NBER WORKING PAPER SERIES

MITIGATING DISASTER RISKS IN THE AGE OF CLIMATE CHANGE

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Working Paper 27066
http://www.nber.org/papers/w27066

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2020, Revised April 2023

We thank Michael Barnett, Patrick Bolton, Jaroslav Borovicka, Kent Daniel, Stefano Giglio, Robert Hall, Lars Peter Hansen, Julian Koelbel, Jeffrey Kubik, Daniel Lashkari, Markus Leippold, Michael Peters, Toan Phan, Esteban Rossi-Hansberg, Bernard Salanié, Tano Santos, Thomas Sargent, Jose Scheinkman, Stijn Van Nieuwerburgh, Laura Veldkamp, Venky Venkateswaran, Olivier Wang, and seminar participants at American Finance Association Annual Meetings, University of Chicago-Institute for Mathematical and Statistical Innovation, China International Conference in Finance, Columbia University, Imperial College London, Indiana University, INQUIRE, NYU Economics, SHOF-ECGI Sustainability Conference, and University of Zurich for helpful comments. We especially thank Jiangmin Xu for his help with the paper. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 27066
April 2020, Revised April 2023
JEL No. E21,E22,E23,G12,G28,G52,H23,H41

ABSTRACT

Emissions abatement alone cannot address the consequences of global warming for weather disasters. We model how society adapts to manage disaster risks to capital stock. Optimal adaptation—a mix of form-level efforts and public spending—varies as society learns about the adverse consequences of global warming for disaster arrivals. Taxes on capital are needed alongside those on carbon to achieve the first best. We apply our model to country-level control of flooding from tropical cyclones. Learning rationalizes empirical findings, including the responses of Tobin's q, equity risk premium, and risk-free rate to disaster arrivals. Adaptation is more valuable under learning than a counterfactual no-learning environment. Learning alters social-cost-of-carbon projections due to the interaction of uncertainty resolution and endogenous adaptive response.

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1 Introduction

Global costs of weather-related disasters have increased sharply in recent decades. While this trend increase is partly due to economic growth and exposure of physical capital (Pielke et al., 2008), recent climate research links climate change to more frequent disasters (National Academy of Sciences, 2016). Emissions abatement will only impact such losses decades down the road and might not fully address the consequences for weather disasters. Hence, adaptations to mitigate natural disaster risks, be it flooding from tropical cyclones or damage from wildfires, need to play a major role going forward.

Since there is considerable uncertainty on the impact of global warming for the frequency of disasters,[1] adaptation naturally depends on households learning about these consequences. In contrast to emissions abatement, which have been the main focus of research using integrated assessment models (Nordhaus, 2017; Golosov, Hassler, Krusell, and Tsyvinski, 2014; Jensen and Traeger, 2014; Cai and Lontzek, 2019; Barnett, Brock, and Hansen, 2020), such adaptation strategies have thus far been relatively under-emphasized both in climate change research and practice (Bouwer et al., 2007).

To address these issues, we begin by introducing learning and adaptation into a continuous-time stochastic general-equilibrium model with disasters along the lines emphasized by Rietz (1988), Barro (2006), and Pindyck and Wang (2013). Output is determined by an AK growth function augmented with capital adjustment costs (e.g., Hayashi, 1982) that give rise to rents for installed capital and the value of capital (Tobin's average q). Disaster shocks following a Poisson process destroy capital stock, affect equilibrium asset prices, and reduce the welfare of households endowed with recursive utility (Epstein and Zin, 1989).

Mitigation of these disaster shocks is modeled via a combination of two adaptation technologies: (1.) adaptation spending at the firm level that reduces the exposure of a firm's capital to the disaster shock (e.g. sandbags and other temporary barriers to protect buildings) and (2.) spending at the aggregate level that requires collective action which reduces the conditional damage of a disaster arrival and tail risk for all agents in the economy (e.g., an early warning system, infrastructure maintenance and preparedness, and other government

[1] According to a survey (Knutson et al., 2020), the most pessimistic climate model projects the frequency of tropical cyclones in $2^o c$ world to be 2.25 times higher than in the pre-industrial era. The most optimistic model projects a slight decrease relative to pre-industrial levels. The median model projects a modest 13% increase relative to pre-industrial.

funded programs.)[2]

Our model generates the following key properties and predictions. First, while the planner's first-best solution features an optimal mix of spending on both adaptation technologies, firms do not internalize the benefits of aggregate risk mitigation and underspend on total risk mitigation in market economies. We prove that an optimal tax on capital to fund government spending on reducing aggregate tail risks restores the first-best solution while still maintaining a balanced budget.

Second, belief that the economy is in the bad state (B) is a key state variable driving optimal adaptation and equilibrium outcomes. "Bad" news (an unexpected arrival) leads to a discontinuous jump (worsening) of belief, as a disaster arrival is a discrete event also serving as a discrete signal.[3] Absent any arrivals, belief drifts gradually towards the good (G) state, as no news is good news when it comes to arrival of disasters in our model.

Third, unexpected disaster arrivals have both direct effects (i.e. capital destruction) and indirect effects due to learning that the world is riskier than anticipated. As a result, the effects of disaster arrivals on economic growth are also time-varying and persistent. Additionally, Tobin's q falls and the stock market risk premium rises upon a disaster arrival. Without the learning channel in our model, asset valuation multiples, e.g., Tobin's q, would not move upon disaster arrivals as predicted by Pindyck and Wang (2013). The disaster arrival effects on growth, valuation, and risk premium are a major difference between our model and the literature.[4]

We then quantify the importance of learning and adaptation for disaster risk mitigation in the context of tropical cyclones, which include hurricanes, typhoons, cyclones, and tropical storms,[5] that are estimated to affect nearly 35% of the global population. Using panel data covering 109 countries over the period of 1950-2010, we calibrate our model via simulation to target moments pertaining to the macroeconomy (aggregate consumption, investment, and output), to financial markets (the risk-free rate, equity risk premium and Tobin's q), and to the arrivals of tropical cyclones and adaptation (e.g., government flood control budgets). We

[2]See Lasage et.al. (2014), Muis et.al. (2015) and Fried (2023) for evidence on the value of flood control adaptations.

[3]Our model generates time-varying disaster arrival rates via learning (also see e.g., Wachter and Zhu, 2019, Colin-Dufresne, Johannes, and Lochstoer, 2016).

[4]See Hong, Karolyi, and Scheinkman (2020) for a review of recent findings on weather disasters and climate risks including the impact of sea-level rise on coastal property prices. Beliefs of the risks are shown to play a role (Bakkensen and Barrage, 2022).

[5]They are referred to as tropical storms or hurricanes in Atlantic, typhoons in the Pacific, and cyclones in Indian Ocean.

confirm findings in the literature that a typical disaster leads to 1% reduction in GDP growth (Hsiang and Jina, 2014). We also present new findings that country-level asset prices (the risk-free rate, Tobin's q, and equity risk premium) also respond strongly to disaster arrivals, thus allowing us to internally calibrate parameters governing the learning process.

The first finding of our quantitative analysis is that large learning effects are needed to rationalize the data. The second finding is that the value of adaptation is much higher than under the counterfactual no-learning environment. That is, a large part of the value of optimal adaptation derives from uncertainty associated with learning about the climate state. The third finding is that there is a significant gap between welfare in a competitive economy (with only private adaptation) and welfare in the first-best economy, which is implementable in a market economy with optimal capital taxes. Our quantitative conclusions are generally robust to two changes to the model: (1) a generalized belief updating process that allows the underlying state to switch between the good and bad states, and (2) different risk preferences.

Having established the importance of adaptation for mitigating disaster risks in a learning environment, we then explain how learning and adaptation influence the social cost of carbon. We consider a tractable extension, incorporating features from the social cost of carbon model of Van den Bremer and Van der Ploeg (2021). Output depends on both capital and fossil fuels. Using fossil fuels increases the stock of carbon in the atmosphere, which leads to lower recovery rates given a disaster arrival, and hence results in more damages akin to integrated assessment models which generally do no feature learning. Moreover, our model also features uncertainty about the frequency of disasters, which our society learns about from disaster arrivals, and then makes adaptation decisions in response.

We obtain the planner's first-best solution for this economy. We then analyze the decentralized competitive market model. Equilibrium outcomes depend on both the belief (about how likely the economy is in the bad state) and the carbon stock. In order to address the climate externality, we show that a combination of three taxes implements the first-best outcome: (1.) a carbon tax on a firm's fossil-fuel usage; (2.) a tax on firm investment and (3.) a tax on capital to fund aggregate adaptation. The optimal carbon tax rate equals the social cost of carbon in the first-best economy as in the literature. The tax rate on firm investment is chosen to ensure that the equilibrium capital accumulation dynamics is the same as in the first-best economy. This new tax margin is not in Golosov et al. (2014) as capital is a flexible input choice in their model as opposed to a state variable in our model. Finally, the capital tax rate in our generalized model is similar to the one in our baseline model without carbon.

We use our calibration of the first-best solution to highlight the role of learning. As Daniel, Litterman, and Wagner (2019) have observed, integrated assessment models without learning invariably yield a social cost of carbon (and hence an optimal carbon tax) that is gradually rising over time as a larger carbon stock is assumed to lead to more damages. In contrast, the social cost of carbon, under our calibration, is declining at the beginning of the transition period due to interaction of resolution of uncertainty and endogenous response of adaptation. In general, the optimal fossil fuel tax and adaptation spending levels over time depend the society's prior belief and the speed of convergence of beliefs to a steady state.

Our work is related to insightful work by Bretschger and Vinogradova (2019), who model optimal abatement with recurrent disasters. In contrast to ours, their model has no capital adjustment costs, no learning, and does not distinguish between adaptation and fossil fuel (abatement). As a result, in their model, the expected growth rate is constant over time as in Pindyck and Wang (2013) and Tobin's q always equals one, both of which are counter to the evidence in our Section 6. Moreover, the social cost of carbon in their model does not depend on the interaction of learning and adaptation.

2 Model

In this section, we develop a model of learning and adaptation to disaster risks in a market economy. Time is continuous and the horizon is infinite. There is a continuum of identical firms and households, both with a unit measure.

2.1 Firms' and Households' Optimization Problems

Firm production. A firm produces output proportional to its capital stock, K_t. That is, its output is AK_t, where $A > 0$ measures productivity. (This is an AK model).

Firm investment, capital accumulation, and arrival of jumps (disasters). Let I_t denote firm investment. The firm's capital stock K_t evolves as:

$$dK_t = (I_{t-} - \delta_K K_{t-})\,dt + \sigma_K K_{t-} d\mathcal{W}_t^K - N_{t-} K_{t-}(1-Z)d\mathcal{J}_t \ , \tag{1}$$

where δ_K is the depreciation rate of capital. The second term captures continuous diffusive shocks to capital, where $\mathcal{W}_t^K$ is a standard Brownian motion and the parameter σ_K is the diffusion volatility. This term is the standard source of shocks for AK models in macroeconomics and sometimes is interpreted as stochastic depreciation shocks. The last term in (1) captures the loss to the firm's capital from a stochastic arrival of a disaster.

The process $\mathcal{J}_t$ in (1) is a Poisson process where each jump arrives at a constant but unobservable rate, which we denote by λ. We will return to discuss the details for the arrival rate λ. There is no limit to the number of these jump shocks. If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at t. The N_{t-} process is chosen by the firm to mitigate its exposures to disasters, which we introduce later.

Without reducing disaster exposures (which implies $N_{t-} = 1$), upon a disaster arrival at t ($d\mathcal{J}_t = 1$), a stochastic fraction $(1 - Z) \in (0, 1)$ of the firm's capital stock K_{t-} is permanently destroyed at t and hence the surviving capital stock is $K_t = ZK_{t-}$. (For example, if the firm incurred no disaster exposure reduction spending at $t-$ and a shock arrived at t destroying 15 percent of capital stock, we would have $Z = 0.85$.) Naturally, anticipating damages caused by these disasters, the firm has incentives to *ex-ante* reduce its exposures to disaster shocks by spending resources (e.g., sandbags to keep a building from flooding during a tropical cyclone.)

Let $\Xi(Z)$ and $\xi(Z)$ denote the cumulative distribution function (cdf) and probability density function (pdf) for the stochastic fraction of capital recovery Z, respectively, conditional on a jump arrival. While the firm takes the distribution of Z as given, the society as a whole can spend resources to influence the distribution of Z by making disasters less damaging to the economy. We introduce the determinants of $\Xi(Z)$ at the aggregate level in Section 2.4.

Reducing a firm's disaster exposure (firm-level adaptation). Let X^e_{t-} denote the firm's adaptation spending to reduces its exposure to a disaster, where the superscript e refers to *exposure* at $t-$. With this spending at $t-$, should a disaster arrive at t, the firm decreases its capital loss from $(1 - Z)K_{t-}$ to $N_{t-}(1 - Z)K_{t-}$, where $N_{t-} \in [0, 1]$ depends on X^e_{t-}. The effect of this spending on capital stock dynamics is captured by the N_{t-} term in (1). Let $x^e_{t-} = X^e_{t-}/K_{t-}$ denote the firm's scaled disaster exposure reduction spending.

To preserve our model's homogeneity property, we assume that N_{t-} is a function of x^e_{t-}:

$$N_{t-} = N(x^e_{t-}) \,. \tag{2}$$

Equations (1) and (2) imply that if we double X^e_{t-} and capital stock K_{t-} simultaneously, the benefit from reducing disaster damages (in units of goods) also doubles. To see why, observe that $N_{t-} = N(x^e_{t-})$ is unchanged with the simultaneous doubling of X^e_{t-} and K_{t-} but the amount of loss reduced by adaptation, is doubled since K_{t-} has doubled.

We require $N'(x^e) \leq 0$ as adaptation spending reduces damages. Additionally, the marginal effect of spending on reducing damages is decreasing in x^e, which implies $N''(x^e) \geq 0$. Finally, by definition, $N(0) = 1$, as no adaptation spending ($x^e = 0$) no damage reduction.

Capital adjustment costs and firm's objective. Following the q theory of investment (Hayashi, 1982; Abel and Eberly, 1994), we assume that when investing $I_t dt$, the firm incurs capital adjustment costs, $\Phi_t dt$. The firm's dividend payout (profit), Y_t, is then given by

$$Y_t = AK_t - (I_t + \Phi_t) - X^e_t \, . \tag{3}$$

Following Hayashi (1982), we specify the adjustment cost Φ_t as $\Phi_t = \Phi(I_t, K_t)$, where

$$\Phi(I_t, K_t) = \phi(i_t)K_t \ , \tag{4}$$

where $i_t = I_t/K_t$ and $\phi(i)$ is increasing and convex. The firm chooses investment I and the adaptation spending X^e to maximize its present value given by:[6]

$$\mathbb{E}\left(\int_0^\infty \frac{\mathbb{M}_t}{\mathbb{M}_0} Y_t dt\right) \ , \tag{5}$$

where $\mathbb{M}$ is the equilibrium stochastic discount factor (SDF) that captures both the time value and risk premium.[7] The equilibrium SDF is the representative consumer's equilibrium marginal rate of substitution (MRS). Let Q_0 denote firm value at $t = 0$, the solution for (5).[8]

Households' preferences. We work with the recursive utility developed by Epstein and Zin (1989) and formulated in continuous time by Duffie and Epstein (1992). The life-time utility of our representative consumer's recursive preferences is given by:

$$V_0 = \mathbb{E}\left[\int_0^\infty f(C_t, V_t)dt\right] \ , \tag{6}$$

where $f(C, V)$ known as the normalized aggregator is given by

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega - 1}} \tag{7}$$

[6]Financial markets are perfectly competitive and complete. While the firm can hold financial positions (e.g., DIS contracts in net zero supply), these financial hedging transactions generate zero NPV for the firm. Therefore, financial hedging policies are indeterminate, a version of the Modigliani-Miller financing irrelevant result. The firm can thus ignore financial contracts without loss of generality.

[7]The firm takes $\mathbb{M}$ as given when solving its problem and the $\mathbb{M}$ process is determined in equilibrium.

[8]Because installing capital is costly, installed capital earns rents in equilibrium so that Tobin's average q, the ratio between the firm's value (Q_0) and the replacement cost of capital (K_0), exceeds one.

and $\omega = (1-\psi^{-1})/(1-\gamma)$. Here ρ is the rate of time preference, ψ is the elasticity of intertemporal substitution (EIS), γ is the coefficient of relative risk aversion. Unlike expected utility, recursive preferences as defined by (6) and (7) disentangle risk aversion from the EIS.[9] To check the robustness of our analysis, we also analyze our model with external habit formation proposed by Campbell and Cochrane (1999) in Subsection 7.7.

2.2 Bayesian Belief Updating about the Disaster Arrival Frequency

Next, we turn to the disaster arrival process. The arrival rate λ while constant is unobservable to the agent.[10] Therefore, an arrival of a disaster not only destroys capital stock, but also serves as a signal from which households and firms update their beliefs about λ.

While the true disaster arrival rate λ is constant by assumption, households and firms do not have complete information about the value of λ. What the households and firms know at time 0 is that the true value of λ is either λ_G or λ_B with $\lambda_B > \lambda_G$. If the true value of λ is λ_B rather than λ_G, capital stock is more likely to be hit by a disaster (i.e., a negative jump). We refer to the low-arrival-rate and high-arrival-rate scenarios as the good (G) state and the bad (B) state, respectively. Additionally, all agents are endowed with the same prior belief π_{0-} that the true value of λ is λ_B. In sum, all agents in our model have the same information sets, share the same prior, and use the same Bayes rule to update beliefs.

Let π_t denote the time-t posterior belief that $\lambda = \lambda_B$:

$$\pi_t = \mathbb{P}_t(\lambda = \lambda_B)\,, \tag{8}$$

where $\mathbb{P}_t(\,\cdot\,)$ is the conditional probability at t. The expected disaster arrival rate at t, λ_t, is:

$$\lambda_t = \mathbb{E}_t(\lambda) = \lambda(\pi_t) = \lambda_B\pi_t + \lambda_G(1-\pi_t)\,, \tag{9}$$

which is a weighted average of λ_B and λ_G. A higher value of π_t corresponds to a belief that the economy is more likely in State B where the jump arrival rate is $\lambda_B > \lambda_G$.

What leads the agent's belief to worsen (increasing π) is jump arrivals. What leads the belief to revise favorably is no jump arrivals. In this sense, no-jump news is good news.

[9] If $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C,V) = \frac{\rho\, C^{1-\gamma}}{1-\gamma} - \rho V\,.$$

[10] In Section OA of the Online Appendix, we generalize our model to a setting where the unobservable disaster arrival rate λ is stochastic and follows a two-state continuous-time Markov chain.

Mathematically, the agent updates his belief using the Bayes rule:[11]

$$d\pi_t = \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) \,, \tag{10}$$

where

$$\sigma_\pi(\pi) = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda(\pi)} = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda_B\pi + \lambda_G(1-\pi)} > 0\,. \tag{11}$$

Here, signals come from $\mathcal{J}_t$. Note that π_t and λ_t are both martingales which can be seen from (10) as $\mathbb{E}_{t-}[d\mathcal{J}_t] = \lambda_{t-}dt$. When a disaster strikes at t, the belief immediately increases from the pre-jump level π_{t-} to $\pi_t^{\mathcal{J}}$ by $\sigma_\pi(\pi_{t-})$, where

$$\pi_t^{\mathcal{J}} = \pi_{t-} + \sigma_\pi(\pi_{t-}) = \frac{\pi_{t-}\,\lambda_B}{\lambda(\pi_{t-})} > \pi_{t-}\,. \tag{12}$$

If there is no arrival ($d\mathcal{J}_t = 0$) over dt, the household becomes more optimistic. In this case,

$$\frac{d\pi_t}{dt} = \mu_\pi(\pi_{t-}) = \pi_{t-}(1-\pi_{t-})(\lambda_G - \lambda_B)\,, \tag{13}$$

using $\mu_\pi(\pi_{t-}) = -\sigma_\pi(\pi_{t-})\lambda(\pi_{t-})$. Equation (13) is a logistic differential equation. Conditional on no jump ($d\mathcal{J}_v = 0$) for $v \in (s,t)$, we obtain the closed-form logistic function for π_t:

$$\pi_t = \frac{\pi_s e^{-(\lambda_B-\lambda_G)(t-s)}}{1+\pi_s(e^{-(\lambda_B-\lambda_G)(t-s)}-1)}\,. \tag{14}$$

In Figure 1, we plot a simulated path for π starting from $\pi_0 = 0.08$. It shows that absent a jump arrival, belief becomes more optimistic and π_t decreases deterministically between two consecutive jumps following the logistic function given in (14). Once a jump arrives at t, the belief worsens moving upward to $\pi_t^{\mathcal{J}}$ given in (12) by a discrete amount $\sigma_\pi(\pi_{t-})$ given in (11).

2.3 Competitive Market Structure and Equilibrium

Next, we turn to the competitive market economy. Financial markets are dynamically complete. Without loss of generality, it is sufficient to assume that the following financial securities exist at all time t: (i) a risk-free asset thats pays interest at the equilibrium rate of r_t and (ii) the aggregate equity market.[12] To ease exposition, we use **boldfaced** letters to refer to aggregate variables so as to differentiate from the corresponding firm-level variables.[13]

[11] See Theorem 19.6 in Liptser and Shiryaev (2001). A similar learning problem is in Dieckmann (2011).

[12] For markets to be dynamically complete, we also need actuarially fair diffusion and jump hedging contracts (for each possible jump contingency) as in Pindyck and Wang (2013). The net demand is zero for all hedging contracts. For expositional simplicity, we omit these hedging contracts and refer readers to Pindyck and Wang (2013) for related detailed analysis.

[13] Because our model economy is populated with a continuum of identical households and firms, the average of a micro-level variable equals the corresponding variable in the aggregate. For example, the average of I_t equals the aggregate $\mathbf{I}_t$. Similarly, the average of i_t equals the aggregate $\mathbf{i}_t$. Our aggregation result is based on the exact law of large numbers (Duffie and Sun, 2006).

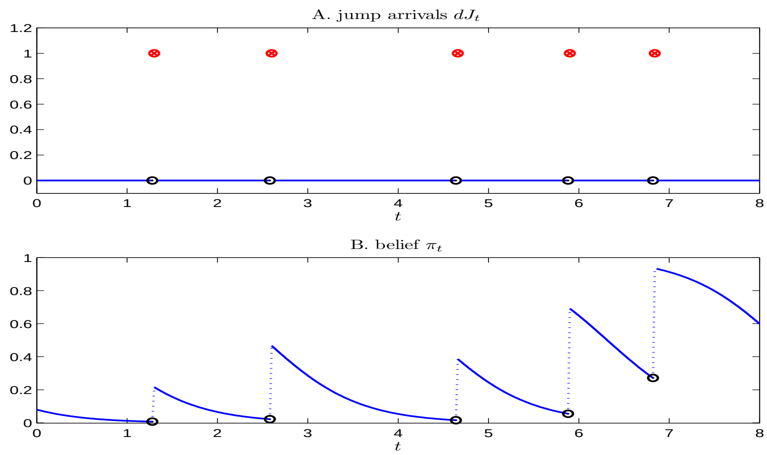

Figure 1: This figure simulates a path for disaster arrival times in Panel A and plots the corresponding belief updating process in Panel B starting with $\pi_0 = 0.08$. The belief decreases deterministically in the absence of jumps but discretely increases upward upon a jump arrival.

Let $\{\mathbf{Q}_t\}$ denote the equilibrium ex-dividend aggregate stock market value and $\{\mathbf{D}_t\}$ denote the aggregate dividends, respectively. The cum-dividend return is then given by

$$\frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} = \mu_{\mathbf{Q}}(\pi_{t-})dt + \sigma_K d\mathcal{W}_t^K + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)d\mathcal{J}_t\,, \tag{15}$$

where $\mu_{\mathbf{Q}}(\pi)$ is the expected stock market return (leaving aside the jump effect). We later verify that the diffusion volatility of the stock market return equals σ_K, the same as the diffusion volatility given in (1). Finally, the last term captures the effect of jumps on returns.

Competitive equilibrium. We define the recursive competitive equilibrium as follows: (a.) Taking the equilibrium risk-free rate r and the equilibrium aggregate stock market return process (15) as given, the representative household chooses consumption C and allocation to the aggregate stock market Γ to maximize lifetime utility given by (6)-(7);[14] (b.) Taking the equilibrium SDF $\{\mathbb{M}_t; t \geq 0\}$ as given, the representative firm chooses investment I and the disaster exposure mitigation spending X^e to maximize its market value given in (5); (c.) The interest rate r, the stock market return process (15), and the SDF $\{\mathbb{M}_t; t \geq 0\}$ are consistent with the households' and firms' optimal decisions and all markets clear in equilibrium.

[14]Since each household is infinitesimally small and has no impact on any aggregate variables, there is no incentive to spend on mitigation. We provide additional discussions later in the paper.

2.4 Source of Externality: Technology Reducing Tail Risk of the Damage Distribution $\Xi(Z)$ for All Firms

Next, we introduce another adaptation technology, which reduces the tail risk of the aggregate disaster distribution $\Xi(Z)$. In contrast to the first type of adaptation technology, which operated at the firm level, this second type of adaptation technology operates at the aggregate level and features an externality (a realistic aspect of adaptation) as its effectiveness depends on collective contributions of all firms in aggregate (i.e. a public good).

We assume that the *aggregate* spending made at $t-$ can curtail left-tail disaster (jump) risks at t if a jump arrives at t.[15] The idea is that changing the distribution of Z for all firms is very costly and requires a spending that is at the order of a fraction of the aggregate capital stock $\mathbf{K}$. Let $\mathbf{X}^d_{t-}$ denote the *aggregate* spending on this distribution-tail-curtailing technology, where the superscript d refers to the notion that this spending is to make the *distribution* of fractional loss $(1-Z)$ less damaging. Let $\mathbf{x}^d_{t-} = \mathbf{X}^d_{t-}/\mathbf{K}_{t-}$ denote this scaled aggregate adaptation spending. Since aggregate risk reduction is a public good, no firm has incentives to spend on this new technology. This is the reason why markets fail.

Specifically, by spending on aggregate tail risk reduction, we change the distribution of the post-jump fractional recovery Z from $\Xi(Z)$ to $\Xi(Z;\mathbf{x}^d_{t-})$. While simultaneously doubling this type of aggregate adaptation spending $\mathbf{X}^d_{t-}$ and the aggregate capital stock $\mathbf{K}_{t-}$ does not change the distribution $\Xi(Z;\mathbf{x}^d_{t-})$, as the ratio $\mathbf{x}^d_{t-} = \mathbf{X}^d_{t-}/\mathbf{K}_{t-}$ remains unchanged, doing so doubles the benefit of this public spending (i.e., the total reduction of damages) *in levels* as the benefit is proportional to $\mathbf{K}_{t-}(1-Z)$ at the aggregate level.[16]

We have completed the description of our market economy model. Before solving it in Section 4, we first analyze the planner's problem. The first-best solution for the planner's model serves as an important benchmark for our analysis of the market economy.

3 Planner's Problem and its First-Best Solution

The social planner chooses consumption $\mathbf{C}$, investment $\mathbf{I}$, and adaptation spendings $\mathbf{X}^d$ and $\mathbf{X}^e$ to maximize the representative household's utility given in (6)-(7) subject to the repre-

[15] Our assumption is motivated by the literature on flood control, where public adaptations reduce the tail event of high inundation levels (Lasage et.al. (2014), Muis et.al. (2015)). Private adaptation is only effective at low inundation levels.

[16] This is similar to the homogeneity assumption for disaster distribution (private adaptation) mitigation spending X^e_{t-}.

sentative firm's production/capital accumulation technology, the adaptation technologies, and the aggregate resource constraint: $\mathbf{C}+\mathbf{I}+\mathbf{\Phi}+\mathbf{X}^d+\mathbf{X}^e=A\mathbf{K}$.

To save on notation, we drop the subscript fb in this section until the end of this section where we summarize the first-best solution.

Dynamic programming. Let $V(\mathbf{K},\pi)$ denote the representative household's value function. The Hamilton-Jacobi-Bellman (HJB) equation for the planner is:

$$\begin{aligned} 0 \;=\; & \max_{\mathbf{C},\mathbf{I},\mathbf{x}^e\,\mathbf{x}^d} f(\mathbf{C},V)+(\mathbf{I}-\delta_K\mathbf{K})V_{\mathbf{K}}(\mathbf{K},\pi)+\mu_\pi(\pi)V_\pi(\mathbf{K},\pi)+\frac{1}{2}\sigma_K^2\mathbf{K}^2V_{\mathbf{KK}}(\mathbf{K},\pi) \\ & +\lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[V\left(\mathbf{K}^{\mathcal{J}},\pi^{\mathcal{J}}\right)-V(\mathbf{K},\pi)\right]\;, \end{aligned} \tag{16}$$

where $\pi^{\mathcal{J}}$ is the post-jump belief given in (12), $\mathbf{K}^{\mathcal{J}}$ is the post-jump capital stock given by

$$\mathbf{K}^{\mathcal{J}}=\left(1-N(\mathbf{x}^e)(1-Z)\right)\mathbf{K}\,, \tag{17}$$

$\mu_\pi(\pi)$ is the expected change of belief absent jumps given in (13), $\lambda(\pi)$ is the jump arrival rate given in (9), and $\mathbb{E}^{\mathbf{x}^d}[\,\cdot\,]$ is the expectation operator with respect to the pdf $\xi(Z;\mathbf{x}^d)$ for the recovery fraction Z for a given level adaptation spending $\mathbf{x}^d$ to reduce aggregate risk.

The first term on the right side of (16) is the household's normalized aggregator (Duffie and Epstein, 1992); the second term captures how investment $\mathbf{I}$ affects $V(\mathbf{K},\pi)$; the third term reflects how belief updating (in the absence of jumps) impacts $V(\mathbf{K},\pi)$; and the fourth term captures the effect of capital-stock diffusion shocks on $V(\mathbf{K},\pi)$. It is worth noting that as the signals in our learning model are discrete (jump arrivals), there is no diffusion-induced quadratic-variation term involving $V_{\pi\pi}$ in the HJB equation (16).

Direct (value destroying) versus learning effects. Finally, the last term (on the second line) of (16) captures the effect of jumps on the expected change in $V(\mathbf{K},\pi)$. This term captures rich economic forces and warrants additional explanations. When a jump arrives at t ($d\mathcal{J}_t=1$), capital falls from $\mathbf{K}_{t-}$ to $(1-Z)\mathbf{K}_{t-}$ absent exposure mitigation spending. By spending $\mathbf{x}^e_{t-}$ to reduce the exposure, the planner reduces the capital loss from $(1-Z)\mathbf{K}_{t-}$ by $N(\mathbf{x}^e_{t-})(1-Z)\mathbf{K}_{t-}$, so that the post-jump capital is $\mathbf{K}^{\mathcal{J}}_t=(1-N(\mathbf{x}^e_{t-})(1-Z))\mathbf{K}_{t-}$ at t.

In sum, a jump triggers two effects on $V(\mathbf{K},\pi)$. First, there is a direct capital destruction effect. As a jump arrival lowers capital stock from $\mathbf{K}_{t-}$ to $\mathbf{K}^{\mathcal{J}}_t=(1-N(\mathbf{x}^e_{t-})(1-Z))\mathbf{K}_{t-}$, the value function decreases from $V(\mathbf{K}_{t-},\pi_{t-})$ to $V(\mathbf{K}^{\mathcal{J}}_t,\pi_{t-})$ even if we ignore the agent's belief updating due to learning. Second, there is a learning (belief-updating) effect. As a jump arrival also cause the belief to increase from π_{t-} to $\pi_t^{\mathcal{J}}$ given in (12), the agent becomes more pessimistic causing the value function to further decrease from $V(\mathbf{K}_t^{\mathcal{J}}, \pi_{t-})$ to $V(\mathbf{K}_t^{\mathcal{J}}, \pi_t^{\mathcal{J}})$. These two effects reinforce each other over time leading to potentially significant losses.

