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### **ABSTRACT**

This paper develops a New Keynesian model with sovereign default risk (NK-Default). We focus on the interaction between monetary policy, conducted according to an interest rate rule that targets inflation, and external defaultable debt issued by the government. Monetary policy and default risk interact since both affect domestic consumption, production, and inflation. We find that default risk amplifies monetary frictions and generates a tension for monetary policy, which increases the volatility of inflation and nominal rates. These monetary frictions in turn discipline sovereign borrowing, slowing down debt accumulation and lowering sovereign spreads. Our framework replicates the positive comovements of spreads with nominal domestic rates and inflation, a salient feature of emerging markets data, and can rationalize the experience of Brazil during the 2015 downturn, with high inflation, nominal rates, and spreads.

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# 1 Introduction

Inflation and sovereign risk are important markers for the credibility of governments in emerging markets. Since the early 2000s, following the steps of advanced economies, many central banks in emerging markets have achieved independence from the central government and have increased their credibility by conquering their historically high inflation. Monetary policy in these countries now largely consists of setting nominal interest rates to target inflation, and the toolkit for central banks is the New Keynesian monetary model with pricing frictions. These models analyze the transmission of interest rate policies to inflation and output but are silent on the interactions with sovereign risk because they have been formulated for advanced open economies, where sovereign risk is not a primary concern.<sup>1</sup> This paper develops a New Keynesian model with sovereign default and analyzes the interactions between monetary policy and sovereign default risk. We find that the efficacy of interest rates rules in managing inflation depends on sovereign risk.

In our framework default risk shapes monetary distortions and affects monetary policy, which is conducted according to a nominal interest rate rule that targets inflation. High default risk leads to low domestic consumption and production as well as high inflation, creating a tension for monetary policy and resulting in an amplification of the volatility of inflation and nominal rates. Monetary policy affects in turn default risk because the presence of monetary distortions curbs government borrowing and leads to lower sovereign spreads. Our *NK-Default* framework combines the workhorse New Keynesian monetary model of Galí and Monacelli (2005) with a standard sovereign default model and can deliver the positive correlations of sovereign spreads with inflation and nominal rates, which we document as a hallmark of emerging markets data.

The small open economy model we consider consists of households, firms, a monetary authority, and a government that borrows internationally. Households value consumption of domestic and foreign goods. They supply labor to intermediate goods firms that produce domestic varieties. The intermediate goods firms are subject to productivity shocks and face frictions in setting their prices, in the tradition of Rotemberg (1982). Final goods firms are competitive and use intermediate goods varieties to produce domestic output, which is consumed by domestic households and exported to the rest of the world. The monetary authority sets nominal interest rates in local currency using an interest rate rule to target domestic inflation. As in standard New Keynesian models, monetary policy and firms' pricing frictions generate monetary frictions, which result in volatile inflation and inefficient production.

The government borrows internationally by issuing long-term bonds denominated in foreign currency and transfers the proceeds from these operations to households. The government lacks commitment to repay its debt and can choose to default. Default is associated with a decline in productivity, which reduces consumption and production and increases inflation. The price of bonds compensates risk-neutral lenders for the risk of default. In this environment, equilibrium default events lead to time-varying default risk and sovereign spreads.

Default risk in our model amplifies monetary frictions and makes it harder for the monetary

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<sup>1</sup>For example, the influential paper by Galí and Monacelli (2005) analyzes monetary policy in the context of perfect financial markets.

authority to stabilize inflation. We measure these frictions with inflation deviations from target and a monetary wedge, recovered from the markup in the New Keynesian Phillips Curve (NKPC). The new insight from our model is that inflation and the monetary wedge also increase with default risk, a mechanism we label *default amplification*. High default risk triggers expectations of future high inflation and recession, and results in high inflation, recession, and a worsening of the monetary wedge now. High expected inflation tends to increase current inflation through the NKPC and low expected consumption lowers current consumption through intertemporal consumption-smoothing. Default risk matters for monetary outcomes.

We consider a Markov problem for the government, which internalizes how its borrowing and default decisions affect the private economy, the monetary policy response, and the resulting monetary frictions. The additional costs from high default risk imposed by monetary frictions result in borrowing wedges in the optimal borrowing condition for the government. Borrowing wedges are positive when monetary wedges are positive and expectations of future inflation are high. In these states, they reduce sovereign borrowing incentives and discipline the government's default risk, a mechanism labeled *monetary discipline*. Lowering debt is useful in our model because the government overborrows and experiences too frequent costly defaults.<sup>2</sup>

We establish that monetary policy interacts with sovereign risk both theoretically, in simplified versions of our model, and quantitatively, in our general model parameterized to Brazil. We also find that these interactions are robust to alternative interest rate rules and to the currency denomination of sovereign debt.

Our theoretical results highlight the tension that default risk presents for monetary policy and identify the mechanisms for default amplification and monetary discipline. We first characterize the default amplification result with simplified preferences that are separable and quasi-linear with respect to foreign goods consumption. We show that if default events are associated with high inflation and low consumption, high default risk leads to increases in inflation and a worsening of the monetary wedge, creating a tension for monetary policy. Following the interest rate rule, nominal rates rise and mute inflation but further increase the monetary wedge. We then move to a two-period example with perfectly rigid prices and characterize these interactions in closed form. Here we establish that default events are indeed associated with low consumption, and high default risk decreases domestic consumption and exports and raises the monetary wedge. We also show that these monetary frictions reduce government borrowing and default risk.

Our NK-Default quantitative model produces patterns for spreads, inflation, and nominal domestic rates that resemble the data of emerging markets. We parameterize our model by setting parameters controlling default, the interest rate rule, and the volatility of productivity shocks such that our model replicates the volatility of sovereign spreads, inflation, and output in Brazil. The model can closely match the target moments and contains additional implications that are consistent with the data. Our model delivers a strong positive comovement of spreads with nominal rates and inflation as in the data.<sup>3</sup> The model is further consistent with the

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<sup>2</sup>Hatchondo et al. (2016) find that sovereign default models with long-term defaultable bonds give the government incentives to overborrow and dilute existing bondholders, and that such dilution incentives are large and important for explaining the sizable spreads in emerging economies.

<sup>3</sup>We document that the positive correlations of spreads with inflation and nominal domestic rates are a robust

evidence on comovements with spreads: a negative correlation with output, and positive ones with the trade balance and the nominal exchange rate, with magnitudes similar to the data. Our model also generates comovements of these variables with output present in emerging markets: output is negatively correlated with inflation, nominal rates, the trade balance, and the nominal exchange rate.

To measure the interactions between monetary and default risk frictions, we compare our benchmark model with two reference models: an *NK-Reference* monetary model without sovereign default risk, similar to Galí and Monacelli (2005), and a real *Default-Reference* model in the tradition of the sovereign default literature, as in Chatterjee and Eyigungor (2012), but with two goods and production.<sup>4</sup> The amplifying effects of sovereign risk on monetary policy lead to higher volatility of inflation and nominal rates in our benchmark NK-Default model, relative to the NK-Reference model. Nominal domestic rates are almost twice as volatile because of default risk. Yet, the disciplining effects of monetary frictions lower equilibrium spreads. The mean spread in our benchmark model is about .5% lower than in the Default-Reference model because the monetary frictions in the NK-Default model slow down debt accumulation. These reference models also fail to deliver the empirically strong positive comovement of spreads with inflation and nominal rates.

We use an event analysis to further quantify the interactions between monetary frictions and default risk. We compare the time paths predicted by our model with the event in Brazil around the 2015 recession and evaluate a counterfactual monetary policy scenario. During the event, output fell in Brazil by about 6%, inflation and the nominal domestic rate increased by about 4%, and spreads increased by about 3%. We apply our model to this event by feeding a sequence of productivity shocks to reproduce the dynamics of output. Our model delivers time paths for inflation, nominal rates, spreads, and nominal exchange rates that resemble those in Brazil. A decline in productivity in our model leads to an increase in the probability of default and hence an increase in sovereign spreads. Inflation rises because lower productivity increases the unit costs of production and because of the rise in default risk. This increase in inflation causes a depreciation of nominal exchange rates. Nominal interest rates increase as monetary policy tightens, in response to the high inflation. We then perform a counterfactual experiment that considers looser monetary policy during the event. In the counterfactual, output decreases less but inflation and spreads increase substantially more. We infer that the increase in nominal rates in Brazil during the event not only controlled inflation but also moderated the debt crisis.

We evaluate the robustness of our results in three extended models. In the first extension, we change the currency denomination of sovereign debt to local currency. We find that the amplifying effects of default risk on monetary policy are robust. The volatilities of inflation and nominal domestic rates in this model with default risk are quite similar to those in our benchmark model and higher than in the NK-Reference model. Sovereign spreads, however, are lower in this model relative to the benchmark (and much lower than in the Default-Reference model) because high depreciation rates in recessions make local currency debt a better hedge.

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feature in 10 emerging markets with central banks that target inflation.

<sup>4</sup>Several studies have incorporated multiple goods and production in real models of sovereign default, such as Mendoza and Yue (2012) and Arellano et al. (2018).

The second and third extensions consider alternative interest rate rules. We evaluate a rule that places more weight on inflation deviations as well as a rule that adds an output gap term. Our results on the interactions between monetary policy and sovereign risk are unaltered. These rules produce less volatile inflation and nominal domestic rates than the benchmark and comparable spreads.

Finally, we evaluate the welfare implications of the benchmark model, reference models, and extended models. Our NK-Default model has two sources of inefficiencies arising from monetary and default risk frictions. A comparison of welfare across these models is shaped by how the details of the sovereign debt market and monetary rules interact with these two frictions. Across the models with default risk, we find that welfare can be higher with monetary frictions than without monetary frictions. In two specifications, when the interest rate rule weights inflation heavily and when debt is denominated in local currency, welfare is higher with monetary frictions. The combination of lower spreads and low volatility of inflation in these monetary economies dominates the higher spreads and nil volatility of inflation in the economy without monetary frictions.

**Related Literature** Our project builds on two distinct literature on emerging markets business cycles: the work on sovereign default and the work on New Keynesian monetary policy. We construct a quantitative sovereign default model, in the tradition of [Eaton and Gersovitz \(1981\)](#), as in [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#) but with long-term debt, as in [Hatchondo and Martinez \(2009\)](#) and [Chatterjee and Eyigungor \(2012\)](#). We expand this framework to incorporate production and an import-export structure. Our domestic monetary environment is close to the workhorse framework of [Galí and Monacelli \(2005\)](#). One methodological difference between our project and standard monetary models is that we use global methods rather than local approximations around the steady state to compute the model. Furthermore, we focus on a simple interest rate rule that captures features of the inflation-targeting policies in emerging markets and do not address optimal monetary policy, along the lines of [Schmitt-Grohé and Uribe \(2007\)](#) or [Corsetti et al. \(2010\)](#).

The literature on sovereign default has recently turned to questions raised by nominal rigidities. Several papers have considered environments with defaultable sovereign debt and downward rigidity of nominal wages. [Na et al. \(2018\)](#) first introduced this friction in a model of sovereign default and emphasize that exchange rate pegs are costly because they prevent devaluations that would adjust real wages to their efficient level. Optimal policy in their environment delivers the joint incidence of devaluations and defaults. [Bianchi et al. \(2018\)](#) study the role of downward rigidity of nominal wages for procyclical fiscal policies, which result from a tradeoff between fiscal policy stimulating demand but possibly increasing default risk. [Bianchi and Mondragon \(2018\)](#) find the incidence of self-fulfilling debt crises for economies that lack monetary independence increases when nominal wages are rigid downward. Our project shares the emphasis in these papers on the interaction between sovereign risk and monetary frictions but differs in important ways. First, price frictions in our model arise from optimal price setting by firms which result in a standard New Keynesian Phillips Curve, where expectations of future inflation matter for current inflation and output. These papers, in contrast, directly impose that

nominal wages are downwardly rigid and abstract from the role of inflation expectations in affecting current inflation and output. Second, our modeling of monetary policy focuses on a positive theory that resembles the practice of many emerging markets central banks, which set interest rates to target inflation.<sup>5</sup>

A large literature, following [Calvo \(1988\)](#), studies the incentives of governments to default on debt that is denominated in local currency with inflation. [Aguiar et al. \(2013\)](#) analyze the tradeoffs generated by monetary policy credibility in a dynamic continuous-time model of self-fulfilling default crises and show that monetary policy credibility helps suppress self-fulfilling debt crises but hinders the benefits of state-contingent payments induced by inflation.<sup>6</sup> [Hur et al. \(2018\)](#) and [Sunder-Plassmann \(2018\)](#) also study the interaction between inflation and defaultable debt denominated in local currency. The former considers exogenous inflation, for given covariance structures with fundamentals, whereas the latter builds on a cash-and-credit model with constant money supply. [Nuno and Thomas \(2019\)](#) build a continuous-time model with local currency debt and default and a discretionary choice of inflation, whereas [Engel and Park \(2019\)](#) analyze how default and inflation incentives shape the composition of sovereign debt between local and foreign currency. In contrast with these papers, we emphasize the joint dynamics of endogenous inflation and sovereign risk, with a monetary authority that uses interest rate rules, hence abstracting from the incentives of using monetary policy to inflate away the debt. We view this work as complementary to ours and important especially for emerging markets that have not been able to achieve central bank independence.

Finally, our model's implications for the terms of trade, nominal and real exchange rates, and government borrowing raise a natural comparison with the work on capital controls and exchange rates in small open economies, such as [Farhi and Werning \(2012\)](#), [Fanelli \(2017\)](#), and [Devereux et al. \(2019\)](#).

## 2 Model

We consider a small open economy composed of households, final good producers, intermediate goods firms, a monetary authority, and a fiscal government. There are three types of goods: final domestic goods, domestic intermediate varieties, and foreign imported goods. The final good is produced using all varieties of differentiated intermediate goods and consumed by both domestic and foreign households. Each intermediate good variety is produced with labor.

Foreign demand for domestic goods (export demand) is given by

$$X_t = \left( \frac{P_t^d}{\varepsilon_t P_t^*} \right)^{-\rho} \zeta,$$

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<sup>5</sup>The statutory objectives, given by the legislature to central banks in inflation-targeting emerging markets, center on controlling inflation. For example, the only objective for the Monetary Policy Committee of the Central Bank of Brazil is the achievement of the inflation targets set by the National Monetary Council.

<sup>6</sup>Concerning the multiplicity of equilibria and the role inflation can play in selecting among them, [Corsetti and Dedola \(2016\)](#) focus on unconventional monetary policy whereas [Bacchetta et al. \(2018\)](#) analyze how interest rate rules can be used to prevent the self-fulfilling crises in the environment of [Lorenzoni and Werning \(2019\)](#).

where  $P_t^d$  is the price of domestic goods in local currency,  $P_t^*$  the price of foreign goods in foreign currency,  $\zeta$  the level of overall foreign demand,  $\rho$  the trade elasticity, and  $\varepsilon_t$  the nominal exchange rate. An increase in  $\varepsilon_t$  represents a depreciation of the home currency. We assume that the law of one price holds, so we can write the price of the foreign good in local currency as  $P_t^f = \varepsilon_t P_t^*$ . The terms of trade  $e_t$  equal

$$e_t = \frac{P_t^f}{P_t^d} = \frac{\varepsilon_t P_t^*}{P_t^d}. \quad (1)$$

Hence, the foreign demand for domestic goods is a function of the terms of trade and the level of overall foreign demand  $\zeta$ :

$$X_t = e_t^\rho \zeta. \quad (2)$$

We normalize the foreign price  $P_t^*$  to one in all periods.

## 2.1 Households

Identical households consume domestic goods  $C_t$  and foreign goods  $C_t^f$  and supply labor  $N_t$ . Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, C_t^f, N_t), \quad (3)$$

where  $u(C_t, C_t^f, N_t)$  is the per-period utility function and  $\beta$  is the discount factor of households.

Taking prices as given, households choose consumption, labor supply, and holdings of domestic bonds  $B_t^d$ . These domestic bonds are denominated in local currency and can only be traded by domestic households. Households own firms and receive their profits  $\Psi_t$ . They also earn labor income  $W_t N_t$  and receive government transfers  $T_t$ . Their budget constraint is given by

$$P_t^d C_t + (1 + \tau_f) P_t^f C_t^f + q_t^d B_{t+1}^d \leq W_t N_t + B_t^d + \Psi_t + T_t \quad (4)$$

where  $q_t^d$  is the nominal price of domestic discount bonds and  $\tau_f$  is a constant consumption tax that households pay on imports. We can characterize households' choices with the following optimality conditions:

$$-\frac{u_{N,t}}{u_{C,t}} = w_t, \quad (5)$$

$$\frac{u_{C^f,t}}{u_{C,t}} = (1 + \tau_f) e_t, \quad (6)$$

$$u_{C,t} = i_t \beta \mathbb{E}_t \left[ \frac{u_{C,t+1}}{\pi_{t+1}} \right]. \quad (7)$$

The real wage is  $w_t = W_t / P_t^d$ , the gross domestic goods inflation, hereafter *inflation*, is  $\pi_t = P_t^d / P_{t-1}^d$ , and the nominal domestic interest rate is the yield of the discount bond price  $i_t \equiv 1 / q_t^d$ .



## 2.2 Final Goods Producers

The final good is produced using a unit measure of differentiated intermediate goods  $y_{jt}$ ,  $j \in [0, 1]$ , under perfect competition,

$$Y_t = \left[ \int_0^1 y_{jt}^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}}, \quad (8)$$

where  $\eta$  is the elasticity of substitution between intermediate goods. The optimization problem of the final goods producers yields the standard demand function

$$y_{jt} = \left( \frac{p_{jt}}{P_t^d} \right)^{-\eta} Y_t, \quad (9)$$

where  $p_{jt}$  is the price of intermediate good  $j$  at time  $t$ . The price of domestic goods  $P_t^d$  is the price index  $P_t^d = \left[ \int_0^1 p_{jt}^{1-\eta} dj \right]^{\frac{1}{1-\eta}}$ .

## 2.3 Intermediate Goods Producers

Each differentiated intermediate good is produced with labor  $n_{jt}$ , using a constant returns to scale production function with productivity  $z_t$ :

$$y_{jt} = z_t n_{jt}. \quad (10)$$

Productivity depend on the aggregate shock  $\tilde{z}_t$  and also on the credit standing of the government  $\Theta_t$  such that  $z_t = z(\tilde{z}_t, \Theta_t)$ . As we see below, when credit standing is good productivity is equal to the shock and when credit standing is bad productivity is lower.

Intermediate goods firms are monopolistically competitive and set the prices for their products, taking as given the demand (9). These firms, however, face price-setting frictions in that they have to pay a quadratic adjustment cost when they change their prices away from the target inflation rate  $\bar{\pi}$ , as in [Rotemberg \(1982\)](#). Taking as given the wage  $W_t$  and the final good price  $P_t^d$ , an intermediate firm  $j$  chooses labor and its price to maximize the present discounted value of profits,

$$\max_{\{n_{jt}, p_{jt}\}} \mathbb{E}_0 \sum_t Q_{t,0} \left\{ p_{jt} y_{jt} - (1 - \tau) W_t n_{jt} - \frac{\varphi}{2} \left( \frac{p_{jt}}{p_{j,t-1}} - \bar{\pi} \right)^2 P_t^d Y_t \right\},$$

subject to the production function (10). Firms discount profits using the stochastic discount factor of households,  $Q_{t,0} = \beta^t \frac{u_{C,t} P_t^d}{u_{C,0} P_0^d}$ , and get a labor subsidy  $\tau$ .<sup>7</sup> The first-order condition for each firm, after imposing symmetry across all firms ( $p_{jt} = P_t^d$ ), results in

$$(1 - \tau) \frac{w_t}{z_t} = \frac{\eta - 1}{\eta} + \frac{1}{\eta} \left\{ \varphi (\pi_t - \bar{\pi}) \pi_t - \mathbb{E}_t \left[ \beta \frac{u_{C,t+1}}{u_{C,t}} \frac{Y_{t+1}}{Y_t} \varphi (\pi_{t+1} - \bar{\pi}) \pi_{t+1} \right] \right\}. \quad (11)$$

This equation is a standard New Keynesian Phillips Curve (NKPC) that relates inflation to a measure of contemporaneous unit cost,  $(1 - \tau)w_t/z_t$ , and expected inflation.

<sup>7</sup>We follow the standard practice in the New Keynesian literature by introducing a constant subsidy designed to alleviate average inefficiencies induced by market power.

## 2.4 The Monetary Authority

The monetary authority conducts policy using a nominal interest rate rule. The nominal domestic rate  $i_t$  depends on a long-run value  $\bar{i}$  and responds to the deviation of inflation from target,  $\pi_t$  relative to  $\bar{\pi}$ , and to monetary shocks  $m_t$  such that

$$i_t = \bar{i} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\alpha_P} m_t. \quad (12)$$

In our benchmark model the interest rate rule targets domestic goods inflation.<sup>8</sup> Later we will also analyze a rule that adds an output gap term.

## 2.5 Government and External Debt

The fiscal government engages in international borrowing using long-term bonds denominated in foreign currency and can default on its debt. To keep long-term debt tractable, we consider random maturity bonds, as in [Hatchondo and Martinez \(2009\)](#). The bond is a perpetuity that specifies a price  $q_t$  and a quantity  $\ell_t$  such that the government receives  $q_t \ell_t$  units of foreign currency in period  $t$ . The following period, a fraction  $\delta$  of the debt matures and the government's debt is the sum of legacy debt that has not matured  $(1 - \delta)B_t$  and the new issuance  $\ell_t$  such that  $B_{t+1} = (1 - \delta)B_t + \ell_t$ . Each unit of debt calls for a payment of  $r^* + \delta$  every period.<sup>9</sup>

The government can default on its debt, and depending on its default history, it is in good or bad credit standing, which is encoded in  $\Theta_t$ . When the government repays the debt,  $D_t = 0$ , credit standing is good  $\Theta_t = 0$  and the government borrows and decides on the level of debt next period  $B_{t+1}$ . When the government defaults,  $D_t = 1$ , it avoids paying the debt but receives a direct utility cost  $v_t$  and a temporary bad credit standing,  $\Theta_t = 1$ . The utility costs  $v_t$  are i.i.d. *enforcement shocks* that control the enforceability of debt. With a bad credit, the government loses access to financial markets and productivity is depressed  $z(\tilde{z}_t, \Theta_t = 1) \leq \tilde{z}_t$ . A government in bad credit standing, regains good credit with probability  $\iota$ , and reenters financial markets with zero debt obligations.

The government transfers  $T_t$  to households which equal the net receipts from its operations. Its budget constraint in local currency conditional on having a good credit standing and repaying its debt is

$$T_t + \tau W_t N_t = \varepsilon_t [q_t (B_{t+1} - (1 - \delta)B_t) - (r^* + \delta)B_t] + \tau_f P_t^f C_t^f, \quad (13)$$

where the net capital inflow from debt operations is multiplied by the nominal exchange rate  $\varepsilon_t$  to convert it to domestic currency. When the government is in bad credit standing, its budget constraint is as in (13) with  $B_t = B_{t+1} = 0$ . The tax rates for labor and foreign goods consumption,  $\tau$  and  $\tau_f$ , are set as in standard New Keynesian models, to correct the markup in goods markets and allow for a static optimal tariff on imports in steady state, such that  $(1 - \tau) = \frac{\eta - 1}{\eta}$  and

<sup>8</sup>We focus on a rule that targets domestic inflation because [Galí and Monacelli \(2005\)](#) find this rule most closely approximates the optimal policy in their setting and due to computational limitations. Targeting a consumer price inflation would require keeping track of the previous period's terms of trade as an additional state variable.

<sup>9</sup>We normalize the debt service payment of the bond to  $r^* + \delta$  so that the default-free bond price for this instrument equals 1.

