NBER WORKING PAPER SERIES

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Working Paper 26745
http://www.nber.org/papers/w26745

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2020

I thank editors and reviewers of previous drafts for many useful comments. Code and updates are at http://johnhcochrane.com. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 26745
February 2020, Revised in October 2020
JEL No. E3,E31,E32,E4,E5,E6,E62,E63

ABSTRACT

I construct a simple model with sticky prices and interest rate targets, closed by fiscal theory of the price level with long-term debt and fiscal and monetary policy rules. Fiscal surpluses rise following periods of deficit, to repay accumulated debt, but surpluses do not respond to arbitrary unexpected inflation and deflation, so fiscal policy remains active. This specification avoids many puzzles and counterfactual predictions of standard active-fiscal specifications. The model produces reasonable responses to fiscal and monetary policy shocks, including smooth and protracted disinflation following monetary or fiscal tightening.

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1 Introduction

This paper advances the fiscal theory of monetary policy, bringing us closer to a realistic model useful for policy analysis.

A "fiscal theory of monetary policy" substitutes active fiscal policy for active monetary policy to complete the determination of inflation and output in standard monetary policy models, in particular models in the new-Keynesian/DSGE tradition that include intertemporal optimization, potentially rational expectations, market clearing, price stickiness or other nominal frictions, and a central bank that follows an interest rate target. Under "active" fiscal policy (Leeper (1991)), the government debt valuation equation determines unexpected inflation, and provides the extra condition needed to uniquely determine equilibria under an interest rate target. This condition replaces the active monetary policy assumption that the central bank sends the economy to an explosively inflationary or deflationary path for all but one value of the inflation rate. I develop the model and I analyze of the effects of fiscal and monetary policy shocks.

The central innovation of this paper is a fiscal policy process in which the government repays deficits with subsequent surpluses, partly or in full, yet fiscal policy remains active. Fiscal surpluses respond to changes in the value of debt that result from accumulated deficits or changes in real interest rates, but surpluses do not respond to changes in the value of debt that result from arbitrary unexpected inflation or deflation. Equivalently, fiscal surpluses follow an s-shaped moving average process, in which a shock to deficits leads to eventual surpluses.

This specification resolves a conundrum of current models, surveyed below, that allow active fiscal policy. In an active-fiscal regime, these models require surpluses to be positively correlated. The government cannot promise fully or even substantially to repay deficits by subsequent surpluses. As a result, in an active-fiscal regime, these models make several related and counterfactual predictions. They predict that inflation is large and volatile; that inflation and deficits are strongly correlated; that higher surpluses raise rather than reduce the debt; that deficits are financed entirely by inflating away outstanding debt and not by borrowing; and that that government bond real returns have stock-like volatility and cyclical variation and therefore stock-like average returns.

In this literature, these pathologies are not present in passive-fiscal specifications. In that case, the surplus responds to the value of debt, and therefore deficits are at least partially repaid by subsequent surpluses. But since the surplus responds to *all* variation in the value of debt, including that brought on by arbitrary unexpected inflation, the regime that can repay debts must have passive fiscal policy.

For this reason, typical models that measure or test regimes find an active-fiscal regime, if at all, only for short periods, usually with high inflation such as the 1970s, in which the counterfactual puzzles are dominated by even more counterfactual predictions of their active-money specifications. To avoid the puzzles, they typically estimate that most of the sample, especially the post-1980 period, is in a passive fiscal regime. By generalizing the active-fiscal regime to allow partial or full repayment of debts, the fiscal specification of this paper can expand the applicability of the active-fiscal regime.

Indeed, I write the model in a form that reflects the observational equivalence between active-fiscal and active-money regimes. *Any* data that this model can fit by active money, it can fit by active fiscal policy. One can fit the entire sample by an active-fiscal regime.

The remaining ingredients are well known. I choose minimal additional ingredients that exhibit a plausible model, to examine the effect of the fiscal policy specification in a transparent and well-understood environment.

I specify long-term nominal government debt with a geometric maturity structure. Long-term debt helps the model to produce a negative response of inflation to unexpectedly higher nominal interest rates, without specifying a negative fiscal shock contemporaneous to the monetary policy shock as happens in standard passive-fiscal new-Keynesian models. (Long-expected and persistent interest rate rises still uniformly raise inflation, as they do in the standard models.) Higher nominal interest rates lower the nominal value of long-term debt. If there is no change the present value of surpluses, debt is undervalued at the original price level. People try to buy more debt, and thereby demand fewer goods and services. The price level declines until debt regains its real market value. Variation in the real interest rate and endogenous surpluses alter this underlying mechanism in interesting ways, but the basic mechanism remains. (This insight is due to Sims (2011).)

I use the standard textbook new-Keynesian IS and Phillips curves, despite their well-known empirical shortcomings. This specification allows me to focus on the effects of the novel fiscal specification, in a familiar playground, but limits the model's direct empirical applicability.

Sticky prices mean that real interest rates vary, even with (as here) no shocks to preferences or technology. Discount rate variation is important to this model's predictions of the effects of fiscal and monetary policy. For example, a rise in nominal interest rates with sticky prices raises real interest rates. Higher real interest rates raise the discount rate for government debt, lowering its present value, and providing an inflationary force.

Other than the central innovation described above, I specify standard fiscal and monetary policy rules. The interest rate target and the surplus respond to inflation and output.

The main results of this paper are model responses to persistent fiscal and monetary policy shocks. A fiscal shock is a shock to deficits, with no contemporaneous change in the monetary policy disturbance. A monetary policy shock is a shock to the interest rate target, with no contemporaneous change to the fiscal policy disturbance.

An unexpected deficit shock leads to a protracted inflation, and via the Phillips curve it leads to an output expansion. When monetary policy endogenously reacts to inflation, monetary policy moderates the initial inflation, by spreading inflation further forward. In this way, monetary policy rules that react to inflation and output have a novel and useful stabilizing role in an active-fiscal regime. The deficit shock does not just lead to a one-period price level jump, as in flexible price models with one-period debt. The smooth inflation response, modulated by monetary policy, is an important step to a realistic description of a fiscal inflation. Today's deficits also lead to a long string of future surpluses which partially repay the accumulated debts, and the surplus responds to debt in equilibrium. Both mechanisms naturally help to buffer the inflationary effect of fiscal shocks, but both observations could lead one to falsely infer a passive fiscal regime.

An unexpected monetary policy shock leads to a protracted disinflation, and an output decline. The protracted disinflation comes largely from the long-term debt effect. It overcomes the "Fisherian" prediction of related models that higher interest rates raise inflation. Endogenous surplus variation amplifies the deflationary impact of monetary policy: Higher interest rates raise future inflation. Higher future inflation induces higher future surpluses, and hence lower current inflation. Endogenous discount rate variation buffers the deflationary effect, however. The rise in nominal rates raises real rates and discount rates, which lowers the present value of debt, an inflationary force.

The results are novel in part because the question is different. The interesting monetary policy shock in an active-fiscal model holds fiscal policy constant in some way – here, the residual in the surplus rule. Conventional new-Keynesian models specify (implicitly) a contemporaneous "passive" fiscal contraction, which produces disinflation. Conventional estimates do not try to orthogonalize monetary and fiscal policy actions.

This paper's point is also methodological. You may have recognized problems with standard new-Keynesian active-money model and you may have seen how fiscal theory can repair such model's pathologies in a simple way. (My list features Cochrane (2011a), and Cochrane (2017a), Cochrane (2018) on zero bound issues.) But you may have been daunted by theoretical controversies that pervade the literature, by the impression that the model would make immediately silly predictions, by estimates in the literature that reject fiscal regimes or find them only in very limited subsamples, or by the impression that you would have to *do* something fundamentally hard and different. Additional identifying restrictions, Bayesian estimation, and Markov-switching may have put you off.

This paper shows by contrast that a completely active-fiscal theory of monetary policy model can be easily built, can surmount classic criticisms, and produces reasonable responses, avoiding pathological predictions such as price level jumps and surpluses that raise rather than lower the value of debt. Indeed, this paper shows by example that constructing a completely fiscal theory of monetary policy models is a nearly trivial modification of active-money new-Keynesian DSGE models.

Observational equivalence, and to transparently make this methodological point, are additional reasons that I keep the model simple, using textbook IS and Phillips curves and simple policy rules that do not well describe the data, and therefore I do not proceed to identification (the hard part), estimation and testing, or even comparing impulse responses to my own atheoretical VAR estimates in Cochrane (2019). Via observational equivalence, one can rewrite any active-money model as active-fiscal, and quickly incorporate any empirical success of the entire literature. Adding a sensible fiscal specification and active fiscal policy is unrelated, really, to the complexities which deliver better empirical performance of the IS, Phillips, and other parts of the conventional model.

Models may be observationally equivalent, but the questions one is led to ask, and the answers one will find, are potentially quite different. Adding the fiscal theory begs a reasonable description of fiscal policy, usually neglected or shoved off to footnotes about lump-sum taxes, and adding fiscal variables to the model's evaluation. The main contribution of this paper is aimed squarely at repairing the damage that a standard form of fiscal elaboration does to this project. A fiscal perspective leads one to ask different questions. For example, one is led to ask for the response of monetary policy shocks holding fiscal policy constant in some sense, whereas an active-money passive-fiscal perspective wants correlated monetary and fiscal policy shocks. Similarly, just which model one should solve – which IS and Phillips curves, and which currently popular ingredients such as financial frictions and heterogeneous constrained agents help the model to fit various phenomena – remains a large, open, and fertile field of investigation, and the answers are likely to change when a detailed fiscal side of the model, and interactions between fiscal affairs and the rest of the model are built in.

This paper is organized to get quickly to the main contribution, but adds ingredients one by one so the reader can understand their role in the final model. Section 2 sets out linearized identities involving debt and inflation. They also allow us to see some mechanisms directly. Section 3 lays out the basic idea of a fiscal theory of monetary policy in a flexible price model, and its shortcomings that motivate more elaborate ingredients. Section 4 explains the fiscal policy process in this simple environment, Section 5 adds long-term debt. Section 6 lays out the full model with price stickiness, long-term debt, and policy rules that respond to endogenous variables. Section 7 presents responses to fiscal and monetary policy shocks. Section 8 explains how the fiscal policy specification resolves, and is necessitated by, a variety of puzzles. Finally, Section 9 reviews the literature in detail, establishing that this fiscal specification is novel, useful, and relating this paper to others in the literature. Though this is not the usual organization, it is easier to understand the motivation of this model and its results, and to compare those to other work, once the reader knows what this model and its results are.

2 Linearized debt identities

I use a set of linearized identities to express the fiscal side of the model including long-term debt and time-varying bond returns. These identities are derived in the Appendix to Cochrane (2019).

Start with the linearized evolution of the real market value of nominal government debt,

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1}. \tag{1}$$

Log debt at period $t+1$, v_{t+1}, is equal to its value at period t, v_t, increased by the log nominal return on the portfolio of government bonds r^n_{t+1} less log inflation π_{t+1}, and less the real primary surplus s_{t+1}. The parameter $\rho = e^{-r}$ is a constant describing the linearization point. I take $\rho = 1$ in the numerical calculation below. We can use this value if we rule out unit roots as well as explosive roots in the solution.

Precisely, s_{t+1} is ρ times the real primary surplus scaled by steady state debt. The other terms of (1) are logarithms. I derive (1) by Taylor-expanding the nonlinear exact debt accumulation equation in the level of the surplus, rather than the log, because surpluses are often negative. For brevity, I refer to s_{t+1} as simply the "surplus." Cochrane (2019) defines the variables as ratios to GDP, and includes a GDP growth term in (1). For simplicity I abstract from stochastic growth here, but it is an important transformation for empirical work.

In the case of one-period debt, the ex-post nominal return equals the interest rate, $r^n_{t+1} = i_t$. Long-term debt shows up in this accounting by the presence of an ex-post nominal return r^n_{t+1} potentially not equal to the interest rate. If long-term bond prices fall, r^n_{t+1} is negative and the market value of debt v_{t+1} falls.

The main model below uses (1) directly, as the matrix solution method implicitly solves it forward and imposes stationarity of the value of debt v_t and all other variables. However, it is useful to solve (1) forward explicitly, as this step allows us to solve simpler models analytically and thereby to understand the mechanisms behind computed solutions of the larger model.

Iterating (1) forward, we have a present value identity,

$$v_t = \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \left(r^n_{t+j} - \pi_{t+j} \right) + \lim_{T\to\infty} \rho^T v_{t+T}. \tag{2}$$

I impose the infinitely-lived consumer's transversality condition, so this identity holds in expectation and the limiting term vanishes. Taking innovations $\Delta E_{t+1} \equiv E_{t+1} - E_t$ and rearranging, we have an innovation identity,

$$\Delta E_{t+1}\pi_{t+1} - \Delta E_{t+1} r^n_{t+1} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} \left(r^n_{t+1+j} - \pi_{t+1+j} \right). \quad (3)$$

A decline in the present value of surpluses must correspond to a lower real value of the debt. This reduction can come about by unexpected inflation, or in the presence of long-term debt by a decline in nominal long-term bond prices and therefore a low return r^n_{t+1}.

The second term on the right hand side captures discount rate effects. If expected real returns on government bonds rise, the present value of surpluses declines, which requires either inflation or negative bond returns.

When does a decline in the present value of surpluses result in inflation vs. a decline in nominal bond prices? To eliminate the bond return on the left-hand side of (3), and to include long-term debt in the model below, we need bond pricing equations. Assuming a geometric maturity structure, in which the face value of maturity j debt declines at the rate ω^j, we have a second linearized identity

$$r^n_{t+1} = \omega q_{t+1} - q_t \quad (4)$$

where q_t is the log price at time t of the portfolio of government bonds. We can solve this identity forward to express the bond price as the inverse of its future returns,

$$q_t = -\sum_{j=0}^{\infty} \omega^j r^n_{t+1+j}. \quad (5)$$

Taking innovations ΔE_{t+1} of this equation we have we have

$$\Delta E_{t+1} r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[\left(r^n_{t+1+j} - \pi_{t+1+j} \right) + \pi_{t+1+j} \right]. \quad (6)$$

An unexpectedly low bond return $\Delta E_{t+1} r^n_{t+1}$ corresponds mechanically to higher expected future nominal returns, as bond prices are the inverse of bond yields. And nominal expected returns equal real returns plus inflation.

We can now use (6) to eliminate the bond return from (3), which gives us an identity linking inflation, surpluses, and discount rates directly:

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_{t+1} \left(r^n_{t+1+j} - \pi_{t+1+j} \right). \quad (7)$$

In the presence of long-term debt $\omega > 0$, a fiscal shock can be met by drawn-out inflation rather than a one-period price level shock. The drawn-out inflation devalues long-term bonds as they come due. The bond return r^n_{t+1} in (3) essentially marks the future inflation to market. Drawn-out inflation is more realistic than one-period price-level jumps or i.i.d. inflation. Real expected return variation adds a discount rate effect.

Identities (3) and (7) allow us to account for the variation of unexpected inflation. Unexpected inflation must stem from revisions in expected surpluses or revisions in discount rates. Cochrane (2019) examines these decompositions in a VAR. I use these identities here to understand the roots of inflation in the model.

3 Simplest FTMP model

To introduce the concept of a fiscal theory of monetary policy, and to establish a benchmark for response functions, start with flexible prices, a constant real interest rate, one-period debt $\omega = 0$, and exogenous policy processes $\{s_t\}$, $\{i_t\}$. The model is composed of only the Fisher equation, i.e. the linearized intertemporal first-order condition

$$i_t = E_t \pi_{t+1}, \quad (8)$$

the debt accumulation equation (1) which simplifies to

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1},$$

and the transversality condition that the value of debt cannot explode.

As above, we can solve the debt accumulation equation forward and take innovations to complete the model with the inflation identity (3) or (7). These relations simplify to

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j}. \quad (9)$$

Unexpected inflation equals the negative of the revision of the present value of real primary surpluses.

I specify "monetary policy" here as control over the nominal interest rate, without changing surpluses. Central banks broadly speaking cannot directly affect taxes or spending. By setting an interest rate target $\{i_t\}$, the bank can still determine expected inflation by (8). The central bank remains powerful in this fiscal model.

I specify "fiscal policy" here as control over real primary surpluses, without changing the nominal interest rate target. Such fiscal policy determines unexpected inflation, the instantaneous response of inflation to a shock $\Delta E_{t+1}\pi_{t+1}$, via (9). That is the entire effect of fiscal policy on inflation. For this latter step to work, we need "active" fiscal policy, that surpluses do not respond to unexpected inflation in such a way that (9) holds for any value of unexpected inflation.

This flexible-price, money-free model is Fisherian: A rise in interest rates, with no change in surpluses, produces a rise in expected inflation one period later via $i_t = E_t\pi_{t+1}$, and it produces no change in current inflation $\Delta E_{t+1}\pi_{t+1} = 0$. In response to a fiscal shock, this model can only produce one period of inflation, a price-level jump that devalues outstanding short-term (by assumption) bonds. If monetary policy chooses to follow this inflation by raising interest rates, we will see a persistent inflation, but that is an endogenous effect of monetary policy not essential to the fiscal policy shock and such persistent inflation does not reduce the initial inflation shock. If we wish to describe events such as the 1970s as a response to fiscal events, it would be more attractive to have a model in which fiscal policy shocks directly lead to a drawn-out inflation response.

Obviously, if we wish to have an empirically realistic model, we need to generalize this simple structure. Therefore, I add sticky prices, long-term debt, policy rules, and a more realistic fiscal policy. But as we will see the basic logic of its solution remains.

The flexible-price new-Keynesian version of this model specifies that surpluses are passive in (9), so the latter holds for any unexpected inflation. This model selects unexpected inflation by an active monetary policy. It can produce a negative inflation response by implicitly assuming a coincident fiscal contraction, a negative shock in (9), achieved by "passive" fiscal authorities. The fiscal theory of monetary policy can produce the same response, but would call it a coordinated fiscal and monetary shock.

4 Fiscal policy

I present here a simple version of the fiscal policy specification in the context of this frictionless model. We add to the model of the last section a specific process for real primary surpluses. The model is

$$i_t = E_t \pi_{t+1} \tag{10}$$

$$s_{t+1} = \alpha v_t^* + u_{s,t+1} \tag{11}$$

$$\rho v_{t+1}^* = v_t^* - \Delta E_{t+1} \pi_{t+1}^* - s_{t+1} \tag{12}$$

$$\rho v_{t+1} = v_t - \Delta E_{t+1} \pi_{t+1} - s_{t+1} \tag{13}$$

$$\Delta E_{t+1} \pi_{t+1}^* = -\beta_s \varepsilon_{s,t+1} \tag{14}$$

$$u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}. \tag{15}$$

Equation (10) repeats the flexible price model with an interest rate target. For now, consider a fixed interest rate target; we will add a monetary policy rule later. Equations (11), (12) and (14) jointly describe the evolution of primary surpluses s_t. We will, bit by bit, see how these work. Equation (13) is the debt evolution equation (1), with one-period debt so $i_t = r_{t+1}^n$, substituting in the Fisher equation $i_t = E_t \pi_{t+1}$ and with a constant real interest rate. Comparing (12) to (13) we can interpret the former: The latent state variable v_t^* accumulates past surpluses and deficits as does the value of debt, but v^* ignores changes in the value of debt that come from unexpected inflation different from the target $\Delta E_{t+1} \pi_{t+1} \neq \Delta E_{t+1} \pi_{t+1}^*$. As we shall see, this specification gives us a fiscal policy that is active, and rules out multiple equilibria, but nonetheless pays off debts accumulated from past deficits in a way that $s_t = u_{s,t}$ plus an AR(1) for $u_{s,t}$ would not do.

