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LEVERAGE DYNAMICS AND FINANCIAL FLEXIBILITY

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ABSTRACT

We develop a q theory of investment with endogenous leverage, payout, hedging, and risk-taking dynamics. The key frictions are costly equity issuance and incomplete markets. We show that the marginal source of external financing on an on-going basis is debt. The firm lowers its debt when making a profit, increases its debt in response to losses and induced higher interest payments, and even taps external equity markets at a cost before exhausting its endogenous debt capacity. The firm seeks to preserve its financial flexibility by prudently managing its leverage and investment. Paradoxically, it is the high cost of equity issuance that causes the firm to keep leverage low, in contrast to the predictions of static Modigliani-Miller tradeoff and Myers-Majluf pecking-order theories. Our model generates leverage and investment dynamics that are consistent with the empirical evidence.

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1 Introduction

In his AFA presidential address DeMarzo (2019) states that "Capital structure is not static, but rather evolves over time as an aggregation of sequential decisions." New decisions are called for in response to realized profits and losses, or changes in the firm's investment opportunity set. We take such a dynamic perspective and propose a tractable model that accounts for CFOs' stated top four concerns, as reported in Graham and Harvey (2001) and Graham (2022): 1.) maintain financial flexibility, 2.) target credit ratings, 3.) preserve internal funds, and 4.) mind earnings volatility.

We show how these considerations come into play in a dynamic model with classic tradeoff considerations, when a firm faces external equity financing costs. In our model (as in standard tradeoff models) debt has a funding advantage over equity but may cause financial distress.[1] The firm seeks to avoid equity issuance costs in the future. It does so by conserving its debt capacity and retaining its earnings in the present, so that it can keep relying on cheaper debt financing in the future. Although debt is costless to issue, more debt today reduces the firm's ability to use debt in future, especially in bad times. This *opportunity cost of debt* plays a crucial role in our analysis.

To bring out the underlying economic mechanism, we develop our model in three steps: First, we consider a dynamic tradeoff model with costless equity financing; Second, we incorporate equity issuance costs; and, Third we integrate investment into the model to describe the joint investment and leverage dynamics predictions.

The model with costless-equity-issuance differs from the standard dynamic tradeoff model of Leland (1994) in two key aspects. First, debt is short term and can be continuously adjusted without cost, as in classic portfolio choice and asset pricing models (e.g., Merton 1971, and Black and Scholes 1973). Second, the firm's earnings before interest and taxes (EBIT) in our model are subject to *discontinuous* jumps.[2] Downward jump shocks are crucial to generate default and positive credit spreads in equilibrium even with short-term debt.[3] Jumps generate negatively skewed and fat-tailed EBIT growth as observed in the data. The firm's debt-to-earnings ratio x is the natural

[1] We follow DeMarzo (2019) to allow for two sources of debt funding advantages: cheaper cost of capital (when shareholders are more impatient than creditors) and the standard tax benefit of debt. As in the capital structure tradeoff theory literature following Modigliani and Miller (1961) and Leland (1994), we assume that shareholders are protected by limited liability and can declare default at any time. The absolute priority rule (APR) applies when the firm defaults. Creditors liquidate the firm and collect the liquidation recovery value, which is assumed to be a fraction of unlevered firm value as in Leland (1994). Creditors ex ante price the firm's default risk.

[2] Malenko and Tsoy (2021) use the same EBIT process to study optimal time-consistent debt policies.

[3] In Leland type of models, default is due to the assumption that the firm cannot adjust its long-term debt. There is no default in diffusion models where debt is instantaneously rolled over (Black and Scholes 1973). We assume an EBIT process that nests the standard geometric Brownian motion (GBM)

state variable in our model, consistent with Graham (2022), who reports that corporate capital structure appears to "focus on Debt/EBITDA or credit ratings rather the classic debt/value or debt/assets".[4]

We show that when equity issuance is costless the firm keeps its leverage at a constant target level at all time, until it chooses to default in response to a large negative earnings shock. Leverage is held at the target by continuously issuing equity when making losses and paying out dividends when making profits. However, such continuous and active equity adjustments are empirically counter-factual. Indeed, it is a well-established finding that firms rarely issue equity, and when they do, they issue lumpy amounts.[5]

We introduce costly equity issuance into the preceding model in a second step. We show that this enhanced model generates empirically plausible joint dynamics of leverage, retained earnings, equity issuance, and payout. Anticipating that its future financing costs rise with leverage (above target), the firm optimally retains its earnings and conserves its debt capacity. The value of financial flexibility in this enhanced model is high given that earnings shocks can be large and permanent.

Figure 1 illustrates how the firm's policies and value function vary with leverage. At any given time, the firm finds itself in one of four mutually exclusive regions, depending on its history of past earnings shocks. These regions are separated by three endogenous thresholds. From the left to the right of Figure 1 we have: 1.) A *payout region*, where the firm pays a lumpy dividend and reaches its *target leverage*; 2.) A *target zone*, where the firm passively responds to earnings shocks, letting its debt balance move up or down in response to negative or positive earnings shocks like a credit-card revolver; 3.) An *equity-issuance region*, where the firm recapitalizes by issuing equity to proactively delever its balance sheet and to bring leverage to the *recapitalization target level*; and 4.) A *default region*, where it is optimal for shareholders to default. Thus, market leverage dynamics ($\Delta\, ML$) are given by:

$$\Delta\, ML = \textbf{equity payout} - \textbf{retained earnings} - \textbf{equity issuance} + (\mathbf{100\%} - ML)\,.$$

This equation states that equity payout increases ML, retained earnings and equity issuance reduce ML, and default means that ML jumps to 100% as equity is protected by limited liability. Each of the four terms on the right side of this equation indicates the key margin the firm uses to manage its leverage and corresponds to the color-matched region in Figure 1.

process in the contingent-claim literature (e.g., Fischer, Heinkel, and Zechner, 1989; Leland, 1994; Goldstein, Ju, and Leland, 2001; and Strebulaev, 2007) as a special case.

[4] We can equivalently convert the debt-earnings ratio into market leverage (ML), measured by the debt-to-firm value ratio.

[5] See Donaldson (1961) and Shyam-Sunder and Myers (1999) among others.

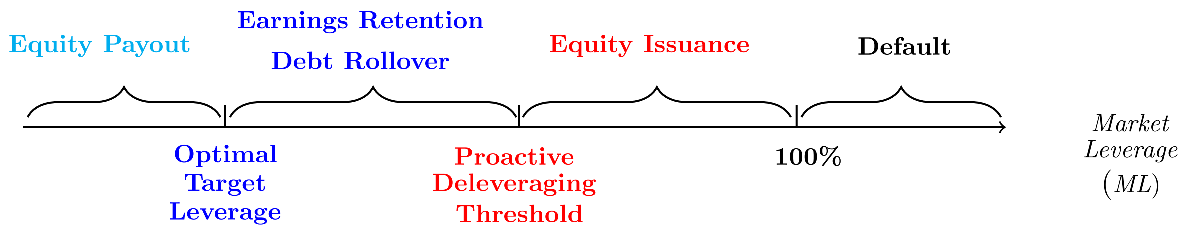


Figure 1: FOUR-REGION SOLUTION FOR MARKET LEVERAGE (ML). In the **equity payout** region, the firm makes a lumpy payout to reach target leverage. In the **earnings retention/debt rollover** region, the firm retains earnings and rolls over its debt as a creditor-card revolver. In the **equity issuance** region, the firm proactively deleverages. In the **default** region, the firm's ML would have exceeded one without default. The three thresholds, **target leverage**, **proactive deleveraging threshold**, and the **100%** ML threshold, are endogenously determined.

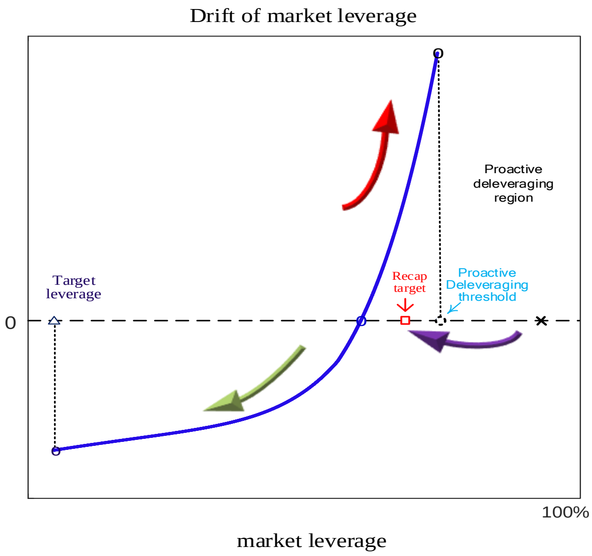
These four regions illustrate the preferred order in which the firm finances itself: First, retained earnings, second, debt, and third, equity. This prediction has a pecking-order flavor (Myers, 1984), but with one crucial difference: The firm issues costly equity before exhausting its financing capacity as a preemptive move to reduce the risk of future default.[6]

Figure 2 depicts the highly non-linear dynamics of leverage when equity issuance is costly. It illustrates how the drift of leverage varies with the level of leverage. When market leverage ML is to the left of the black dot on the horizonal axis, leverage tends to drift down towards target leverage, as shown by the green arrow. When ML is to the right of the black dot, leverage is expected to drift upward, as shown by the red arrow. In that region the firm is likely to be stuck in a *debt death spiral*, as debt begets debt. When ML is to the right of the vertical dashed black line (e.g., due to a downward earnings jump), the firm proactively recapitalizes its balance sheet by issuing equity to bring its leverage down to the *recapitalization target* (the red open square), as shown by the purple arrow. This process continues until the firm is hit by a negative earnings shock that is so large that it causes a default.

A striking and paradoxical observation emerging from our analysis is that *target leverage* is lower the higher are equity issuance costs. Far from encouraging firms to take on cheaper debt, higher equity issuance costs cause the firm to be more prudent with its debt policy. Higher equity issuance costs also lower the firm's debt capacity, which in turn reduces its financial slack. To reduce the risk of getting into a debt death spiral, the firm chooses a more prudent debt policy to preserve its financial flexibility. This is why equity issuance costs are key to explaining the relatively low observed corporate

[6] This prediction is consistent with the findings of Fama and French (2005), DeAngelo, DeAngelo, and Stulz (2010), and DeAngelo, Gonçalves, and Stulz (2018): Firms tap equity markets before exhausting their financing capacity.

Figure 2: Leverage dynamics.



leverage levels. Another key result is that target leverage coincides with the firm's payout boundary. Indeed, the optimality condition for target leverage is the same as the one for the optimal payout decision.

The firm's leverage rarely stays at the *target leverage*. There is a *target zone* for leverage, where it is optimal to let leverage change passively in response to profits and losses. Issuing equity in this zone to bring leverage back down to target would be too costly. The firm simply relies on good earnings realizations to pay back some of its accumulated debt in this zone.

The predictions of our model are in line with empirical findings, in particular the observed negative relation between positive earnings shocks and leverage (what is referred to as the leverage-profitability puzzle).[7] [8] When the firm's earnings are hit by a sufficiently large downward jump shock, passively rolling over debt may no longer be optimal. The firm could then find it optimal to recapitalize and actively bring down

[7] See Titman and Wessels (1988), Myers (1993), and Rajan and Zingales (1995) among others, on the leverage-profitability puzzle. Fama and French (2002), Leary and Roberts (2005), and Lemmon, Roberts, and Zender (2007) show that leverage is sticky in the short run.

[8] Denis and McKeon (2012) find that the evolution of a firm's leverage depends on whether or not it produces a financial surplus. DeAngelo, Gonçalves, and Stulz (2018) show that firms pay down their debt when they receive a positive earnings shock and increase their debt when they have no choice to do otherwise. Their main conclusion is that "Debt repayment typically plays the main direct role in deleveraging." Our results are also consistent with the findings of Korteweg, Schwert, and Strebulaev (2019), who show that firms tend to cover operating losses by drawing down their lines of credit, giving rise to similar leverage dynamics as in our model. Over a longer time horizon our leverage dynamics are also consistent with the findings of DeAngelo and Roll (2015), who emphasize that leverage is far from time invariant.

leverage. Such a recapitalization improves the firm's financial situation at the expense of old shareholders, whose ownership is significantly diluted.[9]

The firm can be either endogenously risk-averse or risk-seeking, depending on the level of its debt-to-earnings ratio. When leverage is low it drifts down to the target level and firm value is concave. When leverage is high, the firm endogenously becomes a risk-seeker as in Jensen and Meckling (1976) and firm value is convex.[10]

In the third step of our analysis we extend the model by endogenizing the earnings process through capital accumulation. We model investment following the classic q theory of investment (Hayashi, 1982, and Abel and Eberly, 1994). This generalized model delivers highly non-linear and non-monotonic predictions about corporate investment, marginal q, and average q, which sharply contrast with the predictions of the classical q theory of investment under Modigliani-Miller assumptions.

We show that when leverage is low (and the firm is risk-averse) investment decreases with leverage. This is due to a debt-overhang effect (Myers, 1977) driven by the firm's concerns to preserve liquidity. In contrast, when leverage is high (and firm value is convex), investment increases with leverage. Moreover, a higher marginal q is associated with lower investment, the opposite of the conventional prediction.

Our calibrated model predicts an average market leverage of 24%, within the range of estimates reported in Strebulaev and Whited (2012) and a proportion of firms with market leverage in excess of 59.4% of only 1%. Even a small fixed equity issuance cost generates a wide target zone, where leverage changes passively and significantly lowers both target and average leverage. When equity issuance is so costly that the firm never wants to issue equity, we show that the firm chooses a leverage target that is very close to zero, in line with the zero-leverage finding in Strebulaev and Yang (2013). We also show that target leverage is sensitive to cash flow risk, in particular jump shocks, but is only moderately sensitive to changes in the corporate income tax rate, and insensitive to changes in the liquidation recovery value of assets. Our model thus generates and explains the prudent leverage management observed for listed companies. It resolves most of the corporate finance puzzles that empirical studies have identified.

[9] This prediction is consistent with the findings of DeAngelo, DeAngelo, and Stulz (2010) that when firms issue equity in order to delever, existing shares are significantly diluted.

[10] Hugonnier, Morellec, and Malamud (2015) show that firm value can be concave or convex depending on the level of cash holdings in a model with lumpy investment and capital supply uncertainty. Della Seta, Morellec, and Zucchi (2020) develop a model showing that short-term debt and rollover losses can cause risk-taking when firms are close to financial distress.

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2 Model

We begin by introducing the firm's earnings before interest and taxes (EBIT) process.

Earnings process. The firm's EBIT Y_t evolves according to the following geometric jump-diffusion process:

$$\frac{dY_t}{Y_{t-}} = \mu dt + \sigma d\mathcal{B}_t - (1 - Z)d\mathcal{J}_t\,, \quad Y_0 > 0\,, \tag{1}$$

where μ is the drift parameter, σ is the diffusion-volatility parameter, $\mathcal{B}$ is a standard Brownian motion process, and $\mathcal{J}$ is a pure jump process with a constant arrival rate λ. We denote the sequence of independent jump arrival moments by $\{T^{\mathcal{J}}\}$. This process (1) is widely used in the macro-finance rare-disasters literature to model aggregate consumption or GDP.[11]

This jump-diffusion process generalizes the geometric Brownian motion process commonly used in the contingent-claim capital structure literature, e.g., Fischer, Heinkel, and Zechner (1989), Leland (1994), and Goldstein, Ju, and Leland (2001).[12] Since the diffusion shock $\mathcal{B}$ is continuous, if a jump does not occur at date t ($d\mathcal{J}_t = 0$), we have $Y_t = Y_{t-}$ (where $Y_{t-} \equiv \lim_{s\uparrow t} Y_s$ denotes the left limit of the firm's earnings).

If a jump arrives at date t ($d\mathcal{J}_t = 1$), EBIT changes from Y_{t-} to $Y_t = ZY_{t-}$, where $Z \in [0, 1]$ is a random variable with a well-behaved cumulative distribution function $F(\,\cdot\,)$. Since the expected percentage EBIT loss upon a jump arrival is given by $(1 - \mathbb{E}(Z))$, the expected EBIT growth rate, g, is given by:

$$g = \mu - \lambda(1 - \mathbb{E}(Z))\ . \tag{2}$$

Equity and debt investors. We assume that all investors (creditors and equityholders) are risk neutral.[13] However, the firm faces external financing costs. We assume that the firm's shareholders are weakly impatient relative to debt investors. Shareholders discount future payouts at the rate of γ, larger than the risk-free rate r: $\gamma \geq r > 0$. The

[11] This framework is widely used in modeling various asset-pricing and macroeconomic phenomena. Examples include Rietz (1988), Barro (2006), Gorbenko and Strebulaev (2010), Barro and Jin (2011), Bhamra and Strebulaev (2011), Gabaix (2012), Gourio (2012), Wachter (2013), and Rebelo, Wang, and Yang (2022), among others. See Décamps, Gryglewicz, Morellec, and Villeneuve (2017) and Gryglewicz, Mancini, Morellec, Schroth, and Valta (2022) for theory and empirical analyses for financially constrained firms with both permanent and transitory earnings shocks.

[12] Malenko and Tsoy (2021) use the same EBIT process with downward jumps to study optimal time-consistent debt policies.

[13] We can generalize our model by allowing investors to be risk averse and well diversified. For example, we can account for investors' risk aversion by using the stochastic discount factor (SDF) to price the firm's free cash flows. For brevity, we leave this extension out.

wedge $\gamma - r$ describes in a simple way the idea that debt is a cheaper source of financing than equity and generates a meaningful payout policy.[14] Before specifying the firm's financing choices and financial distress costs, we introduce the first-best benchmark with no financial distress.

