OPTIMAL BANK REGULATION IN THE PRESENCE OF CREDIT AND RUN-RISK

Anil K. Kashyap
Dimitrios P. Tsomocos
Alexandros P. Vardoulakis

Optimal Bank Regulation In the Presence of Credit and Run-Risk
Anil K. Kashyap, Dimitrios P. Tsomocos, and Alexandros P. Vardoulakis
NBER Working Paper No. 26689
January 2020
JEL No. E44,G01,G21,G28

ABSTRACT

We modify the Diamond and Dybvig (1983) model so that, besides offering liquidity services to depositors, banks also raise equity funding, make loans that are risky, and can invest in safe, liquid assets. The bank and its borrowers are subject to limited liability. When profitable, banks monitor borrowers to ensure that they repay loans. Depositors may choose to run based on conjectures about the resources that are available for people withdrawing early and beliefs about banks' monitoring. We use a new type of global game to solve for the run decision. We find that banks opt for a more deposit-intensive capital structure than a social planner would choose. The privately chosen asset portfolio can be more or less lending-intensive, while the scale of intermediation can also be higher or lower depending on a planner's preferences between liquidity provision and credit extension. To correct these three distortions, a package of three regulations is warranted.

Anil K. Kashyap
Booth School of Business
University of Chicago
5807 S. Woodlawn Avenue
Chicago, IL 60637
and NBER
anil.kashyap@chicagobooth.edu

Dimitrios P. Tsomocos
Said Business School and St. Edmund Hall
University of Oxford
Dimitrios.Tsomocos@sbs.ox.ac.uk

Alexandros P. Vardoulakis
Board of Governors of the Federal Reserve System
alexandros.vardoulakis@frb.gov

1 Introduction

It is well understood that financial intermediaries, hereafter banks, can provide useful services through both the lending that they undertake and the deposits that they offer. Banks use both sides of their balance sheet to perform these services and, in doing so, expose themselves to credit risk and funding (run) risk. The efficiency of the resulting allocations, from a social point of view, depends jointly on the overall scale of the banking sector and the mix of lending and deposit-taking. Yet, most banking models tend to focus on only one service or type of risk, often neglecting how bank fragility endogenously affects intermediation and vice versa. In this paper, we propose a new model where banks create economic surplus for both borrowers and depositors but choose a balance sheet that exposes the economy to excessive run-risk and an inefficient scale of intermediation. We use the model to study alternative regulations that have been proposed since the Global Financial Crisis and derive the optimal regulatory mix.

Our model starts with the setup developed by Diamond and Dybvig (1983) that explains how a bank can provide liquidity services to depositors (savers) with uncertain consumption/funding needs. We modify their model so that the bank also raises equity funding, makes loans that are risky, and can invest in safe, liquid assets. We also assume that banks and borrowers are subject to limited liability so that each of them has incentives to take risks that they do not fully bear the costs of. We suppose, similar in spirit to Diamond (1984), that banks monitor the loans to guarantee that they are repaid. Monitoring is costly and, hence, must be profitable for banks to do so. Even if savers were able to monitor borrowers as efficiently as the bank, delegating this function to banks is optimal, because banks can combine monitoring and deposit-granting, which is itself valuable. However, because the loans are risky and have an uncertain liquidation value, depositors may opt to run on the bank depending on the conjectured ability of the bank to serve early withdrawals and the bank's incentives to continue monitoring borrowers. Finally, the bank is managed by a banker who wants to maximize the profits accruing to her rather than worrying about the utility of depositors (or borrowers). The banker internalizes how her choices affect run-risk and the desire of depositors and borrowers to supply deposits and demand loans, respectively. Depositors and borrowers rationally expect the level of run-risk that will prevail in equilibrium when making these decisions but are atomistic and do not internalize their marginal impact on aggregate banking portfolios and run-risk.[1]

We characterize the run decision by introducing a new specification of the type of global game developed by Goldstein and Pauzner (2005) to analyze Diamond-Dybvig style models. In our setup, depositors get signals about the (liquidation) value of loans if they are called early to help pay depositors. Depositors use a threshold rule in deciding to run, such that, if their signal about the liquidation value is below the cutoff, depositors decide to withdraw even if they do not need to

---

[1]The desirability of regulation does not rely on the fact that the banker internalizes the effect of her actions. See the online appendix for an analysis of the case where the banker does not fully account for the effect of her actions. Indeed, we show in the online appendix that failing to do so results also in suboptimal welfare for the banker, rendering the justification of policy interventions easier.

consume immediately. Conversely, if the signal is above the cutoff, only depositors with urgent consumption needs withdraw, while the rest keep their deposits in the bank. The threshold depends on the deposit contract, the balance sheet, and the profitability of the bank. This is because the deposit contract specifies the additional return from waiting instead of withdrawing early, the balance sheet determines the capacity of the bank to serve early withdrawals, and the profitability of the bank governs its incentives to monitor borrowers.

The proof to establish the uniqueness of the threshold is novel and may be of independent interest for other applications of global games. As in Goldstein and Pauzner (2005), global strategic complementarities for depositors are not present, but in our setup, the property of state monotonicity is also absent. This property obtains when the incentive to take an action is monotonic with respect to the underlying fundamentals (state) of the economy—and is typically required in mainstream global games (see Morris and Shin, 2003). In our model, state monotonicity would obtain if the relative incentive to run becomes stronger as the liquidation value of loans becomes smaller. If the bank has liquid resources to serve all depositors queuing to withdraw, then monotonicity holds. But it fails to hold once the bank runs out of liquid resources and, thus, only a percentage of withdrawals can be met. Intuitively, once liquid assets are exhausted, the payoff from withdrawing is higher when the liquidation value of the bank's assets is higher, because the probability of being at a lucky spot in the queue is higher. Hence, the property of state monotonicity fails.

We employ a new argument to prove uniqueness of the equilibrium, which does not require state monotonicity *everywhere* but only at threshold points for fundamentals. This property is easily satisfied in our framework. Moreover, we provide intuition about generalizing it to other environments where this kind of *perverse* state monotonicity obtains. Hence, the proof we propose could be adapted to other applications in which improvements in fundamentals reduce the incentives to act in some regions and increase them in others.

The modifications to Diamond-Dybvig that we propose are all necessary to study credibly the differences between the private and social optima. First, micro-founding the probability of a run is crucial to understand the determinants of run-risk. In partial equilibrium, one expects that raising capital requirements or liquidity requirements will reduce the probability of a run. We show this is true in our model. But, in general equilibrium, agents will adjust their behavior in response to regulation, and their new choices will feed back to alter run-risk via equilibrium interest rates.

Second, it is important to explicitly model the optimization behavior of bankers, depositors, and borrowers in order to understand how run-risk affects agents choices and their welfare. Also, instead of assuming ad hoc welfare criteria, such as the minimization of run-risk, we model the effect of the choices on the agents' utilities. This allows us to compare privately and socially optimal choices. In general, they will differ, though we can also describe the special case where private banking allocations are constrained efficient.

Third, introducing costly monitoring highlights the importance of the bank's profitability for its stability (over and above the relative liquidity of its assets and the runability of its liabilities). Monitoring is unobservable and costly, so it only occurs if the banker finds it ex post profitable. Policy

interventions that crush banking profits can backfire if the banker loses the incentive to monitor. In this case, depositors can opt to run because they realize bank credit risk has endogenously increased because of the lack of monitoring. Hence, we highlight a novel connection between moral hazard and run-risk, which is important for studying regulations that compress banking profits.[2]

Fourth, by modeling the utility impact of agent choices, it becomes clear that borrowers and depositors care about the amount of lending and the level of deposits that are available. Thus, the overall size of the banking sector matters beyond just the assets' and liabilities' composition. The Modigliani-Miller theorem is violated in various ways in our model, but allowing for endogenous equity issuance helps us understand the implications of these violations for the size of the banking sector and how the banking sector converts deposits into loans.

The model is suitable for positive analysis and can be extended or simplified in various dimensions to study unobservable portfolio choices and risk-shifting, the interaction between inside and outside equity, the role of imperfect competition in banking, the relationship between fire-sales, and run-risk among other things. Some of these extensions are studied in the online appendix, while others are left for future research.

In the body of the paper, we focus on the normative properties of the model and the study of optimal regulation. We have four main findings.

First, there are three independent distortions in private banking choices. Both the private and social planner's equilibria can be characterized by the mix of loans versus liquid assets, the mix of deposits versus equity, and the scale of intermediation. The scale can be summarized in various ways: One helpful one is the proportion of deposits that are converted to loans. In the private equilibrium, the banker wants to maximize her profits from collecting deposits to extend loans. Because she understands how run-risk and how the profit margin between the loan and deposits rates are each determined, she chooses levels of liquidity and equity that are profit maximizing for her. Compared to the banker, the social planner cares about how these choices affect not only banking profits, but also the utility of depositors and borrowers. We show that the other two agents are disadvantaged in the private equilibrium and explain how the asset mix, liability mix, and scale of intermediation can be adjusted to benefit the savers or borrowers.

Second, each distorted choice by the banker has a component tied to a *run externality* and another component that reflects the surplus from intermediation that accrues to either the borrowers or savers. The run externality arises because banks are prone to take risks that raise the probability of a run without internalizing how run-risk directly matters for borrowers' and savers' welfare. This is a consequence of the fact that banks maximize their own profits subject to limited liability, while borrowers and savers are atomistic, so that they do not internalize how their choices affect run-risk.

Third, how the planner seeks to correct the run externalities depends on how much weight is placed on borrowers versus savers, i.e., on the implicit preference between credit extension and liquidity provision. The run-risk arises from choices on both sides of the bank's balance sheet, and

---

[2]This possibility may be of independent interest to researchers as it highlights that runs can occur even at financial institutions that are very liquid but lack the incentives to monitor.

the planner would opt for both a less deposit-intensive capital structure and a less lending-intensive asset allocation in order to reduce run-risk. But, doing so can restrict credit extension and, hence, reduce the surplus to borrowers. Thus, the desire to reduce run-risk is balanced against the benefits from offering deposits and extending loans.

When savers are favored, the planner chooses a more equity-intensive capital structure, a more liquidity-intensive asset allocation, and a lower scale of intermediation. Although more deposit-taking will require higher deposit rates that compress bank profit margins and discourage equity issuance, cutting lending allows the banker to raise loan rates. The higher lending rates offset the higher deposit costs, so that equity issuance is still possible. Those deposits that are not channeled to lending are invested in the liquid assets, which further reduces run-risk and further offsets some of the increase in deposit rates.

In contrast, when borrowers are favored, the planner shifts to a relatively more lending-intensive asset allocation and a higher scale of intermediation, while choosing a more equity-intensive capital structure. Any attempt to increase lending by raising more deposits would compress the profit margin and reduce the incentives to provide equity. Instead, the planner reduces the liquid asset holdings and shifts funding away from deposits to release resources for loan extension and maintain the incentives for equity issuance.

Fourth, individual regulations can be used to move the private choices closer to those that a planner would choose. But no single regulation can deliver the planning outcomes. Capital and liquidity regulations reduce run-risk, but inefficiently restrict credit creation. Moreover, as banks take excessive risk on both sides of their balance sheet, controlling risk on one side may result in risk materializing in the other side. To replicate the planner's preferred allocations, a combination of three regulations are needed to correct the distortions associated with the bank's asset allocation, capital structure, and scale of intermediation. The asset and liability distortions can be corrected using a capital and a liquidity requirement. Capital and liquidity requirements are jointly helpful and should be treated as complementary, since they operate on different intermediation margins. But, both are less effective at boosting the overall scale of intermediation, which the planner may favor in order to expand lending and help borrowers. To raise or lower the scale of intermediation, other regulations, such as deposit subsidies or lending subsidies, would be needed.

Related literature. Our analysis of potential runs can be contrasted to several other approaches that have been developed in the literature. For instance, a bank-run in our setup can occur because the information about fundamentals is very bad. Thus, our analysis can be compared with many prominent papers that analyze information-based runs such as Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998), Uhlig (2010), Angeloni and Faia (2013), and Boissay, Collard and Smets (2016). According to this fundamental view of runs, policy interventions are needed to correct pecuniary externalities arising from missing markets; otherwise the private equilibrium is constrained efficient (Allen and Gale, 2004). Although we have abstracted from pecuniary externalities and relative price effects, the fundamental run probability in our model depends on the bank's liquidity and capital. Thus, regulation could be used to address not only the

adverse consequences of any pecuniary externalities, but also the probability that they may occur.

However, a bank-run can also occur because of a coordination problem among depositors even if the bank is solvent in the long-run. This type of run can be interpreted as being panic-based, and for analyzing this kind of run, it is important to know what determines the panic. In the Diamond-Dybvig model, panics are a multiple equilibrium outcome. There are several ways to tackle the multiplicity of equilibria such that the model is suitable for policy analysis. Cooper and Ross (1998), Peck and Shell (2003), and Diamond and Kashyap (2016) consider that the probability of a bank-run is driven by sunspots. Ennis and Keister (2005) take an axiomatic approach to equilibrium selection and link the probability of a particular equilibrium being played to the appropriately defined incentives of agents. In our earlier working paper, Kashyap, Tsomocos, and Vardoulakis (2014), as well as in Gertler and Kiyotaki (2015) and in Choi, Eisenbach, and Yorulmazer (2016), the probability of a run is determined by an exogenous function of key fundamentals. Despite the fact that these approaches may be, to varying degrees, adequate for certain types of positive analysis, the derivation of optimal policy requires micro-foundations for the incentives to run on the bank.

In order to address such concerns, we follow the global games approach—developed by Carlsson and van Damme (1993) and Morris and Shin (1998) and applied to bank runs by Goldstein and Pauzner (2005)—to derive a unique probability of run. This approach ties run incentives to the fundamentals of the economy, the balance sheet structure of the bank, and the equilibrium payoffs to depositors withdrawing early and late. Rochet and Vives (2004), Vives (2014), and Morris and Shin (2016) also take a global game approach but assume simpler payoffs for depositors that wait and depositors that withdraw, which are exogenous and do not depend on equilibrium allocations and interest rates. The former study the implications of lender of last resort policies, while the latter two examine how capital and liquidity regulations affect run-risk in partial equilibrium. Despite the micro-foundation for run dynamics, these approaches are arguably not adequate to study optimal regulation, because deposit rates, which are important for welfare in general equilibrium, do not directly influence the incentives of depositors to run. This disconnect is because the run decision is made by fund managers to whom depositors have delegated their portfolios, while the managers' contracts and payoffs are not tied to interest rates. Goldstein and Pauzner (2005) explicitly consider the impact of deposit contracts in run incentives but consider a fixed liquidation value for loans and an exogenously set upper dominance region for the global game. We propose a new way to endogenize the liquidation value as well the upper dominance region.

Apart from the new type of global game we consider, we also differ from the aforementioned papers because we endogenize the supply of deposits and equity as well as the loan demand. Hence, we allow the deposit and loan rates to be determined in equilibrium and to respond to regulations. As we show, this is important in order to capture the welfare effects on different agents, to permit an examination of the externalities from private banking choices, and to determine how a planner would want to alter these choices. Ikeda (2018) and Carletti, Goldstein, and Leonello (2019) study the inefficiency of privately optimal choices on bank-run risk, while incorporating the feedback effect on deposit rates. The former paper considers a Rochet-Vives global game and studies the

inefficiencies arising from banks neglecting how their actions affect the deposit rate. The latter considers a Goldstein-Pauzner global game and studies how fire-sale externalities influence the efficiency of private allocations. Both papers suggest that capital and liquidity regulations are jointly useful but abstract from modeling borrowers as well as the surpluses created by intermediation, which as we show are important for the optimal policy response.

Our model could easily be extended to incorporate incomplete contracts and fire-sales. Indeed, we show in the online appendix how incomplete deposit contracts introduce an additional reason why private allocations are inefficient, and we conjecture the same for fire-sale externalities. But, these frictions, irrespective of how important there are by themselves, are not necessary to justify banking regulation in our setup.

We should note that both in our model and the literature we have surveyed so far, deposit contracts are uncontingent, so deposit rates cannot be indexed by the realization of the state of the economy. Instead, contracts could incorporate suspension clauses as in Ennis and Keister (2009) and Keister (2015), which would give rise to different run dynamics. As noted in these papers, suspension of convertibility may not be ex post efficient, but the interaction between run dynamics, ex post intervention and ex ante convertible deposit contracts is an interesting topic for future research (see, also, Keister and Mitkov, 2017).

Finally, our paper speaks to the growing literature studying the complementarity of capital and liquidity regulations, such as Walther (2016) and Kara and Ozsoy (2016) in the presence of fire-sale externalities; Boissay and Collard (2016) when the interbank market cannot efficiently allocate resources; and Van den Heuvel (2017) in a real business cycle model with risk-taking incentives and a Diamond and Kashyap (2016) run specification. The overreaching conclusion from our analysis is that the number of optimal regulatory interventions is equal to the number of distorted intermediation margins rather than the number of externalities in the model. In other words, it doesn't make much difference if the same externality or multiple externalities distort multiple margins in distinct ways. Instead, what matters is whether different margins are or are not distorted.

The remainder of the paper is separated into four parts. In section 2, we describe the model and show the privately optimal choices for the bank, the savers, and the entrepreneurs. In section 3, we study the efficient allocations chosen by a social planner and derive expressions for the wedges between the private and social decisions. In section 4, we explore how regulation can be used to correct the private inefficiencies. Section 5 concludes by summarizing the main findings, reiterating the intuition for them, and describing a few directions for future research. All proofs as well as additional derivations and model extensions are relegated to an online appendix.

2   Model

The model consists of three periods, $t = \{1, 2, 3\}$, features a single consumption good, and includes three types of agents. In particular, there is a continuum of entrepreneurs, a continuum of savers and

one banker. We will be referring to individual entrepreneurs and savers as the entrepreneur (E) and the saver (S), and we will be also using the singular or plural for these agents depending on flow of the discussion. But it should be clear that each of these agents is atomistic and representative of her type, taking aggregate equilibrium variables as given. On the contrary, we assume that the banker (B) internalizes how her choices affect the aggregate equilibrium variables that matter for her profitability.

Each entrepreneur has access to a productive, but illiquid, risky technology and chooses how much to borrow to invest in it. Funds invested at date 1 yield A per unit of investment at date 3 with probability ω (which we call the "good" state of the world) and zero otherwise (the "bad" state). The project delivers no output at date 2 but it can be liquidated. The liquidation value, ξ, is uncertain and independent of the productivity shock (A). The productivity shocks across the projects of individual entrepreneurs are perfectly correlated and all projects have the same liquidation value at $t = 2$. Hence, we are always referring to the representative entrepreneur and the representative project.

The banker manages an institution, which we call a bank, that acts as an intermediary between the entrepreneurs and savers. The bank is funded by raising equity from the banker and deposits from savers.[3] The funds raised at date 1 are invested into either a liquid storage asset or in loan to entrepreneurs. The loan contract with the entrepreneur specifies an uncontingent loan rate and it is backed by the project it funds and the cash flows that it generates. In case the entrepreneur fails to repay the loan obligation, the bank can seize the project and any cash flows that have been generated. Loans are callable, i.e., the bank does not need to wait for the project to mature, but can request full repayment at an earlier point in time.

Moreover, the banker decides whether to monitor the entrepreneur's project at $t = 3$ or not. Monitoring is important because the productivity shock is private information to the entrepreneur. Without monitoring, the entrepreneur would report the bad state of the world and default. Because entrepreneurs have the same investment opportunities and the productivity shocks are perfectly correlated, the banker needs to incur the monitoring cost once for the whole loan portfolio, which we assume is independent of how many loans are extended.

Savers are identical ex ante and each of them has a large endowment at date 1 that is used to fund initial consumption and savings. However, each saver receives an idiosyncratic preference shock at $t = 2$ to consume early or late as in Diamond and Dybvig (1983). Given that the preference shocks are independent and identically distributed, some fraction of savers will need to consume at t=2 and the rest waits to consume at date 3. In other words, savers are ex ante identical, but ex post heterogeneous. We will refer to them as impatient and patient, respectively. The (ex ante) representative saver invests in bank deposits or holds a liquid storage asset. Deposits are demandable, which is important to provide incentives to the banker to monitor as we describe in detail later.

A loan can be recalled and liquidated to pay deposits. Upon being recalled it yields an immediate

---

[3]We assume that savers do not buy equity in the bank in order to simplify the exposition of our baseline model. In the online appendix, we present an extension where the bank can raise both inside equity from bankers and outside equity from other agents. Overall, the main results from our benchmark model continue to hold.

gross return ξ, which is uncertain and follows a uniform distribution $U \sim \left[\underline{\xi}, \overline{\xi}\right]$ with $0 \leq \underline{\xi} < 1 < \overline{\xi}$ and $\Delta_\xi \equiv \overline{\xi} - \underline{\xi}$. The fact that $\overline{\xi}$ can exceed 1 will be important in what follows. When a loan is called, the entrepreneur forfeits the portion of the project that is funded by the loan.[4]

Depending on the value of ξ and the rest of its balance sheet, the bank may not have enough resources to fully pay depositors if they all decide to withdraw. Typically, aside from extremely high or low realizations of ξ, the bank is at risk for self-fulfilling runs: A patient saver will demand her deposits early if she believes that other patient savers will do the same. In order to address the coordination problem and obtain a unique equilibrium, we assume that savers receive noisy signals about the true realization of ξ at $t = 2$. These signals not only provide information about the fundamental ξ, but also about the beliefs of other savers, and so serve to coordinate the patient savers' decisions. In particular, we show that there is a unique threshold ξ^*, such that all patient savers withdraw their deposits when the true realization is below that threshold and keep their deposits in the bank otherwise. We will refer to ξ^* as the run threshold, while the probability of a run is $q = (\xi^* - \underline{\xi})/\Delta_\xi$.

The liquidation value can be justified in several ways. For instance, the incomplete project could have a secondary use in the interim period because it can be used in conjunction with an alternative short-term technology. Or, we could assume that it can be sold to some outside investors as in Shleifer and Vishny (1992). In other words, ξ does not strictly represent the salvage value of the long-term investment, as for example in Cooper and Ross (1998), but rather the liquidation/resale value of long-term investment. $\overline{\xi}$ has to be high enough that the bank can always withstand a panic for some realizations. While $\underline{\xi}$ has to be low enough that the bank may run out of liquidity even if a panic does not occur. We describe the importance of these bounds in section 2.4.[5]

The run threshold depends on the savers' beliefs about what they will receive by being patient as opposed to joining a run. More precisely, they need to form expectations over a variety of possible outcomes that involve their own utility function, the utility function of the entrepreneur, and the production function. Essentially, to judge the risk of the period 3 deposits, the saver needs to infer the bank's profitability and its balance sheet, which will depend on the entrepreneurs' loan demand. In turn, the loan demand depends on the curvature of the entrepreneurs' utility function and the

---

[4]We have assumed that the bank recalls and liquidates the loans when it needs liquidity rather than selling them in a secondary market to outside investors. This is a convenient way to ensure that run-risk adversely affects real economic activity. Alternatively, we could have assumed that the bank sells loans instead of recalling them. Outside investors could price these loans based on their own cash-in-hand or monitoring ability, and thus, the loans' resale value could similarly be described by ξ. But, additional assumptions would be needed to introduce an (direct) effect of run-risk on entrepreneurs' operations.

[5]Our model can easily be adjusted to make the liquidation value depend on the expected value of the loans, i.e., $\xi \cdot \omega(1 + r_I)$, where r_I is the loan rate and ω is the good state where the loan is repaid. Then, ξ would capture the fraction (between 0 and 1) of the expected value that can be obtained at liquidation. The liquidation value would vary because ξ varies. Given that the expected value of loans is higher than one, the two approaches would yield qualitatively similar results. Alternatively, we could have assumed that ξ does not vary, but the probability distribution $\widetilde{\omega}$ varies as in Goldstein and Pauzner (2005). Then, the liquidation value would continuously vary with the realization of the true probability distribution ω. The upper and lower dominance regions in the incomplete information game would still be endogenously determined in these cases. Matta and Perotti (2016) also consider runs resulting from assets' liquidity risk and study how secured debt can adversely impact the incentives to run.

shape of the production function. The potential payoffs for period 3 deposits can then be contrasted to the value of withdrawing early. This comparison will depend on the depositors' attitudes toward risks in period 2 versus 3. Moreover, the run-risk will enter into the welfare calculation of savers and entrepreneurs, and the planner will account for this dependence in choosing allocations.

We will make three assumptions to make these calculations as simple as possible, essentially by making the key schedules that are relevant for these expectations and comparisons linear. In particular, we assume that savers have quasi-linear preferences for consumption, entrepreneurs have linear preferences and no initial endowment, and the production technology is linear. The main cost of assuming linearity is that it leaves savers and entrepreneurs at a point where they break even from their savings and borrowing choices. This means that intermediation generates no surplus for them. The absence of any surplus does not distort the basic properties of the private equilibrium, but it does have powerful implications for the nature of optimal regulation. So we reintroduce curvature into the savers' and entrepreneurs' problems in other ways that make the results less extreme without complicating either the calculations of the expectations that matter for the run threshold or the derivations of distortions that motivate policy interventions. We will clearly identify and explain each of the three assumptions as we describe the rest of the model, but none of them are responsible for any of our main results.

Figure 1 presents the timeline of the model. Sections 2.1-2.4 describe the agents' optimization problems and the determination of the run threshold. We will see that solving for the indirect utility functions of the savers and entrepreneurs facilitates much of the ensuing analysis. So those will be two of the key objects that we derive. Section 2.5 defines and characterizes the private equilibrium.

$t = 1$	$t = 2$	$t = 3$
E borrow and invest in risky projects	S learn their type	State of the world is determined
S invest in demandable bank deposits	S receive noisy signals about ξ	E privately learn the realization
and potentially in the liquid asset	and decide whether to withdraw	B decides whether to monitor
B raises equity and deposits and	B recalls loans and pays withdrawals	Loans and deposits are repaid in the
invests in loans and liquid assets	If $\xi < \xi^*$, a run occurs	good state and default in the bad

Figure 1: Model Timeline

2.1 Savers

The savers are endowed with e_S at $t = 1$ and decide how much of their endowment to invest in bank deposits, D, and how much to hold in the liquid storage technology, LIQ_S, respectively. At $t = 2$, a portion of savers, δ, receive a preference shock to consume immediately, while the rest, $1 - \delta$, want to consume at $t = 3$. Preference shocks are private information, are independent and identically distributed, and are not contractible ex ante.

Deposits are demandable, early withdrawals are serviced sequentially, and the $t = 2$ and $t = 3$ uncontingent interest rates are r_D and $\bar{r}_D$, respectively. This contract structure creates the possibility

of a run, since patient savers may choose to demand their deposits early depending on their own information and their expectations about the actions of other patient savers.

We denote all variables that are not predetermined at $t = 1$ as functions of the liquidation value, ξ, and the portion of savers who decide to withdraw at $t = 2$, $\lambda \in [\delta, 1]$. In equilibrium, either all savers choose to withdraw, $\lambda = 1$, or only the impatient savers withdraw, $\lambda = \delta$. However, the out-of-equilibrium beliefs, which play an important role in the determination of the run probability, depend on the conjectured portion of savers withdrawing. This conjecture can be anywhere between δ and 1.

It is instructive to review each of the possible scenarios. If there is no run, i.e., $\xi \in \left[\xi^*, \bar{\xi}\right]$, only impatient depositors withdraw, and they receive the full amount of promised payment, $D(1 + r_D)$. Patient depositors' repayments will depend on the realization of the technology shock in the next period; they receive their promised payment, $D(1 + \bar{r}_D)$, with probability ω and zero otherwise. In a run, all depositors attempt to withdraw, and there is probability $\theta(\xi, 1)$ that any depositor is repaid.[6]

Savers have quasi-linear preferences for consumption, such that they value consumption linearly at $t = 2$ and $t = 3$. This is the first of the three aforementioned assumptions that make our numerical calculations easier. This assumption greatly simplifies the patient savers' decision about whether to join a run, because it means that all that must be computed is the expected payoff from deposits in period 2 versus 3.

In contrast, if the savers were risk averse, then finding the threshold that determines whether to run is much more complicated. The complication arises because a patient saver needs to compute her expected utility differential between waiting and withdrawing, accounting for all possible out-of-equilibrium beliefs about the actions of other savers. Computing the expected deposit payoffs is a lot simpler than computing the expectation of a nonlinear function of the deposit payoffs. Moreover, quasi-linearity yields a very tractable deposit supply schedule, which substantially simplifies the normative analysis of alternative regulations.

The disadvantage of the linearity assumption is that it greatly degrades the usefulness of a deposit to a saver. Essentially, the deposit becomes a pure financial instrument whose only value is that it pays more interest than the liquid asset. Put differently, the deposit essentially would be equivalent to a bond.

To make deposits more useful, we assume that for impatient savers there is a transactional advantage to having a deposit in normal situations where there is no run (see also Peck and Shell, 2010, for a similar modeling assumption about the utility from the transaction services of deposits). To do this, we suppose that this advantage is described by a concave function V that is increasing in the amount promised and repaid, $D(1 + r_D)$. This additional benefit of deposits partially offsets the stark implications of the linearity assumption for savers' utility, while still making it simple to solve for the run threshold. Modeling things this way leads to no qualitative changes relative to a model with concave utility and no transactions services but makes it substantially easier and faster

---

[6]This probability of receiving payment when fundamentals are ξ and λ savers withdraw is denoted by $\theta(\xi, \lambda)$ and is determined by equation (10), derived in section 2.3. In a run, all savers attempt to withdraw, i.e., $\lambda = 1$. Hence, the probability of being repaid is $\theta(\xi, 1)$.

to solve the model.

The expected utility of an individual saver is given by

$$
\mathbb{U}_S = \overbrace{U\left(e_S - D - LIQ_S\right)}^{\text{Utility at } t=1} + \overbrace{\int_{\underline{\xi}}^{\xi^*} \left[\beta\delta + \beta^2(1-\delta)\right] \cdot \left[\theta(\xi,1) \cdot D(1+r_D) + LIQ_S\right] \frac{d\xi}{\Delta_\xi}}^{\text{run}}
$$

$$
+ \underbrace{\int_{\xi^*}^{\overline{\xi}} \beta\delta\left[D(1+r_D) + LIQ_S\right]\frac{d\xi}{\Delta_\xi}}_{\text{no run,impatient}} + \underbrace{\int_{\xi^*}^{\overline{\xi}} \beta^2(1-\delta)\left[\omega \cdot D(1+\bar{r}_D) + LIQ_S\right]\frac{d\xi}{\Delta_\xi}}_{\text{no run,patient}} + \underbrace{\int_{\xi^*}^{\overline{\xi}} V\left(D(1+r_D)\right)\frac{d\xi}{\Delta_\xi}}_{\text{transaction services}},
$$

$$(1)$$

where $U(\cdot)$ is the utility function for $t=0$ consumption with $U'(\cdot) > 0$ and $U''(\cdot) < 0$; $V(\cdot)$ captures the transaction services of deposits with $V(0) = 0$, $V'(\cdot) > 0$ and $V''(\cdot) < 0$; and $\beta \leq 1$ is the time-discount factor.[7]

Savers choose the level of deposits and their holdings of the liquid asset to maximize (1). An individual saver takes the run threshold, ξ^*, and the probability of being repaid in a run, $\theta(\xi,1)$, as given. These objects depend on the aggregate bank portfolio, and we suppose that the individual saver is atomistic so as to not account for her impact on them. In contrast, a social planner would internalize the effect of the choices. Nevertheless, individual savers have rational expectations and correctly anticipate the equilibrium level of run-risk when making their decisions. Finally, short-selling of deposits and the liquid asset is not allowed, i.e., $D \geq 0$ and $LIQ_S \geq 0$.

The optimal choice of deposits by S yields the following deposit supply (DS) schedule:

$$
U'(e_S - D - LIQ_S) \geq \left[\beta\delta + \beta^2(1-\delta)\right](1+r_D)\int_{\underline{\xi}}^{\xi^*} \theta(\xi,1)\frac{d\xi}{\Delta_\xi}
$$

$$
+ \left[\beta\delta(1+r_D) + \beta^2(1-\delta)\omega(1+\bar{r}_D) + V'\left(D(1+r_D)\right)(1+r_D)\right](1-q), \quad (2)
$$

which holds with strict equality if savers choose to hold deposits in equilibrium, i.e., $D > 0$. Condition (2) says that savers equate the marginal utility of forgone consumption at $t=1$ to the expected marginal utility gain from holding deposits in the future. In a run, all savers withdraw; an individual saver will receive $1 + r_D$ per unit of deposits with probability $\theta(\xi,1)$ for each realization of ξ below the run threshold ξ^*. Otherwise, a run does not occur, which happens with probability $1 - q$, and only impatient savers withdraw in equilibrium. In this case, an individual saver is either impatient with probability δ and receives the period 2 deposit rate $1 + r_D$ or is patient with probability $1 - \delta$ and receives the period 3 deposit rate $1 + \bar{r}_D$ with probability ω. Absent a run, S also enjoys the marginal benefit of transaction services, $V'\left(D(1+r_D)\right)(1+r_D)$.

Savers can choose to self-insure and hold the liquid asset. The optimal liquid holdings, LIQ_S,

---

[7]The transaction services only accrue to the impatient depositors, so the last term must be multiplied by δ. However, given that we have left the function V unspecified, the δ can be subsumed inside V, and by doing that, we simplify the notation in many subsequent equations.

are given by:

$$U'(e_S - D - LIQ_S) \geq \beta\delta + \beta^2(1 - \delta),\tag{3}$$

which holds with strict equality if savers choose to self-insure in equilibrium, i.e., $LIQ_S > 0$. Unless stated otherwise, we consider cases that savers do not want to hold the liquid asset but want to hold deposits, i.e., (2) holds with equality, while (3) is slack. This is not a particularly strong assumption. They will choose to do this when (i) their endowments are not excessive, (ii) the bank offers high enough deposit rates, or (iii) transaction services are sufficiently valuable.

Substituting the deposit schedule (2) into (1) and using the definition of run probability q, we get the indirect utility function,

$$\mathbb{U}_S^* = U(e_s - D) + U'(e_s - D)D + (1 - q)\left[V(D(1 + r_D)) - V'(D(1 + r_D))D(1 + r_D)\right],\tag{4}$$

for the benchmark case that $LIQ_S = 0$.[8] Given our assumptions about $V(\cdot)$, it is easy to show that the third term in (4) is strictly positive.[9] Moreover, the first two terms in (4) must be higher than $\mathbb{U}_S^\alpha$ (the utility level from saving only using liquid assets), otherwise savers could self-insure using the liquid asset and attain that level of utility. Hence, savers are always better-off using the bank compared to autarky.

Even with our simplifying assumptions, this framework produces an equilibrium and a set of decision rules for savers that have very sensible, intuitive properties. In particular, savers use the bank because it offers a better way to save for the future and because it facilitates transactions. The former is captured by the gain/surplus in terms of period 1 consumption (the first two terms in (4)). The latter is captured by the gain/surplus from the transaction services (the last term in (4)).

2.2 Entrepreneurs

Entrepreneurs have the rights to operate real projects that are in perfectly elastic supply, require a unit of funding at $t = 1$, are infinitely divisible when liquidated, and mature at $t = 3$. For simplicity, E does not have an endowment but borrows I from the bank at interest rate r_I to invest in the risky technology. Moreover, E is risk-neutral and derives utility only from consumption at $t = 3$. Finally, E is protected by limited liability when projects mature and loans are due.

The risk neutrality of entrepreneurs together with the absence of initial endowment is our second simplifying assumption. Risk neutrality means that the entrepreneur cares only about the expected

---

[8]For completeness, if $LIQ_S > 0$ then the savers' indirect utility is $\mathbb{U}_S^\alpha + (1 - q)\left[V(D(1 + r_D)) - V'(D(1 + r_D))D(1 + r_D)\right]$, where $\mathbb{U}_S^\alpha$ is the utility in autarky, i.e., savers only use the liquid asset to smooth intertemporal consumption. The autarkic utility is $\mathbb{U}_S^\alpha = U(U'^{-1}(\beta\delta + \beta^2(1 - \delta))) + [\beta\delta + \beta^2(1 - \delta)] \cdot [e_S - U'^{-1}(\beta\delta + \beta^2(1 - \delta))]$. Alternatively, savers could attempt to directly lend to entrepreneurs assuming that they could also monitor them. We present this case in section B.8 of the online appendix.

[9]Given that $D > 0$, the term can be written as $(1 - q)D[V(D)/D - V'(D)]$. Because $V(0) = 0$, $V(D)/D$ is the slope of the straight line starting at zero and passing through D. Given that V is strictly concave, its image is always above the image of the straight line for any point $x \in (0, D)$. Because V' is strictly decreasing, V will necessarily cross the straight line connecting zero and D from above and, hence, the derivative of V at D is strictly smaller than the slope of the straight line.

profits from operating the technology. The absence of an endowment means that the decision to default will depend only on the productivity of the technology, not on entrepreneurs' leverage. Otherwise, the savers' run decision becomes more intertwined with how much the entrepreneur wants to borrow and the relative risk-aversion of the two parties will matter for how risks are shared.

The linearity of the production function for the technology is the last of our three simplifying technical assumptions. This assumption pushes the entrepreneur to borrow as much as possible and simplifies the derivation of the loan demand. Unfortunately, the linearity also means that the loan rate that the bank will choose will depend only on the technology shock. Were the production function to have any curvature, then that curvature would be an important determinant of the level of loan demand and the interest rate on loans. However, in that case, the relative curvature of the production function would non-trivially complicate the loan demand as well as the determination of banking allocations even with risk-neutral savers and entrepreneurs.