To see how this specification determines unexpected inflation, substitute (11) into (12) to obtain

$$\rho v_{t+1}^* = (1 - \alpha) v_t^* - \Delta E_{t+1} \pi_{t+1}^* - u_{s,t+1}.$$

Thus, assuming $\{\Delta E_{t+1} \pi_{t+1}^*\}$ is stationary, v^* grows at less than the steady state interest rate for $\alpha > 0$, and it is stationary for $\alpha > 1 - \rho$. (Choosing $\rho = 0$ conveniently unites the two cases.) Next, differencing (12) and (13) we have

$$(v_{t+1} - v_{t+1}^*) = \rho^{-1} (v_t - v_t^*) - \rho^{-1} \left(\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} \pi_{t+1}^* \right). \tag{16}$$

Since v_t^* does not explode, the condition that debt v_t does not explode requires $v_t = v_t^*$ and $\Delta E_{t+1}\pi_{t+1} = \Delta E_{t+1}\pi_{t+1}^*$.

By making surpluses *not* respond to arbitrary unexpected inflation, debt does not respond to arbitrary unexpected inflation, and fiscal policy chooses a single value of unexpected inflation, which $i_t = E_t\pi_{t+1}$ and an interest rate target by themselves leave undetermined. But surpluses do respond to increases in the value of debt brought about by past deficits. This active-fiscal government can borrow, and promise to pay off the debt.

We can view the government in this model as having an interest rate target i_t and an unexpected inflation target $\Delta E_t\pi_{t+1}^*$. Expected inflation follows from the interest rate target via $i_t = E_t\pi_{t+1}$. We can also start with an arbitrary stochastic inflation target, $\{\pi_t^*\}$. The government implements the inflation target by setting the interest rate target to $i_t = E_t\pi_{t+1}^*$ and by using the unexpected value of the inflation target in the fiscal rule (12).

Equation (14) is not necessary. We could just substitute $-\beta_s\varepsilon_{s,t+1}$ into the state variable evolution (12). But expressing it this way allows us to interpret $\Delta E_{t+1}\pi_{t+1}^*$ as an unexpected inflation target.

The parameter β_s measures how much the unexpected inflation target moves when there is a fiscal shock. It is part of the policy process. Through β_s, a surplus shock changes the unexpected inflation target, which changes the state variable v^*, which changes subsequent surpluses. Thus, (14) describes how much future surpluses respond to pay off a current deficit shock. The β notation thinks of this parameter as a regression coefficient of the stochastic inflation target on the shock in question.

The fiscal and monetary policy parts of the model separate. Inflation determination now reduces to the pair

$$i_t = E_t\pi_{t+1}$$
$$\Delta E_t\pi_{t+1} = -\beta_s\varepsilon_{s,t+1}.$$

Fiscal affairs only enter into inflation determination via the latter condition. Given the equilibrium interest rate and inflation process, we can then find the surplus and value of debt from the equilibrium versions of (11), (12) and (15),

$$
\begin{aligned}
s_{t+1} &= \alpha v_t + u_{s,t+1} \\
\rho v_{t+1} &= v_t - \beta_s \varepsilon_{s,t+1} - s_{t+1} \\
u_{s,t+1} &= \rho_s u_{s,t} + \varepsilon_{s,t+1}.
\end{aligned}
$$

That fiscal events only feed back to affect inflation through the parameter β_s, here and in the sticky price model to follow, is a feature not a bug. This model is constructed to be a transparent and minimal extension of the standard model. It's easy to add additional feedback effects if one wants them, for example by putting government expenditures in utility or by adding distorting taxes. Indeed, an enormous literature, some mentioned in the review section, introduces fiscal policy in DSGE models to study such effects. But these features would cloud the picture of how fiscal price level determination per se works. Let us first really understand fiscal price determination, then add the separate question how spending and taxes alter equilibrium inflation and output. Also, such feedback from fiscal affairs to inflation occurs equally under active monetary policy specifications, so it really is orthogonal to the purposes of this paper.

4.1 Is it reasonable?

Once one considers its possibility, specifying that fiscal policy responds to changes in the value of debt that result from accumulated deficits and (later) from changes in the real interest rate, but does not respond to re-valuation of the debt stemming from arbitrary unexpected inflation or deflation, is not unreasonable or artificial.

Governments often do raise surpluses after a time of deficits. Doing so makes good on the explicit or implicit promise made when borrowing, and sustains the reputation needed for future borrowing. Governments often raise revenue from debt sales, and the value of debt increases after such sales, which essentially proves that investors believe surpluses will rise to pay off new debts. We see many institutions in place to try to guarantee or pre-commit to repayment, rather than default or inflation, and those institutions help the government to borrow in the first place.

But the same government may well refuse to accommodate changes in the value of debt that come from arbitrary unexpected inflation and deflation, and people may well expect such behavior. Should, say, a 50% cumulative deflation break out, likely in a severe recession, does anyone expect the U.S. government to sharply raise taxes or to drastically cut spending, to pay an unexpected, and, it will surely be argued, undeserved, real windfall to nominal bondholders – Wall Street bankers, wealthy individuals, and foreigners, especially foreign central banks? Will not the government regard the deflation as a temporary aberration, prices "disconnected from fundamentals," like a stock market "bubble," that fiscal policy should ignore until it passes? Indeed, is the response to such an event not more likely to be additional fiscal stimulus, deliberate unbacked fiscal expansion, not heartless austerity? Concretely, Cochrane (2017a) and Cochrane (2018) argue that this expectation is why the standard new-Keynesian prediction of a deflationary shock at the zero bound, and the old-Keynesian predictions of a "deflation spiral," did not happen in 2008-2009. Many economists call now for governments to pursue helicopter-drop unbacked fiscal stimulus in response to below-target inflation. Such a policy likewise represents a refusal to passively adapt surpluses to deflation, but to repay debts in normal circumstances. Jacobson, Leeper, and Preston (2019) argue persuasively that the Roosevelt Administration, in its abandonment of the Gold standard, executed such an unbacked fiscal expansion while preserving its reputation for repaying normal-times debt. Conversely, is not fiscal austerity a common response to inflation?

Committing not to accommodate arbitrary unexpected inflation and deflation is as wise as committing to repay debts. The former allows the government to produce a stable price level, just as the latter allows the government to borrow when in need.

We can see institutions and reputations at work to make these commitments. A gold standard is a commitment to raise taxes to buy gold in the event of inflation, or to borrow gold against credible future taxes, rather than to enjoy the bounty of an inflation-induced debt reduction. An inflation target signals the government's fiscal commitment to pay off nominal debts at the inflation target, neither more nor less, as much or possibly more than it signals the government's desired value for coefficients in a central bank Taylor rule. Many economists have suggested an analogous fiscal rule that raises surpluses in response to inflation, and runs deficits in the event of deflation, but still pays off debts incurred from past deficits should the price level come out on target. Such a rule is exactly the sort of policy I describe.

4.2 The surplus moving average

The structure (10)-(15) also allows us to specify surplus process with an s-shaped moving average that can promise full or partial debt repayment, within the AR(1) structure that is convenient for model solution. It is not the only way to specify such a process, but it is an elegant and convenient way to do so. Examining the implied surplus moving average, i.e. the response to a surplus shock, gives insight for how the model works.

To see the basic point most clearly, simplify to $i_t = 0$, and thus $E_t\pi_{t+1} = 0$. In equilibrium, the surplus and debt now follow

$$s_{t+1} = \alpha v_t + u_{s,t+1} \tag{17}$$

$$\rho v_{t+1} = v_t + \beta_s \varepsilon_{s,t+1} - s_{t+1} \tag{18}$$

$$u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}. \tag{19}$$

To find the implied moving average representation of the surplus process, which I denote

$$s_{t+1} = a(L)\varepsilon_{s,t+1} = \sum_{j=0}^{\infty} a_j \varepsilon_{s,t+1-j}, \tag{20}$$

first find the debt v_t process. Substituting (17) in (18),

$$\rho v_{t+1} = (1-\alpha)v_t + \beta_s \varepsilon_{s,t+1} - u_{s,t+1}$$

$$v_{t+1} = \frac{1}{\rho} \frac{1}{\left(1 - \frac{1-\alpha}{\rho} L\right)} \left(\beta_s \varepsilon_{s,t+1} - u_{s,t+1}\right).$$

Substituting back to the s process (17),

$$s_{t+1} = \frac{\alpha}{\rho} \frac{L}{\left(1 - \frac{1-\alpha}{\rho} L\right)} \left(\beta_s \varepsilon_{s,t+1} - u_{s,t+1}\right) + u_{s,t+1}$$

$$s_{t+1} = \left[1 - \frac{\alpha}{\rho} \frac{L}{\left(1 - \frac{1-\alpha}{\rho} L\right)}\right] u_{s,t+1} + \beta_s \frac{\alpha}{\rho} \frac{L}{\left(1 - \frac{1-\alpha}{\rho} L\right)} \varepsilon_{s,t+1}. \tag{21}$$

This representation is convenient for some intuition below. We can go further, gathering the terms of the first polynomial and substituting in the disturbance process, to write

$$s_{t+1} = \frac{\left(1 - \frac{1}{\rho}L\right) a_u(L) + \beta_s \frac{\alpha}{\rho} L}{1 - \frac{1-\alpha}{\rho} L} \varepsilon_{s,t+1}, \tag{22}$$

where, reflecting (19),

$$a_u(L) \equiv \frac{1}{1 - \rho_s L}.$$

Some points below hold for any disturbance process, so I introduce the general notation $a_u(L)$.

We can think of this calculation as the surplus moving average representation implied by the model, as I have derived it. We can also regard this surplus moving average as an "exogenous" surplus process, in the usual way of thinking of such models. We thereby think about (17)-(19) as just a way to write the surplus moving average (22) in state-space form, but all the economics of the model flow from this surplus process directly. Indeed, we can *derive* the representation (17)-(19) as a consequence of the surplus moving average (22). This way of thinking of the model avoids the entire v and v^* business. I offer this observation as an alternative for a reader who finds the v and v^* expression distasteful or confusing, and similarly as reassurance that it really does not do anything unusual in the end.

To be explicit at the cost of some repetition, in this way of thinking, we write the model as

$$\begin{aligned} i_t &= E_t \pi_{t+1} \\ s_{t+1} &= a(L) \varepsilon_{s,t+1}, \\ \Delta E_{t+1} \pi_{t+1} &= -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = -a(\rho) \varepsilon_{s,t+1}, \end{aligned} \tag{23}$$

and $a(L)$ is given by (22). The right hand equality in (23) comes from Hansen, Roberds, and Sargent (1992). The Appendix includes a self-contained derivation for convenience. In the case of the surplus moving average (22), we have

$$a(\rho) = \beta_s$$

so unexpected inflation follows

$$\Delta E_{t+1}\pi_{t+1} = -a(\rho)\varepsilon_{s,t+1} = -\beta_s \varepsilon_{s,t+1}. \tag{24}$$

We just *derived* unexpected inflation, equation (24) of the original specification, from the surplus moving average (22).

Similarly, we can derive the value of debt, and thereby (17) of the original specification. The value of debt is given by

$$v_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} = \frac{a(L) - a(\rho)}{1-\rho L^{-1}} \varepsilon_{s,t+1}. \tag{25}$$

The right hand expression is the Hansen and Sargent (1981) prediction formula, adapted to this model's timing. The Appendix gives a self-contained derivation. Substituting the surplus moving average (22) on the right hand side of (25), we derive relation (17). To show this result, I calculate $\alpha v_t - s_{t+1}$, and I show the result is equal to $-u_{s,t+1}$:

$$\begin{aligned}
\alpha v_t - s_{t+1} &= \left[\alpha \frac{a(L)-a(\rho)}{1-\rho L^{-1}} - a(L)\right]\varepsilon_{s,t+1} \\
&= \left[\left(\frac{\alpha}{1-\rho L^{-1}} - 1\right) a(L) - \alpha \frac{a(\rho)}{1-\rho L^{-1}}\right]\varepsilon_{s,t+1} \\
&= \frac{1}{1-\rho L^{-1}}\left[\left(\alpha - 1 + \rho L^{-1}\right) a(L) - \alpha\beta_s\right]\varepsilon_{s,t+1} \\
&= \frac{1}{1-\rho L^{-1}}\left\{\rho L^{-1}\left(1 - \frac{1-\alpha}{\rho} L\right)\left(\frac{\left(1-\frac{1}{\rho}L\right) a_u(L) + \beta_s \frac{\alpha}{\rho} L}{1-\frac{1-\alpha}{\rho}L}\right) - \alpha\beta_s\right\}\varepsilon_{s,t+1} \\
&= \frac{1}{1-\rho L^{-1}}\left[\left(\rho L^{-1} - 1\right) a_u(L) + \beta_s\alpha - \alpha\beta_s\right]\varepsilon_{s,t+1} \\
&= -a_u(L)\varepsilon_{s,t+1} = -u_{s,t+1}.
\end{aligned}$$

In this reading of (17), the surplus does not "respond" to debt. Surpluses are exogenous and debt is determined by the present value of surpluses. Equation (17) is an observed correlation between the debt and surplus process.

4.3 The s-shaped moving average

However one thinks of the model, by looking at the surplus moving average we understand better how the fiscal policy process works. With $\beta_s = a(\rho) = 0$, this surplus process always pays back any incurred debts. Any positive responses a_j, including $a_0 = 1$ by definition, are met by a corresponding set of negative a_j. (Hansen, Roberds, and Sargent (1992) show $a(\rho) = 0$ is the criterion for present value budget balance with constant interest rates, and indeed its only observable implication.) For example, $a(L) = 1 - \rho^{-1}L$ or $s_{t+1} = \varepsilon_{t+1} - \rho^{-1}\varepsilon_t$ has $a(\rho) = 0$. A shock $\varepsilon_1 = -1$ leads to $s_1 = -1$, $s_2 = +\rho$. The deficit is paid back the next period with interest.

The $\beta_s = a(\rho) = 0$ case reminds us that fiscal theory of the price level does not require that the government refuses to pay its debts, or always inflates away debt. This specification provides a contrary example in which the government never inflates away any debt and always repays debt with subsequent surpluses. The constant price level is nonetheless determined by fiscal theory and deficits and debt vary as much as one likes.

The first term of the surplus moving average expressed as (21) shows more clearly how this model generates debt repayment in the case $\beta_s = 0$. This term of the moving average has a movement in one direction, 1, followed by a string of small negative movements in the opposite direction. They are small, since α is a small number – the fraction of outstanding debt repaid by primary surpluses each period. And they decay over time slowly, with a $(1 - \alpha)$ autocorrelation coefficient. Adding dynamics $u_{s,t+1}$ smears out the shock, giving a stream of deficits which are only slowly followed by a longer-lasting small AR(1) in the opposite direction.

However, we want to study economies that do have some unexpected inflation, in which part of a deficit may be inflated away. The $\beta_s \neq 0$ parameter of this parametric surplus process allows us to introduce unexpected inflation in a convenient way. The second term of (21) adds a small, AR(1)-shaped decay in the direction of the original shock. This term adds a force by which the government does not fully pay back its accumulated deficits, and produces some unexpected inflation which inflates away and devalues some of the outstanding initial debt.

Figure 1 presents two cases of the surplus and debt process (17)-(19), equivalently the surplus moving average (22). The dashed lines present $\rho_s = 0.7$, $\alpha = 0.1$, $\beta_s = 0$. I plot the response to a deficit shock, $\varepsilon_{s,1} = -1$, which tells a cleaner story. The surplus starts

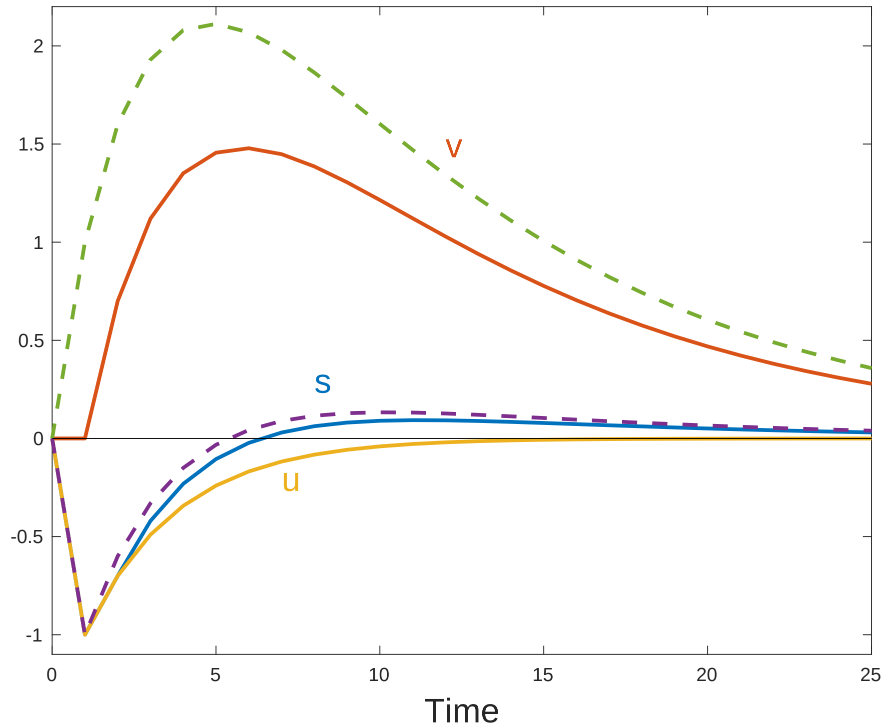

Figure 1: Response of the simple example surplus process to a unit deficit shock. The solid lines present the case $\beta_s = 1.0$. The dashed lines present the case $\beta_s = 0$. Other parameters $\rho = 1$, $\rho_s = 0.7$, $\alpha = 0.1$.

by following the AR(1) pattern of the disturbance $u_{s,t}$. These deficits increase the debt v_t. In turn, the increased debt slowly pushes up the surpluses. Eventually the deficits cross the zero line to surpluses, and positive surpluses start to pay down the debt. The many small positive responses on the right hand side of the graph in this case exactly pay off the initial deficits, $\sum_{j=1}^{\infty} \rho^{j-1} s_j = a(\rho) = 0$, and there is no unexpected inflation.

The solid lines plot the case $\beta_s = 1.0$. In this case $\sum_{j=1}^{\infty} \rho^{j-1} s_j = -a(\rho) = -1$, so the entire initial deficit s_1 is inflated away by a unit unexpected inflation. We see this behavior by the fact that the initial debt response is zero – despite a unit deficit, the two terms on the right hand side of (18) offset. Debt rises subsequently however. The persistent disturbance u_s adds persistent deficits $s_j < 0$ for $j > 1$, and these additional deficits are paid off by subsequent surpluses. The disturbance has cumulative response $\sum_{j=1}^{\infty} \rho^{j-1} u_j = -1/(1-\rho\rho_s) = -3.33$. If we had an AR(1) surplus $s_t = u_{s,t}$, we would see a 3.33% inflation shock at time 0, lowering the initial value of debt by the entire area under the u_t response.

4.4 Thinking about the parameters

The parameter β_s is convenient, as it directly controls unexpected inflation and the cumulative surplus response $a(\rho)$. However, it is best regarded as a reduced-form parameter, rather than an independent policy lever.

The parameter β_s lets the modeler control $\alpha(\rho)$, but in reality the government does the hard work of raising expected future surpluses, and $\beta_s = a(\rho)$ is the result. I interpret impulse-response functions this way, looking at the features of the time path of surpluses that result in unexpected inflation rather than the other way around.

Though one can estimate β_s and specify it in a model, one should not necessarily hold β_s constant as one examines alternative values for other parts of the fiscal and monetary policy specification, and that is what I mean by "reduced form." Usually, keeping β_s constant while moving other parameters of the policy process does not ask an interesting or sensible question. In practical terms, β_s directly controls unexpected inflation. If you hold β_s constant as you move other parameters of the policy process, those parameters cannot produce a different value of unexpected inflation.

