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ABSTRACT

We study the effects of supply and demand shocks in a general disaggregated model with multiple sectors, factors, and input-output linkages, as well as downward nominal wage rigidities and a zero lower bound constraint. In response to shocks, some sectors become tight and operate at full capacity while others become slack and under-utilize the resources available to them. We use the model to understand how the Covid-19 crisis, an omnibus of various supply and demand shocks, affects output, unemployment, and inflation. Throughout the analysis, we focus on the role of the production network and of the elasticities of substitution. We establish that under some conditions, the details of the production network can be summarized by simple sufficient statistics. We use these sufficient statistics to conduct global comparative statics, and illustrate the intuition for our results using a nonlinear ASAD representation of the model. Negative sectoral supply shocks and shocks to the sectoral composition of demand are necessarily stagflationary, whereas negative aggregate demand shocks are deflationary. The effects of the former are stronger and the effects of the latter are weaker with stronger complementarities in production and in consumption. These shocks interact and are amplified or mitigated through nonlinearities. We quantify our results using disaggregated data from the U.S.

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1 Introduction

The outbreak of Covid-19 is an unusual macroeconomic shock. It cannot be easily categorized as a supply shock or as a demand shock, and it does not affect every part of the economy in the same way. While some sectors of the economy, like food retail and consumer manufacturing are running into supply constraints and struggling to keep up with demand, other sectors like commercial manufacturing, restaurants, and many service industries are laying off workers to reduce excess capacity.[1]

To analyze this divergent situation, we use a general disaggregated model and aggregate up from the micro level to the macro level. We allow for an arbitrary number of sectors and factors as well as unrestricted input-output linkages and elasticities of substitution. We incorporate downward nominal wage rigidities and a zero lower bound constraint.

We model the outbreak of the pandemic as a combination of different shocks. One the one hand, the epidemic reduces the quantity of factors available, as workers withdraw from the labor force due to lock-downs or a reduced willingness to work. On the other hand, the epidemic reduces productivity of firms by forcing them to change their production plans. Finally, the epidemic changes final demand across and within periods, as households reduce spending and rebalance their expenditures.

We characterize the responses of aggregate output, inflation, unemployment to these supply and demand shocks. We also characterize the responses of disaggregated variables like individual sales, prices, and quantities of the different sectors, as well as individual income shares, wages, and Keynesian unemployment of the different factors.[2] The determination of which factor markets become slack and which factor markets become tight is endogenous and highly nonlinear. One of the contributions of the paper is to investigate these forms of nonlinearities.

We show that to a first order, changes in aggregate output depend only on sales-shares-weighted changes in productivities and factors supplied in equilibrium. Changes in the

[1] See Gopinath (2020) for a discussion of why the Covid-19 crisis should be thought as a combination of supply and demand shocks.

[2] Keynesian unemployment measures the amount of slack in a given factor market. When the demand for the factor goes down, and when the wage of the factor cannot fall enough because of downward nominal wage rigidities, some of the available supply of the factor, which may itself have been reduced by a negative supply shock, is not utilized in equilibrium. This happens, for example, if in a labor market, wages cannot fall enough and so some workers would like to work at the going wage but cannot find a job. Measured unemployment in the data reflects not only Keynesian unemployment but also frictional unemployment and classical unemployment, as well as workers who are simply locked down.

quantities of factors supplied in turn depend on the structure of the network and on the elasticities of substitution in production and in final demand. This result, which extends Hulten's theorem (1978) to environments with nominal rigidities, is useful to obtain local comparative statics and also as an intermediate step to derive global comparative statics.

We discuss the impact of each of the shocks in turn, starting with negative factor supply shocks, arising for example from lock-downs or closures. We show that downward nominal wage rigidities always weakly *magnify* the impact on output of negative factor supply shocks: in equilibrium, the shocks can endogenously reduce demand more than supply in some factor markets, push them against their downward nominal wage rigidity constraint, and lead to Keynesian unemployment.

With shocks to factor supply only, we also prove a global *sufficient statistic* result. As long as all elasticities of substitution are the same, then conditional on the initial factor income shares, the structure of the input-output network is *globally* irrelevant. In other words, even though there are many sectors and potentially complex and nonlinear supply chains, this information is entirely summarized by the initial factor income shares as long as the elasticities of substitution are uniform.

For this benchmark case, we provide global comparative statics. We show that as long as the uniform elasticity of substitution is less than one, so that there are *complementarities*, the set of Keynesian equilibria is a *lattice*. This implies that there is a unique best (worst) equilibrium with the minimal (maximal) number of slack factor markets and the minimal (maximal) amount of Keynesian unemployment in each factor market. In the best and worst equilibria, a reduction in the supply of a factor lowers spending on the other factors. Therefore, a negative factor supply shock in one market depresses the other factor markets. Similarly, a binding downward nominal wage rigidity constraint in one factor market pushes other factor markets towards their constraint. For this case with complementarities, we can use a simple algorithm similar to the one used by Vives (1990) or Elliott et al. (2014) to compute the best and worst equilibria.

We illustrate graphically how the equilibrium changes in response to shocks with an aggregate supply (AS) and aggregate demand (AD) diagram. A novelty of our model is that supply shocks do not simply shift the AS curve, but they also change its shape, resulting in apparent instability of the AS relationship. The unstable shape of the AS curve reflects the nonlinearities arising from the interaction of complementarities and occasionally-binding downward nominal wage rigidities.

Our sufficient statistic approach can also be extended, along the lines of Baqaee (2015), to cover the case where there are shocks to the composition of demand within the period, say because households demand more groceries and fewer cruises. In this case, changes in household spending across goods changes nominal expenditures in different factor markets, as final demand is distributed to factors through the input-output network. There is more Keynesian unemployment and a larger recession when households shift their spending away from goods that are intensive, directly and indirectly through supply chains, on potentially rigid factors.

Finally, households' intertemporal preferences may change, so that households may choose to spend more in the future and less today. For example, households may be afraid of consuming certain goods in the midst of a pandemic and might, at given prices, prefer to delay spending until the pandemic has abated when consuming these goods become safe again. This acts like a falling tide, lowering nominal expenditures on all factors simultaneously, potentially causing all factor markets to become slack. We call this a negative aggregate demand shock. Like negative supply shocks and shocks to the composition of demand, negative aggregate demand shocks can cause Keynesian unemployment and reduce output. However, unlike those two shocks which are are necessarily stagflationary, aggregate demand shocks are deflationary.

We also provide local comparative statics outside of the benchmark case with uniform elasticities, though global comparative statics are no longer available. We show how elasticities of substitution interact with the input-output network to redirect demand away from some factors and towards others, causing Keynesian unemployment in factor markets where demand goes down more than supply, and further reducing output.

We use a quantitative input-output model of the US economy to gauge the importance of the various theoretical forces that we identify. We show that in the absence of more disaggregated price and quantity data, unavailable at the time of writing, disaggregated supply and demand shocks cannot all be separately identified. What is clear, however, is that to match a simultaneous reduction in output and inflation, the model must include a negative aggregate demand shock since by themselves, supply shocks and changes in the composition of final demand are stagflationary. The data on aggregate nominal expenditure, hours worked by sector, and inflation, can be rationalized with the combination of a negative aggregate demand shock and either negative shocks to sectoral labor supplies or shocks to the sectoral composition of final demand. In the future, the arrival of more reliable disaggregated data will allow an assessment of the relative contributions of these different shocks. This uncertainty non-withstanding, we show that complementarities generate large Keynesian spillovers across sectors and significantly amplify output losses.

We extend our basic framework to allow for endogenous supply and demand shocks. In the first extension, we show that occasionally-binding credit constraints on households reduce nominal expenditures overall, acting like negative demand shocks. In the second extension, we show that occasionally-binding credit constraints on firms, by causing firm failures, act like negative supply shocks. We show how these endogenous supply and demand shocks can then propagate through the production network and affect output, unemployment, and inflation.

Finally, we analyze policy responses to the Covid-19 shock. We study the effect of fiscal policy, monetary policy, and tax incentives such as payroll tax cuts. We show that indiscriminate fiscal stimulus is wasteful and should instead be targeted towards the sectors that use more intensively, directly and indirectly through the network, the factors that are depressed. We also show that complementarities in production reduce the effectiveness of these different policies.

The outline of the paper is as follows. In Section 2, we set up the model, define the equilibrium and notation, and discuss the shocks. In Section 3, we establish the basic comparative statics of the model and illustrate them with a simple example. In Section 4, we establish our sufficient-statistic result and prove some global comparative statics. In Section 5, we conduct a quantitative exercise to understand the importance of the various mechanisms we have emphasized for the Covid-19 crisis. The rest of the paper contains extensions: in Section 6, we extend the model to include occassionally-binding credit constraints on households; in Section 7, we extend the model to include occassionally-binding credit constraints on firms, causing firm failures; and in Section 8 we investigate monetary, fiscal, and tax policy responses. We conclude in Section 9.

Related Literature

The paper is part of the literature on economic effects of the Covid-19 crisis. Below is a brief description of the most closely related papers. In future versions of the paper, we hope to expand our discussion of other related literature.

Guerrieri et al. (2020) show that under certain configurations of the elasticities of substitution, negative labor supply shocks can cause negative demand spillovers. They focus on substitutabilities whereas we focus on complementarities and the associated nonlinearities from occasionally-binding downward wage rigidity.[3]

Bigio et al. (2020) study optimal policies in response to Covid-19 crisis in a two-sector Keynesian model with a directly affected sector and a sector hit by spillovers. We differ in both focus and framework, since we are not focused on optimal policy and instead try to understand the importance of the production structure.[4]

Fornaro and Wolf (2020) study Covid-19 in a New-Keynesian model where the pandemic is assumed to have persistent effects on productive capacity in the future by lowering aggregate productivity growth. The expected loss in future income reduces aggregate demand. They show that a feedback loop can arise between aggregate supply and aggregate demand if productivity growth in turn depends on the level of economic activity.[5] We differ in that we focus on the effects of current supply disruptions.

Barrot et al. (2020) study the effect of Covid-19 using a quantitative production network with complementarities and detailed administrative data from France. Bodenstein et al. (2020) analyze optimal shutdown policies in a two-sector model with complementarities and minimum-scale requirements. Our approach differs due from these papers due our focus on nominal rigidities and Keynesian effects.

Other economics papers studying the effects of Covid-19 include Eichenbaum et al. (2020a,b), Dingel and Neiman (2020), Berger et al. (2020), Alvarez et al. (2020), Atkeson (2020a,b), Bethune and Korinek (2020), Caballero and Simsek (2020), Faria-e Castro (2020), Gourinchas (2020), Jones et al. (2020), Jorda et al. (2020), Kaplan et al. (2020), Krueger et al. (2020), Jorda et al. (2020), Bodenstein et al. (2020), Barro et al. (2020), Fernández-Villaverde and Jones (2020), Hall et al. (2020), Glover et al. (2020), Bonadio et al. (2020), Acemoglu et al. (2020), and Brinca et al. (2020).

This paper is also related to previous work by the authors, including Baqaee (2015), studying the effect of shocks to the composition of demand in a production network with downward wage rigidity, Baqaee (2018) and Baqaee and Farhi (2020a), studying the effects

[3]The economics of these two cases are different because with substituabilities, a negative labor supply shock reduces the share of that labor and reduces aggregate nominal expenditure through intertemporal substitution, if intertemporal substitution outpaces intratemporal substitution, this reduces demand for the other labors despite the increase in their shares. With complementarities instead, the propagation of the shock is not driven by intertemporal substitution but by the fact that a negative labor supply shock increases the share of that labor, which in turn reduces the demand for the other labors; the same logic applies to the endogenous reductions in labor induced by the initial shock, which further reduce the demand for the other labors, etc.

[4]Bigio et al. (2020) study a fully dynamic model specified in continuous time, which allows them to analyze how the effects unfold over time.

[5]This could be because of reduced investment in research and development due to a reduced size of the market à la Benigno and Fornaro (2018)

of entry and exit in production networks, Baqaee and Farhi (2019), studying the effects of nonlinearities in neoclassical production networks, and most relatedly, Baqaee and Farhi (2020b), a companion paper to this which studies the nonlinear effects of negative supply and demand shocks associated with the Covid-19 crisis using a neoclassical model.

Our analysis is also related to production network models with nominal rigidities, like Pasten et al. (2017) and Pasten et al. (2019) who study propagation of monetary and TFP shocks in models with sticky prices, Ozdagli and Weber (2017) who study the interaction of monetary policy, production networks, and asset prices, and Rubbo (2020) and La'O and Tahbaz-Salehi (2020) who study optimal monetary policy with sticky prices.

2 Setup

In this section, we set up the basic model. We break the description of the model in two. First, we discuss the intertemporal problem of how the representative household chooses to spend its income across periods. Second, we discuss the intratemporal problem of how a given amount of expenditures is spent across different goods within a period. We then define the equilibrium notion and discuss the shocks that we will be studying.

2.1 Environment and Equilibrium

There are two periods, the present denoted without stars, and the future denoted with stars, and there is no investment.[6] We take the equilibrium in the future as given. As in Krugman (1998) and Eggertsson and Krugman (2012), this is isomorphic to an infinite-horizon model where after an initial unexpected shock in period 1, the economy returns to a long-run equilibrium with market clearing and full employment. We denote the supply of the future composite final-consumption good by $\bar{Y}_*$, its price by $\bar{p}_*^Y$, and future final income and expenditure by $\bar{I}_* = \bar{E}_* = \bar{p}_*^Y \bar{Y}_*$, which are all taken to be exogenous.

We focus on the present, where there are a set of producers $\mathcal{N}$ and a set of factors $\mathcal{G}$ with supply functions L_f. We denote by $\mathcal{N} + \mathcal{G}$ the union of these sets. We abuse notation and also denote the number of producers and factors by $\mathcal{N}$ and $\mathcal{G}$.

[6]We abstract from investment in the main body of the paper in order to keep the exposition manageable. We show in Appendix A how our approach generalizes to environments with investment.

Consumers. The representative consumer maximizes intertemporal utility

$$(1-\beta)\frac{Y^{1-1/\rho}-1}{1-1/\rho}+\beta\frac{Y_*^{1-1/\rho}-1}{1-1/\rho},$$

where ρ is the intertemporal elasticity of substitution (IES) and $\beta \in [0,1]$ captures households' time-preferences. The intertemporal budget constraint is

$$\sum_{i\in\mathcal{N}} p_i c_i + \frac{\bar{p}_*^Y Y_*}{1+i} = \sum_{f\in\mathcal{G}} w_f L_f + \sum_{i\in\mathcal{N}} \pi_i + \frac{\bar{I}_*}{1+i},$$

where p_i and c_i are the price and final consumption of good i, the nominal interest rate is $(1+i)$, the wage and quantity of factor f are w_f and L_f, and π_i is the profit of producer i. The consumption bundle in the present period is given by

$$Y = \mathcal{D}(c_1,\ldots,c_N;\omega_{\mathcal{D}}),$$

a homothetic final-demand aggregator of the final consumptions c_i of the different goods i. The parameter $\omega_{\mathcal{D}}$ is a preference shifter capturing changes in the composition of final demand.

For future reference, we define the price p^Y of the consumption bundle Y by

$$p^Y = \mathcal{P}(p_1,\ldots,p_N;\omega_{\mathcal{D}}).$$

where $\mathcal{P}$ is the dual price index of the quantity index $\mathcal{D}$. We also denote by

$$E = p^Y Y$$

the present final expenditure. In the rest of the paper, we will refer to Y as *output*.[7]

[7] Output here corresponds to welfare. Changes in output and changes in welfare always coincide to the first order, but they do not always coincide to higher orders. Without changes in the preference shifter $\omega_{\mathcal{D}}$, changes in welfare coincide with changes in real GDP at any order. With changes in the preference shifter, changes in changes in welfare do not coincide with changes in real GDP at the second order. See Baqaee and Farhi (2020b) for detailed discussion.

Producers. Producer i maximizes profits

$$\pi_i = \max_{\{y_i\},\{x_{ij}\},\{L_{if}\}} p_i y_i - \sum_{j \in \mathcal{N}} p_j x_{ij} - \sum_{f \in \mathcal{G}} w_f L_{if}$$

using a production function

$$y_i = A_i F_i \left(\left\{ x_{ij} \right\}_{j \in \mathcal{N}}, \left\{ L_{if} \right\}_{f \in \mathcal{G}} \right),$$

where A_i is a Hicks-neutral productivity shifter, y_i is total output, and x_{ij} and L_{if} are intermediate and factor inputs used by i.

Market equilibrium. Market equilibrium for goods is standard. The market for i is in equilibrium if

$$c_i + \sum_{j \in \mathcal{N}} x_{ji} = y_i.$$

Market equilibrium for factors is non-standard, the wages of factors cannot fall below some exogenous lower bound.[8] We say that factor market f is in equilibrium if the following there conditions hold:

$$(w_f - \bar{w}_f)(L_f - \bar{L}_f) = 0, \quad \bar{w}_f \leq w_f, \quad L_f \leq \bar{L}_f,$$

where

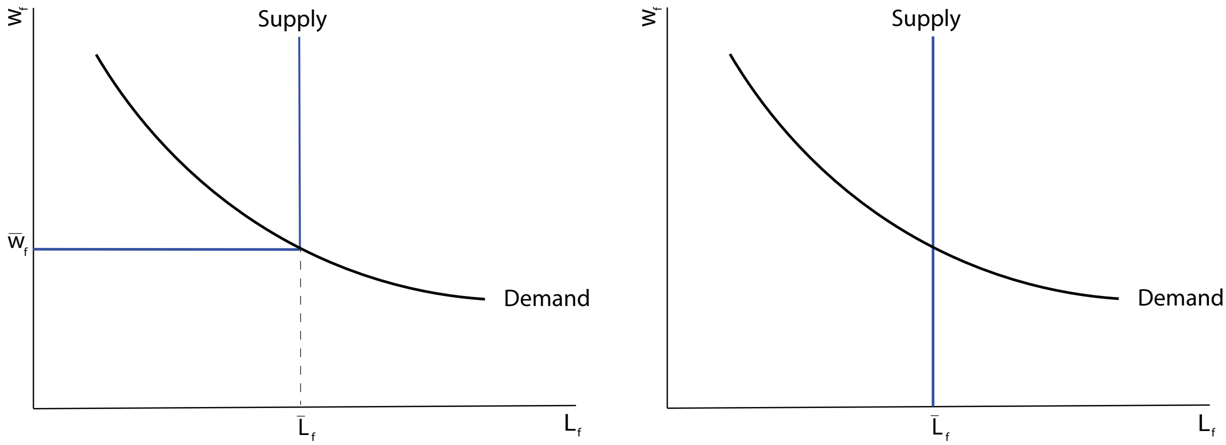
$$L_f = \sum_{i \in \mathcal{N}} L_{if}$$

is the total demand for factor f. The parameters $\bar{w}_f$ and $\bar{L}_f$ are exogenous minimum nominal wage and endowment of the factor.

