understood as the (infinitesimal) change in an variable X in response to (infinitesimal) shocks. For example,

[11]Our results can easily be extended beyond the nested-CES case along the lines of Section 5 in Baqaee and Farhi (2019a).

the supply shocks are $d \log A_i$ and $d \log \bar{L}_f$, and the shocks to the sectoral composition of demand are $d \log \omega_{0j}$. We sometimes write them in vector notation as $d \log A$, $d \log \bar{L}$, and $d \log \omega_0$. Similarly, for discrete changes in a variable, we write $\Delta \log X$.

We proceed in several steps. First, we derive an Euler equation for nominal expenditure which gives changes in current nominal expenditure as a function of changes in the current price index. Second, we derive an aggregation equation which gives changes in output as a function of changes in nominal expenditure and changes in factor income shares. Third and finally, we derive propagation equations which give changes in factor income shares and changes in the price index as a function of changes in nominal expenditure. Putting these steps together gives a complete characterization of local comparative statics.

3.1 Euler Equations for Consumption and Expenditure

The consumption Euler equation for the Ricardian households is

$$Y = \frac{(1-\beta)}{\beta} \left(\frac{p^Y}{\bar{p}^Y/(1+i)} \right)^{-\rho} \frac{\bar{Y}_*}{(1+i)} \left(1 - \sum_{h \in \mathcal{G}} \lambda_h^* (1-L_h)(1-\phi_h) \right). \tag{3.1}$$

Log-linearizing the Euler equation results in an AD curve that relates changes in output $d \log Y$ to changes in the price index $d \log p^Y$:

$$d \log Y = -\rho d \log p^Y + d \log \zeta + d \log \Theta, \tag{3.2}$$

where $d \log \zeta$ and $d \log \Theta$ are intercepts. The first intercept term is

$$d \log \zeta = -\rho \Big(d \log(1+i) + d \log \frac{\beta}{1-\beta} - d \log \bar{p}_*^Y \Big) + d \log \bar{Y}_*. \tag{3.3}$$

With some abuse of terminology, we call $d \log \zeta$ an *aggregate* or *intertemporal demand* shock. A positive aggregate demand shock can come about from a reduction in the nominal interest rate or the discount factor, or an increase in future prices or output (a proxy for forward guidance).[12]

[12] If the crisis lasts for more than one period, and there are no credit-constraints, the Euler equation can still be used to write output in each period as a function of the price index in that period and exogenous shocks. That is, $\Delta \log Y_t = -\rho \Delta \log p_t^Y - \rho \left(\sum_{j=1}^{T} \Delta \log(1 + i_{t+j-1}) + \Delta \log \frac{\beta_*}{\beta_t} - \Delta \log \bar{p}_*^Y \right) + \Delta \log \bar{Y}_* + d \log \Theta$, where t indexes time and $*$ is the terminal period when the economy recovers. Since this is the only dynamic relationship, the rest of the analysis can be combined with this Euler equation instead to determine output in each period before recovery. This approach is only tenable if the periods are short-lived however, since we assume that the nominal wage constraint is exogenous.

The second intercept is

$$d \log \Theta = \frac{\mathbb{E}_{\lambda^*}\left(L_f(1-\phi_f) d \log L_f\right)}{1 - \mathbb{E}_{\lambda^*}\left((1-L_f)(1-\phi_f)\right)},$$

where the expectation uses the factor income shares in the future, λ^*, as the probability distribution. Note that without HtM households, $\phi_f = 1$ for every f, this term is always zero. We call $d \log \Theta$ the *endogenous aggregate demand* shock. This term captures the fact that reductions in employment today reduce spending today, since $1-\phi_f$ of type f workers become constrained. Therefore, as pointed out by Guerrieri et al. (2020), a supply-shock driven reduction in employment can feed back into reduced nominal demand because some households are HtM.

Since $E = p^Y Y$, changes in nominal expenditure $d \log E$ are similarly given by

$$d \log E = d \log(p^Y Y) = (1-\rho) d \log p^Y + d \log \zeta + d \log \Theta. \tag{3.4}$$

Recall that ρ is the intertemporal elasticity of substitution (IES). When $\rho > 1$, increases in prices $d \log p^Y > 0$ reduce nominal expenditure as consumers substitute towards the future. Conversely, when $\rho < 1$, increases in prices $d \log p^Y > 0$ increase nominal expenditure as consumers substitute towards the present. When $\rho = 1$, and there are no HtM households, changes in nominal expenditure are exogenously given by the shocks $d \log E = d \log \zeta$. Although our propositions allow for arbitrary values of ρ, we will focus primarily on the case where $\rho = 1$, which is a focal point for the empirical literature on the IES.

3.2 Aggregation Equation for Output

Next, we express changes in output as a function of changes in nominal expenditure and changes in factor shares.

Proposition 1 (Aggregation)**.** *Changes in output are given by*

$$\begin{aligned} d \log Y &= \sum_{i \in \mathcal{N}} \lambda_i d \log A_i + \sum_{f \in \mathcal{G}} \lambda_f d \log L_f, \\ &= \underbrace{\sum_{i \in \mathcal{N}} \lambda_i d \log A_i + \sum_{f \in \mathcal{G}} \lambda_f d \log \bar{L}_f}_{\Delta \text{ potential output}} + \underbrace{\sum_{f \in \mathcal{L}} \lambda_f \min\left\{d \log \lambda_f + d \log E - d \log \bar{L}_f, 0\right\}}_{\Delta \text{ output gap}}. \end{aligned}$$

The first expression for $d \log Y$ shows that a version of Hulten's (1978) theorem holds for this economy. In particular, to a first-order, changes in output can only be driven by changes in the productivities $d\log A_i$ weighted by their producer's sales share λ_i, or by changes in the quantities of factors $d\log L_f$ weighted by their income shares λ_f.[13]

The second expression uses the fact that while changes in capitals $f \in \mathcal{K}$ are exogenous with $d\log L_f = d\log \bar{L}_f$, changes in labors $f \in \mathcal{L}$ are endogenous with $d\log L_f = d\log \bar{L}_f + \min\left\{d\log\lambda_f + d\log E - d\log\bar{L}_f, 0\right\} \leq d\log\bar{L}_f$. Here we have used the observation that factor f is demand-constrained, with $d\log w_f = 0$ and $d\log L_f = d\log\lambda_f + d\log E$ if, and only if, changes in nominal expenditure on this factor $d\log\lambda_f + d\log E$ are below changes in its potential supply $d\log\bar{L}_f$.

The first term in the second expression is the change in potential output and corresponds to the change in output that would occur in a neoclassical version of the model with flexible wages and full employment of all factors. The second term is the the negative output gap that can open up in the Keynesian version of the model with downward nominal wage rigidities because of Keynesian unemployment in the different factor markets. These Keynesian spillovers depend on endogenous changes in nominal expenditure $d\log E$ (pinned down by the Euler equation for expenditures) and factor income shares $d\log\lambda_f$ (pinned down by the propagation equations in the next subsection). It is only through the determination of these endogenous sufficient statistics that the structure of the network and the elasticities of substitution matter.

Without delving into the details of the disaggregated model, we can already make an observation about the way inflation responds to shocks.

Corollary 1 (Inflation). *At the full-equilibrium steady-state, the change in the price level is given by*

$$d\log p^Y = d\log E - d\log Y = \frac{1}{\rho} d\log\zeta - \frac{1}{\rho}\left(\sum_{f\in\mathcal{G}} \lambda_f \phi_f d\log L_f\right).$$

Hence, reductions in employment are *stagflationary* unless they are accompanied by exogenous negative aggregate demand shocks. In particular, negative supply shocks or shocks to the sectoral composition of demand are both inflationary. This corollary follows from combining the Euler equation (3.4) with Proposition 1.

[13]This expression also shows that changes in the sectoral composition of demand within the period $d\log\omega_0$, or changes in aggregate demand $d\log\zeta$, can only change output through changes in the quantities of factors.

3.3 Propagation Equations for Shares, Prices, and Factor Employment

We now show how changes in factor income shares $d\log\lambda_f$ are determined. For a matrix M, we denote by $M_{(i)}$ its i-th row by $M^{(j)}$ its j-th column. We write $Cov_{\Omega^{(j)}}(\cdot,\cdot)$ to denote the covariance of two vectors of size $1+\mathcal{N}+\mathcal{G}$ using the j-the row of the input-ouput matrix $\Omega^{(j)}$ as a probability distribution.

Proposition 2 (Propagation)**.** *Changes in sales and factor shares are given by*

$$d\log\lambda_k = \theta_0 Cov_{\Omega^{(0)}}\left(d\log\omega_0, \frac{\Psi_{(k)}}{\lambda_k}\right) + \sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j-1)Cov_{\Omega^{(j)}}\left(\sum_{i\in\mathcal{N}}\Psi_{(i)}d\log A_i - \sum_{f\in\mathcal{G}}\Psi_{(f)}\left(d\log\lambda_f - d\log L_f\right), \frac{\Psi_{(k)}}{\lambda_k}\right)$$

almost everywhere, where changes in factor employments are given by

$$d\log L_f = \begin{cases} d\log\bar{L}_f, & \textit{for} \quad f\in\mathcal{K}, \\ \min\left\{d\log\lambda_f + d\log E, d\log\bar{L}_f\right\}, & \textit{for} \quad f\in\mathcal{L}. \end{cases}$$

To understand these equations, it helps to break them down into forward and backward propagation equations. Forward propagation equations describe changes in prices:

$$d\log p_k = -\sum_{i\in\mathcal{N}}\Psi_{ki}d\log A_i + \sum_{f\in\mathcal{G}}\Psi_{kf}d\log w_f,$$

with the change in the wage given by $d\log w_f = d\log\lambda_f + d\log E - d\log L_f$. Changes in prices propagate downstream (forward) through cost functions. A negative productivity shock $\Delta\log A_i$ to a producer i upstream from k increases the price of k in proportion to how much k buys from i directly and indirectly as measured by Ψ_{ki}. Similarly an increase $d\log w_f = d\log\lambda_f - d\log L_f + d\log E$ in the wage of factor f increases the price of k in proportion to the direct and indirect exposure of k to f.

Backward propagation equations describe changes in sales or factor shares given changes in prices:

$$d\log\lambda_k = \theta_0 Cov_{\Omega^{(0)}}\left(d\log\omega_0, \Psi_{(k)}/\lambda_k\right) + \sum_{j\in 1+\mathcal{N}}\lambda_j(\theta_j-1)Cov_{\Omega^{(j)}}\left(-d\log p, \Psi_{(k)}/\lambda_k\right).$$

Changes in sales propagate upstream (backward) through demand. The first term on the right-hand side $\theta_0 Cov_{\Omega^{(0)}}(d\log\omega_0, \Psi_{(k)}/\lambda_k)$ on the right-hand side is the direct effect of shocks to the sectoral composition of final demand on the sales of k. These shocks directly increase the share of k if they redirect demand towards goods j that have high direct and indirect exposures to k relative to the rest of the economy as measured by Ψ_{jk}/λ_k to k .

The second term $\sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j - 1)Cov_{\Omega^{(j)}}(-d\log p, \Psi_{(k)}/\lambda_k)$ on the right-hand side captures the changes in the sales of i from substitutions by producers j downstream from k. If producer j has an elasticity of substitution θ_j below one so that its inputs are complements, then it shifts its expenditure towards those inputs l with higher price increases $d\log p_l$, and this increases the demand for k if those goods l buy a lot from k directly and indirectly relative to the rest of the economy as measured by Ψ_{lk}/λ_k. These expenditure-switching patterns are reversed when θ_j is above one (the inputs of j are substitutes). When θ_j is equal to one (the inputs of j are Cobb-Douglas) these terms disappear.

Combining the backward and forward propagation equations yields Proposition 2. Note that once a factor market f becomes slack, the change in its income share $d\log\lambda_f$ becomes irrelevant for changes in all the other sales and factor shares since they then translate one for one into changes in employment of the factor $d\log L_f$ and leave its wage unchanged with $d\log w_f = 0$.

4 Cobb-Douglas Example

To better understand how the economy responds to supply and demand shocks, in this section, we focus on the Cobb-Douglas special case: $\rho = \theta_j = 1$ for every $j \in \mathcal{N}$. Hence, intertemporal and intersectoral preferences are log, and production functions are Cobb-Douglas. We discuss how the results change when we deviate from the Cobb-Douglas assumption in Section 5.

4.1 A Microfoundation for Demand Shocks

When household preferences are Cobb-Douglas, there is a simple microfoundation for the demand shocks motivated by health concerns. To see this, consider households with log preferences

$$(1-\beta)\left[\sum_{i\in\mathcal{N}} \bar{\Omega}_{0i}\log c_i - H\left(\{c_i\}_{i\in\mathcal{N}}\right)\right] + \beta\sum_{i} \bar{\Omega}_{0i}\log c_i^*,$$

where $\beta \in [0,1]$ captures households' time-preferences, and c_i and c_i^* are current and future consumption of good i. The function $H\left(\{c_i\}_{i\in\mathcal{N}}\right)$ is a homothetic aggregator that captures health concerns of the household associated with consumption today. We assume there are no health concerns in the future. We let the disutility of consumption due to health concerns be

$$H\left(\{c_i\}_{i\in\mathcal{N}}\right) = \sum_i \kappa_i \log c_i,$$

where $\kappa_i \geq 0$ captures the riskiness of consuming good i. As κ_i increases, households choose to spend a smaller fraction of their permanent income on purchasing i. We call an increase in κ_i an individual negative demand shock for sector i (in contrast to aggregate demand shocks which affect spending on all goods produced this period).

The health-risk parameters κ then map into shocks to both the intersectoral composition of demand

$$\Delta \log \omega_{0i} = \Delta \log \frac{\bar{\Omega}_{0i} - \kappa_i}{(1 - \sum_{j\in\mathcal{N}} \kappa_j)\bar{\Omega}_{0i}},$$

and shocks to aggregate demand

$$\Delta \log \zeta = -\Delta \log(1+i) - \Delta \log \frac{\beta}{1-\beta} + \Delta \log \bar{E}_* + \Delta \log(1 - \sum_{j\in\mathcal{N}} \kappa_j).$$

For future reference, when we refer to an aggregate demand shock, we mean a change in $\Delta \log \zeta$ that keeps the intersectoral composition of final demand $\Delta \log \omega_0 = 0$ constant.

4.2 Local Comparative Statics

Using Propositions 1 and 2, we analyze negative supply shocks $d \log \bar{L}_f \leq 0$ and negative demand shocks $d\kappa_i < 0$. For simplicity, set the share of potentially HtM households in every sector to be the same $\phi_i = \phi$. Recall that $\mathcal{S}$ and $\mathcal{D}$ are the equilibrium sets of supply- and demand-constrained factors. We give comparative statics for a given $\mathcal{S}$ and $\mathcal{D}$. We then give conditions for these sets of supply- and demand-constrained factors to indeed arise in equilibrium. We start by considering supply shocks.

Supply Shocks. Consider negative supply shocks on their own. In response to negative supply shocks, aggregate expenditures fall in the present, since some households become HtM. This reduction in spending reduces employment in demand-constrained factor markets and depresses output.

