CONTINGENT LINEAR FINANCIAL NETWORKS

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<u>ABSTRACT</u>

In this paper, we develop a methodology to estimate hidden linear networks when only an aggregate outcome is observed. The aggregate observable variable is a linear combination of the different networks and it is assumed that each network corresponds to the transmission mechanism of different shocks. We implement the methodology to estimate financial networks among US financial institutions. Credit Default Swap rates are the observable variable and we show that more than one network is needed to understand the dynamic behavior exhibited in the data.

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1 Introduction

The 2008 world financial crisis started in what was supposed to be a relatively small, and segmented market. At that time the outstanding value of Sub-Prime debt was a tad below a trillion dollars, while the market value of credits in both the formal and shadow US financial sectors was higher than 23 trillion dollars. So, a default on 4 percent of an isolated market (the high-risk mortgages) was expected to have an equal minuscule effect. Not surprisingly, the Obama administration only requested 800+ billion dollars for the rescue package. It is obvious, today, that the systemic consequences of the sub-prime crisis were unexpectedly larger. The shock propagated to other financial sectors and countries. The world total losses reached several trillion. This has spurred research on systemic risk, and in particular on the estimation of the underlying financial networks governing the propagation of shocks - or as it is sometimes known as financial contagion.

The estimation of systemic risk has been closely linked to the estimation of financial networks. Banks can be interconnected through many different channels. One type of link is related to the exposure banks have to similar microeconomic or industry shocks. For example, two bank's balance sheets can be interconnected because both are lending to the same firm or sector and it suffers a shock. The second type of channel is related to interbank contracts. Two banks can be interrelated because one bank lends to the other, or they hold each other's liabilities. Therefore, a deterioration in the balance sheet of the borrowing bank affects the quality of the assets of the lending bank. Finally, banks can be exposed to similar macroeconomic shocks such as exchange rate, inflation rate, interest rates, economic activity, real estate, etc. In sum, there are many possible ways in which banks are linked to each other.[1] The objective of the empirical literature has been to estimate the "average" linkage among banks, and to determine the contribution each bank has to the overall risk in the economy — a measure of systemic risk.

Observing and understanding the contracts underlying the links between banks is sometimes challenging, however. Direct lending from one bank to the other of course is simple to document. The relationship across other contracts — sometimes complex and through indirect channels — implies that the information required to estimate the financial networks is daunting. First, some trivial aspects, such as the currency of denomination of a contract, is unclear. For example, a financial contract can specify all its flows and payments in US Dollars, but its return can be perfectly correlated with the Euro-Dollar rate. So, even though this is a contract denominated in the local currency, it is actually a de-facto foreign exchange contract. Contracts such as default swaps with tranches, or contracts that are indexed to macroeconomic variables are complex and fully understanding the type of connection that they generate across two financial institutions might be impossible. Second, most of the times the contagion path is outside the regulatory or country jurisprudence. For example, interconnections can occur through firms that might not even reside in the country of anal-

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[1]See Allen and Gale (2000), Freixas et al. (2000) for earlier contributions, and Acemoglu et al. (2015), Allen et al. (2012), Caballero and Simsek (2013), Elliott et al. (2014), and Cabrales et al. (2014) for recent theoretical papers. Empirical papers that estimate financial networks include Billio et al. (2010), Merton et al. (2013), Adrian and Brunnermeier (2016), and Girardi and Ergün (2013).

ysis. For instance, assume a bank lends to Intel, who sells to a South Korean firm, that manufactures a monitor sold to a California firm, who gets a loan from a different bank. In this setting, the two banks are related through the South Korean firm. However, the requirements on reporting to be able to uncover such relationship are impossible — i.e. the US regulator can't force the South Korean company to reveal its clients and suppliers. The complexity, number, and variety of contracts that can be used among financial institutions have several implications in the estimation of networks. In principle, if every contract is fully disclosed and documented the relation between banks could be described precisely. In practice, however, the granularity and detail of the required information are unfeasible, even to regulators and central banks. For this reason, the financial network literature tends to simplify the information demands by approximating the underlying network using aggregate data.

In the literature, two main approaches have been taken. The first one uses information-theoretic principles to fill in the blanks of pairwise data using aggregate data. [2] For example, one can read the total inter-bank debt from Form 10-Ks. After that, the worst-case scenario can be evaluated by inferring the pair-wise debt structure using a maximum entropy principle. Those methods, while used in some stress tests, only give a rough estimate of the debt structures and do not fully utilize the information from time-varying data.

The second approach uses the correlation of prices of financial contracts Onnela et al. (2004) to construct interbank financial networks. The estimates based on correlations can be biased in the presence of heteroskedastic shocks.[3] Furthermore, both of these approaches, do not take into account the possibility that the network between banks is asymmetric, non-linear, and that there might be more than one network at play.

First, for any given network, it is likely that the transmission of shocks between banks is asymmetric and non-linear. In other words, the impact on the balance sheet of two banks depends on the size of the shock — a small shock might imply a small propagation, but a large shock might produce a large propagation. The reason is that larger shocks might trigger clauses in the financial contracts that smaller shocks do not cause.[4] More importantly, regulation of financial institutions is inherently non-linear — bankruptcy for instance, where the bank is allowed to operate freely within a capital requirement range, but quite restrictively outside of it. This implies that the nature of the network might be different depending on the size of the shock. In fact, the average correlation might severely underestimate the transmission of shocks when a larger shock hits the system.

Second, and equally importantly, it is likely that there is more than a single network describing the system. Each type of shock is likely to be transmitted through different contracts and the propagation would be conditional on the shock. For example, imagine that there are three banks, and two are negatively exposed to real estate movements while the third one is positively exposed to real estate, but all three banks are negatively exposed

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[2]See Upper (2011); Elsinger et al. (2013) for maximum entropy and Anand et al. (2015) for minimal density

[3]See Forbes and Rigobon (2002) for the bias in assessing financial contagion when using correlations.

[4]As in Acemoglu et al. (2015)

to interest rate increases. If in the economy there is an increase in interest rates, all three banks move together, but if there is a decline in the price of real estate, only two are affected. The correlation structure, and therefore the network, changes with the shock.

Third, both of the previous mechanisms imply that the propagation of shocks between banks is asymmetric. For example, assume there are two banks, a large national bank (such as CitiBank) and a small regional bank such as (Watertown Savings Bank). A shock that decreases the value of the large bank's assets by 10 percent might have an impact on the small one that is very different from a shock that reduces the small bank's assets by the same proportion. The shock to the large bank might be more meaningful to the small one than vice-versa. When the measurement of systemic risk uses either stock market prices, or credit default swaps, these interactions are contemporaneous. Therefore, the estimation of even a single network is complicated by endogenous variable biases. A correlation is incapable of capturing these differences.

Consequently, the estimation of systemic risk in the financial system is complicated by asymmetries, contemporaneous endogenous biases, and parameter instability. Our methodology makes several assumptions and simplifications to be able to solve the identification problem. First, the underlying networks are unobservable, but the outcome of their interactions is observable. In other words, the changes in bank balance sheets and financial conditions are the outcomes that can be observed and it is the result of many interactions acting jointly. Therefore, the estimation of the contingent financial networks is not only an identification problem but also a computational challenge.

The second assumption is the realization that non-linearity and multi-networks are empirically intertwined. In fact, the non-linearity is tantamount to the network being contingent to the size of shock: larger shocks imply different transmission mechanisms than smaller ones. There are a number of papers modeling financial networks demonstrating the cause of potential shifts in the transmission mechanism. Some insist on the change of risk transmission mechanism is induced by monetary policies (Altunbas et al., 2010). That is because monetary policy has a significant effect on company financial decisions, which in turn could change the structure of financial contracts and result in discontinuities in risk transmission mechanisms. Others believe market volatility could result in a phase transition of non-linear financial networks, which incurs new risk transmissions mechanism (Acemoglu et al., 2015). Multiple networks can be a piecewise approximation of the non-linearity underlying these papers. Therefore, we make two important assumptions about the structure of the network: First, we estimate a collection of linear networks. In other words, in the case of non-linearity, we approximate a non-linear network as a set of linear ones. In the case of different networks for each type of shock, we approximate each one as a linear network. Second, we assume that the overall outcome is a mixture of multiple underlying unobservable networks and the shocks.

The identification problem is solved using the heteroskedasticity observed in the data. The estimation imposes a maximum number of parameters that can be recovered from the data; therefore, we limit the number of networks that can be estimated. This implies that it is likely that in many circumstances we are still not capturing the full complexity in

the financial system. In this case, however, our estimation procedure captures the most "dissimilar" networks. Furthermore, we can evaluate and discuss the informational value gained by allowing an additional network. Finally, we implement the estimation procedure through an intuitive EM algorithm.

We apply our methodology to estimate the US financial network among the 10 largest financial institutions in the country. The data collected are their credit default swaps. In this paper, a first pass to the problem, we concentrate on estimating a small network.

The first contribution of the paper is the estimation of a mixture model of multiple endogenous asymmetric linear networks.[5] The intuition behind the estimation comes from the fact that in a linear network the correlation structure across banks is the result of a linear combination of the covariance implied by each network - where the weights are the relative variances of each shock. When the variance of those shocks changes through the sample, the weights change, leading to variation in the covariances. From that variation of the second moment, it can be recovered the underlying structure of the networks. The identification requires, therefore, finding periods where the shocks experience different volatilities. There are two ways to do so: first, statistical identification where the changes in the variance in the observed data determine the regimes. This procedure is very similar to the one developed by Sentana and Fiorentini (2001). The advantage of this methodology is that it allows estimating the asymmetry of large versus small shocks - where the statistical identification truly captures the differences in the propagation mechanisms. The second alternative is to find periods where macroeconomic shocks experience different variances and use those as heteroskedastic sources. The advantage of this procedure is that the identification is closer to economic events rather than statistical events. So, if the relationships are contingent to the shocks this procedure allows us to estimate the networks that closely capture such linkages.

We study 9 different macroeconomic variables that try to capture different aggregate shocks that typically afflict economies: (1) commodity price shocks (WTI Oil Prices), (2) Inflation (PriceStats daily inflation index), (3) Economic Activity (Non-farm payroll), (4) Stock Market Prices (S&P), (5) Risk (VIX), (6) Housing Prices (Case & Shiller index), (7) liquidity provision (short term interest rates), (8) exchange rates (nominal trade-weighted exchange rate), and (9) the yield curve (the difference between the long and short interest rates). Of course in a finite sample, it is not clear that all shocks occur - for instance high inflation has not happened in a long time in the US. This implies that not all networks can be identified.

When estimating multiple networks, the expectation-maximization (EM) algorithm, which is commonly used for estimating mixture models, is adopted. The EM algorithm alternates between an expectation (E) step, which updates the probability of a network dominating a regime, and a maximization (M) step, which computes each network through heteroskedasticity based on the probability updated in the E step.

The paper is organized as follows: In Section 2, a typical problem of the financial network

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[5]The procedure follows Rigobon (2003).

is formulated and the problem of identification is discussed. We start with a well-studied two-bank case to provide some intuition of the identification problem, and then provide a proof of solution uniqueness in the single network case. We discuss also the identification problem when the data generating process is explained by multiple networks. Section 3 discusses the case when a single network is estimated. We present both the statistical and the macroeconomic strategies of identification. Section 4 presents a test for the number of networks. We find that in the 10 banks case we need 3 networks to be able to explain the data. We present the results of the networks in this section. Finally, Section 5 concludes.