In words, there are two possibilities. One possibility is $w_f \geq \bar{w}_f$ and full employment of the factor with $L_f = \bar{L}_f$. In this case, we say that the wage is flexible and that the market clears. The other possibility is that $w_f = \bar{w}_f$ and less than full employment of the factor with $L_f \leq \bar{L}_f$. We then say that the market is rigid or slack and that it does not clear.

We only consider two cases: the case where $\bar{w}_f$ is equal to its pre-shock market-clearing value, denoted by the set $\mathcal{L} \subseteq \mathcal{G}$; and the case where $\bar{w}_f = -\infty$, making the wage of f flexible and ensuring the market for f always clears, denoted by the subset $\mathcal{K} \subseteq \mathcal{G}$. For concreteness, we call $\mathcal{K}$ the capital factors and $\mathcal{L}$ the labor factors.

[8] In Appendix B, we extend the model to allow for some downward wage flexibility.



(a) Factors in $\mathcal{L}$ (b) Factors in $\mathcal{K}$

Figure 2.1: Equilibrium in the factor markets.

Of course, these are just names, in practice, one may easily imagine that certain capital markets could also be subject to nominal rigidity. For example, we can use sticky capital as a way to model firm failures: imagine firms take out within-period loans to pay for their variable expenses, secured against their capital income. If the firm's capital income declines in nominal terms, then the firm defaults on the loan, exits the market, and its capital becomes unemployed for the rest of the period.[9]

We denote the endogenous set of flexible factor markets by $\mathcal{F} \subseteq \mathcal{G}$. In other words, $f \in \mathcal{F}$ if, and only if, $L_f = \bar{L}_f$. We denote the set of endogenous factor markets that do not clear (rigid factors) by $\mathcal{R} \subseteq \mathcal{G}$. Hence, $f \in \mathcal{R}$ if, and only if, $L_f < \bar{L}_f$. Of course, $\mathcal{K} \subseteq \mathcal{F}$, and $\mathcal{R} \subseteq \mathcal{L}$. Figure 2.1 illustrates the supply and demand curves in the factor markets.

Equilibrium. Given a nominal interest rate $(1+i)$, factor supplies $\bar{L}_f$, productivities A_i, and demand shifters $\omega_{\mathcal{D}}$, an equilibrium is a set of prices p_i, factor wages w_f, intermediate input choices x_{ij}, factor input choices L_{if}, outputs y_i, and final demands c_i, such that: each producer maximizes its profits subject to its technological constraint; consumers maximize their utility; and the markets for all goods and factors are in equilibrium. Without loss of generality, we normalize $\bar{Y}^* = \bar{Y} = 1$ and $p^Y = 1$.

[9]We build on this observation further in Section 7, where we formally introduce an extensive margin of firm exit, and study the importance of increasing returns to scale.

2.2 Comparative Statics

We provide comparative statics with respect to shocks, starting at an initial equilibrium with full employment of all factors. A natural disaster, like the Covid epidemic, can be captured as a combination of negative supply and demand shocks.

Supply shocks. On the one hand, the pandemic causes a reduction in the available endowment of labor $\bar{L}_f$, driven either by government-mandated shutdowns, deaths, or reduced willingness to work.

Similarly, the epidemic might reduced the productivity A_i of the different producers by changing the way firms can operate, for instance by reducing person-to-person interactions.

Demand shocks. Similarly, the pandemic can also change the composition of final demand today, since at given prices, households may shift expenditure away from some goods like restaurants and cruises, and towards other goods like groceries and healthcare. We model this as a change in the preference shifter $\omega_{\mathcal{D}}$.

On the other hand the pandemic can reduce households' willingness to consume today relative to tomorrow: at given prices, households may choose to consume less during the epidemic and more afterwards. We model this as an increase in the discount factor $\beta/(1-\beta)$.

Finally, in an attempt to offset a negative shock, the monetary authority might lower the nominal interest rate $1 + i$. It might be restricted in its ability to do so by the binding of the zero lower bound (ZLB). We also allow the monetary authority to increase future prices $\bar{p}^Y_*$. Other policies might also raise or lower future output $\bar{Y}_*$.

2.3 Input-Output Definitions

To analyze the model, we define some input-output objects such as input-output matrices, Leontief inverse matrices, and Domar weights associated with any equilibrium. To make the exposition more intuitive, we slightly abuse notation by treating factors with the same notation as goods. For each factor f, we interchangeably use the notation L_{if} or $x_{i(N+f)}$ to denote its use by producer i, the notation L_f or y_f to denote total factor supply, and p_f or w_f to refer to its price or wage. This allows us to add factor supply and demand into the input-output matrix along with the supply and demand for goods. Furthermore, we define final demand as an additional good produced by producer 0 according to the final demand aggregator. We interchangeably use the notation c_i or x_{0i} to denote final consumption of good i. We write $1+\mathcal{N}$ for the union of the sets $\{0\}$ and $\mathcal{N}$, and $1+\mathcal{N}+\mathcal{G}$ for the union of the sets $\{0\}$, $\mathcal{N}$, and $\mathcal{G}$. With this abuse of notation, we can stack every market in the economy into a single input-output matrix that includes the household, the producers, and the factors.

Input-output matrix. We define the input-output matrix to be the $(1+\mathcal{N}+\mathcal{G})\times(1+\mathcal{N}+\mathcal{G})$ matrix Ω whose ijth element is equal to i's expenditures on inputs from j as a share of its total income/revenues

$$\Omega_{ij} \equiv \frac{p_j x_{ij}}{p_i y_i} = \frac{p_j x_{ij}}{\sum_{k\in\mathcal{N}+\mathcal{G}} p_k x_{ik}}.$$

The input-output matrix Ω records the *direct* exposures of one producer to another, forward from upstream to downstream in costs, and backward from downstream to upstream in demand.

Leontief inverse matrix. We define the Leontief inverse matrix as

$$\Psi \equiv (I-\Omega)^{-1} = I + \Omega + \Omega^2 + \dots.$$

The Leontief inverse matrix Ψ records instead the *direct and indirect* exposures through the supply chains in the production network. This can be seen from the fact that $(\Omega^n)_{ij}$ measures the weighted sums of all paths of length n from producer i to producer j.

Nominal expenditure and Domar weights. Recall that nominal expenditure is the total sum of all final expenditures

$$E = \sum_{i\in N} p_i c_i = \sum_{i\in N} p_i x_{0i}.$$

We define the Domar weight λ_i of producer i to be its sales share as a fraction of GDP

$$\lambda_i \equiv \frac{p_i y_i}{E}.$$

Note that $\sum_{i\in N}\lambda_i > 1$ in general since some sales are not final sales but intermediate sales. Note that the Domar weight λ_f of factor f is simply its total income share.

The accounting identity $p_i y_i = p_i x_{0i} + \sum_{j\in\mathcal{N}} p_i x_{ji} = \Omega_{0i}E + \sum_{j\in\mathcal{N}} \Omega_{ji}\lambda_j E$ links the Domar weights to the Leontief inverse via

$$\lambda_i = \Psi_{0i} = \sum_{j\in\mathcal{N}} \Omega_{0j}\Psi_{ji},$$

where $\Omega_{0j} = (p_j x_{0j})/(\sum_{k\in\mathcal{N}+\mathcal{G}} p_k x_{0k}) = (p_j c_j)/E$ is the share of good j in final expenditure.

2.4 Nested-CES Economies

For simplicity, we restrict attention to nested-CES economies. That is, we assume every production function and the final demand function can be written as nested-CES functions (albeit with an arbitrary set of nests). Any nested-CES economy can be written in *standard form*, defined by a tuple $(\bar{\omega}, \theta, F)$. The $(1 + \mathcal{N} + \mathcal{G}) \times (1 + \mathcal{N} + \mathcal{G})$ matrix $\bar{\omega}$ is a matrix of input-output parameters. The $(1+\mathcal{N})\times 1$ vector θ is a vector of microeconomic elasticities of substitution. Each good $i \in \mathcal{N}$ is produced with the production function

$$\frac{y_i}{\bar{y}_i} = \frac{A_i}{\bar{A}_i}\left(\sum_{j\in\mathcal{N}+\mathcal{G}} \bar{\omega}_{ij}\left(\frac{x_{ij}}{\bar{x}_{ij}}\right)^{\frac{\theta_i-1}{\theta_i}}\right)^{\frac{\theta_i}{\theta_i-1}},$$

where x_{ij} are intermediate inputs from j used by i. We represent final demand as the purchase of good 0 from producer 0 producing the final good

$$\frac{y_0}{\bar{y}_0} = \left(\sum_{j\in\mathcal{N}+\mathcal{G}} \bar{\omega}_{0j}\frac{\omega_{0j}}{\bar{\omega}_{0j}}\left(\frac{x_{0j}}{\bar{x}_{0j}}\right)^{\frac{\theta_0-1}{\theta_0}}\right)^{\frac{\theta_0}{\theta_0-1}},$$

where ω_{0j} is a demand shifter. In these equations, variables with over-lines are normalizing constants equal to the values at some initial competitive equilibrium and we then have $\bar{\omega} = \bar{\Omega}$.[10] To simplify the notation below, we think of ω_0 as a $1 \times (1 + \mathcal{N} + \mathcal{G})$ vector with k-th element ω_{0k}.

Through a relabelling, this structure can represent any nested-CES economy with an arbitrary pattern of nests and elasticities. Intuitively, by relabelling each CES aggregator to be a new producer, we can have as many nests as desired.

[10] Note that when mapping the original economy to the re-labeled economy, the different nests in final demand are mapped intro different producers j. To simplify the exposition, we have imposed that there are only demand shocks in the outermost nest mapped to producer 0. It is easy to generalize the results to allow for demand shocks in all the nests corresponding to final demand.

3 Local Comparative Statics

In this section, we describe the comparative statics of the basic model and provide some examples. Our results here are local (first-order) comparative statics. Below in Section 4, we provide global comparative statics in important special cases.

Because of downward wage-rigidity, variables like aggregate output and inflation are not differentiable everywhere. Therefore, our local comparative statics should be understood as holding almost-everywhere. Furthermore, there are potentially multiple equilibria, in which case, local comparative statics should be understood as perturbations of a given locally-isolated equilibrium.

We write $d \log X$ for the differential of an endogenous variable X, which can also be understood as the (infinitesimal) change in an endogenous variable X in response to (infinitesimal) shocks. For example, the supply shocks are $d \log A_i$ and $d \log \bar{L}_f$, and the shocks to the composition of demand are $d \log \omega_{0j}$. We sometimes write them in vector notation as $d \log A$, $d \log \bar{L}$, and $d \log \omega_0$.

3.1 Euler Equation for Output

Log-linearizing the Euler equation gives changes in output $d \log Y$ as a function of changes in the price index $d \log p^Y$ and the shocks

$$d \log Y = -\rho d \log p^Y + d \log \zeta, \tag{3.1}$$

where

$$d \log \zeta = -\rho\Big(d \log(1+i) + \frac{1}{1-\beta} d \log \beta - d \log \bar{p}^Y_*\Big) + d \log \bar{Y}_*$$

is an *aggregate demand* shock. A positive aggregate demand shock can come about from a reduction in the nominal interest rate or the discount factor, or an increase in future prices or output (a proxy for forward guidance).

Equation (3.1) implies that without negative aggregate demand shocks $d \log \zeta = 0$, shocks move output and prices in opposite directions. That is, negative supply shocks or shocks to the sectoral composition of demand are necessarily *stagflationary*. Even without working through the rest of the model, we can already see that negative aggregate demand shocks are necessary in order to produce a reduction in both output and prices.[11]

[11] In section 6, we extend the basic model to include some credit-constrained or hand-to-mouth households,

3.2 Euler Equation for Spending

Changes in nominal expenditure $d\log E$ are similarly given by

$$d\log E = d\log(p^Y Y) = (1-\rho)d\log p^Y + d\log\zeta. \tag{3.2}$$

Recall that ρ is the intertemporal elasticity of substitution (IES). When $\rho > 1$, increases in prices $d\log p^Y > 0$ reduce nominal expenditure as consumers substitute towards the future. Conversely, when $\rho < 1$, increases in prices $d\log p^Y > 0$ increase nominal expenditure as consumers substitute towards the present. When $\rho = 1$, changes in nominal expenditure are exogenously given by the shocks $d\log E = d\log\zeta$. Although our propositions allow for arbitrary values of ρ, we will focus primarily on the case where $\rho = 1$, abstracting from intertemporal substitution.

3.3 Output

We can express changes in output as a function of changes in nominal expenditure and changes in factor shares.

Proposition 1. *Changes in output are given by*

$$\begin{aligned} d\log Y &= \sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log L_f, \\ &= \underbrace{\sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log \bar{L}_f}_{\textit{neoclassical effect}} + \underbrace{\sum_{f\in\mathcal{L}} \lambda_f \min\left\{d\log\lambda_f + d\log E - d\log\bar{L}_f, 0\right\}}_{\textit{Keynesian effect}}. \end{aligned}$$

The first expression for $d\log Y$ shows that a version of Hulten's theorem holds for this economy. In particular, to a first-order, changes in output can only be driven by changes in the productivities $d\log A_i$ weighted by their producer's sales share λ_i, or by changes in the quantities of factors $d\log L_f$ weighted by their income shares λ_f.[12]

The second expression uses the fact that while changes in capitals $f \in \mathcal{K}$ are exogenous with $d\log L_f = d\log\bar{L}_f$, changes in labors $f \in \mathcal{L}$ are endogenous with $d\log L_f = d\log\bar{L}_f + \min\left\{d\log\lambda_f + d\log E - d\log\bar{L}_f, 0\right\} \le d\log\bar{L}_f$. Therefore, in the Keynesian model

and this provides an endogenous mechanism for delivering negative aggregate demand shocks.

[12]This expression also shows that changes in the composition of demand within the period $d\log\omega_0$, or changes in aggregate demand $d\log\zeta$, can only change output through changes in the quantities of factors.

with downward nominal wage rigidities, negative shocks can be amplified relative to a neoclassical economy with flexible wages where all factor markets clear at full employment. To a first-order, the response of output in the Keynesian model is the same as the neoclassical model, plus a negative adjustment for Keynesian unemployment which depends on endogenous changes in nominal expenditure $d\log E$ and factor income shares $d\log\lambda_f$. It is only through the determination of these endogenous sufficient statistics that the structure of the network matters.

3.4 Shares, Prices, and Factor Employment

We make use of the following notation. For a matrix M, we denote by $M_{(i)}$ its i-th row by $M^{(j)}$ its j-th column. We write $Cov_{\Omega^{(j)}}(\cdot,\cdot)$ to denote the covariance of two vectors of size $1+\mathcal{N}+\mathcal{G}$ using the j-the row of the input-ouput matrix $\Omega^{(j)}$ as a probability distribution.

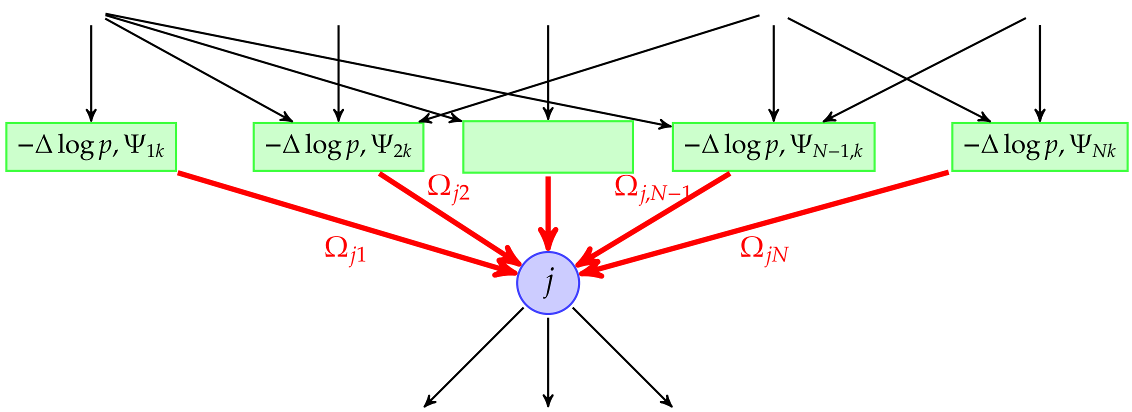


Figure 3.1: Graphical illustration of the IO covariance operator.

Proposition 2. *Changes in sales and factor shares are given by*

$$d\log\lambda_k = \theta_0 Cov_{\Omega^{(0)}}\left(d\log\omega_0, \frac{\Psi_{(k)}}{\lambda_k}\right) + \sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j - 1) Cov_{\Omega^{(j)}}\left(\sum_{i\in\mathcal{N}} \Psi_{(i)} d\log A_i - \sum_{f\in\mathcal{G}} \Psi_{(f)}\left(d\log\lambda_f - d\log L_f\right), \frac{\Psi_{(k)}}{\lambda_k}\right)$$

almost everywhere, where changes in factor employments are given by

$$d\log L_f = \begin{cases} d\log \bar{L}_f, & \text{for} \quad f \in \mathcal{K}, \\ \min\left\{d\log\lambda_f + d\log E, d\log\bar{L}_f\right\}, & \text{for} \quad f \in \mathcal{L}. \end{cases}$$

We can break down these equations into forward and backward propagation equations. Forward propagation equations describe changes in prices:

$$d\log p_k = -\sum_{i\in\mathcal{N}} \Psi_{ki} d\log A_i + \sum_{f\in\mathcal{G}} \Psi_{kf}\left(d\log\lambda_f + d\log E - d\log L_f\right).$$

Changes in prices propagate downstream (forward) through costs. A negative productivity shock $\Delta\log A_i$ to a producer i upstream from k increases the price of k in proportion to how much k buys from i directly and indirectly as measured by Ψ_{ki}. Similarly an increase $d\log w_f = d\log\lambda_f - d\log L_f + d\log E$ in the wage of factor f increases the price of k in proportion to the direct and indirect exposure of k to f.

Backward propagation equations describe changes in sales or factor shares:

$$d\log\lambda_k = \theta_0 Cov_{\Omega^{(0)}}\left(d\log\omega_0, \Psi_{(k)}/\lambda_k\right) + \sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j - 1) Cov_{\Omega^{(j)}}\left(-d\log p, \Psi_{(k)}/\lambda_k\right).$$

Changes in sales propagate upstream (backward) through demand. The first term on the right-hand side $\theta_0 Cov_{\Omega^{(0)}}(d\log\omega_0, \Psi_{(k)}/\lambda_k)$ on the right-hand side is the direct effect of shocks to the composition of final demand on the sales of k. These shocks directly increase the share of k if they redirect demand towards goods j that have high direct and indirect exposures to k relative to the rest of the economy as measured by Ψ_{jk}/λ_k to k .

