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SUSTAINABLE INVESTING IN EQUILIBRIUM

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ABSTRACT

We present a model of investing based on environmental, social, and governance (ESG) criteria. In equilibrium, green assets have negative alphas, whereas brown assets have positive alphas. The ESG investment industry is at its largest, and the alphas of ESG-motivated investors are at their lowest, when there is large dispersion in investors' ESG preferences. When this dispersion shrinks, so does the ESG industry, even if all investors' ESG preferences are strong. Greener assets are more exposed to an ESG risk factor, which captures shifts in customers' tastes for green products or investors' tastes for green holdings. Under plausible conditions, the latter tastes produce positive social impact.

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1. Introduction

Sustainable investing is an investment approach that considers not only financial but also environmental, social and governance (ESG) objectives. This approach initially gained popularity by imposing negative screens under the umbrella of socially responsible investing (SRI), but its scope has expanded significantly in recent years. Assets managed with an eye on sustainability have grown to tens of trillions of dollars, and they seem poised to grow further.[1] Given the rapid growth of ESG-driven investing, it seems important to understand its effects on asset prices and corporate investment.

We analyze both financial and real effects of sustainable investing through the lens of a general equilibrium model. The model features many heterogeneous firms and many heterogeneous agents, yet it is highly tractable, yielding simple and intuitive expressions for the quantities of interest. The model illuminates the key channels through which agents' preferences for sustainability can move asset prices, tilt portfolio holdings, determine the size of the ESG investment industry, and cause real impact on society.

In the model, firms differ in the sustainability of their activities. "Green" firms generate positive externalities on society, "brown" firms generate negative externalities, and there are different shades of green and brown. Agents differ in their preferences for sustainability, or "ESG preferences." These preferences have two dimensions. First, agents derive utility from holdings of green firms and disutility from holdings of brown firms. Second, they care about firms' aggregate social impact. Naturally, they also care about financial wealth.

Agents' tastes for green holdings affect asset prices. The greener the asset, the lower is its CAPM alpha in equilibrium. Green assets have negative alphas, whereas brown assets have positive alphas. Consequently, agents with stronger ESG preferences, whose portfolios tilt more toward green assets and away from brown assets, earn lower expected returns. Yet such agents are not unhappy because they derive utility from their holdings.

The model implies three-fund separation, whereby each agent holds the market portfolio, the risk-free asset, and an "ESG portfolio," which is largely long green assets and short brown assets. Agents with stronger-than-average ESG preferences go long the ESG portfolio, whereas agents with weaker preferences go short. Agents with average ESG preferences hold the market portfolio. If there is no dispersion in ESG preferences, all agents simply hold the

[1] According to the 2018 Global Sustainable Investment Review, sustainable investing assets exceeded \$30 trillion globally at the start of 2018, a 34% increase in two years. As of November 2019, more than 2,600 organizations have become signatories to the United Nations Principles of Responsible Investment (PRI), with more than 500 new signatories in 2018/2019, according to the 2019 Annual Report of the PRI.

market. Even if all agents derive a large amount of utility from green holdings, they end up holding the market if their ESG preferences are equally strong, because asset prices adjust to reflect those preferences. In this equal-preference case, the ESG investment industry does not exist: despite the strong demand for green holdings, a market index fund is all that is needed to satisfy investors. For the ESG industry to exist, some dispersion in ESG preferences is necessary. The larger the dispersion, the larger the industry.

We illustrate the economic significance of the above effects by calibrating a case in which there are two types of investors, those sharing equal concerns about ESG and those not concerned at all. The key free parameter is Δ, the maximum certain return an ESG-concerned investor is willing to forego in exchange for investing in her desired portfolio instead of the market. The negative alpha such investors earn is greatest, and the ESG industry is largest, when dispersion in ESG preferences is greatest (here meaning ESG investors constitute half of total wealth). That worst-case alpha is substantially smaller than Δ, however, because equilibrium prices adjust to ESG demands. For example, when ESG investors have a Δ of 4%, their worst-case alpha is only -2%. This difference between Δ and alpha provides ESG investors with an "investor surplus" in that they give up less return than they are willing to in order to hold their desired portfolio. ESG investing's price impact also lessens its impact on the ESG investment industry's size, which we measure as the aggregate ESG dollar tilt away from the market portfolio. For example, if the ESG industry reaches 33% of the stock market's value when Δ is 1%, then doubling the strength of ESG concerns (raising Δ to 2%) increases that maximum industry size by less than half, to 46% of the market's value.

Sustainable investing leads to positive social impact. We define social impact as the product of a firm's ESG characteristic and the firm's operating capital. By pushing green asset prices up and brown ones down, agents' tastes for green holdings induce more investment by green firms and less investment by brown firms. We show that the aggregate social impact induced by ESG investing is nonnegative, and under weak assumptions it is positive. Aggregate social impact is larger when agents care more about ESG, when they are less risk-averse, when market values have a larger effect on investment, and when the dispersion of ESG characteristics across firms is greater. Aggregate social impact is also larger if ESG investing leads firms to improve their ESG characteristics. While agents' green tastes have both financial and real effects, agents' concerns about aggregate social impact have neither. The latter concerns do not affect behavior because of agents' infinitesimal size: small agents do not internalize the social impact of their actions.

Finally, we introduce the concept of an "ESG factor" that captures unexpected changes in ESG concerns. These concerns can change in two ways: customers may shift their demands for goods of green providers, or investors may change their appreciation for green holdings. Both of these channels contribute to the ESG factor's risk in our model.

The ESG factor affects the relative performance of green and brown assets, both ex post and ex ante. Ex post, the factor's positive realizations boost green assets while hurting brown assets. If ESG concerns strengthen unexpectedly, green assets can outperform brown assets despite having lower expected returns. Ex ante, greener assets have larger exposures to the ESG factor. These exposures increase the market betas of green assets and decrease those of brown assets, as long as the ESG factor comoves positively with the aggregate economy. Consistent with such comovement, Bansal, Wu, and Yaron (2018) find that green stocks outperform brown stocks when the economy is strong, and vice versa. Interpreting their evidence in the context of our model, the ESG factor raises expected returns for green assets and lowers them for brown assets. Exposure to ESG risk is thus another reason why green assets may outperform brown assets over a period of time.

Multiple prior studies report that green assets underperform brown assets, in various contexts. Hong and Kacperczyk (2009) find that "sin" stocks (i.e., stocks of public firms producing alcohol, tobacco, and gaming, which we would classify as brown) outperform non-sin stocks. They argue that social norms lead investors to demand compensation for holding sin stocks. Barber, Morse, and Yasuda (2018) find that venture capital funds that aim not only for financial return but also for social impact earn lower returns than traditional funds. They argue that investors derive nonpecuniary utility from investing in dual-objective funds. Baker et al. (2018) and Zerbib (2019) find that green bonds tend to be priced at a premium, thus offering lower yields than otherwise similar traditional bonds. Both studies argue that the premium is driven by investors' environmental concerns. All of these results are consistent with the effects of tastes for green holdings in our model.

Some studies find the opposite result, based on different definitions of green and brown. Firms perform better if they are better-governed, judging by employee satisfaction (Edmans, 2011) or strong shareholder rights (Gompers, Ishii, and Metrick, 2003), or if they have higher ESG ratings in the 1992–2004 period (Kempf and Osthoff, 2007). These results are consistent with our model if ESG concerns strengthened unexpectedly over the sample period. Another reason why green stocks can outperform brown stocks is that the risk premium induced by the ESG factor can be large enough to overcome green stocks' negative alphas.

Our model is related to prior theoretical studies of sustainable investing. Heinkel, Kraus, and Zechner (2001) build an equilibrium model in which exclusionary ethical investing affects firm investment. They consider two types of investors, one of which refuses to hold shares in polluting firms. The resulting reduction in risk sharing increases the cost of capital of polluting firms, thereby depressing those firms' investment. Fama and French (2007) offer a conceptual framework with two types of investors whose tastes for assets as consumption goods affect asset prices. Baker et al. (2018) build a model featuring two types of investors with mean-variance preferences, where one type also has tastes for green assets. Their model predicts that green assets have lower expected returns and more concentrated ownership, and they find support for these predictions in the universe of green bonds. Pedersen, Fitzgibbons, and Pomorski (2019) consider the same two types of mean-variance investors, but they also add a third type that is unaware of firms' ESG scores. They show that stocks with higher ESG scores can have either higher or lower expected returns, depending on the wealth of the third type of investors. They obtain four-fund separation and derive the ESG-efficient frontier characterizing the tradeoff between the ESG score and the Sharpe ratio.[2]

While some of our results overlap with those of the above studies, our modeling is somewhat different, and we offer additional novel insights. We have a continuum of investors with CARA preferences over wealth and two dimensions of ESG preferences. We show that the size of the ESG investment industry, as well as investors' alphas, crucially depend on the dispersion in investors' ESG tastes. We derive conditions under which greener assets are riskier due to their larger exposures to an ESG risk factor. We show that this factor, along with the market return, prices assets in a two-factor model. We also show that under weak conditions, ESG investing has positive social impact.

Our assumption that some investors derive nonpecuniary benefits from green holdings has a fair amount of empirical support in the mutual fund literature. Mutual fund flows respond to ESG-salient information, such as Morningstar sustainability ratings (Hartzmark and Sussman, 2019) and environmental disasters (Bialkowski and Starks, 2016). Flows to SRI mutual funds are less volatile than flows to non-SRI funds (Bollen, 2007) and less responsive to negative past performance (Renneboog, ter Horst, and Zhang, 2011). Investors in SRI funds also indicate willingness to forgo financial performance to accommodate their social preferences (Riedl and Smeets, 2017).

