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ABSTRACT

We provide the planner's solution to a model where households learn from exogenous natural disaster arrivals about arrival rates and spend to mitigate future damages. Mitigation cannot be decentralized due to positive externalities from curtailing aggregate risks. First-best can be implemented by capital taxes and mitigation subsidies. Willingness-to-pay, toward public health for pandemics or environmental protection for climate disasters, depends on mitigation efficacy. Efficacy can be inferred from damage functions that depend on prior arrivals which determine preparedness. Regulatory risks arise since disaster leads to pessimistic arrival-rate beliefs and taxes or mandates to fund mitigation, which reduce consumption, investment and stock-market value.

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"*Mitigation is the effort to reduce loss of life and property by lessening the impact of disasters. In order for mitigation to be effective we need to take action now—before the next disaster—to reduce human and financial consequences later (analyzing risk, reducing risk, and insuring against risk).*" (FEMA website)

1 Introduction

The arrival of natural disasters can cause not only significant human suffering but also prolonged economic damage. For instance, the literature on weather disasters such as hurricanes points to persistent declines in growth and productivity due to destruction of physical capital (Dell, Jones and Olken (2014), Hsiang and Jina (2014)). Similarly, the literature on pandemics and the 1918 Flu, which is now particularly relevant in light of Covid-19, finds long-term declines in growth and productivity due to a combination of shocks to underlying worker health and other forms of firm intangible capital (Almond (2006), Barro, Ursua, and Weng (2020), and Guimbeau, Menon and Musacchio (2020)). However, the extent of damages depends on investment and greater preparedness, be it greater environmental protection in the case of climate disasters or better public health capabilities in the case of pandemics (see World Health Organization (2018)).

But how much mitigation is adequate? We provide a model to understand the cost and benefit of mitigation, endogenous damage functions depending on preparedness and potential market failures.[1] Our model is set in continuous time. Households learn from natural disaster arrivals about whether Poisson arrival rates are high or low (i.e., what we refer to as a bad versus good state). The bad state corresponds to more frequent arrival rates, such as a greater frequency of natural disasters as climate scientists predict will occur if global temperatures do not level off or if pandemics arrive more frequently.

Arrival of a disaster leads to a jump in belief in the bad state (i.e. perceived risk). Absent any arrivals, this belief drifts down toward the good state (i.e. no news is good news when

[1] In Barro (2006, 2009), he develops rare-disaster models and provides empirical evidence. The roots of this disaster literature go back to Rietz (1988).

it comes to disasters).[2] Hence, our model generates time-varying disaster arrival rates via learning. Learning (Colin-Dufresne, Johannes, and Lochstoer (2016)) and disasters with time-varying arrival rates (as in Gabaix (2012), Gourio (2012), and Wachter (2013)) have been shown to be quantitively important to simultaneously explain business cycles and asset price fluctuations.

Damages are modeled as downward jumps in the tangible or intangible capital stock — conditional on the arrival of the disaster, these damages follow a Pareto distribution and are i.i.d. across arrivals (Gabaix (2009)). While the importance of disasters for integrated assessment of climate changes has notably been raised by Weitzman (2009), our model additionally allows for the fat-tailedness of damages to be mitigated, in the sense of first-order stochastic dominance, by spending today that comes at the cost of consumption and/or investment.

By mitigation spending, we primarily mean ex-ante measures such as locating physical assets away hurricane paths or spending on more resilient building in the case of climate disasters. In the case pandemics, we mean public health spending (e.g., stockpiles of ventilators) as opposed to ex-post mitigation measures such as social distancing to flatten the curve that have been emphasized in light of Covid-19, both empirically (Hatchett, Carter and Lipsitch (2007), Adda (2016) and theoretically (Eichenbaum, Rebelo, and Trabandt (2020)).[3]

Otherwise, our model features familiar technologies from an AK growth model in macro and finance. Households are endowed with the widely-used non-expected utility proposed by Epstein and Zin (1989) and Weil (1990), which separates risk aversion from the elasticity of intertemporal substitution. There are convex adjustment costs to capital that give rise to rents for capital and the value of capital (Tobin's average q) can fluctuate depending on

[2]Households might also learn from extreme temperature signals regarding the state of disaster arrivals. But for simplicity, we focus on just learning from disaster arrivals. There is evidence that households learn or update beliefs regarding climate change based on personal experiences with disasters (see, e.g., Yale Climate Opinion Maps, Anderson and Robinson (2019)).

[3]However, to the extent some social distancing measures are institutionalized and intended to reduce damages against future pandemic occurrences, we would consider these to be ex-ante mitigation and in the purview of our model.

future investment opportunities.

Despite the novelty of introducing both belief updating and mitigation technology, our model is tractable. The planner's solution is characterized by an endogenously derived nonlinear ordinary differential equation for the value function (the certainty equivalent wealth) together with first-order conditions for investment and mitigation spending that depend on household belief regarding disaster arrivals. The boundary conditions are given by solutions when the household belief is permanently in the low or high arrival state.[4]

Because mitigation changes the distribution of damages conditional on arrival, which benefits all firms and households, aggregate risk mitigation cannot be decentralized due to the positive externalities of mitigation. We show in a dynamically complete market setting that the competitive equilibrium corresponds to households and firms optimally choosing no mitigation. We verify that the planner's solution for the no-mitigation technology case also corresponds to the competitive equilibrium solution where there is mitigation technology but private agents optimally choose no mitigation spending.

We then show that the planner's solution is implementable via a variety of schemes. Even though there are complete markets, the competitive economy has an extreme form of underspending on mitigation and over-investment in capital from the societal perspective since firms do not internalize the benefits of aggregate risk mitigation. Taxing capital effectively lowers the firm's marginal product of capital thereby addressing its over-investment motive, which in turn lowers the firm's average q in equilibrium. By using the tax proceeds and fully reimbursing the firm for its mitigation spending, the first-best solution can be achieved while still maintaning budget balance. This differs from optimal Pigouvian taxes to address negative externalities of carbon emissions for climate change (Salanie (2000), Golosov, Hassler, Krusell and Tsyvinski (2014)).

We consider a calibration exercise to highlight the importance of mitigation for welfare analysis. We start with a disaster calibration of an economy without any mitigation tech-

[4]The solutions for the two special cases (at the boundaries) generalize the model in Pindyck and Wang (2013), which originally examined the general equilibrium effects of disasters in a continuous-time production model with Poisson arrivals of disasters, by allowing for mitigation.

nology following the logic in Barro and Jin (2011) and Pindyck and Wang (2013). Their calibrations encompass general disasters including wars and have time-invariant beliefs regarding arrival rates. We focus instead on two natural disasters. The first is cyclones that damage GDP and growth rates of small economies as documented by Hsiang and Jina (2014). The second is in the context of pandemics such as the 1918 Flu or Covid-19. Otherwise, the other parameter values, including risk aversion and time preference, productivity, and stock market return and volatility, are set to capture reasonable scenarios similar to their studies.

Holding fixed these parameters, we introduce a mitigation technology and allow beliefs to account for the possibility that the economy can be in a good or bad state. We use historical data on these two natural disasters to calibrate arrival rates and damages for the good state. The bad state is then defined as an arrival rate that is much higher than in the good state. We can think of this calibration exercise as calculating how valuable mitigation would be should households entertain the possibility that cyclones or pandemics might be more frequent in the future than in historical samples. We see how our welfare analysis depends on then two key parameters: mitigation efficiency and the arrival rate.

For simplicity and empirical relevance, we focus our discussions below on household preferences with elasticity of intertemporal substitution equal to one or above following the literature on long-run risks (Bansal and Yaron, 2004). To make the point on risk management associated with mitigation transparent, we consider a solution with unit elasticity of intertemporal substitution as in Tallarini (2000) (hence nesting log utility as a special case). When no mitigation technology is available, the arrival of disasters even though it leads to an update toward worse household beliefs does not lead to any changes in investment-capital ratio and stock market valuation (e.g., Tobin's average q). This utility specification is the knife-edge case where the substitution and wealth effects exactly offset each other leaving investment and consumption unaffected by beliefs.

In our cost-benefit analysis, we highlight the high non-linearity of mitigation spending particularly near the boundary where prior beliefs are of a low arrival rate but an arrival then leads to updating. The benefit of mitigation is that households enjoy more sustainable growth and higher welfare. The market failure or difference between the planner's solution and the competitive equilibrium solution are significant even when household beliefs regarding disaster arrival rates are low due to this non-linearity in optimal mitigation with household beliefs.

Our analysis is related to recent integrated assessment models (IAMs) of climate change (Nordhaus (2017)) that incorporate general preferences and risks in a stochastic general equilibrium framework (see, e.g., Jensen and Traeger (2014), Hambel, Kraft and Schwartz (2018), Cai and Lontzek (2019), Barnett, Brock and Hansen (2020), Daniel, Litterman and Wagner (2019)). Whereas the integrated assessment literature is focused on abatement to control the path of temperatures and calculating the social cost of carbon, our focus is on mitigating disasters more generally and quantifying willingness-to-pay (WTP) for mitigation technology.

To this end, we use a WTP metric from Pindyck and Wang (2013), for both cyclones and pandemics across a range of assumptions regarding mitigation efficiency and arrival rates of the bad state. The reason is that the expected damage, which is the product of arrival rate and damage conditional on arrival, are significant for both disasters. The caveat to these calculations is that traditional willingness-to-pay calculations to avoid disasters as in our model is sensitive to modeling of multiple disasters and when disasters affect both consumption and loss of life (Martin and Pindyck (2015, 2019)).

Another caveat is that a WTP calculation depends crucially on our estimates of mitigation efficacy in practice. Mitigation efficiency is also an important parameter of interest to practitioners and empirical researchers more generally. Measuring this efficacy absent randomized intervention is difficult primarily because of the endogeneity of mitigation. Our model offers a strategy to gauge this parameter nonetheless.

Damages conditional on arrival are higher when an economy has few prior arrivals and long inter-arrival times since perceived risks and mitigation spending or preparedness are low as a result. Direct damages in our model correspond to long-run destruction of either tangible or intangible capital and productivity. Hence, data on damage conditional on arrival and mitigation spending beforehand can be used to infer mitigation efficacy using exogenous arrivals and inter-arrival times as instruments.

Our model also generates transition or regulatory risk oft-discussed by policy makers regarding climate change (Carney (2015)). These discussions typically center on potential emissions taxes in the future that will strand carbon assets and lead to a drop in stock market value (Bolton and Kacperczyk (2020)). There is a similar notion in our model associated with mitigation of disasters. The arrival of a disaster raises household perceived risks that then increases the value of mitigation and hence taxes to finance the mitigation. This then leads to prolonged drops in consumption, investment, and the stock market. Transitions risks have been less studied compared to the pricing of physical risks for firm cashflows (see Hong, Karolyi, and Scheinkman (2020) for a review of recent findings that include Hong, Li and Xu (2019), Bansal, Kiku and Ochoa (2019), and Suntheim et.al. (2020)). Our model generates a number of testable predictions that relate capital taxes and mitigation subsidies to the arrival of disasters.

Finally, aside from taxes and subsidies, mitigation spending can also be implemented as a government mandate for households to invest in firms that mitigate. This implementation can be interpreted as that of sovereign wealth funds or public pension plans having to invest in firms that have sufficiently high environmental, social, and governance (ESG) criteria (Bolton, Samama, and Stiglitz, 2010). This implementation echoes how government can affect stock prices (Pastor and Veronesi (2012)). Our model hence suggests a rationale for ESG investing that complements an important model in the literature based on households with prosocial preferences who incentivize firms to invest in green through a cost of capital channel in an incomplete market setting (Heinkel, Kraus and Zechner (2001), Hong and Kacperczyk (2009), Pastor, Stambaugh and Taylor (2019)).

2 Model

In this section, we set up the model. Time is continuous and the horizon is infinite.

Aggregate Production and Resource Constraint. Let K denote the aggregate capital stock, which is the sole factor of production. Aggregate output, Y, is given by

$$Y_t = AK_t \,, \tag{1}$$

where $A > 0$ is a constant that defines productivity. This is a version of the AK model in macroeconomics and finance.[5] In our specification, K is the *total* stock of capital; it includes physical capital as traditionally measured, but also human capital and firm-based intangible capital (such as, patents, know-how, brand value, and organizational capital).

In each period, aggregate output is spent in one of the three possible ways—consumption, investment, and mitigation. Let C_t, I_t, and X_t denote consumption, investment, and mitigation spending, respectively.

Following the q theory of investment (Hayashi, 1982 and Abel and Eberly, 1994), we assume that when investing $I_t dt$, the firm also incurs capital adjustment costs, which we denote by $\Phi_t dt$. That is, the total cost of investment per unit of time is $(I_t + \Phi_t)$ including both capital purchase and adjustment costs. Therefore, we have the following aggregate resource constraint:

$$Y_t = C_t + (I_t + \Phi_t) + X_t \,. \tag{2}$$

The most natural interpretation of mitigation spending is public health spending in the context of pandemics or carbon abatement and limiting construction in areas vulnerable to disaster in the context of the climate change literature.[6] We specify the capital adjustment later in this section.

Investment and Capital Accumulation. The capital stock K evolves as:[7]

$$dK_t = I_{t-}dt + \sigma K_{t-} d\mathcal{W}_t - (1 - Z)K_{t-} d\mathcal{J}_t \,. \tag{3}$$

[5]Lucas (1988) develops an AK model where the creation and transmission of knowledge occurs through human capital accumulation. Rebelo (1991) uses AK models to explain how heterogeneity in growth experiences can be the result of cross-country differences in government policy. Jones and Manuelli (2005) provide a survey of the AK growth literature. In finance, Cox, Ingersoll, and Ross (1985) develops an equilibrium model of capital accumulation closely related to the AK model.

[6]Mitigation spending X_t effectively reduces output which tightens resources constraints for consumption and investment: $Y_t - X_t = C_t + (I_t + \Phi_t)$.

[7]This capital accumulation technology has been widely used in macro and finance. For example, see Barro (2009) and Pindyck and Wang (2013).

The first term in (3) is investment I. The second term captures continuous shocks to capital, where $\mathcal{W}_t$ is a standard Brownian motion and the parameter σ is the diffusion volatility (for the capital stock growth). This diffusion shock is the source of shocks for the standard AK models in macroeconomics. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at t.

