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ABSTRACT

The H-1B Visa Reform Act of 2004 dictates an annual allocation of 85,000 visas with 20,000 reserved for advanced-degree applicants. We represent the main requirements of this legislation as formal axioms and characterize visa allocation rules consistent with the axioms. Despite the precise number reserved, we show that the range of implementations satisfying these axioms can change the allocation of advanced-degree visas by as much as 14,000 in an average year. Of all rules satisfying these axioms, the 2019 rule imposed by executive order is most favorable to advanced-degree holders. However, two earlier modifications resulted in larger changes, potentially unintentionally.

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1 Introduction

Since its introduction in the Immigration Act of 1990, the United States (U.S.) H-1B program has enabled American companies to temporarily employ educated foreign workers in occupations that require specialized knowledge.¹ As it is the largest temporary employment program for skilled immigration in the U.S., the H-1B program has drawn sustained attention from economists. This research has addressed the impact of skilled immigration programs on students (Kato and Sparber, 2013; Amuedo-Dorantes and Furtado, 2017), workers (Peri, Shih, and Sparber, 2015; Mayda, Ortega, Peri, Shih, and Sparber, 2018), firms (Doran, Gelber, and Isen, 2014; Kerr, Kerr, and Lincoln, 2015; Mayda, Ortega, Peri, Shih, and Sparber, 2020; Glennon, 2020), innovation (Kerr and Lincoln, 2010; Dimmock, Huang, and Weisbenner, 2021), and more. Papers within this literature have analyzed the consequences of legislated H-1B reforms, and some have assessed of the value of the program as a whole (e.g., Bound, Khanna, and Morales, 2017; Chassamboulli and Peri, 2018).

In this paper, we contribute to this literature by analyzing this program from the perspective of market design. We model the H-1B system in a manner inspired by recent market design research on reserve systems (Kominers and Sönmez, 2016; Dur, Kominers, Pathak, and Sönmez, 2018). Within this framework we formalize the main requirements of H-1B legislation with three simple axioms. This model allows us to characterize the range of visa allocation rules permitted under the minimal requirements of the law and to demonstrate that significantly different visa allocations may occur across this range. Our analysis allows us to identify important policy levers available within the H-1B system—levers that have often been ignored in both academic and political debate, and which have been modified in recent years in important ways.

Our model describes the system put in place by the H-1B Visa Reform Act of 2004.

¹Examples include employment in engineering, computer science, or other STEM fields.

This act set forth visa allocations for two classifications of applicants: 65,000 visas are made available for all eligible applicants and an additional 20,000 visas are reserved for applicants with a master's degree or higher academic credential from eligible U.S. institutions.² Popular discussions refer to these quotas as caps; we call the 20,000 visas for applicants with U.S. advanced degrees the *reserved cap* and the 65,000 visas available for all applicants the *unreserved cap*. Although the size of the reserved cap was precisely specified and prominently advertised, details on how the cap should be implemented were left to the discretion of the U.S. Customs and Immigration Service (USCIS) (Federal Register, 2005, 2018).³ The central focus of our analysis will be understanding how different means of implementation affect the number of visas granted to those who have documented an eligible degree in their H-1B petition (*reserved-category* applicants) and to those who did not (*general-category* applicants).

We model the set of visa allocation rules that are potentially in compliance with the statute as satisfying three simple axioms. First, they *accommodate the reserve policy* by restricting access to reserved visas to reserved-category applicants only. Second, they are *non-wasteful*, a minimal efficiency requirement that ensures visas do not go unused while there is eligible unmet demand. That means a general-category applicant should not be denied a visa unless all unreserved visas are exhausted; similarly, a reserved category applicant should not be denied a visa unless all visas (reserved or unreserved) are exhausted.⁴ Finally, they *respect priorities*, which means an applicant qualified for a visa should not lose it to a candidate who has lower priority for that visa. When a visa allocation rule

²When discussing "applicants" we refer to the workers that would receive these visas. Note, however, that H-1B system is "employer driven," meaning that companies initiate the application process on behalf on the workers that they want to employ (for further discussion, see Kerr and Kerr, 2020). Also note that H-1Bs associated with certain employers (e.g., institutions of higher education, research- or education-related nonprofits, and government research organization) do not count towards the legislated caps.

³According to the Federal Register, "Congress did not specify any procedures for implementation or dictate the manner in which USCIS should allocate H-1B numbers made available pursuant to the new exemption" (Federal Register Vol. 70, No. 86, Thursday, May 5).

⁴Throughout the paper, an applicant refers to one who is qualified for a visa. Of course, unqualified applicants can be denied without violating the non-wastefulness property.

satisfies all three properties, we say it *complies with the statute*.⁵

Within this framework, we compare the four visa allocation rules that have been implemented in the H-1B program since fiscal year (FY) 2005 (summarized in Table 1). These rules differ in how they determine priority—changing between determining priority by arrival time versus by random lotteries—and in the order in which they assign visas toward the reserved or unreserved caps. Despite these differences, these rules all comply with the statute as defined above. Our results characterize how these rules allocate visas between general- and reserved-category applicants and document significant differences. Across the visa rules that comply with the statute, the advantage given to applicants with an advanced degree (henceforth "*skill bias*") may be significantly altered by certain design decisions, and since FY2005 the H-1B program has at times enacted policies that either maximize and minimize this skill bias.

[Table 1 about here]

After establishing our main theoretical results, we reexamine the recent administrative history of the H-1B program. The potential importance of the issues that we model first became apparent in former President Trump's *Buy American and Hire American Executive Order* in 2017, which instructed the U.S. Department of Homeland Security to propose reforms to ensure that H-1B visas are awarded to the most-skilled or highest-paid petition beneficiaries. This declaration led to an adoption of a new visa allocation rule for FY2020, and the process of its enactment demonstrated some degree of understanding of the theoretical forces we have described. For example, a government press release stated:

"Currently, [...] the advanced degree exemption is selected prior to the H-1B cap.

The proposed rule would reverse the selection order and count all registration or

⁵We use the terminology "complies with statute" to summarize compliance with these three axioms, not to formally take a position on the legal status of specific proposals. Of course, the legislation lays out additional requirements that must be satisfied. We believe that the model we present captures the legal requirements of core importance to the final allocation, so we abstract from any other legal requirements to facilitate parsimonious analysis.

petitions towards the number projected as needed to the reach the H-1B cap first. Once a sufficient number of registration or petitions have been selected for the H-1B cap, USCIS would then select registration or petitions towards the advanced degree exemption."

"The proposed process would result in an estimated increase of up to 16 percent (or 5,340 workers) in the number of selected H-1B beneficiaries with master's degree or higher." (USCIS, 2018)

Quotes like these indicate that the USCIS correctly identified the role of processing order and then proposed a reform to it that maximizes skill bias in our analysis. This reform may therefore serve as an example of relatively informed and deliberate policy-making. However, given the sophistication demonstrated by the administration and the public attention surrounding this reform, the events surrounding comparable prior changes are more surprising. Our results establish that the allocation rule initially deployed after the H-1B Visa Reform Act of 2004 achieved the same degree of skill bias as the Trump administration's reform, and that administrative changes that occurred in FY2006 and FY2009 result in larger year-to-year changes to the degree of skill bias than did the FY2020 reform. Unlike the FY2020 reform, these prior reforms were not motivated by a desire to influence skill bias, and indeed were only minimally publicized or publicly scrutinized. Based on the discussions available in the Federal Register, it appears that these earlier reforms were driven simply by logistical considerations, and their significant impact on skill bias may have not been broadly appreciated.

This paper proceeds as follows. The next subsection discusses related literature. Section 2 defines the visa allocation problem, presents our axioms, and formally defines the four visa allocation rules that were summarized in Table 1. Section 3 presents our formal results characterizing the performance of the different visa allocation rules. Section 4 reexamines the recent history of H-1B reforms through the lens of the theory we have established. The last section concludes. All proofs are in the appendix. A secondary appendix contains additional information describing the four visa allocation rules, as presented in the Federal Register.

1.1 Related Literature

Our paper relates and contributes to the literatures on market design and on the H-1B system.

Relation to Literature on Market Design

Within the market-design literature, our work is closely related to the growing literature on reserve systems. The role of processing order of different types of positions was first studied by Kominers and Sönmez (2016) in an abstract framework, and later by Dur, Kominers, Pathak, and Sönmez (2018) in the context of school choice. These paper emphasize that reserve size is insufficient for describing a reserve policy when the order of processing is not explicit. Dur, Kominers, Pathak, and Sönmez (2018) further establishes formal comparative static results comparing changes in reserve sizes to changes in the processing order of reserves. It also establishes two conceptual forces driving these differences, labeled *processing bias* and *random number bias*, that help one understand the importance of reforms to processing order and to the priority system in the H-1B context. Similar forces are relevant in the work of Dur, Pathak, and Sönmez (2020), which considers place-based affirmative action in Chicago and characterizes optimal and constrained optimal implementation of reserve policies in the presence of multiple reserve groups. These forces are also at play in the work of Sönmez and Yenmez (2021), which studies the vertical and horizontal reservation policies present in India's constitutionally mandated affirmative action system.⁶ To eliminate a number of anomalies arising from a mechanism mandated by the Indian Supreme Court in 1995, Sönmez and Yenmez (2021) propose a mechanism that integrates horizontal and vertical reservation policies when all positions are identical.⁷ Sönmez and Yenmez (2020) further extend this analysis for field applications with position heterogeneity. Other related papers on reserve systems include Hafalir, Yenmez, and Yildirim (2013), Aygün and Turhan (2020), Pathak, Sönmez, Ünver, and Yenmez (2020), Delacrétaz (2020), and Aygün and Bo (2021).

