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AN APPLICATION TO THE COVID-19 CRISIS

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Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis

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ABSTRACT

We study supply and demand shocks in a general disaggregated model with multiple sectors, factors, and input-output linkages, as well as downward nominal wage rigidities and a zero lower bound constraint. We use the model to understand how the Covid-19 crisis, an omnibus of supply and demand shocks, affects output, unemployment, inflation, and leads to the coexistence of tight and slack labor markets. Under some conditions, the details of the production network can be summarized by simple sufficient statistics that we use to conduct global comparative statics. Negative sectoral supply shocks and sectoral demand shocks are stagflationary, whereas negative intertemporal demand shocks are deflationary. Complementarities magnify Keynesian spillovers for the former shocks but mitigate them for the latter. We illustrate the intuition using a nonlinear AS-AD representation. In a quantitative model of the US calibrated to current disaggregated data, sectoral supply and demand shocks on their own generate more than 10% inflation, and negative intertemporal demand shocks on their own generate 7% deflation. *Both* types of shocks are necessary to capture the disaggregated data, each explains about half the reduction in real GDP, and putting both together results in 0.3% inflation and as much as 10% Keynesian unemployment in April 2020. Nevertheless, aggregate demand stimulus is only about a third as effective as in a typical recession where all labor markets are slack. More targeted forms of demand stimulus are more effective.

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1 Introduction

Covid-19 is an unusual macroeconomic shock. It cannot easily be categorized as an aggregate supply or demand shock. Rather, it is a messy combination of disaggregated supply and demand shocks. These shocks then propagate through supply chains to create different cyclical conditions in different parts of the economy. Some sectors become tight as they run into supply constraints and struggle to keep up with demand. Other sectors become slack and shed workers to reduce excess capacity because of lack of demand.

To analyze this divergent situation, and take advantage of the available disaggregated data, we use a general disaggregated model and aggregate up from the micro level to the macro level. We allow for an arbitrary number of sectors and factors as well as unrestricted input-output linkages and elasticities of substitution. We incorporate downward nominal wage rigidities and a zero lower bound constraint.

We model the outbreak of the pandemic as a combination of supply and demand shocks. We define demand shocks as changes in households' expenditures for fixed prices and income, and supply shocks as changes in the economy's production possibilities. On the one hand, the epidemic sets off demand shocks by changing final demand within and across periods. Households may rebalance their current expenditures across sectors because they are locked down, fear getting infected by the virus if they consume certain goods, or dislike the steps they would need to take to consume certain goods safely. Households may also reduce their current expenditures overall if they prefer to postpone spending to the future when conditions for consumption are back to normal. On the other hand, the epidemic also triggers supply shocks. It can shrink the economy's productive capacity by reducing the supply of usable labor and productivity in the different sectors due to lock-downs, working from home, social distancing in the workplace, or reduced willingness to work.

We first provide *local* comparative statics for these supply and demand shocks. We characterize the responses of aggregates such as output, inflation, and unemployment as well as of disaggregated variables. In particular, we show how the elasticities of substitution in production and in consumption interact with the input-output network to redirect demand away from some factors and towards others, causing Keynesian unemployment in labor markets where demand goes down more than supply.¹

¹Keynesian unemployment measures the amount of slack in a given factor market. It captures underemployment due to lack of demand for the good that the factor is producing because of downwardly rigid wages. Measured unemployment in the data reflects not only Keynesian unemployment but other forms of

We also provide *global* comparative statics. These global comparative statics allow us to capture the nonlinearities of the model and in particular how the shocks interact with each other and get amplified or mitigated as they get larger. We discuss the impact of each of the shocks in turn. We start with negative shocks to the stock amount of labor that can safely be used in the different sectors.

We prove that for these negative sectoral supply shocks, as long as all elasticities of substitution are all the same, the initial factor income shares are global *sufficient statistics* for the production network of the economy. In other words, even though there are many sectors and potentially complex and nonlinear supply chains, this information is entirely summarized by the initial factor income shares.

In this benchmark case, we are able to provide global comparative statics. We show that as long as the uniform elasticity of substitution is less than one, so that there are *complementarities*, the set of equilibria is a *lattice*. This implies that there is a unique best (worst) equilibrium with the minimal (maximal) number of slack labor markets and the minimal (maximal) amount of Keynesian unemployment in each labor market. In the best and worst equilibria, a reduction in the quantity of labor supplied in a market lowers spending on the other labor markets. Therefore, a negative shock to potential labor in one market depresses the other labor markets. Similarly, a binding downward nominal wage rigidity constraint in one labor market pushes other labor markets towards their constraint.

We illustrate graphically how the equilibrium changes in response to shocks with an aggregate supply (AS) and aggregate demand (AD) diagram. A novelty of our model is that supply shocks do not simply shift the AS curve, but they also change its shape, resulting in apparent instability of the AS relationship. The unstable shape of the AS curve reflects the nonlinearities arising from the interaction of complementarities and occasionally-binding downward nominal wage rigidities.

Our global sufficient statistic approach can also be extended, along the lines of Baqaee (2015), to cover the case where there are shocks to the sectoral composition of demand within the period. In this case, changes in household spending across sectors lead to changes in expenditures on the different factor markets, as final demand is distributed to the factors through the input-output network. There is more Keynesian unemployment and a larger recession when households shift their spending away from goods that are intensive, directly and indirectly through supply chains, on slack labor markets.

supply-driven underemployment due to the pandemic. See Section 2.2 for a discussion.

Although negative sectoral supply and demand shocks reduce output and cause Keynesian unemployment, these shocks are necessarily *inflationary* and hence lead to stagflation. Intertemporal demand shocks that reduce current overall nominal spending for given prices, on the other hand, reduce output, cause Keynesian unemployment, and are *deflationary*. These negative aggregate demand shocks can depress all labor markets.

While stronger complementarities amplify Keynesian unemployment in response to negative sectoral supply shocks and shocks to the sectoral composition of demand, they mitigate Keynesian unemployment in response to negative aggregate demand shocks. Intuitively, with complementarities, negative sectoral supply shocks redirect expenditure towards the tight labor markets that are becoming more expensive, thereby amplifying Keynesian unemployment in the slack labor markets that have downwardly rigid wages. Similarly, complementarities amplify changes in the sectoral composition of demand by causing substitution towards those markets that become more expensive, exacerbating Keynesian unemployment in those slack labor markets that are being deprived of demand. By contrast, negative aggregate demand shocks reduce expenditure on all factors and make tight labor markets cheaper and slack labor markets with downwardly rigid wages relatively more expensive. In the presence of complementarities, this in turn redirects expenditure towards slack labor markets, thereby mitigating Keynesian unemployment.

We use a quantitative input-output model of the US economy with realistic complementarities to gauge the importance of the various theoretical forces that we identify. We calibrate the model to match sectoral data on employment and nominal expenditures. Our analysis shows that *both* supply and demand shocks are necessary to match the data. Whilst negative supply shocks, on their own, can explain the bulk of the reduction in real GDP (around 11%), they would result in far too much inflation (more than 10%). This remains true if they are combined with the shocks to the sectoral composition of demand. On the other hand, negative aggregate demand shocks, on their own, predict too much deflation (around 7%) and too small reductions in real GDP (around 6%). However, combining all these different supply and demand shocks matches both the aggregate and disaggregated patterns in the data, resulting in 13% reduction in output with mild inflation of 0.3% and as much as 10% Keynesian unemployment.

We use the model to classify sectoral labor markets as supply-constrained (tight) or demand-constrained (slack). As an external check on the analysis, we find that subsequent recovery in sectoral labor markets classified as supply-constrained in April is four times stronger than those classified as demand-constrained. This is consistent with the idea that

supply constraints are driven partially by lockdowns or social distancing, and so as states reopened in May, the supply constraints were relaxed and the supply-constrained sector recovered more strongly than the demand-constrained sectors.

We also extend our basic framework to allow for “endogenous” demand and supply shocks. In the first extension, we show that borrowing constraints that bind endogenously for households who lose income reduce aggregate nominal expenditures and act like negative aggregate demand shocks. In the second extension, we show that credit constraints that bind endogenously for firms that lose profits cause firm failures and act like negative supply shocks. These endogenous supply and demand shocks act as multipliers, amplifying reductions in output, increasing Keynesian unemployment, and generating either extra deflationary (for the former) or extra inflationary forces (for the latter).

Finally, we analyze policy responses to the Covid-19 shock. We study the effect of fiscal policy (government spending and transfers), monetary policy, and tax incentives such as payroll tax cuts. While these policies are important and effective at stimulating the economy, they can also be less potent than in more traditional recessions. This is because unlike typical recessions, there is a coexistence of tight and slack labor markets. Stimulating spending on tight labor markets is wasteful and complementarities further worsen this problem. In our quantitative model, the increase output from reverting the decline in aggregate demand (via forward guidance or fiscal policy) in the presence of sectoral shocks (in the Covid-19 recession) is only a third of its value in the absence of sectoral shocks (in a typical recession). More bang for the buck can be achieved by targeting stimulus to the depressed labor markets. For example, government spending can be targeted towards the sectors that rely more intensively, directly and indirectly through the network, on the labor markets that are depressed.

The outline of the paper is as follows. In Section 2, we set up the model, define the equilibrium and notation, and discuss the shocks. In Section 3, we establish the basic comparative statics of the model and illustrate them with a simple example. In Section 4, we establish our sufficient-statistic result and prove some global comparative statics. In Section 5, we conduct a quantitative exercise to understand the importance of the various mechanisms we have emphasized for the Covid-19 crisis. The rest of the paper contains extensions: in Section 6, we extend the model to include occasionally-binding credit constraints on households; in Section 7, we extend the model to include occasionally-binding credit constraints on firms, causing firm failures; and in Section 8 we investigate monetary, fiscal, and tax policy responses. We conclude in Section 9.

Related Literature

The paper is part of the literature on economic effects of the Covid-19 crisis, as well as multi-sector models with nominal rigidities.

Guerrieri et al. (2020) show that under certain configurations of the elasticities of substitution, negative labor supply shocks can cause negative demand spillovers. They focus on substitutabilities, whereas we focus on complementarities and the associated nonlinearities from occasionally-binding downward wage rigidity.² Bigio et al. (2020) study optimal policies in response to the Covid-19 crisis in a two-sector Keynesian model. We differ in both focus and framework, since we are not focused on optimal policy and instead try to understand the importance of the production structure.³ Fornaro and Wolf (2020) study Covid-19 in a New-Keynesian model where the pandemic is assumed to have persistent effects on productive capacity in the future by lowering aggregate productivity growth. The expected loss in future income reduces aggregate demand. They show that a feedback loop can arise between aggregate supply and aggregate demand if productivity growth in turn depends on the level of economic activity.⁴ We differ in that we focus on the effects of current supply disruptions. Caballero and Simsek (2020) study a different kind of spillover, between asset prices and demand shortages.

Our paper also relates to quantitative multi-sector models. Barrot et al. (2020) study the effect of Covid-19 using a quantitative production network with complementarities and detailed administrative data from France. Bonadio et al. (2020) study the effect of Covid-19 in a quantitative international trade model. Bodenstein et al. (2020) analyze optimal shutdown policies in a two-sector model with complementarities and minimum-scale requirements. Our approach differs due from these papers due our focus on nominal rigidities and Keynesian effects. Brinca et al. (2020) use a statistical model to decompose sectoral outcomes to the Covid-19 crisis into demand- and supply-side sources. Kaplan et

²The economics of these two cases are different because with substitutabilities, a negative labor supply shock reduces the share of that labor and reduces aggregate nominal expenditure through intertemporal substitution, if intertemporal substitution outpaces intratemporal substitution, this reduces demand for the other labors despite the increase in their shares. With complementarities instead, the propagation of the shock is not driven by intertemporal substitution but by the fact that a negative labor supply shock increases the share of that labor, which in turn reduces the demand for the other labors; the same logic applies to the endogenous reductions in labor induced by the initial shock, which further reduce the demand for the other labors, etc.

³Bigio et al. (2020) study a fully dynamic model specified in continuous time, which allows them to analyze how the effects unfold over time.

⁴This could be because of reduced investment in research and development due to a reduced size of the market à la Benigno and Fornaro (2018).

al. (2020) combine an SIR model with a multi-sector heterogeneous agent New Keynesian model to study the economic impact of the pandemic.

This paper is also related to other work by the authors, especially Baqaee and Farhi (2020b). Whereas in this paper, we study how exogenous shocks interact with nominal frictions and result in involuntary unemployment, Baqaee and Farhi (2020b) is a companion paper where we analyze the nonlinear mapping from changes in hours and household preferences to real GDP. In this companion paper, we find that the negative supply and demand shocks associated with Covid-19 are large enough that accounting for nonlinearities is quantitatively important.

Our analysis is also related to production network models with nominal rigidities, like Baqaee (2015), who studies the effect of shocks to the sectoral composition of demand in a production network with downward wage rigidity, Pasten et al. (2017) and Pasten et al. (2019) who study propagation of monetary and TFP shocks in models with sticky prices, Ozdagli and Weber (2017) who study the interaction of monetary policy, production networks, and asset prices, and Rubbo (2020) and La'O and Tahbaz-Salehi (2020) who study optimal monetary policy with sticky prices.

2 Setup

In this section, we set up the basic model. We break the description of the model in two. First, we discuss the intertemporal problem of how the representative household chooses to spend its income across periods. Second, we discuss the intratemporal problem of how a given amount of expenditures is spent across different goods within a period. We then define the equilibrium notion and discuss the shocks that we will be studying.

2.1 Environment and Equilibrium

There are two periods, the present denoted without stars, and the future denoted with stars, and there is no investment.⁵ We take the equilibrium in the future as given. As in Krugman (1998) and Eggertsson and Krugman (2012), this is isomorphic to an infinite-horizon model where after an initial unexpected shock in period 1, the economy returns

⁵We abstract from investment in the main body of the paper in order to keep the exposition manageable. We show in Appendix B how our approach generalizes to environments with investment.

to a long-run equilibrium with market clearing and full employment.⁶ We denote the supply of the future composite final-consumption good by \tilde{Y}_* , its price by \bar{p}_*^Y , and future final income and expenditure by $\bar{I}_* = \bar{E}_* = \bar{p}_*^Y \tilde{Y}_*$, which are all taken to be exogenous.

We focus on the present, where there are a set of producers \mathcal{N} and a set of factors \mathcal{G} with supply functions L_f . We denote by $\mathcal{N} + \mathcal{G}$ the union of these sets. We abuse notation and also denote the number of producers and factors by \mathcal{N} and \mathcal{G} .

Consumers. The representative consumer maximizes intertemporal utility

$$(1 - \beta) \frac{Y^{1-1/\rho} - 1}{1 - 1/\rho} + \beta \frac{Y_*^{1-1/\rho} - 1}{1 - 1/\rho},$$

where ρ is the intertemporal elasticity of substitution (IES) and $\beta \in [0, 1]$ captures households' time-preferences. The intertemporal budget constraint is

$$\sum_{i \in \mathcal{N}} p_i c_i + \frac{\bar{p}_*^Y Y_*}{1 + i} = \sum_{f \in \mathcal{G}} w_f L_f + \sum_{i \in \mathcal{N}} \pi_i + \frac{\bar{I}_*}{1 + i},$$

where p_i and c_i are the price and final consumption of good i , the nominal interest rate is $(1 + i)$, the wage and quantity of factor f are w_f and L_f , and π_i is the profit of producer i . The consumption bundle in the present period is given by

$$Y = C(c_1, \dots, c_N; \omega_{\mathcal{D}}),$$

a homothetic final-demand aggregator of the final consumptions c_i of the different goods i . The parameter $\omega_{\mathcal{D}}$ is a preference shifter capturing changes in the sectoral composition of final demand.

For future reference, we define the price p^Y of the consumption bundle Y by

$$p^Y = \mathcal{P}(p_1, \dots, p_N; \omega_{\mathcal{D}}).$$

where \mathcal{P} is the dual price index of the quantity index \mathcal{D} . We also denote by

$$E = p^Y Y$$

⁶As long as there is no investment, our analysis applies to situations where the crisis lasts for multiple periods without change, see footnote 12.

the present final expenditure. In the rest of the paper, we will refer to Y as *output*.⁷

Producers. Producer i maximizes profits

$$\pi_i = \max_{\{y_i\}, \{x_{ij}\}, \{L_{if}\}} p_i y_i - \sum_{j \in \mathcal{N}} p_j x_{ij} - \sum_{f \in \mathcal{G}} w_f L_{if}$$

using a production function

$$y_i = A_i F_i \left(\{x_{ij}\}_{j \in \mathcal{N}}, \{L_{if}\}_{f \in \mathcal{G}} \right),$$

where A_i is a Hicks-neutral productivity shifter, y_i is total output, and x_{ij} and L_{if} are intermediate and factor inputs used by i .

Market equilibrium. Market equilibrium for goods is standard. The market for i is in equilibrium if

$$c_i + \sum_{j \in \mathcal{N}} x_{ji} = y_i.$$

Market equilibrium for factors is non-standard, the wages of factors cannot fall below some exogenous lower bound.⁸ We say that factor market f is in equilibrium if the following three conditions hold:

$$(w_f - \bar{w}_f)(L_f - \bar{L}_f) = 0, \quad \bar{w}_f \leq w_f, \quad L_f \leq \bar{L}_f,$$

where

$$L_f = \sum_{i \in \mathcal{N}} L_{if}$$

is the total demand for factor f . The parameters \bar{w}_f and \bar{L}_f are exogenous minimum nominal wage and endowment of the factor.

In words, there are two possibilities. One possibility is $w_f \geq \bar{w}_f$ and employment of the factor is equal to potential with $L_f = \bar{L}_f$. In this case, we say that the market is tight,

⁷Output here corresponds to welfare. Changes in output and changes in welfare always coincide to the first order, but they do not always coincide to higher orders. Without changes in the preference shifter $\omega_{\mathcal{D}}$, changes in welfare coincide with changes in real GDP at any order. With changes in the preference shifter, changes in changes in welfare do not coincide with changes in real GDP at the second order. See Baqaee and Farhi (2020b) for detailed discussion.

⁸In Appendix C, we extend the model to allow for some downward wage flexibility.

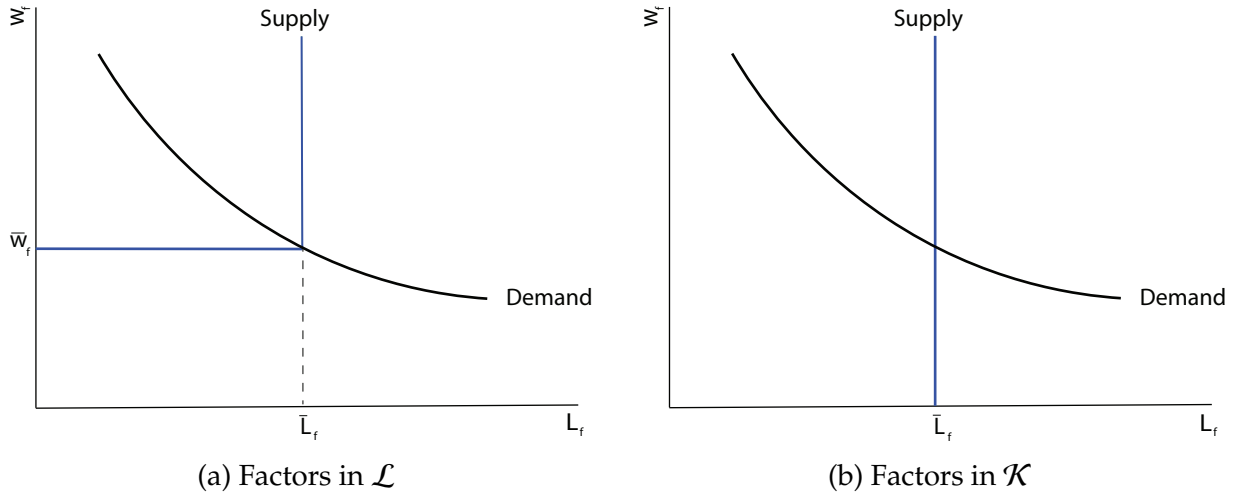


Figure 2.1: Equilibrium in the factor markets.

that it clears, and that it is *supply-constrained*. The other possibility is that $w_f = \bar{w}_f$ and employment of the factor is less than potential $L_f \leq \bar{L}_f$. We then say that the market is slack, that it does not clear, and that it is *demand-constrained*. In this case, we call the underemployment $\bar{L}_f - L_f$ of the factor *Keynesian unemployment* since it is caused by a lack of demand for the good that the factor is producing given the rigid wage.