In this simple model, the persistence ρ_s of the surplus process is the main other parameter. As we have seen, β_s measures how much of the *initial, first period* deficit shock $\varepsilon_{s,1}$ is inflated away. But as persistence ρ_s rises, cumulative deficits rise, and one would naturally expect more inflation from a fiscal shock. It might make more sense to compare the inflationary effects of two values of ρ_s by specifying that the government will inflate away the same fraction of the *overall* deficit shock, not the same fraction of the *first period* deficit shock. The overall deficit shock, is $a_u(\rho) = \sum_{j=1}^{\infty} \rho^{j-1} u_j = -1/(1 - \rho\rho_s)$. Thus, it seems more interesting to specify a larger value $\beta_s = \beta_{s,0}/(1-\rho\rho_s)$; or equivalently that $a(\rho)$ is the same fraction of $a_u(\rho)$ not the same number. We would then naturally conclude that if deficit shocks become more long-lasting, unexpected inflation is larger.

There is no right or wrong here, there are only interesting and uninteresting values of policy parameters to compare with each other. Here and more importantly below, I find that interesting policy experiments invite us to change β_s along with other policy parameters.

Why not take a stand, and parameterize the dependence of β_s on other policy parameters directly, as I suggest here with $\beta_s = \beta_{s,0}/(1-\rho\rho_s)$? At this stage, I do not wish to restrict in general which policy experiments are interesting and which are not interesting, and I wish to leave the model as simple as possible. By varying β_s along with other parameters, I force myself and the reader to think about which variation asks an interesting question.

The "right" answer of course is to derive and constrain policy with a specific model of optimal policy, derived from either economic and political fundamentals, or a specific theoretical or empirical model of how parameters move across changes in policy regimes. But for the present purpose that would be a long distraction, and also it would unwisely tie the analysis to one specific such model.

4.5 Active and passive policy, and an analogy to monetary policy rules

To understand the idea behind the specification (11)-(15), and to see that it is reasonable and not an artificial trick, it is useful to express standard active-money/passive fiscal policy for the same model in an analogous way.

In this simple model, an interest rate target determines expected inflation via the Fisher equation, $i_t = E_t \pi_{t+1}$. But this relation leaves unexpected inflation $\Delta E_{t+1}\pi_{t+1}$ undetermined. The whole question is what else to add to the model to tie down unexpected inflation.

The standard monetary policy rule,

$$i_t = \phi_\pi \pi_t + u_{i,t} \tag{26}$$

is algebraically equivalent to a more insightful form introduced by King (2000),

$$i_t = i_t^* + \phi_\pi (\pi_t - \pi_t^*) \tag{27}$$

$$i_t^* = E_t \pi_{t+1}^*. \tag{28}$$

Together with the equilibrium condition $i_t = E_t \pi_{t+1}$, model dynamics are now

$$E_t \left(\pi_{t+1} - \pi_{t+1}^* \right) = \phi_\pi (\pi_t - \pi_t^*). \tag{29}$$

With $\|\phi_\pi\| > 1$ any value but $\pi_t = \pi_t^*$ and thus $i_t = i_t^*$ leads to an explosive solution.

Adding a rule that nominal explosions are not equilibria, we conclude that $\pi_t = \pi_t^*$ is the unique equilibrium. The same logic at time $t+1$ yields $\pi_{t+1} = \pi_{t+1}^*$, $\Delta E_{t+1}\pi_{t+1} = \Delta E_{t+1}\pi_{t+1}^*$, the crucial determinacy issue we are after. For $\|\phi_\pi\| < 1$, any value of $\Delta E_{t+1}\pi_{t+1}$ leads to converging expectations of future inflation, and we are back to indeterminacy – passive monetary policy.

King's rule (27)-(28) clearly expresses the two separate and distinct functions of the interest rate rule in the active-money model: There is a monetary policy, which sets the observed, equilibrium path of interest rates i_t^*. (Monetary policy i_t^* may also feed back from other variables following rules, it need not be a stochastic peg.) There is a separate inflation target π_t^*, and a separate equilibrium-selection policy in which the central bank threatens hyperinflation or deflation in response to deviations from the inflation target π_t^*. We do not observe the hyperinflations in equilibrium. Monetary policy determines expected inflation. Equilibrium selection policy determines unexpected inflation.

I write the active-fiscal specification (11)-(14) intentionally in an analogous fashion. It likewise separates how surpluses behave in equilibrium from how they react to off-equilibrium inflation. Equilibrium condition (16), which I repeat here for convenience,

$$\rho(v_{t+1} - v_{t+1}^*) = (v_t - v_t^*) - \left(\Delta E_{t+1}\pi_{t+1} - \Delta E_{t+1}\pi_{t+1}^*\right)$$

is a direct analogy to (29).

If the surplus responds to the value of debt, $s_{t+1} = \gamma v_t + u_{s,t+1}$ rather than respond to the latent variable v_t^* in (11), then fiscal policy is passive. Debt follows

$$\rho v_{t+1} = (1-\gamma)v_t - \Delta E_{t+1}\pi_{t+1} - u_{s,t+1}.$$

If $\gamma > 0$, debt grows more slowly than the interest rate, satisfying the transversality condition. (The criterion $\gamma > 1-\rho$ bounds debt itself.) Any value of unexpected inflation $\Delta E_{t+1}\pi_{t+1}$ is an equilibrium, because it leads to surpluses which validate the inflation-induced change in the value of debt.

To include both active and passive possibilities in one model, we could write a more general form

$$s_{t+1}^* = \alpha v_t^* + u_{s,t+1}$$

$$
\begin{aligned}
s_{t+1} &= \alpha v_t^* + \gamma(v_t - v_t^*) + u_{s,t+1} \\
\rho v_{t+1}^* &= v_t^* - \Delta E_{t+1}\pi_{t+1}^* - s_{t+1}^* \\
\rho v_{t+1} &= v_t - \Delta E_{t+1}\pi_{t+1} - s_{t+1}.
\end{aligned}
$$

Now, (16) generalizes to

$$
\rho(v_{t+1} - v_{t+1}^*) = (1-\gamma)(v_t - v_t^*) - \Delta E_{t+1}(\pi_{t+1} - \pi_{t+1}^*). \tag{30}
$$

Again, $\gamma > 0$ means that debt v_t will not explode faster than the interest rate for any unexpected inflation. Since I am not interested in the passive-fiscal specification I leave this generality out of the more detailed model to follow.

Active fiscal policy means that the government will allow debt to explode at rate ρ^{-1} if inflation comes out differently from the inflation target, as the active-money government will force inflation to explode at the rate $\phi_\pi > 1$ if inflation comes out differently from its inflation target in (29).

Expression (30) clearly shows that relationships between surplus, debt and inflation in equilibrium, governed by α, tell us nothing about how surpluses react (or don't) to debt and inflation off equilibrium, governed by γ, as expression (29) clearly shows that relationships between interest rates and inflation in equilibrium tell us nothing about how interest rates would react to inflation off equilibrium governed by ϕ_π.

In particular, in equilibrium $v_t = v_t^*$ we see a positive regression coefficient of surplus on debt, $s_{t+1} = \alpha v_t + ...$ Such regression coefficients have been interpreted as evidence for passive policy, that $\gamma > 0$. Likewise, regression coefficients of equilibrium interest rates on equilibrium inflation have been interpreted as evidence for active monetary policy, that $\phi_\pi > 1$. But, as this example shows, observing such a relationship between equilibrium surpluses and debt is *not* evidence for passive policy. Active fiscal policy does *not* require that the government inflates away rather than repaying debt. In the $\beta_s = 0$ case, debt and deficits vary, the government borrows and then repays, there is no inflation at all, yet active fiscal policy determines the price level. Active fiscal policy only requires that the government refuse to adapt surpluses to changes in the value of debt brought about by arbitrary unexpected inflation and deflation. This fiscal specification maintains that minimum only, while allowing the other commonsense features of fiscal policy that have, mistakenly, been taken as evidence for passive fiscal policy.

In both cases, these expressions stress the separation between the active/passive question, which describes responses to alternative equilibria and unexpected inflation, from monetary policy, which sets the equilibrium interest rate, and from fiscal policy, which borrows, repays, and sometimes inflates away debt. Active fiscal policy substitutes for the monetary equilibrium-selection policy, but can leave monetary policy and observed fiscal policy alone.

More deeply, the King interest rate rule in (29) and the v^* specification of active fiscal policy in (30) show that the active-money vs. active-fiscal regimes are observationally equivalent, for time series on observables, here $\{\pi_t, i_t\}$, drawn from an equilibrum. With $\pi = \pi^*$, we never see the variation $\pi - \pi^*$ or $v - v^*$ necessary to identify the off-equilibrium responses that characterize each regime.

That fact means that identifying which regime we are in takes more effort than a simple time-series test such as regressions of interest rates on inflation or surpluses on debt can provide.

The traditional response to this problem is to add restrictions, which tie off-equilibrium behavior (ϕ_π, γ) to something observable in equilibrium. By doing so, one restricts the range of data that one or the other regime can explain. The literature review in Section 9 explains in some detail. The testable content however is entirely that provided by the identification assumptions.

My point here is to establish that a fiscal regime *can* explain any data that a monetary regime can explain. For that purpose, I do not wish to artificially restrict the ability of either regime to explain data by making additional assumptions. First we open the door to fiscal theory of monetary policy, then, perhaps, we test which side of the door the data land.

In the active-money specification (28), the inflation target $\{\pi_t^*\}$ may not be chosen independently of the interest rate $\{i_t^*\}$ specified by monetary policy. It must respect the private-sector equilibrium condition $i_t = E_t \pi_{t+1}$. That condition determines the expected inflation target, given the interest rate target, or the other way around. Thus, we could write King's rule in this model instead as

$$i_t = E_t \pi_{t+1}^* + \phi_\pi (\pi_t - \pi_t^*)$$

The central bank chooses the path of inflation $\{\pi_t^*\}$ it wishes to see. Expected inflation determines the necessary interest rate target. Or, the central bank picks the interest rate target, and the expected value of the inflation target must follow. We have seen the same restriction in the inflation target π_{t+1}^* of the fiscal specification.

5 Long-term debt

Long-term debt with a geometric maturity structure is a second key ingredient. This section shows how long-term debt modifies the constant real rate, flexible price model.

The model consists of the Fisher equation (8)

$$i_t = E_t \pi_{t+1},$$

and, from (7),

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \Delta E_{t+1} s_{t+1+j} \tag{31}$$

which generalizes (9). (Reminder: The face value of debt declines at the rate ω^j.)

Relation (31) introduces an important link between changes in expected inflation at different dates. Consider a persistent monetary policy shock – a persistent positive unexpected change in $i_t = E_t \pi_{t+1}$, starting at $i_1 = E_1 \pi_2$, with no change in fiscal policy. From (31), inflation follows

$$\Delta E_1 \pi_1 = -\sum_{j=1}^{\infty} \omega^j \Delta E_1 \pi_{1+j}. \tag{32}$$

If the terms $\Delta E_1 \pi_{1+j}$ for $j \geq 1$ are, on average, positive, then $\Delta E_1 \pi_1 < 0$. In this way, a positive persistent monetary policy shock that raises long-run inflation induces a negative initial inflation response. This mechanism temporarily overturns the otherwise Fisherian properties of this flexible-price money-free model. This observation boils down a large effort in more complex models (Sims (2011), Cochrane (2017b), Cochrane (2018)). This mechanism does not require sticky prices, and sticky prices alone do not produce a negative inflation response.

With short-term debt $\omega = 0$, a fiscal shock leads immediately to inflation at time 1 by (31). If the central bank chooses to follow that event with higher interest rates, the inflation will persist, but that is an independent choice and has no effect on immediate inflation $\Delta E_1 \pi_1$.

With long-term debt $\omega > 0$, shocks to the present value of surpluses in (31) now change a weighted average of current and expected future inflation. If a fiscal shock is followed by persistent inflation, the initial inflation will be smaller, or there may even be no immediate inflation. Since monetary policy controls expected inflation, monetary policy chooses whether to smooth forward the fiscal shock, and thereby reduce its immediate impact, or not to do so.

Contrary to the impression one gets with short-term debt models, then, fiscal theory can produce a persistent inflation response to fiscal shocks. And monetary policy plays a crucial role: by accommodating the fiscal shock with higher nominal interest rates, the central bank can turn what would be a price level jump into a slow smooth inflation.

6 Model

Now we are ready to understand and to digest the main model. The model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{33}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{34}$$

$$i_t = \theta_{i\pi} \pi_t + \theta_{ix} x_t + u_{i,t} \tag{35}$$

$$s_{t+1} = \theta_{s\pi} \pi_{t+1} + \theta_{sx} x_{t+1} + \alpha v_t^* + u_{s,t+1} \tag{36}$$

$$\rho v_{t+1}^* = v_t^* + r_{t+1}^n - \pi_{t+1}^* - s_{t+1} \tag{37}$$

$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - s_{t+1} \tag{38}$$

$$E_t \pi_{t+1}^* = E_t \pi_{t+1} \tag{39}$$

$$\Delta E_{t+1} \pi_{t+1}^* = -\beta_s \varepsilon_{s,t+1} - \beta_i \varepsilon_{i,t+1} \tag{40}$$

$$E_t r_{t+1}^n = i_t \tag{41}$$

$$r_{t+1}^n = \omega q_{t+1} - q_t \tag{42}$$

$$u_{i,t+1} = \rho_i u_{i,t} + \varepsilon_{i,t+1} \tag{43}$$

$$u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}. \tag{44}$$

These the same equations as the standard new-Keynesian model, with a set of fiscal equations elaborated and spelled out. I parameterize and solve the model with active fiscal rather than active monetary policy.

Equations (33) and (34) are standard intertemporal IS and Phillips equations, with x_t denoting the output gap, i_t the nominal interest rate and π_t inflation. One can add disturbances to both equations. I leave such disturbances out here as I only present responses to monetary and fiscal policy shocks below. While one can do much to make both equations more realistic, I keep them deliberately simple and standard to focus on the impact of the fiscal novelty on this well-understood textbook structure.

Equations (35) and (36) are monetary and fiscal policy rules. In the fiscal policy rule, surpluses respond to output and inflation. Such a response is natural for both mechanical and policy reasons. Tax receipts are procyclical as tax rate times income rises with income. Spending is countercyclical, due to automatic stabilizer entitlements such as unemployment insurance and food stamps, and also due to deliberate stimulus programs. Imperfect indexation may make surpluses rise automatically with inflation, and inflation may provoke fiscal austerity programs. Beyond fitting data, we are interested in alternative fiscal policy rules that can stabilize inflation or avoid deflation, especially in a period of zero bounds or other constraints on monetary policy, and including or strengthening fiscal policy responses to inflation and output is important to studying these issues. Finally, including policy rules stresses that fiscal theory need not assume fixed or exogenous surplus process.

Equations (36)-(40) generalize the simple fiscal policy specification (11)-(15) to include long-term debt and real interest rate variation. In (37), the state variable v^* responds to real interest rates. Surpluses therefore rise to pay off increases in the value of debt that come from higher real interest costs, but as before surpluses do not respond to changes in the value of debt brought about by arbitrary unexpected inflation. The variable π_t^* is the governments' stochastic inflation target. In (38), which is the debt accumulation identity (1), ex-post nominal returns on the portfolio of government bonds r_{t+1}^n raise or lower the value of debt.

Again, the parameter $\beta_s \neq 0$ in (40) allows for a s-shaped surplus process that does not fully repay debts and interest costs. Now we have β_i as well, which describes how much the state variable v^*, future surpluses, and thereby the value of debt, respond to monetary shocks, or to the deeper shocks that motivate monetary and fiscal policy changes. New-Keynesian models with passive fiscal policy implicitly specify large values for β_i, which we can emulate if we wish to do so.

Equation (41) is the bond pricing equation, which imposes the expectations hypothesis that expected returns on bonds of all maturities are the same. Equation (42) is the linearized identity (4) that relates the return on the government bond portfolio to its price q_t.

In Section 3, we saw that $i_t = E_t\pi_{t+1}$ of that model gave the central bank, which sets the interest rate i_t but cannot affect fiscal policy, control over expected inflation. The bank retains a similar power in this model. Eliminating the output gap x_t from (33)-(34), we have

$$\beta E_t\pi_{t+2} - (1+\beta-\sigma\kappa)E_t\pi_{t+1} + \pi_t = \sigma\kappa i_t. \tag{45}$$

We can write this equation that expected inflation $E_t\pi_{t+1}$ is a two-sided exponentially-weighted moving average of the interest rate i_t,

$$\pi_{t+1} = \frac{\sigma\kappa}{\lambda_1-\lambda_2}\left[i_t + \sum_{j=1}^{\infty}\lambda_1^{-j}i_{t-j} + \sum_{j=1}^{\infty}\lambda_2^{j}E_{t+1}i_{t+j}\right] + \sum_{j=0}^{\infty}\lambda_1^{-j}\Delta E_{t+1-j}\pi_{t+1-j} \tag{46}$$

with weights given by the roots of the lag polynomial (45) (Cochrane (2018) p. 165), plus an exponentially decaying transient response to shocks. This formula naturally generalizes $\pi_{t+1} = i_t + \Delta E_{t+1}\pi_{t+1}$ to sticky prices. Therefore, monetary policy can still determine expected inflation. It just takes a more complex interest rate path to give any particular expected inflation path. The last term of (46) introduces persistent dynamics to responses that only last one period under flexible prices. This standard new-Keynesian model remains Fisherian, like the flexible price case: A rise in interest rates produces a rise in inflation, unless accompanied by a shock to unexpected inflation in the last term. As in the standard solution of this model, if a rise in interest rates is fully expected, it produces a uniform rise in inflation, as the last term is absent.

The IS and Phillips curves (33)-(34) only restrict expected inflation and output. They leave two undetermined expectational errors, needing two forward-looking roots to give a unique equilibrium. As usual, they have one forward and one backward-looking root, so we need one extra forward-looking root. In active-money new-Keynesian models $\theta_{\pi i} > 1$ (roughly speaking) generates the additional explosive root. I specify passive monetary policy with $\theta_{\pi i} < 1$. Fiscal policy provides the extra forward-looking root and determines the additional expectational error. As in the simple case, (37)-(38) imply

$$\rho\left(v_{t+1} - v^*_{t+1}\right) = \left(v_t - v^*_t\right) - \left(\pi_{t+1} - \pi^*_t\right). \tag{47}$$

Hence, stationary solutions and the real transversality condition require $v_t = v^*_t$, $\pi_t = \pi^*_t$. This is the extra forward-looking root provided by active fiscal policy. The computer will discover (47) on its own and impose it, but we can also impose it to simplify the model analytically.

Equation (39) $E_t\pi^*_{t+1} = E_t\pi_{t+1}$ restricts the inflation target. In its place, we could write (37)-(38) as

$$\rho v^*_{t+1} = v^*_t + r^n_{t+1} - E_t\pi_{t+1} - \Delta E_{t+1}\pi^*_{t+1} - s_{t+1}$$
$$\rho v_{t+1} = v_t + r^n_{t+1} - E_t\pi_{t+1} - \Delta E_{t+1}\pi_{t+1} - s_{t+1}.$$

The state variable v^*_t encodes a promise to adapt fiscal policy to arbitrary expected inflation, but insists on only one value for unexpected inflation. Then (47) reads

$$\rho\left(v_{t+1} - v^*_{t+1}\right) = \left(v_t - v^*_t\right) - \left(\Delta E_{t+1}\pi_{t+1} - \Delta E_{t+1}\pi^*_t\right).$$

This writing reinforces the idea that the interest rate and monetary policy are in charge of expected inflation, and fiscal policy here serves only to select unexpected inflation. Choose whichever expression you find prettier.

Again, the model solution separates. Fiscal affairs only influence inflation and output by choosing unexpected inflation $\Delta E_{t+1}\pi_{t+1} = -\beta_s\varepsilon_{s,t+1} - \beta_i\varepsilon_{i,t+1}$. We can solve for inflation and output by pairing this condition with the IS, Phillips, and interest rate rule, (33)-(35), and (43) and ignoring the rest.