2   Modeling Financial Networks

The estimation of financial networks is not new. A significant proportion of papers, however, concentrate on the estimation of symmetric single networks — non-directional, and non-contingent graphs. Our first goal is to show the instability of the network. In this section, we present first, some preliminary evidence based on correlations. We then introduce a linear financial network model, and show: 1. how it can be uniquely identified with covariance data; 2. together with heteroskedastic shocks, how it can explain the volatility in correlation models, and 3. how the heteroskedasticity of shocks can be identified through macroeconomic indices.

2.1   The Time-Varying Correlations

It is desired to find out how and why banks are related to each other, and which banks are "crucial" in the risk transmission mechanisms, and which ones are less relevant. One natural measure for those purposes is the correlation coefficients. In modern portfolio theory, correlation coefficients also play an essential role. MPT quantitatively formalizes the concept of diversification via the statistical notion of covariance, or correlation.

However, most of those applications of linear correlation models make an oversimplified assumption, that the correlation coefficients of any two given financial instruments are time-invariant, or at least time-invariant in the period of analysis.[6] In practice, there are numerous examples where that assumption does not apply. In this paper, we are concerned with the systemic financial risks which primarily propagate through large banks, so we use the Credit Default Swap (CDS) data of the top 10 US banks as an example.[7]

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[6]Although most of the literature estimates the strength of the network transmission by using simple correlations, there are notable exceptions that are worth highlighting. Adrian and Brunnermeier (2016) proposes a new measure of comovement called CoVar — which is defined as the value at risk conditional on the bank being in distress. Girardi and Ergün (2013) extend that measurement to expand the definition of distress. These types of measures are consistent with networks being contingent.

[7]It is important to highlight that CDS might exhibit excessive comovement due to the presence of a government guarantee. (Merton et al., 2013) uses a different approach to measure the credit risk of the banks. They use contingent claims analysis instead of CDS. The CDS are partially guaranteed by government policy — for instance, deposit insurance. Future research should evaluate the robustness of the results presented
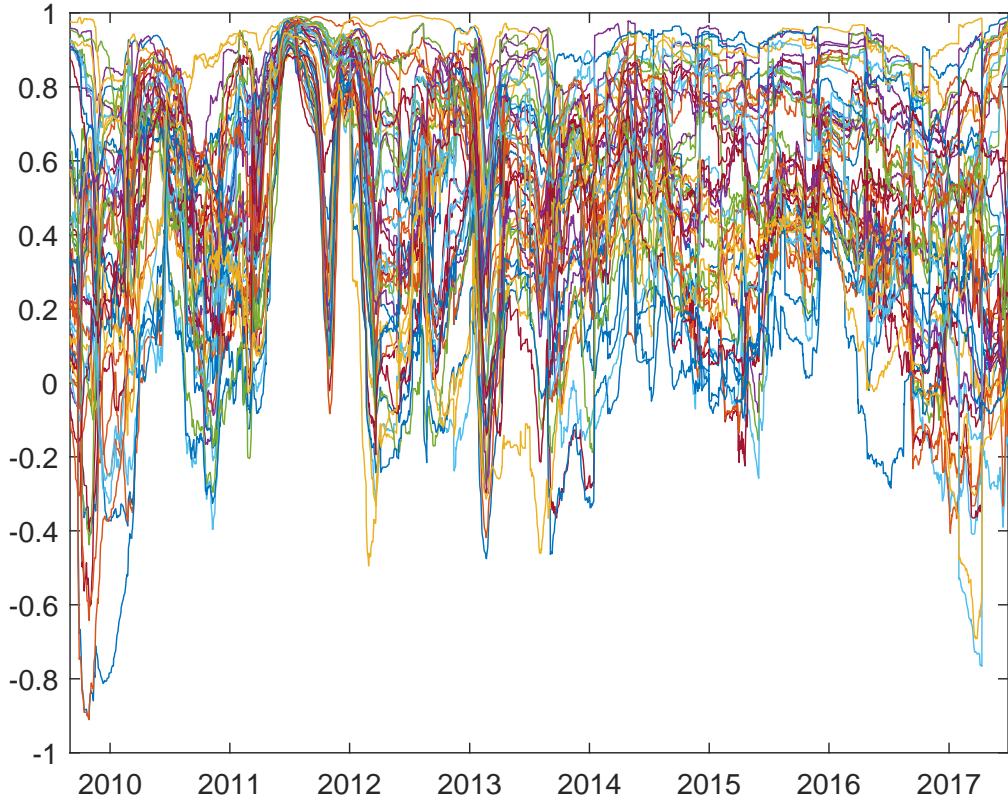
Figure 1: The 45 pairwise correlation coefficients of CDS of the top 10 US banks. The stock exchange id numbers of the 10 banks are JPM, BAC, WFC, C, GS, MS, COF, HSBC, AXP, and CSGN.

The change of volatility could due to either time-varying linear regression coefficients (a.k.a. linear financial networks, as we will define later in the paper), or heteroskedestistic shocks. Figure 1 shows the 45 pairwise correlation coefficients of CDS of the top 10 US banks. It is immediately notable that the correlation cannot be considered time-invariant. At an extreme, the correlation coefficient rises from -0.1 to 0.8 within 100 days. Up to this point, it is unclear whether the time-varying correlation is a result of heteroskedasticity of shocks, or a result of the structural change of risk propagation mechanisms, or both. The two-banks illustrative example we give in Section 2.3.1 shows that all three cases are possible. To distinguish the three cases, we need to develop a linear network model.

in this paper when different measures of financial performance are used.

2.2 A Linear Network Model

Suppose there are N financial institutions (indexed by $n = 1, 2, \cdots, N$) in a contingent financial network, which denoted by the directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. $\mathcal{V}$ is the set of nodes in the graph and each node $v_n \in \mathcal{V}$ corresponds to a financial institution n. Furthermore, each edge $e_{ijm} \in \mathcal{E}$ denotes the mth risk spillover channel between node i and node j. Note that $m \in \{1, 2, \cdots, M\}$ and $i, j \in \{1, 2, \cdots, N\}$.

There are two possible — simple — assumptions on how to implement the contingent networks. One in which the shocks are idiosyncratic and hit each bank individually and then they are propagated in the network, or the second one where the shocks are hitting the system as a whole. The first assumption is one in which the systemic risk of an individual shock affects is determined by the propagation between one bank and the other, while in the second assumption the relative importance of the aggregate shocks is what makes them systemic.

A simple framework where the shocks are idiosyncratic is as follows: Assume that at each time instance t, each node in the contingent financial network receives a shock. When a shock $\epsilon_{m,t}$ hits the market represented by channel m, it has an impact on financial institutions in the network, both directly and indirectly. The direct impact is due to the fact that institution i has some direct exposure in this market or shock, and it is denoted by $\theta_{m,i,t}\epsilon_{m,t}$. Here we assume that two time series $\theta_{m,i,t}$ and $\theta_{m,j,t}$ for any two institutions i and j are independent. As a result, the variance-covariance matrix of direct shock is a diagonal matrix. In other words, the exposure to the shock is orthogonal to each financial institution, although the shock is common.

The second possible way to describe the network is one in which the impact of the macroeconomic shocks is directly affecting all financial institutions. In this case, $\theta_{m,i,t}$ are actually constant in t. In this case, the variance-covariance matrix of the shocks is non-zero on the off-diagonals.

For both of these settings, the indirect impact introduced by a shock $\epsilon_{m,t}$ comes from the fact that financial institutions have exposure through the balance sheet of the other financial institutions. We define Γ_m, $m \in \{1, 2, \cdots, M\}$ as the weighted-directed adjacency matrix of channel m. We assume there cannot be self-loops in the network hence all diagonal entries of Γ_m are zero.

Suppose F_{mt} is a time series of risk measure (or risk factor) specific to channel m, which satisfies

$$F_{mt} = f_m(F_{mt}) + \theta_{m,t}\epsilon_{m,t}. \tag{1}$$

where $f_m : \mathbb{R}^N \to \mathbb{R}^N$ is a vector valued function capturing the risk propagation mechanism. A first order Taylor expansion gives

$$F_{mt} = \alpha_m + \Gamma_m F_{mt} + \theta_{m,t}\epsilon_{m,t}. \tag{2}$$

For the rest of this paper, we assume that our data is demeaned, hence

$$F_{mt} = \Gamma_m F_{mt} + \theta_{m,t}\epsilon_{m,t}. \tag{3}$$

This is a decomposition of any shock to the system between the shock part $(\theta_{m,t}\epsilon_{m,t})$ and the network part (Γ_m). In this model, the time subscript is the same on both sides of the equation. This is a recursive assumption that was first posed by Christiano et al. (1999). Further assume that the risk measure specific to channel m is not observable but the total risk measure, defined as the mixture of risk measures from different channels

$$X_t = \sum_{m=1}^{M} w_{mt} F_{mt}$$

is observable. Here w_{mt} is an indicator random variable. $w_{mt} = 1$ if network m dominates at time t, and $w_{mt} = 0$ otherwise. Let h indicate different heteroskydastic regimes. Now, for each network m and each regime h, the indicator random variable w_{mh} is 1 with probability p_{mh}. is observable. Here w_{mt} is an indicator r

Note that rearranging (3) we obtain

$$F_{mt} = [I - \Gamma_m]^{-1} \theta_{m,t}\epsilon_{m,t}.$$

Hence we observe

$$X_t = \sum_{m=1}^{M} w_{mt} [I - \Gamma_m]^{-1} \theta_{m,t}\epsilon_{m,t}, \tag{4}$$

For convenience we define $A_m = [I - \Gamma_m]^{-1}$.

This describes the observed variable (X_t) as a linear combination of different unobservable networks (Γ_m) and different common shocks $(\theta_{m,t}\epsilon_{m,t})$, M of them. The estimation problem is to uncover the unobserved networks from the moments of the observed variables.

There are two representations of the linear networks we are interested in estimating. Suppose there are N banks and shock propagated through a constant network Γ. We assume that the shocks are independent. So, $E[\epsilon_i\epsilon_j] = 0$ for all i and j.

The first representation assumes that the each shock, ϵ_m, affects each financial institutions through $\theta_{j,m}$, where j is the index for the financial institutions. We however, assume that this effect is independent of the impact another institution receives, even though the aggregate shock is the same one. In other words, the overall model can be described as follows:

$$F_{mt} = A_m \theta_{m,t}^h \epsilon_{m,t} \tag{5}$$
$$E[\epsilon_{i,t}\epsilon_{j,t}] = 0 \tag{6}$$
$$E[(\theta_{i,a,t}^h \epsilon_{i,t}) \times (\theta_{i,b,t}^h \epsilon_{i,t})] = 0 \tag{7}$$

where i and j represent different networks, a and b represent two different financial institutions, and h indicates the different heteroskedastic regimes. We allow the parameters $\theta_{m,t}^h$ to change across regimes. Those shifts are the ones that produce the heteroskedasticity. The shocks, ϵ, are mean zero and unitary variance.

The second representation assumes that the shocks continue to be independent, but that they affect all financial institutions through a constant propagation. The representation is similar to the previous one

$$F_{mt} = \sum_{k=1}^{K} A_m \theta_{km} \epsilon_{m,t}^h \tag{8}$$

$$E[\epsilon_{i,t} \epsilon_{j,t}] = 0 \tag{9}$$

Define

$$B_m = \begin{bmatrix} A_m \theta_{1m} & A_m \theta_{2m} & \cdots & A_m \theta_{Km} \end{bmatrix}$$

Then

$$F_{mt} = B_m \epsilon_{m,t}^h \tag{10}$$

$$E[\epsilon_{i,t} \epsilon_{j,t}] = 0 \tag{11}$$

Here, if we assume that $K = N$, i.e., there are N common shocks affecting N financial institutes, then the second interpretation is equivalent with the first one.