The planner chooses consumption $\mathbf{C}$, investment $\mathbf{I}$, two types of adaptation spendings, $\mathbf{X}^d$ and $\mathbf{X}^e$, to maximize recursive utility given in (6)-(7) by setting the sum of the five terms on the right side of (16) to zero, implied by the optimality argument underpinning the HJB equation for recursive utility (see Duffie and Epstein, 1992). Because of the resource constraint, it is sufficient to focus on $\mathbf{I}$, $\mathbf{X}^d$ and $\mathbf{X}^e$ as control variables.

First-order conditions for investment and two types of adaptation spendings. The first-order condition (FOC) for investment $\mathbf{I}$ is

$$(1 + \Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K})) f_{\mathbf{C}}(\mathbf{C}, V) = V_{\mathbf{K}}(\mathbf{K}, \pi) \ . \tag{18}$$

The right side of (18), $V_{\mathbf{K}}(\mathbf{K}, \pi)$, is the marginal (utility) benefit of accumulating capital stock. The left side of (18) is the marginal cost of accumulating capital, which is given by the product of forgone marginal utility of consumption $f_{\mathbf{C}}(\mathbf{C}, V)$ and the marginal cost of accumulating capital, $(1 + \Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K}))$. Because of capital adjustment costs, increasing $\mathbf{K}$ by one unit requires incurring investment costs more than one unit, which explains the marginal adjustment cost $\Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K})$. Because of non-separability of preferences, $f_{\mathbf{C}}(\mathbf{C}, V)$ depends on not just consumption $\mathbf{C}$ but also the continuation utility V.

The FOC for the scaled aggregate tail risk reduction spending $\mathbf{x}^d$ is

$$f_{\mathbf{C}}(\mathbf{C}, V) = \frac{1}{\mathbf{K}} \lambda(\pi) \int_0^1 \left[\frac{\partial \xi(Z; \mathbf{x}^d)}{\partial \mathbf{x}^d} V\left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}}\right) \right] dZ \ , \tag{19}$$

if the solution is positive, $\mathbf{x}^d > 0$.[17] The planner chooses $\mathbf{x}^d$ to equate the marginal cost of adaptation, which is the forgone marginal (utility) benefit of consumption $f_{\mathbf{C}}(\mathbf{C}, V)$ given on the left side of (19), with the marginal benefit of adaptation given on the right side of (19).[18] By spending $\mathbf{x}^d$ per unit of capital to make the distribution of Z less damaging, the planner changes the pdf $\xi(Z; \mathbf{x}^d)$ for the fractional capital recovery, Z, from $\xi(Z; 0)$ to $\xi(Z; \mathbf{x}^d)$.

Similarly, the FOC for the scaled aggregate disaster exposure reduction spending $\mathbf{x}^e$ is

$$f_{\mathbf{C}}(\mathbf{C}, V) = -\lambda(\pi) N'(\mathbf{x}^e) \mathbb{E}^{\mathbf{x}^d} \left[(1 - Z) V_{\mathbf{K}}\left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}}\right) \right] \ , \tag{20}$$

[17] Otherwise, $\mathbf{x}^d = 0$ as adaptation in reality cannot be negative. When do we see $\mathbf{x}^d = 0$? One scenario is when the technology is very inefficient. In this case, the marginal benefit of spending on disaster distribution mitigation spending is less than one, causing the planner to set $\mathbf{x}^d = 0$.

[18] The second-order condition (SOC) $\lambda(\pi) \int_0^1 \left[\frac{\partial^2 \xi(Z; \mathbf{x}^d)}{\partial (\mathbf{x}^d)^2} V\left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}}\right) \right] dZ < 0$ is satisfied.

if the solution is strictly positive, $\mathbf{x}^e > 0$.[19] That is, the planner optimally chooses $\mathbf{x}^e$ to equate the marginal benefit of reducing the disaster exposure with the marginal cost of doing so. By spending $\mathbf{x}^e_{t-}$ per unit of capital, the planner reduces the post-jump fractional capital loss from $(1-Z)\mathbf{K}_{t-}$ to $\mathbf{K}_{t-} - \mathbf{K}^{\mathcal{J}}_t = N(\mathbf{x}^e_{t-})(1-Z)\mathbf{K}_{t-}$.

Using the homogeneity property to simplify the solution. Our model has the following homogeneity property. If we double capital stock $\mathbf{K}$, it is optimal for the planner to simultaneously double its quantity choices: the two types of adaptation spendings $\mathbf{X}^d$ and $\mathbf{X}^e$, investment $\mathbf{I}$, and consumption $\mathbf{C}$ at all time. As a result, the value function $V(\mathbf{K},\pi)$ is homogeneous with degree $(1-\gamma)$ in $\mathbf{K}$ and given by:

$$V(\mathbf{K},\pi) = \frac{1}{1-\gamma}\left(b(\pi)\mathbf{K}\right)^{1-\gamma}, \tag{21}$$

where $b(\pi)$ is a welfare measure proportional to certainty equivalent wealth under first best to be determined as part of the solution. Using the FOCs (18), (19), (20), substituting the value function $V(\mathbf{K},\pi)$ given in (21) into the HJB equation (16), and simplifying these equations, we obtain the following four-equation ODE system for $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$, and $\mathbf{x}^e(\pi)$:

$$\begin{aligned} 0 = {} & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{b(\pi)}{\rho(1+\phi'(\mathbf{i}(\pi)))}\right)^{1-\psi} - 1\right] + \mathbf{i}(\pi) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\ & + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi)}\left((1-N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}\right) - 1\right], \end{aligned} \tag{22}$$

$$b(\pi) = \left[A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^d(\pi) - \mathbf{x}^e(\pi)\right]^{1/(1-\psi)}\left[\rho(1+\phi'(\mathbf{i}(\pi)))\right]^{-\psi/(1-\psi)}, \tag{23}$$

$$\frac{1}{1+\phi'(\mathbf{i}(\pi))} = \lambda(\pi)\left[\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right]^{1-\gamma} N'(\mathbf{x}^e(\pi))\mathbb{E}^{\mathbf{x}^d(\pi)}\left[(Z-1)(1-N(\mathbf{x}^e(\pi))(1-Z))^{-\gamma}\right], \tag{24}$$

$$\frac{1}{1+\phi'(\mathbf{i}(\pi))} = \frac{\lambda(\pi)}{1-\gamma}\left[\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right]^{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;\mathbf{x}^d(\pi))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}\right]dZ\,. \tag{25}$$

The boundary conditions at $\pi = 0$ and $\pi = 1$ map to a generalized model of Pindyck and Wang (2013), which allows for the two types of adaptation spendings introduced in our model. Note that 1.) the $\pi = 0$ and $\pi = 1$ states are absorbing, in that the economy stays permanently in state G and B respectively, as there is no learning at either state and 2.) $\mathbf{i}$, $\mathbf{x}^d$, $\mathbf{x}^e$, $\mathbf{c}$, and welfare measure b are all constant at the $\pi = 0$ and $\pi = 1$ states. For brevity, we omit the boundary conditions implied by (22)-(25) at $\pi = 0$ and $\pi = 1$.

Next, we summarize our model's solution and provide a proof in Appendix A.1.

[19] Otherwise, $\mathbf{x}^e = 0$ since adaptation cannot be negative.

Proposition 1 *The first-best solution is given by the value function (21), where the welfare measure* $b_{fb}(\pi)$, $\mathbf{i}_{fb}(\pi)$, $\mathbf{x}^d_{fb}(\pi)$, *and* $\mathbf{x}^e_{fb}(\pi)$, *solve the four-equation ODE system (22)-(25).*

4 Competitive Markets Solution

While the planner's (first-best) public adaptation spending is strictly positive, no firms have incentives to reduce the aggregate risk distribution in a market economy. We show that the market solution is equivalent to the planner's solution for the case where only the disaster *exposure* reduction technology is available.

4.1 Firm Adaptation and Investment

A firm maximizes its value given by (5) taking the following SDF $\mathbb{M}_t$ as given:

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -r_{t-}dt - \gamma\sigma_K d\mathcal{W}_t^K + (\eta_t - 1)\left(d\mathcal{J}_t - \lambda(\pi_{t-})dt\right) . \tag{26}$$

The first term on the right side of (26) states the equilibrium restriction that the drift of $d\mathbb{M}_t/\mathbb{M}_{t-}$ equals $-r_{t-}dt$ (Duffie, 2001), where the equilibrium risk-free rate r_{t-} is a function of π_{t-}, $r_{t-} = r(\pi_{t-})$. The second term on the right side of (26) is the diffusion martingale and $\gamma\sigma_K$ is the equilibrium market price of diffusion risk as in Pindyck and Wang (2013), which we verify later. As $\lambda(\pi_{t-})dt = \mathbb{E}_{t-}(d\mathcal{J}_t)$, the last term in (26) is a jump martingale under the physical measure. This implies that when a jump arrives at t, the SDF changes discretely from $\mathbb{M}_{t-}$ to $\mathbb{M}_t^{\mathcal{J}}$ by a multiple of endogenously determined market price of jump risk η_t:

$$\frac{\mathbb{M}_t^{\mathcal{J}}}{\mathbb{M}_{t-}} = \eta_t , \tag{27}$$

which is a function of belief π_{t-} and the realized value of Z: $\eta_t = \eta(\pi_{t-}; Z)$.[20]

Applying the Ito's Lemma to firm value $Q(K_t, \pi_t) = q(\pi_t)K_t$ given in (5) and using (26), we obtain the following HJB equation for Tobin's q, $q(\pi)$, (see Appendix B.1):

$$\begin{aligned} r(\pi)q(\pi) = \max_{i, x^e, x^d} \quad & A - i - \phi(i) - x^e - x^d + (i - \delta_K)q(\pi) + \mu_\pi(\pi)q'(\pi) - \gamma\sigma_K^2 q(\pi) \\ & + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z)\left(q(\pi^{\mathcal{J}})(1 - N(x^e)(1 - Z)) - q(\pi)\right)\right] . \end{aligned} \tag{28}$$

The expectation operator in the last (jump) term (28) takes the aggregate disaster mitigation spending in the economy, $\mathbf{x}^d$, as given. Additionally, there are three optimality conditions.

[20]We provide equilibrium solutions for $r(\pi_{t-})$ and $\eta(\pi_{t-}; Z)$ in Section 5.3 and Subsections 4.3, respectively.

First, (28) implies that $x^d = 0$, as a firm is infinitesimal and hence reducing aggregate disaster risk brings no benefit but only cost to itself.[21] Second, unlike x^d, (28) implies a rather different FOC for the firm's exposure reduction spending x^e:

$$1 = -\lambda(\pi) q(\pi^{\mathcal{J}}) N'(x^e) \mathbb{E}^{\mathbf{x}^d}\left[(1-Z)\eta(\pi; Z)\right] . \tag{29}$$

By spending a dollar at the margin on exposure risk mitigation, the firm reduces the destruction of its capital stock by $-(1-Z)N'(x^e) > 0$ units should a jump arrive. Upon a jump arrival, the gross percentage change of SDF is $\mathbb{M}_t^{\mathcal{J}}/\mathbb{M}_{t-} = \eta(\pi_{t-}; Z)$ and the Tobin's q jumps from $q(\pi)$ to $q(\pi^{\mathcal{J}})$. To obtain the marginal benefit of spending on exposure mitigation X^e, we multiply the marginal reduction of capital stock destruction caused by a jump arrival, $-(1-Z)N'(x^e) > 0$, by $\lambda(\pi)q(\pi^{\mathcal{J}})\eta(\pi; Z)$, and then integrate over all possible values of Z. The resulting expected marginal value of mitigating the disaster exposure, given on the right side of (29), equals one, the marginal cost of mitigating the exposure on the left side of (29).

The FOC for investment implied by (28) is:

$$q(\pi) = 1 + \phi'(i(\pi)) , \tag{30}$$

which is the standard investment optimality condition that equates the marginal q to the marginal cost of investing $1 + \phi'(i(\pi))$. The homogeneity property implies that the average q equals the marginal q as in Hayashi (1982).

4.2 Household Optimization

We show that the household's value function, $J_t = J(W_t, \pi_t)$, is homogeneous with degree $1-\gamma$ in wealth W. That is, $J_t = J(W_t, \pi_t)$ takes the form of:

$$J(W, \pi) = \frac{1}{1-\gamma}(u(\pi)W)^{1-\gamma} , \tag{31}$$

where $u(\pi)$ is a welfare measure that will be endogenously determined.

First, no household spends on disaster exposure or disaster distribution mitigation spendings: $X^d = 0$ and $X^e = 0$, as no one has impact either the aggregate disaster distribution or the aggregate disaster exposure. Second, we solve for the household's optimal consumption C

[21] To be precise, since the firm's adaptation spending x^d has positive marginal cost but zero marginal benefit, the FOC cannot hold with equality and the corner solution $x^d = 0$ is optimal.

and allocation to the risky asset Γ using the following HJB equation:

$$0 = \max_{C,\Gamma} \quad f(C,J) + \mu_\pi(\pi)J_\pi + \lambda(\pi)\int_0^1 \left[J\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right) - J(W,\pi)\right]\xi(Z;\mathbf{x}^d)dZ$$
$$+ \left[r(\pi)W + (\mu_{\mathbf{Q}}(\pi) - r(\pi))\Gamma - C\right]J_W + \frac{\sigma_K^2\Gamma^2 J_{WW}}{2}, \tag{32}$$

where $\mu_{\mathbf{Q}}(\pi)$ is defined in (15), $\pi^{\mathcal{J}}$ is the post-jump belief given in (12), and $W^{\mathcal{J}}$ is the post-jump wealth given by

$$W_t^{\mathcal{J}} = W_{t-} + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\Gamma_{t-}. \tag{33}$$

The aggregate stock market valuation $\mathbf{Q}_t$ is proportional to the aggregate capital stock $\mathbf{K}$: $\mathbf{Q}_t = \mathbf{q}(\pi_t)\mathbf{K}_t$ where $\mathbf{q}(\pi_t)$ is the Tobin's q for $\mathbf{K}$ in equilibrium. When a jump arrives,

$$\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} = \frac{\mathbf{q}(\pi_t^{\mathcal{J}})\mathbf{K}_t^{\mathcal{J}}}{\mathbf{q}(\pi_{t-})\mathbf{K}_{t-}} = \frac{\mathbf{q}(\pi_t^{\mathcal{J}})}{\mathbf{q}(\pi_{t-})}(1 - N(\mathbf{x}_{t-}^e)(1-Z)). \tag{34}$$

Equation (34) states that aggregate stock market value changes from $\mathbf{Q}_{t-} = \mathbf{q}(\pi_{t-})\mathbf{K}_{t-}$ to $\mathbf{Q}_t^{\mathcal{J}} = \mathbf{q}(\pi_t^{\mathcal{J}})\mathbf{K}_t^{\mathcal{J}}$ as a jump arrives for two reasons: 1.) capital stock decreases from $\mathbf{K}_{t-}$ to $\mathbf{K}_t^{\mathcal{J}} = [1 - N(\mathbf{x}_{t-}^e)(1-Z)]\mathbf{K}_{t-}$ by a fraction of $N(\mathbf{x}_{t-}^e)(1-Z)$ and 2.) the aggregate Tobin's q changes from $\mathbf{q}(\pi_{t-})$ to $\mathbf{q}(\pi_t^{\mathcal{J}})$, where $\pi_t^{\mathcal{J}} = \pi_{t-}\lambda_B/\lambda(\pi_{t-})$ is given in (12). For brevity, we drop the time subscripts when it does not cause confusion. That is, we write $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q} = \mathbf{Q}_t^{\mathcal{J}}/\mathbf{Q}_{t-}$.

Substituting (31) into the consumption FOC $f_C(C,J) = J_W(W,\pi)$ and simplifying the expression, we obtain the following consumption rule:

$$C(W,\pi) = \rho^\psi u(\pi)^{1-\psi}W. \tag{35}$$

Consumption is linear in wealth with a π-dependent marginal propensity to consume. Simplifying the household's FOC for the market portfolio allocation Γ, we obtain:

$$\Gamma = -\frac{\mu_{\mathbf{Q}}(\pi) - r(\pi)}{\sigma_K^2}\frac{J_W(W,\pi)}{J_{WW}(W,\pi)} + \frac{\lambda(\pi)}{\sigma_K^2}\mathbb{E}^{\mathbf{x}^d}\left[\left(1 - \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right)\frac{J_W\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right)}{J_{WW}(W,\pi)}\right]. \tag{36}$$

The first term in (36) is the standard Merton's mean-variance demand (absent jumps) and the second term in (36) captures the intertemporal hedging demand as a jump arrival causes both the household's belief π and wealth W as well as the stock market $\mathbf{Q}$ to jump discretely.

4.3 Market Equilibrium

In equilibrium, the household invests all wealth in the stock market, $W_t = \Gamma_t = \mathbf{Q}_t$. We can show that the ratio of the pre-jump and the post-jump SDF $\mathbb{M}_t$ in equilibrium, η_t, is given by

$$\eta_t = \frac{\mathbb{M}_t^{\mathcal{J}}}{\mathbb{M}_{t-}} = \frac{J_W(\mathbf{Q}_t^{\mathcal{J}},\pi_t^{\mathcal{J}})}{J_W(\mathbf{Q}_{t-},\pi_{t-})}. \tag{37}$$

The second equality in (37) states that η_t equals the ratio of the household's post-jump marginal value of wealth $J_W(\mathbf{Q}_t^{\mathcal{J}}, \pi_t^{\mathcal{J}})$ and the pre-jump marginal value of wealth $J_W(\mathbf{Q}_{t-}, \pi_{t-})$. This is because in equilibrium both the household's pre-jump and post-jump wealth are in the stock market: $W_{t-} = \mathbf{Q}_{t-}$ and $W_t^{\mathcal{J}} = \mathbf{Q}_t^{\mathcal{J}}$. Using the homogeneity property, we write η_t as:

$$\eta_t = \eta(\pi_t; Z, \mathbf{x}_{t-}^e) = \left(\frac{u(\pi_t^{\mathcal{J}})}{u(\pi_{t-})}\right)^{1-\gamma} \left(\frac{\mathbf{q}(\pi_t^{\mathcal{J}})}{\mathbf{q}(\pi_{t-})}(1 - N(\mathbf{x}_{t-}^e)(1 - Z))\right)^{-\gamma} . \tag{38}$$

We can further simplify the household's HJB equation (32) as:

$$0 = \frac{\psi^{-1}\rho^{\psi}u(\pi_{t-})^{1-\psi} - \rho}{1 - \psi^{-1}} + \mu_{\mathbf{Q}}(\pi_{t-}) + \mu_{\pi}(\pi_{t-})\frac{u'(\pi_{t-})}{u(\pi_{t-})} - \frac{\gamma\sigma_K^2}{2} + \frac{\lambda(\pi_{t-})}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d}\left(\eta_t \frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right) - 1\right], \tag{39}$$

where η_t is given in (38) and $\mu_{\mathbf{Q}}(\pi_{t-})$ defined in (15) is given by[22]

$$\begin{aligned} \mu_{\mathbf{Q}}(\pi_{t-}) &= r(\pi_{t-}) + \gamma\sigma_K^2 + \lambda(\pi_{t-})\mathbb{E}_{t-}^{\mathbf{x}^d}\left[\eta_t\left(1 - \frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right)\right] & (40) \\ &= \frac{\mathbf{c}(\pi_{t-})}{\mathbf{q}(\pi_{t-})} + \mathbf{i}(\pi_{t-}) - \delta_K + \mu_{\pi}(\pi_{t-})\frac{\mathbf{q}'(\pi_{t-})}{\mathbf{q}(\pi_{t-})} . & (41) \end{aligned}$$

In equilibrium, the household invests all wealth in the stock market, $W_t = \Gamma_t = \mathbf{Q}_t$. Additionally, both the aggregate disaster exposure and distribution adaptation spendings in a laissez-faire economy equal zero: $\mathbf{X}^e = \mathbf{X}^d = 0$.[23] In sum, the solution is given by 1.) the ODE (39) for $u(\pi)$ and the FOCs (35)-(36) for households and 2.) the ODE (28) for $q(\pi)$ and the FOCs (29)-(30) for firms. We can also show that this solution of our market model is the same as that of a planner's problem, where the planner has no access to the adaptation technology that curtails tail risk ($\mathbf{x}^d(\pi) = 0$). This planner's problem is easier to solve. Rather than solving for $u(\pi)$ and $q(\pi)$ in our market economy, it is equivalent to solve for $b(\pi)$ and optimal policies in the planner's economy. Next, we summarize this equivalence result.

Proposition 2 *The market solution is the same as the planner's solution where there is no adaptation technology to change the distribution of the recovery fraction Z ($\mathbf{x}^d(\pi) = 0$).*

See Appendix B.3 for proof. Note that this proposition states that the Welfare Theorem applies when there is no such adaptation technology.

[22] We use the FOC given in (36) and the equilibrium condition $\Gamma_t = W_t$ to obtain (40). Substituting the resource constraint $c(\pi) = A - i(\pi) - \phi(i(\pi)) - \mathbf{x}^e(\pi)$ into the ODE (28) for $q(\pi)$, we obtain (41).

[23] Since households contribute nothing to disaster exposure and distribution mitigation spendings, using the law of large numbers, the aggregate exposure and distribution mitigation spendings are also zero.

5 Optimal Taxation in Market Economy and Asset Prices

In this section, we show that introducing optimal capital taxation into our competitive market economy of Section 2 changes the market-economy solution given in Section 4 to the one implied by the planner's first-best solution given in Section 3. We then derive the asset prices that would hold under a given economy type.

5.1 Firm and Household Optimization under Capital Taxation

The government taxes the firm's capital stock K_t at a rate of $\tau_t = \mathbf{x}^d_{fb,t}$, where $\mathbf{x}^d_{fb,t}$ is the first-best mitigation spending to change the distribution of Z, obtained in Section 3. Then, the government spends $\mathbf{X}^d_t = \tau_t \mathbf{K}_t$ to reduce the tail risk of the disaster distribution.[24] We write the tax rate τ_t as a function of π_t: $\tau_t = \tau(\pi_t) = \mathbf{x}^d_{fb,t} = \mathbf{x}^d_{fb}(\pi_t)$.

Facing a capital tax rate of $\tau(\pi_t)$ and taking the equilibrium SDF $\mathbb{M}_t$ as given, each firm chooses $\{I,\, X^e,\, X^d\}$ to maximize its value given in (5), where its payout, Y_t, is given by[25]

$$Y_t = \left(A - \tau(\pi_t)\right) K_t - I_t - \Phi_t - X^e_t \,. \tag{42}$$

In effect, taxes lower productivity from A to $A - \tau(\pi_t)$. Applying Ito's Lemma to firm value given in (5) and using (26), we obtain the following HJB equation for Tobin's average q, $q(\pi_t)$:

$$\begin{aligned} r(\pi)q(\pi) = \max_{i,\, x^e} \;\; & A - \tau(\pi) - i - \phi(i) - x^e + (i(\pi) - \delta_K)q(\pi) + \mu_\pi(\pi)q'(\pi) - \gamma\sigma_K^2 q(\pi) \\ & + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z, \mathbf{x}^e)\left(q(\pi^{\mathcal{J}})(1 - N(x^e)(1-Z)) - q(\pi)\right)\right] \,. \end{aligned} \tag{43}$$

Note that the tax rate $\tau(\pi)$ appears in (43). The FOCs for i and x^e are given in (29) and (30), respectively, the same as in the no-tax model of Section 4.[26] Next, we prove that incorporating optimal taxes into the competitive-market economy yields the first-best solution.

5.2 Optimal Capital Taxation Restores First-Best

In this section, we show that the household's value function in the competitive economy with optimal taxes is the same as the value function under the first-best. As the household's value function in a market economy depends on wealth W while the planner's value function depends on $\mathbf{K}$, we use the equilibrium result $W_t = \mathbf{q}(\pi_t)\mathbf{K}_t$ in the market economy with taxation to

[24] Equivalently the government can impose via a tax on sales AK_t at the firm level.

[25] The firm does not spend on disaster distribution mitigation ($X^d = 0$), as there is no benefit.

[26] For brevity, we refer readers to Section 4 for the household's problem, as it is in effect the same as in the previous section.

write the household's value function as $J(W_t, \pi_t) = J(\mathbf{q}(\pi_t)\mathbf{K}_t, \pi_t)$. The value functions in the two economies are equal, $V(\mathbf{K}_t, \pi_t) = J(W_t, \pi_t)$, if and only if $b(\pi)$ in the first-best economy equals the product $u(\pi)\mathbf{q}(\pi)$ in the competitive economy with taxes.

Specifically, we show the following results: (1.) the first-order conditions for $\mathbf{i}(\pi)$ and $\mathbf{x}^e(\pi)$ in the competitive economy with an optimal tax rate set at the $\mathbf{x}^d_{fb}$ are the same as those in the planner's economy; (2.) the implied ODE for $u(\pi)\mathbf{q}(\pi)$ in the competitive market economy is the same as the ODE (22) for $b(\pi)$ in the planner's economy; (3.) all the boundary conditions at $\pi = 0$ and $\pi = 1$ in the two economies are the same. Below is a proof.

First, combining the equilibrium aggregate investment FOC, $\mathbf{q}(\pi) = 1+\phi'(\mathbf{i}(\pi))$, implied by (30) with the optimal scaled consumption rule $\mathbf{c}(\pi) = \rho^{\psi}u(\pi)^{1-\psi}\mathbf{q}(\pi) = (\rho\mathbf{q}(\pi))^{\psi}\left[u(\pi)\mathbf{q}(\pi)\right]^{1-\psi}$, implied by (35) and $W = \mathbf{q}(\pi)\mathbf{K}$, we obtain the following expression for consumption:

$$\mathbf{c}(\pi) = \left[\rho(1+\phi'(\mathbf{i}(\pi)))\right]^{\psi}\left[u(\pi)\mathbf{q}(\pi)\right]^{1-\psi} . \tag{44}$$

Using the goods-market clearing condition $\mathbf{c}(\pi) = A - \tau(\pi) - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e(\pi)$ and $b(\pi) = u(\pi)\mathbf{q}(\pi)$, we obtain the following expression:

$$b(\pi) = \left[A - \tau(\pi) - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e(\pi)\right]^{1/(1-\psi)}\left[\rho(1+\phi'(\mathbf{i}(\pi)))\right]^{-\psi/(1-\psi)} , \tag{45}$$

which is the same as the investment FOC, given in (23), for the planner's problem, provided that the capital tax rate equals $\mathbf{x}^d_{fb}(\pi)$: $\tau(\pi) = \mathbf{x}^d_{fb}(\pi)$. Note that (45) summarizes both the consumer's and the firm's optimization FOCs in the market economy with optimal taxes.

Second, substituting (38) for η into the FOC (29) for disaster exposure mitigation x^e in the competitive market economy, we obtain

$$1 = -\lambda(\pi)q(\pi^{\mathcal{J}})N'(x^e)\mathbb{E}^{\mathbf{x}^d}\left[(1-Z)\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\left(\frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}(1-N(\mathbf{x}^e)(1-Z))\right)^{-\gamma}\right] . \tag{46}$$

Using the investment FOC $q(\pi) = 1+\phi'(i(\pi))$, the equilibrium conditions ($q(\pi) = \mathbf{q}(\pi)$ and $i(\pi) = \mathbf{i}(\pi)$), and the $b(\pi) = u(\pi)\mathbf{q}(\pi)$ result for the two economies, we obtain

$$1 = -N'(\mathbf{x}^e(\pi))\lambda(\pi)(1+\phi'(\mathbf{i}(\pi)))\left[\frac{b(\pi^{\mathcal{J}})}{b(\pi)}\right]^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi)}\left[(1-Z)(1-N(\mathbf{x}^e(\pi))(1-Z))^{-\gamma}\right] , \tag{47}$$

which is the same as the planner's FOC (24) for $\mathbf{x}^e$. So far, we have verified that the FOCs for investment and exposure mitigation spending in the two economies are the same.

Third, substituting (41) into (39) and using the consumption rule $c(\pi) = \rho^\psi u(\pi)^{1-\psi} q(\pi)$ implied by the FOC (35), we may rewrite the ODE (39) for the household's $u(\pi)$ as

$$0 = \frac{\rho^\psi u(\pi)^{1-\psi} - \rho}{1-\psi^{-1}} + i(\pi) - \delta_K + \mu_\pi(\pi)\left(\frac{u'(\pi)}{u(\pi)} + \frac{q'(\pi)}{q(\pi)}\right) - \frac{\gamma\sigma_K^2}{2}$$
$$+ \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u\left(\pi^{\mathcal{J}}\right)\mathbf{q}\left(\pi^{\mathcal{J}}\right)}{u(\pi)\mathbf{q}(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}\left((1-N(\mathbf{x}^e)(1-Z))^{1-\gamma}\right) - 1\right]. \tag{48}$$

We obtain (48) by using $\eta(\pi; Z, \mathbf{x}^e)$ given in (38) and $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q}$ given in (34).

Fourth, using the conjecture $b(\pi) = u(\pi)\mathbf{q}(\pi) = u(\pi)(1 + \phi'(\mathbf{i}(\pi)))$, we may simplify the ODE (48) and obtain the following ODE for $b(\pi) = u(\pi)\mathbf{q}(\pi)$:

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(\pi)}{\rho(1+\phi'(\mathbf{i}(\pi)))}\right]^{1-\psi} - 1\right] + \mathbf{i}(\pi) - \delta_K + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} - \frac{\gamma\sigma_K^2}{2}$$
$$+ \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}\left((1-N(\mathbf{x}^e)(1-Z))^{1-\gamma}\right) - 1\right], \tag{49}$$

which is the same as the ODE (22) for $b(\pi)$ in the first-best economy.[27] In sum, we have verified that setting the capital tax at $\tau(\pi) = \mathbf{x}^d_{fb}(\pi)$ in the market economy yields the same allocation as in the first-best economy. Next we summarize this result.

Proposition 3 *Setting the capital tax rate $\tau(\pi_t)$ to $\mathbf{x}^d_{fb}(\pi_t)$ for all firms and then spending all tax proceeds each period to mitigate the tail risk of the disaster distribution: $\tau(\pi_t) = \mathbf{x}^d_{fb}(\pi_t)$, the competitive-market economy attains the first-best resource allocation.*

It is worth noting that the homogeneity property of our model allows us to simplify our analysis by writing the optimal tax rate on capital as a function that only depends on belief, independent of a firm's capital stock: $\tau(\pi) = \mathbf{x}^d_{fb}(\pi)$. However, the market decentralization argument that allows the economy to attain the first-best (after the planner imposes the optimal capital tax to fund the aggregate public adaptation spending) does not depend on the homogeneity property. We provide our intuition in two steps.

First, once the planner taxes all firms possibly using nonlinear tax rates on their capital stocks and uses these tax proceeds to fund the first-best aggregate risk mitigation spending $\mathbf{X}^d_t$, the planner has fixed the market failure. Second, after the externality is addressed and the aggregate climate risk is properly mitigated, it is then optimal for both the representative

[27] Also applying the same arguments to the boundaries at $\pi = 0$ and $\pi = 1$, we can show that the two economies have the same FOCs at the boundaries and, moreover, $b(0) = u(0)\mathbf{q}(0)$ and $b(1) = u(1)\mathbf{q}(1)$.

consumer and producer to choose the first-best consumption and capital investment decisions, respectively.

5.3 Asset Prices

Next, we report and discuss the equilibrium asset pricing implications.