Having determined inflation and output, with $\pi = \pi^*$, $v = v^*$, we can calculate fiscal variables from the remaining equations of the model. Again, in a more general model, these fiscal outcomes will feed back to inflation, output, and interest rates in ways beyond the selection of unexpected inflation, but again simplicity is a virtue here: Fiscal price level determination does not require such feedback, and the mechanisms for such feedback are orthogonal to the price determination question.

As in the simple model, linking the unexpected inflation target directly to policy shocks in (40) is a convenient reduced-form simplification. One can estimate the β parameters, but one should consider changing β_s and β_i change as we change other parameters in order to make interesting comparisons of policy settings.

In section 4, we saw how we might want to modify β_s as we make surplus shocks more persistent, raising ρ_s. Here, we also might want similar modifications as we change policy rules θ. For example, we might want to specify the first part of the unexpected inflation target as

$$\Delta E_{t+1}\pi^*_{t+1} = -\beta_s \Delta E_{t+1} s_{t+1} \tag{48}$$

in place of (40). Now β_s specifies how much of an actual deficit will be met by unexpected inflation, not how much of a deficit shock will be so met. The actual deficit is influenced by its endogenous reaction to inflation and output, which also respond to the shock. To see the effect of (48), use the surplus policy rule (36), and also simplify to $\theta_{sx} = 0$, yielding

$$\Delta E_{t+1} s_{t+1} = \theta_{s\pi} \Delta E_{t+1}\pi_{t+1} + \varepsilon_{s,t+1}.$$

Equation (48) then implies

$$\Delta E_{t+1}\pi_{t+1} = -\beta_s \left(\theta_{s\pi} \Delta E_{t+1}\pi_{t+1} + \varepsilon_{s,t+1}\right)$$

and thus

$$\Delta E_{t+1}\pi_{t+1} = -\frac{\beta_s}{1+\beta_s\theta_{s\pi}} \varepsilon_{s,t+1}.$$

We're back to where we started, but the parameter β_s of the original specification depends on the $\theta_{s\pi}$ parameter.

So, if it is interesting to think of a government policy that splits a constant fraction of shocks to actual deficits between repayment and inflation, rather than splitting shocks to the disturbance part of a policy rule, then we would want to specify (48). Equivalently, we recognize that the parameter β_s is a reduced-form parameter, and we change β_s as we change $\theta_{s\pi}$ in thinking about the effects of alternative policies.

Why not write such a more general structure? First, it makes no difference to the result, it just suggests different parameter values. In the end unexpected inflation is a function of the shocks $\varepsilon_{s,t+1}$ and $\varepsilon_{i,t+1}$. We just end up deriving the coefficients β_s and β_i from that more general structure. Second, while other structures are more plausible, we don't have a single obvious deep theory of just how a government chooses and commits to dividing the fiscal consequences of shocks between state-contingent default via inflation and promises to repay via taxation or spending cuts, that would make sense to impose at this stage of modeling. Third, this specification of fiscal policy is complicated enough for a first attempt. Adding equations to derive the β_s, β_i parameters in a more rigorous way will only obfuscate the model. So, I proceed as specified, with the disclaimer that one should not view the β coefficients in (40) as a deep description of policy, and one should consider changing their values along with changes in other aspects of the policy specification including the coefficients θ and ρ_s, ρ_i, and a warning that I do so in considering the effects of different policy parameters below.

Moreover, the parameters β_i and β_s give the modeler too much control, in a sense. One should regard unexpected inflation as coming from revisions to the present value of surpluses, now including a discount rate term. When one changes β_i or β_s, the surplus process changes, and discount rates may change, until the specified unexpected inflation is again the revision in the present value of surpluses. But it is more economically sensible to think of the surplus and discount rate process as causing the change in inflation. I will look at the surplus and discount rate responses with that intuition in mind.

Long-term debt adds another expectational error, (41), but one more unstable root in (42). Together (41)-(42) solve forward to say that the bond price is the weighted sum of future interest rates,

$$q_t = -E_t \sum_{j=1}^{\infty} \omega^{j-1} i_{t+j-1}.$$

Therefore, the ex-post bond return depends on current and future interest rates,

$$r_{t+1}^n = \omega q_{t+1} - q_t = i_t - \omega \Delta E_{t+1} \sum_{j=1}^{\infty} \omega^{j-1} i_{t+j}.$$

I next compute the impulse-response functions. The surplus responses so computed are the moving average representation of the surplus, $s_t = a(L) \left[\varepsilon_{i,t}\ \varepsilon_{s,t}\right]'$. If one wished, one could avoid the v^*, π^* expression of fiscal policy and simply pair the moving average for surpluses with the equations of the new-Keynesian model (33), (34), (35) and the debt accumulation identity (38), as we did in the simple model. The surplus moving average and debt accumulation equation would result in a unique unexpected inflation, from which the equations of the new-Keynesian model have a unique solution. The surplus moving average would not be algebraically pretty.

7 Responses

The Appendix documents the algebra for numerically solving the model in the standard Blanchard and Kahn (1980) way. I set the model up in VAR(1) form $y_{t+1} = Ay_t + B\varepsilon_{t+1} + C\delta_{t+1}$ with ε denoting structural shocks – monetary and fiscal policy here – and δ denoting expectational errors, i.e. I write $i_t = E_t\pi_{t+1}$ as $\pi_{t+1} = i_t + \delta_{\pi,t+1}$. I solve unstable eigenvalues of A forward and stable eigenvalues backward. We need just as many forward-unstable eigenvalues as there are expectational errors, to uniquely determine the latter as functions of the structural shocks.

Throughout I use parameters $\rho = 1, \sigma = 0.5, \kappa = 0.5, \alpha = 0.2, \omega = 0.7, \rho_i = 0.7, \rho_s = 0.4$. I pick these parameters as vaguely plausible, but to clearly illustrate mechanisms, not to match data.

7.1 Deficit shocks without policy rules

Figure 2 presents the responses of this model to a negative fiscal policy disturbance $u_{s,t}$, i.e. a deficit shock, in the case of no policy rules $\theta = 0$. I specify $\beta_s = 0.25$, allowing a quarter of the deficit shock to be met by inflation and the rest, plus the following deficits, to be financed by borrowing against subsequent surpluses.

With neither monetary policy shock nor rule, the interest rate i_t and therefore long-term nominal bond return r^n_{t+1} do not move. Long-term debt therefore has no influence on these responses, which are the same for any bond maturity ω.

Inflation rises and decays with an AR(1) pattern. The deficit shock results in drawn-out inflation, not just a one-period price-level jump. This drawn-out inflation is entirely the effect of sticky prices. It reflects the last term of (46), the exponentially decaying response to a shock in a sticky-price model. The drawn-out inflation does nothing to reduce the first-period inflation shock.

The nominal interest rate does not move, so the bond return the bond return r^n_1 is

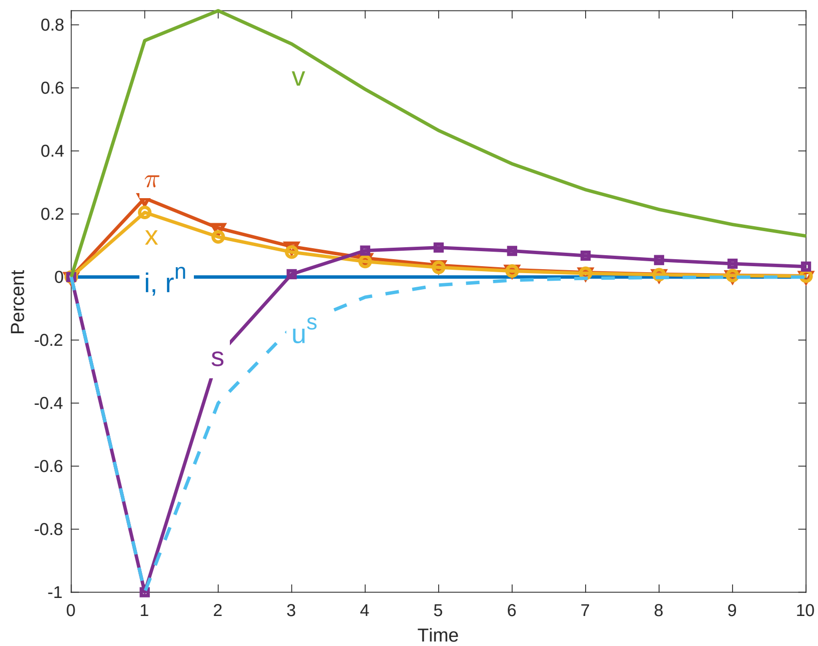

Figure 2: Responses of the sticky-price model to a fiscal shock with no policy rules.

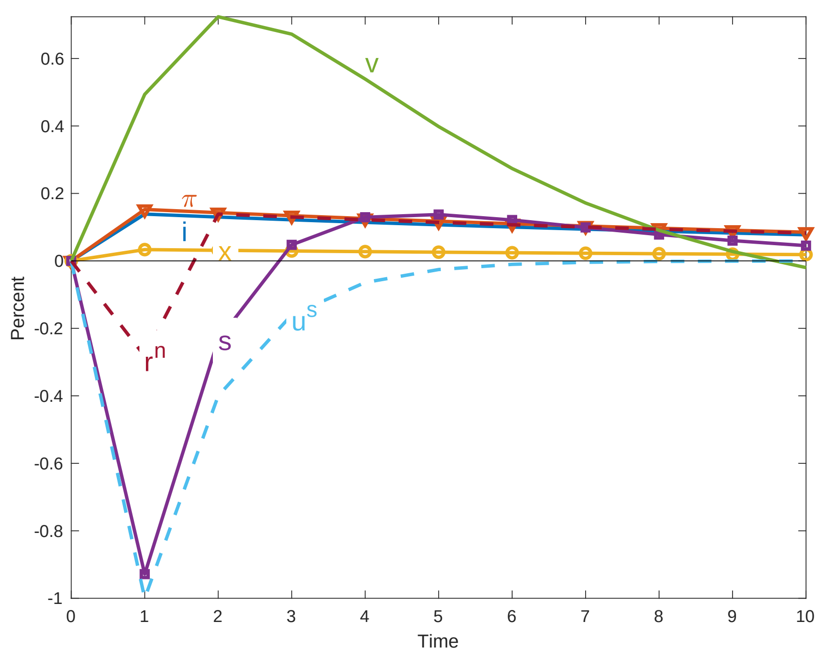

Figure 3: Responses of the sticky-price model to a fiscal shock, with policy rules.

also constant. The real rate falls exactly as inflation rises.

Output rises, following the forward-looking Phillips curve that output is high when inflation is declining, i.e. inflation is high relative to future inflation. This deficit does stimulate, by provoking inflation.

The surplus s_t and the AR(1) surplus disturbance $u_{s,t}$ are not the same. The surplus initially declines, but deficits raise the value of debt overall. A long string of small positive surplus responses on the right side of the graph then partially repays the incurred debt with an s-shaped response pattern. Here we see the major innovation of this model at work – the s-shaped surplus response.

That inflation rises at all comes from the specification $\beta_s = 0.25$. With $\beta_s = 0$, the long run surplus response would be higher, the discounted sum of all future surpluses would be exactly zero, and there would be no inflation. Or, better put, a surplus process with larger long-run positive responses would imply $\beta_s = 0$. Conversely, the government may choose or be of a type that inflates away more of its debts in response to fiscal shocks, which we would model with a higher value of β_s.

Shock and model	$\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j}$	=	$-\sum_{j=0}^{\infty} \Delta E_1 s_{1+j}$	$+\sum_{j=1}^{\infty} (1-\omega^j) \Delta E_1 \left(r^n_{1+j} - \pi_{1+j}\right)$
Fiscal, no θ rules	(0.44)	=	$-(-0.66)$	$+(-0.22)$
Fiscal, yes θ rules	(0.44)	=	$-(-0.50)$	$+(-0.06)$
Monetary, no θ rules	(-1.79)	=	$-(3.26)$	$+(1.47)$
Monetary, yes θ rules	(-0.093)	=	$-(0.53)$	$+(0.62)$

Shock and model	$\Delta E_1 \pi_1$	$-\Delta E_1 r^n_1$	=	$-\sum_{j=0}^{\infty} \Delta E_1 s_{1+j}$	$+\sum_{j=1}^{\infty} \Delta E_1 \left(r^n_{1+j} - \pi_{1+j}\right)$
Fiscal, no θ rules	(0.25)	$-(0.00)$	=	$-(-0.66)$	$+(-0.41)$
Fiscal, yes θ rules	(0.15)	$-(-0.28)$	=	$-(-0.50)$	$+(-0.07)$
Monetary, no θ rules	(-1.37)	$-(-1.37)$	=	$-(3.26)$	$+(3.26)$
$+(0)$					
Monetary, yes θ rules	(-0.36)	$-(-1.06)$	=	$-(0.53)$	$+(1.23)$

Shock and model	$\Delta E_1 r^n_1$	=	$-\sum_{j=1}^{\infty} \omega^j \Delta E_1 \pi_{1+j}$	$-\sum_{j=1}^{\infty} \omega^j \Delta E_1 \left(r^n_{1+j} - \pi_{1+j}\right)$
Fiscal, no θ rules	(0)	=	$-(0.19)$	$-(-0.19)$
Fiscal, yes θ rules	(-0.28)	=	$-(0.29)$	$-(-0.01)$
Monetary, no θ rules	(-1.37)	=	$-(-0.41)$	$-(1.79)$
Monetary, yes θ rules	(-1.06)	=	$-(0.45)$	$-(0.61)$

Table 1: Inflation and bond-return decompositions.

The "Fiscal, no θ rules" rows of Table 1 present the terms of the unexpected inflation decompositions (3) and (7) and the bond return decomposition (6) for these responses.

The cumulative fiscal disturbance is $\Delta E_1 \sum_{j=0}^{\infty} u_{s,j} = 1/(1-\rho_s) = -1.67\%$, which on its own would lead to 1.67% inflation. We see two mechanisms that buffer this fiscal shock. First, the s-shaped endogenous response of surpluses to accumulated debt v^*, pays off one percentage point much of these accumulated deficits, leaving a $\Delta E_1 \sum_{j=0}^{\infty} s_j = -0.66\%$ unbacked fiscal expansion. Second, higher inflation with no change in nominal rate means a lower real interest rate, which raises the value of debt, a deflationary force. This discount rate effect offsets another 0.22% of the fiscal inflation in the top row, leading to 0.44% ω-weighted inflation, and 0.41% in the second row, leading to 0.25% first-period inflation.

7.2 Deficit shocks with policy rules

Next, let us add fiscal and monetary policy reaction functions. I use numerical values

$$i_t = 0.8\,\pi_t + 0.5\,x_t + u_{i,t} \tag{49}$$

$$s_{t+1} = 0.25\,\pi_{t+1} + 1.0\,x_{t+1} + 0.2\,v_t^* + u_{s,t+1} \tag{50}$$

$$u_{i,t+1} = 0.7\,u_{i,t} + \varepsilon_{i,t+1} \tag{51}$$

$$u_{s,t+1} = 0.4\,u_{s,t} + \varepsilon_{s,t+1} \tag{52}$$

These parameters are also intended only as generally reasonable back of the envelope values that illustrate mechanisms clearly in the plots. Estimating policy rules is tricky, as the right hand variables are inherently correlated with errors, and there no reliable instruments.

I specify an interest rate reaction to inflation $\theta_{i\pi}$ less than one, to easily generate a stationary passive-money model. The on-equilibrium monetary-policy parameter $\theta_{i\pi}$ can in principle be measured in this fiscal theory, so regression evidence is relevant. But the evidence for $\theta_{i\pi}$ substantially greater than one in the data, such as Clarida, Galí, and Gertler (2000), is tenuous, needing specific lags, instruments, and a sample period. OLS regressions lead to a coefficient quite close to one – the "Fisher effect" that interest rates rise with inflation dominates the data. With more complex specifications, one can create a passive-money model in which regressions of interest rates on inflation have a coefficient greater than one (Cochrane (2011b)). But, as I am not trying to match regressions and independent estimates of the other parameters of the model, I leave estimation of the policy response functions along with those other parameters for another day.

I use a surplus response to output $\theta_{sx} = 1.0$. The units of surplus are surplus/value of debt, or surplus/GDP divided by debt/GDP, so one expects a coefficient of about this magnitude. For example, real GDP fell 4 percentage points peak to trough in the 2008 recession, while the surplus/GDP ratio fell nearly 8 percentage points. Debt to GDP of 0.5 (then) leads to a coefficient 1.0. Surpluses should react somewhat to inflation, as the tax code is less well indexed than spending. But it's hard to see that pattern in the data. Surpluses were low with inflation in the 1970s and an OLS regression that includes both inflation and output, though surely biased, gives a negative coefficient. (The Appendix presents simple OLS regressions, which give this result and also suggest $\rho_s = 0.4$.) I use $\theta_{s\pi} = 0.25$ to explore what a small positive reaction to inflation can do.

Figure 3 adds these policy responses θ. This plot presents the responses to a deficit shock $\varepsilon_{s,1} = -1$, holding constant the monetary policy *disturbance* $u_{i,t}$ but now allowing surpluses and interest rates to change in response to inflation and output. Table 1 quantifies the corresponding decompositions, in the "fiscal, yes θ" rows.

To produce this example, I did not keep the parameter β_s constant. If we keep β_s constant, then we produce exactly the same unexpected inflation $\Delta E_1 \pi_1 = -\beta_s \varepsilon_{s,1}$ for any choice of the other parameters. As above, β_s is not a deep parameter. The role of β_s is to characterize how much of a fiscal shock the government chooses to meet by inflating away its debt, vs. how much it meets by borrowing against higher future surpluses, but β_s itself does not well characterize this split, especially in this model with long-term debt. For example, suppose the government paid for the fiscal shock with future inflation entirely, raising interest rates and thereby devaluing outstanding long-term bonds. Then we would have a $\beta_s = 0$ even though all of the fiscal shock is met by inflation.

To produce a more comparable simulation across parameter values, I choose the parameter β_s so that the ω-weighted sum of current and expected future inflation relative to the overall size of the fiscal shock $\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} / \sum_{j=0}^{\infty} \rho^j \Delta E_1 u_{s,1+j}$ is the same across the calculation without rules and this calculation with rules. Via the decomposition (7), the numerator is as the total amount of the fiscal shock absorbed by current and future inflation, devaluing long and short term bonds. The denominator is the amount of inflation that the surplus shock would produce on its own absent all policy rules – the αv_t response to debts as well as the θ responses to output and inflation. This scaling produces $\beta_s = 0.61 \times 0.25 = 0.1525$. The graph with an unchanged $\beta_s = 0.25$ is qualitatively similar, except inflation starts at an unchanged 0.25.

Monetary policy reacts to higher inflation and output by raising the nominal interest rate, which was constant with no policy rules. (The nominal interest rate, labeled i, is just below the inflation π line.) This unexpected rise has the standard long-term debt effect: it pushes inflation forward and thereby reduces current inflation. It produces a negative ex-post bond return.

Greater inflation and output also raise fiscal surpluses through the θ_{sx} and $\theta_{s\pi}$ parts of the fiscal policy rule. The surplus line is slightly higher in Figure 3 than in Figure 2. (Look hard. Small changes add up.) These higher subsequent surpluses also reduce the inflationary effects of the fiscal shock.

Finally, the inflation rate is slightly larger than the interest rate, leading to a persistent negative real interest rate. This real rate reduction is also deflationary. You can see it drag down the value of debt despite positive surpluses.

In Table 1, ω-weighted inflation is the same, 0.44%, by construction. Instantaneous inflation 0.15% is about half its previous value 0.25%, since inflation is much more persistent. The cumulative surplus shock is the same, 1.66%, and the s-shape surplus process pays back a bit more, as we saw, leaving 0.50%. Since the interest rate moves with the inflation rate, there is much less real interest rate and discount rate variation, only 0.06% and 0.07%. In the second panel a negative bond return, reflecting future inflation, soaks up the fiscal shock in the mark-to-market accounting.