First-best benchmark. As being (weakly) more patient than equity investors, debtholders value the firm the most. Let $\Pi(Y)$ denote the present value of expected EBITs under first best:

$$\Pi(Y) = \pi Y \,, \tag{3}$$

where π is the EBIT multiple:

$$\pi = \frac{1}{r-g} \,. \tag{4}$$

Equations (3)-(4) map to a Gordon growth valuation model, where g is the expected growth rate of EBIT given in (2). To ensure that $\Pi(Y)$ is finite, we require $r > g$.

Financing choices. The firm's financing choices involve both internal and external sources of funds. Internal funds come from internally generated cash flows that are retained over time. External funds include short-term debt, which we assume to be costless for simplicity,[15] and costly equity. Let X_t denote the firm's debt balance at time t. We assume that debt is short-term, as in most discrete-time dynamic corporate finance models (e.g. Hennessy and Whited, 2007), asset pricing, and macro finance.[16]

Short-term creditors may be exposed to default risk because EBIT is subject to downward jump shocks which are not hedgeable. Thus, following a sufficiently large loss shareholders may default on their debt obligations. Creditors in equilibrium price this default option held by shareholders.

Let $T^{\mathcal{D}}$ denote the firm's optimal default timing and $\mathbf{1}_t^{\mathcal{D}}$ be an indicator function that takes the value of one if and only if the firm defaults at date t and the value zero otherwise

[14] This impatience assumption could be preference based or could arise indirectly because shareholders have other attractive investment opportunities. This relative impatience makes shareholders prefer early payouts, *ceteris paribus.* This assumption has been used in DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin, and Rochet (2007), DeMarzo and Fishman (2007a, 2007b), DeMarzo (2019), and others.

[15] While in practice firms incur transaction costs in issuing debt or securing a line of credit, these costs are small relative to the costs firms incur when they issue equity. What is important for our analysis is that equity issuance costs are higher than debt issuance costs, not whether debt issuance is costly or not. This is why we set debt issuance costs to zero for simplicity.

[16] Black and Scholes (1973) and Merton (1973) use time-varying, short-term, risk-free debt positions in a replicating portfolio to value a call option on a stock. In Merton (1971) the investor optimally holds a levered market portfolio position financed with short-term, risk-free debt (when her risk aversion is sufficiently low or Sharpe ratio is sufficiently high). Lucas (1978) and Breeden (1979) assume that debt is short term in their equilibrium asset pricing models. Kiyotaki and More (1997) and Brunnermeier and Sannikov (2014) also use short-term debt.

($\mathbf{1}_t^{\mathcal{D}} = 1$ if and only if $t = T^{\mathcal{D}}$). We assume for simplicity that following a default the firm is liquidated and that the absolute priority rule (APR) holds in bankruptcy whereby creditors are repaid before shareholders. Let $L_{T^{\mathcal{D}}}$ denote the liquidation value at the moment of default $T^{\mathcal{D}}$. Because the APR applies in bankruptcy and liquidation is inefficient, equityholders only default if it is no longer in their interest to keep the firm going; that is, when the firm's equity value is equal to zero. Creditors then receive the entire liquidation proceeds at $t = T^{\mathcal{D}}$, and the loss given default (LGD) for creditors is $(X_{t-} - L_t)$, the difference between promised repayment X_{t-} and liquidation value L_t.

We model the deadweight losses caused by bankruptcy as in Leland (1994) by specifying that the liquidation recovery value at $T^{\mathcal{D}}$ equals a fraction $\alpha \in (0,1)$ of $\Pi(Y)$, the firm's enterprise value under the first-best given in (3). That is, $L_{T^{\mathcal{D}}}$ is given by:

$$L_{T^{\mathcal{D}}} = \alpha \Pi(Y_{T^{\mathcal{D}}})\,. \tag{5}$$

We may write equivalently that the firm's liquidation recovery value $L_{T^{\mathcal{D}}}$ as

$$L_{T^{\mathcal{D}}} = \ell Y_{T^{\mathcal{D}}}\,, \tag{6}$$

where the parameter ℓ measures the liquidation recovery per unit of earnings:

$$\ell = \alpha \pi. \tag{7}$$

As long as α is not too high, corporate bankruptcy causes deadweight losses and we have a meaningful dynamic tradeoff theory with benefits and costs of debt.

Debt pricing: credit spreads. Let C_{t-} denote the contractually agreed interest payments for the short-term debt issued at date $t-$. We write C_{t-} as

$$C_{t-} = (r + \eta_{t-})X_{t-}\,, \tag{8}$$

where r is the risk-free rate and η_{t-} is a time-varying equilibrium credit spread. To compensate debtholders for the credit losses they may bear if the firm defaults at t, the equilibrium credit spread η_{t-} must satisfy the following zero-profit condition for creditors:

$$X_{t-}(1 + rdt) = (X_{t-} + C_{t-}dt)\,\left[1 - \lambda \mathbb{E}_{t-}\left(\mathbf{1}_t^{\mathcal{D}}\right)dt\right] + \mathbb{E}_{t-}\left(L_t\,\mathbf{1}_t^{\mathcal{D}}\right)\lambda dt\,. \tag{9}$$

The first term on the right side of (9) is equal to the product of the total contractual repayment to creditors, $(X_{t-} + C_{t-}dt)$, and the probability $\left[1 - \lambda \mathbb{E}_{t-}\left(\mathbf{1}_t^{\mathcal{D}}\right)dt\right]$ at time $t-$ that the firm won't default at time t. The second term gives the creditors' expected payoff if the firm defaults. In sum, the equilibrium condition (9) states that creditors' expected rate of return (including both default and no-default scenarios) is equal to the risk-free rate r, which is captured by the left side of (9).

Corporate Taxes. We next introduce the standard tax benefit of debt: Interest payments are tax deductible at the corporate level. The detailed interest tax exemption rules can be quite complicated. To simplify the tax schedule, we only incorporate corporate taxes and leave out personal debt and equity income taxation out of the model.[17] We also ignore tax loss carryforward.

We specify the firm's tax payment (Θ_t) as a function of its interest payment (C_t) and its EBIT (Y_t): $\Theta_t = \Theta(C_t, Y_t)$ that satisfies the following homogeneity property:

$$\Theta(C_t, Y_t) = \theta(c_t) Y_t \,, \tag{10}$$

where $c_t = C_t / Y_t$ and $\theta(c)$ is decreasing in c.

Next, we introduce an important special case of (10), which we use in our quantitative analysis in Section 5. The firm pays income taxes at a constant rate ($\tau > 0$), when malking a profit ($Y_t > C_t$), and pays no taxes when incurring a loss ($Y_t \leq C_t$). This specification captures asymmetric tax treatments for profits and losses in a simple way. The scaled tax payment implied by (10) is:

$$\theta(c) = \tau(1 - c)\mathbf{1}_{c<1} \,, \tag{11}$$

where $\tau > 0$ is the profit tax rate and $\mathbf{1}_{c<1} = 1$ if and only if $c < 1$ and zero otherwise.

Costly external equity issuance. Firms rarely issue external equity, and when they do, they incur significant costs, due to a combination of asymmetric information, managerial incentive issues, and underwriting costs. For convenience we capture adverse selection costs, underwriting fees, and other floatation costs with a simple reduced form function. A large empirical literature has sought to quantify these costs, including the costs arising from the negative stock price reaction in response to the announcement of a new equity issue.[18] Building on the findings of this literature, we assume that the firm incurs both fixed and proportional costs when it issues equity. Fixed costs are what generates the observed infrequent and lumpy equity issuance.

Let N_t denote the firm's (undiscounted) cumulative net amount of external equity financing up to time t, and H_t denote the corresponding (undiscounted) cumulative costs of external equity financing up to time t. To preserve the model's homogeneity

[17] Hennessy and Whited (2005) model taxes with a richer, more realistic schedule. In their influential survey, Graham and Harvey (2001) find very little evidence that firms directly consider personal taxes.

[18] Altinkilic and Hansen (2000) estimate underwrite fee schedules. Asquith and Mullins (1986) measure the indirect costs of external equity using seasoned equity offerings announcements. Hennessy and Whited (2007) use simulated method of moments to infer the magnitude of financing costs and their estimates support the view that external equity issuances involve large indirect costs and firms are sensitive to these costs.

we further assume that the fixed cost is proportional to the firm's earnings Y_t, so that $h_0 Y_t$ denotes the total fixed equity-issuance cost. This form of fixed equity-issuance costs ensures that the firm never grows out of the fixed cost, preserving the first-order consideration of equity issuance costs . The firm also incurs variable equity-issuance costs: For each dollar the firm raises it incurs a marginal cost of h_1. Therefore, when raising net proceeds M_t, the firm incurs a total equity issuance cost of $h_0 Y_t + h_1 M_t$.[19]

Equity payouts. In addition to managing debt and equity issuance, the firm also decides if and when to make distributions to its shareholders. There is an economically meaningful payout policy in our model given that shareholders are *impatient* compared with creditors ($\gamma \geq r$) and/or that debt financing has tax advantages. We denote by U_t the firm's cumulative payout to equityholders up to time t. Then, $dU_t \geq 0$ is the implied non-negative incremental payout process as equityholders are protected by limited liability at all t. We show that the optimal target leverage, a key concept in capital structure theory, is closely tied to the firm's payout to shareholders.

Retained earnings and debt dynamics. Since it is costly to issue equity the firm has an incentive to maintain financial flexibility. This way it avoids having to return to equity markets too often. When the firm is solvent (when $t < T^{\mathcal{D}}$), its debt value X_t evolves according to the following law of motion:

$$dX_t = (C_{t-} + \Theta_{t-} - Y_{t-})\, dt - dN_t + dU_t - \mathbf{1}_t^{\mathcal{D}} \left(X_{t-} - L_t \right) d\mathcal{J}_t \,. \tag{12}$$

The first term on the right side of (12) describes how the value of debt evolves absent equity issuance, payout, and default: if the firm's interest and tax payments ($C_{t-} + \Theta_{t-}$) exceed its contemporaneous EBIT Y_{t-}, the firm's debt balance X_t increases by the net amount: $C_{t-} + \Theta_{t-} - Y_{t-}$. This term captures the firm's behavior under normal circumstances. It pays down debt if making profits and builds up debt otherwise, behaving as a credit card revolver. Importantly, the interest rate at which the firm revolves its credit is endogenously determined.

The second term dN_t and the third term dU_t describe how the firm's net equity issuances and equity payouts change its debt balance X_t, respectively. The last term in (12) captures the consequence of bankruptcy. When a sufficiently large jump loss arrives at t, it causes the firm to default (i.e. $\mathbf{1}_t^{\mathcal{D}} = 1$), so that its market value of debt X_t decreases from its pre-default value X_{t-} to its liquidation value L_t given in (6).

[19] This assumption is similar to the ones made in the cash management literature, e.g., Bolton, Chen, and Wang (2011). Hennessy and Whited (2007) allow for fixed, proportional, and convex costs and find that the fixed and proportional costs are the more important ones.

Combining all four margins, earnings retention, equity issuance, equity payouts, and default, we obtain a complete description of the firm's debt dynamics. In contrast, without equity issuance costs, debt is not a state variable but rather a control variable as we show in Section 3.

Optimality. The firm chooses: i) incremental payouts to shareholders ($dU_t \geq 0$); ii) net external equity issuance $dN_t \geq 0$; and, iii) the default timing $T^{\mathcal{D}}$ to maximize shareholder value:

$$\mathbb{E}_t \left[\int_t^{T^{\mathcal{D}}} e^{-\gamma(s-t)} \left(dU_s - dN_s - dH_s \right) \right] , \tag{13}$$

subject to debt dynamics given in (12), the equilibrium credit risk pricing equation (9), and a transversality condition. Note that since equity issuance is costly (dH whenever $dN > 0$) we need to subtract the equity issuance cost dH in (13). The optimal earnings retention and debt rollover dynamics are implied by (12). Since APR applies in bankruptcy, shareholders receive nothing upon default, which explains why there is no payoff to shareholders from time $T^{\mathcal{D}}$ onward.

Value function and homogeneity property. Both EBIT Y_t and debt X_t are state variables, so that the firm's equity value P_t is given by the function $P(Y_t, X_t)$ that solves the problem defined in (13). Let $V_t = V(Y_t, X_t)$ denote firm value:[20]

$$V(Y_t, X_t) = P(Y_t, X_t) + X_t \,. \tag{14}$$

Let x_t denote the debt-EBIT ratio:

$$x_t \equiv X_t / Y_t \,, \tag{15}$$

and let p_t denote the firm's equity value-EBIT ratio ($p_t = P_t/Y_t$) and v_t denote firm value-EBIT ratio ($v_t = V_t/Y_t = p_t + x_t$). Also let ML_t denote the firm's *market leverage*, the ratio of debt value X_t to the firm's (enterprise) value V_t:

$$ML_t \equiv \frac{X_t}{V_t} = \frac{x_t}{v_t} \,. \tag{16}$$

A common liquidity metric used by practitioners to measure a firm's ability to service its debt is the *interest coverage ratio*, which we denote by ICR_t:

$$ICR_t = \frac{Y_t}{C_t} \,. \tag{17}$$

[20] Because the face value of debt equals the market value of debt before a firm defaults, its enterprise value equals the sum of equity value $P(Y_t, X_t)$ and debt value X_t.

The ICR_t measures the firm's ability to service its debt interest payments out of current EBIT. We can also evaluate the firm's ability to meet the sum of interest and tax payments with its contemporaneous EBIT with the following *liquidity coverage ratio*:

$$LCR_t = \frac{Y_t}{C_t + \Theta_t} \,. \tag{18}$$

When $LCR_t > 1$, the firm's debt balance X_t automatically decreases.[21] ML_t is known as the loan-to-value (LTV) ratio in real estate finance. These ratios are widely used in debt covenants.[22]

3 Dynamic Tradeoff Theory under Costless Equity Issuance

Before analyzing the effect of costly equity issuance, we solve the special case where equity issuance is costless. Our costless equity issuance benchmark, a dynamic formulation of the classical tradeoff model, sets the stage to understand how costly external equity financing fundamentally affects dynamic capital structure and payout policies. (The static tradeoff model is widely taught, see e.g., Berk and DeMarzo, 2020.)

3.1 Solution

First, we summarize equityholders' optimal policies, equity value $p(x)$, and the firm's enterprise value $v(x)$. Let $\overline{x}$ denote the *minimal* level of the debt-EBIT ratio x above which a firm defaults. This implies that in the endogenous insolvent region $x \geq \overline{x}$:

$$v(x) = \overline{x} \quad \text{or equivalently} \quad p(x) = 0 \quad \text{for} \quad x \geq \overline{x} \,. \tag{19}$$

Equityholders choose x to maximize a firm's enterprise value $v(x)$, the sum of its equity and debt values: $v(x) = p(x) + x$, by solving the following problem:

$$\max_{x} \; v(x) \,, \tag{20}$$

[21] Strebulaev (2007) defines financial distress as situations where the firm's ICR_t falls below one and requires the firm to take corrective actions (e.g., inefficient asset sales) to alleviate debt burden under financial distress. We share his view that liquidity measures such as ICR and LCR are critically important for capital structure decisions. However, in Strebulaev (2007) the firm continuously issues equity or makes dividend payments to shareholders and there is no earnings retention. Leverage dynamics in his model are fundamentally different from ours in which earnings retention, debt rollover, equity issuance, and payout jointly determine leverage dynamics.

[22] It is straightforward to introduce other covenants such as cash-flow-based covenants that limit the firm's borrowing. Lian and Ma (2021) document that 80% of debt (for US nonfinancial firms) is backed predominantly by cash flows from operations (cash flow/earnings based lending). Our model is well suited to capture these empirically important cash flow based lending activities.

where $v(x)$ in the endogenous solvent region $x \in [0, \overline{x})$ is given by

$$v(x) = \frac{1}{\gamma - g}\Big[1 + \underbrace{(\gamma - r)x}_{\text{impatience}} - \underbrace{\theta(c(x))}_{\text{taxes}} - \underbrace{\lambda(v(x) - \ell)\Big(\int_0^{\overline{Z}(x)} Z dF(Z)\Big)}_{\text{enterprise's expected loss given default}}\Big] \tag{21}$$

and $\overline{Z}(x)$ is given by

$$\overline{Z}(x) = x/\overline{x}\,, \quad \text{for } \; x \leq \overline{x}\,. \tag{22}$$

Recall that $\overline{x}$ is the equityholders' default threshold, i.e., the minimal value of x satisfying (19). Let x^* denote the optimal target debt-EBIT ratio x for the problem (20).

The denominator on the right side of (21) equals the difference between the equityholders' discount rate γ and the earnings growth rate g. There are four terms inside the square brackets. Absent any debt issuance, we only have the first term and the scaled enterprise value is $v(x) = 1/(\gamma - g)$. The second term in (21) captures the benefit of debt financing because equityholders are impatient relative to debtholders ($\gamma \geq r$). The third term is the tax cost and the last term is the enterprise's expected loss given default.

Finally, the equilibrium credit spread, η^*, is given by

$$\eta^* = \lambda\left[F(\overline{Z}(x^*)) - \left(\frac{\ell}{x^*}\right)\int_0^{\overline{Z}(x^*)} Z dF(Z)\right]. \tag{23}$$

This equation ties the equilibrium credit spread to the firm's default debt-EBIT ratio $\overline{x}$ and its optimal debt-EBIT ratio x^*. The credit spread is lower than the probability of default $\lambda F(\overline{Z}(x^*))$ as creditors' recovery in default is non-negative, $\ell \geq 0$.[23]

Next, extending Myers (1974), we derive the adjusted present value (APV) formula for our model, which provides an alternative, equivalent exposition of our costless-equity-issuance model developed in this section. We provide a derivation of our APV formula including the equilibrium credit spread formula (23) in Appendix A.2.

