THE EARLY HISTORY OF PRICE INDEX RESEARCH

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ABSTRACT

The paper discusses five early approaches to the price (and quantity) index number problem. The five approaches are: (i) the fixed basket approach; (ii) the statistical approach; (iii) the test or axiomatic approach; (iv) the Divisia approach and (v) the economic approach.

The economic approach makes use of the assumption of optimizing behavior under constraint and the approach is discussed under four subtopics: (i) basic theoretical definitions; (ii) the theory of bounds; (iii) exact index numbers and (iv) econometric estimation of preferences.

The paper also discusses several topics raised by Jack Triplett in a recent paper, including: (i) the merits of the test approach to index number theory, (ii) the chain principle and alternatives to it; (iii) the substitution bias and (iv) the new good bias.

Although the paper is for the most part an extensive historical survey, there are a few new results in section 8 on multilateral alternatives to the chain principle. Also in section 6.3, it is shown that the Paasche, Laspeyres and all superlative indexes will satisfy the circularity test to the first order.

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1. Introduction

In order to limit the size of his paper, Jack Triplett has chosen to concentrate on the history of price measurement research during the last three decades. The purpose of this paper is to discuss his survey and augment it by presenting a brief overview of the ancient history of price measurement.

Basically, Triplett takes the economic approach to index number theory as being the correct approach. He distinguishes three main variants of the economic approach: (i) the cost of living index; (ii) the output price index and (iii) the input cost index.

Most of Triplett's review concentrates on three topics: (i) the likely size of the substitution bias (which occurs when we use the Paasche or Laspeyres price indexes to approximate the underlying true economic index); (ii) the appropriateness of using either a Divisia (1926) index or a chain index and (iii) the appropriate treatment of quality change in the construction of price and quantity indexes, which is otherwise known as the new good problem. The largest part of the Triplett paper is devoted to the last topic. This part of the paper is extremely interesting in its own right and is also valuable from the viewpoint of the history of the subject since Triplett was not only an active researcher in hedonic quality adjustment techniques[1], but he was also (and still is) a civil servant and thus was well positioned to comment on Statistical Agency reactions to new measurement techniques.

My overall impression of the paper is quite favorable. Jack Triplett is an empirically oriented economist who knows the underlying theoretical literature very well. He has presented us with a survey of lasting value.

At this point, I would like to list some of his insights that I found particularly interesting or worthy of strong endorsement (points of disagreement will be discussed later):

(i). With the exception of housing, consumer durables are treated as if they were entirely consumed within the year of purchase. This treatment is appropriate in the production accounts but it is not appropriate in the consumer accounts. Along with Triplett, I believe that statistical agencies should make available a rental price treatment of all major categories of consumer durables as supplementary information to their present series. A rental price treatment of purchases of new durable goods would also affect our measure of savings: a consumer's purchase of a durable should be decomposed into a user cost portion and a savings portion and the latter part should be added to conventional savings.[2]

(ii). I am basically sympathetic to his treatment of the continuous time Divisia index. Triplett (1988, 32) points out that we have to approximate the continuous time index by a discrete time approximation. The problem is that there are many discrete approximations that are available but Divisia's theory gives us no guidance as to the specific functional form for the discrete approximation. I shall elaborate on this point in section 5 below.

(iii). I strongly endorse Triplett's (1988, 30-82) comments on the inadequacies of existing data on wages.[3]

(iv). Finally, Triplett's (1988, 77) comments on the problems caused by inadequate documentation of official data series deserves to be stressed.

We conclude this section by presenting an outline of the remainder of this paper.

At least five distinct approaches to price and quantity measurement (or index number theory) can be distinguished in the early literature on the subject: (i) the tabular standard (or the commodity standard or the fixed basket approach); (ii) the statistical approach; (iii) the test approach; (iv) the Divisia index approach and (v) the economic approach. We shall discuss the history of each approach in turn in sections 2-6 below.

In sections 7 to 11 below, we shall discuss a number of issues that are raised by a reading of Triplett's paper - issues that are perhaps somewhat controversial. Section 7 briefly discusses the merits of the test approach to index number theory while section 8 presents an extended discussion of the chain principle. Sections 9 and 10 discuss the possible magnitudes of the substitution bias and the new good bias respectively while section 11 asks whether the theory of the cost of living index has been exhausted.

Section 12 concludes with a list of recommendations directed towards Statistical Agencies.

2. The Fixed Basket Approach

The essence of the fixed basket approach or the tabular standard may be explained as follows. Suppose that there are N goods that consumers in a location can purchase during two periods. In periods 1 and 2, the relevant price vectors are $p^1 = (p_1^1, \ldots, p_N^1)$ and $p^2 = (p_1^2, \ldots, p_N^2)$ respectively. Suppose further (unrealistically) that the quantities purchased of the N goods are constant during the two periods, with the

constant vector of purchases being defined as $q = (q_1, \ldots, q_N)$. Then a natural measure of the average level of prices in period 2 relative to period 1 is $p^2 \cdot q / p^1 \cdot q$ where $p^t \cdot q = \Sigma_{n=1}^{N} p_n^t q_n$ is the inner product of the vectors p^t and q.

The above approach to price measurement has been independently proposed by many people. The earliest known proposer of the method was William Fleetwood, the Bishop of Ely, who wrote the book Chronicon Preciosum in 1707[4]. The constant basket of goods he used to compare the value of money (or conversely, the level of prices) for an Oxford student of 1707 compared to an Oxford student of 1460 was 5 quarters of wheat, 4 hogsheads of beer and 6 yards of cloth.

Perhaps the next independent discovery of the tabular standard was made by the Legislature of Massachusetts in 1780. An account of this discovery is given by Willard Fisher (1913). A tabular standard was used to index the pay of soldiers fighting in the Revolutionary War (a massive inflation had drastically reduced the real value of the fixed nominal pay of the soldiers). The constant quantity basket was 5 bushels of corn, 68 and 4/7 pounds of beef, 10 pounds of sheep's wool and 16 pounds of sole leather.[5]

Joseph Lowe (1823,316) was not an independent discoverer of the constant basket index number formula

(1) $p^2 \cdot q / p^1 \cdot q$,

since he explicitly refers to Fleetwood's book. However, he developed the concept in such detail that he should be considered the father of the consumer price index. Lowe was well aware that the constant basket of

commodities q could vary across demographic groups; on page 332, he presented some representative family budgets for cottagers and for the middle class. On page 97 of the Appendix, he noted that price indexes may be required for other classes of consumers or producers such as farmers and miners while on page 336 of the main text, he advocated the construction of separate "standards" for the labouring class, decomposed into unmarried labourers and married labourers with 2,3, or 4 children. Finally, Lowe (1823,33) also envisaged a national "table of reference" which would price out a constant national consumption vector at the prices of each year t and on pages 94 and 95 of the Appendix, he constructed two such hypothetical tables.

How would the constant vector of commodities q in (1) be determined? Lowe (1823, Appendix 95) answered this question as follows:

"As to quantity, a variation can take place only with increase of of population or change of habits, and any alteration of that kind must be so gradual, that we run very little hazard in assuming a similarity of amount during a given period, which for the sake of precision, we shall suppose to be five years."

Lowe (1823,334) also proposed that the national government should fund the collection of the relevant price and quantity statistics, but if this was not done, the Lowe felt that government agencies should at least provide what data they had at their disposal "on the demand of any respectable association."

Lowe (1823,335-343) listed a host of applications for his proposed tables of reference, including the following: (i) wages, salaries and rents could be indexed to eliminate the anomalies arising out of unforeseen

fluctuations in the value of the country's currency, (ii) they would facilitate salary negotiations, (iii) they could be used to index long term agricultural leases and (iv) bond holders could be paid in real terms if they wanted that option.

Lowe (1823,346) concluded with some pertinent observations on why his proposal had not been implemented up to his time:

"This has, we believe, been owing to two causes; the unfortunate neglect of political economy in the education of our public men; and the interest of government, the greatest of all debtors, to prevent the public from fixing its attention on the gradual depreciation of money that went on during the half century to the late peace."

Scrope (1833,406-407) followed in Lowe's footsteps but was the first to use the term tabular standard to describe the price index defined by (1). However, his treatment was not nearly as detailed as that of Lowe[6], so we will pass on to list others who have endorsed the tabular standard.

If quantities were to remain constant during the two periods under consideration, a whole host of authors endorsed formula (1) to measure price change, including Jevons (1865,122)(1884,122), Sidgwick (1883,67-68) Edgeworth (1925,212)[7] (originally published in 1887), Marshall (1887,363), Bowley (1899)(1901,227)(1928,223), Walsh (1901,540), (1921,543) (1924,544) and Pigou (1912,38). During this period, the precise specification of the constant quantity vector q was a problem which was addressed. Thus Laspeyres (1871) proposed that q should equal $q^1 = (q^1_1, \ldots, q^1_N)$, the base period quantity vector, while Paasche (1874) proposed that q should equal $q^2 = (q^2_1, \ldots, q^2_N)$, the current period quantity vector. Thus (1) can be

specialized to yield the famous Laspeyres and Paasche price indexes, P_L and P_P:

$$(2) \qquad P_L(p^1,p^2,q^1,q^2) = p^2 \cdot q^1/p^1 \cdot q^1;$$

$$(3) \qquad P_P(p^1,p^2,q^1,q^2) = p^2 \cdot q^2/p^1 \cdot q^2 .$$

Given that quantities would not be exactly equal during the two periods under consideration, various authors started to argue that _averages_ of (2) and (3) should be used to measure price change. Thus Sidgwick (1883,68) and Bowley (1901,227) proposed the use of $(1/2)P_L + (1/2)P_P$, while Edgeworth (1925,214) (originally published in 1887) proposed that the q in (1) be set equal to the arithmetic average of the two quantity vectors, $(1/2)q^1 + (1/2)q^2$, (Edgeworth states that this variant was also independently proposed by Alfred Marshall). Bowley (1899) suggested the geometric mean of P_L and P_P, which later came to be known as Irving Fisher's (1922) ideal price index P_F defined as

$$(4) \qquad P_F(p^1,p^2,q^1,q^2) = [p^2 \cdot q^1 \ p^2 \cdot q^2/p^1 \cdot q^1 \ p^1 \cdot q^2]^{1/2} .$$

Walsh (1901,398) proposed that the components q_i of the quantity vector q in (1) should be set equal to the geometric means of the quantities in the two periods. Thus the Walsh price index is

$$(5) \qquad P_W(p^1,p^2,q^1,q^2) = \Sigma_{i=1}^N \ (q_i^1 q_i^2)^{1/2} \ p_i^2/\Sigma_{j=1}^N \ (q_j^1 q_j^2)^{1/2} \ p_j^1 .$$

Finally, Pigou (1912,46) suggested $P_L P_P$ as a measure of price change. Since this price index has rather poor homogeneity properties, Pigou later

modified his measure by taking the square root which yields P_F defined by (4); see Pigou (1932,69).