This complication arises because the realized level of total production depends on the decision of the bank to recall loans, which happens after the realization of the liquidation value of loans, ξ. The loan demand trades off the expected revenue from borrowing to produce against the cost of repaying the loan. And it is much easier to compute the expected profits to entrepreneurs over the realization $\xi \geq \xi^*$ when production is linear rather than when it is a nonlinear function of the number of projects that are not recalled.

In order to make loan demand less mechanical, while still making it straightforward to solve for expected loan demand, we introduce a cost of effort that the entrepreneur incurs upon investing. We suppose that this cost incurred before the payoff from the investment is known; we could also call this cost an adjustment cost. The cost now becomes another factor that matters for loan demand, and the interest rate charged will depend on the marginal adjustment cost that must be paid. We assume that this cost is convex and it pertains to total investment.

The expected utility of an individual entrepreneur can be written as:

$$
\mathbb{U}_E = \int_{\xi^*}^{\overline{\xi}} \{ \omega \cdot [\underbrace{A \cdot (1 - y(\xi, \delta)) \cdot I}_{\text{realized output}} - \underbrace{(1 - y(\xi, \delta)) \cdot I \cdot (1 + r^I)}_{\text{loan obligation}}] - \underbrace{c(I)}_{\text{cost}} \} \frac{d\xi}{\Delta_\xi}, \tag{5}
$$

where $y(\xi, \lambda)$ is the portion of loans that the bank recalls to serve early withdrawals, given by equation (11) derived in section 2.3. If a run does not occur, E repays the outstanding loans in the good state, $(1 - y(\xi, \delta))I$, as long as the per unit payoff, A, is higher than the promised gross loan rate $1 + r_I$. Naturally, E defaults in the bad state when the project pays zero. In a run, all projects funded by bank loans are liquidated, i.e., $y(\xi, 1) = 1$ for all $\xi < \xi^*$, and no production takes place. Finally, $c(I)$ is the effort/adjustment cost with $c(0) = 0$, $c'(\cdot) > 0$ and $c''(\cdot) > 0$.[10] E needs to incur this cost in order to produce before she learns the realization of the state of the world and after run

---

[10]As we show in Corollary 3 later on, this cost matters for policy analysis, because without it the entrepreneur borrows to the point of paying out all proceeds to the bank. In this case, the entrepreneur essentially drops out of the problem. We could also assume that the cost depends only on the portion of investment not recalled, which would complicate the analysis but would create the same motive for a planner to take account of the entrepreneurs as in the setup we analyze.

uncertainty has been resolved. Hence, E will choose not to exert effort and thereby avoid the cost in a run (when all her loans are recalled). Absent a run, given that the net payoff to E is increasing in ξ, she will choose to produce if the following incentive compatibility constraint holds:

$$\omega[A - (1 + r_I)](1 - y(\xi^*, \delta))I - c(I) \geq 0. \tag{6}$$

Entrepreneurs choose the level of investment and, thus, borrow, I, to maximize (5). An individual entrepreneur takes the run threshold, ξ^*, and the aggregate portion of loans recalled, $y(\xi, \delta)$, as given. These objects depend on the aggregate bank portfolio, and we suppose that the individual entrepreneurs are atomistic so that they do not account for their impact on them. A social planner would internalize the effect of the choices. Nevertheless, individual borrowers have rational expectations and correctly anticipate the equilibrium level of run-risk when making their decisions.

The optimal choice of I by E yields the following loan demand (LD) schedule:

$$\int_{\xi^*}^{\overline{\xi}} \{\omega[A - (1 + r^I)](1 - y(\xi, \delta)) - c'(I)\} \frac{d\xi}{\Delta_\xi} = 0. \tag{7}$$

Condition (7) says that E equates the expected profit margin on the remaining projects–given by the difference between the marginal product of investment and the gross loan rate–to the marginal effort cost over the realizations of ξ, where a run does not materialize.

Substituting the loan demand schedule (7) in (5) and using the definition of the run probability, q, we get the following indirect utility function

$$\mathbb{U}_E^* = (1 - q)\left[c'(I)I - c(I)\right], \tag{8}$$

which is the surplus accruing to the entrepreneur when a run does not occur. Given our assumptions about $c(\cdot)$, it is easy to show that (8) is always strictly positive, i.e., E is strictly better-off than in autarky, where $E's$ utility is zero.[11] If effort is costless, then $\mathbb{U}_E^* = 0$, and the loan rate is equated to the marginal product of the project, i.e., $1 + r_I = A$ from (7).

Despite our simplifying assumptions about entrepreneurs' preferences and shape of the production function, the entrepreneurs make choices that are very conventional. Their surplus from operating is increasing in the amount of investment and decreasing in the probability of a run. Hence, entrepreneurs' welfare in equilibrium depends critically on the risk of a run. Our assumptions allow us to capture these relationships in a very tractable way in (8). They also allow us to cleanly identify the distortions that any policies will aim to correct.

---

[11]To show that (8) is strictly positive, it suffices to show that $c'(I) - c(I)/I > 0$ given that $I > 0$; otherwise, E is in autarky (short-selling of projects is not allowed). Because $c(0) = 0$, $c(I)/I$ is the slope of the straight line starting at zero and passing through I. Given that c is strictly convex, its image is always below the image of the straight line for any point $x \in (0, I)$. Because c' is strictly increasing, c will necessarily cross the straight line connecting zero and I from below and, hence, the derivative of c at I is strictly higher than the slope of the straight line.

2.3   Banker and Bank

The banker makes all investment and funding decisions to maximize her own utility. At $t = 1$, she is endowed with e_B and decides how much equity, E, to put into the bank. In addition, she decides how many deposits to raise, D, and decides how much to invest in the liquid assets, *LIQ*, and illiquid loans, *I*, subject to the following balance sheet (*BS*) constraint:

$$I + LIQ = D + E. \tag{9}$$

The balance sheet and profits/dividends after t=2 depend on the realization of ξ and the number of people withdrawing, λ. If a bank-run occurs, i.e., $\xi < \xi^*$, then the bank is liquidated and the proceeds are distributed according to a sequential service constraint. Thus, the probability that any saver is served is equal to

$$\theta(\xi, \lambda) = \frac{LIQ + \xi \cdot I}{\lambda \cdot D \cdot (1 + r_D)}. \tag{10}$$

If there is not a run, i.e., $\xi \geq \xi^*$, the bank will recall and liquidate a portion $y(\xi, \lambda)$ of its loan portfolio to serve the early withdrawals. The amount recalled is given by

$$y(\xi, \lambda) = \frac{\lambda \cdot D \cdot (1 + r_D) - LIQ}{\xi \cdot I}. \tag{11}$$

Our assumptions regarding the distribution of ξ imply that lending is sufficiently attractive such that the bank will opt to hold insufficient liquid assets to service all early deposit withdrawals, even when only the impatient savers withdraw. So the bank is always planning to call some loans. In principle, the bank might also want to liquidate loans beyond the need to serve early withdrawals and carry the proceeds forward using the storage technology. But this would only occur if the realization of ξ is higher than the expected return from holding the loan to maturity, which we have excluded by assumption. As a result, $y(\xi, \lambda)$ will take interior values between zero and one, and it will be decreasing in ξ and increasing in λ.

The dividends the banker receives in the good state of the world at $t = 3$ are equal to the repayment on the remaining loans minus the payment on the remaining deposits. In the bad state, *B* defaults, so the dividends are zero. The banker needs to incur a monitoring cost, *X*, to learn the true state of the world. Alternatively, the banker can forgo the monitoring, in which case the entrepreneur will report that productivity was zero and default, even if the good state of the world materialized. Hence, there is a moral hazard problem, in the spirit of Holmström and Tirole (1997), such that the banker will choose to monitor only if the expected dividends are higher than the monitoring cost.

Given a level for the liquidation value ξ and the number of withdrawals λ, the banker will choose to monitor if the following incentive compatibility constraint is satisfied:

$$\omega \left[(1 - y(\xi, \lambda))I(1 + r_I) - (1 - \lambda)D(1 + \bar{r}_D) \right] - X \geq 0. \tag{12}$$

The first term in (12) is the expected payoff to the banker if she monitors. It is multiplied by ω

because the banker has to decide whether to monitor before she learns the true state of the world, and so she takes an expectation. The second term, X, is the monitoring cost.

Using (11) and the fact that (12) is decreasing in λ, we can derive a threshold $\hat{\lambda}(\xi)$, such that the banker chooses to monitor for $\lambda \leq \hat{\lambda}(\xi)$:

$$\hat{\lambda}(\xi) = \frac{(\xi \cdot I + LIQ)(1 + r_I) - \xi(D(1 + \bar{r}_D) + X/\omega)}{D[(1 + r_D)(1 + r_I) - \xi(1 + \bar{r}_D)]}. \tag{13}$$

Note that this threshold is consistent with all out-of-equilibrium beliefs of savers that govern their decision to withdraw or keep their deposits in the bank, which we describe in section 2.4. In equilibrium, only impatient depositors will withdraw when $\xi \geq \xi^*$. In turn, consistency requires that the bank always chooses to monitor when a run does not occur. Indeed, from the determination of the run threshold in (20), we obtain that $\delta < \hat{\lambda}(\xi^*)$, i.e., banker's monitoring incentives are consistent with the equilibrium behavior of savers.[12]

Overall, the banker's expected utility is given by:

$$\mathbb{U}_B = W(e_B - E) + \int_{\xi^*}^{\bar{\xi}} \{\omega \cdot [\underbrace{(1 - y(\xi, \delta)) \cdot I}_{\text{outstanding loans}} \cdot \underbrace{(1 + r_I)}_{\text{loan rate}} - \underbrace{(1 - \delta) \cdot D}_{\text{patient deposits}} \cdot \underbrace{(1 + \bar{r}_D)}_{\text{deposit rate}}] - \underbrace{X}_{\substack{\text{monit.} \\ \text{cost}}}\} \frac{d\xi}{\Delta \xi}, \tag{14}$$

where $W(\cdot)$ is $B's$ utility function for $t = 0$, with $W'(\cdot) > 0$ and $W''(\cdot)$, which may differ from $U(\cdot)$ of the savers. The banker has also quasi-linear preferences, but, unlike the saver, never needs to consume in the interim period.

2.4 Global Game and Bank-Run Threshold

Having described the choices facing all of the agents in the economy, we can now determine when individual patient savers will want to withdraw their deposits at $t = 2$. The decision depends not only on each saver's belief about the bank's financial health, but also on their beliefs about how other savers' will behave. The source of fundamental uncertainty in our model is the liquidation value ξ. Apart from really high or really low realization of ξ, multiple equilibria arise when savers have complete information about the realization of ξ. The outcomes for the extremely low and high realizations are known as the lower and upper dominance regions, and their existence is critical for obtaining a run threshold (see Lemma 1 for formal derivations).

The lower dominance region is defined by a threshold ξ^{ld} for fundamentals such that every individual patient saver will withdraw her deposits when $\xi \leq \xi^{ld}$, irrespective of what other patient savers do. This region occurs naturally if signal about liquidation value is sufficiently bad, because, if depositors believe the recalled loans have very little value, then they may conclude that even the impatient depositors cannot be fully paid. In that case, the bank is going to fail, so attempting to withdraw is the dominant strategy.

---

[12]In equilibrium, monitoring requires that $\omega[(1 - y(\xi, \delta))I(1 + r_I) - (1 - \delta)D(1 + \bar{r}_D)] - X \geq 0$ for all $\xi \geq \xi^*$. However, it suffices that the constraint only holds for ξ^*, because dividends are increasing in ξ. In turn, this is guaranteed by the fact that $\delta < \hat{\lambda}(\xi^*)$.

The upper dominance region is defined by a threshold ξ^{ud} for fundamentals such that every individual patient saver will not withdraw her deposits when $\xi \geq \xi^{ud}$, irrespective of what other patient savers do. Intuitively, this occurs when the liquidation value is so high that, even if everyone were to run, the bank would be able to pay them. In that case, running makes no sense. Allowing the liquidation value of loan to vary, enables us to obtain endogenously a well-defined upper dominance threshold. In Goldstein and Pauzner (2005), the upper dominance is exogenously assumed because the liquidation value is not allowed to vary. However, our modification complicates the proof of uniqueness for the run threshold, and a new argument is needed, which we present below.

Aside from the two extreme cases, for intermediate values of fundamentals, the bank cannot serve all early withdrawals, and a patient saver may decide to withdraw her deposits if she believes that other patient savers will withdraw as well. In order to resolve this coordination problem, we assume that at $t = 2$, each patient saver i receives a private signal $x_i = \xi + \varepsilon_i$, where ε_i are small error terms that are independent and uniformly distributed over $[-\varepsilon, \varepsilon]$. These signals not only provide information about ξ, but also about other savers' signals, so that inference about their actions is possible. The higher the signal is, the higher is the posterior belief about ξ, and the smaller is the likelihood that other savers receive bad enough signals so that they will opt to withdraw. Both effects reduce the incentive to withdraw. As a result, the incomplete information leads patient savers to coordinate their actions so that they withdraw if fundamentals are below a threshold. These incomplete information games are known in the literature as global games (see also Carlsson and van Damme, 1993).

We seek a symmetric equilibrium characterized by two thresholds (x^*, ξ^*) such that an individual patient saver will withdraw her deposits if her private signal realization x_i is lower than x^* and the bank will experience a run at $t = 2$ and be liquidated, if the realization of ξ is below ξ^*.

Under such a threshold strategy, the number of savers that withdraw at a given level of fundamentals ξ is

$$\lambda(\xi, x^*) = \begin{cases} 1 & \text{if } \xi < x^* - \varepsilon \\ \delta + (1-\delta)Prob\,(x_i \leq x^*) & \text{if } x^* - \varepsilon \leq \xi \leq x^* + \varepsilon \\ \delta & \text{if } \xi > x^* + \varepsilon \end{cases} \tag{15}$$

If the fundamental value ξ is lower than $x^* - \varepsilon$, then all savers receive signals $x_i < x^*$. Hence, all patient savers withdraw, and $\lambda(\xi, x^*) = 1$. The opposite is true for $\xi > x^* + \varepsilon$. In this case, all patient savers receive signals $x_i > x^*$ and keep their deposits in the bank and only the impatient ones withdraw, $\lambda(\xi, x^*) = \delta$. Finally, if fundamentals are close to x^*, i.e., $\xi \in [x^* - \varepsilon, x^* + \varepsilon]$, some patient savers will receive signals that are lower than x^* and, thus, will withdraw their deposits; and others will receive a signal higher than x^* and, thus, will keep their deposits in the bank. Because ε_i, the noise in the private signals, is independently and identically distributed, the law of large numbers holds. This means the number of savers withdrawing for a given level of ξ in the intermediate region is $\lambda(\xi, x^*) = \delta + (1-\delta)Prob\,(x_i \leq x^*) = \delta + (1-\delta)(x^* - \xi + \varepsilon)/2\varepsilon$.

The signal and fundamentals thresholds are derived in two steps. First, given the threshold

strategy x^*, we can derive the threshold for fundamentals, ξ^*, which determines whether the bank is fully liquidated at $t = 2$ or survives to $t = 3$. Because the number of savers withdrawing is decreasing in ξ from (15), the bank is fully liquidated only if $\xi < \xi^*$. That is, ξ^* as a function of x^* is the solution to $\theta\left(\xi^*, \lambda\left(\xi^*, x^*\right)\right) = 1$, which from (10) gives:

$$\xi^* = \frac{\varepsilon\left[(1+\delta)D(1+r_D) - 2 \cdot LIQ\right] + x^*(1-\delta)D(1+r_D)}{2\varepsilon I + (1-\delta)D(1+r_D)}. \tag{16}$$

In other words, for threshold strategy x^*, if ξ is lower than ξ^*, then the numbers of savers withdrawing are owed more than the bank can pay even by liquidating all of its assets. Alternatively, if ξ is higher than ξ^*, fewer savers withdraw, allowing the bank to liquidate fewer assets and survive to $t = 3$.

Next, given the fundamentals threshold ξ^*, an individual saver can compute the signal threshold x^*, below which it is optimal to withdraw conditional on its expectation over the number of savers withdrawing and the private signal she receives. The threshold x^* depends on the utility differential between keeping the deposits in the bank and withdrawing. We denote this differential by $\nu(\xi, \lambda)$ and report in (17) the value it takes for different levels of withdrawals λ when fundamentals are ξ:

$$\nu(\xi, \lambda) = \begin{cases} \omega D(1+\bar{r}_D) - D(1+r_D) & \text{if} \quad \hat{\lambda}(\xi) \geq \lambda \geq \delta \\ -D(1+r_D) & \text{if} \quad \theta(\xi, 1) \geq \lambda > \hat{\lambda}(\xi) \\ -(LIQ + \xi \cdot I)/\lambda & \text{if} \quad 1 \geq \lambda > \theta(\xi, 1) \end{cases} . \tag{17}$$

Consider the first two cases, where there are fewer withdrawals than the maximum that the bank can repay in full, i.e., $\lambda \leq \theta(\xi, 1)$. If $\lambda \leq \hat{\lambda}(\xi)$, the banker chooses to monitor, and S expects to get $\omega D(1+\bar{r}_D)$ if she waits or $D(1+r_D)$ if she withdraws her deposits. Hence, the utility differential is $\omega D(1+\bar{r}_D) - D(1+r_D)$. We call this region of $\lambda's$ a *partial run with monitoring*. Moreover, if $\lambda > \hat{\lambda}(\xi)$, then the banker will not monitor, and S gets nothing if she chooses to wait. Thus, the utility differential is $-D(1+r_D)$. We call this region of $\lambda's$ a *partial run without monitoring*. Finally, if $\lambda > \theta(\xi, 1)$, the bank cannot fully repay all savers that withdraw, and it is liquidated. S gets zero if she waits and is repaid with probability $\theta(\xi, \lambda)$, yielding, in expectation, a payoff of $\theta(\xi, \lambda)D(1+r_D) = (LIQ + \xi \cdot I)/\lambda$. Hence, the utility differential between waiting and withdrawing is equal to $-(LIQ + \xi \cdot I)/\lambda$. We call this region of $\lambda's$ a *full run*. The following Lemma examines in which of these three regions ν may lie for different levels of ξ.

Lemma 1. *Consider $X < \overline{X}$, where $\overline{X} \gg 0$ is an upper threshold for the monitoring cost. For $\xi \leq \hat{\xi}$ only the full run region is non-empty, i.e., $\hat{\lambda}(\xi) < \delta$ and $\theta(\xi, 1) \leq \delta$; $\hat{\xi}$ is the solution to $\theta(\hat{\xi}, 1) = \delta$. For $\xi \in (\hat{\xi}, \xi^{ld}]$, only the partial run without monitoring and full run regions are non-empty, i.e., $\hat{\lambda}(\xi) \leq \delta < \theta(\xi, 1) < 1$; ξ^{ld} is the solution to $\hat{\lambda}(\xi^{ld}) = \delta$. For $\xi \in (\xi^{ld}, \hat{\hat{\xi}})$, all three regions are non-empty, i.e., $\delta < \hat{\lambda}(\xi) < \theta(\xi, 1) < 1$; $\hat{\hat{\xi}}$ is the solution to $\theta(\hat{\hat{\xi}}, 1) = 1$. For $\xi \in [\hat{\hat{\xi}}, \xi^{ud})$, only the partial run with and without monitoring regions are non-empty, i.e., $\delta < \hat{\lambda}(\xi^{ud}) < 1 \leq \theta(\xi, 1)$; ξ^{ud}*

19

is the solution to $\hat{\lambda}(\xi^{ud}) = 1$. *For* $\xi \geq \xi^{ud}$, *only the partial run with monitoring region is non-empty, i.e.,* $\hat{\lambda}(\xi) \geq 1$ *and* $\theta(\xi, 1) > 1$.

Lemma 1 says that when fundamentals are very bad, an individual saver's beliefs are only consistent with aggregate withdrawal behavior in a full run. As fundamentals improve, her beliefs are consistent with other run regions as well, while, when fundamentals are very good, beliefs are only consistent with the partial run with monitoring occurring. In particular, a patient saver will withdraw for $\xi \leq \xi^{ld}$ and will wait for $\xi \geq \xi^{ud}$, irrespective of the actions of other savers.

To understand the decision to withdraw, consider an individual patient saver who receives signal x_i. The saver will use the signal to update her beliefs about the realization of ξ. Given that both ξ and ε_i are uniformly distributed, the posterior distribution of ξ given x_i is $\xi | x_i \sim U[x_i - \varepsilon, x_i + \varepsilon]$. This implies that the utility differential between waiting and withdrawing for a patient saver who receives signal x_i as a function of the cutoff value for running is

$$\Delta(x_i, x^*) = \frac{1}{2\varepsilon} \int_{x_i - \varepsilon}^{x_i + \varepsilon} v(\xi, \lambda(\xi, x^*)) d\xi. \tag{18}$$

In a threshold equilibrium, a patient saver prefers to withdraw, i.e., $\Delta(x_i, x^*) < 0$, for all $x_i < x^*$, and prefers to roll over, i.e., $\Delta(x_i, x^*) > 0$, for all $x_i > x^*$. $\Delta(x_i, x^*)$ is continuous in x_i, because a change in the signal only changes the limits of integration $[x_i - \varepsilon, x_i + \varepsilon]$ and the integrand is bounded. Hence, a patient saver that receives signal $x_i = x^*$ is indifferent between waiting and withdrawing if

$$\Delta(x^*, x^*) = \frac{1}{2\varepsilon} \int_{x^* - \varepsilon}^{x^* + \varepsilon} v(\xi, \lambda(\xi, x^*)) d\xi = 0. \tag{19}$$

Equations (16) and (19) jointly determine the fundamentals' threshold, ξ^*, and the threshold strategy, x^*. We need to show that these thresholds exist and that they are unique. In the body of the paper, we provide the intuition underlying the proof, and we provide the details in the online appendix.

Solving for $x^*(\xi^*)$ from (16) and substituting it in (19), we obtain a single equilibrium condition in terms of fundamentals' threshold ξ^*, i.e., $\bar{\Delta}(\xi^*, \xi^*) = \Delta(x^*(\xi^*), x^*(\xi^*)) = 0$. We need to show that such a ξ^* is unique and that it is indeed a threshold equilibrium. The typical approach in the global games literature considers cases characterized by global strategic complementarities and state monotonicity (see Morris and Shin, 2003, for details). The first property requires that $v(\xi, \lambda)$ is decreasing in λ, i.e., the relative payoff from withdrawing is higher when there are more withdrawals. The second property requires that $v(\xi, \lambda)$ is non-decreasing in ξ and strictly increasing for some ξ, i.e., the relative payoff from waiting is higher when fundamentals are stronger. When these properties obtain, the proof goes as follows: From the existence of lower and upper dominance regions, $\bar{\Delta}$ is negative for low values and positive for high values of ξ, and, thus, there exists a solution ξ^* due to monotonicity. Moreover, from the strict monotonicity property, $\bar{\Delta}$ is strictly increasing, and, thus, the candidate solution ξ^* is unique. This in turn implies a unique strategy threshold $x^*(\xi^*)$. This candidate solution is indeed a threshold equilibrium if (18) is higher than (19) for $x_i > x^*(\xi^*)$
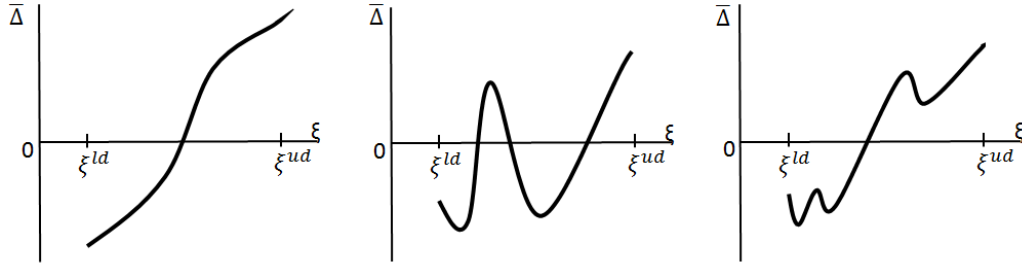
Figure 2: Perverse State Monotonicity

and lower for $x_i < x^*(\xi^*)$, i.e., a patient saver that receives a signal that is higher (lower) than the threshold chooses to wait (withdraw). With global strategic complementarities, these conditions are automatically satisfied because a higher signal means that fundamentals are higher and, thus, there are fewer withdrawals resulting in higher utility differential from waiting.

In our environment, neither of these typical properties hold. First, the model does not exhibit global strategic complementarities because v is increasing in λ in the full run region. As Goldstein and Pauzner (2005) argue, this is a typical property of bank-run models and is due to the fact that the marginal gain from running is lower as more people opt to run because more people are competing for the same liquid resources. They show that one-sided strategic complementarities are adequate to recover the threshold equilibrium under the stricter assumption that noise in private signals in uniformly distributed, which we also assume in our model.

On top of this complication, our model also exhibits what we call *perverse state monotonicity* because v is decreasing in the state ξ in the full run region. Intuitively, this property means that savers have higher incentives to run on a stronger bank than a weaker bank, conditional on a run occurring, because the chances of getting paid are higher. This property occurs because the signals provide information about liquidation value of loans. Presuming that the signals pertain to liquidation values allowed us to endogenously derive the upper dominance region, But, it restricts us from using the usual argument to establish uniqueness.

Because of the perverse state monotonicity, we need to use a new proof that covers three possible cases that are depicted in Figure 2. A first possibility is similar to what is studied in the literature where v is always (weakly) increasing in the state irrespective of whether the model exhibits global strategic complementarities (Rochet and Vives, 2004) or one-sided strategic complementarities (Goldstein and Pauzner, 2005). If this happens to be true, the threshold solving $\bar{\Delta}$ will be unique as depicted in the left panel in Figure 2.[13]

However, v may not be increasing in the state. Instead, it could be a situation, as in the middle panel, where there are multiple solutions, or a situation, as in the right panel, where there is a unique solution, despite the fact that $\bar{\Delta}$ is not increasing in ξ *everywhere* in the domain.

To establish uniqueness in the presence of perverse state monotonicity, we notice that, to rule out

---

[13]See also Matta and Perotti (2019) who model "orderly liquidation" as an alternative to sequential service, whereby (only) the illiquid assets are placed in a mandatory stay and are made available to all depositors, regardless of whether they ran. State monotonicity now obtains and the usual argument for uniqueness applies.

the problematic middle case, $\bar{\Delta}$ does not need to be increasing for all ξ (as is typically required) but instead only needs to be strictly increasing at candidate solutions that solve $\bar{\Delta}(\xi^*, \xi^*) = 0$. We know that such points exist because of continuity and the existence of the extreme regions established in Lemma 1. This weaker requirement is graphically depicted in Figure 3. In the left panel, where there are multiple solutions, $\bar{\Delta}$ will necessarily cross the x-axis both from below and from above because of continuity and the existence of the lower and upper dominance thresholds. This means that the derivative of $\bar{\Delta}$ at the candidate solutions, which are depicted by the dots on the x-axis, can be either positive or negative. On the contrary, in the unique solution in the right panel, the derivative at the candidate solution is *strictly* positive. Hence, the strategy to establish uniqueness comprises of showing that there are no solutions such that the derivative of $\bar{\Delta}$ at a solution is strictly negative, i.e., the set to $\{\bar{\Delta}(\xi^*, \xi^*) = 0 \cap \partial\bar{\Delta}/\partial\xi|\xi = \xi^* < 0\}$ is empty.
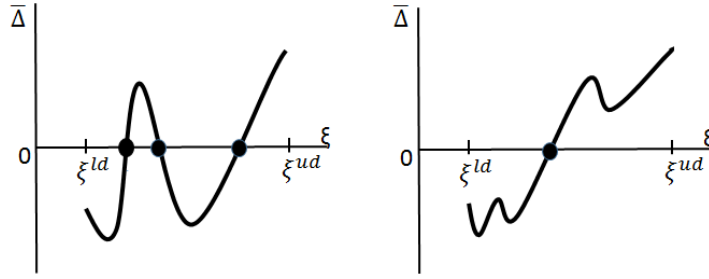
Figure 3: Graphical Representation of Uniqueness Argument

Proposition 1. *Given equilibrium allocations that satisfy the regions in Lemma 1, there exists a unique threshold, x^*, such that patient savers keep their deposits in the bank if $x_i > x^*$, and withdraw if $x_i < x^*$. Moreover, there exists a unique threshold ξ^*, such that the bank does not experience a run if $\xi \geq \xi^*$, and is fully liquidated if $\xi < \xi^*$.*

Proposition 1 establishes the uniqueness of the run threshold provided that there exist equilibrium allocations that satisfy the conditions in Lemma 1. We verify that these conditions hold in the equilibria we examine.[14]

Hereafter, we focus on the case that the noise becomes arbitrarily close to zero. Note that taking the limit $\varepsilon \to 0$ implies that $x^* \to \xi^*$ from (16). The posterior distribution of $\lambda(\xi, x^*)$ for a patient saver who receives signal x^* is uniform over $[\delta, 1]$.[15] As ξ decreases from $x_i + \varepsilon$ to $x_i - \varepsilon$, λ increases from δ to 1. Changing variables in (19) provides the indifference condition, $GG = \int_0^1 v(\xi^*, \lambda)d\lambda = 0$, that determines the unique ξ^*:

$$\int_\delta^{\lambda^*} [\omega D(1 + \bar{r}_D) - D(1 + r_D)]d\lambda - \int_{\lambda^*}^{\theta^*} D(1 + r_D)d\lambda - \int_{\theta^*}^1 \frac{LIQ + \xi^* I}{\lambda}d\lambda = 0, \qquad (20)$$

---

[14] Stronger assumptions may be needed to guarantee that there are not other non-threshold equilibria, which we do not consider in the paper. We are focusing on a threshold equilibrium and our proof establishes that it exists and is unique.

[15] This is true because $Prob(\lambda(\xi, x^*) \leq N|x_i = x^*) = 1 - Prob(\xi \leq x^* + \varepsilon - (N - \delta)/(1 - \delta)2\varepsilon|x_i = x^*) = 1 - (x^* + \varepsilon - (N - \delta)/(1 - \delta)2\varepsilon - x^* + \varepsilon)/(2\varepsilon) = (N - \delta)/(1 - \delta)$, hence $\lambda(\xi, x^*) \sim U[\delta, 1]$.

where $\lambda^* \equiv \hat{\lambda}(\xi^*)$ and $\theta^* \equiv \theta(\xi^*, 1)$.[16] We will use (20) to help demonstrate that the conditions we need to obtain do in fact hold. Define $GG(\xi)$ the value of GG for general ξ. Then, we obtain the following derivative

$$\frac{\partial GG}{\partial \xi} = \omega D(1 + \bar{r}_D)\frac{\partial \hat{\lambda}}{\partial \xi} - \int_{\theta}^{1}\frac{I}{\lambda}d\lambda, \tag{21}$$

which we cannot unambiguously sign for arbitrary values of ξ. The first term strengthens the incentives to wait because the monitoring threshold $\hat{\lambda}$ is increasing in ξ. The second term strengthens the incentives to withdraw because of the perverse state monotonicity. Thus, (21) cannot be used by itself to establish that the solution ξ^* in (20) is unique. Instead, we evaluate (21) only at candidate solutions for ξ^*. Combining (20) and (21) we get

$$\frac{\partial GG^*}{\partial \xi} = \frac{1}{\xi^*}\left[\int_{\theta^*}^{1}\frac{LIQ}{\lambda}d\lambda + \int_{\delta}^{\theta^*}D(1+r_D)d\lambda\right] + \omega D(1+\bar{r}_D)\left[\frac{\partial \lambda^*}{\partial \xi} - \frac{\lambda^* - \delta}{\xi^*}\right] > 0, \tag{22}$$

where $\partial GG^*/\partial \xi \equiv \partial GG/\partial \xi|_{\xi=\xi^*}$ and $\partial \lambda^*/\partial \xi \equiv \partial \hat{\lambda}/\partial \xi|_{\xi=\xi^*}$. The first two terms are necessarily positive. The last term can be written as $\omega D(1+\bar{r}_D)[(\lambda^* - \delta)\xi^* D(1+\bar{r}_D) + (\delta D(1+r_D) - LIQ)(1+r_I)]/[\xi^* D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D))]$ and is greater than zero from Lemma 1. Hence, $\partial GG^*/\partial \xi$ in (22) is strictly positive and the ξ^* solving (20) is unique.

The intuition why our model delivers a strictly positive $\partial GG^*/\partial \xi$ can be seen by rearranging (20) to be:

$$\underbrace{\int_{\delta}^{\lambda^*}\omega D(1+\bar{r}_D)d\lambda}_{\text{Payoff from waiting }\Delta_{wt}^*} = \underbrace{\int_{\delta}^{\theta^*}D(1+r_D)d\lambda + \int_{\theta^*}^{1}\frac{LIQ + \xi^* I}{\lambda}d\lambda}_{\text{Payoff from withdrawing }\Delta_{wd}^*}. \tag{23}$$

Expressed this way, the condition that determines ξ^* says that the payoff differential from waiting, Δ_{wt}^*, must equal the payoff from withdrawing, Δ_{wd}^*, i.e., $\Delta_{wt}^* = \Delta_{wd}^* \equiv \Delta^*$. We are trying to determine whether the left-hand side, Δ_{wt}^*, increases more than the right-hand side, Δ_{wd}^*, as ξ^* increases. This amounts to comparing to point elasticities of Δ_{wt}^* and Δ_{wd}^*.[17]

First, consider the elasticity of the left-hand side of (23), η_{wt}. There is an intuitive explanation for why this is greater than one, i.e. why the saver who receives the threshold signal, believes that the probability of monitoring would increase by more than one percent if she received a one percent higher signal about fundamentals. When ξ^* goes up, the bank can recall fewer loans to service the same level of withdrawals, expected profits go up, and, thus, the region for withdrawals that monitoring is profitable expands. The effect on the probability of monitoring $\lambda^* - \delta$ expands further because the increase in ξ^* has two effects. First, it reduces the need to recall loans to pay impatient savers. Second, the loans that are left in place could in principle serve even more "self-

---

[16]Equation (20) is sufficient to guarantee that a patient saver will not withdraw if a run does not occur, i.e., $\omega D(1 + \bar{r}_D) - D(1 + r_D) > 0$. Thus, only impatient savers withdraw in equilibrium.

[17]The two elasticities are defined as: $\eta_{wt}^* = \omega D(1+\bar{r}_D)\partial(\lambda^* - \delta)/\partial \xi \cdot \xi^*/\Delta^* = \eta_{\lambda^*-\delta}$, where $\eta_{\lambda^*-\delta} \equiv \partial(\lambda^* - \delta)/\partial \xi \cdot \xi^*/(\lambda^* - \delta)$ is the point elasticity of the monitoring probability $\lambda^* - \delta$; and $\eta_{wd}^* = \int_{\theta^*}^{1}(I/\lambda)d\lambda \cdot \xi^*/\Delta^*$, since Δ_{wd}^* is continuous in θ^*. Recall that λ^* is the monitoring threshold for $\xi = \xi^*$, which depends on the recalled loans via (13). The precise mathematical expression connecting the change in the payoff differential between waiting and withdrawing to the point elasticities is $\partial GG^*/\partial \xi = \Delta^*/\xi^* \cdot (\eta_{wt}^* - \eta_{wd}^*)$.

fulfilling" withdrawals ($\lambda > \delta$) by patient savers. So the incentives to monitor will expand more than proportionally to the increase in ξ^*.[18]

Next, turn to the point elasticity of the right-hand side of (23). Because Δ^*_{wd} is *linearly* increasing in the threshold ξ^*, the point elasticity η^*_{wd} is strictly lower than one given that $\theta^* > \delta$, $D > 0$, $r_D \geq 0$ and $LIQ \geq 0$. The fact that the liquidation value depends linearly on the source of strategic uncertainty has been sufficient to establish our uniqueness result. Suppose instead that the liquidation value ξ was a nonlinear function of another underlying state for which savers received noisy signals. Then, the derivative of the liquidation value with respect to that state would matter for the relative change in the point elasticities η^*_{wt} and η^*_{wd}, and more restrictive conditions may be required to establish uniqueness.

Given that the run threshold ξ^* is a critical factor in the welfare analysis, we prove the following corollary that provides some useful information about its characteristics.

Corollary 1. *The run threshold ξ^* is decreasing in the loan rate and investment, while it is increasing in deposits and the period 2 deposit rate. The effects of the liquid asset holdings and of the period 3 deposit rate on ξ^* are ambiguous.*

The overall effect of the banking variables on the run threshold combines the indirect effect of the incentives of the bank to monitor and the direct effect of the payoff differential in (17). Because of one-sided strategic complementarities, higher I or LIQ increases the payoff from withdrawing in the full run region, which pushes ξ^* up, but also strengthens the incentives to monitor, i.e., increases λ^* which pushes ξ^* down. For investment, the first effect is mitigated because of the discounted liquidation value in a full run, $\xi^* I$, and we are able to show that increasing I decreases ξ^* all else being equal. For liquidity, the first effect is relative stronger than for investment, and we cannot unambiguously show which effect dominates. However, Proposition 3 below establishes conditions under which more liquidity reduces ξ^*, and these conditions are easily satisfied in the examples we consider. More deposits reduce the expected profits and, thus, the incentives to monitor, but also increase the payoff from waiting. But the first effect dominates, and ξ^* goes up (all else being equal). Moreover, higher loan rates increase bank profits and, thus, the incentive to monitor, which pushes ξ^* down.