$(1 + \tau_f) = \frac{\rho}{\rho - 1}$ .<sup>10</sup> Using the definition of the terms of trade (1), the government budget constraint in units of domestic goods is

$$t_t + \tau w_t N_t = e_t [q_t (B_{t+1} - (1 - \delta) B_t) - (r^* + \delta) B_t] + \tau_f e_t C_t^f. \quad (14)$$

The government's objective is to maximize the present discounted value of the flow utility derived from consumption and labor by households,  $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_g^t u(C_t, C_t^f, N_t)$ . The government's discount factor  $\beta_g$  can differ from that of the households,  $\beta$ . The government borrows from competitive risk-neutral international lenders that discount the future at a foreign currency rate  $r^*$ . The bond price is such that they break even in expectation, thus receiving compensation for any expected losses from default:

$$q_t = \frac{1}{1 + r^*} \mathbb{E}_t [(1 - D_{t+1})(r^* + \delta + (1 - \delta)q_{t+1})]. \quad (15)$$

In states where the government does not default,  $D_{t+1} = 0$ , each unit of the discount bond makes a payment  $r^* + \delta$ , and the fraction that does not mature has market value  $(1 - \delta)q_{t+1}$ . In states of default, the associated payoff for lenders is zero. We define the government spread as the difference in the yield-to-maturity of the bond and the international rate  $r^*$ , such that

$$\text{spread}_t = (r^* + \delta) \left( \frac{1}{q_t} - 1 \right).$$

### 3 Equilibrium

We consider a Markov equilibrium where the government takes into account that its states and default and borrowing policies affect the allocations of the private equilibrium and the monetary authority's response. The exogenous states are the productivity and monetary shocks  $s = \{\tilde{z}, m\}$ , and the enforcement shock,  $\nu$ . The endogenous states for the private and monetary equilibrium are the level of debt  $B$ , the credit standing  $\Theta$ , and borrowing  $B'$ . Recall that the government chooses borrowing  $B'$  only when it has good credit.

When the government enters the period with good credit standing  $\Theta_{-1} = 0$  and has endogenous state of debt  $B$ , it chooses whether to default or repay the debt. The default decision determines the end-of-the-period credit standing  $\Theta$ . If it enters the period with bad credit  $\Theta_{-1} = 1$ , the government draws a random variable  $\Lambda$  following a Bernoulli distribution. With probability  $\iota$ ,  $\Lambda = 1$  and the government regains good credit  $\Theta = 0$  if it chooses not to default. The evolution of credit standing is given by

$$\Theta(s, \nu, B, \Theta_{-1}) = \begin{cases} 0 & \text{if } (\Theta_{-1} = 0 \text{ and } D = 0) \text{ or} \\ & (\Theta_{-1} = 1 \text{ and } \Lambda = 1 \text{ and } D = 0) \\ 1 & \text{otherwise.} \end{cases} \quad (16)$$

<sup>10</sup>Such tariff neutralizes the potential incentive of the government to use debt to exert market power with respect to the downward-sloping demand for the country's exports. See for example [Corsetti and Pesenti \(2001\)](#) for details.

The private and monetary equilibrium depends on the shocks, the endogenous state for debt  $B$ , the credit standing  $\Theta$ , and the choice for borrowing  $B'$ , because these variables affect government transfers and productivity. Let  $S = \{s, B, \Theta, B'\}$  be the end-of-period state. The private and monetary equilibrium also depends on the government policy functions for future default  $D' = H_D(s', v', B')$ , borrowing  $B'' = H_B(s', B')$ , and the corresponding credit standing  $\Theta'(s', v', B', \Theta)$ , because of the forward looking nature of the equilibrium.

**Definition 1.** *Private and Monetary Equilibrium.* Given state  $\{S\}$ , the government policy functions for default  $H_D(s', v', B')$ , borrowing  $H_B(s', B')$ , the evolution of credit standing  $\Theta'(s', v', B', \Theta)$ , and the transfer function  $t(S)$  consistent with the government budget constraint, the symmetric private and monetary equilibrium consists of

- Households' policies for domestic goods consumption  $C(S)$ , foreign goods consumption  $C^f(S)$ , labor  $N(S)$ , and domestic debt  $B^d(S)$ ,
- Intermediate and final goods firms' policies for labor  $n(S)$ , inflation  $\pi(S)$ , and final domestic goods' output  $Y(S)$  and exports  $X(S)$ ,
- The wage  $w(S)$ , nominal domestic rate  $i(S)$ , and the terms of trade  $e(S)$

such that: (i) the policies for households satisfy their budget constraint and optimality conditions (5), (6), (7); (ii) the policies of intermediate and final goods firms satisfy their optimization problem (8), (9), (10), and (11); (iii) export demand (2) is satisfied; (iv) the nominal domestic rate satisfies the monetary authority's interest rate rule (12); and (v) labor, domestic goods, and domestic bond markets clear, and the balance of payments condition is satisfied.

The labor market clears so that labor demanded by firms equals labor supplied by households  $n = N$ . Domestic bonds are in zero net supply in the economy, reflected in the market clearing condition  $B^d = 0$ . The resource constraint for domestic goods requires that domestic final goods' output equals domestic consumption and exports net of the adjustment costs,

$$C(S) + X(S) + \frac{\varphi}{2}(\pi - \bar{\pi})^2 Y(S) = Y(S) \quad (17)$$

where aggregate output  $Y(S) = z(\tilde{z}, \Theta) N(S)$ .

The balance of payments condition requires that net imports equal net capital inflows, which here equal the government transfer plus the labor subsidy,

$$e(S)C^f(S)(1 + \tau_f) - X(S) = t(S) + \tau w(S)N(S). \quad (18)$$

**The Monetary Wedge.** The presence of price rigidities leads to inefficient use of labor, as monopolistic firms set time-varying markups. We will make use of a *monetary wedge* to measure these distortions in production, defined as

$$1 + \text{monetary wedge} \equiv \frac{z(\tilde{z}, \Theta)}{w(S)} = -\frac{z(\tilde{z}, \Theta)u_C(S)}{u_N(S)}. \quad (19)$$

This wedge captures deviations from production efficiency and depends on the dynamics of current and future inflation, as seen in the NKPC equation (11). Production efficiency requires that the marginal product of labor  $z$  equals the wage  $w$ , which is the marginal rate of substitution between labor and consumption for households  $-u_N/u_C$ .

### 3.1 Government Recursive Formulation

We now describe the recursive problem of the government, which borrows long-term bonds in international financial markets and can default. The bond price,  $q(s, B')$ , is an endogenous function that compensates lenders for default risk. It depends on the shocks  $s$  and the borrowing level  $B'$ , because these affect the probability to default. The bond price schedule that satisfies the break-even condition for lenders depends on the default and borrowing policy functions of the government,

$$q(s, B') = \frac{1}{1+r^*} \mathbb{E} [(1 - H_D(s', \nu', B'))(r^* + \delta + (1 - \delta)q(s', H_B(s', B')))]. \quad (20)$$

By consolidating the equilibrium conditions and the government budget constraint, the private and monetary allocations can be summarized with the decision rules for domestic and foreign goods consumption  $\{C(S), C^f(S)\}$ , labor  $N(S)$ , inflation  $\pi(S)$ , the nominal domestic rate  $i(S)$ , and the terms of trade  $e(S)$ , which satisfy the following system of dynamic equations:

$$C(S) + e(S)^\rho \xi = \left[1 - \frac{\varphi}{2} (\pi(S) - \bar{\pi})^2\right] z(\bar{z}, \Theta) N(S) \quad (21)$$

$$e(S)^\rho \xi = e(S) [C^f(S) + (1 - \Theta)((r^* + \delta)B - q(s, B')(B' - (1 - \delta)B))] \quad (22)$$

$$\frac{u_{C^f}(S)}{u_C(S)} = \frac{\rho}{\rho - 1} e(S) \quad (23)$$

$$(\pi(S) - \bar{\pi}) \pi(S) = \frac{\eta - 1}{\varphi} \left( -\frac{u_N(S)}{z(\bar{z}, \Theta) u_C(S)} - 1 \right) + \frac{\beta}{u_C(S) z(\bar{z}, \Theta) N(S)} F(s, B', \Theta) \quad (24)$$

$$u_C(S) = i(S) \beta M(s, B', \Theta) \quad (25)$$

$$i(S) = \bar{i} \left( \frac{\pi(S)}{\bar{\pi}} \right)^{\alpha_P} m, \quad (26)$$

where the functions  $F$  and  $M$  are the expectations in the firms' pricing condition (NKPC) and the households' Euler condition given by

$$F(s, B', \Theta) = \mathbb{E} [z(\bar{z}', \Theta') N(S') u_C(S') (\pi(S') - \bar{\pi}) \pi(S')], \quad (27)$$

$$M(s, B', \Theta) = \mathbb{E} \frac{u_C(S')}{\pi(S')}. \quad (28)$$

The future state  $S' = (s', B', \Theta'(s', \nu', B', \Theta), H_B(s', B'))$  depends on the future government policies and the evolution of credit standing given by (16). Note that when  $\Theta = 1$ , the expectations on the right-hand-side of (27) and (28) also include the probability of regaining a good credit  $\iota$ .

The equilibrium conditions (21) to (26) are analogous to those arising in the standard New Keynesian small open economy of Galí and Monacelli (2005). The difference in our model is that the government understands that its choice of default  $D$  and borrowing  $B'$  affect the state  $S$  and

the equilibrium. Moreover, the government's choices determine next period's state variables, which means that future allocations and prices also depend on the government's current choices. These future effects are encoded in the functions  $q(s, B')$ ,  $F(s, B', \Theta)$ , and  $M(s, B', \Theta)$ .

We can now set up the recursive problem of the government. Let  $V(s, \nu, B)$  be the value with the option to default such that

$$V(s, \nu, B) = \max_{D \in \{0,1\}} \left\{ (1 - D)W(s, B) + D \left[ W^d(s) - \nu \right] \right\}, \quad (29)$$

where  $W(s, B)$  is the value from repaying debt, and  $W^d(s) - \nu$  is the value from defaulting. Specifically, the value of repaying is

$$W(s, B) = \max_{B'} \left\{ u(C(S), C^f(S), N(S)) + \beta_g \mathbb{E} V(s', \nu', B') \right\} \quad (30)$$

subject to the private and monetary equilibrium, with  $S = \{s, B, \Theta = 0, B'\}$  and characterized by conditions (21) through (26), and also subject to the break-even condition for the bond price schedule (20).

If the government chooses to default, the debt  $B$  is eliminated, productivity is reduced, and the government suffers the utility cost  $\nu$ . After default, with probability  $\iota$ , the government regains access to the international financial markets and re-enters with zero debt. The defaulting value  $W^d$  net of the enforcement cost is given by

$$W^d(s) = \left\{ u(C(S), C^f(S), N(S)) + \beta_g \mathbb{E} \left[ \iota V(s', \nu', B' = 0) + (1 - \iota) W^d(s') \right] \right\} \quad (31)$$

subject to the private and monetary equilibrium with  $S = \{s, B = 0, \Theta = 1, B' = 0\}$ .

It is convenient to write the default decision of the government as a cutoff rule based on the default cost  $\nu$ . Given that default costs are i.i.d., the default decision  $D(s, \nu, B)$  can be characterized by a cutoff cost  $\nu^*(s, B)$  at which the repayment value is equal to the default value such that

$$\nu^*(s, B) = W^d(s) - W(s, B), \quad (32)$$

and the sovereign is indifferent between the two options. Then  $D(s, \nu, B) = 1$ , whenever  $\nu \leq \nu^*(s, B)$  and zero otherwise. Let  $\Phi$  be the cumulative distribution of  $\nu$ , such that default probability given  $s$  equals  $\Phi(\nu^*(s, B))$ .

We now define the recursive equilibrium for the economy.

**Definition 2. Equilibrium.** *Given the aggregate state  $\{s, \nu, B\}$ , a recursive equilibrium consists of government policies for default  $D(s, \nu, B)$  and borrowing  $B'(s, B)$ , and government value functions  $V(s, \nu, B)$ ,  $W(s, B)$ , and  $W^d(s)$  such that*

- *Taking as given future policy and value functions  $H_D(s', \nu', B')$ ,  $H_B(s', B')$ ,  $V(s', \nu', B')$ ,  $W(s', B')$ , and  $W^d(s')$ , government policies for default and borrowing and value functions solve its optimization problem.*
- *Government policies and values are consistent with future policies and values.*

### 3.2 Government Borrowing

To illustrate the forces shaping debt accumulation, we manipulate the government's problem and derive its optimality condition. In this derivation we have assumed that all functions in the government problem are differentiable.<sup>11</sup> Optimal foreign borrowing satisfies the following Euler equation

$$\begin{aligned} u_{Cf} \left[ q + \frac{dq}{dB'} (B' - (1 - \delta)B) \right] (1 - \tau_m^X) - \tau_m^C \\ = \beta_g \mathbb{E} \left\{ (1 - D') u'_{Cf} [r^* + \delta + (1 - \delta)q'] \left( 1 - \tau_m^{X'} \right) \right\}. \end{aligned} \quad (33)$$

which relates the marginal utility of foreign goods consumption across periods to the bond price schedule  $q(s, B')$ , future default and borrowing decisions  $D'$  and  $B''$ , future bond prices  $q(s', B'')$ , and the borrowing wedges  $\tau_m^X$  and  $\tau_m^C$ . Appendix A contains the explicit derivation of this equation.

The borrowing wedges  $\tau_m^X$  and  $\tau_m^C$  arise only due to monetary frictions and capture the Lagrange multipliers associated with the NKPC and domestic Euler equations. These wedges depend on  $B'$  and are present in this Euler equation because the government internalizes the consequences of its borrowing and default decisions for the monetary frictions. When the NKPC and domestic Euler are slack, these multipliers are zero and so are the borrowing wedges.

Equation (33) reflects the government's borrowing incentives, which are affected by three major forces. The first is the incentive to smooth and tilt the time path of foreign goods consumption. This force is present in standard models without default risk, like Galí and Monacelli (2005), which exhibit the following undistorted international Euler equation

$$q u_{Cf} = \beta_g \mathbb{E} \left[ u'_{Cf} (r^* + \delta + (1 - \delta)q') \right]. \quad (34)$$

Here, borrowing smooths the marginal utility of foreign goods consumption against shocks and achieves the right tilting of consumption over time, given  $q$  and  $\beta_g$ .

The second force affecting borrowing is the endogenous bond price schedule  $q$  and the presence of legacy long-term debt  $(1 - \delta)B$  as emphasized by Arellano and Ramanarayanan (2012). Bond prices decrease with borrowing due to the increased risk of default,  $\frac{\partial q}{\partial B'} \leq 0$ , and a higher legacy debt  $(1 - \delta)B$  incentivizes borrowing because lower prices dilute this debt,  $-\frac{\partial q}{\partial B'} (1 - \delta)B \geq 0$ . Such debt dilution incentives leads to *overborrowing*, as established by Hatchondo et al. (2016). A lower discount for the government,  $\beta_g < \beta$ , also leads to overborrowing as discussed by Aguiar et al. (2019).<sup>12</sup>

The third force works through the borrowing wedges  $\tau_m^X$  and  $\tau_m^C$  and is unique to our model with sovereign risk and monetary frictions. Positive borrowing wedges lower incentives to borrow. The borrowing wedges tend to be large when the monetary wedge or inflation is high. As we explore below, when default is likely, the expectations terms in the NKPC and domestic

<sup>11</sup>We do not require this assumption for the computation of the model, nor do we employ the Euler equation derived in this section for the numerical implementation.

<sup>12</sup>Cuadra and Sapriza (2008) and Hatchondo et al. (2009) model such additional discounting as arising from high political turnover.

Euler equations tend to be high, which depress activity and lead to a high monetary wedge and high inflation. As a consequence, the borrowing wedges are high, giving the government an incentive to reduce  $B'$ . Lowering  $B'$  reduces default risk, which lowers the monetary wedge and inflation. An additional benefit of reduced borrowing is that it leads to a depreciation of the terms of trade and a boost in exports which lowers the monetary wedge. In the next section we provide a sharper characterization of these interactions of monetary frictions and default risk and relate them to our two main mechanisms, default amplification and monetary discipline.

## 4 Interactions of Monetary Frictions with Default Risk

This section contains theoretical results on the interactions of monetary frictions and default risk in simplified versions of our framework. We show that default risk creates a tension for the monetary policy rule between costly inflation and inefficient production. In response to high default risk, an interest rate rule that targets inflation is unable to achieve both inflation stability and production efficiency. We also show that default risk amplifies monetary frictions and leads to larger monetary wedges. These monetary frictions in turn discipline the government's borrowing and lower default risk.

### 4.1 Monetary Tradeoffs with Default Risk<sup>13</sup>

In our environment rising default risk generates a tradeoff between costly inflation variability and inefficient production. As a consequence, aggregate responses are amplified by the presence of default risk, resulting in more volatile inflation, nominal domestic rate, and the monetary wedge. To illustrate this tradeoff, we revisit the key equilibrium NKPC pricing condition:

$$(\pi - \bar{\pi}) \pi = \frac{\eta - 1}{\varphi} \left( -\frac{u_N}{zu_C} - 1 \right) + \frac{\beta}{Yu_C} \mathbb{E} (Y' u'_C (\pi' - \bar{\pi}) \pi'). \quad (35)$$

Firms increase inflation when the monetary wedge is low ( $-u_N/zu_C$  is high) or future expected inflation  $\pi'$  and marginal utility of consumption  $u'_C$  are high.<sup>14</sup> We want to analyze how an increase in default risk impacts inflation incentives. Recall that during a default event, inflation is high and consumption is low due to low productivity. Hence, if the risk of a default next period is high, expectations for future inflation and the marginal utility of consumption are high, which increases the expectation term of the NKPC. Such rise in future expectations from high default risk gives firms incentives to raise current inflation.

In response to these inflationary pressures, the monetary policy rule calls for higher interest rates, tight monetary policy. These high nominal domestic rates, however, depress consumption further through the domestic Euler equation, which reduces production and increases the monetary wedge. Therefore, high default risk leads to inefficient low production, high monetary wedge, and muted inflation due to the tight monetary policy.

<sup>13</sup>This subsection draws heavily from the insightful discussion of Luigi Bocola.

<sup>14</sup>We find that  $u'_C$  and  $Y'$  tend to move in opposite directions but that the net effect is dominated by the marginal utility term and that the covariance between  $Y' u'_C$  and  $\pi'$  tend to be positive.



To understand the tradeoff confronting monetary policy, it is useful to consider a case where nominal interest rates do not respond to inflation. Faced with high default risk that increases expectations of future inflation, firms would increase current inflation. The unresponsive monetary policy would support production and dampen the increase in the monetary wedge. With this counterfactual monetary response, improvements in production efficiency come at the cost of additional inflation. Next, we characterize formally this tension between rising inflation and widening monetary wedges, generated by default risk.

We simplify the model by assuming that preferences are quasi-linear in foreign goods consumption and are given by

$$u(C, C^f, N) = \theta \log C + (1 - \theta) C^f - \frac{N^{1+\frac{1}{\zeta}}}{1 + 1/\zeta}. \quad (36)$$

We evaluate the responses of the model to an increase in default risk  $E_{s'}\Phi(v^*(s', B'))$  from higher government borrowing  $B'$ .<sup>15</sup> With these preferences consumption of foreign goods fully adjusts to accommodate net capital inflows from debt operations. Higher borrowing only affects the private and monetary equilibrium through its impact on default risk, which changes the expectation terms in the pricing condition (24) and the domestic Euler (25). Recall, that these expectation terms are the functions  $F(s, B', \Theta)$  and  $M(s, B', \Theta)$  defined in (27) and (28). We assume that  $F(s, B', \Theta)$  and  $M(s, B', \Theta)$  are differentiable and increase with borrowing, and then analyze how changes in borrowing affect the equilibrium using a first-order Taylor expansion.

**Assumption 1.**  $\partial F(s, B', \Theta)/\partial B' \geq 0$  and  $\partial M(s, B', \Theta)/\partial B' \geq 0$ , and the parameters satisfy the restriction  $a_0 \geq (\partial M(s, B', \Theta)/\partial B')/(\partial F(s, B', \Theta)/\partial B')$ , with  $a_0 = \frac{\varphi^\theta}{(\eta-1)(1+\frac{1}{\zeta}(\alpha_C+\rho(1-\alpha_C)))\beta\bar{i}\bar{\pi}}$  where  $\alpha_C$  is the share of domestic consumption in output at the approximating point in the Taylor expansion.

These assumptions imply that expected inflation and marginal utility of consumption rise with default risk as  $B'$  rises. These properties, that hold in our quantitative model, feed through the equilibrium and affect current allocations and prices. The following proposition characterizes up to first-order the effects on default risk, inflation, the nominal domestic rate, and the monetary wedge from increased borrowing under Assumption 1 and preferences given by (36).

**Proposition 1.** *Higher borrowing increases default risk, inflation, the nominal domestic rate, and the monetary wedge.*

*Proof.* See Appendix B.1 □

High borrowing increases default risk, which in turn affects current outcomes for inflation and the monetary wedge. High default risk increases expectations of future inflation and marginal utility. Given monetary policy  $i$ , the NKPC calls for an increase in current inflation and the domestic Euler equation calls for a decline in current consumption, which increases the monetary wedge. The interest rate rule that target inflation generates an increase in the nominal domestic rate which dampens the rise in inflation but amplifies the increase in the

<sup>15</sup>We could consider other shocks that increase default risk, such as news about future productivity or default costs. Our results are robust to these other sources of variation.

monetary wedge. Default risk acts akin to a cost-push shock, simultaneously increasing inflation and depressing output. A monetary policy that targets inflation will generate sizable monetary wedges and their associated loss of output and consumption, due to default risk.

The effects of government borrowing on inflation and the monetary wedge in Proposition 1 crucially relies on the presence of default risk. In environments without default risk, it is easy to show that higher borrowing would have no effect on inflation, the nominal domestic rate, domestic consumption or output. With quasi-linear preferences, fluctuations in borrowing and capital flows would be simply absorbed by foreign goods consumption.

## 4.2 Default Amplification and Monetary Discipline: A Two-Period Example

This section considers a two-period example to characterize sharply the mechanisms for default amplification and monetary discipline. In this setting we can show that default events are indeed associated with low domestic consumption, in line with Assumption 1, and we prove two main results. The first one, which resembles Proposition 1, establishes that high default risk leads to high monetary wedges and inefficient production. The second result shows that the presence of monetary frictions induce the government to borrow less and reduce default risk. All the derivations and proofs are relegated to Appendix B.2.

In period 1, the government starts with no debt and can issue one-period bonds  $B$  in international markets. Productivity is 1 in period 1 and  $z$  in period 2, which is known in period 1. We abstract from inflation costs but preserve pricing frictions by assuming that in period 1 prices are perfectly rigid, while in period 2 they can be changed costlessly, and the nominal domestic rate in period 1 is set at  $i$ . The only shock is the utility cost from default  $\nu$ , which hits the economy in period 2 and shapes the default decision. The shock  $\nu$  is drawn from a distribution with cumulative and probability density functions  $\Phi(\nu)$  and  $\phi(\nu)$ , respectively, with a strictly increasing hazard function  $h(\nu) = \phi(\nu)/(1 - \Phi(\nu))$ . Default in period 2 induces a productivity loss  $z_d \leq z$ . Household preferences are given by (36).

We solve the model working backwards. The equilibrium in period 2 depends on  $B$  and the default decision of the government. Conditional on repaying, the private equilibrium  $\{C_2, C_2^f, N_2, e_2\}$  satisfies the following conditions:

$$C_2 + e_2^p = zN_2, \quad C_2 = \frac{\rho\theta}{(\rho - 1)(1 - \theta)}e_2, \quad N_2^{\frac{1}{\xi}}C_2 = \theta z, \quad e_2^p = e_2(C_2^f + B). \quad (37)$$

The terms of trade  $e_2$  depend only on the level of domestic goods consumption  $C_2$ . Moreover,  $C_2$ ,  $N_2$ , and  $e_2$  are independent of the level of debt  $B$ , which is fully absorbed by  $C_2^f$ . The private equilibrium in default  $\{C_{2d}, C_{2d}^f, N_{2d}, e_{2d}\}$  satisfies the same conditions, with  $B = 0$  and productivity  $z_d$ . The default decision is governed by a threshold  $\nu^*(B)$  that is linear in  $B$  and the probability of default is given by  $\Phi(\nu^*(B))$ . In this case, the bond price schedule and its derivative are

$$q(B) = \frac{1}{1 + r^*}[1 - \Phi(\nu^*(B))], \quad \frac{\partial q(B)}{\partial B} = -\frac{1}{1 + r^*}\phi(\nu^*(B)). \quad (38)$$

The following Lemma characterizes conditions under which default risk increases.

**Lemma 1.** *Default risk  $\Phi(v^*)$  increases with debt  $B$  and decreases with second-period productivity  $z$ .*

We now analyze the problem in period 1. With rigid prices, the efficient labor market condition  $-u_{N_1} = u_{C_1}$  is not satisfied in general. Instead the equilibrium is shaped by the domestic Euler condition,

$$\frac{1}{C_1} = \frac{\beta i}{\tilde{\pi}} \left[ \frac{1 - \Phi(v^*(B))}{C_2} + \frac{\Phi(v^*(B))}{C_{2d}} \right]. \quad (39)$$

which links the marginal utility of domestic goods consumption across periods to default risk  $\Phi(v^*(B))$ , monetary policy  $i$ , and expected period 2 inflation, assumed fixed at a constant  $\tilde{\pi}$ . Note that any value of  $\tilde{\pi}$  does not generate resource costs or distortions in the second period.

