PANDEMIC LOCKDOWN:
THE ROLE OF GOVERNMENT COMMITMENT

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<u>ABSTRACT</u>

This note studies optimal lockdown policy in a model in which the government can limit a pandemic's impact via a lockdown at the cost of lower economic output. A government would like to commit to limit the extent of future lockdown in order to support more optimistic investor expectations in the present. However, such a commitment is not credible since investment decisions are sunk when the government makes the lockdown decision in the future. The commitment problem is more severe if lockdown is sufficiently effective at limiting disease spread or if the size of the susceptible population is sufficiently large. Credible rules that limit a government's ability to lock down the economy in the future can improve the efficiency of lockdown policy.

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1 Introduction

In response to the COVID-19 pandemic, governments across the world implemented lockdown policies to limit the spread of infections. In numerous cases, those policies were eventually extended. For example, on March 22, 2020 New York Governor Andrew Cuomo extended the statewide lockdown from April 19 to April 29. Then on April 16, the lockdown was further extended from April 29 to May 15. By the end of the following day, a total of 23 state governors had extended lockdown policies beyond their initial plans, some by over one month.[1]

In this note, we study the value of government commitment in choosing a lockdown policy. We consider a simple economy that captures policy tradeoffs based on commonly used SIR models of pandemics (Kermack and McKendrick, 1927; Ferguson et al., 2020; Wang et al., 2020). Investors provide capital, the government chooses a lockdown policy, and workers supply labor. A lockdown imposes an upper bound on labor supply while also limiting disease spread and health costs. Our framework is general and subsumes key mechanics of many other macroeconomic SIR models with lockdown or mitigation elements in the literature.[2] An important feature of our model is that investment is made before future lockdown policy is chosen. We think of this feature as capturing the long-term investments that businesses make while anticipating the future trajectory of a lockdown policy.

The optimal policy under government commitment trades off the aggregate output cost with the health benefit associated with lockdown. Aggregate output decreases with the intensity of the lockdown through two channels. First, it decreases directly through lower labor supply, which is curbed by the lockdown. Second, it decreases indirectly through lower investment, which results from investors' expectation of a lower marginal product of capital due to the lockdown. The health benefit of a lockdown is higher if the lockdown technology is more effective at limiting infections or if the share of the initial susceptible population is larger.

Our main result focuses on how the extent of a lockdown is impacted by the government's lack of commitment. A government would like to commit to limit the extent of future lockdown in order to support more optimistic investor expectations in the present. However, such a commit-

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[1]These states include Colorado, Connecticut, Georgia, Idaho, Illinois, Indiana, Kansas, Louisiana, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, Montana, New Mexico, New York, Ohio, Rhode Island, South Carolina, Tennessee, Vermont, Washington, and Wisconsin.

[2]See for example Atkeson (2020a,b), Eichenbaum et al. (2020), Berger et al. (2020), Alvarez et al. (2020), Kaplan et al. (2020), Jones et al. (2020), Glover et al. (2020), and Piguillem and Shi (2020).

ment may be not credible since investment decisions are sunk when the government makes the lockdown decision in the future. In this situation, a government without commitment imposes a more stringent lockdown relative to the optimal policy under commitment. Investors rationally anticipate the government's lack of commitment, causing them to invest less than they would in anticipation of the policy under commitment. Through this mechanism, lack of commitment results in a larger reduction in investment and output during a lockdown than is socially optimal.

We establish conditions under which lack of commitment by the government reduces social welfare. If the lockdown is sufficiently effective at limiting disease spread or if the number of susceptible individuals is sufficiently high, then the optimal policy is time-inconsistent, leading to social welfare losses. Investors provide less capital and the government chooses a more stringent lockdown relative to what would happen under commitment. In contrast, if a lockdown is not very effective or if the size of the susceptible population is low, then the optimal policy under commitment involves no lockdown and is time-consistent.

These results suggest that commitment problems leading to welfare losses during a lockdown are more likely to arise in environments with greater capacity to limit disease spread through lockdown, such as urban areas in advanced economies. A similar commitment problem arises when considering lockdowns early in a pandemic, when the size of the susceptible population is high and herd immunity has not yet developed.

Our results imply that a credible government lockdown policy plan can improve the efficiency of lockdown policy. In principle, such a plan can depend on new information that arrives during a lockdown, such as estimates of disease mortality, the state of the economy, the likelihood of vaccine discovery, or the medical system's capacity. Some of this information may not be contractible, in which case a rigid plan can be too constraining, and policy flexibility is desirable. To capture this value of flexibility, we extend our model to allow the government to learn new noncontractible information before choosing a lockdown policy. In this extended model, we show that rules that impose limits on future lockdown policy can increase social welfare, even though policy flexibility is valuable. The reason is that a government lacking commitment chooses more lockdown in the future than is socially desirable. As such, a marginally binding rule increases social welfare by raising investment and output at no cost of reduced policy flexibility.

We emphasize that our analysis does not imply that lockdowns are socially harmful. In fact, reducing or lifting the lockdown in our model is detrimental if the resulting health costs exceed

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the immediate economic gains. Our model abstracts from policy mistakes involving insufficient degrees of lockdown by assuming that policy is chosen by a benevolent government that maximizes long-run social welfare.[3] Our analysis points to the value of a government plan that defines limits on the extent of *future* lockdown. Such a plan is beneficial if the expected *future* economic gains of those limits—from stimulating investment toward its efficient level—exceed the health costs.

Our analysis relates to the nascent literature on optimal policy in a pandemic, with some recent contributions listed in footnote 2. This literature focuses on various aspects of government policy, including the optimal intensity and timing of lockdowns. We depart from this literature by focusing on the value of government commitment in the context of lockdown policy.