3.2 Adjusted Present Value (APV)

We can rewrite the scaled enterprise value (21) by decomposing it as follows:

$$v(x) = v(0) + pb(x) - pc(x)\,, \tag{24}$$

where the first term in (24) is the unlevered value of the firm

$$v(0) = \frac{1 - \tau}{\gamma - g}\,, \tag{25}$$

[23] For the special case where creditors recover nothing upon default, so that $\ell = 0$, the equilibrium spread η is simply given by $\lambda F(\overline{Z}(x^*))$.

the second term $pb(x)$ in (24) is the present benefit (value) of debt financing given by

$$pb(x) = \frac{(\tau - \theta(c(x))) + (\gamma - r)x}{\gamma - \widehat{g}(x)}, \tag{26}$$

and the third term $pc(x)$ in (24) is the present cost (value) of debt financing given by

$$pc(x) = \frac{\lambda(v(0) - \ell)\int_0^{\overline{Z}(x)} ZdF(Z)}{\gamma - \widehat{g}(x)}. \tag{27}$$

In (26) and (27), $\widehat{g}(x)$ is the expected earnings growth rate conditional on no default:

$$\widehat{g}(x) = \mu - \lambda\left(1 - \int_{\overline{Z}(x)}^{1} ZdF(Z)\right). \tag{28}$$

Note that $\widehat{g}(x)$ is lower than g given in (2) and $g - \widehat{g}(x) = \lambda\int_0^{\overline{Z}(x)} ZdF(Z)$. This wedge reflects the expected reduction of earnings growth due to default.

3.3 Summary and Comparison with the Leland Literature

In sum, the enterprise value formula (21) and the APV formula (24) are two equivalent ways, offering complementary economic perspectives, of describing $v(x)$. When equity issuance is costless, the optimal financing policy is for the firm to set its debt-EBIT ratio at a constant target level x^* at all time. The firm makes dividend payments to shareholders whenever it books a profit and issues equity to repay its outstanding debt whenever it makes a loss (provided that the loss is not too high) to keep $x = x^*$. The firm defaults if the loss is too large, which is the case when the fractional loss $(1 - Z)$ is larger than $1 - \overline{Z}(x^*) = 1 - (x^*/\overline{x})$. In sum, the firm's optimal policy is characterized by two endogenous barriers for x: x^* and $\overline{x} > x^*$.

It is helpful to put these results in context of the existing contingent-claim capital structure literature following Leland (1994), which also assumes costless equity issuance. Note first that the default mechanisms are different: in model the firm can continuously adjust both its debt and equity, so only a sufficiently large jump loss causes the firm to default, whereas in Leland (1994) the firm is not allowed to adjust debt after issuance at time 0 (in effect, the adjustment cost is assumed to be infinite) so that even continuous diffusion shocks can trigger default. Second, the equilibrium credit spread in our model is always positive, as jumps are unpredictable, but the credit spread is zero just prior to default in Leland (1994) and other contingent-claim models with term debt and diffusion shocks.[24] Third, debt is short term in our model but a perpetuity in Leland (1994) (or term debt as in Leland and Toft, 1996).

[24] The prediction of diffusion-based structural credit risk models following Leland (1994) that the credit spread is equal to zero just prior to default is considered to be a major contradiction of credit-risk pricing evidence. See discussions in Duffie and Singleton (2000) and Duffie and Lando (2001).

Key predictions of this costless-equity-issuance benchmark are inconsistent with empirical and survey findings. For a start, companies only intermittently (and rarely) raise outside equity. Also under this benchmark, volatility (or higher-order moments) has no effect on leverage,[25] and financial slack has no value. But, these predictions are inconsistent with key survey findings of Graham and Harvey (2001), namely that CFOs highly value financial flexibility, care about the firm's credit ratings, and about the firm's ability to service its debts; moreover, CFOs consider earnings and cash-flow volatility to be a major factor influencing debt policies.

We show next that if external equity issuance is costly all the model predictions are in line with existing evidence and with the survey findings above.

4 Leverage, Earnings Retention, Equity Issuance, and Payout

With costly equity issuance, the firm's dynamic capital structure decisions become state-contingent; they involve both issuance timing decisions and issuance size margins for debt and equity. Moreover, the firm's capital structure, earnings retention, payout, and default decisions are then all interconnected. In contrast to the benchmark when equity issuance is costless (for which x is a *control variable*), when equity issuance is costly, x becomes a *state variable*.

At each time t, the state variable x is in one of four mutually exclusive regions, separated by three endogenously determined thresholds: a.) the payout boundary $\underline{x}$; b.) the equity-issuance boundary $\widehat{x}$; and c.) the default boundary $\overline{x}$. Naturally, we have $\underline{x} < \widehat{x} \leq \overline{x}$. The four regions are: 1.) the *payout region*: $x < \underline{x}$, where the firm makes a lumpy dividend payout $(\underline{x} - x)$; 2.) the *debt-financing region*: $\underline{x} \leq x < \widehat{x}$, where the firm relies on earnings retention and debt rollover policies to manage its finances; 3.) the *equity-issuance region*: $\widehat{x} \leq x \leq \overline{x}$, where the firm issues equity to reduce its leverage to an endogenously determined recapitalization target for its debt-EBIT ratio, denoted by $\widetilde{x}$; and 4.) the *default region*: $x > \overline{x}$, where it is optimal for the firm's shareholders to default. Figure 3 illustrates these four regions.

4.1 Payout Region

When x is below the endogenous payout boundary $\underline{x}$, the firm makes a lump-sum payment $(\underline{x} - x)Y$ to shareholders, so that the following value-continuity condition holds:

$$p(x) = p(\underline{x}) + \underline{x} - x\,, \quad \text{for} \quad x < \underline{x}\,. \tag{29}$$

[25] To be precise, this is after we take out the effect of systematic volatility on the discount rate. That is, this assumption for the costless-equity-issuance model holds under the risk-neutral measure.

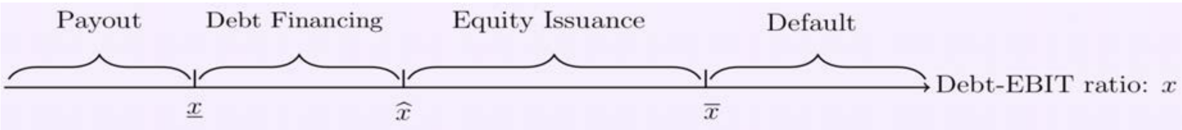


Figure 3: This figure demonstrates the four mutually exclusive regions for debt-EBIT ratio x. For low leverage ($x < \underline{x}$), the firm makes a lumpy dividend payout ($\underline{x} - x$). For $\underline{x} \le x \le \widehat{x}$, the firm behaves as a creditor card revolver. For $\widehat{x} < x \le \overline{x}$, the firm issues external equity to bring down x to an endogenous target level $\widetilde{x} \in (\underline{x}, \widehat{x})$ in the debt-financing region. Finally, for $x > \overline{x}$, the firm defaults. The three thresholds satisfying $\underline{x} < \widehat{x} \le \overline{x}$ are endogenously determined. The payout boundary $\underline{x}$ is also the optimal target debt-EBIT ratio. The recapitalization target $\widetilde{x}$ in the debt financing region is where x_t starts after any seasoned equity issuance regardless of its pre-issuance level of $x_{t-} \in (\widehat{x}, \overline{x})$.

Since (29) holds for x close to $\underline{x}$, we obtain the following smooth-pasting condition for $\underline{x}$:

$$p'(\underline{x}) = -1\,, \tag{30}$$

by taking the limit $x \to \underline{x}$. At $\underline{x}$, the marginal benefit of paying out dividends equals one: $-p'(\underline{x}) = 1$. Since the payout boundary $\underline{x}$ is an optimal choice, we also have the following super-contact condition (see, e.g., Dumas, 1991):

$$p''(\underline{x}) = 0\,. \tag{31}$$

As we show below, the firm is at its optimal target (market) leverage $\underline{x}/v(\underline{x})$ at the payout boundary, where firm value $v(x)$ is at its highest.[26] The intuition for this result is as follows. When choosing the optimal target leverage at its inception, the firm uses debt as its marginal source of financing, given that external equity is more costly than debt. Moreover, since debt issuance costs are zero, the firm must be borrowing up to the level $\underline{x}$ that finances dividend distribution to shareholders at the optimal target leverage. The firm would like to keep x permanently at $\underline{x}$, but this is not feasible (and suboptimal), given that EBIT shocks are unhedgeable and equity issuance is costly.

Next, we turn to the interior region where the firm spends most of its time.

4.2 Earnings Retention and Debt Financing Region

We begin by describing the firm's leverage dynamics in the interior region where $\underline{x} \le x \le \widehat{x}$ (recall that $\widehat{x}$ is the endogenous equity-issuance boundary). We then solve for its equity value $p(x)$ in this region.

[26] This prediction differs from Hennessy and Whited (2005), who conclude that there is no target leverage in their model. Our definition of target leverage is similar to the one in DeAngelo, DeAngelo, and Whited (2011), who emphasize the existence of target leverage in dynamic models.

Leverage dynamics. The firm makes no payouts to shareholders whenever $x > \underline{x}$, as doing so increases its debt burden and the marginal cost of servicing debt, which exceeds one for $x > \underline{x}$. The firm behaves like a *credit card revolver*: It simply lets x evolve in response to profits and losses (after interest and tax payments). It issues no equity and does not default so long as $x \leq \widehat{x}$. Realizing a profit reduces its debt balance, while incurring a loss increases its leverage.

Using Ito's Lemma, we obtain the following process for $x_t = X_t/Y_t$:

$$dx_t = \mu_x(x_{t-})\,dt - \sigma x_{t-} d\mathcal{B}_t + \left(x_t^{\mathcal{J}} - x_{t-}\right) \, d\mathcal{J}_t\,, \tag{32}$$

where $\mu_x(x_{t-})$, the first term in (32), is the drift of x ignoring the effect of jumps:[27]

$$\mu_x(x_{t-}) = [c(x_{t-}) + \theta(c(x_{t-})) - 1] - \left(\mu - \sigma^2\right) x_{t-} \tag{33}$$

and $x_t^{\mathcal{J}}$ is the post-jump debt-EBIT ratio given by

$$x_t^{\mathcal{J}} = \mathbf{1}_t^{\mathcal{D}} \ell + \left(1 - \mathbf{1}_t^{\mathcal{D}}\right) \frac{x_{t-}}{Z}\,. \tag{34}$$

The second term in (32), $-\sigma x_{t-}$, captures the effect of EBIT diffusion shocks on x_{t-}: A positive shock increases the firm's value and lowers x_t explaining the negative sign.[28]

The last term in (32) describes the effect of a jump shock on x. Upon arrival of a jump loss at t, the debt-EBIT ratio changes from x_{t-} to the post-jump level of $x_t^{\mathcal{J}}$. Equation (34) describes the two possible scenarios for $x_t^{\mathcal{J}}$: 1.) if the firm does not default ($\mathbf{1}_t^{\mathcal{D}} = 0$), the debt-EBIT ratio increases from x_{t-} to $x_{t-}^{\mathcal{J}} = x_{t-}/Z$; 2.) if the jump triggers a default ($\mathbf{1}_t^{\mathcal{D}} = 1$), then equity is worth zero, creditors receive the entire firm liquidation proceeds $\ell Y_{T^{\mathcal{D}}}$, so that $x_t^{\mathcal{J}} = \ell$. Next, we characterize equity value $p(x)$ in this region.

Equity value $p(x)$. The equity value function, $p(x)$, satisfies the following ODE:

$$\begin{aligned}(\gamma - \mu)\, p(x) \quad &= \quad [c(x) + \theta(c(x)) - 1 - \mu x]\, p'(x) + \frac{\sigma^2 x^2}{2} p''(x) \\ &\quad +\lambda \left[\int_{\overline{Z}(x)}^{1} Z p(x/Z) dF(Z) - p(x)\right]\,,\end{aligned} \tag{35}$$

for $x \in (\underline{x}, \widehat{x})$. As expected, $p(x)$ is a decreasing function of x in this region, as a lower x brings the firm closer to its target leverage at $\underline{x}$. Firm value $v(x) = p(x) + x$ also decreases with x in this region, given that any $x \in (\underline{x}, \overline{x})$ is above the firm's target leverage.

[27] Since jumps are downward, the drift of x *including* the effect of jumps is different from $\mu_x(x_{t-})$ as the jump induces an additional term: $\lambda \mathbb{E}\left(x_t^{\mathcal{J}} - x_{t-}\right)$.

[28] Note that this is consistent with the observed negative leverage-profitability relation (Titman and Wessels 1988, Myers 1993, and Rajan and Zingales 1995).

The first two terms on the right side of (35) capture the drift and volatility effects of x on $p(x)$. The last term in (35) captures the effect of jumps on equity value.[29] Since jumps link $p(x)$ to $p(x/Z)$ for $Z \geq \overline{Z}(x)$ we need to solve $p(\,\cdot\,)$ globally. The solution method for our jump-diffusion model is different from pure diffusion models, which only require local information around x.

A jump arrival can produce three different scenarios. First, if $Z < \overline{Z}(x_{t-}) = x_{t-}/\overline{x}$, the post-jump debt is so high ($x_t^{\mathcal{J}} = x_{t-}/Z > x_{t-}/\overline{Z}(x_{t-}) = \overline{x}$) that the firm defaults, resulting in $p(x_t^{\mathcal{J}}) = 0$. Second, if the jump loss is small or moderate, the firm simply rolls over its debt (like a credit-card revolver). Third, if the jump loss is somewhat large, the firm issues costly equity to repay its debt and bring its leverage down to a more moderate level, which we refer to as the *recapitalization target.* The firm does not wait until it completely exhausts its entire financing capacity to recapitalize by raising new outside equity. The reason is that this would excessively expose the firm to the risk of a large jump loss. Our dynamic model, thus, does not generate Myers (1984) static pecking-order results. There comes a point where leverage is so high that it is optimal to incur the cost of equity issuance and recapitalize. By recapitalizing the firm maximizes its continuation value. We next characterize the firm's recapitalization decisions.

4.3 Equity Issuance Region

The equity issuance option, although costly, is valuable as it allows the firm to avoid an even worse outcome: a costly default. We next characterize the endogenous equity-issuance region: $\widehat{x} \leq x \leq \overline{x}$. Let M_t denote the net equity issuance proceeds and let $m_t = M_t/Y_t$. We denote by $\widetilde{x}_t$ the "target debt-EBIT ratio from recapitalization":

$$\widetilde{x}_t = x_t - m_t \,. \tag{36}$$

Given that $p(x)$ must be continuous before and after issuance, the following value-matching condition holds:

$$p(x) = p(x-m) - [h_0 + (1+h_1)m] = p(\widetilde{x}) - [h_0 + (1+h_1)m] \,. \tag{37}$$

Note that new shareholders break even (under competitive financial markets), so that $p(x_t)$ is equal to the equity value after issuance, $p(\widetilde{x}_t)$, after paying for the sum of: (a.) net equity issuance m_t and, (b.) the total issuance costs $(h_0 + h_1 m_t)$. Also, conditional on paying the fixed equity issuance cost, the optimal net equity issuance amount, m,

[29] A jump causes its debt-EBIT ratio to increase from x_{t-} to $x_t^{\mathcal{J}} = x_{t-}/Z$ and the equity value to decrease from $p(x_{t-})$ to $p(x_t^{\mathcal{J}}) = p(x_{t-}/Z)$.

must satisfy the following FOC:

$$-p'(\widetilde{x}) = -p'(x-m) = (1+h_1)\,. \tag{38}$$

This condition says that the net marginal benefit of reducing debt at the post-issuance debt-EBIT ratio $\widetilde{x}$, $-p'(\widetilde{x})-1$, equals the marginal cost of equity issuance (h_1).

Inverting (38), we obtain the "recapitalization target" debt-EBIT ratio $\widetilde{x}$, which is independent of the firm's pre-issuance level of x. As long as the marginal equity issuance cost is strictly positive ($h_1 > 0$), the "recapitalization target" $\widetilde{x}$ is larger than the optimal target debt-EBIT ratio $\underline{x}$, as $-p'(\underline{x}) = 1$. The intuition is as follows. External equity is the marginal source of financing when the firm *deleverages* its balance sheet to reach the optimal recapitalization target $\widetilde{x}$. This is in sharp contrast with the economics underpinning the determinants of the firm's optimal target $\underline{x}$ leverage, where debt is the marginal source of financing. Since the marginal cost of debt issuance is lower than the marginal cost of equity issuance, we have $\underline{x} < \widetilde{x}$.

The firm enters into the equity-issuance region only from the earnings-retention-and-debt-rollover region. When it does so, it issues equity to bring down x to the recapitalization target $\widetilde{x}$. Even though the firm spends almost no time in the equity-issuance region, the option of issuing costly equity to deleverage in the future has profound implications for leverage dynamics, as we show in the next section.

Finally, we determine the firm's optimal equity issuance boundary $\widehat{x}$. Equations (37) and (38) together imply that $p(x)$ is linear in x in the equity-issuance region:

$$p(x) = p(\widehat{x}) - (1+h_1)(x-\widehat{x})\,, \quad \widehat{x} \le x < \overline{x}\,. \tag{39}$$

The intuition is as follows. The firm always returns to the recapitalization target $\widetilde{x}$ from any level of x when issuing equity. Also, the marginal equity issuance cost (h_1) is constant.[30] Since $p(x)$ is differentiable at its endogenous equity-issuance boundary $\widehat{x}$, the following value-matching and smoothing-pasting conditions must hold at $\widehat{x}$:

$$p(\widehat{x}+) = p(\widehat{x}-) \quad \text{and} \quad p'(\widehat{x}+) = p'(\widehat{x}-)\,. \tag{40}$$

Equation (39) and the smoothness of $p(x)$ jointly imply that at $\widehat{x}$:

$$p'(\widehat{x}) = -(1+h_1)\,. \tag{41}$$

[30] To be precise, for any $x \in [\widehat{x}, \overline{x})$, we have $p(x) = p(\widetilde{x}) - h_0 - (1+h_1)(x-\widetilde{x})$ .Using the preceding equation, we obtain $p(x) - p(\widehat{x}) = -(1+h_1)(x-\widehat{x})$ given in (39).

