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ABSTRACT

Electric vehicles have a unique potential to transform personal transportation. We analyze the transition to electric vehicles with a dynamic model that captures the falling costs of producing electric vehicles, the decreasing pollution from electricity generation, the increasing substitutability of electric for gasoline vehicles, and the durability of the vehicle stock. Due to the external costs from pollution, inefficiencies under business as usual result from the mix of vehicles as well as the transition timing, the severity of which depends on substitutability. We calibrate the model to the US market and find the magnitude of the inefficiency is rather modest: less than 5 percent of total external costs. The optimal purchase subsidy for electric vehicles and the optimal ban on the production of gasoline vehicles both give about the same efficiency improvement, but the latter leads to a sharp increase in gasoline vehicle production just before the ban. Phasing out gasoline vehicles with a bankable production quota reduces deadweight loss substantially more than the other policies, but may lead to a very large deadweight loss if set incorrectly.

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1 Introduction

Transportation is a substantial source of global and local air pollution (Davis and Killian 2011, Tschofen et al 2019). Several new technologies including electric vehicles, fuel cell vehicles, biofuel powered vehicles, and improved gasoline vehicles hold promise for decreasing this pollution. Of this suite of technologies, electric vehicles are unique in that, over the near term, they are commercially viable and have the potential to yield dramatic reductions in air pollution. The driving range of electric vehicles has increased and their production costs have decreased due to technological advances in batteries, electric motors, and materials. These changes, along with investments in complementary infrastructure such as charging stations, are making electric vehicles viable substitutes for many transportation uses. At the same time, emissions from electricity generation have fallen dramatically thereby reducing the pollution from electric vehicles (Holland et al., 2018). This combination of falling costs, falling emissions, and increasing substitutability has led to calls for a radical transformation of our transportation systems toward electric vehicles including policies that ban gasoline vehicles.[1]

Substantial research has analyzed electric vehicle adoption and policy in static models.[2] However, static models are not well suited to study the transition from a fleet dominated by gasoline vehicles to a fleet dominated by electric vehicles.[3] This paper constructs, analyzes, and simulates a dynamic model of the electric vehicle transition. The model allows us to analyze questions about the timing of electric vehicle adoption and how the timing is affected by policies such as an electric vehicle purchase subsidy or a gasoline vehicle production ban. Furthermore, the model allows a comprehensive welfare assessment of these policies.

Our model accounts for important dynamic aspects of the electric vehicle transition. First, as coal plants retire and investments in renewables and battery storage grow, the

[1] Twelve countries currently have planned fossil-fuel vehicle production bans, e.g., Norway in 2025 and Britain and France in 2040. (See https://en.wikipedia.org/wiki/Phase-out_of_fossil_fuel_vehicles Accessed 12/16/2019)

[2] Examples include Archsmith et al. (2015), Holland et al. (2016), Yuksel et al. (2016), Muehleger and Rapson (2018), Davis (2019), and Xing et al (2019).

[3] There are two papers with dynamic models closely related to ours. Creti et al (2018) have a dynamic model of vehicle adoption but assume a fixed vehicle stock and perfect substitution. Langer and Lemoine (2017) analyze adoption of a green technology but do not explicitly model replacement of a dirty good.

trend of decreasing pollution from electricity generation is predicted to continue. Holland et al. (2018) document an extraordinary decline in air pollution from electricity generation in 2010-2017 and determine the corresponding decline in damages from electric vehicles. Our model allows for declining damages from electric vehicles over time. Second, electric vehicle production costs are projected to continue to drop primarily due to improvements in the production of batteries. Falling costs could be due to learning by doing, scale economies, or exogenous technological change. Our main model allows for exogenously declining production costs of electric vehicles, but we extend the model to incorporate endogenously falling production costs through learning by doing. Third, the substitutability of electric and gasoline vehicles may change over time either because of improvements in range, charging time, or complementary infrastructure such as charging stations (e.g., indirect network effects). Our main model assumes a range of substitutability, but we also analyze substitutability that increases exogenously over time or endogenously through investments in complementary infrastructure. Fourth, vehicles are durable goods that depreciate. Thus we use dynamic optimization techniques to determine how the current stock of vehicles depends on future policies, costs, and consumer preferences.

The model allows us to study the transition from a variety of viewpoints. First, we solve a planner's problem that fully accounts for external costs from both gasoline and electric vehicles. Next we determine the business as usual solution that occurs in the marketplace when the external costs are ignored. Comparing these two solutions reveals two sources of inefficiencies: in the timing of electric vehicle adoption and in the long run vehicle mix. The adoption timing inefficiency is largest at high or low levels of substitutability between electric and gasoline vehicles because the market adopts electric vehicles too late or too early. The vehicle mix inefficiency is largest at intermediate levels of substitutability in which the planner would fully transition to electric vehicles but the market does not. To assess the magnitude of the inefficiencies, we calibrate our model for the US market. Overall, the largest deadweight loss occurs at intermediate levels of substitutability, but the loss is modest: less than 5 percent of total externality costs.

The calibrated model also offers insights into second best policies for improving the efficiency of the transition. Up to now, the most common electric vehicle policy has been subsidies for their purchase. We show that these subsidies can improve the adoption timing and reduce the deadweight loss for all levels of substitutability. Dynamically adjusting subsidies modestly improves their performance. Another policy is a ban on the production of gasoline vehicles. Twelve countries have currently announced bans and similar policies have been proposed for the US. In our model, a production ban can reduce deadweight loss if the planner's solution calls for a complete transition to electric vehicles. An unfortunate feature of the production ban, however, is an inefficient build up of the stock of gasoline vehicles in anticipation of the ban. A third policy, which we introduce and evaluate, is a bankable gasoline vehicle production quota. This policy caps cumulative production of gasoline vehicles and can be implemented by an intertemporal cap-and-trade program. The bankable production quota results in the smallest deadweight loss by a substantial margin, even compared to dynamically adjusted subsidies.

In addition to the general literature on electric vehicles, this paper contributes to several other literatures. A number of papers use dynamic models of transitions between technologies to analyze the design of subsidies (Kalish and Lilien 1983; Meyer et al. 1993; and Langer and Lemoine 2017) and learning by doing (Van Benthem et al. 2008; Chakravorty et al. 2012; Amigues et al. 2016; Bahel and Chakravorty 2016; and Creti et al. 2018). Relative to this literature, which assumes that technologies are perfect substitutes, a distinguishing characteristic of our work is that we allow electric and gasoline vehicles to be imperfect substitutes and also consider substitutability that is exogenously and endogenously increasing. Another literature analyzes issues related to electric vehicle charging infrastructure such as the effect of charging standards and the relationship between charging infrastructure and electric vehicle policy (Springel 2016; Li et al. 2017; Zhou and Li 2018; and Li 2019). We contribute to this literature by describing the relationship between infrastructure development and the substitutability of gasoline and electric vehicles and how this relationship affects the transition to electric vehicles.

2 Model

Consider a continuous time model in which society benefits from the stock of gasoline and/or electric vehicles. The benefit per unit of time in dollars is given by $U(G, X)$ where $G(t)$ denotes the stock of gasoline vehicles and $X(t)$ the stock of electric vehicles at time t.[4] Letting U_G and U_X denote the partial derivatives, we assume U is concave with $U_G > 0$ and $U_X > 0$.[5] The degree of substitutability between gasoline and electric vehicles plays an important role in determining the transition between them. Substitutability can be measured by either the cross-partial derivative U_{GX} or a cross-price elasticity, and the use of these concepts is interchangeable in our model.

The stocks of gasoline and electric vehicles evolve over time by production of new vehicles and retirement of existing vehicles due to events such as accidents and mechanical failure. Let $g(t)$ denote the production of gasoline vehicles and $x(t)$ denote the production of electric vehicles at time t. The state equations for the stocks of vehicles are $\dot{G} = -aG + g$ and $\dot{X} = -aX + x$, where $\dot{G}$ and $\dot{X}$ are time derivatives and a is the vehicle retirement rate. The expected lifetime of a vehicle is $\frac{1}{a}$. If there is no production of new vehicles, then the stock of vehicles decreases exponentially.

Each vehicle has a one-time production cost and ongoing usage costs. Let c_g denote the production cost of a gasoline vehicle, and let δ_g denote the usage costs of driving a gasoline vehicle per unit of time. Usage costs include both private operating costs, e.g., fuel purchases, and external costs, e.g., emissions of air pollution.[6] We assume that both c_g and δ_g are constant over time. At time t, total production costs are given by $c_g g$ and total usage costs are given by $\delta_g G$.

Electric vehicles initially have greater production costs and/or greater usage costs than gasoline vehicles, but that these costs are falling over time. Let the production cost of an electric vehicle at time t be $c_x(t)$ with $\dot{c}_x < 0$ and $\ddot{c}_x \geq 0$. Decreases in c_x over time are due to, for example, exogenous improvements in battery technology.[7] Let usage costs of driving an

[4] For notational convenience, we often suppress writing variables as explicit functions of time.

[5] Below we extend the model to include endogenous changes in benefits from investment in charging infrastructure.

[6] In practice, there may be externalities associated with production as well.

[7] Kittner et al (2017). Below we extend the model to include endogenous reductions in production costs from learning by doing.

electric vehicle per unit of time at time t be $\delta_x(t)$ with $\dot{\delta}_x \leq 0$. Decreases in δ_x over time are due to, for example, decreases in external costs from air pollution as the electricity grid gets cleaner. Let the limits of production and usage costs for electric vehicles be $\lim_{t\to\infty} c_x(t) = \hat{c}_x$ and $\lim_{t\to\infty} \delta_x(t) = \hat{\delta}_x$.

The planner's problem determines the production of gasoline and electric vehicles to maximize discounted benefits net of production and usage costs. Let $r > 0$ be the interest rate and assume the planner starts in an initial steady state with a positive stock G^{ss} of gasoline vehicles but zero electric vehicle stock.[8] The planner's problem is

$$\max_{g,x} \int_0^\infty e^{-rt} \left(U(G, X) - c_g g - c_x x - \delta_g G - \delta_x X \right) dt \tag{1}$$

subject to the constraints

$$\begin{aligned} \dot{G} &= -aG + g \ ; \ G(0) = G^{ss} \\ \dot{X} &= -aX + x \ ; \ X(0) = 0 \\ & \quad g \geq 0 \ ; \ x \geq 0. \end{aligned} \tag{2}$$

In this optimal control problem, the control variables are the production levels g and x, and the state variables are the stocks G and X.

The planner's problem describes the first best solution and also characterizes the outcome of a competitive equilibrium if all externalities are corrected by Pigovian taxes. Modifying the planner's problem allows us to analyze other market outcomes. For example, if we modify the objective function by ignoring the externalities, then the solution characterizes the outcome of a competitive equilibrium in which the externalities are not corrected, the so-called "business as usual" or BAU outcome. As explained in detail below, modifying the planner's problem in other ways allows analysis of policies such as electric vehicle subsidies or gasoline vehicle production bans.

Necessary conditions for the planner's problem are derived in Online Appendix A. These

[8]Assume further that $U_G(0,0) > (a+r)c_g + \delta_g$ which ensures that G^{ss} is positive and that $U_x(G^{ss}, 0) < (a+r)c_x(0) + \delta_x(0) - \dot{c}_x(0)$ which ensures that no electric vehicles are produced at $t = 0$.

conditions include the state equations for G and X and the corresponding adjoint equations

$$\dot{\alpha} = (a + r)\alpha + \delta_g - U_G, \tag{3}$$

and

$$\dot{\beta} = (a + r)\beta + \delta_x - U_X,$$

where α is the adjoint variable for G and β is the adjoint variable for X. Because the objective and state equations are linear in the controls, and the controls must be non-negative, we present the Kuhn-Tucker first order conditions:

$$\begin{aligned} g \geq 0 \quad \alpha - c_g \leq 0 \quad g(\alpha - c_g) = 0 \\ x \geq 0 \quad \beta - c_x \leq 0 \quad x(\beta - c_x) = 0. \end{aligned}$$

These conditions show that the adjoint variables are bounded above by the production costs and equal production costs when g or x is interior.