We may generalize our AK model to allow for multiple factors of production.[8]

Arrival of Disasters. Capital stock is also subject to jump shocks that cause stochastic permanent losses of the existing capital stock. We capture this effect via the third term, where $\mathcal{J}_t$ is a (pure) jump process with a constant but unknown arrival rate, which we denote by λ, to be described shortly.

When a jump arrives ($d\mathcal{J}_t = 1$), it permanently destroys a stochastic fraction $(1 - Z)$ of the capital stock K_{t-}, as Z is the recovery fraction. (For example, if a shock destroyed 15 percent of capital stock, we would have $Z = .85$.) There is no limit to the number of these jump shocks.[9] If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears.

Let $\Xi(Z)$ and $\xi(Z)$ denote the cumulative distribution function (cdf) and probability density function (pdf) for the recovery fraction, Z, conditional on a jump arrival, respectively. The household can influence the distribution of Z by doing mitigation.

Next, we discuss how we model the constant but unknown arrival rate of the jump process, λ. We suppose that the arrival rate can be either low or high. If the rate is high, it is more likely that capital stock will be hit by a disaster (i.e., a negative jump shock). If the rate is low, a disaster is much less likely. We refer to the low-rate and high-rate scenarios as good state (G) and bad state (B), respectively, and use λ_G and λ_B to denote the corresponding jump arrival rate of a jump in the respective state. Naturally, $\lambda_B > \lambda_G$. While the state is constant over time, the household does not observe the state and therefore has to learn about the value of λ over time to assess the likelihood that the arrival rate is high or low.

[8]See Appendix A.3 for details.

[9]Stochastic fluctuations in the capital stock have been widely used in the growth literature with an AK technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (disaster) jumps.

We will discuss the household's learning dynamics shortly.

Mitigation Technology and Adjustment Cost of Capital. We use lower-case variables to denote the corresponding upper-case variables divided by contemporaneous K. For example, $c_t = C_t/K_t$, $i_t = I_t/K_t$, $\phi_t = \Phi_t/K_t$, and $x_t = X_t/K_t$.

To preserve our model's homogeneity, we make two economically sensible simplifying assumptions. First, the distribution for the post-jump recovery fraction Z at t depends on the pre-jump scaled mitigation x_{t-} and hence we may write the cdf as $\Xi(Z; x_{t-})$ and pdf as $\xi(Z; x_{t-})$. That is, if mitigation spending X doubles, the benefit of mitigation also doubles. Let the domain for the admissible values of Z be $(0, \overline{Z})$, where $0 < \overline{Z} < 1$ is a constant. The minimal fraction loss is thus $(1 - \overline{Z})$ upon a jump arrival.

Second, the capital adjustment cost function, $\Phi(I, K)$, is homogeneous with degree one in I and K and thus can be written as:

$$\Phi(I, K) = \phi(i)K \ , \tag{4}$$

where $\phi(i)$ is increasing and convex.[10] Because installing capital is costly, installed capital earns rents in equilibrium so that Tobin's q, the ratio between the value and the replacement cost of capital, exceeds one.[11]

These two assumptions allow us to simplify our two-dimensional problem into a one-dimensional optimization problem. Below we start with the two-dimensional formulation as doing so eases the exposition of economic insights.

Preferences. We use the Duffie and Epstein (1992) continuous-time version of the recursive preferences developed by Epstein and Zin (1989) and Weil (1990), so that a representative consumer has homothetic recursive preferences given by:

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_s, V_s) ds \right] \ , \tag{5}$$

[10]Homogeneous adjustment cost functions are analytically tractable and have been widely used in the q theory of investment literature. Hayashi (1982) showed that with homogeneous adjustment costs and perfect capital markets, marginal and average q are equal.

[11]In Barro (2006, 2009), he also analyzes an endogenous AK growth model with disaster risks but without capital adjustment costs in a discrete-time setting. Therefore, Tobin's average q in his model is always one.

where $f(C,V)$ is known as the normalized aggregator given by

$$f(C,V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}} \ . \tag{6}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and we let $\omega = (1-\psi^{-1})/(1-\gamma)$. Unlike expected utility, recursive preferences as defined by (5) and (6) disentangle risk aversion from the EIS. An important feature of these preferences is that the marginal benefit of consumption is $f_C = \rho C^{-\psi-1}/[(1-\gamma)V]^{\omega-1}$, which depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

If $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C,V) = \frac{\rho C^{1-\gamma}}{1-\gamma} - \rho V. \tag{7}$$

This more flexible utility specification is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the large follow-up long-run risk literature. We show that in our model, the EIS parameter plays an important role as well.

3 Solution

The social planner maximizes the representative household's utility given in (5)-(6) subject to the production/capital accumulation technology and the aggregate resource constraint described in Section 2. Let $V(K,\pi)$ denote the value function.

Next, we derive the representative households' or planner's Bayesian learning rule and then use dynamic programming to solve the optimal policies and value function.

Learning. The household dynamically updates her belief about the arrival rate of disasters. Let π_t denote the time-t posterior belief that $\lambda = \lambda_B$. That is,

$$\pi_t = \mathbb{P}(\lambda_t = \lambda_B | \mathcal{F}_t) \,, \tag{8}$$

where $\mathcal{F}_t$ is the household's information set up to t. At time t, the expected jump arrival rate, denoted by λ_t, is given by

$$\lambda_t = \lambda(\pi_t) = \lambda_B \pi_t + \lambda_G (1 - \pi_t) \,, \tag{9}$$

which is a weighted average of λ_B and λ_G. A higher value of π_t corresponds to a belief that the economy is more likely in State B which has a high jump arrival rate.

What makes the household's belief to worsen (increasing π) is jump arrivals. What makes the household's belief to revise favorably is no jump arrivals. In this sense, no-jump news is good news. In expectation, with rational learning, belief change cannot be predicted, which means belief has to be a martingale.

Mathematically, the household updates her belief by following the Bayes rule:[12]

$$d\pi_t = \sigma_\pi(\pi_{t-}) \left(d\mathcal{J}_t - \lambda_{t-} dt \right) \,, \tag{10}$$

where

$$\sigma_\pi(\pi) = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda(\pi)} = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda_B \pi + \lambda_G(1-\pi)} > 0 \,. \tag{11}$$

Here, signals come from $\mathcal{J}_t$. Because $\mathbb{E}_{t-}[d\mathcal{J}_t] = \lambda_{t-} dt$, (10) implies that the household's belief process π is a martingale. When a disaster strikes at t, the household's belief immediately increases by $\sigma_\pi(\pi)$ from the pre-jump level π to $\pi^{\mathcal{J}}$, where

$$\pi^{\mathcal{J}} = \pi + \sigma_\pi(\pi) = \frac{\pi \lambda_B}{\lambda(\pi)} > \pi \,. \tag{12}$$

If there is no arrival over time interval dt, the household becomes more optimistic. Mathematically, if $d\mathcal{J}_t = 1$, we have $d\pi_t = \mu_\pi(\pi_{t-})dt$, where

$$\mu_\pi(\pi) = -\sigma_\pi(\pi)\lambda(\pi) = \pi(1-\pi)(\lambda_G - \lambda_B) < 0 \,. \tag{13}$$

[12]See Theorem 19.6 in Lipster and Shiryaev (2001).

Now suppose that there is no jump during a finite time interval (s,t), i.e., $dJ_v = 0$ for $s < v \leq t$. By using (13) to integrate π from s to t conditional on no jump, we obtain the following logistic function:

$$\pi_t = \frac{\pi_s e^{-(\lambda_B - \lambda_G)(t-s)}}{1 + \pi_s\left(e^{-(\lambda_B - \lambda_G)(t-s)} - 1\right)} \,. \tag{14}$$

In Figure 1, we plot a simulated path for π starting from $\pi_0 = 0.1$. It shows that absent a jump arrival, belief becomes more optimistic, i.e., π_t decreases deterministically. Once a jump arrives, the belief worsens, i.e., jumps upward by a discrete amount $\sigma_\pi(\pi)$.

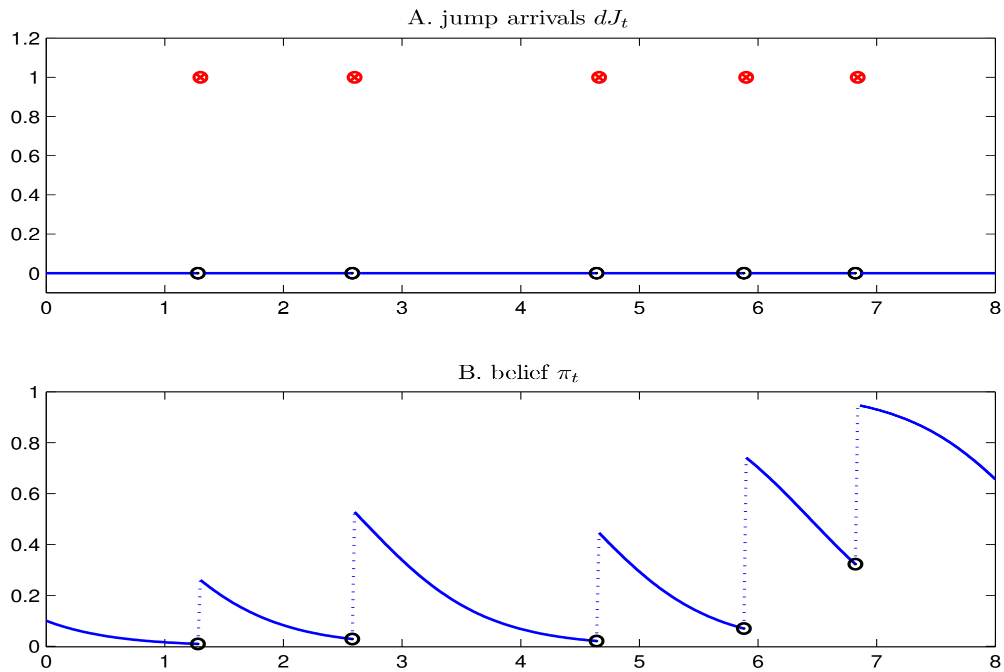

Figure 1: This figure simulates a path for jump arrival times in Panel A and plots the corresponding belief updating process in Panel B starting with $\pi_0 = 0.1$. The belief changes deterministically in the absence of jumps but discretely upward upon a jump arrival.

Dynamic Programming. The Hamilton-Jacobi-Bellman (HJB) equation for the planner's allocation problem is:

$$\begin{aligned} 0 \;=\; & \max_{C,I,x} f(C,V) + IV_K(K,\pi) + \mu_\pi(\pi)V_\pi(K,\pi) + \frac{1}{2}\sigma^2 K^2 V_{KK}(K,\pi) \\ & +\lambda(\pi)\mathbb{E}\left[V\left(ZK,\pi^{\mathcal{J}}\right) - V(K,\pi)\right] \;, \end{aligned} \tag{15}$$

where the expected change of belief in the absence of jumps, $\mu_\pi(\pi)$, is negative and given in (13), the expected arrival rate of a jump, $\lambda(\pi)$, is given in (9), the post-jump belief $\pi^{\mathcal{J}}$ is given in (12) as a function of the pre-jump belief π, and the expectation $\mathbb{E}[\,\cdot\,]$ is with respect to the pdf $\xi(Z;x)$ for the recovery fraction Z for a given level of scaled mitigation x.

The first term on the right side of (15) is the household's normalized aggregator; the second term captures how investment I affects $V(K,\pi)$; the third term reflects how belief updating (in the absence of jumps) impacts $V(K,\pi)$; and the fourth term captures the effect of capital-stock diffusion shocks on $V(K,\pi)$. It is worth noting that as the signals in our learning model are discrete (jump arrivals), there is no diffusion volatility induced quadratic variation term involving $V_{\pi\pi}$ in the HJB equation (15). Instead, the possibility of jumps induces both an expected level and uncertainty effects, both of which are captured by the last term that we discuss in detail soon.

Direct versus Learning Effects. Finally, the last term (appearing on the second line) of (15) describes the effects of jumps on the expected change in $V(K,\pi)$. This term captures rich economic forces and warrants additional explanations. When a jump arrives at t ($d\mathcal{J}_t = 1$), capital stock falls from K_{t-} at time $t-$ to $K_t = ZK_{t-}$ at t, which also causes the household to become more pessimistic. As a result, her belief increases from the pre-jump level of π_{t-} to the post-jump level of $\pi_t = \pi^{\mathcal{J}}$, as given by (12). Therefore, the expected change of the value function conditional on a jump arrival is given by $\mathbb{E}\left[V\left(ZK_{t-},\pi^{\mathcal{J}}\right) - V(K_{t-},\pi_{t-})\right]$. To take into account that the jump arrival is uncertain, we multiply this term by the jump arrival intensity at $t-$, $\lambda(\pi_{t-})$, to obtain the last term in (15).

It is important to note that a jump triggers two effects on the value function. First, there is an direct effect: $(1-Z)$ fraction of the capital stock is permanently destroyed, which lowers the value function from $V(K_{t-},\pi_{t-})$ to $V(ZK_{t-},\pi_{t-})$. Second, there is a learning effect: the household's belief worsens to $\pi_t = \pi^{\mathcal{J}} = \pi_{t-}\lambda_B/\lambda(\pi_{t-}) > \pi_{t-}$, which further lowers the value function from $V(ZK_{t-},\pi_{t-})$ to $V(ZK_{t-},\pi^{\mathcal{J}})$. These two effects reinforce each other leading to potentially significant losses to the household. We further discuss the details below.

The household optimally chooses consumption C, investment I, and mitigation X to maximize her utility by setting the sum of all the five terms on the right side of (15) to zero, as implied by the standard argument underpinning the HJB equation generalized to the setting with recursive utility (see Duffie and Epstein, 1992). Because of the resource constraint, it is sufficient for us to focus on I and x as control variables.

First-Order Conditions for Investment and Mitigation. The first-order condition (FOC) for investment I is

$$(1 + \Phi_I(I, K))f_C(C, V) = V_K(K, \pi) \ . \tag{16}$$

The right side of (16) is the marginal (utility) benefit of investment. The left side of (16) is the marginal cost of investment, which is given by the product of marginal benefit of consumption $f_C(C, V)$ and the marginal capital cost of investing $(1 + \Phi_I(I, K))$, the latter of which includes the marginal unity investment cost and the marginal adjustment cost.