To see this, define the average negative labor shock to the supply-constrained factors

$$d \log \bar{L}_{\mathcal{S}} = \sum_{f\in\mathcal{S}} \frac{\lambda_f}{\lambda_{\mathcal{S}}} d \log \bar{L}_f,$$

where $\lambda_{\mathcal{S}} = \sum_{f\in\mathcal{S}} \lambda_f$. Similarly, the average employment change in the demand-constrained factors is

$$d\log L_{\mathcal{D}} = \sum_{f\in\mathcal{D}} \frac{\lambda_f}{\lambda_{\mathcal{D}}} d\log L_f < \sum_{f\in\mathcal{D}} \frac{\lambda_f}{\lambda_{\mathcal{D}}} d\log \bar{L}_f = d\log \bar{L}_{\mathcal{D}},$$

where $\lambda_{\mathcal{D}} = \sum_{f\in\mathcal{D}} \lambda_f$. Keynesian unemployment is given by $d\log L_{\mathcal{D}} - d\log \bar{L}_{\mathcal{D}}$. Using Proposition 1, we can write

$$d\log Y = \lambda_{\mathcal{S}} d\log \bar{L}_{\mathcal{S}} + \lambda_{\mathcal{D}} d\log \lambda_{\mathcal{D}} + \lambda_{\mathcal{D}} d\log E = \lambda_{\mathcal{S}} d\log \bar{L}_{\mathcal{S}} + \lambda_{\mathcal{D}} d\log E.$$

The second equality follows from Proposition 2 which implies that there are no changes in the share of factors $d\log \lambda_f = 0$. Reductions in nominal spending $d\log E$ reduce output by causing Keynesian unemployment.

Using the Euler Equation (3.4), starting at the full employment allocation, the change in nominal spending today is

$$d\log E = (1-\phi)\lambda_{\mathcal{S}} d\log \bar{L}_{\mathcal{S}} + (1-\phi)\lambda_{\mathcal{D}} d\log E = \frac{(1-\phi)\lambda_{\mathcal{S}} d\log \bar{L}_{\mathcal{S}}}{1-(1-\phi)\lambda_{\mathcal{D}}}.$$

Hence, negative supply shocks reduce nominal GDP by reducing the income of credit-constrained consumers directly and indirectly through a Keynesian-cross type effect. Combining these equations results in the following.

Proposition 3 (Supply shocks)**.** *Suppose $\rho = \theta_0 = \theta_j = 1$, and $\phi_j = \phi$ for all $j \in \mathcal{N}$. Then, in response to negative labor supply shocks $d\log \bar{L}$ we have*

$$d\log Y = \lambda_{\mathcal{S}} d\log \bar{L}_{\mathcal{S}} + \lambda_{\mathcal{D}} d\log L_{\mathcal{D}} = \frac{\lambda_{\mathcal{S}}}{1-(1-\phi)\lambda_{\mathcal{D}}} d\log \bar{L}_{\mathcal{S}}.$$

The direct impact on output of the negative shock to the supply-constrained factors is given by $\lambda_{\mathcal{S}} d\log \bar{L}_{\mathcal{S}}$, and the amplification of this shock through Keynesian channels is given by the multiplier $1/[1-(1-\phi)(1-\lambda_{\mathcal{S}})]$. Naturally, amplification is stronger, the lower is the social insurance parameter $\phi < 1$. Amplification is also stronger when the share of the supply-constrained factors $\lambda_{\mathcal{S}}$ is low.

We now go back and check that our conjectured set of supply-constrained factors is indeed the equilibrium set of supply-constrained factors. A factor f is demand-constrained in equilibrium if, and only if, $f \in \mathcal{L}$ and

$$\frac{(1-\phi)}{1-(1-\phi)\lambda_{\mathcal{D}}} \lambda_{\mathcal{S}} d\log \bar{L}_{\mathcal{S}} < d\log \bar{L}_f$$

That is, as long as the negative shock to factor f is sufficiently small in magnitude compared to the average shock affecting the supply-constrained part of the economy. This condition is harder to satisfy the smaller is the set of supply-constrained factors $\lambda_{\mathcal{S}}$ and the higher is the market completeness parameter ϕ. In particular, if we assume that there are no credit-constrained households $\phi = 1$, then this condition cannot be satisfied and all factors are supply-constrained. In this case, Keynesian frictions would not be triggered.

Demand Shocks. To understand demand shocks, starting at the full employment steady-state without supply shocks, we consider an aggregate demand shock. After that, we consider individual demand shocks.

Proposition 4 (Aggregate demand shocks)**.** *Suppose $\rho = \theta_0 = \theta_j = 1$, and $\phi_j = \phi$ for all $j \in \mathcal{N}$. For an aggregate demand shock, $d \log \zeta$, the change in output is*

$$d \log Y = \lambda_{\mathcal{D}} d \log L_{\mathcal{D}} = \lambda_{\mathcal{D}} d \log E = \frac{\lambda_{\mathcal{D}}}{1 - (1 - \phi)\lambda_{\mathcal{D}}} d \log \zeta.$$

The last equality uses (3.4). Hence, as long as there are some HtM households $\phi \neq 1$, aggregate demand shocks are also amplified by a multiplier $1/(1 - (1 - \phi)\lambda_{\mathcal{D}})$, for similar reasons to supply shocks.[14]

Next consider some individual demand shocks $d\kappa_i$, starting at the full employment steady-state without supply shocks. In this case, reduced demand for good i will ripple up the supply chain and differentially affect different factor markets. To see this, note that in demand constrained sectors, employment falls according to the reduction in nominal spending

$$d \log L_i = d \log \lambda_i + d \log E = d \log \left(\sum_j \Psi_{ji} \left(\bar{\Omega}_{0j} - \kappa_j \right) \right) + d \log \left[\left(1 - \sum_h \lambda_h^* (1 - \frac{L_h}{L_h^*})(1 - \phi) \right) \right] < 0,$$

where the second equality uses the Euler equation for expenditures (3.4). Intuitively, there are two reasons why nominal spending on factor i can fall. First, as emphasized in Baqaee (2015), a negative demand shock $d\kappa_j > 0$ to consumption good j affects demand for factor i by j's network-adjusted factor intensity $\Psi_{ji} > 0$. Intuitively, Ψ_{ji} is the fraction

[14] For supply shocks, the details of the production network do not show up in the results, and only the factor shares mattered. This is a manifestation of a more general result, shown in Appendix C, which establishes that as long as all elasticities of substitution are uniform, for negative supply shocks and aggregate demand shocks, initial factor shares are a global sufficient statistic for the production network. The Cobb-Douglas economy is just an example of this more general result because all the elasticities of substitution are equal to one. This sufficient statistic result does not hold for individual demand shocks.

of j's revenues that are ultimately paid out to factor i, both directly and indirectly. This is the first summand. The second summand captures the fact that demand shocks to any demand-constrained factors will depress the income of credit-constrained consumers, and through this, lower overall expenditures. The equation above is a linear system in $d\log L$, so solving through gives

$$d\log L_i = \frac{-\sum_j \Psi_{ji} d\kappa_j}{\lambda_i(1-\sum_h \kappa_h)} - \left[\frac{1-\phi}{\phi}\sum_{f\in\mathcal{D}} L_f \frac{\sum_j \Psi_{jf} d\kappa_j}{\lambda_i(1-\sum_h \kappa_h)}\right],$$

the first summand is the direct effect of the negative demand shock and the second summand is the negative spillovers from HtM households. In the complete markets case, with $\phi = 1$, only the direct effect matters. However, when there are credit-constrained consumers, the indirect effect also matters.

Combining these observations with Proposition 1 allows us to state the following.

Proposition 5 (Individual demand shocks)**.** *Suppose $\rho = \theta_0 = \theta_j = 1$, and $\phi_j = \phi$ for all $j \in \mathcal{N}$. For individual demand shocks, $d\kappa$, the change in output is*

$$d\log Y = -\frac{1}{(1-\sum_{h\in\mathcal{N}} \kappa_h)}\left[\sum_{i\in\mathcal{D}}\sum_{j\in\mathcal{N}} \Psi_{ji} d\kappa_j + \frac{1-\phi}{\phi}\sum_{i\in\mathcal{D}} \frac{L_i}{L_i^*}\sum_{j\in\mathcal{D}} \Psi_{ji} d\kappa_j\right].$$

The first term is the direct effect of the negative demand shock and the second term are the Keynesian spillovers from the presence of HtM households.

4.3 Global Comparative Statics

In general, the equilibrium of the model may not be unique. However, there is a simple-to-compute unique "best" equilibrium. We can also provide global comparative statics for this equilibrium. To state our result, we endow $\mathbb{R}^{\mathcal{G}}$ with the partial ordering $x \leq y$ if and only if $x_f \leq y_f$ for all $f \in \mathcal{G}$. Recall that we use Δ to denote discrete changes in a variable to distinguish them from infinitesimal local changes denoted by d.

Proposition 6 (Ranking equilibria)**.** *Suppose that $\rho = \theta_0 = \theta_j = 1$ for every $j \in \mathcal{N}$. Then there is a unique best equilibrium: for any other equilibrium, $\Delta\log Y$ and $\Delta\log L$ are lower than at the best equilibrium.*

Proposition 6 provides a straightforward way to compute this best equilibrium using a greedy algorithm along the lines of Vives (1990) or, more recently, Elliott et al. (2014).

We can find the best equilibrium as follows. Solve the model assuming all factor markets are supply-constrained. If one of the wages is below the minimum, call this market demand-constrained and set its wage equal to its lower bound. Recompute the equilibrium assuming that these factor markets are demand-constrained. Continue in this manner until the wage in every candidate supply-constrained market is above its lower bound.

For the best equilibrium, we can conduct global comparative statics for both supply and demand shocks.

Proposition 7 (Global comparative statics)**.** *Under the assumptions of Proposition 6, in the best equilibrium, the following holds:*

1. *Real GDP $\Delta \log Y$ and employment $\Delta \log L$ are increasing and the price level $\Delta \log p^Y$ is decreasing in supply shocks $\Delta \log \bar{L}$.*

2. *Real GDP $\Delta \log Y$, employment $\Delta \log L$, and the price level $\Delta \log p^Y$ are increasing in exogenous aggregate demand shocks $\Delta \log \zeta$.*

3. *Employment $\Delta \log L$ is decreasing in individual demand shocks $\Delta \kappa_i$.*

The global comparative static results in Proposition 7 show that the local-comparative static results hold globally. Furthermore, as anticipated in Section 3, (1) shows that negative labor shocks in some factor markets raise the overall price level and, if there are HtM households, create Keynesian unemployment in other factor markets. On the other hand, (2) shows that negative aggregate demand shocks, whether driven by policy, expectations about the future, or health concerns can create Keynesian unemployment whilst lowering the overall price level. Finally, (3) shows that individual demand shocks lower employment globally.[15]

4.4 Intuition Using AS-AD Representation

The intuition for Proposition 7 can be more effectively illustrated via an AS-AD representation. The best equilibrium of the model is the point at which an aggregate supply and aggregate demand curve intersect. This representation is useful for comparing the

[15]Following Section 4.1, an aggregate demand shock is driven by health-concerns if $\Delta \kappa_i = \bar{\Omega}_{0i} \kappa$ for any $\kappa > 0$. In this case, health-concerns reduce expenditures today without changing the sectoral composition of household spending. For technical reasons, we do not characterize changes in output and inflation when the sectoral composition of final demand changes. This is because when the sectoral composition of final demand changes, changes in real GDP can no longer be measured using $\Delta \log Y$ globally (only locally). See the path-dependence problem discussed in Baqaee and Farhi (2020b).

behavior of our model to the classic single-sector analysis. To draw this representation, for technical reasons, we require that demand shocks are uniform across sectors $\kappa_i = \bar{\Omega}_{0i}\kappa$, so that they do not change sectoral composition of final demand.[16]

To derive the AS curve, fix some level of output Y. There is a price level $p^Y(Y)$ such that: given the implied level of expenditure $E(Y) = p^Y(Y)Y$, the wage of every factor is consistent with the amount of expenditures on that factor; and these wages give rise to prices that are consistent with the AS curve $p^Y(Y)$. To derive the AD curve, we invert the consumption Euler equation (3.1) to express $p^Y(Y)$ as a function of Y, and use the reductions in employment consistent with the AS curve.

An example is plotted in Figure 4.1 at the initial equilibrium in the absence any exogenous shock.[17] The downward slope of the left-side of the AS curve depends on the downward flexibility of factor prices. If the set of capitals is empty ($\mathcal{K} = \emptyset$), then the AS curve is horizontal to the left. If the set of labors is empty ($\mathcal{L} = \emptyset$), then the AS curve is vertical to the left. Of course, in the case when there are no potentially-sticky factor markets, we recover the neoclassical model.

The shape of the AD curve depends on the share of HtM households ϕ. When there are no HtM households, $\phi = 1$, the AD curve is just given by $\Delta \log p^Y = -\Delta \log Y$. However, when there are HtM households, the AD curve becomes kinked. When output is higher than potential, no household is losing income, and so the AD curve is the same as the one with only Ricardian households. However, when output is below potential, the AD curve becomes flatter. Intuitively, when output is below potential, unemployment lowers nominal expenditures, and hence, for a given amount of output Y, reduces the price level.

4.4.1 Negative Supply Shock

As discussed earlier, negative supply shocks in one market can spill over into other markets if there are credit-constrained households. This negative spillover is larger, the larger is the share of households that can potentially become constrained. Figure 4.2 represents a negative supply shock using an AS-AD diagram with and without HtM households.

Start without HtM households. Initially, the AS curve is horizontal to the left since,

[16] As explained before, if demand shocks are not uniform, then discrete changes in real GDP $\Delta \log Y^{GDP}$ and the price level $\Delta \log p^{GDP}$, defined as integrals of the relevant Divisia index, cannot be recovered from $\Delta \log Y$ and $\Delta \log p^Y$.

[17] To plot the examples throughout this section, we simplify the structure of the economy somewhat by assuming there exists a retailer that produces the composite consumption good for the household. This rules out shocks to the sectoral composition of demand, since the only consumption good is produced by this retailer.