Several recent studies have identified multiple aspects of ESG-related risk. Hoepner et al. (2018) find that ESG engagement reduces firms' downside risk, as well as their exposures to a downside-risk factor. Ilhan, Sautner, and Vilkov (2019) show that firms with higher carbon

[2]Theoretical work on sustainable investing also includes Albuquerque, Koskinen, and Zhang (2019), Friedman and Heinle (2016), Gollier and Pouget (2014), and Luo and Balvers (2017). Empirical work includes Geczy, Stambaugh, and Levin (2005), Hong and Kostovetsky (2012), and Cheng, Hong, and Shue (2016), in addition to studies that we cite below in more detail. For surveys of the early literature on sustainable investing, see Bauer, Koedijk, and Otten (2005) and Renneboog, ter Horst, and Zhang (2008).

emissions exhibit more tail risk and more variance risk. Bolton and Kacperczyk (2019) argue that investors demand compensation for exposure to carbon risk in the form of higher returns on carbon-intense firms. Luo and Balvers (2017) find a premium for boycott risk. We complement these studies with a theoretical contribution. We construct a systematic ESG risk factor that has two components, which are linked to unexpected shifts in ESG concerns of firms' customers and market investors. We show that greener assets load more heavily on this factor, and we also link ESG risk exposure to market betas and expected returns.

This paper is organized as follows. Section 2 presents the model. Section 3 explores the model's quantitative implications. Section 4 discusses the ESG factor. Section 5 concludes.

2. Model

The model considers a single period, from time 0 to time 1, in which there are N firms, $n = 1, \dots, N$. Let $\tilde{r}_n$ denote the return on firm n's shares in excess of the riskless rate, r_f, and let $\tilde{r}$ be the $N \times 1$ vector whose nth element is $\tilde{r}_n$. We assume that $\tilde{r}$ is normally distributed:

$$\tilde{r} = \mu + \tilde{\epsilon} \;, \tag{1}$$

where μ contains equilibrium expected excess returns and $\tilde{\epsilon} \sim N(0, \Sigma)$. In addition to financial payoffs, firms produce social impact. Each firm n has an "ESG characteristic" g_n, which can be positive (for "green" firms) or negative (for "brown" firms). Firms with $g_n > 0$ have positive social impact, meaning they generate positive externalities (e.g., cleaning up the environment). Firms with $g_n < 0$ have negative social impact, meaning they generate negative externalities (e.g., polluting the environment). In Section 2.3, we model firms' social impact in greater detail.

There is a continuum of agents who trade firms' shares and the riskless asset. The riskless asset is in zero net supply, whereas each firm's stock is in positive net supply. Let X_i denote an $N \times 1$ vector whose nth element is the fraction of agent i's wealth invested in stock n. Agent i's wealth at time 1 is $\tilde{W}_{1i} = W_{0i}\,(1 + r_f + X_i'\tilde{r})$, where W_{0i} is the agent's initial wealth. Besides liking wealth, agents also derive utility from holding green stocks and disutility from holding brown stocks.[3] Each agent i has exponential (CARA) utility

$$V(\tilde{W}_{1i}, X_i) = -e^{-A_i \tilde{W}_{1i} - b_i' X_i} \;, \tag{2}$$

[3] We frame the discussion in terms of green and brown stocks, but our main ideas apply more broadly to any set of green and brown assets, such as bonds and private equity investments.

where A_i is the agent's absolute risk aversion and b_i is an $N \times 1$ vector of nonpecuniary benefits that the agent derives from her stock holdings. Holding the riskless asset brings no such benefit. The benefit vector has agent-specific and firm-specific components:

$$b_i = d_i g \ , \tag{3}$$

where g is an $N \times 1$ vector whose nth element is g_n and $d_i \geq 0$ is a scalar measuring agent i's "ESG sensitivity." Agents with higher values of d_i care more about the ESG characteristics of their holdings. Agents also care about firms' aggregate social impact, but that component of preferences does not affect agents' portfolio choices. We postpone the discussion of that component until Section 2.3.

2.1. Expected Returns

Due to their infinitesimal size, agents take asset prices (and thus also the return distribution) as given when choosing their optimal portfolios, X_i, at time 0. To derive the first-order condition for X_i, we compute the expectation of agent i's utility in equation (2) and differentiate it with respect to X_i. As we show in the Appendix, agent i's portfolio weights are

$$X_i = \frac{1}{a_i}\Sigma^{-1}\left(\mu + \frac{1}{a_i}b_i\right) \ , \tag{4}$$

where $a_i \equiv A_i W_{0i}$ is agent i's relative risk aversion. For tractability, we assume that $a_i = a$ for all agents. We define w_i to be the ratio of agent i's initial wealth to total initial wealth: $w_i \equiv W_{0i}/W_0$, where $W_0 = \int_i W_{0i} di$. The market-clearing condition requires that x, the $N \times 1$ vector of weights in the market portfolio, satisfies

$$\begin{aligned} x &= \int_i w_i X_i \, di \\ &= \frac{1}{a}\Sigma^{-1}\mu + \frac{\bar{d}}{a^2}\Sigma^{-1}g \ , \end{aligned} \tag{5}$$

where $\bar{d} \equiv \int_i w_i d_i di \geq 0$ is the wealth-weighted mean of ESG sensitivities d_i across agents. Note that $\bar{d} > 0$ unless the mass of agents who care about ESG is zero. Solving for μ gives

$$\mu = a\Sigma x - \frac{\bar{d}}{a}g \ . \tag{6}$$

Premultiplying by x' gives the market equity premium, $\mu_M = x'\mu$:

$$\mu_M = a\sigma_M^2 - \frac{\bar{d}}{a}x'g \ , \tag{7}$$

where $\sigma_M^2 = x'\Sigma x$ is the variance of the market return. In general, the equity premium depends on average ESG preferences, $\bar{d}$, through $x'g$, which is the overall "greenness" of the market portfolio. If the market is net green ($x'g > 0$) then stronger ESG preferences (i.e., larger $\bar{d}$) reduce the equity premium. If the market is net brown ($x'g < 0$), stronger ESG preferences increase the premium. For simplicity, we make the natural assumption that the market portfolio is ESG-neutral,

$$x'g = 0 \ , \tag{8}$$

so that the equity premium is independent of agents' ESG preferences. In this case, equation (7) implies $a = \mu_M/\sigma_M^2$. Combining this with equation (6) and noting that the vector of market betas is $\beta = (1/\sigma_M^2)\Sigma x$, we obtain our first proposition.

Proposition 1. *Expected excess returns in equilibrium are given by*

$$\mu = \mu_M \beta - \frac{\bar{d}}{a} g \ . \tag{9}$$

We see that expected excess returns deviate from their CAPM values, $\mu_M \beta$, due to ESG preferences for holding green stocks.

Corollary 1. *If $\bar{d} > 0$, the expected return on stock n is decreasing in g_n.*

As long as the mass of agents who care about sustainability is nonzero, $\bar{d}$ is positive, and expected returns are decreasing in stocks' ESG characteristics. Because the alpha of stock n is defined as $\alpha_n \equiv \mu_n - \mu_M \beta_n$, equation (9) yields the following corollary.

Corollary 2. *The alpha of stock n is given by*

$$\alpha_n = -\frac{\bar{d}}{a} g_n \ . \tag{10}$$

If $\bar{d} > 0$, green stocks have negative alphas, and brown stocks have positive alphas. Moreover, greener stocks have lower alphas.

As long as some agents care about sustainability, equation (10) implies that the CAPM alphas of stocks with $g_n > 0$ are negative, the alphas of stocks with $g_n < 0$ are positive, and α_n is decreasing with g_n. Furthermore, the negative relation between α_n and g_n is stronger when risk aversion, a, is lower and when the average ESG sensitivity, $\bar{d}$, is higher.

Proposition 2. *The expected excess return on agent i's portfolio is given by*

$$\mathrm{E}(\tilde{r}_i) = \mu_M - \delta_i \left(\frac{\bar{d}}{a^3} g' \Sigma^{-1} g \right) \ , \tag{11}$$

where $\delta_i \equiv d_i - \bar{d}$.

This equation is derived in the Appendix. The term in parentheses is nonnegative, so the sign of the second term depends on the sign of δ_i. Agents with $\delta_i > 0$ earn below-market expected returns because their portfolios tilt toward stocks with negative alphas. In contrast, agents with $\delta_i < 0$ earn above-market returns because they tilt toward positive-alpha stocks.

Corollary 3. *If $\bar{d} > 0$ and $g \neq 0$, agents with larger δ_i earn lower expected returns.*

Under the conditions of this corollary, the term in parentheses in equation (11) is strictly positive. Therefore, agents with stronger ESG sensitivities (i.e., larger δ_i) earn lower expected returns. The conditions are not satisfied if no agents care about ESG ($\bar{d} = 0$) or if all firms are ESG-neutral ($g = 0$); in that case, $\mathrm{E}(\tilde{r}_i)$ is independent of δ_i because all agents hold the market. The effect of δ_i on $\mathrm{E}(\tilde{r}_i)$ is stronger when the average ESG sensitivity is stronger (i.e., when $\bar{d}$ is larger), when risk aversion a is smaller, and when $g'\Sigma^{-1}g$ is larger.

The low expected returns earned by ESG-sensitive agents do not imply these agents are unhappy. As we show in the Appendix, agent i's expected utility in equilibrium is given by

$$\mathrm{E}\left\{V(\tilde{W}_{1i})\right\} \quad = \quad \bar{V}e^{-\frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g} \ , \tag{12}$$

where $\bar{V}$ is the expected utility if the agent has $\delta_i = 0$. We see that expected utility is increasing in δ_i^2 (note from equation (2) that $\bar{V} < 0$), so it is larger for agents with larger absolute values of δ_i. The more an agent's ESG sensitivity d_i deviates from the average in either direction, the more ESG preferences contribute to the agent's utility. High-δ_i investors derive utility from their holdings of green stocks, while low-δ_i investors derive utility from the positive alphas of brown stocks.