Proposition 4 *Tobin's average q for the aggregate capital stock is* $\mathbf{q}(\pi) = 1 + \phi'(\mathbf{i}(\pi))$, *where* $\mathbf{i}(\pi)$ *is the optimal investment-capital ratio. The equilibrium risk-free rate,* $r(\pi)$, *is given by*

$$\begin{aligned} r(\pi) \ &= \rho + \psi^{-1}(\mathbf{i}(\pi) - \delta_K) - \frac{\gamma(\psi^{-1}+1)\sigma_K^2}{2} - \left[(1-\psi^{-1})\left(\frac{u'(\pi)}{u(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right) - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right]\mu_\pi(\pi) \\ &- \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - 1\right] - \lambda(\pi)\frac{\psi^{-1}-\gamma}{1-\gamma}\left[1 - \mathbb{E}^{\mathbf{x}^d}\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\eta(\pi; Z, \mathbf{x}^e)\right)\right], \end{aligned} \tag{50}$$

where $\eta(\pi; Z, \mathbf{x}^e)$ *is given in (38) and* $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q}$ *is the jump-triggered (gross) percentage change of the stock market value given in (34). The stock market risk premium,* $rp(\pi)$, *is*

$$rp(\pi) = \gamma\sigma_K^2 - \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[(\eta(\pi; Z, \mathbf{x}^e) - 1)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right)\right]. \tag{51}$$

These results apply to both the market economy with taxation and the one without.

Out of the six terms in (50), the first three terms are the contributing factors to the equilibrium interest rate in AK models with diffusion shocks. The fourth term captures the effect of belief updating. The fifth term describes how the jump-induced expected change of the marginal value of wealth ($\mathbb{M}^{\mathcal{J}}/\mathbb{M}$) contributes to the risk-free rate. The sixth term captures the additional effect of jumps on the equilibrium risk-free rate due to the household's recursive (non-separable) Epstein-Zin preferences rather than expected utility.[28]

There are two terms for the market risk premium rp given in (51). In addition to the diffusion risk premium (the first term), there is a jump risk premium (the second term), which equals the expectation over the product of the (net) percentage change of marginal value of wealth ($\mathbb{M}$), $(\eta(\pi; Z, \mathbf{x}^e) - 1)$, and the (net) percentage change of the stock market value given in (34), both of which are caused by jump arrivals. A downward jump causes the household's marginal utility to increase ($\eta(\pi; Z, \mathbf{x}^e) = \mathbb{M}^{\mathcal{J}}/\mathbb{M} \geq 1$). As the stock market valuation decreases upon a jump arrival, ($\mathbf{Q}^{\mathcal{J}} < \mathbf{Q}$), the jump risk premium is positive.

[28] To be precise, for recursive utility, $f_{CV} \neq 0$ and therefore the SDF $\mathbb{M}_t$ is not additively separable, which makes jumps to have an additional intertemporal effect. For expected utility ($\gamma = \psi^{-1}$), this term disappears.

6 Application to Tropical Cyclones

We apply our model of learning and adaptation for weather disasters to tropical cyclones and leverage our asset pricing results to highlight the learning channel. Our largest sample contains annual observations for the real GDP per capita growth rate and cyclone landfalls across 109 countries from 1960 to 2010 with 5,410 county-year observations in total.[29]

6.1 Frequencies of Landfalls and Spendings on Flood Control

Let $\text{Landfall}_{i,t}$ be an indicator variable that equals one if and only if country i experienced at least one cyclone landfall that is "tropical storm" or higher in year t. Table 1 reports the sample statistics of cyclone landfalls for each of the four regions.[30] Globally, a country on average experiences a tropical cyclone landfall once every 7.4 years, as the disaster arrival rate is 0.135 per annum (in Table 1.)

Table 1: Summary statistics of cyclone landfalls

Region	(1) Total # of country-year obs.	(2) Total # of cyclone landfall obs.	(3) Freq. of landfalls = (2)/(1): Disaster arrival rate estimate
North Atlantic	1,587	229	0.144
West Pacific	638	326	0.511
North India	719	75	0.104
South Atlantic	2,466	99	0.040
Global	5,410	729	0.135

The primary adaptation for countries in our sample is government flood control budgets. Unlike the landfall data, such data is not readily available. We hand collected data on government flood control budgets based on public sources by focusing on countries in the West Pacific (including Oceania), which according to Table 1 faces the most frequent tropical cyclone landfalls. We are able to obtain through various sources 72 country-year observations of government flood control budgets for a cross section of eight countries.[31] For this cross section, the average annual government flood control budget is around 0.1% (10 basis points) of the country's capital stock with a standard deviation of 0.05% across country-years observations.

[29] These are the same set of countries as in Hsiang and Jina (2014) excluding Taiwan for which there is no GDP data from the World Bank Development Indicator.

[30] We assign the 109 countries into four regions: North Atlantic (including North America, the Caribbean, and West Europe) West Pacific (including Oceania), North India (including North India, Middle East, North Africa, and Central Europe), and South Atlantic (including Latin America and Sub-Saharan Africa).

[31] West Pacific countries include China, Japan, Korea, and the Philippines. Oceania countries include Australia, Indonesia, New Zealand and Papua New Guinea.

There are also private spendings as well on flood control according to field studies, which typically place these private spendings somewhat around 0.03% – 0.05% of capital stock, below the 0.1% of capital stock for public spendings (Lasage et.al., 2014).

To provide some perspectives on these small expenditures on flood control, over this sample period, the output-to-capital ratio is about 30% (with a standard deviation of 17%). The investment-capital ratio is 7% (with a standard deviation of 4%) and the consumption-capital ratio is 22% (with a standard deviation of 13%). The small expenditures on adaptation presumably reflect a belief that the consequences of global warming are relatively mild but they may significantly increase should the frequencies of arrivals increase and the society quickly updates beliefs towards the most pessimistic model projections.

6.2 Damage from Landfalls and Asset Market Reactions

Importantly, we retrieve two key panel regression estimates on the response of growth and asset prices to the arrival of cyclones that highlight the role of learning in financial markets. How policies (e.g., investment and consumption) and asset prices respond to a cyclone arrival depend on beliefs π which change over time (Propositions 1 and 4). A landfall is bad news leading to more pessimistic beliefs for future growth. Asset prices also fall in anticipation of more frequent disasters in the future. In Pindyck and Wang (2013), which is a special case of our no-learning model, disasters lead to a destruction in capital stock $\mathbf{K}$ but the growth rate is identically and independently distributed at all time. That is, even after a disaster arrival destroys a fraction of the country's capital stock, there is no impact at all on either growth projection or asset prices (e.g., Tobin's q, the risk-free rate, and the risk premium) going forward in Pindyck and Wang (2013). This is because there is no learning in their model.

Table 2: Baseline model estimation results

	Dependent variable: Growth rate of real GDP per capita				
	(1) North Atlantic	(2) West Pacific	(3) North India	(4) South Atlantic	(5) Global
Landfall	-0.0061*	-0.0029*	-0.0088***	-0.0275***	-0.0077***
	(-2.01)	(-1.94)	(-3.35)	(-3.69)	(-4.29)
Country FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Time trends	Yes	Yes	Yes	Yes	Yes

We now show that landfalls damage growth and asset prices respond adversely to news of cyclone arrivals, consistent with our model in which learning plays a key role. Table 2 reports the estimates of the impact of a major cyclone making landfall on growth for each region and also the world. The dependent variable is the per capita growth rate. The independent variable is the Landfall indicator. The panel regression has country fixed effects, year fixed effects, and country-specific quadratic time trends. A landfall disaster reduces the expected annual growth rate by 0.61%, 0.29%, 0.88%, and 2.75% in North Atlantic, West Pacific, North India, and South Atlantic respectively, and by 0.77% in the global sample. Since the average annual growth rate in our sample is 1.95% (with a standard deviation of 5.09%), a landfall, which lowers the annual growth rate by 0.77% on average, is quite economically damaging.[32]

Since the data availability for financial variables is quite limited before 1990, and to be consistent with our samples using real GDP growth data, the sample period of macro-financial variables for the cyclone landfall analysis is from 1990 to 2010. Even then, we only have a subset of countries that have the relevant financial variables. Panel A of Table 3 reports the unconditional moments for asset prices pooling all these remaining countries. These moments including a risk-free rate of 1.43% and an equity risk premium of 5.26%, a volatility of equity market returns of 26.57%,, and a Tobin's average q of 2.49.

Table 3: Summary statistics of asset prices

Panel A provides the summary statistics of the financial variables used in our study. RealRF is real interest rate (nominal interest rate minus inflation rate). ERP is equity risk premium (stock market return net of nominal interest rate). TobinQ is Tobin's average q. VolRET is volatility of annual stock market return. Annual risk-free nominal interest rate, inflation rate, and stock market return data at the country level are from the IMF and the World Bank. Panel B reports regression of these asset-pricing moments on cyclone landfalls. Estimates for RealRF and ERP are in percentages. t-statistics with clustered robust standard errors are shown in parentheses below the estimates. The period for the cyclone sample is 1990-2010.

Panel A: Summary Statistics

	Mean	Standard deviation	Median	10 percentile	90 percentile
RealRF (%)	1.43	4.32	1.32	-4.40	6.91
ERP (%)	5.26	24.26	5.61	-27.47	37.36
TobinQ	2.49	4.84	1.51	0.60	3.65
VolRET (%)	26.57	8.26	26.38	15.24	37.52

Panel B: Asset Market Reaction to Landfalls

	RealRF	ERP	TobinQ
Landfall	-0.090**	0.307**	-0.101**
	(-2.34)	(2.48)	(-2.11)

[32]Our estimates are consistent with those reported in Hsiang and Jina (2014), who estimate the marginal effect of windspeed on GDP growth damage.

As before with real GDP growth in Table 2, we use a Panel regression model in Panel B of Table 3 to measure the impact of a cyclone landfall on a country's real interest rate (RealRF), equity risk premium (ERP), or Tobin's average q (TobinQ) by using country and time fixed effects. The panel regression model regresses financial variables on an indicator for cyclone landfall (Landfall) for the whole sample. A cyclone landfall on average reduces Tobin's average q by 0.10, lower the real interest rate by 0.09%, and increases equity risk premium by 0.31% per annum. These estimates are inconsistent with models of disasters absent learning, e.g., Pindyck and Wang (2013), as we discussed earlier.

7 Quantitative Analysis

In this section, we first calibrate our model and then conduct a quantitative analysis.

7.1 Distributional and Functional Form Specifications

As in Barro (2006) and Pindyck and Wang (2013), we assume that the distribution function of the recovery fraction Z upon a cyclone arrival is given by a power law over $Z \in (0, 1)$:

$$\Xi(Z; \mathbf{x}^d) = Z^{\beta(\mathbf{x}^d)} \,, \tag{52}$$

where $\beta(\mathbf{x}^d)$ is the exponent function that depends on scaled disaster distribution mitigation $\mathbf{x}^d$. To ensure that our model is well defined, we require $\beta(\mathbf{x}^d) > \gamma - 1$.

Conditional on a jump arrival, the expected fractional capital loss for a firm is given by

$$\ell(\pi) = N(x^e)(1 - \mathbb{E}^{\mathbf{x}^d}(Z)) = \frac{N(x^e)}{\beta(\mathbf{x}^d) + 1} \,. \tag{53}$$

The larger the value of $\beta(\,\cdot\,)$, the smaller the expected fractional loss $\mathbb{E}^{\mathbf{x}^d}(1-Z)$ even absent the firm's disaster exposure mitigation x^e. To capture the benefit of public mitigation, we assume that $\beta(\mathbf{x}^d)$ is increasing in $\mathbf{x}^d$: $\beta'(\mathbf{x}^d) > 0$. The benefit of public disaster distribution mitigation $\mathbf{x}^d$ is to increase the capital stock recovery (upon the arrival of a disaster) in the sense of first-order stochastic dominance in that $\Xi(Z; \mathbf{x}^d)$ decreases with $\mathbf{x}^d$.

Let $g_t = g(\pi_t)$ denote a firm's expected growth rate including the jump effect. The homogeneity property implies that growth is independent of the aggregate capital $\mathbf{K}$ and

$$g(\pi) = i(\pi) - \delta_K - \lambda(\pi)\ell(\pi) = i(\pi) - \delta_K - \frac{\lambda(\pi)N(x^e)}{\beta(\mathbf{x}^d) + 1} \,. \tag{54}$$

We specify the firm's exposure mitigation technology $N(x^e)$ as follows:

$$N(x^e) = 1 - (x^e)^{\zeta} \,, \tag{55}$$

where $0 < \zeta < 1$. That is, the more exposure mitigation spending x^e the smaller the (fractional) damage, i.e., the lower the level of $N(x^e)$. Additionally, the marginal benefit of x^e on reducing damages diminishes. We use the following linear specification for $\beta(\mathbf{x}^d)$ which governs the public disaster distribution mitigation technology:

$$\beta(\mathbf{x}^d) = \beta_0 + \beta_x \mathbf{x}^d \,, \tag{56}$$

with $\beta_0 \geq \max\{\gamma - 1, 0\}$ and $\beta_x > 0$. The coefficient β_0 is the exponent for the distribution function of the fractional recovery Z in the absence of mitigation. The coefficient β_x is a key parameter and measures the efficiency of the aggregate disaster distribution mitigation technology. Finally, we use a quadratic adjustment cost function (e.g., Hayashi, 1982):

$$\phi(i) = \frac{\theta i^2}{2} \,, \tag{57}$$

where the parameter θ measures how costly it is to adjust capital.

7.2 Calibration and Parameter Choices

Table 4: Parameter Values

Parameters	Symbol	Value
disaster jump arrival rate in State G	λ_G	0.1
disaster (jump) arrival rate in State B	λ_B	0.8
prior of being in State B	π_0	0.08
power law exponent absent adaptation	β_0	39
distribution adaptation technology parameter	β_x	$1,800$
exposure adaptation technology parameter	ζ	0.4
elasticity of intertemporal substitution	ψ	1.5
time rate of preference	ρ	5%
productivity parameter	A	27%
quadratic adjustment cost parameter	θ	17
coefficient of relative risk aversion	γ	8
capital diffusion volatility	σ_K	8%
depreciation rate of capital	δ_K	6%

All parameter values, whenever applicable, are continuously compounded and annualized.

Our model has 13 parameters. We calibrate these parameters by targeting 13 moments described in Section 6. The calibrated values of these parameters are given in Table 4.

The new parameters in our analyses are the three for the learning process (λ_G, λ_B, and π_0) and the other three for the adaptation technologies (β_0, β_x and ζ). In order to determine these six parameters, we use six moments from our panel data on the frequencies of tropical cyclone landfalls, their impact on GDP growth and asset prices (risk-free rate, equity risk premium, and Tobin's average q), and the levels of private and public adaptation spendings that we obtained and reported in Section 6, i.e., around 0.1% and 0.04% of capital stock, respectively. To rationalize the empirical findings, we need quite a large spread in λ_G and λ_B, consistent with the considerable uncertainty in climate science projections (Knutson et al., 2020).

A number of the macro-finance moments we are targeting, such as the risk-free rate rate and equity risk premium (Panel A of Table 3), are similar to those targeted in the asset pricing literature. Hence, our preference parameters, e.g., the EIS ψ and coefficient of relative risk aversion γ, are similar to those used in this literature. For instance, Bansal and Yaron (2004) show that setting the coefficient of relative risk aversion γ to a value between 7 to 10 and an EIS ψ to be larger than one is necessary to match the equity risk premium and the risk-free rate. Similarly, the parameters for the production part of our model, e.g., productivity, capital adjustment costs, and the capital depreciation rate, are chosen to match the aggregate output and production targets discussed in Section 6.1. The calibrated values turn out to be close to those in the literature (e.g., Eberly, Rebelo, and Vincent, 2012), suggesting that our calibration strategy yields sensibly robust parameter values for our quantitative analysis.

Next, we use these parameters to analyze a few economies. In Figure 2, we plot and compare the solutions for three economies: 1.) the planner's first-best solution (solid blue lines), 2.) the market economy (dashed red lines) and 3.) the planner's solution with no learning (dotted black lines). In the next two subsections, we do two pair-wise comparisons.

7.3 Comparing First-Best with Competitive-Market Solutions

In this subsection, we compare the first-best with competitive-market solutions. A key feature that both economies share is learning. In Panel A of Figure 2, we see that public mitigation $\mathbf{x}^d$ (solid blue line) rapidly increases with the disaster arrival rate λ in the first-best economy. In contrast, the market solution features no public mitigation spending (dashed red line) regardless of beliefs due to externalities. Panel B shows that private mitigation $\mathbf{x}^e$ in both economies increases with λ. Since there is no $\mathbf{x}^d$ in the competitive economy, private adaptation has to take up the slack. But the total adaptation spendings given by the sum, $\mathbf{x}^e + \mathbf{x}^d$, are lower in the market economy than in the first-best economy, meaning that the combined risk mitigation is still under-provided in the laissez-faire market economy.

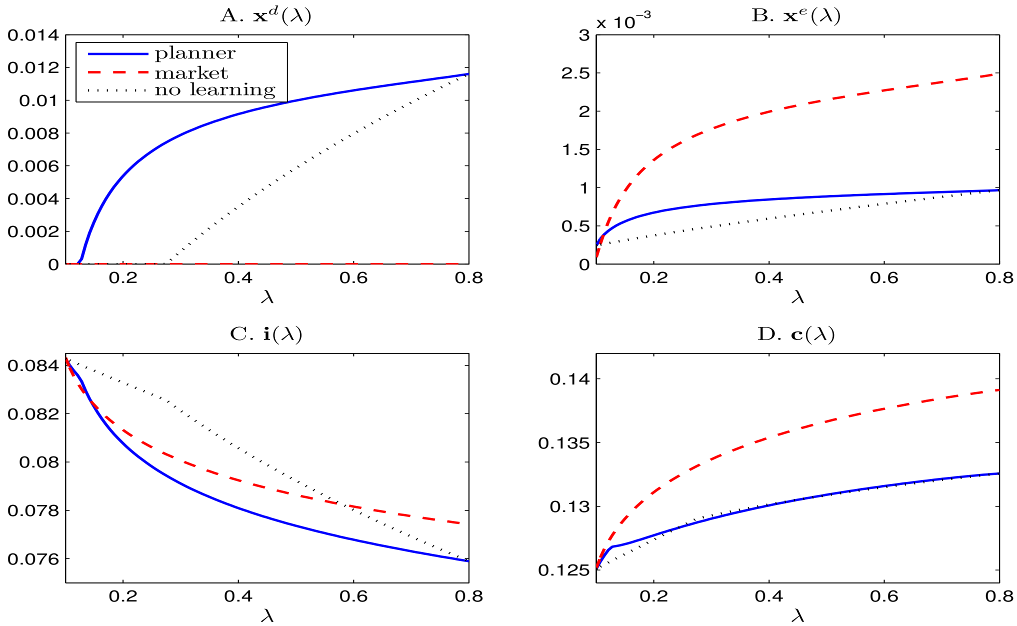

Figure 2: This figure compare the solutions for three economies: 1.) the planner's first-best solution (solid blue lines), 2.) the market economy (dashed red lines), and 3.) the planner's solution with no learning (dotted black lines). The first two economies feature Bayesian learning. The parameters values are given in Table 4.

Now we turn to Figure 3. We define WTP $\zeta_p(\pi)$ and $\zeta_m(\pi)$ as the fraction of capital the market economy with no adaptation is willing to give up to transition to the planner's first-best economy and the market economy with just private adaptation, respectively.[33] Panel A shows that both WTPs increase with belief π.[34] The WTP wedge $\zeta_p(\pi) - \zeta_m(\pi)$ measures

[33]To calculate the WTP measures, $\zeta_p(\pi)$ and $\zeta_m(\pi)$, we use the representative household's value functions (welfare measures proportional to the certainty equivalent wealth) for the three economies. Formally, we use

$$\zeta_p(\pi) = 1 - \frac{\underline{b}(\pi)}{b_{fb}(\pi)} \quad \text{and} \quad \zeta_m(\pi) = 1 - \frac{\underline{b}(\pi)}{\widehat{b}(\pi)} > 0 \,,$$

where b_{fb}, $\widehat{b}$, and $\underline{b}$ are the welfare measures (proportional to certainty equivalent wealth) in the (planner's) first-best economy, the market economy (with access to both adaptation technologies but only private mitigation technology will be adapted in equilibrium), and the market economy (with access to neither adaptation technology), respectively.

[34]We can decompose the WTPs into the risk premium and timing premium components by building on the idea and extending the procedure proposed in Epstein, Farhi, and Strzalecki (2014). We show that for our calibrated baseline, while the timing premium is also important, the risk premium component is the major contributor to the total WTP.

the additional welfare gain of having access to the tail-risk public adaptation technology in a market economy. This additional welfare gain increases with λ and is quite substantial for the real-world relevant range of values for λ.

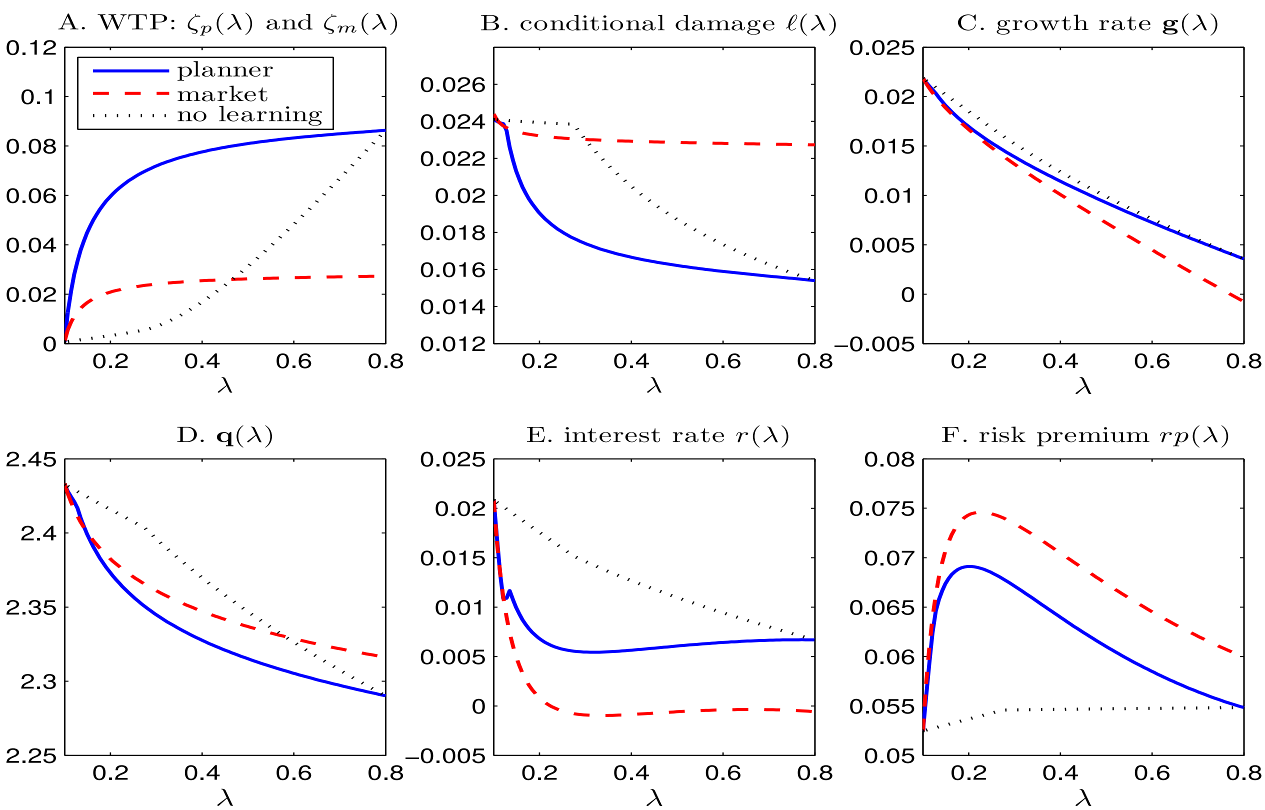

Figure 3: This figure compare the solutions for three economies: 1.) the planner's first-best economy (solid blue lines), 2.) the market economy (dashed red lines), and 3.) the planner's economy with no learning (dotted black lines). The first two economies feature Bayesian learning. The parameters values are given in Table 4.

In Panel B of Figure 3, we show that the conditional damage $\ell(\lambda)$ in both the first-best and market economies decrease with λ. Additionally, the conditional damage $\ell(\lambda)$ in the first-best economy is lower than in the market economy. Moreover, as λ increases, the wedge between $\ell(\lambda)$ in the two economies widens. Because of larger risk mitigation and smaller conditional damage $\ell(\lambda)$ in the first-best economy than in the market economy, the expected growth rate $\mathbf{g}(\lambda)$ is higher in the first-best economy than the market economy (Panel C). This is because the society is more prepared in the first-best economy than in the market economy. The growth-rate difference in the two economies increases with λ and is quantitatively large for the real-world relevant range of λ.

In Panels D, E, and F of Figure 3, we show that in both the first-best and market economies, Tobin's average q decreases as belief worsens, however, the interest rate and risk premium are nonlinear and non-monotonic in λ. This is because while the mean growth prospect gets worse as λ increases, uncertainty is the highest for the intermediate range of λ. As both the mean and higher-order-moment effects are important in the first-best and market economies, the impact of λ on the interest rate and risk premium are nonlinear and non-monotonic.

7.4 Learning versus No-Learning Counterfactual

In this subsection, we assess the value of learning by comparing the solution of our first-best model with learning (of Section 3) with the solution of a counterfactual planner's model with no learning. In the counterfactual no-learning model, we assume that the disaster arrival rate is fixed at a given value of λ and then solve the model. We find that adaptation spendings (in Panels A and B of Figure 2) for $\pi \in (0, 1)$ are larger in our learning model (solid blue lines) than in our counterfactual no-learning model (dotted black lines). As the $\pi = 0$ state (where $\lambda = \lambda_G = 0.1$) and the $\pi = 1$ state (where $\lambda = \lambda_B = 0.8$) are absorbing, the solutions for the first-best learning model (solid blue lines) and the planner's no-learning model (dotted black lines) are the same at $\pi = 0$ and $\pi = 1$ states.That is, adaptation spendings are the highest in the learning model where there is uncertainty over climate states (intermediate values of λ). Investment in our learning model is also lower than in our no-learning counterfactual model (Panel C), but consumption differences in the two economies are limited (Panel D).

7.5 Comparative Statics

In Online Appendix OC, we conduct comparative static analyses with respect to four key parameters: the EIS (ψ), the disaster arrival rate in state B (λ_B), the time rate of preference (ρ), and the coefficient of relative risk aversion (γ). Our main mitigation findings are robust across these four parameter values. The main difference lies in valuation ratios, e.g., the price-dividend ratio.[35] Finally, in the last subsection of Online Appendix OC (Subsection OC.5), we provide a welfare decomposition using our model to further reinforce the importance of learning in determining welfare.

[35] When EIS $\psi = 1$, the price-dividend ratio, $\mathbf{q}/\mathbf{c}$, equals $1/\rho$, the inverse of the time rate of preference, for all levels of π, which is known in the asset-pricing literature, e.g., Wachter (2013). When ψ is greater (less) than one, this $\mathbf{q}/\mathbf{c}$ ratio decreases (increases) with π. That is, equity valuation ratios react negatively to bad (e.g., disaster arrival) news consistent with the reason why the long-run risk literature chooses $\psi > 1$.

7.6 Generalized Learning Model with Stochastic Arrival Rate λ_t

The disaster arrival rate in our baseline model (Section 2), while unobservable, is constant. In Appendix OA, we generalize our baseline model to allow for the unobservable disaster arrival rate to be stochastic, by using a two-state Markov Chain (see, e.g., Wachter and Zhu, 2019). We show that our main quantitative results and conclusions continue to hold in the generalized model of Appendix OA where the transition rates between states G and B are small.

7.7 External Habit Model

In Appendix OB, we replace the Epstein-Zin recursive utility used in our baseline model of Section 2 with another widely-used risk preference—the external habit model proposed by Campbell and Cochrane (1999). For brevity, we focus on the planner's solution. We calibrate our external habit model by targeting the same moments as we do for our baseline model whenever feasible. The quantitative implications on mitigation spendings and welfare in our external habit model are similar to those in our baseline model with Epstein-Zin preferences. However, the two models generate opposite predictions on how investment $\mathbf{i}$ and Tobin's average $\mathbf{q}$ respond as belief becomes more pessimistic (π increases). While both $\mathbf{i}$ and $\mathbf{q}$ increase with π in our habit model, the opposite holds in our baseline Epstein-Zin model. The intuition follows from our discussion regarding comparative statics with respect to ψ.[36]

8 Implications for the Social Cost of Carbon

In this section, we generalize our baseline model (Section 2) to draw out the implications for the social cost of carbon (SCC) as the society learns about the severity of climate disasters and adapts to the challenges it faces. We show how the planner can attain the first-best outcome by optimally using taxes on carbon, investments and capital. We calibrate our generalized model so as to provide quantitative predictions for projections of SCC over time.

8.1 Generalized Model: Fossil Fuels, Carbon Stock, and Disasters

First, we introduce fossil-fuel-usage caused emissions, H_t, as an additional factor of production at the micro level, so that a firm's output equals $AK_t^{\alpha}H_t^{1-\alpha}$, with $0 < \alpha < 1$, as in Golosov

[36]From the long-run risk literature and the comparative static analysis for our baseline Epstein-Zin model with respect to ψ in Section OC of the Online Appendix, we know that an EIS (lower than one) causes the valuation ratios, e.g., the price-dividend ratio, to go up in response to bad news. Our habit model inherits this property, which explains the key differences between the two utility models.

et al. (2014) and Van den Bremer and Van der Ploeg (2021). The stock of (aggregate) atmospheric carbon (in Gitatons) that exceeds the pre-industrial atmospheric carbon stock associated with man-made emissions, which we denote by $\mathbf{S}_t$, evolves:

$$d\mathbf{S}_t = (\mathbf{H}_{t-} - \delta_S \mathbf{S}_{t-})dt + \sigma_S \mathbf{S}_{t-} d\mathcal{W}_t^S \,, \tag{58}$$

where $\mathbf{H}_t$ is the aggregate fossil fuel emissions by all firms: $\mathbf{H}_t = \int H_t^\nu d\nu$, δ_S is the decaying rate of the atmospheric carbon stock, $\mathcal{W}_t^S$ is a standard Brownian motion, and the parameter σ_S is the volatility of atmospheric carbon. Let ϑ denote the correlation coefficient between $\mathcal{W}_t^S$ and the standard Brownian motion $\mathcal{W}_t^K$. We measure both the firm-level H and the aggregate $\mathbf{H}$ in units of carbon and therefore also measure $\mathbf{S}$ in units of carbon (e.g., tons of carbon). The aggregate resource condition is:

$$A\mathbf{K}_t^\alpha \mathbf{H}_t^{1-\alpha} = \mathbf{C}_t + \mathbf{I}_t + \Phi(\mathbf{I}_t, \mathbf{K}_t) + \mathbf{X}_t^d + \mathbf{X}_t^e + p_H \mathbf{H}_t \,, \tag{59}$$

where p_H denotes the price of carbon per ton (tc) in units of consumption good, the numeraire.