Our interpretation that elements of the H-1B reserve system were not fully appreciated relates to the work reported in Pathak, Rees-Jones, and Sönmez (2020). That paper presents an online experiment eliciting a representative U.S. sample's preferences over reserve systems. In scenarios related to school admission and visa allocation, approximately 40% of subjects chose to implement reserve systems in a way that could be fully rationalized by an incorrect belief that processing order is irrelevant. In that experiment, a trivially small fraction of subjects responded in a manner that revealed full understanding of these systems. This widespread misunderstanding of how reserve systems operate can rationalize why the elements of the H-1B system that we study received little public attention prior to their publicization by the Trump administration. Our analysis of the history of H-1B reforms served as a key motivation for the hypotheses tested by Pathak, Rees-Jones, and Sönmez (2020), and that paper's findings lend credence to elements of our interpretation of the history presented in Section 4.

Beyond the specific context of reserve system design, our paper relates to the large literature on market design under various classes of distributional constraints such as lower quotas, upper quotas, and regional quotas. A partial list includes Abdulkadiroğlu (2005), Biro, Fleiner, Irving, and Manlove (2010), Kojima (2012), Budish, Che, Kojima, and Mil-

⁶When either policy is implemented on a stand-alone basis for non-overlapping protected groups, the vertical reservations considered in this work correspond to Over-and-Above allocation rule we study, and horizontal reservation correspond to Exemptions-First allocation rule.

⁷Later rediscovered by a High Court ruling, their proposed mechanism became mandated in the State of Gujarat in August 2020, and endorsed by the Supreme Court for the whole country in December 2020.

grom (2013), Westkamp (2013), Ehlers, Hafalir, Yenmez, and Yildirim (2014), Echenique and Yenmez (2015), Kamada and Kojima (2015, 2017, 2018), Bo (2016), Dogan (2016), Fragiadakis and Troyan (2017), and Abdulkadiroğlu and Grigoryan (2021). Our paper is also related to the literature studying the formal properties of allocation processes in the field. This literature includes studies of entry-level labor markets (Roth, 1984; Roth and Peranson, 1999), school choice (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003; Pathak and Sönmez, 2008, 2013), spectrum auctions (Milgrom, 2000), kidney exchange (Roth, Sönmez, and Ünver, 2004, 2005), internet auctions (Edelman, Ostrovsky, and Schwarz, 2007; Varian, 2007), course allocation (Sönmez and Ünver, 2010; Budish, 2011), cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013), assignment of airport arrival slots (Schummer and Vohra, 2013; Schummer and Abizada, 2017), refugee resettlement (Jones and Teytelboym, 2017; Delacrétaz, Kominers, and Teytelboym, 2016; Andersson, 2017), and more.

Relation to Literature on the H-1B System

As reviewed in the introduction, a significant empirical literature has examined H-1B system (see, e.g., Kerr and Lincoln, 2010; Kato and Sparber, 2013; Doran, Gelber, and Isen, 2014; Kerr, Kerr, and Lincoln, 2015; Peri, Shih, and Sparber, 2015; Amuedo-Dorantes and Furtado, 2017; Bound, Khanna, and Morales, 2017; Chassamboulli and Peri, 2018; Mayda, Ortega, Peri, Shih, and Sparber, 2018, 2020; Glennon, 2020; Dimmock, Huang, and Weisbenner, 2021). Papers within this literature have commonly analyzed the consequences of the H-1B Visa Reform Act of 2004 or the role of H-1Bs in the post-reform regime. While many papers have presented the post-2004 system as if it were governed by a single stable policy, our work demonstrates that this window of time is best thought of as a series of policies that differ significantly in their degree of skill bias. This observation leads to two practical implications for empiricists operating within this literature. First, our results illustrate a potential confounding factor that can arise in empirical analysis. Because the different visa allocation rules lead to different distributions of skill among successful H-1B applicants, controlling for the visa allocation rule can be important when studying the causes or consequences of skilled immigration.

Second, our results indicate that the changes to skill bias occurring due to switches in the assignment rule can serve as a new, and largely untapped, source of policy variation. While papers cited above have leveraged the change in total caps imposed in the 2004 reform or rules regarding the randomization of priority for identification, we are not aware of papers making similar use of changes in processing order or changes in the number of priority orders used. By documenting the changes of these design elements and the fact that some are plausibly exogenously driven by logistical issues, we provide a new source of identifying variation to this literature.

2 Model

There are q slots of immigration visas to be awarded to members of a set I of applicants, where each applicant can be awarded at most one slot. The set of applicants is partitioned into two sets as the *general-category* applicants I_G and the *reserved-category* applicants I_R . While all slots are identical otherwise, $q_r \leq q$ slots are exclusively set aside for the set of reserved-category applicants. We refer these slots as *reserved* slots. The remaining $q_u = q - q_r$ unreserved slots can be awarded to any applicant. To simplify the analysis, we assume that there is excess demand for the visas; that is

$$|I_G| \ge q_u$$
 and $|I_R| \ge q_r$.

This assumption holds for all years since the reserved slots were introduced in the H-1B Visa Reform Act of 2004.

We refer to the triple $E = \langle I, q_u, q_r \rangle$ as a visa allocation problem, and assume that it is fixed throughout our analysis. Let \mathcal{E} denote the set of visa allocation problems.

A priority order π is a linear order on the set of applicants I. Let Π denote the set of priority orders on the set of applicants I.

Motivated by U.S. H-1B visa allocation policies since 2004, we focus on allocation rules that rely on two priority orders $\pi_u, \pi_r \in \Pi$, one for each type of slot. Here,

- π_u identifies the claims of all applicants for the unreserved slots, and
- π_r identifies the claims of the reserved-category applicants for the reserved slots.⁸

These priority orders can depend on factors such as the timing of arrival of applications, exam scores, salary of the applied jobs, or simply a random lottery draw. While the two priority orders can be identical, they can also be different. In the rest of this section, we fix the priority orders $\pi_u, \pi_r \in \Pi$.

2.1 Matchings and Visa Allocation Rules

Given a visa allocation problem $E \in \mathcal{E}$, a matching is a function $\mu : I \to \{r, u\} \cup \{\emptyset\}$ such that

$$|\mu^{-1}(r)| \le q_r$$
 and $|\mu^{-1}(u)| \le q_u$.

Let $\mathcal{M}(E)$ denote the set of all matchings for the visa allocation problem $E = \langle I, q_u, q_r \rangle$. Since $E \in \mathcal{E}$ is fixed throughout our analysis, we will suppress the argument in $\mathcal{M}(E)$ and simply denote it as \mathcal{M} whenever there is no ambiguity.

For any matching $\mu \in \mathcal{M}$ and applicant $i \in I$,

- $\mu(i) = r$ indicates that applicant *i* is awarded a reserved slot,
- $\mu(i) = u$ indicates that applicant *i* is awarded an unreserved slot, and

⁸Since the reserved slots are exclusively set aside for the reserved-category applicants, only the relative priority order of these applicants is relevant under the priority ranking π_r .

• $\mu(i) = \emptyset$ indicates that applicant *i* is not awarded a slot.

Since all slots are identical, each applicant is assumed to be indifferent between receiving a reserved slot and an unreserved slot. We further assume that each applicant strictly prefers receiving a slot to not receiving one.

For any matching $\mu \in \mathcal{M}$, let

- $|\mu| = |i \in I : \ \mu(i) \neq \emptyset|$ denote the number of applicants who are allocated a slot,
- $|\mu_r| = |i \in I : \mu(i) = r|$ denote the number of applicants who are allocated a reserved slot,
- |µ_u| = |i ∈ I : µ(i) = u| denote the number of applicants who are allocated an unreserved slot,
- μ(I_R) = {i ∈ I_R : μ(i) ≠ ∅} denote the set of reserved-category applicants who are each allocated a slot, and
- μ(I_G) = {i ∈ I_G : μ(i) ≠ ∅} denote the set of general-category applicants who are each allocated a slot.

A visa allocation rule is a function $\varphi : \mathcal{E} \to \bigcup_{E \in \mathcal{E}} \mathcal{M}(E)$ that selects a matching $\mu \in \mathcal{M}(E)$ for each visa allocation problem $E \in \mathcal{E}$.

2.2 Desiderata for Matchings and Visa Allocation Rules

We study matchings and visa allocation rules that satisfy the following three axioms.

Definition 1 A matching $\mu \in \mathcal{M}$ accommodates reservation policy if, for any $i \in I_G$,

$$\mu(i) \neq r.$$

Likewise, a visa allocation rule **accommodates reservation policy** if its outcome accommodates reservation policy for each visa allocation problem.

Our first axiom ensures that only qualified applicants can be awarded the slots set aside for the reserved-category applicants.

Definition 2 A matching $\mu \in \mathcal{M}$ is non-wasteful if,

1) for any $i \in I_R$,

 $\mu(i) = \emptyset \implies |\mu| = q, and$

2) for any $i \in I_G$,

$$\mu(i) = \emptyset \implies |\mu_u| = q_u.$$

Likewise, a visa allocation rule is **non-wasteful** if its outcome is non-wasteful for each visa allocation problem.

Our second axiom ensures that each slot is to be allocated, provided that there are eligible applicants.