At this stage, the fixed basket approach to index number theory merged into the test and economic approaches.

3.   The Statistical Approach.

This approach, which originated with Jevons (1965)(1884), assumed that increases in the supply of money increased all prices proportionately except for random fluctuations. Thus with additive errors and a sufficient number of independent observations, an appropriate price index could be obtained by taking the arithmetic mean of the price ratios p_i^2/p_i^1 while with multiplicative errors, an appropriate price index could be obtained by taking the geometric mean of the price ratios. This second alternative was advocated by Jevons, and thus we obtain the Jevons price index P_J:

$$(6) \qquad P_J(p^1,p^2) \equiv \Pi_{i=1}^N \ (p_i^2/p_i^1)^{1/N}.$$

In addition to Jevons, two other prominent economists who advocated the statistical approach to index numbers were Bowley (1901,223-226)(1921,202),(1928,217-223) and Edgeworth (1888)(1896)(1901)(1923)(1925)[8]. Edgeworth mainly advocated the median of the price ratios p_i^2/p_i^1 as the best estimator of price change.

The statistical approach was criticized by Irving Fisher (1911,194-196) who explained in an absolutely convincing manner why all prices cannot move proportionately (due to the existence of fixed price contracts, for example). Fisher's criticisms were ignored by the profession as were those of Walsh (1924). However, Keynes (1930,71-81) effectively demolished the

naive statistical approach by constructing various tables of index numbers which showed systematic differences over time and hence the hypothesis of approximate proportional change in all prices could not be maintained empirically.[9] Bowley (1928;221) also criticized the approach on narrower statistical grounds by indicating that the price movements were not statistically independent.

Although Jevons' naive statistical approach is no longer advocated, as was indicated in Triplett's (1988) paper, statistical sampling of the prices of the various components of a price index is still done today. A problem with many of these sampling procedures is that prices are sampled independently of quantities. Pigou (1932,77) was perhaps the first to propose that values should be sampled in the two periods under consideration, along with the corresponding prices and quantities, and then the Fisher ideal index P_F defined by (4) should be used to construct a measure of price change over the commodities in the sample of values. This sample price index could then be used to deflate the population value ratio over the two periods. Pigou's proposal deserves serious consideration by Statistical Agencies even today.

4.   The Test Approach

The origins of the test approach are rooted in the more or less casual observation of the early workers in the index number field on their favorite index number formulae or those of their competitors.

Thus Jevons (1884,152) (originally published in 1865) recognized that his unweighted geometric mean formula (6) gave index number comparisons between any two years that were independent of the base year. Edgeworth (1896,137) gave a clear general treatment of this base invariance test[10]

which we can phrase as follows. Let $P(p^0, p^t, q^0, q^t)$ be a generic index number formula of the type defined by (2) to (5) above which compares the level of prices in period t to the level of prices in period 0, the base year. Let p^t and q^t be the price and quantity vectors pertaining to year t for t=0,1,..., T. Let i, s and t denote arbitrary years. With the base year equal to 0, the level of prices in year t relative to s is taken to be $P(p^0, p^t, q^0, q^t)/P(p^0, p^s, q^0, q^s)$. If we change the base to year i, then the level of prices in period t relative to s is $P(p^i, p^t, q^i, q^t)/P(p^i, p^s, q^i, q^s)$. The base invariance test demands that these two numbers be equal; i.e., that

$$(7) \qquad P(p^0, p^t, q^0, q^t)/P(p^0, p^s, q^0, q^s) = P(p^1, p^t, q^1, q^t)/P(p^1, p^s, q^1, q^s).$$

Our next test was first proposed by Laspeyres (1871,308), and has come to be known as the <u>strong identity test</u>: if prices in the two periods under consideration remain constant, then even if the quantities change[11], the level of prices should remain unchanged; i.e., we should have

$$(8) \qquad P(p, p, q^1, q^2) = 1$$

where P denotes the index number formula or function, $p = (p_1, ..., p_N)$ denotes the common price vector in both periods and $q^t = (q_1^t, ... q_N^t)$ denotes the quantity vector in period t for t = 1,2.

The statistician Westergaard (1890,218-219) formulated what later[12] became known as the <u>circularity test</u>: the bilateral index number formula should satisfy the following equation:

$$(9) \qquad P(p^1, p^2, q^1, q^2)P(p^2, p^3, q^2, q^3) = P(p^1, p^3, q^1, q^3)$$

where p^t and q^t are the price and quantity vectors pertaining to periods t for t - 1,2,3. The right hand side of (9) computes the price level in period 3 relative to the price level in period 1 in one step, using the bilateral index number formula or function P. The left hand side of (9) computes the level of prices in period 3 relative to period 1 in two steps: in the first step, we use the bilateral formula $P(p^1,p^2,q^1,q^2)$ to compute the level of prices in period 2 relative to period 1 and then in the second step, we use the bilateral formula $P(p^2,p^3,q^2,q^3)$ to compute the level of prices in period 3 relative to period 2. The product of these two steps is supposed to yield the level of prices in period 3 relative to period 1.

The Dutch economist Pierson (1896) informally proposed two tests: (i) invariance to changes in the units of measurement (which Irving Fisher (1911)(1922,420) first called the change of units test and later called the commensurability test) and (ii) the time reversal test which can be stated mathematically as follows:

$$(10) \qquad P(p^2,p^1,q^2,q^1) - 1/P(p^1,p^2,q^1,q^2).$$

Up to this point in time, research or the test or axiomatic approach to index number theory was rather casual and unsystematic. The first systematic researcher on the axiomatic approach was Walsh (1901)(1921)(1924) who proposed a number of tests, including the constant quantities test[13]; i.e., if quantities remain fixed at the vector q during the two periods under consideration, then the appropriate formula for the price index is

$$(11) \qquad P(p^1,p^2,q,q) - p^2 \cdot q/p^1 \cdot q.$$

Another test proposed by Walsh was the <u>strong proportionality</u> (in prices) <u>test</u>; i.e., if λ is a positive scaler and prices in period 2 are equal to λ times the corresponding prices in period 1, then [14]

$$(12) \qquad P(p^1, \lambda p^1, q^1, q^2) = \lambda.$$

A final test proposed by Walsh (1901,389)(1921,540)(1924,506) was his <u>multiperiod</u> <u>identity</u> <u>test</u>[15]; i.e., the bilateral index number function P is to satisfy the following functional equation:

$$(13) \qquad P(p^1, p^2, q^1, q^2) P(p^2, p^3, q^2, q^3) P(p^3, p^1, q^3, q^1) = 1 \ .$$

Note that the prices and quantities in period 4 are exactly equal to the prices and quantities in period 1, p^1 and q^1 respectively. As we shall see later, the test (13) will be useful in evaluating the usefulness of the chain principle.

The next major contributor to the test approach is Irving Fisher (1911)(1921)(1922). Since Fisher's contributions to the test approach are quite well known (in fact, he is often credited with inventing the approach), we will not review his contributions in any detail. However, we do wish to make two comments about his work.

Our first comment is to note that Fisher (1911,403) seems to have been the first to observe that the choice of a functional form $P(p^1, p^2, q^1, q^2)$ for a price index implicitly determines the functional form for the corresponding quantity index $Q(p^1, p^2, q^1, q^2)$; i.e., the product of the two indexes should equal the value ratio for the two periods under consideration. Thus given P, Q is implicitly determined by the following equation:

(14) $\qquad P(p^1,p^2,q^1,q^2) \; Q(p^1,p^2,q^1,q^2) \; = \; p^2 \cdot q^2 / p^1 \cdot q^1.$

Frisch (1930,399) called (14) the _product test_ while Samuelson and Swamy (1974,572) called it the _weak factor reversal test_.

Our second comment about Fisher's contributions to the test approach is relatively unknown. Fisher (1922,140) explained how to "rectify" an arbitrary bilateral index number formula $P(p^1,p^2,q^1,q^2)$ so that the rectified formula P* would satisfy the time reversal test (10): simply define P* as follows:

(15) $\qquad P*(p^1,p^2,q^1,q^2) \; = \; [P(p^1,p^2,q^1,q^2)/P(p^2,p^1,q^2,q^1)]^{1/2}.$

Fisher's time rectification procedure indeed does work as advertised: the P* defined by (15) will satisfy (10). The only problem is that the procedure is clearly due to Walsh (1921,542), who was a discussant for Fisher's (1921) paper which was a preview for Fisher (1922). Unfortunately, Fisher's (1922,183) historical comments on the rectification principle fail to mention Walsh at all. However, on the positive side, Fisher (1922,396-398) generalized Walsh's basic idea by showing how an index number formula could also be rectified to satisfy Fisher's _factor reversal test_[16] or be rectified to simultaneously satisfy the factor and time reversal tests.

Frisch (1930)(1936,5-7) effectively criticized the test approach to index number theory on the grounds that it could be shown that no bilateral index number formula $P(p^1,p^2,q^1,q^2)$ could satisfy all reasonable tests or axioms[17] and when some tests were dropped so as to achieve a consistent set

of tests, there was no general agreement on which subset of tests should be
dropped. Hence the test approach did not seem to lead anywhere.