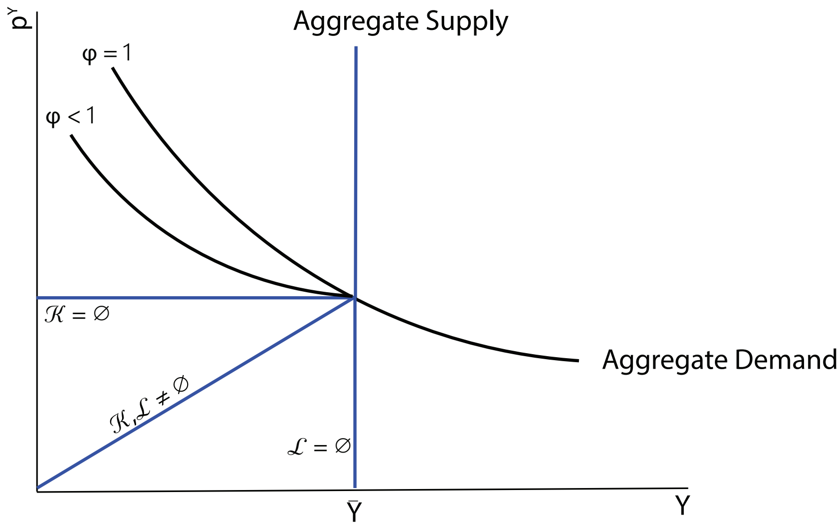


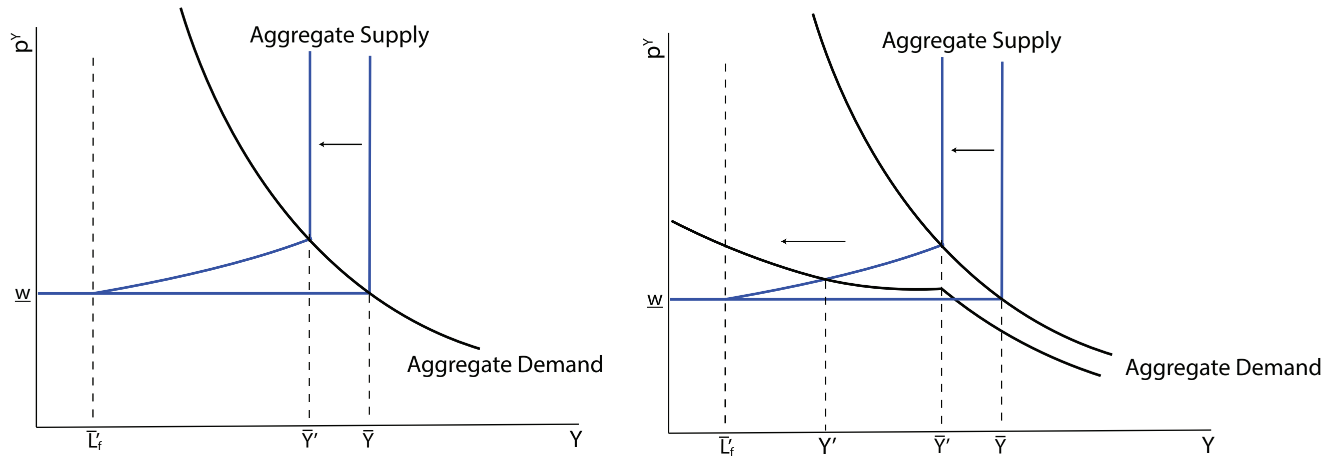
Figure 4.1: AS-AD representation of the equilibrium without shocks. The $\mathcal{K} = \emptyset$ case is when all factors have downwardly rigid wages, and $\mathcal{L} = \emptyset$ case is when all factors have flexible wages. The complete markets case is $\phi = 1$, and $\phi < 1$ is the case with some credit-constrained households.

in this example, there are no capitals ($\mathcal{K} = \emptyset$). The initial level of output is given by $\bar{Y}$ and the new level of labor available in the shocked sector is given by $\bar{L}'_f$. Unlike standard models, in response to the negative supply shock, the shape of the AS curve changes. The negative supply shock shifts the AS curve backwards, but it still intersects the AD curve at a point where output is equal to potential.

Now consider the panel with HtM households. In this case, the AS curve looks the same as before. However, now the AD curve also responds. In particular, the AD curve moves down and becomes kinked at the new level of potential output. The reason the AD curve moves is that the negative supply shock, by causing some constrained households to lose their jobs, reduces nominal expenditures. Whenever output is below potential, this is achieved via lower employment, and hence even lower expenditures, which lowers the AD curve further. Hence, with HtM households, output is now below potential in response to negative supply shocks, but we nevertheless have inflation as predicted by Proposition 7.

4.4.2 Negative Aggregate Demand Shock

Figure 4.3 plots the response of the economy to a negative aggregate demand shock instead, which shifts the AD curve down. Without HtM households, the AD curve would follow the dashed trajectory, lowering output below potential and (weakly) lower inflation. However, with HtM households, the AD curve is flatter when output is below



(a) Without HtM Households

(b) With HtM Households

Figure 4.2: AS-AD representation of the equilibrium with supply shocks. For this illustration, assume $\mathcal{K} = \emptyset$.

potential, and this feedback loop further strengthens the effect of initial negative demand shock, reducing output even further.

4.4.3 Interaction of Supply and Demand Shocks

We end our discussion of the Cobb-Douglas model by considering simultaneous negative supply and demand shocks. For simplicity, we assume there are no HtM households in Figure 4.4. Assuming away HtM households is also likely to be consistent with the first few months of the Covid outbreak in the United States, where massive government support prevented personal incomes from declining in nominal terms.

In Figure 4.4, a negative supply shock shifts the AS curve to the left and a negative demand shock pushes the AD curve down. With supply shocks only, output would fall and inflation would rise. In conjunction with negative demand shocks, output falls even farther and inflation is brought down.

The figure shows that since the reduction in output is partly driven by supply shocks, and some factor markets are supply-constrained, positive aggregate demand stimulus is less potent than it would be in the absence of negative supply shocks. This simple observation will turn out to be a quantitatively important reason to expect aggregate demand stimulus to be less potent during the Covid crisis.

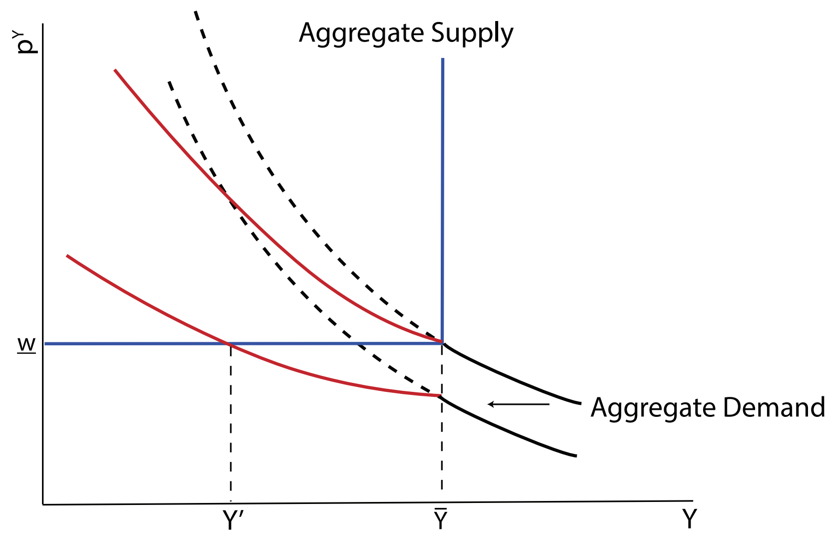


Figure 4.3: AS-AD representation of the equilibrium with demand shocks. We assume $\mathcal{K} = \emptyset$. The dashed lines are the AD curve without HtM households.

5 Complementarities in Production

At the sectoral level, there is strong empirical evidence that elasticities of substitution in production are well below one. In this section, we generalize and extend Section 4 beyond the unit-elasticity Cobb-Douglas assumption. First, we show that the global comparative statics in Section 4 extend to the case with complementarities. Then we show that complementarities have ambiguous effects on supply and demand shocks: complementarities amplify Keynesian spillovers from supply shocks, but mitigate Keynesian spillovers from demand shocks. Therefore, in the presence of both types of shocks, it is not a priori clear whether complementarities amplify or mitigate shocks.

5.1 Global Comparative Statics

We begin by proving that the set of equilibria can still be ranked.

Proposition 8 (Ranking equilibria with complementarities)**.** *Suppose that $\rho = \theta_0 = 1$ and $\theta_j = \theta < 1$ for every $j \in \mathcal{N}$. Then there is a unique best equilibrium: for any other equilibrium, $\Delta \log Y$ and $\Delta \log L$ are lower than at the best and higher than at the worst.*

We consider comparative statics in the supply shocks $\Delta \log \bar{L}$ and exogenous demand shocks $\Delta \log \zeta$ either due to policy or health concerns.

Proposition 9 (Global comparative statics with complementarities)**.** *Under the assumptions of Proposition 8, in the best equilibrium, the comparative statics in Proposition 7 still hold.*

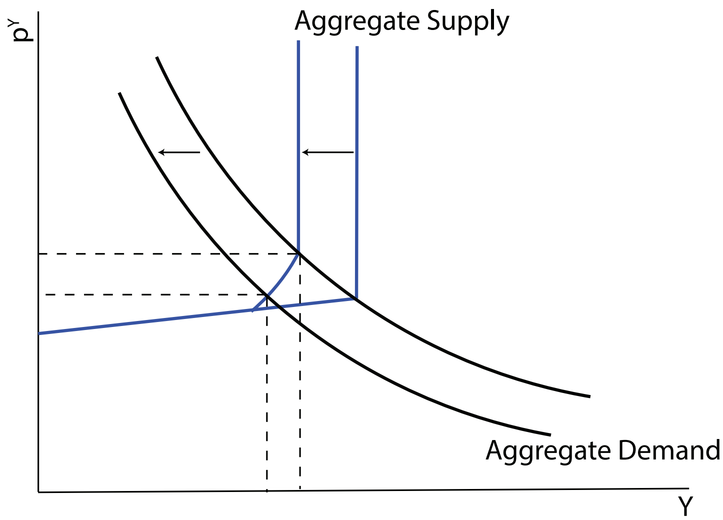


Figure 4.4: Negative supply shock coupled with a negative aggregate demand shock.

The key difference between Proposition 9 and the Cobb-Douglas version in Proposition 7 is that complementarities can amplify negative supply shocks through Keynesian spillovers even without HtM households. Intuitively, if two factors i and j supply the same downstream producer k, so $\Psi_{ki} \neq 0$ and $\Psi_{kj} \neq 0$, then complementarities in k's production function will cause negative supply shocks to i to spillover into reduced demand for j. This can cause factor j to become demand-constrained.

Formally, under the assumptions of Proposition 8, using Proposition 2, for $k \neq j$, we can write

$$\lambda_k \frac{d \log \lambda_k}{d \log L_j} = \frac{(1-\theta)}{\theta} \mathbb{E}_{\Omega^{(0)}}(\Psi_{(k)} \Psi_{(j)}) - \frac{(1-\theta)}{\theta} \sum_{f \in \mathcal{S}} \mathbb{E}_{\Omega^{(0)}}(\Psi_{(k)} \Psi_{(f)}) \left(\frac{d \log \lambda_f}{d \log L_j} \right).$$

Terms of the form $\mathbb{E}_{\Omega^{(0)}}(\Psi_{(k)} \Psi_{(j)})$ are non-negative, and they are equal to zero if, and only if, k and j share no downstream consumers (directly or indirectly) except the household.

The intuition for this equation is the following: the first term captures how an increase in the quantity $d \log L_j$ of factor j raises the income share of k as long as demand for k and j are not orthogonal $\mathbb{E}_{\Omega^{(0)}}(\Psi_{(k)} \Psi_{(j)}) \neq 0$. The second term shows that even if k and j are orthogonal, in that they share no downstream consumers directly or indirectly, the shock to j can change factor prices for supply-constrained factors f, and changes in f's factor price can then be transmitted to k, if f and k are not orthogonal.

Benefits of Flexibility and Reallocation. Proposition 9 also implies that wage flexibility and factor reallocation are desirable in the best equilibrium. These two corollaries may at first seem obvious, but they are by no means universally valid. Since the model with nominal rigidities is inefficient, the theory of the second best means that seemingly desirable attributes like flexibility and reallocation can actually turn out to be harmful in general. However, these propositions guarantee that neoclassical intuitions about flexibility and reallocation are still empirically relevant.

To show that wage flexibility is desirable, we take a factor $f \in \mathcal{L}$ and remove its downward wage rigidity constraint by moving it to $\mathcal{K}$. This amounts to creating a more flexible economy.

Corollary 2. *Under the assumptions of Proposition 8, for the best equilibrium,* $\Delta \log Y$ *and* $\Delta \log L$ *are higher in the more flexible than the less flexible economy.*

In addition to the fact that flexibility is desirable, we can also prove that reallocation is desirable. We consider two factors h and h' that are paid the same wage at the initial equilibrium and that have the same minimum nominal wage. The idea is that these two factors are really the same underlying factor, but that frictions to reallocation prevent one from being used in place of the other. We then consider an economy where these reallocations are allowed to take place.

Corollary 3. *Under the assumptions of Proposition 8, the best equilibrium of the no-reallocation economy has lower output* $\Delta \log Y$ *and employment* $\Delta \log L$ *than the best equilibrium of the reallocation economy.*

5.2 Amplification and Mitigation of Shocks

As mentioned before, complementarities amplify negative supply shocks and mitigate negative demand shocks. To see this clearly, assume there is full social insurance $\phi = 1$ and a single final good (the same intuition holds in the absence of these assumptions). Using Propositions 1 and 2, we have the following.

Proposition 10 (Supply and demand shocks with complementarities)**.** *Suppose there is one final consumption good, the elasticity of substitution in production is* $\theta_j = \theta < 1$*, and there are no credit constraints* $\phi = 1$*. Then, in response to negative labor supply shocks* $d \log \bar{L}$ *and aggregate demand shocks* $d \log \zeta$*, we have*

$$d \log Y = \frac{\sum_{f \in \mathcal{S}} \lambda_f d \log \bar{L}_f}{1 - (1-\theta)(1-\lambda_{\mathcal{S}})} + \left(1 - \frac{(1-\theta)\lambda_{\mathcal{S}}}{1-(1-\theta)(1-\lambda_{\mathcal{S}})}\right)(1-\lambda_{\mathcal{S}}) d \log \zeta,$$

where $\lambda_{\mathcal{S}} = \sum_{f \in \mathcal{S}} \lambda_f$*.*

The first term is the response of real GDP to supply shocks and the second term is the response to aggregate demand shocks. We start by discussing the supply shocks. The denominator $1/(1-(1-\theta)(1-\lambda_{\mathcal{S}}))$ captures the amplifying effect of complementarities. In particular, complementarities across producers can transmit negative supply shocks in one factor market as negative demand shocks to other factor markets. This negative spillover is larger, the stronger are the complementarities.

The intuition is the following: negative supply shocks raise the wages of some factors, redirect demand towards them, and deprive other factor markets of demand who then become constrained. The amount of Keynesian unemployment in the demand-constrained factor markets increases as we lower the elasticity of substitution θ. In fact, the amplification from complementarities $1/(1-(1-\theta)(1-\lambda_{\mathcal{S}}))$ take a similar functional form to the amplification from incomplete markets in Proposition 3.

Now consider the second term, which is the response of real GDP to negative aggregate demand shocks. Fixing expenditure shares, the negative aggregate demand shock reduces output because it reduces employment in the demand-constrained sectors and this effect is equal to $(1-\lambda_{\mathcal{S}})d\log\zeta$. The term in brackets in front captures the mitigating effect of complementarities.

The intuition is the following: in response to a reduction in aggregate demand, the price of supply-constrained sectors falls. Due to complementarities, this causes expenditures to switch towards the demand-constrained factor markets, whose prices cannot fall, and this substitution boosts employment in those factor markets. Intuitively, with complementarities, factor markets with flexible prices, for example capital markets, absorb part of the negative demand shock and redirect demand to demand-constrained sectors.

For brevity, we include more detailed derivations in Appendix H. In this appendix, we also show how the result in Proposition 10 can be demonstrated graphically using AS-AD diagrams.