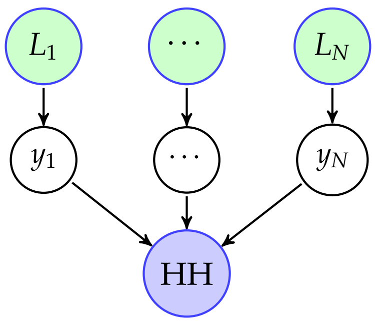
The second term $\sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j - 1) Cov_{\Omega^{(j)}}(-d\log p, \Psi_{(k)}/\lambda_k)$ on the right-hand side captures the changes in the sales of i from substitutions by producers j downstream from k. This is depicted in Figure 3.1. If producer j has an elasticity of substitution θ_j below one so that its inputs are complements, then it shifts its expenditure towards those inputs l with higher price increases $d\log p_l$, and this increases the demand for k if those goods l buy a lot from k directly and indirectly relative to the rest of the economy as measured by Ψ_{lk}/λ_k. These expenditure-switching patterns are reversed when θ_j is above one (the inputs of j are substitutes). When θ_j is equal to one (the inputs of j are Cobb-Douglas) these terms disappear.

Combining the backward and forward propagation equations results in Proposition

2. Note that once a factor market f becomes slack, the change in its income share $d\log\lambda_f$ becomes irrelevant for changes in all the other sales and factor shares since they then translate one for one into changes in employment of the factor $d\log L_f$ and leave its wage unchanged with $d\log w_f = 0$.

3.5 A (Somewhat) Universal Example

In Appendix D, we work through some illustrative examples showing how supply and demand shocks can propagate up and down supply chains to cause Keynesian spillovers across sectors. However, for now, we instead focus on a simpler example which will nevertheless prove to contain an element of universality.



$$Y/\bar{Y} = \left(\sum_i \bar{\lambda}_i (y_i/\bar{y}_i)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},$$
$$y_i/\bar{y}_i = L_i/\bar{L}_i,$$
$$L_i = \min\{\bar{L}_i, \lambda_i E/\bar{w}_i\}$$
$$w_i = \max\{\lambda_i E/\bar{L}_i, \bar{w}_i\}.$$

Figure 3.2: Horizontal Economy. The arrows represent the flow of resources for production. Each sector has its own factor market.

Consider the horizontal economy depicted in Figure 3.2. We call it horizontal because there are no intermediate inputs. Each sector produces linearly with its own labor and sells directly to the household who substitutes across them with an elasticity of substitution $\theta < 1$. Labor cannot be reallocated across sectors, and so there are as many labor markets as there are sectors. We therefore refer to a a sector or to its labor market interchangeably. The different labor markets are all potentially rigid ($\mathcal{L} = \mathcal{G}$ and $\mathcal{K} = \emptyset$). We assume that the intertemporal elasticity of substitution is $\rho = 1$. We introduce negative labor supply shocks $d\log\bar{L}_f \leq 0$ in the different sectors. To start with, suppose that there are neither shocks to the composition of demand ($d\log\omega_{0j} = 0$) nor aggregate demand shocks ($d\log\zeta = 0$).

Recall that $\mathcal{F}$ and $\mathcal{R}$ are the equilibrium sets of flexible and rigid factors. We give comparative statics for a given $\mathcal{F}$ and $\mathcal{R}$. We then give conditions for these sets of flexible and rigid factors to indeed arise in equilibrium.

Define the average negative labor supply shock to the flexible factors

$$d\log\bar{L}_{\mathcal{F}} = \sum_{f\in\mathcal{F}} \frac{\lambda_f}{\lambda_{\mathcal{F}}} d\log\bar{L}_f,$$

where $\lambda_{\mathcal{F}} = \sum_{f\in\mathcal{F}} \lambda_f$. Similarly, define average employment change in the rigid factors

$$d\log L_{\mathcal{R}} = \sum_{f\in\mathcal{R}} \frac{\lambda_f}{\lambda_{\mathcal{R}}} d\log L_f < \sum_{f\in\mathcal{R}} \frac{\lambda_f}{\lambda_{\mathcal{R}}} d\log\bar{L}_f = d\log\bar{L}_{\mathcal{R}},$$

where $\lambda_{\mathcal{R}} = \sum_{f\in\mathcal{R}} \lambda_f$. Keynesian unemployment is given by $d\log L_{\mathcal{R}} - d\log\bar{L}_{\mathcal{R}}$.

By Proposition 2, the change in the share of a factor f is given by

$$d\log\lambda_f = (\theta-1)\Big(\sum_{g\in\mathcal{F}} \lambda_g \left(d\log\lambda_g - d\log\bar{L}_g\right) - \left(d\log\lambda_f - L_f\right)\Big).$$

Summing across all flexible factors and solving the resulting linear equation gives changes in total spending on flexible factors

$$\lambda_{\mathcal{F}} d\log\lambda_{\mathcal{F}} = -\frac{(1-\theta)(1-\lambda_{\mathcal{F}})\lambda_{\mathcal{F}} d\log\bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}.$$

This can be used to deduce average changes in employment in the rigid factors

$$\lambda_{\mathcal{R}} d\log L_{\mathcal{R}} = \sum_{f\in\mathcal{R}} \lambda_f d\log L_f = \sum_{f\in\mathcal{R}} \lambda_f d\log\lambda_f = -\sum_{f\in\mathcal{F}} \lambda_f d\log\lambda_f = -\lambda_{\mathcal{F}} d\log\lambda_{\mathcal{F}}.$$

A negative effective supply shock $d\log\bar{L}_{\mathcal{F}} < 0$ increases the shares of the flexible factors. The shock increases the wages of the flexible factors, which redirects expenditure towards their sectors because of complementarities, which further increases the wages of the flexible factors, etc. ad infinitum. Of course, if spending on flexible sectors increases, then spending on rigid sectors decreases, and this reduces employment in those sectors because wages cannot fall.

Using Proposition 1, we can see that Keynesian channels amplify the output effect of the negative supply shocks to the flexible factors since

$$d\log Y = \lambda_{\mathcal{F}} d\log\bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d\log L_{\mathcal{R}} = \frac{\lambda_{\mathcal{F}} d\log\bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}. \quad (3.3)$$

The direct impact on output of the negative supply shock to the flexible factors is given by $\lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}}$, and the amplification of this shock through Keynesian channels is given by the multiplier $1/[1-(1-\theta)(1-\lambda_{\mathcal{F}})]$. Naturally, amplification is stronger, the lower is the elasticity of substitution $\theta < 1$. Amplification is also stronger when the share of the flexible factors $\lambda_{\mathcal{F}}$ is low.

We now go back and check that our conjectured set of flexible factors is indeed the equilibrium set of flexible factors. A potentially rigid factor f is actually rigid in equilibrium if, and only if,

$$d \log \bar{L}_f > \frac{(1-\theta)\lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}.$$

That is, as long as the negative shock to factor f is sufficiently small in magnitude compared to the average shock affecting the flexible part of the economy. This condition is harder to satisfy the smaller is the set of flexible factors $\lambda_{\mathcal{F}}$ and the higher is the elasticity of substitution θ. In particular, if we had assumed that sectors were substitutes $\theta \geq 1$ instead of being complements with $\theta < 1$, then this condition could not be satisfied and all factors would be flexible.

This condition also shows that Keynesian channels amplify the shock compared to the neoclassical economy with flexible wages since

$$d \log Y = \underbrace{\lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d \log \bar{L}_{\mathcal{R}}}_{\text{neoclassical effect}} + \underbrace{\sum_{f \in \mathcal{R}} \lambda_f \Big(\frac{(1-\theta)\lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} - d \log \bar{L}_f \Big)}_{\text{Keynesian effect}},$$

where the Keynesian effect is always negative. Note that here, as in Proposition 1, Keynesian amplification is defined relative to the output reduction that would take place in response to the negative factor supply shocks *to all the factors* in a neoclassical economy. This notion is related to but different from the Keynesian amplification of the negative factor supply shocks to the flexible factors, that we used to discuss equation 3.3. This different notion is defined relative to the output reduction that would take place in response to the negative factor supply shocks *to the flexible factors* in a neoclassical economy, and it is also informative since the factor supply shocks to the rigid factors have no impact on the equilibrium because these markets are slack.

If in addition to the negative labor supply shocks, there were also shocks to the composition of demand $d \log \omega_0$ and to aggregate demand $d \log \zeta < 0$, then the response of output would become

$$d\log Y = \frac{\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} - \frac{\theta\lambda_{\mathcal{F}} d\log \omega_{0\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} + \left(1 - \frac{(1-\theta)\lambda_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}\right)(1-\lambda_{\mathcal{F}}) d\log \zeta,$$

where $d\log \omega_{0\mathcal{F}} = \sum_{f\in\mathcal{F}} (\lambda_f/\lambda_{\mathcal{F}}) d\log \omega_{0f}$. The second term on the right-hand side captures the fact that if consumers redirect expenditure towards flexible factors and away from rigid factors, then this exacerbates Keynesian unemployment in rigid factors and further reduces output. The third term is the effect of the negative aggregate demand shock. The direct effect of the negative aggregate demand shock, captured by $(1-\lambda_{\mathcal{F}})d\log\zeta$, is to lower employment in rigid factor markets and to reduce output. This direct effect is mitigated because the shock lowers the prices of flexible factors, bringing them closer to those of rigid factors, and triggering expenditure switching away from flexible factors and towards rigid ones, as captured by $-[(1-\theta)\lambda_{\mathcal{F}}/(1-(1-\theta)(1-\lambda_{\mathcal{F}}))](1-\lambda_{\mathcal{F}})d\log\zeta$.

4 A Benchmark with Simple Network Sufficient Statistics

Proposition 2 shows that in general, detailed information about the input-output network is required to compute counterfactuals. However, we now show that in a benchmark case, this information can be summarized by a small number of sufficient statistics, namely the initial factor income shares. In this benchmark, the disaggregated nature of the model remains critical because the different factor markets endogenously experience different cyclical conditions.

This benchmark case with simple network sufficient statistics is useful for several reasons. First, it shows that the analysis in Example 3.5 applies much more broadly to economies with complex input-output networks. Second, it clarifies exactly what ingredients are necessary for the production network to matter beyond the initial factor shares. Third, it allows us to obtain not only local but also global comparative statics.

To obtain the benchmark, we assume that the intertemporal elasticity of substitution is $\rho = 1$, that all the elasticities in production and in final demand are the same with $\theta_j = \theta$ for all $j \in 1 + \mathcal{N}$, and that there are no productivity shocks $\Delta \log A = 0$. To start with, we assume there no shocks to the composition of demand $\Delta\log\omega_0 = 0$. We can then write changes in output $\Delta \log Y(\Delta\log\bar{L}, \Delta\log\zeta, \bar{\Omega})$ as a set-valued function of discrete labor supply shocks $\Delta\log\bar{L}$, aggregate demand shocks $\Delta\log\zeta$, and the initial input-output matrix $\bar{\Omega}$. We use Δ to denote discrete global changes to distinguish them from infinitesimal local changes which we denote with d. We show how the results extend to shocks to the composition of final demand in Section 4.7.

4.1 Global Sufficient Statistics

The next proposition shows that Y depends on the input-output network $\bar{\Omega}$ *only* through the initial factor shares $\bar{\lambda}_f$ for $f \in \mathcal{G}$.

Proposition 3 (Global Sufficient Statistics)**.** *Suppose that the intertemporal elasticity of substitution is $\rho = 1$ and that the elasticities of substitution in production and in final demand are all the same with $\theta_j = \theta$ for every $j \in 1 + \mathcal{N}$. Suppose that there are only factor supply shocks $\Delta \log \bar{L}$ and aggregate demand shocks $\Delta \log \zeta$ but no productivity shocks and no shocks to the composition of demand. Then*

$$\Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}) = \Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}')$$

for every $\bar{\Omega}$ and $\bar{\Omega}'$ such that $\bar{\lambda}_f = \bar{\Psi}_{0f} = \bar{\Psi}'_{0f} = \bar{\lambda}'_f$ for every $f \in \mathcal{G}$. More generally, given the shocks, the initial factor income shares $\bar{\lambda}_f$ are sufficient statistics for equilibrium changes in aggregate output $\Delta \log Y$, the aggregate price index $\Delta \log p^Y$, factor wages $\Delta \log w_f$, factor quantities $\Delta \log L_f$, and factor income shares $\Delta \log \lambda_f$.

An implication is that the local comparative statics for the horizontal economy in Section 3.5 actually apply much more generally. In particular, they apply to *any* production network as long as the elasticities of substitution in production and final demand are uniform. Another implication of this proposition is that the network can only matter globally beyond the initial factor shares if: the elasticities of substitution are different or if there are shocks to the composition of demand or to productivities.

To prove Proposition 3, we show that the equilibrium conditions do not depend on the input-output matrix $\bar{\Omega}$ beyond the initial factor shares $\bar{\lambda}$. First, note that real GDP is given by deflating changes in nominal GDP by the price index

$$\Delta \log Y = \Delta \log \zeta - \Delta \log p^Y.$$

Next, we can show that changes in the price index depend only on the changes in the wages of different factors, since every good is ultimately made up of factors, so that

$$\Delta \log p^Y = \frac{1}{1-\theta} \log \left(\sum_{f \in \mathcal{G}} \bar{\lambda}_f \exp \Big((1-\theta) \Delta \log w_f \Big) \right).$$

Finally, the wage for each factor is determined by the interaction of factor supply and demand, and factor demand can be shown to be isoelastic with elasticity θ in the price of each factor relative to the GDP deflator, so that

$$\Delta \log w_f = \begin{cases} \frac{1}{\theta} \left(\Delta \log \zeta - \Delta \log \bar{L}_f \right) + \frac{\theta-1}{\theta} \Delta \log p^Y, & \text{for} \quad f \in \mathcal{K}, \\ \max \left\{ \frac{1}{\theta} \left(\Delta \log \zeta - \Delta \log \bar{L}_f \right) + \frac{\theta-1}{\theta} \Delta \log p^Y, 0 \right\}, & \text{for} \quad f \in \mathcal{L}. \end{cases}$$

Taken together, these equations pin down which factor markets are endogenously rigid, what wages are in flexible factor markets, and hence what the GDP deflator and real GDP are in equilibrium. Since these equations do not depend on $\bar{\Omega}$ beyond the initial factor income shares, this proves the result.

Letting $\Delta \log L_f$ be the endogenous change in hours by sector, we can further write real GDP directly in terms of changes in hours as

$$\Delta \log Y = \frac{\theta}{\theta - 1} \log \left(\sum_{f \in \mathcal{G}} \bar{\lambda}_f \exp \left(\frac{\theta - 1}{\theta} \Delta \log L_f \right) \right),$$

where

$$\Delta \log L_f = \begin{cases} \Delta \log \bar{L}_f, & \text{for} \quad f \in \mathcal{K}, \\ \min \left\{ \Delta \log \lambda_f + \Delta \log \zeta, \Delta \log \bar{L}_f \right\}, & \text{for} \quad f \in \mathcal{L}. \end{cases}$$

This exact functional form makes it clear that the economy behaves "as if" aggregate output were produced from the different factors via a CES aggregate production function with an elasticity of substitution θ.

4.2 Lattice Structure and Global Comparative Statics

In general, the equilibrium of the Keynesian model is not unique. However, for our benchmark case with uniform elasticities, we can prove there are simple-to-compute unique "best" and "worst" equilibria as long as there are complementarities ($\theta < 1$). We can also provide global comparative statics for these equilibria.

To state our result, we endow $\mathbb{R}^{\mathcal{G}}$ with the partial ordering $x \leq y$ if and only if $x_f \leq y_f$ for all $f \in \mathcal{G}$. Formally, we show that set of equilibrium values of the changes in factor quantities $\Delta \log L$ is a complete lattice under the partial ordering $\leq$.

Proposition 4. *Under the assumptions of Proposition 3, and assuming in addition that $\theta < 1$, there is a unique best and worst equilibrium: for any other equilibrium, $\Delta \log Y$ and $\Delta \log L$ are lower than at the best and higher than at the worst. Furthermore, both in the best and in the worst equilibrium, $\Delta \log Y$ and $\Delta \log L$ are increasing in $\Delta \log \bar{L}$ and in $\Delta \log \zeta$.*

The global comparative static result in the proposition generalize the insight of the horizontal economy in Section 3.5. In particular, negative labor supply shocks in some factor markets create Keynesian unemployment in other factor markets. This would not happen with substitutes ($\theta \geq 1$) where negative labor supply shocks would not lead to any Keynesian unemployment in any factor market.

Proposition 4 also provides a straightforward way to compute this best equilibrium using a greedy algorithm along the lines of Vives (1990) or, more recently, Elliott et al. (2014). We can find the best equilibrium as follows. Solve the model assuming all factor markets are flexible. If one of the wages is below the minimum, call this market rigid and set its wage equal to its lower bound. Recompute the equilibrium assuming that these factor markets are rigid. Continue in this manner until the wage in every candidate flexible market is above its lower bound. The worst equilibrium can be found in the same way but starting from the assumption that all markets are rigid, and checking at every step if a priori rigid markets have employments above their endowments.

In Appendix C, we build on Proposition 4 to prove some other global comparative statics, showing that factor mobility and wage flexibility, in the best equilibrium, are always weakly desirable, as long as the elasticity of substitution is less than one.

4.3 AS-AD Representation

We can represent the best equilibrium as the point at which an aggregate supply and aggregate demand curve intersect. The AD curve, which is a decreasing log-linear relationship, is given by the Euler equation, and relates aggregate output to the price level today. Deriving the AS curve is less straightforward. To do so, fix some level of output Y. There is a price level $p^Y(Y)$ such that: given the implied level of expenditure $E(Y) = p^Y(Y)Y$, the wage of every factor is consistent with the amount of expenditures on that factor; and these wages give rise to prices that are consistent with $p^Y(Y)$.