We characterize the private equilibrium and establish a main result on default amplification.

**Proposition 2.** *Higher default risk  $\Phi(v^*)$  increases the monetary wedge in period 1.*

A higher default risk increases the future marginal utility of consumption, since the expectation in the domestic Euler equation (39) places more weight on default states in which consumption is lower,  $C_{2d} \leq C_2$ , because of lower productivity,  $z_d \leq z$ . In response to this low expected future consumption, consumption in period 1,  $C_1$ , declines. The lower  $C_1$  also leads to a real appreciation,  $e_1$  decreases, which reduces exports. Labor is lower because of lower demand in both domestic and export markets.<sup>16</sup> As a result, the monetary wedge increases.

We now analyze the optimal borrowing for the government and associated default risk. With utility linear in  $C^f$ , only  $v$  shocks, and one period debt, the government's optimal borrowing condition (33) becomes simpler and is given by

$$1 - h(v^*(B))B - \frac{\tau_m^C(B)}{1 - \Phi(v^*(B))} = \beta_g(1 + r^*), \quad (40)$$

where  $\tau_m^C(B)$  is the borrowing wedge. When  $\tau_m^C(B) = 0$ , the government borrows up to the point the marginal benefit of borrowing net of the exposure to default risk, captured by the hazard function scaled by the level of debt,  $1 - h(v^*(B))B$ , is equal to the desire to tilt consumption,  $\beta_g(1 + r^*) < 1$ . A positive monetary wedge reduce the incentive to borrow and default risk because the borrowing wedge  $\tau_m^C(B)$  inherits the sign of the monetary wedge.<sup>17</sup>

Next we set monetary policy to generate a positive monetary wedge and show that it disciplines government borrowing and default risk. To this end, we compare our model to a *flexible* price version. These two models have identical equilibria in the second period, for a given level of debt  $B$ , leading to the same default cutoff  $v^*(B)$  and default probability  $\Phi(v^*(B))$ . In the flexible price model, the domestic Euler condition (39) is replaced by an efficient labor

<sup>16</sup>In the next section, we show that in our full model high default risk continues to be associated with an increase in the monetary wedge but with an exchange rate depreciation.

<sup>17</sup>The expression for  $\tau_m^C(B)$  is given by

$$\tau_m^C(B) = \left( 1 + \frac{u_{N_1}(B)}{u_{C_1}(B)} \right) \frac{[1 + (\rho - 1)e_1(B)^{\rho-1}] \beta i \phi(v^*(B)) (u_{C_{2d}} - u_{C_2})}{u_{C_1}(B) \tilde{\pi}}$$

where the sign of the first term is given by that of the monetary wedge and the second term is positive.

market condition  $-u_{N_1^{\text{flex}}} = u_{C_1^{\text{flex}}}$ . The government's optimal borrowing condition is then given by equation (40) with  $\tau_m^C(B) = 0$  due to the absence of monetary frictions. Let the optimal borrowing and the associated default risk be  $B_{\text{flex}}^*$  and  $\Phi_{\text{flex}}^* = \Phi(v^*(B_{\text{flex}}^*))$ , respectively.

It is useful to define the real interest rate in this flexible-price economy,  $r^{\text{flex}}$ , implicitly determined by the domestic Euler condition  $\frac{1}{C_1^{\text{flex}}} = \beta r^{\text{flex}} \left[ \frac{1 - \Phi_{\text{flex}}^*}{C_2} + \frac{\Phi_{\text{flex}}^*}{C_{2d}} \right]$ . This real rate tend to be low with high default risk  $\Phi_{\text{flex}}^*$  because domestic consumption in default is low. The following assumption sets monetary policy such that the real rate is higher in our model.

**Assumption 2.** *The nominal domestic rate satisfies  $i \geq \tilde{\pi} r^{\text{flex}}$ .*

This high nominal rate can be welfare enhancing for households because, as we show below, it lowers excessive default risk.<sup>18</sup> With Assumption 2, if the government were to choose  $B_{\text{flex}}^*$  in our model, domestic consumption and labor would be lower than in the flexible economy and the monetary wedge would be positive. When  $B \geq B_{\text{flex}}^*$ , default risk increases and, according to Proposition 2, such high borrowing induces an even higher monetary wedge and a positive borrowing wedge  $\tau_m^C(B)$ . These forces are summarized in the next lemma.

**Lemma 2.** *The borrowing wedge  $\tau_m^C(B) \geq 0$  for all  $B \geq B_{\text{flex}}^*$ .*

When the borrowing wedge is positive, equation (40) implies that optimal borrowing  $B^*$  will be lower than  $B_{\text{flex}}^*$ . Lemma 2 shows that  $\tau_m^C(B)$  indeed is positive when  $B$  exceeds  $B_{\text{flex}}^*$ , leading to a lower optimal borrowing with pricing frictions  $B^* \leq B_{\text{flex}}^*$  and lower default risk  $\Phi^* \leq \Phi_{\text{flex}}^*$ . The follow proposition establishes our main theoretical result on monetary discipline.

**Proposition 3.** *Equilibrium default risk is lower with price frictions,  $\Phi^* \leq \Phi_{\text{flex}}^*$ .*

The analysis of this section illustrates that default risk worsens monetary wedges, leading to default amplification, which in turn disciplines government borrowing and default risk. In the next section, we show that these forces are quantitatively important in the general model.

## 5 Quantitative Analysis

We now conduct the quantitative analysis of our model. We document key patterns of inflation, nominal domestic rates, and spreads in emerging market data. We then describe the parameterization of the model, discuss decision rules and impulse responses, and compare the model's implications with the data and reference models.

### 5.1 Spread, Inflation, Nominal Domestic Rate, and Output in the Data

Many emerging markets have adopted inflation-targeting regimes as their monetary policy since the early 2000s. This effort has been largely successful in bringing inflation down to

<sup>18</sup>Households welfare is actually hump-shaped in  $i$ , reaching its maximum at a level that exceeds  $\tilde{\pi} r^{\text{flex}}$  because default risk in equilibrium is excessive.

|              | Mean          |             | Correlation with Spread |                       |            |
|--------------|---------------|-------------|-------------------------|-----------------------|------------|
|              | CPI Inflation | Govt Spread | CPI Inflation           | Nominal Domestic Rate | Output     |
| Brazil       | 5.9           | 2.6         | 59                      | 59                    | -62        |
| Chile        | 3.0           | 1.4         | 30                      | 39                    | -49        |
| Colombia     | 5.2           | 3.2         | 74                      | 76                    | -60        |
| Indonesia    | 6.6           | 2.8         | 17                      | 75                    | -62        |
| Korea        | 2.6           | 1.1         | 44                      | 74                    | -30        |
| Mexico       | 4.3           | 2.3         | 48                      | 27                    | -54        |
| Peru         | 2.8           | 3.0         | 50                      | 55                    | -33        |
| Philippines  | 3.9           | 2.9         | 17                      | 82                    | -26        |
| Poland       | 3.0           | 1.7         | 59                      | 52                    | -11        |
| South Africa | 5.8           | 1.9         | 54                      | 20                    | -49        |
| <b>Mean</b>  | <b>4.4</b>    | <b>2.4</b>  | <b>45</b>               | <b>58</b>             | <b>-38</b> |

Table 1: Emerging Market Inflation Targeters, Key Statistics

single digits.<sup>19</sup> We collect data on inflation, spreads, nominal domestic rates, and output for 10 emerging markets that are inflation targeters. The sample of emerging markets consists of those in the JP-Morgan Emerging Market Bond Index (EMBI). Table 1 reports key statistics on the joint behavior of these data, using quarterly series. The data start in 2004, by which point all countries considered had adopted inflation targeting, and run through 2017. Because of data availability, we focus on inflation based on the consumer price index (CPI) and compute it as the log difference in the index relative to four quarters prior.<sup>20</sup> The spreads are EMBI-based and are measured as the difference in yields between foreign currency government bonds of these emerging markets and a comparable U.S. government bond. Domestic nominal rates are short-term rates in local currency from either interbank markets or government instruments, the shortest maturity available. Output is the four-quarter difference in log gross domestic product. We highlight several salient features of the data that will inform our quantitative work.

Inflation is low for these inflation-targeting emerging markets, on average 4.4%. These single-digit inflation patterns contrast sharply with the historical experience of these countries, which have featured several episodes of hyperinflation. Emerging markets bond yields continue to exhibit a sizable spread, on average 2.4%.

We also report correlations of spreads with inflation, nominal domestic rates, and output. As documented in many studies, spreads are negatively correlated with output for this sample, with an average correlation of  $-38\%$ . Correlations of spreads with nominal rates are strongly

<sup>19</sup>See Roger (2009) and Ha et al. (2019) for more details on the implementation and performance of inflation targeting in emerging markets.

<sup>20</sup>We have confirmed that the main moments for domestic goods (producer price index) inflation are very similar to the CPI ones, for the countries where both are available.

positive, on average 58%. Note that the underlying bonds from which spreads and nominal rates are constructed are in different currencies, and hence these comovements reflect positive correlations between inflation and default risk. The correlation between spreads and inflation is positive, with a sample average of 45%.

## 5.2 Functional Forms and Parameterization

We assume separable preferences between a CES composite of domestic goods  $C$  and imported goods  $C^f$  and labor  $N$  such that the per-period utility function is given by

$$u(C_t, C_t^f, N_t) = \log \left[ H(C_t, C_t^f) \right] - \frac{N_t^{1+1/\zeta}}{1+1/\zeta}, \quad (41)$$

where  $H(C_t, C_t^f)$  is the CES composite  $H(C_t, C_t^f) = \left( \theta C_t^{\frac{\rho-1}{\rho}} + (1-\theta)(C_t^f)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$ . Here  $\theta$  controls the share of imports in consumption and  $\rho$  is the trade elasticity. We can derive the consumer price index (CPI) as the price of the bundle of domestic and foreign goods consumption,  $P^{\text{CPI}} = P^d \left[ \theta^\rho + (1-\theta)^\rho e^{1-\rho} \right]^{\frac{1}{1-\rho}}$ , and the resulting CPI inflation,

$$\pi^{\text{CPI}} = \frac{P^{\text{CPI}}}{P_{-1}^{\text{CPI}}} = \pi \frac{\left[ \theta^\rho + (1-\theta)^\rho e^{1-\rho} \right]^{\frac{1}{1-\rho}}}{\left[ \theta^\rho + (1-\theta)^\rho e_{-1}^{1-\rho} \right]^{\frac{1}{1-\rho}}},$$

where the subscript  $_{-1}$  denotes the previous period's value. The rate of depreciation of the nominal exchange rate is

$$\frac{\varepsilon}{\varepsilon_{-1}} = \frac{e}{e_{-1}} \frac{P^d}{P_{-1}^d} = \frac{e}{e_{-1}} \pi, \quad (42)$$

which depends on inflation and the depreciation of the terms of trade.

We assume that productivity shocks  $\tilde{z}_t$  follow an AR(1) process  $\log \tilde{z}_t = \rho_z \log \tilde{z}_{t-1} + \sigma_z \varepsilon_t$  with  $\varepsilon_t \sim \mathcal{N}(0, 1)$ . Following [Chatterjee and Eyigungor \(2012\)](#), we assume that while in default, productivity suffers a convex penalty  $\max \{0, \lambda_0 \tilde{z} + \lambda_1 \tilde{z}^2\}$  with  $\lambda_0 \leq 0 \leq \lambda_1$ , such that

$$z(\tilde{z}, \Theta) = \begin{cases} \tilde{z} & , \text{ if } \Theta = 0 \\ \tilde{z} - \max \{0, \lambda_0 \tilde{z} + \lambda_1 \tilde{z}^2\} & , \text{ otherwise.} \end{cases}$$

We abstract from monetary shocks  $m$  for the benchmark model parameterization, but incorporate them in a later section to analyze the monetary transmission with counterfactuals.

The model also contains enforcement shocks  $\nu$  that control the relative values of repayment and default. We integrate these shocks into our computational technique following [Dvorkin et al. \(2018\)](#) and [Gordon \(2019\)](#). This computational technique consists of augmenting the model with taste shocks in the discrete choice tradition. The taste shocks slightly perturb the borrowing  $B'$  and the default-repayment choices and help with numerical stability and robust convergence in models with long-term defaultable debt. The taste shocks are shaped by two parameters,  $\varrho_B$

and  $q_D$ . The  $q_B$  parameter controls the shock to the taste for borrowing and is set to  $1e^{-6}$ , which is the smallest value that guarantees convergence for a wide range of parameter values while at the same time keeping choice probabilities quite tight. The shocks to the default-repayment decisions map into the model's enforcement shocks  $\nu$  as a logistic distribution with location 0 and scale 1. The parameter  $q_D$  controls the relative importance of the enforcement shocks for the default decision. Appendix F details the structure of taste shocks and their numerical properties, as well as the algorithm for the computation and simulation of the model.

| <i>Assigned Parameters</i>             |                               |
|--|-------------------------------|
| Frisch elasticity                      | $\zeta = 0.33$                |
| Trade elasticity                       | $\rho = 5$                    |
| Domestic consumption weight            | $\theta = 0.62$               |
| Varieties elasticity                   | $\eta = 6$                    |
| International risk-free rate           | $r^* = 0.5\%$                 |
| Market reentry probability             | $\iota = 4.17\%$              |
| Price adjustment cost                  | $\varphi = 58$                |
| Persistence of productivity shock      | $\rho_z = 0.9$                |
| Export demand level                    | $\xi = 1$                     |
| Interest rate rule intercept           | $\bar{i} = \bar{\pi} / \beta$ |
| <i>Parameters from Moment Matching</i> |                               |
| Private discount factor                | $\beta = 0.9866$              |
| Government discount factor             | $\beta_g = 0.9766$            |
| Inflation target                       | $\bar{\pi} = 1.015$           |
| Interest rate rule coefficient         | $\alpha_P = 1.4$              |
| Volatility of productivity shock       | $\sigma_z = 0.95\%$           |
| Productivity in default                | $\lambda_0 = -0.17$           |
|  | $\lambda_1 = 0.19$            |
| Enforcement shock                      | $q_D = 1e^{-4}$               |

Table 2: Parameter Values

We consider a quarterly model and set parameters based on other studies and as part of a moment-matching exercise, to replicate properties of the data of Brazil. The first set of parameters, assigned directly, includes the Frisch elasticity  $\zeta$ , the trade elasticity  $\rho$ , the weight of domestic goods in consumption  $\theta$ , varieties' elasticity  $\eta$ , the international interest rate  $r^*$ , the probability of return to financial markets after default  $\iota$ , the Rotemberg adjustment cost  $\varphi$ , and the persistence of the productivity shock  $\rho_z$ . For the Frisch elasticity, we choose a value of 0.33

|                            | Data | NK-Default |
|----------------------------|------|------------|
| Mean inflation             | 5.9  | 5.9        |
| Mean nominal domestic rate | 11.2 | 11.1       |
| Volatility of inflation    | 1.8  | 1.8        |
| Volatility of output       | 1.9  | 1.9        |
| Volatility of consumption  | 1.8  | 2.0        |
| Mean spread                | 2.6  | 2.6        |
| Volatility of spread       | 0.9  | 0.9        |
| Output, spread correlation | -62  | -60        |

Table 3: Model Fit

following Galí and Monacelli (2005). This is a conservative value in line with the open economy New Keynesian literature. The trade elasticity  $\rho$  is set at 5, as in Devereux et al. (2019). This number is within the range of estimates in the trade elasticity literature. We set  $\theta$  to get Brazil's imports as a share of consumption in the balanced-trade steady state of 15%, which implies  $\theta = 0.62$ , given the value of the trade elasticity. The elasticity of substitution between varieties  $\eta$  is 6, standard in the literature, inducing a 20% markup. The international risk-free rate is 2% annually, consistent with U.S. Treasury yields, implying  $r^* = 0.5\%$ . We target an average length of market exclusion of roughly six years, which is an average duration of sovereign defaults based on Cruces and Trebesch (2013). We set the Rotemberg adjustment cost using the well-known first-order equivalence between Calvo and Rotemberg pricing frictions: our varieties' elasticity of  $\eta = 6$  and a Calvo frequency of price changes of roughly once per year (once every fourth quarter) imply a value for  $\varphi$  of 58.<sup>21</sup> Given that we are considering a short horizon of the data, it is difficult to precisely estimate the persistence of the productivity process. Instead, we set the persistence parameter  $\rho_z$  to a reference value of 0.9, comparable with many international real business cycle studies. Finally, we set the intercept of the interest rate rule to the steady-state condition  $\bar{i} = \bar{\pi}/\beta$  and normalize the level of export demand  $\zeta$  to 1.

The second set of parameters is chosen to match a set of moments of Brazil. These eight parameters are the discount factor of the private sector  $\beta$  and of the government  $\beta_g$ , the inflation target  $\bar{\pi}$ , the interest rate rule coefficient  $\alpha_p$ , the volatility of the productivity innovations  $\sigma_z$ , the parameters of the default cost function  $\{\lambda_0, \lambda_1\}$ , and the parameter governing the importance of the enforcement shock  $\varrho_D$ . The moments we target are the mean and volatilities for CPI inflation and spreads, mean nominal domestic rate, the volatility of output and consumption, and the correlation between the spread and output. Most parameters affect all moments, yet some moments are more informative of certain parameters. The average CPI inflation rate in the data is the most informative on  $\bar{\pi}$ . The weight on inflation in the interest rate rule  $\alpha_p$  heavily affects the volatility of CPI inflation. The volatility of productivity shocks is the main driver of that of

<sup>21</sup>See, for example, Miao and Ngo (2018) for the mapping between the Calvo and Rotemberg parameters.



output. As in standard sovereign default models, the productivity default cost parameters, the borrower’s discount  $\beta_g$ , and the enforcement shock parameter  $\varrho_D$  are crucial for the dynamics of spreads and the volatility of consumption. The discount factor of the private sector  $\beta$  controls the average real domestic rate. We collect the values of all the parameters in Table 2.

Table 3 contains the results of the moment-matching exercise. CPI inflation, nominal domestic rates, and spreads are reported annualized. The model matches quite closely the moments in the data. In the model and data, CPI inflation is about 5.9%, spreads are 2.6%, and nominal domestic rates are about 11%. The volatilities for CPI inflation, output, and consumption are a bit under 2%, and the volatility of spreads is just under 1%. Output is negatively correlated with spreads, with a correlation close to  $-60\%$ .

### 5.3 Reference Models

To measure the interactions between monetary frictions and default risk, we compare our findings with two reference models. The first reference model, labeled **NK-Reference**, is a version of the Galí and Monacelli (2005) model with nominal rigidities and without default. The second reference model, labeled **Default-Reference**, is our sovereign default model without monetary frictions.

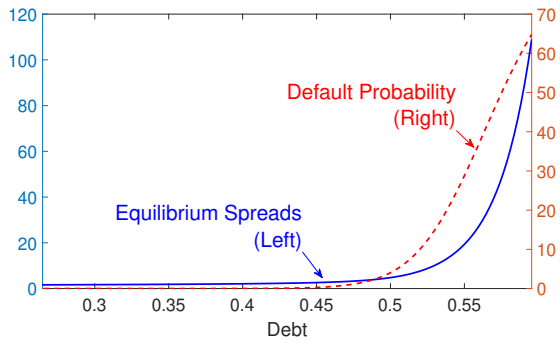
The equilibrium of the NK-Reference model is characterized by conditions (21–26), international Euler condition (34), and an exogenous debt-elastic bond price schedule to close the model, as in Schmitt-Grohé and Uribe (2003). The debt-elastic bond price schedule is  $q^*(B)^{-1} = \beta + \Gamma [\exp(B - \bar{B}) - 1]$ , with  $\Gamma$  set to  $1e^{-5}$ , which gives a very loose borrowing schedule, and  $\bar{B}$  set to give the same average debt level as our baseline. We solve the NK-Reference model with a first-order log-linear approximation of the equilibrium conditions, for the same parameters as the benchmark.

In Appendix E we also solve a version of our model calibrated to have ample borrowing and no default risk in equilibrium, with global methods. It displays very similar properties to the NK-Reference model because absent default risk the borrowing wedges in (33) are quantitatively unimportant.

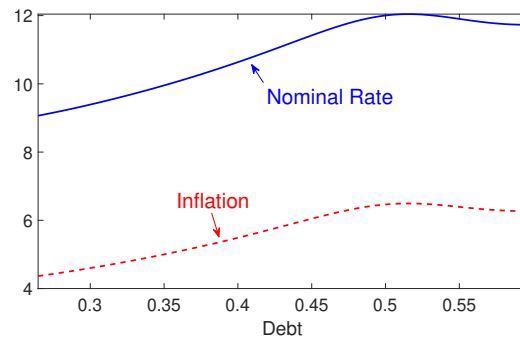
The Default-Reference model is a real sovereign default model with flexible prices. The allocations of this model can be implemented in a monetary model, when monetary policy pursues a “strict inflation target” policy. Under such policy, inflation is always at target and nominal domestic rates equal the real interest rates in the flexible price economy plus the inflation target. The private equilibrium of this model is characterized by conditions (21–23) and an efficient labor allocation  $-u_N/u_C = z$ . We compute the Default-Reference model with the same global methods and for the same parameters as for the baseline NK-Default.

### 5.4 Default Risk Amplifies Monetary Frictions

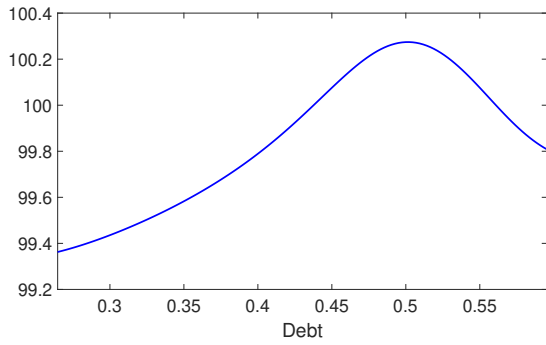
Before analyzing the model-generated time series, we illustrate the main mechanisms relating default risk to monetary distortions. High default risk is associated with large monetary wedges, high inflation, and high nominal rates. The sensitivity of nominal domestic rates to default risk is an additional source of volatility for monetary policy.



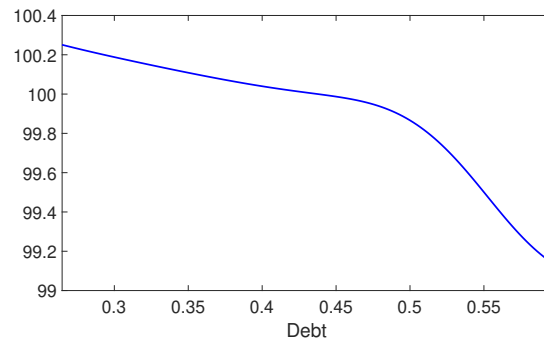
(a) Spreads and Default Risk



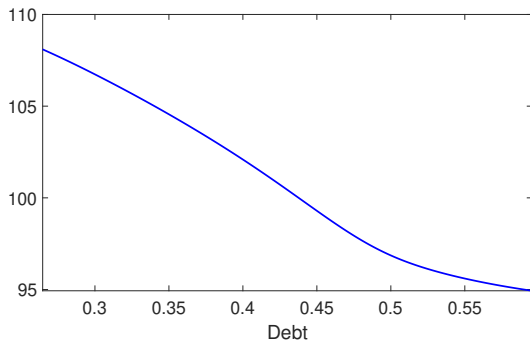
(b) Nominal Domestic Rate and Inflation



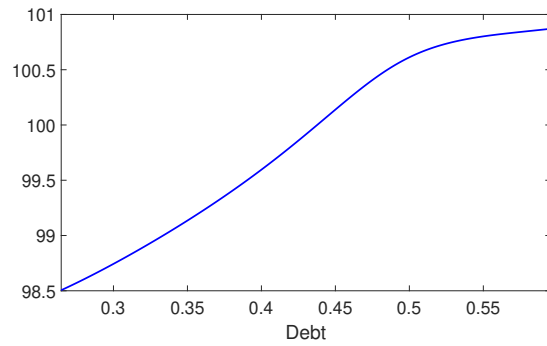
(c) Output



(d) Domestic Goods Consumption



(e) Foreign Goods Consumption



(f) Terms of Trade

Figure 1: Policy Rules

Figure 1 presents policy rules as a function of government debt  $B$  relative to mean exports for the median level of productivity and focus on the behavior conditional on not defaulting. Panel (a) plots the one-period-ahead default probability (right axis) and spreads (left axis) as a function of debt. Default probabilities increase with current debt  $B$  because debt due next period  $B' = H_B(s, B)$  increases with  $B$ , which makes default more likely. We emphasize two regions: a high default zone, for  $B$  roughly above 0.5, and a low default zone, for lower levels of debt. As is typical in sovereign default models, in the high default zone, the probability of default sharply increases in the current debt level. The figure also plots spreads, which with long-term debt reflect not only one-period-ahead default probabilities but also the default risk at all horizons, increasing with debt. Panels (b) through (f) in Figure 1 display key variables of the private and monetary equilibrium. We describe these policy rules in the high default zone first, then turn to the low default zone.