This example produces drawn-out inflation in response to a transitory fiscal shock, not a price level jump. The endogenous policy responses smooth forward and thereby reduce the inflation and output response to the fiscal shock, a novel argument in favor of such rules. This result begins a suggestive story of the 1970s. However, the model does not produce the lower output characteristic of stagflation. That failure is likely rooted in the simplistic and often-criticized nature of this Phillips curve, and also the absence of any interesting supply side of the economy such as the oil shocks and productivity slowdown of the 1970s.

7.3 Monetary policy shocks without policy rules

Figure 4 presents responses to a monetary policy shock $\varepsilon_{i,1}$, with no policy rule response to endogenous variables $\theta = 0$. The nominal interest rate i_t just follows the AR(1) shock process $u_{i,t}$.

Again, the tricky question in this response is what value of β_i to specify – what is the most interesting way to define a monetary policy shock that does not move fiscal policy? I already specify that the monetary policy shock comes with no direct fiscal shock $u_{st} = 0$. I furthermore choose β_i so that the value of the debt v_t is unaffected by the shock, $\Delta E_1 v_1 = 0$, as you can see in Figure 4. Any rise in the value of the debt triggers subsequent surpluses via the αv_t term in the surplus process, so specifying no shock to debt is another way of specifying that the monetary policy shock does not directly change surpluses.

This choice of β_i also sensibly generalizes the case without pricing frictions or policy rules. Taking innovations of the debt accumulation equation (38), we have

$$\rho \Delta E_1 v_1 = \Delta E_1 r_1^n - \Delta E_1 \pi_1 - \Delta E_1 s_1. \tag{53}$$

Recall also the identity (3),

$$\Delta E_1 \pi_1 - \Delta E_1 r_1^n = -\sum_{j=0}^{\infty} \rho^j \Delta E_1 s_{1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_1 \left(r_{1+j}^n - \pi_{1+j} \right). \tag{54}$$

In the simple case we hold surpluses constant $\Delta E_1 s_{1+j} = 0$ and real interest rates are constant. Both terms on the right hand side of (54) are zero, so the left-hand side is also zero: The period 1 price level jump exactly matches the decline in nominal bond prices, $\Delta E_1 r_1^n = \Delta E_1 \pi_1$. In (53), then, the real value of debt does not change.

Inflation π declines initially, and then rises to meet the higher nominal interest rate. This model remains Fisherian in the long run, or to expected interest rate movements. But the rise in inflation is long delayed, and would be hard to detect. Output also declines, following the new-Keynesian Phillips curve in which output is low when inflation is rising, i.e. lower than future inflation. The path of the expected nominal return r_{t+1}^n follows the interest rate i_t, as this model uses the expectations hypothesis. That rise in expected returns and bond yields sends bond prices down, resulting in the sharply

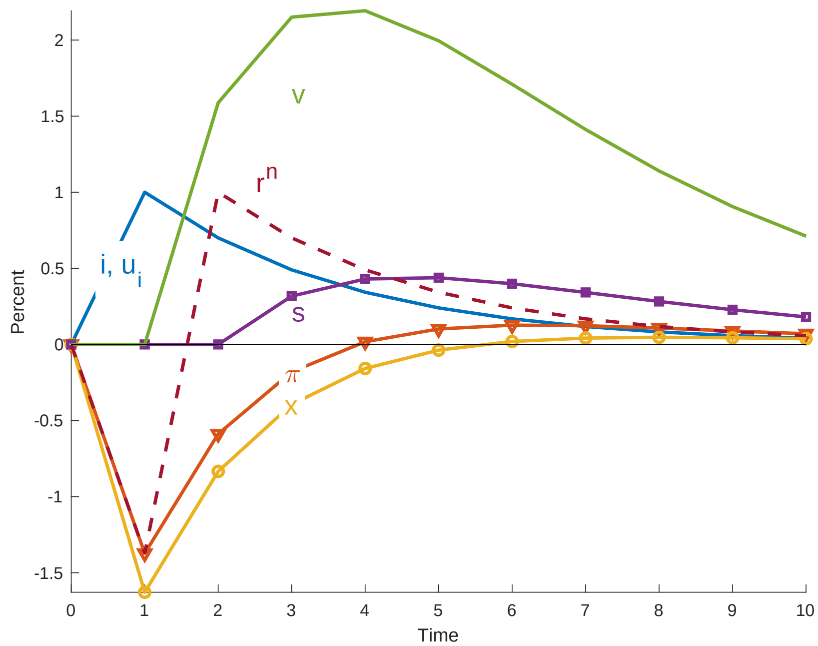

Figure 4: Responses to a monetary policy shock, no policy rules.

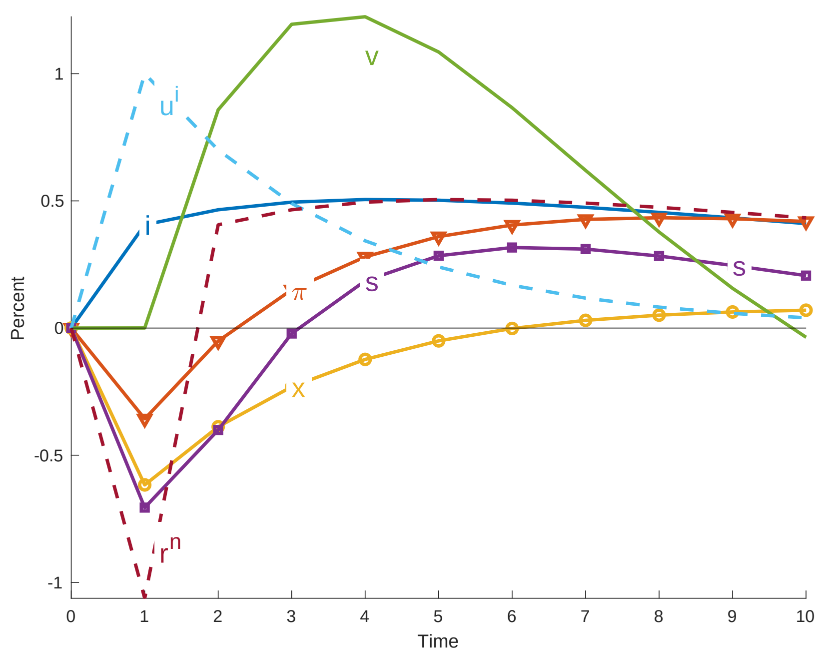

Figure 5: Responses to a monetary policy shock, with policy rules.

negative instantaneous bond return r_1^n. Subtracting inflation from these nominal bond returns, the expected real interest rate rate, expected real bond return, and discount rate rise persistently.

Surpluses are not constant. Here, I define a monetary policy shock that holds constant the fiscal policy *disturbance* $u_{s,t} = 0$, but not surpluses s_t themselves. Even though surpluses do not (yet) respond directly to inflation and output, surpluses respond to the increased value of the debt v that results from higher real returns on government bonds. I include this effect deliberately. It seems a realistic description of an "unchanged fiscal policy" that fiscal authorities will raise surpluses to meet higher real interest costs. One can easily change the model to embody the opposite assumption. This positive surplus response enhances the disinflation. The higher real discount rates resulting from the interest rate, on the other hand, push near-term inflation up.

The "Monetary, no θ rules" rows of Table 1 present inflation decompositions for this case. The ω-weighted sum of inflation is large and negative, -1.79%. In the absence of price stickiness, this number would be zero–monetary policy could rearrange inflation, lowering current inflation by raising future inflation, but monetary policy could not create less inflation overall. Here it does, by two effects: Monetary policy induces a large fiscal tightening, as we see in the rising surplus, amounting to 3.26% deflationary pressure. Sticky prices lead to higher real interest rates, which give an offsetting 1.47% inflationary discount rate effect.

In the middle panel, the unexpected inflation and bond returns exactly offset. This occurs by construction, as I specified the shock not to change the value of debt, as above. By consequence, the surplus and discount rate shocks must also exactly offset.

7.4 Monetary policy shocks with policy rules

Figure 5 plots responses to the monetary policy shock, now adding fiscal and monetary policy rules θ that respond to output and inflation. The monetary policy rule responses to lower inflation and growth push the interest rate i initially below its disturbance u_i. I held down the coefficient $\theta_{i\pi} = 0.8$, rather than a larger value, to keep the interest rate response from being negative, the opposite of the shock. Interest rates that go in the opposite direction from monetary policy shocks are a common feature in new–Keynesian models of this sort. (Cochrane (2018) p. 175 shows some examples.) But such responses are confusing, and my point here is to illustrate mechanisms. The interest rate is then quite flat, the policy rules times rising inflation and output offsetting the declining disturbance u_i. Long-term bonds again suffer a negative return on impact, due to the persistent rise in nominal interest rate. They then follow interest rates with a one period lag, under the model's assumption of an expectations hypothesis. The real rate, the difference between interest rate and inflation, again rises persistently.

Comparing the cases with and without policy rules, the surplus, responding to the output and inflation decline, now declines sharply on impact and persists negatively for a few years, before recovering. The monetary policy change induces a fiscal policy change, primarily by inducing a recession. These deficits contribute an additional inflationary force that offsets the disinflationary force of the interest rate rise. This simulation illustrates an important balance of competing effects behind the usual presumption that higher interest rates lower inflation.

Output and inflation responses have broadly similar patterns, but about half the magnitude of the response without policy rules, and somewhat more persistent dynamics. As an instance of a general pattern, policy rules smooth and therefore help to buffer the inflation and output responses to shocks. This consideration motivating Taylor-type monetary policy rules and automatic stabilizers is a fiscal-theory novelty.

The "Monetary, yes θ rules" rows of Table 1 again quantify these offsetting effects on inflation. The ω-weighted sum of inflation is very small, as the initial negative inflation balances the later positive inflation. However, this is not a pure rearrangement of inflation over time as in the frictionless model. There is instead a substantial 0.53% overall fiscal tightening, offset by a slightly larger 0.62% overall rise in the discount rate. In the one-period accounting of the middle panel, we see the still-present though much smaller -0.36% one-period inflation. The large increase in discount rate which on its own would cause inflation causes bond prices to fall instead.

7.5 Shock definition

These calculations require us to think just how we wish to define and orthogonalize monetary and fiscal policy disturbances. In the simplest model, I defined monetary policy as a movement in interest rates that does not change surpluses. In the context of the latter more general model, that definition does not seem interesting. There I define a monetary policy shock as a movement in the Taylor-rule residual $u_{i,t}$ that does not affect the fiscal *disturbance* $u_{s,t}$. But monetary policy nonetheless has fiscal consequences: Surpluses respond to output, to inflation, to changes in the value of debt induced by varying real interest rates, unexpected inflation, and past surpluses. This is not passive fiscal policy in the traditional definition, since it does not respond to arbitrary unexpected-inflation induced variation in the value of the debt. But it is a likely fiscal response to a monetary policy shock.

Should an analysis of the effects of monetary policy include such systematic fiscal policy responses? In many cases, yes. If one is advising Federal Reserve officials on the effects of monetary policy, they likely want to know what happens if the Fed were to raise interest rates persistently $u_{i,t}$, but the Treasury takes no unusual action. But they would likely want us to include usual fiscal actions and responses, as we include the usual behavioral responses of all agents. They might not want us to assume that fiscal authorities embark on a simultaneous deviation from standard practice, a change in $u_{s,t}$.

Perhaps not, however. Perhaps the Fed officials would like us to keep fiscal surpluses constant in such calculations, so as not to think of "monetary policy" as having effects merely by manipulating fiscal authorities into austerity or largesse. An academic description of the effects of monetary policy might likewise want to turn off predictable fiscal reactions, again to describe the monetary effects of monetary policy on the economy, not via manipulation of fiscal policy. In that case, even if one estimates θ_s response parameters in the data, one should turn them off to answer the policy question.

There is no right and wrong in specifying policy questions, there is only interesting and uninteresting – and transparent vs. obscure. The issue is really just what do we – and the Treasury, and the Federal Reserve – find an *interesting* question, and is the modeler clear on just what assumption has been made. Calculations of the effects of monetary policy must and do, implicitly or explicitly, specify what parts of fiscal policy are held constant or allowed to move. This eternal (and eternally forgotten) lesson is especially important here.

Though orthogonal shocks are interesting for policy experiments, if we are describing history, estimating the model, or thinking about how external shocks affect the economy, we will surely confront monetary $u_{i,t}$ and fiscal $u_{s,t}$ disturbances that occur at the same time, as both authorities respond to similar events. For this reason, the responses I calculate holding one of the fiscal $u_{s,t}$ and monetary $u_{i,t}$ disturbances constant in turn are surely unlikely guidelines to interpreting specific historical events. The classic “monetary policy shock” of the early 1980s involved joint monetary, fiscal (two rounds of tax reform), and regulatory (supply or marginal cost shock) reforms. At a minimum, this fact means that estimating policy shocks with a fiscal sensibility needs one more difficult orthogonalization. And perhaps the Fed officials, since they are seeing events that make them consider raising interest rates, *do* want you to put in whatever fiscal policy disturbance Treasury officials are likely to pursue in the same circumstance.

These calculations are also important rhetorically and methodologically. Yes, one can include such endogenous reactions or policy rules if it is interesting to do so. There is nothing in fiscal theory that requires “exogenous” surpluses. We can model fiscal and monetary policy quite flexibly. We need only one thing – that the fiscal authorities refuse to validate arbitrary inflations and deflations.

8 Puzzles resolved

This section documents the counterfactual predictions of positively correlated surplus processes, and how an s-shaped surplus process resolves them. These observations offer multifaceted and robust evidence how important it is to include an s-shaped surplus process in a fiscal-monetary model, or equivalently how much damage the contrary assumption does to the model’s ability to fit data. For simplicity, I explore these issues using the case of one-period debt and a constant expected return.

8.1 The magnitude of inflation and the correlation of inflation and surpluses

With one-period debt and a constant expected return, the unexpected inflation identity (9) gives us

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\sum_{j=0}^{\infty}\rho^j s_{t+1+j} = -a(\rho)\varepsilon_{s,t+1} = -\frac{1}{1-\rho\rho_s}\varepsilon_{s,t+1}. \tag{55}$$

The last equation is the case of an AR(1)

$$s_{t+1} = \rho_s s_t + \varepsilon_{s,t+1}.$$

Surplus shocks are large, so a positively autocorrelated surplus predicts counterfactually large inflation. Estimates in Table 2 below give a $\sigma\left(\varepsilon_{s,t+1}\right)=5.5\%$ annual standard deviation of surplus shocks and persistence $\rho_s = 0.55$. These estimates imply a 5.5/(1-0.55) = 12.2% standard deviation of unexpected inflation from surplus shocks alone. We don't see anything like this volatility of inflation.

Such large inflation shocks would stand out from all the other sources of inflation and generate a large correlation between deficits and inflation. This prediction is the opposite of the sign we typically see in the data – inflation comes in booms with surpluses, and disinflation in recessions with deficits. More generally, there is little correlation between inflation and deficits, across time or countries.

By contrast, consider the important case $a(\rho)=0$, in which debts are fully repaid. Now a string of positive short-run a_j, representing the fact that surpluses and deficits are persistent, is met by a long strong of negative a_j as illustrated in Figures 1 and Figure 6 below. Such an s-shaped moving average representation removes the prediction that inflation must come with deficits.

In the presence of other shocks, a value $0 < a(\rho) << 1$ can still remove the prediction of a strong correlation between deficits and inflation. (One could go the opposite direction with $a(\rho) < 0$ to generate a prediction of deficits with deflation, but in fact discount rates account for the negative correlation of surpluses with inflation.)

A correlation of deficits with inflation is *possible*. The surplus process does not have to be s-shaped. Many extreme inflations correlate with deficits, and some cross-country experience lines up inflation and devaluation with deficits. For example, in Jiang (2019b), Jiang (2019a), an AR(1) surplus assumption seems to work. But a strong correlation of deficits with inflation is not a *necessary* prediction of the fiscal theory. It results from an auxiliary assumption, or, as we shall see, a mistaken measurement, of a positively autocorrelated surplus process.

8.2 Surpluses and the change in the value of the debt

A positively correlated surplus predicts that a higher surplus produces a *higher* value of the debt, because it signals higher surpluses to come and the debt is the present value of subsequent surpluses. In fact, deficits are financed by borrowing, which raises the value of debt, and surpluses pay down the debt (Canzoneri, Cumby, and Diba (2001)).

However, if today's deficit predicts higher subsequent surpluses – if the moving average is s-shaped, if $a(\rho)$ is small – then the prediction is reversed. Higher surpluses lower the value of the debt, in the model as in reality.

We can state and analyze the point most simply by taking innovations of the flow identity (1) to write

$$\rho\Delta E_{t+1}v_{t+1} = -\Delta E_{t+1}\pi_{t+1} - \Delta E_{t+1}s_{t+1} = [a(\rho) - 1]\varepsilon_{s,t+1} \tag{56}$$

If $a(\rho) > 1$, as is the case with an AR(1), then a surprise surplus at time $t+1$ raises the value of debt, because it implies higher future surpluses. If $a(\rho) = 0$, then a surprise surplus at time $t+1$ implies an exactly offsetting decline in future surpluses, so a positive surplus surprise lowers the value of the debt one for one. If $0 < a(\rho) << 1$, then a positive surplus lowers the value of debt, but not quite one for one.

Inflation is a connected observation. Look at the debt accumulation equation itself, (1),

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1}.$$

This equation seems to state already that a higher s_{t+1} lowers the value v_{t+1}. How does the AR(1) example reverse that prediction? Because with $a(\rho) > 1$, it states that inflation π_{t+1} moves at the same time, in the opposite direction (more surplus, less inflation) and by a greater quantity. In the case $a(\rho) = 0$, inflation is completely unaffected by the surplus shock and the conventional reading of the equation applies. If $0 < a(\rho) << 1$ then a deficit shock corresponds to some inflation, and some increase in the value of debt, prompted by partial repayment.

Now, a higher dividend typically raises the stock market value, since it forecasts higher subsequent dividends. A positively correlated dividend process does make sense. But debt is not equity. The baseline cashflow pattern for any debt issue is s-shaped. If you take out a mortgage, you have an initial deficit and then a string of surpluses. If a corporation borrows, it has an initial cash inflow, and a string of cash outflows. This is the natural pattern to assume of government debt as well.

8.3 Financing deficits – revenue or inflation

When the government runs a deficit, it has to get the resources from somewhere. Usually, we think that the government borrows to finance a deficit. Such borrowing results in a larger value of debt. And to borrow, the government must promise to repay, to run an s-shaped surplus. Equation (56) captures this intuition with $a(\rho) = 0$.

That story can't work for an AR(1), with $a(\rho) = 1/(1 - \rho\rho_s) > 1$, or other surplus process with positive moving average coefficients. So how does the government finance a deficit in this case? By inflation (or more generally, default).

Suppose the government decides at $t + 1$ to run an unexpected deficit. At $t + 1$, surprise inflation π_{t+1} devalues the outstanding nominal debt that must be rolled over. In real terms this is equivalent to a partial default. For $a(\rho) > 1$, the government sells *less* debt v_{t+1} than previously planned. The inflation-induced devaluation is even larger than the current deficit shock. For $a(\rho) = 1$, the inflation-induced devaluation is equal to the deficit, so the deficit is exactly financed by devaluation. And if $0 < a(\rho) < 1$, then the deficit is partially financed by inflation, and partially financed by borrowing.

Most deficits in US and other advanced countries are clearly financed by borrowing more debt, another piece of evidence that $a(\rho)$ is a small number.