Definition 3 A matching $\mu \in \mathcal{M}$ respects priorities if,

- 1) for any $i, j \in I$, $\left(\mu(i) = \emptyset \text{ and } \mu(j) = u\right) \implies j \pi_u i, \text{ and}$
- 2) for any $i, j \in I_R$,

$$(\mu(i) = \emptyset \text{ and } \mu(j) = r) \implies j \pi_r i$$

Likewise, a visa allocation rule is **respects priorities** if its outcome respects priorities for each visa allocation problem. Our third axiom ensures that allocation of both type of slots respects their given priority orders.

It is convenient to collect all three axioms into the following condition.

Definition 4 A matching $\mu \in \mathcal{M}$ complies with the statute if and only if (i) it accommodates reservation policy, (ii) it is non-wasteful, and (iii) it respects priorities. Likewise, a visa allocation rule φ complies with the statute if and only if (i) it accommodates reservation policy, (ii) it is non-wasteful, and (iii) it respects priorities.

2.3 Comparisons Between Visa Allocation Rules

We now explain how we compare visa allocation rules. First, given two visa allocation rules φ and ψ , we say rule φ is **more favorable for the reserved-category applicants** than rule ψ if, for any $E \in \mathcal{E}$, $\mu = \varphi(E)$, and $\nu = \psi(E)$,

$$|\mu(I_R)| \ge |\nu(I_R)|.$$

Definitions of less favorable for the reserved-category applicants, more favorable for generalcategory applicants, and less favorable for the general-category applicants are analogous (see Appendix C for formal presentation of these other definitions).

Next, we say a visa allocation rule φ that complies with the statute is **reserved-category-maximal** if, for any problem $E \in \mathcal{E}$ and rule ψ that complies with the statute, rule φ is more favorable for the reserved-category applicants than rule ψ . Definitions of reserved-category-minimal, general-category-maximal, and general-category minimal are analogous (see Appendix C for formal presentation of these other definitions).

2.4 Post-2004 Visa Allocation Rules in the U.S.

As we have discussed in the Introduction, four visa allocation rules have been used to allocate H-1B visas in the U.S. since the inception of the H-1B Visa Reform Act of 2004. We next describe the matching produced by each of the four rules for a given visa allocation problem $E = \langle I, q_u, q_r \rangle$.

The first two rules rely on an identical priority order for reserved slots and unreserved slots. That is, $\pi_u = \pi_r = \pi$. More precisely, these priority orders depend on the arrival time of H-1B applications, giving priority to earlier applications.⁹ When two applications arrive at the same date, ties are randomly broken.

For the following two visa allocation rules, fix a priority order $\pi \in \Pi$, and let $\pi_u = \pi_r = \pi$.

Exemptions-First Visa Allocation Rule φ^{ef} :

Consider all applicants one-at-a-time based on the priority order π , until either all applicants are considered or all slots are exhausted.

- If the applicant in consideration is a member of the reserved category, allocate her
 - a reserved slot provided that not all reserved slots are exhausted,
 - an unreserved slot provided that there still remains at least one unreserved slot and all reserved slots are exhausted.
- If the applicant is a member of the general category, allocate her an unreserved slot, provided that not all unreserved slots are exhausted.

An applicant who fails to receive a slot at the end of this process is not awarded a slot.

Over-and-Above Visa Allocation Rule φ^{oa} :

⁹Implicit in our interpretation is the assumption that time spent assessing an application for a reservedcategory position does not influence the ranking of the applicant under the priority order π_u for unreserved positions.

Step 1: Consider all applicants one-at-a-time based on the priority order π . Allocate an unreserved slot to the applicant in consideration, provided that not all unreserved slots are exhausted. Proceed to Step 2 either when all applicants are already considered or all unreserved slots are exhausted.

Step 2: Consider all remaining reserved-category applicants one-at-a-time based on the priority order π . Allocate a reserved slot to the applicant in consideration, provided that not all reserved slots are exhausted. Terminate the procedure either when all reserved-category applicants are already considered or all reserved slots are exhausted.

An applicant who fails to receive a slot in either step is not awarded a slot.

For the next two visa allocation rules, fix two priority orders $\pi_u, \pi_r \in \Pi$.

Reserved-Initiated Visa Allocation Rule φ^{ru} :

Step 1: Consider all reserved-category applicants one-at-a-time based on the priority order π_r . Allocate a reserved slot to the reserved-category applicant in consideration, provided that not all reserved slots are exhausted. Proceed to Step 2 either when all reserved-category applicants are already considered or all reserved slots are exhausted.

Step 2: Consider all remaining applicants one-at-a-time based on the priority order π_u . Allocate an unreserved slot to the applicant in consideration, provided that not all unreserved slots are exhausted. Terminate the procedure either when all applicants are already considered or all unreserved slots are exhausted.

An applicant who fails to receive a slot in either step is not awarded a slot.

Unreserved-Initiated Visa Allocation Rule φ^{ur} :

Step 1: Consider all applicants one-at-a-time based on the priority order π_u . Allocate an unreserved slot to the applicant in consideration, provided that not all unreserved slots are exhausted. Proceed to Step 2, either when all applicants are already considered or all unreserved slots are exhausted. Step 2: Consider all remaining reserved-category applicants one-at-a-time based on the priority order π_r . Allocate a reserved slot to the applicant in consideration, provided that not all reserved slots are exhausted. Terminate the procedure, either when all reserved-category applicants are already considered or all reserved slots are exhausted.

An applicant who does not receive a slot in either step is not awarded a slot.

Observe that all four rules allocate all slots sequentially to the extent there are qualified applicants, they all restrict access to unreserved slots for applicants from reserved category, and they all allocate both types of slots based on the relevant priority order. Hence, they all satisfy each of the three axioms we formulated; i.e. they each comply with the statute.

Observation 1 Visa allocation rules φ^{oa} , φ^{ef} , φ^{ru} , and φ^{ur} comply with the statute.

3 Results

In this section, we present our results comparing the outcomes of the four visa allocation rules given in Section 2.4 with each other and with any other rule that complies with the statute. All our results are presented for a fixed visa allocation problem $E = \langle I, q_u, q_r \rangle$ and consequently also for a fixed set of applicants I. Therefore, when we relate our analytical results to actual H-1B visa allocation reforms, we are implicitly assuming away any possible change in the set of applicants due to these reforms.

3.1 Visa Allocation Rules for FY2005-FY2008

From FY2005 through FY2008, visa allocation rules were based on a single priority order that is induced by the arrival date of H-1B petitions. For the purposes of Theorem 1 below, therefore, we also fix $\pi_r = \pi_u = \pi$.

For FY2005 the mechanism of choice for H-1B allocation was the Over-and-Above visa allocation rule φ^{oa} , whereas for FY2006-FY2008 the mechanism of choice was the

Exemptions-First visa allocation rule φ^{ef} . Focusing on rules based on the same priority order for both types of positions, our first result establishes that the rule φ^{oa} is reservedcategory-maximal and general-category-minimal. In contrast, the rule φ^{ef} is reservedcategory-minimal and general-category-maximal.¹⁰

Theorem 1 Given any priority order $\pi \in \Pi$, let $\pi_u = \pi_r = \pi$. Let $\mu^{ef} = \varphi^{ef}(E)$ be the outcome of the Exemptions-First visa allocation rule, $\mu^{oa} = \varphi^{oa}(E)$ be the outcome of the Over-and-Above visa allocation rule, and matching $\mu \in \mathcal{M}$ be any matching that complies with the statute. Then,

- 1. $\mu^{ef}(I_R) \subseteq \mu(I_R) \subseteq \mu^{oa}(I_R)$ and
- 2. $\mu^{oa}(I_G) \subseteq \mu(I_G) \subseteq \mu^{ef}(I_G).$

3.2 FY2009 H-1B Allocation Reform

Prior to FY2009, the use of a single priority order that relies on arrival date of H-1B petitions had resulted in employers spending significant effort and money to send petitions by expedited overnight delivery for receipt on the first day petitions would be allowed. In FY2008, this resulted in more than 150,000 petitions being delivered on the same day and burdening employers, delivery services, and USCIS offices.

Consequently, USCIS abandoned the practice of relying on a single priority order that depends on the arrival of the petitions. Instead, USCIS allowed a period of five days for all petitions to be submitted, and prioritized petitions through two random lotteries, one for the general-category applicants and the other for the reserved-category applicants. We refer the resulting two priority orders as π_u and π_r respectively. As USCIS adopted two priority orders rather than one, they also adopted a new visa allocation rule φ^{ru} , abandoning rule φ^{ef} that they relied upon for FY2006-FY2008.

¹⁰Strictly speaking the result is slightly stronger, since relations hold not only in terms of the number applicants who receive positions from each group, but rather in terms of set inclusion.

We next show that, even though it was the result of the above-described logistical considerations, this reform resulted in a visa allocation rule that is more favorable for the reserved-category applicants and less favorable for the general-category applicants. To have a meaningful comparison of the rules φ^{ru} and φ^{ef} , we assume that

- 1. both rules rely of the same priority order π for allocation of the unreserved slots,
- 2. the rule φ^{ef} also relies on the same priority order π for allocation of the reserved slots,
- 3. whereas the rule φ^{ru} relies on a possibly distinct priority order π^* for allocation of the reserved slots.