We only consider two cases: the case where \bar{w}_f is equal to its pre-shock market-clearing value, denoted by the set $\mathcal{L} \subseteq \mathcal{G}$; and the case where $\bar{w}_f = -\infty$, making the wage of f flexible and ensuring the market for f always clears, denoted by the subset $\mathcal{K} \subseteq \mathcal{G}$. For concreteness, we call \mathcal{K} the capital factors and \mathcal{L} the labor factors.

Of course, these are just names, in practice, one may easily imagine that certain capital markets could also be subject to nominal rigidities. This can be a way to model firm failures: imagine firms take out within-period loans to pay for their variable expenses, secured against their capital income. If the firm's capital income declines in nominal terms, then the firm defaults on the loan, exits the market, and its capital becomes unemployed for the rest of the period.⁹

We denote the endogenous set of supply-constrained factor markets by $\mathcal{S} \subseteq \mathcal{G}$. In other words, $f \in \mathcal{S}$ if, and only if, $L_f = \bar{L}_f$. We denote the endogenous set of demand-constrained factor markets by $\mathcal{D} \subseteq \mathcal{G}$. Hence, $f \in \mathcal{D}$ if, and only if, $L_f < \bar{L}_f$. Of course, $\mathcal{K} \subseteq \mathcal{S}$, and $\mathcal{D} \subseteq \mathcal{L}$. Figure 2.1 illustrates the supply and demand curves in the factor markets.

⁹We build on this observation further in Section 7, where we formally introduce an extensive margin of firm exit, and study the importance of increasing returns to scale.

Equilibrium. Given a nominal interest rate $(1 + i)$, factor supplies \bar{L}_f , productivities A_i , and demand shifters $\omega_{\mathcal{D}}$, an equilibrium is a set of prices p_i , factor wages w_f , intermediate input choices x_{ij} , factor input choices L_{if} , outputs y_i , and final demands c_i , such that: each producer maximizes its profits subject to its technological constraint; consumers maximize their utility; and the markets for all goods and factors are in equilibrium. Without loss of generality, we normalize $\tilde{Y}^* = \bar{Y} = 1$ and $p^Y = 1$.

2.2 Comparative Statics

We provide comparative statics with respect to shocks, starting at an initial equilibrium with full employment of all factors. A natural disaster, like the Covid epidemic, can be captured as a combination of negative supply and demand shocks. We define a demand shock to be a shock that changes the household's expenditure shares on the different goods (across sectors and over time) *at given prices*. We define supply shocks to be shocks that change the possibilities to produce the different goods.

Supply shocks. We define supply shocks to be changes in the economy's production possibility frontier, which could come in the form of either reduced factors or reduced productivity. We call reductions in the available productive endowment of labor \bar{L}_f shocks to *potential labor*. These are reductions that would take place absent any nominal frictions. These reductions could have different drivers. They could be driven directly by government action, like mandated shutdowns and stay at home orders. They could also be due to a reduced willingness to work by employees due to health concerns or policy disincentives such as overly generous unemployment insurance. Finally, reductions in potential labor could also be the result of a reorganization of production. For example, firms could be forced to operate at lower capacity to implement social distancing, such as a restaurant that can only safely serve a fraction of the customers it used to serve. In this case, workers would be involuntarily unemployed due to a reduced physical capacity to employ them and not because there is not enough demand for the good that they produce. This type of supply-driven underemployment would occur even in the absence of downward nominal wage rigidities. For this reason, we do not include this form of underemployment in our definition of Keynesian unemployment.¹⁰

¹⁰To model this formally, we can imagine that $\bar{L}_f = \min\{\tilde{L}_f, S_f\}$, where \tilde{L}_f is the physical endowment of labor and S_f is a "safety" input which, in the initial equilibrium, is not scarce. Since it is not scarce, it commands a price of zero initially. However, the pandemic reduces the supply of S_f so that it binds. At

Similarly, the epidemic might reduced the productivity A_i of the different producers by changing the way firms can operate, for instance by reducing person-to-person interactions.

Demand shocks. Similarly, the pandemic can also change the current sectoral composition of final demand, since at given prices and income, households may shift expenditure away from some goods like cruises and air transportation, and towards other goods like groceries and online retail. We model this as a change in the preference shifter $\omega_{\mathcal{D}}$.

On the other hand the pandemic can reduce households' willingness to consume in the present relative to the future: at given prices and income, households may choose to consume less during the epidemic and more afterwards. We model this as an increase in the discount factor $\beta/(1 - \beta)$.

2.3 Input-Output Definitions

To analyze the model, we define some input-output objects such as input-output matrices, Leontief inverse matrices, and Domar weights associated with any equilibrium. To make the exposition more intuitive, we slightly abuse notation by treating factors with the same notation as goods. For each factor f , we interchangeably use the notation L_{if} or $x_{i(\mathcal{N}+f)}$ to denote its use by producer i , the notation L_f or y_f to denote total factor supply, and p_f or w_f to refer to its price or wage. This allows us to add factor supply and demand into the input-output matrix along with the supply and demand for goods. Furthermore, we define final demand as an additional good produced by producer 0 according to the final demand aggregator. We interchangeably use the notation c_i or x_{0i} to denote final consumption of good i . We write $1 + \mathcal{N}$ for the union of the sets $\{0\}$ and \mathcal{N} , and $1 + \mathcal{N} + \mathcal{G}$ for the union of the sets $\{0\}$, \mathcal{N} , and \mathcal{G} . With this abuse of notation, we can stack every market in the economy into a single input-output matrix that includes the household, the producers, and the factors.

Input-output matrix. We define the input-output matrix to be the $(1 + \mathcal{N} + \mathcal{G}) \times (1 + \mathcal{N} + \mathcal{G})$ matrix Ω whose ij th element is equal to i 's expenditures on inputs from j as a share of its

this point, the supply of useable labor \bar{L}_f falls one-for-one with S_f . In this case, employees would refuse to hire any additional workers since their marginal product is zero. A formal capacity constraint like this is isomorphic to our formulation where we directly shock \bar{L}_f in terms of real GDP, inflation, and hours worked. The only difference is that the increase in the wage w_f would not take place and would instead be captured as a Ricardian rent by the firm.

total income/revenues

$$\Omega_{ij} \equiv \frac{p_j x_{ij}}{p_i y_i} = \frac{p_j x_{ij}}{\sum_{k \in N+\mathcal{G}} p_k x_{ik}}.$$

The input-output matrix Ω records the *direct* exposures of one producer to another, forward from upstream to downstream in costs, and backward from downstream to upstream in demand.

Leontief inverse matrix. We define the Leontief inverse matrix as

$$\Psi \equiv (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \dots$$

The Leontief inverse matrix Ψ records instead the *direct and indirect* exposures through the supply chains in the production network. This can be seen from the fact that $(\Omega^n)_{ij}$ measures the weighted sums of all paths of length n from producer i to producer j .

Nominal expenditure and Domar weights. Recall that nominal expenditure is the total sum of all final expenditures

$$E = \sum_{i \in N} p_i c_i = \sum_{i \in N} p_i x_{0i}.$$

We define the Domar weight λ_i of producer i to be its sales share as a fraction of GDP

$$\lambda_i \equiv \frac{p_i y_i}{E}.$$

Note that $\sum_{i \in N} \lambda_i > 1$ in general since some sales are not final sales but intermediate sales. Note that the Domar weight λ_f of factor f is simply its total income share.

The accounting identity $p_i y_i = p_i x_{0i} + \sum_{j \in N} p_i x_{ji} = \Omega_{0i} E + \sum_{j \in N} \Omega_{ji} \lambda_j E$ links the Domar weights to the Leontief inverse via

$$\lambda_i = \Psi_{0i} = \sum_{j \in N} \Omega_{0j} \Psi_{ji},$$

where $\Omega_{0j} = (p_j x_{0j}) / (\sum_{k \in N+\mathcal{G}} p_k x_{0k}) = (p_j c_j) / E$ is the share of good j in final expenditure.

2.4 Nested-CES Economies

For simplicity, we restrict attention to nested-CES economies. That is, we assume every production function and the final demand function can be written as nested-CES functions (albeit with an arbitrary set of nests). To be precise, any nested-CES economy can be written in *standard form*, defined by a tuple $(\bar{\omega}, \theta, F)$. The $(1 + \mathcal{N} + \mathcal{G}) \times (1 + \mathcal{N} + \mathcal{G})$ matrix $\bar{\omega}$ is a matrix of input-output parameters. The $(1 + \mathcal{N}) \times 1$ vector θ is a vector of microeconomic elasticities of substitution. Each good $i \in \mathcal{N}$ is produced with the production function

$$\frac{y_i}{\bar{y}_i} = \frac{A_i}{\bar{A}_i} \left(\sum_{j \in \mathcal{N} + \mathcal{G}} \bar{\omega}_{ij} \left(\frac{x_{ij}}{\bar{x}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}},$$

where x_{ij} are intermediate inputs from j used by i . We represent final demand as the purchase of good 0 from producer 0 producing the final good

$$\frac{y_0}{\bar{y}_0} = \left(\sum_{j \in \mathcal{N} + \mathcal{G}} \bar{\omega}_{0j} \frac{\omega_{0j}}{\bar{\omega}_{0j}} \left(\frac{x_{0j}}{\bar{x}_{0j}} \right)^{\frac{\theta_0 - 1}{\theta_0}} \right)^{\frac{\theta_0}{\theta_0 - 1}},$$

where ω_{0j} is a demand shifter. In these equations, variables with over-lines are normalizing constants equal to the values at some initial competitive equilibrium and we then have $\bar{\omega} = \bar{\Omega}$.¹¹ To simplify the notation below, we think of ω_0 as a $1 \times (1 + \mathcal{N} + \mathcal{G})$ vector with k -th element ω_{0k} .

Through a relabelling, this structure can represent any nested-CES economy with an arbitrary pattern of nests and elasticities. Intuitively, by relabelling each CES aggregator to be a new producer, we can have as many nests as desired.

3 Local Comparative Statics

In this section, we describe the comparative statics of the basic model and provide some examples. Our results here are local (first-order) comparative statics. In Section 4, we provide global comparative statics in important special cases.

¹¹Note that when mapping the original economy to the re-labeled economy, the different nests in final demand are mapped into different producers j . To simplify the exposition, we have imposed that there are only demand shocks in the outermost nest mapped to producer 0. It is easy to generalize the results to allow for demand shocks in all the nests corresponding to final demand.

Because of downward wage-rigidity, variables like aggregate output and inflation are not differentiable everywhere. Therefore, our local comparative statics should be understood as holding almost-everywhere. Furthermore, there are potentially multiple equilibria, in which case, local comparative statics should be understood as perturbations of a given locally-isolated equilibrium.

We write $d \log X$ for the differential of an endogenous variable X , which can also be understood as the (infinitesimal) change in an endogenous variable X in response to (infinitesimal) shocks. For example, the supply shocks are $d \log A_i$ and $d \log \bar{L}_f$, and the shocks to the sectoral composition of demand are $d \log \omega_{0j}$. We sometimes write them in vector notation as $d \log A$, $d \log \bar{L}$, and $d \log \omega_0$. Similarly, for discrete changes in a variable, we write $\Delta \log X$.

We proceed in several steps. First, we derive an Euler equation for nominal expenditure which gives changes in current nominal expenditure as a function of changes in the current price index. Second, we derive an aggregation equation which gives changes in output as a function of changes in nominal expenditure and changes in factor income shares. Third and finally, we derive propagation equations which give changes in factor income shares and changes in the price index as a function of changes in nominal expenditure. Putting these steps together gives a complete characterization of local comparative statics.

3.1 Euler Equations

We derive two standard Euler equations, one for output and one for nominal expenditure, both of which will play an important role in the analysis.

Log-linearizing the Euler equation gives changes in output $d \log Y$ as a function of changes in the price index $d \log p^Y$ and the shocks

$$d \log Y = -\rho d \log p^Y + d \log \zeta, \quad (3.1)$$

where

$$d \log \zeta = -\rho \left(d \log(1+i) + \frac{1}{1-\beta} d \log \beta - d \log \bar{p}_*^Y \right) + d \log \bar{Y}_*.$$

With some abuse of terminology, we call $d \log \zeta$ an *aggregate* or *intertemporal demand* shock.¹² A positive aggregate demand shock can come about from a reduction in the

¹²If the crisis lasts for more than one period, the Euler equation can still be used to write output in each period as a function of the price index in that period and exogenous shocks. That is, $\Delta \log Y_t =$

nominal interest rate or the discount factor, or an increase in future prices or output (a proxy for forward guidance).

Equation (3.1) implies that without negative aggregate demand shocks $d \log \zeta = 0$, shocks move output and prices in opposite directions. That is, negative supply shocks or shocks to the sectoral composition of demand are necessarily *stagflationary*. Even without working through the rest of the model, we can already see that negative aggregate demand shocks are necessary in order to produce a reduction in both output and prices.¹³

Changes in nominal expenditure $d \log E$ are similarly given by

$$d \log E = d \log(p^Y Y) = (1 - \rho)d \log p^Y + d \log \zeta. \quad (3.2)$$

Recall that ρ is the intertemporal elasticity of substitution (IES). When $\rho > 1$, increases in prices $d \log p^Y > 0$ reduce nominal expenditure as consumers substitute towards the future. Conversely, when $\rho < 1$, increases in prices $d \log p^Y > 0$ increase nominal expenditure as consumers substitute towards the present. When $\rho = 1$, changes in nominal expenditure are exogenously given by the shocks $d \log E = d \log \zeta$. Although our propositions allow for arbitrary values of ρ , we will focus primarily on the case where $\rho = 1$, abstracting from intertemporal substitution.

3.2 Aggregation Equation for Output

We can express changes in output as a function of changes in nominal expenditure and changes in factor shares.

$-\rho \Delta \log p_t^Y - \rho \left(\sum_{j=1}^T \Delta \log(1 + i_{t+j-1}) + \Delta \log \frac{\beta_t}{\beta_t} - \Delta \log \bar{p}_*^Y \right) + \Delta \log \bar{Y}_*$, where t indexes time and $*$ is the terminal period when the economy recovers. Since this is the only dynamic relationship, the rest of the analysis can be combined with this Euler equation instead to determine output in each period before recovery. This approach is only tenable if the periods are short-lived however, since we assume that the nominal wage constraint is exogenous and does not depend on the length of the recession.

¹³In section 6, we extend the basic model to include some credit-constrained or hand-to-mouth households, and this provides an endogenous mechanism for delivering negative aggregate demand shocks.

Proposition 1. *Changes in output are given by*

$$\begin{aligned}
d \log Y &= \sum_{i \in \mathcal{N}} \lambda_i d \log A_i + \sum_{f \in \mathcal{G}} \lambda_f d \log L_f, \\
&= \underbrace{\sum_{i \in \mathcal{N}} \lambda_i d \log A_i + \sum_{f \in \mathcal{G}} \lambda_f d \log \bar{L}_f}_{\Delta \text{ potential output}} + \underbrace{\sum_{f \in \mathcal{L}} \lambda_f \min \{d \log \lambda_f + d \log E - d \log \bar{L}_f, 0\}}_{\Delta \text{ output gap}}.
\end{aligned}$$

The first expression for $d \log Y$ shows that a version of Hulten's (1978) theorem holds for this economy. In particular, to a first-order, changes in output can only be driven by changes in the productivities $d \log A_i$ weighted by their producer's sales share λ_i , or by changes in the quantities of factors $d \log L_f$ weighted by their income shares λ_f .¹⁴

The second expression uses the fact that while changes in capitals $f \in \mathcal{K}$ are exogenous with $d \log L_f = d \log \bar{L}_f$, changes in labors $f \in \mathcal{L}$ are endogenous with $d \log L_f = d \log \bar{L}_f + \min \{d \log \lambda_f + d \log E - d \log \bar{L}_f, 0\} \leq d \log \bar{L}_f$. Here we have used the observation that factor f is demand constrained with $d \log w_f = 0$ and $d \log L_f = d \log \lambda_f + d \log E$ if, and only if, changes in nominal expenditure on this factor $d \log \lambda_f + d \log E$ are below changes in its potential supply $d \log \bar{L}_f$.

The first term in the second expression is the change in potential output and corresponds to the change in output that would occur in a neoclassical version of the model with flexible wages and full employment of all factors. The second term is the the negative output gap that can open up in the Keynesian version of the model with downward nominal wage rigidities because of Keynesian unemployment in the different factor markets. These Keynesian spillovers depend on endogenous changes in nominal expenditure $d \log E$ and factor income shares $d \log \lambda_f$. It is only through the determination of these endogenous sufficient statistics that the structure of the network and the elasticities of substitution matter.

3.3 Propagation Equations for Shares, Prices, and Factor Employment

We now show how changes in factor income shares $d \log \lambda_f$ are determined. Readers can skip this subsection safely since most of the intuition for the rest of the paper can be gleaned from the special case in Section 3.4.

¹⁴This expression also shows that changes in the sectoral composition of demand within the period $d \log \omega_0$, or changes in aggregate demand $d \log \zeta$, can only change output through changes in the quantities of factors.

We make use of the following notation. For a matrix M , we denote by $M_{(i)}$ its i -th row by $M^{(j)}$ its j -th column. We write $Cov_{\Omega^{(j)}}(\cdot, \cdot)$ to denote the covariance of two vectors of size $1 + \mathcal{N} + \mathcal{G}$ using the j -th row of the input-output matrix $\Omega^{(j)}$ as a probability distribution.

Proposition 2. *Changes in sales and factor shares are given by*

$$d \log \lambda_k = \theta_0 Cov_{\Omega^{(0)}} \left(d \log \omega_0, \frac{\Psi_{(k)}}{\lambda_k} \right) + \sum_{j \in 1+\mathcal{N}} \lambda_j (\theta_j - 1) Cov_{\Omega^{(j)}} \left(\sum_{i \in \mathcal{N}} \Psi_{(i)} d \log A_i - \sum_{f \in \mathcal{G}} \Psi_{(f)} (d \log \lambda_f - d \log L_f), \frac{\Psi_{(k)}}{\lambda_k} \right)$$

almost everywhere, where changes in factor employments are given by

$$d \log L_f = \begin{cases} d \log \bar{L}_f, & \text{for } f \in \mathcal{K}, \\ \min \{ d \log \lambda_f + d \log E, d \log \bar{L}_f \}, & \text{for } f \in \mathcal{L}. \end{cases}$$

We can break down these equations into forward and backward propagation equations. Forward propagation equations describe changes in prices:

$$d \log p_k = - \sum_{i \in \mathcal{N}} \Psi_{ki} d \log A_i + \sum_{f \in \mathcal{G}} \Psi_{kf} (d \log \lambda_f + d \log E - d \log L_f).$$

Changes in prices propagate downstream (forward) through costs. A negative productivity shock $\Delta \log A_i$ to a producer i upstream from k increases the price of k in proportion to how much k buys from i directly and indirectly as measured by Ψ_{ki} . Similarly an increase $d \log w_f = d \log \lambda_f - d \log L_f + d \log E$ in the wage of factor f increases the price of k in proportion to the direct and indirect exposure of k to f .

Backward propagation equations describe changes in sales or factor shares:

$$d \log \lambda_k = \theta_0 Cov_{\Omega^{(0)}} \left(d \log \omega_0, \Psi_{(k)}/\lambda_k \right) + \sum_{j \in 1+\mathcal{N}} \lambda_j (\theta_j - 1) Cov_{\Omega^{(j)}} \left(-d \log p, \Psi_{(k)}/\lambda_k \right).$$

Changes in sales propagate upstream (backward) through demand. The first term on the right-hand side $\theta_0 Cov_{\Omega^{(0)}}(d \log \omega_0, \Psi_{(k)}/\lambda_k)$ on the right-hand side is the direct effect of shocks to the sectoral composition of final demand on the sales of k . These shocks directly increase the share of k if they redirect demand towards goods j that have high direct and indirect exposures to k relative to the rest of the economy as measured by Ψ_{jk}/λ_k to k .

The second term $\sum_{j \in 1+N} \lambda_j (\theta_j - 1) \text{Cov}_{\Omega^{(i)}}(-d \log p, \Psi_{(k)}/\lambda_k)$ on the right-hand side captures the changes in the sales of i from substitutions by producers j downstream from k . If producer j has an elasticity of substitution θ_j below one so that its inputs are complements, then it shifts its expenditure towards those inputs l with higher price increases $d \log p_l$, and this increases the demand for k if those goods l buy a lot from k directly and indirectly relative to the rest of the economy as measured by Ψ_{lk}/λ_k . These expenditure-switching patterns are reversed when θ_j is above one (the inputs of j are substitutes). When θ_j is equal to one (the inputs of j are Cobb-Douglas) these terms disappear.