**High Default Zone Policies.** The behavior of variables in the zone of high default risk is largely driven by expectations of allocations and prices during actual default events. As explained in the simplified models in Section 4, the two intertemporal conditions in the private and monetary equilibrium, the NKPC and the domestic Euler equation, link current allocations, prior to default, to expected future allocations in default. Greater default risk is associated with a decline in domestic goods consumption and output, as seen in panels (c) and (d), because of the increasing expectations of future inflation and recessions during defaults. When the risk of default increases, the domestic Euler equation for households calls for a reduction in current consumption and hence output, given the expectations of low future consumption in default and an unresponsive nominal domestic rate. Inflation and nominal rates remain elevated in this zone, as seen in panel (b), because although output and consumption fall, firms do not reduce contemporaneous inflation because of high expected inflation next period, in case of default. Foreign goods consumption in panel (e) declines with debt even though the economy borrows fairly aggressively at increasingly high spreads. The terms of trade in panel (f) are fairly flat as domestic and foreign goods consumption fall with debt at comparable rates.

**Low Default Zone Policies.** When default risk is low, debt affects allocations mainly through its impact on foreign goods consumption and the terms of trade. As debt rises foreign goods consumption falls sharply because the tight bond price schedule limits the availability of borrowing, as seen in panel (e). The decline in foreign goods consumption leads to a depreciation in the terms of trade, which in turn boosts exports.<sup>22</sup> In response to this increased export demand, firms increase production and prices. Nominal domestic rates rise to fight the higher inflation, which depresses domestic consumption. We abstracted from these effects in the simplified models of Section 4, where the quasi-linear preferences disconnect the terms of trade from imported consumption.

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<sup>22</sup>As Blanchard et al. (2017) discuss, in the workhorse open-macro model a reduction in capital inflows is expansionary because it depreciates the exchange rate. Our model features these expenditure switching effects in the low default zone because with default risk capital inflows decline with debt. In the high default zone, the additional forces arising from expectations of future inflation reverse these effects.

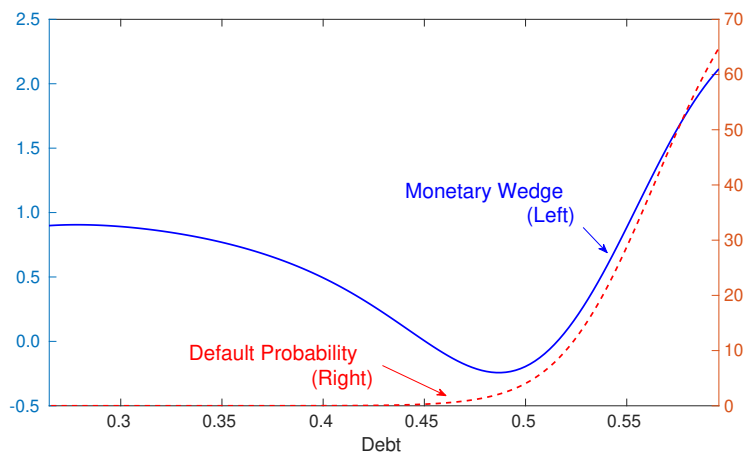


Figure 2: The Monetary Wedge and the Probability of Default

**The Monetary Wedge.** We now turn to describing the monetary wedge. To highlight the interactions between the monetary wedge and default risk, in Figure 2 we plot both the monetary wedge (left axis) and the one-period-ahead default probability (right axis) as a function of debt.

In the high default zone, the monetary wedge increases rapidly with debt because both consumption and output fall: rising default risk sharply depresses domestic consumption, leading to inefficiently low demand and a depressed level of activity. In the low default zone, the monetary wedge is decreasing in the level of debt because the increase in output dominates the decline in domestic consumption. The economy produces more to export because the terms of trade depreciate with the decline in foreign goods consumption.

Default risk is essential for the dependency of the monetary wedge and monetary policy on debt. In Figure 3 we compare the consequences of debt in the NK-Default baseline model with the NK-Reference model without default risk. Panel (a) compares monetary wedges, and panel (b) shows domestic nominal interest rates. In the reference model, debt does not distort the level of activity; the monetary wedge is flat at zero. The NK-Reference model has a very loose borrowing schedule, which allows foreign goods consumption and the terms of trade to be insensitive to debt. Such ample borrowing possibilities disconnect domestic allocations and prices from the indebtedness of the economy. Moreover, absent default, the intertemporal channels in the high default zone of the NK-Default model are not operative.

Panel (b) of Figure 3 shows that nominal domestic rates in the NK-Reference model do not vary with debt, in contrast to our baseline model. Nominal rates in each of the models mirror the dynamics of inflation. The responsiveness of nominal rates and inflation to debt is the additional source of volatility for these variables in our baseline NK-Default model.

## 5.5 Monetary Frictions Discipline Borrowing

We have shown in Section 4.2 that the presence of monetary frictions disincentivizes borrowing in a simplified version. We illustrate these forces in the context of our quantitative results by comparing borrowing and spreads in our NK-Default model to those of the Default-Reference

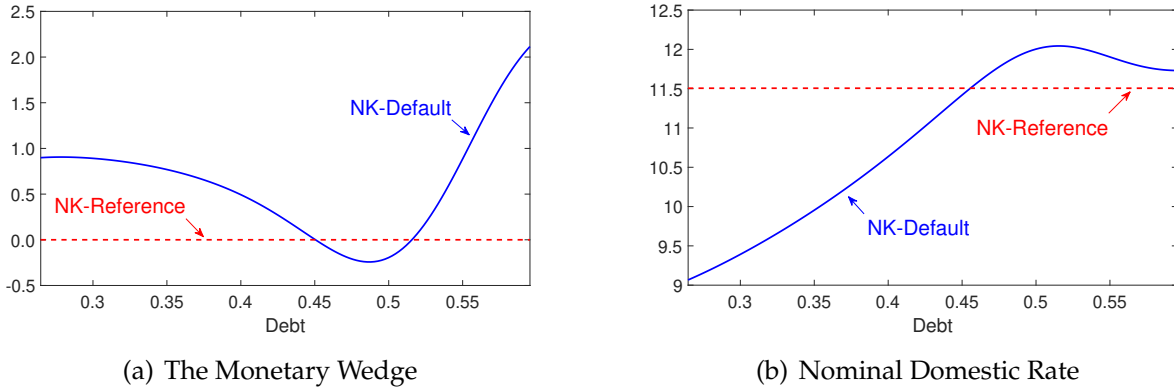


Figure 3: The Monetary Wedge and Nominal Domestic Rates

model.

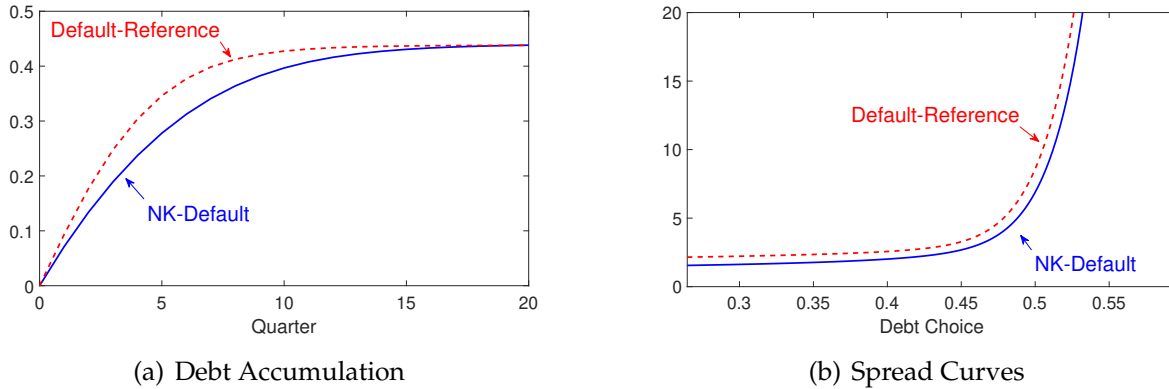


Figure 4: Debt Paths and Spread Curves

Panel (a) of Figure 4 compares the pattern of debt accumulation in the two models: both are simulated starting with zero debt, keeping productivity at median throughout. In both models, the government accumulates debt and settles at a level that is similar. However, in the NK-Default model, this accumulation is slower than in the Default-Reference model because of monetary frictions. Along the transition, borrowing wedges are positive and associated with large monetary wedges, disincentivizing borrowing.

Panel (b) of Figure 4 plots spread schedules in the two models, NK-Default and Default-Reference, for arbitrary  $B'$ . Spreads are lower for each level of  $B'$  in the NK-Default model. Lenders offer a more favorable bond price schedule since slower debt accumulation results in less debt dilution.

## 5.6 Impulse Response Functions

In Section 5.4 we described the policy rules as a function of debt. Here, we analyze responses of the main variables of interest to productivity shocks  $z$ . We construct the impulse response

functions (IRFs) in our nonlinear model following [Koop et al. \(1996\)](#). We simulate a panel of 50,000 units for 5,000 periods. For the first 4,950 periods, the shocks follow their underlying Markov chain so that the cross-sectional distribution converges to the ergodic distribution of the model. In period 4,951, the impact period, normalized to 0 in the plots, we reduce the shock realizations by 1.3%, about half of the standard deviation of the shock. From period 4,952 onward, shocks resume their Markov chain processes. The impulse responses plot the average across the time series.<sup>23</sup> We also contrast the IRFs of our baseline model to those from the two reference models.

Figure 5 plots the responses of output, domestic goods consumption, foreign goods consumption, terms of trade, inflation, the nominal domestic rate, spreads, and debt. The blue lines are the IRFs of our baseline NK-Default model; the dashed red lines are the IRFs of the NK-Reference model. First we describe the responses of our baseline model.

Declines in productivity lead to a decline in output of about 1.14%, which is quite similar to the decline in the shock. Consumption declines a bit more; consumption of domestic and foreign goods falls by 1.33% and 1.25%, respectively. The terms of trade appreciate mildly, especially over the medium run because domestic goods consumption recovers more slowly than foreign goods consumption.

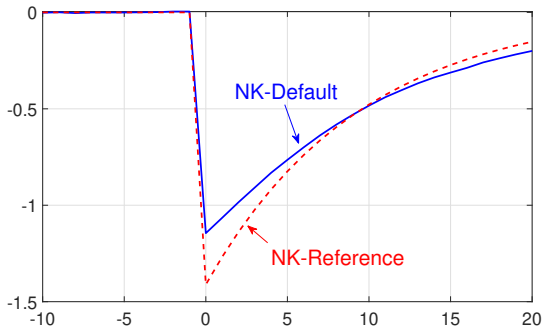
As is typical in sovereign default models, low productivity tightens the bond price schedule because default is more likely in recessions, and with persistent shocks, low productivity makes future recessions more likely. The tight bond price schedule leads to higher spreads and a reduction in debt. Spreads rise about 0.8%, and debt contracts slowly by about 3%. Inflation rises about 1.2% on impact, because of the high unit cost from low productivity and the increased default risk. Nominal domestic rates rise in response to the elevated inflation, about 1.6%. These dynamics illustrate that productivity shocks lead to strong, positive comovements of spreads with inflation and nominal rates.

The responses of the NK-Reference model are also shown in Figure 5. Output and domestic consumption responses are similar to those of our baseline. Foreign goods consumption, in contrast, expands during the downturn. The increase in imported consumption reflects the ample possibilities for insurance with external borrowing. Recall that preferences are non-separable between domestic and foreign goods, and hence smoothing the marginal utility of foreign goods requires an increase in imported consumption, given the drop in domestic consumption. Inflation and nominal rates also rise in the NK-Reference model because of the higher unit cost for production, but their responses are much more muted, about half of the responses in the NK-Default baseline. Terms of trade appreciate sharply in the NK-Reference model because of uncovered interest parity forces: high nominal domestic rates and unchanged foreign interest rates lead to an expected depreciation, which in turn implies that the exchange rate appreciates on impact. The appreciation depresses exports and output, consistent with the more muted rise in inflation. Borrowing expands significantly to support the consumption of foreign goods and the spreads are always zero.

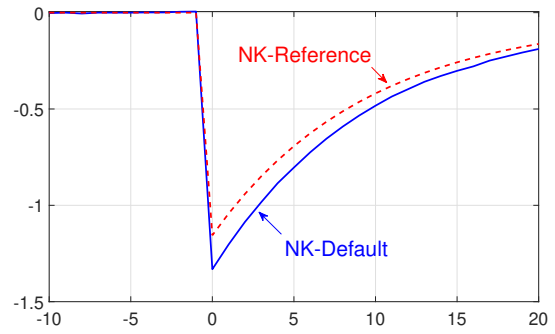
These IRF comparisons highlight the role of default risk in monetary policy. Default risk

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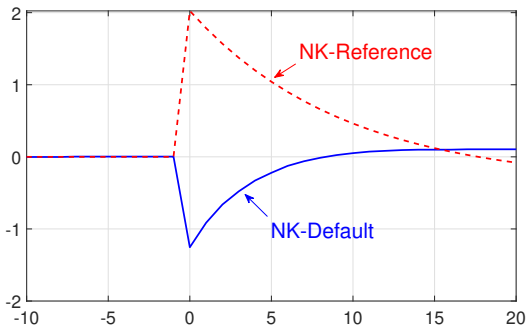
<sup>23</sup>The impulse responses are computed over all units, including those with defaults. Discarding defaults from the cross-sectional average does not alter the properties of the IRFs.



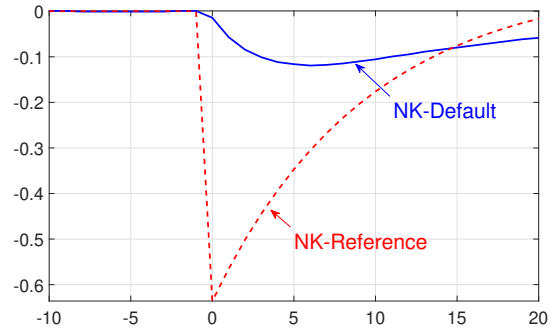
(a) Output ( $Y$ )



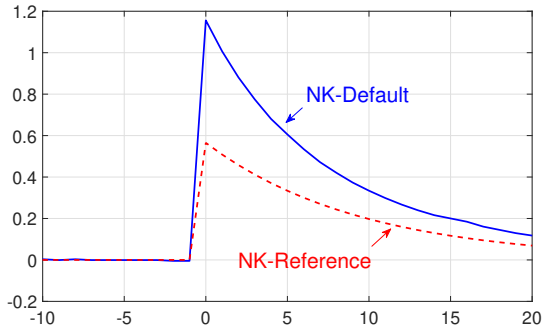
(b) Domestic Goods Consumption ( $C$ )



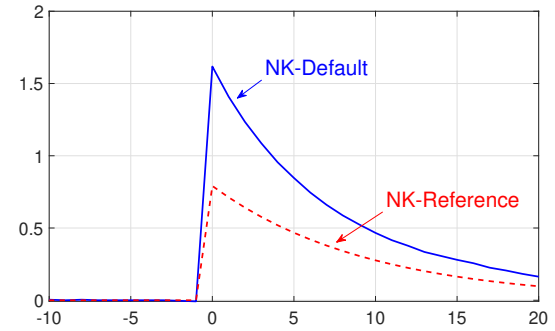
(c) Foreign Goods Consumption ( $C^f$ )



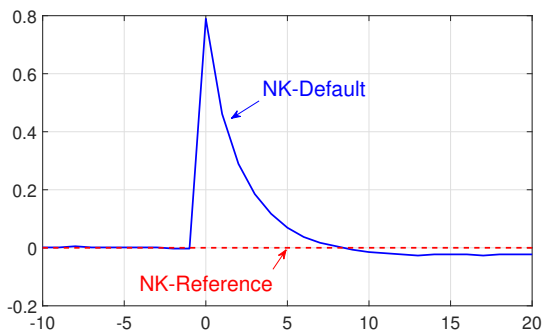
(d) Terms of Trade ( $e$ )



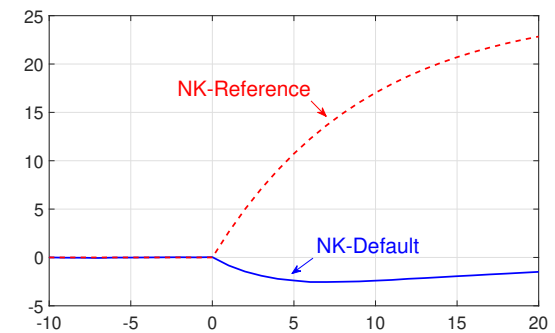
(e) Inflation ( $\pi$ )



(f) Nominal Domestic Rate ( $i$ )

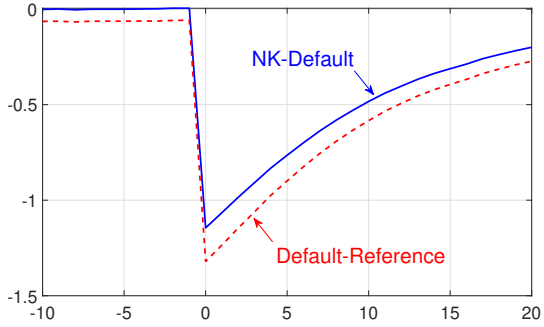


(g) Spread

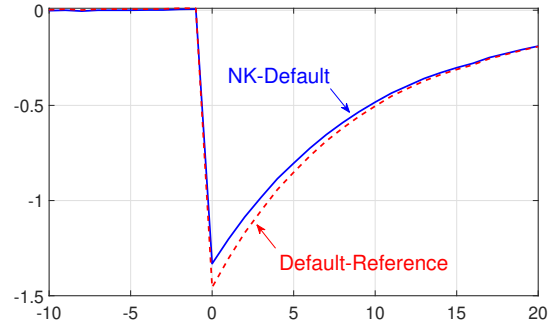


(h) Debt ( $B$ )

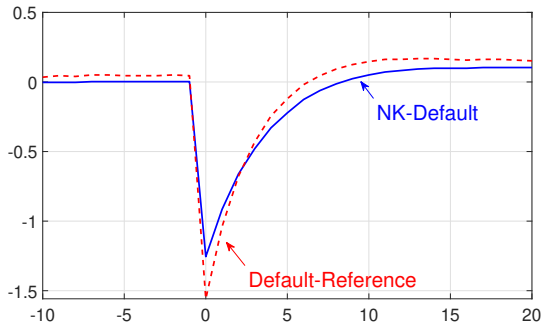
Figure 5: Impulse Responses: Compared with NK-Reference



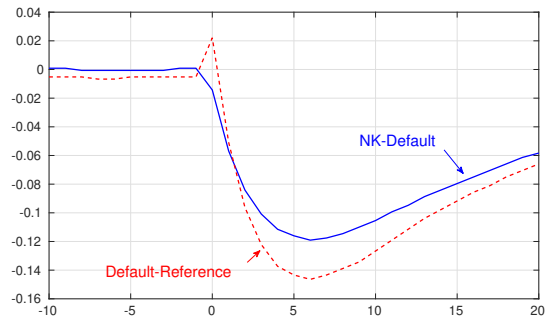
(a) Output ( $Y$ )



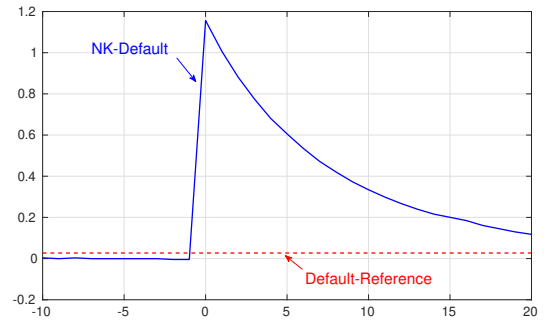
(b) Domestic Goods Consumption ( $C$ )



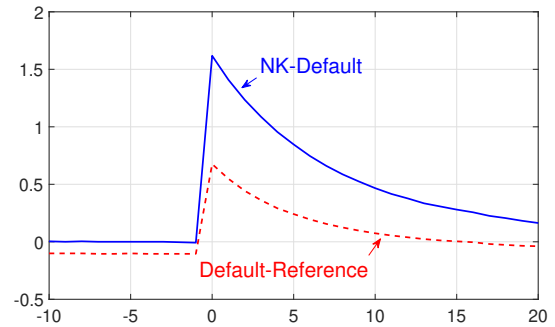
(c) Foreign Goods Consumption ( $C^f$ )



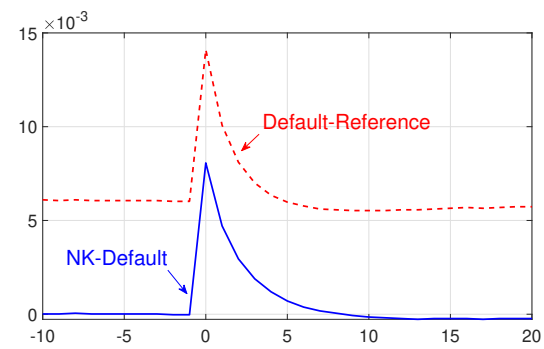
(d) Terms of Trade ( $e$ )



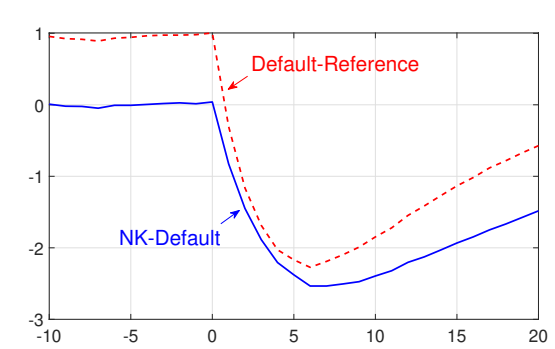
(e) Inflation ( $\pi$ )



(f) Nominal Domestic Rate ( $i$ )



(g) Spread



(h) Debt ( $B$ )

Figure 6: Impulse Responses: Compared with Default-Reference



and tight borrowing conditions raise spreads and induce additional volatility in inflation and aggregate consumption. These larger swings in inflation call for more aggressive monetary policy with stronger responses of nominal domestic rates.

We also compare the IRFs in the NK-Default baseline to the Default-Reference model. Figure 6 plots the IRFs for the same variables, normalizing each panel to the level in period  $-1$  in the NK-Default model. The responses of output and domestic consumption in the Default-Reference model are quite similar to those in our baseline. The decline in foreign goods consumption is somewhat larger because default risk is higher in this model and borrowing conditions are tighter. Spreads are higher on average in the Default-Reference model and increase on impact by about the same amount as in our baseline. Debt is higher on average and also decreases in the downturn. Domestic goods inflation is zero by construction and nominal domestic rates equal the domestic real interest rate implied by the domestic Euler equation. Domestic interest rates rise in the recession because consumption is expected to grow after the downturn; their response, however, is more modest than in our baseline, with a response on impact of about one-third of the baseline. These IRF comparisons show that monetary frictions lead, on average, to lower spreads and debt and to more volatile inflation and nominal rates.

## 5.7 Second Moments

Table 4 reports first and second moments for Brazilian data and simulated data from our baseline NK-Default model and the two reference models. The data consist of the series reported in Section 5.1, for CPI inflation, nominal domestic rates, spread, and output, as well as the trade balance, reported relative to output, and trade-weighted nominal exchange rate depreciation. All moments are reported in percentage points.

Overall, the moments in the baseline model resemble the Brazilian data. The mean CPI inflation, nominal domestic rate, and spread as well as the volatility of inflation, output, and spreads are targets in our moment-matching exercise. The model delivers a volatility of the nominal rate that is close to the data, whereas it underestimates the volatility of the trade balance and misses the high volatility in nominal exchange rates. In our model, exchange rates are about 30% more volatile than output, reflecting the common disconnect between exchange rates and fundamentals in much of international business cycle theory.

The model delivers the positive correlations of inflation and nominal domestic rates with spreads in the data, with magnitudes of about 60%. These correlations arise in our model because, across both state variables, namely, productivity  $z$  and debt  $B$ , inflation and spreads comove positively.