Consider interior production of gasoline vehicles. When $g > 0$, the first order conditions imply that $\alpha = c_g$ and hence because c_g is constant over time we have $\dot{\alpha} = 0$. The adjoint equation for G then implies

$$U_G = (a + r)c_g + \delta_g. \tag{4}$$

To interpret this equation, define the *full marginal cost* of the gasoline vehicle as $(a+r)c_g+\delta_g$, which is the sum of annualized depreciation, investment, and operating costs. If gasoline vehicles are produced, then the annual marginal benefit of a gasoline vehicle equals its full marginal cost.

Interior production of electric vehicles has a similar interpretation. If $x > 0$, then $\beta = c_x$, so taking the time derivative gives $\dot{\beta} = \dot{c}_x$, and the adjoint equation for X implies

$$U_X = (a + r)c_x + \delta_x - \dot{c}_x. \tag{5}$$

This equation is analogous to (4) except it has an additional cost $-\dot{c}_x > 0$, which is the opportunity cost of producing the electric vehicle at time t instead of waiting until it is cheaper to produce in the future. The full marginal cost of an electric vehicle, $(a+r)c_x+\delta_x-\dot{c}_x$, includes this opportunity cost, and, under our assumptions, decreases over time.[9]

In the initial steady state $x(0) = 0$ and $X(0) = 0$, so the initial steady state stock of gasoline vehicles, G^{ss}, is determined by $U_G(G^{ss}, 0) = (a + r)c_g + \delta_g$. The initial steady state production of gasoline vehicles is $g = aG^{ss}$.

Several elements of the model deserve additional explanation. The benefit function U represents the benefits to society from optimally allocating the stocks of vehicles to consumers. Adding a vehicle may require the existing stock of vehicles to be reallocated. This benefit function is consistent with well functioning markets for new and used vehicles. The stocks of gasoline and electric vehicles G and X are only differentiated by fuel type. In practice, many other attributes may matter to consumers including the age of the vehicle. The assumption that usage costs for a gasoline vehicle, δ_g, are constant over time does not necessarily imply that gasoline technology is stagnant. If certain components of the usage costs, e.g., the social cost of carbon, are increasing over time, then we are implicitly assuming that other components of the usage costs (e.g., fuel economy) are changing to offset these increases. On the electric side, we assume usage costs are decreasing due to declines in external costs over time. This does not require modeling vintages of electric vehicles if declines in external costs arise due to improvements in the electricity grid, rather than efficiency improvements to the electric drivetrains. An improvement in the grid leads to a contemporaneous decrease in external costs from the entire fleet of electric vehicles regardless of the ages of the vehicles. Finally, usage costs are independent of the stock of vehicles. This does not hold if expanding the stock of vehicles causes each vehicle to be driven less.

2.1 Terminal steady state

Before analyzing the optimal time paths, consider the terminal steady state in which all improvements to electric vehicles and the electricity grid have been completed, i.e., production costs have converged to $\hat{c}_x$, and usage costs have converged to $\hat{\delta}_x$. Let g^∞ be gasoline vehicle production in the terminal steady state. Note that $g^\infty = 0$ if there exists some T such that $g(t) = 0$ for all $t > T$. Our first proposition gives conditions that determine whether or not

[9]The time derivative of the full marginal cost is $(a + r)\dot{c}_x + \dot{\delta}_x - \ddot{c}_x$.

it is optimal to have gasoline vehicle production in the terminal steady state. All proofs are in the Appendix.

Proposition 1. *Let X^* be defined by $U_X(0, X^*) = (a+r)\hat{c}_x + \hat{\delta}_x$. Gasoline vehicle production is zero in the terminal steady state, i.e., $g^\infty = 0$, if*

$$U_G(0, X^*) < (a+r)c_g + \delta_g.$$

Conversely, $g^\infty > 0$ if $U_G(0, X^*) > (a+r)c_g + \delta_g$.

In the proposition, X^* is the number of electric vehicles which would be optimal if there were no gasoline vehicles in the terminal steady state. The proposition states that gasoline vehicle production is zero if the marginal benefit of a gasoline vehicle, when there are no gasoline vehicles but X^* electric vehicles, is less than the full marginal cost of a gasoline vehicle. Terminal steady state gasoline vehicle production is zero if this marginal benefit is small, but is positive if this marginal benefit is large.[10]

Dividing the first two marginal benefits in Proposition 1 yields a condition on society's marginal rate of substitution (MRS) between gasoline and electric vehicles. In particular, $g^\infty = 0$ if

$$\frac{U_X(0, X^*)}{U_G(0, X^*)} > \frac{(a+r)\hat{c}_x + \hat{\delta}_x}{(a+r)c_g + \delta_g}.$$

i.e., gasoline vehicle production is zero if the MRS at $(0, X^*)$ is smaller than the ratio of full marginal costs. If the vehicles are perfect substitutes and the MRS is one, then the result implies that gasoline vehicle production is zero if the full marginal cost of an electric vehicle is cheaper. If the vehicles are not good substitutes and the MRS is not one, then the full marginal cost of an electric vehicles would need to be substantially cheaper in order for gasoline vehicle production to be zero in the terminal steady state.

[10] If U is derived from an underlying discrete choice model, the marginal benefit $U_G(0, X^*)$ would be interpreted as the utility gain to the individual with the highest gain in valuation from a gasoline vehicle relative to optimally having either no vehicle or an electric vehicle.

2.2 Transition From Gasoline to Electric Vehicles

Now turn to the transition from gasoline to electric vehicles. By assumption, $g(0) = aG^{ss} > 0$ but $x(0) = 0$, i.e., gasoline vehicles are initially produced but electric vehicles are not. This leads to two key transition times: t^g, the time when gasoline vehicle production stops, and t^e, the time when electric vehicle production starts. More precisely t^g is defined such that $g(t) > 0$ for $t \in [0, t^g]$ but $g(t) = 0$ for $t > t^g$.[11] If $t_g = \infty$ then gasoline vehicle production is always nonzero. Similarly, t^e is defined such that $x(t) = 0$ for all $t \in [0, t^e)$ but $x(t) > 0$ for all $t \in [t^e, \infty)$.

For these transition times, there are two possible solutions to the planner's problem. If $t^e < t^g$, then there is a period of time in which gasoline and electric vehicles are both produced. We call this a *simultaneous solution.* If $t^g < t^e$, then there is a period of time in which neither gasoline nor electric vehicles are produced. We call this a *gap solution* due to the gap in vehicle production. Surprisingly, this solution obtains for reasonable parameterizations of the model, namely, if vehicles are perfect substitutes. This result and other details of the gap solution are given in Online Appendix B. Here we focus on specifying the transition times in the simultaneous solution.

The simultaneous solution is characterized first by production of gasoline vehicles only, then by production of both gasoline and electric vehicles, and finally by production of electric vehicles only. Before t^e, the solution has $g = aG^{ss} > 0$ but no electric vehicle production. Electric vehicle production begins at t^e so (5) must hold at this time. Substituting $G(t^e) = G^{ss}$ and $X(t^e) = 0$ into (5) yields an equation which characterizes t^e (see the proposition below). Over the interval $[t^e, t^g]$, both gasoline and electric vehicles are produced so both (4) and (5) must hold and the vehicle stocks (and hence production) are determined by these equations. Note that the costs of electric vehicles are falling so more vehicles are produced and $\dot{X} > 0$. Because the right-hand-side of (4) is constant over time, it follows that $\dot{G} < 0$ over this interval. If $t^g < \infty$, then gasoline vehicle production ceases at t^g. The characterization of t^g involves solving a differential equation (see the proposition below). After t^g the stock of gasoline vehicles simply decreases exponentially, i.e., $G(t) = G(t^g)e^{-a(t-t^g)}$ for every $t > t^g$.

[11] It may be optimal to temporarily cease gasoline vehicle production to draw down the existing stock of gasoline vehicles. In this case, which is explored in Online Appendix C, $g(t)$ need not equal zero for all $t > t^g$.

Because the gasoline vehicle stock is decreasing, the electric vehicle stock increases toward a terminal steady state, as determined by (5).

The following proposition characterizes the transition times:

Proposition 2. *In the simultaneous solution, the transition time t^e is the solution to*

$$U_X(G^{ss},0) = (a+r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e). \tag{6}$$

If $t^g < \infty$, the transition time t^g is the solution to

$$c_g = \int_{t^g}^{\infty} e^{-(a+r)(\tau - t^g)} \left[U_G(G^{sim}(\tau), X^{sim}(\tau)) - \delta_g\right] d\tau \tag{7}$$

where $G^{sim}(t) = G(t^g)e^{-a(t-t^g)}$ and $X^{sim}(t)$ satisfies $U_X(G^{sim}(t), X^{sim}(t)) = (a+r)c_x + \delta_x - \dot{c}_x$ for all $t > t^g$.

The characterization of t^e in (6) shows that electric vehicle production begins when the full marginal cost of the electric vehicle falls such that it exactly equals the marginal benefit of an electric vehicle given a zero stock of electric vehicles. Because gasoline vehicles are in steady state at t^e, equation (6) can also be written in terms of the MRS:

$$\frac{U_X(G^{ss},0)}{U_G(G^{ss},0)} = \frac{(a+r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e)}{(a+r)c_g + \delta_g}.$$

If there are individuals who highly value electric vehicles, the MRS when $X = 0$ could be quite large. In this case it might be optimal to produce electric vehicles even if their full marginal costs substantially exceed the full marginal costs of a gasoline vehicle. Conversely, if electric vehicles are seen as inferior even by the individuals with the highest relative valuations (perhaps due to range anxiety), then the full marginal costs would need to fall below the full marginal costs of a gasoline vehicle before electric vehicles are produced.

The characterization of t^g in (7) shows that gasoline vehicle production stops when the cost of producing a gasoline vehicle equals the present value of the lifetime benefit of driving a gasoline vehicle net of usage costs from that time on. Two points are worth noting about (7). First, the discount factor $e^{-(a+r)(\tau - t^g)}$ reflects both the interest rate, r, and rate of vehicle retirement, a. Second, the path of the gasoline vehicle stock after t^g, given by $G^{sim}(t) = G(t^g)e^{-a(t-t^g)}$, is decreasing exponentially because no new gasoline vehicles are produced after t^g. However, $G(t^g)$ is not equal to G^{ss} because during the interval $[t^e, t^g]$ both gasoline and electric vehicles are produced so the stocks evolve to satisfy both (4) and (5).

2.3 Market outcomes and BAU

A market economy does not have a planner making decisions to produce gasoline and electric vehicles. Rather these decisions are decentralized to firms and consumers through markets and prices. The solution to (1) describes the first best solution to our model, which can be decentralized by Pigovian taxes that fully reflect the externalities. If such taxes are politically infeasible, it may still be possible to improve outcomes by implementing market policies such as a subsidy on the purchase of electric vehicles or a ban on the production of gasoline vehicles. Our framework allows us to analyze the welfare consequences of such policies.

We start by describing the Business As Usual (BAU) market outcome. Under BAU consumers ignore all externalities, and there are no market policies put in place to try to correct for this behavior. Let δ_g^{pvt} and δ_x^{pvt} be the private operating costs of gasoline and electric vehicles, i.e., the usage costs without the externalities. To describe how electric vehicles are adopted over time by a competitive market, we use the optimal control problem

$$\max_{g,x} \int_0^\infty e^{-rt} \left(U(G,X) - c_g g - c_x x - \delta_g^{pvt} G - \delta_x^{pvt} X \right) dt \tag{8}$$

subject to the constraints in (2). This problem is similar to (1), but with the modification that δ_g and δ_x have been replaced by δ_g^{pvt} and δ_x^{pvt}. The solution to (8) gives us the time paths of production (g and x) and vehicle stocks (X and G) under BAU. To find the welfare associated with the BAU outcome, we substitute the time paths into

$$\int_0^\infty e^{-rt} \left(U(G,X) - c_g g - c_x x - \delta_g G - \delta_x X \right) dt, \tag{9}$$

and then evaluate the integral. Notice this function is the objective function of the original planner's problem, which includes the externalities. With the value of this function at the BAU time paths in hand, we can calculate the deadweight loss of BAU relative to the first best.