In recent years, the test approach has sprung to life again, largely
due to the efforts of Wolfgang Eichhorn (1973)(1976) and his students and
colleagues.[18]

5. The Divisia Approach.

Divisia's (1926,39-40) derivation of the price and quantity indexes
associated with his name can be summarized as follows. Let the prices
$p_i(t)$ and the quantities $q_i(t)$, i=1, ..., N, be functions of (continuous)
time t and let expenditure at time t be the value $v(t) = \Sigma_{i=1}^{N} p_i(t)q_i(t)$.
Assuming differentiability, the rate of change of value at time t is:

$$(16) \qquad dv(t)/dt = \Sigma_{i=1}^{N} p_i(dq_i/dt) + \Sigma_{i=1}^{N} q_i(dp_i/dt).$$

Divisia then divided both sides of (16) by $p(t) \cdot q(t) = \Sigma_{i=1}^{N} p_i(t)q_i(t)$ and
equated the right hand side of the resulting equation to $Q'(t)/Q(t) +$
$P'(t)/P(t)$ where $Q(t)$ and $P(t)$ are aggregate quantity and price levels
pertaining to period t and $Q'(t)$ and $P'(t)$ denote their time derivatives.
Thus we have:

$$(17) \qquad \Sigma_{i=1}^{N} \frac{p_i q_i'(t)}{v(t)} + \Sigma_{i=1}^{N} \frac{q_i p_i'(t)}{v(t)} = \frac{Q'(t)}{Q(t)} + \frac{P'(t)}{P(t)} .$$

Divisia then defined $Q(t)$ and $P(t)$ as solutions to the following
differential equations:

$$(18) \qquad \frac{Q'(t)}{Q(t)} = \Sigma_{i=1}^{N} \frac{p_i(t)q'_i(t)}{p(t) \cdot q(t)} ; \frac{P'(t)}{P(t)} = \Sigma_{i=1}^{N} \frac{q_i(t)p'_i(t)}{p(t) \cdot q(t)} .$$

Somewhat surprisingly, virtually the same derivation was made earlier by the English economist, T.L. Bennet (1920,461), except that he did not divide (16) through by $v(t) = p(t) \cdot q(t)$.

The above derivation of the Divisia indexes is very mechanical and is unrelated to economics (i.e., choice under constraint). However, later both Ville (1946) and Hulten (1973) related the Divisia indexes to economic price and quantity indexes under the assumptions of optimizing behavior and a linearly homogenous aggregator function.[19]

Triplett (1988) observes, the problem with the Divisia approach to price measurement is that we generally cannot observe prices and quantities continuously. Thus the continuous time Divisia indexes must be approximated using discrete time data and there are many ways of forming discrete time approximations to say $P(2)/P(1)$[20], where $P(t)$ is the Divisia index for time period t defined by (18) (plus an initial normalization). Diewert (1980,444-445) showed that the Laspeyres and Paasche indexes, P_L and P_P defined by (2) and (3) above, could be regarded as discrete time approximations to $P(2)/P(1)$ as could the Tornqvist-translog P_T defined by

$$(19) \qquad \ln P_T(p^1, p^2, q^1, q^2) = \Sigma_{i=1}^{N} (1/2)(s_i^1 + s_i^2)\ln(p_i^2/p_i^1)$$

where the shares s_i^t are defined as $s_i^t = p_i^t q_i^t / p^t \cdot q^t$, $t=1,2$ and $i=1, \ldots, N$. Since the indexes P_L, P_P and P_T can differ considerably, the Divisia approach does not lead to a practical resolution of the price measurement problem.[21]

To conclude this section on the Bennet-Divisia approach, we note that Bennet (1920,457) suggested the following discrete approximations to

measure <u>differences</u> (rather than the <u>ratios</u> of Divisia) in the aggregate price and quantity levels:

$$(20) \qquad \Delta P = P(2) - P(1) = \Sigma_{i=1}^{N} \ (1/2)(q_i^1 + q_i^2)(p_i^2 - p_i^1) \ ;$$

$$(21) \qquad \Delta Q = Q(2) - Q(1) = \Sigma_{i=1}^{N} \ (1/2)(p_i^1 + p_i^2)(q_i^2 - q_i^1) \ .$$

Bennet also showed that the difference in expenditures for the two periods, $\Sigma_{i=1}^{N} \ p_i^2 q_i^2 - \Sigma_{i=1}^{N} \ p_i^1 q_i^1$, was exactly equal to $\Delta P + \Delta Q$, where ΔP and ΔQ are defined by the right hand sides of (20) and (21).[22]

6.   The Economic Approach.

The economic approach to index number theory[23] relies on the assumption of optimizing behavior on the part of economic agents: utility maximizing or expenditure minimizing behavior on the part of consumers and profit maximizing or cost minimizing behavior on the part of producers.

The first two papers to use an explicit utility maximizing framework appear to be by Bennet (1920) and Konüs (1924). Bennet's paper drew on an earlier paper by Bowley (1919) (he used Bowley's notation and data) and may be regarded as an attempt to determine the approximate magnitude of the substitution bias using the assumption of an quadratic utility function. Bowley (1928,226)(1938) was in turn influenced by Bennet and developed his own quadratic approximations. Bennet's paper was very short and sketchy and did not have the impact that the Konüs paper eventually had. Konüs (1924,16-18) not only presented a very clear definition of the true cost of living for an individual optimizing consumer, he also developed the now well known Paasche and Laspeyres bounds.[24] Konüs (1924,20-21) also showed that the Paasche and Laspeyres price indexes, (3) and (2) above, bound the

true cost of living index even in the general nonhomothetic preferences case, provided that we evaluate the true index at a suitable utility level that is between the base and current period levels.

To complete our brief survey of the early history of the economic approach to index number theory, we shall review the economic approach under four subdivisions: (i) basic theoretical definitions, (ii) the theory of bounds, (iii) exact index numbers and (iv) econometric approaches.

6.1 Basic Theoretical Definitions

As Triplett (1988) noted, there are three main branches of price index theory. (i) For the true cost of living index, see Konüs (1924), Samuelson (1947,156) and Pollak (1971). For related quantity indexes, see Bowley (1928,230), Allen (1949), Malmquist (1953) and Pollak (1971). (ii) For theoretical definitions of the output price index, see Hicks (1940), Fisher and Shell (1972), Samuelson and Swamy (1974;588-592), Archibald (1977) and Diewert (1983b). For related quantity indexes, see Bowley (1921,203), Bergson (1961,31-34), Moorsteen (1961), Fisher and Shell (1972,53), Samuelson and Swamy (1974,588-591), Sato (1976,438) and Hicks (1981,256). (iii) The input cost index is defined by Triplett (1983a,274) and Diewert (1980,459) and corresponding quantity indexes are defined in Diewert (1980,456-460).

There is a fourth branch of price index theory not mentioned by Triplett: (iv) constant utility income deflators. On this last branch of theoretical index number theory, see Diewert and Bossons (1987).

6.2 The Theory of Bounds

Observable bounds to the generally unobservable economic price and quantity indexes were first worked out by Pigou (1912,44-46)(1932,62-63) and Haberler (1927,78-92) independently of Konüs (1924,17-19)   ;o established the Paasche and Laspeyres bounds for the true cost of living. For a generalization of these bounds to nonlinear budget constraints, see Frisch (1936,18).[25] It is clear that a large portion of revealed preference theory that is often attributed to Hicks (1940) and Samuelson (1947,157) had already been developed by Pigou, Konüs, Haberler and Frisch.

Other researchers who established bounds on true indexes in the two observation situation include Leontief (1936,49), Friedman (1938,125), Allen (1949), Malmquist (1953), Moorsteen (1961,464), Pollak (1971), Fisher and Shell (1972,57-62), Samuelson and Swamy (1974,581-591), Archibald (1975) and Diewert (1981,167-179)(1983a,173-210)(1983b,1056-1090).

The above theory of bounds all pertains to the two observation situation. Afriat (1967)(1977) generalized the two observation theory to cover the many observation case.

6.3 Exact Index Numbers

Let an aggregator function[26] $f(q)$ be given where q is an N dimensional quantity vector; i.e., $q = (q_1, \ldots, q_N)$. The cost function C which is generated by f may be defined as

(22) $C(u,p) = \min_q \{p \cdot q : f(q) \geq u\}$;

i.e., $C(u,p)$ is the solution to the problem of minimizing the cost $p \cdot q = \sum_{i=1}^{N} p_i q_i$ of achieving at least the utility (or output) level u, where p=

$(p_1, \ldots, p_N)$ is an exogenous vector of prices facing the consumer (or producer).

An index number formula or function $P(p^1, p^2, q^1, q^2)$ of the type we considered in section 4 is defined to be <u>exact</u>[27] for an aggregator function f if

$$(23) \qquad P(p^1, p^2, q^1, q^2) = C(u, p^2)/C(u, p^1)$$

for some utility or output level u where q^t solves (22) when $p = p^t$ for t=1,2; i.e., P is exact for f (or its dual cost function C) if P equals the relevant economic index under the assumption of optimizing behavior on the part of an economic agent using the aggregator function f. The right hand side of (23) is $P_K(p^1, p^2, u)$, the Konüs price index or true cost of living index for a consumer that has the utility function f and faces the vector of prices p^t in period t for t=1,2.

The English language literature on exact index numbers has its roots in the theory of quadratic approximations. As we indicated earlier, Bennet (1920,460) attempted to determine an appropriate index number formula for the true cost of living of a single "satisfaction" maximizing consumer under the hypothesis that the underlying utility function f(q) was a general quadratic function. Bowley (1928,226)(1938) followed up on Bennet's approach and provided his own second order approximation. Frisch (1936,27-29) criticized Bowley's index number formula and developed an alternative formula which he called the double expenditure method. Wald (1939,329) and Balk (1981,1556) correctly pointed out that Frisch's index number formula was <u>not</u> exact for a general quadratic utility function. However, Frisch (1936,29-30) did correctly show[28] that his index number

formula collapsed to the Fisher ideal price index P_F defined by (4) if one assumed homothetic quadratic preferences so that $f(q) = \Sigma_{i=1}^N \Sigma_{j=1}^N a_{ij}q_iq_j = q \cdot Aq$ where $A \equiv [a_{ij}]$ is a symmetric N by N matrix of parameters that characterize tastes. This is an early example of an exact index number formula.