Combining the backward and forward propagation equations yields Proposition 2. Note that once a factor market f becomes slack, the change in its income share $d \log \lambda_f$ becomes irrelevant for changes in all the other sales and factor shares since they then translate one for one into changes in employment of the factor $d \log L_f$ and leave its wage unchanged with $d \log w_f = 0$.

3.4 A (Somewhat) Universal Example

In Appendix D, we work through some illustrative examples showing how supply and demand shocks can propagate up and down supply chains to cause Keynesian spillovers across sectors. However, for now, we instead focus on a simpler example which will nevertheless prove to contain an element of universality.

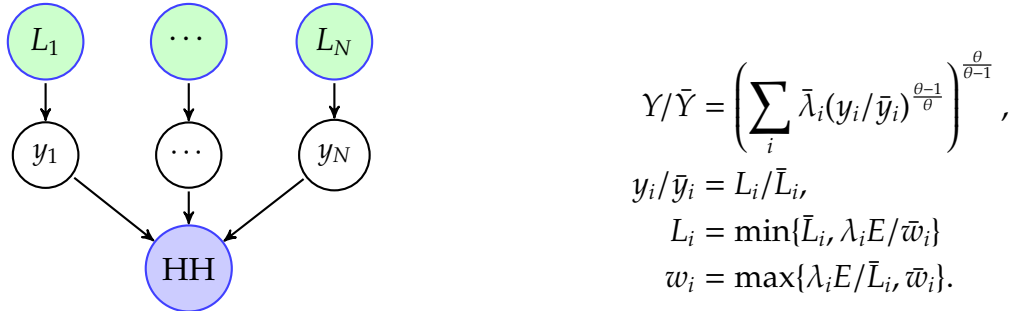


Figure 3.1: Horizontal Economy. The arrows represent the flow of resources for production. Each sector has its own factor market.

Consider the horizontal economy depicted in Figure 3.1. We call it horizontal because there are no intermediate inputs. Each sector produces linearly with its own labor and sells directly to the household who substitutes across them with an elasticity of substitution $\theta < 1$. Labor cannot be reallocated across sectors, and so there are as many labor markets as there are sectors. We therefore refer to a sector or to its labor market interchangeably. The

different labor markets all have downwardly rigid wages ($\mathcal{L} = \mathcal{G}$ and $\mathcal{K} = \emptyset$). We assume that the intertemporal elasticity of substitution is $\rho = 1$. We introduce negative labor shocks $d \log \bar{L}_f \leq 0$ in the different sectors. To start with, suppose that there are neither shocks to the sectoral composition of demand ($d \log \omega_{0j} = 0$) nor aggregate demand shocks ($d \log \zeta = 0$).

Recall that \mathcal{S} and \mathcal{D} are the equilibrium sets of supply- and demand-constrained factors. We give comparative statics for a given \mathcal{S} and \mathcal{D} . We then give conditions for these sets of supply- and demand-constrained factors to indeed arise in equilibrium.

Define the average negative labor shock to the supply-constrained factors

$$d \log \bar{L}_S = \sum_{f \in \mathcal{S}} \frac{\lambda_f}{\lambda_S} d \log \bar{L}_f,$$

where $\lambda_S = \sum_{f \in \mathcal{S}} \lambda_f$. Similarly, define average employment change in the demand-constrained factors

$$d \log L_{\mathcal{D}} = \sum_{f \in \mathcal{D}} \frac{\lambda_f}{\lambda_{\mathcal{D}}} d \log L_f < \sum_{f \in \mathcal{D}} \frac{\lambda_f}{\lambda_{\mathcal{D}}} d \log \bar{L}_f = d \log \bar{L}_{\mathcal{D}},$$

where $\lambda_{\mathcal{D}} = \sum_{f \in \mathcal{D}} \lambda_f$. Keynesian unemployment is given by $d \log L_{\mathcal{D}} - d \log \bar{L}_{\mathcal{D}}$.

By Proposition 2, the change in the share of a factor f is given by

$$d \log \lambda_f = (\theta - 1) \left(\sum_{g \in \mathcal{S}} \lambda_g (d \log \lambda_g - d \log \bar{L}_g) - (d \log \lambda_f - L_f) \right).$$

Summing across all supply-constrained factors and solving the resulting linear equation gives changes in total spending on supply-constrained factors

$$\lambda_S d \log \lambda_S = - \frac{(1 - \theta)(1 - \lambda_S) \lambda_S d \log \bar{L}_S}{1 - (1 - \theta)(1 - \lambda_S)}.$$

This can be used to deduce average changes in employment in the demand-constrained factors

$$\lambda_{\mathcal{D}} d \log L_{\mathcal{D}} = \sum_{f \in \mathcal{D}} \lambda_f d \log L_f = \sum_{f \in \mathcal{D}} \lambda_f d \log \lambda_f = - \sum_{f \in \mathcal{S}} \lambda_f d \log \lambda_f = - \lambda_S d \log \lambda_S.$$

A negative effective supply shock $d \log \bar{L}_S < 0$ increases the shares of the supply-constrained

factors. The shock increases the wages of the supply-constrained factors, which redirects expenditure towards their sectors because of complementarities, which further increases the wages of the supply-constrained factors, etc. *ad infinitum*. Of course, if spending on supply-constrained sectors increases, then spending on demand-constrained sectors decreases, and this reduces employment in those sectors because wages cannot fall.

Using Proposition 1, we can see that Keynesian channels amplify the output effect of the negative supply shocks to the supply-constrained factors since

$$d \log Y = \lambda_S d \log \bar{L}_S + \lambda_D d \log L_D = \frac{\lambda_S d \log \bar{L}_S}{1 - (1 - \theta)(1 - \lambda_S)}. \quad (3.3)$$

The direct impact on output of the negative shock to the supply-constrained factors is given by $\lambda_S d \log \bar{L}_S$, and the amplification of this shock through Keynesian channels is given by the multiplier $1/[1 - (1 - \theta)(1 - \lambda_S)]$. Naturally, amplification is stronger, the lower is the elasticity of substitution $\theta < 1$. Amplification is also stronger when the share of the supply-constrained factors λ_S is low.

We now go back and check that our conjectured set of supply-constrained factors is indeed the equilibrium set of supply-constrained factors. A factor f is demand-constrained in equilibrium if, and only if,

$$d \log \bar{L}_f > \frac{(1 - \theta)\lambda_S d \log \bar{L}_S}{1 - (1 - \theta)(1 - \lambda_S)}.$$

That is, as long as the negative shock to factor f is sufficiently small in magnitude compared to the average shock affecting the supply-constrained part of the economy. This condition is harder to satisfy the smaller is the set of supply-constrained factors λ_S and the higher is the elasticity of substitution θ . In particular, if we had assumed that sectors were substitutes $\theta \geq 1$ instead of being complements with $\theta < 1$, then this condition could not be satisfied and all factors would be supply-constrained.

This condition also shows that Keynesian channels amplify the shock compared to the neoclassical economy with flexible wages since

$$d \log Y = \underbrace{\lambda_S d \log \bar{L}_S + \lambda_D d \log \bar{L}_D}_{\Delta \text{ potential output}} + \underbrace{\sum_{f \in \mathcal{D}} \lambda_f \left(\frac{(1 - \theta)\lambda_S d \log \bar{L}_S}{1 - (1 - \theta)(1 - \lambda_S)} - d \log \bar{L}_f \right)}_{\Delta \text{ output gap}},$$

where the Keynesian effect is always negative. Note that here, as in Proposition 1, Keynesian amplification is defined relative to the output reduction that would take place in response to the negative factor shocks *to all the factors* in the neoclassical economy. This notion is different from the Keynesian amplification of the negative supply shocks to the supply-constrained factors, used for equation 3.3. The latter is defined relative to the output reduction that would take place in response to the negative shocks *to the supply-constrained factors* in a neoclassical economy. This is informative because the negative supply shock to the demand-constrained factors have no impact on the equilibrium because these markets are slack.

If in addition to the negative labor shocks, there were also shocks to the sectoral composition of demand $d \log \omega_0$ and to aggregate demand $d \log \zeta < 0$, then the response of output would become

$$d \log Y = \frac{\lambda_S d \log \bar{L}_S}{1 - (1 - \theta)(1 - \lambda_S)} - \frac{\theta \lambda_S d \log \omega_{0S}}{1 - (1 - \theta)(1 - \lambda_S)} + \left(1 - \frac{(1 - \theta)\lambda_S}{1 - (1 - \theta)(1 - \lambda_S)}\right) (1 - \lambda_S) d \log \zeta,$$

where $d \log \omega_{0S} = \sum_{f \in S} (\lambda_f / \lambda_S) d \log \omega_{0f}$. The second term on the right-hand side captures the fact that if consumers redirect expenditure towards supply-constrained factors and away from demand-constrained factors, then this exacerbates Keynesian unemployment in demand-constrained factors and further reduces output. The third term is the effect of the negative aggregate demand shock. The direct effect of the negative aggregate demand shock, captured by $(1 - \lambda_S) d \log \zeta$, is to lower employment in slack factor markets and to reduce output. This direct effect is mitigated because the shock lowers the prices of tight factors, bringing them closer to those of slack factors, and triggering expenditure switching away from supply-constrained factors and towards demand-constrained ones, as captured by $-[(1 - \theta)\lambda_S / (1 - (1 - \theta)(1 - \lambda_S))](1 - \lambda_S) d \log \zeta$.

4 A Benchmark with Simple Network Sufficient Statistics

Proposition 2 shows that in general, detailed information about the input-output network is required to compute counterfactuals. However, we now show that in a benchmark case, this information can be summarized by a small number of sufficient statistics, namely the initial factor income shares. Of course, the disaggregated nature of the model remains critical because the different factor markets endogenously experience different cyclical conditions.

This benchmark case with simple network sufficient statistics is useful for several reasons. First, it shows that the analysis in Example 3.4 applies much more broadly to economies with complex input-output networks. Second, it clarifies exactly what ingredients are necessary for the production network to matter beyond the initial factor shares. Third, it allows us to obtain not only local but also global comparative statics.

To obtain the benchmark, we assume that the intertemporal elasticity of substitution is $\rho = 1$, that all the elasticities in production and in final demand are the same with $\theta_j = \theta$ for all $j \in 1 + \mathcal{N}$, and that there are no productivity shocks $\Delta \log A = 0$.¹⁵ Write changes in output $\Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega})$ as a set-valued function of discrete labor endowment shocks $\Delta \log \bar{L}$, aggregate demand shocks $\Delta \log \zeta$, and the initial input-output matrix $\bar{\Omega}$. We use Δ to denote discrete global changes to distinguish them from infinitesimal local changes which we denote with d . For now, assume that the household's intersectoral preferences are held constant $\Delta \log \omega_0 = 0$. We show how the results extend to shocks to the sectoral composition of final demand in Section 4.7.

4.1 Global Sufficient Statistics

The next proposition shows that Y depends on the input-output network $\bar{\Omega}$ *only* through the initial factor shares $\bar{\lambda}_f$ for $f \in \mathcal{G}$.

Proposition 3 (Global Sufficient Statistics). *Suppose that the intertemporal elasticity of substitution is $\rho = 1$ and that the elasticities of substitution in production and in final demand are all the same with $\theta_j = \theta$ for every $j \in 1 + \mathcal{N}$. Suppose that there are only factor supply shocks $\Delta \log \bar{L}$ and aggregate demand shocks $\Delta \log \zeta$ but no productivity shocks and no shocks to the sectoral composition of demand. Then*

$$\Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}) = \Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}')$$

for every $\bar{\Omega}$ and $\bar{\Omega}'$ as long as $\bar{\lambda}_f = \bar{\Psi}_{0f} = \bar{\Psi}'_{0f} = \bar{\lambda}'_f$ for every $f \in \mathcal{G}$. More generally, given the shocks, the initial factor income shares $\bar{\lambda}_f$ are sufficient statistics for equilibrium changes in aggregate output $\Delta \log Y$, the aggregate price index $\Delta \log p^Y$, factor wages $\Delta \log w_f$, factor quantities $\Delta \log L_f$, and factor income shares $\Delta \log \lambda_f$.

An implication is that the local comparative statics for the horizontal economy in Section 3.4 actually apply much more generally. In particular, they apply to *any* production

¹⁵In Appendix B, we show how this section can be extended to a version of the model with investment.

network as long as the elasticities of substitution in production and final demand are uniform. Another implication of this proposition is that the network can only matter globally beyond the initial factor shares if: the elasticities of substitution are different or if there are shocks to the sectoral composition of demand or to productivities.

We prove Proposition 3 by showing that the equilibrium conditions do not depend on the input-output matrix $\bar{\Omega}$ beyond the initial factor shares $\bar{\lambda}$. First, note that real GDP is given by deflating changes in nominal GDP by the price index

$$\Delta \log Y = \Delta \log \zeta - \Delta \log p^Y.$$

Next, we can show that changes in the price index depend only on the changes in the wages of different factors, since every good is ultimately made up of factors, so that

$$\Delta \log p^Y = \frac{1}{1-\theta} \log \left(\sum_{f \in \mathcal{G}} \bar{\lambda}_f \exp \left((1-\theta) \Delta \log w_f \right) \right). \quad (4.1)$$

Finally, the wage for each factor is determined by the interaction of factor supply and demand, and factor demand can be shown to be isoelastic with elasticity θ in the price of each factor relative to the GDP deflator, so that

$$\Delta \log w_f = \begin{cases} \frac{1}{\theta} (\Delta \log \zeta - \Delta \log \bar{L}_f) + \frac{\theta-1}{\theta} \Delta \log p^Y, & \text{for } f \in \mathcal{K}, \\ \max \left\{ \frac{1}{\theta} (\Delta \log \zeta - \Delta \log \bar{L}_f) + \frac{\theta-1}{\theta} \Delta \log p^Y, 0 \right\}, & \text{for } f \in \mathcal{L}. \end{cases}$$

Taken together, these equations pin down which factor markets are endogenously demand-constrained, what wages are in supply-constrained factor markets, and hence what the GDP deflator and real GDP are in equilibrium. Since these equations do not depend on $\bar{\Omega}$ beyond the initial factor income shares, this proves the result.

4.2 Lattice Structure and Global Comparative Statics

In general, the equilibrium of the Keynesian model is not unique. However, for our benchmark case with uniform elasticities, we can prove there are simple-to-compute unique “best” and “worst” equilibria as long as there are complementarities ($\theta < 1$). We can also provide global comparative statics for these equilibria.

To state our result, we endow $\mathbb{R}^{\mathcal{G}}$ with the partial ordering $x \leq y$ if and only if $x_f \leq y_f$ for all $f \in \mathcal{G}$. Formally, we show that set of equilibrium values of the changes in factor

quantities $\Delta \log L$ is a complete lattice under the partial ordering \leq .

Proposition 4. *Under the assumptions of Proposition 3, and assuming in addition that $\theta < 1$, there is a unique best and worst equilibrium: for any other equilibrium, $\Delta \log Y$ and $\Delta \log L$ are lower than at the best and higher than at the worst. Furthermore, both in the best and in the worst equilibrium, $\Delta \log Y$ and $\Delta \log L$ are increasing in $\Delta \log \bar{L}$ and in $\Delta \log \zeta$. On the other hand, both in the best and worst equilibrium, $\Delta \log p^Y$ is decreasing in $\Delta \log \bar{L}$ and increasing in $\Delta \log \zeta$.*

The global comparative static result in the proposition generalize the insight of the horizontal economy in Section 3.4. In particular, negative labor shocks in some factor markets raise the overall price level and create Keynesian unemployment in other factor markets. On the other hand, negative aggregate demand shocks can create Keynesian unemployment whilst lowering the overall price level.

Proposition 4 also provides a straightforward way to compute this best equilibrium using a greedy algorithm along the lines of Vives (1990) or, more recently, Elliott et al. (2014). We can find the best equilibrium as follows. Solve the model assuming all factor markets are supply-constrained. If one of the wages is below the minimum, call this market demand-constrained and set its wage equal to its lower bound. Recompute the equilibrium assuming that these factor markets are demand-constrained. Continue in this manner until the wage in every candidate supply-constrained market is above its lower bound. The worst equilibrium can be found in the same way but starting from the assumption that all markets are demand-constrained, and checking at every step if a priori demand-constrained markets have employments above their labor endowments.

4.3 AS-AD Representation

We can represent the best equilibrium as the point at which an aggregate supply and aggregate demand curve intersect. The AD curve, which is a decreasing log-linear relationship, is given by the Euler equation, and relates aggregate output to the price level today. Deriving the AS curve is less straightforward. To do so, fix some level of output Y . There is a price level $p^Y(Y)$ such that: given the implied level of expenditure $E(Y) = p^Y(Y)Y$, the wage of every factor is consistent with the amount of expenditures on that factor; and these wages give rise to prices that are consistent with $p^Y(Y)$ in (4.1).

An example is plotted in Figure 4.1 at the initial equilibrium in the absence any exogenous shock. The downward slope of the left-side of the AS curve depends on the downward flexibility of factor prices. If the set of capitals is empty ($\mathcal{K} = \emptyset$), then the

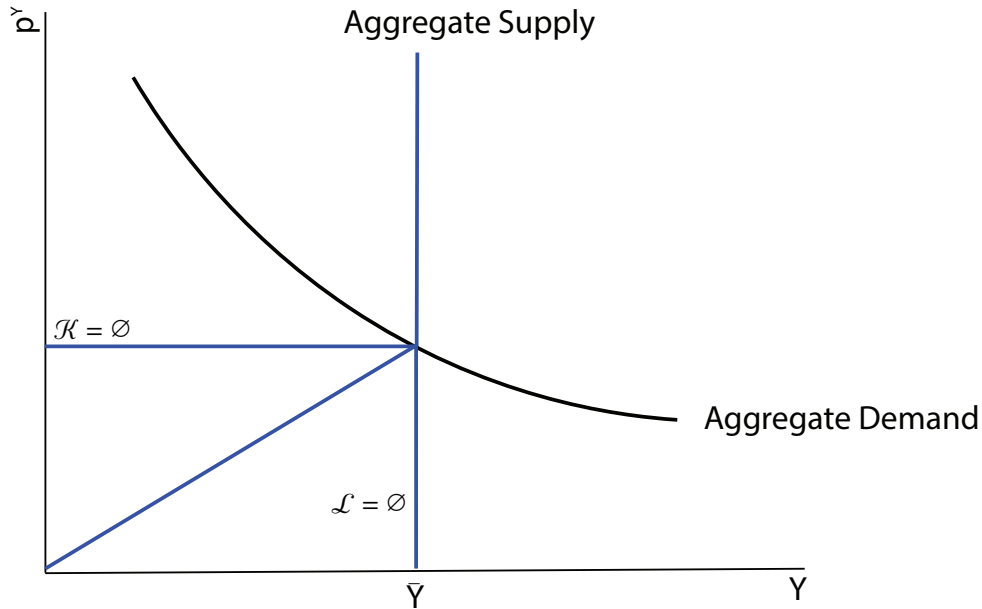


Figure 4.1: AS-AD representation of the equilibrium without shocks. The $\mathcal{K} = \emptyset$ case is when all factors have downwardly rigid wages, and $\mathcal{L} = \emptyset$ case is when all factors have flexible wages.

AS curve is horizontal to the left. If the set of labors is empty ($\mathcal{L} = \emptyset$), then the AS curve is vertical to the left. Of course, in the case when there are no potentially-sticky factor markets, we recover the neoclassical model.

Aggregate demand shocks $d \log \zeta$ shift the AD curve in the usual way, and it is easy to see from this figure that a negative aggregate demand shock reduces present prices and output. The AS curve, on the other hand, does not have a simple closed-form representation, and supply shocks transform the shape of the AS curve in non-obvious ways. In the next few subsections, we use nonlinear AS-AD diagrams to illustrate how different shocks interact with complementarities to affect output and inflation.

4.4 Keynesian Amplification with Complementarities

As discussed earlier, complementarities across producers can transmit negative supply shocks in one factor market as negative demand shocks to other factor markets. This negative spillover is larger, the stronger are the complementarities. In other words, the amount of Keynesian unemployment in the demand-constrained factor markets is decreasing as a function of the elasticity of substitution θ .

In Figure 4.2, we plot an example for a uniform-elasticity economy with two equally-

sized factor markets. Both factors are labors and have downwardly rigid wages. There are no capitals ($\mathcal{K} = \emptyset$). We feed a 20% negative shock the supply of one of the factors. When there are complementarities ($\theta < 1$), the negative supply shock in one factor market causes the downward nominal wage rigidity to bind and triggers Keynesian unemployment in the other factor market. By contrast, with substitutability ($\theta \geq 1$), the downward nominal wage rigidity constraint does not bind in any of the two factor markets and the model behaves exactly like the neoclassical model with flexible wages.