The intuition for (16) is as follows. To increase the capital stock by one unit, which generates a marginal utility benefit of V_K, the household needs to give up $(1 + \Phi_I(I, K))$ units of her consumption in order to purchase one unit of capital and then install it into the firm making it productive. Therefore, the marginal cost of increasing capital stock by one unit is $(1 + \Phi_I(I, K))$ units of marginal benefit of consumption f_C. Unlike in standard expected-utility models, $f_C(C, V)$ depends on not just consumption C but also the continuation utility V, which reflects the non-separability of preferences.

The FOC with respect to mitigation is

$$Kf_C(C, V) = \lambda(\pi) \int_0^{\overline{Z}} \left[\frac{\partial \xi(Z; x)}{\partial x} V\left(ZK, \frac{\pi \lambda_B}{\lambda(\pi)} \right) \right] dZ \ , \tag{17}$$

if the solution is strictly positive, $x > 0$. Otherwise, $x = 0$ as mitigation cannot be negative. The planner optimally chooses x to equate the marginal cost of mitigation, which is the forgone marginal (utility) benefit of consumption $Kf_C(C, V)$ given in the left side of (17), with the marginal benefit of mitigation given in the right side of (17).[13] By doing mitigation x per unit of capital, the planner changes the pdf $\xi(Z; x)$ for the fractional capital recovery,

[13]The second-order condition (SOC) is given by $\lambda(\pi) \int_0^{\overline{Z}} \left[\frac{\partial^2 \xi(Z,x)}{\partial x^2} V\left(ZK, \frac{\pi \lambda_B}{\lambda(\pi)} \right) \right] dZ < 0$, which we verify.

Z, from $\xi(Z;0)$ to $\xi(Z;x)$. We provide detailed discussions about the stochastic dominance properties of $\xi(Z;x)$ and the economic tradeoff shortly.

Using Homogeneity Property to Simplify Solution. We show that the value function $V(K,\pi)$ is homogeneous with degree $(1-\gamma)$ in K and thus we can write $V(K,\pi)$ as follows:

$$V(K,\pi) = \frac{1}{1-\gamma}\left(b(\pi)K\right)^{1-\gamma}, \tag{18}$$

where $b(\pi)$ is the function determined as part of the solution.

Using the FOCs (16) and (17) and substituting the value function $V(K,\pi)$ given in (18) together with the implied policy rules into the HJB equation (15), and simplifying the equations, we obtain the following three-equation ODE system for $b(\pi)$, $i(\pi)$, and $x(\pi)$:

$$\begin{aligned}
0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{b(\pi)}{\rho(1+\phi'(i(\pi)))}\right)^{1-\psi}-1\right]+i(\pi)-\frac{\gamma\sigma^2}{2}+\mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\
&\quad +\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})-1\right], \qquad (19)\\
b(\pi) &= \left[A-i(\pi)-\phi(i(\pi))-x(\pi)\right]^{1/(1-\psi)}\left[\rho(1+\phi'(i(\pi)))\right]^{-\psi/(1-\psi)}, \qquad (20)\\
1 &= \frac{\lambda(\pi)(1+\phi'(i(\pi)))}{1-\gamma}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\int_0^{\overline{Z}}\left[\frac{\partial\xi(Z;x(\pi))}{\partial x}Z^{1-\gamma}\right]dZ\,. \qquad (21)
\end{aligned}$$

Next, we provide the boundary conditions at $\pi=0$ and $\pi=1$ and discuss the intuition. As we show, the model at the two boundaries map to the model in Pindyck and Wang (2013), but generalized to allow for mitigation spending. When $\pi=0$, the economy is permanently in state G. Therefore there is no learning and the solution boils down to solving the three unknowns, $b(0)$, investment $i(0)$, and mitigation spending $x(0)$, via the following three-equation system:

$$\begin{aligned}
0 &= \frac{\left(\frac{b(0)}{\rho(1+\phi'(i(0)))}\right)^{1-\psi}-1}{1-\psi^{-1}}\rho+i(0)-\frac{\gamma\sigma^2}{2}+\frac{\lambda_G}{1-\gamma}\left(\mathbb{E}(Z^{1-\gamma})-1\right), \qquad (22)\\
b(0) &= \left[A-i(0)-\phi(i(0))-x(0)\right]^{1/(1-\psi)}\left[\rho(1+\phi'(i(0)))\right]^{-\psi/(1-\psi)}, \qquad (23)\\
\frac{1}{1+\phi'(i(0))} &= \frac{\lambda_G}{1-\gamma}\int_0^{\overline{Z}}\left[\frac{\partial\xi(Z;x(0))}{\partial x}Z^{1-\gamma}\right]dZ\,. \qquad (24)
\end{aligned}$$

When $\pi = 0$, investment-capital ratio $i(0)$, scaled mitigation spending $x(0)$, and consumption-capital ratio $c(0)$ are all constant at all time.

By applying essentially the same analysis to the other boundary at $\pi = 1$, i.e., when the state is B, we solve for the three unknowns, $b(1)$, $i(1)$, and $x(1)$, via (A.3)-(A.5), another three-equation system in the Appendix.

The economy is on a growth path with constant investment opportunity when $\pi = 0$ or $\pi = 1$. None of these results hold obviously in our general model when $0 < \pi < 1$.

We summarize our model's solution in the following proposition.

Proposition 1 *The planner's solution is given by the triplet, $b(\pi)$, $i(\pi)$, and $x(\pi)$, where $0 \leq \pi \leq 1$, via the three-equation ODE system, (19)-(21) in the interior region $0 < \pi < 1$, together with the boundary conditions (22)-(24) for $\pi = 0$ and (A.3)-(A.5) for $\pi = 1$.*

Expected Fractional Loss, Growth Rate, and Mitigation Technology. We further assume as in Barro and Jin (2011) and Pindyck and Wang (2013) that the cdf of Z is given by the following power function defined over $[0, \overline{Z}\,]$:

$$\Xi(Z; x) = \left(Z/\,\overline{Z}\,\right)^{\beta(x)} \;\; ; \;\; 0 \leq Z \leq \overline{Z} \;\; , \tag{25}$$

where $\beta(x)$ is the exponent function that depends on mitigation x. To ensure that our model is well defined, we require $\beta(x) > \gamma - 1$.

Conditional on a jump arrival, the expected fractional capital loss is given by

$$\ell(\pi) = 1 - \mathbb{E}(Z) = 1 - \frac{\beta(x(\pi))}{\beta(x(\pi)) + 1}\overline{Z}\,. \tag{26}$$

The larger the value of $\beta(\,\cdot\,)$, the smaller the expected fractional loss $\mathbb{E}(1 - Z)$. To capture the benefit of mitigation, we assume that $\beta(x)$ is increasing in x, $\beta'(x) > 0$. The benefit of mitigation is to increase the capital stock recovery (upon the arrival of a disaster) in the sense of first-order stochastic dominance, i.e., $\Xi(Z; x_1) \leq \Xi(Z; x_2)$ for $Z < \overline{Z}$ if $x_1 > x_2$.

Let g_t denote the expected growth rate including the jump effect. The homogeneity property implies that $g_t = g(\pi_t)$, where

$$g(\pi) = i(\pi) - \lambda(\pi)\ell(\pi) = i(\pi) - \lambda(\pi)\left(1 - \frac{\beta(x(\pi))}{\beta(x(\pi)) + 1}\overline{Z}\right)\,. \tag{27}$$

As we show soon, while mitigation $x(\pi)$ may crowd out investment $i(\pi)$, it enhances long-run growth $g(\pi)$ by reducing the expected loss due to jumps.

For our quantitative analysis, we use the following linear specification for $\beta(x)$:

$$\beta(x) = \beta_0 + \beta_1 x\,, \tag{28}$$

with $\beta_0 \geq \max\{\gamma - 1, 0\}$ and $\beta_1 > 0$. The coefficient β_0 is the exponent for recovery Z in the absence of mitigation. The coefficient β_1 is the elasticity of cdf $\Xi(Z)$ with respect to mitigation x, $d\ln\Xi(Z;x)/d\ln x$. This coefficient is a key parameter in our model as it measures the efficiency of the mitigation technology.

Figure 2 shows an instance where $\Xi(Z;x_2)$ first-order stochastically dominates $\Xi(Z;x_1)$, as $x_2 > x_1$ and $\beta'(x) > 0$. For illustration, we pick $\beta(x_2) = 48$ and $\beta(x_1) = 24$.

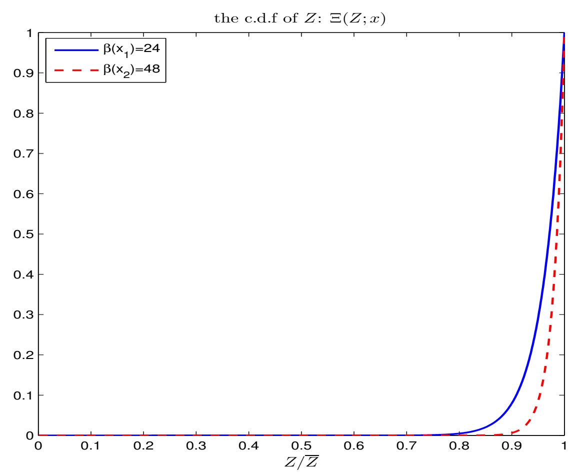

Figure 2: The c.d.f of Z, $\Xi(Z;x)$ for two levels of $\beta(x)$. In this plot, as $\beta(x_1) = 24$ and $\beta(x_2) = 48$, $\Xi(Z;x_2)$ first-order stochastically dominates $\Xi(Z;x_1)$.

Planner's Value Function with No Mitigation Technology. To better connect to the competitive market equilibrium solution, it is useful to summarize the planner's solution when there is no mitigation technology available, i.e., $x = 0$. By using the same argument as we have for the general case, we know that the planner's value function, $\widehat{V}(K,\pi)$, is homogeneous with degree $(1-\gamma)$ in K:

$$\widehat{V}(K,\pi) = \frac{1}{1-\gamma}\left(\widehat{b}(\pi)K\right)^{1-\gamma}, \tag{29}$$

where $\widehat{b}(\pi)$ is a measure of welfare (proportional to the certainty equivalent wealth). By substituting $x(\pi) = 0$ into the solution for the general case and removing the FOC for x, we obtain the solution for $\widehat{b}(\pi)$ together with the optimal investment-capital ratio $i(\pi)$.

In summary, $\widehat{b}(\pi)$ and $i(\pi)$ jointly solve (19)-(20) together with the boundary conditions (22)-(23) and (A.3)-(A.4) with the restriction of no mitigation spending, $x(\pi) = 0$.

4 Competitive Equilibrium and Market Failure

We analyze the decentralized market-equilibrium solution (Appendix B provides details.) Importantly, we show that the market mechanism does not implement the planner's solution in Section 3. This is because aggregate risk mitigation suffers from a free-riding problem as neither households nor firms have incentives to mitigate aggregate risk.

4.1 Market Structure and Problem Formulation

Consider a decentralized competitive equilibrium with (dynamically) complete markets. That is, the following securities can be traded at each point in time: (i) a risk-free asset, (ii) the aggregate stock market (a claim on the value of capital of the representative firm), and (iii) insurance claims for disaster with every possible recovery fraction Z.

Disaster Risk Insurance (DIS). We define DIS as follows: a DIS for the survival fraction in the interval $(Z, Z + dZ)$ is a swap contract in which the buyer makes insurance payments $p(Z; x^*)dZ$, where x^* is the aggregate (scaled) mitigation spending, to the seller and in exchange receives a lump-sum payoff if and only if a shock with survival fraction in $(Z, Z+dZ)$ occurs. That is, the buyer stops paying the seller if and only if the defined disaster event occurs and then collects one unit of the consumption good as a payoff from the seller. The DIS contracts, e.g., the insurance premium payment $p(Z; x^*)$, are priced at actuarially fairly so that investors earn zero profits. $p(Z; x^*)$ depends on not only Z but also x^*. This is because the aggregate mitigation spending x^* changes the distribution for $\Xi(Z)$.

Let $X_{c,t} \geq 0$ and $X_{f,t} \geq 0$ denote the mitigation spending at t by households and firms, respectively. Let H_t denote the household's wealth allocated to the market portfolio at t.

For disaster with recovery fraction in $(Z, Z+dZ)$, $\delta_t(Z)W_t dt$ gives the total demand for the DIS over time period $(t, t+dt)$. Let W_t denote the representative household's wealth.

We define the recursive competitive equilibrium as follows: (1) The representative household chooses consumption C, allocation to the stock market H, various DIS claims $\delta(Z)$, and mitigation spending X_c to maximize utility as given by (5)-(6). (2) The representative firm chooses investment I and mitigation spending X_f to maximize its market value, which is the present discounted value of future cash flows. Private agents take the equilibrium prices of all goods and financial assets including the risk-free rate $r(\pi)$ and the stock-market price process as given. (3) All markets clear.

It is useful to differentiate variables at the micro and macro levels. We use superscript * to denote the equilibrium variables. For example, X_c^* and X_f^* denote the equilibrium mitigation spending by households and firms, and $x_c^* = X_c^*/K$ and $x_f^* = X_f^*/K$. Let $x^* = x_c^* + x_f^*$.

The representative firm solves the following value maximization problem:[14]

$$\max_{I,\, X_f} \ \mathbb{E}\left[\int_0^\infty \frac{\mathbb{M}_s}{\mathbb{M}_0}\left(AK_s - I_s - \Phi_s - X_{f,s}\right) ds\right], \tag{30}$$

where $\mathbb{M}$ is the equilibrium stochastic discount factor that the firm takes as given. Let Q_t denote the solution for (47), the market value of the capital stock. Using the homogeneity property, e.g., $Q(K_t, \pi_t) = q(\pi_t)K_t$, we turn (47) into the following HJB equation:

$$\begin{aligned} 0 = \max_{i,\, x_f} \ & A - i - \phi(i) - x_f - (r(\pi) - i(\pi))q(\pi) + \mu_\pi(\pi)q'(\pi) \\ & - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\int_0^{\overline{Z}} Z^{-\gamma}\xi(Z;x^*)dZ - 1\right)\right] q(\pi) \\ & + \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\int_0^{\overline{Z}} Z^{1-\gamma}\xi(Z;x^*)dZ - 1\right] q(\pi)\,. \end{aligned} \tag{31}$$

The FOC for investment implied by (31) is

$$q(\pi) = 1 + \phi'(i(\pi))\,, \tag{32}$$

[14] Financial markets are perfectly competitive and complete. While the firm can hold financial positions (e.g., DIS contracts), these financial hedging transactions generate zero NPV for the firm. Therefore, financial hedging policies are indeterminate, essentially a version of the Modigliani-Miller result. The firm can thus ignore financial contracts without loss of generality.

which equates the marginal q to the marginal cost of investing $1+\phi'(i)$.