8.4 The risk and premium of government debt

The ex-post real return on government debt in this simple example is

$$r_{t+1} = i_t - \pi_{t+1} = -\Delta E_{t+1}\pi_{t+1} = a(\rho)\varepsilon_{s,t+1}.$$

An AR(1), with $a(\rho) = 1/(1 - \rho\rho_s)$, predicts that the standard deviation of inflation, and therefore of ex-post bond returns, is larger than that of surpluses. As we saw for inflation, a 5.5% annual standard deviation and 0.55 autocorrelation coefficient (Table 2) imply a 5.5/(1-0.55) = 12.2% standard deviation of the real one-year treasury bill returns. In fact, unexpected inflation and real treasury bill returns are vastly less volatile. In the same data, unexpected inflation has a 1.1% standard deviation.

A smaller $a(\rho)$ solves the bond return volatility puzzle. With $a(\rho) = 0$, unexpected inflation in this simple model is zero, and one-year government bonds are risk free in real terms, for any volatility of surpluses themselves.

Surpluses are procyclical, falling in recessions at the same time as consumption falls and the stock market falls. A volatile, procyclical, positively autocorrelated surplus thus generates a high risk premium as well as high risk. But government bonds have a very low expected return as well as low volatility. (Jiang et al. (2019) quantify this observation. They generate a puzzle by incorrectly leaving the value of debt out of their VAR and thus producing much too high an estimate of $a(\rho)$.)

The s-shaped surplus process solves the expected return puzzle as well as the volatile return puzzle, and the low average return of government bonds is one more piece of evidence for the s-shaped surplus process.

Specifically, the present value expression $v_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j}$ and the observation that surpluses s_t are volatile and procyclical suggests that government bonds, a claim to surpluses, should have a stock-like expected return and variance. But with an s-shaped surplus response, government debt becomes like a security whose price rises every time its dividend declines, so even a volatile dividend stream has a steady return. From the debt accumulation equation (1) we can write the one-period real return,

$$r_{t+1} = i_t - \pi_{t+1} = \rho v_{t+1} - v_t + s_{t+1} = [a(\rho) - 1]\varepsilon_{s,t+1} + \varepsilon_{s,t+1}.$$

With $a(\rho) \geq 1$, the innovation in value v_{t+1} reinforces the surplus innovation, so that the rate of return is more volatile than surpluses. With $a(\rho) = 0$, however, a surprise deficit s_{t+1} is met by a rise in the value of debt v_{t+1}, so the overall rate of return is risk free. In fact, government debt is a negative-beta security with a very low expected return. This fact is most easily rationalized by a fiscal theory model with low $a(\rho)$ and procyclical discount rates (Cochrane (2019)).

8.5 Estimates

These stylized facts, putting together various data sources, a simple model, and clear facts, are, I think the most convincing evidence that we must specify an s-shaped surplus process with small $a(\rho)$. However, one should not slight direct estimates. Table 2 presents three vector autoregressions involving surpluses and debt. Here, v_t is the log market value of US federal debt divided by consumption, scaled by the consumption/GDP ratio. I divide by consumption to focus on variation in the debt rather than cyclical variation in GDP. π is the log GDP deflator, g_t is log consumption growth, r_t^n is the nominal return on the government bond portfolio, i_t is the three month treasury bill rate and y_t is the 10 year government bond yield. I infer the surplus s from the linearized identity (1), allowing growth,

$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} - s_{t+1}.$$

Cochrane (2019) describes the data and VAR in more detail.

	s_t	v_t	π_t	g_t	r_t^n	i_t	y_t	$\sigma(\varepsilon)$	$\sigma(s)$
VAR $s_{t+1} =$	0.35	0.043	-0.25	1.37	-0.32	0.50	-0.04	4.75	6.60
std. err.	(0.09)	(0.022)	(0.31)	(0.45)	(0.16)	(0.46)	(0.58)		
$v_{t+1} =$	-0.24	0.98	-0.29	-2.00	0.28	-0.72	1.60		
std. err.	(0.12)	(0.03)	(0.43)	(0.61)	(0.27)	(0.85)	(1.04)		
Small VAR $s_{t+1} =$	0.55	0.027						5.46	6.60
std. err.	(0.07)	(0.016)							
$v_{t+1} =$	-0.54	0.96							
std. err.	(0.11)	(0.02)							
AR(1) $s_{t+1} =$	0.55							5.55	6.60
std. err.	(0.07)								

Table 2: Surplus forecasting regressions. Variables are s = surplus, v = debt/GDP, π = inflation, g = growth, i = 3 month rate, y = 10 year yield. Sample 1947-2018.

The first group of regressions in Table 2 presents the surplus and value regressions in the full VAR. The surplus is moderately persistent (0.35). Most importantly, the surplus responds to the value of the debt (0.043). This coefficient is measured with a t statistic of barely 2, using simple OLS standard errors. However, this point estimate confirms estimates such as Bohn (1998). (Bohn includes additional variables in the regression, which one may interpret as estimates of the θ terms of the policy rule, and which soak up a good deal of residual variance. For this reason, and by using longer samples, Bohn finds much stronger statistical significance though roughly similar point estimates.) Debt is very persistent (0.98), and higher surpluses pay down debt (-0.24).

The second group of estimates presents a smaller VAR consisting of only surplus and debt. The coefficients are similar to those of surplus and debt in the larger VAR, and we will see that this smaller VAR contains most of the message of the larger VAR.

The third estimate is a simple AR(1). Though the small VAR and AR(1) have the same coefficient 0.55 of the surplus on the lagged surplus we will see how the specifications differ crucially on long-run properties.

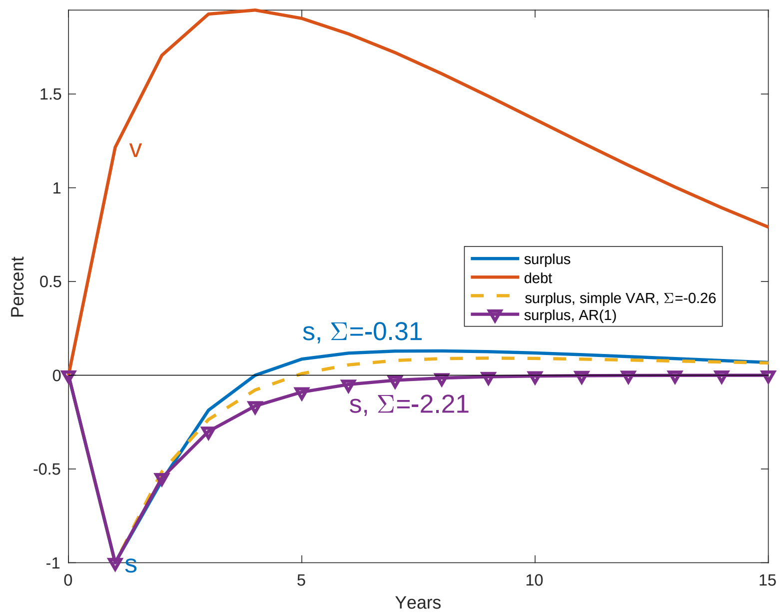

Figure 6: Responses to 1% deficit shocks from VARs. "$\sum =$" gives the sum of the responses.

Figure 6 presents responses of these VARs to a 1% deficit shock at time 1. I allow all variables to move contemporaneously to the deficit shock. The central point shows up right away: *The VAR shows an s-shaped surplus moving average.* The initial 1.0% deficit is followed by two more periods of deficit, for a cumulative 1.75% deficit. But then the surplus response turns positive. The many small positive surpluses chip away at the debt, until the sum of surpluses in response to the deficit shock is only $-a(1) = \sum_{j=0}^{\infty} s_{1+j} = -0.31\%$.

Mechanically, the surplus response function comes from the coefficient by which the surplus responds to the value of debt. The value of debt jumps up initially when surplus jumps down. Shocks to the surplus and value of debt are strongly negatively correlated, itself below a piece of evidence for an s-shaped response: When the government runs a deficit, the value of debt rises, which can only happen if people expect future repayment. Surpluses then respond to the greater value of debt, and slowly bring down the value of debt. Thus, the s-shaped surplus response estimate is robust and intuitive, as the ingredients come from the negative sign of the regression of surplus on debt, the persistent debt response, and the pattern that higher surpluses bring down the value of debt.

The simple VAR shows almost exactly the same surplus response as the full VAR, emphasizing how the response comes just from these intuitive features of that VAR. The point estimate of the sum of coefficients in the simple VAR is smaller, $-a(1) = -0.26$. The simple VAR surplus response crosses that of the full VAR and continues to be larger past the right end of the graph.

The AR(1) response looks almost the same – but it does not rise above zero. It would be very hard to tell univariate and VAR surplus responses apart based on autocorrelations or short-run forecasting ability emphasized in statistical tests. But the long-run implications are dramatically different. For the AR(1), we have $-a(1) = -2.21$. Where a simpleminded constant discount rate model, fed the VAR-estimated surplus process, predicts 0.26%-0.31% inflation in response to a 1% fiscal shock, the AR(1) predicts 2.28% inflation. The volatility of real one-period bonds is entirely unexpected inflation, so the AR(1) also predicts dramatically higher bond return volatility.

8.6 Econometrics

For the purposes of examining present value relations, an estimated AR surplus process is not just a different specification choice that happens to produce a sharply different result for the cumulative sum of responses $a(\rho)$. It is wrong. The valuation equation, simplified for this point to constant discount rate, is

$$v_t = E\left(\sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} | \Omega_t\right) \tag{57}$$

where Ω_t denotes people's information set, which crucially includes the current value of the debt v_t. From (57) we may derive

$$v_t = E\left(\sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} | I_t \subset \Omega_t\right), \tag{58}$$

if the information set I_t includes the value of debt v_t.

Thus, for example, one may make calculations based on a VAR. The VAR variables are a subset of all information. By doing so one does not assume that people have no more information than is included in the VAR.

But this conditioning down does not work if the information set I_t does not include the debt v_t. Then, we have $E(v_t|I_t)$ on the left hand side. Excluding v_t from the VAR and testing a present value relation is simply wrong. Or, more politely, it assumes that people have no more information than we include in the VAR, which is a false assumption.

Several additional econometric considerations justify the current wisdom that it is unwise to try to estimate a surplus process ignoring debt, calculate revisions in its present value, and test relations such as (58).

From (21), the univariate surplus process in my parametric example is

$$s_{t+1} = \frac{1 - \frac{1}{\rho}L}{\left(1 - \frac{1-\alpha}{\rho}L\right)(1 - \rho_s L)}\varepsilon_{s,t+1} + \beta_s \frac{\alpha}{\rho}\frac{L}{1 - \frac{1-\alpha}{\rho}L}\varepsilon_{s,t+1}. \tag{59}$$

In the case $\beta_s = 0$, the numerator coefficient is $1/\rho \geq 1$. This process is not invertible, and *cannot* be recovered from a regression of the surplus on lags of the surplus. An autoregression will lead to

$$s_{t+1} = \frac{1 - \rho L}{\left(1 - \frac{1-\alpha}{\rho}L\right)(1 - \rho_s L)}v_{s,t+1};\ v_{s,t+1} = s_{t+1} - E(s_{t+1}|s_t, s_{t-1}, ...)$$

This wrong estimate has $a(\rho) = \frac{1}{\alpha}\frac{1-\rho^2}{1-\rho_s\rho}$, and a researcher following this procedure would be puzzled to see no inflation in the data. In this case, one must include the value of debt in a VAR in order to correctly estimate the surplus process. (Hansen, Roberds, and Sargent (1992) make this point in general.)

When $\beta_s > 0$, we can write the moving average (59) as

$$s_{t+1} = \frac{\left(1 - \frac{1}{\rho}L\right) + \beta_s \frac{\alpha}{\rho}L(1 - \rho_s L)}{\left(1 - \frac{1-\alpha}{\rho}L\right)(1 - \rho_s L)}\varepsilon_{s,t+1}$$

and hence

$$s_{t+1} = \frac{(1-\lambda_1 L)(1-\lambda_2 L)}{\left(1-\frac{1-\alpha}{\rho}L\right)(1-\rho_s L)}\varepsilon_{s,t+1}; \tag{60}$$

$$\lambda_{1,2} = \frac{1-\alpha\beta_s}{2\rho}\left(1 \pm \sqrt{1+\frac{4\alpha\beta_s\rho\rho_s}{(1-\alpha\beta_s)^2}}\right).$$

The parameter α is small, so the numerator coefficients $\lambda_{1,2}$ are close to one. These decline with β_s, and eventually cross below one, so the surplus moving average becomes invertible. Still, the surplus process has multiple nearly-canceling and near-unit AR and MA roots, which is notoriously difficult to estimate by univariate methods.

In Figure 1 and Figure 6, the AR(1) disturbance $u_{s,t}$ and the two surplus responses s_t are clearly hard to distinguish based on univariate methods. But the response of debt and inflation to the surplus shock captured by β_s, and the response of surplus to debt captured by $\alpha > 0$ are quite different between these processes. A vector autoregressive approach that measures α and β_s has a much better chance of distinguishing these processes.

In sum then, the surplus process specified by this model, like any surplus process we expect of a debt issuer, is of a type common, but all too frequently mistreated in macroeconomics and finance, especially when we are interested in its long-run properties such as $a(\rho)$. In the form (21), we see a big movement in one direction, followed by a long string of offsetting movements in other direction. In (60) we see the classic ARMA with nearly-canceling and near-unit roots.

The subtleties of measuring the surplus response of this model is good news. They mean that we need not be too quickly discouraged by empirical work on surpluses that does not treat these subtleties with care.

For all these reasons, the modern asset pricing literature, following Campbell and Shiller (1988) no longer tries to test present value relations. Instead, it recognizes that relations such as (58) with arbitrary (arbitrage-free, empirical) time-varying expected returns are identities. It remains interesting and productive for economists to measure terms of identities and see which one matters. Challenges remain in reconciling the time-varying dividend or surplus and return forecasts that make sense of present value relations with economic models. But the business of testing present value relations per se – of forecasting the right-hand side of expressions like (58) with information that excludes v_t, and trying to test equality, has vanished, recognized as an attractive but pointless path. This insight is only slowly making its way to present value relations in government finance.

8.7 Summary

Summarizing this whole section, I hope I have put the nail in the coffin that a sensible surplus process must be s-shaped and allow the government to borrow, incurring a deficit today, and promising surpluses in the future. This statement is likely not at all controversial. The more important point is that the fiscal theory can easily accommodate such a surplus process. The main point of contention is the widespread false impression that following such a process implies and requires passive fiscal policy, and that any fiscal theory of the price level model must be at odds with such basic facts. It does not, and it is not.

9 Literature and identification

Finally, I describe how this paper relates to and builds on similar work in the literature

9.1 Models

The fiscal theory of monetary policy, uniting fiscal theory with interest rate targets, sticky prices and other ingredients of the new-Keynesian DSGE paradigm, is a recent development in a long literature on the fiscal theory of the price level. Sims (2011) and Cochrane (2017b) are immediate antecedents, including sticky prices, long-term debt, and computing the response to monetary and fiscal policy shocks. This paper both builds on and simplifies those models. Both models specify perpetuities and an exogenous AR(1) surplus. As a result, they embody the above puzzles.

A variety of more complex, and therefore potentially more realistic models have been built, combining fiscal price determination, detailed fiscal policy, and interest rate targets. Important examples include Davig and Leeper (2006), Leeper, Traum, and Walker (2017), Bianchi and Melosi (2017), Bianchi and Ilut (2017), Bhattarai, Lee, and Park (2016), and Eusepi and Preston (2018).

These models are specified in much more detail than this paper, and are estimated. Most include detailed fiscal policy, distorting taxes, capital, explicit microfoundations, more complex preferences and technologies, nonlinear solution methods, and other elaborations. They include the possibility of active fiscal or active monetary policy, and often Markov switching between the two.

However, none has included the surplus process with an s-shaped moving average, or the equivalent possibility that surplus, while reacting to debt, does not react to arbitrary unexpected inflation. Thus, they all embody the restriction that active fiscal policy cannot specify governments that substantially repay debt, and their fiscal regime embodies counterfactual predictions like the above puzzles. Though I deliberately simplify many elements relative to this literature in the quest of transparency, in that respect this paper is novel.

In simplest terms, and in this paper's notation, these authors write

$$s_{t+1} = \gamma v_t + u_{s,t+1}$$

with positively correlated $u_{s,t+1}$, typically an AR(1). They identify active fiscal policy with $\gamma = 0$, and passive fiscal policy with $\gamma > 0$. They also write monetary policy rules that embody identifying restrictions, unlike the general form (27-28). (The models include other variables such as inflation and output in fiscal policy rules, as I have, and they typically model taxes and spending separately. Often the positive serial correlation comes through a lag rather than the disturbance,

$$s_{t+1} = \rho_s s_t + \gamma v_t + \theta_{s,\pi}\pi_{t+1} + \theta_{s,x}x_{t+1} + \varepsilon_{s,t+1}.$$

The general point is the same however. Some take $\gamma > r$ or $\gamma > r - g$ as the dividing line.)

With this structure, these papers are able to identify active-money vs. active-fiscal regimes. Indeed, many papers incorporate Markov-switching between regimes and measure which regime the economy is in at a point in time (caveat below). Others split the sample and identify regimes in subsamples.

Alas, this approach leads to a conundrum. The active-fiscal regime so constrained produces the above puzzles. Estimates, which look for the least-bad fit, find active fiscal policy at most for short periods characterized by volatile inflation, and small regression coefficients ϕ_π, typically only the 1970s if even then. In such a period the puzzles provide less statistical evidence against the model than the apparently more counterfactual predictions of the models' active-money specifications.

In this structure, one can eliminate the puzzles by adopting $\gamma > 0$. Then, even with positively serially correlated residuals $u_{s,t+1}$ or a single lag $\rho_s s_t$, the surplus has an s-shaped moving average that allows substantial debt repayment and eliminates the puzzles. But doing so also eliminates active fiscal policy in these specifications. Thus, most of the time, the estimates find passive fiscal policy.

At one level, the point of this paper is to write fiscal policy in a way that removes this conundrum. Fiscal policy can be active, yet we see $\gamma > 0$ in equilibrium, or we have a disturbance with an s-shaped moving average, that removes the puzzles. This generalization should at least broaden the applicability of the active-fiscal regime in such models.

More deeply, there is an underlying observational equivalence theorem between active-fiscal and active-money regimes. That theorem goes back at least to Cochrane (1998), and we saw it in section 4.5 here. The regime depends only on how people believe authorities would behave away from the observed equilibrium inflation, and we cannot see, measure, or test such behavior using only time-series data from an equilibrium. Thus, there *is* a way to write any time series that a model can produce equally as the unique equilibrium of an active-fiscal as from an active-money specification. I write fiscal policy in a way that exhibits this observational equivalence, in analogy to King's elegant writing of the standard monetary equilibrium-selection policy. Then the active-fiscal regime can apply *always*, or at least equally well to an active-money regime.

So how does the standard approach achieve a measurement of active-fiscal vs. active-money regimes? Well, by making identification assumptions. There is nothing wrong with identification assumptions. Economics is littered with observational equivalence theorems, and we routinely surmount them with identification assumptions, from the day we try to tell how much of a price change comes from a shift in supply vs. demand.

Unfortunately, in this case, the identification assumptions are implicit, and once stated explicitly they are not compelling. The standard approach assumes that the surplus either responds to all variation in the debt, including that induced by unanticipated inflation $\gamma > 0$, or the surplus responds to no variation of the debt at all $\gamma = 0$. A disturbance $u_{s,t}$ with small $a(\rho)$ would also solve the puzzles in the latter case, but the models also specify that if $\gamma = 0$, the surplus must be positively correlated, i.e. has a large value of $a(\rho)$. These assumptions produce identification all right – they produce the puzzles, so the data can and mostly do reject this regime. But once stated and examined, and – importantly – once one exhibits a reasonable alternative, these identification assumptions seem highly artificial.

Likewise, the standard active-money regime requires identification, essentially that the response of interest rates to off-equilibrium inflation is the same as the response of interest rates to inflation in equilibrium, as section 4.5 clarifies. I argue elsewhere such as Cochrane (2011a) that this identification is just as incredible.