The mechanism underlying the time inconsistency of optimal policy in our setting is in line with the broader insights in the seminal work of Kydland and Prescott (1980), and in particular the literature that studies government commitment in the context of capital taxation (Chari and Kehoe, 1990; Klein et al., 2008; Aguiar et al., 2009). While lack of commitment in our model distorts capital investment as in these frameworks, there are two important differences. First, a lockdown distorts capital investment not directly via taxation, but indirectly by suppressing labor. Second, in our setting, these distortions from lockdown do not increase the government budget, but reduce the long-term health costs of disease spread. Since health costs derive from an underlying SIR model, the value of reducing these costs cannot be represented by a simple concave function, as in a typical model of public goods. This means that the usual methods for comparative statics cannot be applied here.

Our analysis of rules in the presence of noncontractible information relates to the literature on commitment versus flexibility in policymaking (Amador et al., 2006; Athey et al., 2005; Halac and Yared, 2014, 2018). The result that rules can strictly increase social welfare even if flexibility is valuable is consistent with that work. However, in contrast to that work, we obtain this result under milder restrictions on the utility function and the distribution of noncontractible information.

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[3]This assumption may be violated in an extension of our model in which political economy considerations lead the government to overweigh immediate economic gains relative to future health costs of relaxing a lockdown.

2 Model

We consider a simple three-period economy. In the first period, investors provide capital. In the second period, the government chooses a lockdown policy and workers supply labor. In the third period, disease spread follows an SIR model of disease spread and is affected by the lockdown policies of the second period. Lockdown imposes an upper bound on labor supply in the second period while also limiting disease spread and health costs from the third period onward. Importantly, the government chooses an optimal lockdown policy after capital investment is sunk.

2.1 Economic Environment

There are three periods $t = 0, 1, 2$. At $t = 0$, competitive external investors provide capital k. At $t = 1$, a continuum of mass 1 of workers supply up to one unit of labor inelastically subject to a binding upper bound $\ell \in [0, 1]$ representing the degree of lockdown. If $\ell = 1$, there is no lockdown and the maximum amount of labor is supplied. If $\ell = 0$ there is maximal lockdown. A worker's budget constraint is

$$c = w\ell, \tag{1}$$

where c is consumption and w is the market wage. Workers have linear utility over consumption c and receive continuation value V as a function of the future state of the economy.

Capital k combined with labor input ℓ generates output y according to the following production function:

$$y = k^\alpha \ell^{1-\alpha}, \tag{2}$$

where $\alpha \in (0, 1)$. We assume for simplicity that capital depreciates fully. Investors can invest domestically or abroad at a rate of return r^*.[4] As such, in a competitive equilibrium, the marginal product of capital obeys the following no-arbitrage condition:

$$r^* = \alpha k^{\alpha-1} \ell^{1-\alpha} \tag{3}$$

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[4] We consider an open economy for simplicity. The analysis can be easily extended to a closed economy with workers and capitalists.

Labor is competitively supplied so wages equal their marginal product given by

$$w = (1 - \alpha) k^{\alpha} \ell^{-\alpha}. \tag{4}$$

Combining (3) and (4), it follows that in a competitive equilibrium—where capital adjusts to the anticipated level labor supply—consumption given by equation (1) satisfies

$$c = A\ell, \tag{5}$$

where $A = (1 - \alpha)(\alpha/r^*)^{\alpha/(1-\alpha)}$. Note that equation (5) features consumption that is linear in labor input ℓ because capital optimally adjusts to the given level of labor input.

2.2 Disease Spread and Lockdown Policy

We model disease spread as following an SIR model (Kermack and McKendrick, 1927; Ferguson et al., 2020; Wang et al., 2020), which we allow to depend on a lockdown policy, as in Atkeson (2020a), Eichenbaum et al. (2020), and Alvarez et al. (2020). Specifically, we define the state of the economy at time $t = 1, 2$ as $\Omega_t = \{S_t, I_t, R_t, D_t\}$, where $S_t \geq 0$ is the mass of susceptible individuals, $I_t \geq 0$ is the mass of infected and contagious individuals, $R_t \geq 0$ is the mass of recovered individuals, and $D_t \geq 0$ is the mass of deceased individuals. Since the population at date $t = 1$ of worker is normalized to 1 and $D_1 = 0$ without loss of generality, it follows that

$$S_1 + I_1 + R_1 = 1 \quad \text{and} \tag{6}$$
$$S_2 + I_2 + R_2 + D_2 = 1. \tag{7}$$

An SIR model defines a mapping $\Gamma(\cdot)$ that implies a law of motion

$$\Omega_2 = \Gamma(\Omega_1, \ell, \kappa), \tag{8}$$

where the state at date $t = 2$ is a function of the state at date $t = 1$, the degree of lockdown at date $t = 1$, and a parameter $\kappa \in [0, 1]$ capturing the effectiveness of the lockdown technology. Note that implicit in our formulation is the existence of a state Ω_0 and initial lockdown policy at date $t = 0$ that determine Ω_1. Because these are exogenous, we take the state Ω_1 as given without loss

of generality.

Social welfare equals

$$c + V\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right), \tag{9}$$

where $V\left(\cdot\right)$ is a continuation value to society that is a function of the future state. The continuation value $V\left(\cdot\right)$ captures the long-term costs of bad health and mortality associated with disease spread, as guided by the future law of motion of the state Ω_t. Note that through the law of motion for Ω_2 given by equation (8), the continuation value will be impacted by the degree of lockdown, which determines ℓ, and its effectiveness κ.

We make the following intuitive assumption.

Assumption 1. *The value of $V\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right)$ is independent of ℓ if either (i) $\kappa = 0$ or (ii) $S_1 = 0$.*

The first part of Assumption 1 states that the continuation value to society is independent of the degree of lockdown if the lockdown technology is maximally ineffective at limiting disease spread (i.e., if $\kappa = 0$). Since disease spread is independent of the degree of lockdown in this case, future payoffs will not depend on lockdown decisions.

The second part of Assumption 1 states that lockdown also becomes irrelevant if the size of the initial susceptible population is zero (i.e., if $S_1 = 0$). That there are no susceptible individuals means that the entire population is either infected, recovered, or dead, meaning that the disease cannot spread. As such, we assume that disease dynamics are determined only by epidemiological parameters guiding recovery and death rates, which we assume are independent of lockdown.