Let $J_t = J(W_t, \pi_t)$ denote the household's value function. We show that

$$J(W,\pi) = \frac{1}{1-\gamma}(u(\pi)W)^{1-\gamma}\,, \tag{33}$$

where $u(\pi)$ is to be determined. The household solves the following problem:

$$\begin{aligned} 0 = \max_{c,h,\delta,x_c} \; & \frac{\rho\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - \rho}{1-\psi^{-1}} + \left[r(\pi) - \int_0^{\overline{Z}} \delta(Z) p(Z;x^*) dZ + \frac{(\mu_Q(\pi) - r(\pi))h - c - x_c}{w} \right] \\ & + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} - \frac{\gamma\sigma^2}{2} + \lambda(\pi)\left[\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma} \int_0^{\overline{Z}} \left(\frac{w^{\mathcal{J}}}{w}\right)^{1-\gamma} \xi(Z;x^*)dZ - 1 \right]\,, \end{aligned} \tag{34}$$

where $\mu_Q(\pi)$ defined in (B.15) is the expected cum-dividend return (ignoring the jump effect).

The consumption FOC implied by (34) yields the following consumption rule:

$$c(\pi) = \rho^\psi u(\pi)^{1-\psi} w\,. \tag{35}$$

The private sector has no incentives to spend on mitigation:

$$x_c = x_f = 0\,. \tag{36}$$

This is because the benefit of mitigation spending is thinning the fat tail for the disaster damage by increasing the $\beta(x)$ function. As no individual agent influences the distribution $\Xi(Z)$ for the recovery fraction, Z, at the margin, they have no incentives to spend on mitigation spending. In essence, mitigating disaster damages is providing a public good. As a result, market equilibrium features no aggregate mitigation spending: $x^* = x_c^* + x_f^* = 0$.

We thus cannot use the planner's solution given in Section 3 to infer the equilibrium resource allocation and prices as the welfare theorem does not hold in our model.[15] Instead, our market-equilibrium solution is equivalent to the planner's solution when the planner has no access to the mitigation technology. This is summarized in the following proposition.

Proposition 2 *There is no mitigation in competitive equilibrium. The competitive equilibrium solution corresponds to the social planner's solution only when there is no mitigation technology (i.e.* $\beta_1 = 0$*):* $\widehat{V}(K_t,\pi_t) = J(W_t,\pi_t)$*, where* $W_t = q(\pi_t)K_t$.

[15]In contrast, the planner's solution in Pindyck and Wang (2013) can be achieved via market decentralization, as there is no mitigation spending and hence welfare theorem holds in their model.

5 Taxes, Subsidies and Markets

We resurrect the planner's first-best solution in Section 3 in three ways. First, we introduce government mitigation spending, financed via lump-sum taxes, into a competitive market economy. Second, we provide a market-based implementation of the planner's solution by using taxation and subsidies. Finally, we use government mandate.

5.1 Government Mitigation Spending

We show that the planner's solution in Section 3 is attainable via a partially decentralized market setting as follows. The government chooses the optimal path of mitigation spending (financed by time-varying lump-sum taxes) to maximize the household's welfare. Households maximize utility by choosing consumption, portfolio choice, risk management, and mitigation spending policies. Firms maximize market value by choosing investment and potentially financial risk management and mitigation spending policies and face lump-sum taxes.[16]

We relegate our analysis of household's and firm's optimization, which are similar to those treated in Section 4 to Appendix C.

The government chooses mitigation spending X to maximize the household's value function given in (33). In Appendix C, we show that the HJB equation can be simplified to:

$$0 = \max_{x} \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - 1\right] + \frac{A - i^*(\pi) - \phi(i^*(\pi)) - x}{q^*(\pi)} + i^*(\pi) - \rho^{\psi}u(\pi)^{1-\psi} - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\left(\frac{(q^*(\pi))'}{q^*(\pi)} + \frac{u'(\pi)}{u(\pi)}\right) + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right], \tag{37}$$

where the starred variables denote the equilibrium solution. The FOC for x is given by

$$1 = \frac{\lambda(\pi)q^*(\pi)}{1-\gamma}\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\int_0^{\overline{Z}}\left[\frac{\partial\xi(Z;x)}{\partial x}Z^{1-\gamma}\right]dZ\ . \tag{38}$$

As the benevolent planner's value function is equal to the household's, we obtain

$$u(\pi)q(\pi) = b(\pi)\,, \tag{39}$$

[16] As firms are not financially constrained, financial risk management policies are irrelevant.

which follows from the equilibrium condition that the aggregate household wealth is equal to the stock market capitalization, $W_t = Q_t = q(\pi_t)K_t$. By substituting (39) into (38) and using the investment FOC (32), we show that mitigation spending x is the same in this market economy (with government spending) and the social planner's economy of Section 2, as the FOCs for the two problems, (20) and (38), are the same.

By substituting (39) into (35) and using the equilibrium condition $w = q(\pi)$, we obtain

$$c(\pi) = \rho^{\psi} \left(\frac{b(\pi)}{q(\pi)} \right)^{1-\psi} q(\pi) = b(\pi)^{1-\psi} (\rho q(\pi))^{\psi} \,. \tag{40}$$

Using the aggregate resource constraints $A = i(\pi) + \phi(i(\pi)) + c(\pi) + x(\pi)$ and the investment FOC (32), we obtain (20), which means that investment decisions in this market economy (with government spending) and the social planner's economy of Section 2 are the same. Then, the aggregate resource constraint implies that consumption decisions are also the same in the two economies. Finally, substituting these policies rules into (37), we verify that (37) is the same as the HJB equation (19) for the planner's problem.

Next, we implement the first-best solution via taxation and subsidy.

5.2 Taxation and Subsidy: Resurrecting First Best

The government imposes a time-varying proportional tax on each firm's capital stock (or equivalently sales as $Y = AK$ at the firm level) and also fully reimburses the firm's mitigation spending at market price. Let ν denote the tax rate on an individual firm's capital stock K. By setting $\nu_t = x_t^*$, where x_t^* is the socially optimal mitigation spending given in Section 3 and implementing 100% reimbursement of all the firm's mitigation spending, we show that the planner's solution is attained in the competitive market equilibrium.

Given the government taxation and subsidy policy, each firm solves the following problem:

$$\max_{I,\, X_f} \mathbb{E}\left[\int_0^{\infty} \left(\frac{\mathbb{M}_s}{\mathbb{M}_0} \left(AK_s - I_s - \Phi_s - X_{f,s} - \nu_s K_s \right) + p_{0,s} X_{f,s} \right) ds \right] , \tag{41}$$

where $p_{0,s}$ is the time-0 value of the government subsidy to the firm for a unit of its mitigation spending at s for each sample path (e.g., state). The firm makes a tax payment $\nu_s K_s$ and receives a subsidy $p_{0,s}$ for each unit of mitigation spending. Because markets are complete, we know $p_{0,s} = \mathbb{M}_s/\mathbb{M}_0$. Therefore, the firm always breaks even on any level of mitigation spending regardless of the level, as the firm is fully reimbursed for every unit of spending it incurs on mitigation. One possible solution is for firms to choose X_t at socially optimal level of X_t^*. As we show, this is the level of X that is consistent with equilibrium market clearing.

The firm's HJB equation is then

$$0 = \max_i \, (A - \nu(\pi)) - i - \phi(i) - (r(\pi) - i(\pi))q(\pi) + \mu_\pi(\pi)q'(\pi)$$
$$- \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\int_0^{\overline{Z}} Z^{-\gamma}\xi(Z;x^*)dZ - 1\right)\right] q(\pi)$$
$$+ \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\int_0^{\overline{Z}} Z^{1-\gamma}\xi(Z;x^*)dZ - 1\right] q(\pi)\,. \quad (42)$$

The firm's investment FOC is still given by (32). The household optimization is essentially the same as that discussed in Section 4. For brevity, we leave the details out.

In Appendix, we verify that together with household optimization and market clearing, the first-best planner's solution is attainable in equilibrium given the government's subsidy and taxation policies introduced earlier. The next proposition summarizes the key results.

Proposition 3 *In a competitive (and complete) market economy, household consumption and corporate investment attain the first-best solution as the planner does in Section 3, provided that the government is benevolent, in the sense that it optimally chooses mitigation spending to maximize household welfare. Alternatively, effective government taxation and subsidies policies can also attain the first-best outcome.*

In summary, the planner's solution of Section 3 is attained in a market economy where government either chooses mitigation spending (financed by lump-sum taxes) or stipulates effective taxation and subsidy policies. All other decisions are made by private agents in a competitive economy. Next, we implement the first-best solution via government regulation.

6 Calibration Exercise and Parameter Choices

Our calibration exercise is intended to highlight the importance of mitigation for welfare analysis. To this end, we start with a disaster calibration of an economy without any mitigation technology following the logic in Barro and Jin (2011) and Pindyck and Wang (2013). Their calibrations encompass generic disasters including wars and have time-invariant beliefs regarding arrival rates, whereas we will more finely focus on certain natural disasters. But the parameter choices are intended to match certain key macroeconomic moments.

We set the household's annual discount rate ρ at 5%, a common choice for this parameter in macro, public finance, and finance. Estimates of the EIS ψ in the literature vary considerably, ranging from a low value near zero (e.g., Hall, 1988) to values as high as two. Bansal and Yaron (2004) argue that the EIS is above unity and use 1.5 in their long-run risk model. Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. The Appendix to Hall (2009) provides a brief survey of estimates in the literature.

We purposefully set the EIS at $\psi = 1$, which is around the mid point between the two ends of the parameter estimates. The case with $\psi = 1$ is a generalized logarithmic (non-expected) utility allowing for any level of risk aversion and hence includes the logarithmic expected utility ($\gamma = 1/\psi = 1$) as a special case. This unit-EIS case of the Epstein-Zin utility has been used in various macro-finance applications as the wealth and substitution effects offset each other facilitating analysis.[17]

It is worth noting that our numerical solution does not require us to focus on the case with $\psi = 1$. But using this special case eases the exposition of our main results.[18]

As in the q literature, e.g., Hayashi (1982), we use a quadratic function:

$$\phi(i) = \frac{\theta i^2}{2} \ , \tag{43}$$

to model adjustment costs. The parameter θ measures how costly it is to adjust capital. θ will be chosen with other parameters to target certain moments, as we describe next.

[17] Tallarini (2000) uses this special Epstein-Zin utility with unit EIS to study business cycles. Wachter (2013) uses this unit EIS case of the Epstein-Zin utility to obtain closed-form solutions for equilibrium asset pricing in a disaster model with stochastic time-varying arrival intensity. Gourio (2012) and Collin-Dufresne, Johannes, and Lochstoer (2016) discuss the tractable solutions for the unit EIS case, a special case in their respective models.

[18] The main results on the effects of mitigation continue to hold provided that we choose an elasticity of intertemporal substitution that is larger than one. This is a key condition that is required to match asset-pricing moments as first pointed out by Bansal and Yaron (2004).

Table 1: Parameter Values

Parameters	Symbol	Value
time rate of preference	ρ	5%
elasticity of intertemporal substitution	ψ	1
maximal post-jump recovery fraction	$\overline{Z}$	98.1%
power law exponent with no mitigation	β_0	24
jump arrival rate if State is G	λ_G	0.25
coefficient of relative risk aversion	γ	3.1
productivity	A	17.1%
quadratic adjustment cost parameter	θ	30.9
capital diffusion volatility	σ	14.1%
mitigation technology parameter	β_1	842
jump arrival rate if State is B	λ_B	2
Targeted observables without mitigation (State G)		
(real) risk-free rate		0.8%
equity risk premium		6.6%
stock market return volatility		14.5%
consumption-investment ratio		2.84

All parameter values, whenever applicable, are continuously compounded and annualized.

We calibrate the parameters (other than the arrival rate λ_B in state B and the mitigation technology parameter β_1) by targeting the scenario where the arrival rate of disaster is low, i.e., the state is G. This number is taken from a representative and leading study in the climate economics literature using cyclone data by Hsiang and Jina (2014). We define disasters (jump events in our model) as those that trigger predicted long-run GDP loss by more than 1.9%, which is the mean finding among their 6,700 cyclone events. Their GDP loss figure is at the country level. We can extrapolate these numbers for a broader cross-section of weather disasters that plausibly would impact a substantial fraction of global GDP. That is we take these numbers and those below to be representative for a basket of natural disasters tied to temperature and precipitation (Auffhammer, Hsiang, Schlenker, and Sobel (2013)).

Mapping this estimate to our model, we set the largest fractional recovery $\overline{Z}$ to $0.981 = 1 - 1.9\%$. Using the number of cyclones that caused predicted long-run GDP losses less than 1.9%, which is about 5,000, we calibrate the arrival rate in State G to $\lambda_G = (6,700 - 5,000)/6,700 \approx 0.25$ per annum. Given our linear mitigation technology, the expected loss due to realized disasters in the data of around 5.8% GDP corresponds to $\beta_0 = 24$ under the assumption that households had optimistic priors in the past close to $\pi = 0$.

We calibrate the remaining four parameters: risk aversion γ, diffusion volatility σ, adjustment cost parameter θ, and productivity A by targeting the following four moments: the annual risk-free rate 0.8%, the expected annual stock market risk premium 6.6%, the annual stock market return volatility $\sqrt{0.0211} = 14.5\%$, and the consumption-investment ratio, $c(0)/i(0) = 2.84$. Doing so yields the following results: $\sigma = 14.1\%$, $\theta = 30.9$, $\gamma = 3.1$, and $A = 17.1\%$. These parameter values are broadly in line with those used in the literature.[19]

As a robustness check, the implied annual investment-capital ratio is $i(0) = 3.86\%$, the Tobin's q is $q(0) = 1 + \theta \times i(0) = 2.19$. Accounting for the possibility of jumps, we obtain an expected growth rate $g(0) = i(0) - \lambda(0)\ell(0)$ of 2.4% per annum. These un-targeted parameter values are also broadly in line with the values used in the literature.