2.2. Portfolio Tilts

Substituting for μ from equation (9) into equation (4), we obtain an agent's portfolio weights:

Proposition 3. *Agent i's equilibrium portfolio weights are given by*

$$X_i = x + \frac{\delta_i}{a^2}\left(\Sigma^{-1}g\right) \ . \tag{13}$$

Proposition 3 implies three-fund separation as each agent's portfolio can be implemented with three assets: the riskless asset, the market portfolio x, and an "ESG portfolio" whose weights are proportional to $\Sigma^{-1}g$. Agents with $\delta_i > 0$ go long the ESG portfolio; agents with $\delta_i < 0$ short the portfolio. Agent i's portfolio departs from the market portfolio due to the second term in equation (13), which we refer to as agent i's "ESG tilt."

The ESG tilt is zero for agents whose ESG sensitivity is average, in that $d_i = \bar{d}$. Agents with average ESG preferences hold the market portfolio. Interestingly, agents with $d_i = 0$ hold a portfolio that departs from the market in the direction away from ESG. It is suboptimal for an investor to say "I don't care about ESG, so I'm just going to hold the market." Investors who do not care about ESG must tilt away from ESG, otherwise they are not optimizing. The market portfolio is optimal for investors who care about ESG to an average extent, but not for those who do not care about ESG at all.

Corollary 4. *If there is no dispersion in ESG preferences across agents then all agents hold the market portfolio.*

No dispersion in ESG preferences implies $d_i = \bar{d}$, and so zero ESG tilt, for all i. Interestingly, even if all agents derive a large amount of utility from green holdings, they end up holding the market if their preferences are equally strong. The reason is that stock prices adjust to reflect those preferences, making the market everybody's optimal choice. Some dispersion in ESG preferences is necessary for an ESG investment industry to exist.

If the covariance matrix Σ is diagonal, meaning all risk is idiosyncratic, then the ESG portfolio weights are positive for green stocks (whose $g_n > 0$), negative for brown stocks (whose $g_n < 0$), and lower for stocks with more volatile returns. A similar result obtains when Σ has a simple one-factor structure, allowing systematic risk:

$$\Sigma = \sigma^2 \iota\iota' + \eta^2 I_N \ , \tag{14}$$

where ι is an $N \times 1$ vector of ones and I_N is an identity matrix, because in that case[4]

$$\Sigma^{-1} g = \frac{1}{\eta^2}\left(g - \frac{\bar{g}}{\frac{\eta^2}{N\sigma^2} + 1}\iota \right) \ , \tag{15}$$

where $\bar{g} = \iota' g / N$ is the mean g_n across firms. As N gets large, the ESG portfolio goes long stocks that are greener than average ($g_n > \bar{g}$) and short stocks that are browner than average ($g_n < \bar{g}$). The ESG portfolio's positions are smaller when idiosyncratic risk η^2 is higher, because tilting toward the ESG portfolio exposes investors to more idiosyncratic risk.

In general, the ESG tilt depends also on the covariances among stocks. If a stock is positively correlated with a greener stock, the former stock may be shorted by agents who want to hold the greener stock and hedge their risk exposure to it. In principle, even a green stock could be shorted if it is sufficiently correlated with a stock that is even greener.

[4] With Σ given in equation (14), we have $\Sigma^{-1} = \frac{1}{\eta^2}\left(I_N - \frac{1}{\eta^2/\sigma^2 + N}\iota\iota' \right)$.

The ESG tilt is smaller when agents have a higher risk aversion a, because the tilt exposes agents to additional risk. Holding ESG preferences (δ_i) constant, those preferences are reflected less strongly in agents' portfolios if their risk aversion is higher.

2.3. Social Impact

We define the social impact of firm n as

$$S_n \equiv g_n K_n \ , \tag{16}$$

where K_n is the firm's operating capital. Social impact captures the firm's total amount of externalities, which depends both on the nature of the firm's operations (g_n) and on their scale (K_n). The firm chooses its scale at time 0 by investing K_n. We model this choice in a flexible, reduced-form way:

$$K_n = K_{0,n} + \phi \left(\Delta V\right)_n \ . \tag{17}$$

The first term, $K_{0,n}$, summarizes all determinants of the firm's capital that are unrelated to the effect of ESG preferences on market values. The second term, $\phi \left(\Delta V\right)_n$, captures the effect of ESG preferences on investment. The variable $\left(\Delta V\right)_n$ is the firm's time-0 market value attributable to ESG demands: $\left(\Delta V\right)_n \equiv V_n - \hat{V}_n$, where V_n is the firm's market value and $\hat{V}_n$ is its hypothetical value if agents did not care about ESG (i.e., $\overline{d} = 0$). Parameter $\phi \geq 0$ captures the extent to which the changes in market value induced by ESG preferences affect real investment. In the special case where $\phi = 0$, an ESG-induced increase in market value has no effect on the firm's scale of operations. If $\phi > 0$, such an increase in market value causes the firm to invest more, and a decrease in market value causes the firm to invest less, holding non-ESG factors constant.

Proposition 4. *The aggregate social impact induced by ESG-motivated investment is non-negative. It is strictly positive as long as $\phi > 0$, $\bar{d} > 0$, and $g \neq 0$.*

To explain this result, we first let y_n denote the firm's CAPM-implied discount factor, $1/(1 + r_f + \mu_M \beta_n)$. Holding constant the firm's expected time-1 value and beta, we obtain[5]

$$\left(\Delta V\right)_n \ = \ -\alpha_n V_n y_n \ . \tag{18}$$

[5] Our modeling of firm investment in equation (17) can be reconciled with holding constant the firm's expected time-1 value and beta as follows. At time 0, the firm borrows K_n to finance its investment. It invests its baseline capital $K_{0,n}$ in projects with positive net present value, whereas the ESG-induced capital, $\phi \left(\Delta V\right)_n$, is invested in similar projects but with zero net present value, net of repaying lenders.

Combining equations (17) and (18) with equation (10), ESG preferences induce investment by firm n in the amount of

$$\begin{aligned} K_n - K_{0,n} &= -\phi\, \alpha_n V_n y_n & (19) \\ &= \phi \frac{\bar{d}}{a} g_n V_n y_n \;. & (20) \end{aligned}$$

We see that if $\phi > 0$ and $\bar{d} > 0$ then ESG preferences move real investment away from brown firms ($g_n < 0$) and toward green firms ($g_n > 0$). In addition, equation (19) shows that ESG-induced investment is negatively related to α_n as long as $\phi > 0$.

The social impact induced by ESG investment in a given firm is the difference between the firm's actual social impact and its hypothetical impact if agents did not care about ESG. Combining equations (16) through (18), this difference equals

$$S_n(\bar{d}) - S_n(0) = \frac{\bar{d}\phi}{a} V_n y_n g_n^2 \geq 0 \;. \tag{21}$$

This effect is nonnegative. Summing up equation (21) across firms, the effect of ESG investing on aggregate social impact, $S = \sum_{n=1}^{N} S_n$, is given by

$$S(\bar{d}) - S(0) = \frac{\bar{d}\phi}{a} V_y'(g^2) \geq 0 \;, \tag{22}$$

where V_y is the $N \times 1$ vector whose nth element is $V_n y_n$, and g^2 is an $N \times 1$ vector whose nth element is g_n^2. Equation (22) implies Proposition 4. It shows that ESG preferences induce positive social impact unless nobody cares about ESG ($\bar{d} = 0$), all firms are ESG-neutral ($g = 0$), or market values do not affect real investment ($\phi = 0$). Outside those special cases, ESG preferences increase social impact by affecting stock prices in a way that brings about more green investment and less brown investment.

Corollary 5. *The aggregate social impact induced by ESG-motivated investment is larger when $\bar{d}$ is larger, when a is smaller, when ϕ is larger, and when $V_y'(g^2)$ is larger.*

The effect of ESG investing on S is larger when agents care more about ESG (i.e., when $\bar{d}$ is larger), when they are less risk-averse (i.e., when a is smaller), when market values have a larger effect on real investment (i.e., when ϕ is larger), and when the dispersion of g across firms, as measured by $V_y'(g^2)$, is larger. A larger $\bar{d}$, or smaller a, imply a larger effect of ESG preferences on market values, via expected returns (equation (9)). A larger ϕ implies a larger effect of market values on investment through equation (17). Finally, a larger dispersion in g deepens the market value differentials between green and brown firms, leading to larger investment differentials. With green firms investing more and brown firms investing less, aggregate social impact increases. The term $V_y'(g^2)$ is larger also when $\mathrm{Cov}(V_n y_n, g_n^2)$ across firms is larger, that is, when bigger firms tend to be especially green or brown.

Whether ESG investing produces social impact hinges on whether $\phi > 0$. Existing evidence suggests that ϕ is indeed positive. Barro (1990) finds a strong positive relation between aggregate investment and stock prices. Baker and Wurgler (2012) survey papers that relate corporate investment to stocks' alphas. Recall from equation (19) that alphas and investment are negatively related in our model as long as $\phi > 0$. Consistent with $\phi > 0$, Polk and Sapienza find a negative relation between investment and alphas, proxied by negative discretionary accruals. Titman, Wei, and Xie (2004) and van Binsbergen and Opp (2019) also report negative relations between investment and alpha.[6]

One caveat is that none of these empirical papers study ESG-induced changes to market values. We expect ESG-induced alphas to have an especially strong effect on investment, because firms' ESG traits are highly persistent, which makes their ESG-induced alphas highly persistent in our model. Van Binsbergen and Opp (2019) show that when alphas are more persistent, they have stronger effects on investment. Another caveat is that ϕ has a causal interpretation in our model, but the previous papers do not claim to measure causal relations in the data. Closer to our model, Hau and Lai (2013) show empirically that changes in equity prices have a causal effect on investment.