We follow a similar procedure to analyze other market based policies such as taxes or subsidies. In particular, by modifying (8) to reflect the private costs that result from a given policy (e.g., lower purchase costs from a subsidy), the optimization results in the vehicle time paths that would obtain under the policy. We then calculate the deadweight loss of the policy by evaluating these time paths with the integral in (9) and comparing to first best.

One possible source of deadweight loss is due to an inefficient vehicle mix. The following corollary, which follows directly from Proposition 1, provides some guidance on when the BAU terminal steady state has inefficiently positive production of gasoline vehicles.

Corollary 1. *If* $(a+r)c_g + \delta_g^{pvt} < U_G(0, X^*_{BAU}) < U_G(0, X^*) < (a+r)c_g + \delta_g$, *then gasoline vehicles are produced in the terminal steady state under BAU but not under first best.*

The corollary says that the vehicle mix is inefficient in the terminal steady state—i.e., gasoline vehicles are produced under BAU but not under first best—if both marginal benefits are in the interval between $(a+r)c_g+\delta_g^{pvt}$ and $(a+r)c_g+\delta_g$. This condition depends critically on the external costs and the degree of substitutability between gasoline and electric vehicles. If externality costs are large, then this interval is large and it is more likely that the BAU vehicle mix is inefficient. If vehicles are poor substitutes, then the two marginal benefits in the corollary are both large and the vehicle mix is not inefficient because gasoline vehicles are produced under both BAU and first best. If vehicles are good substitutes, then the marginal benefits are both small and the vehicle mix is not inefficient because gasoline vehicles are not produced under either BAU or the first best. The vehicle mix is only inefficient at an intermediate level of substitutability where the marginal benefits are in the interval so that gasoline vehicles are produced in the steady state under BAU but not in first best.

Deadweight loss also may result from the inefficient timing of the transition to electric vehicles. The next proposition shows that this transition time can be too early or too late under BAU.

Proposition 3. *Under BAU, the transition time t^e may be greater or less than under first best.*

The proposition is proved by numerical example below in Figure 2 and Table 2. Here we illustrate the key role played by the degree of substitutability. Figure 1 shows the determination of the transition time t^e for both first best and BAU in the simultaneous solution.[12] The bold curves illustrate the solution to (6) for first best. The value for t^e occurs when the (bold) full marginal cost $(a+r)c_x + \delta_x - \dot{c}_x$ falls to the (bold) marginal benefit $U_X(G^{ss}, 0)$. The thinner curves illustrate the solution to (6) for BAU. Notice that both curves shift down in moving from first best to BAU. The full marginal cost shifts down by the amount of the external costs.[13] To see why the marginal benefit shifts down, first note that BAU ignores external costs and so the initial steady state stock of gasoline vehicles is larger under BAU than in first best: $G^{ss}_{BAU} > G^{ss}$.[14] This higher initial stock shifts down the marginal benefit U_X if $U_{GX} \leq 0$. How much this curve shifts down depends on the substitutability between gasoline and electric vehicles. The figure shows two possibilities. If they are poor substitutes (as in Case 1), then the curve does not shift much and t^e_{BAU} is less than t^e, as shown on the left part of the figure.[15] Conversely, if they are good substitutes (as in Case 2), then $U_X(G^{ss}_{BAU}, 0)$ shifts down a lot in which case t^e_{BAU} can be later than t^e, as shown on the right. Thus the electric vehicle transition can occur too early under BAU if vehicles are poor substitutes, but can occur too late if they are good substitutes.

Although the terminal mix inefficiency and the adoption timing inefficiency may not be the only sources of inefficiency, combining these two results gives insight into the overall magnitude of deadweight loss under BAU. Corollary 1 shows that the terminal vehicle mix inefficiency is largest at intermediate levels of substitutability, but Proposition 3 shows that the adoption timing inefficiency is largest at high or low levels of substitutability. Thus the two inefficiencies are unlikely to exacerbate one another, which suggests that the overall inefficiency may be limited.

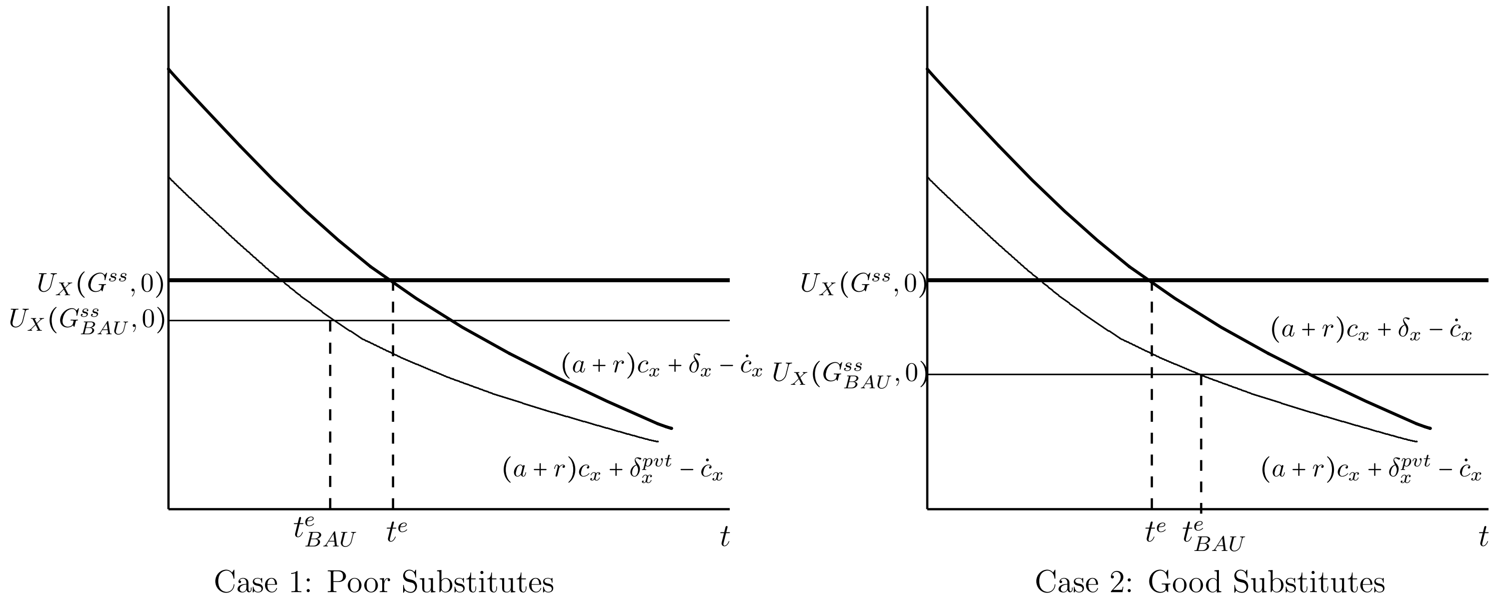
[12] A similar argument holds for the gap solution.

[13] Note that the private and social full marginal costs have the same asymptote since the externality costs go to zero.

[14] More precisely, using (4), the steady state gasoline vehicle stocks are determined by $U_G(G^{ss}, 0) = (a+r)c_g + \delta_g$ and $U_G(G^{ss}_{BAU}, 0) = (a+r)c_g + \delta^{pvt}_g$. Because $U_{GG} < 0$ and $\delta_g > \delta^{pvt}_g$, we have $G^{ss} < G^{ss}_{BAU}$.

[15] For example, if they are additively separable, $U_{GX} = 0$, and the curve does not shift down at all.

Figure 1: First Best vs. Business as Usual t^e



2.4 Extensions

In our main model, the benefit function is constant over time. In practice, electric vehicles may become better substitutes for gasoline vehicles over time as more charging stations are built, charging times decrease, consumers and mechanics become more familiar with the technology, information campaigns promote electric vehicles, and so on. Some of these changes may be reasonably modeled as independent from electric vehicle adoption—e.g., research in battery technologies that reduces charging times—but other changes may be better modeled as jointly determined with electric vehicle adoption—e.g., building charging stations. To model exogenous substitutability, simply note that the benefit function can shift over time and all the preceding analysis holds. To model endogenous substitutability, we introduce complementary infrastructure that affects the benefits of adopting electric vehicles. In particular, let W be the cumulative stock of complementary infrastructure, w be the investment in complementary infrastructure at time t, $\dot{W}=w$ be the state equation, and c_w be the cost per unit of these investments.[16] The benefit function then becomes $U(G,X,W)$

[16]In competitive markets, charging infrastructure investment is subject to the "chicken and egg" problem of two-sided externalities. The planner avoids this externality. See Li et al (2017) for an empirical analysis of the effect of charging stations on the demand for electric cars.

with $U_W > 0$ and $U_{XW} > 0$.[17]

Our main model also assumes that electric vehicle production costs decline exogenously over time. In practice, cost declines may also depend on electric vehicle adoption. Endogenous cost declines can be modeled as learning by doing. More specifically, let the marginal cost of producing electric vehicles decrease in cumulative electric vehicle production, denoted by a new state variable $Z(t)$ with state equation $\dot{Z} = x$.[18] The marginal cost of electric vehicle production then becomes $c_x(Z,t)$.

We extend the theoretical model by analyzing endogenous substitutability and learning by doing in Online Appendix D. Because the results are generally robust to these extensions, our numerical simulations below consider these cases.

3 Model Calibration

To assess the magnitudes of the inefficiencies described above, we present a numerical simulation. This section summarizes the calibration of parameters. Further details are provide in Online Appendix E. To analyze the full roll-out of electric vehicles, the simulation starts in 2005, but the calibration is based on observed values in 2018. Instead of an infinite time horizon, the simulation has a finite terminal time, and, except where noted, the results are not significantly affected by the terminal conditions.[19]

Baseline parameter values are shown in Table 1. The annual externalities and operating costs from gasoline, δ_g, and electric vehicles, δ_x, are calculated from data in Holland et al (2016), Holland et al (2018), and American Automobile Association (2017). The production cost of a gasoline vehicle, c_g, is from the average transaction price for light duty vehicles (Kelly Blue Book 2017). Production costs of an electric vehicle are based primarily on the cost of lithium ion batteries (Kitner et al 2017) and asymptote to the production cost of a gasoline vehicle. The vehicle retirement rate is assumed to be the same for both types of vehicles and is calculated from production and stock information published by the Bureau

[17]In the simulation section, we assume that $U_W(G,0,W) = 0$, i.e., there is no benefit to investments in substitutability when there are no electric vehicles. We make a similar assumption for the exogenous case.

[18]See, for example, Van Benthem et al (2007), Amigues et al (2016), and Bahel and Chakravorty (2016).

[19]We value any terminal vehicle stocks at production costs.

Table 1: Baseline parameter values

Parameter	Value	Description
δ_g	345 + 2726	Annual gasoline vehicle usage cost: externality + operating
δ_x	$605e^{(-0.05t)} + 1535$	Annual electric vehicle usage cost: externality + operating
c_g	35000	Production cost of a gasoline vehicle
c_x	c_g+ $21961e^{-0.06t}$	Production cost of an electric vehicle: 60 kWh battery
G^{2018}	110 million	Stock of gasoline passenger vehicles in 2018
X^{2018}	1 million	Stock of electric passenger vehicles in 2018
ϵ_{Gp_X}	0.01	Cross-price elasticity
a	0.067	Vehicle retirement rate
r	0.05	Interest rate

Notes: All dollar amounts are in 2017 dollars.

of Transportation Statistics (2017) and Bureau of Economic Analysis (2017).