6 Quantitative Application

We now turn to a quantitative calibration. We use a parsimonious stylized quantitative model to disentangle supply and demand shocks, consider the model's predictions about prices, and answer counterfactual questions about the importance of complementarities, the degree of social insurance, and the potency of aggregate demand stimulus. We calibrate our model to match the peak to trough reductions in employment from February, 2020 to May, 2020. We show that complementarities amplify supply shocks and mitigate demand shocks to roughly off-setting effects. We also show that social insurance is crucial for ameliorating the effects of the crisis, significantly raising output, prices, and employment. Finally, we show that the sectorally disparate nature of the Covid-19 crisis has sapped the potency of aggregate demand stimulus compared to a traditional demand-driven recession.

6.1 Calibration

We start by describing our calibration of the model and of the shocks.

Calibrating the economy. There are 66 sectors and sectoral production functions use labor, capital, and intermediates. The share parameters of the functions are calibrated so that at the initial pre-shock allocation, expenditure shares match those in the input-output tables from the BEA. We focus on the short run and assume, following Baqaee and Farhi (2019a), that labor and capital cannot be reallocated across sectors. We construct the input-output matrix using the 2015 annual U.S. input-output data from the BEA, dropping the government, non-comparable imports, and second-hand scrap industries. The dataset contains industrial output and inputs for 66 industries.

We set the elasticity of substitution between labor and capital to 0.5, between value-added and intermediate inputs to 0.6, across intermediates to 0.2. We set the elasticity of substitution across final uses to be $\theta_0 = 1.0$. We also set the intertemporal elasticity of substitution $\rho = 1.0$. These numbers are broadly in line with estimates from Atalay (2017), Herrendorf et al. (2013), Oberfield (2013), and Boehm et al. (2019).

We assume that sectoral labor markets feature downward nominal wage rigidity, whereas sectoral capital markets have flexible rental rates. Goods prices are set competitively and flexibly. Finally, since personal incomes did not decline, due to large government transfer programs, we assume full social insurance and set the fraction of households that become HtM to zero for the initial calibration.

Calibrating the shocks. Covid-19 set off an array of supply and demand shocks. We model the Covid-19 crisis using a combination of shocks to potential labor supplies and shocks to the sectoral composition of demand across sectors and time periods. We begin by describing how we calibrate demand shocks, and then describe how we calibrate supply shocks.

Since both the intertemporal ρ and intersectoral θ_0 elasticities of substitution are equal to one for the household, realized changes in household spending patterns can be directly fed into the model as demand shocks (because household expenditure shares do not depend on relative prices).

Given the demand shocks, in principle, if the model is perfectly correctly specified, we can directly feed changes in hours by sector as the primitive supply shocks. This is because if a labor market is supply constrained, then the only way to match hours in that market is via a reduction in potential employment. On the other hand, if a labor market is demand constrained and has Keynesian unemployment, then any reduction in potential labor supplied up to the realized reduction in hours will have no effect on any outcome. This also means that there is ambiguity about how large supply shocks are in demand-constrained sectors. We resolve this ambiguity by setting supply shocks in demand-constrained sectors to zero.[18]

We describe our data sources for the primitive supply and demand shocks. Data on the sectoral composition of demand comes from the May, 2020 release of personal consumption expenditures from the BEA. Since personal consumption is about 66% of final demand, we downweight these shocks by 2/3. This is equivalent to assuming that the sectoral composition of other components of final demand has not changed. The primitive demand shock to the intertemporal composition of demand (aggregate demand) is chosen to deliver 9.3% reduction in nominal GDP implied by downweighting the reduction in PCE. To calibrate the primitive supply shocks, we compute changes in hours worked by sector from the May, 2020 BLS Economic News release. Figure A.1 in Appendix A shows the sectoral supply and demand shocks.

Of course, since our model is quite stylized with mostly uniform elasticities of substitution, this procedure results in a reasonable but imperfect fit to the employment data. In this calibration, the (size-weighted) average error in hours in non-healthcare sectors is 2.3%.[19] Having calibrated the model, we show predicted changes in macro aggregates, decompose the importance of different shocks, and consider the model's out-of-sample performance.

[18]This choice does not matter for our baseline in terms of aggregate and sectoral output, inflation, and employment but it maximizes the amount of Keynesian unemployment. This choice also affects our counterfactual with only supply shocks.

[19]Our simulations predict counterfactually large reductions in employment by hospitals and ambulatory health care services. However, despite large reductions in expenditures on these sectors (from reduced elective procedures, etc.), in the data, healthcare industries do not show large reductions in employment. Presumably, this reflects the fact that the excess capacity in the healthcare industry is not wasted. Healthcare workers are instead engaged in non-market activities related to the pandemic. Due to the unique role these sectors play in the pandemic, we exclude them here.

6.2 Role of Supply and Demand Shocks

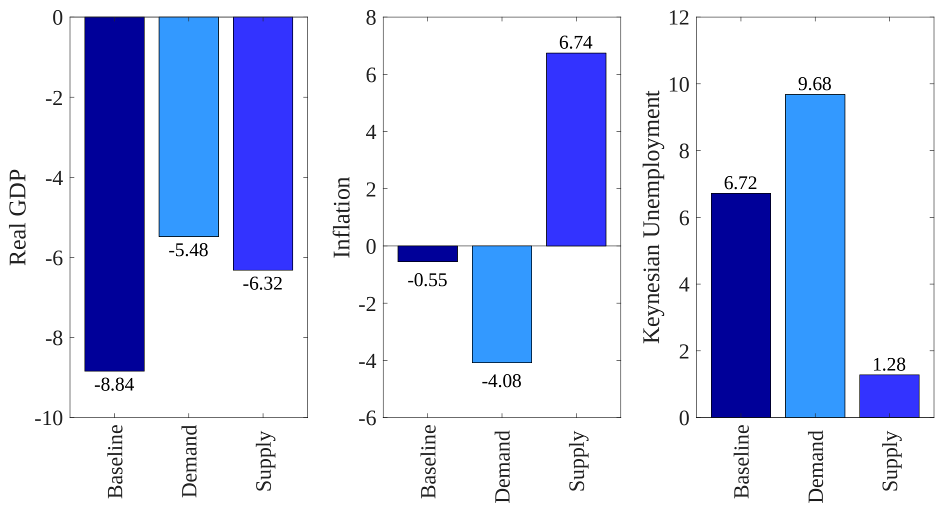


Figure 6.1: Real GDP, inflation, and Keynesian unemployment as a function of shocks for the model with complementarities. The "Baseline" line includes negative demand and supply shocks. The "Supply" bar only includes the sectoral supply shocks. The "Demand" bar only includes the demand shocks.

Figure 6.1 displays the baseline calibration and decomposes it into only supply or only demand shocks. The "Baseline" is the model which includes both the negative demand and negative sectoral supply shocks. The "Supply" bar features only the negative sectoral supply shocks whereas the "Demand" bar features only the demand shocks.

Real GDP. Figure 6.1 shows that negative demand shocks lead to a 5.5% reduction of real GDP and negative supply shocks reduce real GDP by 6.3%.[20] Because of nonlinearities, the effect of the shocks together (-8.9%) is not the same as the sum of the two shocks. Intuitively, reductions in demand in sectors experiencing large negative supply shocks do not reduce output by as much, as illustrated in Figure 4.3.

Inflation. Although the supply shocks on their own generate large reductions in output, Figure 6.1 shows that they also generate very substantial amounts of inflation around 7%. Meanwhile, the demand shocks, on their own, generate substantial deflation of around 4%. The baseline model, on the other hand, predicts an inflation rate of around −0.6%. The baseline model performs relatively well, since most price indices show either moderate inflation or moderate deflation. For instance, CPI inflation for this period was −0.9%

[20] We measure real GDP and the change in inflation using chained Tornqvist approximations to the Divisia index along a linear path.

while PCE inflation was −0.7%.[21] Both supply and demand shocks are needed to make sense of the large output reduction and moderate inflation observed in the data.

At a more disaggregated level, we compare the change in prices in the model to realized changes in producer prices over the sample period. In the model, demand-constrained sectors experience −5.3% inflation and supply-constrained sectors experience inflation of 1.0%. In the data, those sectors that are demand-constrained (according to the model) experienced inflation of −2.4% whereas those identified as supply-constrained had inflation of 1.0%. Figure A.2 in Appendix A shows a scatter plot of prices in the model against the data at the sectoral level.

Since we do not use any information about prices in calibrating the model, the model's performance in terms of prices is an out-of-sample test. Despite being highly stylized, the model performs a reasonable job of separating demand- and supply-constrained sectors although it somewhat overpredicts the magnitude of disaggregated price changes. This may be due to the fact that some capital markets also have nominal rigidities, as explained in Section 2, or that goods prices are also sticky.

Unemployment. We measure Keynesian unemployment by the reduction in hours in labor markets that are demand-constrained.[22] This means that we assume that demand-constrained sectors received no negative supply shocks. Therefore, Figure 6.1 is the maximum amount of Keynesian unemployment consistent with the model. [23]

Figure 6.1 shows that the negative demand shocks, on their own, generate about 9.7% Keynesian unemployment. The "Supply" bar in the figure shows that sectoral supply shocks, on their own, generate 1.3% Keynesian unemployment. Since this calibration has complete markets, this amplification effect is entirely due to complementarities, as discussed in Section 5.2. Together, the supply and demand shocks generate around 7% Keynesian unemployment, which is less than demand shocks on their own, since some of the sectors hit with negative demand shocks are supply-constrained once we account for the negative supply shocks.

[21]The PCE is computed as a Fisher index and it therefore has changing weights reflecting the changing sectoral composition of final demand (unlike the CPI) and is therefore consistent with our model. On the other hand, the PCE does not capture changes in product variety, which could be of concern during lockdowns. Jaravel and O'Connell (2020) show that disappearing goods increased the effective inflation rate in the UK by around 80 basis points. This bias is not large enough to significantly affect our conclusions. We refer the reader to Section D for an extension of the model which allows for disappearing varieties.

[22]Keynesian unemployment is defined as $\sum_{f\in\mathcal{L}}(\bar{\lambda}_f/\bar{\lambda}_{\mathcal{L}})(\Delta\log\bar{L}_f - \Delta\log L_f) \geq 0$, where $\bar{\lambda}_{\mathcal{L}} = \sum_{f\in\mathcal{L}}\bar{\lambda}_f$. This captures the percentage underutilization of efficiency units of labor across labor markets.

[23]In principle, these labor markets may have also experienced negative supply shocks. These reductions in potential output, however, are unobservable since supply is rationed, and, as explained above, we assume that these shocks are not present.

6.3 Tightness and Slackness Across Sectors

Although almost all sectors experienced reductions in hours, in some sectors, these reductions are due to supply constraints whilst in others they are due to demand shortfalls (see Figure A.3 for a complete description). In the baseline, 30 factor markets are demand-constrained and 36 factor markets are supply-constrained.

Supply-constrained sectors include food products and beverages (−8%), food services and accommodations (−39%), construction (−9%), and motion pictures (−54%). We interpret the reduction in hours in these sectors to be driven by state-mandated lockdowns, social distancing orders that limited capacity, and employers' fears of being held legally liable should their employees get sick. These restrictions and fears were severe during March and early April. As social distancing orders are lifted in May and June, some of these industries, may go from being supply-constrained to being demand-constrained instead. Recall that supply-constrained does not necessarily imply that the reductions are driven by reductions in labor supply or workers' willingness to work. Rather, a supply-constrained sector is one where an increase in nominal demand for the good the sector produces would not translate into increased employment.

Demand-constrained sectors include transportation industries, like air transportation (−40%), water transportation (−43%), rail transportation (−19%), and petroleum and coal (−21%) and oil and gas extraction (−18%).[24] These are industries which experienced sharp reductions in nominal spending, either directly by the household, or indirectly through the supply chain.

6.4 Role of Complementarities

Figure 6.2 displays aggregate outcomes in a version of the model where we set all elasticities of substitution $\sigma = \rho = \theta_j = 1$ — that is, the Cobb-Douglas model in Section 4.

Real GDP, inflation, and unemployment. In the Cobb-Douglas model, real GDP declines by around 8% in response to the shocks, which is similar to the response of the benchmark model. However, the breakdown between supply and demand is quite different. The supply shocks, on their own, reduce real GDP by only 4.8% (compared to 6.3%

[24] Our simulations also show that healthcare related industries, like hospitals and ambulatory health care services also experienced reductions in employment of (−19%) and (−16%). However, presumably, this excess capacity in the healthcare industry is not wasted but engaged in non-market activities related to the pandemic.

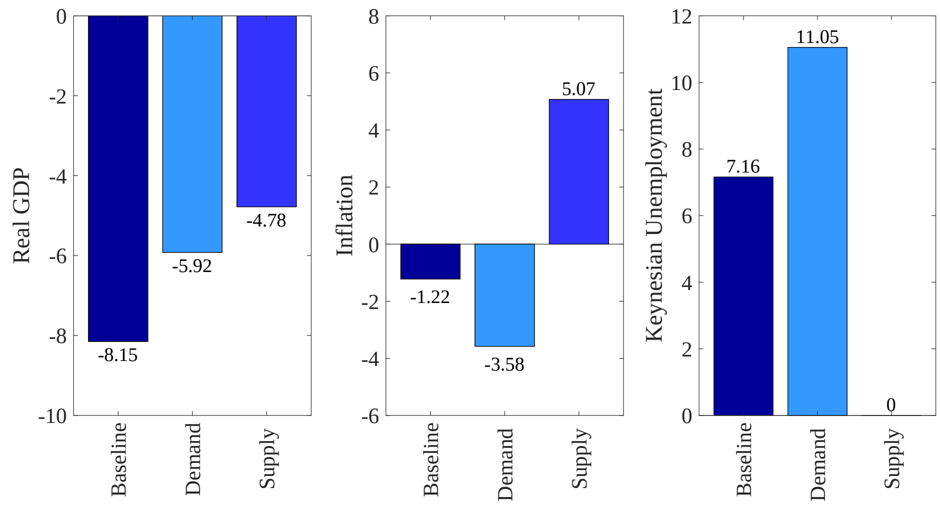


Figure 6.2: Real GDP, inflation, and Keynesian unemployment as a function of shocks without complementarities. The "Baseline" line includes negative demand and supply shocks. The "Supply" bar only includes the sectoral supply shocks. The "Demand" bar only includes the demand shocks.

in the benchmark) while the demand shocks reduce real GDP by 5.9% (compared to 5.5% in the benchmark). Hence, as explained in Section 5.2, complementarities amplify the importance of supply shocks and mitigate the effect of demand shocks, and these effects seem to be roughly off-setting one another.

With only sectoral supply shocks, Keynesian unemployment is now 0% (instead of 1.3% in the benchmark). This follows from the discussion in Section 4: this version of the model has complete markets and no complementarities, so supply shocks in one sector do not change nominal spending on other sectors, and hence do not have Keynesian spillovers.

6.5 Policy Implications

We end this section by considering some policy counterfactuals. Two important policy tools used to combat adverse effects of the Covid pandemic have been stimulative monetary policy and increased social insurance, in the form of transfers like unemployment benefits. We discuss both of these in turn.

Implications for aggregate demand management. Sectorally disparate supply and demand shocks blunt the power of aggregate demand stimulus. Conventional monetary policy, forward guidance, and untargeted fiscal policy boost aggregate demand. However, with heterogeneous supply and demand shocks, reversing the decline in aggregate demand is not enough to offset the negative effect of the shocks.