To model the damage of the aggregate carbon stock $\mathbf{S}_t$, we assume that the distribution of the post-jump fractional recovery Z depends on $\mathbf{S}_t$. We assume that the damage of weather disaster shocks while uncertain increases in expectation with $\mathbf{S}_t$. As in our baseline model, firms and households learn from disaster arrivals over time regarding the severity of climate risk. To maintain the homogeneity structure of our model, we assume that the distribution function of the post-jump *fractional* recovery Z, Ξ, depends on both aggregate adaptation spending, $\mathbf{x}_{t-}$, and the scaled carbon stock, $\mathbf{s}_{t-} = \mathbf{S}_{t-}/\mathbf{K}_{t-}$, i.e., $\Xi(Z; \mathbf{x}_{t-}^d, \mathbf{s}_{t-})$. The higher the carbon stock $\mathbf{s}_{t-}$, the lower the expected recovery caused by a disaster.

The carbon-to-productive-capital ratio $\mathbf{s} = \mathbf{S}/\mathbf{K}$ evolves as follows:

$$d\mathbf{s}_t \;=\; \mu_s(\pi_{t-}, \mathbf{s}_{t-})dt + \mathbf{s}_{t-}\left[\sigma_S d\mathcal{W}_t^S - \sigma_K d\mathcal{W}_t^K + N_{t-}(1-Z)d\mathcal{J}_t\right], \tag{60}$$

where $\mu_s(\pi_{t-}, \mathbf{s}_{t-})$ is given by

$$\mu_s(\pi_{t-}, \mathbf{s}_{t-}) = \mathbf{h}_{t-} - \left(\mathbf{i}_{t-} - \delta_K + \delta_S - \sigma_K^2 + \vartheta\sigma_K\sigma_S\right)\mathbf{s}_{t-} \,. \tag{61}$$

8.2 First-best Solution

Let $V(\mathbf{K}, \mathbf{S}, \pi)$ denote the representative household's value function. The planner chooses $\{\mathbf{C}, \mathbf{I}, \mathbf{x}^e\, \mathbf{x}^d, \mathbf{H}\}$ to solve the following HJB equation:

$$\begin{aligned} 0 = \max \quad & f(\mathbf{C}, V) + (\mathbf{I} - \delta_K \mathbf{K})\, V_{\mathbf{K}} + \mu_\pi(\pi) V_\pi + (\mathbf{H} - \delta_S \mathbf{S}) V_{\mathbf{S}} + \frac{\sigma_K^2 \mathbf{K}^2 V_{\mathbf{KK}}}{2} + \frac{\sigma_S^2 \mathbf{S}^2 V_{\mathbf{SS}}}{2} \\ & + \vartheta\sigma_K\sigma_S \mathbf{KS} V_{\mathbf{KS}} + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[V\left((1 - N(\mathbf{x}^e)(1-Z))\mathbf{K}, \mathbf{S}, \pi^{\mathcal{J}}\right) - V(\mathbf{K}, \mathbf{S}, \pi)\right], \end{aligned} \tag{62}$$

subject to the aggregate resource additivity condition (59).

As in Cai and Lontzek (2019) and Van den Bremer and Van der Ploeg (2021), we define the social cost of carbon (SCC), as the marginal disutility/disvalue of emitting an additional ton of carbon divided by the marginal utility of consumption:

$$m_t \equiv -\frac{V_{\mathbf{S}}(\mathbf{K}_t, \mathbf{S}_t, \pi_t)}{f_{\mathbf{C}}(\mathbf{C}_t, V_t)} \,. \tag{63}$$

We may use the SCC, m_t defined in (63), to express the FOC for the fossil fuel usage $\mathbf{H}_t$ as:

$$(1-\alpha)A\mathbf{K}_t^{\alpha}\mathbf{H}_t^{-\alpha} = p_H + m_t \,. \tag{64}$$

Rewriting (64), we obtain the following expression for the scaled fossil fuel usage $\mathbf{h}_t^{fb}$:

$$\mathbf{h}_t^{fb} = \left(\frac{(1-\alpha)A}{p_H + m_t}\right)^{1/\alpha} \,. \tag{65}$$

We show that the value function V is homogeneous with degree $(1-\gamma)$ in $\mathbf{K}$ and $\mathbf{S}$:

$$V(\mathbf{K}, \mathbf{S}, \pi) = \frac{1}{1-\gamma}\left(b(\pi, \mathbf{s})\mathbf{K}\right)^{1-\gamma} , \tag{66}$$

where $\mathbf{s} = \mathbf{S}/\mathbf{K}$ and $b(\pi, \mathbf{s})$ is a measure of welfare proportional to the household's certainty equivalent wealth under optimality. Using the first-best policies and $b(\pi_t, \mathbf{s}_t)$, we obtain the following expression for the SCC measure m_t defined in (63) in the first-best economy:[37]

$$m(\pi_t, \mathbf{s}_t) = -\frac{b_{\mathbf{s}}(\pi_t, \mathbf{s}_t)}{\rho}\left(\frac{\mathbf{c}(\pi, \mathbf{s}_t)}{b(\pi_t, \mathbf{s}_t)}\right)^{\psi^{-1}} \,. \tag{67}$$

In Online Appendix OD.1, we solve the model characterized by the PDE system for $b(\pi, \mathbf{s})$.

8.3 Market Economy with Optimal Taxes Attains First-best

In this subsection, we show that the planner attains the first-best outcome via optimal taxation and a lump-sum transfer. Let τ_t^x, τ_t^h, and τ_t^i denote the tax rates on a firm's capital K_t, fossil-fuel usage and investment, respectively. Let $\mathbf{L}_t$ denote the lump-sum transfer to the firm, which only depends on the aggregate variables. A firm's payout at t, Y_t, is then given by

$$Y_t = AK_t^{\alpha}H_t^{1-\alpha} - \left(I_t + \Phi_t + X_t^e + X_t^d + p_H H_t\right) - \left[\,\tau_t^x K_t + \tau_t^h H_t + \tau_t^i\left(I_t + \Phi_t\right)\right] + \mathbf{L}_t \,. \tag{68}$$

Taking the equilibrium SDF $\mathbb{M}_t$ and the three tax rates as given, each firm maximizes its value given in (5) by choosing $\{I,\, X^e,\, X^d,\, H\}$.

[37] Using the first-best solution, we can calculate the first-best $\mathbf{c}(\pi, \mathbf{s})$ by using the resource constraint: $\mathbf{c}(\pi, \mathbf{s}) = A\mathbf{h}(\pi, \mathbf{s})^{1-\alpha} - \mathbf{i}(\pi, \mathbf{s}) - \phi(\mathbf{i}(\pi, \mathbf{s})) - \mathbf{x}^d(\pi, \mathbf{s}) - \mathbf{x}^e(\pi, \mathbf{s}) - p_H\mathbf{h}(\pi, \mathbf{s})\,.$

The key idea here is that the planner can attain the first-best by implementing Pigouvian taxes (for investment and fossil fuel usage) in our dynamic model in addition to taxing capital to fund aggregate adaptation spending (as in the baseline model of Section 2).

The FOC for a firm's fossil fuel usage, h, is: $(1-\alpha)Ah(\pi, \mathbf{s})^{-\alpha} = p_H + \tau^h$, where τ^h is the Pigouvian tax on fossil fuel usage h. By setting $\tau_t^h = m(\pi_t, \mathbf{s}_t)$ at all t, where $m(\pi, \mathbf{s})$ is given in (67), we obtain the same FOC for h_t as in the planner's problem. By taxing fossil fuel at SCC under the first-best $m(\pi, \mathbf{s})$, the planner (locally) addresses the externality caused by carbon emissions. However, this is not enough.

This is because our model features capital adjustment costs, which means capital stock K is a state variable rather than a choice variable as in Golosov et al. (2014). This has important implications on the optimal tax plan. To attain the first-best, the planner needs to ensure that both carbon stock $\mathbf{S}$ and capital stock $\mathbf{K}$ to follow the same trajectory as in the first-best economy. Hence, in addition to using a fossil fuel tax τ^h to manage carbon accumulation, the planner also needs to use an investment tax τ^i to manage capital stock accumulation.

Specifically, facing an investment tax τ^i, a firm chooses i to satisfy the FOC:

$$q(\pi, \mathbf{s}) = \left(1 + \phi'(\mathbf{i}(\pi, \mathbf{s}))\right)\left(1 + \tau^i\right). \tag{69}$$

By optimally choosing τ^i, the planner increases a firm's marginal cost of investing from $(1 + \phi'(\mathbf{i}(\pi, \mathbf{s})))$ as in the standard q theory to the term on the right side of (69), which includes the additional marginal cost to the society $\tau^i\left(1 + \phi'(\mathbf{i}(\pi, \mathbf{s}))\right)$. Taking the investment tax as given, the firm optimally equates its marginal q (marginal benefit of investing) on the left side of (69) with its cum-tax marginal cost of investing on the right side.

Similar to the analysis for the market economy under government intervention for our baseline model (Section 5), we show that it is optimal for the planner to tax capital at the rate τ_t^x that equals the aggregate adaptation spending, i.e., $\tau_t^x = \mathbf{x}_t^d$ at all time t and use the aggregate proceeds from capital taxes to fund the aggregate public adaptation spending $\mathbf{X}_t^d$.

Finally, we show that the lump-sum transfer associated with optimal Pigouvian taxes is given by $\mathbf{L}_t = m(\pi_t, \mathbf{s}_t)\mathbf{H}_t^{fb} + \tau^i(\pi_t, \mathbf{s}_t)\left(\mathbf{I}_t^{fb} + \mathbf{\Phi}_t^{fb}\right)$, where $\tau_t^i = \tau^i(\pi_t, \mathbf{s}_t)$ and

$$\tau^i(\pi_t, \mathbf{s}_t) = \frac{\mathbf{s}_t b_{\mathbf{s}}(\pi_t, \mathbf{s}_t)}{b(\pi_t, \mathbf{s}_t) - \mathbf{s}_t b_{\mathbf{s}}(\pi_t, \mathbf{s}_t)}. \tag{70}$$

To ensure that the market economy tracks the capital stock accumulation in the first-best economy, the planner links the optimal tax rate to the welfare measure $b(\pi_t, \mathbf{s}_t)$ and its derivative in the first-best economy, as (70) shows.

To further deepen our understanding of the mechanism, we substitute the lump-sum transfer expression for $\mathbf{L}_t$ into a firm's payout process (68) and obtain:

$$\begin{aligned} Y_t &= AK_t^{\alpha}H_t^{1-\alpha} - \big(I_t + \Phi_t + X_t^e + X_t^d + p_H H_t\big) \\ &\quad - \big[\, \mathbf{x}_{fb}^d(\pi_t, \mathbf{s}_t)K_t + m(\pi_t, \mathbf{s}_t)(H_t - \mathbf{H}_t^{fb}) + \tau^i(\pi_t, \mathbf{s}_t)\big(I_t + \Phi_t - (\mathbf{I}_t^{fb} + \mathbf{\Phi}_t^{fb})\big) \,\big] . \end{aligned} \quad (71)$$

Recall that $X^d = 0$, as a firm is infinitesimal and reducing aggregate disaster risk brings no benefit but only cost to itself as in our baseline model. Now we have expressed a firm's fossil fuel tax payment as the carbon tax rate $m(\pi_t, \mathbf{s}_t)$ multiplied by a firm's *excess* fossil fuel usage relative to the first-best level, $H_t - \mathbf{H}_t^{fb}$. This means when $H_t - \mathbf{H}_t^{fb} < 0$, the firm receives a subsidy. Similarly, a firm's investment tax payment equals the investment tax rate $\tau^i(\pi_t, \mathbf{s}_t)$ multiplied by its total investment costs exceeding the first-best level, $\big(I_t + \Phi_t - \big(\mathbf{I}_t^{fb} + \mathbf{\Phi}_t^{fb}\big)\big)$. This is because the planner only optimally penalizes a firm's deviation from the first-best. Our carbon and investment taxes are Pigouvian taxes in general equilibrium.

Finally, by combining the investment and carbon taxes with the optimal capital tax τ^x to fund the aggregate adaptation spending, the planner fully addresses all externalities and attains the first-best. Next, we summarize the three tax rates chosen by the planner to attain the first-best in a market economy. Online Appendix OD provides a proof.

Proposition 5 *The planner attains the first-best outcome in a market economy by setting: 1.)* $\tau_t^x = \mathbf{x}_{fb}^d(\pi_t, \mathbf{s}_t)$*, where* $\mathbf{x}_{fb}^d(\pi_t, \mathbf{s}_t)$ *is the scaled first-best public (distribution) adaptation; 2.)* $\tau_t^h = m(\pi_t, \mathbf{s}_t)$*, where* $m(\pi_t, \mathbf{s}_t)$ *is SCC in the first-best economy given in (67); and 3.)* $\tau_t^i = \tau^i(\pi_t, \mathbf{s}_t)$*, where* $\tau^i(\pi_t, \mathbf{s}_t)$ *is given in (70).*

8.4 Calibration

Next, we calibrate our generalized model with carbon stock. The solution of this generalized model boils down to a PDE system even after we use the homogeneity property to simplify our analysis. After incorporating fossil fuel and carbon dynamics, our model has 20 parameters (Table 5) while our baseline model of Section 7 has 13 parameters (Table 4).

First, we assume that the β function that describes the disaster damage distribution depends on not only $\mathbf{X}^d$ and $\mathbf{K}$ but also $\mathbf{S}$ in our carbon model as follows:

$$\beta(\mathbf{x}^d, \mathbf{s}) = \beta_0 + \beta_x \mathbf{x}^d - \beta_s \mathbf{s} \, , \quad (72)$$

where $\mathbf{x}^d = \mathbf{X}^d/\mathbf{K}$, $\mathbf{s} = \mathbf{S}/\mathbf{K}$, and β_x and β_s are positive parameters. Compared with (56), we now incorporate the effect of carbon stock $\mathbf{s}$ on disaster damages, which is captured by

β_s. Generalizing our calibration procedure for the baseline model, we determine the eight

Table 5: Parameter Values for Generalized Model with Carbon

Parameters	Symbol	Value
disaster (jump) arrival rate in State G	λ_G	0.1
disaster (jump) arrival rate in State B	λ_B	0.8
prior of being in State B	π_0	0.08
power law exponent absent adaptation	β_0	39
distribution adaptation technology parameter	β_x	$2,500$
exposure adaptation technology parameter	ζ	0.25
damage parameter from atmospheric carbon	β_s	$10,000$
carbon decaying rate	δ_S	3%
return-to-scale parameter	α	0.96
volatility of carbon stock growth	σ_S	7.5%
price of carbon per ton (tC)	p_H	540
initial value of $p_H \mathbf{s}$	$p_H \mathbf{s}_0$	13.6%
elasticity of intertemporal substitution	ψ	1.5
time rate of preference	ρ	5%
productivity parameter for Cobb-Douglas function	A	43%
quadratic adjustment cost parameter	θ	17
coefficient of relative risk aversion	γ	8
capital diffusion volatility	σ_K	8%
depreciation rate of capital	δ_K	6%
correlation between capital and carbon stocks	ϑ	0

All parameter values, whenever applicable, are continuously compounded and annualized.

parameters in the first panel of Table 5 (λ_G, λ_B, β_0, β_x, β_s, ζ, δ_S, and π_0) by targeting the GDP growth, asset prices (risk-free rate, equity risk premium, and Tobin's average q), the levels of private and public adaptation spendings (reported in Section 6), and the steady state of $\mathbf{s}$ at 0.05%.

The four parameters in the second Panel of Table 5 (α, σ_S, p_H, and $\mathbf{s}_0$) are related to fossil fuels and carbon stock dynamics in our model and damage from carbon stock. We use the parameter values from the carbon economics literature for these parameters (see e.g., Van den Bremer and Van der Ploeg, 2021).

For the three preference parameters (the EIS ψ, risk aversion γ, and the time rate of preference ρ) and the four production parameters (productivity A, the quadratic adjustment cost θ, capital diffusion volatility σ_K, and capital depreciation rate δ_K) reported in the last Panel of Table 5, we use the same values as those in Table 4 for our baseline model without carbon (of Section 2). Finally, we set the correlation coefficient between capital shocks and carbon stock shocks to zero: $\vartheta = 0$.

8.5 Social Cost of Carbon (SCC) Projections

In this subsection, we discuss the SCC projections over time using the first-best solution. It is helpful to first plot how SCC $m(\pi, \mathbf{s})$ varies with the two state variables: π and $\mathbf{s}$.

SCC as a function of belief π and s. Panel A of Figure 4 shows that SCC, $m(\pi, \mathbf{s}_0)$, is increasing and concave in π. As the world becomes riskier, i.e., as π increases, SCC increases as we expected. However, the rate at which SCC increases, $m_\pi(\pi, \mathbf{s}_0)$, decreases with π. This is because the society endogenously increases adaptation spendings $\mathbf{x}^d$ and $\mathbf{x}^e$ as π increases (Section 7). Panel B of Figure 4, shows that SCC, $m(\pi, \mathbf{s}_0)$, is increasing in $\mathbf{s}$.

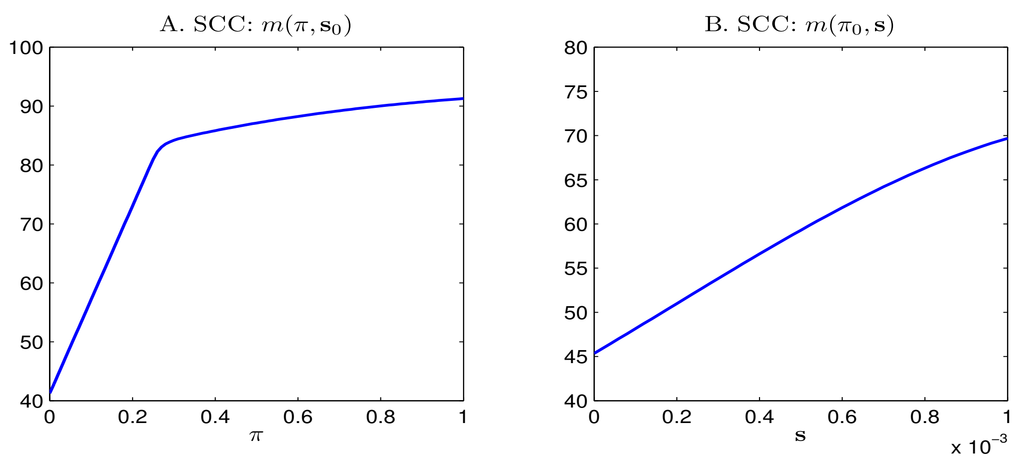

Figure 4: Panel A plots SCC $m(\pi, \mathbf{s})$ as a function of π fixing $\mathbf{s} = \mathbf{s}_0$. Panel B plots SCC as a function of $\mathbf{s}$ fixing $\pi = \pi_0$. The parameters values are in Table 5.

Next, we simulate our model (from year 0) and analyze predicted SCC projections. To highlight the role of learning, we first consider the no-learning counterfactual environment.

SCC projections under no-learning counterfactual. In Figure 5, we see that both the mean and quantiles of SCC increase over time. This is because carbon accumulates over time in expectation and SCC increases with (scaled) carbon stock $\mathbf{s}$. The qualitative prediction of our model is similar to other recent integrated assessment models (Jensen and Traeger 2014; Cai and Lontzek, 2019). However, unlike these models with no adaptation margins, active adaptations in our model reduce the slope of SCC projections over time.

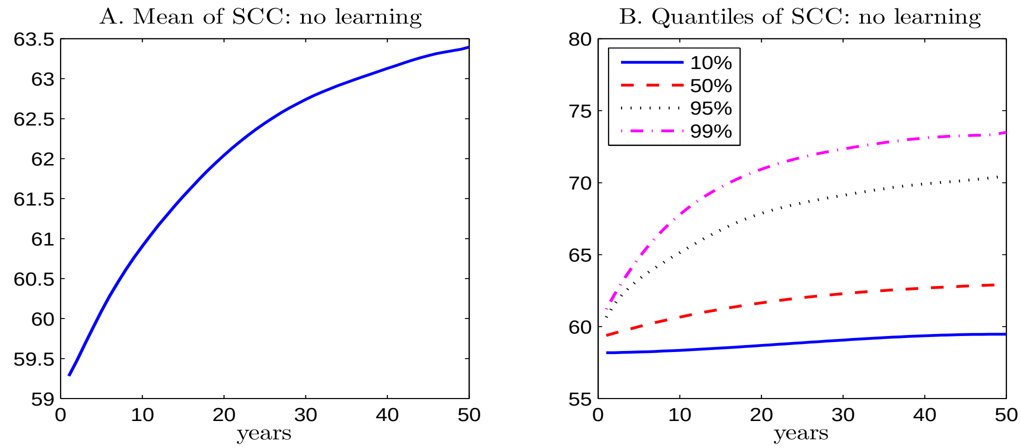

Figure 5: The mean (Panel A) and quantiles (Panel B) of social cost of carbon (SCC) over time in the counterfactual no-learning environment. The parameters values are in Table 5.

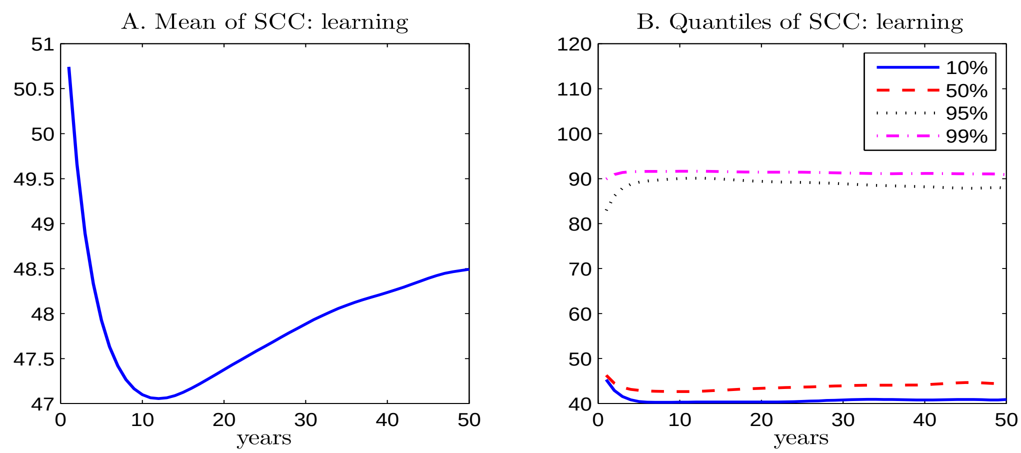

Figure 6: The mean (Panel A) and quantiles (Panel B) of social cost of carbon (SCC) over time in our learning model.The parameters values are in Table 5.

SCC projections in our learning model. Figure 6 shows that the mean of SCC projections in our learning environment first decreases over time, bottoms out around ten years, and then increases over time (Panel A). This prediction fundamentally differs from those in no-learning counterfactual environments, such as the one we just analyzed in Figure 5 and models in the literature, which have no learning.

Why in a learning model does SCC first decreases before increases over time? This is due to the interaction of (1) belief dynamics and (2) the endogenous adaptation response to changing beliefs. First, belief π, being a martingale, spreads out stochastically over time and eventually settles either at state G or at state B with probability one.[38] Second, because of endogenous adaptation response to changing beliefs, SCC is concave in π, which we just discussed using Panel A of Figure 4. Therefore, due to Jensen's inequality, SCC first decreases over time in our learning model.

Quantitatively, our calibrated calculation shows that the learning effect dominates the standard carbon emission effect for the first ten or so years and the standard carbon stock effect dominates for the later years, generating the inverse-hump shaped SCC transition dynamics.[39] The inter-quantile SCC range (for a given pair of quantiles) in our learning model is much wider than the range in the no-learning counterfactual (comparing Figures 5 and 6).

In sum, we show that the interaction of learning and adaptation generates new qualitative predictions and large quantitative effects for SCC projections.

9 Conclusion

We develop a model of adaptation to mitigate weather disaster risks arising from global warming. Optimal adaptation — a mix of private efforts and public spending — depends on learning about the consequences of global warming for disaster arrivals. The planner's solution can be implemented via a combination of taxes on capital and carbon. We apply our model to major tropical cyclones and calibrate the learning process using empirical findings on the response of asset prices to disaster arrivals. There are a number of implications, including the dependence of social-cost-of-carbon projections on the interaction of uncertainty resolution and endogenous response of adaptation. To obtain these results, we made simplifying assumptions on adaptation technologies. Relaxing them, which we leave for future research, would yield additional insights for policymakers.

[38] This result follows from the martingale convergence theorem (Liptser and Shiryaev, 2001).

[39] Note that SCC projections are also about 20% lower in our learning model than in the no-learning counterfactual model (Figure 5). This is consistent with our model's prediction that it is more valuable to adapt when agents learn from disaster arrivals (Section 7.4)).

References

Bakkensen, L.A. and Barrage, L., 2022. Going underwater? Flood risk belief heterogeneity and coastal home price dynamics. *Review of Financial Studies*, 35(8), pp.3666-3709.

Bansal, R. and Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4), pp.1481-1509.

Barnett, M., Brock, W. and Hansen, L.P., 2020. Pricing uncertainty induced by climate change. *Review of Financial Studies*, 33(3), pp.1024-1066.

Barro, R.J., 2006. Rare Disasters and Asset Markets in the Twentieth Century. *Quarterly Journal of Economics*, 121, pp.823-866.

Bouwer, L.M., Crompton, R.P., Faust, E., Happe, P. and Pielke Jr, R.A., 2007. Confronting disaster losses. *Science-New York then Washington*, 318(5851), p.753.

Bretschger, L. and Vinogradova, A., 2019. Best policy response to environmental shocks: Applying a stochastic framework. *Journal of Environmental Economics and Management*, 97, pp.23-41.

Cai, Y. and Lontzek, T.S., 2019. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6), pp.2684-2734.

Campbell, J.Y. and Cochrane, J.H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2), pp.205-251.

Collin-Dufresne, P., Johannes, M. and Lochstoer, L.A., 2016. Parameter learning in general equilibrium: The asset pricing implications. *American Economic Review*, 106(3), pp.664-698.

Daniel, K.D., Litterman, R.B. and Wagner, G., 2019. Declining CO2 price paths. *Proceedings of the National Academy of Sciences*, 116(42), pp.20886-20891.

Dieckmann, Stephan, 2011. Rare Event Risk and Heterogeneous Beliefs: The Case of Incomplete Markets. *Journal of Financial and Quantitative Analysis* 46(2), pp.459-488.

Duffie, D., 2001. Dynamic asset pricing theory, Third edition, *Princeton University Press*.

Duffie, D. and Epstein, L.G., 1992. Stochastic differential utility. *Econometrica*, 60(2), pp.353-394.

Duffie, D. and Sun, Y., 2012. The exact law of large numbers for independent random matching. *Journal of Economic Theory*, 147(3), pp.1105-1139.

Eberly, J., Rebelo, S. and Vincent, N., 2012. What explains the lagged-investment effect?. Journal of Monetary Economics, 59(4), pp.370-380.

Epstein, L.G., Farhi, E. and Strzalecki, T., 2014. How much would you pay to resolve long-run risk?. *American Economic Review*, 104(9), pp.2680-97.

Epstein, L.G. and Zin, S.E., 1989. Substitution, risk aversion, and the temporal behavior of consumption. *Econometrica*, 57(4), pp.937-969.

Fried, S., 2022. Seawalls and stilts: A quantitative macro study of climate adaptation. *The Review of Economic Studies*, 89(6), pp.3303-3344.

Golosov, M., Hassler, J., Krusell, P. and Tsyvinski, A., 2014. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1), pp.41-88.

Hayashi, F., 1982. Tobin's marginal q and average q: A neoclassical interpretation. *Econometrica*, 50: 215-224.

Hong, H., Karolyi, G.A. and Scheinkman, J.A., 2020. Climate finance. *Review of Financial Studies*, 33(3), pp.1011-1023.

Hsiang, S.M. and Jina, A.S., 2014. The causal effect of environmental catastrophe on long-run economic growth: Evidence from 6,700 cyclones (No. w20352). National Bureau of Economic Research.

Jensen, S. and Traeger, C.P., 2014. Optimal climate change mitigation under long-term growth uncertainty: Stochastic integrated assessment and analytic findings. *European Economic Review*, 69, pp.104-125.

Knutson, T., Camargo, S.J., Chan, J.C., Emanuel, K., Ho, C.H., Kossin, J., Mohapatra, M., Satoh, M., Sugi, M., Walsh, K. and Wu, L., 2020. Tropical cyclones and climate change assessment: Part II: Projected response to anthropogenic warming. *Bulletin of the American Meteorological Society*, 101(3), pp.E303-E322.

Lasage, R., Veldkamp, T.I., De Moel, H., Van, T.C., Phi, H.L., Vellinga, P. and Aerts, J.C.J.H., 2014. Assessment of the effectiveness of flood adaptation strategies for HCMC. *Natural Hazards and Earth System Sciences*, 14(6), pp. 1441-1457.

Liptser, R.S. and Shiryaev, A.N., 2001. Statistics of random processes II: Applications. New York: Springer-Verlag, 2ed.

Martin, I.W. and Pindyck, R.S., 2015. Averting catastrophes: The strange economics of Scylla and Charybdis. *American Economic Review*, 105(10), pp.2947-2985.

Muis, S., Gaceneralp, B., Jongman, B., Aerts, J.C. and Ward, P.J., 2015. Flood risk and adaptation strategies under climate change and urban expansion: A probabilistic analysis using global data. *Science of the Total Environment*, 538, pp. 445-457.

National Academy of Sciences (2016), *Attribution of extreme weather events in the context of climate change*. Washington, D.C.: The National Academies Press.

Nordhaus, W.D., 2017. Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7), pp.1518-1523.

Pielke Jr, R.A., Gratz, J., Landsea, C.W., Collins, D., Saunders, M.A. and Musulin, R., 2008. Normalized hurricane damage in the United States: 1900-2005. *Natural Hazards Review*, 9(1), pp.29-42.

Pindyck, R.S. and Wang, N., 2013. The economic and policy consequences of catastrophes. *American Economic Journal: Economic Policy*, 5(4), pp.306-339.

Rietz, T. A., 1988. The equity risk premium: a solution. *Journal of Monetary Economics*, 22(1), pp.117-131.

Van den Bremer, T.S. and Van der Ploeg, F., 2021. The risk-adjusted carbon price. *American Economic Review*, 111(9), pp.2782-2810.

Wachter, J.A., 2013. Can time-varying risk of rare disasters explain aggregate stock market volatility?. *Journal of Finance*, 68(3), pp.987-1035.

Wachter, J.A. and Zhu, Y., 2019. Learning with rare disasters. Wharton School Working Paper.

Appendices

A The First-best Economy in Section 3

A.1 Planner's Resource Allocation

Substituting the value function (21) into the FOC (18) for investment, the FOC (19) for the aggregate disaster distribution adaptation spending, and the FOC (20) for the aggregate disaster exposure adaptation spending, we obtain:

$$b(\pi) = \mathbf{c}(\pi)^{1/(1-\psi)} \left[\rho(1+\phi'(\mathbf{i}(\pi)))\right]^{-\psi/(1-\psi)} \, , \tag{A.1}$$

$$\rho\mathbf{c}(\pi)^{-\psi^{-1}} b(\pi)^{\psi^{-1}-1} = \frac{\lambda(\pi)}{1-\gamma}\left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)}\right)^{1-\gamma} \int_0^1 \left[\frac{\partial \xi(Z;\mathbf{x}^d)}{\partial \mathbf{x}^d}(1-N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}\right] dZ \, , \tag{A.2}$$

$$\rho\mathbf{c}(\pi)^{-\psi^{-1}} b(\pi)^{\psi^{-1}-1} = \lambda(\pi)\left[\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right]^{1-\gamma} N'(\mathbf{x}^e(\pi))\mathbb{E}^{\mathbf{x}^d(\pi)}\left[(Z-1)(1-N(\mathbf{x}^e(\pi))(1-Z))^{-\gamma}\right] \, , \tag{A.3}$$

where the post-jump $\pi^{\mathcal{J}}$ is given in (12) as a function of the pre-jump π. Substituting the resource constraint, $\mathbf{c}(\pi) = A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^d(\pi) - \mathbf{x}^e(\pi)$, into (A.1), we obtain (23). Substituting (A.1) into (A.3), we obtain (24) and substituting (A.1) into (A.2), we obtain (25). Finally, substituting the value function (21) and the FOC (23) into the HJB equation (16), we obtain the ODE (22).