The final calibration step involves specifying a functional form for the benefit function $U(G,X)$. The functional form must satisfy the concavity assumption and ideally be parsimonious with parameters which we can identify from estimates in the literature. In addition, our focus on the introduction of electric vehicles and the end of gasoline vehicle production requires the functional form to admit corner solutions with only gasoline or only electric vehicles.[20] We use the benefit function

$$U(G,X) = A\ln(G + \eta X + \gamma\eta GX), \tag{10}$$

where A, γ, and η are parameters to be calibrated.[21] This function nests both linear and convex indifference curves, and the parameters η and γ can be interpreted as the relative preference for electric vehicles and as the degree of substitutability, respectively. If $\gamma = 0$, vehicles are perfect substitutes and η describes the slope of the linear indifference curves. If $\gamma > 0$ indifference curves are convex, which reflects a social preference for balanced consumption of the two types of vehicles. Further note that either $G = 0$ or $X = 0$ implies unitary demand elasticity. We use observed data on prices and quantities in 2018 and calculate a cross price elasticity using results in Xing et al (2019), $\epsilon_{Gp_X} = 0.01$, to calibrate the benefit

[20]This requirement rules out the simplest versions of widely used functional forms such as Cobb-Douglas or constant elastiticity of substitution (CES).

[21]We also explore sensitivity to the more flexible benefit function $U(G,X) = A(G + \eta X + \gamma\eta GX)^{\beta}$, which has an additional parameter, β, to fit.

function parameters A, η, and γ.

Although we refer to it as baseline, we do not want to place undo emphasis on the estimated cross price elasticity of 0.01. The data in Xing et al (2019) considers the fleet of electric vehicles up to 2014. Although the vehicles in the fleet had a range of characteristics, from relatively expensive vehicles with large batteries and long range (e.g., the Tesla Model S) to relatively inexpensive vehicles with small batteries and short range (e.g., the Nissan Leaf), the vast majority of actual vehicles sold were of the latter type.[22] Our calibration has only a single representative electric vehicle with a fairly large battery and long range. This vehicle is likely to be a better substitute for gasoline vehicles than the baseline cross price elasticity would indicate. Accordingly, in the simulation section we consider a range of possible cross price elasticities, and also allow for exogenously and endogenously increasing substitutability over time.

4 Simulation Results

We use the open-source program BOCOP (2017) to simulate numerical solutions to (1) and (8). BOCOP implements a local optimization method in which the optimal control problem is approximated by a finite dimensional optimization problem using a time discretization.[23] Where possible, results from BOCOP are verified by solving the necessary conditions numerically in Mathematica.

4.1 Time paths of vehicle stocks and production

Figure 2 shows the time paths for production and stocks of electric and gasoline vehicles under two scenarios. In the "First Best" scenario, usage costs include both private costs and external costs. In the "BAU" scenario, usage costs only include the private costs. Panel A of Figure 2 uses the baseline cross-price elasticity of $\epsilon_{Gp_X} = 0.01$. In both scenarios, gasoline vehicle production is not noticeably affected by electric vehicle production. In fact,

[22] Table 1 in Holland et al. (2019) shows the electric vehicle fleet in 2014. Around 70 percent have batteries smaller than 25 kWh.

[23] The optimization problem is solved by IPOPT, (Interior Point OPTimizer) using sparse exact derivatives computed by Automatic Differentiation by OverLoading (ADOL-C).

electric vehicle production only serves to increase the total stock of vehicles. This reflects the poor substitutability of electric vehicles for gasoline vehicles implied by the low cross price elasticity. BAU leads to more gasoline vehicles in the initial steady state because it does not account for the external costs from these vehicles. On the electric side, the terminal stock of electric vehicles is the same in First Best and BAU because external costs from electric vehicles fall to zero over time. Because the vehicles are poor substitutes, electric vehicles are adopted too early under BAU, as described in the discussion of Prop. 3. In particular, the First Best production of electric vehicles begins in 2031, but BAU begins production of electric vehicles in 2028.[24] The deadweight loss of BAU relative to First Best is $18.5 billion.

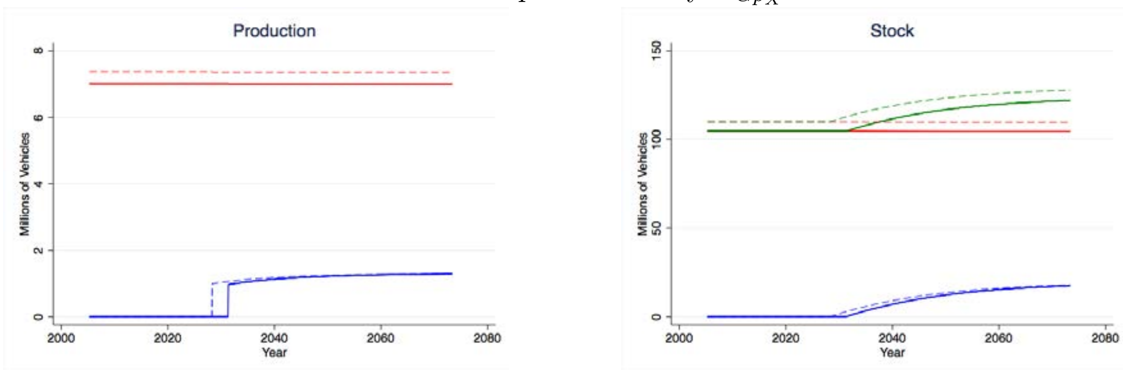
Panel B of Figure 2 shows time paths for $\epsilon_{Gp_X} = 2$. In this case, electric and gasoline vehicles are better substitutes so electric vehicle production results in a substantial reduction in gasoline vehicle production under both First Best and BAU. Electric vehicle adoption causes the stock of gasoline vehicles to decline, but does not decrease the total stock of vehicles. Although production of gasoline vehicles declines, it never ceases so the terminal time has a mix of electric and gasoline vehicle production. Under BAU, the total vehicle terminal stock is approximately equal to First Best, however, BAU has an inefficient vehicle mix: too few electric vehicles and too many gasoline vehicles. BAU electric vehicle production begins too late, which, in conjunction with the fact that BAU electric production begins too early in Panel A, proves Proposition 3. The deadweight lass from BAU in Panel B is $20.9 billion.

In Panel C of Figure 2, substitutability is even better ($\epsilon_{Gp_X} = 5.5$), and electric vehicle production again begins too late under BAU. First Best has a period of simultaneous production of both gasoline and electric vehicles before ceasing production of gasoline vehicles. In contrast, BAU production of gasoline vehicles continues throughout. Because BAU does not cease gasoline vehicle production but First Best does (as described in Corollary 1) the vehicle mix inefficiency is large, and this elasticity results in the largest inefficiency with $29.9 billion in deadweight loss.

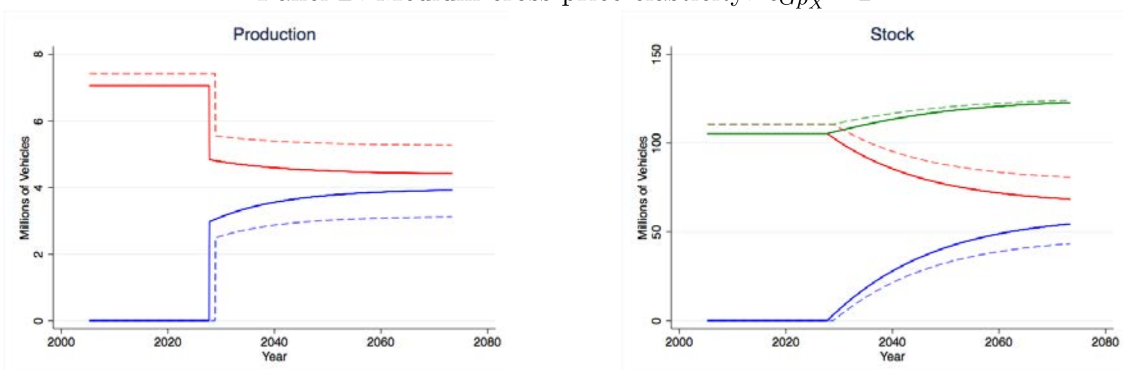
[24]Our calibration is based on the state of the electric vehicle market in 2018. A subsidy scenario, which decreases the production cost of the electric vehicles by $7500 to reflect current subsidies, replicates the market outcome with a stock of 1 million electric vehicles in 2018 but ramps up electric vehicle production more quickly than actually occurred.

Figure 2: First Best and BAU: Vehicle Production and Stocks

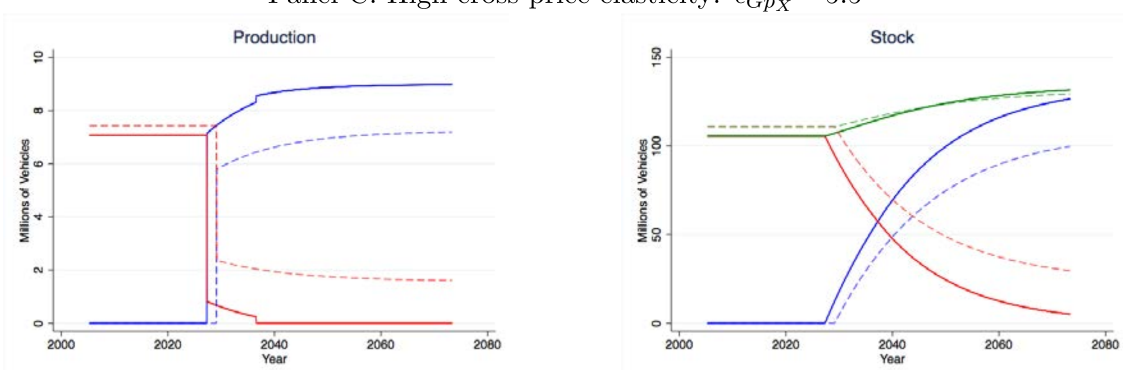
Panel A: Low cross-price elasticity: ϵ_{Gp_X} = 0.01



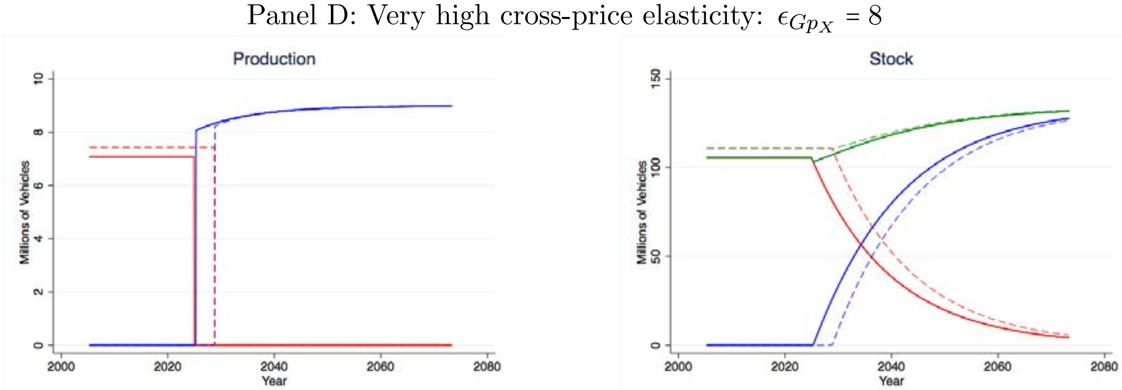
Panel B: Medium cross-price elasticity: ϵ_{Gp_X} = 2



Panel C: High cross-price elasticity: ϵ_{Gp_X} = 5.5



Panel D: Very high cross-price elasticity: ϵ_{Gp_X} = 8



Notes: Red is gasoline vehicles, blue is electric vehicles, and green is total vehicles. Solid is “First Best” and includes all costs. Dashed is “BAU” and ignores all externalities.

Panel D shows the time paths with $\epsilon_{Gp_X} = 8$. This very high degree of substitutability leads to the gap solution in which gasoline vehicle production ceases slightly before electric vehicle production begins. Gasoline vehicle production ceases under both BAU and First Best, but electric vehicle adoption occurs too late under BAU, and the deadweight loss is \$23.7 billion.

Three things are notable about the results in Figure 2. First, consistent with Proposition 3, electric vehicle production can begin too early or too late. Panel A of Figure 2 shows electric vehicle production under BAU can begin too early if electric and gasoline vehicles are poor substitutes. Conversely Panels B-D show production can begin too late if the vehicles are better substitutes. Thus the adoption timing inefficiency is largest when the cross-price elasticity is large or small, and smallest at intermediate elasticities.