To see this, we consider the reduction in real GDP in response to a pure negative demand shock, holding fixed the sectoral composition of final demand and setting supply shocks to zero. In the Cobb-Douglas model, the negative aggregate demand shock associated with Covid, on its own, reduced real GDP by around 5%. Therefore, a large enough aggregate demand stimulus can raise real GDP by around 5% fully offsetting the negative aggregate demand shock. However, with the full set of supply and demand shocks, the same sized aggregate demand stimulus raises real GDP from −8.2% to −5.8%. In other words, the same aggregate demand stimulus only raises real GDP by around 2.2%. Hence, the presence of sectoral shocks cuts the potency of aggregate demand stimulus by *half* in the Cobb-Douglas model.

In the model with complementarities, this effect is even more extreme. Whereas the aggregate demand shock on its own reduces output by 3.9%, with the full set of sectoral supply and demand shocks, reversing the reduction in aggregate demand through stimulus only boosts output by around 1.1%. Hence, the potency of the aggregate demand stimulus is cut almost by a factor of four in the model with complementarities. Intuitively, this is because the increase in aggregate demand raises the price of supply-constrained factors, and complementarities then cause expenditures to switch towards these factors and away from demand-constrained ones. This reduces the stimulative effect of aggregate demand stimulus.

If we think of the model without sectoral shocks as a typical recession, this means that aggregate demand stimulus is significantly less effective in the Covid-19 recession than in a typical recession. The reason is that without sectoral shocks, the reduction in aggregate demand renders all labor markets demand constrained, and starting from there, an increase in aggregate demand increases employment in all labor markets. By contrast, with sectoral shocks, some labor markets are supply constrained, and starting from there, an increase in aggregate demand is partly dissipated in wage increases in supply-constrained labor markets (the more so, the stronger the complementarities across sectors).

Reduced Social Insurance. Figure 6.3 shows how aggregate outcomes change in the model with complementarities and in the Cobb-Douglas model as we vary the share of households that are potentially HtM. As expected from Figure 6.3, the presence of HtM households amplifies the reduction in real GDP, reduces inflation, and causes Keynesian unemployment. For example, in the Cobb-Douglas model, when every single unemployed agent becomes HtM, real GDP falls by 13% rather than 8%, with very significant deflation of 8% rather than 1%, and Keynesian unemployment of 15% rather than 7%. This underscores the important role that transfers have played in mitigating the negative demand effects associated with the Covid-19 crisis. In the absence of these policies, employment and output would be significantly lower.

These numbers are smaller with complementarities, since the endogenous negative aggregate demand shock associated with HtM households is partially absorbed by supply-constrained factor markets. Specifically, in response to the negative endogenous aggregate demand shock, the price of capital declines, which triggers substitution away from capital and towards labor due to complementarities. This is a quantitatively significant stabilizing force in the model. Nevertheless, even in the model with complementarities, social insurance is still very important.

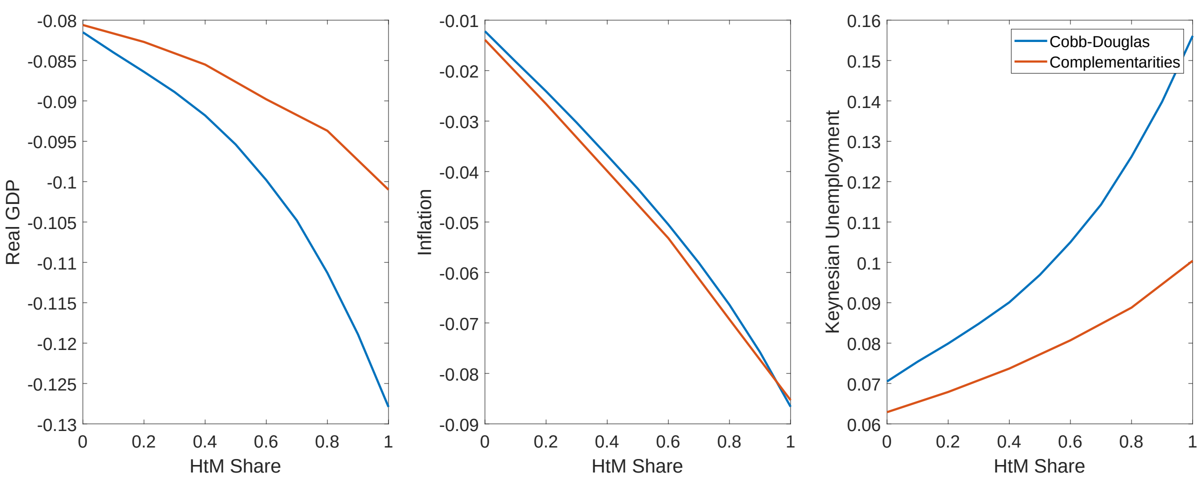


Figure 6.3: Real GDP, inflation, and Keynesian unemployment in the Cobb-Douglas model and the model with complementarities as a function of the share of potentially HtM workers.

7 Extensions

In this section, we briefly summarize extensions of the basic framework that appear in the appendix. Appendix D extends the framework to cover capital market imperfections and bankruptcies. In this appendix, we show that firm exits act like endogenous negative productivity shocks. Accordingly, they are amplified by input-output linkages (just as exogenous productivity shocks are amplified by input-output linkages). Furthermore, exits change relative prices, and these relative price changes can redirect the flow of spending and cause Keynesian spillovers, much as negative supply shocks. Finally, we also show how exits can result in scarring effects since firms that exit today may not be replaced in the future, this lowers output in the future, which reduces aggregate demand today via the Euler equation (a mechanism emphasized by Fornaro and Wolf, 2020). Appendix E generalizes the results in Section 3 to environments with investment and establishes global comparative statics. Finally, Appendix F extends our results to the case where wages are semi-flexible.

8 Conclusion

This paper analytically characterizes and numerically quantifies the impact of different supply and demand shocks in a general disaggregated model with multiple sectors, factors, and input output linkages, as well as occasionally-binding downward nominal wage rigidity, credit-constraints, and a zero lower bound.

Separating supply and demand sources for the crisis are important since they have different implications for the effects of policy. Nevertheless, the analysis in this paper is purely positive. For a normative analysis, we would have to take a stance on the health-related externalities of production and consumption. In particular, it may be that implementing the flexible price allocation is not necessarily optimal one once we account for these externalities. Nevertheless, the results of any normative analysis would rely on understanding the positive forces analyzed in this paper.

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Appendix A Additional Graphs

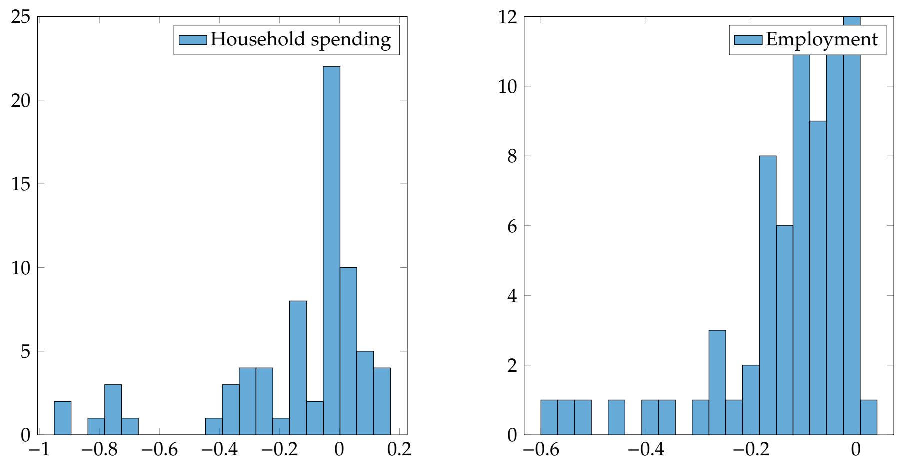

Figure A.1: Reduction in nominal household spending (left panel) and hours worked (right panel) as fractions by sector in May, 2020 compared to February, 2020.

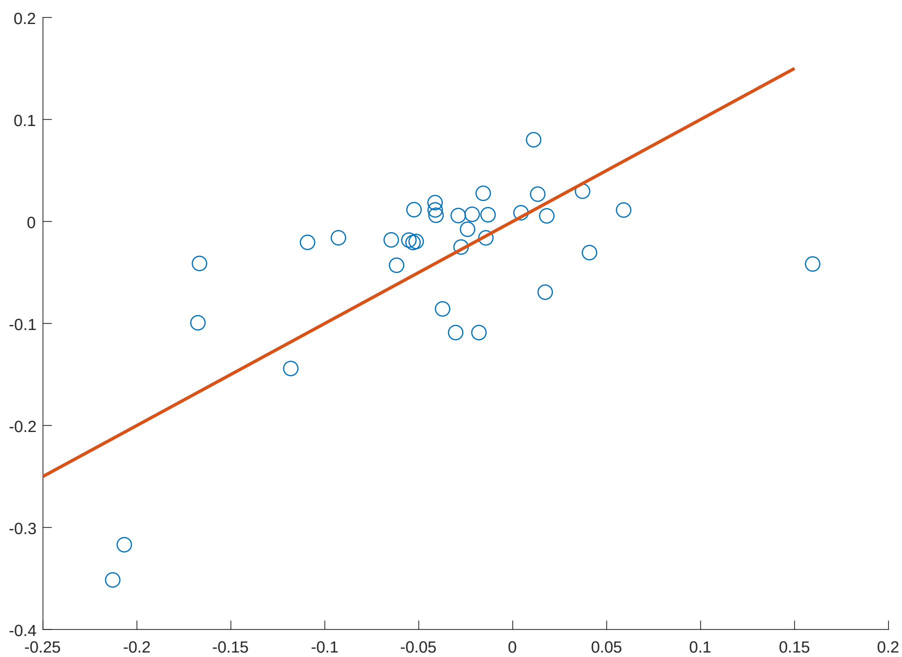

Figure A.2: Changes in model implied prices are on the x-axis and changes in producer prices are on the y-axis. The red line is the 45-degree line.

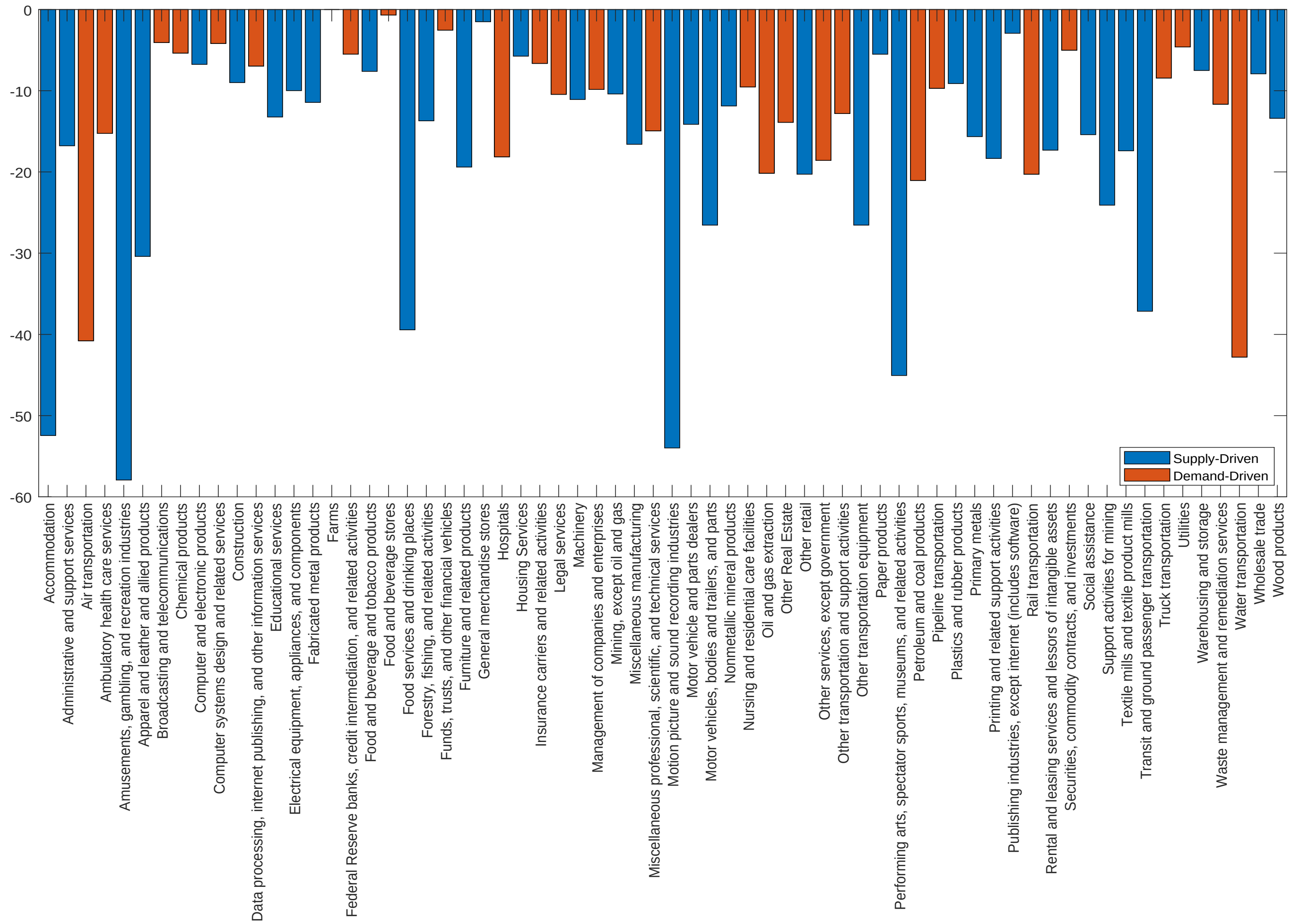

Figure A.3: Model implied percentage reduction in hours by sector from February to May 2020. Sectors below capacity are "demand-driven."