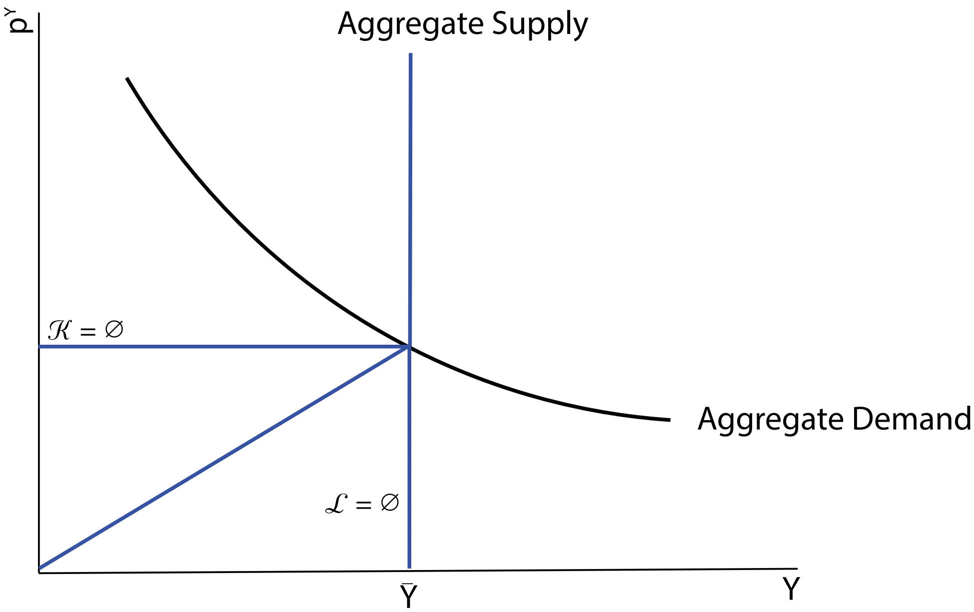


Figure 4.1: AS-AD representation of the equilibrium without shocks. The $\mathcal{K} = \varnothing$ case is when all factors are potentially rigid, and $\mathcal{L} = \varnothing$ case is when all factors are always flexible.

An example is plotted in Figure 4.1 at the initial equilibrium in the absence any exogenous shock. The downward slope of the left-side of the AS curve depends on the downward flexibility of factor prices. If the set of flexible factors is empty ($\mathcal{K} = \varnothing$), then the AS curve is horizontal to the left. If the set of rigid factors is empty ($\mathcal{L} = \varnothing$), then the AS curve is vertical to the left. Of course, in the case when there are no potentially-rigid factor markets, we recover the neoclassical model.

Aggregate demand shocks $d \log \zeta$ shift the AD curve in the usual way, and it is easy to see from this figure that a negative aggregate demand shock reduces present prices and output. The AS curve, on the other hand, does not have a simple closed-form representation, and supply shocks transform the shape of the AS curve in non-obvious ways. In the next few subsections, we use nonlinear AS-AD diagrams to illustrate how different shocks interact with complementarities to affect output and inflation.

4.4 Keynesian Amplification with Complementarities

As discussed earlier, complementarities across producers can transmit negative supply shocks in one factor market as negative demand shocks to other factor markets. This negative spillover is larger, the stronger are the complementarities. In other words, the amount of Keynesian unemployment in the rigid factor markets is decreasing as a function of the elasticity of substitution θ.

In Figure 4.2, we plot an example for a uniform-elasticity economy with two equally-sized factor markets. Both factors are labors and are therefore potentially rigid. There are no capitals ($\mathcal{K} = \emptyset$). We feed a 20% negative shock the supply of one of the factors. When there are complementarities ($\theta < 1$), the negative supply shock in one factor market causes the downward nominal wage rigidity to bind and triggers Keynesian unemployment in the other factor market. By contrast, with substitutability ($\theta \geq 1$), the downward nominal wage rigidity constraint does not bind in any of the two factor markets and the model behaves exactly like the neoclassical one.

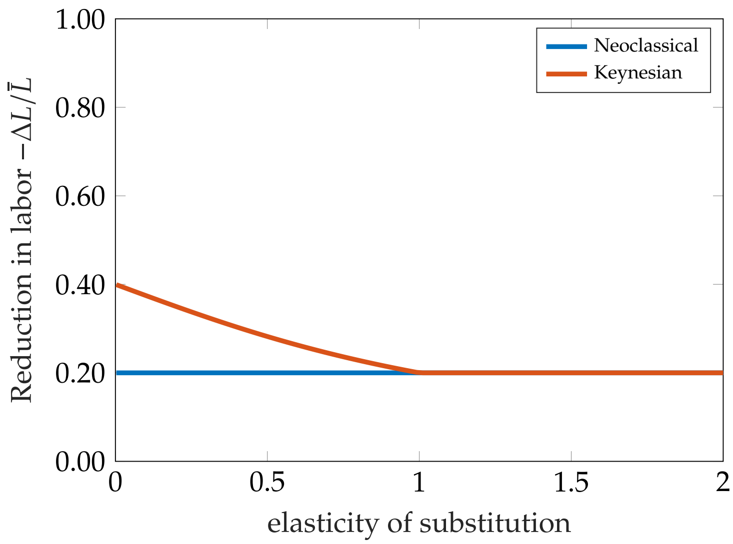


Figure 4.2: The change in the quantity of labor supplied in the neoclassical (flexible wage) and Keynesian example as a function of the elasticity of substitution.

However, the strength of this effect on output is hump-shaped in the elasticity of substitution. In Figure 4.3, we plot the change in output in the Keynesian model with downward wage rigidity against the response of the neoclassical model with flexible wages. As we already discussed, the behavior of output in the two models coincides when $\theta \geq 1$ but diverges as soon as $\theta < 1$. However, the behavior of the two models coincides again as θ approaches zero.

Intuitively, as complementarities become stronger, the marginal product of the rigid factor falls more. Output is more and more determined by the productive capacity of the negatively shocked flexible factor. In other words, as complementarities become stronger,

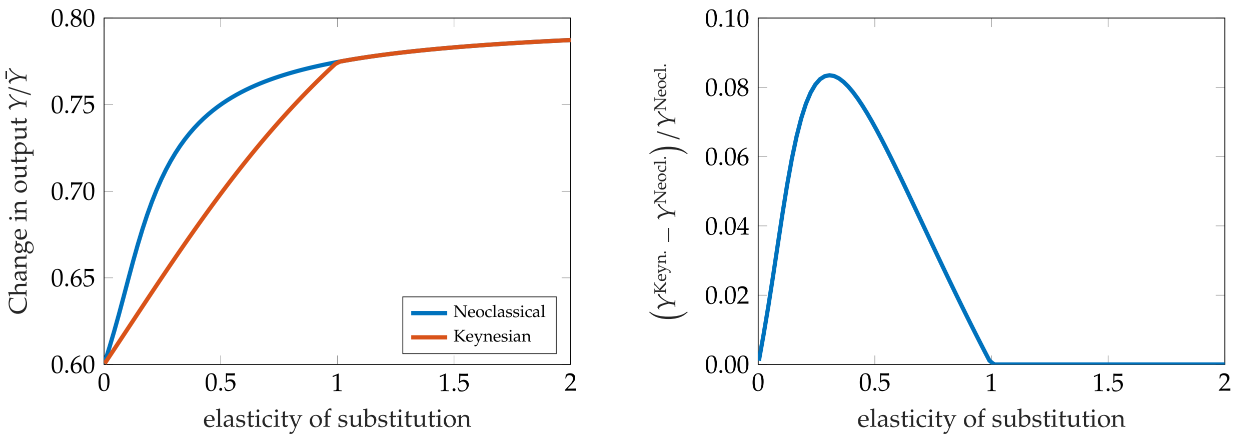


Figure 4.3: The panel on the left shows the change in output, in a neoclassical (flexible wage) and Keynesian example, in response to a reduction in one sector's labor as a function of the elasticity of substitution. The panel on the right shows the percentage difference between the neoclassical and Keynesian models.

the income share of the non-shocked rigid factor falls more in response to the negative shock to the flexible factor, and, as a result, the rigid factor becomes less critical and its Keynesian unemployment matters less for output. In particular, in the perfect complement limit, the unemployed workers in the rigid factor market have a marginal product of zero, and so their loss is irrelevant for output.[13]

Figure 4.4 represents this negative supply shock using an AS-AD diagram. The AS curve is horizontal to the left since there are no capitals ($\mathcal{K} = \emptyset$). The initial level of output is given by $\bar{Y}$ and the new level of labor available in the shocked sector is given by $\bar{L}'_f$. The negative supply shock shifts the AS curve to the left.

In the figure, we draw the new AS curve for different values of the elasticity of substitution θ. Unlike standard models, in this model, the shape of the AS curve itself changes in response to supply shocks. In particular, the negative labor supply shock introduces two kinks into the AS curve. The first kink is the point at which the AS curve becomes horizontal, and the second kink is the point at which the AS curve becomes vertical. The first kink always occurs at the point where $Y = \bar{L}'_f$. Intuitively, this is the level of aggregate output that would cause the shocked sector itself to become rigid. The second kink, on

[13]The non-montonic pattern in Figure 4.3 does not show in a linear approximation, and so does not appear in equation (3.3). Intuitively, as the negative supply shock gets smaller, the hump in Figure 4.3 moves towards the left and is pressed up against the axis, and so the amplification of output reductions is increasing in the degree of complementarities over a bigger range. In the limit of infinitesimal shocks, this range becomes complete.

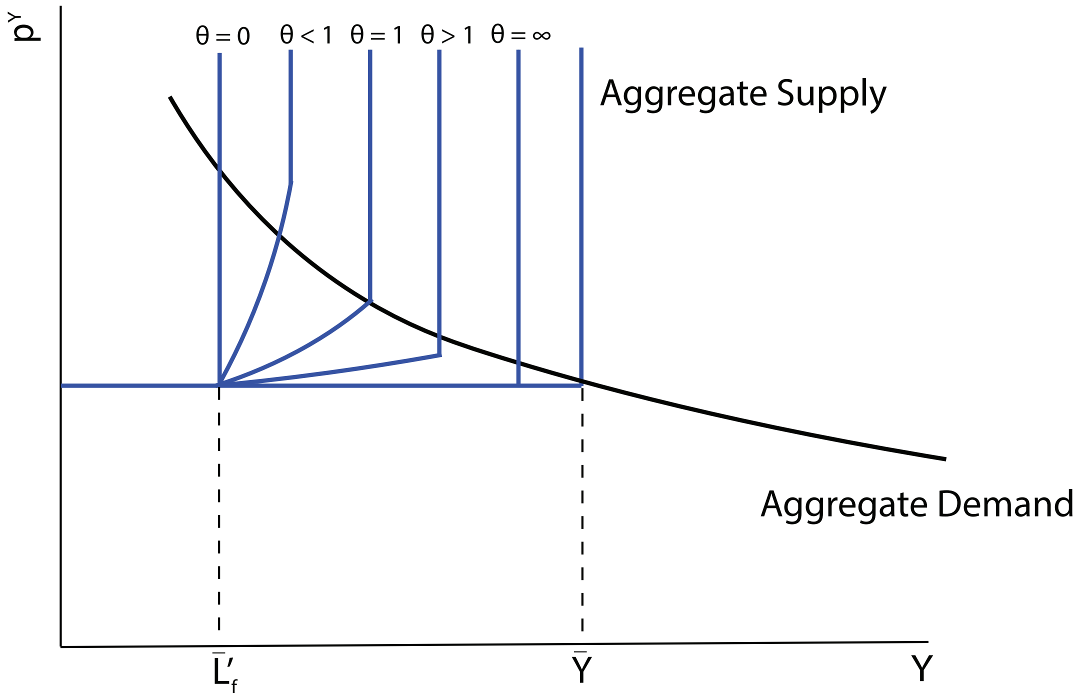


Figure 4.4: The effect of the same negative supply shock to a factor for different values of the elasticity of substitution θ.

the other hand, moves as we vary the elasticity of substitution.

As we lower the elasticity of substitution θ, the kink point at which the AS curve becomes vertical shifts north-westwards. As long as $\theta > 1$, the second kink is below the AD curve, and so the equilibrium is the same as the neoclassical one, because the AS and AD intersect along the neoclassical portion of the AS curve. Intuitively, when $\theta > 1$, no factor market becomes rigid and so downward nominal wage rigidity is never triggered. Once the elasticity of substitution has been lowered to $\theta = 1$, the Cobb-Douglas case, the second kink exactly intersects the AD curve. As θ goes below one, the second kink moves above the AD curve, downward nominal wage rigidities are triggered, and the equilibrium has lower output and higher inflation than the neoclassical model. Finally, as θ goes to zero and we approach the Leontief case, the second kink point moves directly above the first kink point, and so the reduction in output in the neoclassical model and Keynesian model become the same again.

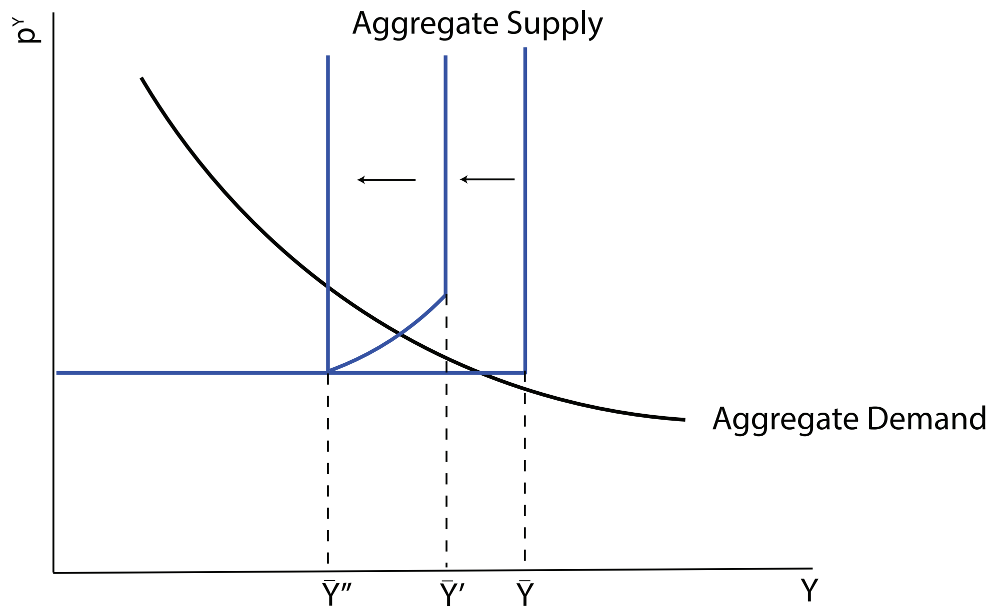


Figure 4.5: Negative labor supply shocks in a two-sector model, $\bar{Y}$ is output without any shocks, $\bar{Y}'$ is output with shocks to only one sector, and $\bar{Y}''$ is output with shocks to both sectors.

4.5 Keynesian Amplification with Heterogeneous Shocks

Next, we consider how heterogeneity in the size of the shock affects the equilibrium. In Figure 4.5, we consider the same example as in Section 4.4, but we now allow for negative supply shocks in both factor markets.

First consider the case where there is only a negative supply shock to one of the factors. The shock shifts the AS curve back and introduces two kinks. It results in Keynesian unemployment and a reduction in output over and above that which takes place in the neoclassical model represented by $\bar{Y}'$.

Now, consider the case where there is also a negative supply shock of the same magnitude to the other factor market, so that the negative supply shock is now uniform across the two factor markets. The kink disappears, output falls to its neoclassical level $\bar{Y}''$, and there are no longer any Keynesian forces in the model: downward nominal wage rigidities do not bind in any factor market, there is no Keynesian unemployment, and there is no Keynesian amplification of output reductions. Once again, this is because the first and second kink are now directly on top of each other.

The lesson is that we should expect Keynesian forces from negative factor supply shocks to be stronger when the shocks are more heterogeneous. If the negative supply shocks are more homogeneous, then it is less likely that supply outstrips demand in any factor market. Indeed, when the shock uniformly affects all factor markets together, then relative factor prices do not change, all factor prices increase, and the nominal rigidities are never triggered.

Covid-19 plausibly caused a heterogeneous shock to labor supply, since it affected labor supply in some sectors much more severely than in others. Whereas many white-collar jobs can be done at home, most blue-collar work require workers to work in close proximity to each other and to their clients.[14] This means that lock-downs disproportionately affect some sectors, and the more heterogeneous are these effects, the more likely they are to trigger Keynesian unemployment.

4.6 Interaction of Supply and Demand Shocks

Next, we show how negative factor supply shocks and aggregate demand shocks interact with one another. In Figure 4.6, we show how the equilibrium responds to a negative labor supply shock together with a negative aggregate demand shock, assuming there are complementarities. We deviate from the example of Section 4.4 by allowing for more than two factors, and by allowing for potentially-rigid labors and always flexibe capitals ($\mathcal{K} \neq \emptyset$).

As expected, the negative aggregate demand shock shifts the AD curve backwards. If there are no supply shocks, then aggregate demand shocks are potent, causing output to fall by a lot. If there are some capitals ($\mathcal{K} \neq \emptyset$), then the aggregate demand shock can also reduce prices a lot.

Now, consider what happens if the negative aggregate demand shock coincides with negative supply shocks. As usual, the negative labor supply shock introduces kinks into the AS curve and shifts the curve backwards. In equilibrium, the effect of the negative AD shock is now much less potent for output. In fact, in the extreme case where the first and second kink are on top of each, the negative aggregate demand shock has no effect on output unless it is very large. However, even though the negative supply shock blunts the importance of aggregate demand for output, aggregate demand shocks remain critical for the determination of prices. In particular, aggregate demand shocks reduce inflation, and without them, it is impossible to deliver a reduction in output without inflation.

[14] See for example Mongey et al. (2020).

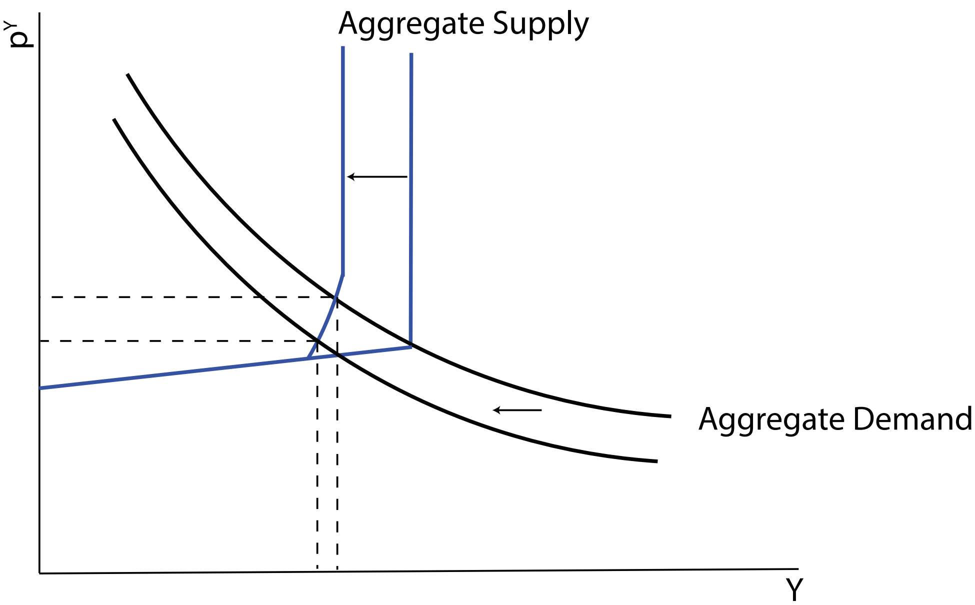


Figure 4.6: Negative labor supply shock coupled with a negative aggregate demand shock.