In sum, we now have fully characterized the model solution summarized in Proposition 1.

A.2 Asset Pricing Implications in the First-best Economy

Duffie and Epstein (1992) show that the SDF $\{\mathbb{M}_t : t \geq 0\}$ implied by the planner's solution is:

$$\mathbb{M}_t = \exp\left[\int_0^t f_V(\mathbf{C}_s, V_s)\, ds\right] f_{\mathbf{C}}(\mathbf{C}_t, V_t) \, . \tag{A.4}$$

Using the FOC for investment (18), the value function (21), and the resource constraint, we obtain:

$$f_{\mathbf{C}}(\mathbf{C}, V) = \frac{1}{1+\phi'(\mathbf{i}(\pi))} b(\pi)^{1-\gamma}\mathbf{K}^{-\gamma} = \frac{1}{\mathbf{q}(\pi)} b(\pi)^{1-\gamma}\mathbf{K}^{-\gamma} \tag{A.5}$$

and

$$f_V(\mathbf{C}, V) = \frac{\rho}{1-\psi^{-1}}\left[\frac{(1-\omega)\mathbf{C}^{1-\psi^{-1}}}{((1-\gamma))^{\omega-1}} V^{-\omega} - (1-\gamma)\right] = -\epsilon(\pi) \, , \tag{A.6}$$

where

$$\epsilon(\pi) = -\frac{\rho(1-\gamma)}{1-\psi^{-1}}\left[\left(\frac{\mathbf{c}(\pi)}{b(\pi)}\right)^{1-\psi^{-1}}\left(\frac{\psi^{-1}-\gamma}{1-\gamma}\right) - 1\right] . \tag{A.7}$$

Using the equilibrium relation between $b(\pi)$ and $\mathbf{c}(\pi)$, we simplify (A.7) as:

$$\begin{aligned} \epsilon(\pi) = {} & \rho + \left(\psi^{-1}-\gamma\right)\left[\mathbf{i}(\pi) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\right] \\ & + \left(\psi^{-1}-\gamma\right)\left[\frac{\lambda(\pi)}{1-\gamma}\left(\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}\left[(1-N(\mathbf{x}^e)(1-Z))^{1-\gamma}\right] - 1\right)\right] \, , \end{aligned} \tag{A.8}$$

where the post-jump belief $\pi^{\mathcal{J}}$ given in (12) is a function of the pre-jump belief π. For expected utility where $\psi = 1/\gamma$, we have $\epsilon(\pi) = \rho$. Using Ito's Lemma and the optimal allocation, we obtain

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -\epsilon(\pi)dt - \gamma\left[(\mathbf{i}(\pi) - \delta_K)dt + \sigma_K d\mathcal{W}_t^K\right] + \frac{\gamma(\gamma+1)}{2}\sigma_K^2 dt + \left((1-\gamma)\frac{b'(\pi)}{b(\pi)} - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right)\mu_\pi(\pi)dt$$
$$+ \left[\frac{\mathbf{q}(\pi)}{\mathbf{q}(\pi^{\mathcal{J}})}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}(1 - N(\mathbf{x}^e)(1-Z))^{-\gamma} - 1\right] d\mathcal{J}_t\,. \tag{A.9}$$

As the expected percentage change of $\mathbb{M}_t$ equals $-r_t$ per unit of time (Duffie, 2001), we obtain the following expression for the equilibrium interest rate:

$$\begin{aligned} r(\pi) \; &= \rho + \psi^{-1}(\mathbf{i}(\pi) - \delta_K) - \frac{\gamma(\psi^{-1}+1)\sigma_K^2}{2} - \left[(1-\psi^{-1})\frac{b'(\pi)}{b(\pi)} - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right]\mu_\pi(\pi) \\ &- \lambda(\pi)\left[\frac{\mathbf{q}(\pi)}{\mathbf{q}(\pi^{\mathcal{J}})}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}((1-N(\mathbf{x}^e)(1-Z))^{-\gamma}) - 1\right] \\ &- \lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1 - \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}((1-N(\mathbf{x}^e)(1-Z))^{1-\gamma})\right)\right]\,. \end{aligned} \tag{A.10}$$

Recall that $\mathbf{D}_t = \mathbf{C}_t$ and $\mathbb{M}_{t-}\mathbf{D}_{t-}dt + d(\mathbb{M}_t\mathbf{Q}_t)$ is a martingale under the physical measure (Duffie, 2001). Applying Ito's Lemma to $\mathbb{M}_{t-}\mathbf{D}_{t-}dt + d(\mathbb{M}_t\mathbf{Q}_t)$ and setting its drift to zero, we obtain

$$\begin{aligned} \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} &= \rho - (1-\psi^{-1})\left[\mathbf{i}(\pi) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\right] \\ &+ \lambda(\pi)\frac{1-\psi^{-1}}{1-\gamma}\left[1 - \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}\left[1 - N(\mathbf{x}^e)(1-Z)\right]^{1-\gamma}\right]\,. \end{aligned} \tag{A.11}$$

We obtain the aggregate Tobin's average $\mathbf{q}$ from (A.11). For the special case with $\psi = 1$ and any risk aversion $\gamma > 0$, the dividend yield (and equivalently the consumption-wealth ratio) is $\mathbf{c}(\pi)/\mathbf{q}(\pi) = \rho$.

B Market Equilibrium Solution in Section 4

B.1 Firm Value Maximization

First, using Ito's Lemma, we obtain the following dynamics for $Q_t = Q(K_t, \pi_t)$:

$$\begin{aligned} dQ_t \;\; &= \;\; \left((I - \delta_K K)Q_K + \frac{1}{2}\sigma_K^2 K^2 Q_{KK} + \mu_\pi(\pi)Q_\pi\right)dt + \sigma_K K Q_K d\mathcal{W}_t^K \\ &+ \left(Q((1-N(x^e)(1-Z))K, \pi^{\mathcal{J}}) - Q(K,\pi)\right)d\mathcal{J}_t\,. \end{aligned} \tag{B.12}$$

No arbitrage implies that the drift of $\mathbb{M}_{t-}(AK_{t-} - I_{t-} - \Phi(I_{t-}, K_{t-}) - X_{t-}^e - X_{t-}^d)dt + d\left(\mathbb{M}_t Q_t\right)$ is zero under the physical measure (Duffie, 2001). Applying Ito's Lemma to this martingale, we obtain

$$\begin{aligned} 0 = \max_{I, x^e, x^d} \; & \mathbb{M}(AK - I - \Phi(I,K) - x^e K - x^d K) + \mathbb{M}\left((I - \delta_K K)Q_K + \frac{1}{2}\sigma_K^2 K^2 Q_{KK} + \mu_\pi(\pi)Q_\pi\right) \\ & + Q\left[-r(\pi) - \lambda(\pi)\left(\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - 1\right)\right]\mathbb{M} - \mathbb{M}\gamma\sigma_K^2 K Q_K \\ & + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z, \mathbf{x}^e)Q((1-N(x^e)(1-Z))K, \pi^{\mathcal{J}}) - Q(K,\pi)\right]\mathbb{M}\,. \end{aligned} \tag{B.13}$$

And then by using the homogeneity property $Q(K, \pi) = q(\pi)K$, we obtain the simplified HJB equation (28). Simplifying the FOC for the exposure mitigation spending implied by (B.13), we obtain (29). Similarly, simplifying the investment FOC implied by (B.13), we obtain (30).

B.2 Household's Optimization Problem

By following the cum-dividend return of the aggregate asset market given in (15), the representative household accumulates wealth as:[40]

$$dW_t = r(\pi_{t-})W_{t-}dt + (\mu_{\mathbf{Q}}(\pi_{t-}) - r)\Gamma_{t-}dt + \sigma_K \Gamma_{t-} d\mathcal{W}_t^K - C_{t-}dt + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1 \right) \Gamma_{t-} d\mathcal{J}_t \,. \quad \text{(B.14)}$$

By using the W_t process given in (B.14), we obtain the HJB equation (32) for the household's value function. The FOCs for consumption C and the market portfolio allocation Γ are given by

$$f_C(C, J) = J_W(W, \pi) \,, \quad \text{(B.15)}$$

$$\sigma_K^2 \Gamma J_{WW}(W, \pi) = -(\mu_{\mathbf{Q}}(\pi) - r(\pi))J_W(W, \pi) + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d} \left[\left(1 - \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} \right) J_W \left(W^{\mathcal{J}}, \pi^{\mathcal{J}} \right) \right] . \quad \text{(B.16)}$$

Subsituting (31) into (B.15), we obtain the optimal consumption rule given by (35). Simplifying the FOC for Γ given by (B.16), we obtain (36).

B.3 Market Equilibrium

First, the firm's (scaled) disaster exposure adaptation spending is positive and equals the aggregate exposure mitigation spending: $x^e = \mathbf{x}^e > 0$. Second, in equilibrium, the household invests all wealth in the market portfolio and holds no risk-free asset, $\Gamma = W$ and $W = \mathbf{Q}$. Simplifying the FOCs, (35) and (36), and using the value function (31), we obtain:

$$c(\pi) = \rho^{\psi} u(\pi)^{1-\psi} \mathbf{q}(\pi) \,, \quad \text{(B.17)}$$

$$\mu_{\mathbf{Q}}(\pi) = r(\pi) + \gamma\sigma_K^2 + \lambda(\pi) \left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))\eta(\pi; Z, \mathbf{x}^e)) \right] . \quad \text{(B.18)}$$

Then substituting (31) into the HJB equation (32), we obtain (39). Using these equilibrium conditions, we simplify the HJB equation (32) as follows:

$$\begin{aligned} 0 \;=\; & \frac{1}{1-\psi^{-1}} \left(\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} - \rho \right) + \left(\mu_{\mathbf{Q}}(\pi) - \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} \right) - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi) \frac{\mathbf{u}'(\pi)}{\mathbf{u}(\pi)} \\ & + \frac{\lambda(\pi)}{1-\gamma} \left[\frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z)\eta(\pi; Z, \mathbf{x}^e))) - 1 \right] . \end{aligned} \quad \text{(B.19)}$$

[40] The first four terms in (B.14) are standard as in the classic portfolio-choice problem with no insurance or disasters. The last term is the loss of the household's wealth from her portfolio's exposure to the market portfolio. (We leave out the disaster insurance demand as they net out to zero in equilibrium and do not change the equilibrium analysis.) Pindyck and Wang (2013) provide a detailed description of their dynamically complete markets setting (with various diffusion and stage-contingent actuarially fair jump hedging contracts.). Our dynamically complete markets setting builds on Pindyck and Wang (2013).

Third, by substituting $\mathbf{c}(\pi) = A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e(\pi)$ into (28), we obtain

$$\begin{aligned} 0 = & \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} - r(\pi) + \mathbf{i}(\pi) - \delta_K + \mu_\pi(\pi)\frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)} - \gamma\sigma_K^2 \\ & - \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))\eta(\pi; Z, \mathbf{x}^e))\right] . \end{aligned} \quad \text{(B.20)}$$

By using the homogeneity property and comparing (15) and (B.12), we obtain

$$\mu_{\mathbf{Q}}(\pi) = \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} + \mathbf{i}(\pi) - \delta_K + \mu_\pi(\pi)\frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)} . \quad \text{(B.21)}$$

Then substituting (B.21) into (B.19), we obtain

$$\begin{aligned} \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} = & \ \rho - (1 - \psi^{-1})\left[\mathbf{i}(\pi) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\left(\frac{\mathbf{u}'(\pi)}{\mathbf{u}(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right)\right] \\ & + \lambda(\pi)\left(\frac{1 - \psi^{-1}}{1 - \gamma}\right)\left[1 - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))\eta(\pi; Z, \mathbf{x}^e))\right] . \end{aligned} \quad \text{(B.22)}$$

Substituting (B.22) into (B.20), we obtain the following expression for the equilibrium risk-free rate:

$$\begin{aligned} r(\pi) = & \ \rho + \psi^{-1}(\mathbf{i}(\pi) - \delta_K) - \frac{\gamma(\psi^{-1} + 1)\sigma_K^2}{2} - \left[(1 - \psi^{-1})\left(\frac{u'(\pi)}{u(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right) - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right]\mu_\pi(\pi) \\ & - \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - 1\right] \\ & - \lambda(\pi)\left[\frac{\psi^{-1} - \gamma}{1 - \gamma}\left(1 - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))\eta(\pi; Z, \mathbf{x}^e))\right)\right] . \end{aligned} \quad \text{(B.23)}$$

Using (15) and (B.18), we obtain the following expression for the market risk premium $rp(\pi)$:

$$rp(\pi) = \mu_{\mathbf{Q}}(\pi) + \lambda(\pi)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right) - r(\pi) = \gamma\sigma_K^2 - \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[(\eta(\pi; Z, \mathbf{x}^e) - 1)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right)\right] , \quad \text{(B.24)}$$

which implies (51).

In sum, we have derived the equilibrium resource allocations and the asset pricing implications summarized in Proposition 2 and Proposition 4.

Online Appendices

OA Model with Stochastic Disaster Arrival Rates

The disaster arrival rate in our baseline model of Section 2, while unobservable, is constant. In this section, we generalize the baseline model to allow for the unobservable disaster arrival rate to be stochastic.[1] We assume that the disaster arrival rate follows a two-state continuous-time Markov chain taking two possible values, λ_G in state G and $\lambda_B > \lambda_G$ in state B. Let φ_G denote the transition rate from state G to state B and φ_B denote the transition rate from state B to state G. That is, over a small time period Δt, the transition probability from the G state to the B state is $\varphi_G \Delta t$ and similarly the transition probability from the B state to the G state is $\varphi_B \Delta t$. Our baseline unobservable constant λ model of Section 2 is a special case of this model with $\varphi_G = \varphi_B = 0$.

OA.1 Model

As in our baseline model, let π_t denote the conditional probability that the economy is in state B. The belief process $\{\pi_t\}$ evolves as:

$$d\pi_t = \mathbb{E}_{t-}[d\pi_t] + \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) , \tag{OA.1}$$

where $\sigma_\pi(\pi)$ is given by (11) and $\lambda_{t-} = \lambda_B \pi_{t-} + \lambda_G(1-\pi_{t-})$ is the expected disaster arrival rate at $t-$ given in (9). Note that the second term is a martingale by construction. Since the economy follows a two-state Markov chain, the expected change of belief is given by

$$\mathbb{E}_{t-}[d\pi_t] = (\varphi_G - (\varphi_B + \varphi_G)\pi_{t-})dt\,. \tag{OA.2}$$

We can thus rewrite (OA.1) as follows:

$$d\pi_t = (\varphi_G - (\varphi_B + \varphi_G)\pi_{t-})dt + \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) . \tag{OA.3}$$

Equation (OA.3) implies that π_t in our generalized model is no longer a martingale. This is in sharp contrast with our baseline model (with constant arrival rate), where belief π_t given in (10) is a martingale. Rewriting the drift term in (OA.3), we see that the expected change of belief π_t in our generalized learning model is given by the difference between $\varphi_G(1-\pi_{t-})$, which is the transition rate out of state G, φ_G, multiplied by $1-\pi_{t-}$, the conditional probability in state G, and $\varphi_B\pi_t$, which is the transition rate out of state B, φ_B, multiplied by π_{t-}, the conditional probability in state B.[2]

We note that the jump martingale term (the second term in (OA.3)) in our generalized model is the same as in the belief updating process (10) for our baseline model. As a result, when a disaster strikes at t, the belief immediately increases from the pre-jump level π_{t-} to $\pi_t = \pi^{\mathcal{J}}$ by $\sigma_\pi(\pi_{t-})$, where $\pi^{\mathcal{J}}$ is given by (12), the same as in our baseline model with unobservable constant arrival rate λ.

[1] Ghaderi, Kilic, and Seo (2022) also develops a Bayesian learning model that builds on Wachter (2013).

[2] As a result, when $\pi_t = 0$ (in the G state for sure), the drift of belief π_t is exactly φ_G, the arrival rate from the G to the B state. Similarly by symmetry, when $\pi_t = 1$ (in the B state for sure), the drift is exactly $-\varphi_B$.

Taking these results together, absent jump arrivals (i.e., $d\mathcal{J}_t = 0$), we obtain the following expression for the rate at which belief changes, $\widehat{\mu}_\pi(\pi_{t-}) = d\pi_t/dt$:

$$\widehat{\mu}_\pi(\pi) = (\varphi_G - (\varphi_B + \varphi_G)\pi) - \pi(1-\pi)(\lambda_B - \lambda_G)\,. \tag{OA.4}$$

Generalizing the unobservable λ from a constant to a stochastic process (two-state Markov chain) does not change the belief updating upon the immediate arrival of a jump. However, belief updating conditional on no jump arrival is different from the baseline case with unobservable constant arrival rate λ.

Next, we calculate the posterior belief π_t at t conditional on no jump arrival over the time interval (s,t), i.e., $dJ_v = 0$ for $s < v \leq t$. Using (OA.3) and integrating $\{\pi_v; v \in (s,t)\}$ from s to t conditional on no jump over the interval (s,t), we obtain the following function:

$$\pi_t \;=\; \pi_s - \frac{2(\delta_0\pi_s^2 + \delta_1\pi_s + \delta_2)(e^{-\sqrt{\delta_1^2-4\delta_0\delta_2}(t-s)} - 1)}{(\sqrt{\delta_1^2 - 4\delta_0\delta_2} + \delta_1 + 2\delta_0\pi_s)(e^{-\sqrt{\delta_1^2-4\delta_0\delta_2}(t-s)} - 1) + 2\sqrt{\delta_1^2 - 4\delta_0\delta_2}}\,, \tag{OA.5}$$

where $\delta_0 = -(\lambda_G - \lambda_B)$, $\delta_1 = \lambda_G - \lambda_B - (\varphi_G + \varphi_B)$, and $\delta_2 = \varphi_G$. For our baseline model ($\varphi_G = \varphi_B = 0$), π_t in (OA.5) can be simplified to (14).

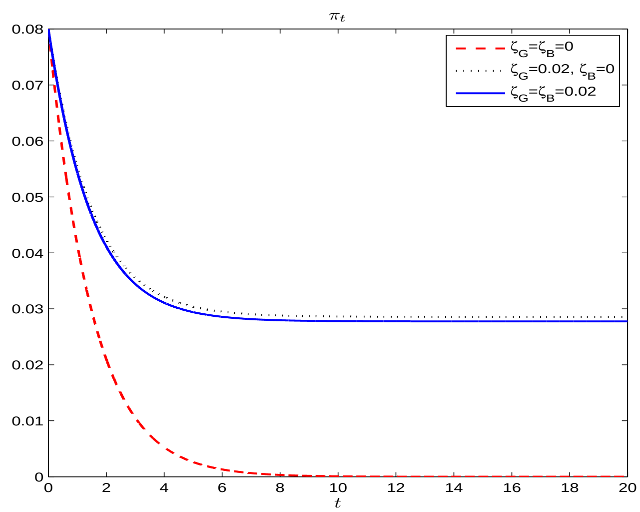

Figure O-1: This figure plots the time series of π_t absent jumps in our generalized model, where the jump arrival rate, λ, is unobservable and follows a two-state Markov chain taking on two possible values ($\lambda_G = 0.1$ and $\lambda_B = 0.8$) with a prior of $\pi_0 = 0.08$ that the current value of λ is λ_B. Our baseline model with constant unobservable λ corresponds to $\varphi_G = \varphi_B = 0$ (the dashed red line).

In Figure O-1, we plot the belief process $\{\pi_t : t \in (0,20)\}$ conditional on no jump arrival, which means $d\mathcal{J}_v = 0$ where $v \in (0,t) = (0,20)$, for three cases: 1.) the stationary case with $\varphi_G = \varphi_B = 2\%$ (the solid blue line); 2.) the case with $\varphi_G = 2\%$ and $\varphi_B = 0$, where the economy is eventually absorbed at the B state, (the dotted black line); and 3.) the baseline constant λ case as $\varphi_G = \varphi_B = 0$ (the dashed red line). The prior for the low value of λ is set at $\pi_0 = 0.08$ for all three cases.

First, for the two cases with stochastic λ, π_t decreases with t even absent jump arrivals. For example, the solid blue line (for the $\varphi_G = \varphi_B = 2\%$ case) shows that π_t slowly decreases to

0.0277 in twenty years absent jump arrivals. For the other case where the B state is absorbing ($\varphi_B = 0$), π_t decreases to 0.0285 at $t = 20$ absent jumps (the dotted black line.) The belief dynamics for these two cases with stochastic λ are similar to the dynamic for our constant unobservable λ model (the dashed red line), which shows that π_t decreases over time to zero and the agent becomes more optimistic (the no-news-is-good-news result), and the only difference is the long-run mean absent jump arrivals. So long as the transition rates φ_G and φ_B are small (which is the practically relevant case), our baseline model (with constant unobservable λ) and the stochastic unobservable λ model generate similar quantitative predictions. For parsimony, we use the constant λ model for our quantitative analysis in the paper.

OA.2 Solution

Using the belief process $\{\pi_t\}$ given in (OA.3), we obtain the following HJB equation for the planner's allocation problem:

$$\begin{aligned} 0 \quad = \quad & \max_{\mathbf{C},\mathbf{I},\mathbf{x}^e\,\mathbf{x}^d} \; f(\mathbf{C},V) + (\mathbf{I} - \delta_K \mathbf{K}) V_{\mathbf{K}}(\mathbf{K},\pi) + \widehat{\mu}_\pi(\pi) V_\pi(\mathbf{K},\pi) + \frac{1}{2}\sigma_K^2 \mathbf{K}^2 V_{\mathbf{KK}}(\mathbf{K},\pi) \\ & + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[V\left((1 - N(\mathbf{x}^e)(1-Z))\mathbf{K}, \pi^{\mathcal{J}}\right) - V(\mathbf{K},\pi)\right] \;, \end{aligned} \tag{OA.6}$$

where $\widehat{\mu}_\pi(\pi)$ is given in (OA.4). The FOCs for aggregate investment $\mathbf{I}$, (scaled) aggregate disaster distribution mitigation spending $\mathbf{x}^d$, and (scaled) aggregate disaster exposure mitigation spending $\mathbf{x}^e$ are the same as those for our baseline model (with constant unobservable λ), which are given in (18), (19), and (20), respectively.

Substituting the value function $V(\mathbf{K},\pi)$ given in (21) and its derivatives into the HJB equation (OA.6), using the three FOCs ((18), (19), and (20)), and simplifying these equations, we obtain the four-equation ODE system for $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$, given in

$$\begin{aligned} 0 \quad = \quad & \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(\pi)}{\rho(1+\phi'(\mathbf{i}(\pi)))}\right]^{1-\psi} - 1\right] + \mathbf{i}(\pi) - \delta_K - \frac{\gamma\sigma_K^2}{2} + \widehat{\mu}_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\ & + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi)}((1 - N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}) - 1\right] . \end{aligned} \tag{OA.7}$$

and (23)-(25) for $\pi \in (0,1)$. The key difference between (OA.7) and the ODE (22) for $b(\pi)$ in our baseline model (with constant but unobservable λ) is that the drift of π absent jumps, $\widehat{\mu}_\pi(\pi)$ given in (OA.4), appears in (OA.7) while $\mu_\pi(\pi)$ given in (13) appears in the ODE (22).[3] The other three equations for $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$ for our stochastic λ model are (23), (24), and (25), the same as those for our baseline model of Section 3. The boundary conditions at the $\pi = 0$ and $\pi = 1$ states are implied by the preceding equations.

Next, we summarize the solution for our generalized learning model.

Proposition 6 *The first-best solution for our generalized learning model is given by the value function (21) and the quartet policy rules,* $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$, *and* $\mathbf{x}^e(\pi)$, *where* $0 \le \pi \le 1$, *via the four-equation ODE system ((OA.7), (23), (24), and (25)).*

[3] The wedge $\widehat{\mu}_\pi(\pi) - \mu_\pi(\pi) = (\varphi_G - (\varphi_B + \varphi_G)\pi)$ precisely captures the effect of stochastic transition between the G and B states.

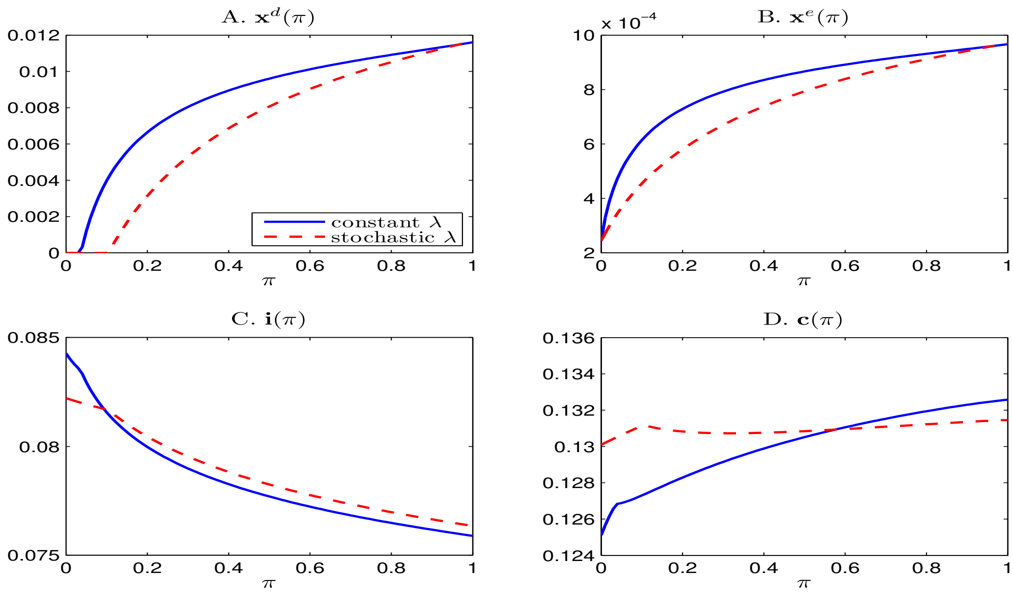

Figure O-2: This figure compares two learning models: the constant λ and the stochastic λ models. The transition rates are $\varphi_G = \varphi_B = 0.02$ for the stochastic λ model (solid blue lines). The transition rates are $\varphi_G = \varphi_B = 0$ for our baseline (constant λ) model (dashed red lines).

OA.3 Quantitative Analysis

Next, we analyze the solutions for our generalized model with stochastic unobservable λ. For the stochastic λ model, we set both the transition rate from state G to B (φ_G) and that from state B to G (φ_B) to 2%, i.e., $\varphi_G = \varphi_B = 1/50 = 2\%$, which imply an average duration of 50 years for both G and B states. In the long run, the economy is in either state G or B with equal (50%) probability. To ease exposition and facilitate comparison with our baseline (constant unobservable λ) model, we keep all other parameter values unchanged.

In Figure O-2, we plot (scaled) public mitigation $\mathbf{x}^d(\pi)$ (Panel A), (scaled) private mitigation $\mathbf{x}^e(\pi)$ (Panel B), investment-capital ratio $\mathbf{i}(\pi)$ (Panel C), and consumption-capital $\mathbf{c}(\pi)$ (Panel D) as functions of belief π for the planner's first-best solutions: the solid blue lines are for the baseline constant λ model and the dashed red lines are for the stochastic λ model.

Panels A and B show that for both public mitigation $\mathbf{x}^d(\pi)$ and private exposure mitigation $\mathbf{x}^e(\pi)$ are significantly lower for the stochastic λ model, and this is intuitive because the agent is exposure to less uncertainty about the belief due the mean reversion of π in the stochastic λ model, which induces less mitigation motivation. Quantitatively, the differences for investment and consumption are of very small (second- and third-order effects, as we can see from the scale for the vertical axes in Panels C and D.) This is because the transition of λ occurs once every fifty years on average.

Note that investment and consumption are even flatter (less responsive to changes of belief) in the stochastic λ model than in the constant λ model. Figure O-3 corroborates the belief mean reversion effect on welfare, growth, and valuation by showing that the welfare measure, the WTP $\zeta_p(\pi)$ (Panel A), the expected growth rate $\mathbf{g}(\pi)$ (Panel C), Tobin's average q, and the risk-free rate $r(\pi)$ are all smoother (flatter) as functions of π in the stochastic λ model than in the constant λ model.

The intuition is as follows. As belief mean reversion in the stochastic λ model, the agent is less optimistic in the low-π state but also less pessimistic in the high-π state, in the stochastic λ model, i.e., compared with the constant λ model. As a result, the planner reduces both consumption and investment in response to changes of belief (so that the planner better smoothes investment/consumption across states and over time.)

In sum, our analysis shows that for plausible values of slow belief mean reversion, the quantitative results of our learning model (with stochastic λ) are similar to those of our learning model (with constant λ). And we confirm the intuition that belief mean reversion reduces the impact of learning on welfare, valuation and policy rules.

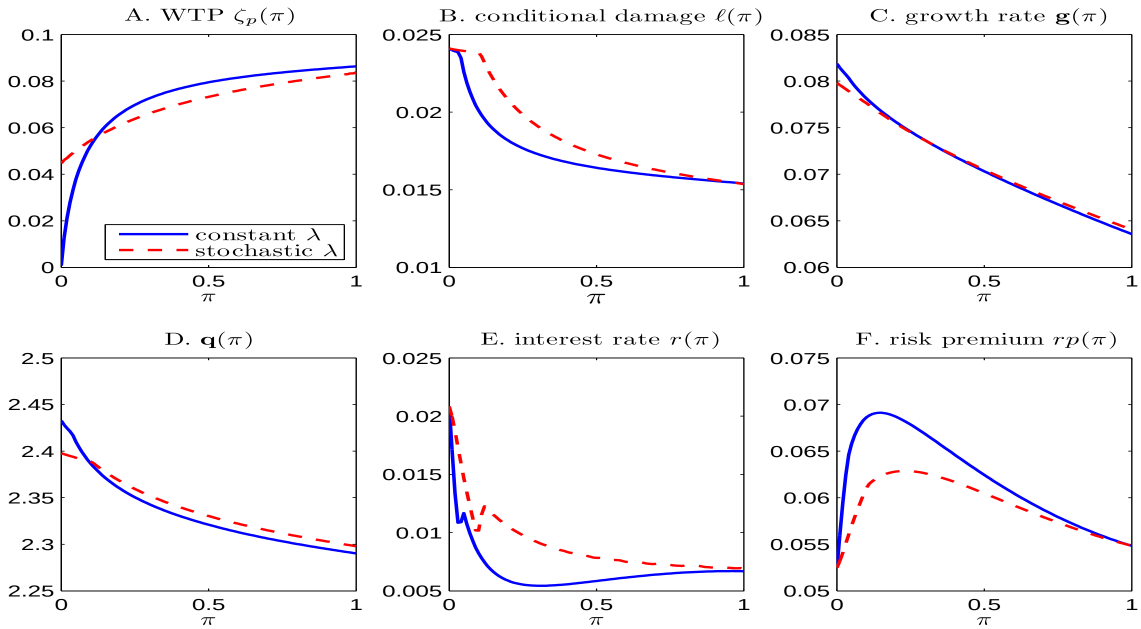

Figure O-3: This figure compares two learning models: the constant λ and the stochastic λ models. The transition rates are $\varphi_G = \varphi_B = 0.02$ for the stochastic λ model (solid blue lines). The transition rates are $\varphi_G = \varphi_B = 0$ for our baseline (constant λ) model (dashed red lines).