The model also features the countercyclicality of inflation, nominal domestic rates, nominal depreciations, spreads, and the trade balance present in the Brazilian data. As explained with the IRFs, inflation and nominal rates tend to rise with low productivity. The nominal exchange rate depreciates because its dynamics are mainly driven by inflation. Spreads are countercyclical because recessions are associated with high default risk. The trade balance is also countercyclical because the high spreads in recessions lead to a reduction in international borrowing. These properties induce the positive correlations of spreads with the trade balance and nominal depreciation rates, that are present in the data. These findings are consistent with [Na et al.](#)

| <i>Mean</i>                    | Data | NK-<br>Default | NK-<br>Reference | Default-<br>Reference |
|--------------------------------|------|----------------|------------------|-----------------------|
| CPI inflation                  | 5.9  | 5.9            | 6.1              | 0.0                   |
| Nominal domestic rate          | 11.2 | 11.1           | 11.5             | 5.3                   |
| Spread                         | 2.6  | 2.6            | —                | 3.2                   |
| <i>Standard Deviation</i>      |      |                |                  |                       |
| CPI inflation                  | 1.8  | 1.8            | 1.0              | 0.6                   |
| Nominal domestic rate          | 2.2  | 2.5            | 1.3              | 1.8                   |
| Spread                         | 0.9  | 0.9            | —                | 0.8                   |
| Output                         | 1.9  | 1.9            | 2.4              | 2.1                   |
| Consumption aggregate          | 1.8  | 2.0            | 2.0              | 2.2                   |
| Trade balance                  | 0.9  | 0.3            | 1.9              | 0.5                   |
| Nominal depreciation rate      | 14.7 | 2.4            | 2.0              | 1.9                   |
| <i>Correlation with Spread</i> |      |                |                  |                       |
| CPI inflation                  | 59   | 60             | —                | −1                    |
| Nominal domestic rate          | 59   | 64             | —                | 18                    |
| Output                         | −62  | −60            | —                | −42                   |
| Trade balance                  | 61   | 35             | —                | 33                    |
| Nominal depreciation rate      | 51   | 45             | —                | −1                    |
| <i>Correlation with Output</i> |      |                |                  |                       |
| CPI inflation                  | −16  | −88            | −81              | 7                     |
| Nominal domestic rate          | −23  | −96            | −98              | −60                   |
| Trade balance                  | −77  | −18            | 62               | −23                   |
| Nominal depreciation rate      | −18  | −62            | −28              | 7                     |

Table 4: Moments: Data, NK-Default, and Reference Models

(2018), who find in their work that default risk is correlated with depreciation rates in emerging countries.

Table 4 also reports the moments from the NK-Reference model, which by construction is silent on default risk. Average CPI inflation and the nominal domestic rate are the same as in the benchmark, largely reflecting the common parameters  $\bar{\pi}$  and  $\bar{i}$ . The volatilities of the nominal interest rate and CPI inflation are, however, only about half of those in the NK-Default baseline. Default risk makes inflation more volatile because it affects inflation and the monetary wedge, as discussed in Section 4 and illustrated in the IRFs. This comparison shows that an emerging market central bank targeting inflation must implement a more volatile interest rate policy as a result of sovereign default risk.

As in our baseline model, the NK-Reference model also features countercyclical inflation, nominal domestic rates, and nominal depreciation, but the cyclicity of the trade balance is positive, in contrast with the data. With well-functioning financial markets, the country increases borrowing in recessions to smooth consumption.

The final column of Table 4 reports moments for the Default-Reference model. Recall that CPI inflation in this model fluctuates only because of changes in the terms of trade, and nominal domestic rates are equal to the real rates implied by consumption dynamics. The means and volatilities of CPI inflation and nominal rates are substantially lower than in the benchmark model, whereas output and consumption volatility are comparable. Mean spreads, however, are higher than in the benchmark by about 0.5%, because of the disciplining role of monetary frictions on borrowing. Another manifestation of the greater propensity to borrow in the Default-Reference model is its more volatile trade balance, with a standard deviation almost twice that of the baseline.

Absent nominal frictions, the Default-Reference model fails to generate the strong positive comovement of spreads with either CPI inflation or the nominal depreciation rate. Here these series are essentially uncorrelated. The model does exhibit a positive but quantitatively modest correlation of spreads with the domestic rate, less than a third of the value in the baseline. Finally, CPI inflation and the nominal depreciation rate are essentially acyclical, in contrast with the strong countercyclical pattern of NK-Default, whereas nominal domestic rates and the trade balance cyclical patterns are negative, as in the baseline model.

## 5.8 Brazil Event and Counterfactual

Now we perform an event analysis and compare our model with the Brazilian 2015 recession. We find that our model produces similar time paths as in the data, with increases in nominal domestic rates, inflation, and spreads. We also conduct a counterfactual experiment where nominal rates are kept low during the recession. In this counterfactual, the downturn is milder but inflation and spreads increase even more.

For the event analysis, we feed in a path of productivity shocks such that the time path of output in the model replicates the one in the data. We fix the initial level of debt to its mean in the limiting distribution. We then compare the predictions of the model for CPI inflation, nominal domestic rates, spreads, and nominal exchange rates with the data.

The dashed blue lines with circle markers in Figure 7 represent the series in the data. Brazil

experienced a recession from 2014 to late 2016, with output contracting from 3% above trend to 3% below trend, a 6% decline in total. It then recovers starting in 2016Q3. During this period, CPI inflation increases by 3.8%, the nominal domestic rate increases by 3.3%, and spreads rise from about 2% to 5%. When output recovers after 2017, inflation, the nominal rate, and spreads all fall.

The solid red lines in Figure 7 are the corresponding series in the model. To match the dynamics of output, the model requires that the underlying productivity shock decreases from 2014 to late 2016. The recession drives up sovereign spreads. The declining productivity and rising spreads lead to an increase in inflation. Monetary policy responds to this high inflation with hikes in the nominal domestic rate. Quantitatively, the model matches the rise in inflation and spreads during the recession, around 4.2% for inflation and 3% for spreads. After the recovery from 2016Q3 onward, the model also reproduces the inflation decrease of about 4% and the drop in spreads of about 3%.

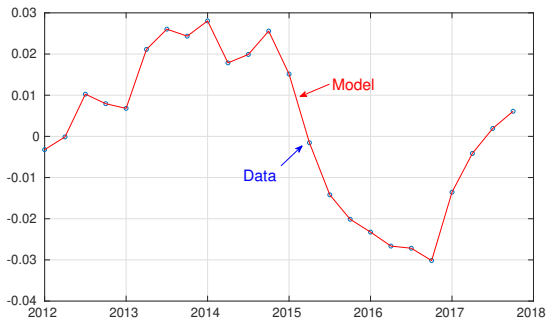
In terms of exchange rates, the model delivers a depreciation in the nominal exchange rate of about 20% during the recession, starting from 2015Q1 onward. In the data, the overall depreciation in the nominal exchange rate during this period is also about 20%, but with a much higher volatility that we miss. Although our model matches the overall depreciation, it fails to replicate the large depreciation in Brazil in 2016.

To evaluate the effects of the hikes in nominal rates on the Brazilian economy during this event, we conduct a counterfactual experiment with a dovish central bank. In this alternative scenario, instead of tightening in response to high inflation, the central bank keeps a low nominal domestic rate, similar to its 2015 level, following the start of the recession.

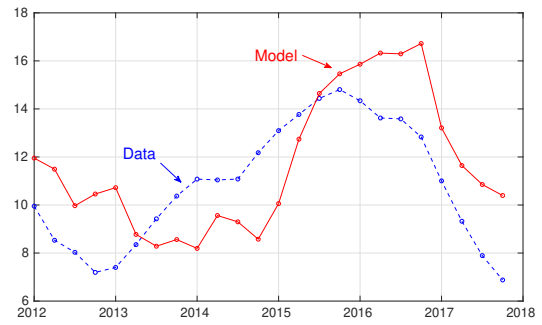
We implement these counterfactual interest rates through the use of the monetary shock  $m$  in the interest rate rule (12). In the parameterization for the main quantitative results, we abstracted from these shocks. We now allow for low probability, i.i.d.  $m$  shocks and compute the model over a wide range of values for  $m$ . In the counterfactual, we feed in the appropriate level for  $m$  such that the nominal domestic rate remains at its 2015 level. We also confirm that the main quantitative properties of our model are unaltered with these small variance monetary shocks and illustrate the transmission of these monetary shocks by constructing impulse response functions, displayed in Appendix C.

The counterfactual series are plotted in black lines in Figure 8 and the baseline results are again in solid red lines. The expansionary monetary shocks in the counterfactual induce lower nominal domestic rates and thus stimulate consumption and output. This higher demand increases the unit cost of production, which in turn generates high inflation. Lower nominal rates also lead to increased spreads, about 2.5% higher, since low rates reduce monetary frictions and incentivize the government to increase borrowing. The higher default risk further increases inflation. Overall, in late 2016, the inflation rate in the counterfactual scenario would be about 2% higher than in the benchmark case. This counterfactual highlights the disciplining role of monetary frictions for sovereign borrowing and default risk.

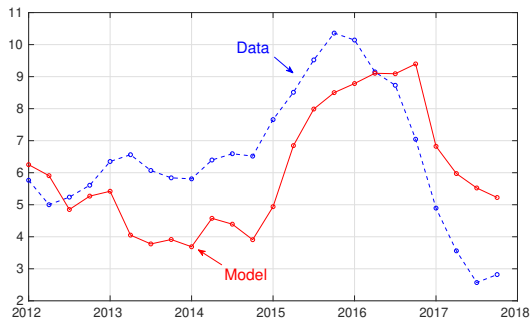
In summary, our model matches the patterns of inflation, nominal domestic rates, and spreads during the Brazilian downturn of 2015. The counterfactual analysis highlights the role of monetary frictions in limiting borrowing and moderating crisis events.



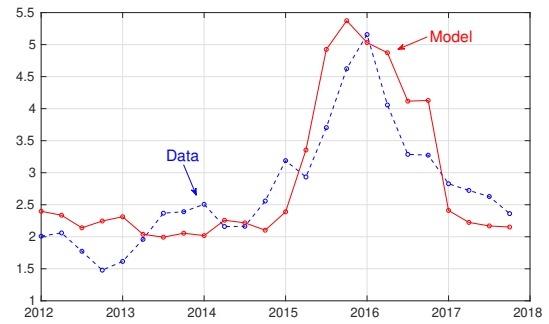
(a) Output



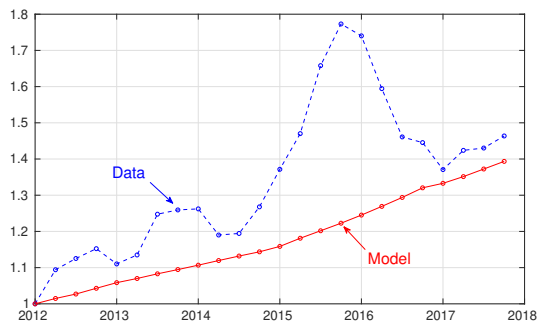
(b) Nominal Domestic Rate



(c) Inflation

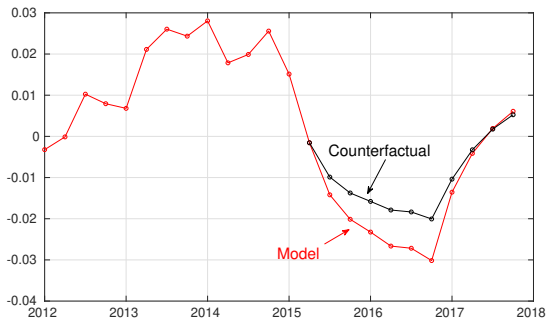


(d) Spread

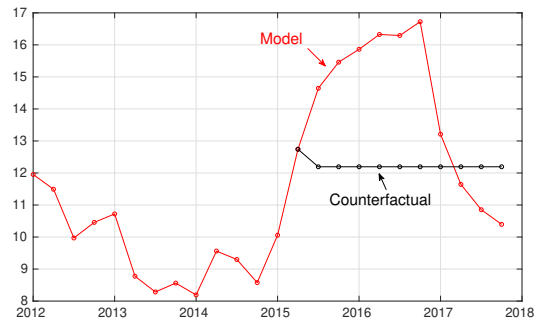


(e) Nominal Exchange Rate

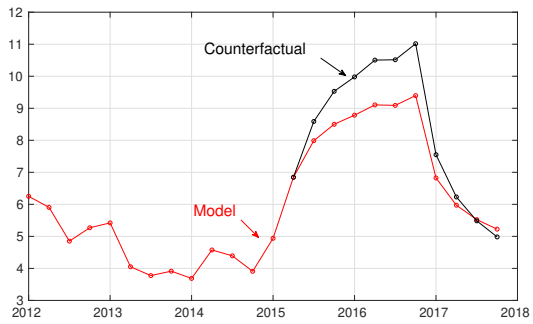
Figure 7: Event Analysis for Brazil



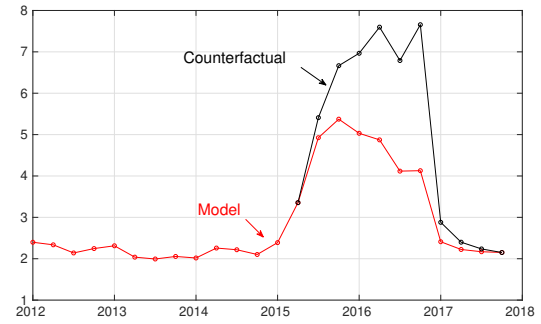
(a) Output



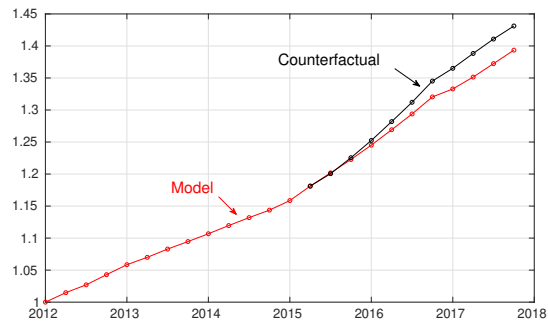
(b) Nominal Domestic Rate



(c) Inflation



(d) Spread



(e) Nominal Exchange Rate

Figure 8: Counterfactual Experiment

## 6 Extensions, Robustness, and Welfare

In this section we study the robustness of our findings to extensions of the baseline model. We analyze two main variations. The first extension considers an environment with government debt denominated in local currency. The second extension considers alternative specifications for the interest rate rule. Here we perform a comparative static exercise over the weight on inflation and also extend the interest rate rule to respond to output. We also compare welfare across these alternative models and our benchmark.

### 6.1 Local Currency Government Debt

Governments in emerging markets increasingly borrow in local currency, as documented in [Du and Schreger \(2016\)](#) and [Ottonello and Perez \(2019\)](#). Here we explore the implications of sovereign debt denominated in local currency for our NK-Default model. Domestic monetary policy in this framework, of course, directly affects the real liabilities of the central government because domestic inflation alters the real value of the debt. In our model, however, inflating away the debt is not a consideration for monetary policy because the nominal interest rate rule responds only to inflation deviations from target. We find that in our environment, the denomination of government debt has only minor effects on the dynamics of inflation, spreads, or nominal domestic rates.

To analyze the case with local currency government debt, we modify the government's budget constraint (13) in the baseline model. The nominal government budget constraint in local currency is

$$T_t + \tau W_t N_t = q_t \left( B_{t+1}^{\text{lc}} - (1 - \delta) B_t^{\text{lc}} \right) - (r^* + \delta) B_t^{\text{lc}} + \tau_f P_t^f C_t^f, \quad (43)$$

where nominal local currency government debt is  $B_{t+1}^{\text{lc}}$ .

We deflate this budget constraint by the price of domestic goods  $P_t^d$  and combine it with the budget constraint of households to obtain the balance of payments condition,

$$e_t^o \bar{\xi} = e_t C_t^f + (r^* + \delta) \frac{B_t}{\pi_t} - q_t \left( B_{t+1} - (1 - \delta) \frac{B_t}{\pi_t} \right), \quad (44)$$

where real government debt, in terms of domestic goods, is  $B_t = B_t^{\text{lc}} / P_{t-1}^d$ . This expression makes it explicit that inflation  $\pi_t$  affects the real value of government debt.<sup>24</sup>

The private and monetary equilibrium of the model with local currency debt consists of equations (21–26) with the balance of payments condition (22) replaced by (44). The bond pricing condition is also modified, as international lenders arbitrage the foreign currency risk-free return  $r^*$  with the foreign currency return on the local currency government debt. This arbitrage implies that lenders need to be compensated not only for default risk, but also for the expected nominal exchange rate depreciation. The bond price for local currency bonds is

$$q_t = \frac{1}{1 + r^*} \mathbb{E} \left[ \frac{e_t}{e_{t+1}} \frac{1}{\pi_{t+1}} (1 - D_{t+1}) (r^* + \delta + (1 - \delta) q_{t+1}) \right]. \quad (45)$$

<sup>24</sup>With abuse of notation  $B_t$  in this extension refers to debt in domestic goods, whereas in the baseline model, it refers to debt in foreign currency.

For our definition of sovereign spreads with local currency debt, we follow [Du and Schreger \(2016\)](#) and measure them with the *local currency credit spread*. This credit spread is the difference in yield-to-maturity between defaultable and default-free bonds, both for instruments that are denominated in the same currency and have equal duration.<sup>25</sup> We compute the model with local currency government debt using the same parameter values as in the baseline model.

The second column of [Table 5](#) reports the results for the model with local currency government debt and shows that the properties of this version of the model are very similar to those in the baseline model. The only significant difference is that mean spreads are lower when debt is denominated in local currency, which we elaborate on below. Importantly, the standard deviations of nominal domestic rates and inflation, as well as their correlations with spreads, are unchanged relative to the baseline. In [Appendix D](#), we also show that the impulse response to productivity shocks and the behavior of the monetary wedge and nominal rate as functions of debt are similar to the ones in the baseline model.

The similar volatilities in nominal domestic rates across debt denomination highlight the robustness of our first finding: default risk amplifies monetary frictions. The volatility in nominal rates of 2.5 is larger than the volatility of 1.3 in the NK-Reference model with no default in [Table 4](#).<sup>26</sup>

Our second main finding, that monetary frictions discipline government default risk, is also robust across debt denomination. The local currency debt specification also delivers a lower mean spread of 1.9 relative to the mean spread of 3.2 in the Default-Reference model in [Table 4](#). In fact, with local currency, spreads are even lower than in the baseline model because the inflation dynamics induce state-contingent debt repayments that are a good hedge for the sovereign: high inflation in recessions means that the real burden of debt falls when income is low. These desirable properties of local currency debt alleviate financial frictions and default risk, leading to a lower average spread.

Average credit spreads are also lower with local currency debt because lenders price the product of nominal devaluations  $\frac{e}{e'} \frac{1}{\pi'}$  and future default risk, encoded in future prices  $q'$ , which implies that the covariance between these two variables alters the bond price. In our model, this covariance is positive: high prices are associated with low nominal devaluation rates, which boosts the average price of local currency bonds (or lowers the mean credit spread). We find, nevertheless, that this effect is quantitatively small.

## 6.2 Variants on Interest Rate Rules

We now turn to our second extension of the baseline model that evaluates variations in the interest rate rule. We consider two variations: one that increases the weight on inflation deviations

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<sup>25</sup>In our model, the foreign currency price of a default-free local currency bond is

$$q_t^* = \frac{1}{1+r^*} \mathbb{E} \left[ \frac{e_t}{e_{t+1}} \frac{1}{\pi_{t+1}} (r^* + \delta + (1-\delta)q_{t+1}^*) \right].$$

<sup>26</sup>For brevity, we focus only on the comparisons with the NK-Reference model with foreign currency because results from an NK-Reference model with local currency debt are almost identical.



| <i>Mean</i>                    | Benchmark | Local<br>currency | Rule with<br>larger $\alpha_P$ | Rule with<br>output gap |
|--------------------------------|-----------|-------------------|--------------------------------|-------------------------|
| CPI inflation                  | 5.9       | 5.9               | 6.0                            | 5.7                     |
| Nominal domestic rate          | 11.2      | 11.3              | 11.2                           | 11.0                    |
| Spread                         | 2.6       | 1.9               | 2.9                            | 2.7                     |
| <i>Standard Deviation</i>      |           |                   |                                |                         |
| CPI inflation                  | 1.8       | 1.9               | 0.9                            | 1.5                     |
| Nominal domestic rate          | 2.5       | 2.5               | 1.6                            | 2.2                     |
| Spread                         | 0.9       | 0.4               | 0.9                            | 0.9                     |
| Output                         | 1.9       | 2.0               | 2.0                            | 2.0                     |
| Consumption aggregate          | 2.0       | 2.0               | 2.1                            | 2.0                     |
| Trade balance                  | 0.3       | 0.4               | 0.4                            | 0.4                     |
| Nominal depreciation rate      | 2.4       | 2.5               | 1.8                            | 2.3                     |
| <i>Correlation with Spread</i> |           |                   |                                |                         |
| CPI inflation                  | 60        | 57                | 45                             | 72                      |
| Nominal domestic rate          | 64        | 61                | 62                             | 76                      |
| Output                         | -60       | -57               | -63                            | -79                     |
| Trade balance                  | 35        | 26                | 34                             | 29                      |
| Nominal depreciation rate      | 45        | 41                | 20                             | 45                      |
| <i>Correlation with Output</i> |           |                   |                                |                         |
| CPI inflation                  | -88       | -86               | -61                            | -84                     |
| Nominal domestic rate          | -96       | -95               | -91                            | -90                     |
| Trade balance                  | -18       | 5                 | -21                            | -12                     |
| Nominal depreciation rate      | -62       | -58               | -22                            | -48                     |

Table 5: Moments: Extended Models

and another that adds an output gap term.

**Larger Weight on Inflation** We have seen that our model generates sizable volatility in inflation and nominal domestic rates relative to both reference models: NK-Reference and Default-Reference. As in standard New Keynesian models, the volatilities of these variables are affected by the weight on inflation deviations in the interest rate rule. We consider a comparative static exercise that increases the weight on inflation  $\alpha_P$  to 2.5, from 1.4 in the benchmark. All other parameters remain unchanged in this experiment.

In Table 5, we report the first and second moments for this comparative static exercise. Higher  $\alpha_P$  lowers the volatility of inflation, nominal domestic rates, and nominal devaluations. Spreads in this model, however, increase modestly on average and display similar variability as in the benchmark. All other volatilities and correlations are similar to the benchmark, including the positive correlations of spreads with nominal rates and inflation. This comparative static exercise maintains the same two-way interactions of monetary policy and default risk in the benchmark: default risk amplifies monetary frictions, as seen by the higher volatility in nominal rates of 1.6 relative to the NK-Reference of 1.3, and monetary frictions continue to discipline default risk, as seen by the lower mean spread of 2.9% relative to the Default-Reference of 3.2%.<sup>27</sup>

**Weight on Output Gap** The monetary policy rule in the baseline model has nominal domestic rates responding only to inflation deviations. In this subsection, we expand the rule to include an output gap term. We modified the baseline model by replacing the interest rate rule in equation (12) with

$$i = \bar{i} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\alpha_P} \left( \frac{Y_t}{Y_t^{\text{flex}}} \right)^{\alpha_Y} m_t. \quad (46)$$

The output gap is defined as the ratio of output in the model  $Y_t$  relative to output in the model with flexible prices  $Y_t^{\text{flex}}$ . Flexible output is defined as the average output in the Default-Reference model conditional on the realization of the shock  $z_t$  and also on whether or not the economy is in default.<sup>28</sup> We set the weight on the output gap to 0.5, which is the value in the rule of Taylor (1993) and maintain all other parameters as in the benchmark model.

In Table 5 we report results for this variation and show that the properties of our model change very modestly. The mean and volatility of spreads remain practically unchanged, the volatilities of inflation and nominal domestic rates are a bit lower, and the correlations of spreads with nominal rates and inflation continue to be positive. In Appendix D, we present impulse response functions to productivity shocks for this model as well as the monetary wedge and nominal rates as a function of debt and show that they behave similarly to the benchmark model. This robustness exercise shows that adding an output gap term to the interest rate rule does not

<sup>27</sup>The difference in the volatility of nominal rates arising from default becomes larger when we recompute the NK-Reference model with a larger weight on inflation of 2.5, where the volatility of nominal rates is 0.9%.