Another early example was given by Wald (1939)(1939,325) who assumed the following general quadratic aggregator function:

$$(24) \qquad f(q) \equiv a_0 + a \cdot q + (1/2)q \cdot Aq \quad , \qquad A=A^t \quad ,$$

where a_0, $a \equiv [d_1, \ldots, d_N]$ and $A \equiv [a_{ij}]$ are respectively a parameter, a vector of parameters and a symmetric N by N matrix of parameters.

Unfortunately, in order to evaluate Wald's general index number formula that is exact for (24), information on income elasticities is required (we shall not write out his general index number formula since it is rather complex). However, if we assume homothetic preferences again (i.e., the aggregator function is a monotonically increasing function of a linearly homogeneous function) so that $a_0 = 0$ and $a = 0_N$ in (24)[29] then all of the consumer's income elasticities equal unity and Wald's general index number formula collapses down to the Fisher price index (4) and again we obtain the exact index number result of Frisch.[30]

Unknown to the above authors, the Frisch-Wald exact index number result had already been obtained by Konüs and Byushgens (1926,167-172) a decade earlier.[31] In this remarkable paper, they introduced duality theory into the economics literature; i.e, they expressed consumer preferences not only by the direct utility function $f(q)$ but also by the corresponding indirect utility function g defined as follows:

(25) $g(p,y) \equiv \max_q \{f(q) : p \cdot q \leq y\}$;

i.e., the indirect utility function $g(p,y)$ gives the maximum utility

attainable as a function of the prices faced by the consumer $p = (p_1, \ldots, p_N)$ and the income or expenditure $y \geq 0$ to be spent on the N goods during

the period under consideration. Konüs and Byushgens assumed that the

direct utility function f was linearly homogeneous in which case the

indirect utility function g can be expressed as follows in terms of the

unit cost function $c(p) \equiv C(1,p)$ where $C(u,p)$ was defined by (22) above:

(26) $g(p,y) = y/c(p)$.

Konüs and Byushgens considered three classes of homothetic preferences

which were defined via the indirect utility function $g(p,y)$ or

equivalently, using (26), via the unit cost function $c(p)$.

The first case they considered had the following unit cost function:

(27) $c(p) \equiv \Sigma_{i=1}^{N} a_i p_i$, $a_i > 0$, i=1, $\ldots$, N.

As is well known[32], the dual direct utility function is the fixed

coefficient or no substitution f defined as follows:

(28) $f(q_1, \ldots, q_N) \equiv \min_i \{ q_i/a_i : i = 1, \ldots, N\}$.

Under these conditions, Konüs and Byushgens (1926,162) showed that the

Laspeyres and Paasche indexes, P_L and P_P defined by (2) and (3) above, will

exactly equal the true cost of living (defined by the right hand side of

(23) for any positive utility level u), provided that the consumer's direct

utility function is defined by (26) and (27).

The second case they considered was the case of Cobb-Douglas preferences[33] which can be characterized by the following unit cost function:

$$(29) \qquad c(p) = \alpha \ \Pi_{i=1}^{N} \ p_i^{\alpha_i} \qquad , \ \alpha_i > 0 \ , \ \Sigma_{i=1}^{N} \ \alpha_i = 1 \ ,$$

Konüs and Byushgens (1926,165) showed that the generalized Jevons index defined by (6) (except that $1/N$ is replaced by α_i) is <u>exact</u> for these Wicksell-Cobb-Douglas preferences, where the unknown parameters α_i can be determined as follows: $\alpha_i = p_i^t q_i^t / p^t \cdot q^t$, the ith expenditure share, i=1, ..., N, for any period t.

In the final case considered by Konüs and Byushgens (1926,168), the consumer's preferences were characterized by the following unit cost function:

$$(30) \qquad c(p) = (p \cdot Bp)^{1/2} = (\Sigma_{i=1}^{N} \ \Sigma_{i=1}^{N} \ b_{ij} p_i p_j)^{1/2}$$

where $B = [b_{ij}]$ is a symmetric N by N matrix of unknown parameters that characterize preferences. They showed that the Fisher ideal index P_F defined by (4) was <u>exact</u> for the preferences characterized by (26) and (30). Konüs and Byushgens (1926,171) also showed that if the inverse of the matrix B existed, say $A = B^{-1}$, then the direct utility function corresponding to (26) and (30) was

$$(31) \qquad f(q) = (q \cdot Aq)^{1/2}$$

which is a monotonic transformation of the homogeneous quadratic utility function considered by Frisch and Wald.

Finally, Konüs and Byushgens (1926,171) exhibited both the system of inverse demand functions, $p/y - Aq/q \cdot Aq$, and the system of ordinary demand functions, $q - yBp/p \cdot Bp$, that correspond to the homogeneous quadratic preferences defined by (31). They also suggested that the unknown parameters appearing in the A or B matrices could be determined given a sufficient number of price and quantity observations. However, note that a knowledge of A or B is not required in order to evaluate the Fisher price index P_F.

After the contributions of Bowley, Frisch and Wald to the theory of exact index numbers, the subject remained dormant until Afriat (1972,44-47), Pollak (1971,117-132) and Samuelson and Swamy (1974,573-574) reexamined the subject. All of these authors examined the three cases considered by Konüs and Byushgens and some other cases as well.

Diewert (1976,134) defined a price index function $P(p^1, p^2, q^1, q^2)$ to be superlative if P was exact for preferences which had a cost function $C(u,p)$ $- uc(p)$ where $c(p)$ is a unit cost function that could provide a second order approximation to an arbitrary twice continuously differentiable linearly homogeneous function. The idea was that a superlative index number formula $P(p^1, p^2, q^1, q^2)$, which could be evaluated using only observable price and quantity data for the two periods under consideration, would correspond to a flexible functional form for a unit cost function $c(p)$. For example, P_F defined by (4) is a superlative price index since it is exact for the c defined by (30) and this c has the required second order approximation property. Another example of a superlative index is the Walsh index P_W defined by (5) since it is exact for the unit cost function which is dual to the aggregator function

$f(q) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} q_i^{1/2} q_j^{1/2}$; see Diewert (1976,132). A third example of a superlative price index is P_T defined by (19) which is exact for a translog unit cost function;[34] see Diewert (1976,121).

Unfortunately, Diewert (1976) defined two (infinite) families of superlative index number formulae and this raised the question as to which formula should be used in empirical applications. Fortunately, Diewert (1978) showed that all choices of a superlative formulae gave the same answer to the second order and hence the choice was usually immaterial. More precisely, Diewert showed that every known superlative index number formula $P(p^1, p^2, q^1, q^2)$ had the same first and second derivatives when evaluated at equal prices (i.e., $p^1 = p^2$) and equal quantities (i.e., $q^1 = q^2$).[35]

As an interesting footnote to the history of economic thought, it should be noted that Diewert was not the first to use the above second order approximation technique; Edgeworth (1901,410-411) used a variant of it to show that the Walsh index P_W defined by (5) approximated to the second order the Edgeworth (1925,213) - Marshall (1887,372) index defined as follows:

(32) $\qquad P_{EM}(p^1, p^2, q^1, q^2) = [\sum_{i=1}^{N} (1/2)(q_i^1 + q_i^2) p_i^2] / [\sum_{j=1}^{N} (1/2)(q_j^1 + q_j^2) p_j^2]$.

The Taylor series expansion technique around an equal price and quantity point was used again by Edgeworth (1923,347) to show that the Laspeyres index P_L defined by (2) and the Fisher index P_F defined by (4) satisfy the circularity test (9) to the first order. Since this Proposition seems to have been forgotten, we sketch a proof of it.

Let the index number function P be either P_L or P_F and define the functions f and g as follows:

$$(33) \qquad f(p^1,p^2,p^3,q^1,q^2,q^3) = P(p^1,p^2,q^1,q^2)P(p^2,p^3,q^2,q^3) \; ;$$

$$g(p^1,p^2,p^3,q^1,q^2,q^3) = P(p^1,p^3,q^1,q^3).$$

For $P=P_L$ or for $P=P_F$, it can be verified that the following equalities hold:

$$(34) \qquad f(p^1,p^2,p^3,q^1,q^2,q^3) - g(p^1,p^2,p^3,q^1,q^2,q^3) \quad = 1 \; ;$$

$$\nabla_{p^1} f(p^1,p^2,p^3,q^1,q^2,q^3) - \nabla_{p^1} g(p^1,p^2,p^3,q^1,q^2,q^3) = -q/p \cdot q \; ;$$

$$\nabla_{p^2} f(p^1,p^2,p^3,q^1,q^2,q^3) - \nabla_{p^2} g(p^1,p^2,p^3,q^1,q^2,q^3) = 0_N \; ;$$

$$\nabla_{p^3} f(p^1,p^2,p^3,q^1,q^2,q^3) - \nabla_{p^3} g(p^1,p^2,p^3,q^1,q^2,q^3) = q/p \cdot q \; ;$$

$$\nabla_{q^i} f(p^1,p^2,p^3,q^1,q^2,q^3) - \nabla_{q^i} g(p^1,p^2,p^3,q^1,q^2,q^3) = 0_N, \; i=1,2,3,$$

provided that the above functions are evaluated at equal prices (i.e., $p^1 = p^2 = p^3 \equiv p$) and equal quantities (i.e., $q^1 = q^2 = q^3 \equiv q$) where $\nabla_{p^i} f = [df/dp_1^i, \ldots, df/dp_N^i]$ is the vector of first order partial derivatives of f with respect to the components of p^i, etc. The meaning of (34) is that the functions f and g approximate each other to the first order when evaluated at an equal price and quantity point. Thus the Laspeyres and Fisher price indexes satisfy the circularity test to the first order.

Using the results in Diewert (1978,898) and the results in the above paragraph, it can be shown that the Paasche, Laspeyres and all superlative

index number formulae will similarly satisfy the circularity test to the first order.

Note that Edgeworth's Proposition helps to explain Fisher's (1922,280) empirical finding that P_F satisfied the circularity test to a very high degree of approximation.