The remaining parameters λ_B (the arrival rate in the bad state) and β_1 (the effectiveness of mitigation) are chosen in the following manner. Currently, arrival rate of cyclones for λ_G of 0.25 is once every four years. We set λ_B to be 2, which means that in the bad state, a cyclone on average arrives twice a year, which seems reasonable given climate change projections. We later perform comparative static analysis with respect to λ_B to better understand the impact of disaster risk. We calibrate the the parameter β_1 for the mitigation technology by targeting the optimal mitigation at zero for State G, i.e., $x(0) = 0$. And doing so yields $\beta_1 = 842$. That is, there is no value-add from the mitigation technology in State G, $b(0) = \widehat{b}(0)$.

7 Cost and Benefit of Mitigation

Measuring the Welfare Gain of Government Mitigation Spending. How much are we worse off if the economy is completely laissez faire? To answer this question, we introduce

[19] As an example, while using a different calibration strategy (for example, they do not target the capital adjustment costs), Barro and Jin (2011) also report the calibrated coefficient of relative risk aversion is their paper is about three. Our estimates are also close to those in Pindyck and Wang (2013), even though they use a different set of moments for the disaster arrival rate and the damage function.

the following willingness to pay (WTP) metric as in Pindyck and Wang (2013).

Let ζ denote the fraction of capital stock that the society is willing to pay to go from the competitive market economy with no mitigation spending to an economy where the government either chooses the optimal regulation or directly the optimal level of mitigation spending, as discussed in Section 5. To make the society indifferent between the two options, the following condition has to hold:

$$V((1-\zeta(\pi))K, \pi) = \widehat{V}(K, \pi) \ . \tag{44}$$

The left side of (44) is the value function under optimal government mitigation mandate or spending in an otherwise market economy with a lower level of capital stock (as a ζ fraction of K is deducted) and the right side is the value function under status quo.

By substituting the value functions given in (18) and (29) into the household's indifference condition (44), we obtain the following equation for $\zeta(\pi)$:

$$\zeta(\pi) = 1 - \frac{\widehat{b}(\pi)}{b(\pi)} > 0 \ . \tag{45}$$

The WTP $\zeta(\pi)$ measures the value creation by government mitigation regulation/spending measured by the percentage increase in the society's certainty-equivalent wealth.

Policy Rules and Tobin's q. The solution to our model in Figure 3 emphasizes the optimal response depending on households' belief about arrival rates of disasters. We plot optimal mitigation spending (Panel A), consumption (Panel B), investment (Panel C) and value of capital (Tobin's average q) (Panel D) as functions of belief π, where $\pi = 0$ means the economy is in State G while $\pi = 1$ means the economy is in State B.

There are two lines — a solid blue line that describes the planner's solution ($\beta_1 = 842$) and a dashed red line that corresponds to the competitive market solution (with no mitigation in equilibrium), which is same as the planner's solution with no mitigation technology ($\beta_1 = 0$).

Panel A shows that mitigation ramps up from zero to around 2.1% of the capital stock (which corresponds to 12.4% of GDP) even when we increase belief π from near zero to around 2% (Panel A). In contrast, when the household believes entirely in the bad scenario ($\pi = 1$), the mitigation spending is a little over 4% (around 24% of GDP.)

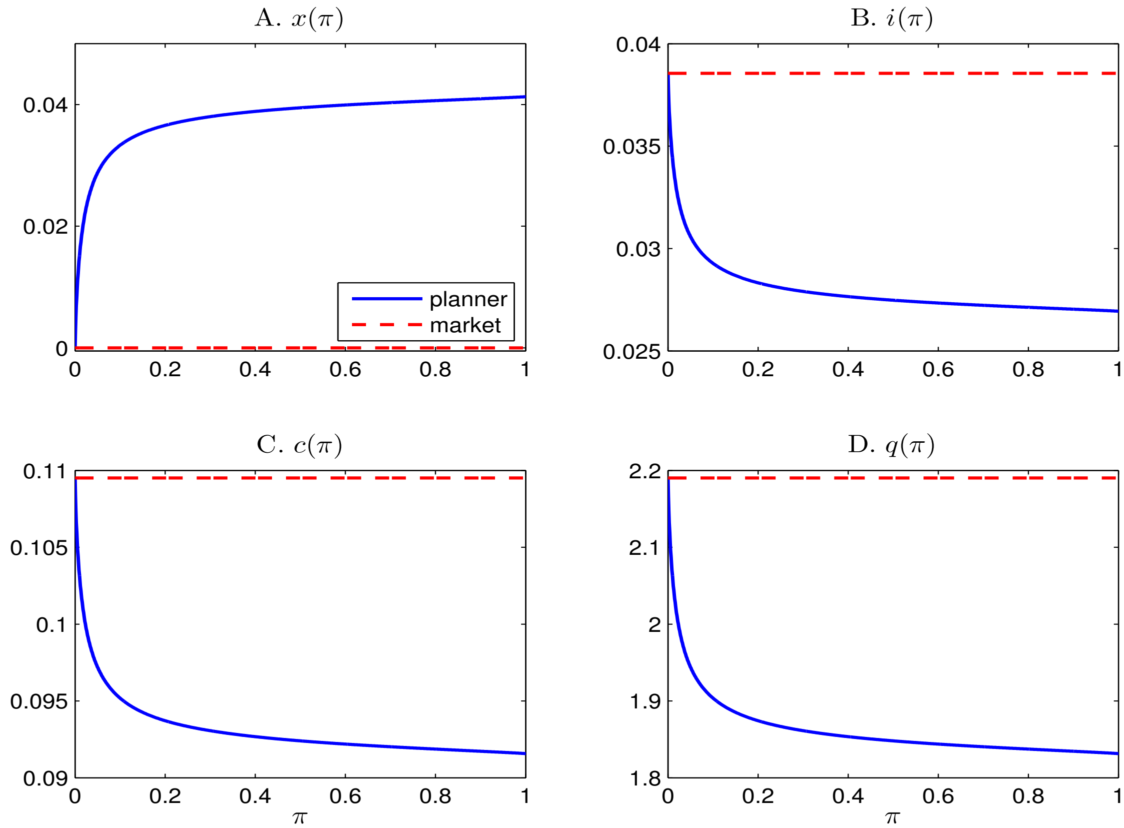

Figure 3: This figure plots (scaled) mitigation $x(\pi)$ (Panel A), investment-to-capital ratio $i(\pi)$ (Panel B), consumption-to-capital $c(\pi)$ (Panel C), and the value of capital (Panel D) as functions of π, belief regarding disaster arrival rates. $\pi = 0$ is the most optimistic belief in a low arrival rate and $\pi = 1$ is the most pessimistic belief in a high arrival rate.

Mitigating now and aggressively comes at the expense of investment, consumption, and the value of capital (Panels B-D). Notice in Panels B-D the dashed red lines corresponding to the competitive-market solution are flat even as beliefs worsen. We have intentionally picked preference parameter values (specifically, we set a key parameter, the EIS, to one, an empirically reasonable choice) so that the investment-capital ratio, the consumption-capital ratio, and capital valuation multiple (e.g., Tobin's average q) all remain constant as the optimal mitigation is zero in competitive equilibrium even as beliefs deteriorate. Doing so allows us to single out the impact of mitigation by analyzing the solid blue lines.[20]

[20]The main results on the effects of mitigation continue to hold provided that we choose an elasticity of intertemporal substitution that is larger than one. For example, Table **??** also reports the results for the case where $\psi = 1.5$. Requiring $\psi \geq 1$ is a key condition that is required to match asset-pricing moments as

A broader message is the endogenous high nonlinearity for both price and quantity variables in our model. This message is in line with studies of financial crisis (e.g., Brunnermeier and Sannikov, 2014) and climate change (e.g., Barnett, Brock, and Hansen, 2020).

Maximizing Welfare and Insuring Economic Growth. To better understand why the government ramps up mitigation spending so quickly and non-linearly even if households put just a little bit weight on the bad scenario, we turn to the welfare costs. In Panel A of Figure 4, we plot the following welfare measure:

$$\tau(\pi) = \frac{b(\pi)}{b(0)} \quad \text{and} \quad \widehat{\tau}(\pi) = \frac{\widehat{b}(\pi)}{\widehat{b}(0)} \,. \tag{46}$$

These are (scaled) certainty-equivalent wealth for the planner's and the competitive-market economies, respectively. Both curves start at one when $\pi = 0$ by definition.

In competitive-market economy (with no government), $\widehat{\tau}(\pi)$, declines dramatically and non-linearly with beliefs (the dashed red line). In contrast, with a benevolent government, $\tau(\pi)$ (the solid blue line) drops much less and stays above $\widehat{\tau}(\pi)$ (the dashed red line) because government mitigation generates substantial downside protection (curtailment or loosely speaking hedging benefits) which leads to higher social welfare.

In Panel B, we plot the WTP $\zeta(\pi)$ given in (45), which is equal to the difference between one and $\widehat{\tau}(\pi)/\tau(\pi)$, the ratio between the two lines in Panel A.[21] For example, even with a mere 2% probability attached to the bad scenario, the household is willing to give up 37.1% of the existing capital stock for the government mitigation spending.

In Panel C, we corroborate the benefit of using the mitigation technology by showing that the conditional damage are less as a result of mitigation. The more pessimistic the society, the greater the benefit of curtailing disaster risks. Note that in the market economy, since there is no mitigation spending, $\ell(\pi)$ remains constant as for the case where the mitigation technology is unavailable.

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first pointed out by Bansal and Yaron (2004).

[21] As we noted earlier, $b(0) = \widehat{b}(0)$. This is because the mitigation technology parameter so that the household optimally chooses no mitigation even with access to the technology in the most optimistic scenario.

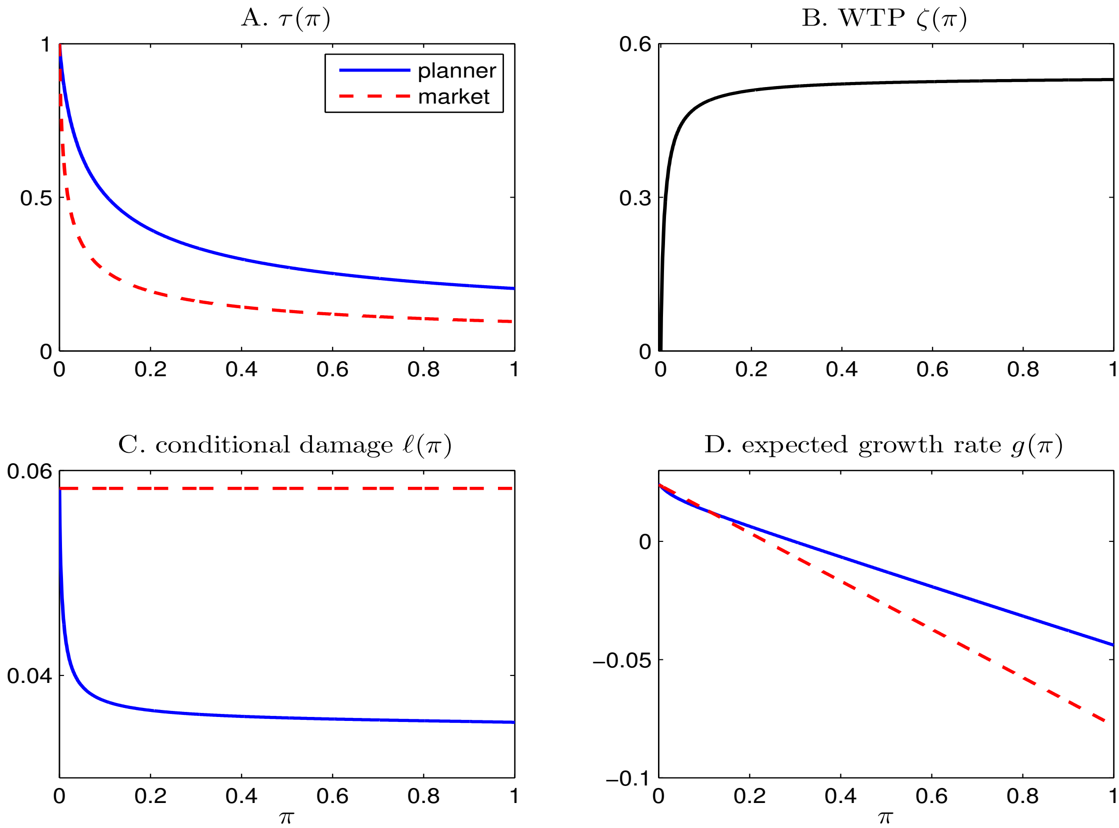

Figure 4: This figure plots the welfare and growth implications of optimal mitigation strategy. Panel A reports a welfare measure proportional to household's certainty equivalent wealth. Panel B reports the households' willingness to pay (specifically, certainty equivalent wealth) for government mitigation spending (in percentage terms). Panels C and D report the conditional damage $\ell(\pi)$ and the expected growth $g(\pi)$, respectively.

In contrast, the effect of mitigation on growth g_t (including the *ex ante* effect of disaster arrivals) is subtler. For low levels of beliefs, the society sacrifices some growth in the short term for sustainable growth in the future as immediate cuts to consumption and investment are substantial and the immediate benefit of curtailing risks on growth are not yet very high (due to a low expected arrival rate $\lambda(\pi)$.) Therefore, growth with government mitigation spending could be lower than in the competitive market with no government.

However, as the society becomes more pessimistic (i.e., more weight on the bad scenario), it is better prepared with government mitigation spending than without when disasters arrive in the future. As a result, government mitigation spending significantly buffers growth slowdown by reducing the expected disaster damages. This plot captures the essence of government mitigation spending to insure sustainable economic growth.

Comparative Statics: Effects of Mitigation Efficacy β_1 and Arrival Rate λ_B. In Table 2, we report the welfare gain of government mitigation for varying levels of mitigation efficacy β_1 for two levels of EIS ($\psi = 1$ in Panel A and $\psi = 1.5$ in Panel B). Recall that our baseline of $\beta_1 = 842$ is set such that the planner chooses no mitigation when $\pi = 0$. But as before, when π rises to 2% or 5%, we see a high WTP as we described previously. When $\psi = 1.5$ as opposed to $\psi = 1$ in our baseline case, the WTP is smaller but still significant at 11.6% and 18% for $\pi = 2\%$ and $\pi = 5\%$, respectively.