Second, the terminal vehicle mix inefficiency is largest if gasoline vehicle production does not cease under BAU but does under First Best. This occurs at intermediate levels of substitutability. At low substitutability, gasoline vehicle production does not cease under either first best or BAU. At high levels of substitutability, gasoline vehicle production ceases under both first best and BAU.

Third, consider our prior observation that the total deadweight loss may be limited because the adoption timing and terminal vehicle mix inefficiencies are largest at different elasticities. Consistent with this observation, the deadweight losses for all elasticities in Figure 2 are relatively small: BAU deadweight loss is less than \$30 billion over 70 years or an annualized value of about \$1.5 billion. For comparison, total annual gasoline vehicle external costs in the initial steady state are approximately \$38 billion per year.[25] Thus deadweight loss is less than 5 percent of initial external costs.

4.2 Second Best Policies

Implementation of the first best through Pigovian taxes that fully reflect externality costs may not be politically feasible. Nonetheless, it may still be possible to improve on BAU outcomes by implementing market-based policies such as a subsidy on the purchase of an electric vehicle or a ban on the production of gasoline vehicles. To analyze such policies,

[25] \$345 per vehicle multiplied by 110 million vehicles.

we modify (8) to account for the policy, and then evaluate this solution using the welfare function (9). To avoid comparing inferior policies, we use a numerical search routine to calculate the policy of each type that minimizes deadweight loss. Unless the policy fully internalizes all external costs, it will not attain the first best solution so we call the optimal policy of each type the *second best policy*.

Table 2 shows the results for three second best policies: an electric vehicle purchase subsidy which is constant over time (EV Subsidy), a policy that bans gasoline vehicle production after a certain time (GV Ban), and a bankable gasoline vehicle production quota (GV Quota) which limits cumulative production of gasoline vehicles. First consider the EV Subsidy. Relative to BAU, the second-best EV Subsidy reduces deadweight loss for all four elasticities. For the low elasticity shown in Panel A, the second-best EV Subsidy is actually a tax of $762 per vehicle. In this case, electric vehicles are adopted too early under BAU and the tax delays their adoption. In the other panels, electric vehicles are adopted too late under BAU and the second-best EV Subsidy is positive and hastens their adoption time. The largest reduction in deadweight loss (about a 40 percent reduction) is in Panel C where the EV Subsidy improves both the adoption timing and the terminal vehicle mix inefficiency.

The second policy (GV Ban) prohibits gasoline vehicle production after a certain time. For low and medium elasticity in Panels A & B, gasoline vehicle production does not cease in the first best, so it is not surprising that the the second best GV Ban never actually bans gasoline production. This policy simply replicates BAU and does not reduce deadweight loss, so it is not shown in Table 2. With the high elasticity in Panel C, gasoline vehicle production does cease in the first best, and the second best GV Ban prohibits gasoline production after 2026. In both panels C and D, the second best GV Ban reduces the adoption timing inefficiency and the terminal vehicle mix inefficiency. Notice that the deadweight loss from the GV Ban is about the same as that from the EV Subsidy.

Unfortunately, the GV Ban leads to perverse incentives. Because vehicles are durable goods, production and purchase decisions are based on future benefits. The anticipation of a future ban causes gasoline vehicle production to optimally spike before the ban occurs.[26]

[26] Production capacity constraints would obviously reduce the ability of producers to spike production of gasoline vehicles. With capacity constraints, producers would produce at capacity until the ban goes in place and would have an incentive to install additional capacity in anticipation of the ban.

Table 2: Second Best Policy Results

Policy	Optimal Parameter	Deadweight Loss ($ billions)	Transition Time t^e (Year)	t^g (Year)	Terminal State G^T	X^T
		Panel A: Low cross-price elasticity: $\epsilon_{Gp_X} = 0.01$				
First Best	n.a.	0	2031.4	n.a.	104.5	17.3
BAU	n.a.	18.5	2028.4	n.a.	109.8	17.7
EV Subsidy	ψ_1= -762	18.0	2030.0	n.a.	109.8	15.7
		Panel B: Medium cross-price elasticity: $\epsilon_{Gp_X} = 2$				
First Best	n.a.	0	2027.8	n.a.	68.7	53.9
BAU	n.a.	20.9	2028.9	n.a.	81.0	42.9
EV Subsidy	ψ_1= 1090	18.3	2026.8	n.a.	76.3	50.2
		Panel C: High cross-price elasticity: $\epsilon_{Gp_X} = 5.5$				
First Best	n.a.	0	2027.3	2036.9	5.6	125.7
BAU	n.a.	28.9	2029.1	n.a.	30.2	99.0
EV Subsidy	ψ_1= 1419	18.6	2026.4	n.a.	13.6	121.2
GV Ban	t^g = 2026.5	18.7	2028.7	2026.5	6.1	125.7
GV Quota	$\mathcal{G}$ = 161	8.7	2026.5	2026.4	5.1	126.8
		Panel D: Very high cross-price elasticity: $\epsilon_{Gp_X} = 8$				
First Best	n.a.	0	2025.3	2025.0	4.5	127.1
BAU	n.a.	23.7	2028.9	2028.8	6.1	125.8
EV Subsidy	ψ_1= 1125	17.0	2025.7	2025.5	4.9	130.3
GV Ban	t^g = 2023.8	18.1	2026.2	2023.8	5.0	127.0
GV Quota	$\mathcal{G}$ = 141	8.3	2024.2	2023.7	4.3	127.9

Notes: For the EV Subsidy policy, the price path formula is $\tilde{c}_x = (c_g - \psi_1) + 21961e^{-0.06t}$, where ψ_1 is selected to minimize deadweight loss. For the GV Ban, t^g indicates the year in which the ban is implemented. For the GV Quota, $\mathcal{G}$ is the cumulative allowed production of gas vehicles. G^T and X^T are the values at the end of the finite horizon.

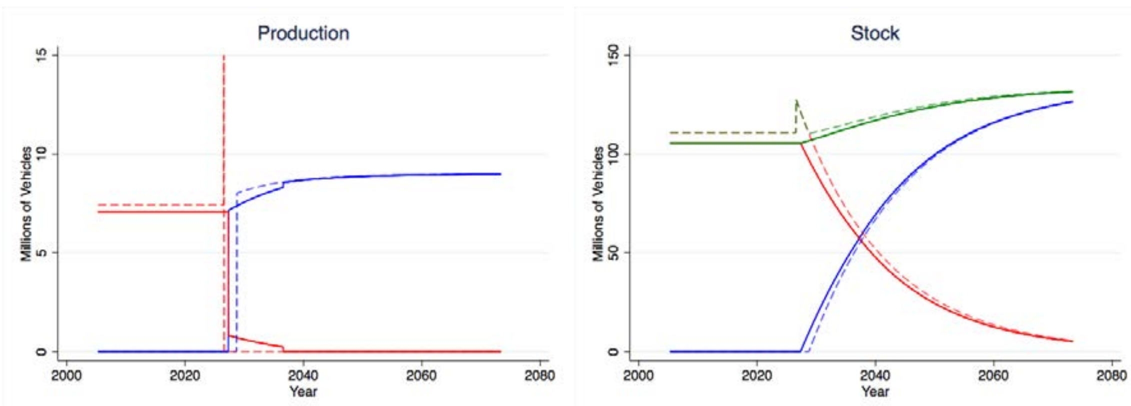
Panel A of Figure 3 shows the production and stocks of vehicles under first best and the second best GV Ban for $\epsilon_{Gp_X} = 5.5$. The second best GV Ban prohibits gasoline production after 2026, and there is a spike in gasoline production and a coincident increase in the gasoline vehicle stock just before that date.

The third policy (GV Quota) is a bankable gasoline vehicle production quota which limits cumulative production of gasoline vehicles. This policy is similar EPA's phase-out of lead in gasoline during the 1980's and could be implemented with an intertemporal cap-and-trade program. The cap-and-trade program would begin with an initial bank of permits and a permit would be retired with the production of each gasoline vehicle. Panel B of Figure 3 shows the production and stocks of vehicles under first best and the second best GV Quota where the optimal size of the initial bank is 161 million for $\epsilon_{Gp_X} = 5.5$. The GV Quota does not lead to a production spike. Table 2 shows that the GV Quota does quite well in comparison to the subsidy or ban polices, as its deadweight loss is less than half that of the other policies. This advantage occurs because the GV Quota introduces a shadow cost on the production of each gasoline vehicle, which leads to a reduction in gasoline vehicle production well before the electric vehicle transition.

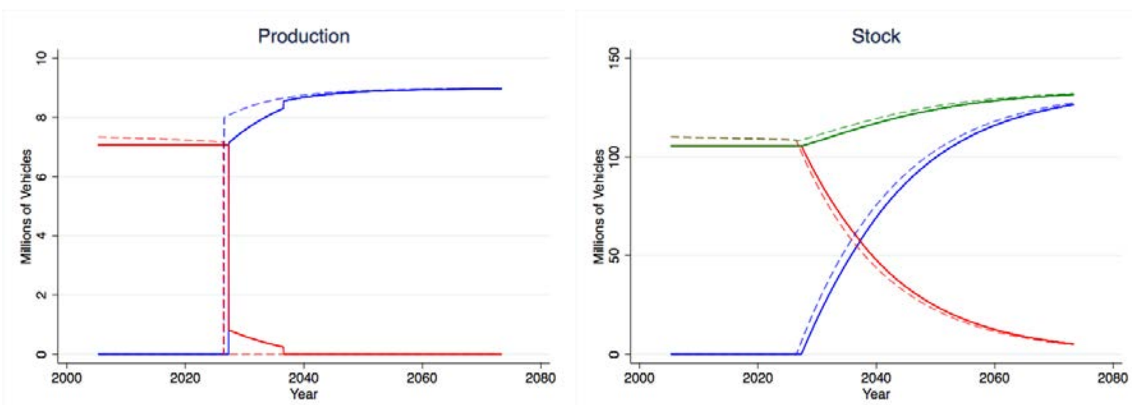
Online Appendix G considers additional second best policies: electric vehicle subsidies that vary over time and a more flexible quota policy. For dynamic subsidies, we allow the subsidies to vary over three parameters: the initial subsidy value, the terminal subsidy value, and the decay rate of the subsidy. We then optimize over the parameters singly or pair-wise to calculate second best dynamic subsidies. The EV Subsidy, which is constant over time, compares well with the dynamic subsidies and attains a deadweight loss within 10% of the best dynamic subsidy we calculate. For a flexible quota policy, we consider a two parameter quota with an initial bank of permits and annual injections of permits. The optimized flexible quota is identical to the GV Quota in the high-substitutability cases in which it is optimal to cease production of gasoline vehicles. In addition, the flexible quota can reduce deadweight loss substantially in the low-substitutability cases in which it is optimal to reduce production of gasoline vehicles but not to cease production completely. With this modification, the gasoline vehicle production quota can be applied to a broader range of situations.

Figure 3: GV Ban and GV Quota: Vehicle Production and Stocks for $\epsilon_{Gp_X} = 5.5$

Panel A: GV Ban



Panel B: GV Quota



Notes: Red is gasoline vehicles, blue is electric vehicles, and green is total vehicles. Solid is "first best" and includes all costs. Dashed is second best "GV Ban" or "GV Quota"

4.3 Extensions

Two of the key drivers of the electric vehicle transition are the substitutability between electric and gasoline vehicles and the fall in production costs of electric vehicles. In this section we modify the preceding model to account for increasing substitutability and learning by doing.