Finally, the effect of the period 3 deposit rate is ambiguous (see (B.16) for a detailed expression). On the one hand, it reduces the incentives to monitor because the higher deposit rate lowers bank profits. On the other hand, when it is higher, it also increases the payoff from waiting in the partial run region with monitoring. Our ability to analyze this case highlights the benefit from explicitly modeling the actual payoffs in the incomplete information game. In our examples, we find that increasing $\bar{r}_D$ pushes ξ^* down and, thus, reduces the probability of a run, all else being equal.

---

[18]To see this mathematically, note that $\partial(\lambda^* - \delta)/\partial\xi \cdot \xi^* = [\lambda^* D(1+r_D) - LIQ](1+r_I)/[D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D))] = [D(1+r_D) - LIQ](1+r_I)/[D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D))] + (\lambda^* - \delta)(1+r_D)(1+r_I)/[(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)]$. The first term represents the increase in the monitoring probability—for a one percent increase in the run threshold—that is due to the fewer loans recalled to pay impatient savers. The second term represents the increase in the monitoring probability that is due to the ability to serve more "self-fulfilling" withdrawals without hurting profits and monitoring incentives. It is easy to see that the percent increase in $\lambda^* - \delta$ from this channel is more that one percent.

The conclusion would be completely reversed if we had considered a reduced-form incomplete information game à la Rochet and Vives (2004) (or in models with *ad hoc* run probability functions). In Rochet and Vives (2004), depositors delegate the decision to withdraw to fund managers who have a very different payoff function than the one described in (17). In particular, they assume that managers receive an exogenous payoff $\mathcal{A} > 0$ if the run does not occur or the bank does not default and suffer a negative payoff $\mathcal{B} < 0$ otherwise. In terms of our model, this formulation would imply that the indifference condition (20), which determines ξ^*, becomes $\int_\delta^{\lambda^*} \mathcal{A} d\lambda + \int_{\lambda^*}^1 \mathcal{B} d\lambda = 0$. As a result, $\bar{r}_D$ affects ξ^* only via the monitoring threshold λ^*. Specifically, if a planner wants to reduce the run probability, then she would have to implement a lower $\bar{r}_D$ to increase bank profits and, thus, the out-of-equilibrium monitoring incentives. This would not be beneficial for depositors. In contrast, because the run threshold also depends on the actual payoffs in our model, the planner can reduce the run probability by setting a higher $\bar{r}_D$, without hurting the depositors. A higher deposit rate could encourage a higher supply of deposits and more intermediation, which could mitigate any negative effect of more capital and liquidity on credit extension.

2.5 Private Equilibrium

The banker will choose both sides of the balance sheet to maximize her own profits as shown in (14). The choices of loans, liquidity, deposits, and equity need to satisfy the global game constraint (20), which determines the run threshold, the deposit supply schedule (2), which determines the deposit rates, and the loan demand schedule (7), which determines the loan rate. The balance sheet identity also holds so only three of the four variables can be freely chosen. The banker will internalize the effect of her actions on the run threshold, on deposit supply, and on loan demand. As a result, conditions (20), (2), and (7) not only need to be satisfied in equilibrium, but will be explicit constraints in the banker's optimization problem—the solution to which defines the *private equilibrium* (PE). Essentially, this means that the banker also chooses directly the level of the run threshold as well as the deposit and loan rate, rather than letting them be determined implicitly in equilibrium.[19] Finally, the banker will never choose allocations that result in lower welfare than her outside option. The bank can either invest in the storage technology or lend to entrepreneurs without accepting savers' deposits. Thus, the banker's outside option is $\overline{\mathbb{U}}_B = \max\left[\mathbb{U}_B^\alpha, \mathbb{U}_B^n\right]$, where $\mathbb{U}_B^\alpha$ is the utility in autarky (where the bank holds only liquid assets) and $\mathbb{U}_B^n$ is the utility when the banker does not take deposits and lends to E using only her own capital.[20]

Definition 1. *The private equilibrium is defined as the set of banking assets, $\{I, LIQ\}$, banking*

---

[19]In many global games applications, the run threshold can be derived in closed form using a condition similar to (20). Given the complexity of our environment, we are not able to solve for the threshold in closed form to substitute into the banker's problem. Instead, we explicitly impose (20) as a constraint that the banker faces and have her optimize also over ξ^*, respecting this constraint and all the implicit relationships in the model.

[20]The utility in autarky is $\mathbb{U}_B^\alpha = W(e_B - \alpha) + \alpha$, where $a > 0$ is the storage investment satisfying $W'(e_B - \alpha) = 1$. Note that, if $W'(e_B) > 1$, then $\alpha = 0$. If the banker lends to entrepreneurs using only her own capital, her utility is $\mathbb{U}_B^n = W(e_B - n) + \omega(1 + r_n)n - X$, where r_n is the interest rate determined by the loan demand $1 + r_n = A - c'(n)/\omega$. Because B internalizes how the choice of n affects the loan rate that entrepreneurs are willing to accept, the optimal $n > 0$ is the solution to $-W'(e_B - n) + \omega A - c'(n) - c''(n)n = 0$.

liabilities, $\{D,E\}$, the run threshold, ξ^*, deposit rates, $\{r_D, \bar{r}_D\}$, and the loan rate, r_I, that maximize banker's utility, $\mathbb{U}_B$, defined in (14) subject to the balance sheet (BS) constraint (9), the global game (GG) constraint (20), the deposit supply (DS) schedule (2), and the loan demand (LD) schedule (7).

Each first-order condition in the private equilibrium takes the form

$$\frac{\partial \mathbb{U}_B}{\partial C} + \sum_{\mathcal{Y}} \psi_{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial C} = 0, \tag{24}$$

where $C \in \{I, LIQ, D, E, \xi^*, r_D, \bar{r}_D, r_I\}$, $\psi_{\mathcal{Y}}$ are the shadow values on constraints $\mathcal{Y} \in \{BS, GG, DS, LD\}$ and $\partial \mathcal{Y}/\partial C$ the partial derivatives capturing the effect of choice C on these constraints. Take for example the lending choice, i.e., $C = I$, then (24) says that the optimal level of lending is determined by having the banker trade off the marginal return accruing to her, $\partial \mathbb{U}_B/\partial I$, against the shadow cost of funds, ψ_{BS}, and the way it affects the run threshold determination, the deposit supply, and the loan demand. The optimality condition for the other variables can be similarly interpreted, and we report the detailed expressions for the partial derivatives in the online appendix.

The optimality condition with respect to E, which is $\psi_{BS} = W'(e_B - E)$, deserves special attention because it highlights another way in which our model differs from others in the literature. This condition says that injecting more equity requires the banker to give up consumption in the initial period in exchange for increasing the funds of the bank.[21] Thus, the shadow cost of funds, ψ_{BS}, is inversely related to the amount of equity the banker puts in the bank. In banking models without endogenous credit and run-risk, the higher funding costs of injecting more equity would feed in higher loan rates and lower investment. In our model, this is not necessarily true: Higher equity can improve the safety of deposits, which can be compatible with lower loan rates and more investment.

We restrict deposit rates to be positive. Hence, (24) will be strictly less than zero when r_D hits the non-negativity constraint. Absent this constraint, the banker may want to offer a period 2 deposit rate that is negative, since this would allow her to reduce the probability of a run. In the numerical examples we present, r_D hits the non-negativity constraint both in the private and planning equilibria, but we have also solved for cases where it can take negative values. The implications of our model for the distortions between the private and planning equilibria as well as the effects and desirability of regulation continue to hold under a negative deposit rate for early withdrawals as long as run-risk is present in equilibrium.

Of course, early deposit rates can be made sufficiently negative so that running on the bank is never profitable. Such run-preventing deposit contracts have been studied for example in Cooper and Ross (1998). In our model, however, runnable deposits are important to discipline the banker, and there are limits to how low the early deposit rate can be set both because of this disciplinary role and because savers can stop using the bank if the rates become too low. Moreover, negative

---

[21]Note that the condition does not include term for the effect of additional equity on constraints *GG*, *DS*, and *LD*. This is true because, E does not appear directly in (20), (2), or (7), but this does not mean that equity is irrelevant for their determination. On the contrary, equity issuance can affect the run probability, the deposit supply, and the loan demand through its joint determination with other equilibrium variables.

rates reduce the transaction value of deposits, and, depending on the functional form of the function *V*, negative rates can be very restrictive. For example, Peck and Shell (2010), who also model transaction services from deposits in a Diamond-Dybvig environment, assume that the transactions services disappears if rates are negative. This stark discontinuity could endogenously generate and justify the non-negative constraint for the early deposit rate, because banking is less profitable otherwise.

Moreover, we consider a function *V* that does not necessarily exclude negative early deposit rates *a priori*. We solve for the run-proof equilibria under negative early deposit rates in section B.10 in the online appendix and show that they are dominated by the equilibrium with positive run-risk that we study. In other words, nothing in our environment necessarily rules out run-free equilibria, but, at least in our examples, agents would not prefer these equilibria.

Note that we are not claiming, in general, that run-proof contracts cannot be optimal, but rather we restrict our attention to cases in which they are not and some run-risk is socially optimal. We think studying these cases is interesting because the literature has adequately studied how to eliminate run-risk, but it has been mostly silent about any potential desirability to reduce it but not eliminate it completely.[22]

While the banker is free to choose any three of the quantities on her balance sheet, to explain how the model works, it is helpful to summarize the choices in terms of three different combinations of the quantities. In particular, we will summarize the private equilibrium in terms of the asset allocation choice, the capital structure choice, and the scale of intermediation. The three intermediation margins can be easily interpreted (see section B.1 in the online appendix for detailed expressions).

The *asset allocation margin* is

$$AAM_{PE} = \left(\frac{d\mathbb{U}_B}{dLIQ} - \frac{d\mathbb{U}_B}{dI} \right) + \frac{d\mathbb{U}_B}{d\xi^*} \left(\frac{d\xi^*}{dLIQ} - \frac{d\xi^*}{dI} \right). \tag{25}$$

Hence, AAM_{PE} captures the decision to shift a unit of risky loans into liquid asset holdings, which consists of the effect on *B*'s utility via bank profitability (the first term) and the effect via the run probability (the second term).

Similarly, the *capital structure margin* is

$$CSM_{PE} = \left(\frac{d\mathbb{U}_B}{dE} - \frac{d\mathbb{U}_B}{dD} \right) - \frac{d\mathbb{U}_B}{d\xi^*} \frac{d\xi^*}{dD}, \tag{26}$$

and captures the decision to replace a unit of deposits with equity (and the resulting impact on the run probability).

---

[22]See also Keister (2015) for a model with flexible deposit contracts, i.e., the payment that a depositor receives is determined by the bank as a best response to realized withdrawals in the intermediate period. Runs in his framework are partial in the sense that the bank can alter payments to stop withdrawals by patient depositors and avoid liquidation once the run state is revealed.

Lastly, the *scale of intermediation margin* is

$$SIM_{PE} = \left(\frac{d\mathbb{U}_B}{dI} + \frac{d\mathbb{U}_B}{dD} \right) + \frac{d\mathbb{U}_B}{d\xi^*} \left(\frac{d\xi^*}{dI} + \frac{d\xi^*}{dD} \right) \tag{27}$$

and captures the decision to raise a unit of deposits in order to expand credit extension (along with the resulting effect on the run probability). This margin is a proxy for the spread between r_I and $\bar{r}_D$ (see equation (B.13) in the online appendix). Intuitively, increasing both I and D pushes r_I down and $\bar{r}_D$ up given that $\partial r_I / \partial I < 0$ and $\partial \bar{r}_D / \partial D > 0$. Instead, if higher I is funded with equity or higher D is used to buy liquid assets, the spread will not necessarily shrink. Thus, the spread can proxy for our notion of intermediation, which is the amount of loans funded by deposits.[23]

To understand how these margins determine the structure of the bank's balance sheet, start in reverse order. Given a level of D and LIQ, the SIM_{PE} determines the level of lending by fixing the intermediation spread; higher I requires a lower spread, all else being equal, and vice versa. Then, AAM_{PE} and CSM_{PE} simultaneously fix LIQ and D, or equivalently, the liquidity ratio $\ell = LIQ/(I+LIQ)$ and the leverage ratio $k = E/(E+D)$, given that $E = I + LIQ - D$ from the balance sheet identity.

Corollary 2. *The liquidity ratio, ℓ, the leverage ratio, k, and the intermediation spread, $r_I - \bar{r}_D$, are sufficient to characterize the banker's optimal choices.*[24]

Before we compare the private equilibrium and the social one, we briefly discuss the assumptions about the banker's behavior that give rise to the private equilibrium. First, the possibility of a run disciplines the banker, who internalizes how her lending and funding choices affect the probability of a run and hence the probability that she will make profits (see, also, Calomiris and Kahn, 1991; Diamond and Rajan, 2000, 2001). These considerations are captured by the terms multiplied by the shadow value on the global game constraint, ψ_{GG}, in (24). Section B.3 in the online appendix shows that the banker chooses allocations that result in higher run-risk when she does not internalize the effect of her actions on run dynamics. The scope for the planner to improve welfare then would be much higher.

Second, the banker internalizes how all of her choices affect the deposit rate demanded by

---

[23]Our model is not scale invariant, or in other words, the equilibrium conditions cannot all be normalized by the balance sheet size $I + LIQ$. There are four reasons that equilibrium allocations depend on size. First, the concave utilities for savers initial consumption and transaction services of deposits depend on the levels of these variables. Second, the convex effort cost by the entrepreneurs depends on the level of investment. Third, the banker's initial level of consumption determines utility in the initial period and lastly, the level of the monitoring cost is a constant. If we relax these assumptions, then the equilibrium will be scale invariant and could be characterized by only the asset allocation and capital structure intermediation margins. We elaborate on this special case later in conjunction with the social efficiency of private allocations.

[24]The statistics in Corollary 2 are not exclusive. For example, one could combine (24) with respect to LIQ and D, on the one hand, and with respect to E and I, on the other, to obtain alternative intermediation margins to AAM and CSM. These margins would be proxied by a reverse ratio, LIQ/D, and a risk-weighted capital ratio, $CR = E/I$, respectively. We focus on the intermediation margins described above as they provide an intuitive way to describe the banker's choices. However, in section 4 we examine the effect of many alternative regulatory tools beyond liquidity and leverage requirements, such as risk-weighted capital requirements, liquidity coverage ratio, and net stable ratio requirements. These regulations will impact the three intermediation margins in different ways; some will be complements, while others will be substitutes.

savers. We suppose this is possible because the debt is demandable and the depositors can fully observe the bank's balance sheet. So, at the moment of entering into a deposit contract, depositors observe the balance sheet and can rationally compute the underlying risks. Hence, if the banker alters the balance sheet, savers can request new deposit contract terms, so that their optimal supply of deposits reflects the new risks. The banker anticipates this behavior and, thus, internalizes the impact of all of her choices on the deposit supply. These considerations are captured by the terms multiplied by the shadow value on the deposit supply schedule, ψ_{DS}, in (24).

Alternatively, one could assume instead that deposit contracts are incomplete and that not all banking choices are perfectly observed. In that case, after collecting deposits, the bank could take more lending risk than the depositors would prefer. This would result in a *commitment problem* for the banker (see, also, Matutes and Vives, 2000; Martinez-Miera and Repullo, 2017). Under these assumptions, the banker realizes that taking more risk increases the cost of raising deposits and would ideally want to promise depositors that she will behave prudently. After the deposit contract has been signed, however, the banker would have an incentive to deviate towards lending more, holding fewer liquid assets, and raising less equity—and the depositors could not do anything about that. Consequently, the banker would only internalize the terms that are specified in the deposit contract, which would, at minimum, be the amount of deposits and the deposit rates. As a result, only the first-order conditions in (24), with respect to $\{D, r_D, \bar{r}_D\}$, would include the terms multiplied by ψ_{DS}. This lack of commitment generates additional distortions, which would be present even in the absence of run-risk. Thus, abstracting from them allows us to isolate the impact of regulation on distortions induced only by runs. See section B.4 in the online appendix for a characterization of equilibria under deposit contract incompleteness and lack of commitment by the banker.

Third, the banker internalizes how all of her choices affect the loan rate that entrepreneurs are willing to accept. These considerations are captured by the terms multiplied by the shadow value on the loan demand schedule, ψ_{LD}, in (24). Alternatively, we could have assumed that the banker is a price-taker with respect to the loan rate, which could be the case in anonymous competitive lending markets. This assumption would imply that $\psi_{LD} = 0$ in (24). Apart from being at odds with the monitoring function of the bank, the price-taking assumption would introduce additional reasons why private and social outcomes may diverge. The planner would account for how interest rates affect loan demand. In order to focus on distortions induced by runs and simplify the analysis, we relegate the characterization of equilibria under price-taking behavior in loan markets to section B.5 in the online appendix.

3   Efficient Allocations

Our bank offers socially useful services but faces the risk of a run. One of the main points of our analysis is that private banking choices generate *run externalities*, which adversely affect the welfare of savers and borrowers. Run-risk is harmful for all agents in the economy—the bank, savers, and borrowers—but the effects on their respective welfare differ. The bank would benefit from low

run-risk, but reducing the risk lowers profits. Savers would prefer lower run-risk and more deposit services, while borrowers would benefit from higher investment accompanied by lower run-risk.

The private choices that the agents make work as follows. Bankers fully internalize how their lending and capital decisions change the probability of a run, the deposit rates, and the loan rate. However, the banker exploits her limited liability and maximizes her own utility, disregarding the direct effect of her actions on the welfare of savers and entrepreneurs. In turn, savers and entrepreneurs are atomistic and do not internalize how their own decisions matter for aggregate bank allocations (and hence on the probability of a run). In order to examine how these externalities distort outcomes, we consider a social planner who internalizes the effects of lending and capital structure decisions on all agents but still is constrained by the market structure of the economy. We will show that the externalities operate via all three intermediation margins derived in section 2.5 and create three independent distortions, which the planner would like to address. Section 3.1 sets up the planner's problem and identifies the sources of differences between the private and social optimization margins. Section 3.2 presents a numerical solution to the model and describes how the private and social planner's allocations differ.

3.1 Social Planner

The social planner will choose all endogenous variables to maximize the following social welfare function:

$$\mathbb{U}_{SP} = \mathbb{U}_B + w_S \mathbb{U}_S + w_E \mathbb{U}_E, \tag{28}$$

where $w_S \geq 0$, $w_S \geq 0$ are the weights assigned to S and E. We have normalized the weight on B to one to facilitate the comparison of the planner's and the private optimality conditions (other normalizations are also possible—for example, assign weights to all agents adding up to one). Agents' utilities are given by (1), (5), and (14). The planner is constrained by the market structure of the economy, i.e., she cannot use lump-sum transfers to allocate resources across agents or complete any missing markets.[25] Moreover, the planner respects the run determination given by (20) as well as the deposit supply and loan demand schedules (2) and (7).[26] Given that the latter two hold with equality in the planner's problem, we can substitute the indirect utilities (4) and (8) in (28) to get

---

[25]Given the absence of lump-sum transfers, we cannot unambiguously construct a welfare criterion to maximize the total surplus. Thus, we assign weights for different agents in a social welfare function and study different constellations of these weights. Although we remain agnostic about the appropriate level of the weights, we discuss the potential political economy considerations of regulation.

[26]In principle, the planner may want to divert from the privately optimal deposit supply and loan demand schedules, which means that (2) and (7) do not need to hold with equality in the planner solution. For example, the planner could choose deposit or loan rates that do not necessarily satisfy all these conditions with equality and implement the resulting allocations by choosing instruments, such as Pigouvian taxes on interest income/expenses, that distorts (2) or (7). Farhi and Werning, 2016, and Bianchi and Mendoza, 2018, consider such taxes to implement the constrained efficient allocations. Note that the taxes can also take negative values, in which case they are interpreted as subsidies. If these conditions do not hold with equality, then the Lagrange multipliers associated with them are zero. Given that our focus is on regulations imposed on banks, we abstract in our baseline analysis from such tools and examine their implications in section B.9 in the online appendix.

the following social welfare function:

$$\mathbb{U}_{SP}^* = \mathbb{U}_B + w_S \mathbb{U}_S^* + w_E \mathbb{U}_E^*. \tag{29}$$

Using (29) instead of (28) adds a lot of tractability in the optimizing conditions and, as we show below, allows us to pinpoint the sources of distortions in the private equilibrium that a planner would like to address. In particular, $\mathbb{U}_S^*$ only depends on D and ξ^* with

$$\frac{\partial \mathbb{U}_S^*}{\partial D} = -U''(e_S - D) - (1-q)V''(D(1+r_D))D(1+r_D)^2 > 0 \tag{30}$$

and

$$\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} = -[V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D)]\frac{1}{\Delta_\xi} < 0. \tag{31}$$

Therefore, a planner that cares about S would like to increase the level of deposits and reduce the probability of a run.

Similarly, $\mathbb{U}_E^*$ only depends on I and ξ^* with

$$\frac{\partial \mathbb{U}_E^*}{\partial I} = (1-q)c''(I)I > 0 \tag{32}$$

and

$$\frac{\partial \mathbb{U}_E^*}{\partial \xi^*} = -[c'(I)I - c(I)]\frac{1}{\Delta_\xi} < 0. \tag{33}$$

Hence, a planner that cares about E would like to increase the level of investment and reduce the probability of a run.

Recall that there is no scope for the planner to improve the welfare of the banker because the banker already internalizes all the effects of her choices on her utility. As we have explained, there are various ways to relax the assumptions that are responsible for this, but then the planner would be fixing run-related distortions as well as distortions unrelated to run-risk.

Definition 2. *The social planner's equilibrium is defined as the set of banking assets, $\{I, LIQ\}$, banking liabilities, $\{D, E\}$, the run threshold, ξ^*, deposit rates, $\{r_D, \bar{r}_D\}$, and the loan rate, r_I, that maximize social welfare, $\mathbb{U}_{SP}^*$, defined in (29) subject to the balance sheet (BS) constraint (9), the global game (GG) constraint (20), the deposit supply (DS) schedule (2), and the loan demand (LD) schedule (7).*

Each first-order condition in the planner's equilibrium takes the form

$$\frac{\partial \mathbb{U}_B}{\partial C} + w_S \frac{\partial \mathbb{U}_S^*}{\partial C} + w_E \frac{\partial \mathbb{U}_E^*}{\partial C} + \sum_{\mathcal{Y}} \zeta_{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial C} = 0, \tag{34}$$

where $\zeta_{\mathcal{Y}}$ are the shadow values on the constraints $\mathcal{Y} \in \{BS, GG, DS, LD\}$ and are different than $\psi_{\mathcal{Y}}$ in the private equilibrium. The planner's optimality conditions differ from the private ones, because the planner explicitly accounts for how her choices affect S and E, and thus will assign different

shadow values, $\zeta_{\mathcal{Y}}$, on the same constraints $\mathcal{Y}$ faced by the banker. Following the same steps as for the private equilibrium, we can derive the same three intermediation margins for the planner's problem (see section B.2 in the online appendix for detailed expressions).

The asset allocation margin for the planner can be written as $AAM_{SP} = AAM_{PE} + AAM_{WD}$, where AAM_{PE} is given by (25) and AAM_{WD} is a wedge, which captures the additional distortions that the planner takes into account and will try to correct, given by:

$$AAM_{WD} = \underbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*} \right) \cdot \left(\frac{\partial \xi^*}{\partial LIQ} - \frac{\partial \xi^*}{\partial I} \right)}_{\text{Run externality from asset allocation}} - \underbrace{w_E(1-q)c''(I)I}_{\text{Surplus to } E \text{ from additional } I} . \qquad (35)$$

The first term in (35) captures the externality from run-risk due to the choice of the asset allocation between liquid assets and loans. If shifting a unit of loans to liquid assets reduces the run probability, i.e., $d\xi^*/dLIQ - d\xi^*/dI$ shown in (B.8) is negative, then because the planner cares about S and E, she would want a more liquid asset mix than in the private equilibrium. The last term in (35) captures the surplus created for the entrepreneur from an additional unit of investment, and this consideration leads the planner to increase the level of investment to favor E. Note that, in principle, a more liquid asset mix and more investment can be consistent if they are accompanied by a bigger balance sheet.

Similarly, the capital structure margin for the planner is $CSM_{SP} = CSM_{PE} + CSM_{WD}$, where CSM_{PE} is given by (26) and the wedge CSM_{WD} is given by:

$$CSM_{WD} = -\underbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*} \right) \frac{\partial \xi^*}{\partial D}}_{\text{Run externality from liabilities mix}} + \underbrace{w_S[U''(e_S - D)D + (1-q)V''(D(1+r_D))D(1+r_D)^2]}_{\text{Surplus to } S \text{ from additional } D}.$$

$$(36)$$

Shifting funding from deposits to equity mitigates the run externality and helps S and E but reduces the surplus to S, because the last term is negative (because utility from both consumption and transactions services is concave).

Finally, the scale of intermediation margin for the planner is $SIM_{SP} = SIM_{PE} + SIM_{WD}$, where SIM_{PE} is given by (27) and the wedge SIM_{WD} is given by:

$$SIM_{WD} = \overbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*} \right) \cdot \left(\frac{\partial \xi^*}{\partial I} + \frac{\partial \xi^*}{\partial D} \right)}^{\text{Run externality from intermediation scale}}$$
$$\underbrace{- w_S[U''(e_S - D)D + (1-q)V''(D(1+r_D))D(1+r_D)^2] + w_E(1-q)c''(I)I}_{\text{Surplus to } S \text{ and } E \text{ from higher intermediation scale}}. \qquad (37)$$

The planner will want less intermediation, if raising an additional unit of deposits to fund investment increases the probability of a run, i.e., $\partial \xi^*/\partial I + \partial \xi^*/\partial D > 0$. Yet, the run externality considerations are offset by the second term, because taking more deposits and making more loans increases the surplus to both S and E.

Having now described the various wedges, we have established three of our four main results.

First, we have seen that there are three distorted margins that differ between the private and socially-optimal allocations. Second, all three wedges feature a component driven by run externalities, which B, S, and E do not internalize, and a component that captures the surplus created for either S and/or E. The planner trades off reducing run-risk in order to tackle the run externalities and improve the surplus accruing to S and/or E when the run does not occur. In doing so, she chooses a different asset allocation, capital structure, and scale of intermediation, which have a direct impact on the surplus and an indirect impact on run-risk.

Our third main result is that correcting these distortions involves tradeoffs because correcting them invariably skews allocations toward either favoring borrowers over savers or vice versa. For instance, if the bank holds more safe assets and makes fewer loans, that switch marginally helps the savers because it makes their deposits safer. Conversely, the opposite choice of more loans and fewer safe assets creates more opportunities for the borrowers but reduces the buffer that helps mitigate the riskiness of deposits. In section 3.2, we present a numerical example of the private and planning equilibria and discuss how the allocations differ in reference to the aforementioned intermediation margins and associated distortions.

In section 4, we show how to implement (decentralize) the planner's solution with regulation. This will take us to our fourth main result that the three distortions in the intermediation margins are independent, and three independent tools are generally needed to replicate the planner's allocations.

Before proceeding, for completeness, the following Corollary shows the conditions under which the private equilibrium is constrained efficient, or in other words all wedges are zero.

Corollary 3. *The private and social planner's equilibria coincide if all of the following conditions hold: (i) $c''(\cdot) = 0$; (ii) $V''(\cdot) = 0$; and (iii) $U''(\cdot) = 0$ or $e_S > \overline{e}_S$ such that $LIQ_S > 0$.*

The Corollary 3 essentially says that, if the bank is creating no welfare gains for S and E, then there is nothing a planner, who respects the market structure, can do to improve outcomes. The reason is that, in this case, savers' and entrepreneurs' welfare are each constant at the autarkic levels and the banker already internalizes everything that matters to her.

Additional inefficiencies could be introduced to the model to justify a role for policy. For example, the liquidation value ξ could be a function of the amount of loan recalled, y, and determined in a fire-sale, which the banker would not internalize. Alternatively, one could assume that the run induces a deadweight loss, which S and E do not internalize and B neglects because she is protected by limited liability.

Even with either of these alternatives, the distortions would manifest themselves through the same three intermediation margins we have described. As long as the asset allocation, capital structure, and scale of intermediation load on the fire-sale price or deadweight loss, then the market failure will operate through all three margins to distort outcomes. Conversely, the number of distortions is determined by the number of intermediation margins that are misaligned and not by the number of market failures in the model. If we added externalities from fire-sales and deadweight losses to the other frictions in our baseline model, we would still have distortions in the asset allo-

cation, capital structure, and scale of intermediation margins, but the wedges would merely include additional terms.

3.2 Numerical Example

In this section, we present our benchmark numerical example and compare the private equilibrium with the social planner's solution under different weights on *E* and *S*. The discussion is organized around the distortions in the three intermediation margins, which we have just described.

3.2.1 Parametrization

The full set of parameters we used to solve the model is shown in Table 1. The parametrization should be taken more as an illustrative example to highlight the mechanisms in the model, rather than as a realistic calibration of the economy that would be suitable for making quantitative statements about the absolute optimal level of banking regulations. We have experimented with various other parameter choices, and the findings that we emphasize are quite robust.

Our model would require some obvious modifications to use it for quantitative policy analysis. For example, all liabilities in our model are unsecured, while, in practice, certain types of deposits are insured. Deposit insurance, even if partial, would reduce the reliance on market discipline exerted by depositors in determining credit risk. Hence, deposit rates would be much lower, as we see in advanced economies. Moreover, it is not clear whether the various capital regulations in practice (Basel requirements, stress tests, restrictions on dividend payouts) are indeed binding and whether one should be calibrating to match a regulated economy rather than an unregulated private equilibrium. Another potential concern is that our model consists of three periods, so one would need to decide how to interpret the length of each period.

Finally, the quasi-linearity of preferences, which simplifies the computation of the run threshold and derivation of policy significantly as well as the finite horizon of the model, make depositors willing to accept a higher probability of a run than if they were risk averse or if there was a continuation value for the bank. One could add convex bankruptcy costs to mimic a higher degree of risk-aversion as well as model the continuation value, but we have not done so because it is not important to make our fundamental analytic points.

With these caveats in mind, let us call attention to some of the considerations that we took into account while choosing the model parameters. Our objective in picking them was to obtain a private equilibrium that is rich enough to describe the mechanisms we want to highlight, rather than to match observable variables. First, the bank is profitable enough, and the initial equity of the banker and her preference for current consumption are such that she voluntarily uses some of her endowment to buy more equity in the bank. So the banker finds intermediation to be profitable. Second, the deposit services provided by the bank lead savers to forgo self-insuring by directly holding the liquid asset. If savers were opting to self-insure, then the banking sector is under-performing as a provider of liquidity and, hence, intermediation, and regulations that makes the

bank more stable would have an additional positive effect. Third, we have chosen the parameters so that the bank makes loans and invests in liquid assets, but also plans to liquidate some loans to serve early withdrawals. The key parameters that are responsible for this outcome are the size of the liquidity preference shock, the distribution of the liquidation values for recalled loans, and the riskiness of investment opportunity. Fourth, we have chosen logarithmic utility for period 1 consumption for both the savers and the banker, but we have assumed that the banker values future consumption more than the savers do. In particular, $U(x) = log(x)$ and $W(x) = \gamma \cdot log(x)$, where $\gamma < 1$.[27] Finally, we set $V(x) = c_D \cdot log(1 + x)$ and $c(x) = c_I \cdot x^{\phi_I}$, with $\phi_I > 1$, which satisfy the general properties required for the transaction services' and effort cost functions.

3.2.2 Intermediation Margins in Private and Social Equilibria

Tables 2 shows the private equilibrium and planning outcomes for small perturbation of the welfare weights away from the private equilibrium.[28] We focus the analysis around the three intermediation margins derived in section 3.1.

Before turning to the details, it is helpful to recall three things we already know about the nature of the distortions that the planner is trying to correct. First, the banker is already internalizing everything that matters for her own welfare. The problem is that she is ignoring the consequences of her choices on the saver and the borrower. Therefore, anything the planner does to take this into account will make the banker worse off. So the planner will be constrained on this front by the need to make sure that the banker will still find it profitable enough to monitor loans. The saver generally wants safer deposits. This can be accomplished by reducing the riskiness of the asset mix or by raising more equity from the banker. The banker will only contribute more equity if the expected dividend yield is high enough. Finally, the borrower would like to get more loans but has a downward sloping demand curve, so more lending will only occur at lower interest rates.

Initially, consider the case where the planner favors E and S equally—for example, $w_E = w_S = 0.1$ in Table 2. The planner would like to increase liquid holdings in the asset mix to address the run externality because $\partial \xi^* / \partial LIQ - \partial \xi^* / \partial I < 0$ in (35). Similarly, the planner would like to increase the amount of equity in the liabilities mix because $\partial \xi^* / \partial D > 0$ in (36). However, to benefit E and S, the planner needs to pay attention to the level of loans and deposits to make sure that they do not drop. The way to achieve these various goals is to grow the overall size of the bank's balance sheet while making assets more liquidity intensive and liabilities more equity intensive. The reduction in run-risk mitigates the upward pressure on the deposit rate, and the volume of lending has to be

high enough so that, even though the loan rate will be lower, expected dividends grow by enough to induce the banker to supply more equity.

One way to think about what is happening in this experiment is to recognize that, in the private equilibrium, the banker is restricting lending to prop up loan rates and limiting deposits to suppress the cost of deposits. The planning allocations correct these problems. In doing so, the banker is made slightly worse off, but the other two agents are much better off. Overall, social welfare rises.

Next, consider the case that the planner wants to favor S, but cares little about E; for example $w_E = 0$ and $w_S = 0.2$. Similar to the first case, the planner would like to improve the liquidity of the asset mix to address the run externality. But, the planner is now less concerned about the surplus accruing to E and can more easily shift some of the investment towards the liquid asset. At the same time, the planner would like to increase equity in the liabilities mix in order to address the run externality but without cutting deposits, which is what matters for the surplus accruing to S. The planner can increase both liquidity and deposits and, at the same time, guarantee that B will inject more equity in the bank by cutting the level of lending. This set of changes is enough to make equity funding attractive for the banker because the loan-deposit spread rises boosting bank profitability. The overall size of the balance sheet goes up, but the scale of intermediation, measured by the amount of lending that is deposit funded, falls.

Finally, consider the other case where the planner wants to favor E but cares little about S—for example, $w_E = 0.2$ and $w_S = 0$. The planner would like to increase lending and, thus, the surplus accruing to E. As a result, the planner shifts liquidity to loans, which makes deposits more expensive. The planner could then substitute away from deposits to equity to fund the higher investment, but this would only be possible if such a shift is accomplished by maintaining enough profitability so that equity injection by the banker remains attractive. Given the optimality of private allocations from B's perspective, such a shift from deposits to equity—with a shift from liquidity to loans at the same time—would not be profitable for the banker. However, because the planner is not concerned with helping S, it is possible to reduce deposit-taking, which lowers the deposit rate and also the necessary amount of liquidity that is needed to be carried for early withdrawals. So the planner shifts allocations so that more deposits are being used to support lending; the scale of intermediation increases, despite the fact that the bank operates with a smaller balance sheet.

3.2.3   Lending, Run-Risk and Welfare in Private and Social Equilibria

The private equilibrium may exhibit over- or under-investment (or lending) compared with the planner's outcomes depending on the weights on E and S. Lending falls and liquid asset holdings rise as more weight is placed on S. Both factors contribute to a higher liquidity ratio $\ell \equiv LIQ/(LIQ + I)$. Lower lending is accompanied by a higher loan rate, and this induces the banker to provide more equity. The leverage ratio $k \equiv E/(E + D)$ improves as a result. Even though this also helps deposits increase somewhat, the amount of deposits that support lending is falling, so that the spread between the loan rate and period 3 deposit rate rises. The opposite is true when the planner places

more weight on E. In this case, investment jumps, but deposits fall.

Naturally, the run probability is lower when more weight is placed on S. But, lower run-risk accompanied by higher ℓ and k can still be consistent with higher investment. This can happen when the bank has a bigger balance sheet and uses more deposit financing to support lending. Notice that this is what happens when the planner cares equally about entrepreneurs and savers.

The enhanced stability of both the asset portfolio and the capital structure of the bank is beneficial to S especially because, as discussed above, it can be accompanied by a higher level of deposit services. Lower run-risk is also beneficial to E but may come at the cost of lower investment and, hence, lower surplus from production. Indeed, when investment falls below its level in the private equilibrium, the entrepreneur will be worse-off. As more weight is placed on E, the level of investment increases, pushing up the run probability and reducing the surplus from deposit services. S's welfare goes down, and, after a point, she is worse-off compared with the private equilibrium. In this example, both S and E can be made better-off when the planner cares about them equally. B is always worse-off, because she already internalized what mattered for her and any deviation from the private equilibrium reduces her welfare.[29] Note that the planner not only increases social welfare $\mathbb{U}_{sp} = \mathbb{U}_B + w_S \mathbb{U}_S^* + w_E \mathbb{U}_E^*$, which depends on the weights, but also the overall surplus in the economy, $\mathbb{S}_{sp} = \mathbb{U}_B + \mathbb{U}_S^* + \mathbb{U}_E^*$. Thus, the planner could improve the welfare of all agents if she were able to make lump-sum transfers across agents.