Theorem 2 Fix any pair of priority orders $\pi, \pi^* \in \Pi$. Assuming $\pi_u = \pi_r = \pi$, let $\mu^{ef} = \varphi^{ef}(E)$ be the outcome of the Exemptions-First visa allocation rule. Assuming $\pi_u = \pi$ and $\pi_r = \pi^*$, let $\mu^{ru} = \varphi^{ru}(E)$ be the outcome of the Reserved-Initiated visa allocation rule. Then,

- 1. $|\mu^{ef}(I_R)| \le |\mu^{ru}(I_R)|$ and
- 2. $\mu^{ru}(I_G) \subseteq \mu^{ef}(I_G).$

3.3 FY2020 H-1B Allocation Reform

In contrast to previous reforms in H-1B visa allocation rules where the changes were officially justified based on logistical considerations, the reform of 2019 was motivated by an officially-stated objective of increasing the fraction of reserved-category applicants who receive H-1B visas. In 2019, USCIS adopted the Unreserved-Initiated visa allocation rule φ^{ur} starting in FY2020, thus abandoning the rule φ^{ru} that was used for over a decade.

Consistent with its officially-stated objective, our next result establishes that the 2019 reform resulted in a visa allocation rule that is more favorable for the reserved-category

applicants and less favorable for the general-category applicants. Moreover, for any pair of fixed priority orders for reserved and unreserved slots, our result also establishes that the rule φ^{ur} is reserved-category-maximal and general-category-minimal, whereas the rule φ^{ru} is reserved-category-minimal and general-category-maximal.

Theorem 3 Fix any pair of priority orders $\pi, \pi^* \in \Pi$. Let $\mu^{ru} = \varphi^{ru}(E)$ be the outcome of the Reserved-Initiated visa allocation rule, and $\mu^{ur} = \varphi^{ru}(E)$ be the outcome of the Unreserved-Initiated visa allocation rule. Let $\mu \in \mathcal{M}$ be any matching that complies with the statute. Then

- 1. $|\mu^{ru}(I_R)| \le |\mu(I_R)| \le |\mu^{ur}(I_R)|$ and
- 2. $\mu^{ur}(I_G) \subseteq \mu(I_G) \subseteq \mu^{ru}(I_G).$

3.4 Comparison of Post-2004 H-1B Visa Allocation Rules

Consider all four post-2004 visa allocation rules φ^{ef} , φ^{oa} , φ^{ru} , and φ^{ur} . Given any pair of priority orders $\pi, \pi^* \in \Pi$, fix

- 1. the priority order for unreserved slots of each of the four rules at $\pi_u = \pi$,
- 2. the priority order for reserved slots of each of the two single-priority rules φ^{ef} and φ^{oa} at $\pi_r = \pi$, and
- 3. the priority order for reserved slots of each of the two dual-priority rules φ^{ru} and φ^{ur} at $\pi_r = \pi^*$.

Observe that Step 1 of the two rules φ^{oa} and φ^{ur} are identical. Therefore, unreserved slots are allocated to the same set of applicants under both rules. Not only do both rules match an identical set of general-category applicants, but they also match an identical number of reserved-category applicants. The set of reserved-category applicants matched in Step 2 potentially differ under these rules since they rely on different priority orders to fill the reserved slots.

Observation 2 Fix a pair of priority orders $\pi, \pi^* \in \Pi$. Assuming $\pi_u = \pi_r = \pi$, let $\mu^{oa} = \varphi^{oa}(E)$ be the outcome of the Over-and-Above visa allocation rule and $\mu^{ef} = \varphi^{ef}(E)$ be the outcome of the Exemptions-First visa allocation rule. Assuming $\pi_u = \pi$ and $\pi_r = \pi^*$, let $\mu^{ru} = \varphi^{ru}(E)$ be the outcome of the Reserved-Initiated visa allocation rule and $\mu^{ur} = \varphi^{ur}(E)$ be the outcome of the Unreserved-Initiated visa allocation rule. Then,

- 1. $|\mu^{oa}(I_R)| = |\mu^{ur}(I_R)|$ and
- 2. $\mu^{oa}(I_G) = \mu^{ur}(I_G).$

The following result immediately follows from Theorems 1-3 and Observation 2.

Corollary 1 Fix a pair of priority orders $\pi, \pi^* \in \Pi$. Assuming $\pi_u = \pi_r = \pi$, let $\mu^{oa} = \varphi^{oa}(E)$ be the outcome of the Over-and-Above visa allocation rule and $\mu^{ef} = \varphi^{ef}(E)$ be the outcome of the Exemptions-First visa allocation rule. Assuming $\pi_u = \pi$ and $\pi_r = \pi^*$, let $\mu^{ru} = \varphi^{ru}(E)$ be the outcome of the Reserved-Initiated visa allocation rule and $\mu^{ur} = \varphi^{ur}(E)$ be the outcome of the Unreserved-Initiated visa allocation rule. Then,

- 1. $|\mu^{ef}(I_R)| \leq |\mu^{ru}(I_R)| \leq |\mu^{ur}(I_R)| = |\mu^{oa}(I_R)|$ and
- 2. $\mu^{oa}(I_G) = \mu^{ur}(I_G) \subseteq \mu^{ru}(I_G) \subseteq \mu^{ef}(I_G).$

3.5 Estimated Outcomes Across Rules

So far, our formal results have not placed any structure on the distribution of the two priority orders π_u and π_r . To quantify the effects of these four different policies in practice, assumptions about that structure are needed. Suppose both priority orders are independent uniform draws, as is the practice in the U.S. since FY2009, from the set of all priority orders.¹¹ For the Over-and-Above and Exemptions-First rules, the derivations utilize priority order π_u only. For the Reserved-Initiated and Unreserved-Initiated rules, the derivations utilize both priority orders.

For the Over-and-Above rule, in Step 1 the q_u unreserved slots are allocated to members of both groups in proportion to the sizes of both groups. Subsequently in Step 2, all q_r reserved slots are awarded to reserved-category applicants, unless of course fewer than q_r reserved-category applicants remain who are unmatched. In that case, each remaining reserved-category applicant is awarded a reserved slot. Therefore, the expected numbers of reserved-category and general-category applicants matched under the Over-and-Above visa allocation rule are:

$$\begin{aligned} |\mu^{oa}(I_R)| &= \frac{|I_R|}{|I|} q_u + \min\left\{q_r, |I_R| - \left(\frac{|I_R|}{|I|} q_u\right)\right\},\\ |\mu^{oa}(I_G)| &= \frac{|I_G|}{|I|} q_u. \end{aligned}$$

The following observation is helpful to derive the expected number of reserved-category and general-category applicants matched under the Exemptions-First visa allocation rule. Under this rule, the reserved cap provides a benefit to reserved-category applicants only if their proportional share is less than the reserved cap. Otherwise all slots (reserved or unreserved) are allocated in proportion to the sizes of both groups. If, on the other hand, the proportional share of the reserved-category applicants is less than the reserved cap, then all q_r reserved slots (which is more than their proportional share of all slots) are awarded to reserved-category applicants, whereas all q_u unreserved slots (which is less than their proportional share of all slots) are awarded to general-category applicants. Therefore, the expected numbers of reserved-category and general-category applicants

¹¹By conducting our calculations under this assumption, we isolate changes in allocations that are attributable to the changes in processing order and changes in the number of priorities used. These calculations do not assess the role of differences in the distribution of priority across groups (as could occur when priority is determined by arrival time).

matched under the Exemptions-First allocation rule are:

$$|\mu^{ef}(I_R)| = \max\left\{q_r, \frac{|I_R|}{|I|}(q_u + q_r)\right\},\|\mu^{ef}(I_G)| = \min\left\{q_u, \frac{|I_G|}{|I|}(q_u + q_r)\right\}.$$

Under the Reserved-Initiated allocation rule, in Step 1 the q_r reserved slots are awarded to the q_r highest π_r -priority reserved-category applicants. Therefore, the remaining $|I_R| - q_r$ reserved-category applicants and $|I_G|$ general-category applicants compete for the q_u unreserved positions in Step 2, each group in expectation receiving their proportional share from these slots. Hence, the expected numbers of reserved-category and generalcategory applicants matched under the Reserved-Initiated allocation rule are:

$$|\mu^{ru}(I_R)| = q_r + \left(\frac{|I_R| - q_r}{|I| - q_r}\right) q_u, |\mu^{ru}(I_G)| = \left(\frac{|I_G|}{|I| - q_r}\right) q_u.$$

Under the Unreserved-Initiated allocation rule, all applicants compete for q_u unreserved slots in Step 1, each group in expectation receiving their proportional share from these slots. Subsequently in Step 2, all q_r reserved slots are awarded to reserved-category applicants, unless of course fewer than q_r reserved-category applicants remain who are unmatched. In that case, each remaining reserved-category applicant is awarded a reserved slot. Therefore, the expected numbers of reserved-category and general-category applicants matched under the Unreserved-Initiated visa allocation rule are:

$$\begin{aligned} |\mu^{ur}(I_R)| &= \frac{|I_R|}{|I|} q_u + \min\left\{q_r, |I_R| - \left(\frac{|I_R|}{|I|} q_u\right)\right\}, \\ |\mu^{ur}(I_G)| &= \frac{|I_G|}{|I|} q_u. \end{aligned}$$

[Table 2 about here]

In Table 2, we use these formulas together with data from the 2019 Federal Register on application rates by general and advanced-degree applicants to quantify the effect of different rules. These data cover the most recently available year and a five-year average. Data from these years are of particular interest because they formed the basis of the calculations in the USCIS government press release announcing the Trump administration's modification of the allocation rule (USCIS, 2018). The table shows that the share of advanced-degree applications has increased over time since the count for FY2017 is greater than the five-year average. The 2006 replacement of Over-and-Above with Exemption-First resulted in a reduction of about 14,000 visa awards to advanced-degree applicants, using the numbers from the five-year average. The 2008 switch to Reserved-Initiated with two separate lotteries increased the number of awards to advanced-degree applicants by about 8,800 to 33,495. As mentioned above, with two lotteries, the scope for changing the number of advanced-degree applicants by changing processing of applicants shrinks. Even though Reserved-Initiated generates the least favorable outcome for advanced-degree applicants, it generates an outcome that is only about 5,339 applications worse for this group than Unreserved-Initiated, the rule which generates the most favorable outcome for advance-degree applicants.¹² This pattern shows that the rule changes in 2006 and 2008 were each quantitatively more significant than the 2019 change. We also observe the same phenomenon using application data from FY2017 as our benchmark.