Table 2: The effect of mitigation efficacy β_1 and the disaster likelihood λ_B on WTP for the mitigation technology: $\zeta(\pi)$. We report the values for $\zeta(\pi)$ at two levels of belief: $\pi = 2\%$ and $\pi = 5\%$ ($\psi = 1$ in Panels A and C, and $\psi = 1.5$ in Panels B and D.)

	Panel A.	$\psi = 1$	Panel B.	$\psi = 1.5$
β_1	$\pi = 2\%$	$\pi = 5\%$	$\pi = 2\%$	$\pi = 5\%$
0	0	0	0	0
200	8.27%	9.66%	0.56%	0.83%
400	24.4%	29.0%	5.40%	8.28%
842	**37.1%**	**44.9%**	11.6%	18.0%
1600	43.7%	53.4%	16.5%	25.0%
	Panel C.	$\psi = 1$	Panel D.	$\psi = 1.5$
λ_B	$\pi = 2\%$	$\pi = 5\%$	$\pi = 2\%$	$\pi = 5\%$
0.25	0	0	0	0
1	3.50%	6.79%	1.58%	3.26%
2	**37.1%**	**44.9%**	11.6%	18.0%
4	86.3%	86.6%	39.6%	44.0%

Needless to say, a near doubling of the baseline mitigation efficacy to 1600 yields much larger estimates than the baseline. But even cutting the efficacy in half still yields sizeable WTP in the mid twenty percent range for $\psi = 1$.

Increasing the EIS from one to 1.5 lowers the WTP as the household is more willing to substitute consumption over time and across states. Even when $\psi = 1.5$, we get a non-trivial WTP at 8.28% for $\pi = 5\%$. When β_1 is 200, which is a fairly inefficient technology, we still get sizeable WTPs when $\psi = 1$. Only when we use $\psi = 1.5$ do we get close to zero WTP. Noe that these are not necessarily small effects if we express these percentages in terms of GDP. Overall, we view these numbers as confirming a high WTP for mitigation spending across a wide range of plausible parameters.

We then conduct a similar comparative static exercise with respect to the disaster likelihood λ_B for two levels of the EIS ($\psi = 1$ in Panels A and C and $\psi = 1.5$ in Panels B and D). In both panels, if $\lambda_B = \lambda_G = 0.25$, naturally there is no change in mitigation policy. As in our calibration, the optimal mitigation when $\pi = 0$ is zero, we thus obtain $\zeta(0) = 0$ in this case. If $\lambda_B = 1$, which is four times λ_G, the WTP is much higher: 3.50% of the existing capital stock (i.e., about 20.5% of GDP) even if belief is a mere 2% and 6.79% of the capital stock (i.e., about 39.7% of GDP) if belief is 5%. These effects are already quite substantial.

For our baseline where $\lambda_B = 8\lambda_G$, as we discussed earlier, the WTP is 37.1% when $\pi = 2\%$ and 44.9% of the capital stock when $\pi = 5\%$. The quantitative effect remains large even when $\psi = 1.5$. For example, if $\lambda_B = 2$, even at $\pi = 2\%$, the WTP is 11.6% of the capital stock.

Our analysis is related to the "cost of business cycle" literature (Lucas (1987)). In contrast to the small welfare gains from eliminating business-cycle risk, we suggest very large welfare gains and call for an active government mitigation to prepare for disaster risks. A key in our model is that learning from disaster arrivals amplifies the risk resembling the long-run risk channel as in Bansal and Yaron (2004) and Collin-Dufresne, Johannes, and Lochstoer (2016).

Climate Disasters versus Pandemics Our baseline calibration is based on cyclones. But we have also conducted a similar set of calculations for pandemics assuming $\overline{Z} = 90.5\%$, $\beta_0 = 6.86$ and $\lambda_G = 0.038$. We then vary β_1 around 308 and λ_B around 0.38. That is, pandemics are more rare than weather or climate disasters but result in greater damage. Interestingly, we find similar results for policy functions and WTP because the expected damage, which is the product of arrival rate and damage conditional on arrival, are significant for both disasters. That is, mitigating weather disaster risk is not so different from mitigation pandemic risk.

8 Conditional Damage Functions

A crucial parameter in our cost and benefit analysis is β_1 — the efficacy of mitigation in ameliorating damages conditional on the arrival of a disaster. A natural way to retrieve this parameter is to see how damages vary depending on mitigation spending, perhaps cross-sectionally or over time for a country. Of course, endogeneity concerns regarding mitigation spending loom large and absent large scale randomized interventions, it is hard to interpret such Panel or time series estimates.

Our model offers a way to gauge this parameter. In our model, natural disaster arrivals follow a Poisson process and agents learn about the arrival rate, which is fairly plausible for many disasters such as hurricanes or viruses. Moreover, we assume that damages conditional on an arrival of a disaster are uncorrelated over time—absent any mitigation spending. This assumption is harder to verify given that mitigation is likely present. Nonetheless, we do believe that conditional on arrival, there is a fairly large tail as far as damages (i.e. different viruses might be more lethal biologically or certain hurricanes are simply more destructive) and that these conditional damages are likely to be random absent mitigation.

Frequency of arrivals and inter-arrival times entirely drive perceived risks and hence mitigation in our model. As a result, damage of a disaster conditional on an arrival is much higher when perceived risks and mitigation are low, i.e., less preparedness. The reduced-form implication is that a disaster that strikes after a long absence of disasters leads to much larger conditional damages than a disaster that strikes following a recent cluster of disasters due to time-varying preparedness. To the extent we can measure that conditional damages decline with higher beliefs, we can then gauge β_1.

We build up the logic of this strategy in steps using Figure 5, where we simulate a jump path with four arrivals at $t = 0.5, 0.81, 1.42, 2.25$ (Panel A).

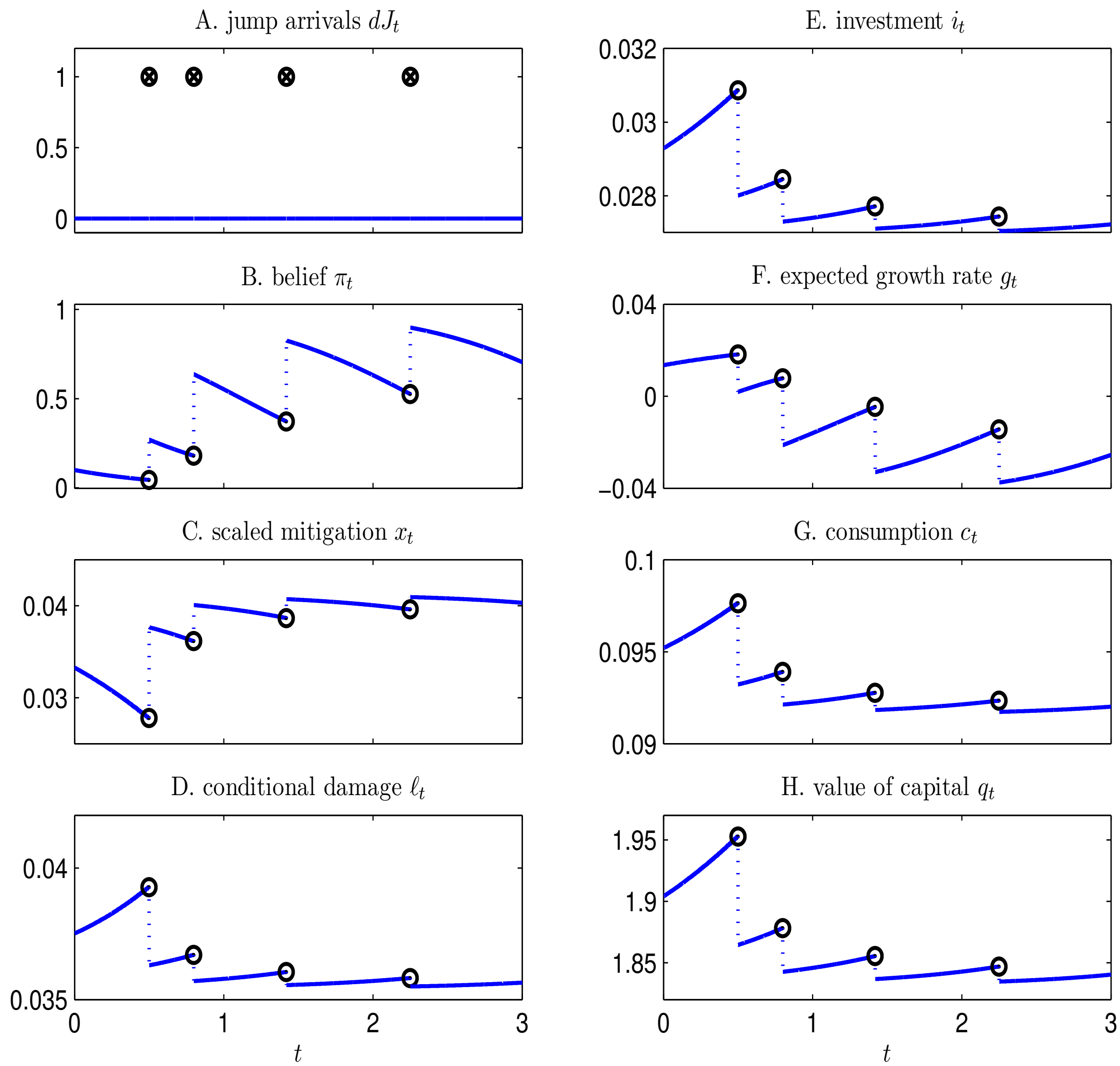

Figure 5: This figure simulates a jump path with four arrivals at $t = 0.5, 0.81, 1.42, 2.25$ (with $\pi_0 = 0.1$) and plots dynamics for belief updating, policies, and valuation.

Upward Revision of Perceived Risks and Mitigation Following Disasters Arrivals. Following each arrival, belief jumps up in Panel B. We start π at a prior $\pi_0 = 0.1$, which then jumps at the first arrival time $t = 0.5$ from the pre-jump level of 0.044 to 0.27. Notice that when there is no jump, beliefs drift downward. But in this jump path, the jumps arrive quickly and there is little reversion. The third and fourth arrivals then lead the belief or perceived risks to rise close to one.

Following each arrival, mitigation also jumps up in Panel C. This follows from rational updating of perceived risks with a disaster arrival that discretely increases the value of mitigation spending. Mitigation spending as a fraction of capital stock x jumps from around 2.8% to 3.8% at the first jump time. But notice that subsequent jumps do not elicit as strong a revision of mitigation spending due to the non-linearity of mitigation as a function of π and close inter-arrival times for these jumps discussed in the cost and benefit analysis.

Conditional Damages $\ell(\pi)$. Panel D shows that damages conditional on arrival are higher when beliefs are low than when beliefs are high. The reason is mitigation spending. For example, at the first jump time $t = 0.5$, the conditional damage decreases from the pre-jump level, 3.9%, to 3.6%, which is accompanied by a substantial increase of x from 2.8% to 3.8%, as discussed earlier. When beliefs are optimistic, corresponding to an economy that has had relatively few prior arrivals and long inter-arrival times, mitigation spending or preparedness is low. Damage in our model corresponds to long-run destruction of either tangible or intangible capital and productivity.

Note that expected damages conditional on arrival are the same across jumps absent mitigation. Hence, difference in damages over time are entirely driven mitigation spending and realized losses $(1 - Z)$. We can estimate $\ell(\pi)$ using our π formula which is entirely dependent on frequency of arrivals and inter-arrival times. Such an estimate is well-identified under the assumptions of our model.

9 Regulatory Risks and Sustainable Investing

Transition Risks. As we mentioned in the Introduction, there is substantial discussion in regulatory circles pertaining to transition risk of climate change. These discussions typically center on potential emissions taxes in the future that will strand carbon assets and lead to a drop in stock market value. There is a similar notion in our model associated with mitigation of disasters. Our model generates a number of testable predictions that relate capital taxes and mitigation subsidies to the arrival of disasters.

To see why, recall from Figure 5 that the arrival of a disaster raises household perceived risks that then triggers mitigation. This mitigation as we have pointed out in Section 5 has to be funded by capital taxes to subsidize mitigation. In other words, we expect that post arrival of a disaster a higher likelihood of corporate taxes and mitigation subsidies.

Notice that the upward revisions in beliefs and mitigation lead to a crowding out of investment (presented in Panel E). The substantial increase in mitigation at $t = 0.5$ conditional on a jump arrival is accompanied by a substantial cut in investment (from 3.1% to 2.8%) and consumption (from 9.8% to 9.3%) of capital stock (see Panels E and G.) As a result, the expected growth rate (including the jump effect) decreases less from 1.8% to 0.2%. This is the channel through which shocks to growth are damped. Finally, note that the drop in value of capital moves from 1.95 to 1.86 in sync with drop in investment, as implied by the q-theoretic investment optimality condition. This then captures the regulatory risk that is much discussed in policy and practitioner circles.

Moreover, subsequent jumps lead to smaller absolute declines in i_t. Similarly, there is a larger downward jump in expected growth rate for the first jump than for the fourth jump. This is because mitigation spending is much higher for the economy that has been hit from a number of recent arrivals and hence there is no more need for mitigation and taxes to fund it.

Regulatory risk arises in our learning model because the arrival of a disaster triggers not just a direct effect as discussed above but also an indirect effect of belief updating that disasters in the future are more likely and hence investment opportunities are worse. Increasing mitigation funded by taxes to curtail the damage of disaster risk causes further prolonged drops in consumption and investment, leading to lower expected growth rates and stock market valuations. This ramped-up mitigation persists until underlying perceived risks is diminished and the economy restores confidence.

Government Sustainability Mandates and Firm Incentives. Finally, aside from taxes and subsidies, mitigation spending can also be implemented as a government mandate for the representative investor (e.g., sovereign wealth funds) to only invest in firms that meet the following sustainable investing or ESG criterion: in order for investors to include a firm in their portfolios, it has to at least spend $X_t^* = x^*(\pi_t)K_t$ on mitigation, where $x^*(\pi_t)$ is the socially optimal first-best level given in Section 3. If the firm fails to do so, its equity cannot be included in the market portfolio. As a result, its share price will be sufficiently low and in equilibrium, the firm will choose to meet the mandate requirement.