We should note that neither the banker nor the planner opt to hold excess liquidity. Holding excess liquidity could be desirable in order to eliminate the probability of a run altogether. Section B.10 in the online appendix investigates such run-proof equilibria. A large literature has focused on run-proof equilibria and the implications for credit intermediation (see Cooper and Ross, 1998; Ennis and Keister, 2006; and Diamond and Kashyap, 2016). We show that such contracts are not optimal in the examples we investigate, because of the wide distribution for the liquidation value of loans and the high profitability of lending.[30]

---

[29]This is not the case under incomplete deposit contracts discussed in section B.4. Then, the planner can also improve B's welfare by forcing her to internalize how her actions matter for the supply of deposits.

[30]Government guarantees, such as deposit insurance or implicit bailout subsidies, may also be useful policy interventions to reduce run-risk. We have abstracted from introducing government guarantees in the model for two reasons. First, it would not unambiguously improve outcomes, as in the original Diamond-Dybvig setup, because it will remove the disciplining role of deposits and adversely affect the decision of the banker to monitor on top of the typical incentives to take more risk (see, for example, Kareken and Wallace, 1978; Cooper and Ross, 2002; and Admati et al., 2012). Second, designing deposit insurance within a global games framework is far from straightforward and beyond the scope of the current paper (Allen et al., 2015, study government guarantees within a global games framework and a simpler banking sector that the one in our paper). Finally, we do not study emergency liquidity assistance from a Lender of Last Resort (Rochet and Vives, 2004) or suspension of convertibility (Ennis and Keister, 2009), which would also require non-trivial modifications in the model we present. See also Keister (2015) for an analysis of efficient bailouts, which are possible with proper prudential regulation. We believe that these are important avenues for future research in models that feature an elaborate banking sector subject to both credit and run-risk like ours.

4 Regulation

We now explore how the planner's solution can be decentralized via various regulatory interventions. We group tools into two categories. The first category includes tools that target the capital structure in order to get more equity in the bank. The second category includes tools that aim to make the asset mix more liquid. Sections 4.1 and 4.2 discuss the effects when the tools are used in isolation. Section 4.3 discusses how the regulations can be optimally combined to implement the planner's solution as a private equilibrium.

4.1 Tools Targeting Capital

We examine two tools that can be used to increase the amount of equity in the bank. The first one is a requirement $\bar{k}$ on the leverage ratio, where here it is helpful to use the balance sheet identity to write it as $k = E/(I + LIQ) \geq \bar{k}$. The second one is a requirement $\overline{CR}$ for the risk-weighted capital ratio, i.e, $CR = E/I \geq \overline{CR}$, where we have assumed a risk-weight of one for loans and zero for liquid assets.

The direct effect of both tools would be to increase the level of equity in the bank's liabilities, which should intuitively push the probability of a run down and help with the run externalities. Both k and CR will affect the probability of a run directly by changing allocations in (20), and indirectly by influencing loan and deposit rates.

It is helpful to keep in mind that the banker will only invest in additional equity if the bank profits rise. Profitability depends on the spread between the loan and period 3 deposit rate, so rising profitability requires either higher loan rates or lower deposit rates. Hence, either lending will have to fall so that the loan rate can rise, or deposits have to be made safer so that the deposit rate can fall.

The following proposition establishes that the direct effect of higher k or CR, i.e., keeping interest rates and other allocations constant, is to reduce run-risk.[31]

Proposition 2. *Set $X = 0$. The partial equilibrium effect of higher requirement $\bar{k}$ or $\overline{CR}$ on q is negative.*

The above proposition establishes that, if one fixes the liquidity ratio, ℓ, as well as interest rates, then tightening $\bar{k}$ or $\overline{CR}$ reduces run-risk in exactly the same way because $k = CR(1 - \ell)$. Hence, in a partial equilibrium setting, one would conclude that these two regulations are equivalent.[32]

However, the general equilibrium effects of the two regulations on the incentive of bankers to hold liquidity and on the deposit supply and loan demand will differ. These differences stem from the way that two regulations impact the three intermediation margins. The intermediation margins under leverage regulation become $AAM_k = AAM_{PE}$, $CSM_k = CSM_{PE} + \psi_k$, and $SIM_k =$

---

[31]We set the monitoring cost equal to zero purely because it simplifies the proof, but nothing qualitatively changes if instead we fix the cost to be a small positive value.

[32]In the online appendix, we further characterize how the two regulations change the boundaries between the three run regions and affect the payoffs for depositors in each of them.

$SIM_{PE} - \psi_k \bar{k}$, where ψ_k is the Lagrange multiplier on the leverage requirement $k \geq \bar{k}$. The intermediation margins under risk-weighted capital regulation become $AAM_k = AAM_{PE} + \psi_{CR}\overline{CR}$, $CSM_k = CSM_{PE} + \psi_{CR}$, and $SIM_k = SIM_{PE} - \psi_{CR}\overline{CR}$, where ψ_{CR} is the Lagrange multiplier on the risk-weighted capital requirement $CR \geq \overline{CR}$.

The leverage regulation allows the bank to costlessly switch from liquid assets to loans, so that substitution is only possible if lending rates fall. Hence, to boost profitability, deposit rates must also fall. In contrast, the risk-weighted capital requirement lowers lending and hence will be associated with a higher loan rate. The higher loan rate means that the bank can take on more deposits (than if lending had risen).

Table 3 reports the results for individually tightening $\bar{k}$ and $\overline{CR}$. The change in the two requirements from their private equilibrium level satisfies the aforementioned relationship, i.e., $\Delta k = \Delta CR(1 - \ell)$, where ℓ is the liquidity ratio in PE. We will consider two cases: both requirements increases by a little and both requirements increase by a lot.

The direct beneficial effect on run-risk continues to dominate when general equilibrium effects are accounted for and q decreases under both regulations. However, the magnitude of the decrease differs and so do the effects on other components of the bank's balance sheet.

First, consider the case where $\bar{k}$ and $\overline{CR}$ are marginally tightened. To compare the effects, notice how the AAM is differentially impacted. Loans decrease much more under CR than k, which improves bank's asset liquidity. Thus, run-risk goes down even further than what we would expect by just increasing capital. The lower run-risk makes deposits cheaper, and the bank increases its deposit-taking. S gains, while both E and B lose.

The results are directionally similar for k, but the effects are much less pronounced because the bank can shift some liquidity to lending and still satisfy the regulation. Because of the additional lending, the bank does not boost liquidity by as much, so the scope to increase deposits is reduced. The gains for S are smaller and the losses for E are too.

Considering a much larger increase in capital, we obtain the same but stronger effects for CR. However, for k, the substitution towards lending becomes much more pronounced. The bank actually increases lending relative to the private equilibrium and substantially reduces its holding of liquid assets.[33] Given those changes, the cost of deposits must fall in order to make the bank profitable enough to support the higher level of capital. This drop occurs because the total deposits fall, and S are made worse-off from this change. The higher levels of lending make E better off.

4.2 Tools Targeting Liquidity

We examine three tools that can be used to increase the amount of liquidity in the bank. The first one is a requirement $\bar{\ell}$ on the fraction of assets that are liquid, $\ell = LIQ/(I + LIQ) \geq \bar{\ell}$. We will refer to this regulation as the liquidity ratio. The second one is a requirement $\overline{LCR}$ on the liquidity

---

[33]This is in contrast to models where the bank cannot raise additional equity, where stricter capital/leverage requirements (mechanically) result in a drop in credit extension (see, for example, Corbae and D'Erasmo, 2014; Clerc et al., 2015; and the references therein).

coverage ratio, which takes the (lowest) liquidation value of the bank's portfolio in a run relative to runnable liabilities, i.e., $LCR = (LIQ + \underline{\xi} \cdot I)/(D(1 + r_D)) \geq \overline{LCR}$. The third one is a requirement $\overline{NSFR}$ on the net stable funding ratio, which is computed as the fraction of illiquid assets funded by relatively stable sources, i.e., $NSFR = (E + (1 - \delta)D)/I \geq \overline{NSFR}$.

Proposition 3. *Set X=0. The partial equilibrium effect of higher requirements $\bar{\ell}$, $\overline{LCR}$, or $\overline{NSFR}$ on q is negative if $\delta > e^{-1}$ or $\ell > \hat{\ell}$.*

This proposition establishes that, if one fixes the leverage ratio, k, as well interest rates, then tightening $\bar{\ell}$, $\overline{LCR}$, or $\overline{NSFR}$ reduces run-risk in exactly the same way, because $LCR = ((1 - \underline{\xi})\ell + \underline{\xi})/((1 - k)(1 + r_D))$ and $NSFR = (k + (1 - \delta)(1 - k))/(1 - \ell)$. Hence, in a partial equilibrium setting, one would conclude that these three regulations are equivalent.

These regulations have the unintended consequence that extra liquidity raises patient savers' incentives to join the full run. This has been noted by others in models with one-sided strategic complementarities (see also Carletti et al., 2019). We show that these perverse incentives do not dominate when the fraction of patient depositors is small enough or when the liquidity ratio in the private equilibrium is above a threshold. In the private equilibrium we examine, $\hat{\ell} < 0$ because the run-risk is big enough to limit the strength of this channel, so the partial equilibrium effect of higher ℓ on q is negative.

But, as was the case for capital regulations, the general equilibrium effects of liquidity regulations may differ as the three regulations alter the intermediation margins in different ways. The intermediation margins under regulation on ℓ become $AAM_\ell = AAM_{PE} + \psi_\ell$, $CSM_\ell = CSM_{PE}$, and $SIM_\ell = SIM_{PE} - \psi_\ell \bar{\ell}$, where ψ_ℓ is the Lagrange multiplier on the requirement $\ell \geq \bar{\ell}$. The intermediation margins under LCR regulation become $AAM_\ell = AAM_{PE} + \psi_{LCR}(1 - \underline{\xi})$, $CSM_{LCR} = CSM_{PE} + \psi_{LCR}\overline{LCR}(1 + r_D)$, and $SIM_{LCR} = SIM_{PE} - \psi_{LCR}(\overline{LCR}(1 + r_D) - \underline{\xi})$, where ψ_{LCR} is the Lagrange multiplier on the requirement $LCR \geq \overline{LCR}$. Finally, the intermediation margins under $NSFR$ regulation become $AAM_{NSFR} = AAM_{PE} - \psi_{NSFR}\overline{NSFR}$, $CSM_{NSFR} = CSM_{PE} + \psi_{NSFR}\delta$, and $SIM_{NSFR} = SIM_{PE} - \psi_{NSFR}\overline{NSFR}$, where ψ_{NSFR} is the Lagrange multiplier on the requirement $NSFR \geq \overline{NSFR}$.

Table 4 reports the results for individually tightening $\bar{\ell}$, $\overline{LCR}$, or $\overline{NSFR}$. The change in the requirements from their private equilibrium level satisfy the aforementioned relationships, i.e., $\Delta LCR = \Delta \ell (1 - \underline{\xi})/((1 - k)(1 + r_D))$ and $\Delta NSFR = \Delta \ell \cdot NSFR/(1 - \ell)$, where k, $NSFR$ and ℓ are the leverage, net stable funding, and liquidity ratios in PE.

First, focus on requirements $\bar{\ell}$ and $\overline{LCR}$, which are very similar. Mandating that the bank must hold more liquidity changes the trade-off between investing in risky loans and liquid assets, as can be seen by the way these regulation alter the AAM. The higher liquid asset holdings allow the bank to raise more deposits without increasing run-risk. Although the amount of deposits raised increases, the portion that is channeled to loans falls. At the same time, the amount of equity goes up.

Why does requiring it to hold more liquidity induce the bank to both raise more deposits and more equity? The bank can raise more deposits and invest them in the liquid asset to satisfy the

regulation. This is preferable to raising equity in order to invest in the liquid asset, because equity is more expensive. Despite the fact that run-risk decreases, the increased demand for deposits pushes up the deposit rate and, thus, makes loans less profitable. The bank will reduce lending to secure higher loan rates to raise profitability. For a large enough fall in lending, the intermediation spread widens so much that it becomes desirable to increase the amount of equity.

The *NSFR* regulation operates via the same channels, but the effects are less pronounced. The reason is that this regulation also partially resembles a risk-weighted capital requirement. In particular, notice that the *NSFR* can be re-written as $1 - \delta + \delta \cdot CR + (1 - \delta) \cdot \ell/(1 - \ell)$ so that it operates through affecting both capital and liquidity. Relative to the other liquidity regulations, the *NSFR* has a more muted effect on deposits. *S* is better-off, while *E* and *B* are worse-off under all liquidity regulations.

4.3   Combined Regulation and Optimal Regulatory Mix

Finally, we examine whether and how regulation can be combined to implement the social planner's solution as a private equilibrium. The social planner solves for allocations without taking into consideration how the optimal behavior of the banker will change, or, in other words, the first-order conditions of the banker (adjusted for regulatory interventions) are not taken as additional constraints in Definition 2. Hence, the planner's allocations are computed without tying the planner to specific tools. We have seen that the difference between privately and socially optimal choices can be characterized by the wedges in (35), (36), and (37). The rest of the section shows how the regulatory tools previously studied can be combined to mimic the allocations preferred by a social planner.

To do this, it is instructive to set up an augmented planner who is endowed with certain tools. Let $\mathbb{T} = \{k, CR, \ell, LCR, NSFR, \tau_D, \tau_I, \tau_E, \tau_{LIQ}\}$ be this set of potential tools. These options are the capital and liquidity tools discussed in sections 4.1 and 4.2 as well as a tax (subsidy) on deposit-taking, τ_D, a tax (subsidy) on lending, τ_I, a tax (subsidy) on equity issuance, τ_E, and a tax (subsidy) on liquid holdings, τ_{LIQ}. The details of the implementation are spelled out in section B.6. We expand the set of tools beyond the capital and liquidity regulations for two reasons. First, for managing the loan-to-deposit spread, these tools are inadequate. Second, as we will now see, it is simpler to describe how the distorted wedges are connected using the tax/subsidy tools than using the other tools. In part, this is because using the liquidity and capital tools requires checking if they are binding or not.[34]

The following Proposition establishes our fourth main result.

Proposition 4. *Given that the wedges* (35)*,* (36)*, and* (37) *are non-zero and linearly independent, three tools are needed to replicate the planner's preferred allocations.*

---

[34]Without loss of generality, we consider simple expressions for the additional tools. The easiest way to model them is to tax (reward) the banker τ_D, τ_I, τ_E and τ_{LIQ} per unit of D, I, E and LIQ, and then return to (extract from) her a lump-sum amount equal to $\tau_D D$, $\tau_I I$, $\tau_E E$ and $\tau_{LIQ} LIQ$, respectively. We could also describe them in ways that are more like actual taxes or subsidies that we see. For example, the tax (subsidy) for deposit-taking, τ_D, could be replaced by tax (subsidy), $\tilde{\tau}_D$, on the interest payments to patient depositors, i.e., $(1 + \bar{r}_D)(1 + \tilde{\tau}_D)$.

i) Any combination of three tools among τ_D, τ_I, τ_E and τ_{LIQ} suffices.

ii) Moreover, if $CR < AAM_{WD}/(AAM_{WD} + CSM_{WD} + SIM_{WD}) < 1 - \ell$, the planner's solution can be implemented with a combination of a risk-weighted capital requirement, $\overline{CR}$, a liquidity regulation or a lending tax, $\overline{\ell}$ or τ_I, and a deposit-taking subsidy, $-\tau_D$.

iii) Finally, if $CR > AAM_{WD}/(AAM_{WD} + CSM_{WD} + SIM_{WD})$, then the planner's solution can be implemented with a combination of a risk-weighted capital requirement, $\overline{CR}$, a lending subsidy, τ_I, and a deposit-taking subsidy, $-\tau_D$.

Proposition 4 shows that typically capital requirements and deposit-taking subsidies are useful ingredients of the optimal regulatory mix.[35] However, the third tool with which they would be combined will vary and will depend on the sign and size of $AAM_{WD} = (w_S \partial \mathbb{U}_S^* / \partial \xi^* + w_E \partial \mathbb{U}_E^* / \partial \xi^*) \cdot (\partial \xi^* / \partial LIQ - \partial \xi^* / \partial I) - w_E(1-q)c''(I)I$. The sign and magnitude of this wedge depends on the weights w_S and w_E. Setting $w_E = 0$, the wedge captures the increase in savers' utility from lower run-risk that follows from requiring higher liquid asset holdings. The higher are these holdings, the easier it is to satisfy the condition in part ii) of Proposition 4. However, for large enough w_E, AAM_{WD} can become negative, so that case iii) of Proposition 4 obtains and a lending subsidy is needed. This is intuitive because the cost to entrepreneurs from lower lending is more important when w_E is higher. Overall, combining capital regulation and deposit-taking subsidies benefits savers and entrepreneurs as they enhance banking stability and encourage intermediation, but the use of tools to restrict or encourage lending depends on which agents are favored most.

Having established the general properties of the optimal regulatory mix, we proceed to examine how it looks within the context of our numerical example, while providing additional insight into the aforementioned results.

Table 5 reports the outcomes of combining regulatory tools and compares them to the planner's solution for a case where the planner favors the saver. We combine one capital regulation and one liquidity regulation with a tool that can push the bank towards the socially optimal level of intermediation. As already mentioned, there are several potential candidates for the third type of tool, but we will consider a simple tax (or subsidy) on deposit-taking. This tool can indirectly control the intermediation spread, which, as discussed in Corollary 2, is a sufficient statistic for the SIM (provided that the AAM and CSM have been fixed by a capital and a liquidity tool).[36]

As discussed, tightening the risk-weighted capital requirement to target the *CSM* increases the amount of equity in the bank. Raising the capital requirement reduces run-risk and results in lower

---

[35]Our examples either fall into cases ii) or iii). In principle, though, we cannot rule out that case i) might be needed.

[36]The planner cannot use two liquidity or capital tools at the same time because they will not be jointly binding. This result is consistent with the analysis in Checchetti and Kashyap (2016), who show that *LCR* and *NSFR* regulations almost surely will never bind at the same time. However, the collinearity of the *CR* and *k* regulations may be specific to our model and may not even hold for high *k* (see the discussion in section 4.1). If the bank could choose between more types of assets with different levels of risk or to hold off-balance sheet assets, this result may not obtain, though this would not likely deliver the planner's allocations. Although we can only speculate at this point, we believe that such modifications are important avenues for future research.

loan extension. If, in addition, we impose a stricter liquidity requirement, we can get closer to the planner in terms of the asset and capital structure margins. But, the scale of intermediation, as proxied by either the loan-deposit rate spread or the amount of deposit funding investment, moves in the opposite direction of what the planner seeks. A subsidy of $\tau_D = -9.17\%$ can be levied to induce the bank to increase the scale of intermediation to the level that the planner prefers (This subsidy is equivalent to a subsidy of $\tilde{\tau}_D = -7.40\%$ on deposit interest expenses). The combination of the three tools can implement the planner's solution as a private equilibrium. More generally, using two tools that are not redundant would also typically improve welfare relative to any single regulation. However, to mimic the planner, three regulations are required.

The planner puts more emphasis on reducing run-risk and increasing the surplus from deposit services for this set of weights. Hence, capital and liquidity regulations are useful, because they reduce run-risk without impeding deposit-taking (see sections 4.1 and 4.2). Moreover, in lieu of Corollary 2 and the fact that the asset allocation and capital structure wedges are independent, both regulations are needed. The complementarity of the capital and liquidity regulations arises in this case because agents' welfare is affected by both the bank's asset and liability mix. If, instead, the regulator had an ad hoc objective to enhance stability by reducing run-risk, then either capital or liquidity regulation could be used to accomplish this, and the tools would be substitutes.

Finally, one might expect that capital and liquidity requirements are not useful when the planner would like to increase the lending in the economy (as would be the case when the planner puts high weights on E). Table 6 shows partial and full implementation of the planner's solution for this kind of case. To mimic the planner's outcome, we need to use deposit and lending subsidies. The complication arises because, as we see from Table 2, the planner wants higher lending but much less liquidity for this set of weights. So raising liquidity requirements will mean that the planner's allocations cannot be achieved. A capital requirement can be helpful, but it will have to be combined with lending or deposit subsidies. The second column in the table shows what happens when a capital requirement, $\overline{CR} = 4.25\%$, and a deposit subsidy, $\tau_D = -2.20\%$, are combined, while the third column shows that two subsidies, $\tau_D - 2.20\%$ and $\tau_I = -4.33\%$, together can boost lending all the way to where the planner prefers.

Hence, we consider how a lending subsidy can be part of regulatory mix. The last column in the table shows that a lending subsidy, $\tau_I = -4.33\%$, can be combined with $\overline{CR} = 4.25\%$ and $\tau_D = -2.20\%$ to replicate the planner's allocation (column 5). In this case, capital regulation is needed because the bank is tempted to maximize its leverage to reap the benefits of the subsidies. This helps a bit with respect to run-risk compared with the case that only the two subsidies are in place (column 4), but, unlike a liquidity regulation, capital regulation does not restrict lending materially, which is the primary objective of the planner for this set of weights.

To conclude, our findings suggest that capital is very useful as part of the optimal regulatory mix irrespective of which agent the planner favors, while liquidity requirements are more useful for savers. Indeed, we further verify this in section B.7 where we set $c_I = 0$, so that the entrepreneurs drop out of the planner's objective (see Corollary 3).

5 Conclusions

Banks perform important services for the real economy using both sides of their balance sheet. However, the private banking equilibria may not be socially optimal and regulating banking activities can improve social welfare. We have examined how many of the regulations that are often mentioned in policy discussions perform in a relatively familiar model of banking. We started from the Diamond and Dybvig (1983) benchmark precisely because it is so thoroughly studied. The modifications that we made trade-off tractability to keep the model relatively simple, against our preference for additional realistic forces that the baseline model excludes.

Our modifications generate endogenous credit risk in banks' portfolios as well as the risk of an endogenous funding run. This simple pair of features interact in interesting and unexpected ways. We draw several general lessons from the model that we believe will carry over to many other models.

First, we identify three general intermediation margins that are distorted: the relative amounts of liquid and illiquid assets, the mix of deposits and equity, and the spread between loan and deposit rates. Second, the way that a bank privately sets these margins diverges from what a social planner would choose, because of the failure to fully account for the risks created by runs.

Third, the way the planner corrects the distortions in the private equilibrium depends on the relative importance that she places on savers versus borrowers. If the social planner cares sufficiently about savers, the planner chooses relatively more liquidity and equity than the banker and would reduce bank profits by boosting deposit rates and lowering loan rates. As a result, the planner reduces run-risk, improves the provision of liquidity, guarantees a more stable extension of credit and real production, and delivers more overall intermediation compared with the private equilibrium. Capital regulation is still desirable even when the planner cares little about savers, but liquidity regulation needs to be replaced with tools that encourage further credit extension and, hence, actually create more risk.

Fourth, more than one regulatory tool is needed to implement the socially optimal allocations. Optimal policy in models without all of these distortions can be misleading. For example, if the liability structure is constrained, say because deposit levels are exogenously determined and equity is fixed, studying asset allocations and distortions becomes much easier. But, regulation, if any is needed, will amount to fixing liquidity ratios. Similarly, shutting down the liquidity demand and liquidity risk makes it easier to focus on the optimal capital structure and level of investment. But, regulation, if again any is needed, would amount to fixing capital ratios. Instead, when both sides of the bank's balance sheet are endogenously determined the distortions from each side interact and a combination of both capital and liquidity requirements emerge in the optimal regulatory mix.

Moreover, our analysis highlights some political economy aspects of regulation that deserve attention. Our bankers internalize how their decisions matter for run-risk, funding structure and the level of intermediation to maximize their own welfare. Their distorted choices, from a social point of view, have real macroeconomic consequences. Regulation improves aggregate welfare but reduces the rents accruing to bankers. If possible, therefore, a bank's incentives to engage in

regulatory arbitrage would be strong. The lack of regulatory arbitrage in the model we have studied is one of its main shortcomings.

There are other interesting avenues to extend our model, some of which we have already mentioned and are analyzed in the online appendix. One further direction would be to allow the issuance of long-term debt together with demandable deposits and equity. Including loss-absorbing debt instruments in the regulatory mix could introduce additional ways to tackle with run-risk and credit risk. But it would not constitute a full remedy by itself due to the disciplinary role that demandable liabilities play. Moreover, our model is flexible enough to incorporate fire-sale dynamics by endogenizing the liquidation value of long-term investment. Although this would introduce pecuniary externalities as an additional reason why private allocations are inefficient, it would not qualitatively overturn our main conclusions. Finally, one could enrich the set of risky investments from which a banker could choose and, thus, increase the scope for asset substitution. Setting the (relative) risk-weights in capital requirements to capture social risks would then be highly important.

References

Admati, Anat R., Peter M. Demarzo, Martin F. Hellwig and Paul Pfleiderer (2012), 'Debt overhang and capital regulation', *Rock Center for Corporate Governance at Stanford University Working Paper No.114* .

Allen, Franklin and Douglas Gale (1998), 'Optimal financial crises', *Journal of Finance* **53**(4), 1245–1284.

Allen, Franklin and Douglas Gale (2004), 'Financial intermediaries and markets', *Econometrica* **72**(4), 1023–1061.

Allen, Franklin, Elena Carletti, Itay Goldstein and Agnese Leonello (2018), 'Government guarantees and financial stability', *Journal of Economic Theory* **177**, 518–557.

Allen, Franklin, Elena Carletti and Robert Marquez (2015), 'Deposits and bank capital structure', *Journal of Financial Economics* **118**(3), 601–619.

Angeloni, Ignazio and Ester Faia (2013), 'Capital regulation and monetary policy with fragile banks', *Journal of Monetary Economics* **60**(3), 311–324.

Bianchi, Javier and Enrique C. Mendoza (2018), 'Optimal time-consistent macroprudential policy', *Journal of Political Economy* **126**(2), 588–634.

Boissay, Frédéric and Fabrice Collard (2016), 'Macroeconomics of bank capital and liquidity regulations', *BIS Working Papers No 596* .

Boissay, Frédéric, Fabrice Collard and Frank Smets (2016), 'Booms and banking crises', *Journal of Political Economy* **124**(2), 489–538.

Calomiris, Charles W. and Charles M. Kahn (1991), 'The role of demandable debt in structuring optimal banking arrangements', *American Economic Review* **81**(3), 497–513.

Carletti, Elena, Itay Goldstein and Agnese Leonello (2019), 'The Interdependence of Bank Capital and Liquidity', *working paper* .

Carlsson, Hans and Eric van Damme (1993), 'Global games and equilibrium selection', *Econometrica* **61**(5), 989–1018.

Chari, V.V. and Ravi Jagannathan (1988), 'Banking panics, information, and rational expectations equilibrium', *Journal of Finance* **43**(3), 749–761.

Checchetti, Stephen G. and Anil K Kashyap (2016), 'What binds? Interactions between bank capital and liquidity regulations', *in The Changing Fortunes of Central Banking, edited by Philipp Hartmann, Haizhou Huang, Dirk Schoenmaker* .

Choi, Dong Beom, Thomas M. Eisenbach and Tanju Yorulmazer (2016), 'Sooner or later: Timing of monetary policy with heterogeneous risk-taking', *American Economic Review: Papers & Proceedings* **106**(5), 490–495.

Clerc, Laurent, Alexis Derviz, Caterina Mendicino, Stephane Moyen, Kalin Nikolov, Livio Stracca, Javier Suarez and Alexandros P. Vardoulakis (2015), 'Capital regulation in a macroeconomic model with three layers of default', *International Journal of Central Banking* **11**(3), 9–63.

Cooper, Russel and Thomas W. Ross (1998), 'Bank runs: Liquidity costs and investment distortions', *Journal of Monetary Economics* **41**(1), 27–38.

Cooper, Russel and Thomas W. Ross (2002), 'Bank runs: Deposit insurance and capital requirements', *International Economic Review* **43**(1), 55–72.

Corbae, Dean and Pablo D'Erasmo (2014), 'Capital requirements in a quantitative model of banking industry dynamics', *working paper* .

Diamond, Douglas W. (1984), 'Financial intermediation and delegated monitoring', *Review of Economic Studies* **51**(3), 393–414.

Diamond, Douglas W. and Anil K Kashyap (2016), 'Liquidity requirements, liquidity choice and financial stability', *NBER Working Papers 22053* .

Diamond, Douglas W. and Philip H. Dybvig (1983), 'Bank runs, deposit insurance, and liquidity', *Journal of Political Economy* **91**(3), 401–419.

Diamond, Douglas W. and Raghuram G. Rajan (2000), 'A theory of bank capital', *Journal of Finance* **55**(6), 2431–2465.

Diamond, Douglas W. and Raghuram G. Rajan (2001), 'Liquidity risk, liquidity creation, and financial fragility: A theory of banking', *Journal of Political Economy* **109**(2), 287–327.

Ennis, Huberto M. and Todd Keister (2005), 'Optimal fiscal policy under multiple equilibria', *Journal of Monetary Economics* **52**(8), 1359–1377.

Ennis, Huberto M. and Todd Keister (2006), 'Bank runs and investment decisions revisited', *Journal of Monetary Economics* **53**(2), 217–232.

Ennis, Huberto M. and Todd Keister (2009), 'Bank runs and institutions: The perils of intervention', *American Economic Review* **99**(4), 1588–1607.

Farhi, Emmanuel and Iván Werning (2016), 'A theory of macroprudential policies in the presence of nominal rigidities', *Econometrica* **84**(5), 1645–1704.

Gertler, Mark and Nobuhiro Kiyotaki (2015), 'Banking, liquidity and bank runs in an infinite-horizon economy', *American Economics Review* **105**(7), 2011–2043.

Goldstein, Itay and Ady Pauzner (2005), 'Demand-deposit contracts and the probability of bank runs', *Journal of Finance* **60**(3), 1293–1327.

Holmström, Bengt and Jean Tirole (1997), 'Financial intermediation, loanable funds, and the real sector', *The Quarterly Journal of Economics* **112**(3), 663–691.

Ikeda, Daisuke (2018), 'Bank runs, prudential tools and social welfare in a global game general equilibrium model', *Bank of England Working Paper No.732* .

Jacklin, Charles J. (1987), Demand deposits, trading restrictions, and risk sharing, *in* E. C.Prescott and N.Wallace, eds, 'Contractual Arrangements for Intertemporal Trade', Minneapolis: Univ. Minnesota Press.

Jacklin, Charles J. and Sudipto Bhattacharya (1988), 'Distinguishing panics and information-based bank runs: Welfare and policy implications', *Journal of Political Economy* **96**(3), 568–592.

Kara, Gazi I. and S. Mehmet Ozsoy (2016), 'Bank regulation under fire sale externalities', *Finance and Economics Discussion Series 2016-026. Board of Governors of the Federal Reserve System (U.S.)* .

Kareken, John H. and Neil Wallace (1978), 'Deposit insurance and bank regulation: A partial equilibrium exposition', *Journal of Business* **51**(3), 413–438.

Kashyap, Anil, Dimitrios P. Tsomocos and Alexandros P. Vardoulakis (2014), 'How does macroprudential regulation change bank credit supply?', *NBER Working Paper No. 20165* .

Keister, Todd (2015), 'Bailouts and financial fragility', *Review of Economic Studies* **83**(2), 704–736.

Keister, Todd and Yuliyan Mitkov (2017), 'Bailouts, bail-ins and banking crises', *working paper* .

Martinez-Miera, David and Rafael Repullo (2017), 'Search for yield', *Econometrica* **85**(2), 351–378.

Matta, Rafael and Enrico Perotti (2016), 'Insecure debt', *CEPR working paper No. 10505* .

Matta, Rafael and Enrico Perotti (2019), 'Mandatory stay as a timely stop to runs', *mimeo* .

Matutes, Carmen and Xavier Vives (2000), 'Imperfect competition, risk taking, and regulation in banking', *European Economic Review* **44**(1), 1–34.

Morris, Stephen and Hyun Song Shin (1998), 'Unique equilibrium in a model of self-fulfilling currency attacks', *American Economic Review* **88**(3), 587–597.

Morris, Stephen and Hyun Song Shin (2003), 'Global Games: Theory and Applications', *In Advances in Economics and Econometrics: Theory and Applications, Eight World Congress, Vol. 1, ed. Mathias Dewatripont, Lars P. Hanse, and Stephen J. Turnovsky. Cambridge: Cambridge University Press* .

Morris, Stephen and Hyun Song Shin (2016), 'Illiquidity component of credit risk', *International Economic Review* **57**(4), 1135–1148.

Peck, James and Karl Shell (2003), 'Equilibrium bank runs', *Journal of Political Economy* **111**(1), 103–123.

Peck, James and Karl Shell (2010), 'Could making banks hold only liquid assets induce bank runs?', *Journal of Monetary Economics* **57**(4), 420–427.

Rochet, Jean-Charles and Xavier Vives (2004), 'Coordination failures and the lender of last resort: Was bagehot right after all?', *Journal of the European Economic Association* **2**(6), 1116–1147.

Shleifer, Andrei and Robert W. Vishny (1992), 'Liquidation values and debt capacity: A market equilibrium approach', *Journal of Finance* **47**, 1343–1366.

Uhlig, Harald (2010), 'A model of a systemic bank run', *Journal of Monetary Economics* **57**(1), 78–96.

Van den Heuvel, Skander (2017), 'The welfare effects of bank liquidity and capital requirements', *mimeo* .

Vives, Xavier (2014), 'Strategic complementarity, fragility and regulation', *Review of Financial Studies* **27**(12), 3547–3592.

Walther, Ansgar (2016), 'Jointly optimal regulation of bank capital and liquidity', *Journal of Money, Credit and Banking* **48**(2-3), 415–448.

Tables and Figures

$e_S = 2.50$	$e_B = 0.10$	$A = 4.50$	$\omega = 80\%$	$c_D = 0.27$
$c_I = 0.1$	$\phi_I = 3$	$X = 0.10$	$\gamma = 0.15$	$\delta = 0.50$
$\beta = 0.70$	$\bar{\bar{\xi}} = 2.20$	$\underline{\xi} = 0.01$		

Table 1: Parametrization.