 $^{^{12}\}mathrm{This}$ number differs from the 5,340 reported in the USCIS press release because of rounding.

4 Revisiting the History of Visa Allocation Rule Modifications

The H-1B Visa Reform Act of 2004 mandated the introduction of the advanced-degree reserve explicitly to favor higher-skilled applicants—i.e., to influence the skill bias reflected in allocations. Our analysis demonstrates that the range of implementations that comply with this statute differ in their degree of skill bias, and that the quantitative differences in skill bias among the four allocation rules that were adopted are non-negligible. In this section we present a brief review of the history and motivation for these changes. While the FY2020 rule change was explicitly motivated by desire to influence the degree of skill bias, the previous reforms were not. Given the focus on logistical considerations that motivated these prior reforms, it appears that their important influence on skill bias was potentially unintended and unappreciated.

Prior to the H-1B Visa Reform Act of 2004, allocation of H-1B visas was carried out annually on a first-in first-out basis. This practice induced a natural priority order between applicants based on the time a petition arrived at a USCIS processing center. The use of this priority introduced a temporal element to the reserve implementation decision. If the USCIS received an advanced-degree application before the unreserved cap had been exhausted, would this visa count towards the reserved or unreserved cap? A USCIS press release offered their interpretation:

"The first 20,000 H-1B beneficiaries who have earned a master's degree or higher from a U.S. institution of higher education are not subject to the annual congressionally mandated H-1B visa cap of 65,000. After those 20,000 slots are filled, USCIS is required to count those cases against the cap for the remainder of the fiscal year."¹³ (USCIS, 2004)

¹³The USCIS's initial interpretation that the reserve cap should be processed first can be justified by the use of the word "until" in 8 USC 1184(g)(5)(C)).

In the terminology introduced in Section 2.4, the USCIS interpreted the legislation to dictate the Exemptions-First visa allocation rule. This rule was adopted by USCIS for FY2006-FY2008.

Despite this interpretation, the USCIS adopted a different visa allocation rule in the first year of implementation (FY2005). The justification offered by the USCIS for this difference was based on how the application timeline intersected with the passage of the Visa Reform Act. In the U.S., the federal government's fiscal year starts on October 1 of the previous calendar year and runs through September 30 of the calendar year. The Act was announced in December 2004 and took effect mid-fiscal-year in March 2005. By the time the additional reserved cap of 20,000 took effect, applications for FY2005 were already evaluated and the unreserved cap of 65,000 was already allocated. The USCIS indicated that they had no way to identify which of the 65,000 awarded applications would qualify for the advanced-degree exemption, and instead they stated:

"[...], for FY 2005, USCIS has determined that the only appropriate way to implement the H-1B Visa Reform Act of 2004 is to apply the 20,000 exemptions prospectively." (Federal Register, 2005)

While the justification for using a different visa allocation rule for FY2005 than the rule adopted for subsequent years had been clearly articulated, the distributional implications of this one-time implementation have not been analyzed prior to our study. The analysis in Section 3.1 shows that, of all the rules that comply with the statute and utilize a single priority, the two rules implemented in FY2005 and FY2006-FY2008 play special roles: the Over-and-Above visa allocation rule is reserved-category-maximal and generalcategory-minimal, whereas the Exemptions-First visa allocation rule is reserved-categoryminimal and general-category-maximal. The analysis of Section 3.5 indicates that this difference is quantitatively important, with the Over-and-Above rule allocating 16.7% more of annual slots to advanced-degree applicants (compared to the Exemptions First rule) in our analysis of FY2013-FY2017 averages. Given that influencing skill bias was a key motivation of the H-1B Visa Reform Act of 2004, the fact that such large changes in skill bias were implemented for logistical convenience partially motivates our belief that the impact of these implementation decisions was not appreciated at the time.

During the period when the Exemptions-First rule was used, changing application patterns created difficulties in using an arrival-time-based priority order. Over the three years from FY2006-FY2008 applications arrived earlier and caps were reached sooner.¹⁴ In FY2008, a number of general-category applications sufficient to meet the 65,000 unreserved cap arrived on the first day that applications were accepted by the USCIS.¹⁵ Anticipating this may happen, employers spent significant effort and money to send petitions by expedited overnight delivery for receipt on the first day petitions would be allowed, resulting in more than 150,000 petitions being delivered on the same day and burdening employers, delivery services, and USCIS offices.¹⁶ This development made clear that the use of arrival time as a priority measure was no longer tenable.

Consequently, in March 2008 USCIS abandoned its use of arrival time for priority. Under their new rule, applications must be submitted in a five day window. Valid applications submitted within this window had priority assigned randomly, and a second independent and random priority was assigned to all advanced-degree applicants.¹⁷ Under the new procedure, the USCIS continued to process the reserved cap first. This group

¹⁴The H-1B petition filing period for any fiscal year typically begins on April 1st of the previous calendar year. The Federal Register reports that in FY2006 the general cap was reached on August 10, 2005 and the advanced-degree cap was reached on January 17, 2006; in FY2007 the general cap was reached on May 26, 2006 and the advanced-degree cap was reached on July 26, 2006; and in FY2008 the general cap was reached on April 1, 2007 and the advanced-degree cap was reached on May 4, 2007. (Federal Register, 2008).

¹⁵In the USCIS's implementation of the arrival-time priority order, day of arrival determined priority and ties were resolved with a random lottery. As a result, though FY2008 still fell under the arrival-time priority regime, in effect priority was fully determined by random lottery among individuals tied for top priority under arrival time.

¹⁶This information comes from the United States District Court Case Walker Macy (2017) vs. U.S. Citizenship and Immigration Services.

¹⁷If sufficient applications from either category was not received within five days, the reform allowed for additional petitions that arrived later. When this condition is triggered it returns some influence of arrival time into the priority order.

of decisions results in the Reserved-Initiated allocation rule introduced in Section 2.4. This rule remained in effect from FY2009-FY2019.

The change in the allocation rule implemented in FY2009 again influenced the degree of skill bias reflected in visa allocations. While it is natural to assume that some change could arise due to differences in arrival times of reserved-category and general-category applications, a large and more subtle effect arises from the mere addition of a separate independent priority for reserved-category applicants. When a single priority is used, the practice of allocating the 20,000 highest-priority reserved-category applicants to the reserve cap results in comparatively low-priority reserved-category applicants competing for the general-cap positions. In contrast, when separate and independent priorities are used, the reserved-category applicants who do not receive a visa under the reserve cap are not selected for low priority when considered for the general cap. This absence of selection results in a greater number of visas going to reserved-category applicants, as we formalized in Section 3.2. The analysis of Section 3.5 again indicates that this difference is quantitatively important, with the Reserved-Initiated rule allocating 10.4% more of annual slots to advanced-degree applicants (compared to the Exemptions First rule) in our analysis of FY2013-2017 averages. Given that this change was transparently made to address the logistical problems with arrival-time-based priority, changing the degree of skill bias through the decisions to use two independent lotteries may have been unintended. We are not aware of any indication that it was an explicit design decision, and the lack of publicity surrounding this effect suggests that the impact was unappreciated.

In stark contrast to these earlier rule modifications, the changes proposed in response to President Trump's *Buy American and Hire American Executive Order* demonstrate an unambiguous and deliberate attempt to influence skill bias through the mechanisms proposed in our theory. This 2017 executive order instructed the U.S. Department of Homeland Security to propose reforms to ensure that H-1B visas are awarded to the most-skilled or highest-paid petition beneficiaries. In response, the USCIS proposed maintaining the general structure of the FY2009-FY2019 Reserved-Initiated visa allocation rule, but reversing the processing order of reserved and unreserved slots: the Unreserved-Initiated rule introduced in Section 2.4. The analysis of Section 3.3 allocation from the FY2020 Unreserved-Initiated visa allocation rule exhibits more skill bias than the FY2009-FY2019 Reserved-Initiated visa allocation rule. Moreover, Section 3.4 demonstrates that its outcome awards the same number of advanced-degree visas as in FY2005 Over-and-Above visa allocation rule, and thus it is reserved-category-maximal even including single-priority-order rules.

5 Conclusion

Former President Trump's 2017 Buy American and Hire American Executive Order led the USCIS to reform the H-1B visa allocation system in 2019. We show that the newlyadopted Unreserved-Initiated rule is the most favorable for advanced-degree applicants among all rules satisfying our three axioms capturing compliance with the statute. Surprisingly, of the three modifications to the H-1B visa allocation rule since the Act of 2004, the 2019 reform causes the smallest year-to-year change in the number of advanced-degree awards. Despite that, the distributional implications of the 2019 reform were much more widely publicized compared to the more consequential (but possibly accidental) changes in FY2006 and FY2009.