OB External Habit Model

In this appendix, we solve the model with external habit (Campbell-Cochrane) preferences (Section 7.7) and provide a quantitative analysis.[4]

OB.1 Model

The representative agent has a non-expected utility over consumption $\{C_t; t \geq 0\}$ relative to a stochastic habit process $\{\mathcal{H}_t; t \geq 0\}$ (Campbell and Cochrane, 1999) given by:

$$\mathbb{E}\left(\int_0^\infty \rho e^{-\rho t} U(C_t, \mathcal{H}_t) dt\right) , \tag{OB.8}$$

[4] An alternative to the external habit model analyzed in this section is to specify an internal habit formation model as in Jermann (1998). Due to space constraints, we leave the internal habit formation model out.

where $\rho > 0$ is the time rate of preference, $U(C, \mathcal{H}) = \frac{(C-\mathcal{H})^{1-\gamma}}{1-\gamma}$, and $\gamma > 0$ is a curvature parameter. It is convenient to work with S_t, the surplus consumption ratio at t defined as

$$S_t = \frac{C_t - \mathcal{H}_t}{C_t} \,. \tag{OB.9}$$

Let s_t be its natural logarithm: $s_t = \ln(S_t)$. As in Campbell and Cochrane (1999) and this literature, we assume that s_t follows a mean-reverting process with stochastic volatility:

$$ds_t = (1-\kappa_s)(\overline{s} - s_t)dt + \delta(s_t)\sigma_K d\mathcal{W}_t^K \,, \tag{OB.10}$$

where $\overline{s} > 0$ is the steady-state value of s_t and κ_s measures the degree of persistence.[5] The function $\delta(s_t)$ in (OB.10) is the same sensitivity function as the one in Campbell and Cochrane (1999). The production side of the economy and the learning model are the same as in our baseline model of Section 2.

Planner's solution. The (log) surplus consumption ratio $\{s_t; t \geq 0\}$ acting as the exogenous preferences shock is the new state variable. Let $V(\mathbf{K}, \pi, s)$ denote the household's value function. The following HJB equation characterizes the planner's optimal resource allocation:

$$\rho V = \max_{\mathbf{C}, \mathbf{I}, \mathbf{x}^e \mathbf{x}^d} \rho \frac{(\mathbf{C}e^s)^{1-\gamma}}{1-\gamma} + (\mathbf{I} - \delta_K \mathbf{K})V_{\mathbf{K}} + \mu_\pi(\pi)V_\pi + (1-\kappa_s)(\overline{s} - s)V_s + \frac{\sigma_K^2 \mathbf{K}^2 V_{\mathbf{KK}}}{2} + \frac{1}{2}\sigma_K^2 \delta(s)^2 V_{ss}$$
$$+ \sigma_K^2 \delta(s) \mathbf{K} V_{\mathbf{K}s} + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[V\left((1 - N(\mathbf{x}^e)(1-Z))\mathbf{K}, \pi^{\mathcal{J}}, s\right) - V(\mathbf{K}, \pi, s)\right] \,. \tag{OB.11}$$

Unlike in our baseline model with the Epstein-Zin utility, the agent now not only takes into account the evolution of s (via the drift term involving V_s and the quadratic-variation term involving V_{ss}), but also has incentives to hedge against shocks to the surplus consumption ratio (via the quadratic-covariation term involving $V_{\mathbf{K}s}$).

We show that the value function $V(\mathbf{K}, \pi, s)$ is homogeneous with degree $(1-\gamma)$ in $\mathbf{K}$:

$$V(\mathbf{K}, \pi, s) = \frac{1}{1-\gamma}\left(b(\pi, s)\mathbf{K}\right)^{1-\gamma} \,, \tag{OB.12}$$

where $b(\pi, s)$ is a measure of welfare proportional to the certainty equivalent wealth under optimality. (To ease comparison, we still use b as the function for the welfare measure here but with the understanding that the b function for external habit model depends on both π and s and differs from the b function for our baseline Epstein-Zin model.)

Importantly, unlike the welfare measure ($b(\pi)$) in our baseline model (Section 3), $b(\pi, s)$ in our external habit model depends on not only belief π but also the (log) surplus consumption ratio s. Our external habit model is technically more challenging than our baseline model with Epstein-Zin utility, as the external habit becomes an additional state variable in addition to capital stock and belief.[6]

In (OB.10), $\delta(s_t)$ is the sensitivity function proportional to the conditional volatility of ds_t in response to $d\mathcal{W}_t^K$, which we assume is given by the following square-root function:

$$\delta(s) = \frac{1}{\overline{S}}\sqrt{1 - 2(s - \overline{s})} - 1 \,, \qquad s \leq s_{\max} \tag{OB.13}$$

[5] We write $1-\kappa_s$ as the rate of mean reversion as in Campbell and Cochrane (2015). The higher the value of κ_s, the more persistent the s_t process. The $\kappa_s = 1$ special case corresponds to a unit-root process.

[6] Because of the homogeneity property of the Epstein-Zin utility, only capital stock and belief are state variables after simplifying the model solution.

and $\delta(s) = 0$ for $s > s_{\max}$, where $s_{\max} = \overline{s} + \frac{1-\overline{S}^2}{2}$ and $\overline{S} = e^{\overline{s}}$.[7]

OB.2 Solution

Substituting the value function given in (OB.12) into the HJB equation (OB.11), we obtain

$$\begin{aligned}
0 \;=\; & \max_{\mathbf{c},\,\mathbf{i},\,\mathbf{x}^e\,\mathbf{x}^d} \frac{\rho}{1-\gamma}\left[\left(\frac{\mathbf{c}(\pi,s)e^s}{b(\pi,s)}\right)^{1-\gamma} - 1\right] + (\mathbf{i}(\pi,s) - \delta_K) + \mu_\pi(\pi)\frac{b_\pi(\pi,s)}{b(\pi,s)} \\
& +(1-\kappa_s)(\overline{s}-s)\frac{b_s(\pi,s)}{b(\pi,s)} - \frac{\gamma\sigma_K^2}{2} + \frac{\sigma_K^2\delta(s)^2}{2}\left(\frac{b_{ss}(\pi,s)}{b(\pi,s)} - \gamma\frac{(b_s(\pi,s))^2}{b(\pi,s)^2}\right) \\
& +(1-\gamma)\sigma_K^2\delta(s)\frac{b_s(\pi,s)}{b(\pi,s)} + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}\left((1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma}\right) - 1\right].
\end{aligned}$$

Using the resource constraint $\mathbf{c} = A - \mathbf{i} - \phi(\mathbf{i}) - \mathbf{x}^d - \mathbf{x}^e$ to simplify the FOC for investment $\mathbf{i}$, we obtain the ODE system in the region where $\pi \in [0,1]$ and $s \in (-\infty, s_{\max})$:

$$\begin{aligned}
0 = & \frac{\rho}{1-\gamma}\left[\left(\frac{b(\pi,s)e^{-s}}{\rho(1+\phi'(\mathbf{i}(\pi,s)))}\right)^{1-\gamma^{-1}} - 1\right] + (\mathbf{i}(\pi,s) - \delta_K) + (1-\kappa_s)(\overline{s}-s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& + \mu_\pi(\pi)\frac{b_\pi(\pi,s)}{b(\pi,s)} - \frac{\gamma\sigma_K^2}{2} + \frac{\sigma_K^2\delta(s)^2}{2}\left(\frac{b_{ss}(\pi,s)}{b(\pi,s)} - \gamma\frac{(b_s(\pi,s))^2}{b(\pi,s)^2}\right) + (1-\gamma)\sigma_K^2\delta(s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi,s)}\left((1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma}\right) - 1\right], \qquad \text{(OB.14)}
\end{aligned}$$

and

$$b(\pi,s) = \left[A - \mathbf{i}(\pi,s) - \phi(\mathbf{i}(\pi,s)) - \mathbf{x}^d(\pi,s) - \mathbf{x}^e(\pi,s)\right]^{\gamma/(\gamma-1)}\left[\rho\mathbf{q}(\pi,s)\right]^{1/(1-\gamma)} e^s, \qquad \text{(OB.15)}$$

$$\frac{1}{\mathbf{q}(\pi,s)} = \lambda(\pi)\left[\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right]^{1-\gamma} N'(\mathbf{x}^e(\pi,s))\mathbb{E}^{\mathbf{x}^d(\pi,s)}\left[(Z-1)(1-N(\mathbf{x}^e(\pi,s))(1-Z))^{-\gamma}\right], \qquad \text{(OB.16)}$$

$$\frac{1}{\mathbf{q}(\pi,s)} = \frac{\lambda(\pi)}{1-\gamma}\left[\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right]^{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;\mathbf{x}^d(\pi,s))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma}\right]dZ, \qquad \text{(OB.17)}$$

where Tobin's q is given by the standard q-theoretic formula: $\mathbf{q}(\pi,s) = 1 + \phi'(\mathbf{i}(\pi,s))$.

Using the resource constraint $\mathbf{c} = A - \mathbf{i} - \phi(\mathbf{i}) - \mathbf{x}^d - \mathbf{x}^e$ to simplify the FOCs for mitigation spendings, $\mathbf{x}^e$ and $\mathbf{x}^d$, we obtain the optimal exposure mitigation and distribution mitigation spending rules, (OB.16) and (OB.17) for $\mathbf{x}^e$ and $\mathbf{x}^d$, respectively. The boundary conditions at the absorbing states ($\pi = 0$ and $\pi = 1$) are implied by the preceding equations.

[7] Additionally, we set $\overline{S} = \sigma_K\sqrt{\frac{\gamma}{1-\kappa_s}}$ as in Campbell and Cochrane (1999).

At $s = s_{\max}$, we have the following boundary condition:

$$0 = \frac{\rho}{1-\gamma}\left[\left(\frac{b(\pi, s_{\max})e^{-s_{\max}}}{\rho(1+\phi'(\mathbf{i}(\pi, s_{\max})))}\right)^{1-\gamma^{-1}} - 1\right] + (\mathbf{i}(\pi, s_{\max}) - \delta_K)$$
$$+ (1-\kappa_s)(\overline{s} - s_{\max})\frac{b_s(\pi, s_{\max})}{b(\pi, s_{\max})} - \frac{\gamma\sigma_K^2}{2} + \mu_\pi(\pi)\frac{b_\pi(\pi, s_{\max})}{b(\pi, s_{\max})}$$
$$+ \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}, s_{\max}\right)}{b(\pi, s_{\max})}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi,s_{\max})}((1-N(\mathbf{x}^e(\pi, s_{\max}))(1-Z))^{1-\gamma}) - 1\right]. \quad \text{(OB.18)}$$

Additionally, $\mathbf{i}(\pi, s_{\max})$, $\mathbf{x}^e(\pi, s_{\max})$ and $\mathbf{x}^d(\pi, s_{\max})$, satisfy (OB.15)- (OB.17) at $s = s_{\max}$.[8]

We summarize our model's solution in the following proposition.

Proposition 7 *The first-best solution for our external habit model is given by the value function (OB.12) and the quartet policy rules, $b(\pi, s)$, $\mathbf{i}(\pi, s)$, $\mathbf{x}^d(\pi, s)$, and $\mathbf{x}^e(\pi, s)$, where $0 \leq \pi \leq 1$ and $-\infty < s \leq s_{\max}$, via the four-equation ODE system (OB.14), (OB.15), (OB.16) and (OB.17), together with (OB.18) and (OB.15)-(OB.17) for $s = s_{\max}$.*

Next, we use the planner's solution to derive our model's asset pricing implications.

OB.3 Asset Pricing Implications

Using the planner's solution, we infer the SDF $\mathbb{M}_t$ process by applying Ito's Lemma to:

$$\mathbb{M}_t = e^{-\rho t}\frac{U_C(C_t, \mathcal{H}_t)}{U_C(C_0, \mathcal{H}_0)} = e^{-\rho t}\left(\frac{C_t S_t}{C_0 S_0}\right)^{-\gamma}. \quad \text{(OB.19)}$$

We then use the no-arbitrage restriction for the SDF to obtain the equilibrium risk-free rate, the market price of risk, and the stock market risk premium.

Then using Ito's lemma, we obtain

$$\frac{d\mathbb{M}_t}{\mathbb{M}_t} = -\rho dt - \gamma\left(\mathbf{i}(\pi, s) - \delta_K - \frac{\sigma_K^2}{2}\right)dt + (1-\kappa_s)(\overline{s} - s_t)\left((1-\gamma)\frac{b_s(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_s(\pi, s)}{\mathbf{q}(\pi, s)} - 1\right)dt$$
$$+ \mu_\pi(\pi)\left((1-\gamma)\frac{b_\pi(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_\pi(\pi, s)}{\mathbf{q}(\pi, s)}\right)dt - \left[(1-\gamma)\frac{b_s(\pi, s)^2}{b(\pi, s)^2} - \frac{\mathbf{q}_s(\pi, s)^2}{\mathbf{q}(\pi, s)^2}\right]\frac{(\sigma_K\delta(s))^2}{2}dt + \frac{\sigma_{\mathbb{M}}(\pi, s)^2}{2}dt$$
$$+ \left[(1-\gamma)\frac{b_{ss}(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_{ss}(\pi, s)}{\mathbf{q}(\pi, s)}\right]\frac{(\sigma_K\delta(s))^2}{2}dt - \sigma_{\mathbb{M}}(\pi, s)d\mathcal{W}_t^K + [\eta(\pi, s; Z, \mathbf{x}^e) - 1]\, d\mathcal{J}_t\,, \quad \text{(OB.20)}$$

where $\eta(\pi, s; Z, \mathbf{x}^e) = \frac{\mathbf{q}(\pi,s)}{\mathbf{q}(\pi^{\mathcal{J}},s)}\left(\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)}\right)^{1-\gamma}(1-N(\mathbf{x}^e(\pi, s))(1-Z))^{-\gamma}$ and

$$\sigma_{\mathbb{M}}(\pi, s) = \left[\left(1 + \frac{\mathbf{q}_s(\pi, s)}{\mathbf{q}(\pi, s)} - (1-\gamma)\frac{b_s(\pi, s)}{b(\pi, s)}\right)\delta(s) + \gamma\right]\sigma_K\,. \quad \text{(OB.21)}$$

[8] Note that as $s \to -\infty$ is not reachable in equilibrium, we can ignore the corresponding boundary conditions for our numerical analysis.

Using the equilibrium restriction that the drift of $\frac{d\mathbb{M}_t}{\mathbb{M}_t}$ equals $-r_{t-}dt$, we obtain:

$$\begin{aligned} r(\pi,s) &= \rho + \gamma\left(\mathbf{i}(\pi,s) - \delta_K - \frac{\sigma_K^2}{2}\right) - (1-\kappa_s)(\bar{s}-s_t)\left((1-\gamma)\frac{b_s(\pi,s)}{b(\pi,s)} - \frac{\mathbf{q}_s(\pi,s)}{\mathbf{q}(\pi,s)} - 1\right) \\ &\quad - \mu_\pi(\pi)\left((1-\gamma)\frac{b_\pi(\pi,s)}{b(\pi,s)} - \frac{\mathbf{q}_\pi(\pi,s)}{\mathbf{q}(\pi,s)}\right) + \left[(1-\gamma)\frac{b_s(\pi,s)^2}{b(\pi,s)^2} - \frac{\mathbf{q}_s(\pi,s)^2}{\mathbf{q}(\pi,s)^2}\right]\frac{(\sigma_K\delta(s))^2}{2} \\ &\quad - \left[(1-\gamma)\frac{b_{ss}(\pi,s)}{b(\pi,s)} - \frac{\mathbf{q}_{ss}(\pi,s)}{\mathbf{q}(\pi,s)}\right]\frac{(\sigma_K\delta(s))^2}{2} - \frac{\sigma_{\mathbb{M}}(\pi,s)^2}{2} - \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi,s;Z,\mathbf{x}^e)) - 1\right]. \end{aligned} \tag{OB.22}$$

Applying Ito's Lemma to firm value $Q(K,\pi,s) = q(\pi,s)K$ and using (OB.20), we obtain:

$$\begin{aligned} r(\pi,s)q(\pi,s) &= \max_{i,\,x^e}\ A - i - \phi(i) - x^e + (i - \sigma_{\mathbb{M}}(\pi,s)\sigma_K)q(\pi,s) + \mu_\pi(\pi)q_\pi(\pi,s) \\ &\quad + \left[(1-\kappa_s)(\bar{s}-s) + \delta(s)\sigma_K^2 - \sigma_{\mathbb{M}}(\pi,s)\delta(s)\sigma_K\right]q_s(\pi,s) + \frac{\sigma_K^2\delta(s)^2}{2}q_{ss}(\pi,s) \\ &\quad + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi,s;Z,\mathbf{x}^e)\left(q(\pi^{\mathcal{J}},s)(1-N(x^e)(1-Z)) - q(\pi,s)\right)\right]. \end{aligned} \tag{OB.23}$$

Finally, using the equilibrium conditions $q(\pi,s) = \mathbf{q}(\pi,s)$ and $x^e(\pi,s) = \mathbf{x}^e(\pi,s)$, we write

$$\begin{aligned} \frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} &= \left[\mu_{\mathbf{Q}}(\pi_{t-},s_{t-}) + \lambda(\pi_{t-})\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right]dt + \left[\frac{\mathbf{q}_s(\pi_{t-},s_{t-})\delta(s_{t-})}{\mathbf{q}(\pi_{t-},s_{t-})} + 1\right]\sigma_K d\mathcal{W}_t^K \\ &\quad + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)(d\mathcal{J}_t - \lambda(\pi_{t-})dt), \end{aligned} \tag{OB.24}$$

where $\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} = \frac{(1-N(\mathbf{x}_{t-}^e)(1-Z))\mathbf{q}\left(\pi_t^{\mathcal{J}},s_{t-}\right)}{\mathbf{q}(\pi_{t-},s_{t-})}$. and

$$\begin{aligned} \mu_{\mathbf{Q}}(\pi_{t-},s_{t-}) &= r(\pi_{t-},s_{t-}) + \sigma_{\mathbb{M}}(\pi_{t-},s_{t-})\left(1 + \delta(s_{t-})\frac{\mathbf{q}_s(\pi_{t-},s_{t-})}{\mathbf{q}(\pi_{t-},s_{t-})}\right)\sigma_K \\ &\quad + \lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\eta(\pi_{t-},s_{t-};Z,\mathbf{x}_{t-}^e)\left(1 - \frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right)\right]. \end{aligned} \tag{OB.25}$$

The market risk premium is

$$\begin{aligned} rp(\pi_{t-},s_{t-}) &= \sigma_{\mathbb{M}}(\pi_{t-},s_{t-})\left(1 + \delta(s_{t-})\frac{\mathbf{q}_s(\pi_{t-},s_{t-})}{\mathbf{q}(\pi_{t-},s_{t-})}\right)\sigma_K \\ &\quad - \lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\left(\eta(\pi_{t-},s_{t-};Z,\mathbf{x}_{t-}^e) - 1\right)\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right]. \end{aligned} \tag{OB.26}$$

Next, we calibrate the model and provide a quantitative analysis.

OB.4 Quantitative Analysis

The key new parameter for the external habit model is the (log) surplus consumption parameter κ_s. We set the persistence parameter for external habit at $\kappa_s = 0.87$ per annum as in Campbell and Cochrane (1999). For other parameter values, we borrow from the baseline model to ease exposition.

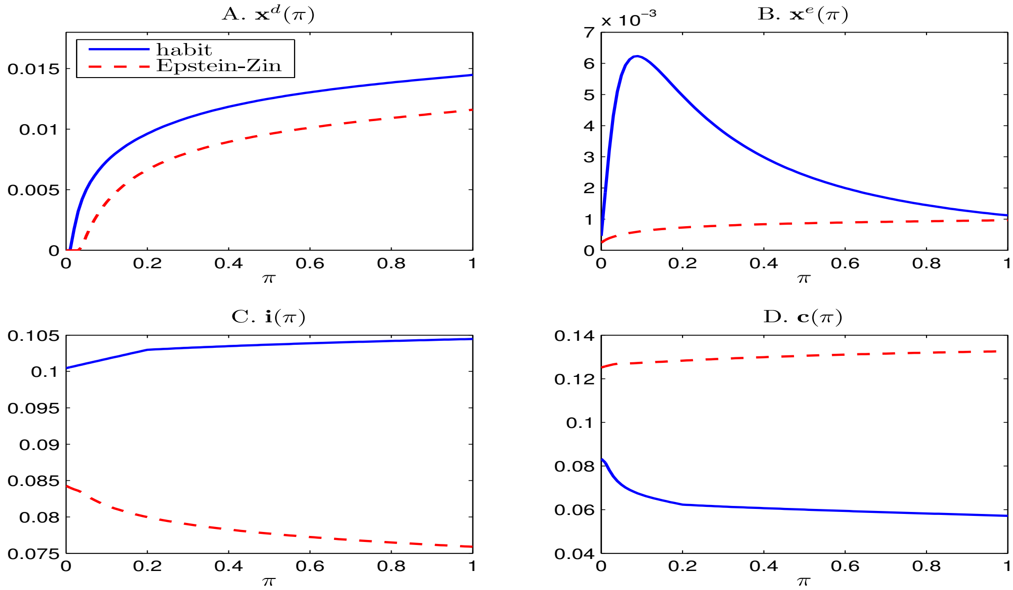

Figure O-4: This figure compares the first-best solutions for the external habit model (solid blue lines) and the baseline model with Epstein-Zin recursive utility (dashed red lines). The parameter values for our baseline (Epstein-Zin) model are summarized in Table 4.

OB.5 Quantitative Results

In Figures O-4 and O-5, we compare the external habit model at the steady state where $S = \overline{S} = 0.63$ with the Epstein-Zin recursive utility model. Panel A of Figure O-4 shows that the distribution mitigation $\mathbf{x}^d(\pi)$ policies for the two utility models are close to each other. However, Panel B of Figure O-4 shows that the exposure mitigation $\mathbf{x}^e(\pi)$ policies for the two models can differ somewhat for intermediate values of π. Nonetheless, our findings based on these two utility models suggest that our main results on how changes of belief impact disaster distribution and exposure adaptation spendings are reasonably robust.

Panel C of Figure O-4 shows that the investment-capital ratio is lower in our Epstein-Zin model than in the external habit model at the steady state where $S = \overline{S} = 0.63$. Panel D of Figure O-4 shows that the consumption-capital ratio is higher in our Epstein-Zin model than in the external habit model, which is expected as the sum of adaptation spending, investment, and consumption is the same and equals the productivity A in these two models.

It is interesting to note that while $\mathbf{i}(\pi)$ decreases with π for the Epstein-Zin utility model, $\mathbf{i}(\pi)$ increases with π in the external habit model. This difference is caused by the long-run risk force in the Epstein-Zin utility specification, where the EIS $\psi > 1$. To generate the prediction that worsening belief (increasing π) lowers Tobin's q and equivalently investment (as investment increases with Tobin's q), we require $\psi > 1$.

The external habit model differs from the baseline Epstein-Zin utility model in two ways. First, risk aversion is significantly enhanced by and also varies with external habit. Second, the EIS implied by our external habit model also generates a time-varying elasticity of intertemporal substitution (EIS). As risk aversion increases with habit stock, the EIS decreases. This is why our model predicts investment (and hence Tobin's q) increases with belief. Figure O-5 reports the WTP, conditional damage $\ell(\pi)$, the expected growth rate $\mathbf{g}(\pi)$, Tobin's average $q(\pi)$, the risk-free rate $r(\pi)$, and the market risk premium $rp(\pi)$. While there are

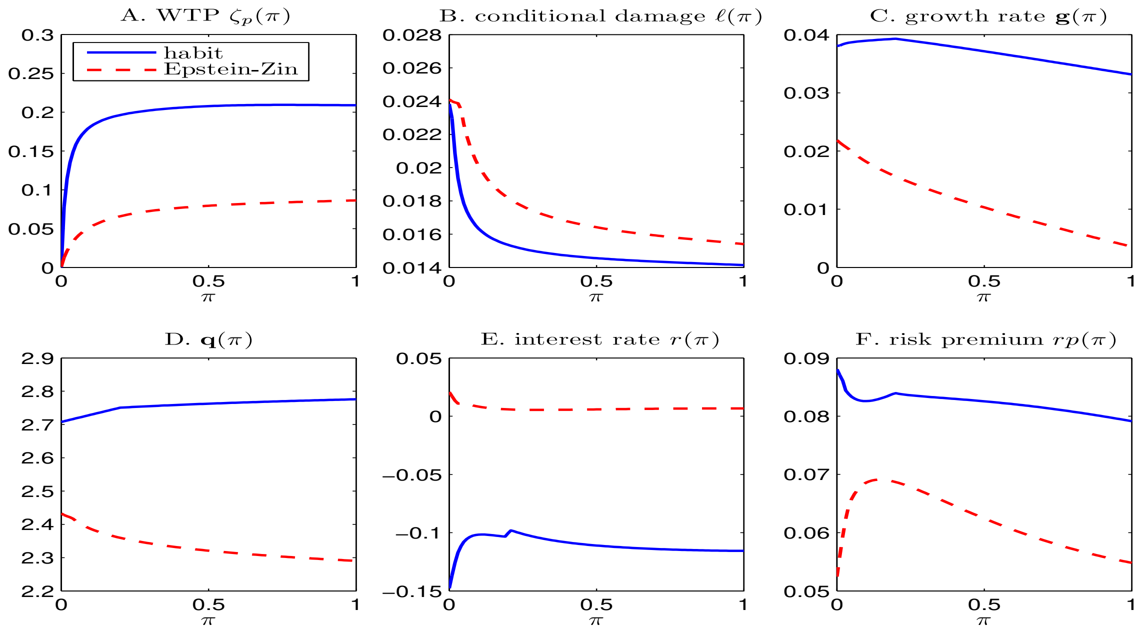

Figure O-5: This figure compares the first-best solutions for the external habit model (solid blue lines) and the baseline model with Epstein-Zin recursive utility (dashed red lines). The parameter values for our baseline (Epstein-Zin) model are summarized in Table 4.

some differences, we see that these two models generate similar results when it comes to the importance of adaptations in reducing conditional damage and insuring growth.

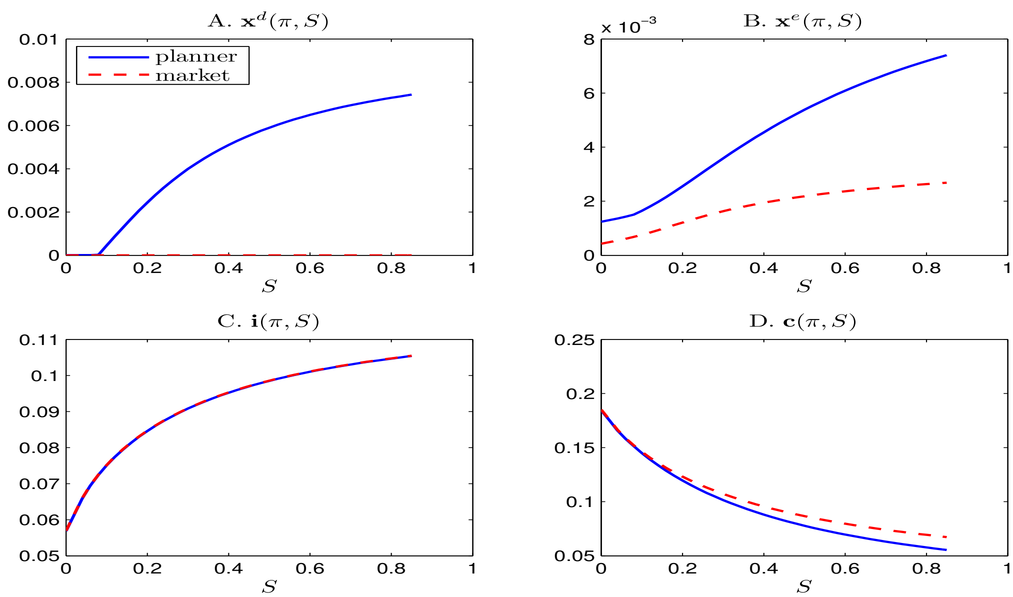

Figure O-6: This figure plots the optimal policies for the first-best economy (solid blue lines) and the market economy (dashed red lines) as functions of surplus consumption ratio S, for the external habit (Campbell-Cochrane) model, where $\pi_0 = 0.08$.

In Figure O-6, we focus on the external habit utility model by comparing two formulations:

the planner's first-best economy (solid blue lines) with the market economy solution (dashed red lines). We plot the two mitigation spending, investment, and consumption policies for varying levels of S, for a given belief $\pi_0 = 0.08$.

Panel A of Figure O-6 shows that there is no public mitigation in a competitive market economy for the same externality argument as in our baseline model with Epstein-Zin utility. This Panel also shows that $\mathbf{x}^d$ increases as the surplus consumption ratio increases. Similarly, both the exposure mitigation spending and investment increase with S (Panels B and C). The intuition for these results is as follows. As we increase S, the marginal utility of consumption (and SDF $\mathbb{M}_t$) decrease, which causes $\mathbf{c}$ to decrease with S (see Panel D). Additionally, the marginal value of investment and that of mitigation (for both types) increase, which causes $\mathbf{x}^d$, $\mathbf{x}^e$, and $\mathbf{i}$ to increase with S as shown in Panels A, B, and C).

Finally, we note that the private mitigation spending $\mathbf{x}^e$ is larger for the market economy than for the planner's economy. This is because the marginal benefit of private mitigation is higher in the market economy as there is no public mitigation. In contrast, as the public mitigation spending $\mathbf{x}^d$ is positive and significant under the planner's economy, the additional value of private mitigation spending in the planner's economy is smaller and hence $\mathbf{x}^e$ is smaller under the planner's economy than under the market economy (a substitution effect.)

In sum, we show that time-varying risk aversion induced by external habit influences optimal mitigation policies, but the general results that we obtain from our baseline model with Epstein-Zin utility remains valid in our external habit model.

OC Comparative Statics

OC.1 Elasticity of Intertemporal Substitution ψ

In Figure O-7, we plot the first-best solutions for three levels of the EIS ψ: $\gamma = 0.125, 1, 1.5$. Panels A and B show that the lower the EIS ψ the higher both public mitigation $\mathbf{x}^d$ and private mitigation $\mathbf{x}^e$ spendings. Quantitatively, these differences are not very large. Panel C shows that the lower the EIS ψ the higher the investment-capital ratio $\mathbf{i}(\pi)$. Panel D shows that the lower the EIS ψ the lower the consumption-capital ratio $\mathbf{c}(\pi)$, as $\mathbf{c} = A - (\mathbf{i} + \mathbf{x}^d + \mathbf{x}^e)$. Panel E shows that the lower the EIS ψ the higher Tobin's average $\mathbf{q}(\pi)$. This follows directly from the comparative static result of changing ψ on $\mathbf{i}$ (Panel C), as Tobin's q is increasing with $\mathbf{i}$: $\mathbf{q}(\pi) = 1 + \phi'(\mathbf{i}(\pi))$. Panel F shows that the lower the EIS ψ the higher the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$, which follows from the comparative effects shown in Panels D and E.

The intuition is as follows. The higher the EIS ψ, the less marginal propensity to consume as in partial equilibrium model consistent with Ramsey/Friedman consumption rule. As a result, the agent spends more on mitigation and also invests more for the future.

Additionally, we show that whether the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ increases or decreases when disaster arrives (which increases (worsens) belief π) crucially depends on whether the EIS ψ is larger or smaller than one. In our baseline case where $\psi = 1.5 > 1$, the equilibrium price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ decreases when a disaster arrives (i.e., when π increases). This result is consistent with Bansal and Yaron (2004) and the subsequent long-run risk literature, who show that the price-dividend ratio decreases in response to a negative growth shock when the EIS parameter ψ is set to be larger than one. Unlike Bansal and Yaron's pure exchange economy, our model features production and hence we need to compute the endogenous dividend $\mathbf{c}$ together with value of capital, Tobin's $\mathbf{q}$, in order to obtain the price-dividend ratio.