	PE	SP for weights (w_E, w_S)				
		(0.00,0.20)	(0.05,0.15)	(0.10,0.10)	(0.15,0.05)	(0.20,0.00)
I	0.862	0.785	0.841	0.873	0.899	0.906
LIQ	0.052	0.221	0.119	0.060	0.012	0.000
D	0.875	0.962	0.919	0.894	0.873	0.867
E	0.038	0.044	0.041	0.039	0.038	0.038
r_I	3.097	3.198	3.131	3.089	3.051	3.042
$\bar{r}_D$	0.717	0.804	0.778	0.767	0.761	0.758
q	0.407	0.386	0.398	0.403	0.407	0.408
ℓ	0.057	0.219	0.124	0.065	0.013	0.000
k	0.042	0.044	0.043	0.042	0.042	0.042
$r_I - (1-\delta)\bar{r}_D$	2.739	2.796	2.742	2.705	2.670	2.663
$I + LIQ$	0.914	1.006	0.960	0.933	0.911	0.906
$I - E$	0.824	0.741	0.800	0.834	0.861	0.867
$E(Div)$	0.745	0.755	0.750	0.747	0.743	0.743
$\Delta\mathbb{U}_E$	-	-1.66%	-0.44%	0.33%	1.02%	1.19%
$\Delta\mathbb{U}_S$	-	3.63%	1.74%	0.71%	-0.10%	-0.30%
$\Delta\mathbb{U}_B$	-	-0.44%	-0.13%	-0.05%	-0.08%	-0.09%
$\Delta\mathbb{U}_{sp}$	-	0.29%	0.11%	0.05%	0.07%	0.14%
$\Delta\mathbb{S}_{sp}$	-	1.53%	1.18%	0.99%	0.84%	0.79%

Table 2: Privately versus Socially optimal solutions. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

49

	PE	Milder increase in		Bigger increase in	
		CR	*k*	*CR*	*k*
I	0.862	0.856	0.862	0.770	0.880
LIQ	0.052	0.064	0.055	0.224	0.019
D	0.875	0.880	0.876	0.940	0.841
E	0.038	0.040	0.040	0.053	0.058
r_I	3.097	3.106	3.099	3.213	3.077
$\bar{r}_D$	0.717	0.713	0.711	0.703	0.605
q	0.407	0.405	0.406	0.387	0.405
ℓ	0.057	0.069	0.060	0.225	0.021
k	0.042	0.044	0.044	0.053	0.065
CR	0.045	0.047	0.047	0.069	0.066
$r_I - (1-\delta)\bar{r}_D$	2.739	2.749	2.743	2.862	2.774
$I + LIQ$	0.914	0.920	0.916	0.993	0.899
$I - E$	0.824	0.816	0.821	0.717	0.821
$E(Div)$	0.745	0.749	0.749	0.779	0.791
$\Delta\mathbb{U}_E$	-	-0.13%	-0.01%	-2.01%	0.51%
$\Delta\mathbb{U}_S$	-	0.17%	0.02%	2.68%	-1.23%
$\Delta\mathbb{U}_B$	-	-0.01%	-0.01%	-0.61%	-1.24%
$\Delta\mathbb{S}_{sp}$	-	0.03%	0.01%	0.05%	-1.96%

Table 3: Single capital regulations. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

	PE	ℓ	*LCR*	*NSFR*
I	0.862	0.746	0.747	0.770
LIQ	0.052	0.258	0.257	0.217
D	0.875	0.959	0.959	0.942
E	0.038	0.045	0.045	0.044
r_I	3.097	3.236	3.236	3.211
$\bar{r}_D$	0.717	0.746	0.745	0.736
q	0.407	0.385	0.385	0.390
ℓ	0.057	0.257	0.256	0.220
k	0.042	0.045	0.045	0.045
LCR	0.069	0.277	0.276	0.238
NSFR	0.552	0.703	0.702	0.670
$r_I - (1-\delta)\bar{r}_D$	2.739	2.863	2.863	2.843
$I + LIQ$	0.914	1.004	1.004	0.987
$I - E$	0.824	0.701	0.701	0.726
$E(Div)$	0.745	0.756	0.757	0.756
$\Delta\mathbb{U}_E$	-	-2.50%	-2.49%	-2.04%
$\Delta\mathbb{U}_S$	-	3.50%	3.48%	2.75%
$\Delta\mathbb{U}_B$	-	-0.51%	-0.51%	-0.35%
$\Delta\mathbb{S}_{sp}$	-	0.49%	0.49%	0.36%

Table 4: Single liquidity regulations. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

	PE	CR	CR & ℓ	CR, ℓ & τ_D
I	0.862	0.829	0.776	0.785
LIQ	0.052	0.116	0.206	0.221
D	0.875	0.899	0.938	0.962
E	0.038	0.047	0.044	0.044
r_I	3.097	3.143	3.204	3.198
$\bar{r}_D$	0.717	0.700	0.735	0.804
q	0.407	0.399	0.392	0.386
q_f	0.200	0.179	0.150	0.147
ℓ	0.057	0.123	0.210	0.219
k	0.042	0.049	0.044	0.044
$r_I - (1-\delta)\bar{r}_D$	2.739	2.792	2.837	2.796
$I + LIQ$	0.914	0.946	0.982	1.006
$I - E$	0.824	0.783	0.732	0.741
$E(Div)$	0.745	0.764	0.754	0.755
$\Delta\mathbb{U}_E$	-	-0.75%	-1.93%	-1.66%
$\Delta\mathbb{U}_S$	-	0.94%	2.57%	3.63%
$\Delta\mathbb{U}_B$	-	-0.15%	-0.31%	-0.44%
$\Delta\mathbb{S}_{sp}$	-	0.04%	0.33%	1.53%

Table 5: Implementation of the planner's solution for $w_E = 0.00$ and $w_S = 0.20$. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

	PE	CR & τ_D	τ_D & τ_I	CR, τ_D & τ_I
I	0.862	0.859	0.900	0.899
LIQ	0.052	0.064	0.010	0.012
D	0.875	0.886	0.872	0.873
E	0.038	0.037	0.038	0.038
r_I	3.097	3.103	3.050	3.051
$\bar{r}_D$	0.717	0.742	0.762	0.761
q	0.407	0.406	0.408	0.407
q_f	0.200	0.197	0.212	0.211
ℓ	0.057	0.069	0.011	0.013
k	0.042	0.040	0.042	0.042
$r_I - (1-\delta)\bar{r}_D$	2.739	2.731	2.669	2.670
$I + LIQ$	0.914	0.923	0.910	0.911
$I - E$	0.824	0.822	0.862	0.861
$E(Div)$	0.745	0.740	0.743	0.743
$\Delta\mathbb{U}_E$	-	-0.07%	1.03%	1.02%
$\Delta\mathbb{U}_S$	-	0.41%	-0.12%	-0.10%
$\Delta\mathbb{U}_B$	-	-0.02%	-0.08%	-0.08%
$\Delta\mathbb{S}_{sp}$	-	0.32%	0.84%	0.84%

Table 6: Implementation of the planner's solution for $w_E = 0.15$ and $w_S = 0.05$. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

Optimal Bank Regulation
In the Presence of Credit and Run-Risk

Anil K Kashyap   Dimitrios P. Tsomocos   Alexandros P. Vardoulakis

Online Appendix

Appendix A reports the proofs to propositions, lemmas and corollaries in the paper, while appendix B reports additional derivations and extensions.

A   Proofs

A.1   Proof of Lemma 1

First, consider that $\xi \leq \hat{\xi} \equiv (\delta D(1+r_D) - LIQ)/I$. Then, $\theta(\xi,1) = (LIQ + \xi \cdot I)/(D(1+r_D)) \leq (LIQ + \hat{\xi}I)/(D(1+r_D)) = \delta$. Also, $\hat{\lambda}(\xi)$ in (13) can be written as $[\theta(\xi,1)(1+r_D)(1+r_I) - \xi(1+\bar{r}_D) - \xi \cdot X/(\omega D)]/((1+r_D)(1+r_I) - \xi(1+\bar{r}_D))$, which is smaller than $\theta(\xi,1)$ as long as $\theta(\xi,1) < 1 + X/[\omega D(1+\bar{r}_D)((1+r_D)(1+r_I) - \xi(1+\bar{r}_D))]$, which is always true. So, $\hat{\lambda}(\xi) < \delta$ as well, and only the full run region is possible. Next, define ξ^{ld} as the solution to $\hat{\lambda}(\xi^{ld}) = \delta$; $\hat{\hat{\xi}}$ as the solution to $\theta(\hat{\hat{\xi}},1) = 1$, yielding $\hat{\hat{\xi}} = (D(1+r_D) - LIQ)/I$; and ξ^{ud} as the solution to $\hat{\lambda}(\xi^{ud}) = 1$, yielding $\xi^{ud} = \hat{\hat{\xi}}/(1 - X/(\omega I(1+r_I))) > \hat{\hat{\xi}}$. Moreover, $\hat{\lambda}(\hat{\hat{\xi}}) > \delta$ and $\partial\hat{\lambda}(\xi)/\partial\xi > 0$, so $\xi^{ld} < \hat{\hat{\xi}} < \xi^{ud}$. Finally, $\xi^{ud} < \bar{\xi}$, because $X < \overline{X} \equiv \omega I(1+r_I)(1 - \hat{\hat{\xi}}/\bar{\xi})$. Using these observations, it is easy to establish the non-empty regions for the remaining $\xi \in (\hat{\xi}, \bar{\xi}]$ in the Lemma.

A.2   Proof of Proposition 1

The proof follows the steps in Goldstein and Pauzner (2005) but includes additional derivations and arguments to tackle the perverse state monotonicity as well as the monitoring incentives and limited liability of the bank.

An equilibrium with threshold x^* exists only if $\Delta(x^*, x^*) = 0$ given by (19). Consider a potential threshold x'. We will show that x' exists and it satisfies (19) at exactly one point, $x' = x^*$.

By the existence of ξ^{ld} and ξ^{ud} defined in Lemma 1, $\Delta(x', x')$ is negative for $x' \leq \xi^{ld} - \varepsilon$ and positive for $x' \geq \xi^{ud} + \varepsilon$. Thus, in order to establish that a threshold equilibrium exists, it suffices to show that $\Delta(x', x')$ is continuous in $x' \in [\xi^{ld} - \varepsilon, \xi^{ud} + \varepsilon]$. It is convenient to write the utility differential $\Delta(x', x')$ as $\Delta(\hat{x} + \Delta x, \hat{x} + \Delta x)$ for some $\hat{x}$ such that Δx is the change in both the signal that

the marginal saver receives and the threshold strategy. Then,

$$\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x) = \frac{1}{2\varepsilon} \int_{\hat{x}+\Delta x-\varepsilon}^{\hat{x}+\Delta x+\varepsilon} v(\xi, \lambda(\xi, \hat{x}+\Delta x)) d\xi$$

$$= \frac{1}{2\varepsilon} \int_{\hat{x}-\varepsilon}^{\hat{x}+\varepsilon} v(\xi+\Delta x, \lambda(\xi+\Delta x, \hat{x}+\Delta x)) d\xi$$

$$= \frac{1}{2\varepsilon} \int_{\hat{x}-\varepsilon}^{\hat{x}+\varepsilon} v(\xi+\Delta x, \lambda(\xi, \hat{x})) d\xi, \quad \text{(A.1)}$$

because $\lambda(\xi+\Delta x, \hat{x}+\Delta x) = \lambda(\xi, \hat{x})$ from (15). In other words, the marginal saver's belief about how many other savers withdraw is unchanged when her private signal and the threshold strategy change by the same amount. Yet, she expects ξ to be higher for $\Delta x > 0$ and lower for $\Delta x < 0$, which is reflected in the calculation of $v(\xi+\Delta x, \lambda(\xi, \hat{x}))$. Thus, we need to show that, for a given distribution of $\lambda' s$, the integral in (A.1) is continuous in Δx.

The integrand $v(\xi+\Delta x, \lambda(\xi, \hat{x}))$ in (A.1) is a piecewise function such that each sub-function is computed over a distribution of λ unaffected by Δx, but the interval for each sub-function depends on Δx. The thresholds $\hat{\lambda}_{\Delta x}$ and $\hat{\theta}_{\Delta x}$, which show the level of withdrawals above which the bank does not monitor and there is a full run (Lemma 1), are functions of certain levels of the liquidation value, which belong in the posterior distribution of ξ and are denoted by $\xi_{\hat{\lambda}_{\Delta x}}$ and $\xi_{\hat{\theta}_{\Delta x}}$, respectively.

Given that the distribution of λs does not change with Δx, we can compute $\xi_{\hat{\lambda}_{\Delta x}}$ as a function of Δx by equating the portion of savers withdrawing at this liquidation value for signal $\hat{x}$, which is $\lambda(\xi_{\hat{\lambda}_{\Delta x}}, \hat{x})$ given by (15), to the threshold $\hat{\lambda}_{\Delta x}(\xi_{\hat{\lambda}_{\Delta x}} + \Delta x)$, which moves with Δx and is given by (13):

$$\lambda(\xi_{\hat{\lambda}_{\Delta x}}, \hat{x}) = \hat{\lambda}_{\Delta x}(\xi_{\hat{\lambda}_{\Delta x}} + \Delta x)$$

$$\Rightarrow \delta + (1-\delta)\frac{\hat{x} - \xi_{\hat{\lambda}_{\Delta x}} + \varepsilon}{2\varepsilon} = \frac{\left[LIQ + \left(\xi_{\hat{\lambda}_{\Delta x}} + \Delta x \right) I \right](1+r_I) - \left(\xi_{\hat{\lambda}_{\Delta x}} + \Delta x \right) (D(1+\bar{r}_D) + X/\omega)}{D \left[(1+r_D)(1+r_I) - \left(\xi_{\hat{\lambda}_{\Delta x}} + \Delta x \right) (1+\bar{r}_D) \right]}.$$

$$\text{(A.2)}$$

Similarly, we can compute $\xi_{\hat{\theta}_{\Delta x}}$ as a function of Δx by equating the portion of savers withdrawing at this liquidation value for signal $\hat{x}$, which is $\lambda(\xi_{\hat{\theta}_{\Delta x}}, \hat{x})$ given by (15), to the threshold $\hat{\theta}_{\Delta x}(\xi_{\hat{\theta}_{\Delta x}} + \Delta x) \equiv \theta(\xi_{\hat{\theta}_{\Delta x}} + \Delta x, 1)$, which moves with Δx and is given by (10):

$$\lambda(\xi_{\hat{\theta}_{\Delta x}}, \hat{x}) = \hat{\theta}_{\Delta x}(\xi_{\hat{\theta}_{\Delta x}} + \Delta x)$$

$$\Rightarrow \delta + (1-\delta)\frac{\hat{x} - \xi_{\hat{\theta}_{\Delta x}} + \varepsilon}{2\varepsilon} = \frac{LIQ + \left(\xi_{\hat{\theta}_{\Delta x}} + \Delta x \right) I}{D(1+r_D)}. \quad \text{(A.3)}$$

To ease notation, we will denote $\hat{\lambda}_{\Delta x}(\xi_{\hat{\lambda}_{\Delta x}} + \Delta x)$ and $\hat{\theta}_{\Delta x}(\xi_{\hat{\theta}_{\Delta x}} + \Delta x)$, that is the value if the threshold at fundamentals $\xi_{\hat{\lambda}_{\Delta x}} + \Delta x$, by $\hat{\lambda}_{\Delta x}$ and $\hat{\theta}_{\Delta x}$, respectively.

Because the number of savers withdrawing decreases as fundamentals improve for given strat-

egy threshold–see equation (15)–and $\hat{\lambda}_{\Delta x} < \hat{\theta}_{\Delta x}$ from Lemma 1, we get that $\xi_{\hat{\theta}_{\Delta x}} < \xi_{\hat{\lambda}_{\Delta x}}$. Thus, using (17), (A.1) can be written as:

$$\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x) = -\frac{1}{2\varepsilon}\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}} \frac{LIQ+(\xi+\Delta x)I}{\lambda(\xi,\hat{x})}d\xi - \frac{1}{2\varepsilon}\int_{\xi_{\hat{\theta}_{\Delta x}}}^{\xi_{\hat{\lambda}_{\Delta x}}} D(1+r_D)d\xi$$

$$+\frac{1}{2\varepsilon}\int_{\xi_{\hat{\lambda}_{\Delta x}}}^{\hat{x}+\varepsilon} \left\{\omega D(1+\bar{r}_D) - D(1+r_D)\right\}d\xi. \tag{A.4}$$

All the integrands ν in (A.4) are bounded and continuous in Δx, the thresholds $\xi_{\hat{\theta}_{\Delta x}}$ and $\xi_{\hat{\lambda}_{\Delta x}}$ change continuously with Δx from (A.3) and (A.2), and the only discontinuity in ν across regions occurs at one discrete point, $\xi_{\hat{\lambda}_{\Delta x}}$. Hence, $\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x)$ is continuous and a threshold equilibrium exists.

We will now establish that the threshold equilibrium is unique. By implicitly differentiating (A.2) and (A.3), we get:

$$\frac{d\xi_{\hat{\lambda}_{\Delta x}}}{d\Delta x} = -\frac{2\varepsilon\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}}{1-\delta+2\varepsilon\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}} < 0 \tag{A.5}$$

because

$$\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}} \equiv \frac{I(1+r_I)-(D(1+\bar{r}_D)+X/\omega)}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]} + \frac{\hat{\lambda}_{\Delta x}(1+\bar{r}_D)}{(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)} > 0,$$

and

$$\frac{d\xi_{\hat{\theta}_{\Delta x}}}{d\Delta x} = -\frac{2\varepsilon I}{(1-\delta)D(1+r_D)+2\varepsilon I} < 0. \tag{A.6}$$

(A.5) and (A.6) tell us that, as fundamentals become better ($\Delta x > 0$), the region where the banker monitors and the region that a run does not occur become bigger.

The derivative of (A.4) with respect to Δx is:

$$d\frac{\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x)}{d\Delta x} = -\frac{1}{2\varepsilon}\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}} \frac{I}{\lambda(\xi,\hat{x})}d\xi - \frac{1}{2\varepsilon}\frac{d\xi_{\hat{\lambda}_{\Delta x}}}{d\Delta x}\omega D(1+\bar{r}_D), \tag{A.7}$$

because $(LIQ+(\xi_{\hat{\theta}_{\Delta x}}+\Delta x)I)/\lambda(\xi_{\hat{\theta}_{\Delta}},\hat{x}) = D(1+r_D)$ from (A.3).

The first term (A.7) is negative and captures the perverse incentives of making withdrawals more profitable when fundamentals are stronger given that the run is underway. The second term in (A.7) is positive and represents the payoff change from decreasing the threshold $\xi_{\hat{\lambda}_{\Delta x}}$ where the banker ceases to monitor. As a result, we cannot unambiguously sign the derivative for any signal x. However, as we discussed in the paper, it suffices to evaluate (A.7) at a *candidate* threshold $\hat{x}$, which as we established above, that exists. If the derivative is positive at candidate threshold, we can conclude that (A.4) does not cross zero from above and, given continuity, the threshold is unique.

Adding and subtracting $1/(2\varepsilon)\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}} \xi_{\hat{\lambda}_{\Delta x}} I/\lambda(\xi,\hat{x})d\xi$ to $\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x) = 0$ in (A.4) we get

that:

$$-\frac{1}{2\varepsilon}\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}}\frac{I}{\lambda(\xi,\hat{x})}d\xi = \frac{1}{\xi_{\hat{\lambda}_{\Delta x}}+\Delta x}\frac{1}{2\varepsilon}\left[\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}}\frac{LIQ+\left(\xi-\xi_{\hat{\lambda}_{\Delta x}}\right)I}{\lambda(\xi,\hat{x})}d\xi + \int_{\xi_{\hat{\theta}_{\Delta x}}}^{\xi_{\hat{\lambda}_{\Delta x}}}D(1+r_D)d\xi\right]$$

$$-\frac{1}{\xi_{\hat{\lambda}_{\Delta x}}+\Delta x}\frac{1}{2\varepsilon}\int_{\xi_{\hat{\lambda}_{\Delta x}}}^{\hat{x}+\varepsilon}\left\{\omega D(1+\bar{r}_D)-D(1+r_D)\right\}d\xi. \tag{A.8}$$

Substituting (A.8) in (A.7) we get:

$$d\frac{\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x)}{d\Delta x} = -\frac{1}{\xi_{\hat{\lambda}_{\Delta x}}+\Delta x}\frac{1}{2\varepsilon}\omega D(1+\bar{r}_D)\left[\frac{d\xi_{\hat{\lambda}_{\Delta x}}}{d\Delta x}\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)+\left(\hat{x}+\varepsilon-\xi_{\hat{\lambda}_{\Delta}}\right)\right]$$

$$+\frac{1}{\xi_{\hat{\lambda}_{\Delta x}}+\Delta x}\frac{1}{2\varepsilon}\left[\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}}\frac{LIQ+\left(\xi-\xi_{\hat{\lambda}_{\Delta x}}\right)I}{\lambda(\xi,\hat{x})}d\xi + \int_{\xi_{\hat{\theta}_{\Delta x}}}^{\xi_{\hat{\lambda}_{\Delta x}}}D(1+r_D)d\xi\right]. \tag{A.9}$$

Using (17), (A.2) and (A.5), the bracketed terms in the first line in (A.9), can be written as:

$$\frac{d\xi_{\hat{\lambda}_{\Delta x}}}{d\Delta x}\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)+\left(\hat{x}+\varepsilon-\xi_{\hat{\lambda}_{\Delta}}\right)=$$

$$-\frac{2\varepsilon}{1-\delta+2\varepsilon\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}}\left[\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}\cdot\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)-\left(\hat{\lambda}_{\Delta x}-\delta\right)+\left(\hat{\lambda}_{\Delta x}-\delta\right)\left(\delta-2\varepsilon\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}\right)\right]. \tag{A.10}$$

Consider the terms in A.10 separately and use the definition of $\hat{\lambda}_{\Delta x}$:

$$\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}\cdot\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)-\left(\hat{\lambda}_{\Delta x}-\delta\right)=$$

$$\frac{(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)I(1+r_I)-(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)(D(1+\bar{r}_D)+X/\omega)+(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)\hat{\lambda}_{\Delta x}D(1+\bar{r}_D))}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]}$$

$$\frac{-\hat{\lambda}_{\Delta x}D(1+r_D)(1+r_I)+\left(\hat{\lambda}_{\Delta x}-\delta\right)\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)D(1+\bar{r}_D)+\delta D(1+r_D)(1+r_I)}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]}$$

$$=\frac{\left(\hat{\lambda}_{\Delta x}-\delta\right)\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)D(1+\bar{r}_D)+(\delta D(1+r_D)-LIQ)(1+r_I)}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]}, \tag{A.11}$$

which is positive from Lemma 1. Hence, there exists small enough noise such that A.10 is negative (bracketed terms positive) and the first line in A.9 is positive.

Now, consider the bracketed terms in the second line A.9, which can be written, by substituting

the definition of $\lambda(\xi,\hat{x})$ from (15), as:

$$\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}} \frac{LIQ + \left(\xi - \xi_{\hat{\lambda}_{\Delta x}}\right)I}{\frac{1+\delta}{2} + \frac{\hat{x}-\xi}{2\varepsilon}}d\xi + \int_{\xi_{\hat{\theta}_{\Delta x}}}^{\xi_{\hat{\lambda}_{\Delta x}}} D(1+r_D)d\xi. \tag{A.12}$$

The first term in (A.9) can be made very close to zero for small enough noise, and hence the second line in (A.9) is positive as well.

This concludes the argument to establish uniqueness of a threshold equilibrium $x' = x^*$ for small noise. See section 2.4 in the paper for a simpler version of this proof in the case of limiting noise, $\varepsilon \to 0$.

To conclude the proof, we need to show that the threshold equilibrium is indeed an equilibrium, i.e., $\Delta(x_i, x^*)$ in (18) is positive for all $x_i > x^*$ and negative for all $x_i < x^*$. A higher (lower) signal indicates not only that the fundamental state is better (worse), but also that fewer (more) patient savers withdraw. Both forces result in lower (higher) incentive to withdraw under global strategic complementarities and state monotonicity. But this is less obvious under one-sided strategic complementarities and perverse state monotonicity: In the run region, the incentive to withdraw increases the fewer savers withdraw and the higher the fundamental state is, which complicates the argument. Goldstein and Pauzner (2005) show that the single-crossing property is sufficient to show that the candidate threshold is indeed an equilibrium in a model without global strategic complementarities but with state monotonicity. We show below that single-crossing is sufficient even under perverse state monotonicity.

First, consider that $x_i < x^*$. Then we can decompose the intervals $[x_i - \varepsilon, x_i + \varepsilon]$ and $[x^* - \varepsilon, x^* + \varepsilon]$ into a common part $c = [x_i - \varepsilon, x_i + \varepsilon] \cap [x^* - \varepsilon, x^* + \varepsilon]$ and two disjoint parts $d^i = [x_i - \varepsilon, x_i + \varepsilon] \backslash c$ and $d^* = [x^* - \varepsilon, x^* + \varepsilon] \backslash c$. Thus, (18) and (19) can be written as:

$$\Delta(x_i, x^*) = \Delta_{\xi \in c}^i + \Delta_{\xi \in d^i}^i, \tag{A.13}$$

$$\Delta(x^*, x^*) = \Delta_{\xi \in c}^* + \Delta_{\xi \in d^i}^*. \tag{A.14}$$

All savers have the same belief about the (deterministic) number of withdrawals for threshold strategy x^*, which are given by $\lambda(\xi, x^*)$ for the level of fundamentals ξ. What changes with the signals is the posterior belief about ξ and, hence, the possible realizations of λ. From (15), $\lambda(\xi, x^*)$ is always one over d^i, thus $\Delta_{\xi \in d^i}^i = \int_{\xi \in d^i} \nu(\xi, 1)d\xi = -\int_{\xi \in d^i}(LIQ + \xi \cdot I)d\xi < 0$. As a result, it suffices to show that $\Delta_{\xi \in c}^i < 0$. We will use the facts that A.14 is zero, and that ν changes sign ("crosses zero") only once and it is positive for higher values of ξ and negative for lower values of ξ in the interval $[x^* - \varepsilon, x^* + \varepsilon]$. Hence, $\Delta_{\xi \in d^*}^* > 0$ and $\Delta_{\xi \in c}^* < 0$, since the fundamentals are higher over d^* than c. The fact that v may be increasing in ξ in the lower segment of c does not matter, because v is still negative in that segment. If $\Delta_{\xi \in c}^i \leq \Delta_{\xi \in c}^*$, then we get the desired result. First, consider the case that all $\xi \in c$ are below the monitoring threshold $\xi_{\hat{\lambda}}$ given by equating (15) and (13), i.e., $\lambda(\xi_{\hat{\lambda}}, x^*) = \hat{\lambda}(\xi_{\hat{\lambda}})$. Then, it is obvious that v is negative over c and $\Delta_{\xi \in c}^i = \Delta_{\xi \in c}^* < 0$. Second, consider the case that ξ_{λ^*} lies within c. Because a saver that receives signal x_i still believes

that the number of withdrawals at each ξ is given by $\lambda(\xi, x^*)$, the (perceived) monitoring threshold, $\xi_{\hat{\lambda}}$, is not affected by the signal. Hence, $\Delta^i_{\xi \in c} = \Delta^*_{\xi \in c} < 0$ in this case as well, which concludes the argument. Essentially, observing a signal x_i below x^* shifts probability from positive values of ν to negative values of ν (recall that noise is uniformly distributed) and, thus, $\Delta(x_i, x^*) < \Delta(x^*, x^*)$. Note that the argument holds trivially if the interval c is empty. The proof for $x_i > x^*$ is similar, which verifies that x^* is indeed a threshold equilibrium.

A.3 Proof of Corollary 1

Totally differentiating (19), we get that $\partial \xi^* / \partial z = -(\partial GG^* / \partial z)/(\partial GG^* / \partial \xi^*)$, where z can be any of I, LIQ, D, r_I, r_D, or $\bar{r}_D$. Recall that $\partial GG^* / \partial \xi > 0$ from (22). Then, $\partial \xi^* / \partial I < 0$, because $\partial GG^* / \partial I = \omega D(1 + \bar{r}_D) \partial \lambda^* / \partial I - \int_{\theta^*}^1 \xi^* / \lambda d\lambda = \omega D(1 + \bar{r}_D)[\partial \hat{\lambda}(\xi^*)/\partial I - (\lambda^* - \delta)/I] + (\theta^* - \delta)D(1 + r_D)/I + \int_{\theta^*}^1 LIQ/(\lambda I)d\lambda > 0$, from $\partial \lambda^* / \partial I - (\lambda^* - \delta)/I = [(\delta D(1 + r_D) - LIQ)(1 + r_I) + (1 - \delta)\xi^* D(1 + \bar{r}_D) + \xi^* X/\omega]/[I \cdot D((1 + r_D)(1 + r_I) - \xi^*(1 + \bar{r}_D)] > 0$ from Lemma 1.

Moreover, $\partial \xi^* / \partial D > 0$ because $\partial GG^* / \partial D = \omega D(1 + \bar{r}_D)[\partial \lambda^* / \partial D + (\lambda^* - \delta)/D] - (\theta^* - \delta)(1 + r_D) < 0$, from $\partial \lambda^* / \partial D = -\xi^*(1 + \bar{r}_D)/[D((1 + r_D)(1 + r_I) - \xi^*(1 + \bar{r}_D)] - \lambda^* / D < 0$.

The partial effect of the loan rate and the early deposit rate are, respectively, negative and positive, because $\partial GG^* / \partial r_I = \omega D(1 + \bar{r}_D) \partial \lambda^* / \partial r_I = \omega D(1 + \bar{r}_D)[(1 + r_D)(1 + \bar{r}_D)\xi^*(1 - \theta^*) + \xi^* X/\omega]/[((1 + r_D)(1 + r_I) - \xi^*(1 + \bar{r}_D)] > 0$ and $\partial GG^* / \partial r_D = \omega D(1 + \bar{r}_D) \partial \lambda^* / \partial r_D - (\theta^* - \delta)D = -\omega D(1 + \bar{r}_D)\lambda^*(1 + r_I)/[((1 + r_D)(1 + r_I) - \xi^*(1 + \bar{r}_D)] - (\theta^* - \delta)D < 0$.

However, the sign of $\partial \xi^* / \partial LIQ < 0$ is ambiguous, because $\partial GG^* / \partial LIQ = \omega D(1 + \bar{r}_D) \partial \lambda^* / \partial LIQ - \int_{\theta^*}^1 1/\lambda d\lambda = \omega D(1 + \bar{r}_D) \cdot [\partial \lambda^* / \partial LIQ - (\lambda^* - \delta)/LIQ] + (\theta^* - \delta)D(1 + r_D)/LIQ + \int_{\theta^*}^1 \xi I/(\lambda LIQ)d\lambda$, and we cannot unambiguously sign $\partial \lambda^* / \partial LIQ - (\lambda^* - \delta)/LIQ = [(\delta D(1 + r_D) - \xi^* I)(1 + r_I) + (1 - \delta)\xi^* D(1 + \bar{r}_D) + \xi^* X/\omega]/[I \cdot D((1 + r_D)(1 + r_I) - \xi^*(1 + \bar{r}_D)]$.

Finally, the sign of $\partial \xi^* / \partial \bar{r}_D$ is also ambiguous, because $\partial GG^* / \partial \bar{r}_D = \omega D(1 + \bar{r}_D) \partial \lambda^* / \partial \bar{r}_D + \omega D(\lambda^* - \delta) = \omega D[(\lambda^* - \delta)(1 + r_D)(1 + r_I) - (1 - \delta)\xi^*(1 + \bar{r}_D)]/[(1 + r_D)(1 + r_I) - \xi^*(1 + \bar{r}_D)]$, which cannot be unambiguously signed.

A.4 Proof of Corollary 3

Given that $c(0) = 0$ and $c'(\cdot) > 0$, $c''(\cdot) = 0$ implies that $c(x) = a_c \cdot x$, with $a_c > 0$. Then, $\partial \mathbb{U}_E^* / \partial \xi^* = -[c'(I)I - c(I)]/\Delta_\xi = 0$. Moreover, the surplus to E, $(1 - q)c''(I)I$, is zero. Similarly, given that $V(0) = 0$ and $V'(\cdot) > 0$, $V''(\cdot) = 0$ implies that $V(x) = a_v \cdot x$, with $a_v > 0$. Then, $\partial \mathbb{U}_S^* / \partial \xi^* = -[V(D(1 + r_D)) - V'(D(1 + r_D))D(1 + r_D)]/\Delta_\xi = 0$. Moreover, the surplus from the transaction services of deposits, $(1 - q)V''(D(1 + r_D))D(1 + r_D)^2$, is zero. Finally, the surplus in terms of period 1 utility, $U''(e_R - D)D$, is zero for $U''(\cdot) = 0$ as well as for $LIQ_S > 0$, because then $\mathbb{U}_S^* = \mathbb{U}_S^\alpha$ given that $V(D(1 + r_D)) - V'(D(1 + r_D))D(1 + r_D) = 0$ from condition (ii). In turn, $LIQ_S > 0$ if savers endowment is higher than some threshold $\bar{e}_S$ at which (3) holds with equality.

A.5 Proof of Proposition 2

First, set $X = 0$ to make the determination of the run threshold in (20) scale invariant. Dividing it by the balance sheet size, $E + D$ (or $I + LIQ$), (20) becomes:

$$GG_{BS} = \int_{\delta}^{\hat{\lambda}} \omega(1-k)(1+\bar{r}_D)d\lambda - \int_{\delta}^{\theta^*} (1-k)(1+r_D) - \int_{\theta^*}^{1} \frac{\xi^*(1-\ell)+\ell}{\lambda}d\lambda = 0, \qquad \text{(A.15)}$$

where

$$\hat{\lambda}_{BS} = \frac{(\xi^*(1-\ell)+\ell)(1+r_I) - \xi^*((1-k)(1+\bar{r}_D))}{(1-k)[(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)]}. \qquad \text{(A.16)}$$

Thus, k affects the payoff differential in a partial run as well as the range that monitoring occurs, $\hat{\lambda} - \delta$, via its effect on bank profitability. Totally differentiating (A.15) with respect to k, while keeping ℓ, r_I, r_D and $\bar{r}_D$ constant we get:

$$\frac{\partial GG_{BS}}{\partial k} = \underbrace{\frac{\partial \hat{\lambda}}{\partial k}\omega(1-k)(1+\bar{r}_D)}_{\text{More monitoring}} - \underbrace{(\hat{\lambda}-\delta)[\omega(1+\bar{r}_D) - (1+r_D)]}_{\substack{\text{Lower payoff} \\ \text{given monitoring}}} + \underbrace{(\theta^* - \hat{\lambda})(1+r_D)}_{\substack{\text{'Higher' payoff} \\ \text{absent monitoring}}}, \qquad \text{(A.17)}$$

where $\partial\hat{\lambda}/\partial k > 0$. Hence, the trade-off from setting a higher requirement $k \geq \bar{k}$ is that monitoring becomes more probable, but the payoff to depositors is smaller given monitoring. Combining the two effects, we get that

$$\frac{\partial GG_{BS}}{\partial k} = \left[\frac{\xi^*(1+\bar{r}_D)}{(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)} + \delta\right]\omega(1+\bar{r}_D) + (\theta^* - \delta)(1+r_D) > 0, \qquad \text{(A.18)}$$

which implies that $\partial\xi^*/\partial k = -(\partial GG_{BS}^*/\partial k)/(\partial GG_{BS}^*/\partial\xi^*) < 0$, i.e., higher k reduces the run probability q.

Finally, note that $CR = k(1-\ell)$. So tightening leverage is equivalent to setting a higher capital requirement, all else being equal, and, thus, higher CR reduces the run probability q.

A.6 Proof of Proposition 3

The proof uses material described in proof A.5. Totally differentiating (A.15) with respect to ℓ, while keeping k, r_I, r_D and $\bar{r}_D$ constant, we get:

$$\frac{\partial GG_{BS}}{\partial \ell} = \underbrace{\frac{\partial \hat{\lambda}}{\partial \ell}\omega(1-k)(1+\bar{r}_D)}_{\text{More monitoring}} - \underbrace{\int_{\theta^*}^{1}\frac{1-\xi^*}{\lambda}d\lambda}_{\substack{\text{Higher payoff} \\ \text{in full run}}}, \qquad \text{(A.19)}$$

where $\partial\hat{\lambda}/\partial\ell > 0$. Hence, the trade-off from setting a higher requirement $\ell \geq \bar{\ell}$ is that monitoring becomes more probable, but the incentives to join a full run increase. Combining the two effects,

we get that

$$\frac{\partial GG_{BS}}{\partial \ell} = (1 - \xi^*) \left[\frac{\omega(1 + \bar{r}_D)(1 + r_I)}{(1 + r_D)(1 + r_I) - \xi^*(1 + \bar{r}_D)} + \log \theta^* \right], \qquad (A.20)$$

which is definitely positive if $\log \theta^* > -1$ given that $\omega(1 + \bar{r}_D) > 1 + r_D$. In turn, this is satisfied under sufficient conditions $\delta > e^{-1}$ given that $\theta^* > \delta$ or $\ell > \hat{\ell} \equiv \left[e^{-1} \cdot (1 - k)(1 + r_D) - \xi^* \right] / (1 - \xi^*)$, which is true for high enough ξ^* in the private equilibrium.

Finally, note that $LCR = ((1 - \underline{\xi})\ell + \underline{\xi}) / (k(1 + r_D))$ and $NSFR = (k + (1 - \delta)(1 - k)) / (1 - \ell)$. So increasing ℓ is equivalent to increasing LCR or $NSFR$, all else being equal, and, thus, higher LCR or $NSFR$ reduce the run probability q.

A.7 Proof of Proposition 4

We show that three tools are needed to replicate the planner's allocations. We start with case i), which considers corrective taxes, and then turn to cases ii) and iii), which consider combinations of taxation and regulatory-ratio tools.

Our conjecture is that three tools are needed. For example, consider τ_I, τ_D and τ_{LIQ}. As shown in section B.6, condition (B.27) is sufficient and necessary to replicate the planner's allocations. This condition takes the following matrix form under the three aforementioned Pigouvian taxation tools

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \tau_I \\ \tau_D \\ \tau_{LIQ} \end{bmatrix} = \begin{bmatrix} AAM_{WD} \\ CSM_{WD} \\ SIM_{WD} \end{bmatrix}, \qquad (A.21)$$

which reduces to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_I \\ \tau_D \\ \tau_{LIQ} \end{bmatrix} = \begin{bmatrix} -(CSM_{WD} + SIM_{WD}) \\ CSM_{WD} \\ -(AAM_{WD} + CSM_{WD} + SIM_{WD}) \end{bmatrix}. \qquad (A.22)$$

Since, generically, $AAM_{WD} \neq 0$, $CSM_{WD} \neq 0$, $SIM_{WD} \neq 0$, $CSM_{WD} + SIM_{WD} = -AAM_{WD} + (w_S \cdot \partial \mathbb{U}_S^* / \partial \xi^* + w_E \cdot \partial \mathbb{U}_E^* / \partial \xi^*) \cdot \partial \xi^* / \partial LIQ \neq 0$ and $AAM_{WD} + CSM_{WD} + SIM_{WD} = (w_S \cdot \partial \mathbb{U}_S^* / \partial \xi^* + w_E \cdot \partial \mathbb{U}_E^* / \partial \xi^*) \cdot \partial \xi^* / \partial LIQ \neq 0$, (A.22) tells us that three tools are needed to replicate the planner's allocations.

Alternatively, we could consider a combination of τ_I, τ_E, and τ_{LIQ}. Following the same methodology, we get $\tau_I = -SIM_{WD} \neq 0$, $\tau_E = -CSM_{WD} \neq 0$, and $\tau_{LIQ} = -(AAM_{WD} + SIM_{WD}) = CSM_{WD} - (w_S \cdot \partial \mathbb{U}_S^* / \partial \xi^* + w_E \cdot \partial \mathbb{U}_E^* / \partial \xi^*) \cdot \partial \xi^* / \partial LIQ \neq 0$. Other possible combinations would also yield the same result, i.e., that three tools are needed to replicate the planner's allocations.