Appendix B Proofs

Proof of Proposition 1. By Shephard's lemma, changes in the price of good i are given by

$$d\log p_i = -d\log A_i + \sum_{j\in\mathcal{N}} \Omega_{ij} d\log p_j + \sum_{f\in\mathcal{G}} d\log w_f,$$

solving this system gives

$$d\log p_i = -\sum_{j\in\mathcal{N}} \Psi_{ij} d\log A_j + \sum_{f\in\mathcal{G}} \Psi_{if} d\log w_f.$$

Furthermore,

$$d\log w_f = d\log\lambda_f + d\log E - d\log L_f.$$

Hence, the change in real GDP is given by

$$\begin{aligned}
d\log Y &= d\log E - \sum_{j\in mathcalN} \Omega_{0j} d\log p_j, \\
&= d\log E + \sum_{j\in\mathcal{N}} \Psi_{0j} d\log A_j - \sum_{f\in\mathcal{G}} \Psi_{0f} d\log w_f, \\
&= d\log E + \sum_{j\in\mathcal{N}} \Psi_{0j} d\log A_j - \sum_{f\in\mathcal{G}} \Psi_{0f}\left(d\log\lambda_f + d\log E - d\log L_f\right), \\
&= d\log E + \sum_{j\in\mathcal{N}} \lambda_j d\log A_j - \sum_{f\in\mathcal{G}} \lambda_f\left(d\log\lambda_f + d\log E - d\log L_f\right), \\
&= \sum_{j\in\mathcal{N}} \lambda_j d\log A_j + \sum_{f\in\mathcal{G}} \lambda_f d\log L_f,
\end{aligned}$$

using the fact that $\Psi_{0i} = \lambda_i$ and $\sum_{f\in\mathcal{G}} \lambda_f = 1$. To complete the proof, note that

$$d\log L_f = \min\{d\log\bar{L}_f, d\log\lambda_f + d\log E - d\log\bar{L}_f\}.$$

□

Proof of Proposition 2. This follows from an application of Proposition 9 in Baqaee and Farhi (2019a). □

Proof of Proposition 6. This is a special case of Proposition 8. □

Proof of Proposition 7. This is a special case of Proposition 9. □

Proof of Proposition 8. Define the function $\Phi(L^0) \mapsto L$ by

$$\begin{aligned}
w_f L_f^0 &= \sum_{j \in \mathcal{N}} \Psi_{jf} \left(\frac{w_f^{1-\sigma}}{\sum_k \Psi_{jk} w_k^{1-\sigma}} \right) p_j c_j, \\
p_j c_j &= \left(\bar{\Omega}_{0i} - \kappa_i \right) E, \\
E &= \frac{(1-\beta) \sum_i (1-\kappa_i)}{\beta} \frac{\bar{E}_*}{1+i} \sum_h \bar{\lambda}_h^* \left(\frac{L_h^0}{L_h^*} (1-\phi_h) + \phi_h \right), \\
\tilde{w}_f &= \min\{\underline{w}_f, w_f\} \mathbf{1}(f \in \mathcal{L}) + w_f \mathbf{1}(f \in \mathcal{K}), \\
L_f &= \min \left\{ \frac{1}{\tilde{w}_f} \sum_{j \in \mathcal{N}} \Psi_{jf} \left(\frac{\tilde{w}_f^{1-\sigma}}{\sum_k \Psi_{jk} \tilde{w}_k^{1-\sigma}} \right) p_j c_j, \bar{L}_f \right\}.
\end{aligned}$$

An equilibrium is when $L^0 = L$. We show that Φ is an increasing function mapping $\prod_{f \in \mathcal{G}} [0, \bar{L}_f]$ into itself.

By Lemma 1, w_{-i} is increasing and w_i is decreasing in L_i^0. This means that $\tilde{w}_{-i}$ is increasing in L_i^0 and $\tilde{w}_i$ is decreasing in i if $w_i > \underline{w}_i$. Hence, L_{-i} is increasing in L_i^0,and L_i is increasing in L_i^0. This proves that Φ is a monotone function, and so we can apply Tarski (1955). ◻

Lemma 1. *For the following system of equations*

$$w_f L_f = \sum_{j \in \mathcal{N}} \Psi_{jf} \left(\frac{w_f^{1-\sigma}}{\sum_k \Psi_{jk} w_k^{1-\sigma}} \right) \Omega_{0j} E,$$

w_{-i} is increasing and w_i is decreasing in L_i.

Proof. Start by noting that

$$Cov_{\Omega^{(0)}}(\Psi_{(f)}, \Psi_{(k)}) = \sum_l \Omega_{0l} \Psi_{lf} \left[\Psi_{lk} - \lambda_k \right],$$

Using this fact, and Proposition 2, we can simplify

$$
\begin{aligned}
\lambda_k d\log\lambda_k &= -(\theta-1)\sum_{f\in\mathcal{G}}\left[-\lambda_f\lambda_k - Cov_{\Omega^{(0)}}(\Psi_{(f)},\Psi_{(k)}) + \mathbf{1}(f=k)\lambda_k\right]\left(d\log\lambda_f - d\log L_f\right)\\
&= \sum_{f\in\mathcal{G}}\left[(1-\theta)\mathbf{1}(f=k)\lambda_k - (1-\theta)\lambda_f\lambda_k - (1-\theta)Cov_{\Omega^{(0)}}(\Psi_{(f)},\Psi_{(k)})\right]\left(d\log\lambda_f - d\log L_f\right)\\
&= \sum_{f\in\mathcal{G}}\left[(1-\theta)\mathbf{1}(f=k)\lambda_k - (1-\theta)\lambda_f\lambda_k - (1-\theta)\sum_l\Omega_{0l}\Psi_{lf}\left(\Psi_{kf}-\lambda_k\right)\right]\left(d\log\lambda_f - d\log L_f\right)\\
&= \sum_{f\in\mathcal{G}}\left[(1-\theta)\mathbf{1}(f=k)\lambda_k - (1-\theta)\sum_l\Omega_{0l}\Psi_{lf}\Psi_{kf}\right]\left(d\log\lambda_f - d\log L_f\right)\\
&= \sum_{f\in\mathcal{G}}\left[(1-\theta)\mathbf{1}(f=k)\lambda_k - (1-\theta)\left[\mathbb{E}_{\Omega^{(0)}}(\Psi_{(f)}\Psi_{(k)})\right]\right]\left(d\log\lambda_f - d\log L_f\right)\\
&= \sum_{f\in\mathcal{G}}\left[(1-\theta)\mathbf{1}(f=k)\lambda_k - (1-\theta)\left[\mathbb{E}_{\Omega^{(0)}}(\Psi_{(f)}\Psi_{(k)})\right]\right]\left(d\log\lambda_f - d\log L_f\right)\\
&= -\sum_{f\in\mathcal{G}}\left[(1-\theta)\left[\mathbb{E}_{\Omega^{(0)}}(\Psi_{(f)}\Psi_{(k)})\right]\right]\left(d\log\lambda_f - d\log L_f\right) + \left[(1-\theta)\lambda_k\right]\left(d\log\lambda_k - d\log L_k\right)\\
&= -(1-\theta)\sum_{f\in\mathcal{G}}\left[\mathbb{E}_{\Omega^{(0)}}(\Psi_{(f)}\Psi_{(k)})\right]\left(d\log\lambda_f - d\log L_f\right) + \left[(1-\theta)\lambda_k\right]\left(d\log\lambda_k - d\log L_k\right)
\end{aligned}
$$

Let

$$B_{kf} = \left[\mathbb{E}_{\Omega^{(0)}}(\Psi_{(f)}\frac{\Psi_{(k)}}{\lambda_k})\right]$$

We know that

$$\sum_f B_{kf} = 1.$$

Hence, letting $x = d\log\lambda/d\log L_i$ be a column vector and e_i the ith basis vector, we can write

$$
\begin{aligned}
\theta x &= -(1-\theta)Bx - (I-(1-\theta)B)\,e_i\\
x &= -\left(\theta I + (1-\theta)B\right)^{-1}(I-(1-\theta)B)\,e_i = -A\,(I-(1-\theta)B)\,e_i.
\end{aligned}
$$

By Lemma 2, $A = (\theta I + (1-\theta)B)^{-1}$ is an M-matrix, hence by A_5 of Theorem 6.2.3 of Berman and Plemmons (1979), $-A\,(I-(1-\theta)B)\,e_i$ has the same signs as $-(I-(1-\theta)B)$. Since $-(I-(1-\theta)B)$ has negative diagonal and positive off-diagonal elements, this means that x_i is negative and x_{-i} is positive, as needed. □

Lemma 2. *The matrix defined in Lemma 2*

$$A = (\theta I + (1-\theta)B)^{-1}$$

is an M-matrix.

Proof. By Theorem 6.2.3 of Berman and Plemmons (1979), it suffices to prove that A^{-1} has all positive elements and that A is a Z-matrix. The fact that A^{-1} has all positive elements is immediate from its definition. To show that A is a Z-matrix, note that we can write

$$\begin{aligned} A &= (\theta I + (1-\theta)B)^{-1}, \\ &= (I - (1-\theta)(I-B))^{-1}, \\ &= \sum_{n=0}^{\infty} (1-\theta)^n (I-B)^n. \end{aligned}$$

Hence, since $I-B$ is an M-matrix, $(1-\theta)(I-B)X$ does not change the sign of the columns of X for any X. Hence, by induction, and the fact that M-matrices are closed under addition, we have that A has the same sign as the elements of $(I-B)$,and hence A is a Z-matrix. □

Proof of Proposition 9. To prove the statements regarding $\Delta \log L$, we use Theorem 3 from Milgrom and Roberts (1994). Since $\Delta \log Y$ is a monotone function of $\Delta \log L$, this also establishes the results about real GDP. It remains to prove the claims regarding inflation. To prove that labor supply shocks (on their own) are inflationary, we need to show that the price level p^Y is decreasing in by $\bar{L}$. To do so, consider some negative labor shocks then

$$\begin{aligned} \Delta \log p^Y &= \Delta \log E - \Delta \log Y \\ &\geq \Delta \log E - \sum_f \bar{\lambda}_f \Delta \log L_f \\ &= \Delta \log \left(\sum_h \bar{\lambda}_h^* \left(\frac{L_h}{L_h^*}(1-\phi_h) + \phi_h \right) \right) - \sum_f \bar{\lambda}_f \Delta \log L_f \\ &\geq \Delta \log \left(\sum_h \bar{\lambda}_h^* \frac{L_h}{L_h^*} \right) - \sum_f \bar{\lambda}_f \Delta \log L_f \\ &= \Delta \log \left(\sum_h \bar{\lambda}_h^* \exp(\log L_h / L_h^*) \right) - \sum_f \bar{\lambda}_f \Delta \log L_f \\ &\geq \sum_h \bar{\lambda}_h^* \Delta \log L_h - \sum_f \bar{\lambda}_f \Delta \log L_f = 0, \end{aligned}$$

as long as $\bar{\lambda}^* = \bar{\lambda}$. The second line follows from the fact that $\log Y$ is log-concave (see Baqaee and Farhi, 2019b).

To prove that aggregate demand shocks, like forward guidance shocks, (on their own) are deflationary, we need to show that the price level p^Y is increasing $E_*/(1+i)$. To do so, consider some shock then

$$\begin{aligned}
\Delta \log p^Y &= \Delta \log E - \Delta \log Y \\
&\geq \Delta \log E - \sum_f \bar{\lambda}_f \Delta \log L_f \\
&= \Delta \log \left(\frac{(1-\beta)\sum_i (1-\kappa_i)}{\beta} \frac{\bar{E}_*}{1+i} \sum_h \bar{\lambda}_h^* \left(\frac{L_h}{L_h^*}(1-\phi_h) + \phi_h \right) \right) - \sum_f \bar{\lambda}_f \Delta \log L_f \\
&\geq \Delta \log \left(\frac{\bar{E}_*}{1+i} \right) - \sum_f \bar{\lambda}_f \Delta \log L_f \\
&\geq \Delta \log \left(\frac{\bar{E}_*}{1+i} \right) - \sum_f \bar{\lambda}_f \Delta \log \left(\frac{\bar{E}_*}{1+i} \right) \\
&\geq 0.
\end{aligned}$$

□

Appendix C The Case of Uniform Elasticities

In this section, we show that when $\theta_0 = \theta_j = \theta$ for every j, for both aggregate demand shocks and factor supply shocks, the details of the production network are summarized by the initial factor shares. The Cobb-Douglas economy in Section 4 is an example of this mmore general phenomenon. Formally, this can be stated as follows.

Proposition 11 (Global Sufficient Statistics)**.** *Suppose that the intertemporal elasticity of substitution is ρ and that the elasticities of substitution in production and in final demand are all the same with $\theta_j = \theta$ for every $j \in 1 + \mathcal{N}$. Suppose that there are only factor supply shocks $\Delta \log \bar{L}$ and aggregate demand shocks $\Delta \log \zeta$ but no productivity shocks and no shocks to the sectoral composition of demand. Then*

$$\Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}) = \Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}')$$

for every $\bar{\Omega}$ and $\bar{\Omega}'$ as long as $\bar{\lambda}_f = \bar{\Psi}_{0f} = \bar{\Psi}'_{0f} = \bar{\lambda}'_f$ for every $f \in \mathcal{G}$. More generally, given the shocks, the initial factor income shares $\bar{\lambda}_f$ are sufficient statistics for equilibrium changes *in aggregate output $\Delta \log Y$, the aggregate price index $\Delta \log p^Y$, factor wages $\Delta \log w_f$, factor quantities $\Delta \log L_f$, and factor income shares $\Delta \log \lambda_f$.*

In other words, with factor supply shocks and aggregate demand shocks, as long as the consumption and production elasticities are the same, the model with a production network is ismorphic to a model without production networks (note that this does not hold for shocks to the composition of demand). Hence, proving propositions in an environment without intermediates also proves it when there are intermediates.

Appendix D Extension: Bankruptcies

The paper so far has abstracted from capital market frictions and bankruptcies. In this section, we briefly discuss how our results can be extended to the case with these frictions. We begin by generalizing our comparative statics to a case with firm exits. We then make three observations: (1) in a production network, the negative effects of demand shocks are amplified if there are exits because of an intermediate-input multiplier; (2) exits, by acting as endogenous negative supply shocks, can change the flow of spending and cause Keynesian spillovers outside of the Cobb-Douglas case; and, (3) firm failures, by potentially destroying intangible firm-specific capital, can reduce output in the future, and by lowering output in the future, reduce aggregate demand today through the Euler equation.

D.1 Local Comparative Statics with Bankruptcies

To capture firm failures, we modify the general structure described in Section 2 as follows. We assume that output in sector $i \in \mathcal{N}$ is a CES aggregate of identical producers j each with constant returns production functions $y_{ik} = A_i f_i(x_{ij}^k)$, where x_{ij}^k is the quantity of industry j's output used by producer k in industry i. Assuming all firms within an industry use the same mix of inputs, sectoral output is

$$y_i = \left(\int y_{ik}^{\frac{\sigma_i - 1}{\sigma_i}} dk \right)^{\frac{\sigma_i}{\sigma_i - 1}} = M_i^{\frac{1}{\sigma_i - 1}} A_i f_i(x_{ij}), \tag{D.1}$$

where x_{ij} is the quantity of input j used by industry i, M_i is the mass of producers in industry i, $\sigma_i > 1$ is the elasticity of substitution across producers, and A_i is an exogenous productivity shifter. From this equation, we see that a change in the mass of operating firms acts like a productivity shock and changes the industry-level price. Therefore, if shocks outside sector i trigger a wave of exits, then this will set in motion endogenous negative productivity shock $(1/(\sigma_i - 1))\Delta \log M_i$ in sector i.

Suppose that each firm must maintain a minimum level of revenue in order to continue operation.[25, 26] We are focused on a short-run application, so we do not allow new entry, but of course, this would be important for long-run analyses.[27]

The mass of firms that operate in equilibrium is therefore given by

$$M_i = \min \left\{ \frac{\lambda_i E}{\bar{\lambda}_i \bar{E}} \bar{M}_i, \bar{M}_i \right\},$$

where $\bar{M}_i$ is the exogenous initial mass of varieties, $\lambda_i E$ is nominal revenue earned by sector i and $\bar{\lambda}_i \bar{E}$ is the initial nominal revenue earned by i. If nominal revenues fall relative to the baseline, then the mass of producers declines to ensure that sales per producer remain constant. In order to capture government-mandated shutdowns of certain firms, we allow for shocks that reduce the exogenous initial mass of producers $\Delta \log \bar{M}_i \leq 0$.