In the identification process, we cannot differentiate between these two models. Klein and Vella (2009) used the second interpretation, while ours uses the first interpretation. For details of the two above interpretations, see the summary in (Lewbel, 2012).

As will become clear in the next section, the model as currently specified cannot be estimated using standard methods. We rely on an method called identification through heteroskedasticity to solve the problem of estimation.[8] Next subsection discusses the solution to the identification problem. A crucial ingredient is the presence of heteroskedasticity. Therefore, we assume that there are regimes of economic environments, denoted by $h \in \{1, 2, \cdots, H\}$. With some unknown probability, each time instance t belongs to a regime $\mathcal{R}_h$. If time $t \in \mathcal{R}_h$, then the shock at that time instance is distributed $\theta_{m,t} \epsilon_{m,t} \sim N(0, \Sigma_{mh})$. We assume the different regimes imply different exposures.

2.3 The identification problem

The identification procedure we use in this paper is related to the identification through heteroskedasticity developed in Rigobon (2003). The intuition of the identification can be developed in a two by two endogenous system of equations.

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[8]See Rigobon (2003), Sentana and Fiorentini (2001). For the description of the original procedure see Wright (1928). For further developments see Lewbel (2012), Lewbel (2018a), and Lewbel (2018b). For applications on crises see Caporin et al. (2018) and for applications on monetary policy see Nakamura and Steinsson (2018), Rigobon and Sack (2003), Rigobon and Sack (2004), and Rigobon and Sack (2008).

2.3.1 A Single Network Endogenous Model

Assume two banks are related according to the following system of equations and shocks

$$
\begin{align}
x_t &= \beta y_t + \epsilon_t \tag{12}\\
y_t &= \alpha x_t + \eta_t \tag{13}
\end{align}
$$

with reduced form,

$$
\begin{align}
x_t &= \frac{1}{1-\alpha\beta}\left(\beta\eta_t + \epsilon_t\right)\\
y_t &= \frac{1}{1-\alpha\beta}\left(\eta_t + \alpha\epsilon_t\right)
\end{align}
$$

where η_t and ϵ_t are the structural shocks and α and β are describing the network.

As it is, this model cannot be estimated from the data. Equations (12) and (13) describe the behavior of the data entirely with 4 parameters/variables: two shocks ϵ and η and two parameters α and β. These four constitute the unknowns of the system. The problem of identification arises because there are three equations in four unknowns. The observable variables x and y have zero mean and in the data only three moments can be estimated; all from the variance-covariance matrix. What are the solutions to the problem? In economics, solutions tend to create circumstances in which an additional equation is added to the system of equations. For instance, the exclusion restriction in the instrumental variable approach boils down to assuming that one parameter is zero (the exclusion assumption). Randomized controlled trials assume that all the variation is due to the treatment — again, this is implicitly assuming that there is no feedback effect ($\beta = 0$). This is a very reasonable assumption when the experiment is properly designed. All these solutions are making a parameter assumption (usually that a parameter is equal to zero). The identification through heteroskedasticity has a slightly different flavor. The easiest way to explain how identification through heteroskedasticity works is to show the system of equation. Assume that the parameters are stable and that the data has heteroskedasticity. For simplicity assume that there are two heteroskedastic regimes. In this case, it is possible to estimate one covariance matrix in each regime.

$$
\begin{align}
\Omega_1 &= \begin{bmatrix} var(x_{t,1}) & covar(x_{t,1}, y_{t,1}) \\ & var(y_{t,1}) \end{bmatrix} = \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix} \sigma_{\epsilon,1}^2 + \beta^2\sigma_{\eta,1}^2 & \alpha\sigma_{\epsilon,1}^2 + \beta\sigma_{\eta,1}^2 \\ & \alpha^2\sigma_{\epsilon,1}^2 + \sigma_{\eta,1}^2 \end{bmatrix}\\
\Omega_2 &= \begin{bmatrix} var(x_{t,2}) & covar(x_{t,2}, y_{t,2}) \\ & var(y_{t,2}) \end{bmatrix} = \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix} \sigma_{\epsilon,2}^2 + \beta^2\sigma_{\eta,2}^2 & \alpha\sigma_{\epsilon,2}^2 + \beta\sigma_{\eta,2}^2 \\ & \alpha^2\sigma_{\epsilon,2}^2 + \sigma_{\eta,2}^2 \end{bmatrix}
\end{align}
$$

There are six unknowns in the system. The two parameters (α and β), and four variances ($\sigma_{\epsilon,1}^2$, $\sigma_{\epsilon,2}^2$, $\sigma_{\eta,1}^2$, and $\sigma_{\eta,2}^2$). As can be seen, there are six equations in six unknowns. This means that the system of equations is just identified.

Notice that even though in each regime the system is under-identified (fewer equations than unknowns) the system as a whole is identified. The key assumptions are two: that the structural shocks are indeed structural (they are uncorrelated) and that the parameters are stable. In the end, the parameter stability allows the heteroskedasticity to add additional equations — which helps solve the identification problem.

The intuition of the two endogenous variables case is as follows. First, the graphical representation of the joint residuals in this model always takes the form of a rotated ellipse. Second, the rotation is summarized by the variance-covariance matrix.

In equations (12) and (13), the only meaningful moment we can compute to estimate the degree of contagion is the covariance matrix. An important question is then, what does the covariance matrix represent? The errors in these models are distributed as a multinomial and their contours are ellipses. To fix concepts, let us start with a simple endogenous system of equations (12) and (13). The 95th percentile of the errors is distributed as a rotated ellipse. We can solve for two independent normal distributions from the structural equations as follows (with some abuse of notation)

$$\phi_1 = \frac{x_t - \beta y_t}{\sigma_\epsilon} \quad \sim \quad N(0,1)$$

$$\phi_2 = \frac{y_t - \alpha x_t}{\sigma_\eta} \quad \sim \quad N(0,1)$$

Because ϕ_1 and ϕ_2 are independent with mean zero and variance one, it is possible to describe the ζ confidence interval as $\phi_1^2 + \phi_2^2 = \zeta$. This is exactly an ellipse. Substituting

$$\left(\frac{x_t - \beta y_t}{\sigma_\epsilon}\right)^2 + \left(\frac{y_t - \alpha x_t}{\sigma_\eta}\right)^2 = \zeta \tag{14}$$

The two axes of the ellipse cannot be computed in closed-form solution, but they depend on the slope of the curves (structural parameters) as well as the relative variances of the shocks. In Figure 2 a graphical representation is shown. Suppose that the blue curve represents the supply and the red is the demand (when there are no shocks). Then x_t represent quantity and y_t represent price. Furthermore, the points reflect some random realization of structural shocks that leads to a point far from the depicted schedules. The ellipse represents the 90th percentile. In this particular case β is assumed to be negative (representing the "demand"), while α is positive. In Figure 2, the variance of the demand shocks is larger than the variance of the shocks to the supply, hence, the ellipse is closely aligned with the supply curve. In the limit, if the variance of the demand is infinitely large, the ellipse would coincide exactly with the supply curve.
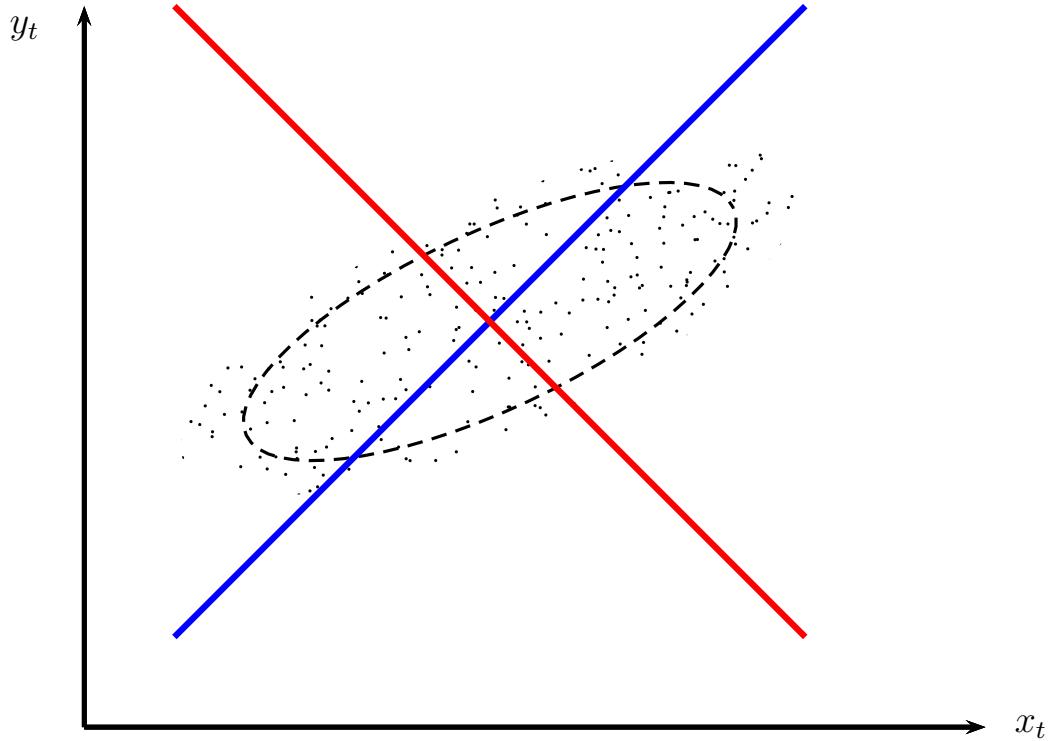
Figure 2: Distribution of Errors

The form of the ellipse is also summarized by the covariance matrix computed in the reduced form. Additionally, most of the methodologies we study are based on the covariance matrix. Therefore, all the sources of bias can be tracked to it. Finally, as mentioned previously, in this model the only statistic that can be computed from the data — that allows us to recover the structural parameters — is the covariance matrix.
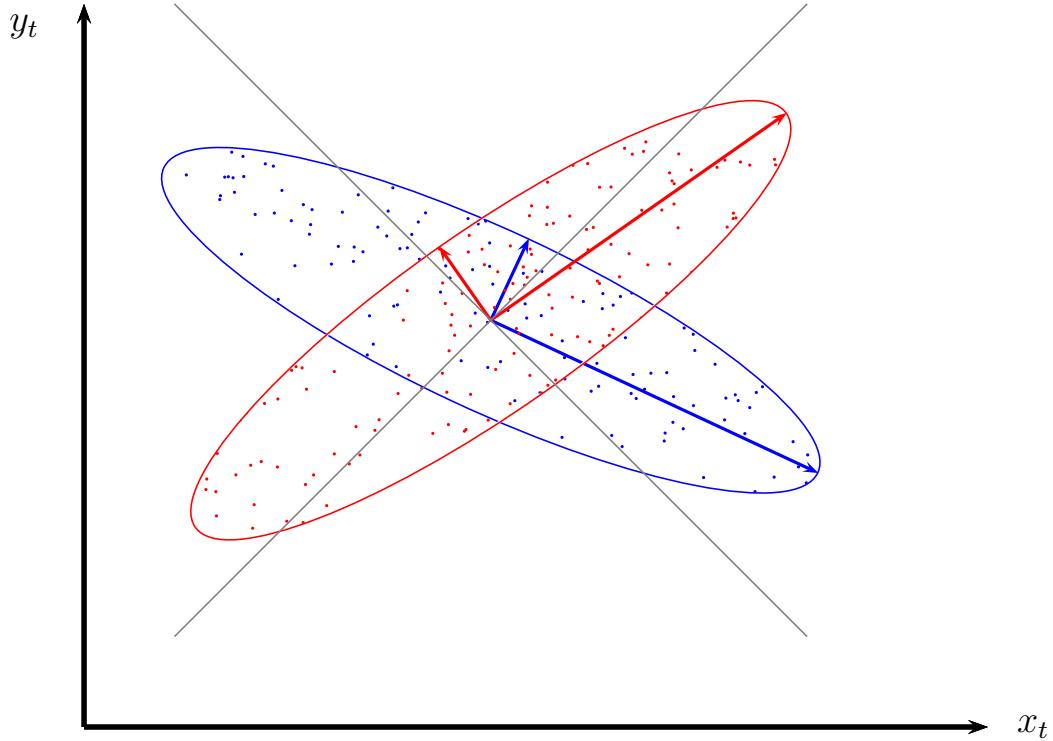
Figure 3: Identification Through Heteroskedasticity