Turning to combinations of taxation and regulatory-ratio tools, consider the following mix: A capital requirement $\overline{CR}$, a liquidity requirement $\bar{\ell}$ and a subsidy on deposit-taking $-\tau_D$. Condition

(B.27) becomes

$$
\begin{bmatrix} \overline{CR} & 1 & 0 \\ 1 & 0 & 1 \\ -\overline{CR} & -\overline{\ell} & -1 \end{bmatrix} \begin{bmatrix} \lambda_{CR} \\ \lambda_{\ell} \\ \tau_D \end{bmatrix} = \begin{bmatrix} AAM_{WD} \\ CSM_{WD} \\ SIM_{WD} \end{bmatrix}, \tag{A.23}
$$

which reduces to

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{CR} \\ \lambda_{\ell} \\ \tau_D \end{bmatrix} = \frac{1}{1 - \overline{CR}(1 - \overline{\ell})} \begin{bmatrix} \overline{\ell} \cdot AAM_{WD} + CSM_{WD} + SIM_{WD} \\ AAM_{WD} - \overline{CR} \cdot (AAM_{WD} + CSM_{WD} + SIM_{WD}) \\ -\overline{\ell} \cdot AAM_{WD} - (1 - \overline{\ell}) \cdot \overline{CR} \cdot CSM_{WD} - SIM_{WD} \end{bmatrix}.
$$
$$\tag{A.24}$$

Thus, λ_{CR} and λ_{ℓ} are positive, i.e., the capital and liquidity requirement are binding, if $CR < AAM_{WD}/(AAM_{WD} + CSM_{WD} + SIM_{WD}) < 1 - \ell$.

Finally, consider a regulatory mix consisting for a capital requirement $\overline{CR}$, a lending subsidy $-\tau_I$, and a deposit-taking subsidy $-\tau_D$. Condition (B.27) becomes

$$
\begin{bmatrix} \overline{CR} & 1 & 0 \\ 1 & 0 & 1 \\ -\overline{CR} & -1 & -1 \end{bmatrix} \begin{bmatrix} \lambda_{CR} \\ \tau_I \\ \tau_D \end{bmatrix} = \begin{bmatrix} AAM_{WD} \\ CSM_{WD} \\ SIM_{WD} \end{bmatrix}, \tag{A.25}
$$

which reduces to

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{CR} \\ \tau_I \\ \tau_D \end{bmatrix} = \begin{bmatrix} AAM_{WD} + CSM_{WD} + SIM_{WD} \\ AAM_{WD} - \overline{CR} \cdot (AAM_{WD} + CSM_{WD} + SIM_{WD}) \\ -(AAM_{WD} + SIM_{WD}) \end{bmatrix}. \tag{A.26}
$$

Thus, $\lambda_{CR} > 0$, i.e., capital requirements are binding because $AAM_{WD} + CSM_{WD} + SIM_{WD} = (w_S \partial \mathbb{U}_S^* / \partial \xi^* + w_E \partial \mathbb{U}_E^* / \partial \xi^*) \cdot \partial \xi^* / \partial LIQ > 0$ under the assumptions in Proposition 3. Moreover, a lending subsidy requires $AAM_{WD} < \overline{CR} \cdot (AAM_{WD} + CSM_{WD} + SIM_{WD})$, which is trivially guaranteed for high enough w_E making $AAM_{WD} < 0$. Otherwise, a tax on lending would be needed, which is equivalent to a liquidity requirement examined above.

B Extensions and Additional Derivations

B.1 Intermediation Margins in Private Equilibrium

The first-order conditions (24) together with the four constraints in $\mathcal{Y}$ can be combined to characterize the private equilibrium as follows. If (24) gives an interior $r_D > 0$, then it is used to determine r_D as a function of all other variables in $\mathcal{C}$; otherwise, set $r_D = 0$. Then, use (2), (7), (9), and (20) to express (implicitly) $\bar{r}_D$, r_I, E, and ξ^* in terms of I, LIQ and D. The next step is to express the shadow values on the four constraints $\mathcal{Y}$ in terms of I, LIQ, and D. The shadow value of funds is

determined by the first-condition with respect to E,

$$\psi_{BS} = W'(e_B + D - I - LIQ), \tag{B.1}$$

where we have substituted $E = I + LIQ - D$.

The shadow value on the deposit supply schedule can be obtained from (24) with respect to $\bar{r}_D$, which yields

$$\psi_{DS} = -\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \psi_{GG}\frac{\partial GG}{\partial \bar{r}_D}\right)\frac{\partial DS^{-1}}{\partial \bar{r}_D}. \tag{B.2}$$

The choice of $\bar{r}_D$ matters for the banker via the effect on profits and on the run dynamics. The shadow value determined in (B.2) captures the sum of these effects as the deposit rate moves along the deposit supply schedule. Because the three variables of interest—I, LIQ, and D—affect the loan demand directly as well as indirectly via $\bar{r}_D$, their overall effect on the deposit supply will be scaled by the shadow value ψ_{DS} in their respective first-order conditions.

The shadow value on the loan demand schedule can be obtained from (24) with respect to r_I, which yields

$$\psi_{LD} = -\left(\frac{\partial \mathbb{U}_B}{\partial r_I} + \psi_{GG}\frac{\partial GG}{\partial r_I}\right)\frac{\partial LD^{-1}}{\partial r_I}. \tag{B.3}$$

Similar to (B.2), condition (B.3) says that the shadow value on the loan demand is measured by how a change in the loan rate along the loan demand schedule affects banker's utility.

Equivalently, combining (24) for $\mathcal{C} = \xi^*$, (B.2) and (B.3), the shadow value on the global game constraint is given by

$$\psi_{GG} = -\frac{d\mathbb{U}_B}{d\xi^*} \cdot \frac{dGG^{-1}}{d\xi^*}, \tag{B.4}$$

where $d\mathbb{U}_B/d\xi^*$ is the total effect of the run threshold ξ^* on banker's utility, which captures the partial direct effect—$\partial \mathbb{U}_B/\partial \xi^*$ in (B.70)—and the partial indirect effects via the deposit and loan rate, in (B.70) and (B.70), respectively:

$$\frac{d\mathbb{U}_B}{d\xi^*} = \left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS^{-1}}{\partial \bar{r}_D}\frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD^{-1}}{\partial r_I}\frac{\partial LD}{\partial \xi^*}\right). \tag{B.5}$$

Similarly, $dGG/d\xi^*$ is the total effect of the run threshold ξ^* on the utility differential determining the run behavior, which captures the partial direct effect—$\partial GG/\partial \xi^*$ in (22)—and the partial indirect effects via the deposit and loan rate, in (B.94) and (B.102), respectively:

$$\frac{dGG}{d\xi^*} = \frac{\partial GG}{\partial \xi^*} - \frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS^{-1}}{\partial \bar{r}_D}\frac{\partial DS}{\partial \xi^*} - \frac{\partial GG}{\partial r_I}\frac{\partial LD^{-1}}{\partial r_I}\frac{\partial LD}{\partial \xi^*}. \tag{B.6}$$

Overall, ψ_{GG} measures the effect of a change in the run threshold, which is consistent with optimal run behavior, i.e., along the global game constraint, on B's welfare.

Combining (24) with respect to LIQ and I and substituting in (B.2), (B.3), and (B.4), we obtain

10

the asset allocation margin in the private equilibrium (AAM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial LIQ} - \frac{\partial \mathbb{U}_B}{\partial I} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS^{-1}}{\partial \bar{r}_D}\left(\frac{\partial DS}{\partial LIQ} - \frac{\partial DS}{\partial I}\right) - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD^{-1}}{\partial r_I}\left(\frac{\partial LD}{\partial LIQ} - \frac{\partial LD}{\partial I}\right)}^{\text{Effect of asset mix on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dLIQ - d\mathbb{U}_B/dI}$$

$$+ \underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS^{-1}}{\partial \bar{r}_D}\frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD^{-1}}{\partial r_I}\frac{\partial LD}{\partial \xi^*}\right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*: \, d\mathbb{U}_B/d\xi^*} \cdot \underbrace{\left(\frac{\partial \xi^*}{\partial LIQ} - \frac{d\xi^*}{dI}\right)}_{\text{Effect of asset mix on } \xi^*} = 0, \qquad (B.7)$$

where $d\xi^*/dLIQ$ and $d\xi^*/dI$ are obtained from total differentiation of (20), hence

$$\frac{d\xi^*}{dLIQ} - \frac{d\xi^*}{dI} = -\left[\frac{\partial GG}{\partial LIQ} - \frac{\partial GG}{\partial I} - \frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS^{-1}}{\partial \bar{r}_D}\left(\frac{\partial DS}{\partial LIQ} - \frac{\partial DS}{\partial I}\right) - \frac{\partial GG}{\partial r_I}\frac{\partial LD^{-1}}{\partial r_I}\left(\frac{\partial LD}{\partial LIQ} - \frac{\partial LD}{\partial I}\right)\right] \cdot \frac{dGG^{-1}}{d\xi^*}.$$
$$(B.8)$$

The asset allocation margin in (B.7) captures the decision to substitute a unit of loans with a unit of liquid assets. The banker in the private equilibrium weighs the effect of the change in the asset mix on the bank profitability (the first line) and on the run threshold, which determines run-risk, because both affect her welfare (the second line). The asset mix matters for bank profits because of portfolio effects (first two terms in first line), but also because of the way it influences the profit margin via the loan rate and deposit rates (remaining terms in first line). The latter (general equilibrium) effect via rates captures how the asset mix matters for the loan rate or deposit rate that entrepreneurs and depositors are willing to accept. Similarly, the asset mix changes the payoffs governing the run dynamics directly and indirectly via the loan and deposit rates (captured by (B.8)), which in turn affect the run threshold influencing B's welfare directly and indirectly via the loan and deposit rates.

Similarly, combining (24) with respect to E and D and substituting in (B.2), (B.3), and (B.4), we obtain the capital structure margin in the private equilibrium (CSM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial E} - \frac{\partial \mathbb{U}_B}{\partial D} + \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS^{-1}}{\partial \bar{r}_D}\frac{\partial DS}{\partial D} + \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD^{-1}}{\partial r_I}\frac{\partial LD}{\partial D}}^{\text{Effect of liabilities mix on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dE - d\mathbb{U}_B/dD}$$

$$- \underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS^{-1}}{\partial \bar{r}_D}\frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD^{-1}}{\partial r_I}\frac{\partial LD}{\partial \xi^*}\right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*: \, d\mathbb{U}_B/d\xi^*} \cdot \underbrace{\frac{d\xi^*}{dD}}_{\substack{\text{Effect of liabilities} \\ \text{mix on } \xi^*}} = 0, \qquad (B.9)$$

where

$$\frac{d\xi^*}{dD} = -\left[\frac{\partial GG}{\partial D} - \frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS^{-1}}{\partial \bar{r}_D}\frac{\partial DS}{\partial D} - \frac{\partial GG}{\partial r_I}\frac{\partial LD^{-1}}{\partial r_I}\frac{\partial LD}{\partial D}\right] \cdot \frac{dGG^{-1}}{d\xi^*}. \qquad (B.10)$$

Finally, combining (24) with respect to I and D and substituting in (B.2), (B.3), and (B.4), we

obtain the margin for the scale of intermediation in the private equilibrium (SIM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial I} + \frac{\partial \mathbb{U}_B}{\partial D} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\left(\frac{\partial DS}{\partial I} + \frac{\partial DS}{\partial D}\right) - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial I} + \frac{\partial LD}{\partial D}\right)}^{\text{Effect of intermediation scale on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dI + d\mathbb{U}_B/dD}$$

$$+ \underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}\right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*: d\mathbb{U}_B/d\xi^*} \cdot \underbrace{\left(\frac{d\xi^*}{dI} + \frac{d\xi^*}{dD}\right)}_{\substack{\text{Effect of intermediation} \\ \text{scale on } \xi^*}} = 0, \qquad \text{(B.11)}$$

where

$$\frac{d\xi^*}{dI} + \frac{d\xi^*}{dD} = -\left[\frac{\partial GG}{\partial I} + \frac{\partial GG}{\partial D} - \frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\left(\frac{\partial DS}{\partial I} + \frac{\partial DS}{\partial D}\right) - \frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial I} + \frac{\partial LD}{\partial D}\right)\right] \cdot \frac{dGG}{d\xi^*}^{-1}.$$
$$\text{(B.12)}$$

Note that, expanding the first two terms in (B.11), we get that

$$\frac{\partial \mathbb{U}_B}{\partial I} + \frac{\partial \mathbb{U}_B}{\partial D} = \omega\left\{[(1-q) - \delta(1+r_D)log(\bar{\xi}/\xi^*)/\Delta_\xi](1+r^I) - (1-\delta)(1+\bar{r}_D)\right\}, \qquad \text{(B.13)}$$

where q is the run probability. Hence, the third margin capturing the scale of intermediation can be proxied by the intermediation spread between the loan rate, r_I, and the late deposit rate, $\bar{r}_D$.

The three intermediation margins pin down the three free variables I, LIQ, and D. The remaining variables, E, ξ^*, r_I, and $\bar{r}_D$, are implicitly functions of the three free variables via constraints (9), (20), (7) and (2), which are always binding in equilibrium . Hence, there are three degrees of freedom and the private equilibrium is characterized by (B.7), (B.9) and (B.11).

B.2 Intermediation Margins in Social Planner's Equilibrium

As for the private equilibrium, we can use the first-order conditions (34) in the planning problem together with the four constraints in $\mathcal{Y}$ to characterize the planning allocations. In particular, we use the first-order condition with respect to r_D to determine its value—which is zero as in the PE— and (2), (7), (9), and (20) to express (implicitly) $\bar{r}_D$, r_I, E, and ξ^* in terms of I, LIQ, and D. As discussed, we consider a planner that respects the deposit supply and loan demand schedule, because we want to focus on regulation to affect bank's behavior. See section B.9 for a more powerful planner, who can levy distortionary taxes to affect the private deposit supply and loan demand schedules.

Up to this point everything is analogous to the characterization of PE allocations in section B.2. But, the planner also cares about the direct effect on S and E welfare as captured in the social welfare function (29). This influences the functional form of the Lagrange multipliers on constraints $\mathcal{Y}$, which are denoted by $\zeta_{\mathcal{Y}}$ instead of $\psi_{\mathcal{Y}}$.

The functional forms of ζ_{BS}, ζ_{DS}, and ζ_{LD} are the same as for ψ_{BS}, ψ_{DS}, and ψ_{LD} given by (B.1), (B.2) and (B.3), with the exception that the latter two are functions of ζ_{GG} instead of ψ_{GG}. The reason is that E, $\bar{r}_D$, and r_I do not appear directly in (29). Note, this does not mean that the

equilibrium values of these Lagrange multipliers are the same in the private and planning solutions. But, the multiplier ζ_{GG} on constraint (20) will have a different functional form compared to (B.4), because ξ^* appears in the indirect utilities:

$$\zeta_{GG} = -\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_3^D}\frac{\partial DS}{\partial r_3^D}^{-1}\frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r^I}\frac{\partial LD}{\partial r^I}^{-1}\frac{\partial LD}{\partial \xi^*} + w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{dGG}{d\xi^*}^{-1}$$

$$= \psi_{GG} - \left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{dGG}{d\xi^*}^{-1}$$

$$= \psi_{GG}\left[1 + \left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{d\mathbb{U}_B}{d\xi^*}\right], \tag{B.14}$$

where the terms in red are the additional terms in the planner's problem. Because $\partial \mathbb{U}_S^*/\partial \xi^* = -[V(D) - V'(D)D]\Delta_\xi < 0$, $\partial \mathbb{U}_E^*/\partial \xi^* = -[c'(I)I - c(I)]\Delta_\xi < 0$, and, from (B.5), $d\mathbb{U}_B/d\xi^* < 0$,[1] internalizing the run externalities makes the Lagrange multiplier on the constraint (20) higher (when evaluated at the PE allocations). We can now derive the wedges between the private and social intermediation margins.

Combining (34) with respect to LIQ and I together with (25), we can derive the following wedge in the Asset Allocation Margin (also reported in (35)):

$$AAM_{WD} = \underbrace{\left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\cdot\left(\frac{\partial \xi^*}{\partial LIQ} - \frac{\partial \xi^*}{\partial I}\right)}_{\text{Run externality from asset allocation}} - \underbrace{w_E(1-q)c''(I)I}_{\text{Surplus to E from additional } I} \tag{B.15}$$

As discussed, the first term in (B.15) captures how a shift in the asset allocation from loans to liquid asset holdings affects savers' and entrepreneurs' welfare via its effect on the run probability. Both savers and entrepreneurs are worse-off when run-risk goes up, i.e., $\partial \mathbb{U}_S^*/\partial \xi^* < 0$ and $\partial \mathbb{U}_E^*/\partial \xi^* < 0$. Thus, if run-risk decreases when the asset allocation shifts towards liquid assets, then the planner would want a more liquid asset mix. The second term in (B.15) captures the surplus created for the entrepreneur from an additional unit of investment. This term is negative (including the minus sign) if the planner puts weight on E and if E extracts some surplus to start with, i.e., $c'' > 0$, which is true for a strictly convex function.

Examining the run externalities in more detail the term $\partial \xi^*/\partial LIQ - \partial \xi^*/\partial I$ captures exactly the effects of substituting a unit of liquid assets with a unit of loans on the probability of a run and is given by (B.8). If it is negative, correcting the run externalities requires a more liquid asset mix. The first and third terms inside the bracket are unambiguously positive, respectively. The first term captures the direct effect of shifting the asset allocation towards more liquid assets on the incentive to run. Combining (B.83) and (B.82) we get that $\partial GG/\partial LIQ - \partial GG/\partial I = \partial GG/\partial \ell$, which

---

[1]$\partial \mathbb{U}_B/\partial \xi^* < 0$ from (B.70); $\partial \mathbb{U}_B/\partial \bar{r}_D < 0$ from (B.73); $\partial DS/\partial \bar{r}_D > 0$ from (B.97); $\partial DS/\partial DS/\xi^* < 0$ from (B.94); $\partial \mathbb{U}_B/\partial r_I > 0$ from (B.71); $\partial LD/\partial r_I < 0$ from (B.103); $\partial LD/\partial \xi^*$ from (B.102) can be positive or negative, so the effect through the loan demand cannot be unambiguously determined. However, in our examples, the first two terms dominate, and overall, a higher run threshold reduces banker's utility all else being equal.

is positive as we prove in Proposition 3 (recall that $\ell \equiv LIQ/(I+LIQ)$ is the share of liquid assets in the asset portfolio). Intuitively, a more liquid asset portfolio (directly) decreases the incentives to run. The third term captures the indirect effect via the loan rate. In particular, $\partial GG/\partial r_I > 0$ from (B.79) and (B.87), $\partial LD/\partial r_I = -\omega \int_{\xi^*}^{\overline{\xi}}(1-y)Id\xi/\Delta_\xi < 0$ from (B.103), and, combining (B.98) and (B.99), we get that $\partial LD/\partial LIQ - \partial LD/\partial I = \omega[A-(1+r_I)]log(\overline{\xi}/\xi^*)/\Delta_\xi[LIQ+I-\delta D(1+r_D)]/I^2 - (1-q)c''(I) > 0$. Thus, the third term in (B.8)—including the minus sign—is positive. Intuitively, a higher loan rate reduces the incentives to run, because it increases bank profits and, hence, the region where the banker decides to monitor. In turn, a more liquid asset portfolio increases the loan demand, because of convex investment costs and fewer loans being recalled, pushing up the loan rate entrepreneurs are willing to pay.

Finally, the second term captures the indirect effect via the late deposit rate. If we could show that this is always positive, then the whole expression would be negative given that $dGG/d\xi^*$ needs to be positive to have $\psi_{GG} > 0$ in the private equilibrium. In particular, combining (B.91) and (B.90) we get that $\partial DS/\partial LIQ - \partial DS/\partial I = q[1-(\xi^*+\underline{\xi})/2-]/D > 0$, while $\partial DS/\partial \bar{r}_D = \omega(1-q)(1-\delta) > 0$ from (B.97). Intuitively, a more liquid asset portfolio increases the demand for deposits because it increases the probability of being repaid in a run, θ, and, thus, pushes down the rate depositors demand. However, the effect of the deposit rate on the incentives to run, $\partial GG/\partial \bar{r}_D$, could be unambiguously determined:

$$\frac{\partial GG}{\partial \bar{r}_D} = \underbrace{(\hat{\lambda}-\delta)\omega D}_{\substack{\text{Higher payoff} \\ \text{given monitoring}}} + \underbrace{\frac{\partial \hat{\lambda}}{\partial \bar{r}_D}\omega D(1+\bar{r}_D)}_{\substack{\text{Lower chance} \\ \text{of monitoring}}} = \omega D \frac{(\lambda-\delta)(1+r_D)(1+r_I)-(1-\delta)\xi^*(1+\bar{r}_D)}{(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)}.$$

(B.16)

In other words, a higher deposit rate increases the payoff from waiting given monitoring, which reduces the incentives to run, but also reduces the chances that monitoring takes place, i.e., $\partial \hat{\lambda}/\partial \bar{r}_D < 0$ from (B.81), which increases the incentives to run. We haven't been able to sign the overall effect analytically, but, in all the examples we have studied, we find that $\partial GG/\partial \bar{r}_D > 0$, or in other words a higher deposit rate reduces the run probability, all else being equal.[2] This is an important and novel channel in our model (see the discussion at the end of section 2.4).

Turning to the capital structure margin, we combine (34) with respect to E and D together with (26) to get the following wedge (also reported in (36)):

$$CSM_{WD} = -\underbrace{\left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{\partial \xi^*}{\partial D}}_{\text{Run externality from liabilities mix}} + \underbrace{w_R\left[U''(e_R-D)D + (1-q)V''(D(1+r_D)D(1+r_D)^2\right]}_{\text{Surplus to S from additional } D}$$

(B.17)

Similar to the wedge in the asset allocation margin, the wedge in the capital structure margin features two components. The first term captures how a shift in the liabilities mix from deposits to equity affects savers' and entrepreneurs' welfare via its effect on the run probability. Both savers and

---

[2]Recall that $\partial \xi^*/\partial \bar{r}_D = -(\partial GG/\partial \bar{r}_D)/(\partial GG/\partial \xi^*)$.

entrepreneurs are worse-off when run-risk goes up. Thus, if run-risk decreases when the liabilities mix shifts towards equity, then the planner would want lower leverage.[3] The second term captures the surplus created for savers from an additional unit of deposits and is negative if the planner puts weigh on S and if S extracts some surplus to start with, which is true for strictly concave functions U and V. In others words, shifting the capital structure away from deposits entails a welfare cost because of the lower surplus from deposit services to savers.

With respect to the run externalities, the term $\partial \xi^*/\partial D$ can be decomposed into a direct effect on the incentives to run and two indirect effects via the deposit and loan rate depicted in (B.10). Similar to above, the direct effect and the indirect effect via the loan rate can be unambiguously signed, but the indirect effect via the deposit rate cannot. In particular, expanding (B.84), we get $\partial GG/\partial D = -\omega(1+\bar{r}_D)[\delta(1+r_D)(1+r_I)+(1-\delta)\xi^*(1+\bar{r}_D)]-(1+r_D)(\theta^*-\delta) < 0$, while $\partial DS/\partial D < 0$ from (B.92), and $\partial LD/\partial D < 0$ from (B.100). But, as mentioned, we find that $\partial GG/\partial \bar{r}_D > 0$, which means that the indirect effect via the deposit rate operates in the opposite direction compared with both the direct effect and indirect effect via the loan rate. Nevertheless, the indirect effect via the deposit rate does not dominate the other two effects and $\partial \xi^*/\partial D > 0$, which means that run-risk increases when deposits go up, all else being equal.

Lastly, combining first-order condition (34) with respect to I and D together with (27), we obtain the following wedge in the scale of intermediation margin (also reported in (37)):

$$SIM_{WD} = \overbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*} \right) \cdot \left(\frac{\partial \xi^*}{\partial I} + \frac{\partial \xi^*}{\partial D} \right)}^{\text{Run externality from intermediation scale}}$$
$$\underbrace{- w_S[U''(e_S-D)D+(1-q)V''(D(1+r_D))D(1+r_D)^2] + w_E(1-q)c''(I)I}_{\text{Surplus to } S \text{ and } E \text{ from higher intermediation scale}}. \tag{B.18}$$

Similar to the other two wedges, (B.18) has a component that captures the externality from the intermediation scale on the run probability and a component that captures the surplus created for S and E. The latter is unambiguously positive as more intermediation, i.e., more deposits channeled to lending, increases the surplus to both savers and entrepreneurs. The impact of the intermediation scale of the run probability captured by $\partial \xi^*/\partial I + \partial \xi^*/\partial D$ (see (B.12) for the detailed expression) comprises of a direct effect as well as indirect effects via the deposit and loan rate, similar to the other two wedges. All three components are hard to sign analytically without knowing the ratio of lending to deposits, I/D, in the private equilibrium.

Overall, the planner balances the run externalities and the additional surpluses to savers and entrepreneurs when deciding how the asset allocation, the capital structure, and the scale of intermediation should differ from the private equilibrium.

---

[3]Note that the equity capital choice does not directly enter the global game constraint GG. Thus, the effect of shifting the capital structure from deposit to equity is written as $\partial \xi^*/\partial E - \partial \xi^*/\partial D = -\partial \xi^*/\partial D$.

B.3 Disciplining Role of Runs

This section explores what is the role of the run in disciplining the banker. In particular, we compute the private equilibrium in which the banker does not internalize the effect of her actions on the run probability, i.e., $\psi_{GG} = 0$ in (24). Table B.1 reports the results. The banker chooses allocations that result in higher run-risk and all agents are worse-off, while capital and liquidity are much lower than in the socially optimal outcomes even when E is favored. As expected, there is a much bigger scope for regulation if B neglected her impact on run-risk and all agents could be made better-off.

	PE $\psi_{GG} > 0$	PE $\psi_{GG} = 0$	SP for weights (w_E, w_S) (0.00,0.20)	(0.10,0.10)	(0.20,0.00)
I	0.862	0.827	0.785	0.873	0.906
LIQ	0.052	0.000	0.221	0.060	0.000
D	0.875	0.803	0.962	0.894	0.867
E	0.038	0.024	0.044	0.039	0.038
r_I	3.097	3.119	3.198	3.089	3.042
$\bar{r}_D$	0.717	0.581	0.804	0.767	0.758
q	0.407	0.425	0.386	0.403	0.408
ℓ	0.057	0.000	0.219	0.065	0.000
k	0.042	0.028	0.044	0.042	0.042
$r_I - (1-\delta)\bar{r}_D$	2.739	2.828	2.796	2.705	2.663
$I + LIQ$	0.914	0.827	1.006	0.933	0.906
$I - E$	0.824	0.803	0.741	0.834	0.867
$E(Div)$	0.745	0.703	0.755	0.747	0.743
$\Delta \mathbb{U}_E$	-	-1.11%	-1.66%	0.33%	1.19%
$\Delta \mathbb{U}_S$	-	-2.10%	3.63%	0.71%	-0.30%
$\Delta \mathbb{U}_B$	-	-0.89%	-0.44%	-0.05%	-0.09%

Table B.1: Private equilibrium allocations when the banker does and does not internalize the effect of her action on run-risk versus Socially optimal solutions. The welfare changes are computed over the level of welfare in the private equilibrium where the banker internalizes run-risk, which is normalized to one for each agent.

B.4 Incomplete Deposit Contracts and Lack of Commitment

This section studies the case of incomplete deposit contracts. First, we show how the wedges in the intermediation margins between the private and social solutions change. Second, we report how the numerical solution for the private equilibrium under incomplete contracts compares to the private and social outcomes discussed in section 3.2.

Under incomplete deposit contracts, the banker only internalizes the effect of terms specified in the deposit contract on the deposit supply. At minimum, these terms include the amount of deposits, D, and the deposit rates, r_D and $\bar{r}_D$. As a result, the banker would be tempted to deviate

when choosing the rest of the balance sheet after she has entered into a deposit contract and received the deposits. The banker does understand that taking more risk increases the cost of raising deposits and would ideally want to promise depositors that she will behave prudently. But, after the deposit contract has been signed, the banker has an incentive to deviate towards lending more, holding fewer liquid assets, and raising less equity.

Depositors have rational expectations and ex ante require that the banker offers higher deposit rates to compensate for the anticipated risk-taking due to the lack of commitment. As a result, the deposit supply schedule has the same functional form to the benchmark environment. The difference in the private equilibrium comes from the fact that the banker will not include the effect of I, LIQ and ξ^* on DS in the respective first-order conditions, i.e., the respective (24) will not include the terms multiplied by ψ_{DS}.

The Lagrange multipliers ψ_{DS} and ψ_{LD} will have the same functional form derived in (B.2) and (B.3), respectively. However, the functional form of ψ_{GG} will be different from the one in (B.4):

$$
\widehat{\psi}_{GG} = -\frac{\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}}{\frac{\partial GG}{\partial \xi^*} - \frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}}. \tag{B.19}
$$

So the difference between the multipliers in the social equilibrium, ζ_{GG} given by (B.14), and private equilibrium, $\widehat{\psi}_{GG}$ given by (B.19), does not only come from the presence of run externalities, but also from externalities arising from contract incompleteness:

$$
\zeta_{GG} - \widehat{\psi}_{GG} =
$$

$$
\underbrace{-\left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{dGG^{-1}}{d\xi^*}}_{\text{Run externality}} + \underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \widehat{\psi}_{GG}\frac{\partial GG}{\partial \bar{r}_D}\right)\frac{\partial DS^{-1}}{\partial \bar{r}_D}\frac{\partial DS}{\partial \xi^*}\frac{dGG^{-1}}{d\xi^*}}_{\text{Incomplete contract externality}}, \tag{B.20}
$$

where $dGG/d\xi^*$ is given as before by (22). Note that, under complete contracts, the difference between ζ_{GG} and ψ_{GG}, given by (B.4), is only due to the run externality, i.e., the first term in (B.20).

The three expressions in (B.21), (B.22), and (B.23) below report the wedges in the three intermediation margins, separating the externalities stemming from incomplete contracts.

The wedge is the asset allocation margin is:

$$
\widehat{AAM}_{WD} = \underbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*} \right) \cdot \left(\frac{\partial \xi^*}{\partial LIQ} - \frac{\partial \xi^*}{\partial I} \right)}_{\text{Run externality from asset allocation}} - \underbrace{w_E(1-q)c''(I)I}_{\text{Surplus to E from additional } I}
$$

$$
+ \left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \widehat{\psi}_{GG} \frac{\partial GG}{\partial \bar{r}_D} \right) \frac{\partial DS}{\partial \bar{r}_D}^{-1} \frac{\partial DS}{\partial \xi^*} \cdot
$$

$$
\underbrace{\cdot \left[\frac{\partial GG}{\partial LIQ} - \frac{\partial GG}{\partial I} - \frac{\partial GG}{\partial r_I} \frac{\partial LD}{\partial r_I}^{-1} \left(\frac{\partial LD}{\partial LIQ} - \frac{\partial LD}{\partial I} \right) \right] \frac{dGG^{-1}}{d\xi^*}}_{\text{(Indirect) Incomplete contract externality}}
$$

$$
\underbrace{- \left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \zeta_{GG} \frac{\partial GG}{\partial \bar{r}_D} \right) \frac{\partial DS}{\partial \bar{r}_D}^{-1} \left(\frac{\partial DS}{\partial LIQ} - \frac{\partial DS}{\partial I} \right).}_{\text{(Direct) Incomplete contract externality}} \tag{B.21}
$$

The terms in the first line are the same ones in (B.15) under complete deposit contracts. The second line captures the effect of the asset allocation on the run threshold, which in turn was not part of the deposit contract that the banker would internalize. The last line captures the direct effect of incomplete contracts on the asset allocation margin, i.e., the planner internalizes how the asset mix affects the deposit supply schedule.

Examining the direct effect first, note that $-(\partial \mathbb{U}_B/\partial \bar{r}_D + \zeta_{GG} \cdot \partial GG/\partial \bar{r}_D) \cdot (\partial DS/\partial \bar{r}_D)^{-1}$ is the planner's Lagrange multiplier on the deposit supply schedule, which we expect to be positive as long as the planner wants to encourage the supply of deposits by offering a higher deposit rate. Also, we have shown that $(\partial DS/\partial I - \partial DS/\partial LIQ) < 0$, which means that having a less liquid asset mix will adversely affect the supply of deposits. The planner internalizes this, but the banker may have an incentive to deviate and take more asset risk after the deposit contract has been signed. The overall direct effect is negative, which means the planner (and a banker that can commit) would like to implement a more liquid asset allocation.

Turning to the indirect externality, $-(\partial \mathbb{U}_B/\partial \bar{r}_D + \widehat{\psi}_{GG} \cdot \partial GG/\partial \bar{r}_D) \cdot (\partial DS/\partial \bar{r}_D)^{-1}$ is the multiplier on DS in the private problem, which we expect to be positive for the same reasons as above. The overall effect from all the terms is negative (see the benchmark model where we sign the other terms). This means that the planner (and a banker that can commit) would, similarly to the direct effect, also want a more liquid asset allocation. Hence, the incomplete contract externality results in more asset risk in private allocations.

Similarly, the wedges for the capital structure becomes:

$$\widehat{CSM}_{WD} = -\underbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*} \right) \frac{\partial \xi^*}{\partial D}}_{\text{Run externality from liabilities mix}} + \underbrace{w_R \left[U''(e_R - D)D + (1-q)V''(D(1+r_D)D(1+r_D)^2 \right]}_{\text{Surplus to S from additional } D}$$

$$\underbrace{-\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \widehat{\psi}_{GG} \frac{\partial GG}{\partial \bar{r}_D} \right) \frac{\partial DS^{-1}}{\partial \bar{r}_D} \frac{\partial DS}{\partial \xi^*} \cdot \left[\frac{\partial GG}{\partial D} - \frac{\partial GG}{\partial r_I} \frac{\partial LD^{-1}}{\partial r_I} \frac{\partial LD}{\partial D} - \frac{\partial GG}{\partial \bar{r}_D} \frac{\partial DS^{-1}}{\partial \bar{r}_D} \frac{\partial DS}{\partial D} \right] \frac{dGG^{-1}}{d\xi^*}}_{\text{(Indirect) Incomplete contract externality}} \cdot$$

(B.22)

Note that in (B.22), the additional terms stem only for the indirect incomplete contract externality through the run threshold because the banker internalizes the effect of D on the deposit supply schedule.

Finally, the wedge in the scale of intermediation margin becomes:

$$\widehat{SIM}_{WD} = \overbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*} \right) \cdot \left(\frac{\partial \xi^*}{\partial I} + \frac{\partial \xi^*}{\partial D} \right)}^{\text{Run externality from intermediation scale}}$$

$$\underbrace{-w_S[U''(e_S - D)D + (1-q)V''(D(1+r_D))D(1+r_D)^2] + w_E(1-q)c''(I)I}_{\text{Surplus to } S \text{ and } E \text{ from higher intermediation scale}}$$

$$+\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \widehat{\psi}_{GG} \frac{\partial GG}{\partial \bar{r}_D} \right) \frac{\partial DS^{-1}}{\partial \bar{r}_D} \frac{\partial DS}{\partial \xi^*} \cdot$$

$$\underbrace{\cdot \left[\frac{\partial GG}{\partial I} + \frac{\partial GG}{\partial D} - \frac{\partial GG}{\partial r_I} \frac{\partial LD^{-1}}{\partial r_I} \left(\frac{\partial LD}{\partial I} + \frac{\partial LD}{\partial D} \right) - \frac{\partial GG}{\partial \bar{r}_D} \frac{\partial DS^{-1}}{\partial \bar{r}_D} \frac{\partial DS}{\partial D} \right] \frac{dGG^{-1}}{d\xi^*}}_{\text{(Indirect) Incomplete contract externality}}$$

$$\underbrace{-\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \zeta_{GG} \frac{\partial GG}{\partial \bar{r}_D} \right) \frac{\partial DS^{-1}}{\partial \bar{r}_D} \frac{\partial DS}{\partial I}}_{\text{(Direct) Incomplete contract externality}} \cdot$$

(B.23)

Along the same lines, the direct incomplete contract externality in (B.23) is only due to the choice of I.

Overall, the externalities from incomplete contracts are an additional source of divergence between private and social intermediation margins. Table B.2 compares the private equilibrium under incomplete deposit contracts to both the private equilibrium under complete contracts and the socially optimal allocations. Comparing the private equilibria under complete and incomplete contracts, we can see that the banker has an incentive to choose a less liquid asset portfolio and more leveraged capital structure. The inability to commit, results in higher run-risk, and the banker needs to cut deposit-taking in order to sustain a not-too-low profit margin. All agents are worse-off. As

a result, the planner cannot only fix the run externalities, but also the inefficiencies arising from incomplete deposit contracts. In other words, the planner can "enable" the banker to commit, which can be beneficial for all agents including the banker.

| | PE *Complete* | PE *Incomplete* | SP for weights (w_E, w_S) | | |
			(0.00,0.20)	(0.10,0.10)	(0.20,0.00)
I	0.862	0.825	0.785	0.873	0.906
LIQ	0.052	0.000	0.221	0.060	0.000
D	0.875	0.797	0.962	0.894	0.867
E	0.038	0.028	0.044	0.039	0.038
r_I	3.097	3.121	3.198	3.089	3.042
$\bar{r}_D$	0.717	0.550	0.804	0.767	0.758
q	0.407	0.424	0.386	0.403	0.408
ℓ	0.057	0.000	0.219	0.065	0.000
k	0.042	0.035	0.044	0.042	0.042
$r_I - (1-\delta)\bar{r}_D$	2.739	2.846	2.796	2.705	2.663
$I + LIQ$	0.914	0.825	1.006	0.933	0.906
$I - E$	0.824	0.797	0.741	0.834	0.867
$E(Div)$	0.745	0.714	0.755	0.747	0.743
$\Delta\mathbb{U}_E$	-	-1.14%	-1.66%	0.33%	1.19%
$\Delta\mathbb{U}_S$	-	-2.12%	3.63%	0.71%	-0.30%
$\Delta\mathbb{U}_B$	-	-0.85%	-0.44%	-0.05%	-0.09%

Table B.2: Private equilibrium allocations under complete and incomplete deposit contracts versus Socially optimal solutions. The welfare changes are computed over the level of welfare in the private equilibrium under complete contracts, which is normalized to one for each agent.