4.4 Default Region

The firm only uses default as a last resort. When the firm's debt is so large that it exceeds its endogenous default boundary $\overline{x}$, it defaults. Because of limited liability, upon default shareholder value equals zero:

$$p(x) = 0\,, \text{ when } \; x \geq \overline{x}\,, \tag{42}$$

and creditors collect the firm's liquidation value $\ell Y_{T^{\mathcal{D}}}$.

Substituting $p(\overline{x}) = 0$ into the linear equity valuation equation (39), we obtain the following relation between the default boundary $\overline{x}$ and the equity-issuance boundary $\widehat{x}$:

$$\overline{x} - \widehat{x} = \frac{p(\widehat{x})}{1 + h_1}\,. \tag{43}$$

4.5 Summary

The equity value $p(x)$ satisfies (29) in the payout region ($x < \underline{x}$), the ODE (35) in the earnings retention/debt rollover region ($\underline{x} < x < \widehat{x}$), (39) in the equity-issuance region ($\widehat{x} < x < \overline{x}$), and $p(x) = 0$ in the default region ($x > \overline{x}$), subject to the boundary conditions given by (30)-(31) for the payout threshold $\underline{x}$ (also the optimal target), (40)-(41) for the equity-issuance threshold $\widehat{x}$, and (43) for the default threshold $\overline{x}$. Finally, the recapitalization target, $\widetilde{x}$, (the post-equity-issuance level of x) solves (38).

In terms of leverage dynamics, the firm optimally behaves like a credit-card revolver, and leverage changes passively in response to the firm's realized profits and losses, in the earnings retention/debt rollover region where the firm finds itself most of the time. That is, leverage increases following a negative earnings shock and decreases following a positive shock, consistent with the documented negative relation between leverage and profitability in the cross-section.[31]

It is only when the firm has been doing well, and when the marginal enterprise value of debt equals zero, that the firm makes a payout to shareholders ($dU_t > 0$). If the firm has been accumulating losses, its leverage eventually may be so high that it chooses to recapitalize in order to bring down leverage (thereby significantly diluting the ownership of its existing shareholders).[32]

[31] This is consistent with the evidence on the profitability puzzle in Titman and Wessels (1988), Myers (1993), Rajan and Zingales (1995), and Welch (2004). Danis, Rettl, and Whited (2014) find a positive relation in the cross-section between leverage and profitability for firms that make significant payouts to shareholders. Their finding is also consistent with our predictions, since in our model firms with $x_t < \underline{x}$ increase their borrowing (thereby increasing their leverage) to pay out the difference $(\underline{x} - x_t)Y_t$ to shareholders.

[32] In contrast, in Leland (1994) and related models without equity issuance costs, the firm continuously engages in equity issuance and payouts. Only the free cash flow $(dU_t - dN_t)$ is economically meaningful

5 Quantitative Analysis

We turn to the analysis of the quantitative implications of our model in this section. This analysis not only reveals the empirical relevance of our model but also brings out key intuitions.

5.1 Parameter Choices: Baseline Case

We set the annual risk-free rate r at 6% as in Leland (1994) and the subsequent contingent-claim capital-structure literature. We set shareholders' discount rate to $\gamma = 6.5\%$, which reflects an impatience wedge of $\gamma - r = 0.5\%$ for shareholders. We set the corporate tax rate τ in (11) at the current US federal rate 21%. Note that the firm pays taxes only when it books a profit. We leave aside personal taxation and other considerations for simplicity.

As in the rare-disaster literature (e.g., Barro 2006, Gabaix 2012, and Wachter 2013), we assume that the cumulative distribution function $F(Z)$ for the recovery fraction Z is governed by a power law:[33]

$$F(Z) = Z^{\beta}, \quad 0 \leq Z \leq 1. \tag{44}$$

The lower is the exponent β, the more fat-tailed is the distribution of $(1 - Z)$.

We calibrate the diffusion volatility parameter σ, and the two jump parameters, β and λ, by matching the model-implied variance, skewness, and excess kurtosis of the annual EBIT logarithmic growth rate to respectively 0.223, -0.252, and 0.326 for the COMPUSTAT data.[34] We obtain a diffusion volatility (σ) of 40.6%, a jump arrival rate ($\lambda^{\mathbb{P}}$) of 1.25 per annum under the physical measure, and $\beta = 6.6$, which implies that each jump arrival causes an expected (percentage) EBIT loss of $(1 - \mathbb{E}(Z)) = 1/(\beta + 1) = 13.2\%$. That is, the EBIT on average jumps downward about once every 9.6 months ($1/\lambda^{\mathbb{P}} = 1/1.25$ years) and the expected percentage loss of EBIT equals $\lambda^{\mathbb{P}}(1 - \mathbb{E}(Z)) = 1.25 \times 13.2\% = 16.5\%$. These numbers indicate that the EBIT process

in these models. Absent default, $dU_t - dN_t = (Y_{t-} - C_{t-})\, dt$ and the gross flows dU_t and dN_t are indeterminate.

[33] See Gabaix (2009) for a survey on power laws in economics.

[34] The dataset is Compustat North American Fundamentals for publicly traded firms in North America, which contains balance sheets reported annually by companies between 1970 and 2020. We drop firms with missing total assets, negative sales, negative cash and short-term investments, negative capital expenditures, negative debt in current liabilities, negative depreciation and amortization, negative R&D expenses, missing SIC, zero common shares outstanding, or negative common equity. We focus on firms incorporated in the United States. Financial firms (SIC code 6000-6999) and utility firms (SIC codes 4900-4999) are excluded from the sample. The final sample comprises 91,657 firm-year observations. We calculate the logarithmic EBIT growth rates and winsorize at 5% on both sides.

Table 1: Parameter Values

This table summarizes the parameter values for our baseline analysis. Whenever applicable, parameter values are annualized.

Parameters	Symbol	Value
risk-free rate	r	6%
shareholders' discount rate	γ	6.5%
corporate tax rate	τ	21%
diffusion volatility	σ	40.6%
(risk-neutral) jump arrival rate	λ	2.5
jump recovery parameter	β	6.6
(risk-neutral) drift absent jumps	μ	34%
liquidation recovery scaled by EBIT	ℓ	3.85
equity issue fixed cost	h_0	0.3
equity issue proportional cost	h_1	0.06

is subject to both significant jump and diffusion shocks. In addition, we assume that the jump arrival rate under the risk-neutral measure, λ, is twice as large as $\lambda^{\mathbb{P}}$, which implies that $\lambda = 2 \times 1.25 = 2.5$ and $\lambda(1 - \mathbb{E}(Z)) = 33\%$ on a risk-adjusted basis, in line with estimates in the credit-risk literature (e.g., Huang and Huang, 2012).

We set the expected EBIT growth rate g to 1% per annum under the risk-neutral measure. This choice also yields a reasonable firm value-to-EBIT multiple for an all-equity financed firm of $\pi = 1/(r - g) = 20$ (given that $r - g = 5\%$).[35]

For the equity issuance costs, we set $h_1 = 0.06$, which is in line with empirical estimates (Hennessy and Whited, 2007; Eckbo, Masulis and Norli, 2007) and within the range of values used in the literature. As in Huang and Huang (2012), we target the recovery value for creditors when the firm defaults to 51% of the debt face value. The equity issuance probability is 7.3% per annum (based on the same COMPUSTAT sample). Calibrating to these two targets, we obtain: 1.) a fixed equity issuance cost parameter of $h_0 = 0.3$, and 2.) a liquidation recovery parameter of $\ell = 3.85$, within the range of estimates in Lian and Ma (2021).

We begin by deriving the quantitative solution for the benchmark with costless equity issuance. We then turn to the analysis of the solution with equity issuance costs.

[35] Using the formula for g given in (2), we obtain $\mu = g + \lambda\mathbb{E}(1 - Z) = 1\% + 33\% = 34\%$.

5.2 Costless Equity Issuance

When equity issuance is costless, firm value $v(x)$ varies with its target debt-EBIT ratio x as displayed in Figure 4. As one would expect, $v(x)$ (the solid blue line) first increases and then decreases with x.[36] At the optimum $x^* = 5.67$ firm value reaches the maximum value of $v(x^*) = 15.49$; the optimal target market leverage is then $ML^* = x^*/v(x^*) = 5.67/15.49 = 37\%$, as indicated by the red dot. Figure 4 exhibits a *dynamic version* of the classic static tradeoff theory solution (see, e.g., Figure 1 of Myers' 1984 AFA Presidential Address), which pits the benefits of debt against the expected costs of default. Our model is fully dynamic and features a realistic growing cash-flow process. Accordingly, the control variable is the debt-EBIT ratio rather than the level of debt as in the static tradeoff theory in Myers (1984).

The dashed black line depicts the constant enterprise value $v(x) = 20$ under the first-best setting where the Modigliani-Miller financing irrelevance theorem holds. The gap between the flat MM (dashed black) line and the solid blue line reflects the value loss due to tax, impatience, and bankruptcy frictions. Note that cash-flow volatility has no impact on market leverage. This is because firms do not value financial flexibility in the absence of equity issuance costs, which is empirically counterfactual.

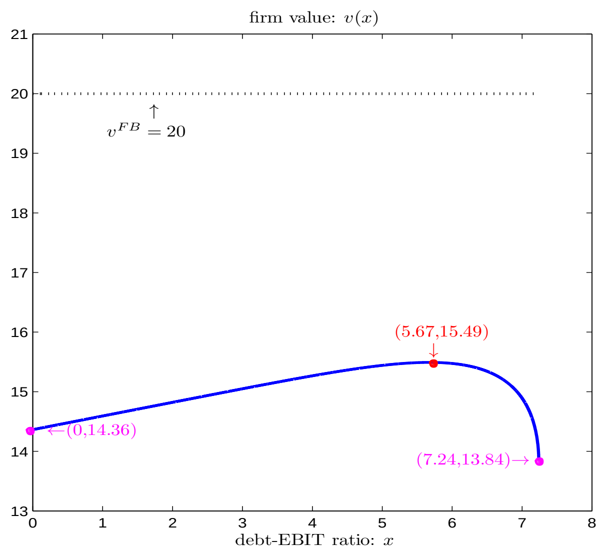


Figure 4: Classical capital structure tradeoff theory: No equity issuance costs ($h_0 = h_1 = 0$). This figure plots firm value $v(x)$ as a function of debt-EBIT ratio x. Firm value is maximized by setting $x = x^* = 5.67$ and equals $v(x^*) = 15.49$. The corresponding optimal target market leverage is $ML^* = 37\%$. The firm's enterprise value under the first-best MM setting is $v^{FB} = 20$. All parameter values other than $h_0 = h_1 = 0$ are given in Table 1.

[36] If the firm were all equity financed ($x = 0$) (a suboptimal policy) it would be worth $v(0) = 14.36$, which is 7% lower than $v(x^*) = 15.49$ under optimal leverage, as displayed in Figure 4.

Adjusted present value (APV) calculations. To further sharpen our understanding of the model, we do an APV calculation using the formulae in Section 3.2. The value of debt financing per unit of EBIT, $pb(x)$, evaluated at the optimal x^* is:

$$pb(x^*) = \frac{\tau c(x^*)}{\gamma - \widehat{g}(x^*)} = \frac{21\% \times 0.356}{6.5\% - 0.91\%} = 1.34, \tag{45}$$

and the PV of distress costs, $pc(x)$, evaluated at x^* is:[37]

$$pc(x^*) = \frac{\lambda(v(0) - \ell)\int_0^{\overline{Z}(x^*)} ZdF(Z)}{\gamma - \widehat{g}(x^*)} = \frac{2.5 \times (14.36 - 3.85)\int_0^{0.37} ZdF(Z)}{6.5\% - 0.91\%} = 0.20\,. \tag{46}$$

Thus, in this calculation, the benefit of debt financing is about 6.5 times the cost of financial distress, in line with the standard tradeoff theory predictions that the benefit of debt financing significantly outweighs the cost of financial distress.

Using the APV formula given in (24), we obtain (scaled) firm value at x^* is given by $v(x^*) = v(0) + pb(x^*) - pc(x^*) = 14.35 + 1.34 - 0.20 = 15.49$. (Recall that (scaled) firm value under all-equity financing is: $v(0) = (1-\tau)/(\gamma - g) = 14.35$.)

We next show how introducing equity issuance costs into the model yields predictions on target leverage levels and leverage dynamics that are in line with empirical evidence and with the survey evidence of Graham and Harvey (2001).

5.3 Costly Equity Issuance

Figure 5 plots the firm's enterprise value $v(x)$, net marginal cost of debt $-v'(x)$, market leverage (ML), and the interest coverage ratio (ICR).

Enterprise value $v(x)$ and the net marginal cost of debt $-v'(x)$. We plot the firm's enterprise value $v(x)$ and marginal cost of debt $-v'(x)$ in Panels A and B respectively. Note that $v(x)$ is maximized at the target debt-to-EBIT ratio $\underline{x} = 1.86$, which is also the optimal payout boundary; $v(x)$ decreases with x for $x \geq \underline{x}$, as leverage exceeds the target value. At origination the firm optimally raises just enough debt so that its debt-to-EBIT ratio x is set at its optimal target $\underline{x} = 1.86$. Any excess funds the firm has available are disbursed as a lumpy dividend to *impatient* shareholders.

The target debt-to-EBIT ratio corresponds to a target market leverage of $ML(\underline{x}) = \underline{x}/v(\underline{x}) = 12.4\%$, which is significantly lower than the value of $ML^* = 37\%$ under costless equity issuance. Once the firm is off the ground running, it makes dividend payments to shareholders, bringing x back to $\underline{x} = 1.86$, whenever its cumulative profits cause x_t to fall below $\underline{x} = 1.86$.

[37] In this calculation, $\gamma = 6.5\%$, $r = 6\%$, $v(0) = 14.36$, $\ell = 3.85$, $x^* = 5.67$, $c(x^*) = 0.356$, $\overline{Z}(x^*) = 0.37$, and $\widehat{g}(x^*) = 0.91\%$.

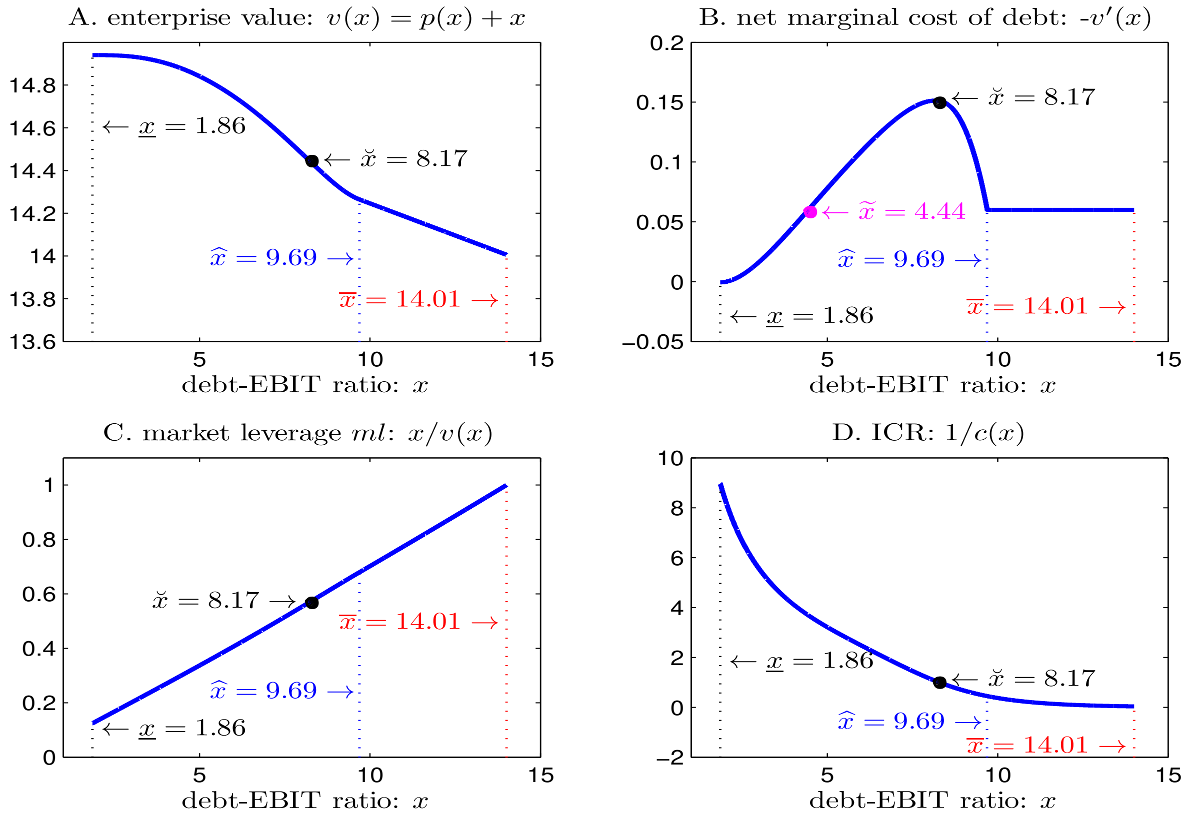


Figure 5: Enterprise value, $v(x) = p(x) + x$, net marginal cost of debt, $-v'(x)$, market leverage ML, and interest coverage ratio ICR. The endogenous target debt-EBIT ratio, also the payout boundary, is $\underline{x} = 1.86$ where market leverage is $ML(\underline{x}) = \underline{x}/v(\underline{x}) = 12.4\%$. The endogenous (upper) default boundary is $\overline{x} = 14.01$, where $p(\overline{x}) = 0$. The equity-issuance boundary is $\widehat{x} = 9.69$, where $p(\widehat{x}) = 4.58$ and market leverage is $ML(\widehat{x}) = \widehat{x}/v(\widehat{x}) = 67.9\%$. The firm's post-equity-issuance target level of x is $\widetilde{x} = 4.44$, with an implied market leverage of $ML(\widetilde{x}) = \widetilde{x}/v(\widetilde{x}) = 29.9\%$. And the inflection point is $\breve{x} = 8.17$ and market leverage is $ML(\breve{x}) = \breve{x}/v(\breve{x}) = 56.5\%$. All parameter values are given in Table 1.