Now, I may have inferred these papers' implicit identification assumptions ($\gamma = 0$ and positively autocorrelated surplus) incorrectly. They are complex models and they use the full set of time series, along with complex formal estimation techniques, to identify regimes. (In the face of well-known observational equivalence and non-identification theorems, it would have helped a lot for the authors to state what the identification assumptions are!) But by definition, if a paper can identify active fiscal vs. active money regimes, in the face of an observational equivalence theorem, that paper must add some restriction on the kinds of data that one or the other regime can describe. The fact that the computer programs run, that the standard errors are not infinite, that posteriors are not exactly equal to priors, that the test for one regime vs. another reports an answer, means that the modelers have added *some* restriction that the active-fiscal model cannot describe some aspect of time series in a way that the active-money model can, and vice versa.

I have written the model with no identification assumptions, to emphasize that an active-fiscal regime can match any data that the model can match in an active-money regime. The reader naturally may want to take the next step: What is an alternative, more reasonable set of identifying assumptions, of restrictions on the model's ability to describe time series, imposed by either regime, by which one may test for active-fiscal vs. active-money regimes? And please conduct such a test.

I do not add alternative identification assumptions in the present paper for two reasons. First, finding credible identifying restrictions is an orthogonal and substantial contribution, and I don't know how to do it. Second, my writing on this issue argues that it is a fool's errand. Cochrane (2011a), Cochrane (2017a) and Cochrane (2018) argue that the standard active-money equilibrium selection policy ($\phi_\pi(\pi - \pi^*)$ in the notation of section 4.5 here) simply makes no sense, and the identification restrictions to make that reaction visible make no sense either. Central banks following interest rate targets do not have an "equilibrium selection policy" that intentionally destabilizes the economy, they do not threaten hyperinflation or otherwise to blow up the economy if inflation differs from their target, (crucially) nobody expects such behavior, and one cannot observe such an off-equilibrium threat from what they do or what they say. If active-money doesn't make any sense, there is no point in testing for it. All we have (so far) is active fiscal policy. So, if you only have one sensible theory, don't restrict it, use it. For these purposes, just use the $v = v^*, \pi = \pi^*$ equilibrium condition as a theoretical footnote to answer determinacy. The model has a unique equilibrium, centrally described by how unexpected inflation reacts to shocks, $\Delta E_{t+1}\pi_{t+1} = \beta_i \varepsilon_{i,t+1} + \beta_s \varepsilon_{s,t+1}$. Estimate it, simulate it, run policy experiments, enjoy it. Don't bother with formal time-series tests of the unobservable question, off-equilibrium reactions to $v \neq v^*$, or $\pi \neq \pi^*$, or restricting the model artificially to appear to test such a thing.

This approach does not mean that the off-equilibrium foundations are empty of economic or empirical content. Yes, we should examine carefully and critically the institutions that commit the government not to respond to unexpected inflation, described at some length above. Historical episodes and analysis of policy institutions and choices are deeply informative about the fiscal-monetary regime. (Many excellent papers by Tom Sargent and coauthors fall in this class, starting with the classic Sargent (1983) to the recent Hall and Sargent (2014) and Bassetto and Sargent (2020).) Economic theory describing equilibrium foundations is deeply important. (Good recent examples include Atkeson, Chari, and Kehoe (2010), Bassetto (2002) and Bassetto (2005).) But do not expect to easily test such off-equilibrium behavior by finding restrictions in time-series drawn from an equilibrium.

The reader may not be convinced by the latter approach, and it is not necessary that he or she be convinced. I only explain why this paper does not add restrictions linking off-equilibrium behavior to something observable and to attempt a test. Readers are invited to add identifying assumptions, defend them, and test models that include them.

How do these papers, including my own Cochrane (2017b), not notice the sharply counterfactual implications of their active-fiscal regime? They don't find them because they don't look for them. None of the cited papers examines whether the fiscal-regime estimates do or do not display or resolve the above list or related puzzles. (Jiang et al. (2019), discussed below, notice, but proclaim a puzzle rather than resolve it with an s-shaped surplus.)

Not noticing is natural. Estimates, especially formal estimates of complex models with time-varying regimes, settle on parameters that do least damage to the data, but that does not mean they don't do a lot of damage to data. Most obviously, most models reject the fiscal regime for the bulk of their data, since the fiscal regime is tied to counterfactual puzzles. So the models, at their estimated parameters, don't display the puzzles. Just why the model rejects a fiscal regime for the 1980s, say, belongs to deep diagnostic exercises that most authors do not pursue.

Moreover, this is not their question and goal. The papers are aimed at measuring impulse response functions at estimated parameters, not evaluating measures of model fit. And formal measures of fit are not revealing of these sorts of stylized fact problems. Any model that isn't sprinkled with additional shocks will fail an overall fit test. Many of the models' structural equations are not exactly true and may provide more statistical evidence against models than these intuitively appealing facts. Even if one computes a measure of fit, it's easy to fail to reject the overall fit of a model despite failing the sign of un-examined point estimates and correlations, if the standard errors are large enough. You have to look for specific qualitative failures to see such failures.

(A technical note: the foundations of determinacy are actually more subtle than they seem. As with monetary policy rules, the θ coefficients raise the possibility of more complex parameter regions. Determinacy relates to whole-model eigenvalues, not one parameter. More deeply, Markov-switching raises a technical and definitional issue. The current value of fiscal γ and monetary ϕ_π responses do not necessarily control the long-run behavior of the economy. If currently $\gamma = 0$, $\phi_\pi > 1$ but people expect a switch to $\gamma > 0$, $\phi_\pi < 1$, then we are really in an active fiscal regime. Whether inflation or the real value of government debt explodes in expectation if the economy starts at an off-equilibrium inflation rate depends on Markov transition probabilities as well as current parameter values. Davig and Leeper (2006) point out that an economy which appears to have both policies passive can be determinate, if people expect at some future date to switch to one or the other active regime. Much new-Keynesian literature at the zero bound, where monetary policy must be locally passive, gains determinacy by imagining a switch to active monetary policy at the end of the zero bound. In sum, even periods that papers identify as active money, or active fiscal, based on time-varying estimates of γ and ϕ_π may actually have inflation determined by active fiscal, or active monetary policy. This is a labeling issue more than a model issue however.)

Some details:

Leeper, Traum, and Walker (2017) is a detailed a sticky-price model allowing fiscal theory solution, aimed at evaluating the output effects of fiscal stimulus. But they specify fiscal policy as an AR(1) (p. 2416) along with one-period debt. Their paper includes the above-mentioned indirect mechanism that buffers the AR(1) surplus conundrum somewhat: A deficit leads to inflation, which raises output, which raises tax revenues, and leads to higher surpluses. But that mechanism is not necessarily large enough to generate substantial repayment of large debts. Some of their surplus responses change sign, but one cannot tell how much debt is repaid by subsequent surpluses, and how this differs across regimes, just by eyeballing the plots. Their paper is focused on a different issue, of course, so there is no reason they should analyze this question.

Bianchi and Melosi (2017) specify that taxes follow an AR(1) that responds to output. Their model switches between a passive-fiscal regime in which surpluses respond to debt and an active-fiscal regime that does not (their equation (6) p. 1041). Government spending also follows an AR(1) that responds to output (p. 1040). Their paper is centrally about the absence of deflation in response to a preference shock, and how expectations of a switch between regimes affects responses to shocks. They don't analyze monetary policy shocks.

Bianchi and Ilut (2017) come to an appealing conclusion: The inflation of the 1970s came from loose fiscal policy, and the disinflation of the 1980s followed a fiscal reform. They augment a new-Keynesian model with a fiscal block and the geometric term structure for government debt that I use here. They also posit monetary and fiscal rules that feed back from interest rates and output. Their tax rate feeds back from the quantity of debt as well as output – $\tilde{\tau}_t = \rho_{t,\xi_t^{sp}}\tilde{\tau}_{t-1} + ... + \delta_{b,\xi_t^{sp}}\tilde{b}^m_{t-1} + ... + \varepsilon_{\tau,t}$ in their equation (5). They specify Markov-switching between active fiscal and active money regimes, between $\delta_{b,\xi_t^{sp}} > 0$ and $\delta_{b,\xi_t^{sp}} = 0$, finding passive fiscal policy in the 1980s and active fiscal policy in the 1970s Parts of the resultant puzzle is a feature in their analysis: They find that the change to active fiscal policy led to an increase in debt as well as to a decline in inflation, because only a government that commits to repay can borrow large quantities. The low value of debt in the 1970s results from an active-fiscal government that cannot commit to repay debts. (This is my interpretation, not theirs.) This story provides a nice counterexample to the usual story that active fiscal policy pairs large debts with a greater danger of inflation. But this result only holds because they rule out the possibility that an active fiscal policy can promise to repay debts, i.e. that we can see a coefficient $\gamma > 0$ ($\delta_{b,\xi_t^{sp}} > 0$) in an active-fiscal equilibrium.

Bhattarai, Lee, and Park (2016) likewise add fiscal policy to a DSGE model. They split the sample pre and post-Volcker. They find both monetary and fiscal policy passive pre-Volker, and thus "equilibrium indeterminacy in the pre-Volcker era," modeled as sunspot shocks. They include standard fiscal and monetary policy rules (their equations (1) and (2)). The fiscal rule moves tax rates in response to output, debt, and lagged tax rates. As in all these papers, the government either responds or does not to the entire value of debt. Fiscal policy is active if and only if the coefficient of taxes on debt is smaller than r. (p. 972.)

Eusepi and Preston (2018) is an active-money passive-fiscal model, in which irrational expectations generate violations of Ricardian equivalence. Even so, they specify fiscal policy that responds to debt, making it passive, and an i.i.d. shock (Equation (6)).

While most of this literature sees a change in regime around 1980, there is notably little agreement which regime held in which sub-period. This fact should warn us that the regime identification is tenuous.

9.2 Direct estimates

Bohn (1998), finding a positive coefficient γ of surplus on debt, writes "The positive response of the primary surplus to changes in debt also shows that U. S. fiscal policy is satisfying an intertemporal budget constraint." This statement might be interpreted in favor of passive fiscal policy, but it is not. Under the fiscal theory interpretation, we see a positive coefficient in equilibrium. One might quibble that complete repayment of debts depends also on the correlation of inflation and fiscal shocks, β_s above; i.e. a surplus response can coexist with partial repayment, and the equation at hand is a valuation equation not a budget constraint. But all of this subtlety long-postdates Bohn's paper. Bohn does not actually say anything about active vs. passive fiscal policy.

Canzoneri, Cumby, and Diba (2001) exploit a different prediction to try to test active vs. passive fiscal policy. They point out that an AR(1) surplus process, discounted at a constant rate, with active fiscal policy, predicts that higher surpluses raise the value of debt, where in the data higher surpluses pay down the debt - essentially focusing on the opposite regression of debt on surplus. They are unusually aware that their estimate embodies an identifying assumption, that there is an underlying observational equivalence theorem, and that an s-shaped moving average surplus process can resolve the puzzle. They argue that the s-shaped surplus process is unreasonable, writing

> "NR [active-fiscal] regimes offer a rather convoluted explanation that requires the correlation between today's surplus innovation and future surpluses to eventually turn negative. We will argue that this correlation structure seems rather implausible in the context of an NR regime, where surpluses are governed by an exogenous political process."

I think it is clear with over two decades of ex-post wisdom that an s-shaped surplus process, which results from any borrowing with promise of repayment, is natural and sensible, no matter how one specifies the government's reaction to unexpected inflation and deflation. Even "political processes" must obey the constraints imposed by market prices, one of which is that people will not lend you money if you do not credibly promise to pay it back. Their evidence that surpluses pay down debts, and deficits increase the value of debt, is straightforward evidence that the surplus process does have this s-shaped moving average. The passive-fiscal regime they advocate has such an s-shaped moving average, induced by $\gamma > 0$. So s-shaped moving averages are not implausible per se, by their own estimates.

Cochrane (2001) advocates a surplus process with an s-shaped moving average representation, in order to fix some of the pathologies of the usual AR(1) specification and in response to the Canzoneri, Cumby, and Diba (2001) puzzle in particular. That paper also shows an example in which one incorrectly measures an AR(1) surplus by leaving debt out of the VAR. There, I wrote the surplus as a sum of two AR(1)s, in which the short term component may represent cyclical deficits and the long term component can represent the long-lasting smaller surpluses that repay deficits. This parametric form also produces a pretty s-shaped response function, visually almost identical to Figure 1, and it would be easy to incorporate in a model such as this one. But this parametric form does not neatly encode or generalize the $a(\rho) = 0$ special case, it does not show as clearly how fiscal policy serves to enforce one specific value of unexpected inflation, and it does not so clearly illustrate how responses to arbitrary inflation are the only key feature of active vs. passive fiscal policy, and it does not express observational equivalence as well.

Jiang et al. (2019) document the puzzle that with a positively correlated surplus process, government bonds should have volatile and procyclical real returns, and hence a large expected return. They estimate a VAR that excludes the value of debt as a forecasting variable, contra the advice and counterexample in Cochrane (2001), contra Hansen, Roberds, and Sargent (1992), and contra the above discussion that leaving out such debt is a mistake. They pronounce a puzzle to be fixed by very large liquidity premiums in government bonds. They do not resolve the predictions of volatile government bond returns, volatile inflation, surpluses that are positively correlated with inflation, and surpluses that raise the value of debt in their model. These predictions remain even with large but steady liquidity premiums in government bonds.

9.3 Active fiscal policy with repayment

An active fiscal policy in which the government nonetheless can promise debt repayment, or run an s-shaped surplus process, has some precedent, at least implicit.

Benhabib, Schmitt-Grohé, and Uribe (2002) advocate that governments undertake unbacked fiscal expansion to escape a deflationary liquidity trap, and commit to such expansion ahead of time to avoid such traps. This is a good example of a policy that repays debt in normal times, but commits not to react to multiple equilibrium inflation.

Jacobson, Leeper, and Preston (2019) clearly describe the Roosevelt administration's actions in 1933 as a refusal to accommodate deflation, and to convince people that the government would undertake an unbacked fiscal expansion. By separating the budget into a "regular" budget and an "emergency" budget, the government was able to commit that the former would be repaid, that future borrowing after the emergency had passed would be repaid, but that the emergency budget was unbacked and should result in inflation. Bianchi and Melosi (2019) model this distinction between "regular budget" which is passive-fiscal and an "emergency budget" of unbacked fiscal expansion to fight deflation.

Eggertsson (2008) gives a similar account of the history. His theoretical account is different, focusing on expectations of monetary policy after the end of the zero bound, but even that account must have the same fiscal underpinnings, "passively" achieved: The government does not pay off the deflation-induced windfall to bondholders.

The United Kingdom's fiscal policy in centuries leading up to the famous restoration of gold parity in the 1920s is a good example in the opposite direction. The government suspended convertibility to gold during wars, and there was some inflation. Rather than pay off its nominal debt at the new lower value, the government refused to respond to that inflation, and repaid debt as if the inflation had not happened. By doing so, it established a reputation that enhanced its ability to borrow vast amounts in the first place, and held down inflation during the period of suspended convertibility. Though the UK also suffered the economic consequences of high marginal tax rates during restoration, viewed in intertemporal terms, it was not as senseless a policy as Keynes famously argued. Whether wise or not, it was a policy followed for centuries and not some-other-planet artificial policy for us to assume holds today, at least in part, spirit or expectation.

9.4 Additional models with detailed fiscal policy

This literature linking fiscal policy to nominal events is a drop in the ocean of quantitative DSGE models in which fiscal policy feeds back to output. Leeper, Plante, and Traum (2010) is an excellent example of such dynamic fiscal policy modeling. (In additional to voluminous economic literature, the authors point to a large-scale IMF effort to develop such models.) The authors estimate a model with government spending, transfers, and distorting taxation on labor, capital, and consumption. The model includes many feedback mechanisms from fiscal to real events. The model is however entirely real, avoiding all the price-level determination issues at the heart of this paper. Models of this sort are ripe for merging with the active-fiscal sort of specification in this paper to include nominal issues and monetary policy.

Leeper, Plante, and Traum (2010) includes a sophisticated estimation of a fiscal VAR. The authors focus on which components of fiscal policy adjust, and find that all fiscal instruments except labor taxes react strongly to debt, and all components including labor taxes contribute to long-run intertemporal financing. (See their Table 2, coefficients of various variables on debt.) The reaction to debt mirrors the coefficient $s_{t+1} = \gamma v_t + ...$ in the very simple VAR of this paper, and produces the sort of s-shaped moving average I impose here. On its own, such a reaction function would produce passive fiscal policy. But one can easily construct a model with a real/nominal distinction in which *equilibrium* fiscal policy follows the sort of empirical characterization in this paper.

9.5 The Fisherian conundrum

I focus much discussion of the effect of monetary policy shocks on how the model can overcome the "Fisherian" prediction that higher interest rates without a contemporary fiscal contraction lead to uniformly higher inflation, and the model can instead produce a negative inflation response to unexpectedly higher interest rates. The Fisherian prediction is a troubling and robust prediction of rational expectations models (Uribe (2018), Cochrane (2018)), and tough to eradicate. This paper shows how long-term debt and fiscal policy which responds positively to output and inflation *can* overcome the Fisherian prediction. That is not a novel contribution, as this ingredient is included in Sims (2011) and my effort to explain that paper, Cochrane (2017b). It is at least implicit in any paper that has a maturity structure of nominal debt, including Bianchi and Ilut (2017).

By working hard to create a negative inflation response to monetary policy I do not mean to imply thereby that we know the Fisherian prediction is false. Evidence that higher interest rates reduce inflation, especially without contemporaneous fiscal contractions, is weak. Uribe adds evidence on the other side. Different parameterizations of this model are also consistent with Fisherian responses.

10 The way forward

This model is still simple and unrealistic. I advance it to show what *can* be done, and to build intuition for mechanisms that will appear in larger models, but are clouded by the interaction of even more effects.

The obvious next step is to incorporate this sort of fiscal policy into more detailed and realistic fiscal-monetary models, of the sort cited above. If I were in charge, I would estimate them entirely in an active-fiscal regime. The point of this paper is that it is nearly trivial to adapt any new-Keynesian DSGE model to active-fiscal underpinnings. Eventually one wants even more ambitious models incorporating habits or other dynamic preferences and investment adjustment costs, heterogeneity, variation in risk premia, labor market and investment frictions, the latest in financial frictions, zero bounds, and so forth. Clearly, one wants detailed fiscal modeling including distorting taxes and inflationary effects in tax and spending policies by which fiscal affairs feed back to real affairs and inflation.

A major point of this paper is that one *can* construct such models, and quite easily from a technical standpoint. But finding the *right* model is not so easy, as that specification search has not been so easy for standard new-Keynesian models.

My monetary policy rule is simplistic, needing at least lags of the interest rate and other variables, expected future variables, and an effective lower bound, plus matching policy rule regressions in data. Specifying and estimating the fiscal policy rule is a challenge of similar order, not yet started, and made even more challenging by the fact that any sensible rule, such as this one, has subtle but crucial long-run responses, or a latent state variable. On the other hand, much of the fiscal policy rule can be estimated from structural knowledge of the tax code, the nature of automatic stabilizers, and visible spending decisions such as stimulus programs in recessions, where the monetary policy rule consists only of modeling the human decisions of central bankers. Estimating the parameters θ of the fiscal policy may be easier than running regressions with delicate instruments for right hand variables correlated with error terms.

If one wants to test monetary vs. fiscal price level determination, one needs to come up with credible identifying assumptions, as outlined above. Or, one can join in my announcement that such tests are pointless and analyze models entirely in the active-fiscal regime.

Like the rest of the model, this surplus process can and should be generalized towards realism in many ways. News about future surpluses and historical episodes are likely not well modeled by AR(1) shocks to the disturbance $u_{s,t}$. The choice to finance deficits by inflating existing debt vs. borrow against future surpluses is likely to change over time and in response to state variables as well.