6.4 Econometric Estimation of Preferences

In this variant of the economic approach to index numbers, the parameters that characterize consumer preferences are estimated. Preferences may be represented by: (i) the direct utility function; (ii) the indirect utility function; (iii) the distance function[36] or (iv) the cost or expenditure function. Once any one of these functions in known, the other three functions can be calculated, at least in principle. In particular, the cost function $C(u,p)$ can be calculated and hence the Konüs price index defined by the right hand side of (23) can be calculated.

Triplett (1988) gives an excellent summary of most of the post 1948 research in this area; additional references to the recent literature can be found in chapter 7 of Deaton and Muellbauer (1980).

It seems appropriate to add a few references to the early history of this approach to index number theory.

The earliest effort at a strategy for determining the parameters which characterize preferences was made by Bennet (1920,462). He assumed a quadratic direct utility function and showed how the consumer's system of demand functions could be obtained in the three good case (although he did not quite exhibit a closed form solution). He then made some comments on how many observations would be required in the general N good case in order

to determine all $1 + N + (1/2)N(N+1)$ of the parameters of the quadratic utility function $f(q)$ defined by (24).[37]

We have already seen that Konüs and Byushgens (1926) were contributors to the econometric approach, since they derived the demand functions corresponding to Cobb-Douglas preferences and homogeneous quadratic preferences. Konüs and Byushgens (1926,172) noted that one price-quantity observation would suffice to determine the parameters of Cobb-Douglas preferences while $(N+1)/2$ observations would be required to determine all $N(N+1)/2$ parameters for the homogeneous quadratic functional forms, (30) or (31). They also noted that statistical determination of the homogeneous quadratic preferences would be difficult.

Another early contributor to the econometric approach to index numbers was Wald (1937)(1939,325), who assumed quadratic preferences; recall (24). In addition, he assumed that the demand functions regarded as functions of income (or expenditure) were known functions for the two periods under consideration. (Alternatively, just a knowledge of the income elasticities of demand at the two observed price and quantity points would suffice). With these assumptions, Wald was able to derive an exact index number formula, so his approach is actually a blend of the exact and econometric approaches.

Wald's blended approach seems worthy of further study. However, the pure econometric approach has severe limitations. The problem with the latter approach is that in order to provide a second order approximation to general preferences, we require approximately $N^2/2$ parameters in the N good case. Since N is perhaps equal to 50,000 for a typical consumer (a supermarket alone has 15,000 to 20,000 separate items), the required number

of parameters to be estimated is approximately 1.25 billion, which would require price and quantity observations for about 25,000 periods.[38]

This concludes our survey of the ancient history of index numbers. We turn now to a discussion of some of the more controversial issues raised by Triplett.

7. On the Test Approach to Index Number Theory

Triplett (1988) does not discuss the test approach to index number theory and perhaps the reader may well feel that this is quite appropriate due to the length of his paper. Although I am basically in agreement with Triplett that the economic approach to index number theory is the most compelling approach, it should be mentioned that the test approach has some advantages. In particular, the test approach does not suffer from the following limitations of the economic approach: (i) the economic approach is based on optimizing behavior, an assumption which may not be warranted in general; (ii) the economic approach generally relies on separability assumptions[39] about the underlying aggregator functions, assumptions which are unlikely to be true in general and (iii) in deriving capital rental prices, the economic approach is usually based on an ex ante expectations about future prices, expectations which cannot be observed, whereas the test approach can be based on ex post accounting data, which can be observed.[40]

8. On the Chain Principle and Multilateral Indexes.

Triplett (1988) contrasts the chain principle with the fixed base principle for constructing a series of index numbers which extends over three or more periods. Given price and quantity data, p^i, q^i, i=1,2,3 for

three periods and a bilateral price index function $P(p^1, p^2, q^1, q^2)$ that

depends only on the data for two periods, the <u>fixed</u> <u>base</u> <u>sequence</u> of

aggregate price levels for the 3 periods would be

$$(35) \qquad 1, \ P(p^1, p^2, q^1, q^2), \ P(p^1, p^3, q^1, q^3)$$

while the <u>chain</u> <u>sequence</u> of price levels would be

$$(36) \qquad 1, \ P(p^1, p^2, q^1, q^2), \ P(p^1, p^2, q^1, q^2) P(p^2, p^3, q^2, q^3).$$

Historically, the fixed base principle was the first to be used

empirically. In the English language literature, the chain principle was

first proposed by Alfred Marshall (1887,373)[41], basically as a method for

overcoming the difficulties in comparing prices over two distant periods,

due to the invention of new commodities.[42]

Irving Fisher (1911,203), who gave the chain system its name, noted

that the chain system was invariant to changes in the base period and he

also saw the advantage of the method in dealing with the new good problem

as the following quotation indicates[43]:

> "It may be said that the cardinal virtue of the successive base
>
> or chain system is the facility it affords for the introduction of new
>
> commodities, the dropping out of obsolete commodities, and the
>
> continued readjustment of the system of weighting to new commodities."
>
> (Fisher, 1911,204).

While it is true that the use of the chain principle has an advantage

in dealing with the introduction of new commodities, it has the following

severe disadvantage noted by Triplett (1988): it does not satisfy Walsh's

Multiperiod Identity Test, (13) above.[44] Thus as Triplett shows, if prices

and quantities systematically oscillate around constant values, the use of the chain method will give biased results. This same point has been made by Szulc (1983) and Hill (1988).

The above difficulty with the chain method was not adequately appreciated by Diewert (1978,895) who argued for the use of the chain principle on the grounds that it would reduce the spread between the Laspeyres and Paasche indexes, (2) and (3) above, and between all known superlative indexes, since price and quantity changes will generally be smaller between adjacent periods than between distant periods. He argued that the spread between the Paasche and Laspeyres indexes would be greater than between the superlative indexes because P_L and P_P only approximate each other to the first order, while superlative indexes approximate each other to the second order (recall our discussion of the Edgeworth second order approximation technique in section 6.3 above). Diewert (1978,894) also presented the results of some numerical experiments using Canadian per capita consumption data for 13 commodity classes over the years 1947-1971. These results showed that the chain method did in fact lead to a smaller spread between P_L defined by (2),P_P defined by (3), P_F defined by (4) and P_T defined by (19) than when a fixed base year, 1947, was used.

Although the chain method will give poor results with oscillating data, Szulc (1983) and Hill (1988) show theoretically that chaining will tend to reduce the spread between the Laspeyres and Paasche price indexes, provided that prices and quantities trend monotonically over the time periods in question. Thus Diewert's (1978,894) empirical results could be rationalized by the hypothesis that monotonic trends in the data outweighed oscillatory movements.

While the chain system fails to satisfy Walsh's Identity Test (13), Hill (1988) showed that the fixed base system fails to satisfy an analogue to the Strong Identity Test (8). Consider the base period to be period 0 and suppose that the bilateral price and quantity indexes, P and Q respectively, are given and they satisfy the Product Test (14). Then under the fixed base system, price P* and quantity Q* comparisons between periods t and t+1 are made as follows:

$$(37) \qquad P*(p^0,p^t,p^{t+1},q^0,q^t,q^{t+1}) = P(p^0,p^{t+1},q^0,q^{t+1})/P(p^0,p^t,q^0,q^t) \ ;$$

$$(38) \qquad Q*(p^0,p^t,p^{t+1},q^0,q^t,q^{t+1}) = Q(p^0,p^{t+1},q^0,q^{t+1})/Q(p^0,p^t,q^0,q^t) \ .$$

The problem is that P* and Q* need not satisfy counterparts of (8) even if the underlying bilateral indexes P and Q do satisfy (8); i.e., it will not generally be the case that

$$(39) \qquad P*(p^0,p^t,p^{t+1},q^0,q^t,q^{t+1}) = 1 \qquad if \ p^t = p^{t+1} \ .$$

Hill (1988,6-7) cited the fixed base Paasche quantity index, $Q(p^0,p^t,q^0,q^t) = p^0 \cdot q^t/p^0 \cdot q^0$, normally used in the national accounts, as an example of a fixed base index which generates a P* which fails to satisfy the Hill Identity Test (39).[45] Thus both the chain and the fixed base systems fail to satisfy theoretically appropriate identity tests.

We turn now to a discussion of possible alternatives to the use to either the fixed base or chain systems. The first person to propose alternatives was Walsh (1901,431), but it is convenient to start our discussion by reviewing some proposals due to Irving Fisher (1922,297-320).

Fisher's (1922,298) first alternative was to list each and every possible binary comparison and thus the index number user could simply pick out the binary comparisons of interest. Let us (unimaginatively) call this the all binary comparisons method. Fisher actually implemented this method for his data on 36 primary commodities for 6 years using the Fisher ideal price index P_F defined by (4). Thus there were 6(6-1) = 30 bilateral comparisons. Fisher (1922,301) then used these bilateral comparisons to determine whether the Base Invariance Test (7) was satisfied to a high degree of approximation:[46] in Fisher's (1922,302) words, "the differences due to differences of base are trifling."

However, Fisher (1922,299-305) recognized that it was not practical or worthwhile to list every possible binary comparison: for twenty periods, there would be 380 separate index numbers. Thus Fisher was led to other classes of alternatives to the use of either the fixed base or the chain systems, which we will call multiperiod systems, or in the context of regional comparisons, multilateral systems. If there are data for T periods, these multilateral methods make use of the data for all T periods simultaneously to construct a series of T price levels and T quantity levels.

Fisher's (1922,305) first multiperiod or multilateral method was the blend system, which works as follows. Suppose we have price and quantity data, $p^t = (p_1^t, \ldots, p_N^t)$ and $q^t = (q_1^t, \ldots, q_N^t)$ for t=1,2, ..., T and a bilateral index number formula $P(p^1, p^2, q^1, q^2)$. Then use each period as the base to construct a series of T aggregate price levels using the bilateral function P. Normalize the resulting series so that the first price is

unity.[47] The resulting T series or normalized price levels may be written as follows:

(40)   1, $P(p^1,p^2,q^1,q^2)/P(p^1,p^1,q^1,q^1)$, ..., $P(p^1,p^T,q^1,q^T)/P(p^1,p^1,q^1,q^1)$;

   1, $P(p^2,p^2,q^2,q^2)/P(p^2,p^1,q^2,q^1)$, ..., $P(p^2,p^T,q^2,q^T)/P(p^2,p^1,q^2,q^1)$;

   $\vdots$

   1 ,$P(p^T,p^2,q^T,q^2)/P(p^T,p^1,q^T,q^1)$, ..., $P(p^T,p^T,q^T,q^T)/P(p^T,p^1,q^T,q^1)$.