In addition to this intuitive assumption, we make the following technical assumption. In the statement of this assumption and for the remainder of our paper, we consider comparative statics with respect to variations in the susceptible population S_1 that are accommodated by variations in the recovered population R_1.

Assumption 2. *The function $V\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right)$ is differentiable in ℓ and the derivative of $V\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right)$ with respect to ℓ, conditional on any $\ell \in (0, 1)$, is (i) continuous in κ and (ii) continuous in S_1.*

Assumption 2 is a technical assumption that guarantees that the continuation value is well-behaved. This assumption allows us to prove our results, which rely on the marginal payoffs from lockdown changing gradually with respect to parameters κ and S_1.

Assumptions 1 and 2 together are sufficient to support our theoretical conclusions. Note that these assumptions are satisfied in many recent macroeconomic models with SIR modules in which disease dynamics respond smoothly to lockdown policies. In these frameworks, the probability of a person's transition from the susceptible state to the infected state is continuously decreasing in the effectiveness of the lockdown technology κ and continuously increasing in the size of the susceptible population S_1. See Eichenbaum et al. (2020) and Alvarez et al. (2020) for examples of models consistent with these assumptions.

2.3 Timeline

The order of events is as follows:

1. At $t = 0$, investors choose investment k;

2. At $t = 1$, the government chooses lockdown policy ℓ, workers supply labor subject to the lockdown policy, output y is produced, and workers and investors consume their respective shares of income; and

3. At $t = 2$, the disease spread progresses according to the transition function Γ.

A key feature of our model is that investment is made before the lockdown policy is chosen. We think of this feature as capturing the long-term investments that businesses make while anticipating the future trajectory of a lockdown policy. In support of this idea, recent survey evidence shows that businesses that expect a more prolonged crisis are more likely to expect to shut down (Bartik et al., 2020). We will explore in detail the implications of this sequencing of investment and lockdown decision for the optimal policy under commitment compared to that under lack of commitment.

3 Optimal Policy under Commitment

Suppose that the government can commit to a lockdown policy ℓ prior to investment decisions. This means that capital optimally adjusts to anticipated labor supply, which, in turn, is determined by the lockdown policy. Substituting consumption under the capital no-arbitrage condition from

equation (5) into (9), the government under commitment solves the following problem:

$$\max_{\ell \in [0,1]} \left\{ A\ell + V\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right) \right\} \tag{10}$$

Importantly, substituting the capital no-arbitrage condition before solving for the optimal degree of lockdown means that the government under commitment takes into account the reaction of investment to the anticipation of its policies. Define $V_\ell(\Gamma(\Omega_1, \ell, \kappa)) \equiv dV(\Gamma(\Omega_1, \ell, \kappa))/d\ell$ as the total derivative of the continuation value with respect to labor input. The first-order necessary condition associated with an interior solution to the problem of the government under commitment is simply

$$-V_\ell\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right) = A. \tag{11}$$

In choosing the degree of lockdown, the government weighs two opposing forces, as in Gourinchas (2020) and Hall et al. (2020). On one hand, it considers the future *health benefits* in terms of reduced mortality from inhibiting the disease spread, as captured by the marginal change in the continuation value $-V_\ell(\Gamma(\Omega_1, \ell, \kappa))$. On the other hand, it considers the *economic costs* captured by foregone marginal product of labor given by A. In turn, the economic costs are twofold. First, conditional on the level of capital, lockdown has a direct impact on output by limiting labor supply. Second, lockdown has an indirect impact on output by reducing the marginal product of capital which reduces investment. The government's ability to commit gives it the ability to take into account both of these factors, leading it to choose the optimal lockdown in anticipation of investors' reaction to the policy.

We also consider two potential corner solutions to the government's problem under commitment: complete lockdown and no lockdown. Under complete lockdown, $\ell = 0$ and

$$-V_\ell\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right) > A. \tag{12}$$

Conversely, under no lockdown, $\ell = 1$ and

$$-V_\ell\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right) < A. \tag{13}$$

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4 Optimal Policy under Lack of Commitment

Under lack of commitment, the government takes capital k as given when choosing the lockdown policy at date $t = 1$. We can substitute for consumption c in equation (9) using equations (1), (2), and (4) to write the program for the the government under lack of commitment at date $t = 1$ as

$$\max_{\ell \in [0,1]} \left\{ (1 - \alpha) k^\alpha \ell^{1-\alpha} + V \left(\Gamma \left(\Omega_1, \ell, \kappa \right) \right) \right\}. \tag{14}$$

Importantly, not substituting the capital no-arbitrage condition before solving for the optimal degree of lockdown means that the government under no commitment does not take into account the reaction of investment to the anticipation of its policies. The derivative of the government objective function with respect to ℓ is

$$(1 - \alpha)^2 k^\alpha \ell^{-\alpha} + V_\ell \left(\Gamma \left(\Omega_1, \ell, \kappa \right) \right). \tag{15}$$

This expression makes clear that a government lacking commitment undervalues the economic cost of lockdown. This is because it takes capital decisions as sunk and does not internalize the impact of lockdown on ex ante investor expectations.[5] Investors take this lack of commitment into account when choosing investment. Therefore, the capital no-arbitrage condition applies with respect to the optimal behavior of government at the time of it choosing a lockdown policy. To see what this means, we substitute the capital no-arbitrage condition in equation (3), which accounts for optimal investor behavior, into equation (15) and rewrite the equilibrium derivative of the government objective function with respect to ℓ:

$$(1 - \alpha) A + V_\ell \left(\Gamma \left(\Omega_1, \ell, \kappa \right) \right) \tag{16}$$

This derivative shows that in equilibrium, the marginal cost of lockdown for a government lacking commitment is $(1 - \alpha) A$. This is below the marginal cost of lockdown for a government under commitment, which is equal to A. At the same time, the marginal benefit from lockdown is the same regardless of government commitment and given by $-V_\ell(\Gamma(\Omega_1, \ell, \kappa))$.