4.7 Shocks to the Composition of Demand

So far, we abstracted away from shocks to the composition of final demand $\Delta \log \omega_0$. However, building on Baqaee (2015), our sufficient statistics approach can be extended to cover these shocks as well.

For each factor $f \in \mathcal{G}$, we translate the changes in the final expenditure share *parameters* into changes of factor income share *parameters* by defining

$$\Delta \log \bar{\lambda}_f = \sum_{j \in \mathcal{N}} \bar{\Omega}_{0j} \exp(\Delta \log \omega_{0j}) \bar{\Psi}_{jf}.$$

These changes in factor income share parameters are *not* the *equilibrium* changes in factor income shares, but they are useful because they encode how shocks to final demand propagate backward (upstream) to affect the demand for the different factors. They depend on the network-adjusted factor intensities $\bar{\Psi}_{jf}$ of the different sectors $j \in \mathcal{N}$ for the different factors $f \in \mathcal{G}$, which measure how much each sector j uses each factor f directly and indirectly through its supply chain.

In fact, as we shall see below, these changes in the factor income share parameters $\Delta \log \bar{\lambda}_f$, together with the initial factor income shares $\bar{\lambda}_f$, are global sufficient statistics for the response of the equilibrium to the shocks. Compared to the situation without shocks to the composition of demand, the list of network sufficient statistics must therefore be expanded beyond the initial factor income share. In other words, with shocks to the composition of demand, we need to know more information about the network than without these shocks, but this information can still be summarized by simple sufficient statistics.[15]

The extension of the results of Section 4.1 is as follow:[16]

$$\Delta \log Y = \Delta \log \zeta - \Delta \log p^Y,$$

$$\Delta \log p^Y = \frac{1}{1-\theta} \log \left(\sum_{f \in \mathcal{G}} \bar{\lambda}_f \exp\left((1-\theta) \Delta \log w_f \right) \right),$$

$$\Delta \log w_f = \begin{cases} \frac{1}{\theta}\left(\Delta \log \bar{\lambda}_f + \Delta \log \zeta - \Delta \log \bar{L}_f\right) + \frac{\theta-1}{\theta} \Delta \log p^Y, & \text{for} \quad f \in \mathcal{K}, \\ \max\left\{ \frac{1}{\theta}\left(\Delta \log \bar{\lambda}_f + \Delta \log \zeta - \Delta \log \bar{L}_f\right) + \frac{\theta-1}{\theta} \Delta \log p^Y, 0 \right\}, & \text{for} \quad f \in \mathcal{L}. \end{cases}$$

We can use these results to prove global comparative statics as in Section 4.2. For example, starting at an initial equilibrium with no shocks, changes in the composition of demand will cause factor market $f \in \mathcal{L}$ to become rigid if, and only if, the network-adjusted demand shock to that factor is negative so that $\Delta \log \bar{\lambda}_f = \sum_{j \in \mathcal{N}} \bar{\Omega}_{0j} \exp(\Delta \log \omega_{0j}) \bar{\Psi}_{jf} < 0$.

5 Quantitative Application

We now turn to quantifying the model. Although the quantitative model does not have uniform elasticities, the intuitions that we developed in the context of this benchmark case will prove useful in understanding the results.

5.1 Setup

We start by describing our calibration of the model and of the shocks.

[15]Since $\sum_{f \in \mathcal{G}} \bar{\Psi}_{jf} = 1$ for all $j \in \mathcal{N}$, we only need to keep track of $\mathcal{N}(\mathcal{G}-1)$ additional sufficient statistics to conduct comparative statics.

[16]Here $\Delta \log Y$ should be interpreted as the change in the consumption quantity index and $\Delta \log p^Y$ as the change in the corresponding ideal price index. These notions correspond to the changes in welfare and in a welfare price index but not to changes in real GDP and the GDP deflator as they are measured in the data. The latter can be computed as path-integrals and they only coincide with the former to the first order of approximation. These distinctions are irrelevant for changes in disaggregated variables such as wages or employments of the different factors.

Calibrating the economy. There are 66 sectors and sectoral production functions use labor, capital, and intermediates. The share parameters of the functions are calibrated so that at the initial pre-shock allocation, expenditure shares match those in the input-output tables from the BEA. We focus on the short run and assume, following Baqaee and Farhi (2019), that labor and capital cannot be reallocated across sectors. We construct the input-output matrix using the annual U.S. input-output data from the BEA, dropping the government, non-comparable imports, and second-hand scrap industries. The dataset contains industrial output and inputs for 66 industries.

The economy has a nested CES form, and the nesting structure is the following. In each sector, labor and capital are combined with elasticity η, intermediates are combined with elasticity θ, intermediates and value-added are combined with an elasticity ϵ, and final output from different sectors are combined with an elasticity σ to form real GDP. In other words, we allow for differences in the elasticities of substitution, but we do not allow them to vary by sector, because such disaggregated estimates are not available.

Based on the empirical literature, we set the elasticity of substitution between labor and capital to be $\eta = 1$, between value-added and intermediate inputs to be $\epsilon = 0.5$, across intermediates to be $\theta = 0.001$. We set the elasticity of substitution across final uses to be 0.8. These numbers are broadly in line with Atalay (2017), Herrendorf et al. (2013), Oberfield (2013), and Boehm et al. (2019), and our numerical findings are fairly robust to variations in these particular numbers (in particular to the low-value of θ).

We assume that the different sectoral labor markets feature occasionally-binding downward nominal wage rigidity constraints. Unless stated otherwise explicitly, we assume that the different sectoral capital markets have flexible rental rates.

Calibrating the shocks. Covid-19 set off an array of supply and demand shocks. Identifying these shocks would be challenging even with accurate disaggregated data. This difficulty is compounded by the fact that quality disaggregated data is not, or not yet, available. We therefore take a more modest approach and consider two polar scenarios, informed by data on changes in aggregate nominal expenditure and changes in hours worked by sector.

For changes in nominal expenditure, we use data from the Opportunity Insights (OI) Economic Tracker.[17] As of April 2020, OI shows that total consumer spending in the US has fallen by a little more than 20%. Since private consumption is only about 2/3

[17]See https://tracker.opportunityinsights.org/

of nominal GDP, and government expenditures has, if anything, increased, we target a reduction in aggregate nominal expenditure of around 12%. For changes in hours worked by sector, we use the May 2020 BLS Economic News release.

Our first "labor-supply-shocks " scenario rationalizes changes in aggregate nominal expenditure and hours worked by sector using a negative shock to aggregate demand and heterogeneous negative shocks to sectoral labor supplies. Our second "composition-of-demand-shocks" scenario rationalizes changes in aggregate nominal expenditure and hours worked by sector using a negative shock to aggregate demand and heterogeneous shocks to the sectoral composition of final demand. In both scenarios, we pick the aggregate demand shock to exactly match the reduction in aggregate nominal expenditure $\Delta \log \zeta = -0.12$. In addition, in the first (second) scenario, we pick the shocks to sectoral labor supplies (sectoral composition of demand) to match changes in hours by sector.

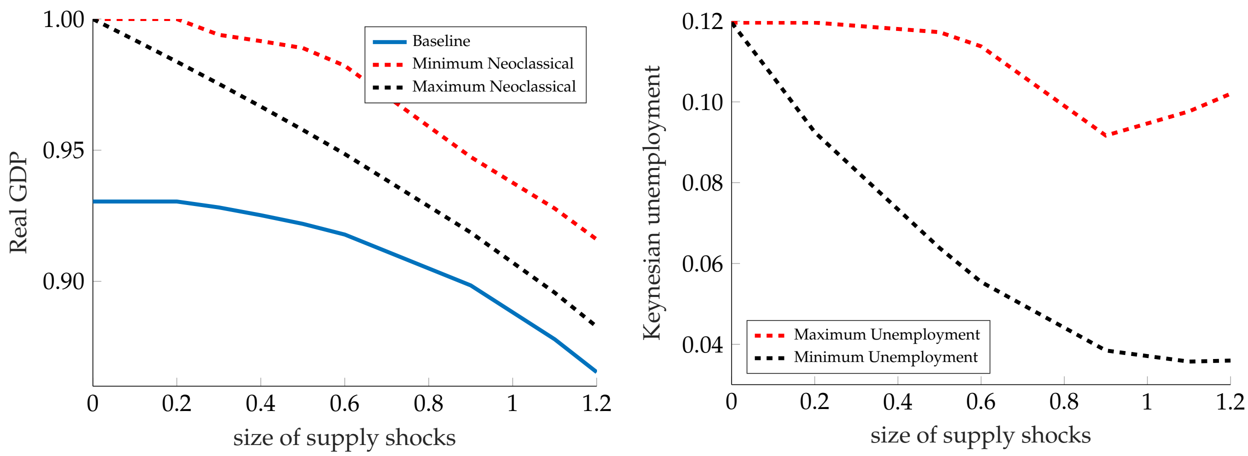
In the absence of additional disaggregated data, it is a priori difficult to say which of these two scenarios, or which combination of these two scenarios, is more likely to be at play in the data. At the time of writing, reliable disaggregated data on changes in the sectoral composition of final demand or on sectoral prices and quantities is not yet available. We plan to revise our analysis when this data is released.

Having calibrated these two scenarios, we then conduct counterfactual experiments to illustrate the quantitative importance of the qualitative points that we made in Sections 3 and 4.

5.2 Labor Supply Shocks

In the labor-supply-shocks calibration, illustrated in Figure 5.1, we match the data on changes in aggregate nominal expenditure and changes in hours worked by sector by feeding a negative aggregate demand shock and negative shocks to sectoral labor supplies. The jagged lines in this figure, and in subsequent figures, are due to the occasionally-binding nature of downward wage rigidity.

The "Baseline" line in Panel 5.1a plots real GDP (relative to its value at the initial equilibrium) with a −12% shock to aggregate demand and heterogeneous negative shocks to sectoral labor supplies. The x-axis scales the negative shocks to sectoral labor supplies: when $x = 0$, there are no shocks to sectoral labor supplies, and then the −12% aggregate demand shock lead to a 7% reduction of real GDP; when $x = 1$, the shocks to sectoral labor supplies match the reductions in hours by sector observed in the data, and then the combination of these shocks and of the −12% aggregate demand shock lead to a 12% reduction in real GDP. One way to think about this is that at the height of the lock-down, the negative labor supply shock are large with x around 1, and then as the economy reopens, x shrinks but the aggregate demand shock remains unchanged.



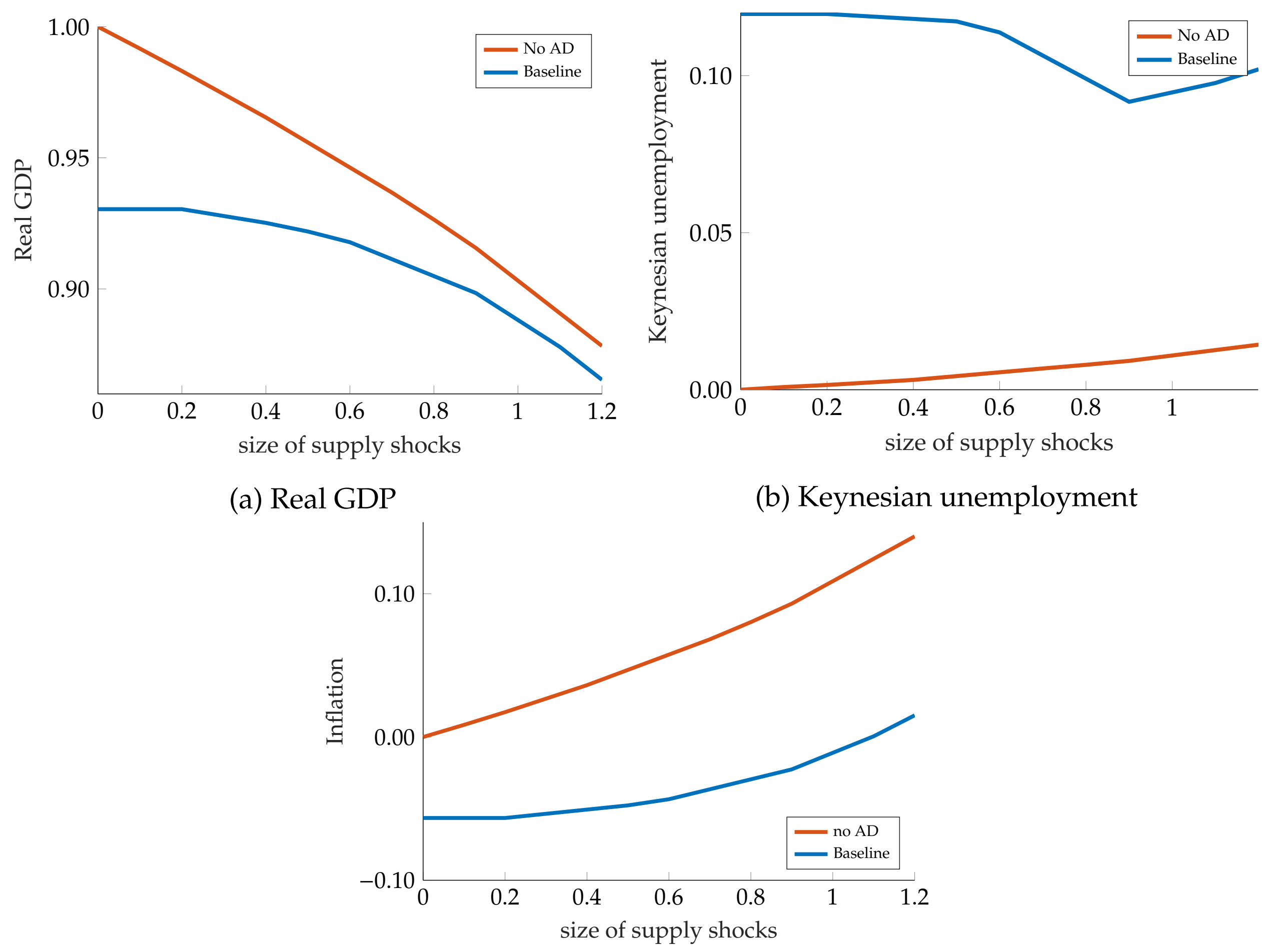
(a) Real GDP

(b) Keynesian unemployment

Figure 5.1: "Baseline" is the model with a negative shock to aggregate demand and negative shocks to sectoral labor supplies. The x-axis scales the negative labor supply shocks.

This figure illustrates some of the identification challenges inherent in separating supply and demand shocks from each other. Negative labor supply shocks to markets that are slack in equilibrium do not affect observed outcomes (prices or quantities). The model that keeps these shocks and the model that discards them are observationally equivalent: both models rationalize the data on changes in aggregate nominal expenditure and hours worked by sectors; and both models feature the same changes in real GDP. However, these two models have different decompositions of changes in real GDP into neoclassical and Keynesian effects and different amounts of Keynesian unemployment. The model that keeps the negative labor supply shocks to slack markets minimizes the amount of Keynesian unemployment and therefore features the maximum neoclassical effect consistent with the data; the model that discards these shocks maximizes the amount of Keynesian unemployment and therefore features the minimum neoclassical effect consistent with the data. These outcomes are illustrated by the 'Maximum Neoclassical' and 'Minimum Neoclassical' lines in Panel 5.1a, and by the "Minimum Unemployment" and "Maximum Unemployment" lines in Panel 5.1b, where Keynesian unemployment is computed as the share-weighted average of unemployed labor across labor markets.[18]

Having made this identification point, and in order to streamline the figures, we focus from now on on the model that discards the negative labor supply shocks to slack markets, minimizes the neoclassical effect in the reduction of real GDP, and maximizes the amount of Keynesian unemployment.



(a) Real GDP

(b) Keynesian unemployment

(c) Inflation

Figure 5.2: Real GDP, inflation, and Keynesian unemployment as a function of the size of the negative labor supply shocks. The "Baseline" line includes an initial negative aggregate demand shock whilst the "No AD" line only features the negative labor supply shocks.

Figure 5.2 compares the model's response to negative shocks to sectoral labor supplies with or without the negative aggregate demand shock. Panel 5.2a shows that the aggregate

[18]More formally, Keynesian unemployment is defined as $\sum_{f\in\mathcal{L}}(\bar{\lambda}_f/\bar{\lambda}_{\mathcal{L}})(\Delta\log\bar{L}_f - \Delta\log L_f) \geq 0$, where $\bar{\lambda}_{\mathcal{L}} = \sum_{f\in\mathcal{L}}\bar{\lambda}_f$. It captures the percentage underutilization of efficiency units of labor across labor markets.

demand shock becomes less important for real GDP as the negative labor supply shocks become larger. This is because when the shocks to sectoral labor supplies are fully scaled with $x = 1$, the reductions in real GDP are comparable.

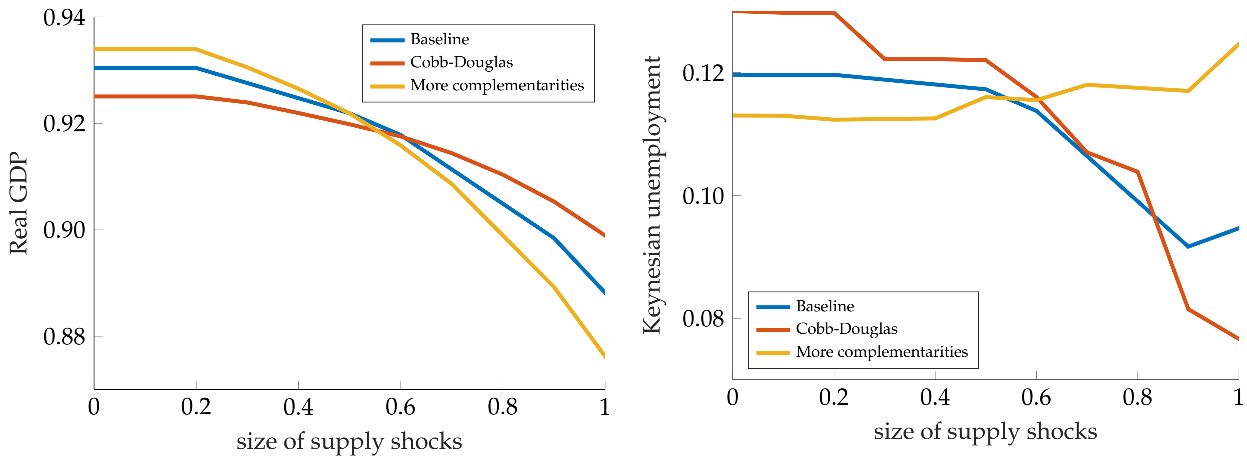
Panel 5.2b plots the (maximal) Keynesian unemployment under the two scenarios. When there is a negative aggregate demand shock, there are many slack labor markets, and, by assumption, those markets receive no exogenous negative labor supply shocks. When there is no negative aggregate demand shock, there are fewer slack labor markets, and hence more exogenous negative labor supply shocks and less Keynesian unemployment. When the shocks to sectoral labor supplies are fully scaled with $x = 1$, the reductions in real GDP and hours worked by sector are comparable under the two scenarios, but the breakdowns between Keynesian and non-Keynesian effects are very different.