The intuition behind the identification through heteroskedasticity comes from the rotation of the residual ellipses. When the variances change, for the same parameters, the ellipses rotate. In Figure 3, we show two cases: One when the shocks to the demand dominate (red), and one when the shocks to the supply dominate (blue). In particular, when the shocks to the demand dominate, then the ellipse approximates the supply curve. In fact, it is identical to the supply curve if the variance of the demand is infinite relative to the supply. Conversely, when the supply shocks are larger, then the long axis of the ellipse tilts toward the demand curve. It is this rotation of the ellipses when the relative variances shift that provides the identification.

It is instructive to re-state the underlying assumptions: structural shocks are uncorrelated (quite uncontroversial) and parameters need to be stable across the regimes (so, this is a good technique to measure spillovers).

2.3.2 Identification in the Contingent Financial Network

Suppose there are N banks and M types of shock (or networks). Suppose further that the networks $\Gamma_1, \Gamma_2, \cdots, \Gamma_M$ are constant over time but the variance of shock ϵ is time-varying, i.e., there is heteroskedasticity in the time series data. Let Σ_h, a diagonal matrix, be the variance of shock ϵ in regime $h \in \{1, 2, \cdots, H\}$, be unknown constants. We are implicitly assuming that the shocks across different banks are uncorrelated – or that a decomposition

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can be written to describe the data generating process as such. In fact, this assumption is standard in economics, where structural shocks are usually assumed to be independent with each other.

The model parameters we need to identify are

$$\theta = \begin{bmatrix} \Gamma_m \\ \Sigma_h \end{bmatrix},$$

and the number of parameters for each network m is $N(N-1) + N \sum_h w_{mh}$. The first term comes from the networks. There are $N(N-1)$ elements in each network (diagonal are ones). The second tem comes from the variance of the structural shocks. The shock affects N banks for each regime — and there are $\sum_h w_{mh}$ regimes in which the specific network dominates.

On the other hand, observed moments (moment constraints) are given by the variance covariance matrix of X_i in each regime h. Without loss of generality, we can assume that $\forall m$, $\mathbb{E}[F_{mt}] = 0$. The following constraints are obtained

$$E_{t \in \mathcal{R}_h} X_t X_t^\top - \sum_{m=1}^M w_{mh} \left((I - \Gamma_m)^{-1} \Sigma_{mh} (I - \Gamma_m)^{-T} \right) = 0,$$

where $E_{t \in \mathcal{R}_h} X_t^2$ denotes the expected value of the matrix $X_t X_t^\top$ given that t belongs to regime h. Because $w_{mh} \in \{0, 1\}$ and $\sum_m w_{mh} = 1$, the above constraints can be written as

$$E_{t \in \mathcal{R}_h} X_t X_t^\top - \left((I - \Gamma_m)^{-1} \Sigma_{mh} (I - \Gamma_m)^{-T} \right) = 0, \forall w_{wh} = 1. \tag{15}$$

In the above, each matrix $E_{t \in \mathcal{R}_h} X_t X_t^\top - \sum_{m=1}^M \left((I - \Gamma_m)^{-1} \Sigma_{mh} (I - \Gamma_m)^{-T} \right)$ is a N-by-N symmetric matrix, therefore each regime h provides $\frac{N(N+1)}{2}$ moment constraints. In total, $\sum_h w_{mh}$ regimes implies $\frac{N(N+1)}{2} \sum_h w_{mh}$ moment constraints.

The condition for an exact or over-identified model is

$$N(N-1) + N \sum_h w_{mh} \leq \frac{N(N+1)}{2} \sum_h w_{mh} \tag{16}$$

which implies that the data must have at least

$$\sum_h w_{mh} \geq 2 \tag{17}$$

for just-identification and

$$\sum_h w_{mh} \geq 3 \tag{18}$$

for over-identification. i.e., we need each network to dominate in at least 3 regimes to achieve over-identification. This identification is the order condition: how many equations are needed

for the system to have less unknowns than knows. In the single network case, this is simple: the single network will dominate all regimes, hence $H = \sum_h w_{mh}$. In the multiple network case, we need to figure out how many different mixture of networks (M) present in the data, and which network dominate which regimes. Those two questions are fundamental questions for the identification of many mixture models, and we will discuss them in more detail later in Section 4

2.4   Solution Uniqueness

When there is a single network, parameters in the above model can be estimated via the Generalized Method of Moments (GMM). We define the score function

$$g(\Gamma, \Sigma_h) = \left\{ X_t X_t^\top - \left((I - \Gamma)^{-1} \Sigma_h (I - \Gamma)^{-\top} \right) \right\}_{t \in \mathcal{R}_h, h=1, \cdots, H} \tag{19}$$

where $\{\cdot\}_{h=1, \cdots, H}$ denote the H-column vector whose entry are evaluated at $h = 1, \cdots H$. Estimation is achieved by solving the optimization problem

$$\min_{A, \Sigma_h} E_{t \in \mathcal{R}_h} g(\Gamma, \Sigma_h)^\top V^{-1} E_{t \in \mathcal{R}_h} g(\Gamma, \Sigma_h)$$

$$\text{s.t. } g(\Gamma, \Sigma_h) = \left\{ X_t X_t^\top - \left((I - \Gamma)^{-1} \Sigma_h (I - \Gamma)^{-\top} \right) \right\}_{t \in \mathcal{R}_h, h=1, \cdots, H} \tag{20}$$

$$\Sigma_h \text{ are diagonal.}$$

GMM requires that the score function is zero iff the system parameters are correct. We can prove that this requirement is satisfied when there is only one network, i.e. $M = 1$. To prepare for the proof, we define $\Sigma = \left[\begin{array}{cccc} w_{11} diag(\Sigma_1) & w_{12} diag(\Sigma_2) & \cdots & w_{MH} diag(\Sigma_H) \end{array} \right]$, where $diag(\cdot)$ denotes the diagonal entries of the matrix in the form of a column vector. In addition, define $\Omega_h = \mathbb{E}_{t \in \mathcal{R}_h} X_t X_t^\top$

Lemma 1. *If (i). $M = 1$, (ii). Γ and Σ are both full rank and (iii). $H \geq 2$, then $\Omega_h - \left((I - \Gamma)^{-1} \Sigma_h (I - \Gamma)^{-\top} \right) = 0$ has an unique solution.*

Intuition of the proof:

The identification problem is equivalent with a tensor decomposition problem (for details of this equivalence, refer to the Appendix). According to the Kruskal's rank condition, if

$$Krank(a_r) + Krank(a_r) + Krank(\sigma_r) \geq 2R + 2$$

then the tensor decomposition problem has a unique solution. In the above equation, $Krank$ stands for Kruskal rank, which is equivalent with matrix rank given assumption (ii). Hence we have $Krank(a_r) = N$ and $Krank(\sigma_r) = \min\{H, MN\}$. Furthermore, R is the rank of the tensor in (28), which equals MN. Inserting the numbers into the Kruskal's rank condition and considering assumption (i) and (iii), we obtain the solution uniqueness.

3 Single Network Estimation under Endogeneity

Throughout we use Credit Default Swap (CDS) as a measure of the risk of bank bonds. The mechanism of CDS can partly justify the additivity assumption (4): suppose F_{mt} is a CDS that only covers one specific type of shocks (e.g. interest rate). Then to cover all types of shocks (interest rate, GDP, stock index, etc.), one has to purchase multiple CDS for all those types of shocks, and the overall cost is the addition of each individual CDS, i.e., $X_t = \sum_{m=1}^{M} F_{mt}$.

The credit default swaps (CDSs) have been used widely in the literature studying risk contagion. Most notably, the CDSs of sovereign debt has been examined by various papers to understand international risk propagation. E.g., Kalbaska and Gątkowski (2012) investigates the eurozone contagion via a regression on CDS spread changes. In general, if a risk contagion mechanism exists among certain countries, their CDS spread will co-move. From the trend of CDS spread, the author reaches a conclusion that Sovereign debt risk is mainly limited to EU countries. Similarly, Caporin et al. (2018) also studies European debt crisis by examines the CDS spread. Through a Bayesian quantile regression that incorporates shock heteroskedasticity, they conclude that the increases in the correlation of CDS come from heteroskedasticity instead of structural changes of risk propagation mechanism.

The CDSs of banks have also been widely used in studying interbank risk propagation. As pointed out by Eichengreen et al. (2012), the CDS spreads of major banks co-move and reflects market economic prospects. Furthermore, during the Subprime Crisis and the following crisis of Lehman Brothers, the common trend is more dominant, i.e., the absorption ratio (Kritzman et al., 2010) is higher. The fact that major financial crises are reflected by CDS regime changes is the exact characteristic we want to study. In our contingent linear financial network model, a significant regime shift is required for unique identification. As shown in (Eichengreen et al., 2012), the common factor accounted for 62% of the variance of major bank CDSs before the 2007 breakout of Subprime Crisis and raised to 77% during the crisis, signaling a major regime shift. Apart from stronger co-movements, we also want to discover whether the change of regime is a sole result of heteroskedasticity of shocks, or a result of both the heteroskedasticity of shocks and the structural change of risk propagation mechanisms.

Suppose there is a set of shocks that we are interested in. A question arises whether or not we can measure them directly rather than model them as exogenous variables. E.g., suppose we are interested in the risk propagation caused by the fluctuation of interest rate, then we may use the time series of interest rates to fit the model. However, one should note that it is not the interest rate at each time instant that affects CDS, it is the expectation of interest rate over future time instances that does. Furthermore, it is the change of expectation, rather than the change of interest rate itself, that acts as a shock. Therefore, in the lack of a model for such expectation, it is better to model them as exogenous shocks and obtain their variance through the identification process.