Given this mandate, each firm solves the following problem:

$$\max_{I, X_f} \mathbb{E}\left[\int_0^\infty \frac{\mathbb{M}_s}{\mathbb{M}_0}\left(AK_s - I_s - \Phi_s - X_{f,s}\,\mathbf{1}_{X_{f,s}\geq X^*}\right) ds\right] , \tag{47}$$

where $\mathbf{1}_{\mathcal{A}}$ is the indicator function. In equilibrium, the firm chooses minimally necessary mitigation spending by setting $X_{f,s} = X^*$ in order to be included in the investors' portfolio. Its investment is also given by the investment FOC, (32). But, we show that the first-best investment is attained in this setting unlike in the market economy of Section 4.

As markets are complete, the household/investor's problem can be written as:[22]

$$\max_{C,\delta,h,x_c} \mathbb{E}\left(\int_0^\infty \frac{\mathbb{M}_s}{\mathbb{M}_0} C_s ds\right) , \tag{48}$$

subject to the following budget constraint:

$$\mathbb{E}\left(\int_0^\infty \frac{\mathbb{M}_s}{\mathbb{M}_0} C_s ds\right) \leq \mathbb{E}\left(\int_0^\infty \frac{\mathbb{M}_s}{\mathbb{M}_0} D_s^* ds\right) , \tag{49}$$

where $D_s^* = AK_s - I_s^* - \Phi_s^* - X_s^*$ is the equilibrium dividend payments.[23]

Finally, the government chooses the mandate X^* to maximize the investor's utility $J(W, \pi; X^*)$, which is equivalent to maximizing (48) as markets are complete. By setting X^* at the first-best level, both households and firms maximize their objectives and market clear. In equilibrium, the individual investor's C_s is equal to the aggregate consumption C_s^*, which enters the equilibrium SDF $\mathbb{M}_s$. In summary, the equilibrium for the three-party optimization with government regulation is the planner's solution in Section 3.

10 Conclusion

We provide the planner's solution to a model where households learn from exogenous natural disaster arrivals about arrival rates and spend to mitigate potential future damages.

[22] We use the Cox and Huang (1989) martingale approach to solve optimal portfolio choice problem.

[23] We leave out the household's mitigation choice in the stated optimization problem as it is immediate to see that investors have no incentives to do mitigation spending out of their own pockets, i.e., $x_c = 0$ for the same free-rider argument.

Mitigation—by curtailing aggregate risk and insuring sustainable growth—is a public good in our model. The planner's solution can be implemented via a capital tax and mitigation subsidy scheme. Our model provides an integrated assessment of the cost and benefit of mitigation efforts such as public health spending or environmental protection via an aggregate risk management rationale. Our model also delivers a number of testable implications pertaining to damage functions, regulatory risks and sustainable investments. Future research avenues include an estimation of our model using damage and mitigation spending data.

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Appendices

A Derivation for the Main Results

A.1 Planner's Solution

Substituting the value function (18) into the FOC (16) for investment and the FOC (17) for mitigation spending, we obtain:

$$b(\pi) \;=\; c(\pi)^{1/(1-\psi)}\left[\rho(1+\phi'(i(\pi)))\right]^{-\psi/(1-\psi)}\;, \tag{A.1}$$

$$\rho c(\pi)^{-\psi^{-1}} b(\pi)^{\psi^{-1}-1} \;=\; \frac{\lambda(\pi)}{1-\gamma}\left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)}\right)^{1-\gamma}\int_0^{\overline{Z}}\left[\frac{\partial\xi(Z;x)}{\partial x}Z^{1-\gamma}\right]dZ\,, \tag{A.2}$$

where the post-jump belief $\pi^{\mathcal{J}}$ is given in (12) as a function of the pre-jump belief π. Then substituting the resource constraint, $c(\pi) = A - i(\pi) - \phi(i(\pi)) - x(\pi)$, into (A.1), we obtain (20). By substituting (A.1) into (A.2), we obtain (21). Finally, substituting the value function, (18), and (20) and (21) into the HJB equation (15) and simplifying, we obtain the ODE given in (19).

By applying essentially the same argument to the right boundary, $\pi = 1$, we obtain the solution for $b(1)$, $i(1)$, and $x(1)$ by jointly solving the following three equations:

$$0 \;=\; \frac{\left(\frac{b(1)}{\rho(1+\phi'(i(1)))}\right)^{1-\psi}-1}{1-\psi^{-1}}\rho + i(1) - \frac{\gamma\sigma^2}{2} + \frac{\lambda_B}{1-\gamma}\left(\mathbb{E}(Z^{1-\gamma})-1\right)\;, \tag{A.3}$$

$$b(1) \;=\; (A - i(1) - \phi(i(1)) - x(1))^{1/(1-\psi)}\left(\rho(1+\phi'(i(1)))\right)^{-\psi/(1-\psi)}\;, \tag{A.4}$$

$$\frac{1}{1+\phi'(i(1))} \;=\; \frac{\lambda_B}{1-\gamma}\int_0^{\overline{Z}}\left[\frac{\partial\xi(Z;x)}{\partial x}Z^{1-\gamma}\right]dZ\;. \tag{A.5}$$

Using the same argument, we obtain the three equations, (22)-(24), for the left boundary, $\pi = 0$. Solving these three equations yields $b(0)$, $i(0)$, and $x(0)$.

A.2 Asset Pricing Implications of Planner's Problem

By using the results in Duffie and Epstein (1992), we obtain the following stochastic discount factor (SDF), $\{\mathbb{M}_t : t \geq 0\}$, implied by the planner's solution:

$$\mathbb{M}_t = \exp\left[\int_0^t f_V(C_s, V_s)\,ds\right] f_C(C_t, V_t)\;. \tag{A.6}$$

Using the FOC for investment (16), the value function (18), and the resource constraint, we obtain:

$$f_C(C, V) = \frac{1}{1+\phi'(i(\pi))}b(\pi)^{1-\gamma}K^{-\gamma}\,, \tag{A.7}$$

and

$$f_V(C,V) = \frac{\rho}{1-\psi^{-1}}\left[\frac{(1-\omega)C^{1-\psi^{-1}}}{((1-\gamma))^{\omega-1}}V^{-\omega} - (1-\gamma)\right] = -\epsilon(\pi)\,, \tag{A.8}$$

where

$$\epsilon(\pi) = -\frac{\rho(1-\gamma)}{1-\psi^{-1}}\left[\left(\frac{c(\pi)}{b(\pi)}\right)^{1-\psi^{-1}}\left(\frac{\psi^{-1}-\gamma}{1-\gamma}\right) - 1\right]. \tag{A.9}$$

Using the equilibrium relation between $b(\pi)$ and $c(\pi)$, we simplify (A.9) as:

$$\epsilon(\pi) = \rho + \left(\psi^{-1}-\gamma\right)\left[i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} + \frac{\lambda(\pi)}{1-\gamma}\left(\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right)\right], \tag{A.10}$$

where the post-jump belief $\pi^{\mathcal{J}}$ is given in (12) as a function of the pre-jump belief π.

Using Ito's Lemma and the optimal allocation, we have

$$\begin{aligned}\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} &= -\epsilon(\pi)dt - \gamma\left[i(\pi)dt + \sigma d\mathcal{W}_t\right] + \frac{\gamma(\gamma+1)}{2}\sigma^2 dt + \left((1-\gamma)\frac{b'(\pi)}{b(\pi)} - \frac{i'(\pi)\phi''(i(\pi))}{1+\phi'(i(\pi))}\right)\mu_\pi(\pi)dt \\ &+ \left[\frac{1+\phi'\left(i(\pi)\right)}{1+\phi'\left(i(\pi^{\mathcal{J}})\right)}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma} Z^{-\gamma} - 1\right] d\mathcal{J}_t\,. \end{aligned} \tag{A.11}$$

As the expected rate of percentage change of $\mathbb{M}_t$ equals $-r_t$ (Duffie, 2001), we obtain the following expression for the interest rate:

$$\begin{aligned} r(\pi) &= \rho + \psi^{-1}i(\pi) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \left[(1-\psi^{-1})\frac{b'(\pi)}{b(\pi)} - \frac{i'(\pi)\phi''(i(\pi))}{1+\phi'(i(\pi))}\right]\mu_\pi(\pi) \\ &- \lambda(\pi)\left[\frac{1+\phi'\left(i(\pi)\right)}{1+\phi'\left(i(\pi^{\mathcal{J}})\right)}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{-\gamma}) - 1\right] \\ &- \lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1 - \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})\right)\right]\,. \end{aligned} \tag{A.12}$$

Since the dividend D_t is equal to C_t in equilibrium and $\mathbb{M}_{t-}D_{t-}dt + d(\mathbb{M}_tQ_t)$ is a martingale (Duffie, 2001), by using Ito's Lemma and setting its drift to zero, we obtain

$$\begin{aligned}\frac{c(\pi)}{q(\pi)} &= r(\pi) + \gamma\sigma^2 + \lambda(\pi)\left[\frac{1+\phi'\left(i(\pi)\right)}{1+\phi'\left(i(\pi^{\mathcal{J}})\right)}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\left(\mathbb{E}(Z^{-\gamma}) - \frac{q(\pi^{\mathcal{J}})}{q(\pi)}\mathbb{E}(Z^{1-\gamma})\right)\right] \\ &- i(\pi) - \mu_\pi(\pi)\frac{q'(\pi)}{q(\pi)} \\ &= \rho - (1-\psi^{-1})\left[i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\right] + \lambda(\pi)\omega\left[1 - \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})\right], \end{aligned} \tag{A.13}$$

where $\omega = (1-\psi^{-1})/(1-\gamma)$. We can calculate Tobin's q from (A.13).

There are two special cases. First, if $\psi = 1$, for any value of γ, the consumption-wealth ratio is constant and $c(\pi)/q(\pi) = \rho$. This is the key result pointed out by Tallarini (1999). The other special case is the expected utility case, $\omega = 1$.

A.3 Constant-Elasticity-of-Substitution Production Function

Suppose there are two factors of production, K_1 and K_2, and the aggregate (composite) capital stock K is given by the following constant-elasticity-of-substitution (CES) function:

$$K(K_1, K_2) = \left(\chi_1 K_1^{\frac{\alpha-1}{\alpha}} + \chi_2 K_2^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}}, \tag{A.14}$$

where α is the elasticity of substitution between K_1 and K_2. The optimal demand for K_2 given the level of K_1 then solves $\max_{K_2} \{K(K_1, K_2) - u_2 K_2\}$, where u_2 denotes the cost of renting a unit of the second factor of production. The optimality condition implies that the ratio K_2/K_1 is constant at all t, as $K_2 = \chi_1^{\frac{\alpha}{\alpha-1}} \left[(\alpha u_2/\chi_2)^{\alpha-1} - \chi_2 \right]^{-\frac{\alpha}{\alpha-1}} K_1$. Therefore, without loss of generality, we can interpret (3) as the accumulation equation for composite capital stock, K, and (1) for output.

B Market Equilibrium Solution

Competitive Equilibrium. (i) the net supply of the risk-free asset is zero; (ii) the demand for the claim to the representative firm is equal to unity, the normalized aggregate supply; (iii) the net demand for the DIS of each possible recovery fraction Z is zero; and (iv) the goods market clears, i.e., $Y_t = C_t + I_t + \Phi_t + X_{c,t} + X_{f,t}$ at all $t \geq 0$.

B.1 Household's Optimization Problem

The equilibrium aggregate stock value is $Q_t^* = q^*(\pi_t) K_t$. We conjecture and later verify that the cum-dividend return of the aggregate stock market is given by

$$\frac{dQ_t^* + D_{t-}dt}{Q_{t-}^*} = \mu_Q(\pi_{t-})dt + \sigma d\mathcal{W}_t - \left(1 - Z \frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})} \right) d\mathcal{J}_t \,, \tag{B.15}$$

where $\pi_t^{\mathcal{J}} = \lambda_B \pi_{t-}/\lambda(\pi_{t-})$ is the post-jump belief and $\mu_Q(\pi)$ is the expected cum-dividend return (ignoring the jump effect). In (B.15), the diffusion volatility is equal to σ, the same parameter as in (3), which we verify later. Also, by using the homogeneity property, we have conjectured a specific form for the change of the cum-dividend return should a jump occur, which we also verify later.

When a disaster occurs at time t, wealth changes discretely from W_{t-} to $W_t^{\mathcal{J}}$, where

$$W_t^{\mathcal{J}} = W_{t-} - \left(1 - Z \frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})} \right) H_{t-} + \delta_{t-}(Z) W_{t-} \,. \tag{B.16}$$

The second term is the loss of the portfolio's market value upon the arrival of a disaster and the last term is the repayment from the DIS contract entered at $t-$. While the mitigation spending X_c makes disasters less damaging for the society, it does not generate any direct benefit for the household upon a jump arrival. This is at the core of the market failure.

The household accumulates wealth as:

$$\begin{aligned} dW_t &= r(\pi_{t-})W_{t-}dt + (\mu_Q(\pi_{t-}) - r)H_{t-}dt + \sigma H_{t-}d\mathcal{W}_t - C_{t-}dt - X_{c,t-}dt \\ &\quad - \left(\int_0^{\overline{Z}} \delta_{t-}(Z)p(Z;x_{t-}^*)dZ\right) W_{t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t - \left(1 - Z\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right) H_{t-}d\mathcal{J}_t \,. \end{aligned} \tag{B.17}$$

The first four terms in (B.17) are standard in the classic portfolio-choice problem with no insurance or disasters (Merton, 1971). The fifth term $X_{c,t-}dt$ is the cost of the household's mitigation spending. The sixth term is the total DIS premium paid by the households before the arrival of disasters. Note that this term captures the financial hedging cost. The seventh term describes the DIS payments by the DIS seller to the household when a disaster occurs. The last term is the loss of the household's wealth from her portfolio's exposure to the stock market.