Panel 5.2c shows that the negative aggregate demand shock significantly reduces the amount of inflation, regardless of the size of the negative shocks to sectoral labor supplies. Without this negative aggregate demand shock, the negative labor supply shocks are stagflationary: they lead to simultaneous reductions in real GDP and increases in inflation. The negative aggregate demand shock superimposes a deflationary force, and is indeed large enough to generate mild deflation when the negative shocks to sectoral labor supplies are fully scaled with $x = 1$.

The intuition for these results can be grasped from the AS-AD diagram in Figure 4.6. The negative aggregate demand shock shifts the AD curve down and creates slack in the economy. Given this negative aggregate demand shock, progressively introducing the negative labor supply shocks shifts the AS curve left and tends to reduce the amount of slack. When the negative labor supply shocks become large, the negative aggregate demand shock does not matter much any more for output and unemployment. However, the negative aggregate demand shock significantly reduces inflation, even when the negative labor supply shocks are large.

Finally, in Figure 5.3 we investigate the role of complementarities by deviating from our baseline elasticities $(\sigma, \theta, \epsilon, \eta) = (0.8, 0.001, 0.5, 1)$. We consider two alternative versions of the model, one with less complementarities than in our baseline where we raise all elasticities of substitution to 1 (Cobb-Douglas), and one with more complementarities where we lower elasticities of substitutions to $(\sigma, \theta, \epsilon, \eta) = (0.5, 0.001, 0.4, 1)$. Panel 5.3a shows the GDP response and panel 5.3b shows the (maximal) Keynesian unemployment.

For small negative labor supply shocks, complementarities attenuate the reduction in output and the increase in Keynesian unemployment. The intuition for this result was



(a) Real GDP

(b) Keynesian unemployment

Figure 5.3: Comparison of the response of the model with negative supply shocks and negative aggregate demand shocks for different values of the elasticities of substitution.

discussed in the context of Example 3.5. The negative aggregate demand shock lowers the price of flexible labors relative to rigid ones, which, in the presence of complementarities, causes expenditures to switch towards the rigid labors, which in turn stabilizes them. The strength of these effects increases with the degree of complementarities.

However, this pattern reverses as negative labor supply shocks become larger and so complementarities start amplifying the reduction in output and the increase in Keynesian unemployment. In the Cobb-Douglas model, Keynesian unemployment falls monotonically as the supply shock gets larger. Intuitively, as we remove more and more labor from slack labor markets, the amount of Keynesian unemployment falls. However, with complementarities, the pattern is non-monotonic, because this direct effect is overtaken by an opposing effect when negative labor supply shocks are large enough. Intuitively, when some sectors are hit with large negative supply shocks, expenditure on the corresponding labor markets increases, which reduces expenditure on the other labor markets and therefore increases the amount slack. These negative demand spillovers are larger when the labor supply shocks are bigger and the complementarities are stronger. The effect of complementarities for given heterogeneous labor supply shocks is illustrated in the AS-AD diagram in Figure 4.4. When we increase the degree of complementarities, the AS curve shifts to the left, and this increases Keynesian unemployment and reduces real GDP.[19]

[19]The elasticity of substitution between labor and capital η plays a somewhat different role under the

5.3 Shocks to the Composition of Demand

Having studied the "labor-supply-shocks" scenario, we now turn to the "composition-of-demand-shocks" scenario. As before, we feed a -12% aggregate demand shock to match the reduction in aggregate nominal expenditure. We also feed a change to the composition of final demand across sectors to match the observed reduction in hours worked by sector. By construction, there are no negative labor supply shocks, and so the neoclassical effect effect of the shocks on real GDP is equal to zero.

	Real GDP	Inflation	Unemployment
Sticky Labor	-0.08	-0.04	0.15
Sticky Labor and Capital	-0.12	0.00	0.16

Table 1: Outcomes for the model with shocks to the composition of demand and a negative aggregate demand shock. The change in real GDP and inflation are calculated as discrete-approximations to the chained (Divisia) index.

Table 1 considers how demand shocks affect real GDP, inflation, and unemployment in two different versions of the model. Our baseline model corresponds to the "Sticky Labor" case and assumes that while all sectoral labor markets feature occasionally-binding downward nominal wage rigidity constraints, all sectoral capital markets feature flexible prices and full utilization. The "Sticky Labor and Capital" case allows sectoral capital markets, just like sectoral labor markets, to feature under-utilization. This is achieved by allowing for downward rigidity in the sectoral capital markets. As explained earlier, a possible micro-foundation for these constraints and the associated under-utilization of capital is firm failures triggered by an inability to repay loans taken to finance capital and the ensuing destruction or temporary idleness of the capital of failed firms.

Both versions of the model match the reduction in aggregate nominal expenditure and do about equally well in matching the cross-sectional reduction in hours worked by sector. The "Sticky Labor" model features a reduction in output of 8%, a large amount of Keynesian unemployment of 15%, and mild deflation of -4%. However, the model with

assumption that the different sectoral capital markets always feature flexible prices and full utilization. In response to a given set of negative shocks to sectoral labor supplies, stronger complementarities between labor and capital raise the share of the labors compared to that of the capitals and mitigates Keynesian unemployment. This shock absorber effect is not very robust. It disappears when the sectoral capital markets are also subject to downward rigidities, which, as explained earlier, could be micro-founded as arising from inability to repay loans and ensuing firm failures. The same applies when endogenous capacity utilization is modeled as a decision motivated by a tradeoff between utilization and depreciation.

sticky labor and capital experiences a greater reduction in output of 12%, since it not only experiences a reduction in employment but also a reduction in the utilization of capital. By implication, it also experiences less deflation with inflation of 0% since the rental rates of the sectoral capital markets cannot go down.

In both versions of the model, the only reason we avoid stagflation is because of the negative aggregate demand shock. In its absence, shocks to the composition of demand would reduce output and lead to inflation. The negative aggregate demand shock further reduces output and introduces deflationary forces which surpass that inflationary forces from shocks to the composition of demand.

6 Extension I: Constrained Consumers

In the first extension, we show how adding constrained consumers to the model provides an endogenous source for aggregate demand shocks. We consider heterogeneous consumers who can save but cannot borrow. This means that if their income declines, then their consumption declines one-for-one with their income. However, if their income increases, holding fixed prices, these households would choose to save some of this income rather than spend all of it today. We assume that all agents have the same preferences which allows us to easily aggregate their consumptions.

6.1 Comparative Statics

Suppose there are two types of consumers: Ricardian and non-Ricardian. Index non-Ricardian consumers by $h \in \mathcal{H}$, and suppose that consumers of type h own a fraction χ_{hf} of factor $f \in \mathcal{G}$. The representative Ricardian consumer $r \notin \mathcal{H}$ behaves similarly to the representative household in Section 2. On the other hand, for the non-Ricardian households $h \in \mathcal{H}$ expenditures are determined by their contemporaneous earnings if those earnings fall more than what they would need to satisfy their Euler equation. We start with local comparative static results and end with a brief comment on global results.

Denote the endogenous set of households that are constrained in equilibrium by $\mathcal{H}^c \subseteq \mathcal{H}$. Hence $h \in \mathcal{H}^c$ if and only if

$$\sum_{f \in \mathcal{G}} \frac{\chi_{hf}\lambda_f}{\sum_{g \in \mathcal{G}} \chi_{hg}\lambda_g} \left(d\log\lambda_f + d\log E \right) < (1-\rho) d\log p^Y + d\log\zeta. \tag{6.1}$$

In words, household h is constrained if their nominal income today is less than how much they wish to consume, as given by their Euler equation.[20]

The key step in this extension is the determination of changes in aggregate nominal expenditure. In the interest of space, we omit the derivations and jump directly to the result, which applies almost everywhere given the set of constrained households $\mathcal{H}^c$:

$$d\log E = \frac{Cov_\lambda\left(\chi_{\mathcal{H}^c}, d\log\lambda\right)}{1 - \mathbb{E}_\lambda\left(\chi_{\mathcal{H}^c}\right)} + (1-\rho)d\log p^Y + d\log\zeta.$$

The covariance uses the factor shares λ_f for $f \in \mathcal{G}$ as the probability distribution and computes the covariance of the income share of constrained households for each factor $\chi_{\mathcal{H}^c_f} = \sum_{h\in\mathcal{H}^c}\chi^h$ with the changes in the factor shares $d\log\lambda_f$.[21]

Therefore, even in the absence of exogenous aggregate demand shocks $d\log\zeta$ such as changes in nominal interest rates or in the desire to save, there is now an endogenous aggregate demand shock. In particular, if the income shares of factors owned by constrained households shrink, then this imparts a negative aggregate demand shock that shrinks nominal expenditures today $d\log E < 0$. Since it is precisely those households whose income falls that become constrained, this endogenously reduces expenditures, and the reductions in expenditures exasperates unemployment.

The accompanying propagation equations determining changes in factor shares $d\log\lambda_f$ as a function of changes in aggregate nominal expenditure $d\log E$ are exactly the same as in Proposition 2. And the aggregation determining $d\log Y$ as a function of the $d\log\lambda_f$'s and $d\log E$ is exactly the same as in Proposition 1.

Furthermore, all the global results of Section 4 generalize to this case with constrained consumers: the set of equilibria has a lattice structure with a best and a worst equilibria, these equilibria are decreasing in $\Delta\log\bar{L}$ and $\Delta\log\zeta$, and there are benefits from wage flexibility.

[20] Note that we assume that the reduction in the income of type h households is borne uniformly by all households of type h. That is, we do not allow some type-h households to get fired (becoming constrained) while others stay on the job.

[21] We could allow for the possibility that some households are exogenously hand-to-mouth. The equation for changes in aggregate nominal expenditure $d\log E$ would stay the same, but we would no longer have to verify that (6.1) holds for those households.

6.2 Example with Constrained Households

Consider again the "somewhat universal" horizontal economy analyzed in Section 3.5. Suppose each labor factor $f \in \mathcal{L}$ is wholly owned by a type of consumer facing potentially binding borrowing constraints and whose only source of income is f. We assume that the intertemporal elasticity of substitution is $\rho = 1$. We consider shocks to the composition of demand $d\log\omega_{0j}$ and to factor supplies $d\log\bar{L}_f$ but we abstract from shocks to productivities and to aggregate demand.

Household h becomes constrained if it owns a factor that has become rigid because its wage has hit the downward nominal wage rigidity constraint. As a result, changes in nominal expenditure are given by

$$d\log E = \frac{\sum_{f\in\mathcal{R}}\lambda_f d\log\lambda_f}{\lambda_{\mathcal{F}}} = -d\log\lambda_{\mathcal{F}}.$$

Hence, an increase in the expenditure share of flexible sectors, by depriving credit-constrained workers of income, reduces nominal expenditures today one-for-one. This is the endogenous negative demand shock.

We then proceed as in Section 3.5 to get

$$\begin{aligned} d\log Y &= \lambda_{\mathcal{F}} d\log\bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d\log L_{\mathcal{R}} \\ &= \lambda_{\mathcal{F}} d\log\bar{L}_{\mathcal{F}} + (1-\lambda_{\mathcal{F}})(1-\theta) d\log\bar{L}_{\mathcal{F}} - \theta\lambda_{\mathcal{F}} d\log\omega_{0\mathcal{F}}, \end{aligned}$$

The first term on the second line is the direct effect of the negative shock to the supplies flexible factors. The second term captures the amplification from the fact that the shock redistributes demand away from the rigid sectors, causing unemployment. The final term captures the fact that a change in the composition of demand towards flexible sectors further reduces employment.

Surprisingly, when we compare the reduction in output to the one we obtained without credit-constrained consumers in Section 3.5, we see that adding endogenously credit-constrained households to the model attenuates the effect on output. Without credit constraints, in response to a negative supply shock $d\log\bar{L}_{\mathcal{F}} < 0$, expenditures on $\mathcal{F}$ increase, which further raises the price of factors in $\mathcal{F}$, which further increases expenditures due to complementarities, and so on. However, with credit constraints, this feedback loop short-circuits. In response to a negative supply shock $d\log\bar{L}_{\mathcal{F}} < 0$, expenditures on $\mathcal{F}$ increase, but this reduces the income of constrained households, reducing total income and spending.

This example also shows how the model can generate recessions without inflation, even in the absence of exogenous aggregate demand shocks. To see this, note that the change in today's price level is

$$d\log p^{Y} = d\log E - d\log Y = -\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}}.$$

Hence, changes in the composition of demand have no effect on the price level. This is because a change in the composition of demand causes output and nominal expenditures to decline at the same rate, leaving the overall price level unchanged. So, if there are only shocks to the composition of demand then we can get $d\log p^{Y} = 0$ and $d\log Y < 0$. This is impossible in the absence of potentially credit-constrained consumers, because the Euler equation necessitates that output and the price level move in opposite directions. The difference here is that consumers who become credit-constrained reduce their spending, lowering nominal demand for all factors, including the flexibly priced ones.

The example above shows how the presence of credit-constrained households can generate recessions without any accompanying inflation, even in the absence of negative aggregate demand shocks. Of course, since wages are constrained to never fall, it is not surprising that the model cannot generate deflation. In Appendix B, we extend the model to allow for some downward wage flexibility and show how this can help to generate a recession and deflation at the same time, without any exogenous negative aggregate demand shocks.

7 Extension II: Firm Failures

Whereas credit-constraints can act as endogenous demand negative shocks, exits and firm-failures can act as endogenous supply shocks.

To capture firm failures, we modify the general Keynesian structure described in Section 2 as follows. We assume that output in sector $i \in \mathcal{N}$ is a CES aggregate of identical producers j each with constant returns production functions $y_{ik} = A_i f_i(x_{ij}^k)$, where x_{ij}^k is the quantity of industry j's output used by producer k in industry i. Assuming all firms within an industry use the same mix of inputs, sectoral output is

$$y_i = \left(\int y_{ik}^{\frac{\sigma_i - 1}{\sigma_i}} dk \right)^{\frac{\sigma_i}{\sigma_i - 1}} = M_i^{\frac{1}{\sigma_i - 1}} A_i f_i(x_{ij}),$$

where x_{ij} is the quantity of input j used by industry i, M_i is the mass of producers in industry i, $\sigma_i > 1$ is the elasticity of substitution across producers, and A_i is an exogenous productivity shifter. From this equation, we see that a change in the mass of operating firms acts like a productivity shock and changes the industry-level price. Therefore, if shocks outside sector i trigger a wave of exits, then this will set in motion endogenous negative productivity shock $(1/(\sigma_i - 1))\Delta \log M_i$ in sector i.

Suppose that each firm must maintain a minimum level of revenue in order to continue operation.[22, 23] We are focused on a short-run application, so we do not allow new entry, but of course, this would be important for long-run analyses.[24]

The mass of firms that operate in equilibrium is therefore given by

$$M_i = \min \left\{ \frac{\lambda_i E}{\bar{\lambda}_i \bar{E}} \bar{M}_i, \bar{M}_i \right\},$$

where $\bar{M}_i$ is the exogenous initial mass of varieties, $\lambda_i E$ is nominal revenue earned by sector i and $\bar{\lambda}_i \bar{E}$ is the initial nominal revenue earned by i. If nominal revenues fall relative to the baseline, then the mass of producers declines to ensure that sales per producer remain constant. In order to capture government-mandated shutdowns of certain firms, we allow for shocks that reduce the exogenous initial mass of producers $\Delta \log \bar{M}_i \leq 0$.

[22] One possible micro-foundation is each producer must pay its inputs in advance by securing within-period loans and that these loans have indivisibilities: only loans of size greater than some minimum level can be secured. This minimum size is assumed to coincide with the initial costs $\bar{\lambda}_i \bar{E} / \bar{M}_i$ of the producer.

[23] Another possible micro-foundation is as follows. Producers within a sector charge a CES markup $\mu_i = \sigma_i / (\sigma_i - 1)$ over marginal cost. These markups are assumed to be offset by corresponding production subsidies. Producers have present nominal debt obligations corresponding to their initial profits $(1 - 1/\mu_i)\bar{\lambda}_i \bar{E} / \bar{M}_i$. The same is true in the future. If present profits $(1 - 1/\mu_i)\lambda_i E / M_i$ fall short of the required nominal debt payment $(1 - 1/\mu_i)\bar{\lambda}_i \bar{E} / \bar{M}_i$, then the firm goes bankrupt and exits. Alternatively, we can imagine that there is no future debt obligation but that firms cannot borrow.

[24] See Baqaee (2018) and Baqaee and Farhi (2020a) for production networks with both entry and exit.

7.1 Local Comparative Statics

We can generalize Propositions 1 and 2 to this context. The only difference is that we must replace $d\log A_i$ by $d\log A_i + (1/(\sigma_i - 1))d\log M_i$, where

$$d\log M_i = d\log \bar{M}_i + \min\{d\log \lambda_i + d\log E - d\log \bar{M}_i, 0\}.$$

This backs up the claim that the $d\log M_i$'s act like endogenous negative productivity shocks. They provide a mechanism whereby a negative demand shock, say in the composition of demand or in aggregate demand $d\log \zeta$, triggers exits which are isomorphic to negative supply shocks.

As in the other examples, the general lesson is that the output response, to a first-order, is again given by an application of Hulten's theorem along with an amplification effect which depends on how the network redistributes demand and triggers Keynesian unemployment in some factors and firm failures in some sectors.

7.2 Illustrative Example

Consider once again the horizontal economy analyzed in Section 3.5. We assume that there are no shocks to aggregate demand ($d\log \zeta = 0$). Since $\rho = 1$, this ensures that nominal expenditure is constant ($d\log E = 0$). We also assume that there are no exogenous shocks to productivities ($d\log A_i = 0$), no shocks to factor supplies ($d\log \bar{L}_f = 0$), and no shocks to the composition of demand ($d\log \omega_{0j} = 0$). Finally, we assume that all sectors have the same within-sector elasticity of substitution $\sigma_i = \sigma > 1$.

We focus on exogenous shocks $d\log \bar{M}_i \leq 0$ capturing government-mandated shutdowns. We show how endogenous failures can amplify these negative supply shocks. The insights are more general and also apply to labor supply shocks. Similarly, failures can be triggered by negative aggregate demand shocks, and the resulting endogenous negative supply shocks can result in stagflation with simultaneous reductions in output and increases in inflation.