We can generalize Propositions 1 and 2 to this context. The only difference is that we must replace $d \log A_i$ by $d \log A_i + (1/(\sigma_i - 1)) d \log M_i$, where

$$d \log M_i = d \log \bar{M}_i + \min\{d \log \lambda_i + d \log E - d \log \bar{M}_i, 0\}.$$

This backs up the claim that the $d \log M_i$'s act like endogenous negative productivity shocks. They provide a mechanism whereby a negative demand shock, say in the composition of demand or in aggregate demand $d \log \zeta$, triggers exits which are isomorphic to negative supply shocks.

As in the other examples, the general lesson is that the output response, to a first-order, is again given by an application of Hulten's theorem along with an amplification effect which depends on how the network redistributes demand and triggers Keynesian unemployment in some factors and firm failures in some sectors.

Having generalized the local comparative statics, we now make three observations about the way bankruptcies can propagate and affect aggregates. In order to simplify the

[25]One possible micro-foundation is each producer must pay its inputs in advance by securing within-period loans and that these loans have indivisibilities: only loans of size greater than some minimum level can be secured. This minimum size is assumed to coincide with the initial costs $\bar{\lambda}_i \bar{E} / \bar{M}_i$ of the producer.

[26]Another possible micro-foundation is as follows. Producers within a sector charge a CES markup $\mu_i = \sigma_i / (\sigma_i - 1)$ over marginal cost. These markups are assumed to be offset by corresponding production subsidies. Producers have present nominal debt obligations corresponding to their initial profits $(1 - 1/\mu_i)\bar{\lambda}_i \bar{E} / \bar{M}_i$. The same is true in the future. If present profits $(1 - 1/\mu_i)\lambda_i E / M_i$ fall short of the required nominal debt payment $(1 - 1/\mu_i)\bar{\lambda}_i \bar{E} / \bar{M}_i$, then the firm goes bankrupt and exits. Alternatively, we can imagine that there is no future debt obligation but that firms cannot borrow.

[27]See Baqaee (2018) and Baqaee and Farhi (2020a) for production networks with both entry and exit.

exposition, we abstract away from HtM households for the rest of the section.

D.2 Intermediate Multiplier of Bankruptcies

If there are increasing returns, then firm failures can also affect supply today directly. As the economy scales down, marginal cost goes up. Our formulation of industry-level production functions (D.1) have this property due to the love-of-variety effect. Hence, firm exits act like negative TFP shocks, and if there are intermediate inputs, then these endogenous negative TFP shocks are amplified.

To see this, consider a Cobb-Douglas model where $\rho = \theta_0 = \theta_j = 1$ and negative demand shocks. In this case, since there are no HtM households, the effect on output is given by

$$d\log Y = \sum_{i\in\mathcal{N}} \lambda_i \frac{1}{\sigma_i - 1} d\log M_i = \sum_{i\in\mathcal{N}} \lambda_i \frac{1}{\sigma_i - 1}(d\log\lambda_i + d\log E).$$

Using the fact that $d\log\lambda_i + d\log E = -\sum_{j\in\mathcal{N}} \Psi_{ji} d\kappa_j/\lambda_i$, we can write

$$d\log Y = -\sum_{i\in\mathcal{N}} \frac{1}{\sigma_i - 1} \sum_{j\in\mathcal{N}} \Psi_{ji} d\kappa_j.$$

Hence, the higher is the use of intermediate inputs, the larger are the elements of the Leontief inverse Ψ, and the larger is the negative effect on output. Intuitively, a reduction in demand causes exits at every step in the supply chain, and so the longer the supply chains, the more costly the exits.

D.3 Bankruptcies and Expenditure Switching

In the previous example, we deliberately chose a Cobb-Douglas economy since the expenditure shares do not respond to relative prices. If the elasticities of substitution are not all equal to one, the endogenous TFP shocks associated with exits, by changing expenditure shares and the flow of spending, can trigger additional cascades of unemployment and failure.

To make this concrete, consider a simple example economy without intermediate inputs where each sector uses only its own labor. Assume that there are no shocks to aggregate demand ($d\log\zeta = 0$). Set the intertemporal elasticity of substitution $\rho = 1$ and share of HtM households $1 - \phi = 0$ to ensure that nominal expenditure is constant ($d\log E = 0$). We also assume that there are no exogenous shocks to productivities ($d\log A_i = 0$), no shocks to potential labor ($d\log \bar{L}_f = 0$), and no shocks to the sectoral composition of demand ($d \log \omega_{0j} = 0$). Finally, we assume that all sectors have the same within-sector elasticity of substitution $\sigma_i = \sigma > 1$.

We focus on exogenous shocks $d \log \bar{M}_i \leq 0$ capturing government-mandated shutdowns. We show how endogenous failures can amplify these negative supply shocks. The insights are more general and also apply to shocks to potential labor. Similarly, failures can be triggered by negative aggregate demand shocks, and the resulting endogenous negative supply shocks can result in stagflation with simultaneous reductions in output and increases in inflation.

We start by analyzing the case where sectors are complements, and then consider the case where they are substitutes. For brevity, we jump directly to the final result and leave the derivations in a different appendix — Appendix G.

Shut-down shock with complements. Assume that sectors are complements ($\theta < 1$) and consider the government-mandated shutdown of some firms in only one sector i. The change in output is given by

$$d \log Y = \lambda_i \frac{1}{\sigma - 1} d \log \bar{M}_i + \frac{(1-\theta)(1-\lambda_i)\frac{\sigma}{\sigma-1}}{1-(1-\theta)(1-\lambda_i)\left(1 - \frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)} \lambda_i \frac{1}{\sigma-1} d \log \bar{M}_i.$$

The first term on the right-hand side is the direct reduction in output from the shut-down in sector i. The second term capture the further indirect equilibrium reduction in output due to firm failures and Keynesian unemployment in the other sectors. Intuitively, the shutdown in sector i raises the relative price of i, and because of complementarities, demand in the rest of the sectors falls. This reduction in nominal spending causes unemployment and additional exits in the other sectors.

Shut-down shock with substitutes. Consider the same experiment as above but assume now that sectors are substitutes ($\theta > 1$). Shut-downs in i raise the price of i relative to other sectors, and cause substitution away from i. As long as the elasticity of substitution within the sector $\sigma > 1$ is large enough and the elasticity of substitution across sectors $\theta > 1$ is not too large, the shut-down in sector i causes unemployment in sector i, but no additional firm failures in sector i. Furthermore, the other sectors maintain full employment and experiences no failures. In this case the response of output is given by

$$d \log Y = \lambda_i \frac{1}{\sigma - 1} d \log \bar{M}_i + \frac{(\theta_0 - 1)(1-\lambda_i)}{1-(\theta_0-1)\lambda_i} \lambda_i \frac{1}{\sigma-1} d \log \bar{M}_i,$$

where the first term on the right-hand side is the direct effect of the shutdown and the second term is the amplification from the indirect effect of the shutdown which results in Keynesian unemployment in i.

D.4 Scarring Effect of Bankruptcies

One of the primary concerns about firm failures is that it results in the destruction of irreversible investments. This lowers output in the future, and through the Euler equation, depresses spending today.[28] In other words, the destruction of irreversible investments can act like an endogenous negative aggregate demand shock. To see this, for simplicity, assume there are no HtM agents and suppose that when firms exit in the first period $d\log M$, they do not return in the next period.

In particular, by the Envelope theorem, output in the future falls by

$$d\log Y_* = \sum_{i\in\mathcal{N}} \frac{\lambda_i^*}{\sigma_i - 1} d\log M_i = \sum_{i\in\mathcal{S}} \frac{\lambda_i^*}{\sigma_i - 1} d\log \bar{M}_i + \sum_{i\in\mathcal{D}} \frac{\lambda_i^*}{\sigma_i - 1} \left(d\log\lambda_i + d\log E\right).$$

The endogenous changes in $d\log Y_*$ then mean that the previously exogenous aggregate demand shock $d\log\zeta$, defined by (3.3) now contains an endogenous term

$$d\log\zeta = -\rho\Big(d\log(1+i) + \frac{1}{1-\beta} d\log\beta - d\log\bar{p}_*^Y\Big) + d\log Y_*.$$

However, the rest of the model remains the same. We can combine the Euler equation in (3.4), with the aggregation and propagation equations in Propositions 1 and 2 (remembering that $d\log A_i$ should be replaced by $d\log A_i + d\log M_i/(\sigma_i - 1)$).

Intuitively, the effect of these failures is very similar to the presence of HtM households in terms of its implications for the AD curve. That is, failures shift the AD curve downwards and flatten its slope, much as incomplete markets do in Figure 4.2b.

Appendix E Extension: Investment

To model investment, we add intertemporal production functions into the model. An investment function transforms goods and factors in the present period into goods that can be used in the future. In this case, instead of breaking the problem into an intertemporal

[28] This mechanism is the same as the one emphasized by Benigno and Fornaro (2018), except here it corresponds to the destruction of irreversible investment instead of reduced investment in innovation.

and intratemporal problem, we must treat both problems at once. In this section, we first discuss the general local comparative statics with investment, extending the results in Section 3, then we discuss a special case with simple sufficient statistics and global comparative statics, extending the results in Section 5.1.

In the body of the paper, we assumed that prices in the future p_*^Y were fixed, which meant that nominal expenditures in the future were also fixed $p_*^Y Y_* = E_*$. In the version of the model with investment, output in the future Y_* is not exogenous, so assuming p_*^Y is no longer equivalent to assuming E_* is fixed. Therefore, we consider both situations.

E.1 General local comparative statics

When we add investment to the model, we can still use Proposition 1 without change. However, we can no longer use the Euler equation to pin down nominal expenditures today, since nominal GDP today includes investment expenditures and output tomorrow can no longer taken to be exogenous. Instead, to determine $d\log E$, we must use a version of Proposition 2. For this subsection, we assume that nominal expenditures in the future period are fixed and we denote the future period by $*$.

In particular, let λ_i^I denote the intertemporal sales share — expenditures on quantity i as a share of the net present value of household income. Furthermore, let $\bar{\Omega}^I$ represent the intertemporal input-output matrix, which includes the capital accumulation equations. Then, letting intertemporal consumption be the zero-th good, and abstracting from shocks to the sectoral composition of demand for simplicity, we can write

$$d\log\lambda_k^I = \sum_j \lambda_j^I(\theta_j - 1)Cov_{\Omega^{I,(j)}}\left(\sum_{i\in\mathcal{N}} \Psi_{(i)}^I d\log A_i - \sum_{f\in\mathcal{G}} \Psi_{(f)}^I \left(d\log\lambda_f^I - d\log L_f\right), \frac{\Psi_{(k)}^I}{\lambda_k^I}\right)$$

almost everywhere, where changes in factor employments are given by

$$d\log L_f = \begin{cases} d\log \bar{L}_f, & \text{for} \quad f\in\mathcal{K}, \\ \min\left\{d\log\lambda_f^I - d\log\lambda_*^I, d\log\bar{L}_f\right\}, & \text{for} \quad f\in\mathcal{L}. \end{cases}$$

This follows from the fact that nominal expenditures on each factor f is given by $d\log\lambda_f^I + d\log E^I$, where E^I is the net-present value of household income. However, since nominal expenditures in the future are fixed, we have $d\log E_* = d\log\lambda_*^I + d\log E^I = 0$. This allows us to write nominal expenditures on each factor as $d\log\lambda_f^I - d\log\lambda_*^I$.

E.2 Global Comparative Statics

We can extend the results in Section 5.1 to the model with investment. To do so, we assume that the intertemporal elasticity of substitution ρ is the same as the intersectoral elasticities of substitution $\rho = \theta_j = \theta$ for every $j \in \mathcal{N}$. In this case, the initial factor shares are, once again, a sufficient statistic for the production network. In particular, Proposition 11 still applies. Furthermore, we can also prove that the set of equilibria form a lattice under some additional assumptions.

Proposition 12. *Suppose that the intertemporal elasticity of substitution, the elasticities of substitution in production and in final demand are all the same θ. Suppose that there are only shocks to potential factor supplies $\Delta \log \bar{L}$. If future nominal expenditure is fixed, then assuming in addition that $\theta < 1$, there is a unique best and worst equilibrium: for any other equilibrium, $\Delta \log L$ are lower than at the best and higher than at the worst. Furthermore, both in the best and in the worst equilibrium, $\Delta \log L$ are increasing in $\Delta \log \bar{L}$.*

Intuitively, a negative shock to potential factor supply today potentially reduces output tomorrow by reducing resources available for consumption tomorrow. Since nominal expenditures tomorrow are fixed, this raises the price level tomorrow. If the elasticity of substitution θ is less than one, then the increase in the price level tomorrow reduces expenditures on non-shocked factor markets and potentially causes them to become slack.

In Proposition 12, we assume that nominal expenditures in the final period are fixed. If instead we assume that the nominal price level in the future is fixed, rather than nominal expenditures, then Proposition 12 applies regardless of the value of the elasticity of substitution θ.

Appendix F Extension: Semi-Flexible Wages

In practice, we might imagine that wages can fall albeit not by enough to clear the market. The possibility that wages may fall obviously has important implications for inflation. Indeed, we show that with shocks to the sectoral composition of demand, and even without shocks to aggregate demand, we can get simultaneous reductions in output *and* inflation.

For each factor $f \in \mathcal{L}$, suppose the following conditions hold

$$\frac{L_f}{\bar{L}_f} = \begin{cases} \left(\frac{w_f}{\bar{w}_f}\right)^{\gamma_f}, & \text{if} \quad w_f \leq \bar{w}_f, \\ 1, & \text{if} \quad w_f > \bar{w}_f. \end{cases}$$

The parameter γ_f controls the degree of downward wage flexibility. If $\gamma_f = \infty$, then the wage is perfectly rigid downwards. If $\gamma_f = 0$, then the wage is fully flexible, and we recover the neoclassical case.

$$\left(\frac{w_f}{\bar{w}_f} - \left(\frac{L_f}{\bar{L}_f}\right)^{\frac{1}{\gamma_f}}\right)\left(L - \bar{L}_f\right) = 0, \quad L_f \leq \bar{L}_f, \quad \left(\frac{L_f}{\bar{L}_f}\right)^{\frac{1}{\gamma_f}} \leq \frac{w_f}{\bar{w}_f}.$$

F.1 Generalizing the Results

The only change to Proposition 1 is that we now have

$$d\log Y = \sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log \bar{L}_f + \sum_{f\in\mathcal{L}} \frac{\gamma_f}{1+\gamma_f}\lambda_f \min\left\{d\log\lambda_f + d\log E - d\log \bar{L}_f, 0\right\},$$

and the only change to Proposition 2 is that we now have

$$d\log L_f = \begin{cases} \frac{\gamma_f}{1+\gamma_f}\left(d\log\lambda_f + d\log E\right) + \frac{1}{1+\gamma_f} d\log \bar{L}_f & \text{if} \quad f\in\mathcal{D} \\ d\log\bar{L}_f & \text{if} \quad f\in\mathcal{S}. \end{cases} \tag{F.1}$$

F.2 Illustrative Example

We now construct an example showing how allowing for some degree of downward wage flexibility allows the model to generate a recession *and* deflation at the same time, without relying on aggregate demand shocks. We return to the example of Section 5. However, this time, suppose that wages have some degree of downward flexibility $0 < \gamma < \infty$ common across all factor markets $f \in \mathcal{L}$.