Figure 4 shows the 100-day moving volatility of the 10 banks of interest. The first thing to notice is that they exhibit a significant regime shift of heteroskedasticity over time. Around

(a) JPM   (b) BAC   (c) WFC

(d) C   (e) GS   (f) MS

(g) COF   (h) HSBC   (i) AXP

(j) CSGN
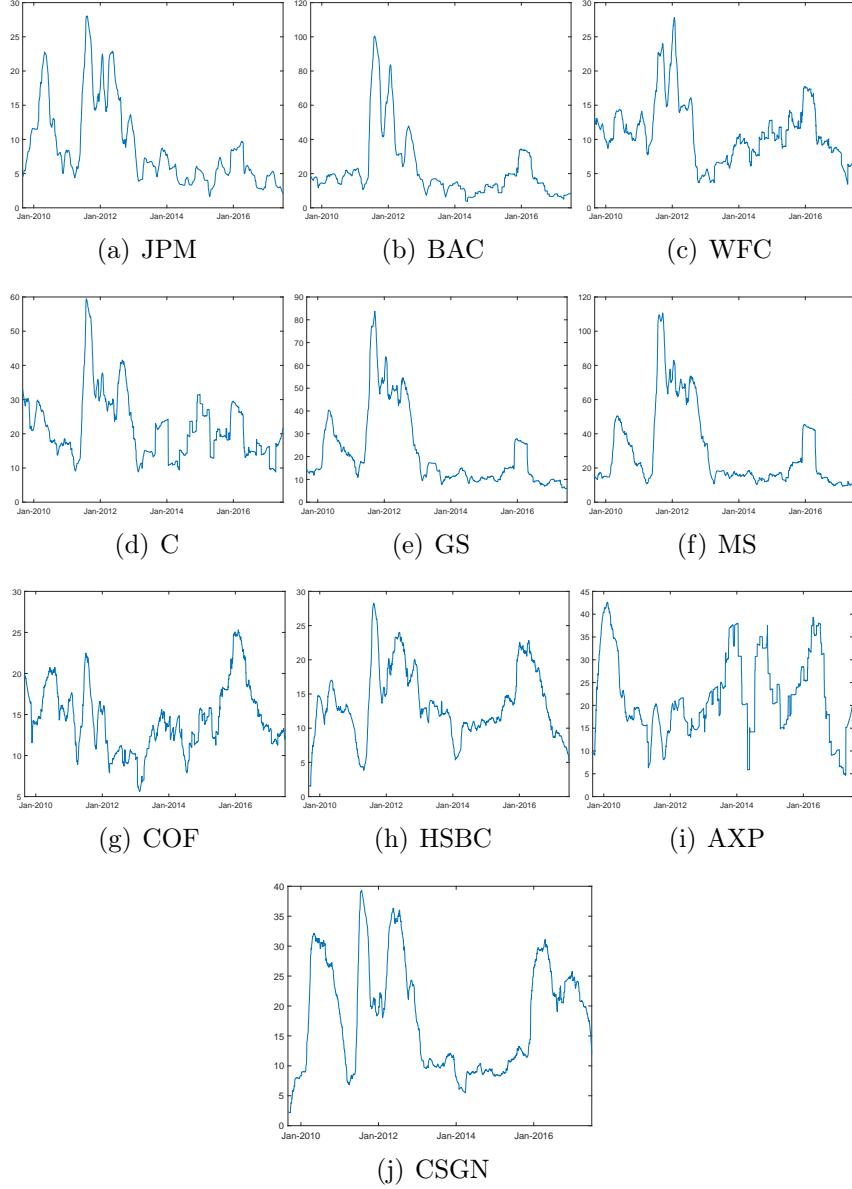
Figure 4: 100 days moving volatility of the 10 banks of interest. CDS data of 10 largest banks in US are collected from Sep. 1, 2009 to June. 20 2017. The stock exchange id tickers of the 10 banks are JPM, BAC, WFC, C, GS, MS, COF, HSBC, AXP, and CSGN.

the time of Nov. 2009, the volatilities of CDS of all 10 banks are high, while the volatilities are significantly lower in mid-2010. In addition, their volatility co-move most of the time, despite regime shifts. The fact that they co-move, allow us to identify a network that is well connected. Apart from similarities in the overall trend, the CDS volatilities exhibit a certain level of bank-specific movements. For example, in early-2016, the CDS of Capital One Financial Corporation has very high volatility that is comparable with that around late 2009. In contrast, the CDS of Morgen Stanley is at a relatively low historical level. That kind of bank-specific characteristic could allow as explore the different risk propagation mechanisms among different banks.

3.1   Selection of Regimes

As mentioned in previous sections, our model is not uniquely identifiable if the variance of shock is constant. When the variance of shock is not constant, we can divide our data into different heteroskedastic regimes. Because the network parameters are constant over regimes, additional regimes are offering our more additional constraints than additional unknowns. With enough heteroskedastic regimes, our model can be uniquely identified (Rigobon, 2003). In practice, however, dividing date into heteroskedastic regimes is not trivial. An effective regime division method should achieve heteroskedasticity among regimes and maintaining homogeneity within regimes. In addition, the regime divided should have some reasonable interpretations in macroeconomics. There are two broad categories of methods to identify heteroskedastic regimes: regime divided by statistic properties of the data itself, and regime divided by other exogenous variables (in our case, it makes sense to use macroeconomic factors). An identification process using regimes defined by statistic properties is named statistic identification, and an identification process using regimes defined by macroeconomic factors is named macroeconomic identification. In this section, we will introduce those two categories of regime divisions in detail, and then apply both to our problem of identifying contingent linear financial networks.

The first category of regime dividing methods is by statistic properties of the data itself. Because we want to separate the variance of shock, it makes sense to look at the quantile level of CDS data volatility. To maximize the separation of unobserved networks, one could use the quantile level of volatility of CDS to define regimes. For example, if there are two banks A and B in the network, one can define four regimes: bank A's volatility is at top 20% quantile level while B is not; bank B's volatility is at top 20% quantile level while A is not; both banks' volatility is at top 20% quantile level, and neither banks' volatility is at top 20% quantile level. It is clear that the overall financial network cannot be the same in different regimes.

However, regimes divided by a fixed quantile level is usually very unbalanced, i.e., the low volatility regimes have far more data points than the high volatility regimes. From an economic perspective, this is fine because exceptionally high volatility only occurs during crises. For identification purposes, however, this is not optimal, and many regimes have so few data points to be used in the identification. Alternatively, we could use unsupervised

learning techniques, such as K-mean and Gaussian Mixture Model to divide data into groups according to their volatility levels. Intuitively, a clustering algorithm group data points at different time instances into subsets, and maximize the similarities of the volatility vector in each subset. Due to the inherent limitation of most clustering algorithms, the global optimal grouping is usually very difficult to find, and the algorithm is sometimes trapped at local optimal solutions. If the dimension of the volatility vector is very high, i.e., we are identifying the network for a large number of banks, then it is even harder for the algorithm to find the global optimal solutions. In this case, we can reduce the dimension of the volatility vector by applying PCA prior to clustering.

The second category of regime dividing methods is by macroeconomic factors. While the above selection of regimes could maximize the separation of unobserved networks, it does not have any economic interpretation. To draw a connection with the macroeconomic environment, one could use the quantile level of macroeconomic factors instead of quantile levels of CDS volatility. The macroeconomic factors we choose include inflation rate, oil/commodity index, security index, market volatility indicators, currency index, and interest rate/yield. Because the CDS data is available daily, ideally we would like all the above macroeconomic factors to be daily as well. Daily inflation data, in particular, is available through the MIT Billion Price Project (Cavallo and Rigobon, 2016) and the PriceStats data platform.

3.2 Estimates: Statistical Identification

	JPM	BAC	WFC	C	GS	MS	COF	HSBC	AXP	CSGN
JPM		0.00	0.12	-0.04	0.28	0.00	-0.05	0.20	-0.02	0.01
		(0.02)	(0.06)	(0.04)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)	(0.01)
BAC	0.31		1.00	-0.15	0.35	0.25	0.04	-0.04	0.00	-0.06
	(0.26)		(0.12)	(0.05)	(0.16)	(0.11)	(0.08)	(0.13)	(0.05)	(0.16)
WFC	0.13	0.15		0.05	-0.04	-0.01	0.46	-0.07	0.03	-0.09
	(0.06)	(0.03)		(0.04)	(0.04)	(0.02)	(0.05)	(0.04)	(0.03)	(0.03)
C	-0.76	0.21	1.00		0.20	0.13	0.40	-0.35	0.06	-0.08
	(0.22)	(0.05)	(0.14)		(0.08)	(0.04)	(0.09)	(0.09)	(0.05)	(0.06)
GS	0.35	0.02	-0.04	0.05		0.55	-0.07	0.22	-0.05	-0.06
	(0.06)	(0.04)	(0.08)	(0.03)		(0.03)	(0.02)	(0.04)	(0.02)	(0.03)
MS	0.17	0.11	-0.19	0.25	1.00		0.04	-0.09	0.03	0.16
	(0.05)	(0.04)	(0.12)	(0.03)	(0.00)		(0.06)	(0.06)	(0.03)	(0.05)
COF	-0.06	0.00	0.87	0.10	-0.15	0.01		0.05	0.01	0.07
	(0.09)	(0.03)	(0.07)	(0.03)	(0.05)	(0.02)		(0.04)	(0.04)	(0.04)
HSBC	0.28	0.01	-0.07	-0.03	0.26	-0.07	0.03		-0.05	0.26
	(0.08)	(0.02)	(0.07)	(0.02)	(0.03)	(0.02)	(0.02)		(0.04)	(0.05)
AXP	0.50	-0.07	0.05	0.41	-0.50	0.17	0.55	-0.12		-0.09
	(0.12)	(0.03)	(0.13)	(0.05)	(0.14)	(0.09)	(0.10)	(0.13)		(0.08)
CSGN	0.07	0.01	-0.29	0.05	-0.13	0.23	0.01	0.64	0.12	
	(0.09)	(0.06)	(0.13)	(0.04)	(0.05)	(0.04)	(0.06)	(0.14)	(0.03)	

Table 1: Estimates of the network structure. We use a Lasso cost function with $\lambda = 10$. Standard deviations (in brackets) are obtained by bootstrapping (2000 resamples) across regimes. In this case, regimes are decided by CDS quantile. There are $H = 20$ regimes.
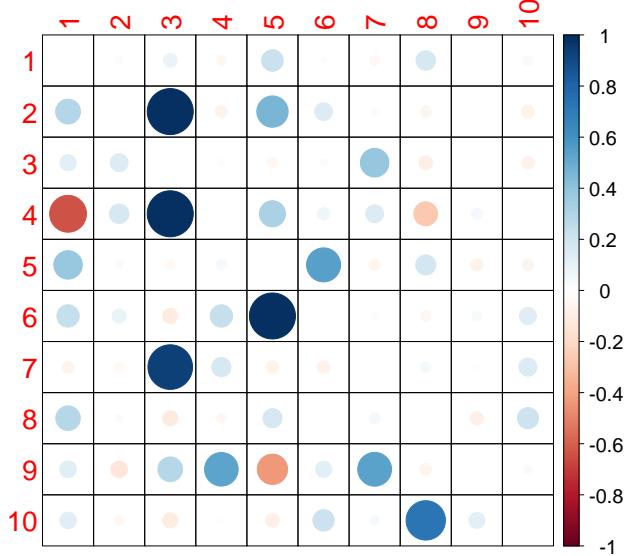
Figure 5: Visualization of the single network identified by statistical regimes

We now turn to the estimation results for our linear network model. As defined in Section 2, we estimate the linear shock propagation channel among the top 10 US banks. In our estimates, we assume that the linear structural parameters that we estimate is always between -1 and 1. i.e.,

$$\gamma_{i,j,m} \in [-1,1], \forall i,j,m. \tag{21}$$

Mathematically, this constraint will remove any non-unique solutions due to columns permutations. Economically, this means that the bank which receives a shock directly is most affected. This approach of removing permutation solutions will be problematic if a lot of the estimated structural parameters are on the boundary (i.e., the constraint (21) is binding for a lot of parameters). However, at least for the CDS spread dataset that we study, only 2 parameters are on the boundary in statistical identification, and no parameter is on the boundary in macroeconomic identification.