Should losses accumulate such that x_t exceeds $\underline{x} = 1.86$, the firm's optimal response is to let x_t drift passively in response to realized EBIT shocks (after servicing its debt). This is in contrast to the firm's optimal response under costless equity issuance, when it immediately issues equity to bring x_t back to $\underline{x} = 1.86$. This *auto-pilot leverage management policy* is optimal for a wide range of debt-EBIT ratios: $x \in (\underline{x}, \widehat{x}) = (1.86, 9.69)$. This corresponds to a range for market leverage of 12.4% to 67.9%. Indeed, the firm issues costly equity only when x_t reaches or exceeds $\widehat{x} = 9.69$, where $p(\widehat{x}) = 4.58$ and market leverage is $ML(\widehat{x}) = \widehat{x}/v(\widehat{x}) = 67.9\%$.

This range of *passive market leverage* represents the *target zone* for optimal leverage widely discussed in the empirical literature (Graham and Harvey, 2001; Fama and French, 2005; and Leary and Roberts, 2005). Any level of market leverage in this range can be seen as an optimal outcome depending on the history of earnings shocks (Baker and

Wurgler, 2002). Also, the prediction that the firm seeks to cap its market leverage at a level (67.9% in our numerical example) far below its maximum leverage capacity is consistent with the survey evidence that CFOs attach great importance to maintaining their credit ratings and therefore seek to keep leverage within limits (Graham and Harvey, 2001).[38] Our target zone prediction for optimal market leverage is also consistent with the evidence in DeAngelo and Roll (2015) who state that "Target-leverage models that place little or no weight on maintaining a particular ratio do a good job replicating the substantial instability of the actual leverage." A fundamental difference between our target-zone model and contingent-claim capital structure models (e.g., Goldstein, Ju, and Leland 2001), is our prediction of no equity issuance in the target zone. Indeed, the latter models predict frequent equity issuances to service outstanding debt (following earnings underperformance).

Even though enterprise value is maximized at the target leverage $ML(\underline{x}) = 12.4\%$, the firm barely spends any time at $ML(\underline{x}) = 12.4\%$. This is because keeping its market leverage at 12.4% at all time is too expensive. A key observation here is that paradoxically low target leverage is not due to debt being more costly, but rather to the firm's objective to avoid future costly equity issuance that may be needed to service debt should the firm find itself in financial distress.

When the debt-to-EBIT ratio x_t exceeds the equity issuance boundary $\widehat{x}$ but remains below the default boundary $\overline{x}$ (when $x_t \in (\widehat{x}, \overline{x}) = (9.69, 14.01)$ or equivalently when market leverage $ML(x) \in (ML(\widehat{x}), ML(\overline{x})) = (67.9\%, 100\%)$) it is optimal for the firm to *recapitalize* its balance sheet by issuing costly external equity and bring its debt-to-EBIT ratio to the *recapitalization target* $\widetilde{x} = 4.44$ (the pink solid dot). Note that at the recapitalization target market leverage is $ML(\widetilde{x}) = \widetilde{x}/v(\widetilde{x}) = 29.9\%$, regardless of the pre-equity-issuance value of x. Concretely, suppose that the firm's market leverage is 60% and that the firm is hit by a negative large earnings shock with a fractional loss $(1-Z) = 28\%$. Then its market leverage instantly increases to 85% absent recapiltalization. It is then optimal for the firm to immediately recapitalize its balance sheet by issuing external equity of 7.57 times its EBIT, so that its market leverage returns to the *recapitalization target* of 4.44. This deleveraging is costly and highly dilutive to existing shareholders' ownership, who end up holding only 20% after the recapitalization.

This recapitalization prediction is consistent with the evidence in DeAngelo and Roll (2015), who find that "many firms have leverage ratios above 0.500 at some point, but almost no firms keep leverage ratios consistently above 0.500 for long periods of

[38] A firm is much less likely to issue debt if doing so increases the likelihood of its credit rating being downgraded (Kisgen, 2006).

time." They suggest that "something akin to distress costs must encourage rebalancing downward from very high leverage." Our recapitalization prediction is a confirmation of their conjecture, as it is optimal for the firm to reduce the likelihood of inefficient liquidation by preemptively recapitalizing its balance sheet as soon as its market leverage exceeds $ML(\widehat{x}) = 67.9\%$. Shareholders reluctantly issue new equity to deleverage the balance sheet because the alternative is worse: a higher risk of default. This explains why in equilibrium firms rebalance downward from very high leverage even though this is highly dilutive and costly for shareholders.

While the aim of the firm's recapitalization is to reduce leverage, it is suboptimal to bring market leverage back down to target leverage $ML(\underline{x})$, which is only 12.4% in our numerical solution. There are two leverage targets in our model: i) optimal target ($\underline{x}$), and ii) recapitalization target ($\widetilde{x}$). When leverage is low the firm lets leverage drift automatically towards its optimal target market leverage $ML(\underline{x}) = 12.4\%$ by cumulatively retaining earnings over time. Second, when leverage is high the firm must actively deleverage to bring leverage back under control. The optimal recapitalization target market leverage is then $ML(\widetilde{x}) = 29.9\%$, which is higher than target leverage $ML(\underline{x}) = 12.4\%$. This is because the marginal source of financing to reach these two distinct targets is different: retained earnings for the optimal target ($ML(\underline{x})$) and external equity for the recapitalization target ($\widetilde{x}$).

Concavity and convexity of $v(x)$. An important result that emerges from our model, depicted in Panel B of Figure 5, is that the firm's enterprise value can be either concave or convex in x depending on the level of x. While always positive $-v'(x)$ is not monotonic: it is first increasing in x, reaching a maximum value at the inflection point $\breve{x} = 8.17$, and then decreasing in x. Even when x is in the leverage target zone, $v(x)$ is concave in the region $(\underline{x}, \breve{x}) = (1.86, 8.17)$ and convex in the region $(\breve{x}, \widehat{x}) = (8.17, 9.69)$. The inflection point, where $v''(x) = 0$, is given by $\breve{x}$. Market leverage at the inflection point is $ML(\breve{x}) = \breve{x}/v(\breve{x}) = 8.17/14.45 = 56.5\%$. The firm is endogenously risk-averse with respect to x in the region $(1.86, 8.17)$ where market leverage is moderate. In contrast, in the region $(8.17, 9.69)$ where market leverage is high the firm is close to the point when it may have to recapitalize by issuing costly external equity. In this region, increasing risk with respect to x creates value for shareholders due to the wealth transfer from creditors to shareholders. This is true even though debt is always fairly priced. Thus, as in Jensen and Meckling (1976), a risk-seeking motive emerges when leverage is high. But the risk-seeking behaviour is for a different reason and is related to Hugonnier, Morellec, and Malamud (2015). In sum, when the firm faces costly external equity financing it is endogenously risk averse with respect to leverage risk when leverage is low but risk loving when leverage is high.

Panel B also confirms two key results discussed earlier: 1) In the equity issuance region $(\widehat{x}, \overline{x}) = (9.69, 14.01)$ the (net) marginal cost of debt always equals the marginal cost of issuing equity: $-v'(x) = h_1 = 0.06$; and, 2.) firm value $v(x)$ is maximized at the payout boundary $\underline{x}$, as $v'(x) = 0$ only when $x = \underline{x}$. Finally, Panels C and D plot market leverage ML and the interest coverage ratio ICR as functions of x, respectively. As expected, market leverage ML increases with x, and ICR increases significantly as x decreases.

6 What Determines Leverage?

We explore next the comparative statics properties of market leverage with respect to the following parameters: 1.) the equity issuance costs (h_0 and h_1), 2.) tax rate τ and liquidation recovery ℓ, and 3.) the parameters of the EBIT process (σ, λ, and β). We report the mean and quantile distribution for market leverage ML in Table 2 for each set of parameters. The row labeled "baseline" reports the results of our baseline solution (see Table 1 for the baseline parameter values.).

6.1 First-order Effects of Equity Issuance Costs

How sensitive are the firm's financial policies to changes in equity issuance costs? For a start to answer this question it is helpful to recall first that market leverage is constant at 37% when the firm faces no equity issuance costs, and second that the average market leverage is 23.9% in the baseline calibration of a fixed issuance cost parameter value $h_0 = 0.3$ and and a variable cost parameter value $h_1 = 0.06$. We show here that financial policies are extremely sensitive to changes in fixed issuance cost parameter values. To be sure, even a tiny fixed issuance cost parameter value of $h_0 = 0.001$ (just slightly greater than zero) induces the firm to become much more prudent in its leverage policy. As is shown in Table 2, median ML when $h_0 = 0.001$ is much closer to median ML when $h_0 = 0.3$, 28.6% instead of 20.3% (for our baseline). Moreover, at the 95% quantile, ML under the two parameter values is barely indistinguishable, 48.3% instead of 47.6% (for our baseline). Hence, the conservative leverage policy prediction of our model depends mostly on the existence of a fixed equity issuance cost. The firm chooses a low target leverage and is able to maintain leverage at a moderate level most of the time, as shown shown in Table 2, by relying on internally generated cash flows to reduce its leverage when it deviates from target following a negative earnings shock. This is possible as long

Table 2: THE EFFECT OF EQUITY ISSUANCE COSTS ON THE DISTRIBUTIONS OF MARKET LEVERAGE ML. Parameter values for the baseline case are given in Table 1. For all other rows, we vary one parameter (shown in the first column) from the baseline case.

	Mean	Std. dev.	5%	50%	95%
baseline	0.239	0.112	0.129	0.203	0.476
$h_0 = 0.001$	0.304	0.098	0.182	0.286	0.483
$h_0 = 2$	0.141	0.099	0.075	0.101	0.351
$h_0 \to \infty$	0.044	0.025	0.034	0.037	0.076

as leverage remains low, for then the drift of x is negative. This is the case most of the time. It is only in somewhat rare circumstances (when the firm is hit by a large negative earnings shock (or a series of negative shocks) that leverage enters a death spiral, where x drifts up. In sum, even a tiny fixed equity issuance cost generates a wide target zone where leverage management is on auto-pilot and were leverage tends to drift down.

Table 2 also illustrates the dynamics of market leverage when the firm is shut out of equity markets ($h_0 \to \infty$). In this extreme situation median leverage is close to zero (3.7%). Since the firm does not have an option to recapitalize, its survival is predicated on leverage staying out of the death spiral zone. This is achieved by relying on debt financing only when the firm cannot avoid it and by sticking to a very conservative dividend policy. However, should the firm be in the unfortunate situation where debt enters a death spiral then market leverage can grow higher than when the firm has access to equity markets.

6.2 Limited Effects of Taxes and Financial Distress Costs

How much do changes in the corporate tax rate affect corporate leverage policies? We explore this question by changing the tax rate from the baseline level of 21% to respectively 10% and 30%. Overall, we find that changes in the corporate tax rate have a limited effect on market leverage; a much smaller effect than in the contingent-claims capital structure literature following Leland (1994). When the tax rate τ is lowered from 21% to 10% median market leverage declines from 20.3% to 15.8%. And when the tax rate τ is increased from 21% to 30% median market leverage increases from 20.3% to only 23.1%. These small responses of corporate leverage policies to significant changes in corporate tax rates are in line with the empirical evidence (see Graham 2003).[39]

[39] The quantile distribution shifts up slightly. Decreasing the tax rate τ from 21% to 10% brings down the average ML from 23.9% to 19.8%, also a moderate effect.

Table 3: THE EFFECTS OF TAXES (τ) AND LIQUIDATION RECOVERY (ℓ) ON THE DISTRIBUTIONS OF MARKET LEVERAGE ML. Parameter values for the baseline case are given in Table 1. For all other rows, we vary one parameter (shown in the first column) from the baseline case.

	Mean	Std. dev.	5%	50%	95%
baseline	0.239	0.112	0.129	0.203	0.476
$\tau = 10\%$	0.198	0.106	0.102	0.158	0.428
$\tau = 30\%$	0.266	0.115	0.147	0.231	0.506
$\ell = 2$	0.233	0.109	0.127	0.198	0.464
$\ell = 8$	0.261	0.121	0.139	0.224	0.518

Changes in the default recovery value parameter ℓ have an even smaller effect on leverage as we can see from Table 3. When the recovery value parameter ℓ is increased from 2 to 8, median market leverage only increases from 19.8% to 22.4%. The reason why corporate leverage policies are not very sensitive to changes recovery value in default is that default is a low probability event. Revisiting Miller (1977)'s famous "rabbit" versus "horse" analogy, we see that neither tax rates nor distress costs are horses in our model once we take external equity financing costs into account.

6.3 Significant Effects of Jump Risk

Jump risk involves both the arrival rate of an earnings shock λ and the distribution of fractional losses, which is parameterized by β. Table 4 shows that a reduction in the expected fractional loss $\mathbb{E}(1-Z) = 1/(\beta+1)$ (conditional on a jump arrival) from 13.2% to 5% substantially increases average market leverage from 23.9% to 40%. At the 95th percentile ML also increases significantly from 47.6% to 68.9%. A reduction in the (risk-adjusted) arrival rate λ (i.e., under the risk-neutral measure) from 2.5 (once every 4.8 months) to 1 (once every year) has a somewhat smaller effect. As Table 4 reports, average ML increases from 23.9% to just 29.6% as a result of this significant reduction in the arrival rate. When we decrease λ further to 0.25 per annum, however, average ML significantly increases to 38%.

Overall, the effects of disaster risk, measured via the jump arrival rate λ or the power-law exponent (β) for the tail distribution on market leverage (both its level and distribution) are quantitatively significant. The reason is that the firm has less flexibility in adjusting its leverage policy in response to large shocks, so that it responds by significantly increasing its target leverage in response to a decrease in the incidence of

Table 4: THE EFFECTS OF JUMP RISK (β, λ) ON THE DISTRIBUTIONS OF MARKET LEVERAGE ML. Parameter values for the baseline case are given in Table 1. For the other two rows, we vary one parameter (shown in the first column) from the baseline case.

	Mean	Std. dev.	5%	50%	95%
baseline	0.239	0.112	0.129	0.203	0.476
$\beta = 19$	0.400	0.144	0.233	0.365	0.689
$\lambda = 1$	0.296	0.123	0.169	0.259	0.553
$\lambda = 0.25$	0.378	0.149	0.215	0.338	0.685

shocks and/or the expected size in the fractional losses as captured by the higher value of the parameter β relative to the benchmark value ($\beta = 19$ instead of $\beta = 6.6$).

7 q-Theory of Investment and Leverage Dynamics

We introduce investment into our main model to be able to explore the joint leverage, investment, and payout dynamics predictions of our model and relate these to empirical evidence. Specifically, we generalize our model by endogenizing the EBIT process via a capital accumulation process and capital adjustment costs as in the classical q-based models of investment.[40]

7.1 Model and Solution

Capital Accumulation, Production, and Endogenous EBIT. Let K and I denote the level of the capital stock and gross investment, respectively. At each time t, the firm's unlevered free cash flow net of capital expenditures is given by:

$$Y_t = AK_t - I_t \,, \tag{47}$$

where A is a constant that measures the firm's productivity.[41]

[40] The $q-$theory of investment is natural for our model as it has the homogeneity property of our main model, so that the debt-capital ratio is the natural state variable. Our theory of dynamic corporate finance and investment is a complement to Hennessy and Whited (2005, 2007) and other discrete-time dynamics models of investment, which assume decreasing returns to scale.

[41] We can interpret this linear production function as one that features a constant-returns-to-scale production function also involving other factors of production. For instance, suppose the firm has a Cobb-Douglas production function with capital and labor as inputs. Let w denote the constant wage rate for a unit of labor and $\widehat{A}$ denote the firm's productivity with this Cobb-Douglas production function. The firm then has the following embedded static tradeoff within its dynamic optimization problem: $\max_{N_t} \widehat{A}K_t^{\nu}N_t^{1-\nu} - wN_t$, which yields optimal labor demand: $N_t^* = \left(\frac{(1-\nu)\widehat{A}}{w}\right)^{1/\nu} K_t$, proportional to

We assume that the capital stock evolves according to the following process:[42]

$$dK_t = \Psi(I_{t-}, K_{t-})dt + \sigma_K K_{t-} d\mathcal{B}_t^K - (1-Z)K_{t-}d\mathcal{J}_t^K \,. \tag{48}$$

Following Lucas and Prescott (1971), Hayashi (1982), Abel and Eberly (1994), and Jermann (1998), we assume that $\Psi(I_{t-}, K_{t-})$ is homogeneous of degree one in I and K:

$$\Psi\left(I, K\right) = \psi(i) \cdot K, \tag{49}$$

where $i = I/K$ and $\psi'(i) > 0$ ($\psi(i)$ is concave and continuously differentiable). The second term in (48) describes the Brownian shock, where σ_K is the diffusion-volatility parameter and $\mathcal{B}^K$ is a standard Brownian motion. These continuous shocks can be thought of as stochastic capital depreciation shocks as in Barro (2006).