Fisher then suggested that the price level for period t, P^t say, should be an arithmetic average of the period t price level in each of the T series in (40); i.e., we have $P^1 = 1$ and for $t = 2,3, ..., T$:

(41) $P^t = (1/T) \, \Sigma_{k=1}^{T} \, P(p^k,p^t,q^k,q^t)/P(p^k,p^1,q^k,q^1)$.

Note that the _multiperiod_ or _multilateral_ period t _aggregate price level_ P^t defined by (41) is a function of the data for all T periods; i.e., $P^t = P^t(p^1, ..., p^T,q^1, ...,q^T)$. The corresponding _multiperiod_ or _multilateral period_ t _aggregate quantity level_ Q^t may be defined as

(42) $Q^t(p^1,...,p^T,q^1,...,q^T) = p^t \cdot q^t / P^t(p^1,...,p^T,q^1,...,q^T)$.

Fisher's (1922,307) second multilateral method was the _broadened base system_. In this method, the period t price level P^t was defined as follows:

(43) $P^t(p^1,...,p^T,q^1,...,q^T) = (1/T)(\Sigma_{k=1}^{T} \, q^k) \cdot p^t$, $t=1,...,T$.

The corresponding quantity levels Q^t can be defined numerically using the multilateral product test relations (42). This method was used by the Economic Commission for Latin America in the early 1960's. It is described in Ruggles (1967,185) and is referred to as the market basket method. Note that (43) is a T period generalization of the two period Edgeworth-Marshall bilateral formula, P_{EM}, defined by (32) above.

Fisher was not the first to suggest the market basket method: Walsh (1901,431) suggested the same method but called it Scrope's method with arithmetic weights. Walsh (1901,399) also suggested the following system of multilateral price levels P^t, which he called Scrope's method with geometric weights:

$$(44) \qquad P^t = \Sigma_{i=1}^N \ (\Pi_{k=1}^T \ q_i^k)^{1/T} \ p_i^t \qquad , \qquad t=1,\ldots,\ T.$$

The corresponding Q^t can be defined by (42) as usual. It is obvious that (44) is the multilateral generalization of Walsh's bilateral index P_W, defined by (5) above.

The above multilateral methods construct price and quantity levels for each period. It is also possible to devise multilateral methods where the multilateral price index P^{st} gives the level of prices in period t relative to the level of prices in period s and is a function of all of the price and quantity information for the T periods; i.e., $P^{st} = P^{st} \ (p^1,\ldots,p^T,q^1,\ldots,q^T)$. Thus Walsh (1924,509) defined the following multiperiod generalization of the Fisher ideal index (4):

$$(45) \qquad P^{st} = [p^t \cdot q^1 \ p^t \cdot q^2 \ldots p^t \cdot q^T / p^s \cdot q^1 \ p^s \cdot q^2 \ldots p^s \cdot q^T]^{1/T}, \ s,t = 1,\ldots,T.$$

The corresponding quantity indexes can be defined as follows:

(46) $\quad Q^{st}(p^1,\ldots,p^T,q^1,\ldots,q^T) \equiv p^t\cdot q^t/p^s\cdot q^s \; P^{st}(p^1,\ldots,p^T,q^1,\ldots,q^T).$

Gini (1931,10) also defined the multilateral price indexes ((45) and calle

the method the <u>successive weights system</u>. Both Walsh and Gini noted that

(45) collapsed to P_F defined by (4) if T=2 and s=1 and t=2.

Finally, Gini (1931,12) did propose a new multilateral method which h

called the <u>circular weight system</u>, because the resulting system of price

indexes P^{st} satisfied a multilateral analogue to the circular test (9)[48].

Given any bilateral price index function $P(p^1,p^2,q^1,q^2)$, define Gini's

multilateral level of prices in period t relative to the period s level as

follows:

$$(47) \qquad P^{st} = \left\{ \frac{P(p^1,p^t,q^1,q^t)}{P(p^1,p^s,q^1,q^s)} \; \frac{P(p^2,p^t,q^2,q^t)}{P(p^2,p^s,q^2,q^s)} \cdots \frac{P(p^t,p^t,q^T,q^t)}{P(p^T,p^s,q^T,q^s)} \right\}^{1/T}$$

for s,t=1, ..., T. The corresponding Q^{st} can be defined by (46). This

method was later proposed (using P_F as the P) by Elteto and Köves (1964) an

Szulc (1964) in the multiregional context and is known as the <u>EKS system</u>.

Gini (1931,13-24) tested out his circular weight system (as well as

some other alternatives) using the Fisher ideal P_F defined by (4) as his

bilateral P, for 8 time period observations on 5 Italian cities. Thus

Gini's computations were both multiperiod (between time periods) and

multilateral (between locations).

The <u>ratio</u> type price indexes, (45) proposed by Walsh and (47) propose

by Gini, can be converted into price <u>levels</u> $P^t(p^1,\ldots,p^T,q^1,\ldots,q^T)$ as

follows: corresponding to (45), define the period t price level P^t as

$$(48) \qquad P^t = [p^t\cdot q^1 \; p^t\cdot q^2 \ldots p^t\cdot q^T]^{1/T} \quad , \quad t=1,\ldots,T$$

and corresponding to (47), define P^t as

(49) $\qquad P^t = [P(p^1,p^t,q^1,q^t)P(p^2,p^t,q^2,q^t)\ldots P(p^T,p^t,q^T,q^t)]^{1/T}$.

In each case, it can be verified that $P^{st} = P^t/P^s$.

If we take the Gini-EKS price levels defined by (49) and divide each of them through by P^1, it can be seen that the resulting normalized price levels P^t/P^1 are closely related to Fisher's (normalized) blended price levels defined by (41): for the Fisher price levels, we take the arithmetic means of the numbers $P(p^k,p^t,q^k,q^t)/P(p^k,p^1,q^k,q^1)$, k=1, ..., T, while for the Gini-EKS price levels, we take the geometric mean of the same T numbers.

Walsh (1901,399)(1924,509) noted the primary disadvantage of using the multiperiod full information price level functions $P^t(p^1,\ldots,p^T,q^1,\ldots,q^T)$: if the number of periods increases, all of the indexes have to be recomputed.[49] This is not necessarily a fatal objection since it is normal practice for statistical agencies to periodically issue historical revisions and there is no reason why the revisions could not be accomplished using multiperiod indexes.

However, at present, it does not seem prudent to enthusiastically endorse a multiperiod system of index numbers since not enough research has been done on the axiomatic properties of the various multilateral or multiperiod alternatives.[50] Moreover, it would be desirable to develop multiperiod exact and superlative index number formulae and then examine the axiomatic properties of the resulting indexes. In the bilateral case, the Fisher ideal price index P_F emerges as the natural choice of a functional form since it seems to satisfy more reasonable tests than any

other known formula and it is superlative as well. We need a multilateral
counterpart to this "ideal" bilateral functional form.

To sum up: a comparison of the fixed base, chain, all binary
comparisons and multiperiod systems leads to no clear choice at this stage
However, if a definite choice has to be made, I would vote for the chain
system used with the bilateral Fisher ideal index P_F.

9.   Is the Substitution Bias Small?

The substitution bias in the consumer price index is the discrepancy
between the Laspeyres or Paasche price indexes, P_L and P_P defined by (2)
and (3) above, and the consumer's true cost of living index, defined by th
right hand side of (23). There is an analogous substitution bias in the
output price index.

The following quotation indicates Triplett's judgment on the size of
the substitution bias:

> "Though it has long been a staple of economists' educations, the
> substitution bias in a fixed-weight price index for consumption
> is just not very large." (Triplett, 1988,26).

To support the above opinion, Triplett cites the relatively close agreemen
between the Laspeyres and Paasche price indexes for U.S. aggregate
consumption data. Triplett's judgement would be correct if in fact these
indexes were true microeconomic Paasche and Laspeyres indexes, but they ar
not: microeconomic samples of price ratios p_i^{t+1}/p_i^t for various goods i are
combined with base period expenditure shares that are obtained from
periodic consumer expenditure surveys. The resulting aggregate indexes ar
not quite P_L and P_P defined by (2) and (3) above.

It may well be that Triplett is correct in his judgement, but the evidence to support his position has not yet been presented.

Due to the computer revolution, it is now possible to undertake some experiments which could help to determine the extent of the substitution bias. Retail outlets that have computerized price and quantity information on their sales could be sampled. Detailed microeconomic price and quantity vectors p^t and q^t could be constructed and the Laspeyres, Paasche and Fisher indexes defined by (2)-(4) above could be calculated and compared with corresponding official consumer or producer price indexes that covered the same range of goods. Such firm oriented experiments could provide useful information on the size of the substitution bias.[51]

10. Is the New Good Bias Small?

Triplett (1988) has a nice discussion of recent methods for adjusting for quality change that have been used by the Bureau of Labor Statistics. In keeping with the historical nature of this extended comment on Triplett, we shall first briefly review the ancient literature on methods for quality adjustment.

Some of the early researchers on price measurement were aware of the problem of quality change but the pace and direction of the change did not seem large enough to warrant an explicit treatment.[52]

However, by the latter part of the nineteenth century, Sidgwick (1883,68) realized that not only were improvements in the quality of goods leading to a bias in price comparisons, but also the growth of international and interregional trade (due primarily to transportation improvements) led to the systematic introduction of "entirely new kinds of things" and this too led to a bias in price comparisons. As the following

quotation indicates, Sidgwick (1883,68) thought that utility theory would play a role in eliminating these biases:

"Here again there seems to be no means of attaining more than a rough and approximate solution of the problem proposed; and to reach even this we have to abandon the prima facie exact method of comparing prices, and to substitute the essentially looser procedure of comparing amounts of utility or satisfaction."