Preliminaries. Changes in the sales of i are given by

$$d\log \lambda_i = (1 - \theta_0)(1 - \lambda_i)\Big(d\log p_i - \sum_{j\in\mathcal{N}} \lambda_j d\log p_j\Big), \tag{7.1}$$

where changes in the price of i depend on changes in the wage in i and on the endogenous reduction in the productivity of i driven by firm failures

$$d \log p_i = d \log w_i - \frac{1}{\sigma - 1} d \log M_i. \tag{7.2}$$

The change in wages in i are given by

$$d \log w_i = \max\{d \log \lambda_i - d \log \bar{L}_i, 0\}, \tag{7.3}$$

and changes in the mass of producers in i are given by

$$d \log M_i = \min\{d \log \lambda_i, d \log \bar{M}_i\}. \tag{7.4}$$

We consider the effect of shutdown shocks $d \log \bar{M}_i$ starting with the case where sectors are complements and then the case where they are substitutes. The effect of negative labor supply shocks $d \log \bar{L}_i$ is similar.

Shut-down shock with complements. We assume that sector are complements ($\theta < 1$), and we consider the government-mandated shutdown of some firms in only one sector i. We can aggregate the non-shocked sectors into a single representative sector indexed by $-i$. We therefore have $d \log \bar{M}_i < 0 = d \log \bar{M}_{-i}$.

The closures of firms in i raise its price ($d \log p_i > 0$), which because of complementarities, increases its share ($d \log \lambda_i > 0$). It therefore does not trigger any further endogenous exit in this shocked sector ($d \log M_i = d \log \bar{M}_i$). In addition, the wages of its workers increases ($d \log w_i > 0$). The shock reduces expenditure on the other sectors ($d \log \lambda_{-i} < 0$), and this reduction in demand triggers endogenous exits ($d \log M_{-i} < 0$), pushes wages against their downward rigidity constraint ($d \log w_{-i} = 0$) and creates unemployment ($d \log L_{-i} < 0$), both of which endogenously amplify the reduction in output through failures and Keynesian effects.

Using equations (7.1), (7.2), (7.3), and (7.4), we find

$$d \log \lambda_i = -\frac{(1-\theta)(1-\lambda_i)}{1-(1-\theta)(1-\lambda_i)\left(1-\frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)} \frac{1}{\sigma-1} d \log \bar{M}_i > 0,$$

$$d \log M_{-i} = d \log L_{-i} = -\frac{\lambda_i}{1-\lambda_i} d \log \lambda_i < 0,$$

and finally

$$d\log Y = \lambda_i \frac{1}{\sigma-1} d\log \bar{M}_i + \frac{(1-\theta)(1-\lambda_i)\frac{\sigma}{\sigma-1}}{1-(1-\theta)(1-\lambda_i)\left(1-\frac{1}{\sigma-1}\frac{\lambda_i}{1-\lambda_i}\right)} \lambda_i \frac{1}{\sigma-1} d\log \bar{M}_i.$$

The first term on the right-hand side is the direct reduction in output from the shut-down in sector i. The second term capture the further indirect equilibrium reduction in output via firm failures and unemployment in the other sectors.

Shut-down shock with substitutes. Consider the same experiment as above but assume now that sectors are substitutes ($\theta > 1$). We conjecture an equilibrium where sales in sector i do not fall more quickly than the initial shock $d\log\lambda_i - d\log\bar{M}_i > 0$. Sector i loses demand following the exogenous shutdown of some of its firms, and this results in unemployment in in the sector ($d\log L_i < 0$) but no endogenous firm failures ($d\log M_i = d\log\bar{M}_i$) . On the other hand, sector $-i$ maintains full employment and experiences no failures.

To verify that this configuration is indeed an equilibrium, we compute

$$d\log\lambda_i = \frac{(\theta-1)(1-\lambda_i)}{1-(\theta-1)\lambda_i} \frac{1}{\sigma-1} d\log \bar{M}_i.$$

We must verify that

$$0 > d\log\lambda_i > d\log\bar{M}_i.$$

The first inequality is verified as long as $\theta > 1$ is not too large. The second inequality is verified if $\sigma > 1$ is large enough and $\theta > 1$ is not too large.

If these conditions are violated, then we can get a jump in the equilibrium outcome. Intuitively, in those cases, the shutdown triggers substitution away from i, and that substitution is so dramatic than it causes more firms to shutdown, and the process feeds on itself ad infinitum. Any level of $d\log L_i < 0$ and $d\log M_i < d\log\bar{M}_i$ can then be supported as equilibria. Although we do not focus on it, this possibility illustrates how allowing for firm failures with increasing returns to scale can dramatically alter the model's behavior.

Assuming the regularity conditions above are satisfied, the response of output is given by

$$d\log Y = \lambda_i \frac{1}{\sigma-1} d\log\bar{M}_i + \frac{(\theta_0-1)(1-\lambda_i)}{1-(\theta_0-1)\lambda_i} \lambda_i \frac{1}{\sigma-1} d\log\bar{M}_i,$$

where the first term on the right-hand side is the direct effect of the shutdown and the second term is the amplification from the indirect effect of the shutdown which results in Keynesian unemployment in i.

8 Extension III: Policy

In this section, we briefly review how policy can affect outcomes. We analyze three different types of policy: monetary, fiscal, and tax policy.

8.1 Monetary Policy

A monetary expansion in this model comes in the form of positive aggregate demand shock

$$d\log\zeta = -\rho\left(d\log(1+i) + \frac{d\log\beta}{1-\beta} - d\log\bar{p}^{Y}\right) + d\log\bar{Y} > 0,$$

this can come about either via lower nominal interest rates, or failing that, forward guidance about the price level in the future. If nominal rates are stuck at the zero-lower bound, then an increase in future prices, by lowering the real interest rate, will stimulate spending today. A positive aggregate demand shock increases nominal expenditures since

$$d\log E = \frac{Cov_{\lambda_{\mathcal{G}}}\left(\chi_{\mathcal{H}^c}, d\log\lambda_{\mathcal{G}}\right)}{1-\mathbb{E}_{\lambda_{\mathcal{G}}}\left(\chi_{\mathcal{H}^c}\right)} + (1-\rho)d\log p^{Y} + d\log\zeta.$$

If nominal expenditures $d\log E$ are sufficiently high, then the economy can maintain full employment regardless of the shocks by guaranteeing that nominal wages do not have to fall in equilibrium

$$\min_{f\in\mathcal{G}}\{d\log\lambda_i + d\log E - d\log\bar{L}_f\} > 0.$$

This is obviously the optimal policy for the monetary authority to pursue, if it is feasible.

Setting aside full-employment policy, we can also consider how output responds to a given monetary stimulus $d\log\zeta > 0$. Since the model is non-linear, the effectiveness of monetary policy depends on what other shocks have hit the economy. A canonical example is if the monetary stimulus coincides with a set of negative supply shocks. In this case, complementarities in production act to reduce the effectiveness of monetary policy.

To see this, consider again the horizontal economy described in Example 3.5. We assume that sectors are complements with $\theta < 1$ and that the intertemporal elasticity of substitution is $\rho = 1$. For simplicity, suppose that there are no constrained households. We hit the economy with negative factor supply shocks $d\log \bar{L} < 0$,. Suppose that in addition, through forward-guidance, the monetary authority is able to raise $d\log\zeta > 0$.

Then, working through the same equations as in the original example, we find that the overall effect on output is

$$d\log Y = \frac{\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} + \left(1 - \frac{(1-\theta)\lambda_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}\right)(1-\lambda_{\mathcal{F}}) d\log\zeta.$$

The first term is the effect of the negative supply shock, amplified by the nominal rigidities, exactly as in Example 3.5. The second term is the effect of the monetary stimulus. The term $(1-\lambda_{\mathcal{F}})d\log\zeta$ is the direct effect of the stimulus on the employment of rigid factors. However, this direct effect is mitigated. This is because monetary stimulus raises the prices of flexible factors in absolute terms and relative to those of rigid factors, and since flexible and rigid factors are complements, this causes expenditures to switch towards flexible factors and away from rigid factors. This force attenuates the effectiveness of monetary policy, and it would not appear if not for the heterogeneity in cyclical conditions across factor markets.

8.2 Payroll Tax Cuts

In this section, we briefly consider the effect of payroll tax cuts used by many governments in the wake of Covid-19. If correctly targeted, payroll tax cuts can alleviate the demand short-fall in slack factor markets. By selectively cutting taxes (or subsidizing) unemployed sectors, a policymaker can actually implement the first-best outcome.[25]

Even without going all the way to the first best, these policies can be helpful. However, as with monetary policy, complementarities in production also reduce the effectiveness of a given intervention. To see this, consider once more the horizontal, economy of Example 3.5. We assume that sectors are complements with $\theta < 1$ and that the intertemporal elasticity of substitution is $\rho = 1$. For simplicity, we assume that there are no constrained households.[26] We hit the economy with negative factor supply shocks $d\log\bar{L}_f < 0$. In

[25] See e.g. Correia et al. (2013); Farhi et al. (2014).

[26] With exogenously constrained households, payroll tax cuts can be helpful through a different channel if they increase their income by boosting their wages, thereby effectively redistributing away from households with low marginal propensities to consume and towards households with high marginal propensities to consume.

addition, we assume that the government institutes a gross payroll subsidy $d\log s_{\mathcal{R}}$ on the rigid factors financed by a tax on the resulting profits. In this case, the output response is

$$d\log Y = \frac{\lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} + \left(1 - \frac{(1-\theta)\lambda_{\mathcal{F}}}{1-(1-\theta)(1-\lambda_{\mathcal{F}})}\right)(1-\lambda_{\mathcal{F}})d\log s_{\mathcal{R}}.$$

As usual, the first term is the effect of the negative supply shock to the flexible sectors, amplified by the nominal rigidities. The second term is the effect of the payroll subsidy. Naturally, a subsidy on rigid factors increases output, and the term $(1-\lambda_{\mathcal{F}})d\log s_{\mathcal{R}}$ is the direct effect of the increase in employment. However, the subsidy on rigid factors also reduces the price of rigid sectors relative to flexible ones. Since factors are complements, this means that expenditures shift towards flexible factors and away from rigid ones, attenuating the effect of the payroll subsidy.

8.3 Fiscal Policy

Finally, we consider the effect of changes in size of and composition of government spending. We assume that $G = 0$ and denote by dG the change in government expenditure and by Ω_k^G the shares of the different sectors in government expenditure. We assume that government spending is deficit-financed, and that the debt is repaid with taxes in the future. We assume that only a fraction $\alpha_{\text{Ricardian}}$ of these future taxes falls on Ricardian households, and the rest falls either on non-Ricardian households or on future generations.

We denote the average marginal propensity to consume by $\overline{MPC} = \mathbb{E}_{\lambda}(1-\chi_{\mathcal{H}^c})MPC_{\text{Ricardian}} + \mathbb{E}_{\lambda}(\chi_{\mathcal{H}^c})$, where $MPC_{\text{Ricardian}} = 1-\beta$ is the marginal propensity to consume for Ricardian households and 1 is the marginal propensity to consume of non-Ricardian households.

The only changes in the analysis concern the determination of changes in nominal expenditure and the propagation equations for sales and factor shares. Changes in nominal expenditure are given by

$$d\log E = \frac{Cov_{\lambda}\left(\chi_{\mathcal{H}^c}, d\log\lambda\right)}{1-\mathbb{E}_{\lambda}\left(\chi_{\mathcal{H}^c}\right)} + (1-\rho)d\log p^Y + d\log\zeta + \frac{1-\alpha_{\text{Ricardian}}MPC_{\text{Ricardian}}}{1-\overline{MPC}}dG.$$

Changes in sales and factor shares are given by

$$\lambda_f d\log\lambda_f = \sum_{k\in\mathcal{N}} \Psi_{kf}\Omega_{0k} d\log\omega_{0k} + \sum_{k\in\mathcal{N}} \Psi_{kf}(\Omega_k^G - \Omega_{0k})\frac{dG}{E}$$
$$+ \sum_{j\in 1+\mathcal{N}} \lambda_j(\theta_j - 1) Cov_{\Omega^{(j)}}\left(\sum_{k\in\mathcal{N}} \Psi_{(k)}\left(d\log A_k\right) + \sum_{g\in\mathcal{G}} \Psi_{(g)}\left(d\log L_g - d\log\lambda_g\right), \Psi_{(f)}\right),$$

where $d\log L_g = d\log \bar{L}_g$ for $f \in \mathcal{K}$ and $d\log L_g = \min\left\{d\log\lambda_g + d\log E, d\log\bar{L}_g\right\}$ for $f \in \mathcal{L}$. We can then combine these formulas with Proposition 1 to get the change in aggregate output exactly as before. This generalizes Baqaee (2015) beyond the Cobb-Douglas special case.

These results show how changes in government spending can stimulate output in two different ways. The first reason is the standard Keynesian-cross argument: an increase in government spending stimulates the incomes of households, who then proceed to consume more. This boosts nominal expenditure by $dG(1 - \alpha_{\text{Ricardian}} MPC_{\text{Ricardian}})/(1 - \overline{MPC})$, which is higher, the higher is the average marginal propensity to consume $\overline{MPC}$ and the lower is the fraction $\alpha_{\text{Ricardian}}$ of future taxes that fall on Ricardian consumers.[27] Interestingly, in the context of the pandemic, the fiscal multiplier could be lower in a partial lock-down if Ricardian households have a low marginal propensity to consume because they want to postpone consumption until the lock-down is fully lifted.

The second reason is slightly more subtle. By choosing the composition of government spending wisely, the government can target its spending to boost the demand of sectors whose factor markets are depressed. This effect is captured by $\sum_{k\in\mathcal{N}} \Psi_{kf}(\Omega_k^G - \Omega_{0k})\frac{dG}{E}$. Intuitively, fiscal policy can move the AD curve by changing both the size and composition of government expenditures. To the extent that the government cannot perfectly target depressed factor markets, some of the government expenditures will end up wastefully increasing the wages of flexible factors instead of stimulating employment, thereby lowering the fiscal multiplier. Furthermore, fiscal multipliers are further dampened in economies with complementarities since to some extent, government spending always ends up increasing the wages of some flexible factors, causing private expenditure to be redirected towards those factors and away from rigid factors.

[27] See e.g. Farhi and Werning (2016) for a discussion.

9 Conclusion

This paper analytically characterizes and numerically quantifies the impact of different supply and demand shocks in a general disaggregated model with multiple sectors, factors, and input output linkages, as well as occasionally-binding downward nominal wage rigidity and zero lower bound constraints.

We find that complementarities in production and consumption create large Keynesian spillovers across sectors and magnify output losses. We also find that the different shocks interact and get amplified or mitigated through powerful nonlinearities.

Disaggregated data on the effects of the Covid-19 crisis are as yet scarce. In future versions of the paper, as more data becomes available, we hope to revise our quantitative analysis. For now, we find that shocks to the sectoral composition of final demand or shocks to sectoral labor supply can both be consistent with the data on hours, spending, and inflation, but only if these shocks are accompanied by a large negative aggregate demand shock. This is because shocks to sectoral labor supply or to the sectoral composition of final demand are necessarily stagflationary, and on their own, generate counterfactual increases in prices.

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Appendix A Investment

To model investment, we can simply add intertemporal production functions into the model. An investment function transforms goods and factors in the present period into goods that can be used in the future. In that case, instead of breaking the problem into an intertemporal and intratemporal problem, we must treat both problems at once. In this case, Proposition 1 still applies without change.

However, we can no longer use the Euler equation to pin down nominal expenditures today, since nominal GDP today includes investment expenditures and output tomorrow can no longer taken to be exogenous. Instead, to determine $d\log E$, we must use a version of Proposition 2.

In particular, let λ_i^I denote the intertemporal sales share — expenditures on quantity i as a share of the net present value of household income. Furthermore, let $\bar{\Omega}^I$ represent the intertemporal input-output matrix, which includes the capital accumulation equations. Then, letting intertemporal consumption be the zero-th good, and abstracting from shocks to the composition of demand for simplicity, we can write

$$d\log\lambda_k^I = \sum_{j\in\mathcal{N}} \lambda_j^I(\theta_j - 1)Cov_{\Omega^{I,(j)}}\left(\sum_{i\in\mathcal{N}} \Psi_{(i)}^I d\log A_i - \sum_{f\in\mathcal{G}} \Psi_{(f)}^I \left(d\log\lambda_f^I - d\log L_f\right), \frac{\Psi_{(k)}^I}{\lambda_k^I}\right)$$

almost everywhere, where changes in factor employments are given by

$$d\log L_f = \begin{cases} d\log\bar{L}_f, & \text{for} \quad f\in\mathcal{K}, \\ \min\left\{d\log\lambda_f + d\log E, d\log\bar{L}_f\right\}, & \text{for} \quad f\in\mathcal{L}. \end{cases}$$

Changes in nominal expenditures today are given by

$$d\log E = \sum_{f\in\mathcal{G}} \frac{\lambda_f^I}{\beta} d\log\lambda_f^I + d\log Y_*,$$

but now $d\log Y_*$ is endogenous. In particular, we have

$$d\log Y_* = \sum_{i\in\mathcal{N}_*} \lambda_{*,i} d\log A_{*,i} + \sum_{i\in\mathcal{G}_*} \lambda_{*,i} d\log L_{*,i},$$

where asterisks denote future variables. The set $\mathcal{G}_*$ is the set of factors from the perspective of the future period. This includes both factor endowments in the future as well as endogenously accumulated capital stocks.

To complete the characterization, we note that for each endogenously accumulated factor $f \in \mathcal{G}_*$, we have

$$d\log L_f = d\log \lambda_f^I + (1-\beta)d\log E + \beta d\log Y_* - d\log p_f,$$

and $d\log p_f$ given by the usual set of forward propagation equations

$$d\log p_f = \sum_{i \in \mathcal{N}} \Psi_{fi}^I d\log A_i + \sum_{j \in \mathcal{G}^I} \Psi_{fj}^I d\log p_j,$$

where $\mathcal{G}^I$ is the set of factor endowments across all periods. The price of these factor endowments are, in turn, given by

$$d\log p_j = \max\{d\log \lambda_j^I + (1-\beta)d\log E + \beta d\log Y_2 - d\log \bar{L}_f, 0\},$$

if j is a sticky factor and $d\log p_j = d\log \lambda_j^I + (1-\beta)d\log E + \beta d\log Y_2 - d\log \bar{L}_f$ otherwise.

Appendix B Some Downward Wage Flexibility

In practice, we might imagine that wages can fall albeit not by enough to clear the market. The possibility that wages may fall obviously has important implications for inflation. Indeed, we show that with shocks to the composition of demand, and even without shocks to aggregate demand, we can get simultaneous reductions in output *and* inflation.