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[5]Note that for there to be a commitment problem, it is necessary that the government puts less weight on the welfare of outside investors than on domestic workers. Our insights would remain qualitatively unchanged if the government's weight on investors were positive but less than that on workers.

The FOC associated with an interior solution to the problem of the government under lack of commitment is simply

$$-V_\ell \left(\Gamma \left(\Omega_1, \ell, \kappa \right) \right) = \left(1 - \alpha \right) A. \tag{17}$$

As previously, we also consider two potential corner solutions to the government's problem under lack of commitment: complete lockdown and no lockdown. Under complete lockdown, $\ell = 0$ and

$$-V_\ell \left(\Gamma \left(\Omega_1, \ell, \kappa \right) \right) > \left(1 - \alpha \right) A. \tag{18}$$

Conversely, under no lockdown, $\ell = 1$ and

$$-V_\ell \left(\Gamma \left(\Omega_1, \ell, \kappa \right) \right) < \left(1 - \alpha \right) A. \tag{19}$$

Denote by ℓ^c the optimal lockdown policy under full commitment and by ℓ^n the equilibrium lockdown under lack of commitment. Then we obtain the following result.

Proposition 1 (Time Inconsistency). *Lockdown under no commitment is weakly larger than lockdown under full commitment: $\ell^n \leq \ell^c$. Moreover, lockdown under no commitment is strictly larger than lockdown under full commitment if either level of lockdown is interior: $\ell^n < \ell^c$ if $\ell^c \in (0,1)$ or $\ell^n \in (0,1)$.*

Proof. See Appendix A.1. □

Proposition 1 shows that an implication of lack of government commitment is that a suboptimal lockdown policy may be chosen. The reason for this is that, absent commitment, the government undervalues the economic cost of lockdown, leading to more lockdown and lower output than would be optimal from an ex ante perspective.

In the next proposition, we examine how the implications of lack of government commitment are impacted by the effectiveness of the lockdown technology with respect to limiting disease spread, as indexed by κ. We focus on cases in which the optimal policy under commitment involve some lockdown for some values of $\kappa \in (0,1)$. We then provide conditions under which the government under lack of commitment deviates from the commitment policy.

Proposition 2 (Effect of Lockdown Technology). *Suppose that there exists a lockdown technology for which the optimal policy under commitment involves some lockdown, that is, $\ell^c < 1$ for some $\kappa \in (0,1)$.*

Then the following is true:

1. *If the lockdown technology has low effectiveness, then the policy under full commitment and under lack of commitment involves no lockdown. That is, $\exists \underline{\kappa} \in (0,1)$ such that $\ell^c = \ell^n = 1$ if $\kappa \leq \underline{\kappa}$.*

2. *If the lockdown technology has intermediate effectiveness, then the policy under full commitment is no lockdown and under lack of commitment is positive lockdown. That is, $\exists \overline{\kappa} \in (\underline{\kappa}, 1]$ such that $\ell^c = 1 > \ell^n$ if $\kappa \in (\underline{\kappa}, \overline{\kappa})$.*

Proof. See Appendix A.2. □

The following proposition considers policy under commitment and lack of commitment as a function of the initial number of susceptible individuals S_1.

Proposition 3 (Effect of Initial Health Status). *Suppose that there exists a population share of susceptible individuals for which the optimal policy under commitment involves some lockdown, that is, $\ell^c < 1$ for some $S_1 \in (0,1)$. Then the following is true:*

1. *If the initial number of susceptible individuals is low, then the policy under full commitment and under lack of commitment involves no lockdown. That is, $\exists \underline{S_1} \in (0,1)$ such that $\ell^c = \ell^n = 1$ if $S_1 \leq \underline{S_1}$.*

2. *If there is an intermediate number of susceptible individuals, then the policy under full commitment is no lockdown and under lack of commitment is positive lockdown. That is, $\exists \overline{S_1} \in (\underline{S_1}, 1]$ such that $\ell^c = 1 > \ell^n$ if $\kappa \in (\underline{S_1}, \overline{S_1})$.*

Proof. The proof is analogous to that of Proposition 2 and is thus omitted. □

If the lockdown technology is sufficiently ineffective at preventing disease (Proposition 2) or if the fraction of susceptible individuals is sufficiently low (Proposition 3), then there is no problem of lack of commitment. Both under commitment and under lack of commitment the economic cost of any lockdown dwarfs the mortality benefits, and having no lockdown is optimal. These results change if the lockdown technology has intermediate effectiveness. In this circumstance, while it is optimal for the government under commitment to not lockdown the economy, the government under lack of commitment which undervalues the cost of lockdown will prefer to lockdown the economy.[6]

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[6]A natural question concerns comparative statics for $\kappa > \overline{\kappa}$ and $S_0 > \overline{S}_0$. Establishing these comparative statics would require additional assumptions beyond those made above.

5 Rules that Limit Future Lockdown

We have established that a government under lack of commitment may choose more severe lock-down than a government under full commitment. As a result, lack of commitment can lead to an economic contraction at date $t = 1$ that is deeper than is socially optimal.

In this environment, a credible lockdown policy plan can be socially optimal. Formally, suppose that rather than choosing a policy $\ell \in [0, 1]$, the policy decision ℓ is exogenously constrained to the optimum under commitment, $\ell = \ell^c$. Such a constraint on policy improves investor expectations of the future and can improve the efficiency of lockdown policy.