Beyond shedding light on this recent history, our results also inform debates on potential future reforms. One class of reform proposals involves changes to the priority structure used; for example, prioritizing applicants based on their wage or based on firms' willingness to pay for a visa in an auction. Our theoretical results on single-priority systems remain valid if these priorities are used, and thus provide guidance on how the reserve system may be concurrently modified to better pursue the goals motivating such a reform. When considering the importance placed on being highly paid versus highly educated, the over-and-above allocation rule or the exemptions-first allocation rule would reflect two extremes. The enacted rule can be chosen to best reflect the optimal tradeoff as assessed by the policy maker, offering a means of fine-tuning that is not easily achieved through modification of the single priority alone. Another class of reform proposals involves changes to reserved-category classification; for example, having caps specific for regions or occupations. Our theoretical results provide immediate guidance on how the processing order of these caps, and how the choice to apply a single priority or to implement independent priorities, would influence the rate of admission of the workers targeted by such a cap. For more detailed discussion of these (and other) reform proposals, see Kerr and Kerr (2020).

An important caveat to the interpretation of our results is that they do not, taken alone, indicate the optimal program design. Instead, our results merely clarify how several design decisions affect the degree of preferential treatment extended to reserved-category applicants. The optimal degree of preferential treatment—or skill bias, when the relevant reserved category is based on advanced-degree status—must be determined through judgments and analyses quite different from our own, which are themselves the subject of a great deal of debate. The Trump administration was relatively clear in their stated desire to maximize skill bias, and the reserved-category-maximal rule they imposed is optimal contingent on accepting that goal. The degree of skill bias that was viewed as optimal at other points in the H-1B program's history is often less clear, beyond the fact that the imposition of reserved caps itself reflects a clear desire for some degree of this bias. While we remain silent on these underlying normative issues, we note that our findings provide a means of determining the normative values that implicitly rationalize administrations' choices going forward. Of course, doing so requires imposing the assumption that administrations deliberately vary the design elements that we have highlighted to achieve their desired policy outcomes. This assumption appears to not have held at times in the past, but it may hold in the future now that these issues have been clearly analyzed.

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Years	Allocation Rule	Priorities Used	Reserves Processed	Publicized
FY2005	Over-and-Above (φ^{oa})	One	Last	Ν
FY2006-08	Exemptions-First (φ^{ef})	One	First	Ν
FY2009-19	Reserved-Initiated (φ^{ru})	Two	First	Ν
FY2020	Unreserved-Initiated (φ^{ur})	Two	Last	Y

Table 1: H-1B Visa Allocation Rules

Notes: This table summarizes the four visa allocation rules used by the USCIS since the H-1B Visa Reform Act of 2004. The "Allocation Rule" column provides the label for this rule used in our theory. Section 2.4 provides a complete description of each rule. The following two columns indicate 1) whether the rule applies one single priority across both caps or two independent priorities for each cap, and 2) whether the rule processes the reserved cap first or last. The final column indicates which rules were significantly publicized upon adoption (by prominent press releases and comments/discussion in the Federal register). Documentation for each allocation rule from the Federal Register is recorded in the Appendix.

Table 2: Number of Slots Awarded to Reserved-Category Applicants Undera 65,000 General Cap and a 20,000 Reserve Cap

	# of Applicants		Reserved-Category Allocation				
	General	Reserved					
	Category	Category	φ^{oa}	φ^{ef}	φ^{ru}	φ^{ur}	
5-yr Average (FY2013-17)	137,017	55,900	38,834	24,630	33,495	38,834	
FY2017	$111,\!080$	87,380	48,619	$37,\!425$	$44,\!542$	$48,\!619$	

Notes: Calculations are based on data from the 2019 Federal Register, assuming same arrival time distribution between reserved-category and general-category applicants, and identical lottery distribution for π_r and π_u .

A Appendix: Proofs

A.1 Proof of Theorem 1

Proof. Fix a visa allocation problem $E \in \mathcal{E}$ and $\pi \in \Pi$. Let $\pi_u = \pi_r = \pi$. Let $\mu^{ef} = \varphi^{ef}(E)$ be the outcome of the Exemptions-First visa allocation rule, $\mu^{oa} = \varphi^{oa}(E)$ be the outcome of the Over-and-Above visa allocation rule, and matching $\mu \in \mathcal{M}$ be any matching that complies with the statute. That is, μ is any non-wasteful matching that accommodates reservation policy and respects priorities.

For any $k \leq q$, let $J_G(k)$ denote the set of general-category applicants and $J_R(k)$ denote the set of reserved-category applicants among k highest π -priority applicants. Hence, $|J_G(k)|$ denotes the number of general-category applicants, and $|J_R(k)|$ denotes the number of reserved-category applicants among k highest π -priority applicants. By definition, for any $k \leq q$,

$$|J_G(k)| + |J_R(k)| = k.$$

Consider the q_u highest π -priority applicants. Of these applicants, $|J_G(q_u)|$ are from the general-category and $|J_R(q_u)|$ are from the reserved-category.

We first relate the set of applicants $\mu^{oa}(I_G)$ and $\mu(I_G)$. Since μ is non-wasteful and respects priorities,

$$|\mu(I_G)| \ge |J_G(q_u)|. \tag{1}$$

Since general-category applicants receive $|J_G(q_u)|$ unreserved and 0 reserved slots under μ^{oa} ,

$$|\mu^{oa}(I_G)| = |J_G(q_u)|.$$
(2)

Equation (1) and equation (2) imply $|\mu^{oa}(I_G)| \leq |\mu(I_G)|$, which in turn implies

$$\mu^{oa}(I_G) \subseteq \mu(I_G),\tag{3}$$

since both matching μ and matching μ^{oa} respect priorities.

We next relate the set of applicants $\mu(I_R)$ and $\mu^{oa}(I_R)$. Equation (1) implies

$$|\mu(I_R)| \le q - |J_G(q_u)| = q - (q_u - |J_R(q_u)|) = q_r + |J_R(q_u)|.$$
(4)

Since reserved-category applicants receive $|J_R(q_u)|$ unreserved slots and q_r reserved slots under matching μ^{oa} ,

$$|\mu^{oa}(I_R)| = q_r + |J_R(q_u)|.$$
(5)

Equation (4) and equation (5) imply $|\mu(I_R)| \leq |\mu^{oa}(I_R)|$, which in turn implies

$$\mu(I_R) \subseteq \mu^{oa}(I_R),\tag{6}$$

since both matching μ and matching μ^{oa} respect priorities.

Next consider the q highest π -priority applicants. Of these applicants, $|J_G(q)|$ are from the general-category and $|J_R(q)|$ are from the reserved-category.

Claim: $|\mu(I_R)| \ge \max\{q_r, |J_R(q)|\}.$

Proof of the Claim: Since matching μ is non-wasteful and it accommodates reservation policy,

$$|\mu(I_R)| \ge q_r. \tag{7}$$

Consider an applicant $i \in I_R$ who is one of the q highest π -priority applicants. Towards a contradiction, suppose $\mu(i) = \emptyset$. Since matching μ is non-wasteful, it accommodates reservation policy and respects priorities, all q_r reserved slots must be awarded to reserved-category applicants who have higher π -priority than applicant i. But since all q_u unreserved slots are also exhausted by non-wastefulness, at least one of the applicants who receive an unreserved slot must have lower π -priority than applicant i, for otherwise applicant i would not be one of the q highest π -priority applicants. This contradicts matching μ respecting priorities (for unreserved slots), yielding the desired contradiction. Hence, $\mu(i) \neq \emptyset$, and thus

$$|\mu(I_R)| \ge |J_R(q)|. \tag{8}$$

Equations (7) and (8) imply

$$|\mu(I_R)| \ge \max\{q_r, |J_R(q)|\},\$$

completing the proof of the Claim.

Observe that

$$|\mu^{ef}(I_R)| = \max\{q_r, |J_R(q)|\} \text{ and } |\mu^{ef}(I_G)| = q - \max\{q_r, |J_R(q)|\}.$$
(9)

Therefore, the first part of equation (9) and the Claim imply $|\mu^{ef}(I_R)| \leq |\mu(I_R)|$, which in turn implies

$$\mu^{ef}(I_R) \subseteq \mu(I_R),\tag{10}$$

 \diamond

since both matching μ and matching μ^{ef} respect priorities.

Finally, observe that the Claim further establishes

$$|\mu(I_G)| \le q - \max\{q_r, |J_R(q)|\}.$$
(11)

Equation (11) and the second part of equation (9) imply $|\mu(I_G)| \leq |\mu^{ef}(I_G)|$, which in turn implies

$$\mu(I_G) \subseteq \mu^{ef}(I_G),\tag{12}$$

since both matching μ and matching μ^{ef} respect priorities.

Together, relations (3), (6), (10), and (12) complete the proof of the theorem. \blacksquare

A.2 Proof of Theorem 2

Proof. Fix a visa allocation problem $E \in \mathcal{E}$ and any pair of priority orders $\pi, \pi^* \in \Pi$. Assuming $\pi_u = \pi_r = \pi$, let $\mu^{ef} = \varphi^{ef}(E)$ be the outcome of the Exemptions-First visa allocation rule. Assuming $\pi_u = \pi$ and $\pi_r = \pi^*$, let $\mu^{ru} = \varphi^{ru}(E)$ be the outcome of the Reserved-Initiated visa allocation rule.

Let J_R^* be the set of q_r highest π^* -priority reserved-category applicants, and J_R be the set of q_r highest π -priority reserved-category applicants. We have

$$\mu_r^{ru}(I) = J_R^*$$
 and $\mu_r^{ef}(I) = J_R$.