Unfortunately, the mathematical apparatus of consumer theory was not sufficiently developed at that time to enable Sidgwick to make any specific progress on the new good problem.

In a brilliant paper, Marshall (1887,373) not only proposed the tabular standard, the chain system and the Edgeworth-Marshall index number formula (32), he also made the first real progress on the appropriate treatment of new goods, as the following quotation indicates:

"This brings us to consider the great problem of how to modify our unit so as to allow for the invention of new commodities. The difficulty is insuperable, if we compare two distant periods without access to the detailed statistics of intermediate times, but it can be got over fairly well by systematic statistics. A new commodity almost always appears at first at something like a scarcity price, and its gradual fall in price can be made to enter year by year into readjustments of the unit of purchasing power, and to represent fairly well the increased power of satisfying our wants which we derive from the new commodity." (Marshall, 1887,373).

As the above quotation indicates, Marshall was well aware of the product cycle and he felt that the early introduction of new commodities

into the consumer price index in the context of the chain system would capture most of the benefits due to the introduction of new commodities. As we shall see later, not quite all of the benefits are captured using Marshall's suggested method, since his method incorrectly ignores the new good in the first period that it makes its appearance.

Marshall (1. ,373,374) also realized that improvements in transportation led to the general availability of location specific goods, such as fish at the seaside or strawberries at a farm. Marshall correctly felt that these "old" goods that suddenly became available at many locations should be regarded as "new" goods and treated in the same way as a genuinely new good. His words on this important observation are worth quoting:

"This class of consideration is of much more importance than at first sight appears; for a great part of modern agriculture and transport industries are devoted to increasing the periods of time during which different kinds of food are available. Neglect of this has, in my opinion, viated the statistics of the purchasing power of many in medieval times with regard to nearly all kinds of foods except corn; even the well-to-do would hardly get so simple a thing as fresh meat in winter." (Marshall, 1887,374).

Marshall's suggested treatment of the new good problem was acknowledged and adopted by many authors including Irving Fisher (1911,204) and Pigou (1912,47). As we saw earlier in section 8, Divisia (1926,45) working from his independent perspective also suggested the use of the chain method as a means of dealing with the new good problem.

The next important contributor to the discussion of new goods in price measurement was Keynes. Keynes (1930,94) described in some detail one of the most common methods for dealing with the new good problem: simply ignore any new or disappearing goods in the two time periods under consideration and calculate the price index on the basis of the goods that are common to the two situations. The corresponding quantity index was to be obtained residually by deflating the relevant value ratio by this narrowly based price index. Keynes called this method the <u>highest common factor</u> method. This method would be identical to Marshall's chain method if the two time periods were chosen to be adjacent ones. However Keynes (1930,105-106) advocated his method in the context of a fixed base system of index numbers and he specifically rejected the chain method for 3 reasons: (i) each time a new product is introduced, a chain index does not take into account the benefits of the expanded choice set, and thus over long periods of time, the chain price index will be biased upwards and the corresponding quantity index will be biased downwards; (ii) the chain index fails Walsh's multiperiod identity test, (13) above, and (iii) the chain method was statistically laborious.

Keynes' last objection to the chain method is no longer relevant in this age of computers. Moreover, Keynes was unable to offer any positive alternative to the chain method for comparing situations separated by long periods of time as the following quotation indicates:

> "We cannot hope to find a ratio of equivalent substitution for gladiators against cinemas, or for the conveniences of being able to buy motor cars against the conveniences of being able to buy slaves." (Keynes, 1930,96).

However Keynes' first objection to the chain method (which was later echoed by Pigou (1932,72)[53]) was certainly valid (as was his second objection). A satisfactory theoretical solution to Keynes' first objection did not occur until Hicks adapted the analytical apparatus of consumer theory to the problem.

When new consumer goods make their appearance for the first time, say in period 2, their prices and quantities can be observed. In period 1, the quantities of the new goods are all obviously zero but what are the corresponding prices? Hicks (1940,114) provided a theoretical solution:

> "They are those prices which, in the 1 situation, would just make the demands for these commodities (from the whole community) equal to zero. These prices cannot be estimated, but we can observe that between the two situations the demands for these commodities will have increased from zero to certain positive quantities; and hence it is reasonable to suppose that the 'prices' of these commodities will usually have fallen relatively to other prices. This principle is sufficient to give us a fairly good way of dealing with the case of new goods."

Of course, in the context of the producer price index, the appropriate period 1 shadow prices for the new goods are those prices which just induce each period 2 producer of the new goods to produce zero quantities in period 1.

Hicks' basic idea was used extensively by Hofsten (1952,95-97) who dealt not only with new goods, but also adapted the Hicksian methodology to deal with disappearing goods as well. Hofsten (1952,47-50) also presents a nice discussion of various methods that have been used to adjust for

quality change, similar to Triplett's (1988) discussion of quality change measurement techniques.

Frank Fisher and Karl Shell (1972,22-26) laid out the formal algebra for constructing the period 1 Hicksian "demand reservation prices" defined in the above quotation by Hicks. Diewert (1980,498-501) used the Hicksian framework to look at the bias in the Fisher price index P_F defined by (4) when the reservation prices were incorrectly set equal to zero and compared this index to the Fisher price index that simply ignored the existence of the new goods in the two periods under consideration (which is Marshall's method).[54] Diewert (1980,501-503) also made some suggestions for estimating the appropriate Hicksian reservation prices in an econometric framework.

Is the new good bias large or small? One can only answer this question in the context of the price measurement procedures used by individual statistical agencies. In Diewert (1987,779), some simple hypothetical examples were given which showed that traditional fixed base procedures could generate much higher measures of price increase than would be generated using the chain method.[55] However, what is needed is empirical evidence.

Numerical computation of alternative methods based on detailed firm data on individual prices and quantities where new goods are carefully distinguished would cast light on the size of the new good bias. Thus the firm oriented experiments suggested at the end of the previous section to cast light on the size of the substitution bias could also be used to study the size of the new good bias.

Another line of empirical work which would be of interest would be to collect industry price and quantity data on various major new goods (e.g., microwave ovens, video recorders, home computers, satellite dishes, etc.) and then attempt to rework the relevant price indexes in the light of this extra data.

11. Has the Theory of the Cost of LIving Index been Exhausted?

Triplett (1988,25) appears to answer the above question in the affirmative as the following quotation indicates:

"The COL index has been subjected to far more research, both theoretical and empirical, than any other price index topic in the history of index numbers. It seems to me that much of the fruit has been picked from this tree."

It seems to me that the harvest is not yet over.

A large gap in our current statistical system is in the area of the consumer's allocation of time. Many years ago, Becker (1965) showed how the consumer's time constraint could be integrated into traditional consumer theory and he applied his new framework to cast light on a wide variety of applied economic problems. Additional applications can be found in a recent book edited by Juster and Stafford (1985). In order to implement Becker's theory, information on the consumer's allocation of time is required, broken up into: (i) time at work[56], (ii) time commuting to work, (iii) time spent shopping, (iv) time spent at housework, and (v) time spent at various leisure activities. Since many productivity improvements involve efficiencies in the consumer's use of time (e.g., a new subway line, an automated banking machine, electronic scanning of prices at the

supermarket, etc.), it seems appropriate for Statistical Agencies to consider the implementation of a version of Becker's framework.

Another area of household statistics which requires further theoretical development and empirical implementation is the area of income statistics: labour income should be decomposed into price and quantity components, income taxes should be taken into account in an appropriate manner and capital gains should be recognized as components of income. The point here is that most of the household measurement theory has concentrated on the commodity demand side and there has not been enough emphasis on the household factor supply and income sides.[57]

12. Conclusion

Jack Triplett has presented the profession with an excellent survey of the price measurement literature since 1961. In sections 2 to 6 above, we augmented his survey by providing an overview of the pre 1961 literature on price measurement and index number theory in general. In sections 7 to 11, we discussed various topics where we did not seem to be in close agreement with some of the views expressed by Triplett in his paper. In some cases, we also provided a historical survey of these controversial topics: in section 8, we reviewed the early literature on the chain principle and various alternatives to it, and in section 10, we reviewed the early literature on the new good problem.

It seems appropriate to conclude by listing three recommendations for Statistical Agencies where we agree with Triplett and then we shall list an additional four recommendations where we may be in some disagreement.

(1). Statistical Agencies should be encouraged to provide users with adequate printed documentation.

(2). The decomposition of labor income (on the household side) and labor payments (on the firm side) into price and quantity components needs improvement: weighted index numbers should be used for quantities rather than unweighted manhours and manhours should be disaggregated into various occupational, educational and demographically homogeneous categories.

(3). In the context of the cost of living index, the flow of services concept should be extended to other classes of consumer durables in addition to housing.

(4). My preferred method for decomposing a value ratio into price and quantity components is the use of a superlative index number formula in the context of the chain method.

(5). Since it is usually impossible to collect complete price and quantity information for each value cell in the relevant accounting framework, it will be necessary to resort to some sort of sampling principle. The appropriate objects to sample are values within the relevant cell in the first period. These sampled values would then be broken up into detailed prices and quantities which would then be observed in the following period as well. Finally, Fisher ideal price indexes should be constructed using these sampled values for the two periods and the corresponding quantity indexes should be constructed by deflating the relevant population value ratios by these (sample) price indexes. The entire procedure is explained in some detail by Pigou (1932,75-77).

(6). The sizes of the substitution bias and the new good bias are still in doubt. The empirical experiments described in sections 9 and 10 above would be useful in determining the size of these biases.

(7). More empirical and theoretical work needs to be done on the household supply side; see the suggestions on incorporating Becker's theory of the allocation of time and on the construction of household real income indexes made in section 11 above.

Footnotes

1. For a complementary historical assessment of hedonic price indexes from another pioneer in the field, see Griliches (1988).

2. The following quotation from Triplett is particularly insightful:

    "The episode of CPI housing underscores the fact that soundness of a government statistical series requires not just good staff analysis, but as well the formation of a consensus among users - a conceptually correct measurement from the standpoint of economic or statistical theory that is not accepted by, or is not understood by, broad user groups does not meet all the requirements for a public economic measurement." (Triplett, 1988, footnote 15).