In principle, such a plan can depend on new information that arrives during a lockdown, such as estimates of disease mortality, the state of the economy, the likelihood of vaccine discovery, or the medical system's capacity. To capture this idea, suppose that a state variable $\theta \in [\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} < \overline{\theta}$, is realized after investment k has been made at date $t = 0$ and before policy ℓ is chosen at date $t = 1$. Suppose that θ is drawn from a probability density function (pdf) $f(\theta)$ over $\theta \in [\underline{\theta}, \overline{\theta}]$. Conditional on θ, social welfare can be written as[7]

$$c + V\left(\Gamma\left(\Omega_1, \ell, \kappa\right), \theta\right). \tag{20}$$

In this extended model, the optimal policies under commitment and no commitment depend on the realization of θ and are denoted by $\ell^c(\theta)$ and $\ell^n(\theta)$, respectively. An argument analogous to that in Proposition 1 implies that $\ell^c(\theta) \geq \ell^n(\theta)$. In other words, conditional on θ, the government lacking commitment chooses a weakly larger lockdown than the government under full commitment. If θ represents contractible information, then a credible plan that imposes the constraint $\ell = \ell^c(\theta)$ can increase social welfare since it forces the government without commitment to choose the policy under full commitment.

In practice, some of the information in θ may not be contractible, in which case a rigid plan can be too constraining, and flexibility is desirable. In this case, we can show that bounded discretion in the form of a rule $\underline{\ell} > 0$ that constrains the government to a policy choice $\ell \in [\underline{\ell}, 1]$ is socially desirable. Formally, let us suppose that $\ell^n(\theta)$ is a decreasing function of θ that is continuous in a neighborhood below $\overline{\theta}$. Therefore, higher values of θ are associated with more lockdown.

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[7]While we introduce the state variable θ as an argument outside of the disease transition function $\Gamma(\cdot)$, this is without loss of generality and we could allow for θ to have a direct effect on disease spread by allowing it to index $\Gamma(\cdot)$.

Moreover, let us suppose that the pdf $f(\theta)$ is strictly positive and is continuous in a neighborhood below $\bar{\theta}$. We can use analogous arguments as in the literature on commitment versus flexibility in policymaking (Amador et al., 2006; Athey et al., 2005; Halac and Yared, 2014, 2018) to show that rules that put a limit on lockdown can boost social welfare, even if the rule cannot depend explicitly on the realization of θ.[8]

Proposition 4 (Value of Rules). *Consider an economy where lockdown under full commitment and under lack of commitment is never maximal, namely $\ell^c(\theta) \geq \ell^n(\theta) > 0$ for all θ, and where optimal lockdown under lack of commitment is sometimes interior, namely $\ell^n(\theta) < 1$ for some θ. Then a rule that imposes a lower bound $\underline{\ell}$ on labor supply strictly increases social welfare under lack of commitment.*

Proof. See Appendix A.3. □

Proposition 4 shows that the introduction of rules increases social welfare even if there is a value to flexibility. The intuition is that a government lacking commitment chooses more lockdown in the future than is socially desirable. As such, a marginally binding rule increases social welfare by raising investment and output at no cost of reduced policy flexibility. A key part of this argument is that extreme levels of future lockdown are assumed to never be optimal under commitment given current information. Thus, a rule that makes such extreme choices infeasible in the future can improve investor expectations and mitigate the economic costs of a lockdown. Our environment could be extended to one in which this assumption is violated, and extreme choices are sometimes optimal in the future even under commitment. In this environment, a limit on future lockdowns with an escape clause under extreme conditions could be optimal.[9]

6 Concluding Remarks

We have analyzed the value of government commitment in choosing a lockdown policy. A government would like to commit to limit the extent of future lockdown in order to support more optimistic investor expectations in the present. However, such a commitment is not credible since investment decisions are sunk when the government makes the lockdown decision. Our results suggest that welfare losses due to lack of commitment are more likely to arise in environments

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[8]Because the function $V(\cdot)$ is not concave and the pdf $f(\cdot)$ can have a flexible structure, the following proposition does not follow directly from previous work.

[9]See Halac and Yared (forthcoming) for a discussion of threshold contracts with escape clauses.

with greater capacity to limit disease spread through lockdown, such as urban areas in advanced economies. These problems may also arise early in a pandemic, when the size of the susceptible population is high and herd immunity has not yet developed. Our analysis highlights the value of lockdown to mitigate the health costs of pandemics, together with the importance of defining the limits of future lockdowns. Through their impact on business expectations, such limits can improve the efficiency of lockdown policy.

Our analysis leaves several interesting avenues for future research. First, in establishing our results we have assumed that the future health benefit of lockdown is the same in the economy under full commitment and under no commitment. This is a good approximation to an economy reaching the end of a pandemic in which there is no lockdown in the future. However, the dynamic analysis of an economy earlier in a pandemic requires the government to consider the path of future lockdown policy, which will depend on the government's degree of commitment in the future. Such an analysis is challenging since a government considers the impact of current policy on disease spread as well as on the incentives of future governments.

Second, we have evaluated the effect of rules that limit lockdowns assuming that governments adhere to such rules. In practice, rules may be broken and the private sector may be uncertain about the government's commitment to respecting them. In the context of capital taxation, Phelan (2006) and Dovis and Kirpalani (2019) show that this consideration leads the private sector to dynamically update its beliefs about a government's ability to commit. We conjecture that in our framework, this uncertainty could cause investors to react to lockdown extensions by becoming increasingly pessimistic about the government's ability to commit to lifting a future lockdown. This could lead to further declines in investment and economic activity in response to lockdown extensions.

Finally, our analysis ignores the availability of monetary and fiscal policy tools, as in Guerrieri et al. (2020). In our framework, these tools could not only mitigate the immediate economic costs of a pandemic, but also boost investment, thus counteracting future economic costs from underinvestment due to the government's lack of commitment. We leave the exploration of how optimal lockdown policy interacts with monetary and fiscal policy under lack of government commitment as an interesting subject of further research.