The third term in (48) describes the risk with respect to discrete downward jumps in the level of capital stock K the firm is exposed to, where $\mathcal{J}^K$ is a jump process with a constant arrival rate $\lambda_K > 0$. Let $T_K^{\mathcal{J}}$ denote the (recurrent) jump arrival time until the firm defaults. If a jump does not occur at time t, so that $d\mathcal{J}_t^K = 0$, we have $K_t = K_{t-}$, where $K_{t-} \equiv \lim_{s\uparrow t} K_s$ is the left limit of K_t. If a jump occurs at time t, so that $d\mathcal{J}_t = 1$, the capital stock drops from K_{t-} to $K_t = ZK_{t-}$. As before, $F(Z)$ denotes a well-behaved cumulative distribution function for $Z \in [0, 1)$. The firm's liquidation value at the moment of default $T_K^{\mathcal{D}}$, is given by

$$L_{T_K^{\mathcal{D}}} = \ell_K K_{T_K^{\mathcal{D}}} \,, \tag{50}$$

where, ℓ_K is the market recovery value per unit of capital. As before, default generates deadweight losses if ℓ_K is sufficiently low.

To preserve the homogeneity property of our model, we also assume that external equity financing costs are proportional to the capital stock K_t, so that $h_0^K K_t$ denotes the fixed equity-issuance cost and $h_1^K M_t$ refers to the proportional equity-issuance cost, where $h_0^K \geq 0$ and $h_1^K \geq 0$, and M_t denotes the net proceeds from the equity issue.

When investment is added to the model, the firm's capital stock K_t replaces Y_t as the state variable. Accordingly, Tobin's average q (enterprise value scaled by K) is denoted by v_t, and the investment-capital ratio by i_t:

$$v_t = V_t/K_t = V(K_t, X_t)/K_t \quad \text{and} \quad i_t = I_t/K_t \,.$$

---

the capital stock K_t. Using this expression for N_t^*, we then obtain the revenue net of labor cost AK_t, where $A = \frac{\nu}{1-\nu}\left((1-\nu)\widehat{A}\right)^{1/\nu} w^{\frac{\nu-1}{\nu}}$. The price of capital is normalized to one.

[42] Pindyck and Wang (2013) use this process in a general-equilibrium setting to quantify the economic cost of catastrophes. Brunnermeier and Sannikov (2014) and Barnett, Brock, and Hansen (2019) use the same capital accumulation process given in (48) with no jumps.

We also let $p_t = P_t/K_t = P(K_t, X_t)/K_t$. Finally, note that since the firm is levered, *marginal* q, denoted by q_m, is given by the marginal equity value of capital:

$$q_m = \frac{\partial V(K,X)}{\partial K} = v(x) - xv'(x) = p(x) - xp'(x)\,. \tag{51}$$

Solution with costly external equity. The solution for $p(x)$ again features four mutually exclusive regions when equity issuance is costly: i) a payout region; ii) a target zone where earnings are retained and debt is rolled over; iii) a recapitalization with equity issuance region; and iv) a default region. In the target zone $p(x)$ satisfies the following ODE:

$$\begin{aligned}\gamma p(x) \;=\; \max_i \quad & -\left[A - i - c(x) - \theta(c(x))\right] p'(x) + \psi(i)(p(x) - xp'(x)) + \frac{\sigma_K^2 x^2}{2} p''(x) \\ & + \lambda_K \left[\int_{\overline{Z}(x)}^1 Zp(x/Z) dF(Z) - p(x) \right], \end{aligned} \tag{52}$$

where $c(x) = C(K,X)/K$ is the coupon payment per unit of capital.

There are two key differences between the ODE (52) for $p(x)$ in our q-based investment model and the ODE (35) for $p(x)$ in the EBIT-based model. First, the scaling variable is capital K rather than EBIT Y, so that the relevant ratio is the EBIT-to-capital ratio $A - i(x)$ as opposed to unity in the EBIT-based model. Second, the (exogenous) earnings drift parameter μ in the EBIT-based model is replaced by $\psi(i(x))$, which captures capital adjustment costs. Indeed, the firm actively adjusts its investments (and asset sale, if $i < 0$) in response to earnings shocks in the target zone.

Interestingly, the investment policy of the levered firm depends on both marginal q and the marginal cost of debt, given that debt is the marginal source of financing for investment in the target zone. This is apparent in the following FOC for investment:

$$\frac{1}{\psi'(i(x))} = \frac{q_m(x)}{-p'(x)} = \frac{p(x) - xp'(x)}{-p'(x)}. \tag{53}$$

Importantly, since $\psi(\,\cdot\,)$ is a concave function, the optimal i increases with $q_m(x)/{-p'(x)}$, the ratio of marginal q to marginal cost of debt.[43]

For brevity, we omit the value functions in the other three regions, the equity issuance conditions, and the boundary conditions, as they are similar to those in the EBIT-based model after adjusting for the definition for x.

[43] Marginal q and the marginal cost of debt $-p'(x)$ are correlated in our model. Also, as we show, the marginal cost of debt is greater than one, $-p'(x) \geq 1$.

7.2 Quantitative Analysis

For brevity we leave out any tax considerations from this analysis. Accordingly, the benefit of debt financing is entirely driven by shareholder impatience given that $\gamma > r$.

Table 5: PARAMETER VALUES FOR THE q MODEL

This table summarizes the parameter values for our q theory of investment and leverage dynamics in Section 7. Whenever applicable, parameter values are annualized.

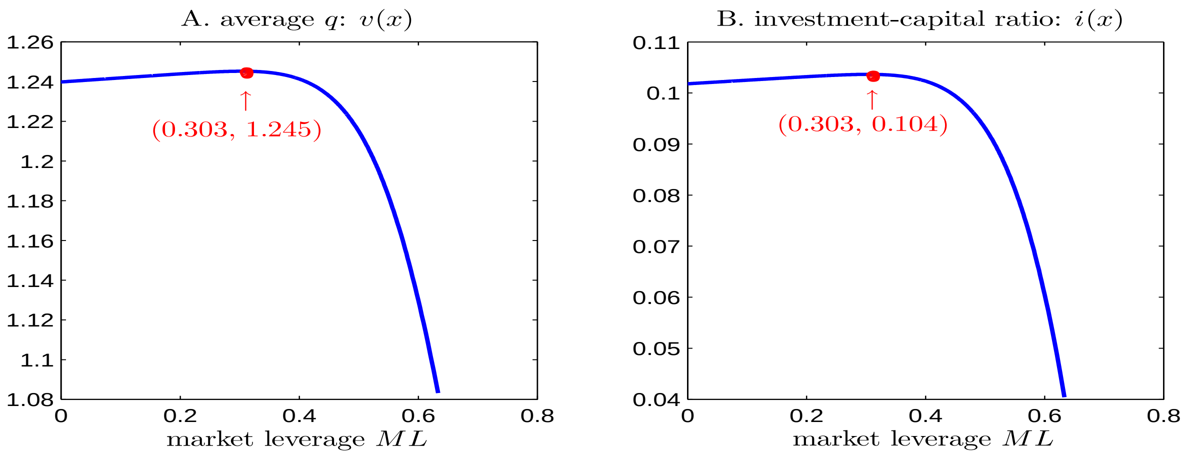
Parameters	Symbol	Value
shareholders' discount rate	γ	6.5%
risk-free rate	r	6%
diffusion volatility	σ_K	40.6%
jump arrival rate	λ_K	2.5
jump recovery parameter	β_K	6.57
capital productivity	A	47.8%
liquidation recovery scaled by capital	ℓ_K	0.28
adjustment cost parameter	ξ	1.9
equity issue fixed cost	h_0^K	0.002
equity issue proportional cost	h_1^K	0.06

We use the following functional form for $\psi(\,\cdot\,)$:

$$\psi\left(i\right) = i - \frac{\xi}{2} i^2 \,, \tag{54}$$

with $\xi > 0$. As in the EBIT-based model, we assume that the distribution of $Z \in [0, 1)$ is given by $F(Z) = Z^{\beta_K}$, where $\beta_K > 0$ is the power-law parameter. To ease comparison with the EBIT-based model specification, we use the same parameter values for the risk-free rate, the diffusion and jump parameters (β_K, σ_K, and λ_K) and the marginal cost of equity issuance h_1^K: $r = 6\%$, $\gamma = 6.5\%$, $\lambda_K = 2.5$, $\sigma_K = 40.6\%$, $\beta_K = 6.57$, and $h_1^K = 0.06$. As for the other five parameters, we set $h_0^K = 0.2\%$, $\xi = 1.9$, $A = 47.8\%$, and $\ell_K = 0.28$ so as to target: i) an average investment-capital ratio i of 10%; ii) a mean average q of 1.24; iii) average market leverage ML of 23.9%; and, iv) a debt recovery upon default of 51% of the debt face value. These parameter values are broadly in line with the values used in dynamic corporate finance and q theory literatures. The quantitative predictions of this capital accumulation-based model specification can be related to the empirical findings in Lian and Ma (2021), especially their comparison of asset-value-based versus cash-flow-based borrowing constraints.

Figure 6: Optimal capital structure, investment and Tobin's average q: Classic tradeoff theory with costless equity issuance: $h_0^K = h_1^K = 0$. Panels A and B plot the firm's average q, $v(x)$, and investment i with respect to market leverage $ML = x/v(x)$. The firm optimally keeps its market leverage at the constant value-maximizing level: $ML^* = 30.3\%$ at all time until it defaults when a jump arrives and causes its EIBT to decrease by more than $1 - Z^* = 69.7\%$. The optimal investment-capital ratio is $i^* = 10.4\%$ per annum and the corresponding average q is $q^* = 1.245$. All parameter values are given in Table 5.



7.2.1 Tradeoff Theory with Costless Equity Issuance.

When there are no equity issuance costs, average q at x^*, $v(x^*)$, satisfies the same Gordon-growth formula given in (20)-(21). Compared with the EBIT-based model with no equity issuance cost, there is an additional condition for investment given by:

$$\frac{1}{\psi'(i^*)} = v(x^*)\,. \tag{55}$$

Equation (55) is a standard FOC in the q theory of investment. It equates the marginal cost of investing, $1/\psi'(i^*)$, with marginal q (which is equal to average q in this case).

In Panels A and B of Figure 6, we plot respectively average q and investment i with respect to market leverage ML. Optimal market leverage is $ML^* = 30.3\%$ (corresponding to a debt-to-capital ratio of $x^* = 0.38$), and the optimal investment-capital ratio is $i^* = 10.4\%$ per annum. Finally, Tobin's average q is $q^* = 1.245$ at all time until the firm defaults (it does so when a jump loss arrives that causes its capital to decrease by more than $1 - Z^* = 69.7\%$). As in the EBIT-based model, the firm constantly issues equity and pays out a dividend to shareholders so as to maintain market leverage at the target $ML^* = 30.3\%$ at all time before it defaults, which is clearly counter-factual.

Table 6: STATIONARY DISTRIBUTIONS OF x AND MARKET LEVERAGE ML. ALL PARAMETER VALUES ARE GIVEN IN TABLE 5.

	Mean	Std. dev.	5%	50%	95%
x	0.298	0.089	0.234	0.259	0.512
ML	0.239	0.073	0.188	0.208	0.412

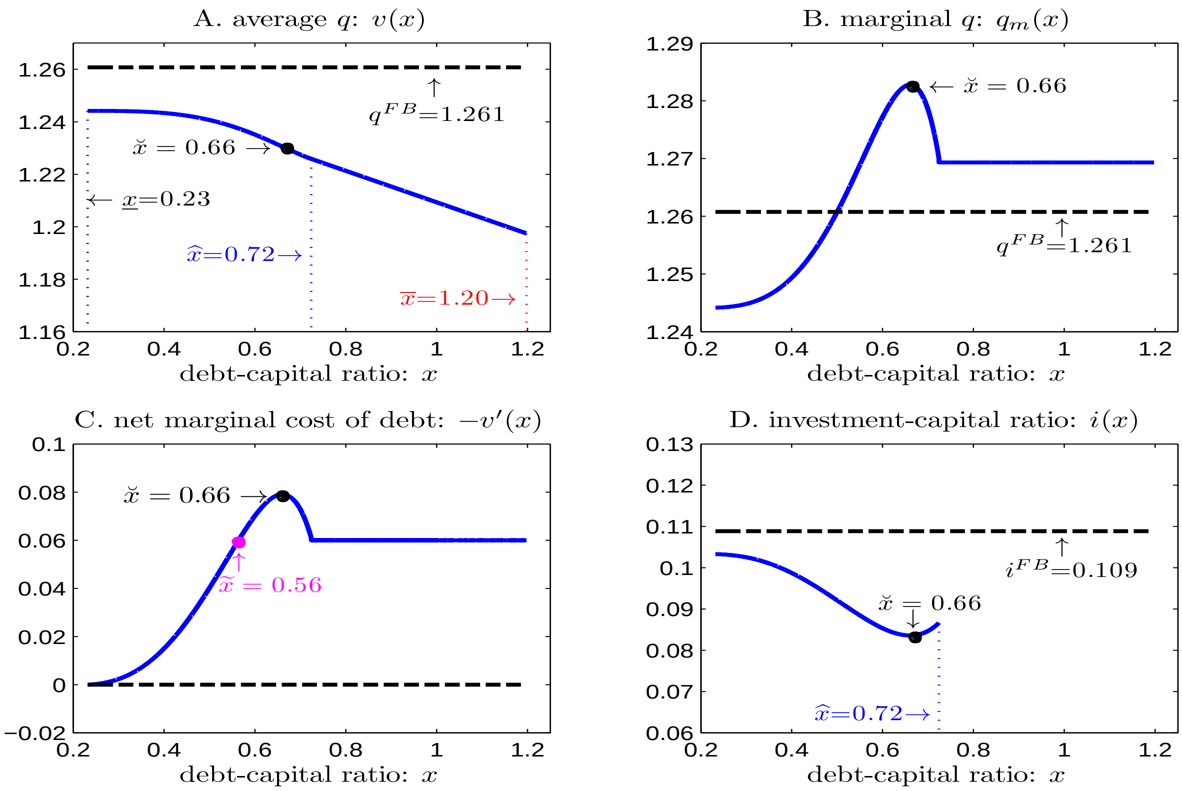


Figure 7: Tobin's average q ($v(x)$), marginal q, ($q_m(x)$), (net) marginal cost of debt ($-v'(x)$), and investment-capital ratio ($i(x)$). The first-best investment-capital ratio is $i^{FB} = 0.109$ and the first-best average q is $q^{FB} = 1.261$. The endogenous (lower) payout boundary is $\underline{x} = 0.23$ and the endogenous (upper) default boundary is $\overline{x} = 1.20$. The equity issuance boundary is $\widehat{x} = 0.72$ and the recapitalization target is $\widetilde{x} = 0.56$. Finally, the $v(x)$ inflection point is $\breve{x} = 0.66$, below which average q, $v(x)$, is concave and above which $v(x)$ is convex. All parameter values are given in Table 5.

7.2.2 Costly-equity-issuance Case

Table 6 reports the predicted distributions for the debt-to-capital ratio x and market leverage ML. The stationary distribution of x is similar to that for the EBIT-based model. Firms manage leverage prudently when equity issuance is costly. Average market leverage is 23.9%, the median market leverage is 25.9%, and only 5% of firms have leverage higher than 51.2%.

We plot the q model solution for $v(x)$ in Figure 7. There are four mutually exclusive regions divided by three endogenous thresholds (as in the EBIT-based model): 1.) the payout threshold, $\underline{x} = 0.23$, which corresponds to a target leverage of $ML(\underline{x}) = 18.7\%$; 2.) the equity-issuance threshold $\widehat{x} = 0.72$, which corresponds to a market leverage of $ML(\widehat{x}) = 59.1\%$; and 3.) the default threshold $\overline{x} = 1.20$. When market leverage is in the range of $(ML(\widehat{x}), ML(\overline{x})) = (59.1\%, 100\%)$, the firm recapitalizes by issuing equity to bring its market leverage down to $ML(\widetilde{x}) = 45.6\%$, where $v(\widetilde{x})$ is equal to the marginal cost of equity financing h_1 (i.e., $-v'(\widetilde{x}) = h_1 = 0.06$).

Average q versus marginal q. Panel A of Figure 7 plots the average q, $v(x) = p(x)+x$, which is nonlinear and decreases with x when the firm faces equity issuance costs. The gap between the first-best value $v^{FB} = 1.261$ and $v(x)$ reflects the expected discounted costs of equity issuance and financial distress.

Panels B of Figure 7 shows that the marginal q, $q_m(x)$, is non-monotonic: Marginal q increases in the region where $x \in (\underline{x}, \breve{x}) = (0.23, 0.66)$ and where firm value $v(x)$ is concave; it reaches the maximal value of 1.282 at the inflection point $\breve{x} = 0.66$ (for $v(x)$); it decreases in the region where $x \in (\breve{x}, \widehat{x}) = (0.66, 0.72)$ and where $v(x)$ is convex.

Note that marginal q is always larger than the average q:

$$q_m(x) - v(x) = -xv'(x) \geq 0\,. \tag{56}$$

This follows from $-v'(x) \geq 0$ and $x > 0$. An additional unit of capital reduces leverage, so that the firm is less financially constrained, and marginal q is larger than the average value of capital (average q). The solid blue line in Panel B is above the solid blue line in Panel A for all levels of x. Marginal q and average q coincide only at the payout boundary where $x = \underline{x}$: $q_m(\underline{x}) = v(\underline{x}) = 1.244$.

Endogenous risk aversion versus risk seeking. Panel C of Figure 7 shows that the marginal cost of debt, $-v'(x)$, increases in the region where $x \in (\underline{x}, \breve{x}) = (0.23, 0.66)$; it reaches the maximal value of 0.08 at the inflection point $\breve{x} = 0.66$; and it decreases in the region where $x \in (\breve{x}, \widehat{x}) = (0.66, 0.72)$. Note that $-v'(x)$ has the same monotonicity property as $q_m(x)$. This is because $q_m'(x) = -xv''(x)$ and the firm is levered ($x > 0$). In sum, the firm is endogenously risk averse ($v''(x) < 0$) in the region where $x \in (\underline{x}, \breve{x})$ and endogenously risk seeking ($v''(x) > 0$) in the region where $x \in (\breve{x}, \widehat{x})$.