3. See Griliches (1970), Triplett (1983b) and Jorgenson (1988) for a lengthier discussion of the issues. It is interesting to note that Bowley (1901,228) (1928,236-237) anticipated many of the modern objections to the use of unweighted average wages as the price of labor and unweighted manhours as the quantity of labor as the following quotation from Bowley (1901,228) illustrates: "The application of index-numbers to wage statistics does not involve any fresh principles. It is not permissible to ignore weights in this case; for an unweighted average would not allow for the general tendency to increase numbers where wages are rising."

4. A good account of Fleetwood's contributions with extensive quotations can be found in Ferger (1946).

5. See Fisher (1913,437)

6. Scrope does not refer Lowe (or anybody else for that matter) since as a Member of Parliament, he seemed to be writing an extended political speech. However, it seems likely that he knew of Lowe's work since his proposals

and terminology were so similar to those of Lowe. Scrope (1833,320) also proposed a comprehensive system of social security for workers "when rendered incapable of labour by illness, age or accident." Another early proposer of the tabular standard who does not refer to Fleetwood or Lowe is the American economist Willard Phillips who made his proposal in 1828; see Jastram (1951) for a description of the contributions of Phillips.

7. Edgeworth (1925,331) called the method the consumption standard rather than the tabular standard.

8. Bowley was not exclusively an advocate of the statistical approach as we shall see later. Also, initially Edgeworth took a broader view of the index number theory (recall his endorsement of the tabular standard if quantities remained constant), but after this initial broadness, he became a very strident defender of the statistical approach; in particular, Edgeworth's (1923) criticisms of Walsh's test approach became quite heated.

9. Keynes (1930,72) also used some rather colorful language to criticize the statistical approach as the following quotation indicates: "I have long believed that this is a will-o'-the-wisp, a circle-squaring expedition which has given an elusive taint, difficult to touch or catch, to the theory of price index numbers traditional in England. This is not equally true of America. Nevertheless, whilst the Americans have not worshipped the mythical creature, they have not (with the exception, perhaps, of Mr. Walsh) actively combatted him or dragged him out of the twilit cave where Edgeworth judiciously kept him."

10. Edgeworth mistakenly attributed this test to Westergaard (1890), but Westergaard's circular test is slightly different as we shall see.

11. A weaker identity test would require both prices and quantities to remain constant. No one could object to this weaker identity test, but it is certainly possible to object to the strong identity test.

12. See Fisher (1922,413).

13. Alternative names for this test might be the constant basket test or the tabular standard test. Walsh (1901,540)(1921,543) does not provide a name for this test.

14. Walsh (1901,385) noted that the strong proportionality test implies the strong identity test; i.e., (12) implies (8).

15. Walsh (1924,506) finally called this test the circular test but it seems best not to confuse Walsh's test (13) with Westergaard's test (9). If P satisfies the time reversal test (10), then (9) implies (13). However, if P satisfies (13), then P satisfies the weak identity test $P(p,p,q,q) = 1$ as well as (10) and (9). Upon noting that the index number formula $P(p^1,p^2,q^1,q^2) = 0$ satisfies (9) but not (10), we see that (9) and (13) are not equivalent tests; i.e., Walsh's test (13) is more restrictive than Westergaard's test (9).

16. This test postulates that the functional form that works for the price index $P(p^1,p^2,q^1,q^2)$ should also do the job as a quantity index, provided that the role of prices and quantities is interchanged. This transforms (14) into Fisher's strong factor reversal test:
$$P(p^1,p^2,q^1,q^2)P(q^1,q^2,p^1,p^2) = p^2 \cdot q^2 / p^1 \cdot q^1.$$

17. Frisch (1930,404-405) tried to show that a bilateral index number formula P could not simultaneously satisfy the base, commensurability, determinateness and factor reversal tests while Wald (1937,180,182) and Samuelson (1974,18-20) showed that P could not simultaneously satisfy the

proportionality, circular, and factor reversal tests. Many other
impossibility theorems were obtained by Eichhorn (1976) and Eichhorn and
Voeller (1976).

18. In particular, see Eichhorn and Voeller (1976).

19. Ville assumed the maximization of a linearly homogeneous utility
function subject to a budget constraint while Hulten assumed cost
minimization subject to a linearly homogeneous production function
constraint. Results equivalent to the Ville-Hulten results were also
obtained by Samuelson and Swamy (1974,578-574).

20. Hofsten (1952) appears to be the first researcher who noticed this
difficulty with the Divisia price index.

21. See also Samuelson and Swamy (1974,579) for alternative suggestions on
how to approximate the continuous time Divisia indexes with discrete data.

22. The right hand side of (21) can be regarded as an approximation to the
arithmetic average of the compensating and equivalent variations defined by
Hicks (1942). Thus the measurement of _differences_ in aggregate quantities
led to consumer led to consumer surplus theory while the measurement of
ratios led to the economic theory of index numbers.

23. For surveys, see Frisch (1936), Samuelson (1947,146-163), Ruggles
(1967), Pollak (1971), Samuelson and Swamy (174), Afriat (1977), Deaton and
Muellbauer (1980,167-184) and Diewert (1980)(1981)(1983a)(1983b)(1987).

24. See for example, Diewert (1983a,171).

25. The English translation of the original Russian article by Konüs
(1924) did not become available until 1939. Thus Frisch (1936,25) did not
have access to the original Konüs article and he mistakenly attributed the
Konüs limits or bounds on the true cost of living to Haberler (1927).

26. Diewert (1976,115) introduced this term to cover both the production and utility function context.

27. The first mention of the word "exact" in this index number context appears to be by Samuelson (1947,155). Afriat (1972,45) also uses the term exact.

28. See Balk (1981,1556) for a clear proof of Frisch's result.

29. With these restrictions, $f(q) - q \cdot Aq - h[g(q)]$ where $h(x) - x^2$ and $g(q) = [q \cdot Aq]^{1/2}$. Thus in this case, f is a monotonically increasing function of the linearly homogeneous function $g(q)$ over the set of q such that $q \cdot Aq \geq 0$.

30. For a nice exposition and evaluation of the Frisch and Wald quadratic approximation results as well as of some proposals by Samuelson (1974,16), see Balk (1981).

31. The Konüs and Byushgens (1926) paper was published in Russian and non Russian speaking economists seemed to be unaware of the paper until it was cited (incorrectly) by Schultz (1939,8).

32. See Pollak (1971,105).

33. The Cobb-Douglas functional form was perhaps first used in the economics literature by Wicksell (1958,98) in the nineteenth century.

34. The translog unit cost function was introduced by Christensen, Jorgenson and Lau (1971).

35. Vartia (1978) provided an alternative derivation of this result.

36. Malmquist (1953) was the first to use the distance function in index number theory.

37.   Bennet (1920,462) recognized that not all of the parameters of a quadratic utility function could be identified (due to the unobservable nature of utility) since he wrote of determining the parameters only up to a ratio.

38.   These perhaps overly pessimistic calculations lead to the following impossibility theorem: economists will never know the truth, even to the second order.

39.   For a definitive treatment of separability concepts and duality theory, see Blackorby, Primont and Russell (1978).

40.   See the theoretical discussion in Diewert (1980,475-476) and the empirical results in Harper, Berndt and Wood (1985) on alternative rental price formulae.

41.   Walsh (1901, 207) attributed the chain principle to Julius Lehr (1885) 45-46), who was motivated to introduce the principle in order to deal with new goods.

42.   Divisia (1926,44-47) saw his method as being a variant of the chain method, with the basic discrete period being one year (which would minimize seasonal fluctuations). Divisia (1926,45) also thought that the chain method was the only logical way to make price comparisons over long periods due to the introduction of new goods and the discovery of new inventions. As new goods in his time, he mentioned machine guns (to replace the bow and arrow), submarines, aircraft, the potato and steam engine.

43.   However, later Fisher (1922,308-309) preferred the fixed base system. Pigou (1932,71) criticized Fisher's later position and endorsed the chain principle for the usual reason: with the introduction of new commodities, the chain principle is the only way to make comparisons between distant

periods.

44. Walsh (1901,401) (1924,506) made the same criticism of the chain system. Szulc (1983,540) also uses Walsh's Test in his evaluation of fixed base versus chain index numbers.

45. Actually, Hill (1988,7) uses the more general Proportionality Test, $P*(p^0,p^t,\lambda p^t,q^0,q^t,q^{t+1}) = \lambda$ for $\lambda > 0$, in place of (39).

46. Fisher (1922,280-283) also found that P_F satisfied Walsh's Multiperiod Identity Test (13) to a high degree of approximation.

47. Actually, Fisher (1922,301) used a slightly different normalization.

48. In fact, all of the multilateral indexes defined in this section satisfy the multilateral circularity test.

49. Walsh (1901,399) also made the following theoretical objection: "Besides, how is a past variation between two years several years ago to be affected by present variations?"

50. For a start on this topic, see Diewert (1987)(1988).

51. Such firm based experiments would yield information on the substitution bias in output price indexes. Perhaps some day in the future when consumers use credit or banking cards to pay for all of their purchases, we could obtain an accurate paper trail that could be used to construct true microeconomic Laspeyres, Paasche and Fisher price indexes.

52. Thus Lowe (1823, Appendix 87) states: "In regard to the quality of our manufactures, we must speak with more hesitation, and can hardly decide whether the balance be in favour of the present or of a former age; for if our fabrics are now much more neat and convenient, they are in a considerable degree less durable."

53.   Pigou, (1932,71) also has a nice criticism of Keynes' highest common factor method which was later repeated by Hofsten (1952,59).   Pigou also criticized Fisher's (1922,308-312) later preference for the fixed base method.

54.   The second index has a smaller bias than the first index.

55.   Since 1978, the U.S. Bureau of Labor Statistics has used a probability sampling approach in the consumer price index which probably reduces some of this fixed weight bias, but the bias is not eliminated.

56.   Riddell (1983) discusses the issues involved in incorporating leisure time (defined to be time spent not at work) in measures of the standard of living and he calculates some of his new measures of economic welfare using Canadian data.

57.   See Diewert and Bossens (1987) for an introduction to these topics.

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