Table 1 shows the estimates of the 10-by-10 network structure using statistical identification. We use a Lasso cost function with $\lambda = 10$. Standard deviations (in brackets) are obtained by bootstrapping (2000 resamples) across regimes. In this case, as the name "statistical identification" suggested, regimes are decided by CDS quantile levels discussed in Section 3.1. The matrix in Table 1 can be regarded as a weighted directed graph. Each column shows where the shock is originated from, and each row shows where the shock propagates to. For example, the structural parameter in the 4th column (Citigroup) and the 1st row (JP Morgan) represents the channel where shock propagates from Citigroup to JP Morgan. In an earlier example in Section 2.1, we used the correlation between Citigroup to JP Morgan as an example to show that the correlation between banks can be very volatile. Here the structural parameter, on the other hand, is reliably estimated with bootstrapping standard deviation of only 0.04.

The estimated matrix of structural parameters is asymmetric in general. However, this does not mean any causal relationship between each pair of banks. In our original model, without (21), any column permutation of Γ will give a new solution and change the direction of edges of the weighted directional graph in Table 1. Now with constraint (21), the directions of the edges of the graph are pinned down by the constraint, but not by any inherent causality in the data.

Figure 5 gives a visualization of the same network. The values of structural parameters are represented by the color and size of the corresponding circles. Positive structural parameters are displayed in blue and negative structural parameters are displayed in red. In addition, color intensity and the area of the circles are proportional to the absolute values of the structural parameters. The visualization helps identify patterns in the risk transmission mechanism. For example, Wells Fargo is exposed to a number of different shocks, while Bank of America is more resilient to shocks transmitted from other banks.

3.3 Estimates: Macro Identification

	JPM	BAC	WFC	C	GS	MS	COF	HSBC	AXP	CSGN
JPM		0.10	0.28	-0.10	0.29	-0.05	-0.06	0.15	0.00	0.01
		(0.06)	(0.07)	(0.02)	(0.08)	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)
BAC	0.34		0.84	0.03	0.35	0.12	0.07	-0.01	-0.01	0.01
	(0.25)		(0.21)	(0.08)	(0.54)	(0.33)	(0.15)	(0.24)	(0.05)	(0.19)
WFC	0.30	-0.02		0.15	-0.23	0.14	0.30	0.11	0.03	-0.08
	(0.10)	(0.05)		(0.03)	(0.05)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)
C	0.10	0.29	0.95		0.48	-0.21	0.10	0.28	0.04	-0.31
	(0.16)	(0.07)	(0.28)		(0.17)	(0.13)	(0.12)	(0.15)	(0.12)	(0.11)
GS	0.94	-0.07	-0.43	0.22		0.30	0.23	0.20	-0.08	0.01
	(0.10)	(0.09)	(0.09)	(0.05)		(0.06)	(0.07)	(0.06)	(0.04)	(0.06)
MS	0.59	0.09	0.06	0.12	0.99		0.17	-0.30	0.01	0.24
	(0.24)	(0.10)	(0.20)	(0.06)	(0.08)		(0.09)	(0.13)	(0.05)	(0.12)
COF	0.03	-0.06	0.87	0.13	0.25	-0.26		0.00	0.06	0.23
	(0.09)	(0.06)	(0.10)	(0.07)	(0.09)	(0.04)		(0.08)	(0.05)	(0.04)
HSBC	0.40	0.00	0.13	-0.21	0.24	0.11	0.01		-0.02	-0.03
	(0.09)	(0.05)	(0.09)	(0.06)	(0.06)	(0.06)	(0.03)		(0.06)	(0.07)
AXP	0.11	-0.23	0.11	0.71	-0.60	0.25	0.28	-0.09		0.23
	(0.34)	(0.06)	(0.27)	(0.16)	(0.16)	(0.11)	(0.18)	(0.24)		(0.12)
CSGN	0.38	-0.02	0.08	0.01	0.07	0.20	-0.11	0.54	0.11	
	(0.13)	(0.06)	(0.12)	(0.07)	(0.06)	(0.12)	(0.07)	(0.18)	(0.05)	

Table 2: Estimates of the network structure. We use a Lasso cost function with $\lambda = 10$. Standard deviations (in brackets) are obtained by bootstrapping (2000 resamples) across regimes. In this case, regimes are decided by macroeconomic indicators. There are $H = 20$ regimes.
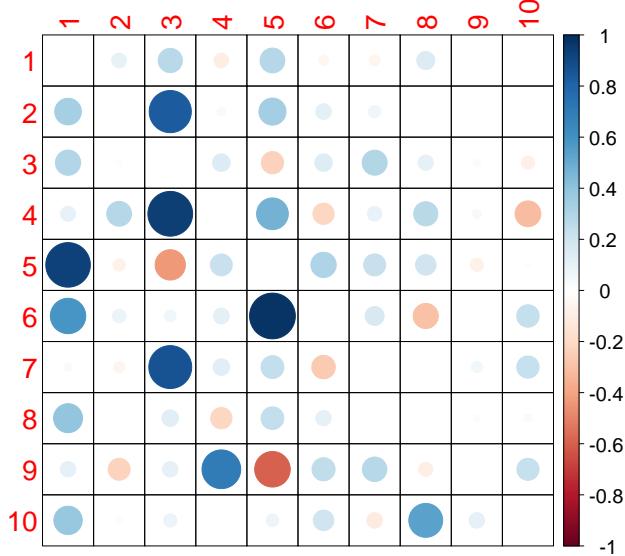
Figure 6: Visualization of the single network identified by macroeconomic regimes

If we select regimes by the quantile levels of macroeconomic factors rather than the quantile levels of CDS spreads, we obtain estimates of the network by macro identification. Table 1 and Figure 5 shows the result of macro identification. Similarly to the statistical identification, Table 2 shows the estimates of the 10-by-10 network structure with a Lasso cost function with $\lambda = 10$. Standard deviations (in brackets) are obtained by bootstrapping (2000 resamples) within regimes. Furthermore, Figure 6 gives a visualization of the network. The values of structural parameters are represented by the color and size of the corresponding circles. Positive structural parameters are displayed in blue and negative structural parameters are displayed in red.

In macro identification, standard deviations of structural parameters are estimated by bootstrapping within each regime, as opposed to bootstrapping across regimes in the entire dataset for statistical identification. This is because the regimes and networks are interchangeable. When we bootstrap the whole dataset, what constitutes network 1 changes. and in fact, the computer has no way to pick network 1 always, and this will exacerbate the standard errors. In the end, the estimates are the mixture of the networks.

Comparing Figure 5 with Figure 6 we notice that the two networks are almost identical. This is because the linear network identified using our approach is the same no matter how regimes are selected. In the last subsection and this subsection, we are using the same dataset, and the model we adopt is a linear model, hence the underlying linear financial network is the same. The bootstrapping standard deviations, on the other hand, are very different between statistical identification and macroeconomics identification. Both methods give consistent estimators but the estimators converge at different rates as the number of data points increases.

4 Multiple Contingent Network Estimation

This section presents a test for the multiple network assumption. In particular, we propose a testing procedure that compares the consistency of the estimates of structural variables in a single network case versus a multiple network case. In a point-wise test, one can obtain the distribution of differences and carry out the test without any assumptions on distributions. However, a point-wise test cannot provide a summary of the results. If we further assume that structural variables follow independent but not necessarily identical Gaussian distributions, we can test the sum of normalized residuals, which follows a Chi-squared distribution.

4.1 F-test for Network Contingency and the Rejection of the Single Network Hypothesis

In this subsection, we construct an intuitive F-test that is used to reject the single network hypothesis. Suppose one observes two sequences of data $\{X_t\}_{t \in D_1}$ and $\{X_t\}_{t \in D_2}$, and estimates structural parameters Γ_{D_1} and Γ_{D_2}. We want to know whether the two sets of structural parameters are consistent. Let $\gamma_{i,j,1}$ and $\gamma_{i,j,2}$ denote the ith row, jth column entry of the network estimated from data set D_1 and D_2 respectively.

We begin with a number of assumptions

Asymptotic Assumptions: (a) The parameter space Θ of $\theta = \begin{bmatrix} \Gamma \\ \Sigma_h \end{bmatrix}$ is a compact subset of $\mathbb{R}^d$, and the true value θ_0 lies in the interior of the parameter space Θ. (b) The moment function $\theta \to g(\theta)$ defined in (19) identifies θ_0: $g(\theta) = 0$ iff $\theta = \theta_0$. (c) The empirical moment function $\theta \to \hat{g}(\theta)$ converges uniformly in probability to the moment function $\theta \to g(\theta)$, namely $\sup_{\theta \in \Theta} \|\hat{g}(\theta) - g(\theta)\| \to_p 0$ (d) The empirical Jacobian $\hat{G}(\theta) = \frac{\partial}{\partial \theta} \hat{g}(\theta)$ is continuous and is uniformly consistent for the Jacobian matrix, $G(\theta) = \frac{\partial}{\partial \theta} g(\theta)$, i.e., $\sup_{\theta \in \Theta} \|\hat{G}(\theta) - G(\theta)\| \to_p 0$ (e) The matrix $G(\theta_0)^\top G(\theta_0)$ is positive definite. (f) The empirical moment function evaluated at the true parameter value obeys a central limit theorem:
$$\sqrt{n}\hat{g}(\theta_0) \sim N(0, \Omega)$$
asymptotically, where n is the number of samples.

Note those assumptions are inherently the same with the assumptions in the original GMM paper by Hansen (1982).

Under the null hypothesis that

$$H_0 : \gamma_{i,j,1} \text{ and } \gamma_{i,j,2} \text{ are the same}$$

their difference

$$\gamma_{i,j,1} - \gamma_{i,j,2} \tag{22}$$

should follows a distribution that with zero mean. If the estimated value of $\hat{\gamma}_{i,j,1} - \hat{\gamma}_{i,j,2}$ lies in the 0.05 left or right quantiles of the bootstrapping distribution, we can reject the null hypothesis and claim that with 90% confidence

$$H_1 : \Gamma_{D_1} \text{ and } \Gamma_{D_2} \text{ are different}$$

The above point-wise test has the advantage of distribution-free. However, without a summarizing statistics, one cannot draw conclusions on the overall network. Suppose further that $\gamma_{i,j,1} - \gamma_{i,j,2}$ follows Gaussian distribution $\mathcal{N}(\bar{\gamma}_{i,j}, \sigma_{i,j}^2)$. Under the null hypothesis that

$$H_0 : \Gamma_{D_1} \text{ and } \Gamma_{D_2} \text{ are the same} \tag{23}$$

their squared difference $(\gamma_{i,j,1} - \gamma_{i,j,2})^2 / \sigma_{i,j}^2$ follows a Chi-squared difference with degree of freedom 1.

In addition, under the independence assumption, the sum of squared differences

$$\sum_{i \neq j} (\gamma_{i,j,1} - \gamma_{i,j,2})^2 / \sigma_{i,j}^2 \sim \mathcal{K}(N(N-1))$$

If the estimated value of $\sum_{i \neq j} (\gamma_{i,j,1} - \gamma_{i,j,2})^2 / \sigma_{i,j}^2$ lies in the 0.1 right quantiles of the bootstrapping distribution, we can reject the null hypothesis and claim that with 90% confidence

$$H_1 : \Gamma_{D_1} \text{ and } \Gamma_{D_2} \text{ are different}$$

4.2   Estimates: the Sensitivity of Network Structures to Macroeconomic Environments

Using the statistical test derived in the last subsection, we are able to compare the network contingency given any two sets of data. In this subsection, we divide our dataset according to quantile levels of macroeconomic factors and test the network contingency to those factors. This procedure is analogous to a sensitivity test of our model. For example, we first look at the network contingency to the inflation rate.