The HJB equation for the household in our decentralized market setting is given by

$$\begin{aligned} 0 = \max_{C,H,\delta,X_c} \; & f(C,J) + \mu_\pi(\pi)J_\pi + \frac{\sigma^2 H^2 J_{WW}}{2} + \lambda(\pi)\int_0^{\overline{Z}} \left[J\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right) - J(W,\pi)\right]\xi(Z;x^*)dZ \\ & + \left[r(\pi)W + (\mu_Q(\pi) - r(\pi))H - \left(\int_0^{\overline{Z}} \delta(Z)p(Z;x^*)dZ\right) W - C - X_c\right] J_W \,, \end{aligned} \tag{B.18}$$

where $\pi^{\mathcal{J}}$ is the post-jump belief given in (12) and $W^{\mathcal{J}}$ is the post-jump wealth given in (B.16). The term, $\left(\int_0^{\overline{Z}} \delta_t(Z)p(Z;x^*)dZ\right) W_t dt$, is the total DIS premium payment in the time interval $(t, t+dt)$. The FOCs for consumption C and the market portfolio allocation H are given by

$$f_C(C,J) = J_W(W,\pi) \tag{B.19}$$

$$\sigma^2 H J_{WW}(W,\pi) = -(\mu_Q(\pi) - r(\pi))J_W(W,\pi) + \lambda(\pi)\mathbb{E}\left[\left(1 - Z\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right) J_W\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right)\right] . \tag{B.20}$$

The second term in (B.20) captures the the jump effect on the household's portfolio choice. The DIS demand $\delta(Z)$ for each Z is given by

$$p(Z;x^*)J_W(W,\pi) = \lambda(\pi)J_W\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right)\xi(Z;x^*) \,. \tag{B.21}$$

The left side of (B.21) is the marginal (utility) cost when the household purchases a unit of DIS contract and the right side of (B.21) is the marginal (utility) benefit. This FOC follows from the point-by-point optimization in (B.18) for the DIS demand and hence it holds for all levels of Z.

Substituting (33) into the FOC (B.19) yields the following consumption rule:

$$C(W,\pi) = \rho^\psi u(\pi)^{1-\psi} W \,, \tag{B.22}$$

which is linear in W but nonlinear in belief π in general.

B.2 Firm Value Maximization

Using (33) and (B.22), we conjecture and then verify later that the SDF is given by

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -r(\pi_{t-})dt - \gamma\sigma d\mathcal{W}_t + \left[\left(\frac{u^*(\pi_t^{\mathcal{J}})}{u^*(\pi_{t-})}\right)^{1-\gamma}\left(\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right)^{-\gamma} Z^{-\gamma} - 1\right](d\mathcal{J}_t - \lambda(\pi_{t-})dt)\,. \quad \text{(B.23)}$$

The equilibrium drift of $d\mathbb{M}_t/\mathbb{M}_{t-}$ is $-r(\pi_{t-})$ (Duffie, 2001). The last term is a jump martingale and the terms inside the square bracket follow from (A.6), (33), and (B.22).

By using Ito's Lemma, we obtain the following dynamics for $Q_t = Q(K_t, \pi_t)$:

$$dQ = \left(IQ_K + \frac{1}{2}\sigma^2 K^2 Q_{KK} + \mu_\pi(\pi)Q_\pi\right)dt + \sigma K Q_K d\mathcal{W}_t + \left(Q(ZK, \pi^{\mathcal{J}}) - Q(K,\pi)\right)d\mathcal{J}_t\,. \quad \text{(B.24)}$$

No arbitrage implies the drift of $\mathbb{M}_{t-}(AK_{t-} - I_{t-} - \Phi(I_{t-}, K_{t-}) - X_{f,t-})dt + d\left(\mathbb{M}_t Q_t\right)$ is zero. By applying Ito's Lemma to this martingale, we obtain

$$\begin{aligned} 0 = \max_{I,\,X_f}\ & \mathbb{M}_{t-}(AK - I - \Phi(I,K) - X_f)dt + \mathbb{M}_{t-}\left(Q_K + \frac{1}{2}\sigma^2K^2Q_{KK} + \mu_\pi(\pi)Q_\pi\right)dt \\ & + Q\left[-r(\pi) - \lambda(\pi)\mathbb{E}\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} Z^{-\gamma} - 1\right)\right]\mathbb{M}_{t-}dt - \mathbb{M}_{t-}\gamma\sigma^2 KQ_K dt \\ & + \lambda(\pi)\mathbb{E}\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} Z^{-\gamma}Q(ZK,\pi^{\mathcal{J}}) - Q(K,\pi)\right]\mathbb{M}_{t-}dt\,. \end{aligned} \quad \text{(B.25)}$$

And then by using $Q(K,\pi) = q(\pi)K$, we obtain (31).

B.3 Market Equilibrium

First, mitigation spending for both households and firms is zero: $x_c = x_f = 0$. Second, in equilibrium, the household (1) invests all wealth in the stock market and holds no risk-free asset, $H = W$ and $W = Q^*$; (2) has zero disaster hedging position, $\delta(Z) = 0$ for all Z. Simplifying the FOCs, (B.19), (B.20), and (B.21), and using the preceding equilibrium conditions, we obtain we obtain:

$$c^*(\pi) = \rho^\psi u(\pi)^{1-\psi} q^*(\pi)\,, \quad \text{(B.26)}$$

$$\mu_Q(\pi) = r(\pi) + \gamma\sigma^2 + \lambda(\pi)\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\left[\mathbb{E}(Z^{-\gamma}) - \frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\mathbb{E}(Z^{1-\gamma})\right]\,, \quad \text{(B.27)}$$

$$p(Z;0) = \lambda(\pi)\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} Z^{-\gamma}\xi(Z;0)\,. \quad \text{(B.28)}$$

Using these equilibrium conditions, we simplify the HJB equation (B.18) as follows:

$$\begin{aligned} 0 \;=\; & \frac{1}{1-\psi^{-1}}\left(\frac{c^*(\pi)}{q^*(\pi)} - \rho\right) + \left(\mu_Q(\pi) - \frac{c^*(\pi)}{q^*(\pi)}\right) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} \\ & + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right]\,. \end{aligned} \quad \text{(B.29)}$$

Third, by substituting $c^*(\pi) = A - i^*(\pi) - \phi(i^*(\pi))$ into (31), we obtain

$$0 = \frac{c^*(\pi)}{q^*(\pi)} - r(\pi) + i(\pi) + \mu_\pi(\pi)\frac{q^*_\pi(\pi)}{q^*(\pi)} - \gamma\sigma^2$$
$$- \lambda(\pi)\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\left[\mathbb{E}(Z^{-\gamma}) - \frac{q(\pi^{\mathcal{J}})}{q(\pi)}\mathbb{E}(Z^{1-\gamma})\right]. \tag{B.30}$$

By using the homogeneity property and comparing (B.15) and (B.24), we have

$$\mu_Q(\pi) = \frac{c^*(\pi)}{q^*(\pi)} + i^*(\pi) + \mu_\pi(\pi)\frac{(q^*(\pi))'}{q^*(\pi)}. \tag{B.31}$$

And then substituting (B.31) into (B.29), we obtain

$$\begin{aligned}\frac{c^*(\pi)}{q^*(\pi)} &= \rho - (1-\psi^{-1})\left[i^*(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\left(\frac{u'(\pi)}{u(\pi)} + \frac{(q^*(\pi))'}{q^*(\pi)}\right)\right] \\ &\quad +\lambda(\pi)\left(\frac{1-\psi^{-1}}{1-\gamma}\right)\left[1 - \left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})\right].\end{aligned} \tag{B.32}$$

Finally, substituting (B.32) into (B.30), we obtain the following equilibrium interest rate:

$$\begin{aligned} r(\pi) &= \rho + \psi^{-1}i^*(\pi) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \left[(1-\psi^{-1})\left(\frac{u'(\pi)}{u(\pi)} + \frac{(q^*(\pi))'}{q^*(\pi)}\right) - \frac{(q^*(\pi))'}{q^*(\pi)}\right]\mu_\pi(\pi) \\ &\quad - \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\mathbb{E}(Z^{-\gamma}) - 1\right] \\ &\quad - \lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1 - \left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})\right)\right].\end{aligned} \tag{B.33}$$

B.4 Equivalence between Market Solution and Planner's Problem with No Mitigation Technology

The value function for the planner's problem $V(K, \pi)$ with no mitigation technology (i.e., $x = 0$), is equal to the household's value function under competitive equilibrium, $J(W, \pi)$. As the household's wealth is equal to the total stock market capitalization, i.e., $W = q(\pi)K$ in equilibrium, $b(\pi)$ in the planner's problem is equal to $u(\pi)q(\pi)$ in the decentralization formulation. The optimal consumption in the planner's problem (20) with no mitigation is the same as (B.26) in the decentralized market formulation. The resource constraints $A = i(\pi) + \phi(i(\pi)) + c(\pi)$ then implies that investment is also the same in the two formulations.

By substituting $b(\pi) = u(\pi)q(\pi)$ into ODE (19) for the planner's problem, we have

$$\begin{aligned} 0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - 1\right] + i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\left(\frac{u'(\pi)}{u(\pi)} + \frac{q'(\pi)}{q(\pi)}\right) \\ &\quad + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)q(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right],\end{aligned} \tag{B.34}$$

which is consistent with the market solution given in (B.32). By substituting $b(\pi) = u(\pi)q(\pi)$ and $q(\pi) = 1 + \phi'(i(\pi))$ into (A.12), we verify that the interest rate process is the same in the two formulations, e.g., (B.33) and (A.12) are the same in equilibrium.

In sum, we have verified that the resource allocation in the decentralized market formulation features no mitigation in equilibrium due (a free-rider's problem) and hence is the same as in the social planner's problem with no mitigation spending.

C Derivations for Results in Section 5

As in Section 4.1, we include the following securities (traded at each point in time): (i) a risk-free asset, (ii) a claim on the value of firm's capital, and (iii) insurance claims for disasters with every possible recovery fraction Z.

We define the economy as follows: (a) The representative household dynamically chooses consumption C_t, investments in the risk-free asset and risky equity, and various DIS claims to maximize utility as given by (5) and (6); (b) The representative firm chooses the level of investment I_t to maximize its market value taking the equilibrium SDF as given; (c) The government chooses mitigation spending X_t to maximize the representative household's utility as given by (5) and (6); (d) All markets clear. We use superscript * to denote the equilibrium variables and/or processes.

In this section, we verify that the market mechanism delivers the first-best consumption and investment policies provided that the mitigation spending is chosen by a benevolent government.

C.1 Household Optimization

As in Section B, the household accumulates wealth as:

$$\begin{aligned} dW_t &= r(\pi_{t-})W_{t-}dt + (\mu_Q(\pi_t) - r)H_{t-}dt + \sigma H_{t-}d\mathcal{W}_t - C_{t-}dt \\ &\quad - \left(\int_0^{\overline{Z}} \delta_{t-}(Z)p(Z;x^*_{t-})dZ\right) W_{t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t - \left(1 - Z\frac{q^*(\pi^{\mathcal{J}}_{t-})}{q^*(\pi_{t-})}\right) H_{t-}d\mathcal{J}_t \,, \end{aligned} \tag{C.35}$$

where x^* is chosen by the government. As it is in the household's interest to choose no mitigation spending, we leave this term out of (C.35). The HJB equation for the household in this setting is:

$$\begin{aligned} 0 = \max_{C,H} \; & f(C,J) + \left[r(\pi)W + (\mu_Q(\pi) - r(\pi))H - \left(\int_0^{\overline{Z}} \delta(Z)p(Z;x^*)dZ\right) W - C\right] J_W \\ & + \mu_\pi(\pi)J_\pi + \frac{\sigma^2 H^2 J_{WW}}{2} + \lambda(\pi)\int_0^{\overline{Z}} \left[J\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}\right) - J(W,\pi)\right]\xi(Z;x^*)dZ \,, \end{aligned} \tag{C.36}$$

Additionally, the FOCs for consumption, market portfolio allocation, and DIS demand are the same as (B.19)-(B.21). Let $J(W,\pi)$ denote the household's value function, given in (33).

Imposing the equilibrium outcome on the households' side, we obtain (B.26), (B.27), and (B.28). Using (33) and these conditions to simplify (C.36), we obtain the following ODE for $u(\pi)$:

$$\begin{aligned} 0 \quad = \quad & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{u(\pi)}{\rho}\right)^{1-\psi}-1\right]+\left(\mu_Q(\pi)-\rho^{\psi}u(\pi)^{1-\psi}\right)-\frac{\gamma\sigma^2}{2} \\ & +\mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)}+\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})-1\right]. \end{aligned} \tag{C.37}$$

C.2 Firm Value Maximization

Taking the SDF in (B.23) as given, the firm chooses investment I to solve:

$$\max_{I,X_f}\ \mathbb{E}\left[\int_0^\infty \frac{\mathbb{M}_t}{\mathbb{M}_0}\left(AK_t-I_t-\Phi_t-X_{f,t}-X_t^*\right)dt\right], \tag{C.38}$$

where X^* is chosen by the government and hence exogenous to the firm. By applying Ito's Lemma to $\mathbb{M}_{t-}(AK_{t-}-I_{t-}-\Phi(I_{t-},K_{t-})-X^*_{t-})dt+d\left(\mathbb{M}_tQ_t\right)$, which is a martingale due to no arbitrage (Duffie, 2001), we obtain the following ODE for $q_t=Q_t/K_t=q(\pi_t)$:

$$\begin{aligned} r(\pi)q(\pi) = {} & A-i-\phi(i)-x^*+i(\pi)q(\pi)+\mu_\pi(\pi)q'(\pi) \\ & -\left[\gamma\sigma^2+\lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\int_0^{\overline{Z}}Z^{-\gamma}\xi(Z;x^*)dZ-1\right)\right]q(\pi) \\ & +\lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\int_0^{\overline{Z}}Z^{1-\gamma}\xi(Z;x^*)dZ-1\right]q(\pi). \end{aligned} \tag{C.39}$$

By differentiating (C.39) with respect to i, we obtain the investment FOC:

$$q(\pi)=1+\phi'(i). \tag{C.40}$$

By using the aggregate resource constraint, $c^*(\pi)=A-i^*(\pi)-\phi(i^*(\pi))-x^*$, we obtain the following expression for the equilibrium expected return of the aggregate stock market:

$$\mu_Q(\pi)=\frac{A-i^*(\pi)-\phi(i^*(\pi))-x^*}{q^*(\pi)}+i^*(\pi)+\mu_\pi(\pi)\frac{(q^*(\pi))'}{q^*(\pi)}. \tag{C.41}$$

By substituting (C.41) into (C.37), we obtain (38), the simplified HJB equation for the government.

Using the same analysis as in Section B, we can show that the equilibrium dividend yield, $c^*(\pi)/q^*(\pi)$, is given by (B.32) and the equilibrium interest rate is given by (B.33).

Finally, we require all the variables at the micro level to equal the corresponding variables at the macro level, e.g., $c(\pi)=c^*(\pi)$, $i(\pi)=i^*(\pi)$, and $u(\pi)=u^*(\pi)$.