So far we have taken firms' ESG characteristics (g_n) as fixed and considered how ESG investing affects firms' scale of operations (K_n). Equation (16) suggests another way that ESG-motivated investors can create social impact: by leading firms to increase their g_n, even if K_n is unchanged. For example, investors could lead a firm to adopt practices that are more environmentally friendly. If some agents care about greenness, all of firm n's stockholders have an incentive to push for a higher g_n. Those who value greenness benefit directly, and all current stockholders benefit from the higher stock price caused by the higher g_n. While we do not model this channel formally, we note that investors can increase g_n through two corporate-governance mechanisms. First, investors can directly engage with managers to increase their firms' g_n. Dyck et al. (2019), for example, find evidence consistent with this mechanism, which works even if ESG investing has no effect on stock prices. Second, if managers' compensation is linked to the firm's stock price, then managers have an incentive to increase that price by increasing g_n (Corollary 1). Since this mechanism works through stock prices, it should be stronger when investors care more about ESG and when they are

[6]Also related, Baker, Stein, and Wurgler (2003) show that investment is most sensitive to mispricing in equity-dependent firms. Bakke and Whited (2010) find that mispricing does not affect investment, whereas Warusawitharana and Whited (2016) find that the response of investment to mispricing is small but positive.

less risk-averse, similar to Corollary 5. Of course, the effects of ESG investing on K_n and g_n are not mutually exclusive; instead, they interact positively, increasing social impact beyond its predicted value in equation (22).

We now enrich the model by allowing agents to care about aggregate social impact, S. Specifically, we assume each agent i's utility is increasing in S:

$$U(\tilde{W}_{1i}, X_i, S) = V(\tilde{W}_{1i}, X_i) + h_i(S) \ , \tag{23}$$

where $h_i'(S) > 0$ and V is the original utility function from equation (2). (The additive specification is not needed; our results are identical if S enters utility multiplicatively.) Agents now care about sustainability in two ways: they derive utility from their holdings as well as from firms' aggregate social impact.

Proposition 5. *If agents derive utility also from aggregate social impact (equation (23)), all of our results in Propositions 1 through 4 and Corollaries 1 through 5 continue to hold.*

According to Proposition 5, the inclusion of S in the utility function does not affect any of our prior results. The key to understanding this proposition is that agents are infinitesimally small. Small agents take stock prices, and hence S, as given when choosing their portfolios. Therefore, agents' preference for S does not affect their portfolio choice. When an agent tilts toward green stocks, she generates a positive externality on other agents via the $h_i(S)$ term in their utility. Being small, though, she does not internalize this effect. As the preference for S does not affect portfolio choice, it does not affect equilibrium asset prices, firms' real investment, or S, either.

Corollary 6. *Social impact is caused not by agents' preference for it, but by their tastes for stocks with nonzero ESG characteristics.*

Social impact is caused by the inclusion of X_i, not S, in the utility function in equation (23). Even if agents care strongly about S, this preference per se has no effect on social impact because it does not affect agents' portfolio choice. For ESG preferences to induce social impact, agents must derive utility from the ESG characteristics of their holdings, X_i. As shown earlier, the latter preference affects agents' portfolio choice, which in turn affects asset prices, which then differentially affects the investment of green and brown firms.

3. Quantitative Implications

To explore the model's quantitative implications, we consider a special case with two types of agents: ESG investors, for whom $d_i = d > 0$, and non-ESG investors, for whom $d_i = 0$. ESG investors thus consume nonpecuniary benefits dg, whereas non-ESG investors consume no benefits (see equation (3)). Let λ denote the fraction of total wealth belonging to ESG investors, so that $1 - \lambda$ is the corresponding fraction for non-ESG investors.

3.1 Expected Returns and Portfolio Tilts

In this setting, $\bar{d} = \lambda d$, so from equation (9) the vector of expected excess returns becomes

$$\mu = \mu_M \beta - \lambda \frac{d}{a} g \ . \tag{24}$$

As λ increases, expected returns on green stocks decrease, whereas expected returns on brown stocks increase. In this comparative static sense, growing interest in ESG increasingly pushes stock prices in the direction of their ESG characteristics.

The portfolio weights for each type of investor follow directly from equation (13), with $\delta_i = (1 - \lambda)d$ for an ESG investor and $\delta_i = -\lambda d$ for a non-ESG investor:

$$X_{esg} = x + (1 - \lambda) \frac{d}{a^2} \Sigma^{-1} g \tag{25}$$

$$X_{non} = x - \lambda \frac{d}{a^2} \Sigma^{-1} g \ . \tag{26}$$

Both ESG tilts depend on λ in an interesting way. As $\lambda \to 0$, all investing is non-ESG, and all capital is invested in the market portfolio x because $X_{non} \to x$. As $\lambda \to 1$, all investing is ESG, and again, all capital is invested in the market because $X_{esg} \to x$. In other words, whether $\lambda \to 0$ or $\lambda \to 1$, all portfolios converge to the market portfolio. When $\lambda \to 0$, everybody holds the market because there are no ESG investors. When $\lambda \to 1$, everybody holds the market because ESG preferences are fully embedded in market prices.

From equation (11), the difference in expected excess returns earned by the two types of investors is

$$\mathrm{E}(\tilde{r}_{esg}) - \mathrm{E}(\tilde{r}_{non}) = -\frac{\lambda d^2}{a^3} g' \Sigma^{-1} g \ . \tag{27}$$

An ESG investor thus earns a lower expected return than a non-ESG investor. The performance gap is larger when there is a greater presence of ESG investors (i.e., when λ is larger). In this comparative static sense, growth in ESG investing deepens the performance gap. The gap is also larger when the two types of investors are further apart in their ESG preferences (i.e., when d is larger), when risk aversion a is smaller, and when $g'\Sigma^{-1}g$ is larger.

3.2. Parameter Specifications

We further simplify our setting by assuming that Σ has the simple one-factor structure given in equation (14) and setting $\beta = \iota$, $x = (1/N)\iota$, $\iota' g = 0$ (i.e., $x'g = 0$), and $g'g = 1$. With these assumptions, as the number of assets (N) grows large, the mean and variance of market returns, the certainty-equivalent return of ESG investors, and other aggregate quantities of interest do not depend on N, as will be evident below.

This simple setting has five free parameters: λ, a, σ, η, and d. We present results over the entire $(0, 1)$ range of λ. We specify a and σ so that the return on the market portfolio has a mean of $\mu_M = 0.08$ and a standard deviation of $\sigma_M = 0.20$, corresponding roughly to annual empirical estimates. To translate these values of μ_M and σ_M into implied values for a and σ, first note that the above assumptions imply the variance of the market, $x'\Sigma x$, is

$$\sigma_M^2 \;=\; \frac{1}{N^2}\,\iota'\left(\sigma^2\iota\iota' + \eta^2 I_N\right)\iota \;=\; \sigma^2 + \frac{\eta^2}{N}\;, \tag{28}$$

so we set $\sigma^2 = \sigma_M^2$, taking the limit as N grows large. Next, recall from equation (7) that $a = \mu_M/\sigma_M^2$. We set $\eta^2 = (0.7/0.3)\sigma^2$, so that the common market factor explains 30% of the variance of each individual stock's return.

The remaining free parameter, d, reflects the strength of ESG preferences. We calibrate this parameter by choosing Δ, the maximum rate of return that an ESG investor is willing to sacrifice, for certain, in order to invest in her desired portfolio rather than in the market portfolio. The sacrifice is greatest when there are no other ESG investors, i.e., when $\lambda \approx 0$, because that is when the ESG investor's portfolio most differs from the market portfolio.

Specifically, we define $\Delta \equiv r_{esg}^* - r_M^*$, where r_{esg}^* is the ESG investor's certainty equivalent excess return when investing in the optimal ESG portfolio, and r_M^* is the same investor's corresponding certainty equivalent if forced to hold the market portfolio instead. For a given Δ, the corresponding value of d is

$$d = \sqrt{2\Delta a^3 \eta^2}\;. \tag{29}$$

We derive this equation, along with the expressions for r_{esg}^* and r_M^*, in the Appendix. In the following analysis, we consider four values of Δ: 1, 2, 3, and 4% per year.

3.3. Certainty Equivalents and Utility

Figure 1 plots the value of $r^*_{esg} - r^*_M$ as λ goes from 0 to 1, for each value of Δ. At $\lambda = 0$, $r^*_{esg} - r^*_M$ takes its maximum value of Δ. As λ approaches 1, the optimal ESG portfolio approaches the market portfolio, and thus $r^*_{esg} - r^*_M$ approaches zero. The value of r^*_M is unaffected by λ, so the ESG investor's certainty equivalent for her desired portfolio, r^*_{esg}, declines as λ increases. As λ increases, stock prices are affected more, so ESG investors must pay more for the green stocks they desire. The resulting drop in r^*_{esg} need not imply, however, that an ESG investor is made less happy by an increased presence of ESG investors. With the latter, there is also greater social impact of ESG investing, as discussed in Section 2.3. The additional utility that the ESG investor derives from the greater social impact can exceed the drop in utility corresponding to the lower r^*_{esg}, which does not incorporate the positive externality of social impact.

Non-ESG investors, on the other hand, do prefer to be lonely in their ESG preferences. Formally, a non-ESG investor's certainty equivalent excess return from holding the optimal non-ESG portfolio, r^*_{non}, is increasing in λ, as we show in the Appendix. Intuitively, a non-ESG investor is happiest when all other investors are ESG ($\lambda = 1$), because that scenario maximizes deviations of prices from pecuniary fundamentals, and the non-ESG investor exploits such deviations to her advantage. The non-ESG investor's preference for loneliness in ESG preferences is even stronger if she derives any utility from aggregate social impact, as in equation (23), because social impact is maximized when $\lambda = 1$.

3.4. Correlation Between the ESG Return and the Market Return

The correlation between the return on an ESG investor's portfolio and the return on the market portfolio is derived in the Appendix:

$$\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right) = \frac{\sigma}{\sqrt{\sigma^2 + \frac{2\Delta}{a}(1-\lambda)^2}}\ . \tag{30}$$

Figure 2 plots the value of $\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right)$ as λ goes from 0 to 1, for each of the same Δ values as earlier. The correlation takes its lowest value at $\lambda = 0$. For $\Delta = 0.01$, that value is nearly 0.9, whereas for $\Delta = 0.04$, it is just over 0.7. As Δ increases, indicating that ESG investors feel increasingly strongly about ESG, those investors' portfolios become increasingly different from the market portfolio in terms of $\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right)$, and this effect is strongest when $\lambda = 0$. However, as λ approaches 1, so does $\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right)$. When ESG investors hold an increasingly large fraction of wealth, market prices adjust to their preferences, and all portfolios converge to the market portfolio.