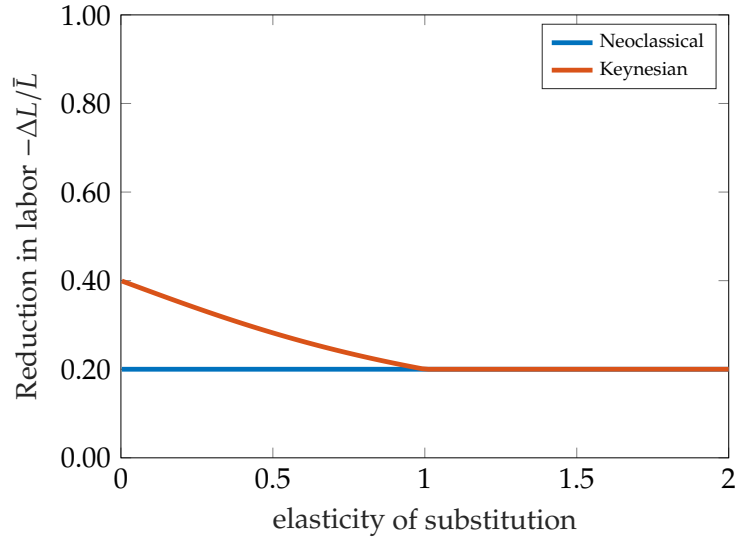


Figure 4.2: The change in the quantity of labor supplied in the neoclassical (flexible wages) and Keynesian (downwardly rigid wages) example as a function of the elasticity of substitution.

However, the strength of this effect on output is hump-shaped in the elasticity of substitution. In Figure 4.3, we plot the change in output in the Keynesian model with downward wage rigidity against the response of the neoclassical model with flexible wages. As we already discussed, the behavior of output in the two models coincides when $\theta \geq 1$ but diverges as soon as $\theta < 1$. However, the behavior of the two models coincides again as θ approaches zero.

Intuitively, as complementarities become stronger, the marginal product of the demand-constrained factor falls more. Output is more and more determined by the productive capacity of the negatively shocked supply-constrained factor. In other words, as complementarities become stronger, the income share of the non-shocked demand-constrained factor falls more in response to the negative shock to the supply-constrained factor, and, as a result, the demand-constrained factor becomes less critical and its Keynesian un-

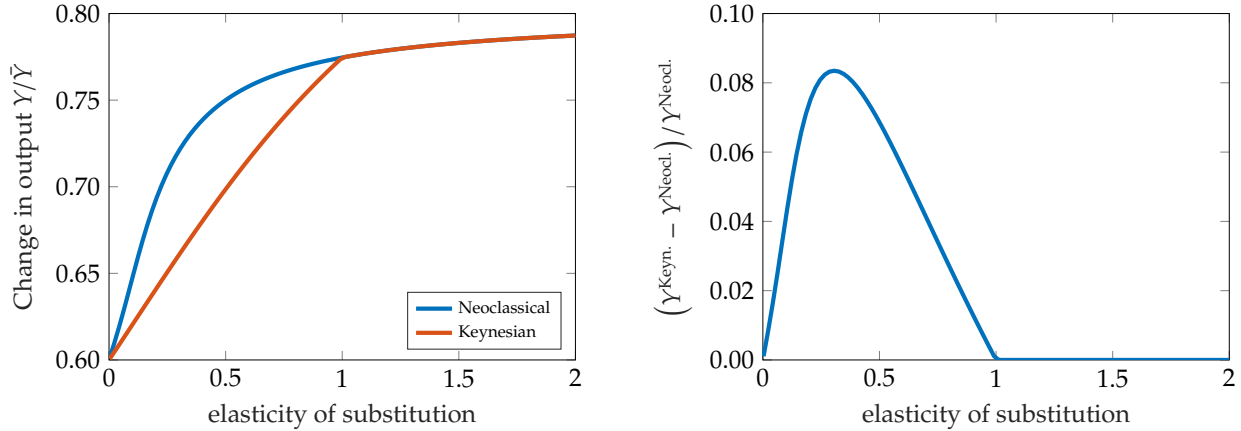


Figure 4.3: The panel on the left shows the change in output, in a neoclassical (flexible wages) and Keynesian (downwardly rigid wages) example, in response to a reduction in one sector’s labor as a function of the elasticity of substitution. The panel on the right shows the percentage difference between the neoclassical and Keynesian models.

employment matters less for output. In particular, in the perfect complement limit, the unemployed workers in the demand-constrained factor market have a marginal product of zero, and so their loss is irrelevant for output.¹⁶

Figure 4.4 represents this negative supply shock using an AS-AD diagram. The AS curve is horizontal to the left since there are no capitals ($\mathcal{K} = \emptyset$). The initial level of output is given by \bar{Y} and the new level of labor available in the shocked sector is given by \bar{L}'_j . The negative supply shock shifts the AS curve to the left.

In the figure, we draw the new AS curve for different values of the elasticity of substitution θ . Unlike standard models, in this model, the shape of the AS curve itself changes in response to supply shocks. In particular, the negative supply shock introduces two kinks into the AS curve. The first kink is the point at which the AS curve becomes horizontal, and the second kink is the point at which the AS curve becomes vertical. The first kink always occurs at the point where $Y = \bar{L}'_j$. Intuitively, this is the level of aggregate output that would cause the shocked sector itself to become demand constrained. The second kink, on the other hand, moves as we vary the elasticity of substitution.

As we lower the elasticity of substitution θ , the kink point at which the AS curve

¹⁶The non-monotonic pattern in Figure 4.3 does not show in a linear approximation, and so does not appear in equation (3.3). Intuitively, as the negative supply shock gets smaller, the hump in Figure 4.3 moves towards the left and is pressed up against the axis, and so the amplification of output reductions is increasing in the degree of complementarities over a bigger range. In the limit of infinitesimal shocks, this range becomes complete.

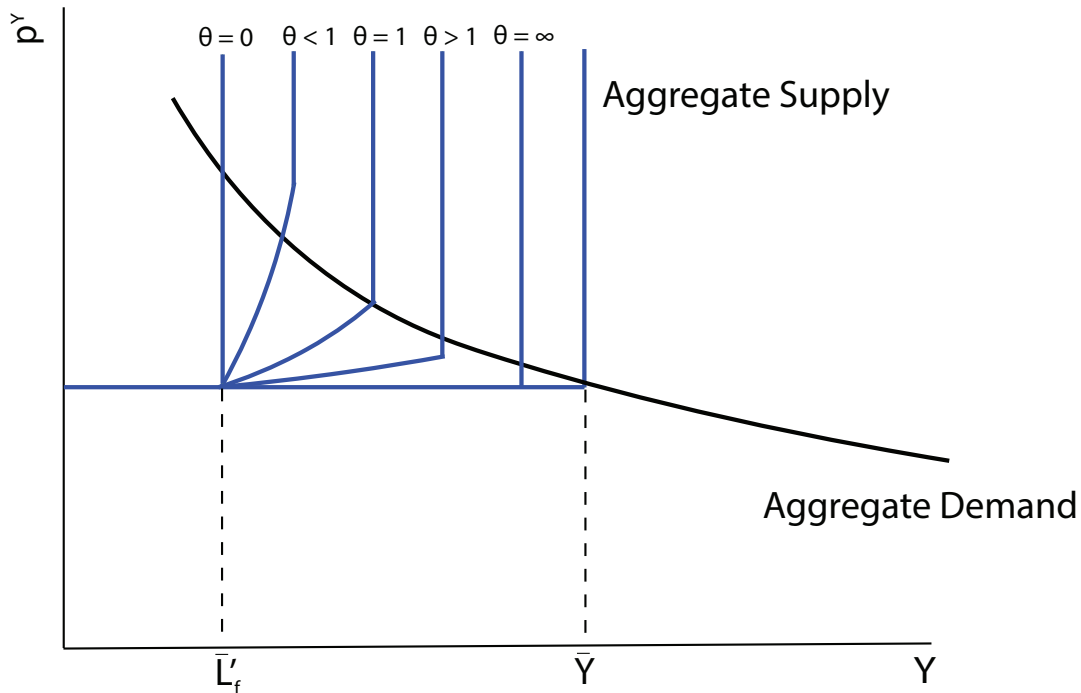


Figure 4.4: The effect of the same negative supply shock to a factor for different values of the elasticity of substitution θ .

becomes vertical shifts north-westwards. As long as $\theta > 1$, the second kink is below the AD curve, and so the equilibrium is the same as the neoclassical one, because the AS and AD intersect along the neoclassical portion of the AS curve. Intuitively, when $\theta > 1$, no factor market becomes demand-constrained and so downward nominal wage rigidity is never triggered. Once the elasticity of substitution has been lowered to $\theta = 1$, the Cobb-Douglas case, the second kink exactly intersects the AD curve. As θ goes below one, the second kink moves above the AD curve, downward nominal wage rigidities are triggered, and the equilibrium has lower output and higher inflation than the neoclassical model. Finally, as θ goes to zero and we approach the Leontief case, the second kink point moves directly above the first kink point, and so the reduction in output in the neoclassical model and Keynesian model become the same again.

4.5 Keynesian Amplification with Heterogeneous Shocks

Next, we consider how heterogeneity in the size of the shock affects the equilibrium. In Figure 4.5, we consider the same example as in Section 4.4, but we now allow for negative supply shocks in both factor markets.

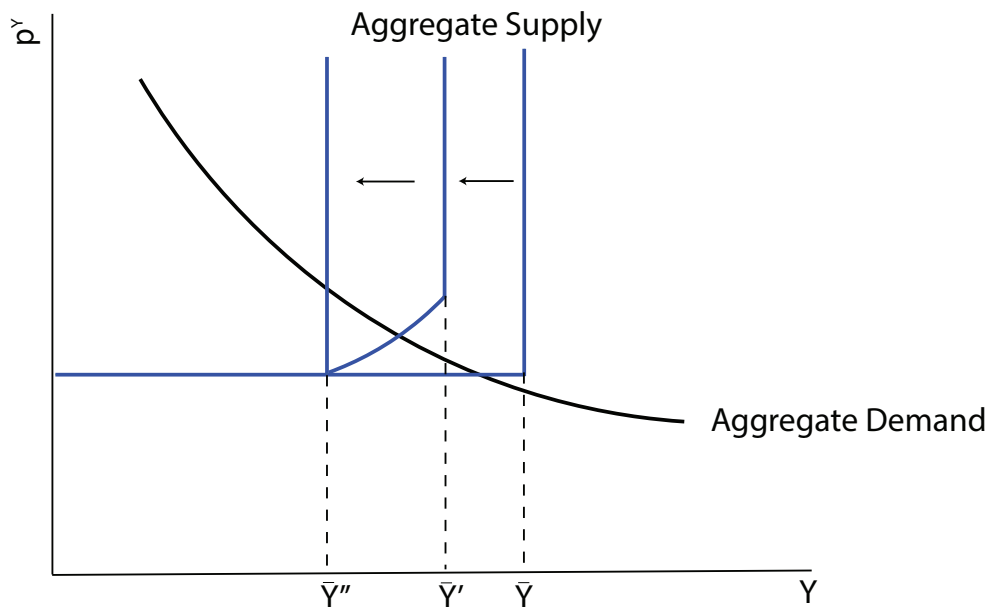


Figure 4.5: Negative supply shocks in a two-sector model, \bar{Y} is output without any shocks, \bar{Y}' is potential output with shocks to only one sector, and \bar{Y}'' is potential output with shocks to both sectors.

First consider the case where there is only a negative supply shock to one of the factors. The shock shifts the AS curve back and introduces two kinks. It results in Keynesian unemployment and a reduction in output over and above the reduction in potential \bar{Y}' .

Now, consider the case where there is also a negative supply shock of the same magnitude to the other factor market, so that the negative supply shock is now uniform across the two factor markets. The kink disappears, output falls to its potential level \bar{Y}'' , and there are no longer any Keynesian forces in the model: downward nominal wage rigidities do not bind in any factor market, there is no Keynesian unemployment, and there is no Keynesian amplification of output reductions. Once again, this is because the first and second kink are now directly on top of each other.

The lesson is that we should expect Keynesian spillovers from negative factor supply shocks to be stronger when the shocks are more heterogeneous. If the negative supply shocks are more homogeneous, then it is less likely that supply outstrips demand in any factor market. Indeed, when the shock uniformly affects all factor markets together, then relative factor prices do not change, all factor prices increase, and the nominal rigidities are never triggered.

Covid-19 plausibly caused a heterogeneous shock to supply, since it affected potential

labor in some sectors much more severely than in others. Whereas many white-collar jobs can be done at home, most blue-collar work require workers to work in close proximity to each other and to their clients.¹⁷ This means that lock-downs, social-distancing, and liability considerations disproportionately affect some sectors, and the more heterogeneous are these effects, the more likely they are to trigger Keynesian unemployment.

4.6 Interaction of Supply Shocks and Aggregate Demand Shocks

Next, we show how negative supply shocks and aggregate demand shocks interact with one another. In Figure 4.6, we show how the equilibrium responds to a negative supply shock together with a negative aggregate demand shock, assuming there are complementarities. We deviate from the example of Section 4.4 by allowing for more than two factors, and by allowing for labors with downwardly rigid wages and capitals with flexible wages ($\mathcal{K} \neq \emptyset$).

As expected, the negative aggregate demand shock shifts the AD curve backwards. If there are no supply shocks, then aggregate demand shocks are potent, causing output to fall by a lot. If there are some capitals ($\mathcal{K} \neq \emptyset$), then the aggregate demand shock can also reduce prices a lot.

Now, consider what happens if the negative aggregate demand shock coincides with negative supply shocks. As usual, the negative supply shock introduces kinks into the AS curve and shifts the curve backwards. In equilibrium, the effect of the negative AD shock is now much less potent for output. In fact, in the extreme case where the first and second kink are on top of each, the negative aggregate demand shock has no effect on output unless it is very large. However, even though the negative supply shock blunts the importance of aggregate demand for output, aggregate demand shocks remain critical for the determination of prices. In particular, aggregate demand shocks reduce inflation, and without them, it is impossible to deliver a reduction in output without inflation.

Figure 4.6 anticipates our finding that matching the data, which features large reductions in employment and muted movements in inflation, will require a combination of both supply and demand shocks.

¹⁷See for example Mongey et al. (2020).

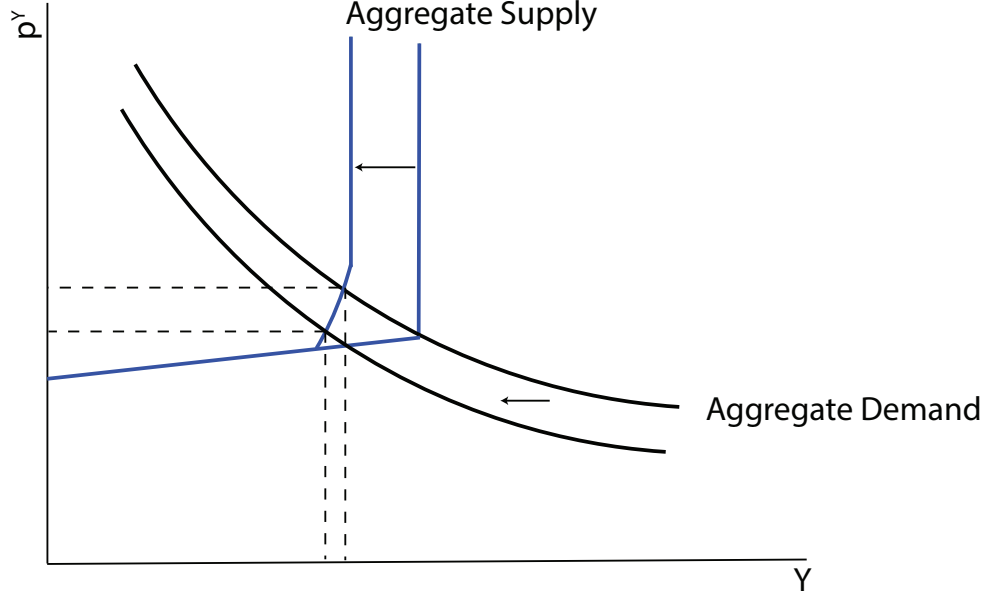


Figure 4.6: Negative supply shock coupled with a negative aggregate demand shock.

4.7 Shocks to the Sectoral Composition of Demand

So far, we abstracted away from shocks to the sectoral composition of final demand $\Delta \log \omega_0$. However, building on Baqaee (2015), our sufficient statistics approach can be extended to cover these shocks as well.

For each factor $f \in \mathcal{G}$, we translate the changes in the final expenditure share *parameters* into changes of factor income share *parameters* by defining

$$\Delta \log \bar{\lambda}_f = \sum_{j \in \mathcal{N}} \bar{\Omega}_{0j} \exp(\Delta \log \omega_{0j}) \bar{\Psi}_{jf}.$$

These changes in factor income share parameters are *not* the *equilibrium* changes in factor income shares, but they are useful because they encode how shocks to final demand propagate backward (upstream) to affect the demand for the different factors. They depend on the network-adjusted factor intensities $\bar{\Psi}_{jf}$ of the different sectors $j \in \mathcal{N}$ for the different factors $f \in \mathcal{G}$, which measure how much each sector j uses each factor f directly and indirectly through its supply chain.

In fact, as we shall see below, these changes in the factor income share parameters $\Delta \log \bar{\lambda}_f$, together with the initial factor income shares $\bar{\lambda}_f$, are global sufficient statistics for the response of the equilibrium to the shocks. Compared to the situation without

shocks to the sectoral composition of demand, the list of network sufficient statistics must therefore be expanded beyond the initial factor income share. In other words, with shocks to the sectoral composition of demand, we need to know more information about the network than without these shocks, but this information can still be summarized by simple sufficient statistics.¹⁸

The extension of the results of Section 4.1 is the following:¹⁹

$$\Delta \log Y = \Delta \log \zeta - \Delta \log p^Y,$$

$$\Delta \log p^Y = \frac{1}{1-\theta} \log \left(\sum_{f \in \mathcal{G}} \bar{\lambda}_f \exp \left((1-\theta) \Delta \log w_f \right) \right),$$

$$\Delta \log w_f = \begin{cases} \frac{1}{\theta} (\Delta \log \bar{\lambda}_f + \Delta \log \zeta - \Delta \log \bar{L}_f) + \frac{\theta-1}{\theta} \Delta \log p^Y, & \text{for } f \in \mathcal{K}, \\ \max \left\{ \frac{1}{\theta} (\Delta \log \bar{\lambda}_f + \Delta \log \zeta - \Delta \log \bar{L}_f) + \frac{\theta-1}{\theta} \Delta \log p^Y, 0 \right\}, & \text{for } f \in \mathcal{L}. \end{cases}$$

We can use these results to prove global comparative statics as in Section 4.2. For example, starting at an initial equilibrium with no shocks, changes in the sectoral composition of demand will cause factor market $f \in \mathcal{L}$ to become demand-constrained if, and only if, the network-adjusted demand shock to that factor is negative so that $\Delta \log \bar{\lambda}_f = \sum_{j \in \mathcal{N}} \bar{\Omega}_{0j} \exp(\Delta \log \omega_{0j}) \bar{\Psi}_{jf} < 0$.

4.8 Benefits of Wage Flexibility and of Factor Reallocation

We end this section with two propositions: that wage flexibility and factor reallocation are desirable. These two propositions may at first seem obvious, but they are by no means universally valid. Since the model with nominal rigidities is inefficient, the theory of the second best means that seemingly desirable attributes like flexibility and reallocation can actually turn out to be harmful in general. However, to the extent that the benchmark case with uniform elasticities is likely to be realistic, then these propositions guarantee that neoclassical intuitions about flexibility and reallocation are still empirically relevant.

¹⁸Since $\sum_{f \in \mathcal{G}} \bar{\Psi}_{jf} = 1$ for all $j \in \mathcal{N}$, we only need to keep track of $\mathcal{N}(\mathcal{G} - 1)$ additional sufficient statistics to conduct comparative statics.

¹⁹Here $\Delta \log Y$ should be interpreted as the change in the consumption quantity index and $\Delta \log p^Y$ as the change in the corresponding ideal price index. These notions correspond to the changes in welfare and in a welfare price index but not to changes in real GDP and the GDP deflator as they are measured in the data. The latter can be computed as path-integrals and they only coincide with the former to the first order of approximation. These distinctions are irrelevant for changes in disaggregated variables such as wages or employments of the different factors.

To show that wage flexibility is desirable, we take a factor $f \in \mathcal{L}$ and remove its downward wage rigidity constraint by moving it to \mathcal{K} . This amounts to creating a more flexible economy.