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Appendix

A Proofs

A.1 Proof of Proposition 1

Proof. To prove the first part of the statement, suppose by contradiction that $\ell^c < \ell^n$. The government under full commitment must weakly prefer choosing ℓ^c to ℓ^n, meaning

$$Al^c + V\left(\Gamma\left(\Omega_1, \ell^c, \kappa\right)\right) \geq Al^n + V\left(\Gamma\left(\Omega_1, \ell^n, \kappa\right)\right). \tag{21}$$

Moreover, the government under lack of commitment must weakly prefer choosing ℓ^n over ℓ^c, conditional on the level of capital k chosen by investors in anticipation of the lack of commitment:

$$(1-\alpha)\, k^\alpha \left[\ell^n\right]^{1-\alpha} + V\left(\Gamma\left(\Omega_1, \ell^n, \kappa\right)\right) \geq (1-\alpha)\, k^\alpha \left[\ell^c\right]^{1-\alpha} + V\left(\Gamma\left(\Omega_1, \ell^c, \kappa\right)\right). \tag{22}$$

Substitution of equation (3) into (22) implies that equation (22) can be rewritten as

$$(1-\alpha)\, Al^n + V\left(\Gamma\left(\Omega_1, \ell^n, \kappa\right)\right) \geq (1-\alpha)\, Al^n \left[\frac{\ell^c}{\ell^n}\right]^{1-\alpha} + V\left(\Gamma\left(\Omega_1, \ell^c, \kappa\right)\right) \tag{23}$$

Since $\ell^c < \ell^n$ and $\alpha \in (0,1)$, it follows that

$$\ell^n \left[\frac{\ell^c}{\ell^n}\right]^{1-\alpha} = \left[\frac{\ell^n}{\ell^c}\right]^\alpha \ell^c > \ell^c. \tag{24}$$

Substitution of (24) into (23) yields

$$(1-\alpha)\, Al^n + V\left(\Gamma\left(\Omega_1, \ell^n, \kappa\right)\right) > (1-\alpha)\, Al^c + V\left(\Gamma\left(\Omega_1, \ell^c, \kappa\right)\right). \tag{25}$$

Combining (21) and (25), we get

$$(1-\alpha)\, A\left(\ell^n - \ell^c\right) > A\left(\ell^n - \ell^c\right), \tag{26}$$

which is a contradiction. Therefore, $\ell^n \leq \ell^c$. To prove the second part of the statement, consider $\ell^c \in (0,1)$ or $\ell^n \in (0,1)$. Suppose by contradiction that $\ell^c = \ell^n \in (0,1)$. Since the optimum is

interior, the FOC for the government under commitment is necessary for optimality:

$$A + V_\ell \left(\Gamma \left(\Omega_1, \ell^c, \kappa \right) \right) = 0. \tag{27}$$

Analogously, the FOC for the government under no commmitment following (16) is:

$$(1 - \alpha) A + V_\ell \left(\Gamma \left(\Omega_1, \ell^n, \kappa \right) \right) = 0. \tag{28}$$

For equations (27) and (28) to simultaneously hold under $\ell^c = \ell^n$ would require

$$A = (1 - \alpha) A, \tag{29}$$

which clearly represents a contradiction. We conclude that $\ell^n < \ell^c$. ☐

A.2 Proof of Proposition 2

Proof. The proof proceeds in three steps.

Proof of Step 1. We establish that there exists $\underline{\kappa}' \in (0, 1)$ for which $l^c < 1$ and $l^n < 1$ are not solutions to the government's problem if $\kappa \leq \underline{\kappa}'$. Suppose that $\kappa = 0$. Then the optimal policy under full commitment and lack of commitment is no lockdown. To see this, the benefit of lockdown is given by the marginal continuation value, which by Assumption 1 satisfies $V_\ell(\Gamma(\Omega_1, \ell, 0)) = 0$ given $\kappa = 0$, while the cost of lockdown is given by the foregone economic output, which equals A for the government with commitment and $(1 - \alpha)A$ for the government without commitment. Since the cost of lockdown is strictly positive with or without commitment, lockdown is never optimal. Now suppose that $\kappa = \varepsilon$ for $\varepsilon > 0$ arbitrarily small. We now show that under $\kappa = \varepsilon$ the optimal policies under both commitment and lack of commitment necessarily admit no lockdown. Consider first the case of a government with commitment. Suppose by way of contradiction that the optimal policy is $\ell^c < 1$ for some $\varepsilon_c > 0$. The FOC required for optimality of this policy is that

$$A + V_\ell \left(\Gamma \left(\Omega_1, \ell^c, \varepsilon_c \right) \right) \leq 0. \tag{30}$$

For any $\ell^c \in [0, 1)$, the left hand side of (30) approaches $A > 0$ as $\varepsilon_c \to 0$ by Assumptions 1 and 2. However, this contradicts (30) for ε_c sufficiently small. This establishes that $\ell^c = 1$ is the unique

solution for $\varepsilon_c > 0$ sufficiently small. Let $\bar{\varepsilon}_c > 0$ denote the highest value of ε_c for which inequality (30) is violated for all $\ell^c \in [0,1]$, and define $\bar{\varepsilon}_c = 1$ if it is never violated for any $\varepsilon_c \in [0,1]$ and $\ell^c \in [0,1]$. Now consider the case of lack of commitment. An exactly analogous argument, with A replaced by $(1-\alpha)A$ proves the claim that $\ell^n = 1$ is the unique solution for $\varepsilon_n > 0$ sufficiently small. Let $\bar{\varepsilon}_n > 0$ denote the highest value of ε_n for which the analog of inequality (30) for the government under no commitment (i.e., with A replaced by $(1-\alpha)A$) is violated for all $\ell^c \in [0,1]$, and define $\bar{\varepsilon}_n = 1$ if it is never violated for any $\varepsilon_n \in [0,1]$ and $\ell^n \in [0,1]$. By continuity, $\ell^c = \ell^n = 1$ is the unique solution if $\kappa \leq \underline{\kappa}'$ for $\underline{\kappa}' = \min\{\bar{\varepsilon}_c, \bar{\varepsilon}_n\} \in (0,1]$.