3.5. ESG versus Non-ESG Expected Portfolio Returns

The difference in expected excess returns on the portfolios of the two investor types is

$$\mathrm{E}\{\tilde{r}_{esg}\} - \mathrm{E}\{\tilde{r}_{non}\} = -2\lambda\Delta \ , \tag{31}$$

as shown in the Appendix. Figure 3 plots this difference as λ goes from 0 to 1. The difference is zero at $\lambda = 0$, but it declines linearly as λ increases. At $\lambda = 1$, ESG preferences are fully reflected in prices, and the difference is at its largest in magnitude. In that scenario, the difference is -2% when $\Delta = 0.01$, but it is -8% when $\Delta = 0.04$. ESG investors thus earn significantly lower returns than non-ESG investors when the former account for a large fraction of wealth (i.e., λ is large) and their ESG preferences are strong (i.e., Δ is large).

3.6. Alphas and the Investor Surplus

The alphas on the ESG and non-ESG investors' portfolios are derived in the Appendix:

$$\alpha_{esg} = -2\lambda(1-\lambda)\Delta \tag{32}$$

$$\alpha_{non} = 2\lambda^2\Delta \ . \tag{33}$$

Panel A of Figure 4 plots α_{esg} as λ goes from 0 to 1. ESG investors earn zero alpha at both extremes of λ. Their portfolio differs most from the market portfolio when $\lambda = 0$, but all stocks have zero alphas in that scenario, because there is no impact of ESG investors on prices. At the other extreme, when $\lambda = 1$, many stocks have non-zero alphas, due to the price impacts of ESG investors, but ESG investors hold the market, so again they earn zero alpha. Otherwise, ESG investors earn negative alpha, which is greatest in magnitude when $\lambda = 0.5$. At that peak, $\alpha_{esg} = -0.5\%$ when $\Delta = 0.01$, but $\alpha_{esg} = -2\%$ when $\Delta = 0.04$.

Interestingly, these worst-case alphas are substantially smaller than the corresponding Δ's. For example, when ESG investors are willing to give up 2% certain return to hold their portfolio rather than the market (i.e., $\Delta = 0.02$), their worst-case alpha is only -1%. The reason is that equilibrium stock prices adjust to ESG demands. These demands push the market portfolio toward the portfolio desired by ESG investors, thereby bringing those investors' negative alphas closer to zero. Through this adjustment of market prices, ESG investors earn an "investor surplus" in that they do not have to give up as much return as they are willing to in order to hold their desired portfolio.

The magnitude of this investor surplus is easy to read off Panel B of Figure 4, which plots α_{esg} as a function of Δ. For any given value of λ, investor surplus is the difference between the corresponding solid line and the dashed line, which has a slope of -1. The surplus increases with Δ because the stronger the ESG investors feel about greenness, the more they move market prices. The relation between the surplus and λ is richer. Formally, investor surplus $\mathcal{I} \equiv \alpha_{esg} + \Delta$ follows quickly from equation (32):

$$\mathcal{I} = \Delta[1 - 2\lambda(1 - \lambda)] \ . \tag{34}$$

Because $0 \leq \lambda \leq 1$, the value in brackets is always between 0.5 and 1, so $\mathcal{I}$ is always between $\Delta/2$ and Δ. It reaches its smallest value of $\Delta/2$ when $\lambda = 0.5$ and its largest value of Δ when $\lambda = 0$ or 1. For example, when $\Delta = 0.02$, $\mathcal{I}$ ranges from 1% to 2% depending on λ.

Figure 5 plots α_{non} as a function of λ and Δ. Like ESG investors, non-ESG investors earn zero alpha when $\lambda = 0$ or $\Delta = 0$. However, α_{non} increases in both λ or Δ. This alpha can be as large as 8% when $\lambda = 1$ and $\Delta = 0.04$. A non-ESG investor earns the highest alpha when all other investors are ESG (i.e., $\lambda = 1$) and when those investors' ESG preferences are strong (i.e., Δ is large) because the price impact of ESG is then particularly large. By going long brown stocks, whose alphas are positive and large, and short green stocks, whose alphas are negative and large, the non-ESG investor earns a large positive alpha.

Given the simplifying assumption that all assets have unit betas, the differences between the alphas plotted in Figures 4 and 5 are equal to the differences in expected returns plotted in Figure 3. Specifically, from equations (31) through (33), $\alpha_{esg} - \alpha_{non} = \mathrm{E}\{\tilde{r}_{esg}\} - \mathrm{E}\{\tilde{r}_{non}\}$.

3.7. Size of the ESG Investment Industry

We define the size of the ESG investment industry by the aggregate amount of ESG-driven investment that deviates from the market portfolio, divided by the stock market's total value. In general, this aggregate ESG tilt is given by

$$T = \int_{i:d_i>0} w_i T_i \, di \ , \tag{35}$$

where

$$T_i = \frac{1}{2}\iota' |X_i - x| \ . \tag{36}$$

The aggregate ESG tilt, T, is a wealth-weighted average of agent-specific tilts, T_i, across all agents who care at least to some extent about ESG (i.e., $d_i > 0$). Each T_i is one half of the sum of the absolute values of the N elements of agent i's ESG tilt, $|X_i - x|$. We compute absolute values of portfolio tilts because ESG-motivated investors both over- and under-weight stocks relative to the market. We divide by two because we do not want to double-count: for each dollar that an agent moves into a green stock, she must move a dollar out of another stock. The value of T_i is formally equivalent to agent i's active share (Cremers and Petajisto, 2009), with the market portfolio as the benchmark, but its interpretation is different: instead of measuring the activeness of the agent's portfolio, T_i measures the portfolio's ESG-induced tilt away from the market.

With two types of agents, the expression for T simplifies to

$$T \;=\; \frac{1}{2}\lambda\iota'|X_{esg} - x| = \lambda(1-\lambda)\sqrt{\frac{\Delta}{2a}}\,\iota'|g|\;, \tag{37}$$

as we show in the Appendix. The aggregate tilt depends on the absolute values of the elements of g. To evaluate $\iota'|g|$ in this quantitative exercise, we further assume that the elements of g are normally distributed across stocks, in addition to the previous assumptions that these elements have a mean of zero and a variance of $1/N$ (recall $x'g = (1/N)\iota'g = 0$ and $g'g = 1$). Then $\iota'|g| = N\mathrm{E}(|g_n|) = \sqrt{2N/\pi}$. Therefore,

$$T = \lambda(1-\lambda)\sqrt{\frac{\Delta N}{a\pi}}\;. \tag{38}$$

We set the number of assets here to $N = 100$. That number is considerably smaller than the actual number of stocks in the U.S. market, but recall that we assume equal market weights across stocks. We reduce N as a concession to the fact that the actual distribution of firm size in the U.S. market is quite disperse. Another reason to choose a small N is that we do not impose any investment constraints. As investors go long and short, the sum of the absolute values of their short positions increases with N, without bounds. In reality, however, investors often face short-sale or margin constraints that would prevent this from happening. Choosing a smaller N helps offset the effect of a growing number of short positions on T. Given the arbitrariness inherent in the choice of N, we are more interested in the dependence of T on λ and Δ than in the magnitude of T per se. The overall level of T depends on N, but the patterns with respect to λ and Δ do not.

Figure 6 plots T for different values of λ and Δ. In Panel A, λ goes from 0 to 1. At both $\lambda = 0$ and $\lambda = 1$, we have $T = 0$ because all investors hold the market portfolio. The maximum value of T in equation (38) always occurs at $\lambda = 0.5$, the maximum of $\lambda(1-\lambda)$. In Panel B, Δ goes from 0 to 0.04. Larger values of Δ produce larger values of T. This relation between Δ and T is concave (see also equation (38)). For example, the ESG industry peaks at 46% of the stock market's value when $\Delta = 0.02$, but doubling the strength of ESG preferences (raising Δ to 0.04) increases that maximum industry size by less than half, to 65% of the market's value. We see that the price impact of ESG preferences weakens their impact on the size of the ESG investment industry.

4. The ESG factor

In this section, we extend our model to show how firms' ESG characteristics can emerge as sensitivities to a risk factor—the ESG factor. The strength of ESG concerns can change over time, both for investors in firms' shares and for the customers who buy the firms' goods and services. If ESG concerns strengthen, customers may shift their demands for goods and services to greener providers (the "customer" channel), and investors may derive more utility from holding the stocks of greener firms (the "investor" channel). Both channels contribute to the ESG factor's risk in our framework.

To model the investor channel, we assume that the average ESG sensitivity $\bar{d}$ shifts unpredictably from time 0 to time 1. To model the customer channel, we need to model firm profits. Let $\tilde{u}_n$ denote the financial payoff (profit in our one-period setting) that firm n produces at time 1, for each dollar invested in the firm's stock at time 0. We assume a two-factor structure for the $N \times 1$ vector of these payoffs:

$$\tilde{u} - \mathrm{E}_0\{\tilde{u}\} = \tilde{z}_h h + \tilde{z}_g g + \tilde{\zeta} \ , \tag{39}$$

where $\mathrm{E}_0\{\ \}$ denotes expectation as of time 0, the random variables $\tilde{z}_h$ and $\tilde{z}_g$ have zero means, and $\tilde{\zeta}$ is a mean-zero vector that is uncorrelated with $\tilde{z}_h$, $\tilde{z}_g$, and $\bar{d}$ and has a diagonal covariance matrix, Λ. The shock $\tilde{z}_h$ can be viewed as a macro output factor, with the elements of h being firms' sensitivities to that pervasive shock. The shock $\tilde{z}_g$ represents the effect on firms' payoffs of unanticipated shifts in customers' ESG concerns. A positive $\tilde{z}_g$ shock increases the payoffs of green firms but hurts those of brown firms.