Corollary 5. *Under the assumptions of Proposition 3 at the best equilibria, $\Delta \log Y$ and $\Delta \log L$ are higher in the more flexible than the less flexible economy.*

In addition to the fact that flexibility is desirable, we can also prove that reallocation is desirable. We consider two factors h and h' that are paid the same wage at the initial equilibrium and that have the same minimum nominal wage. The idea is that these two factors are really the same underlying factor, but that frictions to reallocation prevent them from being flexibly reallocated one into the other. We then consider a reallocation economy where these reallocations are allowed to take place. This amounts to a renormalization of the input-output matrix and of the shocks.

Corollary 6. *Under the assumptions of Proposition 3, the best equilibrium of the no-reallocation economy has lower output $\Delta \log Y$ and employment $\Delta \log L$ than the best equilibrium of the reallocation economy.*

5 Quantitative Application

We now turn to quantifying the model. Although the quantitative model does not have uniform elasticities, the intuitions that we developed in the context of this benchmark case will prove useful in understanding the results. We calibrate our model to match the peak to trough reductions in employment from February 2020 to April 2020.

5.1 Setup

We start by describing our calibration of the model and of the shocks.

Calibrating the economy. There are 66 sectors and sectoral production functions use labor, capital, and intermediates. The share parameters of the functions are calibrated so that at the initial pre-shock allocation, expenditure shares match those in the input-output tables from the BEA. We focus on the short run and assume, following Baqaee and Farhi (2019), that labor and capital cannot be reallocated across sectors. We construct the input-output matrix using the 2015 annual U.S. input-output data from the BEA, dropping the

government, non-comparable imports, and second-hand scrap industries. The dataset contains industrial output and inputs for 66 industries.

The economy has a nested CES form, and the nesting structure is the following. In each sector, labor and capital are combined with elasticity η , intermediates are combined with elasticity θ , intermediates and value-added are combined with an elasticity ϵ , and final output from different sectors are combined with an elasticity σ to form final demand. In other words, we allow for differences in the elasticities of substitution, but we do not allow them to vary by sector, because such disaggregated estimates are not available.

Based on the empirical literature, we set the elasticity of substitution between labor and capital to be $\eta = 0.5$, between value-added and intermediate inputs to be $\epsilon = 0.6$, across intermediates to be $\theta = 0.2$. We set the elasticity of substitution across final uses to be $\sigma = 1.0$. These numbers are broadly in line with Atalay (2017), Herrendorf et al. (2013), Oberfield (2013), and Boehm et al. (2019). We show how our results depend on these parameters. We assume that sectoral labor markets feature downward nominal wage rigidity, whereas sectoral capital markets have flexible rental rates. Prices are set competitively and flexibly.

Calibrating the shocks. Covid-19 set off an array of supply and demand shocks. Identifying these shocks would be challenging even with accurate disaggregated data. This difficulty is compounded by the fact that quality disaggregated data is not yet available. We model the Covid-19 crisis using a combination of shocks to potential labor supplies and shocks to the sectoral composition of demand across sectors and across time periods. We begin by describing how we calibrate demand shocks, and then describe how we calibrate supply shocks.

Since both the intertemporal ρ and intersectoral σ elasticities of substitution are equal to one for the household, realized changes in household spending patterns can be directly fed into the model as demand shocks. In particular, changes in the sectoral composition of household spending across different sectors give us the primitive shocks to the sectoral composition of demand. Furthermore, the change in nominal GDP gives us the primitive shock to aggregate demand, since households expect the economy to recover in the future with stable (pre-shock) prices.

The last set of shocks that remain to be specified are the primitive potential labor supply shocks. In principle, if our model is perfectly correctly specified, we can directly feed changes in hours by sector as the primitive supply shocks. This is because if a labor

market is supply constrained, then the only way to match hours in that market is via a reduction in potential employment. On the other hand, if a labor market is demand constrained and has Keynesian unemployment, then any reduction in potential labor supplied up to the realized reduction in hours will have no effect on any outcome. There is therefore some ambiguity as to how large the supply shocks are in these markets which could be anywhere between zero and the observed reduction in hours. We resolve this ambiguity by setting these shocks to zero.²⁰

We describe our data sources for the primitive supply and demand shocks. The primitive demand shock to the sectoral composition of demand are given by the April 2020 release of personal consumption expenditures from the BEA. Since personal consumption is about 80% of final demand (net of government sectors), we downweight these shocks by around 80%. This is equivalent to assuming that the sectoral composition of other components of final demand has not changed. The primitive demand shock to the intertemporal composition of demand (aggregate demand) is chosen to match the 13% reduction in nominal GDP, projected by the CBO, for the second quarter of 2020. To calibrate the primitive supply shocks, we compute changes in hours worked by sector from the April 2020 BLS Economic News release. Figure 5.1 shows the sectoral supply and demand shocks. Figure A.1 shows reduction in hours by sector in the model and classifies the demand- and supply-constrained sectors.

Of course, since our model is quite stylized, with only a few elasticities of substitution (not chosen to fit the data), this procedure results in a reasonable but imperfect fit to the employment data. In the baseline calibration, the size-weighted average absolute error in hours by sector is 2.4%. Having calibrated the model, we show predicted changes in macro aggregates, decompose the importance of supply and demand shocks, and discuss the role of elasticities of substitution.

5.2 Role of Supply and Demand Shocks

Figure 5.2 displays the baseline calibration of the model as well as versions with only supply or only demand shocks. The baseline model includes both the negative supply and negative demand shocks. To better analyze the results, in displaying the baseline, we impose the a negative 13% aggregate demand shock but we scale the sectoral shocks

²⁰This choice does not matter for our baseline in terms of aggregate and sectoral output, inflation, and employment but it maximizes the amount of Keynesian unemployment. This choice also affects our counterfactual with only supply shocks.

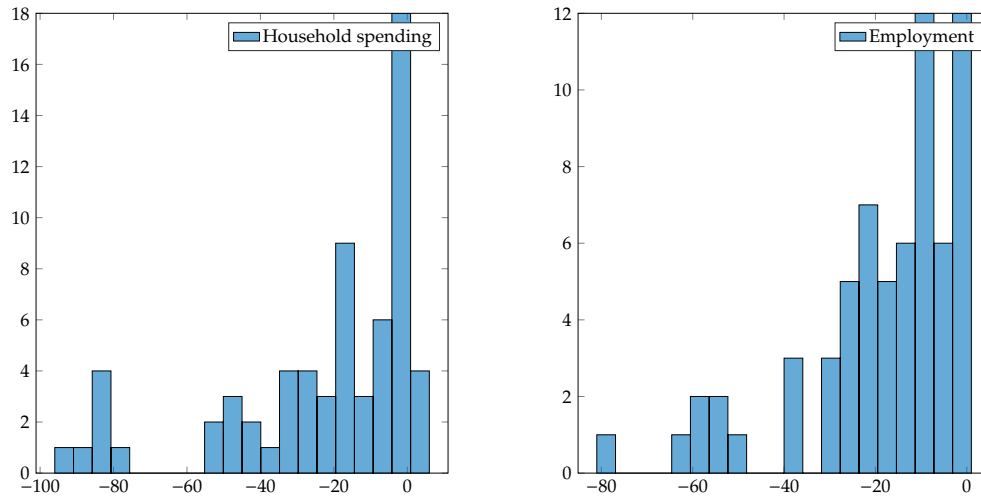


Figure 5.1: Percentage reduction in nominal household spending (left panel) and hours worked (right panel) by sector from February to April 2020.

(sectoral supply shocks and shocks to the sectoral composition of demand) along the x-axis by the “size of sectoral shocks” variable. When “size of sectoral shocks” is zero, there are no sectoral shocks, and there is only the aggregate demand shock. When “size of sectoral shocks” is one, the sectoral shocks are fully scaled at their calibrated values and they interact with the aggregate demand shock.

In each panel, we also plot the outcome with only the supply shocks or only the demand shocks. The “Supply” features only the negative sectoral supply shock whereas the “Demand” line features only the aggregate demand shock and the shock to the sectoral composition of demand.

Real GDP. Figure 5.2a shows that when “size of sectoral shocks” is zero, the -13% aggregate demand shock leads to a 5.5% reduction of real GDP in the baseline model; when “size of sectoral shocks” is one, the combination of supply and demand shocks reduce real GDP by 13% . Demand shocks, on their own, reduce real GDP by around 7% . On the other hand, supply shocks, on their own, reduce real GDP by around 11% .

Because of nonlinearities, the cumulative effects of the shocks on real GDP is not the sum of the effects of the different shocks. By implication, decomposing the reductions in real GDP into the effects of the different shocks depends on the order in which the decomposition is performed. For example, starting with aggregate demand shocks and

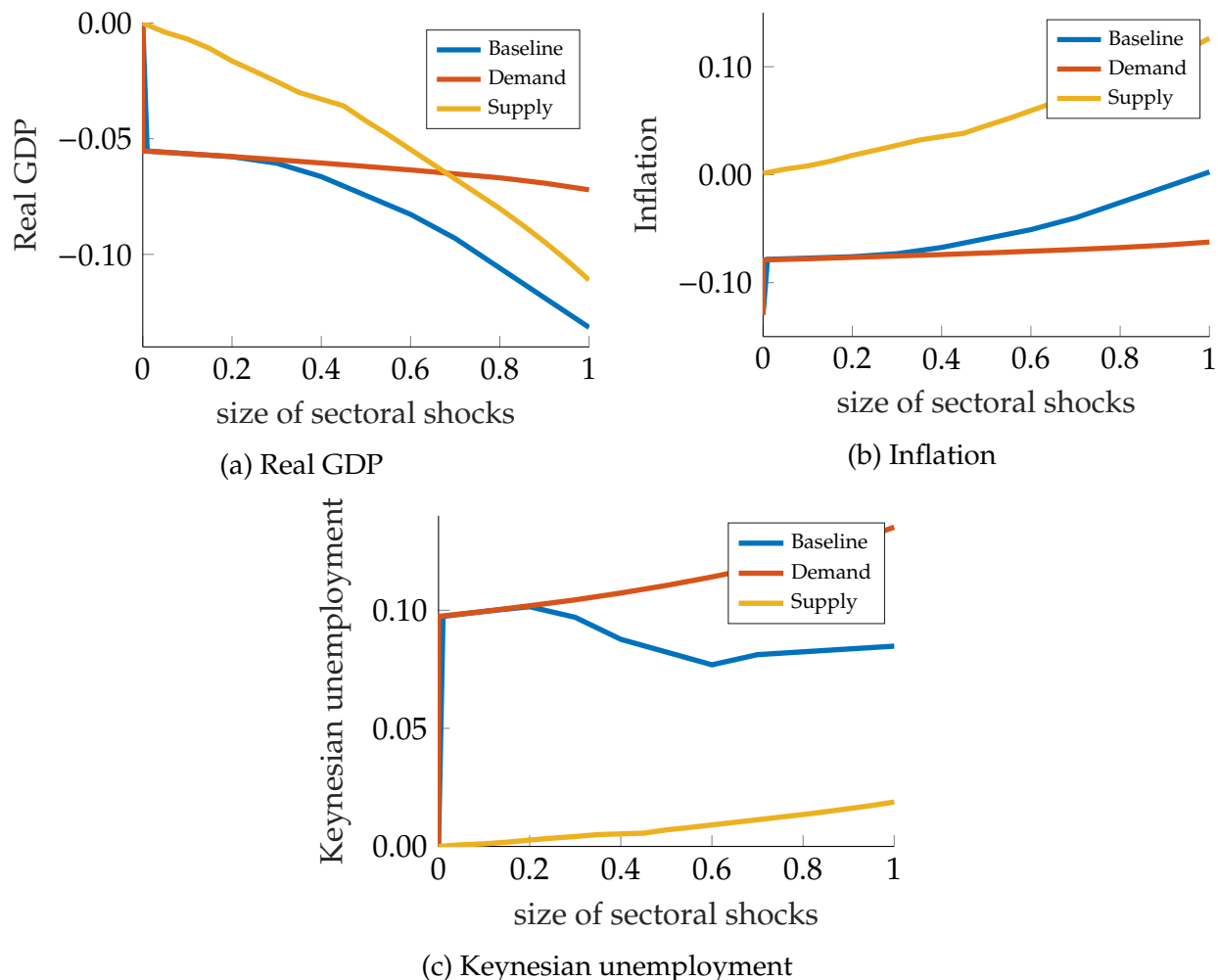


Figure 5.2: Real GDP, inflation, and Keynesian unemployment as a function of the size of sectoral shocks. The “Baseline” line includes a negative aggregate demand shock as well as sectoral supply shocks and shocks to the sectoral composition of demand. The “Demand” line includes a negative aggregate demand shock and shocks to the sectoral composition of demand. The “Supply” line only includes the sectoral supply shocks. The x-axis scales the sectoral shocks from zero to their calibrated values.

then adding sectoral supply shocks and shocks to the sectoral composition of demand leads to conclusion that the former and the latter account respectively for about half of the reduction in real GDP.²¹

Figure 5.2a also hints at how the negative sectoral supply shocks blunt the power of aggregate demand shocks (as see in Figure 4.6). The aggregate demand shock, without

²¹We measure real GDP and the change in inflation using discrete Tornqvist approximations to the chained (Divisia) index.

any sectoral shocks, lowers real GDP by 6%. However, with fully scaled sectoral supply shocks, the aggregate demand shock, only lowers real GDP by an additional 2% (from -11% to -13%). A similar conclusion applies regarding the impact of the aggregate demand shocks with not only the sectoral supply shocks but also the shocks to the sectoral composition of demand.

This last observation also has important implications for aggregate demand management policies. Conventional monetary policy, forward guidance, and untargeted government spending act like a positive aggregate demand shock. With the sectoral shocks, reversing the decline in aggregate demand only simulates real GDP by a third of the effect obtained without the sectoral shocks. If we think of the model without sectoral shocks as a typical recession, this means that aggregate demand stimulus is a third as effective in the Covid-19 recession as in a typical recession. The reason is that without sectoral shocks, the reduction in aggregate demand renders all labor markets demand constrained, and starting from there, an increase in aggregate demand increases employment in all labor markets. By contrast, with sectoral shocks, some labor markets are demand constrained and some are supply constrained, and starting from there, an increase in aggregate demand is partly dissipated in wage increases in supply-constrained labor markets (the more so, the stronger the complementarities across sectors).

Inflation. Although the supply shocks on their own generate large reductions in output, Panel 5.2b shows that they also generate very substantial amounts of inflation around 11%. Meanwhile, the demand shocks, on their own, would predict substantial deflation of around 6.5%. The baseline model, on the other hand, predicts an inflation rate of 0.3%. The baseline model performs relatively well, since most price indices show either moderate inflation or moderate deflation. For instance, CPI inflation from February to April was -0.9% while PCE inflation was -0.7%.²² Both supply and demand shocks are needed to make sense of the large output reduction and moderate deflation observed in the data.

²²The PCE is computed as a Fisher index and it therefore has changing weights reflecting the changing sectoral composition of final demand (unlike the CPI) and is therefore consistent with our model. On the other hand, the PCE does not capture changes in product variety, which could be of concern during lockdowns. Jaravel and O'Connell (2020) show that disappearing goods increased the effective inflation rate in the UK by around 80 basis points. This bias is not large enough to significantly affect our conclusions. We refer the reader to Section 7 for an extension of the model which allows for disappearing varieties.

Unemployment. Finally, Panel 5.2c plots Keynesian unemployment. Keynesian unemployment is measured using the reduction in hours in (non-healthcare) labor markets that are demand-constrained.²³ This means that we assume that demand-constrained sectors received no negative supply shocks. Therefore, Figure 5.2c is the maximum amount of Keynesian unemployment consistent with the model.²⁴

Fig 5.2c shows that the negative aggregate demand shock, on its own, generates about 10% unemployment. In the baseline model, Keynesian unemployment initially falls and then rises as we increase the size of the sectoral supply shocks. Intuitively, when “size of sectoral shocks” is zero, all labor markets are slack because of the negative aggregate demand shock. As we scale up the negative sectoral supply shocks (together with the shocks to the sectoral composition of demand), some labor markets become supply-constrained, and hence Keynesian unemployment falls. Once the negative supply shocks become large enough, substitution towards these sectors increases Keynesian unemployment.

The “Supply” line in the figure reveals that sectoral supply shocks, on their own, do generate Keynesian unemployment because of complementarities. However, the amount of Keynesian unemployment generated by this mechanism is relatively limited. The “Demand” line shows that shocks to the sectoral composition of demand increase Keynesian unemployment by a more significant amount.

Tightness and slackness across sectors. Although almost all sectors experienced reductions in hours, in some sectors, these reductions are due to supply constraints whilst in others they are due to demand shortages (see Figure A.1 for a complete description).

Supply-constrained sectors include food services and accommodations (−64%), construction (−20%), and motion pictures (−60%). We interpret the reduction in hours in these sectors to be driven by state-mandated lockdowns, social distancing orders that limited capacity, and employers’ fears of being held legally liable should their employees get sick. These restrictions and fears were severe during March and early April. As social distancing orders were lifted in May and June, some of these industries, like food services and accommodations, may go from being supply-constrained to being demand-constrained

²³Keynesian unemployment is defined as $\sum_{f \in \mathcal{L}} (\bar{\lambda}_f / \bar{\lambda}_{\mathcal{L}}) (\Delta \log \bar{L}_f - \Delta \log L_f) \geq 0$, where $\bar{\lambda}_{\mathcal{L}} = \sum_{f \in \mathcal{L}} \bar{\lambda}_f$. This captures the percentage underutilization of efficiency units of labor across labor markets. In computing this number, we assume $\Delta \log \bar{L}_f = \Delta \log L_f$ in the healthcare sectors due to the unique role they play in this health crisis. In particular, “excess capacity” in health care services is not socially wasteful in a pandemic.

²⁴In principle, these labor markets may have also experienced negative supply shocks. These reductions in potential output, however, are unobservable since supply is rationed, and, as explained above, we assume that these shocks are not present.

instead.

Demand-constrained sectors include transportation industries, like air transportation (−49%), water transportation (−45%), and rail transportation (−27%), petroleum and coal (−25%) and oil and gas extraction (−24%).²⁵

As a check on the model, we compare recovery in hours by sector from April to May using the May 2020 release of employment data from the BLS. We find that those sectors that the model classified as being demand-constrained in April 2020 recovered on average 1.8%, whereas those sectors classified as supply-constrained recovered by 7.5%. This is consistent with the idea that supply-constraints are partly driven by lockdowns, so as states reopened, the supply-constrained sectors had stronger recoveries.

5.3 Role of Elasticities of Substitution

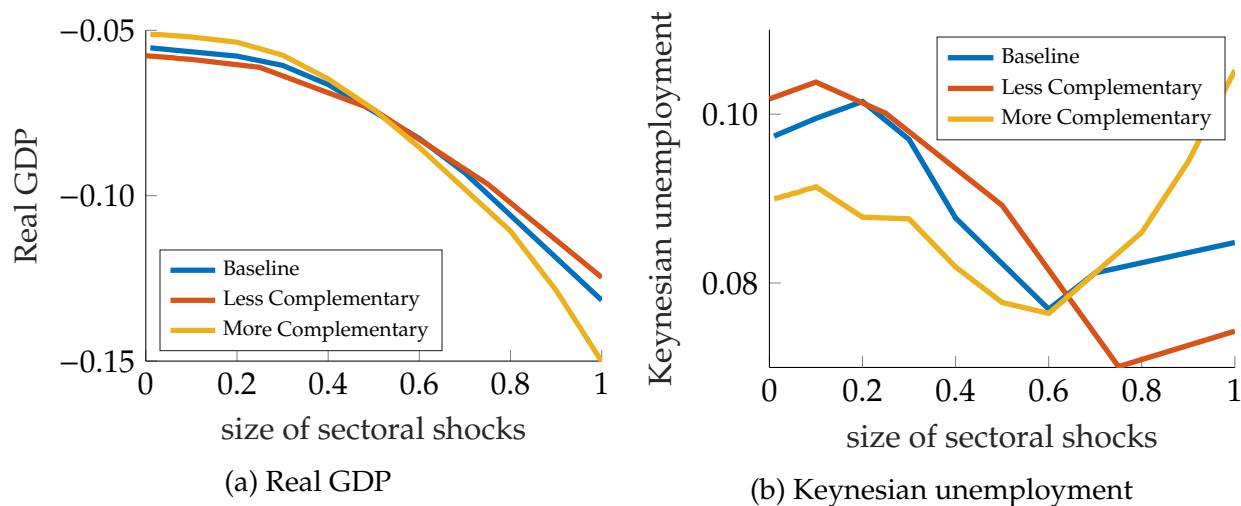


Figure 5.3: Comparison of the response of the model with the aggregate demand shock, sectoral supply shocks, and shocks to the sectoral composition of demand, for different values of the elasticities of substitution.