<sup>28</sup>We average output across debt to smooth any potential differences in output arising from varying tightness in financial frictions across debt levels because such tightness is not the same across models. Results nevertheless are similar if we condition flexible output on all states and choices  $S = \{s, B, \Theta, B'\}$ .

alter the interactions between monetary policy and default risk: default risk amplifies monetary frictions, and monetary frictions discipline default risk.

### 6.3 Welfare

The paper has established that monetary policy interacts with sovereign default risk. We have shown that in our benchmark model, as well as in the extensions presented in Table 5, interest rate rules affect not only inflation but also the properties of sovereign spreads. Here we discuss how welfare varies across these economies and focus on household welfare, as defined in (3). The benchmark economy has two sources of inefficiency: pricing frictions and lack of commitment to repay its debt, or default risk. Monetary policy affects both frictions, and therefore welfare comparisons across monetary rules depend on how these rules interact with the two frictions. Across model economies, the volatility of inflation and the mean spread are measures of the costs of price frictions and default risk.

|   | Welfare (%) |
|---|-------------|
| NK-Default (monetary frictions)           |             |
| Benchmark                                 | -.019       |
| Rule with higher $\alpha_p$               | +.009       |
| Rule with output gap                      | -.007       |
| Local currency debt                       | +.018       |
| Default-Reference (no monetary frictions) | .000        |

Table 6: Welfare, Relative to Default-Reference

In Table 6 we compare welfare across models with different monetary and financial environments. We report gains and losses in consumption equivalence terms relative to the Default-Reference model, which has no pricing frictions but features default risk.<sup>29</sup> We evaluate welfare at the mean level of productivity and zero debt. As is standard in the business cycle literature, including monetary economies, welfare differences are small across models. Nevertheless, we find that, for some monetary rules, welfare is higher with pricing frictions than without.

The top part of the table reports welfare for the NK-Default model, our benchmark together with the alternative monetary rules considered in the previous section. It shows that welfare in our benchmark specification is lower than in the Default-Reference model. Consumption equivalent welfare is .019% lower in our benchmark model because, although our model has lower spreads, it has higher inflation variability. This welfare ranking depends crucially on the coefficient on inflation in the interest rate rule  $\alpha_p$ . With a larger  $\alpha_p$  coefficient, welfare becomes higher in our NK-Default model than in the Default-Reference model, even though this latter model does not suffer from pricing frictions: for  $\alpha_p = 2.5$ , consumption equivalent welfare

<sup>29</sup>Consumption equivalent,  $C^E$ , is computed from household welfare  $V^h$  using log utility as  $C^E = \exp[(1 - \beta)V^h]$ .

is .009% higher. The ranking arises because the disciplining benefits of monetary frictions on borrowing compensate for the costs arising from inflation volatility. We find that welfare in our benchmark model is actually non-monotonic with respect to the coefficient  $\alpha_P$ , reaching a maximum at  $\alpha_P = 2.5$ . The rule with an output gap term performs better than the benchmark but worse than the Default-Reference model and the high  $\alpha_P$  case because inflation remains quite volatile.

The table also illustrates the importance of the denomination of sovereign debt for the welfare effects of monetary policy. Welfare is about .018% higher when sovereign debt is denominated in local currency in our environment with pricing frictions than in the Default-Reference model. Interestingly, our results show that local currency debt is superior to foreign currency debt by about 0.037% of consumption, as seen by the difference in welfare between the local currency debt model and the benchmark. The benefits of local currency debt come from lower spreads and the insurance-like properties discussed in Section 6.1.

Finally, we compare these welfare results with those in Galí and Monacelli (2005). A main finding in that paper is that the optimal monetary policy completely stabilizes domestic inflation and replicates the flexible price model. These results do not hold in our model when the government lacks commitment to repay its debt. With default risk, welfare can be higher with rules that do not completely stabilize inflation, as seen by the higher welfare in the model with a higher  $\alpha_P$  coefficient, relative to the Default-Reference model without monetary frictions (or equivalently strict inflation targeting). Nevertheless, our results resemble those in Galí and Monacelli (2005) in that an interest rate rule that targets domestic inflation is useful for welfare. In that paper, such an interest rate rule achieves welfare levels close to the optimal policy because they work well to stabilize domestic inflation. In our framework with default risk, the same interest rate rule is useful to alleviate both frictions because it stabilizes inflation and disciplines default risk.

## 7 Conclusion

We have developed a framework that combines two important aspects of current policy in emerging markets: sovereign risk in government debt and inflation-targeting as monetary policy. Our work embeds sovereign risk in a New Keynesian model, which is the workhorse framework used for central bank policy. We find that the monetary transmission is altered when economies face sovereign default risk. Monetary policy that targets inflation requires larger fluctuations in nominal domestic rates when the risk of default is a concern. Monetary frictions also discipline sovereign risk and slow down debt accumulation. These results are robust to salient extensions of the benchmark environment, including debt denominated in local currency and rich interest rate rules. Quantitatively, the model replicates key moments in emerging market data, including the comovements of spreads with nominal domestic rates and inflation, and it provides insights into the conduct of monetary policy during Brazil's 2015 recession. These results show that the current standard paradigm in central banks is incomplete and points toward incorporating sovereign risk.

In the last decade, many emerging markets have been successful in bringing down inflation

and giving central banks the independence to maintain stable prices. New Keynesian monetary theory has been an important pillar in guiding such processes, but such theory was developed for countries with deep and well-functioning financial markets. The growing literature on the interaction between monetary theory and financial frictions has identified that some of the lessons are modified, yet many open questions remain. An important question relates to the optimality of interest rate rules and on whether these rules should depend not only on inflation but also on financial conditions. Aoki et al. (2018) show that a combination of capital controls and monetary policy works best for economies with pricing frictions and financial frictions arising in banks with balance sheet concerns. Arellano et al. (2019) show that in economies with sovereign risk, an interest rate rule that also responds to sovereign spreads goes a long way toward preventing overborrowing while maintaining the benefits of low inflation. Low inflation, unfortunately, has not been uniformly achieved in emerging markets, especially those with chronic financial crises, such as Argentina and Turkey. An important open question involves the links between financial imperfections and the inability of governments to achieve a successful monetary policy.

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# ONLINE APPENDIX TO “MONETARY POLICY AND SOVEREIGN RISK IN EMERGING ECONOMIES (NK-DEFAULT)”

BY CRISTINA ARELLANO, YAN BAI, AND GABRIEL MIHALACHE

## A Deriving the Government Borrowing

In this appendix, we derive the government’s optimal borrowing equation (33) in Section 3.2. To illustrate the government’s borrowing incentives, we assume that all the policy functions are differentiable with respect to state  $B$  and that the first-order conditions are sufficient for the government’s optimization problem. Conditional on not defaulting, the government chooses  $\{C, C^f, N, \pi, B'\}$  to solve the following problem:

$$W(s, B) = \max u(C, C^f, N) + \beta_g \mathbb{E} \left\{ \int_{v^*(s', B')} W(s', B') d\Phi(v') + \int_{v^*(s', B')}^{v^*(s', B')} [W^d(s') - v'] d\Phi(v') \right\} \quad (47)$$

subject to the constraints arising from the Private and Monetary Equilibrium

$$[\lambda] \quad C + [C^f + (\delta + r^*)B - q(s, B') (B' - (1 - \delta)B)]^{\frac{\rho}{\rho-1}} = \left[1 - \frac{\varphi}{2} (\pi - \bar{\pi})^2\right] zN, \quad (48)$$

$$[\lambda_e] \quad \frac{u_{C^f}}{u_C} = \frac{\rho}{\rho-1} [C^f + (\delta + r^*)B - q(s, B') (B' - (1 - \delta)B)]^{\frac{1}{\rho-1}}, \quad (49)$$

$$[\kappa] \quad \bar{i} \left(\frac{\pi}{\bar{\pi}}\right)^{\alpha_p} m \beta M(s, B', 0) = u_C, \quad (50)$$

$$[\gamma] \quad \frac{\eta - 1}{\varphi} \left(-\frac{u_N}{zu_C} - 1\right) + \beta \frac{1}{u_C z N} F(s, B', 0) = (\pi - \bar{\pi}) \pi, \quad (51)$$

where the default cutoff  $v^*(s', B')$  equals  $W^d(s') - W(s', B')$  and the functions  $F(s, B', 0)$  and  $M(s, B', 0)$  are defined in (27) and (28) with  $\Theta = 0$ . Here we have substituted the terms of trade  $e$  and the nominal domestic rate  $i$  using the balance of payments condition (22) and the interest rate rule (26). Let  $\lambda, \lambda_e, \kappa,$  and  $\gamma$  be the Lagrange multipliers on the resource constraint (48), the relative demand condition (49), the domestic Euler condition (50), and the NKPC condition (51), respectively. Note that a positive  $\kappa$  is associated with a smaller left-hand side of (50) than its right-hand side. Similarly, a positive  $\gamma$  is associated with a smaller left-hand side of (51) than its right-hand side.

The first-order conditions over  $C, C^f, N, \pi,$  and  $B'$  are

$$u_C - \lambda - \lambda_e \frac{u_{CC^f} u_C - u_{CC} u_{C^f}}{(u_C)^2} + \kappa u_{CC} + \gamma \frac{u_{CC}}{u_C} \left[ -\frac{\eta - 1}{\varphi} \frac{u_N}{z u_C} + \beta \frac{F(s, B', 0)}{z N u_C} \right] = 0, \quad (52)$$

$$u_{C^f} - \lambda \frac{\rho}{\rho - 1} e + \lambda_e \left( \frac{\rho e^{2-\rho}}{(\rho - 1)^2} - \frac{u_{C^f C^f} u_C - u_{CC^f} u_{C^f}}{(u_C)^2} \right) + \kappa u_{CC^f} + \gamma \frac{u_{CC^f}}{u_C} \left[ -\frac{\eta - 1}{\varphi} \frac{u_N}{z u_C} + \beta \frac{F(s, B', 0)}{z N u_C} \right] = 0, \quad (53)$$

$$u_N + \lambda \left[ 1 - \frac{\varphi}{2} (\pi - \bar{\pi})^2 \right] z - \gamma \left( \frac{\eta - 1}{\varphi} \frac{u_{NN}}{z u_C} + \beta \frac{F(s, B', 0)}{N^2 z u_C} \right) = 0, \quad (54)$$

$$- \lambda \varphi (\pi - \bar{\pi}) z N - \kappa \alpha_P \frac{u_C}{\pi} + \gamma (2\pi - \bar{\pi}) = 0, \quad (55)$$

$$\left[ q + \frac{dq}{dB'} (B' - (1 - \delta)B) \right] \left\{ \lambda \frac{\rho}{\rho - 1} e - \lambda_e \frac{1}{\rho - 1} \frac{\rho}{\rho - 1} e^{2-\rho} \right\} - \beta i \frac{\partial M}{\partial B'} \kappa - \beta \frac{1}{u_C z N} \frac{\partial F}{\partial B'} \gamma \quad (56)$$

$$= \beta_g \mathbb{E}(1 - D')(r^* + \delta + (1 - \delta)q(s', B'')) \left\{ \lambda' \frac{\rho}{\rho - 1} e' - \lambda'_e \frac{1}{\rho - 1} \frac{\rho}{\rho - 1} (e')^{2-\rho} \right\}.$$

We can replace the multipliers  $\lambda$  and  $\lambda_e$  with  $(C, C^f, N, \pi)$  using the first-order conditions of (52) and (53). We can replace the term  $\lambda \frac{\rho}{\rho - 1} e - \lambda_e \frac{1}{\rho - 1} \frac{\rho}{\rho - 1} e^{2-\rho}$  on the Euler equation (56) with the borrowing wedge  $\tau_m^X$ :

$$\tau_m^X = G^X [u_C \kappa + ((\eta - 1)/\varphi + (\pi - \bar{\pi})\pi) \gamma],$$

where  $G^X = \frac{\frac{u_{C^f C^f} u_C - u_{CC^f} u_{C^f}}{u_C u_{C^f}} \left( \frac{u_{CC} - u_{CC^f}}{u_C^2} - \frac{u_{CC^f}}{u_C u_{C^f}} \right)}{e^{1-\rho}/(\rho-1) - \frac{u_{C^f C^f} u_C - u_{CC^f} u_{C^f}}{u_C u_{C^f}} + \frac{u_{CC^f} u_C - u_{CC} u_{C^f}}{u_C^2}} - \frac{u_{CC^f}}{u_C u_{C^f}}$ . Also we define the borrowing wedge  $\tau_m^C$  as

$$\tau_m^C = \beta i \frac{\partial M}{\partial B'} \kappa + \beta \frac{1}{u_C z N} \frac{\partial F}{\partial B'} \gamma.$$

Plugging the two borrowing wedges  $\tau_m^X$  and  $\tau_m^C$  into the equation (56), we obtain the Euler equation (33) in Section 3.2.  $\square$

## B Proofs

### B.1 Proof of Proposition 1

The proof consists of two parts. In the first part, we prove that higher borrowing  $B'$  increases future default risk. In the second part we show that, under Assumption 1, current inflation, the nominal domestic rate, and the monetary wedge increase with  $B'$ .

We first present the Private and Monetary Equilibrium under the quasi-linear preferences in (36). In state  $S = (s, B, \Theta, B')$ , the equilibrium satisfies the following conditions

$$C + (C(\rho - 1)(1 - \theta)/\rho\theta)^\rho = z(\bar{z}, \Theta)N \left[ 1 - \frac{\varphi}{2}(\pi - \bar{\pi})^2 \right], \quad (57)$$

$$(\pi - \bar{\pi})\pi = \frac{\eta - 1}{\varphi} \left( \frac{CN^{1/\zeta}}{\theta z(\bar{z}, \Theta)} - 1 \right) + \beta \frac{C}{\theta z(\bar{z}, \Theta)N} F(s, B', \Theta), \quad (58)$$

$$\frac{\theta}{C} = i\beta M(s, B', \Theta), \quad (59)$$

$$i = \bar{i} \left( \frac{\pi}{\bar{\pi}} \right)^{\alpha_p} m, \quad (60)$$

$$C^f = (C(\rho - 1)(1 - \theta)/\rho\theta)^{\rho-1} + (1 - \Theta) [q(s, B')(B' - (1 - \delta)B) - (r^* + \delta)B], \quad (61)$$

where we used the definition of exports,  $X = e^\rho \xi$  with  $\xi = 1$ , and the relations  $u_{C^f}/u_C = \rho e/(\rho - 1)$ ,  $u_{C^f} = 1 - \theta$ ,  $u_C = \theta/C$ , and  $-u_N = N^{1/\zeta}$ . The  $F$  and  $M$  function are given by (27) and (28), respectively. In the following, we consider how  $B'$  affects default risk and the Private and Monetary Equilibrium.

**Higher  $B'$  increases default risk**  $E_{s'} [\Phi(v^*(s', B'))]$  We first show that the government's value under repayment  $W(s, B)$  is decreasing in  $B$ . Take two values  $B_0$  and  $B_1$  with  $0 < B_0 < B_1$ . We want to show  $W(s, B_0) > W(s, B_1)$  for any given  $s$ . First, for a given  $B'$ , the current debt  $B$  is absorbed by  $C^f$  and does not affect  $\{C, N, \pi, i\}$ . We can therefore write  $\{C, N, \pi, i\}$  as functions of  $(s, B')$  and  $C^f$  as a function of  $(s, B, B')$ , conditional on repayment. Hence, every  $B'$  that is feasible under  $B_1$  is also feasible under  $B_0$  since  $(C, N, \pi, i)$  remains the same under the same  $B'$ , while  $C^f$  can take any real value. Moreover,

$$(1 - \delta)q(s, B')B_0 + (r^* + \delta)B_0 < (1 - \delta)q(s, B')B_1 + (r^* + \delta)B_1$$

since  $q(s, B') \geq 0$  and  $r^* + \delta > 0$ . This implies  $C^f(s, B_0, B') > C^f(s, B_1, B')$  for any given  $(s, B')$ . Let  $B'_1$  and  $B'_0$  be the optimal borrowing levels associated with  $B_1$  and  $B_0$ , respectively. The following inequalities hold

$$\begin{aligned} W(s, B_1) &= u \left( C(s, B'_1), C^f(s, B_1, B'_1), N(B'_1) \right) + \beta_g \mathbb{E}V(s', B'_1) \\ &< u \left( C(s, B'_1), C^f(s, B_0, B'_1), N(s, B'_1) \right) + \beta_g \mathbb{E}V(s', B'_1) \\ &\leq u \left( C(s, B'_0), C^f(s, B_0, B'_0), N(s, B'_0) \right) + \beta_g \mathbb{E}V(s', B'_0) \\ &= W(s, B_0). \end{aligned}$$

Note that the first inequality holds because  $C^f(s, B_0, B') > C^f(s, B_1, B')$ , and the second inequality holds because under  $B_0$ ,  $B'_1$  is feasible yet  $B'_0$  is the optimal choice. Hence for  $B_0 < B_1$ ,  $W(s, B_0) > W(s, B_1)$  for any given  $s$ .

The default cutoff given by  $v^*(s, B) = W^d(s) - W(s, B)$  increases with  $B$  since the repaying value  $W(s, B)$  decreases with  $B$  for any given  $s$  and the defaulting value  $W^d(s)$  is independent of  $B$ . This makes the default probability  $\Phi(v^*(s, B))$  increase with  $B$  for any given  $s$ . As a result, the default risk given by  $E_{s'} [\Phi(v^*(s', B'))]$  increases with  $B'$ .

**Higher  $B'$  increases inflation, the nominal rate, and the monetary wedge** We consider the case when the the economy is in good credit standing, with  $\Theta = 0$ .  $B'$  impacts  $\{C, N, \pi, i\}$  exclusively through its effect on the  $F$  and  $M$  functions. We approximate the system of equations (57–61) and the monetary wedge given by

$$\text{weg} = \frac{\theta z(\bar{z}, \Theta)}{CN^{1/\zeta}} - 1$$

with a first-order Taylor expansion around the equilibrium and the monetary wedge  $(\bar{C}, \bar{N}, \bar{\pi}, \bar{i}, \bar{\text{weg}})$  associated with  $\bar{B}'$ . We focus on a level of borrowing  $\bar{B}'$  where inflation is close to target and the monetary wedge is close to zero. We solve for deviations of the equilibrium variables. In the solution, holding shocks constant  $d\bar{z} = dm = 0$ , the deviation of inflation  $d\pi$ , nominal domestic rates  $di$ , and the monetary wedge  $d\text{weg}$  are

$$d\pi = a_1 \left[ -\frac{1}{a_0} dM + dF \right] \quad (62)$$

$$di = \alpha_P \frac{\bar{i}}{\bar{\pi}} d\pi \quad (63)$$

$$d\text{weg} = a_2 \left[ \frac{\beta \bar{i}}{\theta} dM + \alpha_P dF \right] \quad (64)$$

where the positive constants  $a_1$  and  $a_2$  are a convolution of parameters,

$a_1 = \frac{\alpha_C}{\bar{\pi}^2 + \alpha_P \frac{\eta-1}{\varphi} (1 + \frac{1}{\zeta} (\alpha_C + \rho(1-\alpha_C)))} > 0$ ,  $a_2 = \frac{(1 + \frac{1}{\zeta} (\alpha_C + \rho(1-\alpha_C))) \alpha_C}{1 + \alpha_P \frac{\eta-1}{\varphi} (1 + \frac{1}{\zeta} (\alpha_C + \rho(1-\alpha_C)))} > 0$ , and  $\alpha_C = \bar{C}/\bar{N} > 0$ . The deviations of inflation, nominal rates, and the monetary wedge derived in (62–64) together with Assumption 1 prove the result.  $\square$

## B.2 Proofs for Two-Period Example

Here we collect all the proofs for Section 4.2.

### B.2.1 Proof of Lemma 1

*Proof.* Simplifying the system of equations of the private equilibrium (37) under no default, we can show the second period's consumption  $C_2$  solves the following condition

$$C_2 + [C_2(\rho - 1)(1 - \theta)/(\rho\theta)]^\rho = z(\theta z/C_2)^\zeta. \quad (65)$$

Let the optimal solution be  $C_2(z)$ , which increases with  $z$ . The consumption in default  $C_{2d}(z_d)$  satisfies equation (65) with the productivity given by  $z_d$ . Assuming  $z_d \leq z$  and given that  $C_2(z)$  is strictly increasing, we infer that  $C_{2d}(z_d) \leq C_2(z)$ . Define a function  $\mathcal{U}(C, \bar{z})$  as

$$\mathcal{U}(C, \bar{z}) \equiv \log(C) + [C(\rho - 1)(1 - \theta)/(\rho\theta)]^{\rho-1} - \frac{(\theta \bar{z}/C)^{1+\zeta}}{1 + 1/\zeta}.$$

It is easy to show that the repayment value in period 2 is linear in  $B$  and given by  $W_2(B) = \mathcal{U}(C_2(z), z) - B$ . The defaulting value is given by  $W_2^d - v = \mathcal{U}(C_{2d}(z_d), z_d) - v$  for any utility cost  $v$ .

The default cutoff  $v^*(B; z)$  equalizes the defaulting value and the repayment value, that is,

$$\mathcal{U}(C_2(z), z) - B = \mathcal{U}(C_{2d}(z_d), z_d) - v^*$$

so that it is linear in  $B$  and the difference in payoffs between default and repayment

$$v^*(B; z) = \mathcal{U}(C_{2d}(z_d), z_d) - \mathcal{U}(C_2(z), z) + B.$$

Default is preferred if and only if the default cost is small enough, i.e.,  $v \leq v^*(B; z)$ . The default probability is therefore given by  $\Phi(v^*(B; z))$  with the following derivatives

$$\begin{aligned} \frac{\partial \Phi(v^*(B; z))}{\partial B} &= \phi(v^*(B; z)) \frac{\partial v^*(B; z)}{\partial B} = \phi(v^*(B; z)) \geq 0 \\ \frac{\partial \Phi(v^*(B; z))}{\partial z} &= \phi(v^*(B; z)) \frac{\partial v^*(B; z)}{\partial z} = -\phi(v^*(B; z)) \frac{\partial \mathcal{U}(C_2(z), z)}{\partial z} \leq 0, \end{aligned}$$

where we used the relation  $\frac{\partial v^*(B; z)}{\partial z} = -\frac{\partial \mathcal{U}(C_2(z), z)}{\partial z} \leq 0$  since the function  $\mathcal{U}(C_2(z), z)$  increases with  $z$ . In summary, default risk increases with debt  $B$  and decreases with productivity  $z$ .  $\square$

## B.2.2 Proof of Proposition 2

*Proof.* Here we want to prove that higher default risk increases the monetary wedge in period 1. The private allocations  $(C_1, N_1)$  and the terms of trade of  $e_1$  satisfy the following three equations

$$C_1 + e_1^\rho = N_1, \tag{66}$$

$$C_1 = \frac{\rho\theta}{(\rho-1)(1-\theta)} e_1, \tag{67}$$

$$\frac{1}{C_1} = \frac{\beta i}{\tilde{\pi}} \left[ \frac{1 - \Phi(v^*(B))}{C_2} + \frac{\Phi(v^*(B))}{C_{2d}} \right]. \tag{68}$$

Differentiating the above system of equations, we have

$$d \log C_1 = -\frac{C_1 \beta i}{\tilde{\pi}} \left( \frac{1}{C_{2d}} - \frac{1}{C_2} \right) d\Phi,$$

$$d \log N_1 = \left( \frac{C_1}{N_1} + \rho \frac{e_1^\rho}{N_1} \right) d \log C_1.$$

The monetary wedge in period 1 is  $weg_1 = \theta / [C_1 N_1^{1/\zeta}] - 1$ . Using this expression and the above two equations for  $d \log C_1$  and  $d \log N_1$ , we can solve for the derivative of monetary wedge with respect to default risk

$$\frac{dweg_1}{d\Phi} = \left[ \frac{1}{\zeta} \left( \frac{C_1}{N_1} + \rho \frac{e_1^\rho}{N_1} \right) + 1 \right] \frac{\theta \beta i}{\tilde{\pi} N_1^{1/\zeta}} \left( \frac{1}{C_{2d}} - \frac{1}{C_2} \right) \geq 0.$$

A higher default risk raises the monetary wedge, since  $C_{2d} \leq C_2$ .  $\square$

### B.2.3 Proof of Lemma 2

*Proof.* To prove Lemma 2, we first characterize the equilibrium  $C$  and  $N$  under our model and under the flexible-price economy. Using the system of equations (66–68), we can show that  $C_1(B)$  and  $N_1(B)$  satisfy

$$C_1(B) = \frac{1}{\beta(i/\tilde{\pi})} \frac{1}{\frac{1-\Phi(v^*(B))}{C_2} + \frac{\Phi(v^*(B))}{C_{2d}}}, \quad (69)$$

$$N_1(B) = C_1(B) + \left( \frac{(\rho-1)(1-\theta)}{\rho\theta} \right)^\rho C_1(B)^\rho. \quad (70)$$

Similarly, the equilibrium allocations in the flexible-price economy satisfy the following two equations under the optimal choice of  $B_{\text{flex}}$ :

$$C_1^{\text{flex}} = \frac{1}{\beta r^{\text{flex}}} \frac{1}{\frac{1-\Phi(v^*(B_{\text{flex}}^*))}{C_2} + \frac{\Phi(v^*(B_{\text{flex}}^*))}{C_{2d}}}, \quad (71)$$

$$N_1^{\text{flex}} = C_1^{\text{flex}} + \left( \frac{(\rho-1)(1-\theta)}{\rho\theta} \right)^\rho (C_1^{\text{flex}})^\rho. \quad (72)$$

When  $B \geq B_{\text{flex}}^*$ ,  $C_1(B) \leq C_1^{\text{flex}}$  for two reasons. First, according to Proposition 2, when  $B \geq B_{\text{flex}}^*$ , the default risk is higher, i.e.  $\Phi(v^*(B)) \geq \Phi(v^*(B_{\text{flex}}^*))$ . This together with the condition  $C_{2d} \leq C_2$  implies that the future marginal utility is higher in our model, i.e.

$$\frac{1-\Phi(v^*(B))}{C_2} + \frac{\Phi(v^*(B))}{C_{2d}} \geq \frac{1-\Phi(v^*(B_{\text{flex}}^*))}{C_2} + \frac{\Phi(v^*(B_{\text{flex}}^*))}{C_{2d}}.$$

Note that  $C_2$  is independent of  $B$  due to quasi-linear preferences in  $C^f$ . Second, according to Assumption 2, the real interest rate is higher in our model,  $i/\tilde{\pi} \geq r^{\text{flex}}$ . Comparing equation (69) with (71), we see that  $C_1(B) \leq C_1^{\text{flex}}$  when  $B \geq B_{\text{flex}}^*$  due to both the higher default risk and the higher interest rate in our model. Comparing (70) with (72), we can show that  $N_1(B) \leq N_1^{\text{flex}}$  because  $C_1(B) \leq C_1^{\text{flex}}$ .