Proof of Step 2. We establish that there exists $\underline{\kappa} \in [\underline{\kappa}', 1)$ for which $l^c = 1$ and $l^n = 1$ are solutions to the government's problem if $\kappa \leq \underline{\kappa}$. Define $\underline{\kappa}$ as the highest value of κ such that for all $\kappa \leq \underline{\kappa}$ and all $\ell \in [0,1]$, the following condition holds

$$(1-\alpha) A + V\left(\Gamma\left(\Omega_1, 1, \kappa\right)\right) \geq (1-\alpha) A \ell^{1-\alpha} + V\left(\Gamma\left(\Omega_1, \ell, \kappa\right)\right). \tag{31}$$

The left hand side of (31) corresponds to equilibrium welfare for government under no commitment in an equilibrium under no lockdown, and the right hand side of (31) corresponds to the value of deviating to some ℓ. We begin by establishing that $\underline{\kappa} \geq \underline{\kappa}'$. This follows by part (i) since for $\kappa \leq \underline{\kappa}'$, the unique equilibrium under no commitment admits no lockdown, which means that (31) must hold. We now show that $\underline{\kappa} < 1$. The condition of the proposition states that the policy under full commitment admits some positive lockdown for some $\kappa \in (0,1)$. More specifically, it must be the case that under such a value of κ, the choice of $\ell^c < 1$ dominates choosing no lockdown, namely

$$A\ell^c + V\left(\Gamma\left(\Omega_1, \ell^c, \kappa\right)\right) \geq (1-\alpha) A + V\left(\Gamma\left(\Omega_1, 1, \kappa\right)\right). \tag{32}$$

Note that if (32) holds then (31) is violated for $\ell = \ell^c$. Suppose not and suppose that

$$(1-\alpha) A + V\left(\Gamma\left(\Omega_1, 1, \kappa\right)\right) \geq (1-\alpha) A \left[\ell^c\right]^{1-\alpha} + V\left(\Gamma\left(\Omega_1, \ell^c, \kappa\right)\right). \tag{33}$$

Combining equations (32) and (33) yields

$$(1-\alpha) A \left(1 - \left[\ell^c\right]^{1-\alpha}\right) > A \left(1 - \ell^c\right), \tag{34}$$

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which cannot hold since $\alpha \in (0, 1)$ and $\ell^c < 1$. Therefore, by continuity of $V(\cdot)$ in Assumption 2, it follows that $\underline{\kappa} < 1$.

Proof of Step 3. We establish that there exists $\overline{\kappa} \in (\underline{\kappa}, 1)$ for which $l^c < 1$ and $l^n = 1$ are not solutions to the government's problem if $\kappa \in (\underline{\kappa}, \overline{\kappa})$. Suppose that $\kappa = \underline{\kappa} + \varepsilon$ for $\varepsilon > 0$ arbitrarily small. We can establish that $\ell^n < 1$. Suppose it were the case that $\ell^n = 1$. Because (31) is violated at $\kappa = \underline{\kappa} + \varepsilon$, then there exists some ℓ such that the government under lack of commitment can deviate and market itself strictly better off. Therefore, $\ell^n < 1$. Now consider the value of ℓ^c and suppose it were the case that $\ell^c < 1$. For the government under commitment to prefer $\ell^c < 1$ to no lockdown, it is necessary that

$$[V(\Gamma(\Omega_1, \ell^c, \underline{\kappa} + \varepsilon)) - V(\Gamma(\Omega_1, 1, \underline{\kappa} + \varepsilon))] > A(1 - \ell^c). \tag{35}$$

for some $\ell^c < 1$. Consider the left hand side of (35) as $\varepsilon \to 0$, holding ℓ^c fixed. It follows from the definition of $\underline{\kappa}$ in equation (31) that

$$(1 - \alpha) A \left(1 - [\ell^c]^{1-\alpha}\right) \geq \lim_{\varepsilon \to 0} [V(\Gamma(\Omega_1, \ell^c, \underline{\kappa} + \varepsilon)) - V(\Gamma(\Omega_1, 1, \underline{\kappa} + \varepsilon))]. \tag{36}$$

Combining equations (35) and (36) implies that

$$(1 - \alpha) A \left(1 - [\ell^c]^{1-\alpha}\right) > A(1 - \ell^c) \tag{37}$$

which is a contradiction. Therefore, $\ell^c = 1$. The existence of $\overline{\kappa}$ such that $\ell^c = 1 > \ell^n$ if $\kappa \in (\underline{\kappa}, \overline{\kappa})$ thus follows from continuity. $\square$

A.3 Proof of Proposition 4

Proof. Consider a rule $\underline{\ell}(\varepsilon) = \ell^n (\overline{\theta} - \varepsilon)$ for $\varepsilon > 0$ arbitrarily small. We will establish that such a rule strictly increases social welfare. Let $\ell^n(\theta)$ denote the policy under no commitment in the absence of a rule and let $\ell^r(\theta, \varepsilon)$ denote the policy under no commitment subject to a rule. After introducing a rule, the change in social welfare conditional on $\theta < \overline{\theta} - \varepsilon$ is zero since the policy under no commitment is unchanged. The change in social welfare come from $\theta \in [\overline{\theta} - \varepsilon, \overline{\theta}]$ and

equals

$$\int_{\overline{\theta}-\varepsilon}^{\overline{\theta}} \left[A \left(\ell^r \left(\theta, \varepsilon \right) - \ell^n \left(\theta \right) \right) + V \left(\Gamma \left(\Omega_1, \ell^r \left(\theta, \varepsilon \right), \kappa \right), \theta \right) - V \left(\Gamma \left(\Omega_1, \ell^n \left(\theta \right), \kappa \right), \theta \right) \right] f \left(\theta \right) d\theta. \quad (38)$$