In Figure 5.3 we investigate the role of complementarities by deviating from our baseline elasticities $(\sigma, \theta, \epsilon, \eta) = (1, 0.001, 0.4, 0.5)$. We consider two alternative versions of

²⁵Our simulations also show that healthcare related industries, like hospitals and ambulatory health care services also experienced reductions in employment of (−31%) and (−29%). However, despite large reductions in expenditures on these sectors (from reduced elective procedures, etc.), in the data, healthcare industries do not show large reductions in employment. Presumably, this reflects the fact that the excess capacity in the healthcare industry is not wasted. Healthcare workers are instead engaged in non-market activities related to the pandemic.

the model, one with less complementarities than in our baseline $(\sigma, \theta, \epsilon, \eta) = (1, 1, 1, 0.5)$, and one with more complementarities $(\sigma, \theta, \epsilon, \eta) = (0.6, 0.001, 0.3, 0.5)$. Panel 5.3a shows the GDP response and panel 5.3b shows the Keynesian unemployment.

Complementarities mitigate the reduction in output and the increase in Keynesian unemployment when sectoral supply shocks are small but amplify them when the sectoral shocks become large enough. The intuition can be understood from Example 3.4. As we saw there, complementarities mitigate Keynesian spillovers from negative aggregate demand shocks but amplify Keynesian spillovers from negative sectoral supply shocks and shocks to the sectoral composition of demand.²⁶

6 Extension I: Credit Constraints and Household Spending

In the first extension, we show how adding potentially borrowing-constrained households to the model gives rise to endogenous aggregate demand shocks. Intuitively, if, in response to a shock, the nominal income of these households drops and they become borrowing-constrained, they reduce their nominal expenditures more than an unconstrained household. The resulting negative aggregate demand can act as endogenous multipliers, amplifying reductions in output, creating more Keynesian unemployment, and generating deflationary forces.

To simplify the analysis, we consider heterogeneous households who can save but cannot borrow. We assume that all households have the same preferences which allows us to easily aggregate their consumptions.

6.1 Local Comparative Statics

Suppose there are two types of households: unconstrained and potentially-constrained. Index potentially-constrained households by $h \in \mathcal{H}$, and suppose that households of type h own a fraction χ_{hf} of factor $f \in \mathcal{G}$. The representative unconstrained household

²⁶The elasticity of substitution between labor and capital η plays a somewhat different role under the assumption that the different sectoral capital markets always feature flexible prices and full utilization. In response to a given set of negative shocks to sectoral labor supplies, stronger complementarities between labor and capital raise the share of the labors compared to that of the capitals and mitigates Keynesian unemployment. This shock absorber effect disappears when the sectoral capital markets are also subject to downward rigidities, which, as explained earlier, could be micro-founded from firms' inability to repay loans and ensuing firm failures. The same applies when endogenous capacity utilization is modeled as a decision motivated by a tradeoff between utilization and depreciation.

$r \notin \mathcal{H}$ behaves similarly to the representative household in Section 2. On the other hand, for the potentially-constrained households $h \in \mathcal{H}$ expenditures are determined by their contemporaneous earnings if those earnings fall more than what they would need to satisfy their Euler equation. We start with local comparative static results and end with a brief comment on global results.

Denote the endogenous set of households that are constrained in equilibrium by $\mathcal{H}^c \subseteq \mathcal{H}$. Hence $h \in \mathcal{H}^c$ if and only if

$$\sum_{f \in \mathcal{G}} \frac{\chi_{hf} \lambda_f}{\sum_{g \in \mathcal{G}} \chi_{hg} \lambda_g} (d \log \lambda_f + d \log E) < (1 - \rho) d \log p^Y + d \log \zeta. \quad (6.1)$$

In words, household h is constrained if their nominal income today is less than how much they wish to consume, as given by their Euler equation.²⁷

The key step in this extension is the determination of changes in aggregate nominal expenditure. In the interest of space, we omit the derivations and jump directly to the result, which applies almost everywhere given the set of constrained households \mathcal{H}^c :

$$d \log E = \frac{\text{Cov}_\lambda(\chi_{\mathcal{H}^c}, d \log \lambda)}{1 - \mathbb{E}_\lambda(\chi_{\mathcal{H}^c})} + (1 - \rho) d \log p^Y + d \log \zeta.$$

The covariance uses the factor shares λ_f for $f \in \mathcal{G}$ as the probability distribution and computes the covariance of the income share of constrained households for each factor $\chi_{\mathcal{H}^c} = \sum_{h \in \mathcal{H}^c} \chi^h$ with the changes in the factor shares $d \log \lambda_f$.²⁸

Therefore, even in the absence of exogenous aggregate demand shocks $d \log \zeta$ such as changes in nominal interest rates or in the desire to save, there is now an endogenous aggregate demand shock. In particular, if the income shares of factors owned by constrained households shrink, then this imparts a negative aggregate demand shock that shrinks nominal expenditures today $d \log E < 0$. Since it is precisely those households whose income falls that become constrained, this endogenously reduces expenditures, and the reductions in expenditures exasperates unemployment.

The accompanying propagation equations determining changes in factor shares $d \log \lambda_f$

²⁷Note that we assume that the reduction in the income of type h households is borne uniformly by all households of type h . That is, we do not allow some type- h households to get fired (becoming constrained) while others stay on the job and are unconstrained.

²⁸We could allow for the possibility that some households are exogenously hand-to-mouth. The equation for changes in aggregate nominal expenditure $d \log E$ would stay the same, but we would no longer have to verify that (6.1) holds for those households.

as a function of changes in aggregate nominal expenditure $d \log E$ are exactly the same as in Proposition 2. And the aggregation determining $d \log Y$ as a function of the $d \log \lambda_f$'s and $d \log E$ is exactly the same as in Proposition 1.²⁹

6.2 Global Comparative Statics with an Example

To connect these endogenous demand shocks to the AS-AD representation and provide global comparative statics, we return to the example of Section 4.4. Recall that there are two labor markets, one of which is hit with a negative shock to potential labor supply from \bar{L}_f to \bar{L}'_f so that $\Delta \log \bar{L}_f < 0$. Recall also that the intertemporal elasticity of substitution is set to $\rho = 1$. We also assume that the elasticity of substitution θ across sectors is less than one. Finally, we assume that each labor is owned by a potentially-constrained household and identify households with the labor that they supply, so that $\mathcal{H} = \mathcal{L}$ and household $g \in \mathcal{H}$ supplies labor $g \in \mathcal{L}$. A household becomes constrained if, and only if, its labor becomes demand-constrained with the shock.

In this case, it is convenient to rely on the determination of nominal expenditure via the *Keynesian cross* at work in the model:

$$E = \sum_{g \notin \mathcal{H}^c} (1 - \beta) \left(\lambda_g E + \frac{\bar{\lambda}_{g^*} \bar{I}_*}{1 + i} \right) + \sum_{g \in \mathcal{H}^c} \lambda_g E, \quad (6.2)$$

where recall that \bar{I}_* is nominal income in the future period and $\bar{\lambda}_{g^*}$ is the income share of labor g in the future. The first set of summands on the right-hand side of the equation are nominal expenditures by unconstrained households, where intertemporal nominal income is $\lambda_g E + \bar{\lambda}_{g^*} \bar{I}_* / (1 + i)$ and $1 - \beta$ is both the marginal and the average propensity to consume of these households. The second set of summands on the right-hand side of the equation are nominal expenditures by the constrained households, where $\lambda_g E$ is present nominal income which coincides with the consumption of these households.

In Figure 6.1, we plot an AS-AD diagram for this example. The AS curve remains unchanged relative to before. To construct the AD curve, we use $p^Y = E/Y$, with nominal expenditures E given by (6.2). The expenditures shares λ_f , in (6.2), are those that are consistent with the implied level of expenditures $p^Y(Y)Y$, where $p^Y(Y)$ is the AS curve.³⁰

²⁹The same local comparative static equations also apply if some households are exogenously assumed to be hand-to-mouth.

³⁰In drawing the AD curve, we assume that household f , whose wages have increased due to the negative supply shock, is always Ricardian, which amounts to selecting the equilibrium with the highest level of

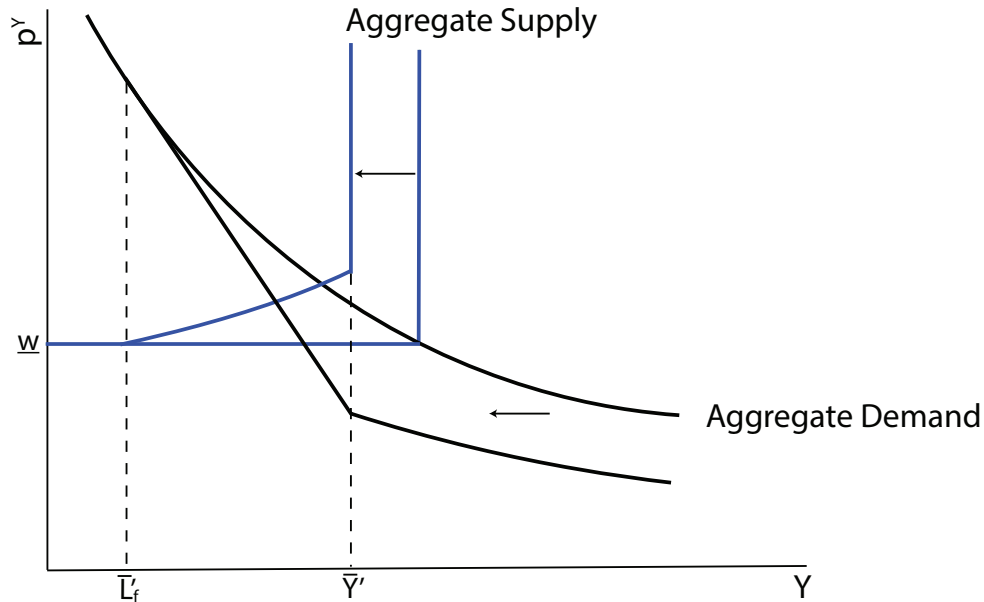


Figure 6.1: AS-AD representation of the example in Section 6.2. The shock is a reduction in potential labor in one of the labor markets.

The reduction in potential labor from \bar{L}_f to \bar{L}'_f shifts the AS curve exactly as in the case where all households are Ricardian studied in Section 4.4, potential output is reduced from \bar{Y} to \bar{Y}' , and output drops below potential output as Keynesian unemployment appears in the non-shocked sector. The difference with potentially-constrained households is that now the AD curve also shifts. This endogenous negative aggregate demand shocks has the double effect of further reducing output and mitigating inflation. Intuitively, as the non-shocked labor market becomes demand-constrained, the income the households that supply it drops. As a result, these households become borrowing constrained and reduce their nominal expenditure.³¹

nominal expenditure and the least amount of Keynesian unemployment in each labor market. Indeed, if there are no Ricardian households, and all households can potentially be constrained, then the model has multiple equilibria. In that case, our diagram corresponds to selecting the equilibrium with the maximum amount of nominal expenditures.

³¹More precisely, the AD curve not only shifts inward but it also changes shape by acquiring two kinks. The first kink occurs when output Y is so low that all labor markets become demand-constrained with all wages equal to their lower bound. For $Y < \bar{L}$, all labor markets are demand-constrained and so labor income shares are constant. Furthermore, since f is Ricardian, (6.2) implies that nominal expenditures are equal to their pre-shock level. This means the AD curve traces out the same curve as the pre-shock level for $Y \in [0, \bar{L}]$. When $Y \in (\bar{L}', \bar{Y}')$, the non-shocked labor markets are slack, and workers in those markets are constrained. This lowers nominal expenditures due to the Keynesian-cross type logic of (6.2), and hence lowers the AD curve relative to what it would have been under the complete markets scenario. Once $Y > \bar{Y}'$, output is high enough that no factor market becomes slack. This is where the second kink point occurs,

7 Extension II: Credit Constraints and Firm Failures

In the previous section, we saw that borrowing constraints that endogenously bind for households who lose income trigger endogenous negative aggregate demand shocks. In this section, we show that credit constraints that endogenously bind for firms that lose profits can lead to cascades of exits and firm-failures and catalyze endogenous negative supply shocks. These supply shocks can act as endogenous multipliers, amplifying reductions in output, creating more Keynesian unemployment, and generating inflationary pressures.

To capture firm failures, we modify the general Keynesian structure described in Section 2 as follows. We assume that output in sector $i \in \mathcal{N}$ is a CES aggregate of identical producers j each with constant returns production functions $y_{ik} = A_i f_i(x_{ij}^k)$, where x_{ij}^k is the quantity of industry j 's output used by producer k in industry i . Assuming all firms within an industry use the same mix of inputs, sectoral output is

$$y_i = \left(\int y_{ik}^{\frac{\sigma_i-1}{\sigma_i}} dk \right)^{\frac{\sigma_i}{\sigma_i-1}} = M_i^{\frac{1}{\sigma_i-1}} A_i f_i(x_{ij}),$$

where x_{ij} is the quantity of input j used by industry i , M_i is the mass of producers in industry i , $\sigma_i > 1$ is the elasticity of substitution across producers, and A_i is an exogenous productivity shifter. From this equation, we see that a change in the mass of operating firms acts like a productivity shock and changes the industry-level price. Therefore, if shocks outside sector i trigger a wave of exits, then this will set in motion endogenous negative productivity shock $(1/(\sigma_i - 1))\Delta \log M_i$ in sector i .

Suppose that each firm must maintain a minimum level of revenue in order to continue operation.^{32, 33} We are focused on a short-run application, so we do not allow new entry,

and beyond this point, nominal expenditures stop changing, and the aggregate demand curve once again follows $\Delta \log p^Y = -\Delta \log Y$.

³²One possible micro-foundation is each producer must pay its inputs in advance by securing within-period loans and that these loans have indivisibilities: only loans of size greater than some minimum level can be secured. This minimum size is assumed to coincide with the initial costs $\bar{\lambda}_i \bar{E} / \bar{M}_i$ of the producer.

³³Another possible micro-foundation is as follows. Producers within a sector charge a CES markup $\mu_i = \sigma_i / (\sigma_i - 1)$ over marginal cost. These markups are assumed to be offset by corresponding production subsidies. Producers have present nominal debt obligations corresponding to their initial profits $(1 - 1/\mu_i)\bar{\lambda}_i \bar{E} / \bar{M}_i$. The same is true in the future. If present profits $(1 - 1/\mu_i)\lambda_i E / M_i$ fall short of the required nominal debt payment $(1 - 1/\mu_i)\bar{\lambda}_i \bar{E} / \bar{M}_i$, then the firm goes bankrupt and exits. Alternatively, we can imagine that there is no future debt obligation but that firms cannot borrow.

but of course, this would be important for long-run analyses.³⁴

The mass of firms that operate in equilibrium is therefore given by

$$M_i = \min \left\{ \frac{\lambda_i E}{\bar{\lambda}_i \bar{E}} \bar{M}_i, \bar{M}_i \right\},$$

where \bar{M}_i is the exogenous initial mass of varieties, $\lambda_i E$ is nominal revenue earned by sector i and $\bar{\lambda}_i \bar{E}$ is the initial nominal revenue earned by i . If nominal revenues fall relative to the baseline, then the mass of producers declines to ensure that sales per producer remain constant. In order to capture government-mandated shutdowns of certain firms, we allow for shocks that reduce the exogenous initial mass of producers $\Delta \log \bar{M}_i \leq 0$.

7.1 Local Comparative Statics

We can generalize Propositions 1 and 2 to this context. The only difference is that we must replace $d \log A_i$ by $d \log A_i + (1/(\sigma_i - 1))d \log M_i$, where

$$d \log M_i = d \log \bar{M}_i + \min\{d \log \lambda_i + d \log E - d \log \bar{M}_i, 0\}.$$

This backs up the claim that the $d \log M_i$'s act like endogenous negative productivity shocks. They provide a mechanism whereby a negative demand shock, say in the composition of demand or in aggregate demand $d \log \zeta$, triggers exits which are isomorphic to negative supply shocks.

As in the other examples, the general lesson is that the output response, to a first-order, is again given by an application of Hulten's theorem along with an amplification effect which depends on how the network redistributes demand and triggers Keynesian unemployment in some factors and firm failures in some sectors.

7.2 Illustrative Example

Consider once again the horizontal economy analyzed in Section 3.4. We assume that there are no shocks to aggregate demand ($d \log \zeta = 0$). Since $\rho = 1$, this ensures that nominal expenditure is constant ($d \log E = 0$). We also assume that there are no exogenous shocks to productivities ($d \log A_i = 0$), no shocks to potential labor ($d \log \bar{L}_f = 0$), and no

³⁴See Baqaee (2018) and Baqaee and Farhi (2020a) for production networks with both entry and exit.

shocks to the sectoral composition of demand ($d \log \omega_{0j} = 0$). Finally, we assume that all sectors have the same within-sector elasticity of substitution $\sigma_i = \sigma > 1$.

We focus on exogenous shocks $d \log \bar{M}_i \leq 0$ capturing government-mandated shut-downs. We show how endogenous failures can amplify these negative supply shocks. The insights are more general and also apply to shocks to potential labor. Similarly, failures can be triggered by negative aggregate demand shocks, and the resulting endogenous negative supply shocks can result in stagflation with simultaneous reductions in output and increases in inflation.

Preliminaries. Changes in the sales of i are given by

$$d \log \lambda_i = (1 - \theta_0)(1 - \lambda_i) \left(d \log p_i - \sum_{j \in \mathcal{N}} \lambda_j d \log p_j \right), \quad (7.1)$$

where changes in the price of i depend on changes in the wage in i and on the endogenous reduction in the productivity of i driven by firm failures

$$d \log p_i = d \log w_i - \frac{1}{\sigma - 1} d \log M_i. \quad (7.2)$$

The change in wages in i are given by

$$d \log w_i = \max\{d \log \lambda_i - d \log \bar{L}_i, 0\}, \quad (7.3)$$

and changes in the mass of producers in i are given by

$$d \log M_i = \min\{d \log \lambda_i, d \log \bar{M}_i\}. \quad (7.4)$$

We consider the effect of shutdown shocks $d \log \bar{M}_i$ starting with the case where sectors are complements and then the case where they are substitutes. The effect of negative labor shocks $d \log \bar{L}_i$ is similar.

Shut-down shock with complements. Assume that sectors are complements ($\theta < 1$) and consider the government-mandated shutdown of some firms in only one sector i . We can aggregate the non-shocked sectors into a single representative sector indexed by $-i$. We therefore have $d \log \bar{M}_i < 0 = d \log \bar{M}_{-i}$.

The closures of firms in i raise its price ($d \log p_i > 0$), which because of complementari-

ties, increases its share ($d \log \lambda_i > 0$). It therefore does not trigger any further endogenous exit in this shocked sector ($d \log M_i = d \log \bar{M}_i$). In addition, the wages of its workers increases ($d \log w_i > 0$). The shock reduces expenditure on the other sectors ($d \log \lambda_{-i} < 0$), and this reduction in demand triggers endogenous exits ($d \log M_{-i} < 0$), pushes wages against their downward rigidity constraint ($d \log w_{-i} = 0$) and creates unemployment ($d \log L_{-i} < 0$), both of which endogenously amplify the reduction in output through failures and Keynesian effects.

Using equations (7.1), (7.2), (7.3), and (7.4), we find

$$d \log \lambda_i = -\frac{(1-\theta)(1-\lambda_i)}{1-(1-\theta)(1-\lambda_i)\left(1-\frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)}\frac{1}{\sigma-1}d \log \bar{M}_i > 0,$$

$$d \log M_{-i} = d \log L_{-i} = -\frac{\lambda_i}{1-\lambda_i}d \log \lambda_i < 0,$$

and finally

$$d \log Y = \lambda_i \frac{1}{\sigma-1}d \log \bar{M}_i + \frac{(1-\theta)(1-\lambda_i)^{\frac{\sigma}{\sigma-1}}}{1-(1-\theta)(1-\lambda_i)\left(1-\frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)}\lambda_i \frac{1}{\sigma-1}d \log \bar{M}_i.$$

The first term on the right-hand side is the direct reduction in output from the shut-down in sector i . The second term capture the further indirect equilibrium reduction in output via firm failures and Keynesian unemployment in the other sectors.