The monetary wedge defined in (19) is

$$\text{monetary wedge} = \frac{\theta}{N_1^{1/\zeta} C_1} - 1.$$

Given that  $N_1(B)^{1/\zeta} C_1(B)/\theta \leq (N_1^{\text{flex}})^{1/\zeta} C_1^{\text{flex}}/\theta = 1$ , the monetary wedge in our model is non-negative. Define the borrowing wedge  $\tau_m^C(B)$  as

$$\begin{aligned} \tau_m^C(B) &= \left( 1 + \frac{u_{N_1}(B)}{u_{C_1}(B)} \right) \frac{[1 + (\rho-1)e_1(B)^{\rho-1}] \beta i \phi(v^*(B)) (u_{C_{2d}} - u_{C_2})}{u_{C_1}(B) \tilde{\pi}} \\ &= \frac{\text{monetary wedge}}{1 + \text{monetary wedge}} \frac{[1 + (\rho-1)e_1(B)^{\rho-1}] C_1(B) \beta i \phi(v^*(B))}{\tilde{\pi}} \left( \frac{1}{C_{2d}} - \frac{1}{C_2} \right) \end{aligned} \quad (73)$$

which is non-negative as long as the monetary wedge is non-negative. Hence, for  $B \geq B_{\text{flex}}^*$ , the monetary wedge is non-negative and  $\tau_m^C(B) \geq 0$ .  $\square$

### B.2.4 Proof of Proposition 3

*Proof.* We first derive the government's Euler equation in the flexible-price economy. The government chooses  $B$ ,  $C_1$ ,  $C_1^f$ , and  $N_1$  to maximize  $u(C_1, C_1^f, N_1) + \beta_g \max \{W_2(B), W_2^d - v\}$  subject to the resource constraint  $C_1 + \left(C_1^f - \frac{1}{1+r^*}[1 - \Phi(v^*(B))]\right)B^{\frac{\rho}{\rho-1}} = N_1$ . The first-order condition on  $B$  gives rise to

$$1 - h(v^*(B_{\text{flex}}^*))B_{\text{flex}}^* = \beta_g(1 + r^*).$$

In our baseline model, the government chooses  $B$ ,  $C_1$ ,  $C_1^f$ , and  $N_1$  to solve the following problem

$$\max u \left( C_1(B), C_1^f(B), N_1(B) \right) + \beta_g \left\{ [1 - \Phi(v^*(B))]W_2(B) + \int_{-\infty}^{v^*(B)} (W_2^d - v')d\Phi(v') \right\},$$

subject to

$$\begin{aligned} C_1 + \left[ C_1^f - \frac{1}{1+r^*}[1 - \Phi(v^*(B))]B \right]^{\frac{\rho}{\rho-1}} &= N_1, \\ C_1 &= \frac{\rho\theta}{(\rho-1)(1-\theta)} \left[ C_1^f - \frac{1}{1+r^*}[1 - \Phi(v^*(B))]B \right]^{\frac{1}{\rho-1}}, \\ \frac{1}{C_1} &= i\beta M(B), \end{aligned}$$

where the expected future marginal utility function  $M(B)$  is given by

$$M(B) = \frac{1}{\tilde{\pi}} \left\{ [1 - \Phi(v^*(B))] \frac{1}{C_2} + \Phi(v^*(B)) \frac{1}{C_{2d}} \right\}.$$

Taking first order conditions on  $B$ , we can show that equation (40) holds with  $\tau_m^C(B)$  given by equation (73).

We now prove that default probability is lower in the baseline model,  $\Phi^* \leq \Phi_{\text{flex}}^*$ , by contradiction. Suppose when  $\Phi^* > \Phi_{\text{flex}}^*$  for  $B^* > B_{\text{flex}}^*$ . According to Lemma 2,  $\tau_m^C(B^*) \geq 0$ . Given that we focus on hazard functions strictly increasing in  $v$ , we can show the following inequalities hold:

$$1 - h(v^*(B^*))B^* - \frac{\tau_m^C(B^*)}{1 - \Phi(v^*(B^*))} \leq 1 - h(v^*(B^*))B^* < 1 - h(v^*(B_{\text{flex}}^*))B_{\text{flex}}^* = \beta_g(1 + r^*).$$

Hence we reach a contradiction on the optimality of  $B^*$ . In equilibrium, it has to be the case that  $B^* \leq B_{\text{flex}}^*$  and  $\Phi^* \leq \Phi_{\text{flex}}^*$ .  $\square$

## C Impulse Response Functions to Monetary Shock

In the event analysis we perform a counterfactual to Brazil by altering the path of nominal domestic rates using monetary shocks  $m$ . In this appendix, we present detailed impulse response

functions for high monetary shocks  $m$  for aggregate output, domestic consumption, imports, inflation, nominal rates, terms of trade, debt, and spreads for our benchmark model augmented to have monetary shocks, with  $\log m \sim \mathcal{N}(0, 0.001)$ . Figure 9 plots the responses to a 1% increase in the  $m$  shock above its mean. The solid blue lines are for the benchmark NK-Default model, and the dashed red lines are for the NK-Reference model. In our model, tight monetary policy depresses output, domestic and foreign goods consumption, and inflation, and leads to a decline in borrowing and sovereign spreads.

## D Extensions and Robustness

This appendix contains additional plots for the extensions and robustness exercises in Section 6. We report impulse response functions to low-productivity realizations, mirroring Figure 5, for the three extensions considered: local currency government debt in Figure 10, an interest rate rule with a higher weight on inflation in Figure 11, and the interest rate rule with an output gap term in Figure 12. In all three figures, the solid blue line is our baseline NK-Default model, and the dashed red line is the extended model.

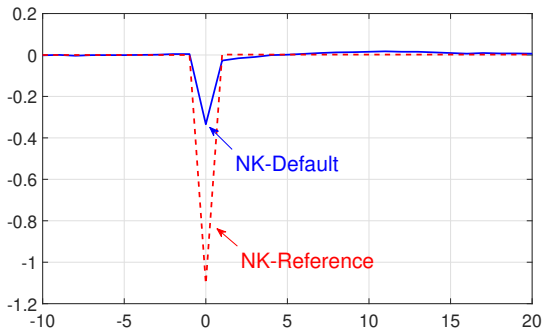
Figure 13 plots the monetary wedge (left panels) and the nominal domestic rate (right panels) against the level of debt for the benchmark model, in solid blue lines, and for extended models, in dashed red lines. This figure shows that the analysis in Section 5.4 applies to the extended models, as the behavior of the monetary wedge and nominal rates in the two high and low default zones, respectively, is similar to that in the baseline model.

## E NK-Reference with Global Methods

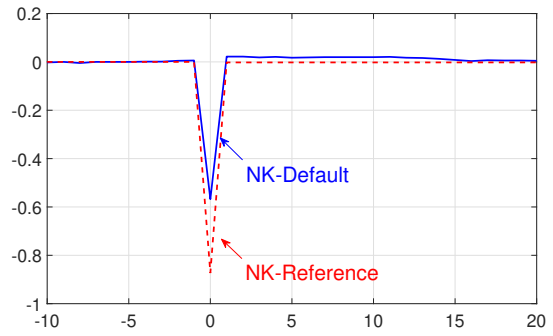
In the main text, we use as the NK-Reference model a version of Galí and Monacelli (2005) solved using local methods with very loose borrowing constraints. This appendix contains results from a version of our baseline model reparameterized to have loose borrowing constraints, no default risk in equilibrium, and a global methods solution. We show that despite seemingly different international borrowing Euler equations (33) and (34), arising from households borrowing in the NK-Reference model while the government is borrowing in our model, the results from these models are very similar. The reason is that, with loose borrowing constraints, the borrowing wedges  $\tau^X$  and  $\tau^C$  are close to zero.

We reparameterize the NK-Default model to match key properties of the NK-Reference environment: spreads with zero mean and zero volatility, and consumption volatility in line with the data. To implement these alternative targets, we start with the baseline parameter values but set  $\lambda_0 = -0.145$ ,  $\varrho_D \rightarrow 0$ , and  $\beta_g = \beta$ . These parameters result in ample borrowing opportunities and spreads with mean and volatility less than 1 basis point. Table 7 compares the second moments of NK-Reference and the reparameterized NK-Default model, which we label Global NK-Reference. Their behavior is very similar: the volatilities of CPI inflation and the nominal domestic rate are close and much lower than in the baseline model, whereas the ratio of trade balance to output is strongly procyclical and four times more volatile than in baseline, a tell

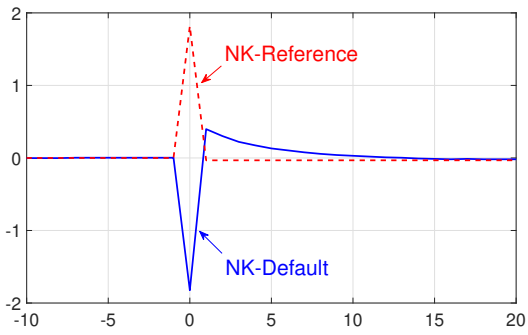




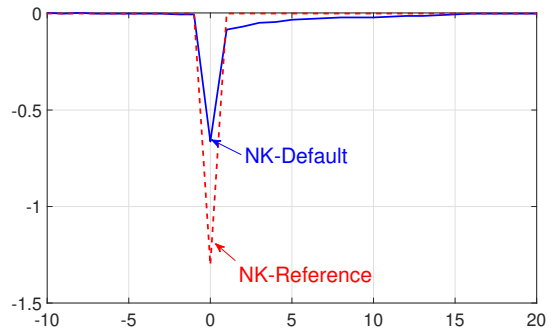
(a) Output ( $Y$ )



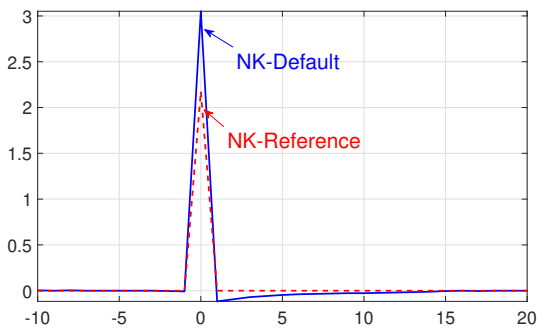
(b) Domestic Goods Consumption ( $C$ )



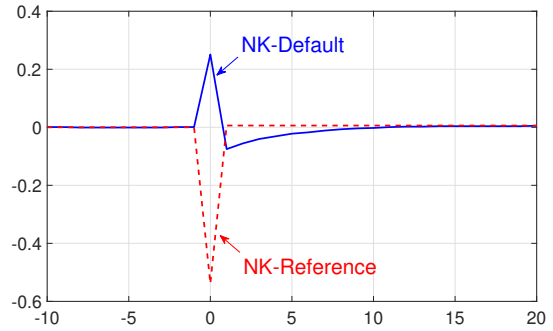
(c) Foreign Goods Consumption ( $C^f$ )



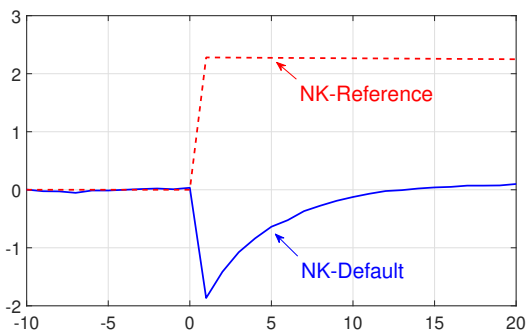
(d) Inflation ( $\pi$ )



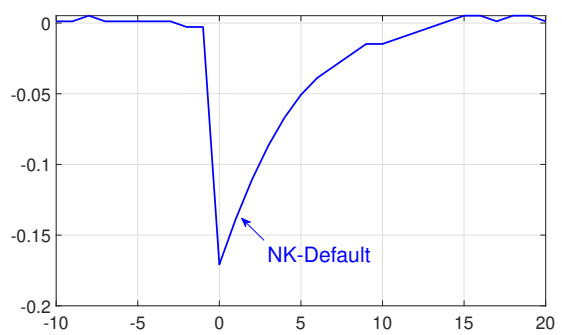
(e) Nominal Domestic Rate ( $i$ )



(f) Terms of Trade ( $e$ )

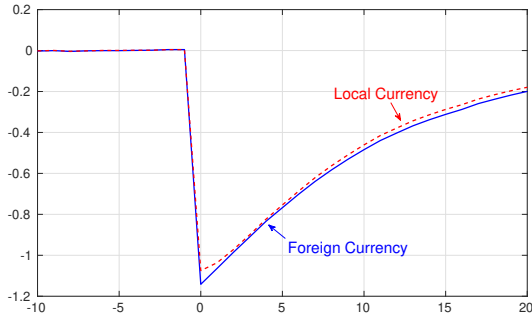


(g) Debt ( $B$ )

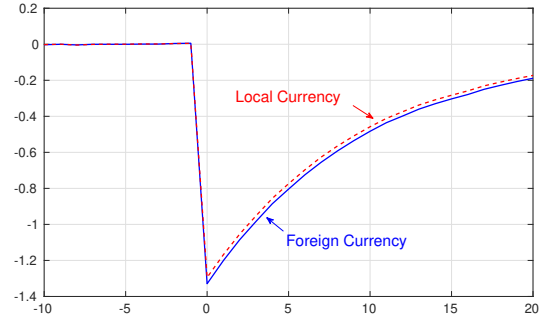


(h) Spread

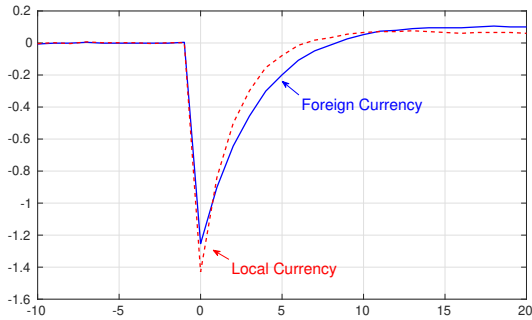
Figure 9: Impulse Responses to Monetary Shock



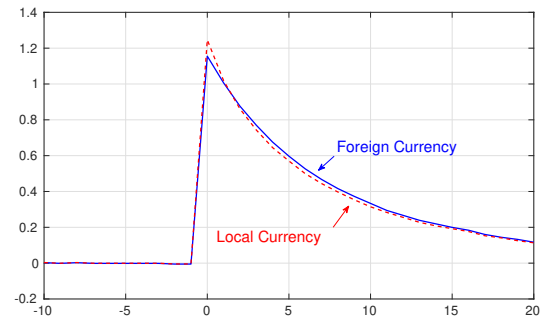
(a) Output ( $Y$ )



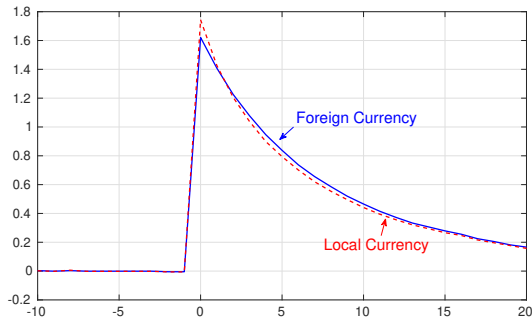
(b) Domestic Goods Consumption ( $C$ )



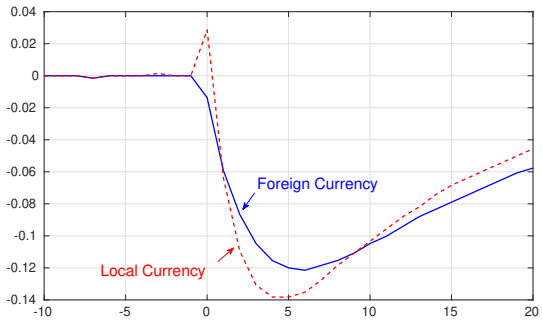
(c) Foreign Goods Consumption ( $C^f$ )



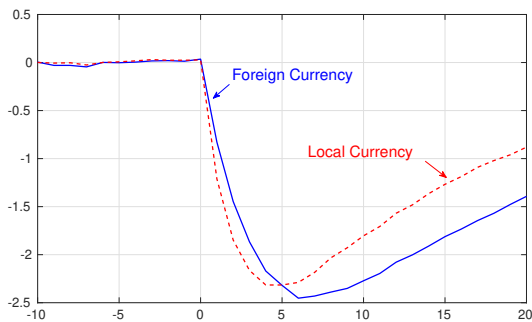
(d) Inflation ( $\pi$ )



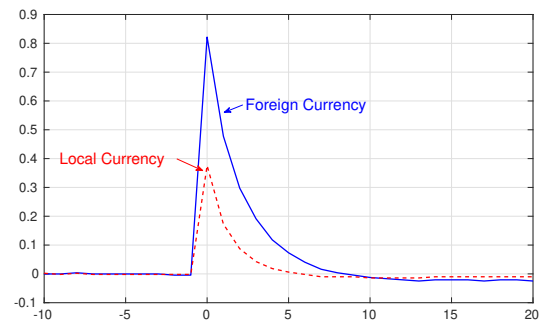
(e) Nominal Domestic Rate ( $i$ )



(f) Terms of Trade ( $e$ )

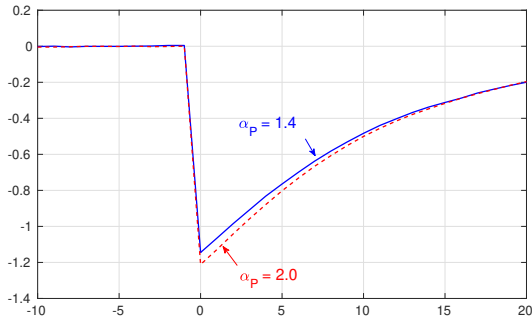


(g) Debt ( $B$ )

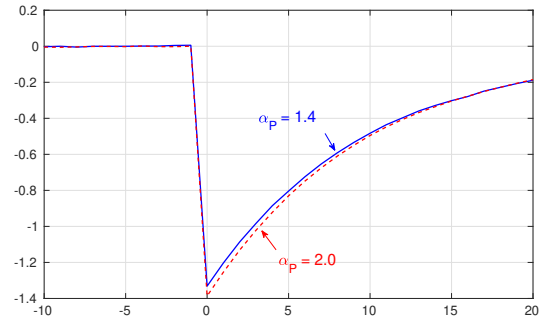


(h) Spread

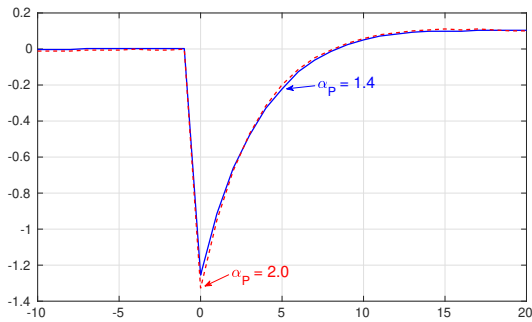
Figure 10: Local Currency: Impulse Responses to Productivity Shock



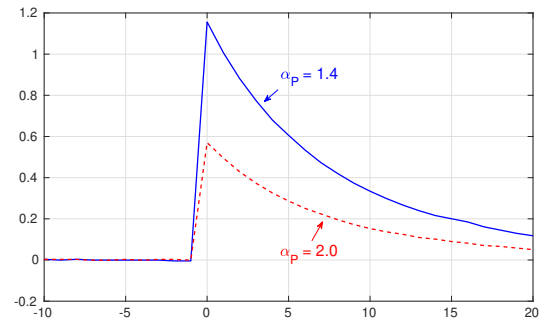
(a) Output ( $Y$ )



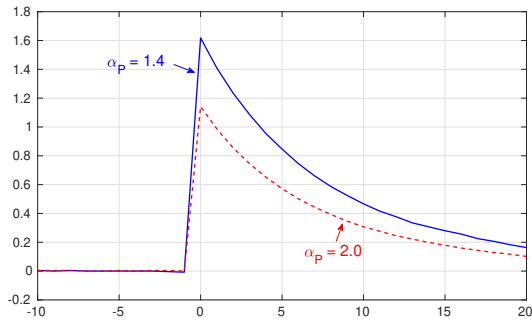
(b) Domestic Goods Consumption ( $C$ )



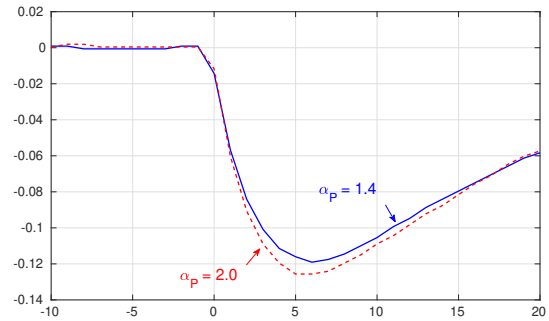
(c) Foreign Goods Consumption ( $C^f$ )



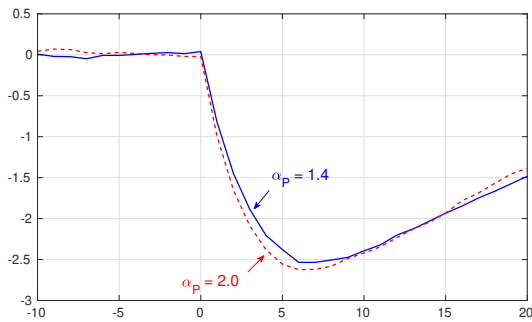
(d) Inflation ( $\pi$ )



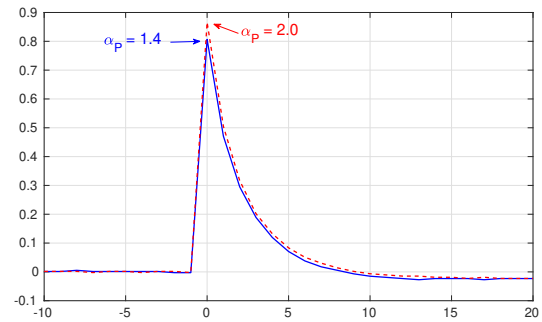
(e) Nominal Domestic Rate ( $i$ )



(f) Terms of Trade ( $e$ )