Let S^* be the set of q_u highest π -priority applicants in $I \setminus J_R^*$ and S be the set of q_u highest π -priority applicants in $I \setminus J_R$. Define

$$S_R^* = S^* \cap I_R,$$
 $S_G^* = S^* \cap I_G,$ and
 $S_R = S \cap I_R,$ $S_G = S \cap I_G.$

Observe that

$$\mu^{ru}(I_R) = J_R^* \cup S_R^*, \qquad \mu^{ru}(I_G) = S_G^*,$$
$$\mu^{ef}(I_R) = J_R \cup S_R, \qquad \mu^{ef}(I_G) = S_G.$$

Let $g \in S_G^*$. That is, applicant g is one of the general-category recipients of an unreserved slot under matching μ^{ru} . By construction of the set S_G^* ,

$$|\{i \in I \setminus J_R^* : i \pi g\}| < q_u,$$

for otherwise applicant g would not be assigned one of the unreserved slots in μ^{ru} .

Since $|J_R \setminus J_R^*| = |J_R^* \setminus J_R|$ and

$$j \pi j^*$$
 for all $j \in J_R \setminus J_R^*$ and $j^* \in J_R^* \setminus J_R^*$

we must have

$$|\{i \in I \setminus J_R : i \pi g\}| < q_u,$$

which in turn implies $g \in S_G$. Therefore, $S_G^* \subseteq S_G$, and hence

$$\mu^{ru}(I_G) \subseteq \mu^{ef}(I_G),\tag{13}$$

showing the second desired relation.

Recall that we have $|J_R^*| = |J_R| = q_r$. Therefore the relation (13), together with the non-wastefulness of matchings μ^{ru} and μ^{M-G} imply

$$|\mu^{ef}(I_R)| \le |\mu^{ru}(I_R)|, \tag{14}$$

showing the first desired relation and completing the proof. \blacksquare

A.3 Proof of Theorem 3

Proof. Fix a visa allocation problem $E \in \mathcal{E}$ and any pair of priority orders $\pi, \pi^* \in \Pi$. Let $\pi_u = \pi$ and $\pi_r = \pi^*$. Let $\mu^{ru} = \varphi^{ru}(E)$ be the outcome of the Reserved-Initiated visa allocation rule, and $\mu^{ur} = \varphi^{ru}(E)$ be the outcome of the Unreserved-Initiated visa allocation rule. Let $\mu \in \mathcal{M}$ be any matching that complies with the statute. That is, μ is any non-wasteful matching that accommodates reservation policy and respects priorities.

Let J_G denote the set of general-category applicants and J_R denote the set of reserved-

category applicants among q_u highest $\pi\text{-priority}$ applicants. By definition,

$$|J_G| + |J_R| = q_u. (15)$$

We first relate the set of applicants $\mu^{ur}(I_G)$ and $\mu(I_G)$. Since μ is non-wasteful and it respects priorities,

$$|\mu(I_G)| \ge |J_G|. \tag{16}$$

Since general-category applicants receive $|J_G|$ unreserved and 0 reserved slots under μ^{ur} ,

$$|\mu^{ur}(I_G)| = |J_G|.$$
(17)

Equation (16) and equation (17) imply $|\mu^{ur}(I_G)| \leq |\mu(I_G)|$, which in turn implies

$$\mu^{ur}(I_G) \subseteq \mu(I_G),\tag{18}$$

since both matching μ and matching μ^{ur} respect priorities.

We next relate the set of applicants $\mu(I_R)$ and $\mu^{ur}(I_R)$. Equation (16) and equation (15) imply

$$|\mu(I_R)| \le q - |J_G| = q - (q_u - |J_R|) = q_r + |J_R|.$$
(19)

Since reserved-category applicants receive $|J_R|$ unreserved slots and q_r reserved slots under matching μ^{ur} ,

$$|\mu^{ur}(I_R)| = q_r + |J_R|.$$
(20)

Equation (19) and equation (20) imply

$$|\mu(I_R)| \le |\mu^{ur}(I_R)|.$$
 (21)

We finally show that

$$\mu(I_G) \subseteq \mu^{ru}(I_G)$$
 and $|\mu^{ru}(I_R)| \le |\mu(I_R)|$

Let J_R^* be the set of q_r highest π^* -priority reserved category applicants. By construction

$$|J_R^*| = q_r \tag{22}$$

and

$$\mu^{ru}(i) = r \quad \text{for any } i \in J_R^*.$$
(23)

Let S^* be the set of q_u highest π -priority applicants in $I \setminus J_R^*$. By construction,

$$|S^*| = q_u. \tag{24}$$

Define $S_R^* = S^* \cap I_R$ and $S_G^* = S^* \cap I_G$. Observe that $\mu^{ru}(I) = J_R^* \cup S^*$. Here,

$$\mu^{ru}(I_R) = J_R^* \cup S_R^*$$
 and $\mu^{ru}(I_G) = S_G^*$. (25)

Claim: $\mu(I_G) \setminus S_G^* = \emptyset.$

Proof of the Claim: Suppose that there exists an applicant $g \in \mu(G) \setminus S_G^*$. Since $g \notin \mu^{ru}(I_G)$ by relation (25), we must have

$$i \pi g$$
 for any $i \in S^*$,

for otherwise matching μ^{ru} would fail to respect priorities. Therefore,

$$S^* \subset \mu(I),\tag{26}$$

since matching μ respects priorities as well. Moreover, since matching μ respects priorities and accommodates reserve policy,

$$J_R^* \subset \mu(I)$$

by construction of the set of applicants J_R^* . Hence,

$$S^* \cup J^*_R \cup \{g\} \subset \mu(I).$$

But since $|S^*| = q_u$ by equation (24) and $|J_R^*| = q_r$ by equation (22),

$$S^* \cap J_R^* = \emptyset$$
 and $g \notin (S^* \cup J_R^*) \implies |S^* \cup J_R^* \cup \{g\}| = q_u + q_r + 1.$

Hence, we have a contradiction as

$$|\mu(I)| \ge q_u + q_r + 1 = q + 1.$$

This proves the claim.

As a result,

$$\mu(I_G) \subseteq S_G^* = \mu^{ru}(I_G). \tag{27}$$

Since μ is non-wasteful,

$$|\mu(I_R)| = \min\{q - |\mu(I_G)|, |I_R|\}.$$

If $|\mu(I_R)| = |I_R|$, then all reserved-category applicants in μ are matched under μ and

$$|\mu^{ru}(I_R)| \le |\mu(I_R)|.$$

Otherwise,

$$|\mu(I_R)| = q - |\mu(I_G)| \ge q - |\mu^{ru}(I_G)| = |\mu^{ru}(I_R)|,$$

 \diamond

where the inequality follows from relation (27). In either case,

$$|\mu^{ru}(I_R)| \le |\mu(I_R)|. \tag{28}$$

Equations (18), (21), (27), and (28) complete the proof. \blacksquare

B Appendix: Documentation of Visa Allocation Rules

This appendix contains excerpts from the Federal Register, the official journal of the federal government of the United States, related to H-1B allocation.

B.1 2005 Federal Register, Vol. 70, No. 86, May 5

"The H-1B Visa Reform Act of 2004 was enacted after the start of FY 2005 and after the receipt of all petitions necessary to reach the existing 65,000 H-1B cap for FY 2005. The amendment [...] authorizing the cap exemption of 20,000 H-1B nonimmigrant aliens with U.S. master's or higher degrees, did not become effective until March 8, 2005. Congress did not specify any procedures for implementation or dictate the manner in which USCIS should allocate H-1B numbers made available pursuant to the new exemption. Congress specifically did not require USCIS to 'reopen' its review of the H-1B petitions already received and re-characterize the petitions that would have qualified for the next exemption had it been in effect at the time the petitions were received. Thus, in order to give full effect to the newly created exemption, it is reasonable to do so only going forward only, applying the exemption to up to 20,000 petitions seeking work start dates during FY 2005." (Page 23777).

"[...], for FY 2005, USCIS has determined that the only appropriate way to implement the H-1B Visa Reform Act of 2004 is to apply the 20,000 exemptions prospectively." (Page 23777). "For FY 2006 and future fiscal years, USCIS will accept and adjudicate properly filled H-1B petitions on a first-in, first-out basis and will track those H-1B petitions that qualify for the U.S. master's or higher degree exemption under the H-1B Visa Reform Act of 2004 as cases are received and adjudicated. [...] Similarly, H-1B nonimmigrant aliens that are exempt under the H-1B Visa Reform Act of 2004 will not be counted towards the fiscal year numerical limit of 65,000. USCIS will continue to exempt such aliens until USCIS has allocated all 20,000 H-1B exemption numbers authorized [...] Thereafter, any H-1B petition granted for an H-1B nonimmigrant alien who has earned a U.S. master's or higher degree, unless otherwise exempt, will be counted against the fiscal year numerical limitations." (Page 23777)

B.2 2008 Federal Register, Vol. 73, No. 57, March 24

"In order to ensure that the 65,000 and 20,000 caps are not exceeded, USCIS monitors the number of H-1B petitions it receives. The first day on which petitions may file H-1B petitions can be as early as six month ahead of the employment start date. Therefore, a petition requesting an employment start date of October 1, the first day of the next fiscal year, may file the H-1B petition as early as April 1 of the current fiscal year. When the USCIS determines, based on the number of H-1B petitions it has received, that the applicable cap will be reached, it announces to the public the final day on which it will accept such petitions for adjudication in that fiscal year. USCIS refers to this day as the "final receipt date." USCIS then randomly selects the number of petitions necessary to reach the cap from the petitions received on the final receipt date." (Page 15390)