There are many steps to take. But each step is also an unexplored opportunity.

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Online Appendix to “A Fiscal Theory of Monetary Policy with Partially Repaid Long-Term Debt”

A Model solution algebra

This Appendix sets out the algebra to solve the model (33)-(44). I express the model in the form

$$y_{t+1} = By_t + C\varepsilon_{t+1} + D\delta_{t+1} \tag{61}$$

where y is a vector of variables, ε are the structural shocks, and δ are expectational errors in the equations that only tie down expectations. (The general case has a leading term Ay_{t+1}, but we do not need that here.) We eigenvalue decompose the transition matrix B, we solve unstable roots forward and stable roots backward to determine the expectational errors δ as a function of the structural shocks ε. Then, we can compute impulse-response functions. This is the standard Blanchard and Kahn (1980) method, applied to this problem.

For ease of reference, the model (33)-(44) is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) \tag{62}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{63}$$

$$i_t = \theta_{i\pi}\pi_t + \theta_{ix}x_t + u_{i,t} \tag{64}$$

$$s_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sx}x_{t+1} + \alpha v_t^* + u_{s,t+1} \tag{65}$$

$$\rho v_{t+1}^* = v_t^* + r_{t+1}^n - \pi_{t+1}^* - s_{t+1} \tag{66}$$

$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - s_{t+1} \tag{67}$$

$$E_t\pi_{t+1}^* = E_t\pi_{t+1} \tag{68}$$

$$\Delta E_{t+1}\pi_{t+1}^* = -\beta_s\varepsilon_{s,t+1} - \beta_i\varepsilon_{i,t+1} \tag{69}$$

$$E_t r_{t+1}^n = i_t \tag{70}$$

$$r_{t+1}^n = \omega q_{t+1} - q_t \tag{71}$$

$$u_{i,t+1} = \rho_i u_{i,t} + \varepsilon_{i,t+1} \tag{72}$$

$$u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}. \tag{73}$$

As outlined in the text, we can difference (66)-(67) to yield

$$\rho\left(v_{t+1}-v_{t+1}^{*}\right)=\rho^{-1}\left(v_{t}-v_{t}^{*}\right)-\rho^{-1}\left(\pi_{t+1}-\pi_{t}^{*}\right).$$

Thus with $\rho \leq 1$, stationary solutions must have $v_t = v_t^*$, $\pi_t = \pi_t^*$. We could leave the $*$ in and let the matrix solution method figure that out, but it's easier to do that manually now. We just impose that the $*$ values equal the non-* variables.

Now (65) with $v_t^* = v_t$, (13), and (69) just describe how surpluses and debt evolve given the other variables. Debt does not explode because $\alpha > 0$ in (11). The model looks like passive fiscal policy with one crucial exception: it allows only one value of unexpected inflation in (69). The point of fiscal theory is to determine unexpected inflation in the place of a Taylor principle threat, and that is just all this fiscal theory does. For solving the model, then, we can simplify by first solving for $\{\pi_t, x_t, r_t^n\}$ given $\{\varepsilon_{s,t}, \varepsilon_{i,t}\}$ and then calculating

$$s_{t+1}=\theta_{s\pi}\pi_{t+1}+\theta_{sx}x_{t+1}+\alpha v_{t}+u_{s,t+1} \tag{74}$$

$$\rho v_{t+1}=v_{t}+r_{t+1}^{n}-\pi_{t+1}-s_{t+1}$$

$$u_{s,t+1}=\rho_{s}u_{s,t}+\varepsilon_{s,t+1}. \tag{75}$$

Again, we could leave it all in and the computer will figure out that this block is independent, but it's prettier and adds intuition to simplify the model algebraically first.

Adding δ shocks in place of expectations and rearranging the equations we now have

$$\begin{aligned}
x_{t+1} &= x_t + \sigma i_t - \sigma\pi_{t+1} + \delta_{x,t+1} + \sigma\delta_{\pi,t+1}\\
\beta\pi_{t+1} &= \pi_t - \kappa x_t + \beta\delta_{\pi,t+1}\\
i_t &= \theta_{i\pi}\pi_t + \theta_{ix}x_t + u_{i,t}\\
\delta_{\pi,t+1} &= -\beta_s\varepsilon_{s,t+1} - \beta_i\varepsilon_{i,t+1}\\
r_{t+1}^n &= i_t + \delta_{r^n,t+1}\\
\omega q_{t+1} &= q_t + r_{t+1}^n\\
u_{i,t+1} &= \rho_i u_{i,t} + \varepsilon_{i,t+1}
\end{aligned}$$

Eliminating i_t, $\delta_{\pi,t}$, r^n_{t+1},

$$
\begin{aligned}
x_{t+1} &= \left(1 + \frac{\kappa\sigma}{\beta} + \sigma\theta_{ix}\right) x_t + \sigma\left(\theta_{i\pi} - \frac{1}{\beta}\right)\pi_t + \sigma u_{i,t} + \delta_{x,t+1} \\
\pi_{t+1} &= -\frac{\kappa}{\beta}x_t + \frac{1}{\beta}\pi_t - \beta_s\varepsilon_{s,t+1} - \beta_i\varepsilon_{i,t+1} \\
\omega q_{t+1} &= \theta_{ix}x_t + \theta_{i\pi}\pi_t + q_t + u_{i,t} + \delta_{r^n,t+1} \\
u_{i,t+1} &= \rho_i u_{i,t} + \varepsilon_{i,t+1}.
\end{aligned}
$$

or in matrix form,

$$
\begin{aligned}
\begin{bmatrix} x_{t+1} \\ \pi_{t+1} \\ q_{t+1} \\ u_{i,t+1} \end{bmatrix} &= \begin{bmatrix} 1 + \sigma\theta_{ix} + \sigma\kappa/\beta & \sigma\theta_{i\pi} - \sigma/\beta & 0 & \sigma \\ -\kappa/\beta & 1/\beta & 0 & 0 \\ \theta_{ix}/\omega & \theta_{i\pi}/\omega & 1/\omega & 1/\omega \\ 0 & 0 & 0 & \rho_i \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ q_t \\ u_{i,t} \end{bmatrix} \\
&+ \begin{bmatrix} 0 & 0 \\ -\beta_i & -\beta_s \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{x,t+1} \\ \delta_{r^n,t+1} \end{bmatrix}
\end{aligned}
$$

Now, we solve the model as

$$
\begin{aligned}
y_{t+1} &= By_t + \ C\varepsilon_{t+1} + D\delta_{t+1} \\
y_{t+1} &= Q\Lambda Q^{-1}y_t + C\varepsilon_{t+1} + D\delta_{t+1} \\
Q^{-1}y_{t+1} &= \Lambda Q^{-1}y_t + Q^{-1}C\varepsilon_{t+1} + Q^{-1}D\delta_{t+1} \\
z_{t+1} &= \Lambda z_t + Q^{-1}C\varepsilon_{t+1} + Q^{-1}D\delta_{t+1} \qquad (76)
\end{aligned}
$$

Let G_f select rows with eigenvalues greater than or equal to one, and G_b select rows with eigenvalues less than one. For example, if the first and third eigenvalues are greater than or equal to one,

$$
G_f = \begin{bmatrix} 1 & 0 & 0 & 0 & ... \\ 0 & 0 & 1 & 0 & ... \end{bmatrix},
$$

$$G_b = \left[\begin{array}{ccccc} 0 & 1 & 0 & 0 & ... \\ 0 & 0 & 0 & 1 & ... \end{array} \right].$$

The z_t corresponding to eigenvalues greater than one must be zero, so we can find the expectational errors δ_{t+1} in terms of the structural shocks ε_{t+1},

$$0 = G_f Q^{-1} C \varepsilon_{t+1} + G_f Q^{-1} D \delta_{t+1}$$
$$\delta_{t+1} = -\left(G_f Q^{-1} D\right)^{-1} G_f Q^{-1} C \varepsilon_{t+1}$$

For this to work there must be as many rows of G_f as columns of δ, i.e. as many eigenvalues greater or equal to one as there are expectational errors. Substituting in (76), we have the evolution of the transformed z variables,

$$z_{t+1} = \Lambda z_t + Q^{-1} \left[I - D \left(G_f Q^{-1} D\right)^{-1} G_f Q^{-1} \right] C \varepsilon_{t+1},$$

and then the original variables obey

$$y_t = Q z_t.$$

I include two small refinements. First, for computation it is better to force the elements of z_t that should be zero to be exactly zero. Machine zeros $(1e{-}14)$ multiplied by explosive eigenvalues eventually explode. Thus, I find the non-zero z only by simulating forward the nonzero elements of z,

$$G_b z_{t+1} = G_b \Lambda z_t + G_b Q^{-1} \left[C - D \left(G_f Q^{-1} D\right)^{-1} G_f Q^{-1} C \right] \varepsilon_{t+1}.$$

B Basic policy rule regressions

I report here basic policy rule regressions. I do not report them in the paper or use their values, since I do not attempt the hard topic of identification – whether the correlation between interest rates and inflation, say, represents the policy rule feedback or the response of inflation to an interest rate shock. I present them to show the data, and to give reassurance that a fiscal policy rule which loads on output and (to a lesser extent) on inflation is not unreasonable, and to examine the size of the correlations.

	ρ	b	c	ρ_u	R^2
		$i_t = a + \rho i_{t-1} + bx_t + c\pi_t + u_{i,t}$			
OLS		0.19	0.98	0.72	0.52
s.e.		(0.16)	(0.21)		
Single OLS		0.13		0.90	0.01
s.e.		(0.23)			
Single OLS			0.97	0.69	0.50
s.e.			(0.21)		
$1-\rho L$		0.29	0.48		0.29
s.e.		(0.10)	(0.24)		
With lag	0.81	0.30	0.22	0.16	0.86
s.e.	(0.06)	(0.06)	(0.11)		
$b/(1-\rho)$		1.55	1.15		
		$s_t = a + \rho s_{t-1} + bx_t + c\pi_t + u_{s,t}$			
	ρ	b	c	ρ_u	R^2
OLS		1.62	-0.38	0.37	0.37
s.e.		(0.34)	(0.35)		
Single OLS		1.64		0.38	0.35
s.e.		(0.32)			
Single OLS			-0.49	0.53	0.03
s.e.			(0.44)		
$1-\rho L$		1.45	-0.24		0.26
s.e.		(0.33)	(0.37)		
With lag	0.39	1.27	-0.38	-0.06	0.50
s.e.	(0.10)	(0.27)	(0.27)		
$b/(1-\rho)$		2.06	-0.62		

Table 3: Policy rule regressions. i= interest rate, x = GDP gap, s = surplus. Sample 1949-2018.

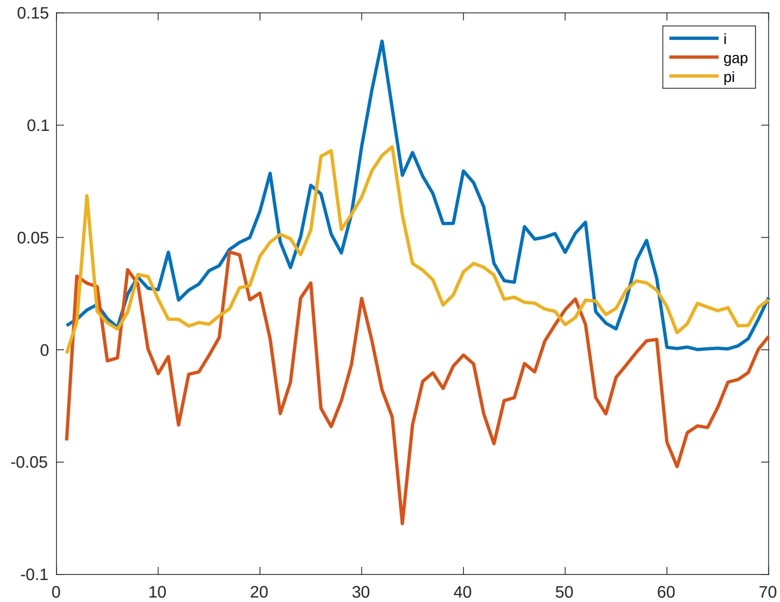

Figure 7: Interest rate, output gap, and inflation

Figure 8: Surplus, output gap, and inflation

Table 3 presents regressions of the interest rate and surplus on inflation and the output gap. Figure 7 presents the interest rate, output gap and inflation data underlying the monetary policy rule regressions, and Figure 8 presents the surplus, output gap and inflation data underlying the surplus policy rule regressions. The data are annual, and the same as used in Cochrane (2019).

These are simple regressions, that make no effort to surmount the profound identification problems of estimating policy rules. In the model, inflation and output gap respond contemporaneously to a surplus and interest rate shocks, so the right hand variable and error terms of the policy rule regressions are correlated. I include the regressions to characterize the data, and to give some sense of reasonable magnitudes.

Though there are thousands of monetary policy rule regressions in the literature, I start with that regression in part to frame the contrast with surplus policy rule regressions in the same data set. The OLS regressions show a small 0.19 output gap response and a large, nearly unit inflation response. The residual is strongly serially correlated. Figure 7 shows that the inflation response is, of course, driven by the rise and fall of inflation in the 1970s and 1980s. The single regression coefficients are just about the same as the multiple regression coefficients, with the output gap providing very little explanatory power.

One may wish to focus on the business cycle frequencies. The next two rows do that, and address serial correlation of the error, in two different ways. Using the error serial correlation ρ_i of the OLS regression, the regression labeled "$1-\rho L$" runs $(1-\rho L)i_t$ on $(1-\rho L)x_t + (1-\rho L)\pi_t$. This specification mirrors that of the policy rule, (35) and (43). If the regression is correctly specified, the quasi-differenced coefficients should be the same. Here, the main effect is to lower the inflation response to about 0.5. The nearly unit inflation response does reflect the low frequency rise and fall of inflation rather than business cycle movement.

Adding a lagged interest rate, in the last regression estimates a partial adjustment model, common in the monetary policy shock literature. The coefficients are reduced, but the implied long run coefficients are much larger. One needs this sort of model to produce a coefficient greater than one on inflation, as Clarida, Galí, and Gertler (2000) famously found. Stationary data do not easily produce a coefficient that leads to explosive behavior.

Overall, these regressions reflect the great uncertainty and sensitivity to specification typical of the literature.

The OLS regression estimate of the surplus rule shows most of all a strong association with the output gap. This association stands out in Figure 8. It is clear at business cycle frequencies and also in the long dip of potential GDP (and, not reported, unemployment) in the 1970s and 1980s. The tables are turned. Here the output gap is the strong correlation, and the inflation coefficient is insignificant and results in very small R^2.

This surplus is the ratio of surplus to value of the debt, or equivalently (surplus/GDP) to (value/GDP). Thus, the coefficient that a 1% rise in GDP gap results in a 1.62 percentage point rise in surplus means, if debt/GDP = 0.5, a 0.81 percentage point rise in surplus/GDP ratio.

One expects the coefficient of surplus on inflation to be positive, due to an imperfectly indexed tax code. The coefficient is -0.38, though insignificant. One can see in Figure 8 that the 1970s, with high inflation, had lower surpluses. This observation however reinforces the central weak point of such regressions. The negative correlation of surpluses with inflation is likely the response of inflation to surplus shocks, not the rule.

Any serious estimation of policy rules, which this is not, must take the identification problem seriously. To measure the interest rate or surplus policy rules, we must find movements in inflation and output gap which are not correlated with the interest rate or surplus disturbances $u_{i,t}$ and $u_{s,t}$. This is a different task than the usual one, of measuring directly the economy's response to monetary or fiscal policy shocks. There, one must find movements in $u_{i,t}$ and $u_{s,t}$ that are not correlated with changing expectations of future inflation, output, etc.

This is not a hopeless task. The Romer and Romer (1989) approach could look for such shocks. Romer and Romer looked for shocks that were a response to inflation, but not to output, in order to measure the response of output to such shocks. We need to measure the systematic part of policy, not the economy's response to policy. So, we either need narrative measurements of the systematic component, or we can use the monetary shock to measure the fiscal response function. Likewise Ramey (2011) pioneered the use of military spending as an exogenous shock to $u_{s,t}$. We can use this to measure the monetary response function.

The structural or narrative approach may be much more fruitful for the fiscal response function than it is for the monetary response function. Much of the strong response of surpluses to output, and the response we wish to measure to inflation, are generated by the tax code and automatic stabilizers. Those can be modeled to generate θ_s parameters. Additional fiscal decisions are measurable too, in acts of congress.

Identification and estimation within the structure of a model may also be fruitful. The task is not as hopeless as it seems from the Cochrane (2011a) critique of monetary policy rule estimation in new-Keynesian models. That paper concerned the difficulties of measuring off-equilibrium responses from data in an equilibrium, which really is hard. The θ responses here are all relations between variables that we do see in equilibrium.

C Hansen-Sargent formulas

Here is the algebra for the Hansen, Roberds, and Sargent (1992) formula (23)

$$\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = a(\rho)\varepsilon_{t+1}$$

where

$$s_{t+1} = a(L)\varepsilon_{t+1} = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j}.$$

We just write out the terms,

$$\begin{aligned}\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \Delta E_{t+1}(&a_0\varepsilon_{t+1} + a_1\varepsilon_t + a_2\varepsilon_{t-1} + ...\\ &+ \rho a_0 \varepsilon_{t+2} + \rho a_1 \varepsilon_{t+1} + \rho a_2 \varepsilon_t + ...\\ &+ \rho^2 a_0 \varepsilon_{t+3} + \rho^2 a_1 \varepsilon_{t+2} + \rho^2 a_2 \varepsilon_{t+1} + ...)\end{aligned}$$

Only the ε_{t+1} terms are nonzero so

$$\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = a_0\varepsilon_{t+1} + \rho a_1 \varepsilon_{t+1} + \rho a_2 \varepsilon_{t+1} + ... = a(\rho)\varepsilon_{t+1}$$

Here is a derivation of the Hansen and Sargent (1981) prediction formula equation

(25). If

$$s_{t+1} = a(L)\varepsilon_{t+1}$$

then

$$E_t \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} = \frac{a(L) - a(\rho)}{1 - \rho L^{-1}} \varepsilon_{t+1}. \tag{77}$$

We just write out the right hand terms and re-group them

$$\begin{aligned} E_t \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} = E_t(& a_0\varepsilon_{t+1} + a_1\varepsilon_t + a_2\varepsilon_{t-1} + a_3\varepsilon_{t-2} + ... \\ & + \rho a_0\varepsilon_{t+2} + \rho a_1\varepsilon_{t+1} + \rho a_2\varepsilon_t + \rho a_3\varepsilon_{t-1} + ... \\ & + \rho^2 a_0\varepsilon_{t+3} + \rho^2 a_1\varepsilon_{t+2} + \rho^2 a_2\varepsilon_{t+1} + \rho^2 a_2\varepsilon_t + ...) \end{aligned}$$

The terms inside the expectation are given by

$$\sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} = \frac{1}{1 - \rho L^{-1}} s_{t+1} = \frac{1}{1 - \rho L^{-1}} a(L)\varepsilon_{t+1} \tag{78}$$

So to generate the conditional expectation E_t we need to subtract off the terms in ε_{t+1} and forward. They are

$$\begin{aligned} & a_0\varepsilon_{t+1} + \rho a_0\varepsilon_{t+2} + \rho a_1\varepsilon_{t+1} + \rho^2 a_0\varepsilon_{t+3} + \rho^2 a_1\varepsilon_{t+2} + \rho^2 a_2\varepsilon_{t+1} + ... \\ & = \left(a_0 + \rho a_1 + \rho^2 a_2 + ...\right) \varepsilon_{t+1} + \rho \left(a_0 + \rho a_1 + \rho^2 a_2 + ...\right) \varepsilon_{t+2} + ... \\ & = a(\rho) \frac{1}{1 - \rho L^{-1}} \varepsilon_{t+1}. \end{aligned}$$

Subtracting these terms from (78), we get (77).