To assess how shifts in ESG preferences affect asset prices, we need to price stocks not only at time 0, as we have done so far, but also at time 1, after the preference shift in $\bar{d}$ occurs. To make this possible in our simple framework, we split time 1 into two times, 1^- and 1^+, that are close to each other. We calculate prices p_1 as of time 1^-, by which time ESG preferences have shifted and all risk associated with $\tilde{u}$ has been realized. Stockholders receive $\tilde{u}$ at time 1^+. During the instant between times 1^- and 1^+, these payoffs are riskless. For economy of notation, we assume the risk-free rate $r_f = 0$.

There are two generations of agents, Gen-0 and Gen-1. Gen-0 agents live from time 0 to time 1^-; Gen-1 agents live from time 1^- to 1^+. Gen-1 agents have identical sensitivities of $d_i = \bar{d}_1$, a condition that gives them finite utility, given the absence of both risk and position constraints during their lifespan. Neither a nor g change across generations. At time 1^-, Gen-0 agents sell stocks to Gen-1 agents at prices p_1, which depend on Gen-1 ESG preferences $\bar{d}_1$ and the financial payoff $\tilde{u}$. This simple setting maintains single-period payoff uncertainty while also allowing risk stemming from shifts in ESG preferences to enter via both channels described earlier.

4.1. Greener Stocks Are More Exposed to the ESG Factor

Given that the payoff $\tilde{u}_n$ is known at the time when the price $p_{1,n}$ is computed, $p_{1,n}$ is equal to $\tilde{u}_n$ discounted at the expected return implied by equation (9) with β_n set to zero:

$$p_{1,n} \;=\; \frac{\tilde{u}_n}{1 - \frac{g_n}{a}\bar{d}_1} \;\approx\; \tilde{u}_n + \frac{g_n}{a}\bar{d}_1 \; . \tag{40}$$

The approximation above holds well for typical discount rates, which are not too far from zero.[7] Representing it as an equality for all assets gives

$$p_1 = \tilde{u} + \frac{1}{a}\bar{d}_1 g \; , \tag{41}$$

which is the vector of payoffs to Gen-0 agents. Its expected value at time 0 equals

$$\mathrm{E}_0\{p_1\} = \mathrm{E}_0\{\tilde{u}\} + \frac{1}{a}\mathrm{E}_0\{\bar{d}_1\}g \; . \tag{42}$$

Note that $p_1 - \mathrm{E}_0\{p_1\}$ equals the vector of unexpected returns for Gen-0 agents, because $\tilde{u}_n$ is the firm's payoff per dollar invested in its stock at time 0. From equations (39) through (42), these unexpected returns, $\tilde{\epsilon} = \tilde{r} - \mathrm{E}_0\{\tilde{r}\}$, are given by

$$\begin{aligned} \tilde{\epsilon} \;&=\; \tilde{z}_h h + \tilde{f}_g g + \tilde{\zeta} & (43) \\ &=\; B\tilde{f} + \tilde{\zeta} \; , & (44) \end{aligned}$$

where $B = [h \;\, g]$, $\tilde{f} = [\tilde{z}_h \;\, \tilde{f}_g]'$, with $\mathrm{E}_0\{\tilde{f}\} = 0$, and $\tilde{f}_g$ denotes the "ESG factor" given by

$$\tilde{f}_g = \tilde{z}_g + \frac{1}{a}\left[\bar{d}_1 - \mathrm{E}_0\{\bar{d}_1\}\right] . \tag{45}$$

The two components of $\tilde{f}_g$ correspond to the two ESG risk channels discussed earlier: $\tilde{z}_g$ represents the customer channel while the other term represents the investor channel. Through both channels, greener stocks are more exposed to ESG factor risk (equation (43)). While the customer channel follows closely from the structure assumed in equation (39), the investor channel emerges from the equilibrium dependence of stock prices on $\bar{d}$.

The elements of $\tilde{f}_g g$ in equation (43) drive a wedge between expected and realized returns for ESG-motivated agents in Gen-0. We thus have the following proposition.

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[7] For arbitrary rates $\rho_1 \approx 0$ and $\rho_2 \approx 0$, we have $\frac{1+\rho_1}{1-\rho_2} = \frac{(1+\rho_1)(1+\rho_2)}{1-\rho_2^2} \approx (1+\rho_1)(1+\rho_2) \approx 1 + \rho_1 + \rho_2$, neglecting ρ_2^2 and $\rho_1\rho_2$. Setting $\tilde{u}_n = 1 + \rho_1$ and $\frac{g_n}{a}\bar{d}_1 = \rho_2$ gives the approximation in equation (40).

Proposition 6. *Green (brown) stocks perform better (worse) than expected if ESG concerns strengthen unexpectedly via either the customer channel or the investor channel.*

As noted earlier, green stocks generally have lower expected returns than brown stocks. If $\tilde{f}_g$ is positive, however, such an outcome boosts the realized performance of green stocks while hurting that of brown stocks. If one computes average returns over a sample period when ESG concerns consistently strengthened more than investors expected, so that the average of $\tilde{f}_g$ over that period is strongly positive, then green stocks could outperform brown stocks, contrary to what is expected.

4.2. The ESG Factor's Effects on Betas and Expected Returns

We also analyze the ex ante effect of the ESG factor's risk on market betas and expected returns. From equation (44), the return covariance matrix is

$$\Sigma = B\Omega B' + \Lambda \;, \tag{46}$$

where $\Omega = \text{Cov}\{\tilde{f}, \tilde{f}'\}$. The vector of market betas is therefore

$$\begin{aligned} \beta &= \frac{1}{\sigma_M^2}\Sigma x \\ &= \frac{1}{\sigma_M^2} B\beta_f + \frac{1}{\sigma_M^2}\Lambda x \\ &= \beta_h h + \beta_g g + \frac{1}{\sigma_M^2}\Lambda x \;, \end{aligned} \tag{47}$$

with

$$\beta_f \;=\; \begin{bmatrix} \beta_h \\ \beta_g \end{bmatrix} = \Omega B' x = \begin{bmatrix} \text{Cov}\{\tilde{\epsilon}_M, \tilde{z}_h\}/\sigma_M^2 \\ \text{Cov}\{\tilde{\epsilon}_M, \tilde{f}_g\}/\sigma_M^2 \end{bmatrix}, \tag{48}$$

and where $\tilde{\epsilon}_M \equiv x'\tilde{\epsilon}$ is the unexpected market return. In words, a stock's market beta depends on the stock's loading on the macro factor (h_n) times that factor's loading on the market (β_h), plus the stock's loading on the ESG factor (g_n) times that factor's loading on the market (β_g), plus a term reflecting idiosyncratic risk. From equation (43),

$$\tilde{\epsilon}_M = (x'h)\tilde{z}_h + (x'g)\tilde{f}_g + x'\tilde{\zeta} \;, \tag{49}$$

so

$$\beta_g = (x'h)\text{Cov}\{\tilde{z}_h, \tilde{f}_g\}/\sigma_M^2 + (x'g)\text{Var}(\tilde{f}_g)/\sigma_M^2 \;. \tag{50}$$

The overall stock market surely loads positively on the macro factor, $\tilde{z}_h$, meaning $x'h > 0$. Also, recall from equation (8) that $x'g = 0$, so the second term in equation (50) drops out. Given how β_g enters in equation (47), we have the following proposition.

Proposition 7. *If* $\text{Cov}\{\tilde{z}_h, \tilde{f}_g\} > 0$ *then ESG factor risk raises the market betas of green stocks but lowers the market betas of brown stocks.*

The premise of Proposition 7—a positive covariance between the macro factor and the ESG factor—seems plausible given the evidence of Bansal, Wu, and Yaron (2018) that green stocks outperform brown stocks in good times but underperform in bad times. Those authors argue that green stocks are similar to luxury goods in that they are in higher demand when the economy does well and thus financial concerns matter less. Given that, Proposition 7 implies that green stocks are riskier due to their exposure to ESG risk. Given Proposition 1, green stocks must offer higher expected returns to compensate for that risk.

Corollary 7. *If* $\text{Cov}\{\tilde{z}_h, \tilde{f}_g\} > 0$ *then ESG factor risk raises the expected returns on green stocks but lowers the expected returns on brown stocks.*

The ex ante effect of ESG risk on expected return thus opposes the effect of the direct ESG-based utility of holding the stock.

Importantly, the results in Proposition 7 and Corollary 7 do not rely on the factor structure assumed in equation (39). Even if we dispose of that structure by setting $\tilde{z}_h = \tilde{z}_g = 0$, thereby shutting down the customer channel, both Proposition 7 and Corollary 7 continue to hold. The investor channel alone is sufficient to generate both results. In that case, green stocks are riskier only because they are more exposed to unexpected shifts in $\bar{d}$.

Finally, if we relax the assumption from equation (8) that $x'g = 0$, the role of ESG factor risk depends on the overall greenness of the market portfolio. If the market is net green, so that $x'g > 0$, then the second term in equation (50) is positive, further increasing the covariance between the market return and the ESG factor. As the economy becomes greener, $x'g$ rises, pushing up β_g in equation (50). The greenifying of the economy thus makes green stocks increasingly exposed to the market, and brown stocks decreasingly so.

Corollary 8. *If* $\text{Cov}\{\tilde{z}_h, \tilde{f}_g\} > 0$ *and* $x'g$ *increases, so does the difference between green and brown stocks' market betas.*

In other words, as the market becomes greener, the market exposures of green stocks rise whereas those of brown stocks fall, resulting in a growing difference between the two.