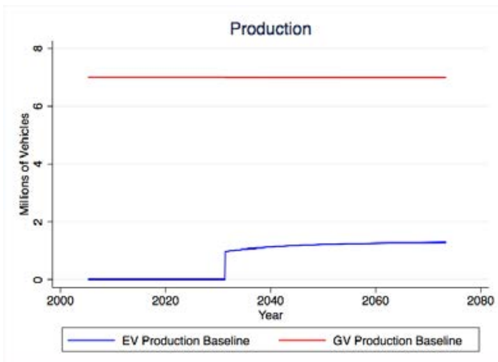
Suppose at first that substitutability is exogenously increasing over time. In our benefit function in (10), substitutability is determined by the parameters η and γ: one-to-one perfect substitutes require that η be one and γ be zero. Thus increasing substitutability arises as η increases toward one and γ decreases toward zero. However if η and γ move independently, benefits for a given fleet of vehicles may be decreasing. To avoid this result, we assume that η and γ move together such that benefits increase over time.[27] In particular, we assume γ decreases exponentially to zero, and η increases toward one such that the marginal rate of substitution is held constant at some level of gasoline and electric vehicles. In essence this holds the indifference curve constant at a point but shifts it inward everywhere toward the line which is tangent at the point (see Online Appendix F for details). With this formulation we have $\gamma = \gamma_o e^{-\phi t}$, where ϕ captures the speed at which substitutability increases and γ_o implies the baseline cross price elasticity of 0.01 at time zero. For $\phi = 0.1$, the cross price elasticity takes 75 years to reach 8. This decreases to 37 years for $\phi = 0.20$.[28] The First Best time paths of production for various values of ϕ are given in Figure 4. For low values of ϕ (Panels A and B) gasoline production is never halted. For intermediate values of ϕ there is a period of simultaneous production of gasoline and electric vehicles. For high values of ϕ, the first best features a gap solution.

Next consider a substitutability that increases endogenously through investment in complementary infrastructure (e.g. charging stations). As above, the parameters η and γ need to be jointly determined so we let γ decay exponentially toward zero as a function of the cumulative stock of investments W. Figure 5 shows the first best vehicle production and stocks for very high cost and low cost charging infrastructure. In Panel A, charging infrastructure has very high costs, no investment occurs, and electric vehicles are simply adopted based on

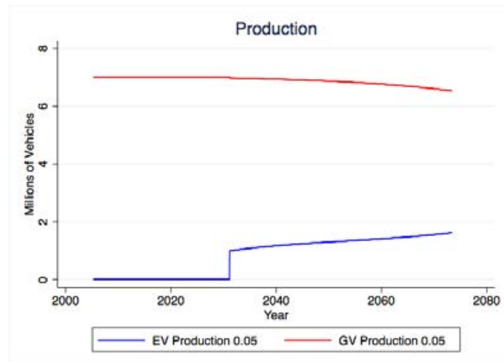
[27]This means that, for a fixed X and G, we have $U_t > 0$.

[28]These calculations assume G stays fixed at 110.

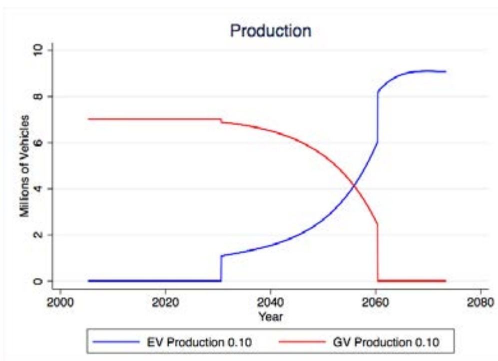
Figure 4: Exogenously increasing substitutability: First Best



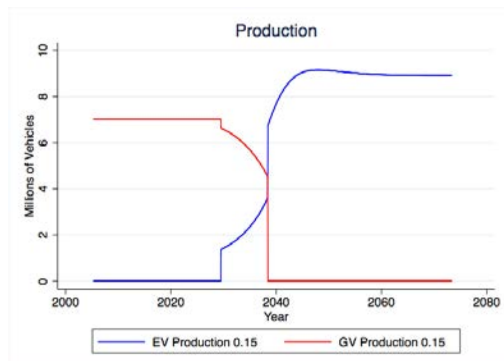
(a) Baseline ($\phi = 0$)



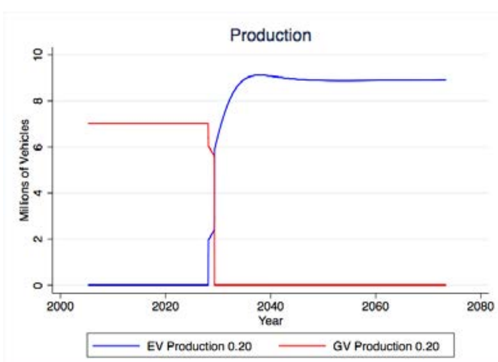
(b) $\phi = 0.05$



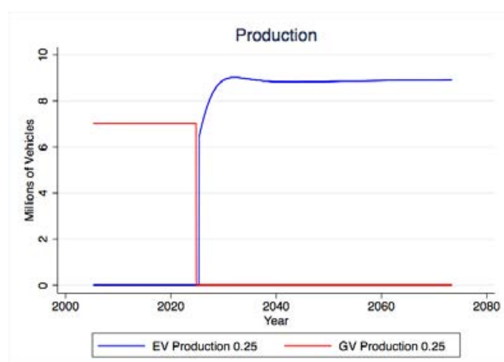
(c) $\phi = .10$



(d) $\phi = 0.15$



(e) $\phi = .20$

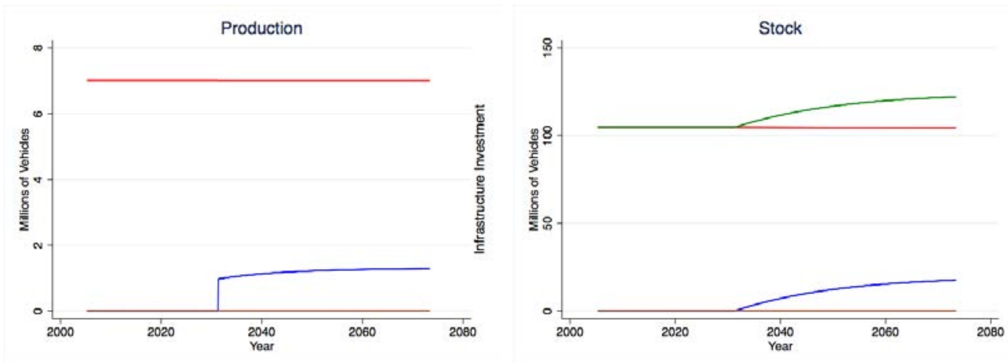


(f) $\phi = 0.25$

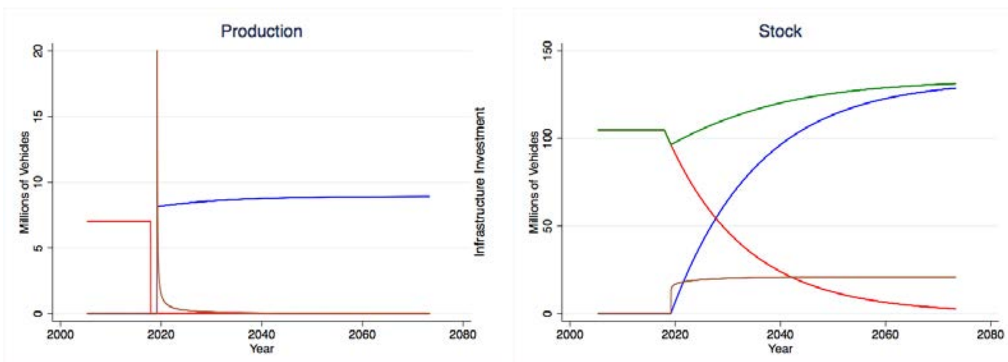
Notes: Red is gasoline vehicles, blue is electric vehicles. Figure shows different values of ϕ where $\gamma = \gamma_0 e^{-\phi t}$, γ_0 corresponds the cross price elasticity of 0.01, and η is adjusted as γ changes in the same manner as in Figure 5.

Figure 5: Endogenously Increasing Substitutability: First Best.

Panel A: Very High Cost Infrastructure



Panel B: Low Cost Infrastructure



Notes: Red is gasoline vehicles, blue is electric vehicles, green is total vehicles, and brown is charging infrastructure.

their low level of substitutability. In Panel B, charging infrastructure is low cost and hence is installed. Investment in charging infrastructure is very rapid and quickly makes electric vehicles nearly perfect substitutes for gasoline vehicles. Indeed, investment is so rapid that the gap solution occurs as evidenced by the decline in total vehicle stock before electric car production begins.

The second best analysis for increasing substitutability is shown in Table 3 for the exogenous substitutability with $\phi = 0.1$ and 0.2 and the endogenous substitutability with low cost charging infrastructure. Comparing Panels A & B shows that with a faster increase in substitutability the optimal subsidy is larger and the transition occurs sooner. With endogenously increasing substitutability (Panel C) the transition occurs even earlier. As under constant substitutability, both the second best EV Subsidy and GV Ban result in small reductions in deadweight loss, but the GV Quota can result in a large reduction in deadweight loss.

Table 3: Increasing Substitutability

Policy	Optimal Parameter	Deadweight Loss ($ billions)	Transition Time t^e (Year)	t^g (Year)	Terminal State G^T	X^T
Panel A: Exogenous $\phi = 0.1$						
First Best	n.a.	0	2030.6	2060.4	36.1	91.1
BAU	n.a.	19.2	2028.3	2064.0	45.9	81.2
EV Subsidy	$\psi_1 = 357$	19.0	2027.6	2063.4	44.3	83.8
GV Ban	$t^g = 2057.6$	18.9	2028.3	2057.6	40.9	86.5
GV Quota	$\mathcal{G} = 341$	13.0	2028.1	2055.3	29.3	98.8
Panel B: Exogenous $\phi = 0.2$						
First Best	n.a.	0	2028.1	2029.3	5.9	125.0
BAU	n.a.	18.0	2027.8	2030.8	6.8	124.6
EV Subsidy	$\psi_1 = 734$	16.9	2026.5	2030.1	6.4	126.9
GV Ban	$t^g = 2028.2$	17.3	2029.3	2028.2	6.5	124.9
GV Quota	$\mathcal{G} = 167$	7.9	2026.6	2027.9	5.5	125.9
Panel C: Endogenous (infrastructure)						
First Best	n.a.	0	2019.2	2017.9	2.8	128.2
BAU	n.a.	18.7	2021.0	2019.9	3.4	128.1
EV Subsidy	$\psi_1 = 766$	15.8	2019.6	2018.5	3.0	130.5
GV Ban	$t^g = 2017.1$	16.2	2019.9	2017.1	3.1	128.4
GV Quota	$\mathcal{G} = 91$	7.8	2018.3	2016.9	2.7	128.8

Notes: For the EV Subsidy policy, the price path formula is $\tilde{c}_x = (c_g - \psi_1) + 21961e^{-0.06t}$, where ψ_1 is selected to minimize deadweight loss. For GV Ban t^g indicates the year in which the ban is implemented. For GV Quota, $\mathcal{G}$ is the cumulative allowed production of gas vehicles. G^T and X^T are the values at the end of the finite horizon.

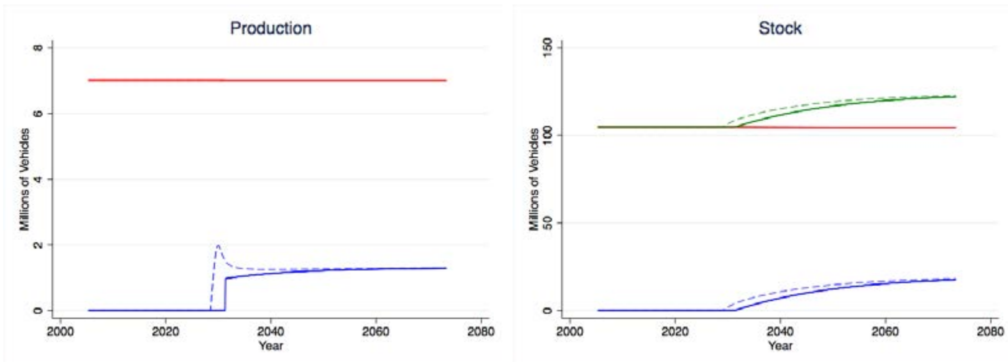
To model falling costs from learning by doing, we estimate the relationship between the cost of lithium ion batteries and cumulative electric vehicle production Z and a time trend (see Online Appendix E.) The resulting cost function for electric vehicles is $c_x(Z,t) = c_g + 21961e^{-0.06t-\theta \ln(Z+1)}$, where $\theta = 0.16$ is an estimated parameter which captures the speed of learning. Comparing this cost function to the baseline cost function shows that they are identical when $Z = 0$ and both asymptote to c_g. Thus the key difference in the cost functions is that learning by doing drives down the electric vehicle production costs faster.