For each factor $f \in \mathcal{L}$, suppose the following conditions hold

$$\frac{L_f}{\bar{L}_f} = \begin{cases} \left(\frac{w_f}{\bar{w}_f}\right)^{\phi_f}, & \text{if} \quad w_f \le \bar{w}_f, \\ 1, & \text{if} \quad w_f > \bar{w}_f. \end{cases}$$

The parameter ϕ_f controls the degree of downward wage flexibility. If $\phi_f = \infty$, then the wage is perfectly rigid downwards. If $\phi_f = 0$, then the wage is fully flexible, and we recover the neoclassical case.

$$\left(\frac{w_f}{\bar{w}_f} - \left(\frac{L_f}{\bar{L}_f}\right)^{\frac{1}{\phi_f}}\right)\left(L - \bar{L}_f\right) = 0, \quad L_f \le \bar{L}_f, \quad \left(\frac{L_f}{\bar{L}_f}\right)^{\frac{1}{\phi_f}} \le \frac{w_f}{\bar{w}_f}.$$

B.1 Generalizing the Results

The only change to Proposition 1 is that we now have

$$d\log Y = \sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log \bar{L}_f + \sum_{f\in\mathcal{L}} \frac{\phi_f}{1+\phi_f}\lambda_f \min\left\{d\log\lambda_f + d\log E - d\log\bar{L}_f, 0\right\},$$

and the only change to Proposition 2 is that we now have

$$d\log L_f = \begin{cases} \frac{\phi_f}{1+\phi_f}\left(d\log\lambda_f + d\log E\right) + \frac{1}{1+\phi_f} d\log\bar{L}_f & \text{if} \quad f\in\mathcal{R} \\ d\log\bar{L}_f & \text{if} \quad f\in\mathcal{F}. \end{cases} \tag{B.1}$$

B.2 Illustrative Example

We now construct an example showing how allowing for some degree of downward wage flexibility allows the model to generate a recession *and* deflation at the same time, without relying on aggregate demand shocks. We return to the example of Section 3.5. However, this time, suppose that wages have some degree of downward flexibility $0 < \phi < \infty$ common across all factor markets $f \in \mathcal{L}$.

We now get

$$d\log Y = \lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d\log L_{\mathcal{R}},$$

where $\lambda_{\mathcal{R}} = \sum_{f\in\mathcal{R}} \lambda_f = 1 - \lambda_{\mathcal{F}}$ is the total share of the rigid factors and $d\log L_{\mathcal{R}}$ is the "representative" employment reduction in the rigid sectors

$$d\log L_{\mathcal{R}} = \sum_{f\in\mathcal{R}} \frac{\lambda_f}{\lambda_{\mathcal{R}}} d\log L_f < \sum_{f\in\mathcal{R}} \frac{\lambda_f}{\lambda_{\mathcal{R}}} d\log\bar{L}_f = d\log\bar{L}_{\mathcal{R}}.$$

In turn, this employment reduction is given as a function of the change $d\log\lambda_{\mathcal{F}}$ in the share of the flexible sectors by

$$\lambda_{\mathcal{R}} d\log L_{\mathcal{R}} = -\frac{\phi}{1+\phi}\lambda_{\mathcal{F}} d\log\lambda_{\mathcal{F}} + \frac{1}{1+\phi}\lambda_{\mathcal{R}} d\log\bar{L}_{\mathcal{R}},$$

and the the change $d\log\lambda_{\mathcal{F}}$ in the share of the flexible sectors is given by

$$\lambda_{\mathcal{F}} d\log\lambda_{\mathcal{F}} = \frac{\lambda_{\mathcal{F}} d\log\omega_{0\mathcal{F}} - (1-\theta)\lambda_{\mathcal{F}}(1-\lambda_{\mathcal{F}})\left[d\log\bar{L}_{\mathcal{F}} - \frac{1}{1+\phi} d\log\bar{L}_{\mathcal{R}}\right]}{1 - \frac{\phi}{1+\phi}(1-\theta)(1-\lambda_{\mathcal{F}})}.$$

Starting with the last equation, we see that once again, the share of flexible factors increases if the shock to the composition of demand redirects expenditure towards these factors or if the labor supply shocks for those factors is larger than the ones hitting the rigid factors. This reduces the shares of rigid factors, creates unemployment, and further reduces output through Keynesian effects. Indeed, putting everything together, we get

$$
\begin{aligned}
d\log Y &= \lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}} \\
&+ \frac{\frac{\phi}{1+\phi}(1-\theta)\lambda_{\mathcal{F}}(1-\lambda_{\mathcal{F}})d\log \bar{L}_{\mathcal{F}} + \left(1-\frac{1}{1+\phi}(1-\theta)\right)\lambda_{\mathcal{R}} d\log \bar{L}_{\mathcal{R}} - \frac{\phi}{1+\phi}\theta\lambda_{\mathcal{F}} d\log \omega_{0\mathcal{F}}}{1-\frac{\phi}{1+\phi}(1-\theta)(1-\lambda_{\mathcal{F}})}.
\end{aligned}
$$

The difference between the case where wages have some downward flexibility ($\phi < \infty$) and the case where they do not ($\phi = \infty$) is that now the wages of the rigid factors falls, and this mitigates the increase in unemployment and the reduction in output. However, there is also a countervailing amplification effect: the labor supply shocks to the rigid factors now also over and above ensuring that they are rigid. This is because these shocks now reduce the wages of the rigid factors, which further redirects expenditure away from them because of complementarities, and further reduces employment of the rigid factors. Of course, allowing for some degree of wage flexibility can endogenously change the sets of flexible and rigid factors, and so we do no try to push the comparison any further.

Instead, we turn our attention to inflation. Using $d\log p^Y = d\log E - d\log Y$, the effect on inflation is

$$
d\log p^Y = -\frac{1}{1+\phi} d\log \lambda_{\mathcal{F}} - \lambda_{\mathcal{F}} d\log \bar{L}_{\mathcal{F}} - \frac{1}{1+\phi}\lambda_{\mathcal{R}} d\log \bar{L}_{\mathcal{R}}.
$$

The first term is negative, since the share of flexible factors expands in response to the negative demand shock, capturing the fact that as demand switches to flexible factors, the price of sticky sectors starts to decline, generating deflation. In the simple case where there are no negative supply shocks $d\log \bar{L} = 0$ but the composition of demand has shifted, we get that output *and* inflation both fall.

Appendix C Benefits of Wage Flexibility and of Factor Reallocation

We end this section with two propositions: that wage flexibility and factor reallocation are desirable. These two propositions may at first seem obvious, but they are by no means universally valid. Since the model with nominal rigidities is inefficient, the theory of the second best means that seemingly desirable attributes like flexibility and reallocation can actually turn out to be harmful in general. However, to the extent that the benchmark case with uniform elasticities is likely to be realistic, then these propositions guarantee that neoclassical intuitions about flexibility and reallocation are still empirically relevant.

To show that wage flexibility is desirable, we take a factor $f \in \mathcal{L}$ and remove its downward wage rigidity constraint by moving it to $\mathcal{K}$. This amounts to creating a more flexible economy.

Corollary 5. *Under the assumptions of Proposition 3 at the best equilibria,* $\Delta \log Y$ *and* $\Delta \log L$ *are higher in the more flexible than the less flexible economy.*

In addition to the fact that flexibility is desirable, we can also prove that reallocation is desirable. We consider two factors h and h' that are paid the same wage at the initial equilibrium and that have the same minimum nominal wage. The idea is that these two factors are really the same underlying factor, but that frictions to reallocation prevent them from being flexibly reallocated one into the other. We then consider a reallocation economy where these reallocations are allowed to take place. This amounts to a renormalization of the input-output matrix and of the shocks.

Corollary 6. *Under the assumptions of Proposition 3, the best equilibrium of the no-reallocation economy has lower output* $\Delta \log Y$ *and employment* $\Delta \log L$ *than the best equilibrium of the reallocation economy.*

Appendix D More Examples

In this section, we use some analytical examples to show how the network structure can matter. We show how shocks to the composition of demand and substitutability in supply chains can also act to reduce output. Throughout all these examples, we assume that the intertemporal elasticity of substitution is $\rho = 1$ so that nominal expenditure is exogenous $d \log E = d \log \zeta$.

D.1 Cobb-Douglas Economy

We first consider how demand shocks affect output and employment in a Cobb-Douglas production-network economy where all elasticities of substitution in production and in final demand are equal to one ($\theta_j = 1$ for all j). This example recalls findings in Baqaee (2015). We allow for shocks to productivities $d\log A_i$, labor supplies $d\log \bar{L}_f$, composition of demand $d\log \omega_{0i}$, and aggregate demand $d\log \zeta$.

Proposition 2 implies that factor shares change only due to changes in the composition of demand:

$$d\log \lambda_f = Cov_{\Omega^{(0)}}\left(d\log \omega_0, \frac{\Psi_{(f)}}{\lambda_f}\right) = \sum_j \Omega_{0j} d\log \omega_{0j} \frac{\Psi_{jf}}{\lambda_f}.$$

The parameter Ψ_{jf} is a network-adjusted measure of use factor f by producer j. The covariance captures the fact that a shock that redirects expenditure away from j reduces the share of factor f if j is more intensive in its use of factor f than the rest of the economy.

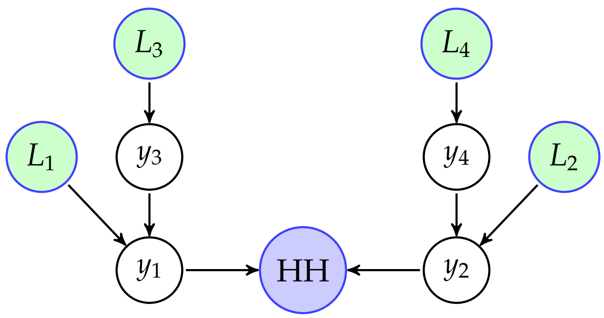
Plugging back into Proposition 1 yields response of output

$$\begin{aligned} d\log Y = &\underbrace{\sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log \bar{L}_f,}_{\text{neoclassical effect}} \\ &\underbrace{+\sum_{f\in\mathcal{L}} \lambda_f \min\left\{Cov_{\Omega^{(0)}}\left(d\log \omega_0, \frac{\Psi_{(f)}}{\lambda_f}\right) + d\log \zeta - d\log \bar{L}_f, 0\right\}}_{\text{Keynesian effect}}. \end{aligned}$$

The terms on the first line summarize the impact of the shock if the economy were neoclassical with no downward nominal wage rigidity. The terms on the second line are negative and capture the additional endogenous reduction in output through Keynesian channels: output is additionally reduced if the composition of demand shifts away from sectors whose network-adjusted use of labors with small shocks is high, or if there is a negative aggregate demand shock. Conditional on shares λ_f, the input-output network matters *only* in so far as it translates changes in household demand into changes in factor demands.

In the Cobb-Douglas example, demand shocks $d\log \omega_0$ and $d\log \zeta$ only propagate backward along supply chains to cause unemployment upstream. On the other hand, supply shocks $d\log \bar{L}_f$ and $d\log A_i$ only propagate forward along supply chains but do not cause any unemployment downstream. In fact, since these shocks do not change factor shares, supply shocks do not cause any unemployment in any of the factors, and so these shocks do not trigger the Keynesian channels. The next example shows how deviating from Cobb-Douglas changes these conclusions.

D.2 Substitutable Supply Chains



$$
\begin{aligned}
(Y/\bar{Y})^{\frac{\theta_0-1}{\theta_0}} &= \bar{\lambda}_1 (y_1/\bar{y}_1)^{\frac{\theta_0-1}{\theta_0}} + \bar{\lambda}_2 (y_2/\bar{y}_2)^{\frac{\theta_0-1}{\theta_0}}, \\
y_i/\bar{y}_i &= L_i/\bar{L}_i, \quad i \in \{3,4\}, \\
y_i/\bar{y}_i &= (L_i/\bar{L}_i)^{1-\omega}(y_{i+2}/\bar{y}_{i+2})^{\omega}, \quad i \in \{1,2\} \\
L_i &= \min\{\lambda_i E, \bar{L}_i\} \qquad , \\
w_i &= \max\{\lambda_i E/\bar{L}_i, \bar{w}_i\}.
\end{aligned}
$$

Figure D.1: Horizontal Economy. The arrows represent the flow of resources for production. Each sector has its own factor market.

Our second example shows how production networks can feature Keynesian unemployment in response to negative supply shocks even without complementarities. However, doing so requires having non-uniform elasticities of substitution (otherwise network-irrelevance applies). In particular, once elasticities of substitution are non-uniform, labor supply shocks can create unemployment upstream and downstream. In contrast to Example 3.5, where complementarities create unemployment in the non-shocked supply chains, in this example, substitutabilities create unemployment *within* the shocked supply chain. We assume away productivity shocks and demand shocks so that $d\log A_i = 0$ for all i, $d\log \omega_{0j} = 0$ for all j, and $d\log \zeta = 0$.

We consider the example in Figure D.1, where the household consumes the output of sectors 1 and 2 with elasticity of substitution $\theta_0 > 1$. The initial expenditure shares are λ_1 and $\lambda_2 = 1 - \lambda_1$ for sectors 1 and 2 respectively. The two downstream sectors have Cobb-Douglas production functions combining sector-specific labor with an upstream input, with respective shares $1-\omega$ and ω. The upstream supplier of 1 is 3 and the one for 2 is 4. The two upstream suppliers produce using industry-specific labor. The sales shares of sector 3 and 4 are given by $\lambda_3 = \omega\lambda_1$, and $\lambda_4 = \omega\lambda_2$. The factor shares of labors in the different sectors are given by $(1-\omega)\lambda_1$, $(1-\omega)\lambda_2$, λ_3, and λ_4. We denote by p_i the price of i and by w_i the wage of workers in i.

We will only consider negative labor supply shocks $d\log \bar{L}_1 \leq 0$ and $d\log \bar{L}_3 \leq 0$ to 1 and 3, and we will maintain the assumption that $d\log \bar{L}_2 = d\log \bar{L}_4 = 0$. Hence the quantity of 1 will decrease, its relative price will increase, and because $\theta_0 > 1$, consumers will substitute expenditure towards good 2. This in turn implies that wages in 2 and 4 will increase. There will not be any unemployment in 2 and 4. However, there may be unemployment in 1 and/or 3 and we focus our attention on these sectors.

Preliminaries. To conduct the analysis, we rely on Proposition 2 which implies that changes in the sales share of sector 1 are given by

$$d\log \lambda_1 = (\theta_0 - 1)(1 - \lambda_1)(d\log p_2 - d\log p_1), \tag{D.1}$$

where $d\log p_1 = (1-\omega)d\log w_1 + \omega d\log p_3$ and $d\log p_3 = d\log w_3$. Changes in the sales share of sector 2 are then given by

$$d\log \lambda_2 = -\frac{\lambda_1}{1-\lambda_1} d\log \lambda_1, \tag{D.2}$$

and since $d\log y_2 = 0$ and $d\log E = 0$, we also have $d\log p_2 = d\log \lambda_2$. Finally, we have $d\log \lambda_3 = d\log \lambda_1$ and $d\log \lambda_2 = d\log \lambda_4$.

Negative downstream labor supply shock. To start with, suppose that $d\log \bar{L}_1 < d\log \bar{L}_3 = 0$. That is, the downstream producer in supply chain 1 is negatively affected.

Then the only equilibrium features

$$d\log \lambda_3 - d\log \bar{L}_3 < d\log w_3 = 0 < d\log \lambda_1 - d\log \bar{L}_1 = d\log w_1.$$

The wage in sector 1 increases and the wage in sector 3 hits its downward rigidity constraint. There is full employment in sector 1 but there is unemployment in sector 3. w_1 increases but w_3 falls. This is because the negative labor supply shock in 1 causes the price of 1 to rise, which causes consumers to redirect expenditures away from 1 since $\theta_0 > 1$, which in turn reduces the demand for 1 and for 3.

This can be verified by substituting these expressions into equation (D.1) and (D.2) to get

$$d\log \lambda_1 = \frac{(\theta_0 - 1)(1-\lambda_1)(1-\omega) d\log \bar{L}_1}{1 + (\theta_0 - 1)\left[(1-\lambda_1)(1-\omega) + \lambda_1\right]} > d\log \bar{L}_1$$

as needed.[28]

Using this expression for $d \log \lambda_1$ and plugging back into Proposition 1 gives

$$d \log Y = \underbrace{\lambda_1(1-\omega) d \log \bar{L}_1}_{\text{neoclassical effect}} + \underbrace{\frac{\omega(\theta_0 - 1)(1-\lambda_1)}{1 + (\theta_0 - 1)\left[(1-\lambda_1)(1-\omega) + \lambda_1\right]} \lambda_1 (1-\omega) d \log \bar{L}_1}_{\text{Keynesian effect}}.$$

Here the first term on the right-hand side coincides with the impact of the negative labor supply shock in the neoclassical model. The second term on the right-hand side is negative and captures the additional reduction in output through Keynesian channel via increases in unemployment in sector 3. Hence, the negative supply shock is transmitted upstream as a negative demand shock. The shock has its greatest impact for intermediate values of ω, balancing the fact that a higher ω magnifies the negative demand effect but lowers the negative supply effect.

Overall this example shows that, once we deviate from Cobb-Douglas, then expenditure switching causes supply shocks to travel in either direction along the supply chain, reducing employment in other parts of the economy, and amplifying the effect of the original shock.

Negative downstream labor supply shock. Similarly, the shock can be transmitted in the opposite direction. To see this, suppose instead that $d \log \bar{L}_3 < d \log \bar{L}_1 = 0$.

The only equilibrium features

$$d \log \lambda_1 - d \log \bar{L}_1 < d \log w_1 = 0 < d \log \lambda_3 - d \log \bar{L}_3 = d \log w_3.$$

This time, it is the downstream sector that suffers the negative demand shock and experiences unemployment. This can be verified by substituting these expressions into equations (D.1) and (D.2) to get as needed

$$d \log \lambda_1 = \frac{(\theta_0 - 1)(1-\lambda_1)\omega d \log \bar{L}_3}{1 + (\theta_0 - 1)\left[(1-\lambda_1)\omega + \lambda_1\right]} > d \log \bar{L}_3.$$

[28] In fact, this would continue to be the case even if the upstream supplier was also negatively affected $d \log \bar{L}_3 < 0$, as long as this negative shock is not too large in magnitude.

Using this expression for $d \log \lambda_3 = d \log \lambda_1$ and plugging back into Proposition 1 gives

$$d \log Y = \underbrace{\lambda_3 d \log \bar{L}_3}_{\text{neoclassical effect}} + \underbrace{\frac{(\theta_0 - 1)(1 - \lambda_1)(1 - \omega)}{1 + (\theta_0 - 1)\left[(1 - \lambda_1)\omega + \lambda_1\right]} \lambda_3 d \log \bar{L}_3}_{\text{Keynesian effect}} .$$

Once again, the first term on the right-hand side coincides with the impact of the negative labor supply shock in the neoclassical model. The second term on the right-hand side is negative and captures the additional reduction in output through Keynesian channel via increases in unemployment in sector 1. The negative supply shock is now transmitted downstream where it reduces demand.