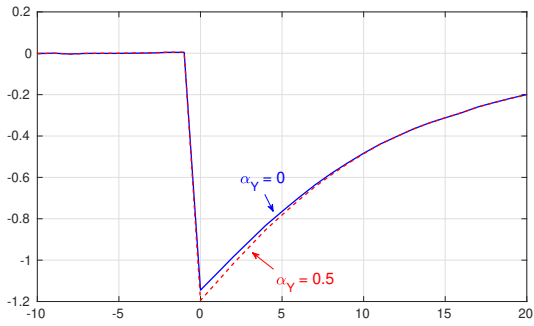


(g) Debt ( $B$ )

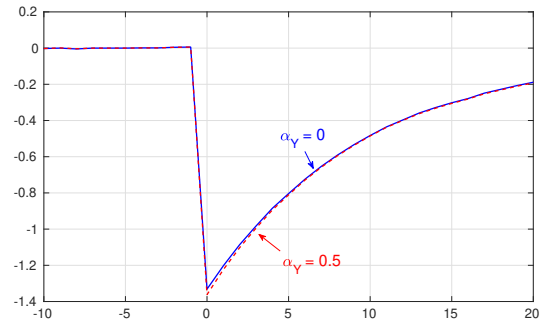


(h) Spread

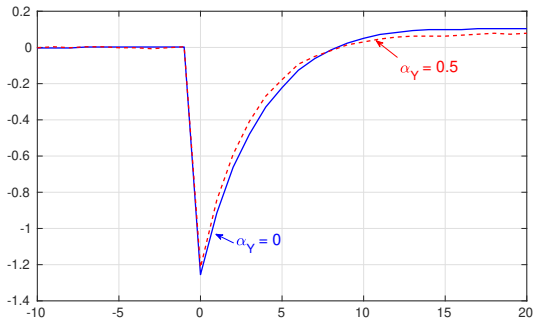
Figure 11: Rule with Higher  $\alpha_p$ : Impulse Responses to Productivity Shock



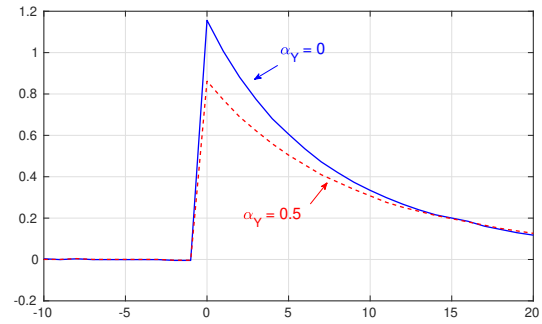
(a) Output ( $Y$ )



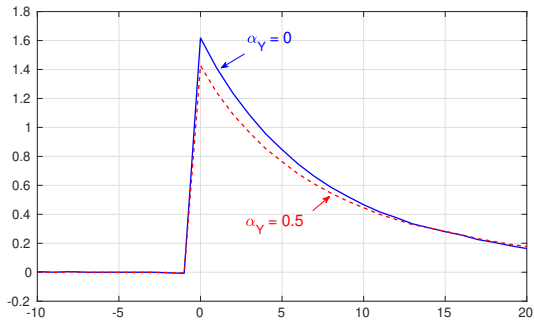
(b) Domestic Goods Consumption ( $C$ )



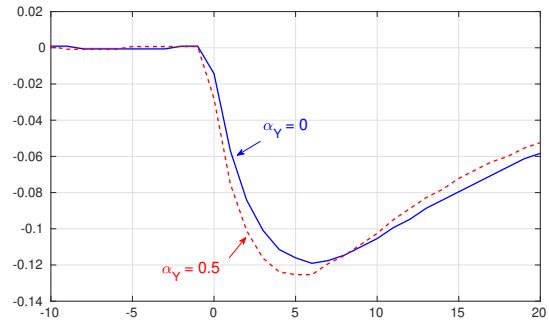
(c) Foreign Goods Consumption ( $C^f$ )



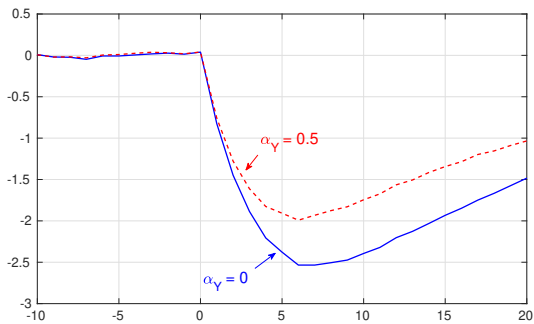
(d) Inflation ( $\pi$ )



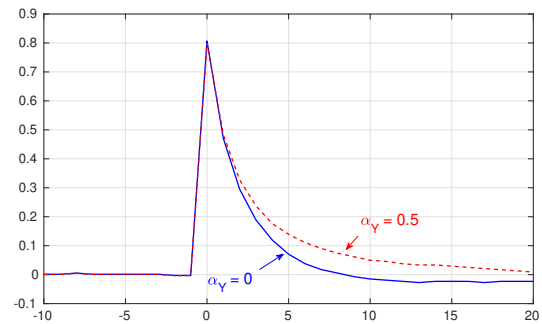
(e) Nominal Domestic Rate ( $i$ )



(f) Terms of Trade ( $e$ )

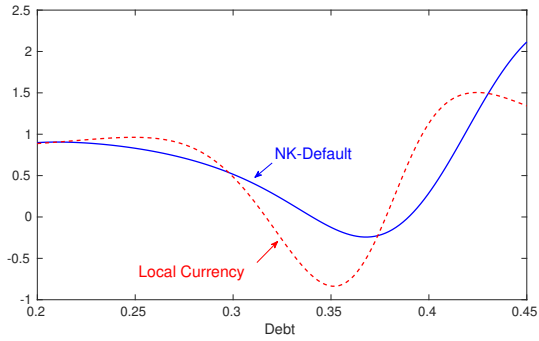


(g) Debt ( $B$ )

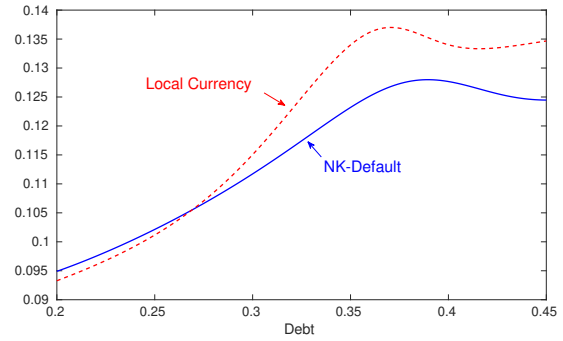


(h) Spread

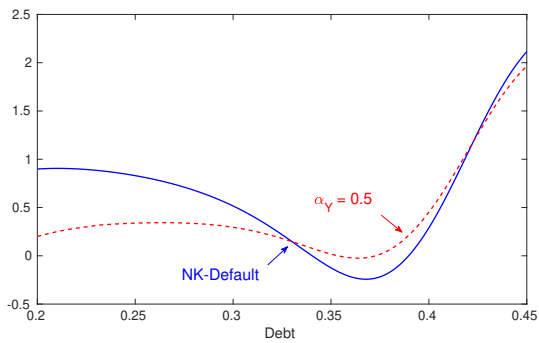
Figure 12: Rule with Output Gap: Impulse Responses to Productivity Shock



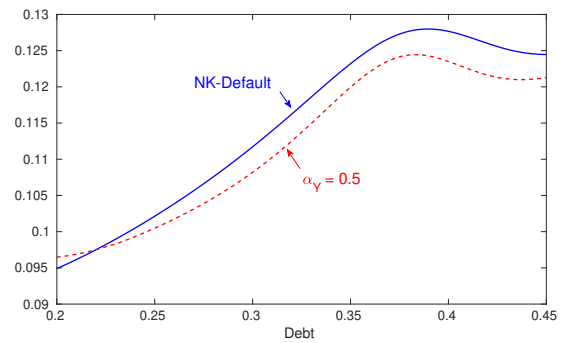
(a) Monetary Wedge: Local Currency



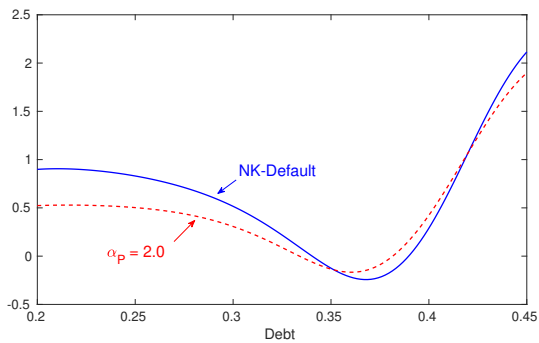
(b) Nominal Domestic Rate: Local Currency



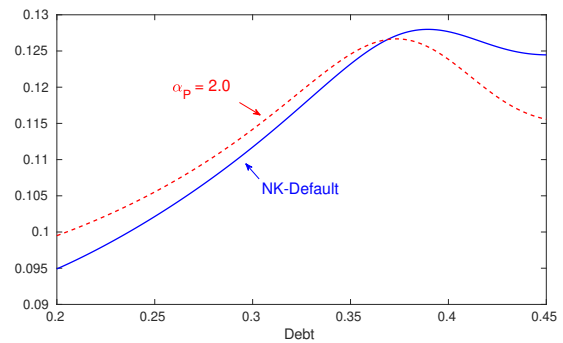
(c) Monetary Wedge:  $\alpha_\gamma = 0.5$



(d) Nominal Domestic Rate:  $\alpha_\gamma = 0.5$



(e) Monetary Wedge:  $\alpha_p = 2.0$



(f) Nominal Domestic Rate:  $\alpha_p = 2.0$

Figure 13: Robustness: Monetary Wedges and Nominal Domestic Rates

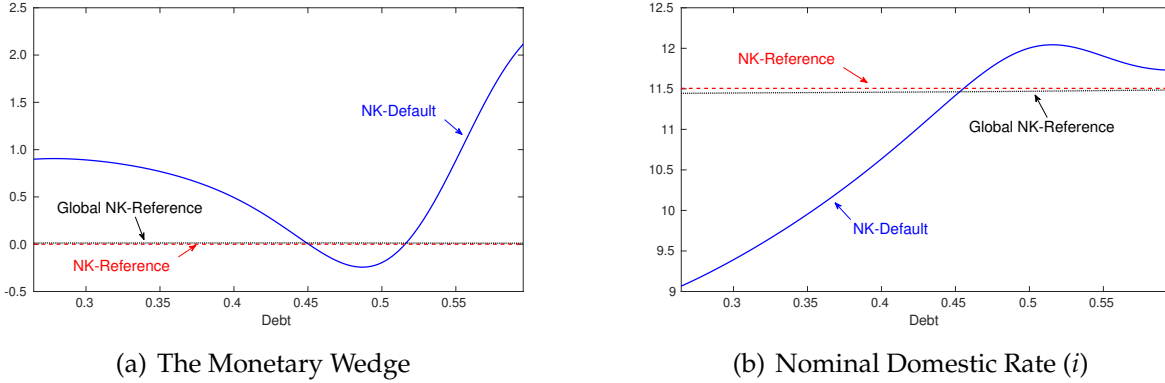


Figure 14: Debt and the Monetary Wedge, Reparameterized NK-Default

tale sign of the greatly relaxed financial frictions. In Figure 14, we also show that the monetary wedge and the nominal interest rate function from the Global NK-Reference model are very similar to the NK-Reference model, in contrast to the resulting functions with default risk.

## F Numerical Implementation<sup>30</sup>

### F.1 Computation with Taste Shocks

We compute the model using discrete choice methods, following [Dvorkin et al. \(2018\)](#) and [Gordon \(2019\)](#), who adapt tools frequently used in structural applied work for the study of sovereign default with long-term debt. [Chatterjee and Eyigungor \(2012\)](#) follow a related strategy, also perturbing the borrowing choice ( $B'$ ), in order to address the convergence problems inherent in models with long-term debt.

We restrict the choice of  $B'$  to be in a discrete set and associate each option with an i.i.d. taste shock distributed Gumbel (Extreme Value Type I). The government's problem becomes

$$\mathcal{W}(s, B, \langle \epsilon_{B'} \rangle) = \max_{B'} \{ \mathcal{J}(s, B, B') + \varrho_B \epsilon_{B'} \} \quad (74)$$

with

$$\mathcal{J}(s, B, B') \equiv u \left[ C(s, B, B'), C^f(s, B, B'), N(s, B, B') \right] + \beta_g \mathbb{E}_{s'|s} \mathcal{V}(s', B') \quad (75)$$

and where  $\varrho_B$  is a constant governing the relative importance of the taste shocks for the choice of  $B'$  and  $\langle \epsilon_{B'} \rangle$  is a vector of taste shocks, one for each possible value of  $B'$  on the grid. As  $\varrho_B \rightarrow 0$  we recover the unperturbed initial problem, with poor numerical convergence properties, whereas as  $\varrho_B \rightarrow +\infty$  the taste shocks dominate and the choice of  $B'$  becomes uniform i.i.d.. Ex ante,

<sup>30</sup>The authors would like to thank Stony Brook Research Computing and Cyberinfrastructure, and the Institute for Advanced Computational Science at Stony Brook University for access to the high-performance Lired and SeaWulf computing systems, the latter of which was made possible by a \$1.4M National Science Foundation grant (#1531492).

|                                | NK-Reference | Global |
|--------------------------------|--------------|--------|
| <i>Mean</i>                    |              |        |
| CPI inflation                  | 6.1          | 6.1    |
| Nominal domestic rate          | 11.5         | 11.5   |
| Spread                         | —            | 0.0    |
| <i>Standard Deviation</i>      |              |        |
| CPI inflation                  | 1.0          | 1.1    |
| Nominal domestic rate          | 1.3          | 1.2    |
| Spread                         | —            | 0.0    |
| Output                         | 2.4          | 2.4    |
| Consumption aggregate          | 2.0          | 1.6    |
| Trade balance                  | 1.8          | 1.3    |
| Nominal depreciation rate      | 2.0          | 2.2    |
| <i>Correlation with Output</i> |              |        |
| CPI inflation                  | −81          | −36    |
| Nominal domestic rate          | −98          | −78    |
| Trade balance                  | 62           | 78     |
| Nominal depreciation rate      | −28          | −1     |

Table 7: NK-Reference Results Using Global Methods

before taste shocks are realized, the choice probabilities are given by

$$\Pr(B' = x|s, B) = \frac{\exp[\mathcal{J}(s, B, x)/q_B]}{\sum_{\tilde{x}} \exp[\mathcal{J}(s, B, \tilde{x})/q_B]} = \frac{\exp[(\mathcal{J}(s, B, x) - \bar{\mathcal{J}}(s, B))/q_B]}{\sum_{\tilde{x}} \exp[(\mathcal{J}(s, B, \tilde{x}) - \bar{\mathcal{J}}(s, B))/q_B]} \quad (76)$$

with  $\bar{\mathcal{J}}(s, B) = \max_{B'} \mathcal{J}(s, B, B')$  and the government's value is

$$W(s, B) = \mathbb{E}_{\langle \epsilon_{B'} \rangle} \{ \mathcal{W}(s, B, \langle \epsilon_{B'} \rangle) \} = \bar{\mathcal{J}}(s, B) + q_B \log \left\{ \sum_{B'} \frac{\mathcal{J}(s, B, B') - \bar{\mathcal{J}}(s, B)}{q_B} \right\}. \quad (77)$$

The value  $\bar{\mathcal{J}}(s, B)$  is what the government would achieve if all the taste shocks were zero (or if the problem were unperturbed), whereas  $W(s, B)$  is the expected value before the realization of the taste shocks. Panel (a) of Figure 15 plots an example of choice probabilities, associated with state  $\langle z = 1, B = 0.25 \rangle$ . The probability mass is tightly centered around the  $B'$  that maximizes  $\mathcal{J}(s, B, B')$ , with about 70% of the mass is concentrated in 2–3 neighboring grid points.

Additionally, we perturb the default decision in a similar fashion. At the start of each period, the government observes default decision taste shocks and decides accordingly:

$$\mathcal{V}(s, B) = \mathbb{E}_{\epsilon_{\text{Repay}}, \epsilon_{\text{Default}}} \max \left\{ W(s, B) + q_D \epsilon_{\text{Repay}}, W^d(s) + q_D \epsilon_{\text{Default}} \right\}. \quad (78)$$

As a consequence, if state  $\langle s, B \rangle$  is realized, the government chooses default with probability

$$\Pr(D = 1|s, B) = \frac{\exp[W^d(s)/q_D]}{\exp[W^d(s)/q_D] + \exp[W(s, B)/q_D]}. \quad (79)$$

For values of  $q_D$  greater than zero, the default probability is everywhere nondegenerate, although often numerically indistinguishable from zero or one. This induces bond price schedules that are smooth in the borrowing choice  $B'$ , further aiding numerical convergence. Panel (b) of Figure 15 plots “borrowing Laffer curves” ( $q(s, B')B'$ ) for various levels of the productivity shock.

In the model augmented with taste shocks, the expression for the bond price schedule becomes

$$q(s, B') = \frac{1}{1 + r^*} \mathbb{E}_{s'|s} \Pr(D = 0|s', B') \left\{ r^* + \delta + (1 - \delta) \sum_{B''} \Pr(B''|s', B') q(s', B'') \right\}. \quad (80)$$

The expectation functions  $M(s, B')$  and  $F(s, B')$  are updated analogously.

Note that the enforcement shocks  $\nu$  in the model map into the default-repayment taste shocks as follows:

$$\begin{aligned} \mathcal{V}(s, B) &= \mathbb{E}_{\epsilon_{\text{Repay}}, \epsilon_{\text{Default}}} \max \left\{ W(s, B) + q_D \epsilon_{\text{Repay}}, W^d(s) + q_D \epsilon_{\text{Default}} \right\} \\ &= \mathbb{E}_{\epsilon_{\text{Repay}}, \epsilon_{\text{Default}}} \max \left\{ W(s, B), W^d(s) + q_D \underbrace{(\epsilon_{\text{Default}} - \epsilon_{\text{Repay}})}_{\nu} \right\}. \end{aligned}$$

The  $\epsilon$  terms are i.i.d. Gumbel (Extreme Value Type I) with the location parameter given by minus the Euler-Mascheroni constant and scale parameter 1 and, as a result, their difference  $\nu$  follows the logistic distribution with location parameter 0 and scale 1. The parameter  $q_D$  controls the relative importance of the enforcement shock for the default decision.



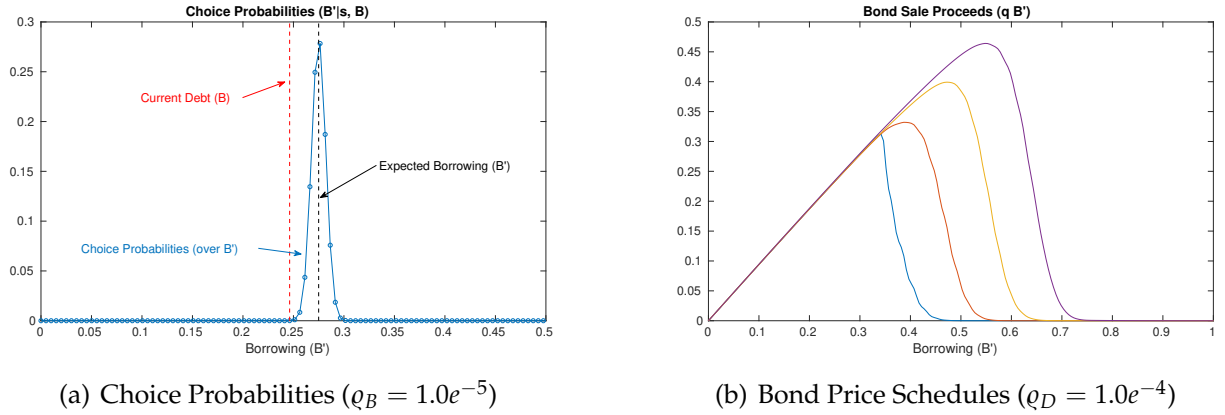


Figure 15: Computation with Taste Shocks

## F.2 Algorithm

The model is subject to an AR(1) productivity shock  $z$ , which we discretize over a grid with  $\#z = 21$  points spanning  $\pm 3$  standard deviations of its unconditional distribution. We also allow for a zero-probability shock to the interest rate rule  $m$ , which we discretize over  $\#m = 7$  points spanning  $\pm 1.5\%$ . The  $m$  shock is i.i.d., with  $\Pr(m = 1) = 1$  and  $\Pr(m \neq 1) = 0$ . We use these zero probability shocks to study the consequences of unexpected monetary tightening in the quantitative analysis. We confirm that the results are identical if instead we consider a Normally-distributed  $m$  shock with low variance. The  $B$  grid consists of  $\#B = 250$  points equally spaced over  $[0, 0.45]$ .

The algorithm proceeds as follows:

1. We start with initial guesses for the value functions  $V_0, W_0^d$  and the bond price schedule  $q_0$ , together with guesses for the  $F_0$  and  $M_0$  functions and the default and borrowing policies. We assume the probability of default is 1 and  $B' = B$  with probability one, everywhere in the state space.
2. We solve for for the private and monetary equilibrium (PME) everywhere in the state space, for arbitrary  $B'$ . We restrict attention to  $B'$  values that do not induce capital inflows or outflows that are “too large,” for which a private and monetary equilibrium might not exist and confirm that this restriction does not bind in equilibrium:

$$| -(r^* + \delta)B + q(s, B') [B' - (1 - \delta)B] | \leq 0.1 \quad (\text{Capital Inflow Bounds})$$

We solve the private and monetary equilibrium via root-finding using Powell’s hybrid method, on a system of two equations in two unknowns,  $C^f$  and  $N$ :

- (a) Using the current guess of  $\langle C^f, N \rangle$  and the capital inflow, we compute the terms of trade  $e$  from the balance of payments condition.
- (b) We compute the implied level of exports  $X$  associated with the terms of trade  $e$ .

- (c) Given  $C^f$  and  $e$ , we can recover domestic consumption  $C$  from the relative consumption condition.
- (d) Given  $C$  and the government's borrowing choice  $B'$ , we compute the domestic nominal rate  $i$  from the domestic Euler equation.
- (e) Given  $i$ , we use the interest rate rule to compute the level of PPI inflation  $\pi$ .
- (f) We use these quantities to compute equation residuals for the New Keynesian Phillips curve and the domestic resource constraint.

The solution to the PME yields policy functions  $C(s, B, B')$ ,  $C^f(s, B, B')$ ,  $N(s, B, B')$ ,  $\pi(s, B, B')$ ,  $i(s, B, B')$ ,  $e(s, B, B')$ .

3. We solve the PME in default similarly. In particular, in default trade is balanced and the capital inflow term is zero, and productivity is penalized. The solution constitutes policy functions in default:  $C_d(s)$ ,  $C_d^f(s)$ ,  $N_d(s)$ ,  $\pi_d(s)$ ,  $i_d(s)$ ,  $e_d(s)$ .
4. Using PME results, we compute the value of the government in each state ( $V$ ) and in default ( $W^d$ ) and derive choice probabilities for the  $B'$  policy and default probabilities.
5. Given borrowing and default policies (probabilities), we update the bond price schedule  $q$  and the expectation functions  $M$  and  $F$ .
6. We check for the convergence of the bond price schedule, value functions, and expectation functions. We stop if values are closer than  $1e^{-7}$  and prices are closer than  $1e^{-5}$  in the sup norm, otherwise we fully update and iterate.

**Simulation.** Model statistics are computed over a simulation of 50,000 periods in length, excluding periods in default and the 20 periods (5 years) following the return to market. Without recovery, the sovereign returns to market without obligations and accumulates debt fast over the following few periods. If we include these transitional debt dynamics in the sample used to compute model moments, we find the results are largely unaltered, with the exception of the cyclical patterns of the trade balance. By including all periods outside of default, the trade balance becomes acyclical, while with our selection criterion the trade balance is countercyclical, as in the data.

**Reference and Extended Models** The computation of the Default-Reference model—which we can think of as either the real version of our model or the model under strict inflation targeting—is almost identical, with one exception: in computing the PME, we use an undistorted labor market condition,  $-u_N/u_C = z$ , instead of the NKPC in the system of two equations in two unknowns. The computation of the extended models is similar to the benchmark: For the version with local currency debt we solve a system of three equations in three unknowns ( $C^f$ ,  $e$ ,  $\pi$ ) since now  $\pi$  appears in the balance of payments condition, breaking the block-recursivity exploited in the solution to the benchmark. The algorithm is otherwise unaltered. The model with an output gap term in the interest rate rule requires that we first compute the Default-Reference model,

simulate it, and save average output levels by productivity shock realization, then load in these statistics for the model with the extended rule. The interest rate rule then responds to deviations of output from these average output levels in the Default-Reference environment.