B.5 Loan Market and Price-Taking Behavior

This section presents the private equilibrium outcomes when the banker acts as a price-taker in the loan market, i.e., she takes r_I as given and does not internalize the effect of the other choices in $\mathcal{C}$ on the loan demand schedule (7). Technically, this means that the first-order conditions (24) in the private equilibrium should not include the terms multiplied by ψ_{LD}. This would introduce an additional reason why the privately and socially optimal allocations diverge on top of the run externalities and surplus considerations present in the three wedges in (35), (36), and (37). Indeed, one can derive expressions similar to (B.21), (B.22), and (B.23) where, instead of the terms for the incomplete deposit contract externalities, there would be terms capturing the externalities from pricing taking behavior in the loan market.

Table B.3 compares the private equilibrium when the banker is a price-taker in the loan market with both the private equilibrium when the banker internalizes the loan demand schedule and the socially optimal allocations. Comparing the two private equilibria, the banker extends more loans to entrepreneurs, which reduces the loan rate and the profit margin, when she does not internalize how

her choice affects the loan demand by entrepreneurs. Naturally, the banker is worse-off compared with the private equilibrium where she fully internalizes her actions. Entrepreneurs and savers are better-off—the former because they get more loans and the latter because the banker raises more deposits to fund lending, which pushes up the deposit rate. It may seem that social welfare is higher in the private equilibrium with price-taking behavior compared with the socially optimal outcomes. This is not true. Consider, for example, that the social welfare weights are $w_E = w_S = 0.1$. The difference between social welfare in the planner's solution and the private equilibrium with risk-taking behavior is 1.06%, i.e., the planner does better. The planner still cares about the banker and, thus, she chooses allocations that favor the banker, who did not internalize how her actions affected loan demand. The concerns about the banker's welfare diminish as more weight is placed on S and E, which can be seen from the last column where the social welfare weights are $w_E = w_S = 0.5$. Social welfare is higher by 0.05% in the planner solution compared with the private equilibrium with price-taking behavior.

| | PE | PE | SP for weights (w_E, w_S) | | | |
		Price-taking	(0.00,0.20)	(0.10,0.10)	(0.20,0.00)	(0.50,0.50)
I	0.862	0.976	0.785	0.873	0.906	0.944
LIQ	0.052	0.000	0.221	0.060	0.000	0.056
D	0.875	0.933	0.962	0.894	0.867	0.957
E	0.038	0.043	0.044	0.039	0.038	0.043
r_I	3.097	2.966	3.198	3.089	3.042	3.015
$\bar{r}_D$	0.717	0.981	0.804	0.767	0.758	0.996
q	0.407	0.400	0.386	0.403	0.408	0.395
ℓ	0.057	0.000	0.219	0.065	0.000	0.056
k	0.042	0.044	0.044	0.042	0.042	0.043
$r_I - (1-\delta)\bar{r}_D$	2.739	2.475	2.796	2.705	2.663	2.516
$I + LIQ$	0.914	0.976	1.006	0.933	0.906	1.000
$I - E$	0.824	0.933	0.741	0.834	0.867	0.901
$E(Div)$	0.745	0.740	0.755	0.747	0.743	0.739
$\Delta \mathbb{U}_E$	-	3.57%	-1.66%	0.33%	1.19%	2.57%
$\Delta \mathbb{U}_S$	-	2.33%	3.63%	0.71%	-0.30%	3.37%
$\Delta \mathbb{U}_B$	-	-1.60%	-0.44%	-0.05%	-0.09%	-1.57%

Table B.3: Private equilibrium allocations with and without price-taking banker's behavior versus Socially optimal solutions. The welfare changes are computed over the level of welfare in the private equilibrium where the banker internalizes the loan demand schedule, which is normalized to one for each agent.

B.6 Tools-Augmented Planner

We consider a tools-augmented planner who is endowed with the set of tools $\mathcal{T} \in \mathbb{T}$ and wants to replicate the social planner's allocations $\mathcal{C}_{sp}$, as a private equilibrium. The tools-augmented planner's problem is akin to a Ramsey planner's problem in the public finance literature.

We will consider two types of tools. First, restrictions on regulatory ratios denoted by $\mathcal{T}_R$. Second, Pigouvian taxes imposed directly on $B's$ payoffs and denoted by $\mathcal{T}_P$. For each $\mathcal{T}_R$, there is a regulatory constraint $RC(\mathcal{T}_R, C) \geq 0$, which ties the tool with the endogenous variables C, while for each $\mathcal{T}_P$, there is an additional term in B's utility, $\mathbb{U}_B(\mathcal{T}_P, C)$. It is important to note that the regulatory constraints, RC, are defined as inequalities, i.e., the planner can tighten them but not loosen them while there are no restrictions on Pigouvian taxes which can be positive or negative. Let $\psi_{\mathcal{T}_R}$ be the multipliers that the banker in the private equilibrium assigns to constraint $RC(\mathcal{T}_R, C) \geq 0$.

Under regulation, the optimization margins change to:

$$AAM_{\mathbb{T}} : AAM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}_R} \left[\frac{\partial RC(\mathcal{T}_R, C)}{\partial LIQ} - \frac{\partial RC(\mathcal{T}_R, C)}{\partial I} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T}_P, C)}{\partial LIQ} - \frac{\partial \mathbb{U}_B(\mathcal{T}_P, C)}{\partial I} \right\} = 0,$$
(B.24)

$$CSM_{\mathbb{T}} : CSM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}_R} \left[\frac{\partial RC(\mathcal{T}_R, C)}{\partial E} - \frac{\partial RC(\mathcal{T}_R, C)}{\partial D} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T}_P, C)}{\partial E} - \frac{\partial \mathbb{U}_B(\mathcal{T}_P, C)}{\partial D} \right\} = 0,$$
(B.25)

$$SIM_{\mathbb{T}} : SIM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}_R} \left[\frac{\partial RC(\mathcal{T}_R, C)}{\partial I} + \frac{\partial RC(\mathcal{T}_R, C)}{\partial D} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T}_P, C)}{\partial I} + \frac{\partial \mathbb{U}_B(\mathcal{T}_P, C)}{\partial D} \right\} = 0.$$
(B.26)

We will show that in order to implement the equilibrium allocations of the social planner, denoted by C_{sp}, it is not necessary to solve the full problem of the tools-augmented planner. Instead, it suffices that there are tools, $\mathcal{T} = \{\mathcal{T}_R, \mathcal{T}_P\}$, that first satisfy the regulatory constraints $RC(\mathcal{T}_R, C_{sp}) = 0$ at the planner's allocations, and, second, the intermediation margins in the associated equilibrium are the same as the intermediation margins of the planner. Essentially, this means that the additional terms in (B.24), (B.25), and (B.26) need to equal the wedges derived in (35), (36), and (37). In matrix form, this can be written as:

$$\begin{bmatrix} \Delta_{\mathcal{T}_R} \\ \Delta_{\mathcal{T}_P} \end{bmatrix} \cdot \begin{bmatrix} \Psi_{\mathcal{T}_R} & \bar{\mathcal{T}}_P \end{bmatrix} = WD_{sp},$$
(B.27)

where $\Psi_{\mathcal{T}_R}$ is the $\mathcal{T}_R x1$ vector of the multiplier on the $\mathcal{T}_R$ regulatory constraints, WD_{sp} is the $3x1$ vector of the wedges in the three intermediation margins evaluated at the planner's equilibrium values, $\Delta_{\mathcal{T}_R} \equiv \Delta RC(\mathcal{T}_R, C_{sp})$ is the $3x\mathcal{T}_R$ matrix of the partial derivatives of the relevant variables for each intermediation margin on the $\mathcal{T}_R$ regulatory constraints, $\bar{\mathcal{T}}_P$ is the $\mathcal{T}_P x1$ vector of Pigouvian tools, and $\Delta_{\mathcal{T}_P}$ is the $3x\mathcal{T}_P$ matrix of the coefficient on the tools $\mathcal{T}_P$ in the partial derivatives of the utility terms, i.e., $\Delta_{\mathcal{T}_P} \cdot \bar{\mathcal{T}}_P \equiv \Delta \mathbb{U}_B(\mathcal{T}, C_{sp})$.

Given that there are at most three distorted wedges in intermediation margins, only three independent tools are needed, i.e, $\#\mathcal{T}_R + \#\mathcal{T}_P = 3$. First, consider that there are three tools $\mathcal{T}_P$ and none

$\mathcal{T}_R$. As long as $\Delta_{\mathcal{T}_P}$ is invertible, three Pigouvian tools $\mathcal{T}_P$ are sufficient to implement the planner's solution, i.e., (B.27) has a solution even if regulatory-ratio tools $\mathcal{T}_R$ are not considered. Alternatively, the planner's allocations can be implemented with just three regulatory-ratio tools, if first, the matrix $\Delta_{\mathcal{T}_R}$ is invertible and, second, all elements in $\Psi_{\mathcal{T}_R}$ are positive. But, three regulatory-ratio tools (capital or liquidity) may not be linearly independent, because choosing two of them may replicate the value of the third. For example, a capital and a liquidity tool can be jointly binding, but two liquidity tools cannot. Additionally, some of resulting multipliers $\psi_{\mathcal{T}_R}$ may be negative, because the planner may want to encourage instead of restrict activity (recall that the regulatory constraints RC are inequalities). Indeed, this is the case we study in section 4.3. For these reasons, we combine a capital and a liquidity tool with a (Pigouvian) subsidy on deposit interest expenses to implement the planner's allocations when savers are favored, while a capital tool is combined with (Pigouvian) deposit and lending subsidies when entrepreneurs are favored. When both regulatory-ratio and Pigouvian tools are used, it suffices that the matrix $\Delta_{\mathcal{T}} \equiv [\Delta_{\mathcal{T}_R} \ \Delta_{\mathcal{T}_R}]'$ is invertible in order to implement the planner's allocations.

We now show that (B.27) is a necessary and sufficient condition such that the social planner's solution described in section 3.1 can be decentralized as a private equilibrium by using regulatory tools $\mathcal{T} = \{\mathcal{T}_R, \mathcal{T}_P\} \in \mathbb{T}$. The tools-augmented planner not only chooses optimally allocations $\mathcal{C}$, but also the level of tools $\mathcal{T} \in \mathbb{T}$ and the multipliers $\psi_{\mathcal{T}_R}$, which are the shadow values that the banker assigns to constraints $RC(\mathcal{T}_R, \mathcal{C}) \geq 0$ in the new equilibrium. Her problem is:

$$\max_{\mathcal{C}, \mathcal{T}, \psi_{\mathcal{T}_R}} \mathbb{U}_{\text{sp}}^{\mathbb{T}} \quad s.t. \ \mathscr{Y}(\mathcal{C}) = 0, \ RC(\mathcal{T}_R, \mathcal{C}) \geq 0, \ AAM_{\mathcal{T}}(\mathcal{T}_R, \mathcal{C}, \Psi_{\mathcal{T}}) = 0, \ CSM_{\mathcal{T}}(\mathcal{T}_R, \mathcal{C}, \Psi_{\mathcal{T}}) = 0,$$

$$SIM_{\mathcal{T}}(\mathcal{T}_R, \mathcal{C}, \Psi_{\mathcal{T}}) = 0. \tag{B.28}$$

Note that the additional utility terms $\mathbb{U}_B(\mathcal{T}_P, \mathcal{C})$ due to Pigouvian taxation tools do not appear in the utility that the tools-augmented planner maximizes, because she engages in lump-sum transfers of equal size.

The first-order condition with respect to $\mathcal{C}$ (similar to first-order condition (34)) are:

$$\frac{\partial \mathbb{U}_B}{\partial \mathcal{C}} + w_S \frac{\partial \mathbb{U}_S^*}{\partial \mathcal{C}} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \mathcal{C}} + \sum_{\mathscr{Y}} \zeta_{\mathscr{Y}} \frac{\partial \mathscr{Y}}{\partial \mathcal{C}}$$

$$+ \sum_{\mathcal{T}_R} \zeta_{\mathcal{T}_R} \frac{\partial RC}{\partial \mathcal{C}} + \zeta_{AAM} \frac{\partial AAM_{\mathcal{T}}}{\partial \mathcal{C}} + \zeta_{CSM} \frac{\partial CSM_{\mathcal{T}}}{\partial \mathcal{C}} + \zeta_{SIM} \frac{\partial SIM_{\mathcal{T}}}{\partial \mathcal{C}} = 0, \tag{B.29}$$

where $\zeta_{\mathcal{T}_R}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM} are the multipliers the tool-augmented planner assigns to regulatory constraints and the three regulation-distorted intermediation margins.

The first-order conditions with respect to the level of tools $\mathcal{T}_R$ and $\mathcal{T}_P$, respectively, are:

$$\zeta_{\mathcal{T}_R} \frac{\partial RC}{\partial \mathcal{T}_R} + \zeta_{AAM} \frac{\partial AAM_{\mathcal{T}}}{\partial \mathcal{T}_R} + \zeta_{CSM} \frac{\partial CSM_{\mathcal{T}}}{\partial \mathcal{T}_R} + \zeta_{SIM} \frac{\partial SIM_{\mathcal{T}}}{\partial \mathcal{T}_R} = 0, \tag{B.30}$$

and

$$\zeta_{AAM} \frac{\partial AAM_{\mathcal{T}}}{\partial \mathcal{T}_P} + \zeta_{CSM} \frac{\partial CSM_{\mathcal{T}}}{\partial \mathcal{T}_P} + \zeta_{SIM} \frac{\partial SIM_{\mathcal{T}}}{\partial \mathcal{T}_P} = 0, \tag{B.31}$$

Finally, choosing optimally the multipliers $\psi_{\mathcal{T}_R}$ yields:

$$\zeta_{AAM} \frac{\partial AAM_{\mathcal{T}}}{\partial \psi_{\mathcal{T}_R}} + \zeta_{CSM} \frac{\partial CSM_{\mathcal{T}}}{\partial \psi_{\mathcal{T}_R}} + \zeta_{SIM} \frac{\partial SIM_{\mathcal{T}}}{\partial \psi_{\mathcal{T}_R}} = 0. \tag{B.32}$$

The solutions of the social and tools-augmented planners coincide if the optimality conditions (34) and (B.29) coincide, i.e., if $\zeta_{AAM} = \zeta_{CSM} = \zeta_{SIM} = 0$ and $\zeta_{\mathcal{T}_R} = 0$ for all tools $\mathcal{T}_R$.

To prove sufficiency, note that augmenting (B.32) and (B.31) and representing them in combat form yields $\Delta_{\mathcal{T}}' \cdot [\zeta_{AAM} \; \zeta_{CSM} \; \zeta_{SIM}]' = 0$. Given that $\Delta_{\mathcal{T}}$ should be invertible for (B.27) to yield a solution, its transpose is also invertible, and the only solution is $\zeta_{AAM} = \zeta_{CSM} = \zeta_{SIM} = 0$. Thus, all $\zeta_{\mathcal{T}_R} = 0$ in (B.30) are also zero, and (34) and (B.29) coincide.

To prove necessity, suppose that (B.27) does not hold, i.e., $\Delta_{\mathcal{T}}$ is not invertible and some or all multipliers $\zeta_{\mathcal{T}_R}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM} do not need to be zero. Using conditions (B.29), we can derive intermediation margins $AAM_{TAP} = AAM_{SP} + AAM_{TAP,WD}$, $CSM_{TAP} = CSM_{SP} + CSM_{TAP,WD}$ and $SIM_{TAP} = SIM_{SP} + SIM_{TAP,WD}$ for the tool-augmented planner, where the wedges are linear combination of the multipliers $\zeta_{\mathcal{T}_R}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM}. The social planner's and tools-augmented planner's solutions coincide if wedges $AAM_{TAP,WD}$, $CSM_{TAP,WD}$, and $SIM_{TAP,WD}$ are all zero, which in principle is possible by varying $\zeta_{\mathcal{T}_R}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM}. However, equations (B.30), (B.31), and (B.32) remove as many degrees of freedom and are, thus, generically satisfied when all multipliers are zero—a contradiction.

B.7 Perfectly Elastic Demand Curve

This section studies the special case of a perfectly elastic demand curve, which obtains for $c_I = 0$. Then, from (7), $1 + r_I = A$, and, from (8), $\mathbb{U}_E^* = 0$. There are two implications of abstracting from a downward sloping demand curve. First, the planner does not consider the welfare of entrepreneurs as they make zero profits. Second, the banker cannot manipulate her profit margin by adjusting the volume of lending to affect the loan rate and all of the adjustment in the intermediation spread is happening via the deposit rate.

Table B.4 reports the results from implementing single and combined regulatory tools, which are consistent with the results in the baseline model when the planner places more weight on savers. Tightening the leverage requirement forces the banker to raise more equity and results in some substitution away from liquid assets toward loans. The difference with the case of a downward sloping loan demand curve is that here the banker can extend more lending without pushing the loan rate down and, hence, eroding her profit margin. As in the general case, funding more loans with equity reduces the need for deposits, which also pushes down the deposit rate improving the intermediation margin and enabling the banker to raise more equity. Expected dividends are higher,

but the banker is worse-off, because she had to contribute more equity than what was optimal for her in the private equilibrium.

The results for liquidity regulation are similar to the baselines ones. The only difference is that the reduction in lending does not boost loan rates, and hence the banker cannot increase deposit-taking as much as she would be able to under a downward sloping demand curve. Contrary to the baseline case, the deposit rate falls because the lower run probability dominates the effect from the somewhat higher deposit demand. In the baseline case, lower lending could support a higher spread and, thus, the banker could increase deposit-taking a lot, undoing the effect of lower run-risk on deposit rates and pushing them to levels above their private equilibrium ones.

Comparing column "ℓ" and "$k\&\ell$", we see that leverage and liquidity regulations can be combined to improve welfare. The margin effect of adding a leverage requirement on top of the liquidity requirement is small in this example but goes in the direction of the baseline results. Finally, a subsidy on deposits, $\tau_D = -3.61\%$, is needed to fully implement the planner's allocations, as is the case in the baseline model when the planner puts higher weight on savers.

	PE	k	ℓ	$k\&\ell$	$k,\ell\&\tau_D$
I	0.550	0.558	0.503	0.503	0.510
LIQ	0.109	0.103	0.193	0.193	0.195
D	0.627	0.625	0.663	0.663	0.672
E	0.031	0.036	0.033	0.033	0.034
r_I	2.300	2.300	2.300	2.300	2.300
$\bar{r}_D$	0.754	0.734	0.690	0.689	0.721
q	0.510	0.506	0.504	0.503	0.501
ℓ	0.165	0.156	0.277	0.277	0.277
k	0.047	0.055	0.047	0.048	0.048
$r_I - (1-\delta)\bar{r}_D$	1.923	1.933	1.955	1.955	1.939
$I + LIQ$	0.658	0.661	0.695	0.696	0.706
$I - E$	0.519	0.522	0.470	0.470	0.477
$E(Div)$	0.244	0.256	0.248	0.249	0.249
$\Delta\mathbb{U}_E$	-	0.00%	0.00%	0.00%	0.00%
$\Delta\mathbb{U}_S$	-	-0.05%	0.962%	0.965%	1.23%
$\Delta\mathbb{U}_B$	-	-0.05%	-0.06%	-0.06%	-0.08%
$\Delta\mathbb{S}_{sp}$	-	-0.01%	0.90%	0.90%	1.16%

Table B.4: Equilibrium allocation under perfectly elastic loan demand. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

B.8 Direct Lending

This section derives the conditions for direct lending to entrepreneurs by savers and computes the equilibrium outcomes for the parametrization in section 3.2.

Direct lending requires the individual savers to be able to monitor the entrepreneur. Denote by X_S the monitoring cost to an individual saver, which we assume can be higher or equal to the monitoring cost of the banker, i.e., $X_S \geq X$. At $t = 1$, an individual saver can lend to the entrepreneur, I_{dl}, at interest rate r_{dl}. In the intermediate period, she would liquidate all of her loans if she turns out to be impatient. Otherwise, the saver waits until the final period and receives the percentage repayment on the loans she made. The saver's utility under direct lending is given by

$$\mathbb{U}_S^{dl} = U(e_R - I_{dl}) + \beta\delta \int_{\underline{\xi}}^{\overline{\xi}} \xi \cdot I_{dl} \frac{d\xi}{\Delta\xi} + \beta^2(1 - \delta)\sum_s (\omega I_{dl}(1 + r_{dl}) - X_S)\frac{d\xi}{\Delta\xi}. \tag{B.33}$$

The entrepreneur will choose I_{dl} to maximize her utlity $\mathbb{U}_E^{dl} = (1 - \delta)[\omega I_{dl}(1 + r_{dl}) - c(I_{dl})]$; with probability δ, an individual entrepreneur has her project liquidated and receives zero utility, while, with probability $1 - \delta$, the saver does not liquidate the project, and the entrepreneur incurs the effort cost and defaults in the bad state. E's optimizing behavior yields the following loan demand schedule:

$$1 + r_{dl} = A - c'(I_{dl})/\omega. \tag{B.34}$$

Because each individual saver is sufficiently small, she takes the loan rate as given and, thus, the loan supply schedule is:

$$1 + r_{dl} = \frac{1}{\omega\beta^2(1 - \delta)}\left[U'(e_R - I_{dl}) - \beta\delta E(\xi)\right]. \tag{B.35}$$

The intersection of the loan demand and loan supply schedule in (B.34) and (B.35) yields the equilibrium loan rate and loan amount. Finally, S's and E's levels of welfare in equilibrium are given by $\mathbb{U}_S^{dl*} = U(e_R - I_{dl}) + U'(e_R - I_{dl})I_{dl} - \beta^2(1 - \delta)X_S$ and $\mathbb{U}_E^{dl*} = (1 - \delta)\left(c'(I_{dl})I_{dl} - c(I_d l)\right)$. Table B.5 reports the equilibrium in the loan market together with how S's and E's levels of welfare compare across three cases: the bank intermediation private equilibrium reported in section 3.2, the direct lending equilibrium, and the autarkic outcome when S uses only the storage technology. Savers are better-off under bank intermediation as they enjoy the transaction services of deposits, and they do not need to pay the monitoring cost. However, for $X = X_S$, savers are better-off lending directly to E compared with autarky. There is a level of the monitoring cost X_S that this stops being true ($X_S/X > 1.46$ in our example). Direct lending is higher than bank lending because the bank strategically curtails credit extension to secure higher loan rates and increase her profit margins. Note that the monitoring cost does not affect the marginal choice of I_{dl} because it is not incurred per unit of loans extended but rather applies to the whole portfolio.

B.9 Additional Distortionary Tools

This section extends the analysis in section 3 by allowing the planner to use tools to distort the deposit supply and loan demand schedules of savers and entrepreneurs. We consider generic tools, τ_{DS} for the deposit supply schedule and τ_{LD} for the loan demand schedule, and discuss how they

	Loan rate	Loan amount	$\%\Delta\mathbb{U}_S$	$\%\Delta\mathbb{U}_E$
Intermediation	3.097	0.862	-	-
Direct lending	1.272	0.928	-3.20%	0.38%
Autarky	-	-	-4.34%	-7.61%

Table B.5: Equilibrium allocation under bank intermediation, direct lending, and autarky with storage. The welfare changes are computed over the level of welfare in the private equilibrium with bank intermediation, which is normalized to one for each agent.

can be implemented in practice.

The deposit supply schedule (2) that the planner faces becomes:

$$U'(e_S - D - LIQ_S) = \left[\beta\delta + \beta^2(1-\delta)\right](1+r_D)\int_{\underline{\xi}}^{\xi^*}\theta(\xi,1)\frac{d\xi}{\Delta_\xi}$$

$$+ \left[\beta\delta(1+r_D) + \beta^2(1-\delta)\omega(1+\bar{r}_D) + V'(D(1+r_D))(1+r_D)\right](1-q) + \tau_{DS}. \tag{B.36}$$

The planner can distort the willingness of savers to hold deposits at given deposit rates by varying the level of the distortionary tool τ_{DS}. In other words, the planner can set τ_{DS}, which implies that (B.36) stops being a constraint in her optimization problem defined in Definition 2 and, thus, $\zeta_{DS} = 0$ in (34). The intervention can be implemented, for example, either as a tax on the supply of deposits at $t = 1$ or as a tax on the interest income accruing to late depositors at $t = 3$ when the bank is solvent. In the first case, the tax can be computed as $-\tau_{DS}/U'(e_R - D - LIQ_S)$, while in the second, as $-\tau_{DS}/(\beta^2(1-\delta)\omega(1+\bar{r}_D)(1-q))$. If $\tau_{DS} < 0$, then a tax is levied, while $\tau_{DS} > 0$ implies a subsidy. We assume that the planner rebates the tax proceeds back to the same agents in the same period in a lump-sum fashion in order to neutralize any income effects.

Similarly, the loan demand schedule (7) becomes:

$$\int_{\xi^*}^{\bar{\xi}}\{\omega[A - (1+r^I)](1-y(\xi,\delta)) - c'(I)\}\frac{d\xi}{\Delta_\xi} + \tau_{LD} = 0. \tag{B.37}$$

The planner can distort the willingness of entrepreneurs to borrow by varying the level of the distortionary tool τ_{LD}, such that, if $\tau_{LD} \neq 0$, then $\zeta_{LD} = 0$ in (34). The intervention can be implemented with a tax on loan repayment in the good state of the world, which can be computed as $-\tau_{LD}/(\int_{\xi^*}^{\bar{\xi}}(1-y(\xi,\delta))(1+r_I)d\xi/\Delta_\xi)$. If $\tau_{LD} < 0$, then a tax is levied, while $\tau_{LD} > 0$ implies a subsidy.

Given that the planner may choose to distort the deposit supply and loan demand schedules, we cannot use the indirect utilities (4) and (8), which imply the social welfare function (29). Instead, the planner maximizes the more elaborate social welfare function (28), which considers the direct utilities (1) and (5).

Table B.6 reports the planning equilibria under two sets of weights and three configurations: the

benchmark one where the planner respects the deposit supply and loan demand schedule, a second where she distorts the deposit supply, and a third where she distorts the loan demand. When the planner can distort the deposit supply schedule, she can convince savers to supply deposits even if this is not optimal for them. For example, a distortionary subsidy on deposits makes savers want to supply more deposits and accept lower deposit rates, which is beneficial for the banker and entrepreneurs: lower deposit rates increase the profit margin of the banker, who is then willing to extend more loans at a lower loan rate. The planner goes all the way down to extracting all the surplus from depositors and pushing them to their participation constraint, i.e., their utility in autarky.[4] The opposite is true when the planner distorts the loan demand schedule, i.e., $\tau_{LD} \neq 0$. A distortionary subsidy on investment/borrowing to E allows the planner to increase lending without having to attract E by offering a lower loan rate. Higher lending requires more deposits, which pushes deposit rates up and enhances transaction services. Yet, the planner can compensate the banker with a higher loan rate, which provides incentives for injecting more equity. The planner goes all the way to extracting all the surplus from entrepreneurs and pushing them to their participation constraint.

The higher the weight on entrepreneurs is, the higher is their utility under a positive τ_{DS}, which distorts the deposit supply schedule urging savers to supply more deposits for lower deposit rates. Similarly, the higher the weight on savers is, the higher is their utility under a positive τ_{LD}, which distorts the loan demand schedule urging entrepreneurs to borrow more for higher loan rates. In all cases, the planner still cares about the banker and can transfer some of the surplus back to her. As the weight on other agents becomes higher, this transfer will become smaller. Note that this was not possible in our benchmark analysis because both the banker and the planner had to respect the deposit supply and loan demand schedules. Hence, the banker was always losing from the planner's interventions.

B.10 Negative Interest Rates and Run-Proof Banking

In our benchmark results in section 3.2, we assumed that the bank cannot set (charge) negative interest rates for early withdrawals, i.e., $r_D \geq 0$. In this section, we relax this assumption. On the one hand, a negative r_D may allow the bank to eliminate all run-risk, i.e., the bank is run-proof despite issuing demandable deposits. On the other hand, a negative r_D would reduce the utility from transaction services and the bank would need to offer a higher late deposit rate, $\bar{r}_D$; otherwise, savers may choose to self-insure by holding the liquid asset and stop using the bank.

There are two subtle assumptions we have made that offer the best chance for negative rates to eliminate run-risk without hurting welfare. The first assumption is that $V(0) = 0$, i.e., if the bank sets $r_D = -100\%$, then savers get zero utility. This is important because, if $V(0) \to -\infty$, savers would become explosively worse-off as r_D became more and more negative, and they would require an explosively high $\bar{r}_D$ to supply deposits. As a result, intermediation would be impossible under

---

[4]Given that the planner distorts the deposit supply schedule (2), savers' welfare in equilibrium is not given by (4) but rather (1). Hence, there is no guarantee that savers are strictly better-off under bank intermediation compared to autarky (see footnote 4).

	PE	SP for $(w_E, w_S) = (0.05, 0.15)$			SP for $(w_E, w_S) = (0.15, 0.05)$		
		No tools	$\tau_{DS} \neq 0$	$\tau_{LD} \neq 0$	No tools	$\tau_{DS} \neq 0$	$\tau_{LD} \neq 0$
I	0.862	0.841	1.190	1.002	0.899	1.211	0.991
LIQ	0.052	0.119	0.000	0.000	0.012	0.000	0.000
D	0.875	0.919	1.164	0.956	0.873	1.184	0.946
E	0.038	0.041	0.026	0.045	0.038	0.027	0.045
r_I	3.097	3.131	2.706	3.312	3.051	2.678	3.316
$\bar{r}_D$	0.717	0.778	0.668	1.047	0.761	0.689	1.008
q	0.407	0.398	0.422	0.392	0.407	0.420	0.393
ℓ	0.057	0.124	0.000	0.000	0.013	0.000	0.000
k	0.042	0.043	0.022	0.045	0.042	0.022	0.046
$r_I - (1-\delta)\bar{r}_D$	3.239	3.242	2.872	3.288	3.170	2.833	3.312
$I + LIQ$	0.914	0.960	1.190	1.002	0.911	1.211	0.991
$I - E$	0.824	0.800	1.164	0.956	0.861	1.184	0.946
$E(Div)$	0.745	0.750	0.858	0.864	0.743	0.859	0.864
$\Delta\mathbb{U}_E$	0.076	-0.44%	11.87%	-7.61%	1.02%	12.97%	-7.61%
$\Delta\mathbb{U}_S$	1.050	1.74%	-4.34%	3.35%	-0.10%	-4.34%	2.87%
$\Delta\mathbb{U}_B$	0.327	-0.13%	14.13%	10.11%	-0.08%	14.02%	10.16%
τ_{DS}	-	-	0.164	-	-	0.173	-
τ_{LD}	-	-	-	0.122	-	-	0.120

Table B.6: Privately versus Socially Optimal Solutions when additional distortionary tools are available. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

very negative r_D. The second assumption is that the liquid asset cannot offer the transaction services of deposits (or if it does, its services are inferior to the ones offered by deposits). If deposits and the liquid assets were perfect substitute for transactions, then savers would very quickly switch to holding the liquid assets once r_D became sufficiently negative. Instead, in our environment, savers would still be willing to hold deposits along with the liquid asset, even under considerably negative r_D. The reason is that savers' utility under bank intermediation is strictly higher than in autarky for $r_D > -100\%$ (see the discussion at the end of section 2.1).

We derive below the conditions under which run-proof banking is possible as well as the corresponding private and social equilibrium allocation for our benchmark parametrization. Our analysis has focused on the externalities induced by the banker's behavior when there is positive run-risk in equilibrium. Thus, the natural candidate for comparison is run-proof banking, which is possible given the two aforementioned assumptions.[5]

Given Lemma 1, banking is run-proof if $\hat{\lambda}(\underline{\xi})$—given by combining (13) and (10) for the lowest

---

[5]This does not mean that negative interest rates are only compatible with run-proof banking. Although quantitatively the results in section 3.2 will differ if we allow for negative r_D, the wedges derived in section 3.1 will maintain their functional form and, hence, the sources of divergence between the privately and socially optimal solutions will be the same.

realization of the liquidation value $\xi = \underline{\xi}$ and the highest number of withdrawals $\lambda = 1$—is not lower than one. Hence, liquid asset holdings should satisfy:

$$\hat{\lambda}(\underline{\xi}) = \frac{\theta(\underline{\xi}, 1) - \frac{\underline{\xi}(1+\bar{r}_D)}{(1+r_D)(1+r_I)} - \frac{\underline{\xi} \cdot X/\omega}{D(1+r_D)(1+r_I)}}{1 - \frac{\underline{\xi}(1+\bar{r}_D)}{(1+r_D)(1+r_I)}} \geq 1 \Rightarrow \theta(\underline{\xi}, 1) \geq 1 + \frac{\underline{\xi} \cdot X/\omega}{D(1+r_D)(1+r_I)}$$

$$\Rightarrow LIQ \geq D(1+r_D) - \underline{\xi}\left(I - \frac{X}{\omega(1+r_I)}\right). \tag{B.38}$$

Beyond that level, it is inefficient to hold liquid assets, so (B.38) holds with equality.

If $LIQ \geq \delta D(1+r_D)$ or

$$(1-\delta)D(1+r_D) \geq \underline{\xi}\left(I - \frac{X}{\omega(1+r_I)}\right), \tag{B.39}$$

then the liquid asset holdings of the bank are higher than the predictable early withdrawals and the bank will transfer the excess liquidity, $\widehat{LIQ} = (1-\delta)D(1+r_D) - \underline{\xi}(I - X/(\omega(1+r_I)))$ in the last period. Otherwise, the bank will need to recall a fraction $\hat{y}(\underline{\xi}) = [\underline{\xi}(I - X/(\omega(1+r_I))) - (1-\delta)D(1+r_D)]/(\underline{\xi} \cdot I)$ of loans to pay early withdrawals. Note that, if $\underline{\xi}$ is very small, then (B.39) is satisfied for a larger range of negative r_D; at the limit, as $\underline{\xi} \to 0$, (B.39) is satisfied for all $r_D > -100\%$, i.e., the bank does not need to recall any loans. For the rest of this section, we assume that $\underline{\xi} \to 0$ in order to simplify the algebra.

In the absence of run-risk, the loan demand schedule can be written as

$$1 + r_I = A - c'(I)/\omega. \tag{B.40}$$

The deposit supply schedule is more elaborate, because the excess liquidity $\widehat{LIQ}$ will be distributed pro-rata to the $(1-\delta)$ patient savers when the bank defaults in the bad state of the world.[6] Thus, the percentage payment, $\mathcal{R}$, to patient savers in the bad state is $\mathcal{R} = \widehat{LIQ}/((1-\delta)D(1+\bar{r}_D))$ and the total payoff from holding deposits in the bad state is $\mathcal{R} \cdot D(1+\bar{r}_D)$. Individual savers take the percentage payment in the bad state as given when choosing the amount of deposits. The deposit

---

[6]In the bad state, the bank defaults if $\widehat{LIQ} < (1-\delta)D(1+\bar{r}_D)$, which is true because $\bar{r}_D > r_D$.

supply schedule is, thus, given by the following first-order condition with respect to D:

$$\overbrace{-U'(e_S - D - LIQ_S)}^{\substack{\text{Consumption cost} \\ \text{of depositing}}} + \overbrace{\beta\delta(1 + r_D)}^{\substack{\text{Marginal payoff to} \\ \text{impatient } S}} + \overbrace{\beta^2(1-\delta)\omega(1+\bar{r}_D)}^{\substack{\text{Marginal payoff to} \\ \text{patient } S \text{ in good state}}} + \overbrace{\beta^2(1-\delta)(1-\omega)\mathcal{R}\cdot(1+\bar{r}_D)}^{\substack{\text{Marginal payoff to} \\ \text{patient } S \text{ in bad state}}}$$

$$+ \underbrace{V'(D(1+r_D))(1+r_D)}_{\substack{\text{Marginal payoff from} \\ \text{transaction services}}} = 0$$

$$\Rightarrow \omega(1-\delta)(\bar{r}_D - r_D) = \frac{U'(e_S - D - LIQ_S) - V'(D(1+r_D))(1+r_D) - \left(\beta\delta + \beta^2(1-\delta)\right)(1+r_D)}{\beta^2}, \tag{B.41}$$

where we have substituted the definition of $\mathcal{R}$.