Shut-down shock with substitutes. Consider the same experiment as above but assume now that sectors are substitutes ($\theta > 1$). We conjecture an equilibrium where sales in sector i do not fall more quickly than the initial shock $d \log \lambda_i - d \log \bar{M}_i > 0$. Sector i loses demand following the exogenous shutdown of some of its firms, and this results in unemployment in in the sector ($d \log L_i < 0$) but no endogenous firm failures ($d \log M_i = d \log \bar{M}_i$). On the other hand, sector $-i$ maintains full employment and experiences no failures.

To verify that this configuration is indeed an equilibrium, we compute

$$d \log \lambda_i = \frac{(\theta-1)(1-\lambda_i)}{1-(\theta-1)\lambda_i}\frac{1}{\sigma-1}d \log \bar{M}_i.$$

We must verify that

$$0 > d \log \lambda_i > d \log \bar{M}_i.$$

The first inequality is verified as long as $\theta > 1$ is not too large. The second inequality is verified if $\sigma > 1$ is large enough and $\theta > 1$ is not too large.

If these conditions are violated, then we can get a jump in the equilibrium outcome. Intuitively, in those cases, the shutdown triggers substitution away from i , and that substitution is so dramatic that it causes more firms to shutdown, and the process feeds on itself ad infinitum. Any level of $d \log L_i < 0$ and $d \log M_i < d \log \bar{M}_i$ can then be supported as equilibria. Although we do not focus on it, this possibility illustrates how allowing for firm failures with increasing returns to scale can dramatically alter the model's behavior.

Assuming the regularity conditions above are satisfied, the response of output is given by

$$d \log Y = \lambda_i \frac{1}{\sigma - 1} d \log \bar{M}_i + \frac{(\theta_0 - 1)(1 - \lambda_i)}{1 - (\theta_0 - 1)\lambda_i} \lambda_i \frac{1}{\sigma - 1} d \log \bar{M}_i,$$

where the first term on the right-hand side is the direct effect of the shutdown and the second term is the amplification from the indirect effect of the shutdown which results in Keynesian unemployment in i .

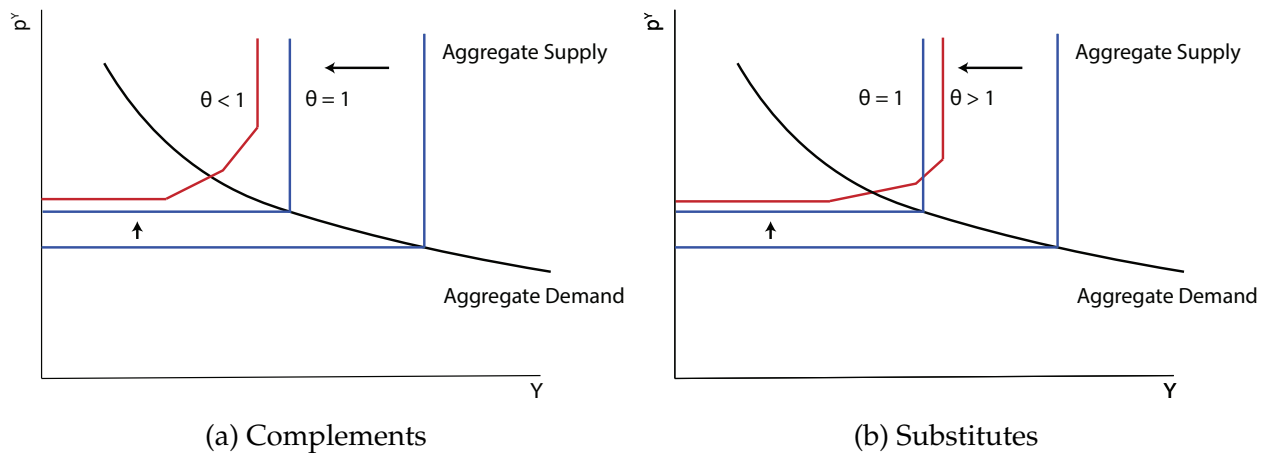


Figure 7.1: AS-AD representation of the example in Section 7.2. The shock is mandatory firm closures in some sector.

AS-AD representation. Figure 7.1 depicts this example using an AS-AD diagram. In the Cobb-Douglas case, in response to an exogenous shutdown shock, the AS curve shifts but maintains its shape. Intuitively, either output is low enough that all factor markets become slack, in which case the price level hits its lower bound, or output is high enough

that all factor markets clear and output is equal to its potential. In equilibrium, output is at potential and there is not Keynesian unemployment.

Outside of the Cobb-Douglas case, the AS curve also changes shape, and there is Keynesian unemployment no matter whether sectors are complements or substitutes as captured by the fact that the AD curve intersects the AS curve in an upward-sloping portion to the left of its vertical portion. When sectors are complements, Keynesian unemployment occurs in non-shocked sectors that lose sales to the shocked sector. When sectors are substitutes, Keynesian unemployment occurs in the shocked sector, as the shocked sector loses sales to other sectors.

8 Extension III: Policy

Finally, we briefly review how policy can affect outcomes. We analyze three different types of policy: monetary policy, tax incentives (payroll tax cuts), and fiscal policy (government spending and transfers).

8.1 Monetary Policy

A monetary expansion in this model comes in the form of positive aggregate demand shock

$$d \log \zeta = -\rho \left(d \log(1+i) + \frac{d \log \beta}{1-\beta} - d \log \bar{p}^Y \right) + d \log \bar{Y} > 0,$$

this can come about either via lower nominal interest rates, or failing that, forward guidance about the price level in the future. If nominal rates are stuck at the zero-lower bound, then an increase in future prices, by lowering the real interest rate, will stimulate spending today. A positive aggregate demand shock increases nominal expenditures since

$$d \log E = \frac{\text{Cov}_{\lambda_{\mathcal{G}}}(\chi_{\mathcal{H}^c}, d \log \lambda_{\mathcal{G}})}{1 - \mathbb{E}_{\lambda_{\mathcal{G}}}(\chi_{\mathcal{H}^c})} + (1 - \rho) d \log p^Y + d \log \zeta.$$

If nominal expenditures $d \log E$ are sufficiently high, then the economy can maintain full employment regardless of the shocks by guaranteeing that nominal wages do not have to fall in equilibrium

$$\min_{f \in \mathcal{G}} \{d \log \lambda_i + d \log E - d \log \bar{L}_f\} > 0.$$

This is obviously the optimal policy for the monetary authority to pursue, if it is feasible.

Setting aside full-employment policy, we can also consider how output responds to a given monetary stimulus $d \log \zeta > 0$. Since the model is non-linear, the effectiveness of monetary policy depends on what other shocks have hit the economy. A canonical example is if the monetary stimulus coincides with a set of negative supply shocks. In this case, complementarities in production act to reduce the effectiveness of monetary policy.

To see this, consider again the horizontal economy described in Example 3.4. We assume that sectors are complements with $\theta < 1$ and that the intertemporal elasticity of substitution is $\rho = 1$. For simplicity, suppose that there are no constrained households. We hit the economy with negative shocks to potential factor supplies $d \log \bar{L}_f < 0$. Suppose that in addition, through forward-guidance, the monetary authority is able to raise $d \log \zeta > 0$.

Then, working through the same equations as in the original example, we find that the overall effect on output is

$$d \log Y = \frac{\lambda_S d \log \bar{L}_S}{1 - (1 - \theta)(1 - \lambda_S)} + \left(1 - \frac{(1 - \theta)\lambda_S}{1 - (1 - \theta)(1 - \lambda_S)}\right) (1 - \lambda_S) d \log \zeta.$$

The first term is the effect of the negative supply shock, amplified by the nominal rigidities, exactly as in Example 3.4. The second term is the effect of the monetary stimulus. The term $(1 - \lambda_S) d \log \zeta$ is the direct effect of the stimulus on the employment of demand-constrained factors. However, this direct effect is mitigated. This is because monetary stimulus raises the prices of supply-constrained factors in absolute terms and relative to those of demand-constrained factors, and since supply-constrained and demand-constrained factors are complements, this causes expenditures to switch towards supply-constrained factors and away from demand-constrained factors. This force attenuates the effectiveness of monetary policy, and it would not appear if not for the heterogeneity in cyclical conditions across factor markets. We saw in our quantitative model in Section 5 that this effect can be very powerful.

8.2 Payroll Tax Cuts

In this section, we briefly consider the effect of payroll tax cuts used by many governments in the wake of Covid-19. If correctly targeted, payroll tax cuts can alleviate the demand short-fall in slack factor markets. By selectively cutting taxes (or subsidizing) unemployed sectors, a policymaker can actually implement the first-best outcome.³⁵

³⁵See e.g. Correia et al. (2013); Farhi et al. (2014).

Even without going all the way to the first best, these policies can be helpful. However, as with monetary policy, complementarities in production also reduce the effectiveness of a given intervention. To see this, consider once more the horizontal economy of Example 3.4. We assume that sectors are complements with $\theta < 1$ and that the intertemporal elasticity of substitution is $\rho = 1$. For simplicity, we assume that there are no constrained households.³⁶ We hit the economy with negative factor supply shocks $d \log \bar{L}_f < 0$. In addition, we assume that the government institutes a gross payroll subsidy $d \log s_D$ on the demand-constrained factors financed by a tax on the resulting profits. In this case, the output response is

$$d \log Y = \frac{\lambda_S d \log \bar{L}_S}{1 - (1 - \theta)(1 - \lambda_S)} + \left(1 - \frac{(1 - \theta)\lambda_S}{1 - (1 - \theta)(1 - \lambda_S)} \right) (1 - \lambda_S) d \log s_D.$$

As usual, the first term is the effect of the negative supply shock to the supply-constrained sectors, amplified by the nominal rigidities. The second term is the effect of the payroll subsidy. Naturally, a subsidy on demand-constrained factors increases output, and the term $(1 - \lambda_S) d \log s_D$ is the direct effect of the increase in employment. However, the subsidy on demand-constrained factors also reduces the price of slack sectors relative to supply-constrained ones. Since factors are complements, this means that expenditures shift towards supply-constrained factors and away from demand-constrained ones, attenuating the effect of the payroll subsidy.

8.3 Fiscal Policy

Finally, we consider the effect of changes in size of and sectoral composition of government spending as well as transfers to non-Ricardian households. We assume that $G = 0$ and denote by dG the nominal change in government expenditure and by Ω_k^G the shares of the different sectors in government expenditure. We assume that government spending is deficit-financed, and that the debt is repaid with taxes in the future. We assume that only a fraction $\alpha_{\text{Ricardian}}$ of these future taxes falls on Ricardian households, and the rest falls either on non-Ricardian households or on future generations. We denote by dT the nominal transfer from Ricardian to non-Ricardian households.

³⁶With exogenously constrained households, payroll tax cuts can be helpful through a different channel if they increase their income by boosting their wages, thereby effectively redistributing away from households with low marginal propensities to consume and towards households with high marginal propensities to consume.

We denote by $\overline{MPC} = \mathbb{E}_\lambda(1 - \chi_{\mathcal{H}^c})MPC_{\text{Ricardian}} + \mathbb{E}_\lambda(\chi_{\mathcal{H}^c})$ the average marginal propensity to consume, where $MPC_{\text{Ricardian}} = 1 - \beta$ is the marginal propensity to consume for Ricardian households and 1 is the marginal propensity to consume of non-Ricardian households.

The only changes in the analysis concern the determination of changes in nominal expenditure and the propagation equations for sales and factor shares. Changes in nominal expenditure are given by

$$d \log E = \frac{\text{Cov}_\lambda(\chi_{\mathcal{H}^c}, d \log \lambda)}{1 - \mathbb{E}_\lambda(\chi_{\mathcal{H}^c})} + (1 - \rho)d \log p^Y + d \log \zeta \\ + \frac{1 - \alpha_{\text{Ricardian}}MPC_{\text{Ricardian}}}{1 - \overline{MPC}}dG + \frac{1 - MPC_{\text{Ricardian}}}{1 - \overline{MPC}}dT.$$

Changes in sales and factor shares are given by

$$\lambda_f d \log \lambda_f = \sum_{k \in \mathcal{N}} \Psi_{kf} \Omega_{0k} d \log \omega_{0k} + \sum_{k \in \mathcal{N}} \Psi_{kf} (\Omega_k^G - \Omega_{0k}) \frac{dG}{E} \\ + \sum_{j \in 1+N} \lambda_j (\theta_j - 1) \text{Cov}_{\Omega^{(j)}} \left(\sum_{k \in \mathcal{N}} \Psi_{(k)} (d \log A_k) + \sum_{g \in \mathcal{G}} \Psi_{(g)} (d \log L_g - d \log \lambda_g), \Psi_{(f)} \right),$$

where $d \log L_g = d \log \bar{L}_g$ for $f \in \mathcal{K}$ and $d \log L_g = \min \{d \log \lambda_g + d \log E, d \log \bar{L}_g\}$ for $f \in \mathcal{L}$. We can then combine these formulas with Proposition 1 to get the change in aggregate output exactly as before. This generalizes Baqaee (2015) beyond the Cobb-Douglas special case.

These results show how changes in government spending can stimulate output in two different ways. The first reason is the standard Keynesian-cross argument: an increase in government spending stimulates the incomes of households, who then proceed to consume more. This boosts nominal expenditure by $dG(1 - \alpha_{\text{Ricardian}}MPC_{\text{Ricardian}})/(1 - \overline{MPC})$, which is higher, the higher is the average marginal propensity to consume \overline{MPC} and the lower is the fraction $\alpha_{\text{Ricardian}}$ of future taxes that fall on Ricardian consumers.³⁷ Interestingly, in the context of the pandemic, the fiscal multiplier could be lower in a partial lock-down if Ricardian households have a low marginal propensity to consume because they want to postpone consumption until the lock-down is fully lifted. Similar observations apply to transfers which stimulate nominal expenditure by $dT(1 - MPC_{\text{Ricardian}})/(1 - \overline{MPC})$.

The second reason is slightly more subtle. By choosing the sectoral composition of

³⁷See e.g. Farhi and Werning (2016) for a discussion.

government spending wisely, the government can target its spending to boost the demand of sectors whose factor markets are depressed. This effect is captured by $\sum_{k \in \mathcal{N}} \Psi_{kf}(\Omega_k^G - \Omega_{0k}) \frac{dG}{E}$. Intuitively, fiscal policy can move the AD curve by changing both the size and sectoral composition of government expenditures.

To the extent that the government cannot perfectly target depressed factor markets, some of the government expenditures will end up wastefully increasing the wages of supply-constrained factors instead of stimulating employment, thereby lowering the fiscal multiplier. Furthermore, fiscal multipliers are further dampened in economies with complementarities since to some extent, government spending always ends up increasing the wages of some supply-constrained factors, causing private expenditure to be redirected towards those factors and away from demand-constrained factors. Once again, similar observations apply to transfers. Just like for monetary policy, we saw in our quantitative model in Section 5 that this effect can be very powerful.

9 Conclusion

This paper analytically characterizes and numerically quantifies the impact of different supply and demand shocks in a general disaggregated model with multiple sectors, factors, and input output linkages, as well as occasionally-binding downward nominal wage rigidity and zero lower bound constraints.

We find that both demand and supply shocks are necessary to make sense of the data. Whereas negative supply shocks, on their own, can cause significant reductions in real GDP they cause far too much inflation. On the other hand, negative demand shocks, on their own, are incapable of generating large enough reductions in real GDP and cause too much deflation. Both shocks together result in a large reduction in GDP and muted reaction in inflation. Using the model, we can classify sectors into ones that are demand-constrained and ones that supply-constrained, and we find that both types of sectors are important.

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Appendix A Additional Graph

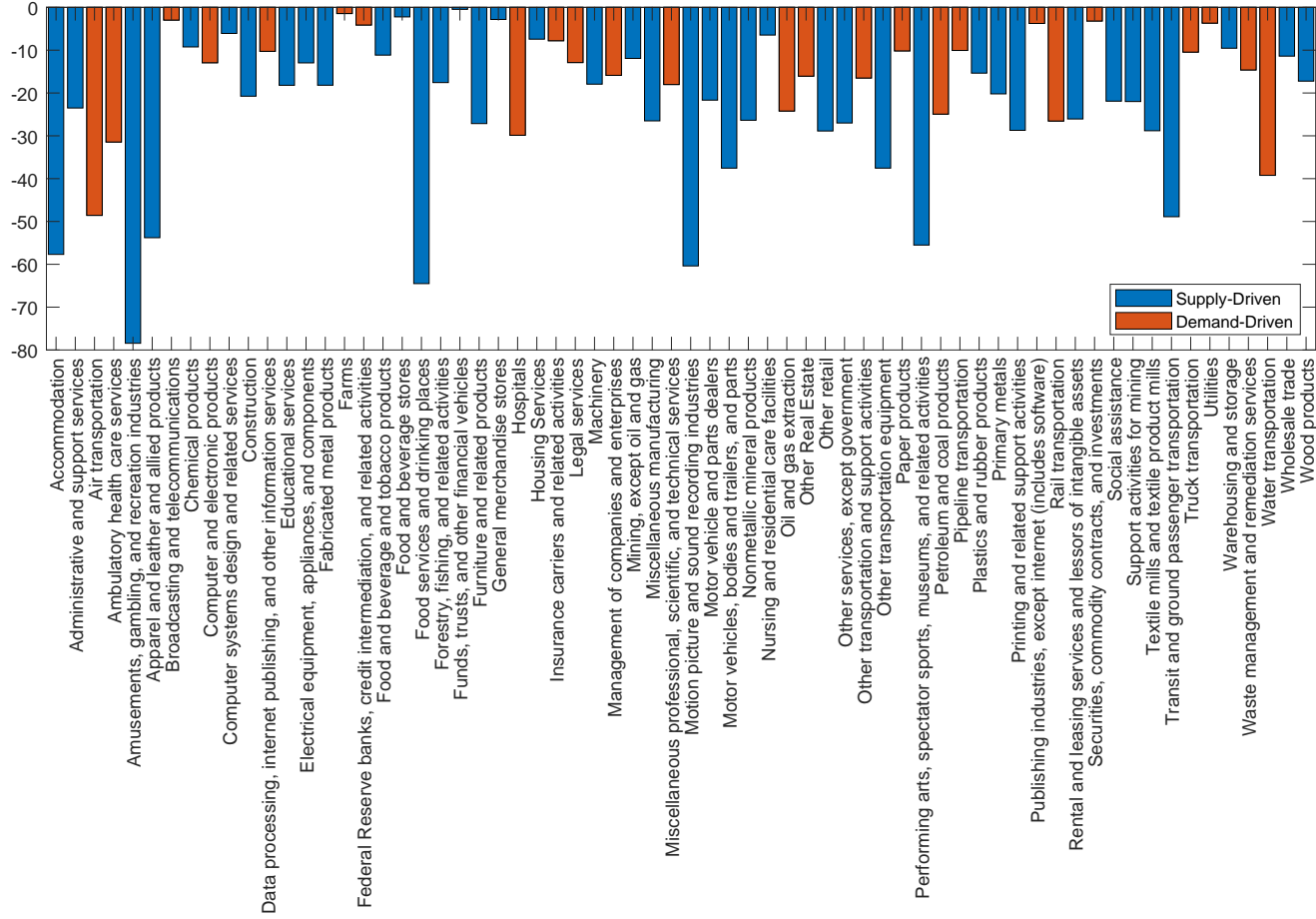


Figure A.1: Model implied percentage reduction in hours by sector from February to April 2020. Sectors below capacity are “demand-driven.”

Appendix B Investment

To model investment, we add intertemporal production functions into the model. An investment function transforms goods and factors in the present period into goods that can be used in the future. In this case, instead of breaking the problem into an intertemporal and intratemporal problem, we must treat both problems at once. In this section, we first discuss the general local comparative statics with investment, extending the results in Section 3, then we discuss a special case with simple sufficient statistics and global comparative statics, extending the results in Section 4.

In the body of the paper, we assumed that prices in the future p_*^Y were fixed, which meant that nominal expenditures in the future were also fixed $p_*^Y Y_* = E_*$. In the version of the model with investment, output in the future Y_* is not exogenous, so assuming p_*^Y is no longer equivalent to assuming E_* is fixed. Therefore, we consider both situations.

B.1 General local comparative statics

When we add investment to the model, we can still use Proposition 1 without change. However, we can no longer use the Euler equation to pin down nominal expenditures today, since nominal GDP today includes investment expenditures and output tomorrow can no longer taken to be exogenous. Instead, to determine $d \log E$, we must use a version of Proposition 2. For this subsection, we assume that nominal expenditures in the future period are fixed and we denote the future period by $*$.