Figure 6 compares the first best with exogenous production cost decreases (our baseline) to the first best with learning by doing for the four cross-price elasticities. Comparing the first best outcomes implicitly assumes that any learning spillovers are internalized and do not result in additional inefficiencies. In all four panels, learning by doing results in an earlier transition to electric vehicles by as much as ten years. In all four panels, production of electric vehicles spikes when production starts. This increase in production causes costs to fall quickly down the learning curve. The optimal spike trades off inefficiently large production with the ability to produce electric vehicles sooner at lower costs and thus depends on the discount rate as well as the substitutability of electric vehicles for gasoline vehicles. In panel A, the spike in electric car production does not perceptibly affect gasoline vehicle production. In panel B, the spike causes gasoline vehicle production to plummet but then recover to its steady state level. In Panels C & D, the spike causes gasoline production to cease and the high levels of substitutability leads to the gap solution. The second best analysis for $\epsilon_{Gp_X} = 5.5$, shown in Table 4, largely conforms to the previous findings: i.e., shows modest reductions in deadweight loss from the EV Subsidy or GV Ban and a substantial reduction in deadweight loss from the GV Quota.

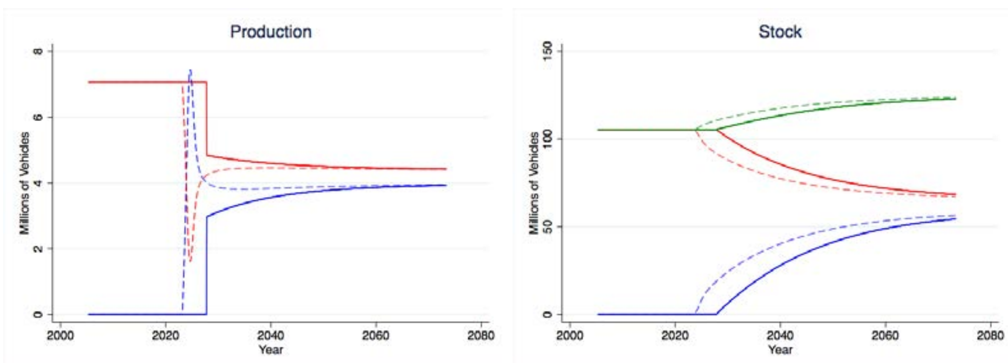
Explicitly modeling investments in complementary infrastructure and learning by doing in electric vehicle production costs shows two things. First, the simple models described here lead to spikes in investment and production. Because these spikes are likely unrealistic, carefully modeling capacity constraints may be an important step in future analyses. Second, the fundamental nature of the time paths is not dramatically altered by investment in infrastructure or learning by doing. The extended models probably suggest greater substitutability and faster declines in costs than we assume in our baseline calculations.

Figure 6: Learning by Doing: Vehicle Production and Stocks for various cross-price elasticities, ϵ_{Gp_X} and $\theta = 0.16$

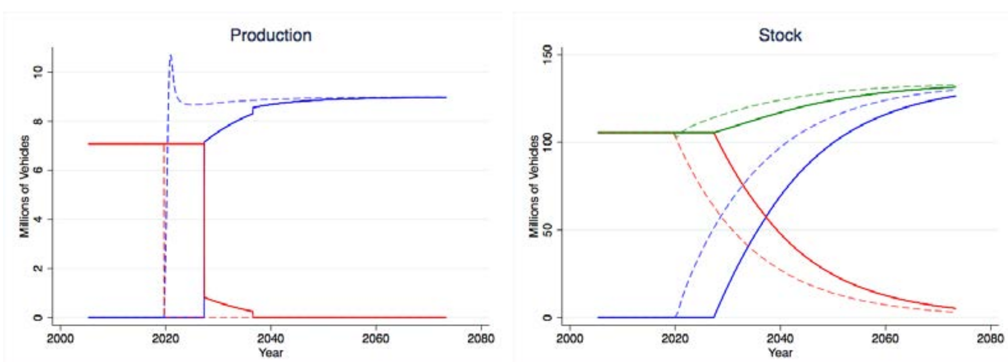
Panel A: $\epsilon_{Gp_X} = 0.01$



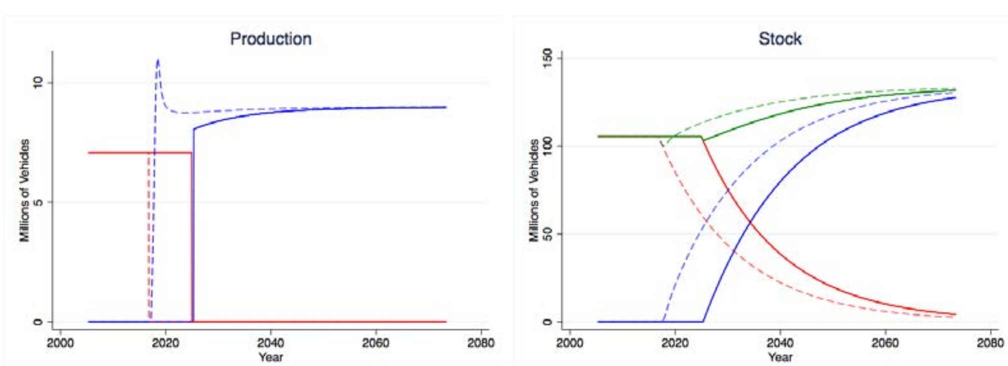
Panel B: $\epsilon_{Gp_X} = 2$



Panel C: $\epsilon_{Gp_X} = 5.5$



Panel D: $\epsilon_{Gp_X} = 8$



Notes: Red is gasoline vehicles, blue is electric vehicles, and green is total vehicles. Solid lines are first best with our baseline exogenous price decreases, and dashed lines are first best with learning by doing.

Table 4: Learning by Doing $\epsilon_{Gp_X} = 5.5$

Policy	Optimal Parameter	Deadweight Loss ($ billions)	Transition Time t^e (Year)	t^g (Year)	Terminal State G^T	X^T
		Panel A: $\theta = 0$				
First Best	n.a.	0	2027.3	2036.9	5.6	125.7
BAU	n.a.	28.9	2029.1	n.a.	30.2	99.0
EV Subsidy	$\psi_1 = 1419$	18.6	2026.4	n.a.	13.6	121.2
GV Ban	$t^g = 2026.5$	18.7	2028.7	2026.5	6.1	125.7
GV Quota	$\mathcal{G} = 161$	8.7	2026.5	2026.4	5.1	126.8
		Panel B: $\theta = 0.16$				
First Best	n.a.	0	2019.3	2019.8	3.2	129.5
BAU	n.a.	28.8	2021.0	n.a.	25.9	104.6
EV Subsidy	$\psi_1 = 931$	20.0	2018.5	n.a.	15.0	119.3
GV Ban	$t^g = 2018.6$	16.7	2019.9	2018.6	3.5	129.7
GV Quota	$\mathcal{G} = 105$	9.3	2018.3	2018.7	3.1	130.1

Notes: For the EV Subsidy policy, the price path formula is $\tilde{c}_x = (c_g - \psi_1) + 21961e^{-0.06t}$, where ψ_1 is selected to minimize deadweight loss. For GV Ban, t^g indicates the year in which the ban is implemented. For GV Quota, $\mathcal{G}$ is the cumulative allowed production of gas vehicles. G^T and X^T are the values at the end of the finite horizon.

4.4 Sensitivity

We analyze the sensitivity of the simulation results in three ways. First consider the effect of several parameters on the terminal steady state production of gasoline vehicles. The social cost of carbon (SCC) is used to determine the external costs associated with global pollution from electric and gasoline vehicles and is an important element in policy discussions of electric vehicles. The value of a statistical life (VSL) is used to determine the external costs of local pollution. The final parameter is the limiting value of the production costs for electric vehicles ($\hat{c}_x$). Table 5 shows, for various elasticities, the smallest non-negative values for these parameters such that gasoline vehicle production ceases in the first best terminal steady state. For low cross-price elasticities, either a high VSL or a high SCC would be required in order for gasoline vehicle production to cease. For example, if $\epsilon_{Gp_X} = 2$, the VSL would have to exceed $171 million or the SCC would need to exceed $516 per ton in order for gasoline vehicle production to cease. These levels greatly exceed most credible estimates. At higher cross price elasticities (above six), gasoline vehicle production would cease even with VSL of zero or a SCC of zero because the gasoline and electric vehicles are such good

Table 5: Parameters That Lead to $g^{\infty} = 0$ in Terminal Steady State

Cross-price Elasticity	Value needed for $G^{\infty} = 0$ in Terminal Steady State		
ϵ_{Gp_X}	VSL	SCC	$\hat{c}_x/c_g$
0.01	88647	257413	0.00
1	476	1401	0.05
2	171	516	0.59
3	81	253	0.80
4	37	127	0.92
5	12	54	0.99
5.16	9	45	1
5.5	3	28	1.02
6	0*	0*	1.04
7	0*	0*	1.08
8	0*	0*	1.10

Notes: Baseline values are $9 million for VSL, $45 for SCC, and 1 for $\hat{c}_x/c_g$. Cross-price elasticities in bold correspond to cases in Figure 2. "0*" indicates that the smallest non-negative value is zero.

substitutes. The final column shows the smallest production cost ratio, $\hat{c}_x/c_g$, which would lead to gasoline vehicle production ceasing. If the vehicles are poor substitutes ($\epsilon_{Gp_X} = 0.01$) electric vehicles would need to be essentially free in order for gasoline vehicle production to cease. At higher cross-price elasticities, gasoline vehicle production ceases even if electric vehicles are more expensive than gasoline vehicles.

Next, Table 6 shows how the second best results change when we vary the SCC. For $\epsilon_{Gp_X} = 5.5$, Panels A and B analyze modest decrements and increments to our baseline SCC while Panel C shows a very high SCC of $200. For all SCC's, the second best EV Subsidy and GV Ban can reduce deadweight loss, but the GV Quota reduces deadweight loss most. Electric vehicle adoption is insensitive to the modest changes in SCC, but t^g is sensitive with even modest changes in the SCC leading to decades difference in the time when gasoline vehicle production ceases. For the very high SCC, which is on the upper end of the range of some current estimates, the First Best electric vehicle transition occurs quite early (2021) and the BAU deadweight loss is substantial. Correspondingly, there are substantial gains from any of the second best policies.

Finally, we study what happens if policy is set based on an incorrect estimate of the cross-

Table 6: Sensitivity Analysis for the SCC ($\epsilon_{Gp_X} = 5.5$)

Policy	Optimal Parameter	Deadweight Loss ($ billions)	Transition Time t^e (Year)	t^g (Year)	Terminal State G^T	X^T
Panel A: Low SCC=$34.20						
First Best	n.a.	0	2027.7	2055.2	6.6	124.6
BAU	n.a.	20.4	2029.1	n.a.	30.2	99.0
EV Subsidy	$\psi_1 = 1180$	13.4	2026.8	n.a.	16.4	117.4
GV Ban	$t^g = 2027.4$	13.7	2029.3	2027.4	6.4	125.3
GV Quota	$\mathcal{G} = 168$	6.6	2027.3	2027.3	5.5	126.4
Panel B: High SCC=$56.27						
First Best	n.a.	0	2027.0	2029.3	5.2	126.1
BAU	n.a.	38.6	2029.1	n.a.	30.2	99.0
EV Subsidy	$\psi_1 = 1649$	24.4	2025.9	n.a.	10.9	124.9
GV Ban	$t^g = 2025.8$	24.5	2028.2	2025.8	5.8	125.9
GV Quota	$\mathcal{G} = 154$	11.1	2025.7	2025.6	4.8	127.1
Panel C: Very High SCC=$200						
First Best	n.a.	0	2021.6	2021.1	3.2	127.7
BAU	n.a.	256.2	2029.1	n.a.	30.2	99.0
EV Subsidy	$\psi_1 = 3445$	155.1	2022.7	2022.6	4.1	138.0
GV Ban	$t^g = 2018.3$	157.8	2022.9	2018.3	3.9	128.2
GV Quota	$\mathcal{G} = 91$	59.2	2018.7	2017.8	2.8	129.4

Notes: For the EV Subsidy policy, the price path formula is $\tilde{c}_x = (c_g - \psi_1) + 21961e^{-0.06t}$, where ψ_1 is selected to minimize deadweight loss. For GV Ban, t^g indicates the year in which the ban is implemented. For GV Quota, $\mathcal{G}$ is the cumulative allowed production of gas vehicles. G^T and X^T are the values at the end of the finite horizon. Baseline SCC ($45.23), High SCC ($56.27) , and Low SCC ($34.20) correspond to the values used in Holland et al 2018. Very High SCC ($200) is similar to the value used in Moore and Diaz (2015).