We first establish that (38) is bounded from below by

$$\int_{\overline{\theta}-\varepsilon}^{\overline{\theta}} \left[A \left(\ell^n \left(\overline{\theta} - \varepsilon \right) - \ell^n \left(\theta \right) \right) + V \left(\Gamma \left(\Omega_1, \ell^n \left(\overline{\theta} - \varepsilon \right), \kappa \right), \theta \right) - V \left(\Gamma \left(\Omega_1, \ell^n \left(\theta \right), \kappa \right), \theta \right) \right] f \left(\theta \right) d\theta.$$

$$(39)$$

If for a given $\theta \in \left[\overline{\theta} - \varepsilon, \overline{\theta} \right]$ we have that $\ell^r \left(\theta, \varepsilon \right) = \ell^n \left(\overline{\theta} - \varepsilon \right)$, then

$$A\ell^r \left(\theta, \varepsilon \right) + V \left(\Gamma \left(\Omega_1, \ell^r \left(\theta, \varepsilon \right), \kappa \right), \theta \right) = A\ell^n \left(\overline{\theta} - \varepsilon \right) + V \left(\Gamma \left(\Omega_1, \ell^n \left(\overline{\theta} - \varepsilon \right), \kappa \right), \theta \right). \quad (40)$$

Now suppose that for a given $\theta \in \left[\overline{\theta} - \varepsilon, \overline{\theta} \right]$, $\ell^r \left(\theta, \varepsilon \right) > \ell^n \left(\overline{\theta} - \varepsilon \right)$. The government under no commitment must weakly prefers choosing $\ell^r \left(\theta, \varepsilon \right)$ in equilibrium to $\ell^n \left(\overline{\theta} - \varepsilon \right) < \ell^r \left(\theta, \varepsilon \right)$:

$$\left(1 - \alpha \right) A\ell^r \left(\theta, \varepsilon \right) + V \left(\Gamma \left(\Omega_1, \ell^r \left(\theta, \varepsilon \right), \kappa \right), \theta \right)$$

$$\geq \left(1 - \alpha \right) A\ell^r \left(\theta, \varepsilon \right) \left(\frac{\ell^n \left(\overline{\theta} - \varepsilon \right)}{\ell^r \left(\theta, \varepsilon \right)} \right)^{1-\alpha} + V \left(\Gamma \left(\Omega_1, \ell^n \left(\overline{\theta} - \varepsilon \right), \kappa \right), \theta \right). \quad (41)$$

Since $\ell^n \left(\overline{\theta} - \varepsilon \right) < \ell^r \left(\theta, \varepsilon \right)$ and $\alpha \in \left(0, 1 \right)$, it follows that

$$\ell^r \left(\theta, \varepsilon \right) \left(\frac{\ell^n \left(\overline{\theta} - \varepsilon \right)}{\ell^r \left(\theta, \varepsilon \right)} \right)^{1-\alpha} = \ell^n \left(\overline{\theta} - \varepsilon \right) \left(\frac{\ell^r \left(\theta, \varepsilon \right)}{\ell^n \left(\overline{\theta} - \varepsilon \right)} \right)^{\alpha} > \ell^n \left(\overline{\theta} - \varepsilon \right). \quad (42)$$

Substitution of equation (42) into (41) implies that

$$A\ell^r \left(\theta, \varepsilon \right) + V \left(\Gamma \left(\Omega_1, \ell^r \left(\theta, \varepsilon \right), \kappa \right), \theta \right) \geq A\ell^n \left(\overline{\theta} - \varepsilon \right) + V \left(\Gamma \left(\Omega_1, \ell^n \left(\overline{\theta} - \varepsilon \right), \kappa \right), \theta \right). \quad (43)$$

Conditions (40) and (43) thus imply that the expression in equation (38) is bounded from below by (39).

Now consider the value of (39). We can show that it is positive for $\varepsilon > 0$ arbitrarily small. Consider $\theta \in \left[\overline{\theta} - \varepsilon, \overline{\theta} \right]$. For a given $\varepsilon > 0$, define $\overline{v} \left(\varepsilon \right) > 0$ as the highest value $\overline{v} \left(\varepsilon \right)$ such that

$\overline{v}(\varepsilon) < \overline{\theta} - \underline{\theta} - \varepsilon$ and also

$$A \left(\ell^n \left(\theta - v \right) - \ell^n \left(\theta \right) \right) + V \left(\Gamma \left(\Omega_1, \ell^n \left(\theta - v \right), \kappa \right), \theta \right) - V \left(\Gamma \left(\Omega_1, \ell^n \left(\theta \right), \kappa \right), \theta \right) > 0 \qquad (44)$$

for all $v \in [0, \overline{v}(\varepsilon))$ and all $\theta \in [\overline{\theta} - \varepsilon, \overline{\theta}]$. To see why $\overline{v}(\varepsilon)$ exists, consider the first order condition that defines $\ell^n(\theta)$

$$(1 - \alpha) A + V_\ell \left(\Gamma \left(\Omega_1, \ell^n \left(\theta \right), \kappa \right), \theta \right) = 0 \qquad (45)$$

It follows that

$$A + V_\ell \left(\Gamma \left(\Omega_1, \ell^n \left(\theta \right), \kappa \right), \theta \right) > 0, \qquad (46)$$

which means that social welfare is strictly increasing in ℓ in a neighborhood around $\ell^n(\theta)$. The existence of $\overline{v}(\varepsilon)$ follows from the fact that $\ell^n(\theta - v)$ is strictly decreasing in θ. Note that that $\overline{v}(\varepsilon) > \varepsilon$ if $\varepsilon = 0$. Moreover, by continuity, there exists some $\varepsilon > 0$ such that $\overline{v}(\varepsilon) > \varepsilon$. Thus, (44) holds for $v = \varepsilon - \left(\overline{\theta} - \theta \right) < \overline{v}(\varepsilon)$, which means that

$$\left[A \left(\ell^n \left(\overline{\theta} - \varepsilon \right) - \ell^n \left(\theta \right) \right) + V \left(\Gamma \left(\Omega_1, \ell^n \left(\overline{\theta} - \varepsilon \right), \kappa \right), \theta \right) - V \left(\Gamma \left(\Omega_1, \ell^n \left(\theta \right), \kappa \right), \theta \right) \right] > 0 \qquad (47)$$

for all $\theta \in [\overline{\theta} - \varepsilon, \overline{\theta}]$. This means that (39) is strictly positive for $\varepsilon > 0$. Therefore, the perturbation strictly increase welfare. $\qquad \square$