In particular, let λ_i^I denote the intertemporal sales share — expenditures on quantity i as a share of the net present value of household income. Furthermore, let $\bar{\Omega}^I$ represent the intertemporal input-output matrix, which includes the capital accumulation equations. Then, letting intertemporal consumption be the zero-th good, and abstracting from shocks to the sectoral composition of demand for simplicity, we can write

$$d \log \lambda_k^I = \sum_j \lambda_j^I (\theta_j - 1) \text{Cov}_{\Omega^I, (j)} \left(\sum_{i \in \mathcal{N}} \Psi_{(i)}^I d \log A_i - \sum_{f \in \mathcal{G}} \Psi_{(f)}^I (d \log \lambda_f^I - d \log L_f), \frac{\Psi_{(k)}^I}{\lambda_k^I} \right)$$

almost everywhere, where changes in factor employments are given by

$$d \log L_f = \begin{cases} d \log \bar{L}_f, & \text{for } f \in \mathcal{K}, \\ \min \{ d \log \lambda_f^I - d \log \lambda_*^I, d \log \bar{L}_f \}, & \text{for } f \in \mathcal{L}. \end{cases}$$

This follows from the fact that nominal expenditures on each factor f is given by $d \log \lambda_f^l + d \log E^l$, where E^l is the net-present value of household income. However, since nominal expenditures in the future are fixed, we have $d \log E_* = d \log \lambda_*^l + d \log E^l = 0$. This allows us to write nominal expenditures on each factor as $d \log \lambda_f^l - d \log \lambda_*^l$.

B.2 Global Comparative Statics

We can extend the results in Section 4 to the model with investment. To do so, we assume that the intertemporal elasticity of substitution ρ is the same as the intersectoral elasticities of substitution $\rho = \theta_j = \theta$ for every $j \in \mathcal{N}$. In this case, the initial factor shares are, once again, a sufficient statistic for the production network. In particular, Proposition 3 still applies. Furthermore, we can also prove that the set of equilibria form a lattice under some additional assumptions.

Proposition 7. *Suppose that the intertemporal elasticity of substitution, the elasticities of substitution in production and in final demand are all the same θ . Suppose that there are only shocks to potential factor supplies $\Delta \log \bar{L}$. If future nominal expenditure is fixed, then assuming in addition that $\theta < 1$, there is a unique best and worst equilibrium: for any other equilibrium, $\Delta \log L$ are lower than at the best and higher than at the worst. Furthermore, both in the best and in the worst equilibrium, $\Delta \log L$ are increasing in $\Delta \log \bar{L}$.*

Intuitively, a negative shock to potential factor supply today potentially reduces output tomorrow by reducing resources available for consumption tomorrow. Since nominal expenditures tomorrow are fixed, this raises the price level tomorrow. If the elasticity of substitution θ is less than one, then the increase in the price level tomorrow reduces expenditures on non-shocked factor markets and potentially causes them to become slack.

In Proposition 7, we assume that nominal expenditures in the final period are fixed. If instead we assume that the nominal price level in the future is fixed, rather than nominal expenditures, then Proposition 7 applies regardless of the value of the elasticity of substitution θ .

Appendix C Some Downward Wage Flexibility

In practice, we might imagine that wages can fall albeit not by enough to clear the market. The possibility that wages may fall obviously has important implications for inflation.

Indeed, we show that with shocks to the sectoral composition of demand, and even without shocks to aggregate demand, we can get simultaneous reductions in output *and* inflation.

For each factor $f \in \mathcal{L}$, suppose the following conditions hold

$$\frac{L_f}{\bar{L}_f} = \begin{cases} \left(\frac{w_f}{\bar{w}_f}\right)^{\phi_f}, & \text{if } w_f \leq \bar{w}_f, \\ 1, & \text{if } w_f > \bar{w}_f. \end{cases}$$

The parameter ϕ_f controls the degree of downward wage flexibility. If $\phi_f = \infty$, then the wage is perfectly rigid downwards. If $\phi_f = 0$, then the wage is fully flexible, and we recover the neoclassical case.

$$\left(\frac{w_f}{\bar{w}_f} - \left(\frac{L_f}{\bar{L}_f}\right)^{\frac{1}{\phi_f}}\right)(L - \bar{L}_f) = 0, \quad L_f \leq \bar{L}_f, \quad \left(\frac{L_f}{\bar{L}_f}\right)^{\frac{1}{\phi_f}} \leq \frac{w_f}{\bar{w}_f}.$$

C.1 Generalizing the Results

The only change to Proposition 1 is that we now have

$$d \log Y = \sum_{i \in \mathcal{N}} \lambda_i d \log A_i + \sum_{f \in \mathcal{G}} \lambda_f d \log \bar{L}_f + \sum_{f \in \mathcal{L}} \frac{\phi_f}{1 + \phi_f} \lambda_f \min \{d \log \lambda_f + d \log E - d \log \bar{L}_f, 0\},$$

and the only change to Proposition 2 is that we now have

$$d \log L_f = \begin{cases} \frac{\phi_f}{1 + \phi_f} (d \log \lambda_f + d \log E) + \frac{1}{1 + \phi_f} d \log \bar{L}_f & \text{if } f \in \mathcal{D} \\ d \log \bar{L}_f & \text{if } f \in \mathcal{S}. \end{cases} \quad (\text{C.1})$$

C.2 Illustrative Example

We now construct an example showing how allowing for some degree of downward wage flexibility allows the model to generate a recession *and* deflation at the same time, without relying on aggregate demand shocks. We return to the example of Section 3.4. However, this time, suppose that wages have some degree of downward flexibility $0 < \phi < \infty$ common across all factor markets $f \in \mathcal{L}$.

We now get

$$d \log Y = \lambda_S d \log \bar{L}_S + \lambda_{\mathcal{D}} d \log L_{\mathcal{D}},$$

where $\lambda_{\mathcal{D}} = \sum_{f \in \mathcal{D}} \lambda_f = 1 - \lambda_S$ is the total share of the demand-constrained factors and $d \log L_{\mathcal{D}}$ is the “representative” employment reduction in the demand-constrained sectors

$$d \log L_{\mathcal{D}} = \sum_{f \in \mathcal{D}} \frac{\lambda_f}{\lambda_{\mathcal{D}}} d \log \bar{L}_f < \sum_{f \in \mathcal{D}} \frac{\lambda_f}{\lambda_{\mathcal{D}}} d \log \bar{L}_f = d \log \bar{L}_{\mathcal{D}}.$$

In turn, this employment reduction is given as a function of the change $d \log \lambda_S$ in the share of the supply-constrained sectors by

$$\lambda_{\mathcal{D}} d \log L_{\mathcal{D}} = -\frac{\phi}{1+\phi} \lambda_S d \log \lambda_S + \frac{1}{1+\phi} \lambda_{\mathcal{D}} d \log \bar{L}_{\mathcal{D}},$$

and the the change $d \log \lambda_S$ in the share of the supply-constrained sectors is given by

$$\lambda_S d \log \lambda_S = \frac{\lambda_S d \log \omega_{0S} - (1 - \theta) \lambda_S (1 - \lambda_S) \left[d \log \bar{L}_S - \frac{1}{1+\phi} d \log \bar{L}_{\mathcal{D}} \right]}{1 - \frac{\phi}{1+\phi} (1 - \theta) (1 - \lambda_S)}.$$

Starting with the last equation, we see that once again, the share of supply-constrained factors increases if the shock to the sectoral composition of demand redirects expenditure towards these factors or if the labor shocks for those factors is larger than the ones hitting the demand-constrained factors. This reduces the shares of demand-constrained factors, creates unemployment, and further reduces output through Keynesian effects. Indeed, putting everything together, we get

$$d \log Y = \lambda_S d \log \bar{L}_S + \frac{\frac{\phi}{1+\phi} (1 - \theta) \lambda_S (1 - \lambda_S) d \log \bar{L}_S + \left(1 - \frac{1}{1+\phi} (1 - \theta) \right) \lambda_{\mathcal{D}} d \log \bar{L}_{\mathcal{D}} - \frac{\phi}{1+\phi} \theta \lambda_S d \log \omega_{0S}}{1 - \frac{\phi}{1+\phi} (1 - \theta) (1 - \lambda_S)}.$$

The difference between the case where wages have some downward flexibility ($\phi < \infty$) and the case where they do not ($\phi = \infty$) is that now the wages of the demand-constrained factors falls, and this mitigates the increase in unemployment and the reduction in output. However, there is also a countervailing amplification effect: the labor supply shocks to the demand-constrained factors now also matter. This is because these shocks now reduce the wages of the demand-constrained factors, which further redirects expenditure away from them because of complementarities, and further reduces employment of the

demand-constrained factors. Of course, allowing for some degree of wage flexibility can endogenously change the sets of supply-constrained and demand-constrained factors, and so we do not push the comparison any further.

Instead, we turn our attention to inflation. Using $d \log p^Y = d \log E - d \log Y$, the effect on inflation is

$$d \log p^Y = -\frac{1}{1+\phi} d \log \lambda_S - \lambda_S d \log \bar{L}_S - \frac{1}{1+\phi} \lambda_D d \log \bar{L}_D.$$

The first term is negative, since the share of supply-constrained factors expands in response to the negative demand shock, capturing the fact that as demand switches to supply-constrained factors, the price of sticky sectors starts to decline, generating deflation. In the simple case where there are no negative supply shocks $d \log \bar{L} = 0$ but the sectoral composition of demand has shifted, we get that output *and* inflation both fall.

Appendix D More Examples

In this section, we use some analytical examples to show how the network structure can matter. We show how shocks to the sectoral composition of demand and substitutability in supply chains can also act to reduce output. Throughout all these examples, we assume that the intertemporal elasticity of substitution is $\rho = 1$ so that nominal expenditure is exogenous $d \log E = d \log \zeta$.

D.1 Cobb-Douglas Economy

We first consider how demand shocks affect output and employment in a Cobb-Douglas production-network economy where all elasticities of substitution in production and in final demand are equal to one ($\theta_j = 1$ for all j). This example recalls findings in Baqaee (2015). We allow for shocks to productivities $d \log A_i$, labor supplies $d \log \bar{L}_f$, sectoral composition of demand $d \log \omega_{0i}$, and aggregate demand $d \log \zeta$.

Proposition 2 implies that factor shares change only due to changes in the sectoral composition of demand:

$$d \log \lambda_f = \text{Cov}_{\Omega^{(0)}} \left(d \log \omega_0, \frac{\Psi^{(f)}}{\lambda_f} \right) = \sum_j \Omega_{0j} d \log \omega_{0j} \frac{\Psi_{jf}}{\lambda_f}.$$

The parameter Ψ_{jf} is a network-adjusted measure of use factor f by producer j . The covariance captures the fact that a shock that redirects expenditure away from j reduces the share of factor f if j is more intensive in its use of factor f than the rest of the economy.

Plugging back into Proposition 1 yields response of output

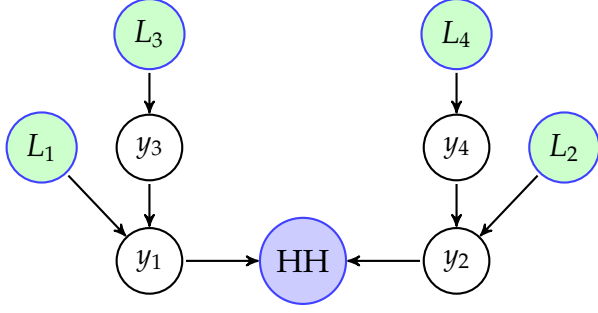
$$\begin{aligned}
 d \log Y = & \underbrace{\sum_{i \in N} \lambda_i d \log A_i + \sum_{f \in \mathcal{G}} \lambda_f d \log \bar{L}_f}_{\Delta \text{ potential output}} \\
 & + \underbrace{\sum_{f \in \mathcal{L}} \lambda_f \min \left\{ \text{Cov}_{\Omega^{(0)}} \left(d \log \omega_0, \frac{\Psi_{(f)}}{\lambda_f} \right) + d \log \zeta - d \log \bar{L}_f, 0 \right\}}_{\Delta \text{ potential output}}.
 \end{aligned}$$

The terms on the first line summarize the impact of the shock if the economy were neoclassical with no downward nominal wage rigidity. The terms on the second line are negative and capture the additional endogenous reduction in output through Keynesian channels: output is additionally reduced if the sectoral composition of demand shifts away from sectors whose network-adjusted use of labors with small shocks is high, or if there is a negative aggregate demand shock. Conditional on shares λ_f , the input-output network matters *only* in so far as it translates changes in household demand into changes in factor demands.

In the Cobb-Douglas example, demand shocks $d \log \omega_0$ and $d \log \zeta$ only propagate backward along supply chains to cause unemployment upstream. On the other hand, supply shocks $d \log \bar{L}_f$ and $d \log A_i$ only propagate forward along supply chains but do not cause any unemployment downstream. In fact, since these shocks do not change factor shares, supply shocks do not cause any unemployment in any of the factors, and so these shocks do not trigger the Keynesian channels. The next example shows how deviating from Cobb-Douglas changes these conclusions.

D.2 Substitutable Supply Chains

Our second example shows how production networks can feature Keynesian unemployment in response to negative supply shocks even without complementarities. However, doing so requires having non-uniform elasticities of substitution (otherwise network-irrelevance applies). In particular, once elasticities of substitution are non-uniform, labor



$$\begin{aligned}
(Y/\bar{Y})^{\frac{\theta_0-1}{\theta_0}} &= \bar{\lambda}_1(y_1/\bar{y}_1)^{\frac{\theta_0-1}{\theta_0}} + \bar{\lambda}_2(y_2/\bar{y}_2)^{\frac{\theta_0-1}{\theta_0}}, \\
y_i/\bar{y}_i &= L_i/\bar{L}_i, \quad i \in \{3, 4\}, \\
y_i/\bar{y}_i &= (L_i/\bar{L}_i)^{1-\omega}(y_{i+2}/\bar{y}_{i+2})^\omega, \quad i \in \{1, 2\} \\
L_i &= \min\{\lambda_i E, \bar{L}_i\}, \\
w_i &= \max\{\lambda_i E/\bar{L}_i, \bar{w}_i\}.
\end{aligned}$$

Figure D.1: Horizontal Economy. The arrows represent the flow of resources for production. Each sector has its own factor market.

supply shocks can create unemployment upstream and downstream. In contrast to Example 3.4, where complementarities create unemployment in the non-shocked supply chains, in this example, substitutabilities create unemployment *within* the shocked supply chain. We assume away productivity shocks and demand shocks so that $d \log A_i = 0$ for all i , $d \log \omega_{0j} = 0$ for all j , and $d \log \zeta = 0$.

We consider the example in Figure D.1, where the household consumes the output of sectors 1 and 2 with elasticity of substitution $\theta_0 > 1$. The initial expenditure shares are λ_1 and $\lambda_2 = 1 - \lambda_1$ for sectors 1 and 2 respectively. The two downstream sectors have Cobb-Douglas production functions combining sector-specific labor with an upstream input, with respective shares $1 - \omega$ and ω . The upstream supplier of 1 is 3 and the one for 2 is 4. The two upstream suppliers produce using industry-specific labor. The sales shares of sector 3 and 4 are given by $\lambda_3 = \omega\lambda_1$, and $\lambda_4 = \omega\lambda_2$. The factor shares of labors in the different sectors are given by $(1 - \omega)\lambda_1$, $(1 - \omega)\lambda_2$, λ_3 , and λ_4 . We denote by p_i the price of i and by w_i the wage of workers in i .

We will only consider negative labor supply shocks $d \log \bar{L}_1 \leq 0$ and $d \log \bar{L}_3 \leq 0$ to 1 and 3, and we will maintain the assumption that $d \log \bar{L}_2 = d \log \bar{L}_4 = 0$. Hence the quantity of 1 will decrease, its relative price will increase, and because $\theta_0 > 1$, consumers will substitute expenditure towards good 2. This in turn implies that wages in 2 and 4 will increase. There will not be any unemployment in 2 and 4. However, there may be unemployment in 1 and/or 3 and we focus our attention on these sectors.

Preliminaries. To conduct the analysis, we rely on Proposition 2 which implies that changes in the sales share of sector 1 are given by

$$d \log \lambda_1 = (\theta_0 - 1)(1 - \lambda_1)(d \log p_2 - d \log p_1), \quad (\text{D.1})$$

where $d \log p_1 = (1 - \omega)d \log w_1 + \omega d \log p_3$ and $d \log p_3 = d \log w_3$. Changes in the sales share of sector 2 are then given by

$$d \log \lambda_2 = -\frac{\lambda_1}{1 - \lambda_1} d \log \lambda_1, \quad (\text{D.2})$$

and since $d \log y_2 = 0$ and $d \log E = 0$, we also have $d \log p_2 = d \log \lambda_2$. Finally, we have $d \log \lambda_3 = d \log \lambda_1$ and $d \log \lambda_2 = d \log \lambda_4$.

Negative downstream labor supply shock. To start with, suppose that $d \log \bar{L}_1 < d \log \bar{L}_3 = 0$. That is, the downstream producer in supply chain 1 is negatively affected.

Then the only equilibrium features

$$d \log \lambda_3 - d \log \bar{L}_3 < d \log w_3 = 0 < d \log \lambda_1 - d \log \bar{L}_1 = d \log w_1.$$

The wage in sector 1 increases and the wage in sector 3 hits its downward rigidity constraint. There is full employment in sector 1 but there is unemployment in sector 3. w_1 increases but w_3 falls. This is because the negative labor supply shock in 1 causes the price of 1 to rise, which causes consumers to redirect expenditures away from 1 since $\theta_0 > 1$, which in turn reduces the demand for 1 and for 3.

This can be verified by substituting these expressions into equation (D.1) and (D.2) to get

$$d \log \lambda_1 = \frac{(\theta_0 - 1)(1 - \lambda_1)(1 - \omega)d \log \bar{L}_1}{1 + (\theta_0 - 1)[(1 - \lambda_1)(1 - \omega) + \lambda_1]} > d \log \bar{L}_1$$

as needed.³⁸

Using this expression for $d \log \lambda_1$ and plugging back into Proposition 1 gives

$$d \log Y = \underbrace{\lambda_1(1 - \omega)d \log \bar{L}_1}_{\Delta \text{ potential output}} + \underbrace{\frac{\omega(\theta_0 - 1)(1 - \lambda_1)}{1 + (\theta_0 - 1)[(1 - \lambda_1)(1 - \omega) + \lambda_1]} \lambda_1(1 - \omega)d \log \bar{L}_1}_{\Delta \text{ output gap}}.$$

Here the first term on the right-hand side coincides with the impact of the negative labor supply shock in the neoclassical model. The second term on the right-hand side is negative and captures the additional reduction in output through Keynesian channel via increases in unemployment in sector 3. Hence, the negative supply shock is transmitted upstream

³⁸In fact, this would continue to be the case even if the upstream supplier was also negatively affected $d \log \bar{L}_3 < 0$, as long as this negative shock is not too large in magnitude.

as a negative demand shock. The shock has its greatest impact for intermediate values of ω , balancing the fact that a higher ω magnifies the negative demand effect but lowers the negative supply effect.

Overall this example shows that, once we deviate from Cobb-Douglas, then expenditure switching causes supply shocks to travel in either direction along the supply chain, reducing employment in other parts of the economy, and amplifying the effect of the original shock.

Negative downstream labor supply shock. Similarly, the shock can be transmitted in the opposite direction. To see this, suppose instead that $d \log \bar{L}_3 < d \log \bar{L}_1 = 0$.

The only equilibrium features

$$d \log \lambda_1 - d \log \bar{L}_1 < d \log w_1 = 0 < d \log \lambda_3 - d \log \bar{L}_3 = d \log w_3.$$

This time, it is the downstream sector that suffers the negative demand shock and experiences unemployment. This can be verified by substituting these expressions into equations (D.1) and (D.2) to get as needed

$$d \log \lambda_1 = \frac{(\theta_0 - 1)(1 - \lambda_1)\omega d \log \bar{L}_3}{1 + (\theta_0 - 1)[(1 - \lambda_1)\omega + \lambda_1]} > d \log \bar{L}_3.$$

Using this expression for $d \log \lambda_3 = d \log \lambda_1$ and plugging back into Proposition 1 gives

$$d \log Y = \underbrace{\lambda_3 d \log \bar{L}_3}_{\Delta \text{ potential output}} + \underbrace{\frac{(\theta_0 - 1)(1 - \lambda_1)(1 - \omega)}{1 + (\theta_0 - 1)[(1 - \lambda_1)\omega + \lambda_1]} \lambda_3 d \log \bar{L}_3}_{\Delta \text{ output gap}}.$$

Once again, the first term on the right-hand side coincides with the impact of the negative labor supply shock in the neoclassical model. The second term on the right-hand side is negative and captures the additional reduction in output through Keynesian channel via increases in unemployment in sector 1. The negative supply shock is now transmitted downstream where it reduces demand.