More specifically, let X_t, a 10-by-1 vector time series, denote the CDS spread of top 10 US banks, and S_t denote the time series of a macroeconomic factor. Similarly to Section 3, macroeconomic factors in this paper include Price Index, WTI, S&P Index, VIX Volatility, 3-month Interest Rate, US Dollar Index, and 10-year minutes 3-month Interest Yield Curve. The dataset is divided into two parts

$$D_1 = \{X_t | S_t \leq \text{median}[S_t]\}, D_2 = \{X_t | S_t > \text{median}[S_t]\}$$

and both the pointwise test and Chi-squared test in Section 4.2 is carried out on these two sub-datasets. The result of tests are summarized in Table 3. In this table, we do not list the

corresponding p-values because the J-statistics is large enough to reject the null hypothesis with p-value less than 0.01 in all scenarios.

Macro Indicator	Mean diff	Mean std	Num. rejection	Jstat
Inflation	0.0923	0.1336	13	105
WTI	0.1859	0.1235	40	314
S&P	0.2015	0.148	31	411
VIX	0.1541	0.1419	21	181
i_3M	0.1634	0.1248	29	235
USD	0.1831	0.1381	27	245
i_10Y-i_3M	0.1533	0.1462	18	236

Table 3: Structural changes driven by macroeconomic indicators

Among the seven macroeconomic factors, network structural changes are most significant when asset prices are used to split the sample. Inflation and Volatility (as measured by the VIX) have the least rejections. Nevertheless, in all seven cases we reject the hypothesis of single network.

4.3   Estimating mixture models using EM-Algorithm

We first repeat the main characteristics of our mixture model. Suppose the data X_t in each regime $t \in \mathcal{R}_h$ is generated by

$$X_t = (1 - \Gamma_m)^{-1} \epsilon_t \tag{24}$$

with probability p_{mh}, where Γ_m are a set of parameters to be identified. Let w_{mh} denote the indicator random variable that equals to 1 if network m dominates in regime h. So $w_{mh} = \begin{cases} 1 & \text{w.p. } p_{mh} \\ 0 & \text{o.w.} \end{cases}$ Also, for now, we assume that the number of mixtures M is already known.

Furthermore, assume that the shock ϵ_t follows a distribution

$$\epsilon_t \sim \mathcal{N}(0, \Sigma_h + \Delta) \tag{25}$$

where Σ_h is a diagonal matrix characterizing the supposedly uncorrelated shocks ϵ_t. In the actual data generating process, however, the shocks may not be perfectly uncorrelated, and this deviation is captured by Δ where each entry

$$\begin{cases} \delta_{ij} = \delta_{ji} \sim \mathcal{N}(0, \sigma_\delta^2), & \text{if } i \neq j \\ \delta_{ij} = 0, & \text{otherwise} \end{cases}$$

Here because we are over-identifying we will not get exact zero covariances in the structural shocks even though that is our identifying restriction. Later, we will use this for the

rejection of the number of networks. Hence, we need to formalize the distribution of δ_{ij} here when we define the mixture model.

By construction, the variance-covariance matrix $\Omega_h = \mathbf{Var}(X_t; t \in \mathcal{R}_h)$ follows

$$[(1 - \Gamma_m)\Omega_h(1 - \Gamma_m)^\top]_{ij, i \neq j} \sim \mathcal{N}(0, \sigma^2) \tag{26}$$

In summary, we are looking at a mixture model with parameters Γ_m, hidden random variables w_{ij}, and observed random variables Ω_h.

EM-Algorithm

If we assume that only one of Σ_{mh} dominates in each regime, then the model becomes a mixture with discrete hidden variables, in which case we can estimate Γ_m fast and in a scalable manner. i.e., we estimate each Γ_m separately.

E-step:

Update $p_{m,h}$, the probability that network m dominates in regime h.

M-step:

$$\min_{\Gamma_m} \prod_h \left\{ p_{m,h} \exp\left[\sum_{i \neq j, h} \left(\left[(I - \Gamma_m)\,\Omega_h\,(I - \Gamma_m)^\top \right]_{i,j} \right)^2 \right] \right\}$$

4.4 The number of networks

To obtain the optimal number of networks, we apply the Bayesian information criterion. The Bayesian information criterion is defined as

$$\mathrm{BIC} = \ln(n)k - 2\ln(L).$$

where n is the number of samples, k is the number of parameters, and L is the maximum log-likelihood. Due to the Gaussian assumption in (26), the negative log-likelihood function is equivalent to the total mean squared error normalized by variance σ_δ^2, up to an additive constant. Because BIC is used for the purpose of model selection, the additive constant is irrelevant.

As shown in Figure 7, the optimal number of networks selected by Bayesian information criteria is 3.

4.5 Network Centrality

In our macro-identification process, we divide regimes according to macroeconomic shocks. It turns out some of those shocks play important roles in the network switching process, while others only contribute to the network switching marginally. In Table 4, regimes are grouped by networks in which they dominate. e.g., in regimes 1, 2, 3, 6, 14, 16, 17 and 18,
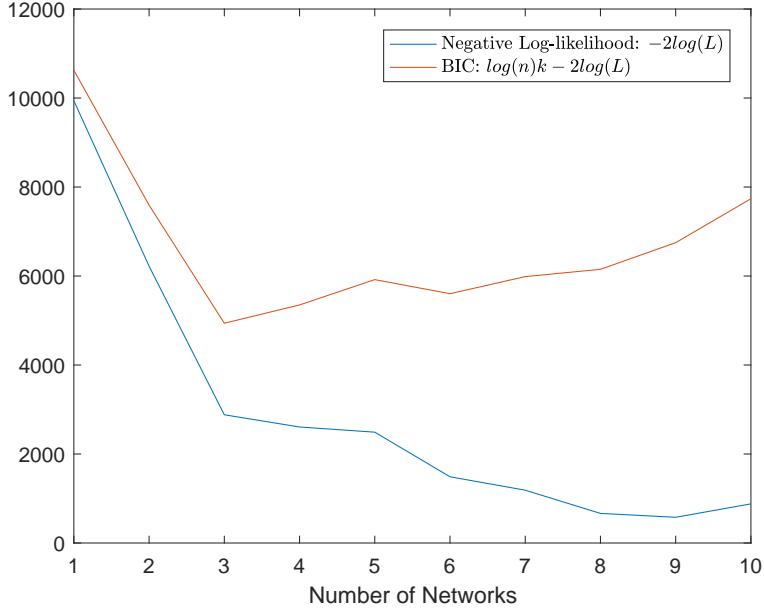
Figure 7: Negative log-likelihood and BIC vs. No. of networks

network 1 dominates. On top of that, we list whether or not each macroeconomic factor is active in each regime.[9] Active factors are denoted by 1 and others by 0.

From Table 4, we can summarize what each network represents. Network 1, which includes the regime where no macroeconomic factor is active, can be interpreted as normal times. Network 2, on the other hand, is very likely to be an asset price shocks, because S&P and exchange rates are both moving. Network 2 also reflects some short-run monetary policy that is usually reflected in asset prices. Finally, Network 3 is clearly an uncertainty shock, as it corresponds to VIX and yield curve activities. We also find that those shocks have different transmission mechanisms within the networks as can be seen in Figure 9.

Now that we know there are three networks in the interbank financial system, a systemic risk measure, namely the Katz centrality, will be calculated for each of the networks. The Katz centrality (Katz, 1953; Junker and Schreiber, 2008) of a network with adjacency matrix Γ is defined as

$$\overrightarrow{C}_{\text{Katz}} = ((I - \alpha A^T)^{-1} - I)\overrightarrow{1}, \tag{27}$$

where α is a damping factor that satisfies $0 \leq \alpha < 1/|\lambda_{\max}|$. In our paper, we choose $\alpha = 0.5$.

The Katz centrality is a generalization of the degree centrality. Intuitively, a node in the graph is more important if it more often receives shocks from other nodes. Furthermore, the Katz centrality considers both the direct impact from other nodes as well as the cascade impact many steps ago. It assumes that both direct and indirect impacts affect the importance of a node, given that indirect impacts are discounted by a factor of α after each step. Apart

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[9]By active macroeconomic factor, we mean that the specific macroeconomic factor is in its top 25% quantile in that regime.

		Inflation	WTI	S&P	VIX	i_3M	USD	i_10Y-i_3M
Network 1	regime 1	0	0	0	0	0	0	0
	regime 2	1	0	0	0	0	0	0
	regime 3	0	1	0	0	0	0	0
	regime 6	1	0	0	1	0	0	0
	regime 14	0	0	0	1	1	1	0
	regime 16	1	0	0	0	0	0	1
	regime 17	0	1	0	0	0	0	1
	regime 18	1	1	0	0	0	0	1
Network 2	regime 4	0	0	1	0	0	0	0
	regime 8	1	0	1	0	1	0	0
	regime 9	0	0	1	0	0	1	0
	regime 10	1	0	1	0	0	1	0
	regime 11	0	0	0	1	0	1	0
	regime 12	0	0	1	0	1	1	0
	regime 13	1	0	1	0	1	1	0
Network 3	regime 5	0	0	0	1	0	0	0
	regime 7	0	1	0	1	0	0	0
	regime 15	0	0	0	0	0	0	1
	regime 19	0	0	0	1	0	0	1
	regime 20	1	0	0	1	0	0	1

Table 4: Network vs. Macro shocks
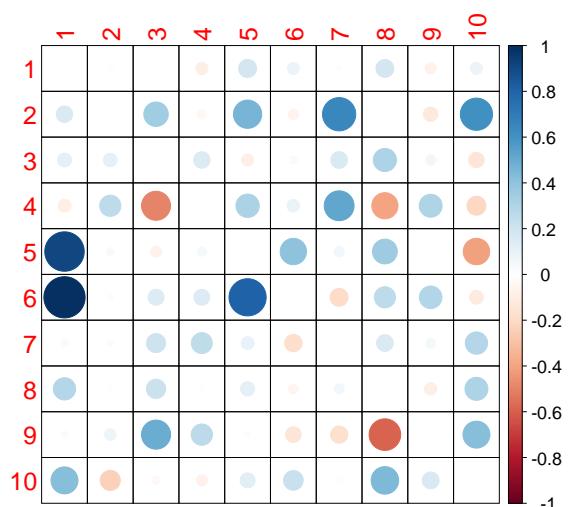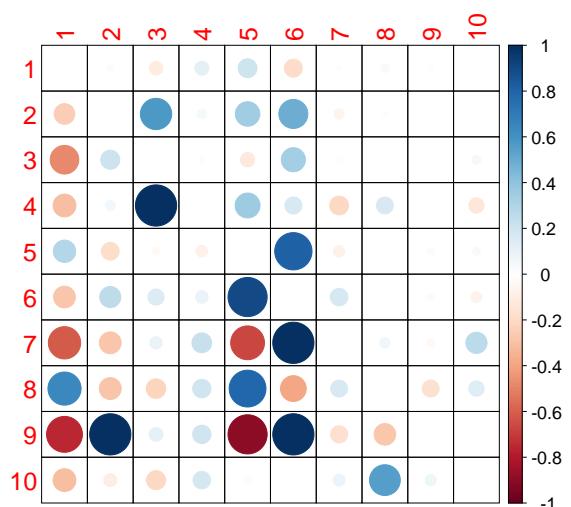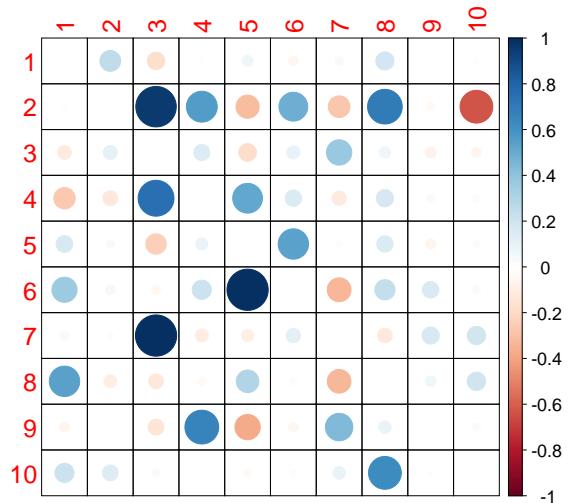
Figure 8: Estimated 3 networks