Corporate investment $i(x)$. Panel D of Figure 7 shows that $i(x)$ is at its highest level when the firm is at its payout boundary: $\underline{i} = i(\underline{x}) = 0.103$, decreases with x in the concave region for $v(x)$, reaches the minimal value of 0.083 at the inflection point $\breve{x} = 0.66$, and increases with x in the convex region for $v(x)$. There is no meaningful investment policy $i(x)$ in the equity-issuance region, where $x > \widehat{x} = 0.72$, as the firm immediately leaves this region by recapitalizing and bringing x back to the *recapitalization target*: $\widetilde{x} = 0.56$.

Investment and marginal q move in exactly the opposite direction with respect to x in both the concave and convex regions for $v(x)$.[44] Note that this is the opposite relation from that in the neoclassical q-theory of investment (e.g., Hayashi, 1982) where investment increases with marginal q. The intuition for this result is as follows: First, in a world where the firm is financially constrained (e..g, due to costly equity issuance), marginal q not only depends on the firm's investment opportunity but also on the firm's balance sheet (e.g., leverage). Second, investment for a financially constrained firm is determined by the ratio between marginal q and the marginal cost of debt ($-v'(x) > 0$), as shown in (53). Differentiating (53) with respect to x, and using the concavity of $\psi(\,\cdot\,)$, we obtain:[45]

$$i'(x) = -\frac{(\psi'(i(x)))^2}{\psi''(i(x))}\frac{\partial}{\partial x}\left[\frac{q_m(x)}{-p'(x)}\right] = \underbrace{-\frac{(\psi'(i(x)))^2}{\psi''(i(x))}\frac{p(x)}{(p'(x))^2}}_{\text{positive as } \psi(\,\cdot\,) \text{ is concave}} p''(x), \tag{57}$$

so that investment is decreasing in x when $p''(x) < 0$ and increasing in x when $p''(x) > 0$.

Finally, the firm immediately recapitalizes by issuing equity in the region where $x \in (\widehat{x}, \overline{x}) = (0.72, 1.20)$, to bring its debt-capital ratio back to $\widetilde{x} = 0.56$. This equity issuance region is represented by the flat blue line in Panels B and C. The firm only enters into this region following a jump loss, and immediately exits the region, barely spending any time there.

Summary. As the solution shown in Figure 7 illustrates, investment and leverage dynamics exhibit both *debt overhang* (Myers, 1977) and *risk shifting* (Jensen and Meckling, 1976) features, but not for the usual reasons. Investment policy serves two roles, value creation (through capital accumulation) and prudent leverage management (via underinvestment) to avoid a costly equity issue or default.

[44] Bolton, Chen, and Wang (2011) derive similar results about investment and marginal q. Key features of our model that differ from that paper include 1.) debt capacity and credit risk pricing which are endogenous in our model but exogenous in BCW; 2.) shocks are permanent in our model but transitory (i.i.d.) in BCW, and 3.) the convex region for $v(x)$ where the firm is a risk seeker, which is not present in BCW.

[45] In the concave $v(x)$ region, marginal q increases with x at a slower rate than the marginal cost of financing; investment thus decreases with marginal q. In the convex $v(x)$ region, marginal q decreases with x at a slower rate than the marginal cost of financing, so that investment $i(x)$ increases with marginal q.

Debt overhang is driven by liquidity management considerations here. When leverage is low or moderate ($\underline{x} \leq x \leq \breve{x}$), the firm's marginal cost of debt is low. But the marginal cost of debt rises with leverage as there is then a greater chance that the firm will be pushed into a costly recapitalization or a default. As a result the higher the firm's debt-capital ratio x, the more the firm underinvests: $i'(x) < 0$.

Beyond the inflection point of $v(x)$, risk-shifting incentives kick in, but not because losses are borne disproportionately by debt holders (debt is short term and always fairly priced). Risk shifting is caused by the fixed costs of equity issuance that the firm is likely to incur in the foreseeable future. Anticipating that the firm's leverage will be much lower after recapitalization (and as a result the capital stock will be much more valuable; i.e., less debt overhang) the firm's incentive to mitigate underinvestment when $x \geq \breve{x}$ becomes stronger. It is thus optimal for the firm to preemptively reduce its underinvestment and build a larger capital stock in the convex region for $v(x)$.

Finally, we note that all other key predictions of the EBIT-based model remain valid in the q-based investment model.

8 Related Literature

Our emphasis on costly equity financing is a key departure from the contingent-claim capital structure theory literature following Fischer, Heinkel, and Zechner (1989) and Leland (1994, 1998).[46] This literature typically represents corporate debt as perpetual risky or term debt with a geometric amortization schedule, and assumes that this term debt is costly to adjust, but that equity is costless to issue. Therefore, the marginal source of financing in these models on a on-going basis is by assumption equity. The firm retains no earnings, continuously issues equity, and makes payouts to shareholders, which are all counterfactual predictions. Moreover, this class of models predicts that active changes in debt policy are such that firms increase debt in response to higher firm value to achieve a higher target, which is also not borne out empirically.[47] Finally, most of the models taking this approach have no investment decisions, which is another important focus of our paper.

Abel (2018) considers a dynamic tradeoff model with short-term debt and a stationary earnings process that is constant for a random duration of time and changes to a new value (drawn from a time-invariant distribution) at a stochastic moment governed by a

[46] Kane, Marcus, and McDonald (1984) is an early important contribution. Goldstein, Ju, and Leland (2001) generalize Leland (1994) to allow for dynamic recapitalization with costly debt issuance. Strebulaev (2007) emphasizes the role of liquidity ratios on dynamic capital structure decisions.
[47] For example, see Section 6 in DeAngelo, DeAngelo, and Whited (2011).

Poisson process. He assumes that the firm can issue equity at no cost and hence there is no need to retain earnings. Although Abel (2018) shares our assumption of short-term debt financing, he assumes that earnings shocks are transitory, whereas in our model earnings shocks are *permanent* (as in Black-Scholes-Merton and the contingent-claims literature). Unlike Abel (1998), we mainly focus on the effects of costly equity issuance on leverage dynamics, which we also extend to a q-theoretic setting featuring investment dynamics.

Another related set of dynamic capital structure models is the debt ratcheting literature based on debt dilution incentives. DeMarzo (2019) and DeMarzo and He (2021) analyze equilibrium leverage dynamics in a Leland-style trade-off model with stochastically amortizing term debt, in which the firm can continuously adjust leverage. Because of the incentives to dilute old debt by accumulating new debt, firms never choose to actively reduce leverage in their model. Moreover, the lack of commitment not to dilute existing debt dissipates all value gains from adjusting leverage in equilibrium (the intuition is similar to the Coase conjecture in the context of durable-goods monopolists.) Unlike these models with non-exclusive term debt, our model features short-term debt that is rolled over and for that reason cannot be diluted.[48] .

Our model is also related to the discrete-time, dynamic, capital structure models with investment (with decreasing returns to scale) developed by Cooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2005, 2007), Gamba and Triantis (2008), and DeAngelo, DeAngelo, and Whited (2011). An important methodological difference of our analysis is the continuous-time formulation of our problem, which yields sharper predictions and characterizations of the firm's non-linear, non-monotonic, state-contingent, path-dependent leverage, payout, equity issuance, and corporate investment policies.[49] For example, we can precisely characterize market leverage dynamics into four mutually exclusive regions, with intuitive boundary conditions. Our analysis reveals that a single target leverage is a misleading notion to describe corporate policies. A more accurate description for leverage policy is that there is a target zone for leverage and that the leverage target for payout is different from the optimal recapitalization target. Besides the different assumptions about returns to scale, other differences are assumptions about

[48] This is because either the firm declares default or rolls over its debt. The term debt in debt-ratcheting models with a shrinking maturity to zero (in the limit) yields a different type of short-term debt from the short-term debt that is continuously rolled over in our model. There is no time gap between the issuance of the debt and its repayment in our model such that new shorter-maturity debt can be issued in-between this gap that dilutes outstanding debt, as in debt-ratcheting models. Another difference is that debt issuance in ratcheting model is smooth but is stochastic in our model.

[49] Moreno-Bromberg and Rochet (2018) in their textbook and Brunnermeier and Sannikov (2016) in their survey discuss the advantages of a continuous-time modeling approach.

productivity shocks, which in these models are often assumed to be transitory, whereas our model features permanent shocks. Another important difference is our assumption of downward jump shocks for leverage dynamics and financial management. As a result of these differences, we generate new predictions that are absent in these models. Our constant returns to scale assumption (combined with capital adjustment costs) naturally yields the prediction that firm size grows exponentially with a stochastic, endogenous drift. This prediction along with endogenous default generates an empirically plausible (fat-tailed) power distribution for firm size[50], which the decreasing-returns-to-scale-based investment models cannot generate.

Another related literature is the continuous-time, corporate liquidity, and risk management literature following Decamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011), Hugonnier, Malamud, and Morellec (2015), and Abel and Panageas (2020). This literature focuses on cash and corporate liquidity management but ignores leverage dynamics.[51] Also earnings shocks in these models are transitory, so that they cannot generate empirically plausible firm size distributions.

More broadly, our model is also related to the dynamic contracting and optimal dynamic security design literature following DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin, and Rochet (2007), DeMarzo and Fishman (2007), Biais, Mariotti, Rochet, and Villeneuve (2010), and Malenko (2019).[52] The optimal contracts in these models can be implemented via a combination of debt, (inside and outside) equity, and corporate liquidity.

9 Conclusion

We have developed a tractable dynamic capital structure model for a firm facing costly external equity that delivers sharp predictions that are in line with empirical evidence. Although parsimonious, our model yields rich and empirically valid predictions that contribute to our understanding of the key role played by equity issuance costs in determining corporate leverage dynamics, earnings retention, payouts, equity issuance, and investment policies observed in practice. Our model also explains why corporate leverage on average is low, and is subject to substantial time-series variation. Leverage dynamics are highly nonlinear and non-monotonic – leverage stochastically drifts towards

[50] Luttmer (2007) develops a tractable model of balanced growth consistent with this size distribution of firms. Gabaix (2009) and Luttmer (2010) survey the literature on the power law and firm dynamics.
[51] When they include debt, these models assume an exogenous debt capacity with risk-free debt. There is no notion of target leverage nor of nonlinear/nonmonotonic leverage dynamics (see Bolton, Chen, and Wang 2011).
[52] Biais, Mariotti, and Rochet (2013) and Sannikov (2013) provide surveys of this literature.

the target level when it is relatively low, but is likely to diverge into a debt death-spiral when it is relatively high, forcing the firm to delever through a substantial recapitalization.

Paradoxically, the explanation for the observed low corporate leverage is not that debt is costly, but that equity is costly. One would think that when equity is costly firms would want to rely more on debt. But that is a static intuition. From a dynamic perspective firms seek to avoid costly equity issuance in the future and therefore maintain financial slack today by keeping leverage low. When equity issuance is extremely costly, the firm barely takes on any debt.

There has been a long-running and still unresolved empirical debate between the tradeoff and pecking-order theories of capital structure since Myer's 1984 AFA presidential address.[53] Our model, in effect, combines insights from both theories by adding equity issuance costs as in Myers and Majluf (1984) to a dynamic tradeoff model with persistent earnings shocks. Our predictions, however, differ from both these static theories in very important ways, such as the highly nonlinear, non-monotonic, path-dependent leverage dynamics, endogenously risk averse and risk seeking behaviors, and the occasional recapitalization through equity issuance to bring down leverage.

[53] See Fama and French (2002) and Frank and Goyal (2008) for example.

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Internet Appendices

A Costless Equity Issuance Benchmark

In this appendix, we first solve the costless-equity-issuance model via backward induction in three steps and then derive the adjusted present value (APV) formula.

A.1 Solution

Step 1: Equity holders' default decision when debt is due. Since default is costly and equity/debt issuance is costless, shareholders will only use default as a last resort.[54] Given the debt-EBIT ratio x_{t-} at time $t-$, suppose that a jump arrives at t causing the firm's EBIT to drop from Y_{t-} to $Y_{t-}Z$, where $Z \in [0,1]$ is a random draw from the distribution function $F(Z)$. The post-jump debt-EBIT ratio then mechanically increases to $x_t^{\mathcal{J}} = x_{t-}/Z$. Let $\overline{x}$ denote the post-jump debt-EBIT ratio x at which the firm is indifferent between defaulting or not following a jump arrival. As the firm's EBIT growth dY_t/Y_{t-} is *i.i.d.*, this default threshold $\overline{x}$ is invariant over time. Let $\overline{Z}(x_{t-})$ denote the corresponding default threshold for the EBIT recovery fraction upon a jump arrival. Rewriting $x_t^{\mathcal{J}} = x_{t-}/Z$ with $x_t^{\mathcal{J}} = \overline{x}$ and $Z = \overline{Z}(x_{t-})$, we obtain

$$\overline{Z}_t = \overline{Z}(x_{t-}) = x_{t-}/\overline{x}\,, \quad \text{for } \; x_{t-} \leq \overline{x} \tag{58}$$

which characterizes the default threshold.

Since it is costless to adjust both debt and equity at any time before default, the firm's total value $v(x) = p(x) + x$ must be constant for all values of $x \leq \overline{x}$. This is because the firm can always adjust x to x^*, the level that maximizes $v(x)$, which we formalize in Step 3. Moreover, since equity value is zero at default ($p(\overline{x}) = 0$) and $v(x)$ is constant for all $x \leq \overline{x}$, we have

$$v(x_{t-}) = \overline{x}\,, \quad \text{for } \; x_{t-} \leq \overline{x}\,. \tag{59}$$

Step 2: Equilibrium credit spread and the firm's enterprise value. Given the default threshold $\overline{Z}_t = \overline{Z}(x_{t-})$ and the firm's liquidation recovery value $L_t = \ell Y_t$, the expected liquidation value upon default is given by

$$\mathbb{E}_{t-}\left(L_t\, \mathbf{1}_t^{\mathcal{D}}\right) = \mathbb{E}_{t-}\left(\ell Y_t\, \mathbf{1}_{Z<\overline{Z}_t}\right) = \ell Y_{t-}\, \mathbb{E}_{t-}(Z\, \mathbf{1}_{Z<\overline{Z}_t}) = \ell Y_{t-} \int_0^{\overline{Z}_t} Z dF(Z)\,. \tag{60}$$

The equilibrium credit spread η_{t-} satisfies the following pricing condition:

$$X_{t-}(1 + rdt) = (X_{t-} + C_{t-}dt)\, \left[1 - \lambda \mathbb{E}_{t-}\left(\mathbf{1}_t^{\mathcal{D}}\right) dt\right] + \mathbb{E}_{t-}\left(L_t\, \mathbf{1}_t^{\mathcal{D}}\right) \lambda dt\,, \tag{61}$$

where C_{t-} is the equilibrium interest payment for the debt issued at date $t-$:

$$C_{t-} = (r + \eta_{t-})X_{t-}\,, \tag{62}$$

Further, combining (60) with the competitive debt market pricing condition (61), we obtain the following equation for the credit spread $\eta_{t-} = \eta(x_{t-})$:

$$\eta(x_{t-}) = \lambda \left[F(\overline{Z}(x_{t-})) - \left(\frac{\ell}{x_{t-}}\right) \int_0^{\overline{Z}(x_{t-})} Z dF(Z)\right]\,. \tag{63}$$

[54] It is optimal for equity holders to repay the existing debt when debt matures absent a jump arrival. This is because diffusion shocks are locally continuous and it is more efficient to issue equity and/or roll over debt in response to diffusion shocks.

This equation ties the equilibrium credit spread to the firm's default strategy and the pre-default debt-EBIT ratio x_{t-}. The credit spread is lower than the probability of default $\lambda F(\overline{Z}(x_{t-}))$ as creditors' recovery in default is non-negative, $\ell \geq 0$.[55]

The homogeneity property and the costless-equity-issuance assumption together imply that the optimal leverage policy is time invariant. A solvent firm that sets the debt level to x at all time then has an enterprise value of $v(x) = p(x) + x$ given by:

$$v(x) = \frac{1}{\gamma - g}\left[1 + (\gamma - r)x - \theta(c(x)) - \lambda(v(x) - \ell)\left(\int_0^{\overline{Z}(x)} ZdF(Z)\right)\right], \tag{64}$$

where g is the expected EBIT growth rate given by $g = \mu - \lambda(1 - \mathbb{E}(Z))$, $\theta(\cdot)$ is the scaled tax payment given by $\theta(c) = \tau(1-c)\mathbf{1}_{c<1}$, $\ell = \alpha\pi = \alpha/(r-g)$ is the liquidation recovery per unit of EBIT, and the default threshold $\overline{Z}(x)$ is given in (58).

Step 3: Optimal leverage choice. Equityholders choose x to maximize the firm's enterprise value by solving the following problem:

$$\max_x \ v(x)\,. \tag{65}$$

Note that this optimization problem takes into account equityholders' default option after debt is issued and the *ex ante* equilibrium credit spread. Let x^* denote the optimal level of x for the optimization problem defined in (65).

Next, we derive an adjusted present value (APV) formula (Myers, 1974), which decomposes the costs and benefits of debt financing in an intuitive way, and offers an equivalent representation of $v(x)$ given in (64) and (65).