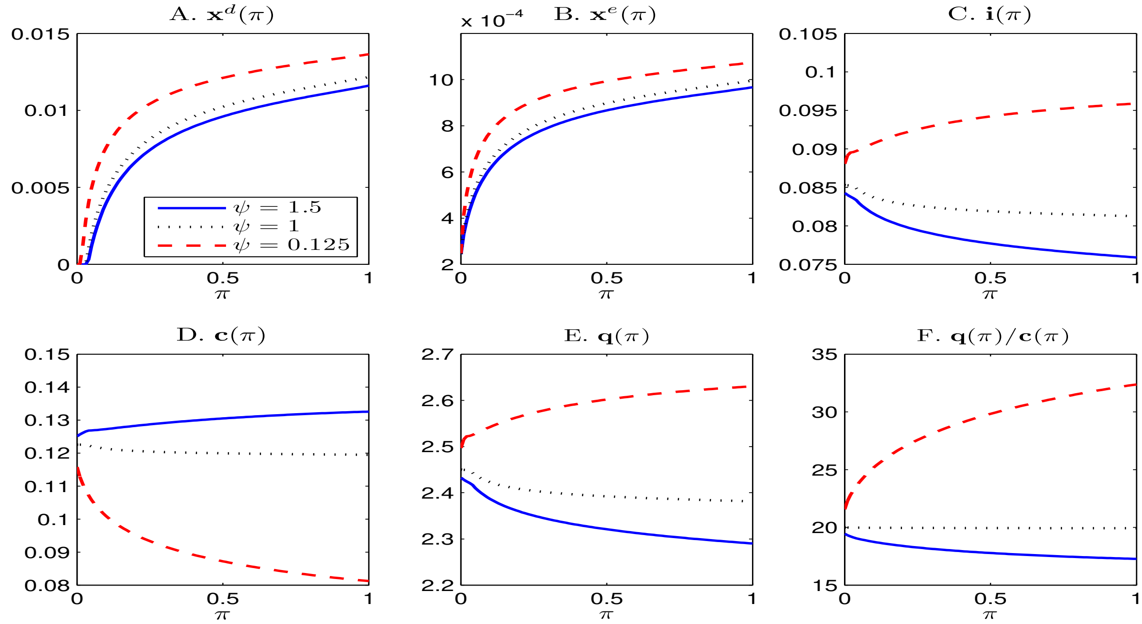

Figure O-7: This figure plots the first-best solution for three values of the EIS ψ: $1/\gamma = 0.125, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

However, we obtain the same results for the effect of EIS on the price-dividend ratio.

For the unity EIS ($\psi = 1$) Epstein-Zin utility case, which is a generalized version of expected logarithmic utility (with a flexible choice of risk aversion parameter γ), the wealth and the substitution effects exactly offset each other. As a result, the equilibrium price-dividend ratio remains constant, i.e., $\mathbf{q}(\pi)/\mathbf{c}(\pi) = 1/\rho = 20$ at all levels of π (See the dotted line in Panel F.) Finally, with $\psi = 1/\gamma = 0.125 < 1$, the wealth effect is stronger than the substitution effect. For this case, as belief worsens (increases), the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ increases, which is empirically counterfactual. This is one reason (among others) why Epstein-Zin utility with an EIS larger than one ($\psi > 1$) is a more appealing utility specification than commonly used expected utility for asset pricing.

In Figure O-8, we show that the the quantitative effects of EIS ψ on the WTP is large (Panel A). In Panel B, the lower the EIS ψ, the lower the conditional damages $\ell(\pi)$. This is because the agent with a lower EIS mitigates less as we show in Panels A and B of Figure O-8. As a result, the lower EIS the lower the conditional damages $\ell(\pi)$. Figure O-9 of Panel A shows that the lower the EIS ψ, the higher the expected growth rate $\mathbf{g}(\pi)$. This result follows from 1.) the lower the EIS the higher investment result (as shown in Panel C in Figure O-7) and 2.) the lower the EIS the higher damage $\ell(\pi)$ (as shown in Panel B of Figure O-8.)

Note that the effects of the EIS on the interest rate is ambiguous which depends on the agent's belief (Panel B). Panel C of Figure O-9 shows the higher the EIS the lower mitigation in equilibrium the higher risk premium.

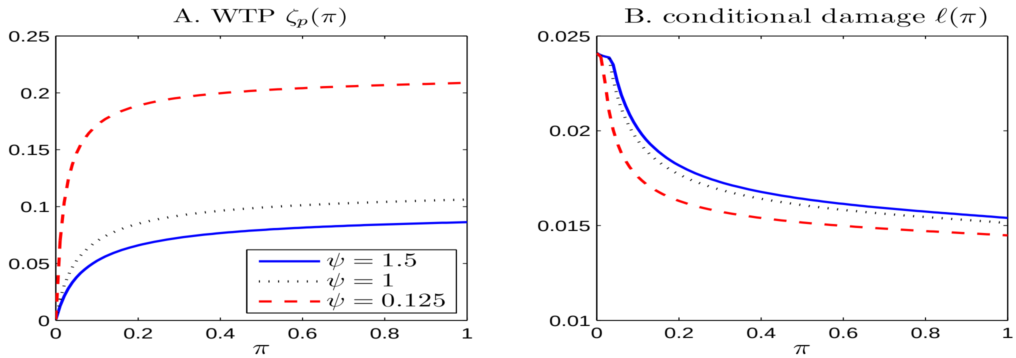

Figure O-8: This figure plots the planner's first-best solution for three values of the EIS ψ: $1/\gamma = 0.125, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

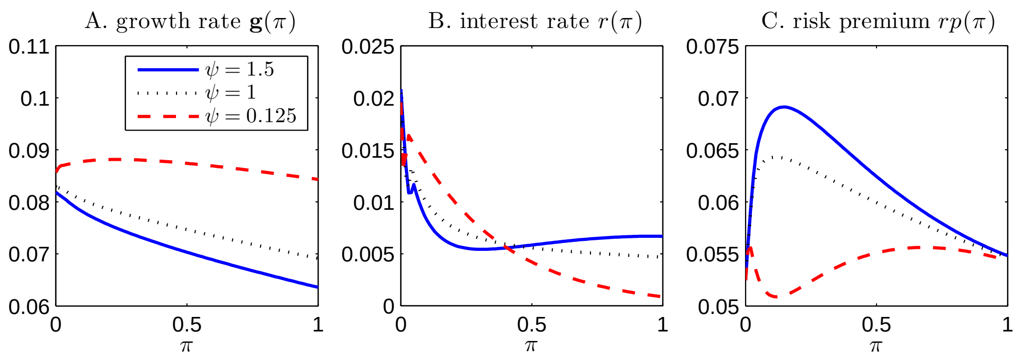

Figure O-9: This figure plots the planner's first-best solution for three values of the EIS ψ: $1/\gamma = 0.125, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

OC.2 Disaster Arrival Rate λ_B in State B

In Figure O-10, we plot the first-best solutions for three levels of the disaster arrival rate in state B: $\lambda_B = 0.4, 0.8, 1$. Panel A shows that the higher the disaster arrival rate λ_B in state B, the higher the public mitigation spending $\mathbf{x}^d$. Moreover, the more pessimistic the agent's belief the stronger this effect. Note that the wedge between the lines for two different levels of λ widens as π increases.

Panel B shows that increasing the arrival rate λ_B has a highly nonlinear effect on the private mitigation spending $\mathbf{x}^e$. Increasing λ_B from 0.4 to 0.8 significantly increases the mitigation spending (for sufficiently large values of π.) However, further increasing λ_B from 0.8 to 1 has limited effects on the mitigation spending. Panel C shows that as λ_B increases, investment falls. The higher the belief level π (the more pessimistic the agent) the larger the impact of λ_B on $\mathbf{i}$. Panel D shows that the impact of λ_B on consumption $\mathbf{c}$ is ambiguous due to the general equilibrium effect.

In Figure O-11, we show that λ_B has a large effect on the WTP ζ_p (Panel A). For example, when the belief changes from $\pi = 0$ to $\pi = 1$, the WTP increases from about 0 to 13% when $\lambda_B = 1$. In contrast, when $\lambda_B = 0.4$, the WTP barely changes from 0 to 2% in response to the

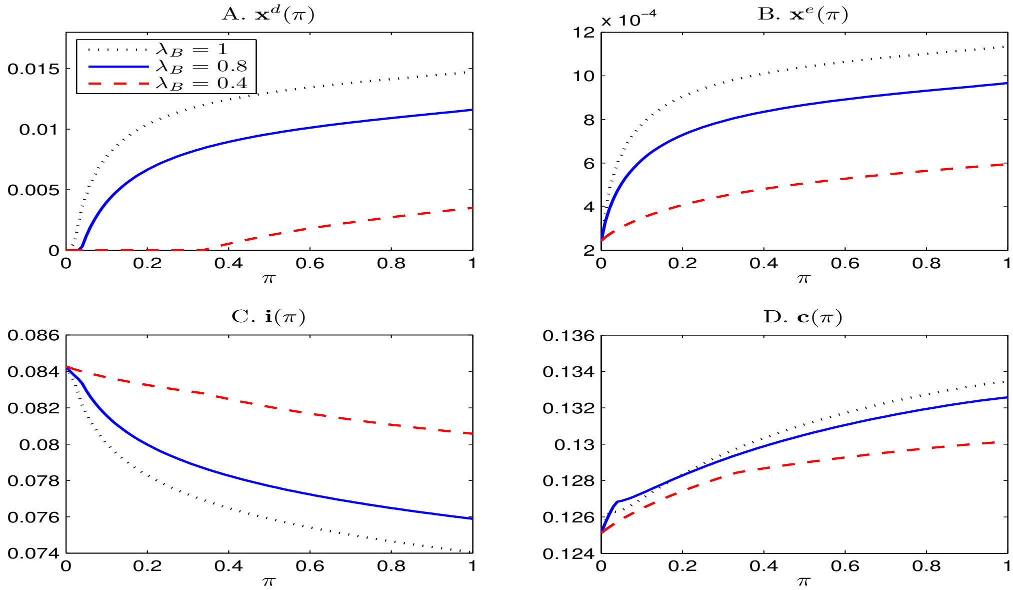

Figure O-10: This figure plots the planner's first-best solution for three values of the annual disaster arrival rate λ_B: $0.4, 0.8, 1$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

same change of the belief. Panel B shows that the higher the arrival rate λ_B the smaller the conditional damage $\ell(\pi)$. This is intuitive as mitigation spending is higher when λ_B is larger. However, as investment is lower when λ_B is larger, the impact of λ_B on the growth rate $\mathbf{g}(\pi)$ is minimal as the two channels (investment and conditional damage) offset each other (Panel C). Panel D shows that the higher the arrival rate λ_B the lower Tobin's $\mathbf{q}$, tracking the impact of λ_B on $\mathbf{i}(\pi)$ as $\mathbf{q}(\pi) = 1 + \theta \mathbf{i}(\pi)$. Panel E and Panel F show that the quantitative effects of λ_B on the risk-free rate r and the market risk premium rp are moderate at best.

OC.3 Time Rate of Preference ρ

In our baseline calculation, we set the time rate of preference ρ at 5% per annum, a commonly used value. Next, we compare our baseline model results with two other economies with lower discount rates: $\rho = 4.5\%$ and $\rho = 6\%$.

Panels A and B of Figure O-12 show that the higher the time rate of preference ρ, the less the planner spends on both types of mitigation spendings, $\mathbf{x}^d$ and $\mathbf{x}^e$. Similarly, Panel C of Figure O-12 shows that the higher the time rate of preference ρ, the less the planner invests and Panel D shows that the higher the time rate of preference ρ the more the agent consumes. The quantitative effects on consumption are large. For example increasing ρ from 4.5% to 6% roughly increases consumption $\mathbf{c}$ from 12% to 15% per annum.

In Figure O-13, we show that the quantitative effects of the time rate of preference ρ on the WTP is significant (Panel A). For example, when we change from $\pi = 0$ to $\pi = 1$, the WTP increases from about 0 to 6.7% when $\rho = 6\%$, and increases from 0 to 10% when $\rho = 4.5\%$.

The higher the time rate of preference ρ the higher the conditional damage $\ell(\pi)$ (Panel B) and the lower the Tobin's $\mathbf{q}$ (Panel D) as the agent is less patient and puts a smaller weight on the future. Since these two forces push towards the same direction, the higher the discount

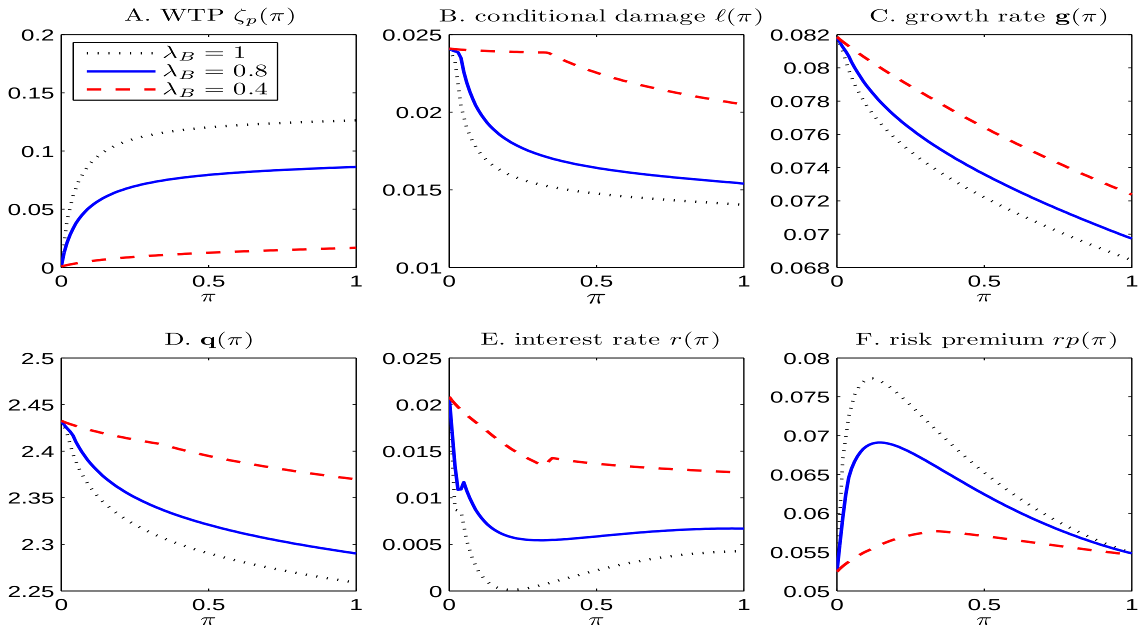

Figure O-11: This figure plots the planner's first-best solution for three values of the annual disaster arrival rate λ_B: $0.4, 0.8, 1$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

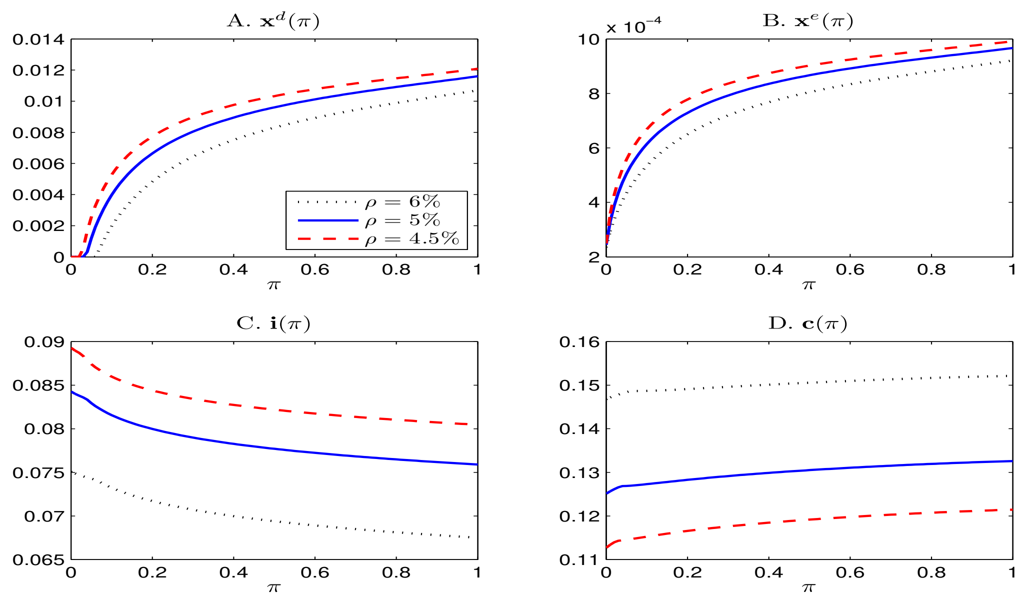

Figure O-12: This figure plots the planner's first-best solution for three values of the annual time rate of preference ρ: $4.5\%, 5\%, 6\%$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

rate ρ the lower growth rate $\mathbf{g}$ (Panel C).

Finally, Panel E shows that the quantitative effect of ρ on the risk-free rate r is moderate at best and Panel F shows that the effect of ρ on the market risk premium rp is very small.

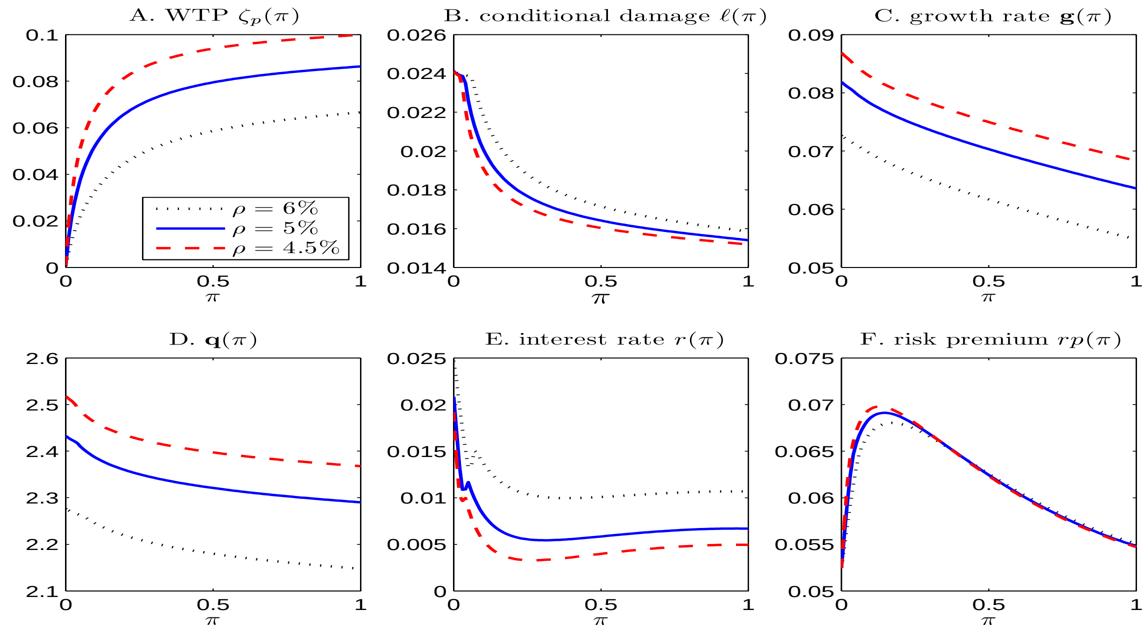

Figure O-13: This figure plots the planner's first-best solution for three values of the annual time rate of preference ρ: $4.5\%, 5\%, 6\%$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

OC.4 Coefficient of Relative Risk Aversion γ

In our baseline calculation, we set the coefficient of relative risk aversion γ at 8, which is within the range of widely used values (e.g., 2 to 10). Next, we compare our baseline model results to two other economies with $\gamma = 4$ and $\gamma = 10$.

Panel A of Figure O-14 shows that the higher the coefficient of relative risk aversion γ, the more the planner spends on distribution mitigation $\mathbf{x}^d$ and the less the planner spends on exposure mitigation $\mathbf{x}^e$. The higher the coefficient of relative risk aversion γ the less the planner invests (Panel C), the more the agent consumes (Panel D).

In Figure O-15, we show that the quantitative effects of increasing risk aversion from $\gamma = 4$ to $\gamma = 10$ on the WTP is large (Panel A). For example, as we increase γ from 4 to 10, the WTP ζ_p increases from 6.6% to 9.8% when the agent's belief is $\pi = 1$.

The higher the coefficient of relative risk aversion γ the lower the conditional damage $\ell(\pi)$ (Panel B of Figure O-15) and the lower the growth rate $\mathbf{g}(\pi)$ (Panel C of Figure O-15). This is because a more risk-averse agent mitigates more but invests less. Quantitatively, the negative effect of increasing γ via investment on growth dominates the positive effect of increasing γ via mitigation. As a result, the net effect of increasing γ on growth is negative.

Finally, Panels E and F of Figure O-15 show that the quantitative effects of γ on the risk-free rate r and the market risk premium rp are very large, as we expect (in line with standard asset pricing results.)

Panel A of Figure O-14 shows that the higher the coefficient of relative risk aversion γ, the more the planner spends on distribution mitigation $\mathbf{x}^d$ and the less the planner spends on exposure mitigation $\mathbf{x}^e$. The higher the coefficient of relative risk aversion γ the less the planner invests (Panel C), the more the agent consumes (Panel D).

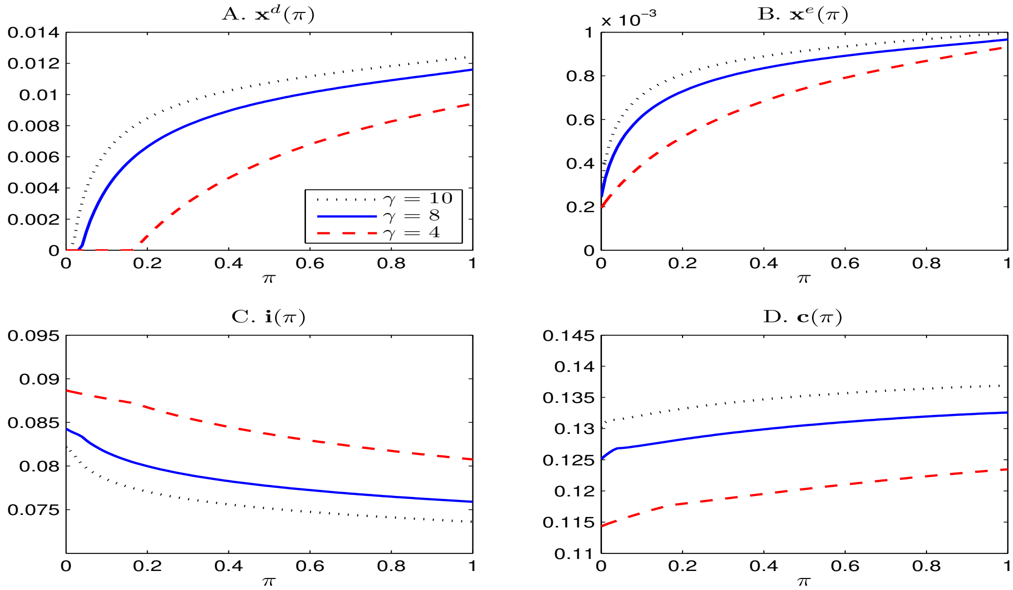

Figure O-14: This figure plots the planner's first-best solution for three values of coefficient of relative risk aversion γ: $4, 8, 10$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

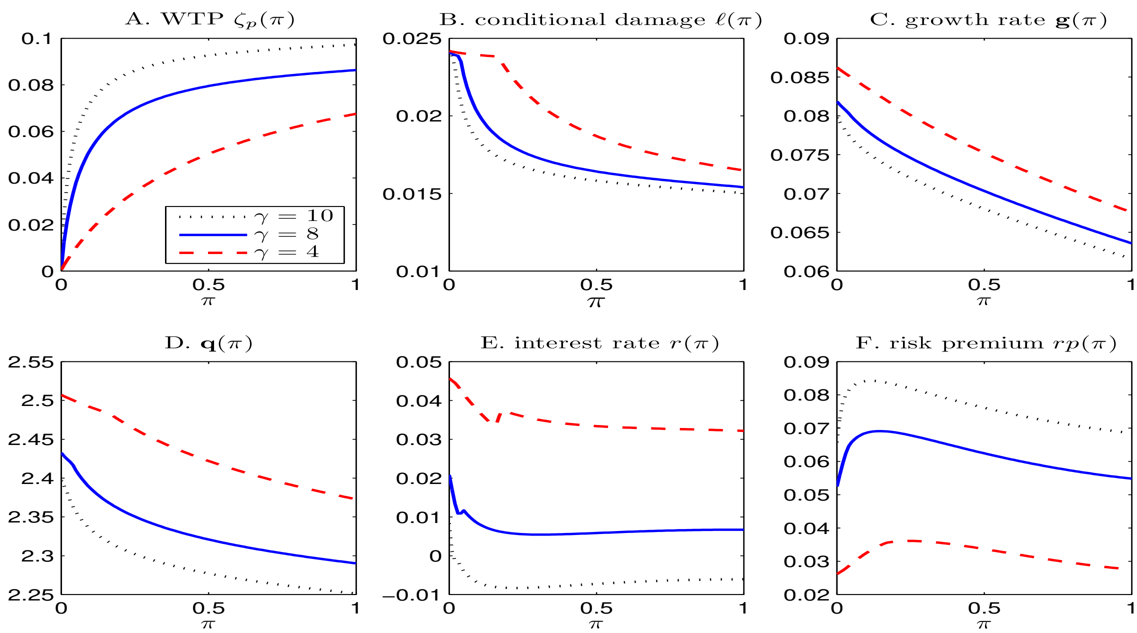

Figure O-15: This figure plots the planner's first-best solution for three values of the coefficient of relative risk aversion γ: $4, 8, 10$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

OC.5 Welfare Decomposition

In Figure O-18, we plot the welfare percentage gain, where welfare is measured in terms of willingness to pay (WTP), if we were to transition from our baseline first-best economy with

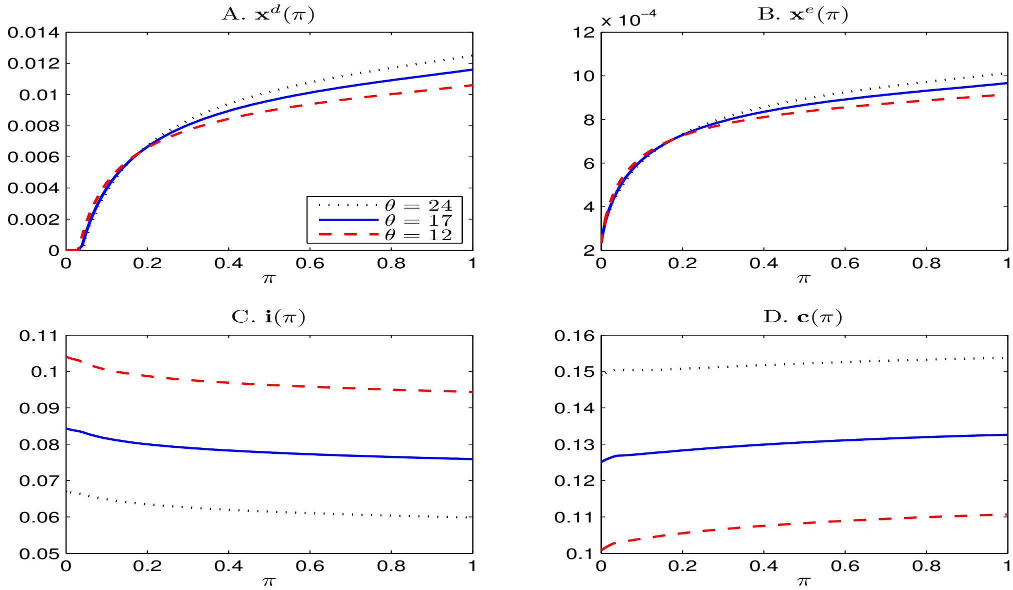

Figure O-16: This figure plots the planner's first-best solution for three values of adjustment cost θ: $12, 17, 24$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

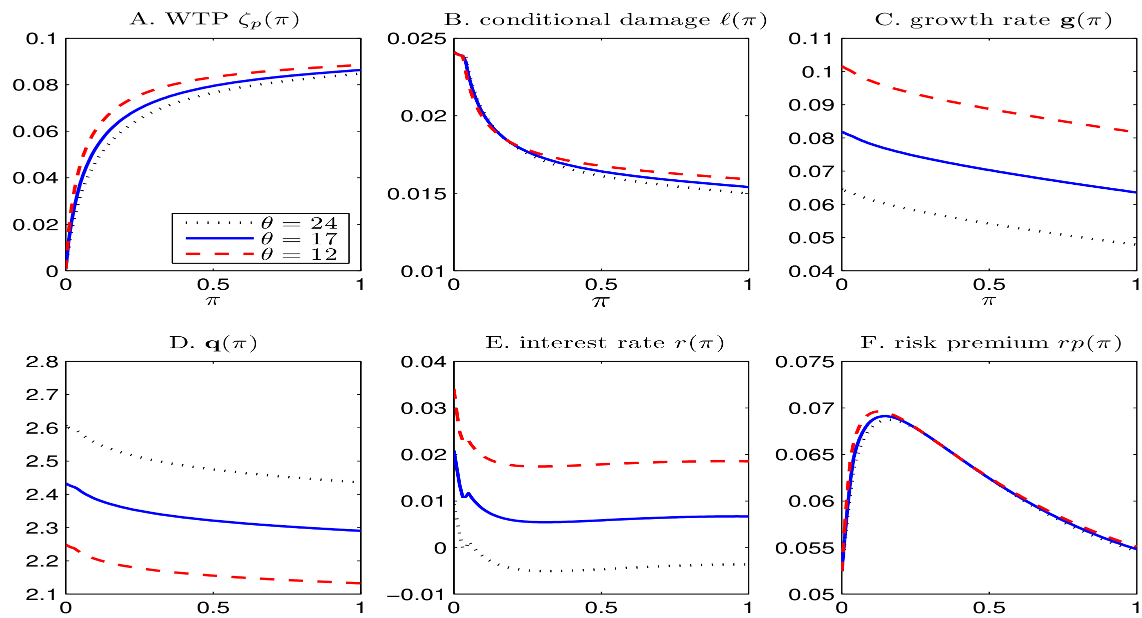

Figure O-17: This figure plots the planner's first-best solution for three values of adjustment cost θ: $12, 17, 24$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 4.

learning (analyzed in Section 3) to various newly constructed economies.

In Panel A, the newly constructed economy features no disaster shocks at all, i.e., $\lambda_G = \lambda_B = 0$, the percentage gain for the consumer's WTP (in units of certainty equivalent wealth) increases from about 7% when $\pi = 0$ to about 30% when $\pi = 1$, if the economy transitioned from our baseline economy to this economy with no disaster shocks at all. In Panel B, we shut down diffusion shocks in the newly constructed economy by setting $\sigma_K = 0$. The percentage WTP gain decreases from about 43% when $\pi = 0$ to about 39% when $\pi = 1$, if the economy transitioned from our baseline economy to this economy with $\sigma_K = 0$. In Panel C, the newly constructed economy features neither disaster (jump) shocks nor diffusion shocks, i.e., $\lambda_G = \lambda_B = \sigma_K = 0$. The percentage welfare gain for the representative consumer's WTP increases from about 47% when $\pi = 0$ to about 61% when $\pi = 1$, as we transition from our baseline economy (in Section 2) to this newly constructed economy with no risk at all.