A negative r_D makes self-insuring through holding the liquid asset more appealing to savers. As a result, LIQ_S can be positive, in contrast to the benchmark equilibrium we have studied, and (3) will hold with equality. We assume that this is the case and verify our conjecture in equilibrium, under the same parametrization used for the benchmark equilibrium. Hence,

$$U'(e_S - D - LIQ_S) = \beta\delta + \beta^2(1-\delta). \tag{B.42}$$

Using the balance sheet constraint (9) and (B.38), we can express equity in terms of the lending, deposit and early deposit rate choices:

$$E = I + Dr_D. \tag{B.43}$$

Moreover, using (B.38), (B.40), (B.41), (B.42), and (B.43), the utility of the banker can be re-written as:

$$\mathbb{U}_B^{nr} = W(e_B - E) + \omega\left[(1+r_I)I + \widehat{LIQ} - (1-\delta)D(1+\bar{r}_D)\right] - X$$

$$= W(e_B - I - Dr_D) + \omega\left[A\cdot I - c'(I)I/\omega + \beta^{-2}V'(D(1+r_D))(1+r_D)\right.$$

$$\left. + \left(\beta^{-1}\delta + (1-\delta)\right)r_D\right] - X. \tag{B.44}$$

The private equilibrium is characterized by the choice of I, D, and r_D that result in the highest $\mathbb{U}_B^{nr}$ in (B.44), i.e., by the following first-order condition:

$$-W'(e_B - I - Dr_D) + \omega A - c''(I)I - c'(I) = 0, \tag{B.45}$$

$$-W'(e_B - I - Dr_D)r_D + \omega\beta^{-2}V''(D(1+r_D))(1+r_D)^2 = 0, \tag{B.46}$$

and

$$-W'(e_B - I - Dr_D)D + \omega\beta^{-2}V''(D(1+r_D))D(1+r_D) + \omega\left(\beta^{-1}\delta + (1-\delta)\right) = 0. \quad \text{(B.47)}$$

Table B.7 reports the equilibrium outcomes such that the bank is run-proof. In order to satisfy (B.38), the banker has an incentive to set a negative deposit rate for early withdrawals; otherwise, she would need to hold as many liquid asset as the amount of deposits and could not use any to extend loans. Eliminating the run reduces the risk premium savers demand to hold deposits, but it also reduces the transaction services to savers and hence the convenience yield that the bank extracts. In particular, there is a trade-off between r_D and $\bar{r}_D$. Given that deposits are safer, savers are willing to accept a negative late deposit rate $\bar{r}_D$ in exchange for a less negative r_D. A more negative r_D allows the banker to channel more deposits to loans and still eliminate all run-risk. But, savers would demand higher compensation in terms of $\bar{r}_D$, which reduces banking profits. The banker balances these two effects and offers deposit rates that are making savers indifferent between supplying deposits and self-insuring ($LIQ_S > 0$ in the run-proof equilibrium). Overall, lending and intermediation are lower in the run-proof private equilibrium, while the liquid asset holdings are substantially higher compared to our benchmark PE where we restrict $r_D \geq 0$. Still, the level deposits are comparable across the two private equilibria. For our parametrization, all agents are worse-off in the run-proof equilibrium. Nevertheless, this does not always need to be always the case. For example, if $\underline{\xi}$ was high enough, then the bank could be run-proof without the need to hold a lot of liquid assets, which hurt lending, or charge negative deposit rates for early withdrawals, which diminish transaction services.

Substituting (B.40), (B.41), and (B.42) in (5) and (1), and setting $q = 0$, we get the following indirect utility functions:

$$\mathbb{U}_E^{nr,*} = c'(I)I - c(I) \quad \text{(B.48)}$$

and

$$\mathbb{U}_S^{nr,*} = \mathbb{U}_S^{\alpha} + V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D). \quad \text{(B.49)}$$

The run-proof socially optimal equilibrium allocation maximize a social welfare function $\mathbb{U}_{sp}^{nr} = \mathbb{U}_B^{nr} + w_E\mathbb{U}_E^{nr,*} + w_S\mathbb{U}_S^{nr,*}$ given (B.44), (B.48), and (B.49). As a result, the first-order conditions characterizing the planner's solution are

$$-W'(e_B - I - Dr_D) + \omega A - c''(I)I - c'(I) + w_E\partial\mathbb{U}_E^{nr,*}/\partial I = 0, \quad \text{(B.50)}$$

$$-W'(e_B - I - Dr_D)r_D + \omega\beta^{-2}V''(D(1+r_D))(1+r_D)^2 + w_S\partial\mathbb{U}_S^{nr,*}/\partial D = 0, \quad \text{(B.51)}$$

and

$$-W'(e_B - I - Dr_D)D + \omega\beta^{-2}V''(D(1+r_D))D(1+r_D) + \omega\left(\beta^{-1}\delta + (1-\delta)\right) + w_S\partial\mathbb{U}_S^{nr,*}/\partial r_D = 0, \quad \text{(B.52)}$$

	PE	PE	Run-proof SP for weights (w_E, w_S)		
	Restrict $r_D \geq 0$	*Run-proof*	(0.05,0.15)	(0.10,0.10)	(0.15,0.05)
I	0.853	0.114	0.104	0.107	0.111
LIQ	0.061	0.683	0.693	0.689	0.686
D	0.876	0.739	0.739	0.739	0.739
E	0.039	0.058	0.058	0.058	0.058
r_I	3.108	3.495	3.496	3.496	3.495
r_D	0.000	-0.076	-0.062	-0.067	-0.071
$\bar{r}_D$	0.726	-0.334	-0.344	-0.340	-0.337
q	0.408	0.000	0.000	0.000	0.000
ℓ	0.067	0.857	0.869	0.865	0.861
k	0.042	0.073	0.073	0.073	0.073
$r_I - (1-\delta)\bar{r}_D$	2.382	3.829	3.839	3.836	3.833
$I + LIQ$	0.915	0.797	0.797	0.797	0.797
$I - E$	0.815	0.056	0.046	0.049	0.053
LIQ_S	0.000	0.081	0.080	0.081	0.081
$\Delta \mathbb{U}_E$	-	-7.33%	-7.33%	-7.33%	-7.33%
$\Delta \mathbb{U}_S$	-	-1.26%	-1.19%	-1.21%	-1.23%
$\Delta \mathbb{U}_B$	-	-40.94%	-43.81%	-42.85%	-41.90%

Table B.7: Private equilibrium and socially optimal run-proof allocations. The welfare changes are computed over the level of welfare in the private equilibrium where deposit rates are restricted to be positive and banking is not run-proof. All equilibria are for $\underline{\xi} = 0$.

where $\partial \mathbb{U}_E^{nr,*}/\partial I = c''(I)I > 0$, $\partial \mathbb{U}_S^{nr,*}/\partial D = -V''(D(1+r_D))D(1+r_D)^2 > 0$, and $\partial \mathbb{U}_S^{nr,*}/\partial r_D = -V''(D(1+r_D))D^2(1+r_D) > 0$.

When the planner puts more weight on the saver, she choose a less negative r_D and, hence, needs higher LIQ to implement the run-proof equilibrium, which pushes lending down compared with the private equilibrium or cases where less weight is put on S. Both the banker and entrepreneurs are worse-off compared with the private equilibrium—the welfare loss for the entrepreneur is small because lending and investment are already low in the private run-proof equilibrium, and the higher investment is, the bigger the surplus to E due to the convex effort cost. Overall, all agents are worse-off, even for the planner's allocations, compared with the benchmark private equilibrium indicating that run-proof banking is not optimal for the example we present.

B.11 Outside Equity

This section extends the baseline model such that the bank has an alternative source of funding apart from the equity contributed by the banker and the deposits offered by savers. In particular, we consider a separate group of agents, who we call *outside investors* and who may choose to buy equity at a certain price from the bank at $t = 1$ in exchange for a share of the dividends at $t = 3$. These investors do not have a preference for early consumption nor do they value the transaction

services of deposits contrary to savers. We assume that their preferences are the same as for bankers, but contrary to them, investors do not have the ability to monitor entrepreneurs and, hence, manage a bank themselves. We will refer to the equity contributed by the banker and investors as *inside equity* and *outside equity*, respectively.[7]

Denote by P the price of one share of outside equity and by O the number of shares issued and distributed to outside investors. Inside equity is equally divided into E shares, i.e., the banker first injects inside equity, normalizing the price of each (inside equity) share to one, and then decides how many shares to issue to outside investors and at what price. Thus, the total number of shares is $E + O$ and the total equity capital $E + P \cdot O$. It is convenient to denote bank dividends as $Div(\xi, \delta) = \omega[(1 - y(\xi, \delta)) \cdot I \cdot (1 + r_I) - (1 - \delta) \cdot D \cdot (1 + \bar{r}_D)]$ and dividends per share as $DPS(\xi, \delta) = Div(\xi, \delta)/(E + O)$. Then, investors choose how much of their period 1 endowment, e_O, to invest in equity, in order to maximize

$$\mathbb{U}_O = W(e_O - P \cdot O) + \int_{\xi^*}^{\overline{\xi}} O \cdot DPS(\xi, \delta) \frac{d\xi}{\Delta_\xi}, \tag{B.53}$$

which, taking the dividends per share as given, yields the following outside equity supply (*ES*) schedule:

$$-P \cdot W'(e_O - P \cdot O) + \int_{\xi^*}^{\overline{\xi}} DPS(\xi, \delta) \frac{d\xi}{\Delta_\xi} \leq 0, \tag{B.54}$$

holding with equality for $O > 0$. Finally, substituting (B.54) in (B.53) we get the following indirect utility for outside investors:

$$\mathbb{U}_O^* = W(e_O - P \cdot O) + P \cdot O \cdot W'(e_O - P \cdot O). \tag{B.55}$$

On top of the previous choices in $\mathcal{C}$, the banker will also choose the level of outside equity, O, and the price, P, that are consistent with the equity supply schedule (B.54). Hence, B's choice set becomes $\widetilde{\mathcal{C}} = \mathcal{C} \cup \{O, P\}$. Because B will receive only a fraction $E/(E + O)$ of the dividends, her utility becomes:

$$\widetilde{\mathbb{U}}_B = W(e_B - E) + \int_{\xi^*}^{\overline{\xi}} E \cdot DPS(\xi, \delta) \frac{d\xi}{\Delta_\xi}$$

$$= W(e_B - E) + \int_{\xi^*}^{\overline{\xi}} \left\{ \frac{E}{E + O} \cdot \omega \cdot [(1 - y(\xi, \delta)) \cdot I \cdot (1 + r_I) - (1 - \delta) \cdot D \cdot (1 + \bar{r}_D)] - X \right\} \frac{d\xi}{\Delta_\xi}. \tag{B.56}$$

---

[7]We have assumed a different investor base for outside equity to keep the extension simple. Note that outside equity and deposit markets can be endogenously segmented, i.e., there is no need to exogenously restrict outside investors or depositors to supply deposits or equity, respectively. The decision to abstain from these markets would be consistent with equilibrium equity prices and deposit rates. Intuitively, savers would require a lower price to purchase equity, because equity is less useful for early consumption, because it is worthless in a run and secondary market trading can be frictional, and because it does not provide transaction services. Moreover, an all-equity funding structure would not be possible even if the bank preferred it due to the disciplinary role of runnable debt, which addresses the critique raised by Jacklin (1987). Similarly, outside investors would require a higher deposit rate to supply deposits, since they do not price their transaction services. See, also, Allen, Carletti, and Marquez (2015) for a model of segmented bank equity and deposit markets.

Note that the banker bears the full cost of monitoring.

The functional form of the deposit supply and loan demand schedules, (2) and (7) respectively, are unaffected by the introduction of outside equity. However, the monitoring threshold $\hat{\lambda}$, given by (13) for certain ξ, will change to $\hat{\tilde{\lambda}}$, because the banker will monitor if her share of, rather than the total, dividends is higher than the monitoring cost:

$$\frac{E}{E+O} \cdot \omega \cdot [(1-y(\xi,\hat{\tilde{\lambda}})) \cdot I \cdot (1+r_I) - (1-\hat{\tilde{\lambda}}) \cdot D \cdot (1+\bar{r}_D)] - X \geq 0$$

$$\Rightarrow \hat{\tilde{\lambda}}(\xi) = \frac{(\xi \cdot I + LIQ)(1+r_I) - \xi\left(D(1+\bar{r}_D) + \frac{E+O}{E}\frac{X}{\omega}\right)}{D[(1+r_D)(1+r_I) - \xi(1+\bar{r}_D)]}. \tag{B.57}$$

In other words, outside equity reduces the threshold for withdrawals under which the banker has incentives to monitor, i.e., $\partial\hat{\tilde{\lambda}}/\partial O < 0$. The functional form of the global game constraint (20) does not change but λ^* is replaced with $\tilde{\lambda}^* \equiv \hat{\tilde{\lambda}}(\xi^*)$ in the limits of integration:

$$\widetilde{GG} = \int_{\delta}^{\tilde{\lambda}^*} [\omega D(1+\bar{r}_D) - D(1+r_D)]d\lambda - \int_{\tilde{\lambda}^*}^{\theta^*} D(1+r_D)d\lambda - \int_{\theta^*}^{1} \frac{LIQ+\xi^*I}{\lambda}d\lambda = 0. \tag{B.58}$$

Finally, the balance sheet incorporates the funds form raising outside equity:

$$\widetilde{BS}: \quad I+LIQ = D+E+P\cdot O. \tag{B.59}$$

B chooses variables in $\widetilde{C}$ to maximize $\widetilde{\mathbb{U}}_B$ in (B.56) subject to $\widetilde{\mathcal{Y}} = \{\widetilde{BS}, \widetilde{GG}, DS, LD, ES\}$ given by (B.59), (B.58), (2), (7), and (B.54). The private equilibrium is characterized by the following first-order conditions:

$$\frac{\partial\widetilde{\mathbb{U}}_B}{\partial\widetilde{C}} + \sum_{\widetilde{\mathcal{Y}}} \psi_{\widetilde{\mathcal{Y}}} \frac{\partial\widetilde{\mathcal{Y}}}{\partial\widetilde{C}} = 0, \tag{B.60}$$

where ψ_{ES} is the Lagrange multiplier on the equity supply schedule and $\partial ES/\partial\widetilde{C}$ are the partial derivatives reported in (B.106)-(B.115) in section B.12.[8] Thus, the banker also internalizes how her choices affect the supply of outside equity by investors. At the same time, issuing outside equity has a direct effect on the disutility from injecting inside equity as can be seen by $\partial\widetilde{\mathbb{U}}_B/\partial E = \partial\mathbb{U}_B/\partial E + O/(E+O)\int_{\underline{\xi}}^{\bar{\xi}} DPS(\xi,\delta)d\xi/\Delta_\xi$, where $\partial\mathbb{U}_B/\partial E < 0$ is given by (B.69). Hence, if the issuance of outside equity is positive, the disutility for the banker from injecting equity is lower because it increases her share of dividends. We now turn to the choice of O and P, which are new

---

[8] The partial derivatives $\partial DS/\partial C$ and $\partial LD/\partial C$ are given by (B.90)-(B.97) and (B.98)-(B.105), respectively, while all $\partial DS/\partial O$, $\partial DS/\partial P$, $\partial LD/\partial O$, and $\partial DS/\partial P$ are zero. For choices $c' = \{I, LIQ, D, r_I, r_D, \bar{r}_D\}$, $\partial\widetilde{\mathbb{U}}_B/\partial c' = E/(E+O) \cdot \partial\mathbb{U}_B/\partial c'$, where $\partial\mathbb{U}_B/\partial c'$ are given by (B.66), (B.67), (B.68), (B.71), (B.72) and (B.73), while $\partial\widetilde{\mathbb{U}}_B/\partial\xi^* = \partial\mathbb{U}_B/\partial\xi^* + O \cdot DPS(\xi^*,\delta)$ using (B.70). The partial derivatives $\partial\widetilde{GG}/\partial C$ have the same functional form given by (B.82)-(B.89) with the exception that $\partial\hat{\lambda}/\partial C$ is replaced by $\partial\hat{\tilde{\lambda}}/\partial C$. In turn, the latter partial derivatives have the same function form with the former given by (B.74)-(B.76) and (B.79)-(B.81) with the exception of the partial derivatives with respect to E and ξ^*, which need to account for outside equity. That is, $\partial\hat{\tilde{\lambda}}/\partial E = \xi \cdot O/E^2 \cdot X/\omega/\{D[(1+r_D)(1+r_I) - \xi(1+\bar{r}_D)]\}$ and $\partial\hat{\tilde{\lambda}}/\partial\xi^* = \partial\hat{\lambda}/\partial\xi^* - O/E \cdot X/\omega/\{D[(1+r_D)(1+r_I) - \xi(1+\bar{r}_D)]\}$ using (B.78). The remaining partial derivatives are discussed in the main text.

to our benchmark analysis.

The first-order conditions (B.60) with respect to O and P are:

$$\frac{\partial \widetilde{\mathbb{U}}_B}{\partial O} + \psi_{BS} \cdot P + \psi_{GG} \frac{\partial \widetilde{GG}}{\partial O} + \psi_{ES} \frac{\partial ES}{\partial O} = 0 \tag{B.61}$$

and

$$\psi_{BS} \cdot O + \psi_{ES} \frac{\partial ES}{\partial P} = 0. \tag{B.62}$$

Substituting (B.62) in (B.61) we get:

$$-\underbrace{\left(\frac{\partial \widetilde{\mathbb{U}}_B}{\partial O} + \psi_{GG} \frac{\partial \widetilde{GG}}{\partial O} \right)}_{\substack{\text{Cost of issuing} \\ \text{outside equity}}} = \underbrace{\psi_{BS} \left(P - O \cdot \frac{\partial ES}{\partial P}^{-1} \frac{\partial ES}{\partial O} \right)}_{\substack{\text{Benefit of issuing} \\ \text{outside equity}}}, \tag{B.63}$$

where $\partial \widetilde{\mathbb{U}}_B / \partial O = -E/(E+O) \int_{\xi^*}^{\overline{\xi}} DPS(\xi, \delta) d\xi / \Delta_\xi < 0$, $\partial ES/\partial O < 0$ from (B.110), $\partial ES/\partial P < 0$ from (B.115), and $\partial \widetilde{GG}/\partial O = \omega D(1 + \bar{r}_D) \partial \hat{\tilde{\lambda}}/\partial O < 0$, because $\partial \hat{\tilde{\lambda}}/\partial O = -\xi/E \cdot X/\omega/\{D[(1 + r_D)(1 + r_I) - \xi(1 + \bar{r}_D)]\} < 0$.

The cost of issuing outside equity consists of two components. First, outside equity reduces the share of dividends accruing to the banker, and second, it makes monitoring less likely, which adversely affects the probability of a run, all else being equal. Note that this does not mean that issuing outside equity increases the run probability in equilibrium, since other variables will adjust and the bank may operate with more capital and liquidity reducing the run probability, as we show in the numerical results below. The benefit of issuing outside equity stems from raising additional funds given the shadow value of funding ψ_{BS}. The banker does not take P as given, but internalizes how her choice of O affects P via ES and, hence, accurately captures the marginal funding benefit from issuing outside equity.

The social planner faces the same choices $\widetilde{C}$ and constraints $\widetilde{Y}$ as the banker in the private equilibrium but wants to maximize the social welfare function $\widetilde{\mathbb{U}}_{sp}^* = \widetilde{\mathbb{U}}_B + w_S \mathbb{U}_S^* + w_E \mathbb{U}_E^* + w_S \widetilde{\mathbb{U}}_O^*$, where the utilities are defined in (B.56), (4), (8), (B.55), and $w_O \geq 0$ is the weight assigned to outside investors.

Table B.8 reports the private and social equilibrium outcomes when the bank issues outside equity and the planner assigns zero weight to outside investors, such that we can have a more straightforward comparison to the case that there is no outside equity funding. We, first, compare the private equilibrium outcomes. Issuing outside equity allows the banker to expand the balance sheet, but reduces her share of profits. To compensate for this, the banker decreases lending to improve the profit margin and also injects more inside equity. Relying more on equity financing allows the banker to channel more deposits into the liquid asset resulting in a smaller scale of intermediation. The higher liquidity and capital ratios outweigh the negative effect of outside equity on monitoring incentives and result in lower run-risk compared to the PE without issuance of outside

equity. Deposits become safer and the bank can attract deposits offering lower deposit rates, which further improves the profit margin. Although it is not reported in the table, the banker enjoys higher utility in the PE where she issues outside equity compared with the benchmark equilibrium. Comparing the privately and socially optimal outcomes when the bank issues outside equity, we derive the same conclusion as in the benchmark case. The planner chooses more liquidity and capital to favor S resulting in less intermediation, but also lower run-risk. On the contrary, the planner cuts the liquid asset holdings to support more lending and favor E resulting in higher run-risk compared with the private equilibrium.[9] Finally, comparing the socially optimal outcomes with and without the issuance of outside equity, we find that the planner can implement lower run-risk and achieve higher social welfare when outside equity funding is allowed.

We should note that these observations do not rely on the fact that the planner assigns zero weight on outside investors. If $w_O > 0$, the planner internalizes how the issuance of outside equity matters for outside investors' welfare via (B.55). Hence, the first-order conditions with respect to O and P—given by (B.61) and (B.61) in PE and SP for $w_O = 0$—incorporate additional terms:

$$\frac{\partial \widetilde{\mathbb{U}}_B}{\partial O} + \psi_{BS} \cdot P + \psi_{GG} \frac{\partial \widetilde{GG}}{\partial O} + \psi_{ES} \frac{\partial ES}{\partial O} + w_O \frac{\partial \mathbb{U}_O^*}{\partial O} = 0 \qquad (B.64)$$

and

$$\psi_{BS} \cdot O + \psi_{ES} \frac{\partial ES}{\partial P} + w_O \frac{\partial \mathbb{U}_O^*}{\partial P} = 0, \qquad (B.65)$$

where $\partial \mathbb{U}_O^*/\partial O = -P^2 \cdot O \cdot W''(e_O - P \cdot O) > 0$ and $\partial \mathbb{U}_O^*/\partial P = -P \cdot O^2 \cdot W''(e_O - P \cdot O) > 0$.

Table B.9 reports the privately and socially optimal outcomes when the planner assigns $w_O > 0$. In sum, the planner offers a better price P to outside investors compared with the case that $w_O = 0$, whose welfare improves. But, the rest of the findings remain unchanged.

---

[9]One inconsequential difference is that for $(w_E, w_S) = (0.1, 0.1)$ the planner chooses lower lending compared with the private equilibrium in the presence of outside equity funding, which results in lower utility for E. The reason is that issuing outside equity can help savers further, and, as we show, social welfare is higher for this set of weights. If, instead, we set $(w_E, w_S) = (0.12, 0.08)$, we find that lending as well as liquidity and capital are higher resulting in higher welfare for both E and S compared with the private equilibrium (as is the case in absence of outside equity funding for $(w_E, w_S) = (0.1, 0.1)$).

| | PE | | SP for weights (w_E, w_S) | | | |
| | | | (0.1,0.1) | | (0.2,0.0) | |
	No OE	*OE*	*No OE*	*OE*	*No OE*	*OE*
I	0.862	0.834	0.873	0.829	0.906	0.908
LIQ_1	0.052	0.131	0.060	0.168	0.000	0.020
D	0.875	0.908	0.894	0.937	0.867	0.875
E	0.038	0.041	0.039	0.043	0.038	0.039
O	0.000	0.002	0.000	0.002	0.000	0.001
$E/(E+O)$	1.000	0.962	1.000	0.959	1.000	0.965
r_I	3.097	3.142	3.089	3.154	3.042	3.047
$\bar{r}_D$	0.717	0.697	0.767	0.748	0.758	0.727
P	-	9.902	-	9.378	-	10.502
q	0.407	0.394	0.403	0.386	0.408	0.401
ℓ	0.057	0.136	0.065	0.169	0.000	0.022
k	0.042	0.059	0.042	0.060	0.042	0.058
$r_I - (1-\delta)\bar{r}_D$	2.739	2.794	2.705	2.780	2.663	2.683
$I+LIQ$	0.914	0.965	0.933	0.997	0.906	0.928
$I-E-P\cdot O$	0.824	0.777	0.834	0.769	0.867	0.855
$E(Div)$	0.745	0.789	0.747	0.796	0.743	0.779
$\Delta\mathbb{U}_E$	-	-	0.33%	-0.62%	1.19%	1.93%
$\Delta\mathbb{U}_S$	-	-	0.71%	2.55%	-0.30%	-1.34%
$\Delta\mathbb{U}_B$	-	-	-0.05%	0.59%	-0.09%	-0.19%
$\Delta\mathbb{U}_O$	-	-	-	0.36%	-	-0.05%
$\Delta\mathbb{U}_{sp}$			0.05%	0.06%	0.14%	0.19%

Table B.8: Privately versus Socially optimal solutions when the bank issues outside equity (*OE*) and $w_O = 0$. The welfare changes are computed over the levels of welfare in the respective private equilibrium, which are normalized to one. We have set $e_O = 0.09$ such that outside investors are willing to buy equity at the price offered by the bank, which is true as long as $e_O > 0.062$.

	PE	SP for weights (w_E, w_S, w_O)			
		(0.1,0.1,0.0)	(0.2,0.0,0.0)	(0.1,0.1,0.1)	(0.2,0.0,0.1)
I	0.834	0.829	0.908	0.824	0.907
LIQ_1	0.131	0.168	0.020	0.179	0.023
D	0.908	0.937	0.875	0.942	0.876
E	0.041	0.043	0.039	0.043	0.039
O	0.002	0.002	0.001	0.002	0.001
$E/(E+O)$	0.962	0.959	0.965	0.956	0.963
r_I	3.142	3.154	3.047	3.161	3.048
$\bar{r}_D$	0.697	0.748	0.727	0.749	0.726
P	9.902	9.378	10.502	9.200	10.392
q	0.394	0.386	0.401	0.385	0.401
ℓ	0.136	0.169	0.022	0.178	0.024
k	0.059	0.060	0.058	0.061	0.058
$r_I - (1-\delta)\bar{r}_D$	2.794	2.780	2.683	2.787	2.685
$I + LIQ$	0.965	0.997	0.928	1.003	0.930
$I - E - P \cdot O$	0.777	0.769	0.855	0.763	0.853
$E(Div)$	0.789	0.796	0.779	0.800	0.781
$\Delta\mathbb{U}_E$	-	-0.06%	1.93%	-0.17%	1.91%
$\Delta\mathbb{U}_S$	-	1.22%	-1.34%	1.41%	-1.30%
$\Delta\mathbb{U}_B$	-	-0.06%	-0.19%	-0.07%	-0.19%
$\Delta\mathbb{U}_O$	-	0.05%	-0.05%	0.10%	-0.02%

Table B.9: Privately versus Socially optimal solutions when the bank issues outside equity. The welfare changes are computed over the levels of welfare in the private equilibrium, which are normalized to one. We have set $e_O = 0.09$ such that outside investors are willing to buy equity at the price offered by the bank, which is true as long as $e_O > 0.062$.

B.12 Derivatives

This section reports the partial derivatives of the banker's utility $\mathbb{U}_B$ in (14), the monitoring threshold $\hat{\lambda}$ in (13), the global game constraint GG in (20), the deposit supply schedule DS in (2), the loan demand schedule LD in (7), and the (outside) equity supply schedule ES in (B.54), with respect to the choice variables in $\mathbb{C}$. When it is unambiguous, we also report the sign of the derivatives.

Partial derivatives $\partial \mathbb{U}_B / \partial \mathcal{C}$.

$$\frac{\partial \mathbb{U}_B}{\partial I} = \omega (1-q)(1+r_I) > 0. \tag{B.66}$$

$$\frac{\partial \mathbb{U}_B}{\partial LIQ} = \omega (1+r_I) \log (\overline{\xi}/\xi^*)/\Delta_\xi > 0. \tag{B.67}$$

$$\frac{\partial \mathbb{U}_B}{\partial D} = -\omega \left[\delta (1+r_D)(1+r_I) \log (\overline{\xi}/\xi^*)/\Delta_\xi + (1-\delta)(1-q)(1+\bar{r}_D) \right] < 0. \tag{B.68}$$

$$\frac{\partial \mathbb{U}_B}{\partial E} = -W'(e_B - E) < 0. \tag{B.69}$$

$$\frac{\partial \mathbb{U}_B}{\partial \xi^*} = - \left[\omega \left((\xi^* I - \delta D(1+r_D) + LIQ)/\xi^* (1+r_I) - (1-\delta)D(1+\bar{r}_D) \right) - X \right] /\Delta_\xi < 0. \tag{B.70}$$

$$\frac{\partial \mathbb{U}_B}{\partial r_I} = \omega \left[(1-q)I - (\delta D(1+r_D) - LIQ) \log (\overline{\xi}/\xi^*)/\Delta_\xi \right] > 0. \tag{B.71}$$

$$\frac{\partial \mathbb{U}_B}{\partial r_D} = -\omega \delta D(1+r_I) \log (\overline{\xi}/\xi^*)/\Delta_\xi < 0. \tag{B.72}$$

$$\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} = -\omega (1-\delta)(1-q)D < 0. \tag{B.73}$$

Partial derivatives $\partial \hat{\lambda} / \partial \mathcal{C}$.

$$\frac{\partial \hat{\lambda}}{\partial I} = \xi^* (1+r_I)/ \left[D \left((1+r_D)(1+r_I) - \xi^* (1+\bar{r}_D) \right) \right] > 0. \tag{B.74}$$

$$\frac{\partial \hat{\lambda}}{\partial LIQ} = (1+r_I)/ \left[D \left((1+r_D)(1+r_I) - \xi^* (1+\bar{r}_D) \right) \right] > 0. \tag{B.75}$$

$$\frac{\partial \hat{\lambda}}{\partial D} = -\xi^*(1+\bar{r}_D)/\left[D\left((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right)\right] - \hat{\lambda}/D < 0. \tag{B.76}$$

$$\frac{\partial \hat{\lambda}}{\partial E} = 0. \tag{B.77}$$

$$\frac{\partial \hat{\lambda}}{\partial \xi^*} = \left[I(1+r_I)-(D(1+\bar{r}_D)+X/\omega)\right]/\left[D\left((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right)\right]$$
$$+ \hat{\lambda}(1+\bar{r}_D)/\left[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right] > 0. \tag{B.78}$$

$$\frac{\partial \hat{\lambda}}{\partial r_I} = (\xi^*I+LIQ)/\left[D\left((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right)\right]$$
$$- \hat{\lambda}(1+r_D)/\left[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right] > 0. \tag{B.79}$$

$$\frac{\partial \hat{\lambda}}{\partial r_D} = -\hat{\lambda}(1+r_I)/\left[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right] < 0. \tag{B.80}$$

$$\frac{\partial \hat{\lambda}}{\partial \bar{r}_D} = -(1-\hat{\lambda})\xi^*/\left[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right] < 0. \tag{B.81}$$

Partial derivatives $\partial GG/\partial \mathcal{C}$ (see proof of Corollary 1).

$$\frac{\partial GG}{\partial I} = \omega D(1+\bar{r}_D)\frac{\partial \hat{\lambda}}{\partial I} - \int_{\xi^*}^1 \frac{\xi^*}{\lambda}d\lambda > 0. \tag{B.82}$$

$$\frac{\partial GG}{\partial LIQ} = \omega D(1+\bar{r}_D)\frac{\partial \hat{\lambda}}{\partial LIQ} - \int_{\xi^*}^1 \frac{1}{\lambda}d\lambda \gtrless 0. \tag{B.83}$$

$$\frac{\partial GG}{\partial D} = \omega D(1+\bar{r}_D)\left[\frac{\partial \hat{\lambda}}{\partial D} + (\hat{\lambda}-\delta)/D\right] - (\theta^*-\delta)(1+r_D) < 0. \tag{B.84}$$

$$\frac{\partial GG}{\partial E} = 0. \tag{B.85}$$

$$\frac{\partial GG}{\partial \xi^*} = \omega D(1+\bar{r}_D)\frac{\partial \hat{\lambda}}{\partial \xi^*} - \int_{\xi^*}^1 \frac{I}{\lambda}d\lambda > 0. \tag{B.86}$$

41

$$\frac{\partial GG}{\partial r_I} = \omega D(1+\bar{r}_D)\frac{\partial \hat{\lambda}}{\partial r_I} > 0. \tag{B.87}$$

$$\frac{\partial GG}{\partial r_D} = \omega D(1+\bar{r}_D)\frac{\partial \hat{\lambda}}{\partial r_D} - D(\theta^* - \delta) < 0. \tag{B.88}$$

$$\frac{\partial GG}{\partial \bar{r}_D} = \omega D(1+\bar{r}_D)\frac{\partial \hat{\lambda}}{\partial \bar{r}_D} + \omega D(\hat{\lambda} - \delta) \gtrless 0. \tag{B.89}$$

Partial derivatives $\partial DS/\partial \mathcal{C}$.

$$\frac{\partial DS}{\partial I} = [\beta \delta + \beta^2(1-\beta)] \cdot q \cdot \frac{\xi^* + \underline{\xi}}{2} \cdot \frac{1}{D} > 0. \tag{B.90}$$

$$\frac{\partial DS}{\partial LIQ} = [\beta \delta + \beta^2(1-\beta)] \cdot q \cdot \frac{1}{D} > 0. \tag{B.91}$$

$$\frac{\partial DS}{\partial D} = U''(e_S - D) - [\beta \delta + \beta^2(1-\beta)] \cdot q \cdot \left(LIQ + I \cdot \frac{\xi^* + \underline{\xi}}{2} \right) \frac{1}{D^2}$$
$$+ (1-q)V''(D(1+r_D))(1+r_D)^2 < 0. \tag{B.92}$$

$$\frac{\partial DS}{\partial E} = 0. \tag{B.93}$$

$$\frac{\partial DS}{\partial \xi^*} = \left\{ [\beta \delta + \beta^2(1-\beta)]\frac{LIQ + \xi^* I}{D} - \delta\beta(1+r_D) \right.$$
$$\left. -(1-\delta)\beta^2\omega(1+r_D) - V'(D(1+r_D))(1+r_D) \right\} \Delta_\xi^{-1} < 0. \tag{B.94}$$

$$\frac{\partial DS}{\partial r_I} = 0. \tag{B.95}$$

$$\frac{\partial DS}{\partial r_D} = (1-q)\left[\beta \delta + V'(D(1+r_D)) + V''(D(1+r_D))D(1+r_D) \right] > 0. \tag{B.96}$$

$$\frac{\partial DS}{\partial \bar{r}_D} = \omega \cdot \beta^2(1-\delta)(1-q) > 0. \tag{B.97}$$

Partial derivatives $\partial LD/\partial C$.

$$\frac{\partial LD}{\partial I} = \omega(A - (1 + r_I))\frac{\delta D(1 + r_D) - LIQ}{I^2}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} - (1 - q)c''(I) \lesseqgtr 0. \tag{B.98}$$

$$\frac{\partial LD}{\partial LIQ} = \omega(A - (1 + r_I))\frac{1}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} > 0. \tag{B.99}$$

$$\frac{\partial LD}{\partial D} = -\omega(A - (1 + r))\frac{\delta(1 + r_D)}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} < 0. \tag{B.100}$$

$$\frac{\partial LD}{\partial E} = 0. \tag{B.101}$$

$$\frac{\partial LD}{\partial \xi^*} = -\left[\omega(A - (1 + r_I))\frac{\xi^* I - \delta D(1 + r_D) + LIQ}{\xi^* I} - c'(I)\right]\Delta_\xi^{-1} \gtreqless 0. \tag{B.102}$$

$$\frac{\partial LD}{\partial r_I} = -\omega\left[(1 - q) - \frac{\delta D(1 + r_D)}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi}\right] < 0. \tag{B.103}$$

$$\frac{\partial LD}{\partial r_D} = -\omega(A - (1 + r))\frac{\delta D}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} < 0. \tag{B.104}$$

$$\frac{\partial LD}{\partial \bar{r}_D} = 0. \tag{B.105}$$

Partial derivatives $\partial ES/\partial \tilde{C}$ for extension in section B.11 .

$$\frac{\partial ES}{\partial I} = \frac{1}{E + O}\omega(1 - q)(1 + r_I) > 0. \tag{B.106}$$

$$\frac{\partial ES}{\partial LIQ} = \frac{1}{E + O}\omega(1 + r_I)\log(\bar{\xi}/\xi^*)/\Delta_\xi > 0. \tag{B.107}$$

$$\frac{\partial ES}{\partial D} = -\frac{1}{E + O}\omega\left[\delta(1 + r_D)(1 + r_I)\log(\bar{\xi}/\xi^*)/\Delta_\xi + (1 - \delta)(1 - q)(1 + \bar{r}_D)\right] < 0. \tag{B.108}$$

43

$$\frac{\partial ES}{\partial E} = -\frac{1}{E+O} \int_{\xi^*}^{\overline{\xi}} DPS(\xi, \delta) \frac{d\xi}{\Delta_\xi} < 0. \tag{B.109}$$

$$\frac{\partial ES}{\partial O} = P^2 \cdot W''(e_O - P \cdot O) - \frac{1}{E+O} \int_{\xi^*}^{\overline{\xi}} DPS(\xi, \delta) \frac{d\xi}{\Delta_\xi} < 0. \tag{B.110}$$

$$\frac{\partial ES}{\partial \xi^*} = -DPS(\xi^*, \delta)/\Delta_\xi < 0. \tag{B.111}$$

$$\frac{\partial ES}{\partial r_I} = \frac{1}{E+O} \omega \left[(1-q)I - (\delta D(1+r_D) - LIQ) \log(\overline{\xi}/\xi^*)/\Delta_\xi \right] > 0. \tag{B.112}$$

$$\frac{\partial ES}{\partial r_D} = -\frac{1}{E+O} \omega \delta D(1+r_I) \log(\overline{\xi}/\xi^*)/\Delta_\xi < 0. \tag{B.113}$$

$$\frac{\partial ES}{\partial \bar{r}_D} = -\frac{1}{E+O} \omega(1-\delta)(1-q)D < 0. \tag{B.114}$$

$$\frac{\partial ES}{\partial P} = P \cdot O \cdot W''(e_O - P \cdot O) - W'(e_O - P \cdot O) < 0. \tag{B.115}$$