A.2 Adjusted Present Value (APV)

Debt financing benefits have two components: tax shields and cheaper debt financing. Let $pb(x)$ denote the (scaled) present value of debt financing benefits:

$$pb(x_t) = \frac{1}{Y_t}\mathbb{E}_t\left[\int_t^{T^{\mathcal{D}}} e^{-\gamma(s-t)}[\underbrace{(\tau - \theta(c(x_s)))\,Y_s}_{\text{tax shields}} + \underbrace{(\gamma - r)X_s}_{\text{cheaper debt}}]ds\right]. \tag{66}$$

Similarly, let $pc(x)$ denote the (scaled) present value of financial distress costs. Upon default, the firm's creditors receive a liquidating payoff $L_{T^{\mathcal{D}}} = \ell Y_{T^{\mathcal{D}}}$ but the firm permanently loses its unlevered continuation value $V(Y_{T^{\mathcal{D}}}, 0)$. Therefore, the realized cost of financial distress at $T^{\mathcal{D}}$ is the difference between $V(Y_{T^{\mathcal{D}}}, 0)$ and $L_{T^{\mathcal{D}}}$, and the PV of distress cost is then given by:

$$pc(x_t) = \frac{1}{Y_t}\mathbb{E}_t\left[e^{-\gamma(T^{\mathcal{D}}-t)}\left(V(Y_{T^{\mathcal{D}}}, 0) - \ell Y_{T^{\mathcal{D}}}\right)\right]. \tag{67}$$

To gain further insight into the APV formula, it is helpful to introduce the truncated EBIT process, $\widehat{Y}_t$ such that for $t < T^{\mathcal{D}}$, $\widehat{Y}_t = Y_t$, but for $t \geq T^{\mathcal{D}}$, $\widehat{Y}_t = 0$. Let $\widehat{g}_{t-} = \widehat{g}(x_{t-})$ denote the

[55] For the special case where creditors recover nothing upon default ($\ell = 0$), $\eta = \lambda F(\overline{Z}(x_{t-}))$.

expected EBIT growth rate for this truncated EBIT $\widehat{Y}_t$ process:[56]

$$\widehat{g}(x_{t-}) = \frac{1}{dt}\mathbb{E}_{t-}\left(\frac{d\widehat{Y}_t}{\widehat{Y}_{t-}}\right) = \mu - \lambda\left(1 - \int_{\overline{Z}(x_{t-})}^{1} ZdF(Z)\right) . \tag{68}$$

Solving (66), we obtain the value of debt financing benefits:

$$pb(x) = \frac{(\tau - \theta(c(x))) + (\gamma - r)x}{\gamma - \widehat{g}(x)} , \tag{69}$$

The numerator on the right side of (69) captures the two flow benefits of debt financing: 1.) $(\gamma - r)x$ due to shareholders' impatience and 2.) the tax shield, which is $(\tau - \theta(c(x)))$ per period. When the firm makes a profit ($Y > C$ and equivalently $c(x) < 1$), the tax shield is $\tau - \tau(1 - c(x)) = \tau c(x)$, where $c(x)$ is the scaled interest payment. This is the standard tax deduction of interest payments. When the firm incurs a loss, the firm pays no taxes ($\theta(c(x)) = 0$) and hence the tax shield is maximized at τ per unit of EBIT. The denominator uses the shareholders' discount rate γ and the growth rate for the truncated EBIT process, as debt financing benefits accrue to shareholders but stop when the firm defaults.

Using (58) and (59) and solving (67), we obtain the present value of financial distress costs:

$$pc(x) = \frac{\lambda(v(0) - \ell) \int_0^{x/v(x)} ZdF(Z)}{\gamma - \widehat{g}(x)} , \tag{70}$$

where $\widehat{g}(x)$ is given in (68). The numerator on the right side of (70) is the flow loss of firm value upon default. The firm only defaults and hence incurs losses when $Z < \overline{Z}(x) = x/v(x)$, which is the upper limit of the integral. The denominator for $pc(x)$ is the same as that for $pb(x)$.

Rewriting (64), we obtain the following intuitive APV formula for firm value:

$$v(x) = v(0) + pb(x) - pc(x) , \tag{71}$$

where the first term in (71) is the unlevered value of the firm

$$v(0) = \frac{1 - \tau}{\gamma - g} , \tag{72}$$

the second term $pb(x)$ is given by (69), and the third term $pc(x)$ is given by (70). The APV formula (71) is an implicit function of $v(x)$, which takes into account the endogenous default threshold $\overline{Z}(x) = x/v(x)$, the equilibrium interest payment $c(x)$, and the tax payment $\theta(c(x))$.

A.3 Summary

In sum, the APV formula (71) states that the firm's value $v(x)$ is given by the sum of the unlevered enterprise value (the first term) and the PV of debt financing benefits (the second

[56] Note that $\widehat{g}_{t-} = \widehat{g}(x_{t-})$ is lower than the EBIT growth rate $g = \frac{1}{dt}\mathbb{E}_{t-}\left(\frac{dY_t}{Y_{t-}}\right) = \mu - \lambda\left(1 - \int_0^1 ZdF(Z)\right) = \mu - \lambda(1 - \mathbb{E}(Z))$ for an unlevered firm. This is because $\widehat{Y}_t = 0$ when a jump with $Z < \overline{Z}(x_{t-})$ arrives and triggers the levered firm to default. In contrast, the firm if unlevered would continue to operate regardless of the size of Z.

term) minus the PV of financial distress costs. (the third term). The optimal debt-EBIT ratio x^* solves the problem defined in (65), which is to choose x to maximize the implicit equation $v(x)$ given in (71) , (25), (69), and (70).

Optimal market leverage is $ML^* = x^*/v(x^*)$, where $v(x^*)$ is the firm value under optimal leverage. The firm only defaults when a jump arrives and the realized recovery fraction Z is lower than $\overline{Z}(x^*) = x^*/v(x^*) = x^*/\overline{x}$. The *ex post* default threshold $\overline{x}$ equals $v(x^*)$, which follows from 1.) firm value is constant for all values of x (as shareholders can always costlessly adjust leverage and therefore are indifferent between any two levels of x in this interval); 2.) by continuity, $v(x^*) = v(\overline{x}) = \overline{x} + p(\overline{x})$; and 3.) equity is worthless upon default: $p(\overline{x}) = 0$.

Finally, with no equity issuance costs, diffusion volatility σ has no effect on leverage. This prediction is inconsistent with the survey finding that CFOs consider earnings and cash flow volatility to be a major factor influencing debt policies (Graham and Harvey, 2001).

Before analyzing the impact of costly equity issuance on financing policies, we highlight a few key differences between our costless equity issuance model and the standard contingent-claim capital structure model of Leland (1994) and Goldstein, Ju, and Leland (2001).

First, debt is short term in our model but a perpetuity in Leland (1994) and term debt in Leland and Toft (1996). Second, the default mechanisms are different: in our costless-equity-issuance model the firm can adjust both its debt and equity continuously, so only a sufficiently large EBIT (jump) loss causes the firm to default, whereas in Leland (1994) the cost of adjusting debt after issuance at time 0 is essentially infinite and hence diffusion shocks can trigger default. Finally, the equilibrium credit spread in our model is always positive, as jumps are unpredictable but is zero just prior to default in Leland (1994) and other contingent-claim models with term debt and diffusion shocks.

B Simulation

We illustrate the joint dynamics of equity issuance, debt financing, payouts, and default along a simulated path. This allows us to see how leverage dynamics may unfold over time.

Panel A of Figure 8 displays the sample path of Y_t starting with $Y_0 = 1$. Panel B singles out the eight jumps (EBIT losses $(1 - Z_t)$) of the Y_t sample path in Panel A. These jumps occur at times $t = 0.55, 0.8, 1.23, 1.6, 2.11, 2.78, 3.28$, and 3.76. The corresponding percentage losses are $(1 - Z) = 0.33, 0.47, 0.1, 0.41, 0.2, 0.76, 0.15$, and 0.41. We include eight jumps of varying sizes to illustrate the richness of leverage dynamics. The last jump at $t = 3.76$ causes the firm to default.

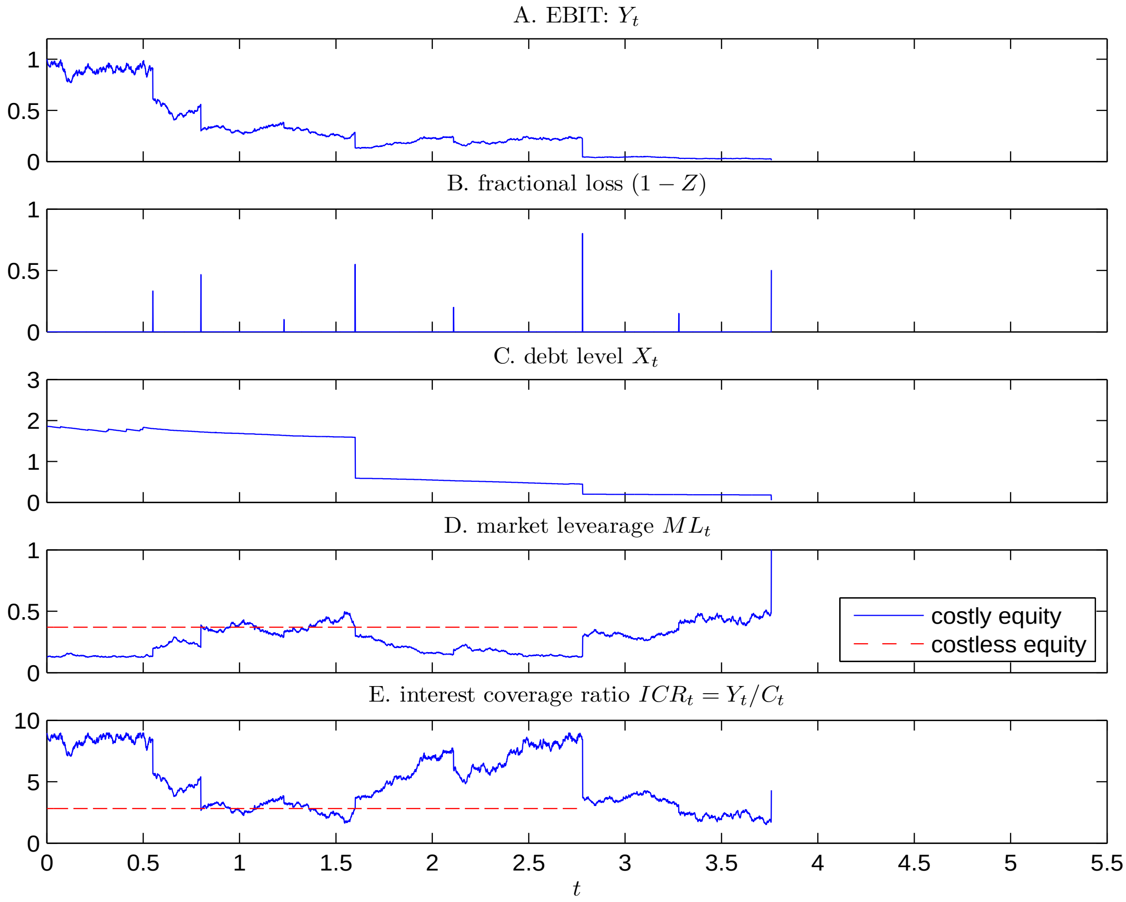

Figure 8: Simulation. Panel A displays a sample path of EBIT Y_t starting with $Y_0 = 1$. Panel B plots eight realized EBIT (percentage) losses for the path in Panel A. Panel C plots the firm's debt (X_t) dynamics. Panels D and E plot the implied market leverage (ML) and interest coverage ratio (ICR), respectively. The red dashed lines in Panels D and E correspond to the costless equity issuance case where $ML = 37\%$ and $ICR = 2.81$ for all t before default. The blue solid lines in Panels D and E are for the costly equity issuance case.

Dynamic evolution of debt X_t. The blue solid line in Panel C plots the evolution of debt X_t in response to realized jump-diffusion shocks to EBIT when equity issuance costs are strictly positive. The firm begins by setting its initial debt-to-EBIT ratio to the optimal target level $x_0 = \underline{x} = 1.86$. Since $Y_0 = 1$, this means that the firm borrows $X_0 = \underline{x}Y_0 = 1.86$ at $t = 0$ and makes a one-time dividend payment to shareholders.

Debt X_t drops discontinuously three times. These events represent two recapitalizations and a default, when the face value of debt is replaced by the liquidation value of the firm's assets. The first discrete debt adjustment occurs at $t = 1.6$ when the firm's EBIT drops by 55% (the fourth jump shock). The pre-jump debt-to-EBIT ratio is at 5.42, (corresponding to a 37% ML); the jump drop in EBIT then causes the debt-to-EBIT ratio to increase to 12, which exceeds the equity-issuance threshold of $\widehat{x} = 9.69$. The firm responds by recapitalizing its balance sheet, raising net proceeds $M_{1.6} = 1$, paying off some of its debt and bringing it down from 1.59 to $X_{1.6} = 0.59$. The gross amount of equity raised is 1.10, which includes issuance costs of $h_0 Y_{1.6} + h_1 M_{1.6} = 0.10$, causing an *equity dilution* of 80% for old shareholders. That is, the new shareholders brought in by the equity issue own 80% of the firm, leaving old shareholders with only 20%.[57] The third discrete debt adjustment occurs at $t = 3.76$ when the EBIT drops by 50%. Given that the pre-jump market leverage is 48.4%, this shock brings the debt-to-EBIT ratio to 14.22, now exceeding the default threshold of $\overline{x} = 14.01$, so that the firm defaults and equity holders are completely wiped out.

The other five jump shocks are small enough that the firm can absorb them through increased borrowing. Debt passively follows $dX_t = (C_{t-} + \Theta_{t-} - Y_{t-})dt$, so that X_t is continuous and smooth. Even when its EBIT drops by 47% at $t = 0.8$, the firm can respond through increased borrowing since its pre-jump market leverage is relatively low (at $ML_{0.8-} = 0.21$).

Costless equity issuance. When the firm faces no equity issuance costs, both market leverage and interest-coverage ratio are constant over time, as shown by the red dashed lines in Panels D and E, with $ML_t = x^*/v(x^*) = 37\%$ and $ICR_t = 1/c^* = 2.81$ until the firm defaults. The firm defaults at $t = 2.78$ because it is the first moment when the realized drop in EBIT (which is 80%) exceeds the default threshold $(1 - \overline{Z}^*) = 63\%$. The ICR may seem a bit low, but that is precisely because equity issuance is costless. We show next that when external equity is costly the firm aims for a much higher ICR.

Dynamics of market leverage ML_t and interest-coverage ratio ICR_t. Panels D and E respectively plot the ML and ICR dynamics under costly equity issuance (the blue solid lines) providing a sharp contrast against the ML and ICR dynamics when equity issuance is costless (flat red dashed lines.) Target market leverage at $t = 0$ is only 12.4%, which is about one third of the target market leverage (37%) when equity issuance is costless. Similarly, the corresponding ICR at $t = 0$ is 8.97, which is 3.2 times the ICR (2.81) when equity issuance is costless.

At $t = 0.55$, the 33% EBIT drop causes market leverage to leap from the pre-jump level of 0.13 to the post-jump level of 0.20. The firm absorbs this shock through increased borrowing, but the permanent negative earnings shock results in a large drop in market value.[58]

In contrast, the jump shock at $t = 1.6$, which causes a 55% decrease in EBIT, is so large that the firm responds with a recapitalization to bring market leverage down to $\widetilde{x}/v(\widetilde{x}) = 29.9\%$.

[57] The second discontinuous debt reduction occurs at time $t = 2.78$, when EBIT decreases by 80% (the sixth jump shock). Following the recapitalization, X_t decreases from 0.44 to 0.20 and again shareholders are significantly diluted.

[58] The jump at $t = 0.8$ causes a $(1 - Z) = 47\%$ drop of EBIT, and market leverage increases mechanically from the pre-jump level of 0.21 to the post-jump level of 0.39. The same passive debt rollover occurs at $t = 1.23$, $t = 2.11$, and $t = 3.28$.

Table 7: STATIONARY DISTRIBUTIONS OF x AND MARKET LEVERAGE ML. ALL PARAMETER VALUES ARE GIVEN IN TABLE 1.

	Mean	Std. dev.	5%	50%	95%
x	3.55	1.62	1.92	3.03	6.96
ML	0.239	0.112	0.129	0.203	0.476
$p(x)$	11.33	1.72	7.69	11.91	13.02

Absent the equity issue, the firm's debt-to-EBIT ratio would have risen to 12 and market leverage would have been 85%, which is suboptimally high.[59] Even after actively delevering, the post-SEO recapitalization target ($ML = 29.9\%$) is higher than the optimal target leverage of 12.4%. In sum, a firm facing external equity issuance costs lets its market leverage bounce around in response to EBIT shocks in the target zone, passively rolling over its debt most of the time, and only resorts to equity issuance when its leverage spirals out of control (when $ML \geq ML(\widehat{x}) = 67.9\%$). Beyond that point the firm actively deleverages to bring its ML to 29.9%.

Next we compute the long-run distribution of x and ML using 100,000 simulated paths.[60]

Distribution of debt-to-EBIT ratios and market leverage. Table 7 reports the mean, the standard deviation, and the quantile distribution of x and $ML(x)$. Under our baseline parameter values, average market leverage is 24%, which is within the range of 20% – 25% for COMPUSTAT firms reported in Strebulaev and Whited (2012), and median leverage is 20%, with a standard deviation of 11%. The vast majority of firms has low market leverage. For example, only 5% of firms have market leverage larger than 48%.

These calculations further illustrate how incorporating external equity issuance costs into a classical dynamic tradeoff theory dramatically changes leverage dynamics, makes the firm much more conservative with its debt policy, and yields more empirically plausible predictions about market leverage and ICR.

[59] Similarly, the jump at $t = 2.78$ causes a 80% decrease of EBIT, which prompts the firm to issue equity to bring down market leverage again to $\widetilde{x}/v(\widetilde{x}) = 29.9\%$.

[60] Each path starts with the target leverage $ML(\underline{x}) = 12.4\%$ and ends when either the firm defaults or it reaches 100 years.