"However, because demand for other H categories has not been as great as for the H-1B classification, USCIS has only had to apply the random selection procedures to H-1B petitions subject to the overall 65,000 cap or the 20,000 cap on master's degree exemption." (Page 15391) "On Monday, April 2, 2007, the first available filing day for fiscal year (FY) 2008, USCIS received H-1B petitions totaling nearly twice the 65,000 cap. This was the first time since the random selection process regulations were promulgated that USCIS received more petitions than available cap numbers on the first available filing day. [...] The high volume of filings scheduled for delivery on April 2 caused logistical problems for overnight couriers and on the two USCIS service centers where filings could be made. Using the petitions received on April 2, and April 3, USCIS conducted the random selection process and thereafter rejected all petitions that were not randomly selected." (Page 15391)

"Just as with the 65,000 cap, the 20,000 cap on the master's degree exemptions has been exhausted earlier and earlier for each fiscal year since the cap exemption was added to the law. For FY 2006, the 20,000 cap was reached on January 17, 2006. For FY 2007, the cap was reached on July 26, 2006, less than four months after petition filings began on April 1, 2006. For FY 2008, the cap was reached on May 4, 2007, just over one month after petition filings began on April 2, 2007. For each of these fiscal years, USCIS announced a final receipt date and conducted the random selection process." (Page 15391)

"USCIS believes that the trend of exhausting the 20,000 cap on master's degree exemptions at an earlier date will continue. Should both 20,000 and 65,000 caps be reached on the same day that numbers become available (e.g., April 1 of the preceding fiscal year) no regulatory mechanism is in place to facilitate administration of the 20,000 cap in relation to the 65,000 cap." (Page 15391)

"This problem would be exacerbated were the 20,000 cap to be reached prior to or at the same time as the 65,000 cap, since all petitions not selected random selection process for the 20,000 cap would be considered twice—at the time of the random selection for the 20,000 cap and, thereafter, for the 65,000 cap." (Page 15391)

"This rule provides that USCIS will include petitions filed on all of those first five business days in the random selection process if USCIS receives a sufficient number of petitions to reach the applicable numerical limit (including limits on exemptions) on any one of the five business days on which USCIS may accept petitions. [...] This rule also provides that, if both the 65,000 and 20,000 caps are reached within the first five business days available for filing H-1B petitions for a given fiscal year, USCIS must first conduct the random selection process for petitions subject to the 20,000 cap on master's degree exemptions before it may begin the random selection process of petitions to be counted towards the 65,000 cap. After conducting the random selection for petitions subject to the 20,000 cap, USCIS then must add any non-selected petitions to the pool of petitions subject to the 65,000 cap and conduct the random selection process for this combined group of petitions. Therefore, those petitions that otherwise would be eligible for the master's degree exemption that are not selected in the first random selection will have another opportsloty to be selected for an H-1B number in the second random selection process. This rule also clarifies that those petitions not selected in either random selection will be rejected." (Page 15392)

B.3 2019 Federal Register, Vol. 84, No. 2, Thursday, January 31

"The statute is ambiguous as to the precise manner by which beneficiaries with a master's or higher degree from a U.S. institution of higher education must be counted toward the numerical allocations. The statue states that the 65,000 numerical limitation does not apply until 20,000 qualifying beneficiaries are exempted, but is otherwise silent as to whether they must be exempted prior to, concurrently with, or subsequent to the 65,000 numerical limitation being counted and/or reached, or some combination therefor. This ambiguity was recognized by DHS when it initially determined how the exemption should be administered." (Page 895)

"DHS believes this approach is most consistent with the overall statutory framework

as it counts all petitions filed by cap-subject petitioners until the numerical limitation is reached, and otherwise precludes additional petitions, allows for an additional 20,000 petitions." (Page 895)

"DHS believes that administering the numerically limited cap exemption in a way that does not reduce the odds of selection for beneficiaries with a U.S. advanced degree under the regular cap is most appropriate and maximizes the overall odds of selection for such beneficiaries under the numerical allocations." (Page 895)

"DHS also disagrees that the statue requires that initial H-1B visas be allocated to petitions in the order received." (Page 896)

"While DHS agrees that Congress has not limited the H-1B classification to the 'best and brightest" foreign nationals, nothing in the statute or legislative history precludes DHS from administering the cap allocation in a way that increases the odds of selection for beneficiaries with a master's or higher degree from a U.S. institution of higher education." (Page 896)

"Rather, this final rule simply creates a registration process to streamline the existing H-1B cap selection process, and reverses the order in which submissions are counted toward the H-1B numerical allocations, but does not change the overall number of foreign workers that may be hired under existing statutory authority." (Page 897)

"Reversing the cap selection order is expected to result in a greater number of beneficiaries with master's or higher degrees from U.S. institutions of higher education being selected under the numerical allocations and is in line with the executive order's directive to 'help ensure that H-1B visas are awarded to the most-skilled or highest-paid petition beneficiaries." Furthermore, master's or higher degree holders still maintain their own selection pool." (Page 912)

"It was clearly Congress's intent to prioritize such workers by creating a 20,000 cap exemption only for them." (Page 912) "DHS is not able to increase the H-1B cap allocations, as the cap allocations are statutory and set by Congress." (Page 913)

"Under the current process, when the number of cap-subject petitions filed with US-CIS during the first five days that such petitions may be filed exceeds the numerical limits, a certain number of petitions projected as needed to meet the 20,000 advanced degree exemption are randomly selected first from the 55,900 advanced degree petitions eligible for the advanced degree exemption. Of the remaining 172,918 petitions, 35,900 (21 percent) of H-1B beneficiaries with master's degree or higher from a U.S. institution of higher education remain in the pool to be selected in the 65,000 regular cap limit. Then, USCIS randomly selects a certain number of petitions projected as needed to meet the 65,000 regular cap limit from the remaining pool, which includes H-1B beneficiaries with bachelor's degrees and beneficiaries with a master's or higher degree from a U.S. institution of higher education no selected under the advanced degree exemption. DHS estimates than an additional 13,495 petitions otherwise eligible for the advanced degree exemption but not selected under the advanced degree exemption would be randomly selected in the regular cap. Therefore, USCIS currently selects an estimated total of 33,495 petitions filed for beneficiaries with a master's or higher degree from a U.S. institution of higher education, which accounts for 17 percent of 192,918 Form I-129 petitions." (Page 928)

"Under the new change to the H-1B cap-subject selection process, those seeking to file an H-1B cap-subject petition will have to submit an electronic registration for each beneficiary, unless the registration requirement is suspended. Only those with selected registrations will be eligible to file an H-1B cap-subject petition during an associated filing period for that fiscal year. As previously stated, DHS continues to assume 192,918 registration will be received annually. Under the new selection process, when registration is required, USCIS would first select a certain number of registrations projected as needed to meet the 65,000 regular cap limit from the 192,918 registrations. All 55,900 H-1B beneficiaries with a master's or higher degree from a U.S. institution of higher education (29 percent) will therefore be included in the pool for selection. DHS estimates that up to 18,825 advanced degree registrations that could be selected during the selection for the regular cap." (Page 928)

"Next, USCIS will select a certain number of registrations projected to meet the 20,000 advanced degree exemption from the remaining pool of 37,065 advanced degree registrations. In total, USCIS is likely to select an estimated 38,835 registrations for petitioners seeking to file H-1B petitions under the advanced degree exemption. These registrations account for 20 percent of the 192,918 registrations. Therefore, DHS estimates USCIS could accept up to 5,340 (or 16 percent) more H-1B cap-subject petitions annually for beneficiaries with a master's or higher degree from a U.S. institution of higher education." (Page 929)

C Comparisons Between Visa Allocation Rules

This section provides precise definitions for our comparisons of visa allocation rules. Given two visa allocation rules φ and ψ , we say

• rule φ is more favorable for the reserved-category applicants than rule ψ if, for any $E \in \mathcal{E}$, $\mu = \varphi(E)$, and $\nu = \psi(E)$,

$$|\mu(I_R)| \ge |\nu(I_R)|.$$

Similarly,

• rule φ is less favorable for the reserved-category applicants than rule ψ if,

for any $E \in \mathcal{E}$, $\mu = \varphi(E)$, and $\nu = \psi(E)$,

$$|\mu(I_R)| \le |\nu(I_R)|;$$

• rule φ is more favorable for the general-category applicants than rule ψ if, for any $E \in \mathcal{E}$, $\mu = \varphi(E)$, and $\nu = \psi(E)$,

$$|\mu(I_G)| \ge |\nu(I_G)|;$$

• rule φ is less favorable for the general-category applicants than rule ψ if, for any $E \in \mathcal{E}$, $\mu = \varphi(E)$, and $\nu = \psi(E)$,

$$|\mu(I_G)| \le |\nu(I_G)|.$$

A visa allocation rule φ that complies with the statute is

- reserved-category-maximal if, for any problem $E \in \mathcal{E}$ and rule ψ that complies with the statute, rule φ is more favorable for the reserved-category applicants than rule ψ ;
- reserved-category-minimal if, for any problem $E \in \mathcal{E}$ and rule ψ that complies with the statute, rule φ is less favorable for the reserved-category applicants than rule ψ ;
- general-category-maximal if, for any problem $E \in \mathcal{E}$ and rule ψ that complies with the statute, rule φ is more favorable for the general-category applicants than rule ψ ;
- general-category-minimal if, for any problem $E \in \mathcal{E}$ and rule ψ that complies with the statute, rule φ is less favorable for the general-category applicants than

rule ψ .