4.3. Two-Factor Pricing

Under the above setting, the ESG factor, with its mean shifted to a non-zero value, also produces near-zero alphas in a two-factor model in which the market return is the other factor. This result obtains when the market portfolio is neither green nor brown ($x'g = 0$) and is also well diversified, in that $x_n \approx 0$ for all n (a large-N scenario). As shown in the Appendix, those conditions imply that excess returns are closely approximated by the regression relation

$$\tilde{r} = \theta \tilde{r}_M + g(\tilde{f}_g + \mu_g) + \tilde{\nu} \ , \tag{51}$$

with $\mathrm{E}\{\tilde{\nu}|\tilde{r}_M, \tilde{f}_g\} = 0$, $\theta = (1/x'h)h$, and $\mu_g = \mu_M \beta_g - \bar{d}/a$. We thus have our final proposition:

Proposition 8. *Each stock has zero alpha with respect to a two-factor model with the market factor and the ESG factor, with stock n's loading on the ESG factor equal to g_n.*

With equation (51), we provide a parsimonious characterization of both expected and realized returns, offering potential guidance for empirical investigations.

5. Conclusion

We analyze both financial and real effects of sustainable investing in a highly tractable general equilibrium model. The model produces a number of empirical implications regarding asset prices, portfolio holdings, the size of the ESG investment industry, and the social impact of sustainable investing. We summarize those implications below.

First, ESG preferences move asset prices. Stocks of greener firms have lower ex ante CAPM alphas, especially when risk aversion is low and the average ESG sensitivity is high. Green stocks have negative alphas, whereas brown stocks have positive alphas. Greener stocks are more exposed to a systematic ESG risk factor, which captures unexpected changes in ESG concerns of customers and investors. If either kind of ESG concerns strengthen unexpectedly over a given period of time, green stocks can outperform brown stocks over that period, despite having lower alphas. Another reason why green stocks can outperform is that under a plausible additional asumption, their exposure to the ESG factor increases their market betas, opposing the effect of green tastes on stocks' expected returns.

Second, portfolio holdings exhibit three-fund separation. Investors with stronger-than-average ESG sensitivities hold portfolios that have a green tilt away from the market portfolio, whereas investors with weaker-than-average ESG sensitivities have a brown tilt. These tilts are larger when risk aversion is lower. Investors with stronger ESG sensitivities earn lower expected returns, especially when risk aversion is low and the average ESG sensitivity is high.

Third, the size of the ESG investment industry—the aggregate dollar amount of ESG-driven investment that deviates from the market portfolio, scaled by total market value— is increasing in the heterogeneity of investors' ESG preferences. If there is no dispersion, there is no ESG industry because everyone holds the market.

Finally, sustainable investing leads to positive social impact by inducing more investment by green firms and less investment by brown firms. Greener firms invest more, especially when risk aversion is low, the average ESG sensitivity is high, and when stock prices have a larger effect on firms' investment.

While the model's return-related predictions have already been examined empirically by prior studies, the predictions related to holdings, industry size, and real impact of sustainable investing remain largely untested, presenting opportunities for future empirical work. One challenge is that our model aims to describe the world of the present and the future, but not necessarily the world of the past, especially the distant past. Although the "sin" aspects of investing have been recognized for decades, the emphasis on ESG criteria is a recent phenomenon. How the model fits in various time periods is a question for empirical work.

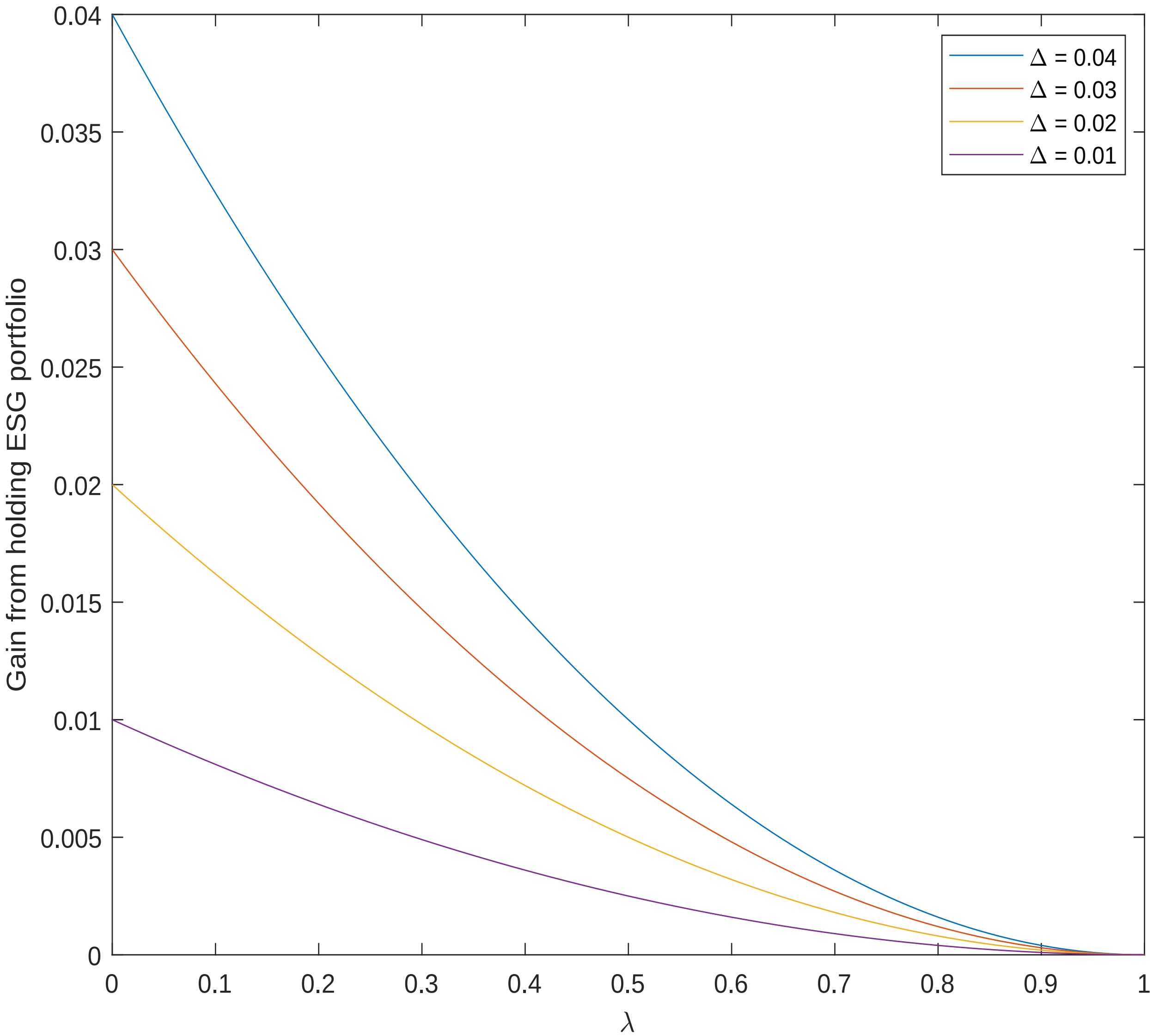


Figure 1. ESG investors' gain in certainty equivalent return. The figure plots the value of $r^*_{esg} - r^*_M$, where r^*_{esg} is an ESG investor's certainly-equivalent return when holding the optimal ESG portfolio, and r^*_M is the investor's certainty-equivalent return if forced to hold the market portfolio instead. Results are plotted against λ, the fraction of wealth belonging to ESG investors, and for different values of Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio.

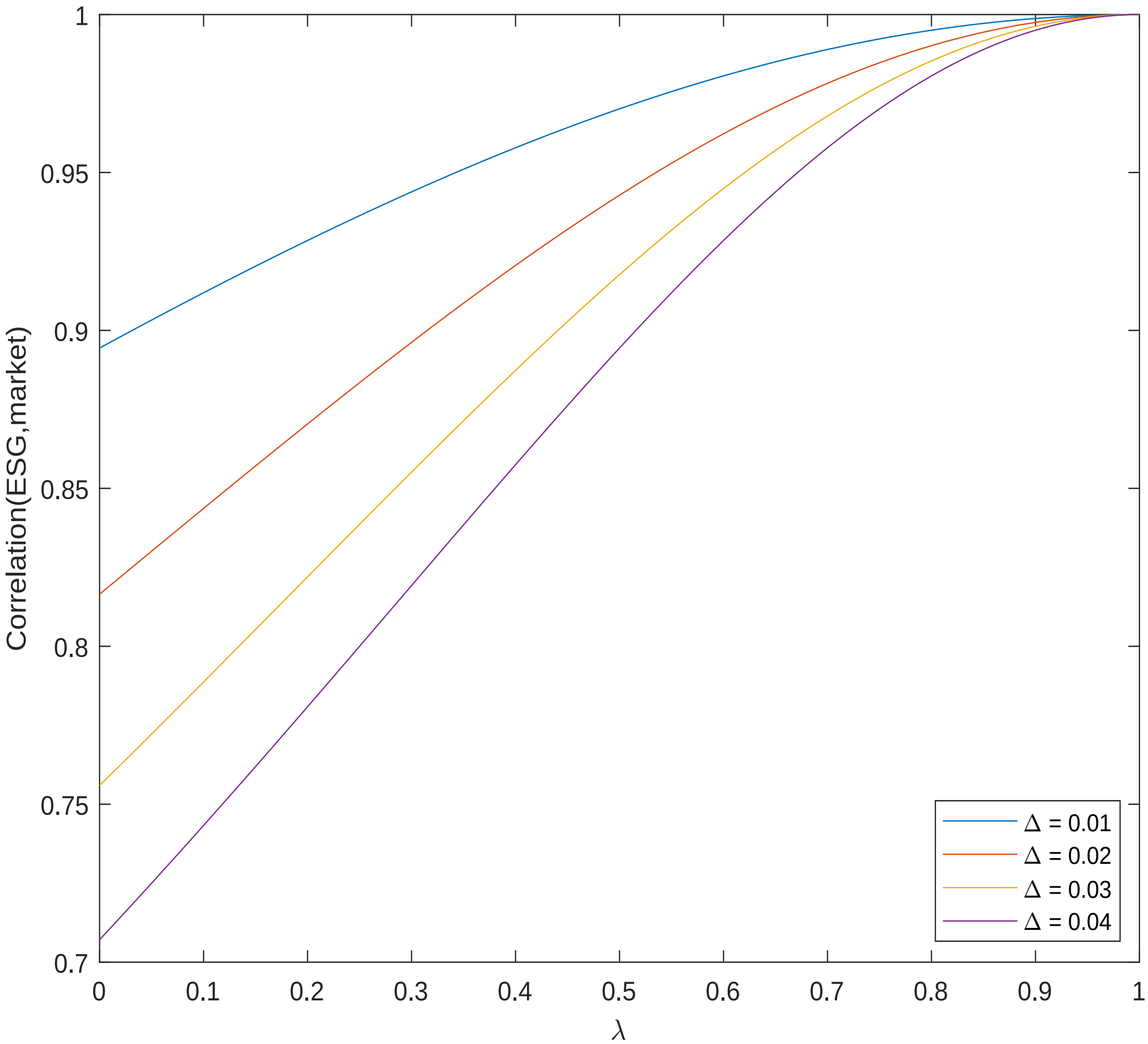


Figure 2. Correlation of ESG investor's portfolio return with the market return. The figure plots the correlation between the returns on the ESG investor's portfolio and the market portfolio. Results are plotted against λ, the fraction of wealth belonging to ESG investors, and for different values of Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio.