Table 7: Deadweight Loss ($ Billions): Incorrect Cross-Price Elasticity

Assumed Elasticity	Policy Parameter	Actual Elasticity 0.01	2	5.5	8
	Panel A: EV Subsidy				
ϵ_{Gp_X} = 0.01	$-762	18.0	25.3	41.3	36.9
ϵ_{Gp_X} = 2	$1090	21.1	18.3	19.2	17.0
ϵ_{Gp_X} = 5.5	$1419	22.4	18.6	18.6	17.4
ϵ_{Gp_X} = 8	$1125	21.2	18.3	19.1	17.0
	Panel B: GV Ban				
ϵ_{Gp_X} = 0.01 or 2	n.a.	18.5	20.9	28.9	23.7
ϵ_{Gp_X} = 5.5	year 2026.5	1242.3	143.9	18.7	20.5
ϵ_{Gp_X} = 8	year 2023.8	1538.0	180.2	21.5	18.1
	Panel C: GV Quota				
ϵ_{Gp_X} = 0.01 or 2	n.a.	18.5	20.9	28.9	23.7
ϵ_{Gp_X} = 5.5	161 million	1385.7	147.7	8.7	12.2
ϵ_{Gp_X} = 8	141 million	1730.5	195.6	12.6	8.3

price elasticity. Panel A of Table 7 shows the deadweight loss that results from different EV Subsidies. Suppose, for example, that a policy maker estimates that the cross-price elasticity is 0.01. The appropriate policy is a negative subsidy (a tax) of $-762, which results in an $18.0 billion deadweight loss if the cross-price elasticity is indeed 0.01. However, this same subsidy would result in a higher deadweight loss of $41.3 billion if the actual cross-price elasticity is 5.5. This is more than double what the deadweight loss would have been if the cross-price elasticity had been correctly estimated (as shown in the main diagonal of Panel A). In contrast, Panels B and C show that setting a gasoline production ban or quota based on the incorrect cross-price elasticity can increase the deadweight loss by a factor of almost 100. Although we do not analyze uncertainty explicitly, this suggests that a price mechanism (such as a subsidy) may dominate a quantity mechanism (such as a quota) in a model in which there is uncertainty about the cross-price elasticity. Note that the deadweight losses in the first row of Panel B and C are simply the deadweight losses from BAU. Comparing these deadweight losses with the other deadweight losses in the table shows that basing policy on the incorrect cross-price elasticity can be much worse than no policy at all.

5 Conclusion

This paper studies the transition from gasoline vehicles to electric vehicles using a theoretical model and numerical simulations calibrated to the U.S. market. The theoretical model shows that BAU electric vehicle production can begin too early or too late depending on the substitutability of gasoline vehicles for electric vehicles. Similarly, the vehicle mix inefficiency depends on substitutability but is highest at intermediate levels of substitutability in which the first best transitions to electric vehicles but BAU does not. These offsetting effects suggest that inefficiencies may not be large. Our numerical simulations assess the magnitudes of these inefficiencies for the U.S. and find modest levels of deadweight loss: less than five percent of total external costs.

Electric vehicle purchase subsidies have been implemented by a number of countries. For example, the U.S. provides a tax credit of $7500 for the purchase of an electric vehicle and several states offer additional subsides. Our numerical simulations show that such subsidies can reduce deadweight loss. We calculate optimal purchase subsidies ranging from $-750 (for very low substitutability) to $3500 (for a very high SCC), which are well below the current U.S. subsidy. These purchase subsidies are constant over time, but more complicated dynamic subsidies do not yield much additional efficiency improvement. The constant subsidies also seem to be robust in the sense that the additional deadweight loss from using an incorrect cross-price elasticity is not extreme.

Gasoline production bans are increasingly attracting the attention of policy makers. We find that a gasoline vehicle production ban can reduce deadweight loss if the cross-price elasticity is such that ceasing gasoline vehicle production would be first best. If gasoline vehicle production does not cease in first best, the ban cannot reduce deadweight loss, so the second-best ban simply replicates BAU. The ban can also result in very large deadweight loss if it is based on an incorrect cross-price elasticity. In particular, deadweight loss can be almost two orders of magnitude larger than BAU if the regulator bans gasoline vehicle production when the true cross-price elasticity is small. Even in the case in which the production ban is effective, it leads to an inefficient spike in production in anticipation of the ban due to the durability of the vehicles. For those countries that have proposed a production ban, the dates of implementation range from 2025 to 2040. Our results for the U.S. suggest that the optimal time to ban production of gasoline vehicles varies quite a bit depending on the model assumptions and parameters. On the one hand, if substitutability is very low, then it is never optimal to ban gasoline vehicles. On the other hand, with a very high SCC or endogenous rollout of inexpensive charging infrastructure, then our simulations optimally implement the ban around 2018.

A bankable gasoline vehicle production quota has not yet been part of the policy discussion surrounding electric vehicles, but our results point out several advantages of the policy. A bankable quota results in the smallest deadweight loss of all policies considered by a substantial margin. In contrast to a production ban, a bankable quota does not lead to an inefficient spike in production because it introduces a shadow value on every vehicle produced. It can also be modified to be effective even in the case in which gasoline vehicle production does not cease in the first best. Like the production ban, however, it is sensitive to incorrect estimates of the cross-price elasticity. Finally, a bankable quota is similar to policies that were effective to phase out leaded gasoline, which may help boost its acceptability to policy makers.

Appendix

Proofs of Propositions 1-2

Before proving Propositions 1-2, we state and prove the following lemma, which allows us to solve for the adjoint variable for gasoline vehicles from the adjoint equation:

Lemma 1. *The adjoint equation (3) for gasoline vehicles is solved by the function*

$$\alpha(t) = e^{(a+r)t}\left[\int_{t_0}^{t} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau)]d\tau + K\right] \tag{11}$$

for an arbitrary constant K.
With an initial condition $\alpha(t_0) = \alpha_0$, the adjoint equation is solved by

$$\alpha(t) = e^{(a+r)t}\left[\int_{t_0}^{t} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau))]d\tau + \alpha_0 e^{-(a+r)t_0}\right] \tag{12}$$

With a terminal condition, the adjoint equation is solved by

$$\alpha(t) = e^{(a+r)t}\int_{t}^{\infty} e^{-(a+r)\tau}[U_G(G(\tau), X(\tau)) - \delta_g]d\tau. \tag{13}$$

Proof. The adjoint equation (3) is a first-order differential equation of the form

$$\dot{\alpha} - (a + r)\alpha = f(t)$$

where $f(t) = \delta_g - U_G(G(t), X(t))$ is a function of t. Using the integrating factor method, the solution to a differential equation of this form, which can be easily verified, is

$$\alpha(t) = e^{(a+r)t}\left[\int_{t_0}^{t} e^{-(a+r)\tau} f(\tau)d\tau + K\right],$$

where K is an arbitrary constant.

If $\alpha(t_0) = \alpha_0$, we set (11) equal to α_0 and solve for K, which implies $K = \alpha_0 e^{-(a+r)t_0}$. Substitution in (11) yields (12).

For the terminal condition, instead of using the transversality conditions, we use the terminal condition $\alpha(T) = \alpha_1$ with an arbitrary end period T. We then determine the constant K_T with this arbitrary end period and take the limit of K_T as $T \to \infty$. With this terminal condition in (11) we can solve for K_T which yields

$$K_T = \alpha_1 e^{-(a+r)T} - \int_{t_0}^{T} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau))]d\tau.$$

We then take the limit as $T \to \infty$ of K_T and substitute the result into (11) which gives

$$\alpha(t) = e^{(a+r)t}\left[\int_{t_0}^{t} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau)]d\tau - \int_{t_0}^{\infty} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau))]d\tau\right]$$

which simplifies to (13). Note that the solution does not depend on the arbitrary constant α_1. □

Proof of Proposition 1

Proof. To prove the condition for banning gasoline vehicles, we show that the solution satisfies the first order conditions with $g^\infty = 0$. As $t \to \infty$, if $g = 0$ and x is such that (5) is satisfied, then for some T_1 we have $(G, X) \approx (0, X^*)$. We can then evaluate the adjoint variable using (13) from Lemma 1, which shows that for $t > T_1$

$$\alpha(t) = e^{(a+r)t}\int_{t}^{\infty} e^{-(a+r)\tau}[U_G(G(\tau), X(\tau)) - \delta_g]d\tau.$$

$$\approx e^{(a+r)t}\int_{t}^{\infty} e^{-(a+r)\tau}[U_G(0, X^*) - \delta_g]d\tau.$$

$$= (U_G(0, X^*) - \delta_g)/(a + r) < c_g$$

which implies $\alpha(t) < c_g$. Together with $g = 0$, the remaining first order condition is satisfied.

A proof by contradiction demonstrates the condition under which gasoline vehicles are not banned. Suppose $g^\infty = 0$ which implies that $g(t) = 0$ for all $t > T_1$ for some T_1. But this implies that for some T_2 with $t > T_2 > T_1$ we have $G \approx 0$ and $X \approx X^*$ so that $U_G(G, X) \approx U_G(0, X^*)$

where ≈ means "arbitrarily close to", i.e., within an ϵ-ball. Again using (13) from Lemma 1, we have that for $t > T_2$

$$\alpha(t) = e^{(a+r)t} \int_t^\infty e^{-(a+r)\tau}[U_G(G(\tau), X(\tau)) - \delta_g]d\tau.$$

$$\approx e^{(a+r)t} \int_t^\infty e^{-(a+r)\tau}[U_G(0, X^*) - \delta_g]d\tau.$$

$$= (U_G(0, X^*) - \delta_g)/(a + r) > c_g$$

which contradicts the first order condition $\alpha \leq c_g$.

□

Proof of Proposition 2

Proof. To characterize t^e note that $x > 0$ over the interval $[t^e, t^g]$ so (5) must hold including at t^e. But at t^e we have $G(t^e) = G^{ss}$ and $X(t^e) = 0$. Substituting these into (5) $U_X(G^{ss}, 0) = (a + r)c_x(t^e) + \delta_x(t^e) + \dot{c}_x(t^e)$ which can then be solved for t^e and characterizes t^e. (6) follows directly.

To characterize t^g, we focus on the adjoint variable α. During $[t^e, t^g]$, we have g interior, so that $\alpha = c_g$. After t_g, α evolves according to the adjoint equation (3) which we can solve using (13) from Lemma 1 to have

$$\alpha(t) = e^{(a+r)t} \int_t^\infty e^{-(a+r)\tau}[U_G(G^{sim}(\tau), X^{sim}(\tau)) - \delta_g]d\tau. \tag{14}$$

But at t^g, we have $\alpha(t^g) = c_g$ which implies the result

$$c_g = \int_{t^g}^\infty e^{-(a+r)(\tau - t^g)}(U_G(G^{sim}(\tau), X^{sim}(\tau)) - \delta_g)d\tau.$$

□

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