Note that the WTP at $\pi = 1$ for this transition is 61%, lower than the sum of the WTP gain in Panel A (30%) and the WTP gain in Panel B (39%) by 8%. This more than 10% reduction of the WTP gain is due to the *interaction* between diffusive shocks and jump shocks in our learning model. That is, the total impact on WTP of shutting down both jump shocks ($\lambda_G = \lambda_B = 0$) and diffusion shocks ($\sigma_K = 0$) together is smaller than the sum of (1.) the effect on WTP by shutting down the jump shocks ($\lambda_G = \lambda_B = 0$ only and (2.) the effect on WTP by shutting down the diffusion shocks ($\sigma_K = 0$) only.

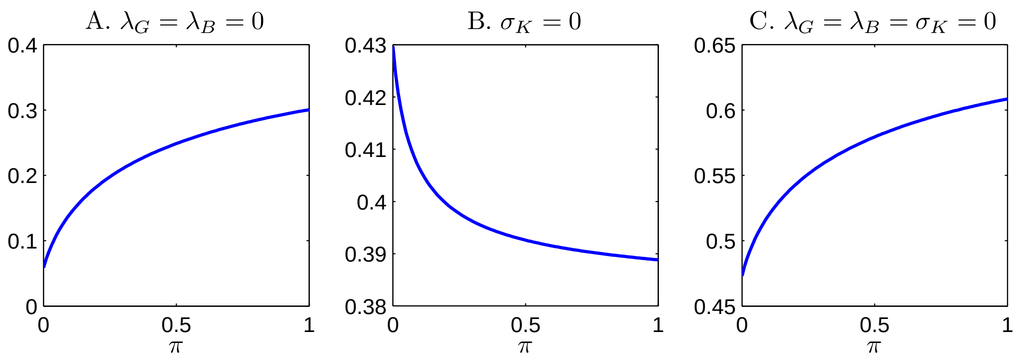

Figure O-18: **Willingness-to-pay (WTP) calculations.** Panel A plots the WTP percentage gain by changing $\lambda_G = 0.1$ and $\lambda_B = 0.8$ to $\lambda_B = \lambda_G = 0$. Panel B plots the WTP percentage gain by changing $\sigma_K = 8\%$ in our baseline economy to $\sigma_K = 0$. Panel C plots the WTP percentage gain by changing $\lambda_G = 0.1$, $\lambda_B = 0.8$, and $\sigma_K = 8\%$ to $\lambda_B = \lambda_G = \sigma_K = 0$.

OD A Generalized Model with Carbon Stock

In this appendix, we solve our generalized model with carbon stock.

OD.1 The PDE System

Using the FOCs and substituting the value function $V(\mathbf{K}, \mathbf{S}, \pi)$ given in (66) into the HJB equation (62), and simplifying the expressions, we obtain the following five-equation PDE system for $b(\pi,\mathbf{s})$, $\mathbf{i}(\pi,\mathbf{s})$, $\mathbf{x}^d(\pi,\mathbf{s})$, $\mathbf{x}^e(\pi,\mathbf{s})$, and $\mathbf{h}(\pi,\mathbf{s})$:

$$
\begin{aligned}
0 = & \frac{\rho}{1-\psi^{-1}}\left[\left[\frac{b(\pi,\mathbf{s})-\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}{\rho(1+\phi'(\mathbf{i}(\pi,\mathbf{s})))}\right]^{1-\psi}-1\right]+(\mathbf{i}(\pi,\mathbf{s})-\delta_K)\,\frac{b(\pi,\mathbf{s})-\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}-\frac{\gamma\sigma_K^2}{2}+\mu_\pi(\pi)\frac{b_\pi(\pi,\mathbf{s})}{b(\pi,\mathbf{s})} \\
& +(\mathbf{h}(\pi,\mathbf{s})-\delta_S\mathbf{s}+\gamma(\sigma_K^2-\vartheta\sigma_K\sigma_S)\mathbf{s})\frac{b_s(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}+\frac{(\sigma_S^2-2\vartheta\sigma_K\sigma_S+\sigma_K^2)\mathbf{s}^2}{2}\left(\frac{b_{\mathbf{ss}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}-\gamma\frac{(b_{\mathbf{s}}(\pi,\mathbf{s}))^2}{b(\pi,\mathbf{s})^2}\right) \\
& +\frac{\lambda(\pi)}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d(\pi,\mathbf{s})}\left(\frac{(1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z))b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)}{b(\pi,\mathbf{s})}\right)^{1-\gamma}-1\right], \qquad \text{(OD.27)}
\end{aligned}
$$

$$
\begin{aligned}
b(\pi,\mathbf{s}) = & \left[A\mathbf{h}(\pi,\mathbf{s})^{1-\alpha}-\mathbf{i}(\pi,\mathbf{s})-\phi(\mathbf{i}(\pi,\mathbf{s}))-\mathbf{x}^d(\pi,\mathbf{s})-\mathbf{x}^e(\pi,\mathbf{s})-p_H\mathbf{h}(\pi,\mathbf{s})\right]^{1/(1-\psi)} \\
& \left[\rho(1+\phi'(\mathbf{i}(\pi,\mathbf{s})))\frac{b(\pi,\mathbf{s})}{b(\pi,\mathbf{s})-\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}\right]^{-\psi/(1-\psi)}, \qquad \text{(OD.28)}
\end{aligned}
$$

$$
\frac{b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})-\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}=\frac{p_H-(1-\alpha)A\mathbf{h}(\pi,\mathbf{s})^{-\alpha}}{1+\phi'(\mathbf{i}(\pi,\mathbf{s}))}, \qquad \text{(OD.29)}
$$

$$
\begin{aligned}
\frac{1}{1+\phi'(\mathbf{i}(\pi,\mathbf{s}))} = & \lambda(\pi)\mathbb{E}^{\mathbf{x}^d(\pi,\mathbf{s})}\left[\frac{(Z-1)N'(\mathbf{x}^e(\pi,\mathbf{s}))\left(b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)-\mathbf{s}^{\mathcal{J}}b_{\mathbf{s}}\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)\right)}{b(\pi,\mathbf{s})}\right. \\
& \left.\times\left(\frac{(1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z))b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)}{b(\pi,\mathbf{s})}\right)^{-\gamma}\right], \qquad \text{(OD.30)}
\end{aligned}
$$

$$
\frac{1}{1+\phi'(\mathbf{i}(\pi,\mathbf{s}))}=\frac{\lambda(\pi)}{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;\mathbf{x}^d(\pi,\mathbf{s}))}{\partial\mathbf{x}^d}\left(\frac{(1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z))b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)}{b(\pi,\mathbf{s})}\right)^{1-\gamma}\right]dZ\,, \qquad \text{(OD.31)}
$$

where $\mathbf{s}^{\mathcal{J}}=\frac{\mathbf{s}}{1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z)}$ is the post-jump carbon-stock-to-productive-capital ratio $\mathbf{s}$.[9]

OD.2 Competitive Market Equilibrium Solution

Firm's Optimization Problem. Taking the equilibrium risk-free rate r_t and the market price of (diffusion and jump) risks, the firm maximizes its market value, $Q(K,\pi,\mathbf{s})$ given in (5), where $\{Y_t\}$ is the firm's payout process given in (68).

Applying Ito's Lemma to firm value $Q(K,\pi,\mathbf{s})=q(\pi,\mathbf{s})K$, we obtain:

$$
\begin{aligned}
r(\pi,\mathbf{s})q(\pi,\mathbf{s}) = & \max_{i,\,x^e,\,x^d,h}\ Ah^{1-\alpha}-p_Hh-i-\phi(i)-x^e-x^d+\left(i-\delta_K-\eta_{\mathbb{M}}^k(\pi,\mathbf{s})\sigma_K\right)q(\pi,\mathbf{s}) \\
& +\mu_\pi(\pi)q_\pi+\frac{(\sigma_S^2-2\vartheta\sigma_S\sigma_K+\sigma_K^2)\mathbf{s}^2}{2}q_{\mathbf{ss}} \\
& +\left[\mu_s(\pi,\mathbf{s})+\vartheta\sigma_S\sigma_K-\sigma_K^2-(\eta_{\mathbb{M}}^s(\pi,\mathbf{s})\vartheta\sigma_S-\eta_{\mathbb{M}}^k(\pi,\mathbf{s})\sigma_K)\right]\mathbf{s}q_{\mathbf{s}} \\
& +\lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi,\mathbf{s};Z,\mathbf{x}^e)\left(q(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}})(1-N(x^e)(1-Z))-q(\pi,\mathbf{s})\right)\right].
\end{aligned}
$$

Note that $x^d=0$ as no firm spends on public mitigation. The FOCs for i and x^e are

$$
q(\pi,\mathbf{s}) = 1+\phi'(i(\pi,\mathbf{s}))\,, \qquad \text{(OD.32)}
$$

$$
1 = -\lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[(1-Z)\eta(\pi,\mathbf{s};Z,\mathbf{x}^e)q(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}})N'(x^e)\right], \qquad \text{(OD.33)}
$$

[9] Recall that $\mathbf{s}$ is a mean-reverting process. Because $\pi=0$ and $\pi=1$ are absorbing states, we can obtain the boundary conditions at $\pi=0$ and $\pi=1$ by substituting $\pi=0$ and $\pi=1$ into (OD.27)-(OD.31).

We have a new FOC for the firm's fossil fuel usage, h, which is

$$(1-\alpha)Ah(\pi,\mathbf{s})^{-\alpha} = p_H\,. \tag{OD.34}$$

In equilibrium, the aggregate scaled fossil fuel, $\mathbf{h}(\pi,\mathbf{s})$, is constant and given by

$$\mathbf{h}(\pi,\mathbf{s}) = \left(\frac{(1-\alpha)A}{p_H}\right)^{\frac{1}{\alpha}}\,. \tag{OD.35}$$

Household's Optimization Problem. The household maximizes the value function J_t. We show that the value function is $J_t = J(W_t,\pi_t,\mathbf{s}_t)$ is given by

$$J(W,\pi,\mathbf{s}) = \frac{1}{1-\gamma}(u(\pi,\mathbf{s})W)^{1-\gamma}\,, \tag{OD.36}$$

where $u(\pi,\mathbf{s})$ is a welfare measure to be determined. The HJB equation is given by

$$\begin{aligned}
0 = \max_{C,\Gamma,X^e,X^d} \quad & f(C,J) + \mu_\pi(\pi)J_\pi + \lambda(\pi)\int_0^1 \left[J\left(W^{\mathcal{J}},\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right) - J(W,\pi,\mathbf{s})\right]\xi(Z;\mathbf{x}^d)dZ \\
& + \left[r(\pi,\mathbf{s})W + (\mu_{\mathbf{Q}}(\pi,\mathbf{s}) - r(\pi,\mathbf{s}))\Gamma - C\right]J_W + \mu_s(\pi,\mathbf{s})J_{\mathbf{s}} + \frac{(\sigma_S^2 - 2\vartheta\sigma_S\sigma_K + \sigma_K^2)\mathbf{s}^2 J_{\mathbf{ss}}}{2} \\
& + \frac{\left(\left(\frac{\mathbf{q}(\pi,\mathbf{s})-\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\sigma_K\right)^2 + 2\vartheta\frac{\mathbf{q}(\pi,\mathbf{s})-\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\frac{\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\sigma_K\sigma_S + \left(\frac{\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\sigma_S\right)^2\right)\Gamma^2 J_{WW}}{2} \\
& + \left(\frac{s\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}(\sigma_S^2 - \vartheta\sigma_K\sigma_S) + \frac{\mathbf{q}(\pi,\mathbf{s}) - s\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}(\vartheta\sigma_S\sigma_K - \sigma_K^2)\right)\Gamma\mathbf{s}J_{W\mathbf{s}}\,.
\end{aligned} \tag{OD.37}$$

Next, we show that by optimally choosing a tax on capital stock, a tax on fossil fuel usage, and a tax on investment, the government can attain the first-best outcome.

OD.3 Optimal Taxation in Market Economy Restores First-Best

In this appendix, we use the second formulation of optimal taxes in the main text, where the planner taxes a firm's fossil fuel usage and investment if they exceed the respective first-best levels. Anticipating that all three taxes are Markovian in $\mathbf{s}$ and π, we write these tax rates as $\tau^x(\pi,\mathbf{s})$, $\tau^h(\pi,\mathbf{s})$, and $\tau^i(\pi,\mathbf{s})$. Applying Ito's Lemma to firm value $Q(K_t,\pi_t,\mathbf{s}_t) = q(\pi_t,\mathbf{s}_t)K_t$ and using the SDF $\mathbb{M}_t$ given in (26), we obtain the following HJB equation for $q(\pi_t,\mathbf{s}_t)$:

$$\begin{aligned}
r(\pi,\mathbf{s})q(\pi,\mathbf{s}) = \max_{i,x^e,x^d,h} \quad & Ah^{1-\alpha} - \tau^x(\pi,\mathbf{s}) - \tau^h(\pi,\mathbf{s})(h-\mathbf{h}) - \tau^i(\pi,\mathbf{s})\left[i + \phi(i) - (\mathbf{i} + \phi(\mathbf{i}))\right] \\
& - p_H h - i - \phi(i) - x^e - x^d + \left(i - \delta_K - \eta_{\mathbb{M}}^k(\pi,\mathbf{s})\sigma_K\right)q(\pi,\mathbf{s}) \\
& + \left[\mu_s(\pi,\mathbf{s}) + \vartheta\sigma_S\sigma_K - \sigma_K^2 - (\eta_{\mathbb{M}}^s(\pi,\mathbf{s})\vartheta\sigma_S - \eta_{\mathbb{M}}^k(\pi,\mathbf{s})\sigma_K)\right]\mathbf{s}q_{\mathbf{s}} \\
& + \frac{(\sigma_S^2 - 2\vartheta\sigma_S\sigma_K + \sigma_K^2)\mathbf{s}^2}{2}q_{\mathbf{ss}} + \mu_\pi(\pi)q_\pi \\
& + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi,\mathbf{s};Z,\mathbf{x}^e)\left(q(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}})(1 - N(x^e)(1-Z)) - q(\pi,\mathbf{s})\right)\right]\,.
\end{aligned} \tag{OD.38}$$

As in our baseline model, firms have no incentives to spend on disaster distribution adaptation: $x^d = 0$. The FOC for x^e is the same as (OD.33) for the market economy without taxes

(Subsection OD.2). The FOC for h is given by $(1-\alpha)Ah(\pi,\mathbf{s})^{-\alpha} = p_H + \tau^h(\pi,\mathbf{s})$ and the FOC i is given by $q(\pi,\mathbf{s}) = (1+\phi'(i(\pi,\mathbf{s})))\,(1+\tau^i(\pi,\mathbf{s}))$.

Next, we show that the household's value in the market economy with taxes $J(W_t,\pi_t,\mathbf{s}_t)$ (Subsection 8.3) equals that in the first-best economy (Subsection 8.2). Using the equilibrium result in the market economy: $W_t = \mathbf{q}(\pi_t,\mathbf{s}_t)\mathbf{K}_t$, wewrite $J(W_t,\pi_t,\mathbf{s}_t) = J(\mathbf{q}(\pi_t,\mathbf{s}_t)\mathbf{K}_t,\pi_t,\mathbf{s}_t)$.

Combining the investment FOC, $\mathbf{q}(\pi,\mathbf{s}) = (1+\phi'(\mathbf{i}(\pi,\mathbf{s})))\frac{b(\pi,\mathbf{s})}{b(\pi,\mathbf{s})-\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}$, with the consumption FOC, $\mathbf{c}(\pi,\mathbf{s}) = \rho^{\psi}u(\pi,\mathbf{s})^{1-\psi}\mathbf{q}(\pi,\mathbf{s}) = (\rho\mathbf{q}(\pi,\mathbf{s}))^{\psi}\left[u(\pi,\mathbf{s})\mathbf{q}(\pi,\mathbf{s})\right]^{1-\psi}$, we obtain:

$$\mathbf{c}(\pi,\mathbf{s}) = \left[\rho(1+\phi'(\mathbf{i}(\pi,\mathbf{s})))\frac{b(\pi,\mathbf{s})}{b(\pi,\mathbf{s})-\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}\right]^{\psi}\left[u(\pi,\mathbf{s})\mathbf{q}(\pi,\mathbf{s})\right]^{1-\psi} . \tag{OD.39}$$

Using $b(\pi,\mathbf{s}) = u(\pi,\mathbf{s})\mathbf{q}(\pi,\mathbf{s})$ and the resource additivity condition, we obtain:

$$\begin{aligned} b(\pi,\mathbf{s}) \;=\; & [Ah(\pi,\mathbf{s})^{1-\alpha} - \tau^x(\pi,\mathbf{s}) - \tau^h(\pi,\mathbf{s})(h(\pi,\mathbf{s})-\mathbf{h}(\pi,\mathbf{s})) - \tau^i(\pi,\mathbf{s})\left[i(\pi,\mathbf{s})+\phi(i(\pi,\mathbf{s})) - (\mathbf{i}(\pi,\mathbf{s})+\phi(\mathbf{i}(\pi,\mathbf{s})))\right] \\ & -p_H h(\pi,\mathbf{s}) - \mathbf{i}(\pi,\mathbf{s}) - \phi(\mathbf{i}(\pi,\mathbf{s})) - \mathbf{x}^e(\pi,\mathbf{s})]^{1/(1-\psi)}\left[\rho(1+\phi'(\mathbf{i}(\pi,\mathbf{s})))\frac{b(\pi,\mathbf{s})}{b(\pi,\mathbf{s})-\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}\right]^{-\psi/(1-\psi)} \end{aligned} \tag{OD.40}$$

Under optimal taxes, (OD.40) is the same as the investment FOC, given in (OD.28), in the first-best economy. This is because (OD.40) summarizes both the consumer's and the firm's FOCs in the market economy with optimal taxes.

OD.4 Asset Prices in the Planner's First-best Economy

We derive asset-pricing implications in the first-best economy. Using Ito's Lemma, we obtain:

$$\begin{aligned} d\mathbf{s}_t \;&=\; d\left(\frac{\mathbf{S}_t}{K_t}\right) = \frac{d\mathbf{S}_t}{K_{t-}} - \frac{\mathbf{S}_{t-}dK_t}{K_{t-}^2} + \frac{\mathbf{S}_{t-}dK_t^2}{K_{t-}^3} - \frac{< d\mathbf{S}_t, dK_t >}{K_{t-}^2} \\ &=\; \mu_s(\pi_{t-},\mathbf{s}_{t-})dt + \mathbf{s}_{t-}\left[\sigma_S d\mathcal{W}_t^S - \sigma_K d\mathcal{W}_t^K + N_{t-}(1-Z)d\mathcal{J}_t\right] , \end{aligned} \tag{OD.41}$$

where $\mu_s(\pi_{t-},\mathbf{s}_{t-}) = \mathbf{h}_{t-} - (\mathbf{i}_{t-} - \delta_K + \delta_S - \sigma_K^2 + \vartheta\sigma_K\sigma_S)\,\mathbf{s}_{t-}$. Duffie and Epstein (1992) show that the SDF $\{\mathbb{M}_t : t \geq 0\}$ implied by the planner's solution is given by:

$$\mathbb{M}_t = \exp\left[\int_0^t f_V(\mathbf{C}_s,V_s)\,ds\right] f_{\mathbf{C}}(\mathbf{C}_t,V_t) . \tag{OD.42}$$

Using the FOC for investment, the value function, and the resource constraint, we obtain:

$$f_{\mathbf{C}}(\mathbf{C},V) = \frac{1}{1+\phi'(\mathbf{i}(\pi,\mathbf{s}))}\frac{b(\pi,\mathbf{s})-\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}b(\pi,\mathbf{s})^{1-\gamma}\mathbf{K}^{-\gamma} = \frac{1}{\mathbf{q}(\pi,\mathbf{s})}b(\pi,\mathbf{s})^{1-\gamma}\mathbf{K}^{-\gamma}, \tag{OD.43}$$

and

$$f_V(\mathbf{C},V) = \frac{\rho}{1-\psi^{-1}}\left[\frac{(1-\omega)\mathbf{C}^{1-\psi^{-1}}}{((1-\gamma))^{\omega-1}}V^{-\omega} - (1-\gamma)\right] = -\epsilon(\pi,\mathbf{s}) , \tag{OD.44}$$

where $\epsilon(\pi,\mathbf{s}) = -\frac{\rho(1-\gamma)}{1-\psi^{-1}}\left[\left(\frac{\mathbf{c}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})}\right)^{1-\psi^{-1}}\left(\frac{\psi^{-1}-\gamma}{1-\gamma}\right) - 1\right]$.

Using Ito's Lemma and the optimal allocation rules, we obtain

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -\epsilon(\pi,\mathbf{s})dt - \gamma\left[\left(\mathbf{i}(\pi,\mathbf{s})-\delta_K\right)dt + \sigma_K d\mathcal{W}_t^K\right] + \left[(1-\gamma)\frac{b_\pi(\pi,\mathbf{s})}{b(\pi,\mathbf{s})} - \frac{\mathbf{q}_\pi(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\right]\mu_\pi(\pi)dt$$
$$+\left[(1-\gamma)\frac{b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})} - \frac{\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\right]\left[(\mu_s(\pi,\mathbf{s}) + \mathbf{s}\gamma(\sigma_K^2 - \vartheta\sigma_S\sigma_K))dt + \sigma_S d\mathcal{W}_t^S - \sigma_K d\mathcal{W}_t^K\right]$$
$$+\frac{\gamma(\gamma+1)}{2}\sigma_K^2 dt + \frac{(\sigma_S^2 - 2\vartheta\sigma_S\sigma_K + \sigma_K^2)\mathbf{s}^2}{2}\left[(1-\gamma)\left(\frac{b_{\mathbf{ss}}}{b} - \frac{\gamma b_{\mathbf{s}}^2}{b^2} - \frac{b_{\mathbf{s}}}{b}\frac{\mathbf{q}_{\mathbf{s}}}{\mathbf{q}}\right) - \frac{\mathbf{q}_{\mathbf{ss}}}{\mathbf{q}} + \frac{2\mathbf{q}_{\mathbf{s}}^2}{\mathbf{q}^2}\right]dt$$
$$+\left[\eta(\pi,\mathbf{s};Z,\mathbf{x}^e) - 1\right]d\mathcal{J}_t\,,$$

where $\eta(\pi,\mathbf{s};Z,\mathbf{x}^e) = \frac{\mathbf{q}(\pi,\mathbf{s})}{\mathbf{q}(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}})}\left(\frac{b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)}{b(\pi,\mathbf{s})}\right)^{1-\gamma}(1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z))^{-\gamma}$ and $\mathbf{s}^{\mathcal{J}} = \frac{\mathbf{s}}{1-N(\mathbf{x}^e(\pi,\mathbf{s}))(1-Z)}$ is the post-jump ratio carbon-productive-capital ratio $\mathbf{s}$.

As the expected percentage change of $\mathbb{M}_t$ equals $-r_t$ per unit of time (Duffie, 2001), we obtain the following expression for the interest rate:

$$\begin{aligned} r(\pi,\mathbf{s}) &= \rho + \psi^{-1}\left(\mathbf{i}-\delta_K\right) - \frac{\gamma(\psi^{-1}+1)\sigma_K^2}{2} - \left[(1-\psi^{-1})\frac{b_\pi}{b} - \frac{\mathbf{q}_\pi}{\mathbf{q}}\right]\mu_\pi(\pi) \\ &- \left[(1-\gamma)\frac{b_{\mathbf{s}}}{b} - \frac{\mathbf{q}_{\mathbf{s}}}{\mathbf{q}}\right]\left(\mu_s(\pi,\mathbf{s}) + \mathbf{s}\gamma(\sigma_K^2 - \vartheta\sigma_S\sigma_K)\right) - \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}\left(\eta(\pi,\mathbf{s};Z,\mathbf{x}^e)\right) - 1\right] \\ &+ \left(\psi^{-1}-\gamma\right)\left[(\mathbf{h}-\delta_S\mathbf{s})\frac{b_{\mathbf{s}}}{b} + \frac{\sigma_S^2\mathbf{s}^2}{2}\left(\frac{b_{\mathbf{ss}}}{b} - \frac{\gamma b_{\mathbf{s}}^2}{b^2}\right) + (1-\gamma)\vartheta\sigma_K\sigma_S\mathbf{s}\frac{b_{\mathbf{s}}}{b}\right] \\ &- \frac{(\sigma_S^2 - 2\vartheta\sigma_S\sigma_K + \sigma_K^2)\mathbf{s}^2}{2}\left[(1-\gamma)\left(\frac{b_{\mathbf{ss}}}{b} - \frac{\gamma b_{\mathbf{s}}^2}{b^2} - \frac{b_{\mathbf{s}}}{b}\frac{\mathbf{q}_{\mathbf{s}}}{\mathbf{q}}\right) - \frac{\mathbf{q}_{\mathbf{ss}}}{\mathbf{q}} + \frac{2\mathbf{q}_{\mathbf{s}}^2}{\mathbf{q}^2}\right] \\ &- \lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1 - \mathbb{E}^{\mathbf{x}^d}\left(\left(\frac{(1-N(\mathbf{x}^e)(1-Z))b\left(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}}\right)}{b(\pi,\mathbf{s})}\right)^{1-\gamma}\right)\right)\right]\,. \end{aligned}$$

Applying Ito's Lemma to $Q(K,\pi,\mathbf{s}) = q(\pi,\mathbf{s})K$, we obtain the PDE for Tobin's q, $q(\pi,\mathbf{s})$:

$$\begin{aligned} r(\pi,\mathbf{s})q(\pi,\mathbf{s}) &= A\mathbf{h}^{1-\alpha} - p_H\mathbf{h} - i - \phi(i) - x^e - \mathbf{x}^d + \left(i - \delta_K - \eta_{\mathbb{M}}^k(\pi,\mathbf{s})\sigma_K\right)q(\pi,\mathbf{s}) + \mu_\pi(\pi)q_\pi \\ &+ \left[\mu_s(\pi,\mathbf{s}) + \vartheta\sigma_S\sigma_K - \sigma_K^2 - (\eta_{\mathbb{M}}^s(\pi,\mathbf{s})\vartheta\sigma_S - \eta_{\mathbb{M}}^k(\pi,\mathbf{s})\sigma_K)\right]\mathbf{s}q_{\mathbf{s}} + \frac{(\sigma_S^2 - 2\vartheta\sigma_S\sigma_K + \sigma_K^2)\mathbf{s}^2}{2}q_{\mathbf{ss}} \\ &+ \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi,\mathbf{s};Z,\mathbf{x}^e)\left(q(\pi^{\mathcal{J}},\mathbf{s}^{\mathcal{J}})(1-N(\mathbf{x}^e)(1-Z)) - q(\pi,\mathbf{s})\right)\right]\,, \end{aligned} \quad \text{(OD.45)}$$

where

$$\eta_{\mathbb{M}}^k(\pi,\mathbf{s}) = \gamma\sigma_K + \left[(1-\gamma)\frac{\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})} - \frac{\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\right](\sigma_K - \vartheta\sigma_S) \quad \text{(OD.46)}$$

and

$$\eta_{\mathbb{M}}^s(\pi,\mathbf{s}) = \gamma\sigma_K + \left[(1-\gamma)\frac{\mathbf{s}b_{\mathbf{s}}(\pi,\mathbf{s})}{b(\pi,\mathbf{s})} - \frac{\mathbf{s}\mathbf{q}_{\mathbf{s}}(\pi,\mathbf{s})}{\mathbf{q}(\pi,\mathbf{s})}\right]\left(\sigma_K - \frac{\sigma_S}{\vartheta}\right)\,. \quad \text{(OD.47)}$$

Finally, using the equilibrium conditions $q(\pi,\mathbf{s}) = \mathbf{q}(\pi,\mathbf{s})$ and $x^e(\pi,\mathbf{s}) = \mathbf{x}^e(\pi,\mathbf{s})$, we write

$$\begin{aligned} \frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} &= \left(\mu_{\mathbf{Q}}(\pi_{t-},\mathbf{s}_{t-}) + \lambda(\pi_{t-})\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right)dt + \sigma_K d\mathcal{W}_t^K \\ &+ \frac{\mathbf{s}_{t-}\mathbf{q}_{\mathbf{s}}(\pi_{t-},\mathbf{s}_{t-})}{\mathbf{q}(\pi_{t-},\mathbf{s}_{t-})}\left[\sigma_S d\mathcal{W}_t^S - \sigma_K d\mathcal{W}_t^K\right] + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)(d\mathcal{J}_t - \lambda(\pi_{t-})dt)\,, \end{aligned}$$

where $\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} = \frac{(1-N(\mathbf{x}_{t-}^e)(1-Z))\mathbf{q}\left(\pi_t^{\mathcal{J}}, \mathbf{s}_{t-}^{\mathcal{J}}\right)}{\mathbf{q}(\pi_{t-}, \mathbf{s}_{t-})}$ and

$$\mu_{\mathbf{Q}}(\pi_{t-}, \mathbf{s}_{t-}) = r(\pi_{t-}, \mathbf{s}_{t-}) + \eta_{\mathbb{M}}^k(\pi_{t-}, \mathbf{s}_{t-})\sigma_K + (\eta_{\mathbb{M}}^s(\pi_{t-}, \mathbf{s}_{t-})\vartheta\sigma_S - \eta_{\mathbb{M}}^k(\pi_{t-}, \mathbf{s}_{t-})\sigma_K)\frac{\mathbf{s}_{t-}\mathbf{q}_{\mathbf{s}}(\pi_{t-}, \mathbf{s}_{t-})}{\mathbf{q}(\pi_{t-}, \mathbf{s}_{t-})}$$
$$+ \lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\eta(\pi_{t-}, \mathbf{s}_{t-}; Z, \mathbf{x}_{t-}^e)\left(1 - \frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right)\right]. \quad \text{(OD.48)}$$

The market risk premium is given by

$$rp(\pi_{t-}, \mathbf{s}_{t-}) = \eta_{\mathbb{M}}^k(\pi_{t-}, \mathbf{s}_{t-})\sigma_K + (\eta_{\mathbb{M}}^s(\pi_{t-}, \mathbf{s}_{t-})\vartheta\sigma_S - \eta_{\mathbb{M}}^k(\pi_{t-}, \mathbf{s}_{t-})\sigma_K)\frac{\mathbf{s}_{t-}\mathbf{q}_{\mathbf{s}}(\pi_{t-}, \mathbf{s}_{t-})}{\mathbf{q}(\pi_{t-}, \mathbf{s}_{t-})}$$
$$- \lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\left(\eta(\pi_{t-}, \mathbf{s}_{t-}; Z, \mathbf{x}_{t-}^e) - 1\right)\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right]. \quad \text{(OD.49)}$$

References for Online Appendices

Bansal, R. and Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4), pp.1481-1509.

Campbell, J.Y. and Cochrane, J.H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2), pp.205-251.

Campbell, J.Y. and Cochrane, J.H., 2015. The Fragile Benefits of Endowment Destruction. *Journal of Political Economy*, 123(5), pp.1214-1226.

Ghaderi, M., Kilic, M. and Seo, S.B., 2022. Learning, Slowly Unfolding Disasters, and Asset Prices. *Journal of Financial Economics,* 143, pp.527-549.

Jermann, U. J., 1998. Asset pricing in production economies. *Journal of Monetary Economics*, 41(2), pp.257-275.

Pindyck, R.S. and Wang, N., 2013. The economic and policy consequences of catastrophes. *American Economic Journal: Economic Policy*, 5(4), pp.306-339.

Wachter, J.A. and Zhu, Y., 2021. A model of two days: discrete news and asset prices. Wharton School Working Paper.