	Network 1	Network 2	Network 3
JPM	0.041 (0.005)	0.008 (0.013)	0.055 (0.015)
BAC	0.150 (0.018)	0.176 (0.017)	0.195 (0.045)
WFC	0.059 (0.006)	0.046 (0.013)	0.066 (0.011)
C	0.124 (0.008)	0.152 (0.022)	0.054 (0.039)
GS	0.100 (0.012)	0.122 (0.017)	0.139 (0.021)
MS	0.181 (0.016)	0.188 (0.021)	0.221 (0.025)
COF	0.117 (0.018)	0.054 (0.039)	0.075 (0.032)
HSBC	0.037 (0.016)	0.066 (0.036)	0.075 (0.015)
AXP	0.084 (0.020)	0.144 (0.033)	0.027 (0.016)
CSGN	0.107 (0.012)	0.044 (0.050)	0.095 (0.036)

Table 5: Estimates of network Katz centrality. Centrality values are normalized to sum up to 1. We use a discount factor $\alpha = 0.5$. Standard deviations (in brackets) are obtained by bootstrapping (2000 resamples) across regimes. In this case, regimes are decided by macroeconomic indicators. There are $H = 20$ regimes.

from original applications in social networks and biological networks, Katz centrality has also been applied to evaluate systemic risk in financial networks, see (Thurner and Poledna, 2013; Temizsoy et al., 2017). The Katz centrality of the top 10 banks in the US in the three estimated networks is shown in Table 5.

To understand the economic intuition of the Katz centrality, we first look at an alternative definition of it

$$C_{\text{Katz}}(i) = \sum_{k=1}^{\infty} \sum_{n=1}^{N} \alpha^k (\Gamma^k)_{ni}$$

From this definition, it is easy to see that the Katz centrality calculates the summation of an infinite series of impacts given a uniform shock to each bank. In our estimates, centrality values are normalized to sum up to 1, therefore each value $\tilde{C}_i$ just means the portion of shock that is transmitted through bank i.

In summary, we find the data can be explained by 3 networks in the financial network among the top 10 banks in the US with our criteria. We reject the hypothesis of 1 network using an F-test and then use the Bayesian information criteria to conclude that 3 networks are optimal. With 4 or more networks, the model complexity penalty term in the BIC would standout and reject the models. We are only applying our identification method on financial networks though, other applications of our identification method could give 4 or more networks as the optimal solution.

The three different networks we estimates imply different economic behaviors. The first network represents normal time, the second network represents an equity market shock, and the third network represents a VIX shock. Furthermore, the Katz centrality in different networks is not the same. Interestingly, some banks are always systemically important (i.e., Bank of America and Morgan Stanly), but depending on the type of shocks other banks might change their rank of centrality.
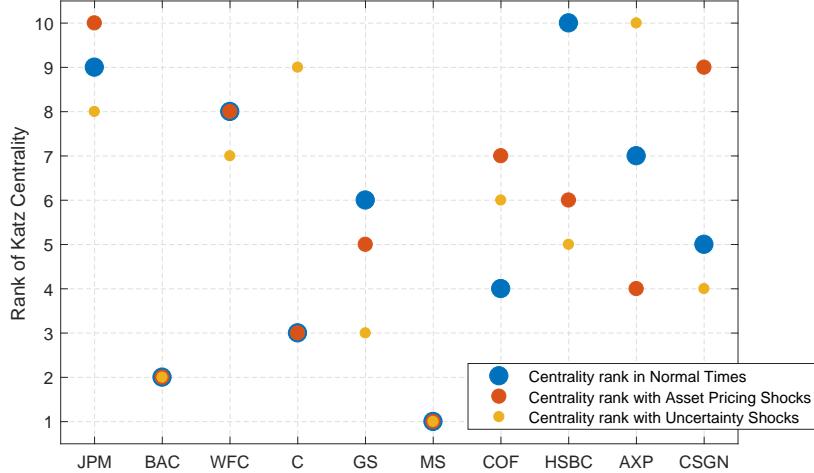
Figure 9: Rank of the estimated Katz centrality of in the 3 estimated networks.

Without the multi-network assumption in this paper, we would have incorrectly estimated a single network. In that case, the centrality rank assuming 1 network is given by JPM: 10, BAC: 2, WFC:9, C:7, GS:3, MS:1, COF:5, HSBC:8, AXP:6, CSGN:4. The resulting centrality measures, or any other systemic risk measures, will only be the weighted average of different scenarios in general. As a result, we lack the ability to identify banks that is systematic with respect to some specific shocks. Furthermore, with WFC for example, the centrality rank assuming 1 network is only 9th place, but it is actually more important in all three networks. Such an incorrect estimate of systemic risks could result in suboptimal decisions for financial practitioners, policymakers and regulators.

5   Conclusions

Understanding the interconnections within the financial system has been a first-order concern in developed economies since the 2008 global financial crises. In fact, macroeconomic prudential regulation needs to determine which banks and financial institutions are systemically important so they can be supervised closely. In the network language, it would mean that such financial institutions have a large centrality. Most of this analysis has been done either by concentrating on symmetric responses (computation based on correlations) or by observing a subset of financial contracts. This approach has been quite fruitful. In our view, however, both approaches might be incomplete.

On the one hand, the estimation of networks tends to obviate the directionality of the effects. In other words, Bank of America might have a large impact on American Express, but the opposite might not be true. Network analysis in the literature tends to obviate this feature. The second approach, which concentrates on the detail description of the contingent contracts across banks, could represent a solution to this problem. On the other

hand, it is virtually impossible to observe all possible contracts. Therefore, a market price is conceivably a more reliable measure of the actual exposure. This ambivalence implies that each procedure has a weakness that we have tried to address in the current paper. Furthermore, as has been shown in the literature, the nature of the transmission mechanisms changes with the shocks hitting the economy – meaning that the network is contingent on the state of the economy. We argue that the estimation of an asymmetric and contingent network requires a different identification method. We develop a methodology based on identification through heteroskedasticity. Applying this estimation method on CDS data of 10 large banks, we construct a financial network model and find that the data generating the model is consistent with three networks, each one contingent on one different macroeconomic shock.

Our results indicate that the systemically important bank depends on the type of shock that hits the economy – which is ultimately transmitted through a different network. Without our contingent financial network model, it is not possible to identify the importance of each bank in the financial system when a specific type of shock hits the economy. Indeed, we reject that the data is explained by a single network – suggesting that a policy designed based on that network would be inappropriate when a different transmission mechanism governs the dynamics of the system.

From the regulatory point of view, understanding the relative rankings on the financial institutions and how such ranking shifts in the sample is important. Our data is short and therefore we are limited in our ability to observe shocks that have not happened yet. For instance, we have not observed large positive productivity shocks, or relatively high inflation rates, or even high interest rates. Therefore, our conclusions are conditional to the sample we have seen. Within that sample, though, it is easy to identify tranquil times, periods when there are shocks to asset prices, and times where uncertainty is high. In those settings, which bank is systemically important changes. Central banks and regulators need to pay attention to these changes if the proper macro-prudential regulation is to be designed. In fact, regulators with misleading information about the financial network may not be able to make the most appropriate policy decisions to minimize the impact of those shocks.

Furthermore, the application of our identification method for contingent networks is not limited to policymaking. For example, asset management practitioners could use our method to estimate the contingent network and allocate assets according to the dominate shock in a period of time, and macroeconomists could use our method to evaluate the impact of macroeconomic interventions. In general, how to model, estimate and intervene in shock contingent networks is still an open and important topic for future research.

Appendices

A Equivalent Formulation via Tensor Decomposition

We first show that the identification problem is equivalent to a tensor decomposition problem.

In previous sections, we identify multiple layers by matching the second moments

$$\Omega_h = \sum_{m=1}^{M}(1 - \Gamma_m)^{-1}\Sigma_{mh}(1 - \Gamma_m)^{-\top}$$

We define a new N-by-MN matrix

$$\bar{A} = \begin{bmatrix} (1 - \Gamma_1)^{-1} & (1 - \Gamma_2)^{-1} & \cdots & (1 - \Gamma_M)^{-1} \end{bmatrix}$$

and a MN-by-MN diagonal matrix

$$\bar{\Sigma}_h = \begin{bmatrix} \Sigma_{1h} & & & \\ & \Sigma_{2h} & & \\ & & \ddots & \\ & & & \Sigma_{Mh} \end{bmatrix}.$$

Then we can write the moment matching equation as

$$\Omega_h = \bar{A}\bar{\Sigma}_h\bar{A}^\top.$$

Because the matrix $\bar{\Sigma}_h$ is diagonal, we can further write

$$\Omega_h = \sum_{r=1}^{MN} \vec{a}_r \sigma_{rh} \vec{a}_r^\top$$

where $\vec{a}_r$ is the rth column of $\bar{A}$ and σ_{rh} is the rth diagonal entry of $\bar{\Sigma}_h$. Because vector outer products can be written as tensor products, we can also write

$$\Omega_h = \sum_{r=1}^{MN} (\vec{a}_r \otimes \vec{a}_r) \, \sigma_{rh}$$

where $\otimes$ is the tensor product. Now if we stack all the second moments Ω_h along a third dimension, we obtain a N-by-N-by-H tensor $[\Omega_h]$ and it holds that

$$[\Omega_h] = \sum_{r=1}^{MN} \vec{a}_r \otimes \vec{a}_r \otimes \vec{\sigma}_r \tag{28}$$

where $\vec{\sigma}_r = \begin{bmatrix} \sigma_{r1} & \sigma_{r2} & \cdots & \sigma_{rH} \end{bmatrix}^\top$.

We can obtain estimates of A_m by taking the rank-MN tensor decomposition of $[\Omega_h]$.

According to the Kruskal's rank condition, if

$$Krank(a_r) + Krank(a_r) + Krank(\sigma_r) \geq 2R + 2$$

then the tensor decomposition problem has a unique solution. In the above equation, $Krank$ stands for Kruskal rank, which is equivalent with matrix rank given assumption (ii). Hence we have $Krank(a_r) = N$ and $Krank(\sigma_r) = \min\{H, MN\}$. Furthermore, R is the rank of the tensor in (28), which equals MN. Inserting the numbers into the Kruskal's rank condition and considering assumption (i) and (iii), we obtain the solution uniqueness.

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