We now get

$$d\log Y = \lambda_{\mathcal{S}} d\log \bar{L}_{\mathcal{S}} + \lambda_{\mathcal{D}} d\log L_{\mathcal{D}},$$

where $\lambda_{\mathcal{D}} = \sum_{f\in\mathcal{D}} \lambda_f = 1 - \lambda_{\mathcal{S}}$ is the total share of the demand-constrained factors and $d\log L_{\mathcal{D}}$ is the "representative" employment reduction in the demand-constrained sectors

$$d\log L_{\mathcal{D}} = \sum_{f\in\mathcal{D}} \frac{\lambda_f}{\lambda_{\mathcal{D}}} d\log \bar{L}_f < \sum_{f\in\mathcal{D}} \frac{\lambda_f}{\lambda_{\mathcal{D}}} d\log \bar{L}_f = d\log \bar{L}_{\mathcal{D}}.$$

In turn, this employment reduction is given as a function of the change $d\log\lambda_{\mathcal{S}}$ in the share of the supply-constrained sectors by

$$\lambda_{\mathcal{D}} d\log L_{\mathcal{D}} = -\frac{\gamma}{1+\gamma}\lambda_{\mathcal{S}} d\log\lambda_{\mathcal{S}} + \frac{1}{1+\gamma}\lambda_{\mathcal{D}} d\log \bar{L}_{\mathcal{D}},$$

and the the change $d\log\lambda_{\mathcal{S}}$ in the share of the supply-constrained sectors is given by

$$\lambda_{\mathcal{S}} d\log\lambda_{\mathcal{S}} = \frac{\lambda_{\mathcal{S}} d\log\omega_{0\mathcal{S}} - (1-\theta)\lambda_{\mathcal{S}}(1-\lambda_{\mathcal{S}})\left[d\log\bar{L}_{\mathcal{S}} - \frac{1}{1+\gamma}d\log\bar{L}_{\mathcal{D}}\right]}{1-\frac{\gamma}{1+\gamma}(1-\theta)(1-\lambda_{\mathcal{S}})}.$$

Starting with the last equation, we see that once again, the share of supply-constrained factors increases if the shock to the sectoral composition of demand redirects expenditure towards these factors or if the labor shocks for those factors is larger than the ones hitting the demand-constrained factors. This reduces the shares of demand-constrained factors, creates unemployment, and further reduces output through Keynesian effects. Indeed, putting everything together, we get

$$\begin{aligned} d\log Y &= \lambda_{\mathcal{S}} d\log\bar{L}_{\mathcal{S}} \\ &+ \frac{\frac{\gamma}{1+\gamma}(1-\theta)\lambda_{\mathcal{S}}(1-\lambda_{\mathcal{S}})d\log\bar{L}_{\mathcal{S}} + \left(1-\frac{1}{1+\gamma}(1-\theta)\right)\lambda_{\mathcal{D}} d\log\bar{L}_{\mathcal{D}} - \frac{\gamma}{1+\gamma}\theta\lambda_{\mathcal{S}} d\log\omega_{0\mathcal{S}}}{1-\frac{\gamma}{1+\gamma}(1-\theta)(1-\lambda_{\mathcal{S}})}. \end{aligned}$$

The difference between the case where wages have some downward flexibility ($\gamma < \infty$) and the case where they do not ($\gamma = \infty$) is that now the wages of the demand-constrained factors falls, and this mitigates the increase in unemployment and the reduction in output. However, there is also a countervailing amplification effect: the labor supply shocks to the demand-constrained factors now also matter. This is because these shocks now reduce the wages of the demand-constrained factors, which further redirects expenditure away from them because of complementarities, and further reduces employment of the demand-constrained factors. Of course, allowing for some degree of wage flexibility can endogenously change the sets of supply-constrained and demand-constrained factors, and so we do not push the comparison any further.

Instead, we turn our attention to inflation. Using $d\log p^Y = d\log E - d\log Y$, the effect on inflation is

$$d\log p^Y = -\frac{1}{1+\gamma}d\log\lambda_{\mathcal{S}} - \lambda_{\mathcal{S}} d\log\bar{L}_{\mathcal{S}} - \frac{1}{1+\gamma}\lambda_{\mathcal{D}} d\log\bar{L}_{\mathcal{D}}.$$

The first term is negative, since the share of supply-constrained factors expands in response to the negative demand shock, capturing the fact that as demand switches to supply-constrained factors, the price of sticky sectors starts to decline, generating deflation. In the simple case where there are no negative supply shocks $d\log\bar{L} = 0$ but the sectoral composition of demand has shifted, we get that output *and* inflation both fall.

Appendix G Derivations for Example with Failures

Preliminaries. Changes in the sales of i are given by

$$d\log\lambda_i = (1-\theta_0)(1-\lambda_i)\Big(d\log p_i - \sum_{j\in\mathcal{N}}\lambda_j d\log p_j\Big), \tag{G.1}$$

where changes in the price of i depend on changes in the wage in i and on the endogenous reduction in the productivity of i driven by firm failures

$$d\log p_i = d\log w_i - \frac{1}{\sigma-1}d\log M_i. \tag{G.2}$$

The change in wages in i are given by

$$d\log w_i = \max\{d\log\lambda_i - d\log\bar{L}_i, 0\}, \tag{G.3}$$

and changes in the mass of producers in i are given by

$$d\log M_i = \min\{d\log\lambda_i, d\log\bar{M}_i\}. \tag{G.4}$$

We consider the effect of shutdown shocks $d\log\bar{M}_i$ starting with the case where sectors are complements and then the case where they are substitutes. The effect of negative labor shocks $d\log\bar{L}_i$ is similar.

Shut-down shock with complements. Assume that sectors are complements ($\theta < 1$) and consider the government-mandated shutdown of some firms in only one sector i. We can aggregate the non-shocked sectors into a single representative sector indexed by $-i$. We therefore have $d\log\bar{M}_i < 0 = d\log\bar{M}_{-i}$.

The closures of firms in i raise its price ($d\log p_i > 0$), which because of complementarities, increases its share ($d\log\lambda_i > 0$). It therefore does not trigger any further endogenous exit in this shocked sector ($d\log M_i = d\log\bar{M}_i$). In addition, the wages of its workers increases ($d\log w_i > 0$). The shock reduces expenditure on the other sectors ($d\log\lambda_{-i} < 0$), and this reduction in demand triggers endogenous exits ($d\log M_{-i} < 0$), pushes wages against their downward rigidity constraint ($d\log w_{-i} = 0$) and creates unemployment ($d\log L_{-i} < 0$), both of which endogenously amplify the reduction in output through failures and Keynesian effects.

Using equations (G.1), (G.2), (G.3), and (G.4), we find

$$d\log\lambda_i = -\frac{(1-\theta)(1-\lambda_i)}{1-(1-\theta)(1-\lambda_i)\left(1-\frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)}\frac{1}{\sigma-1}d\log\bar{M}_i > 0,$$

$$d\log M_{-i} = d\log L_{-i} = -\frac{\lambda_i}{1-\lambda_i}d\log\lambda_i < 0,$$

and finally

$$d\log Y = \lambda_i\frac{1}{\sigma-1}d\log\bar{M}_i + \frac{(1-\theta)(1-\lambda_i)\frac{\sigma}{\sigma-1}}{1-(1-\theta)(1-\lambda_i)\left(1-\frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)}\lambda_i\frac{1}{\sigma-1}d\log\bar{M}_i.$$

The first term on the right-hand side is the direct reduction in output from the shut-down in sector i. The second term capture the further indirect equilibrium reduction in output via firm failures and Keynesian unemployment in the other sectors.

Shut-down shock with substitutes. Consider the same experiment as above but assume now that sectors are substitutes ($\theta > 1$). We conjecture an equilibrium where sales in sector i do not fall more quickly than the initial shock $d\log\lambda_i - d\log\bar{M}_i > 0$. Sector i loses demand following the exogenous shutdown of some of its firms, and this results in unemployment in in the sector ($d\log L_i < 0$) but no endogenous firm failures ($d\log M_i = d\log\bar{M}_i$) . On the other hand, sector $-i$ maintains full employment and experiences no failures.

To verify that this configuration is indeed an equilibrium, we compute

$$d\log\lambda_i = \frac{(\theta-1)(1-\lambda_i)}{1-(\theta-1)\lambda_i}\frac{1}{\sigma-1}d\log\bar{M}_i.$$

We must verify that

$$0 > d\log\lambda_i > d\log\bar{M}_i.$$

The first inequality is verified as long as $\theta > 1$ is not too large. The second inequality is verified if $\sigma > 1$ is large enough and $\theta > 1$ is not too large.

If these conditions are violated, then we can get a jump in the equilibrium outcome. Intuitively, in those cases, the shutdown triggers substitution away from i, and that substitution is so dramatic than it causes more firms to shutdown, and the process feeds on itself ad infinitum. Any level of $d\log L_i < 0$ and $d\log M_i < d\log\bar{M}_i$ can then be supported as equilibria. Although we do not focus on it, this possibility illustrates how allowing for

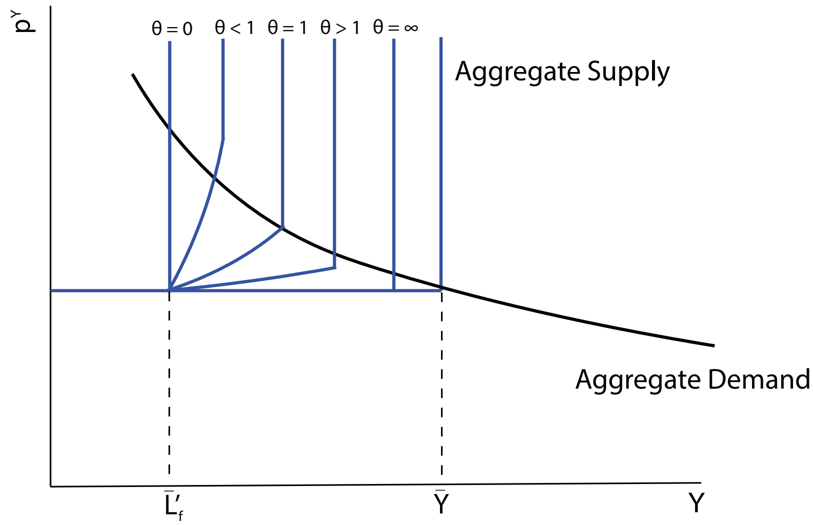

Figure H.1: The effect of the same negative supply shock to a factor for different values of the elasticity of substitution θ.

firm failures with increasing returns to scale can dramatically alter the model's behavior.

Assuming the regularity conditions above are satisfied, the response of output is given by

$$d\log Y = \lambda_i \frac{1}{\sigma - 1} d\log \bar{M}_i + \frac{(\theta_0 - 1)(1 - \lambda_i)}{1 - (\theta_0 - 1)\lambda_i} \lambda_i \frac{1}{\sigma - 1} d\log \bar{M}_i,$$

where the first term on the right-hand side is the direct effect of the shutdown and the second term is the amplification from the indirect effect of the shutdown which results in Keynesian unemployment in i.

Appendix H More on Complementarities

In this appendix, using AS-AD diagrams, we illustrate how complementarities amplify negative supply shocks and mitigate negative demand shocks.

H.1 Amplification of Supply Shocks

Figure H.1 represents a supply shock to some factor f using an AS-AD diagram. The AS curve is horizontal to the left since there are no capitals ($\mathcal{K} = \emptyset$). The initial level of output is given by $\bar{Y}$ and the new level of labor available in the shocked sector is given by $\bar{L}'_f$. The negative supply shock shifts the AS curve to the left.

In the figure, we draw the new AS curve for different values of the elasticity of substitution θ. As we lower the elasticity of substitution θ, the kink point at which the AS curve becomes vertical shifts north-westwards. As long as $\theta > 1$, the second kink is below the AD curve, and so the equilibrium is the same as the neoclassical one, because the AS and AD intersect along the neoclassical portion of the AS curve. Intuitively, when θ is above one, no factor market becomes demand-constrained and so downward nominal wage rigidity is never triggered. Once the elasticity of substitution has been lowered to $\theta = 1$, the Cobb-Douglas case, the second kink exactly intersects the AD curve. When θ goes below one, the second kink moves above the AD curve, downward nominal wage rigidities are triggered, and the equilibrium has lower output and higher inflation than the neoclassical model. Finally, as θ goes to zero and we approach the Leontief case, the second kink point moves directly above the first kink point, and so the reduction in output in the neoclassical model and Keynesian model become the same again.

The mechanism is Figure H.1 is only operative if there are some factors that are supply-constrained. If every factor is demand-constrained, then this amplification effect is non-functioning. Also, note that greater reductions in output must be accompanied with more inflation.

H.2 Mitigation of Demand Shocks

Whereas complementarities amplify negative supply shocks, the same forces act to mitigate negative aggregate demand shocks. To see this, consider Figure H.2, where we draw how the equilibrium responds to a negative demand shock as we lower the elasticity of substitution. The AS curve is upward sloping whenever Y is less than $\bar{Y}$ if there are some factors with flexible prices.

In response to a reduction in aggregate demand, the price of supply-constrained sectors falls. Due to complementarities, this causes expenditures to switch towards the demand-constrained factor markets, whose prices cannot fall, and this substitution boosts employment in those factor markets. Intuitively, with complementarities, factor markets with flexible prices, for example capital markets, can absorb the negative demand shock and redirect demand to demand-constrained sectors.

The mechanism is Figure H.2 is only operative if there are some factors that are supply-constrained. If every factor is demand-constrained, then this shock absorber effect is non-functioning.

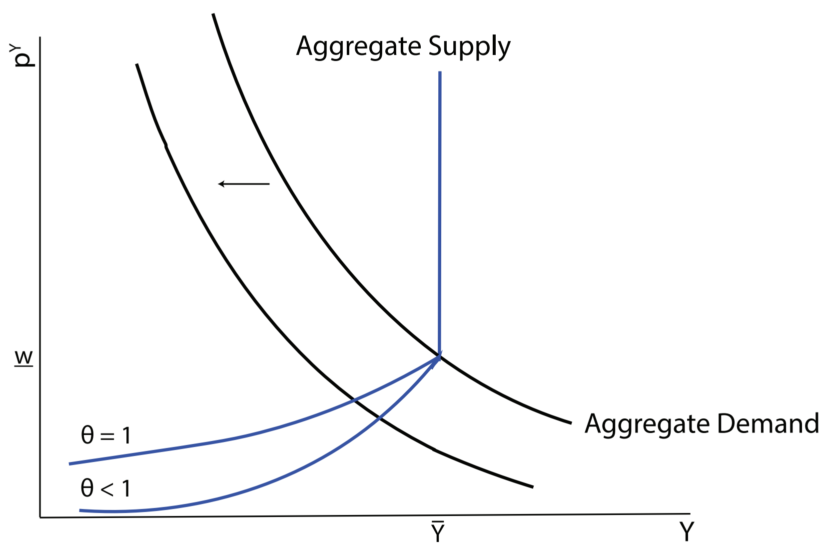

Figure H.2: The effect of the same negative demand shock for different values of the elasticity of substitution θ with $\theta' < \theta$.