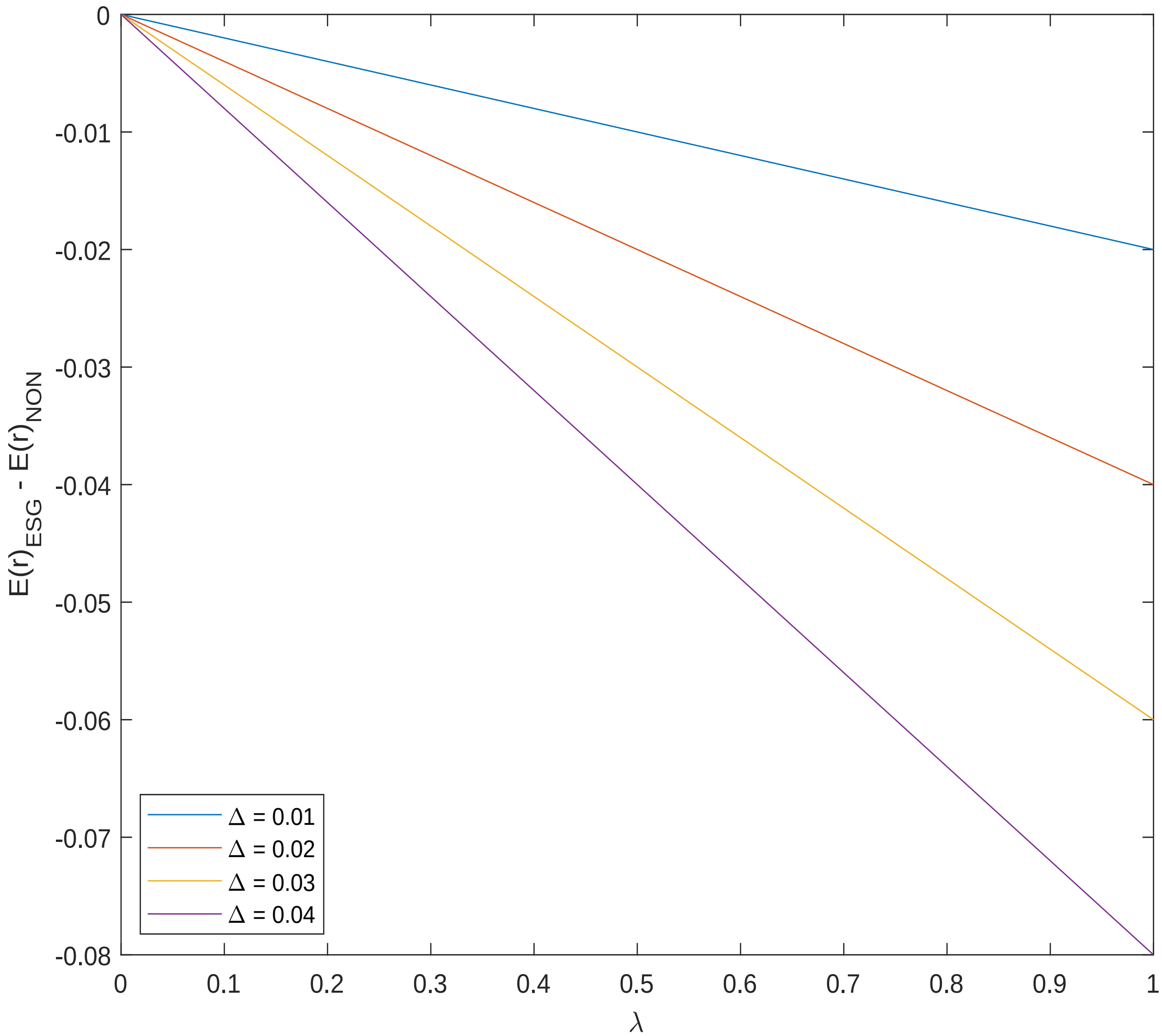
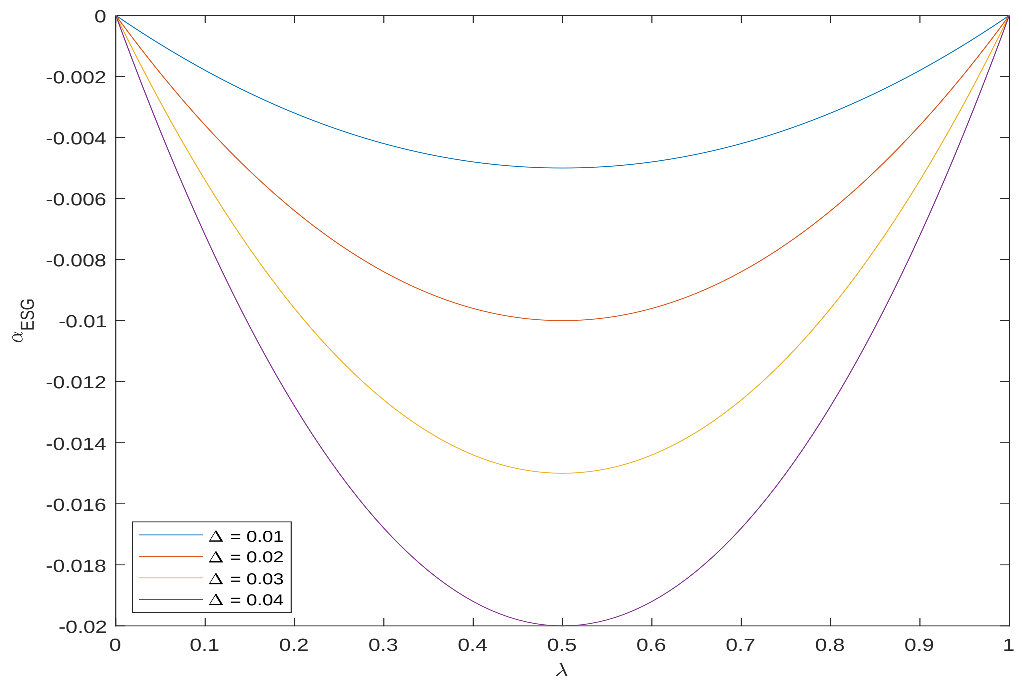


Figure 3. ESG versus non-ESG expected portfolio return. The figure plots the expected excess return on the portfolio of ESG investors minus the corresponding value for non-ESG investors. Results are plotted against λ, the fraction of wealth belonging to ESG investors, and for different values of Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio.

Panel A. The Role of λ



Panel B. The Role of Δ

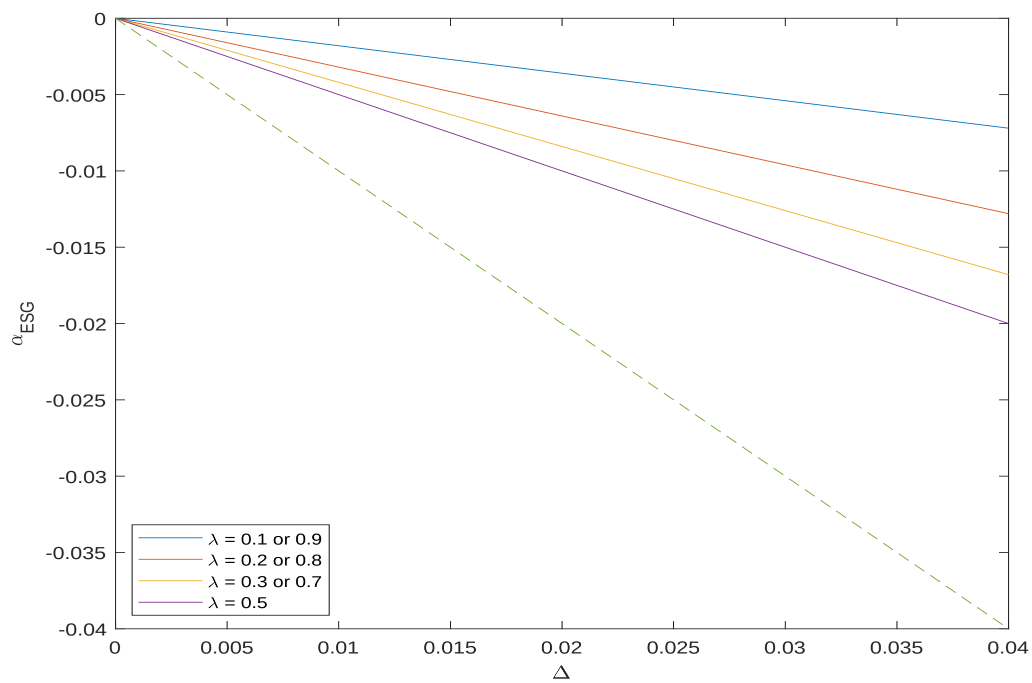
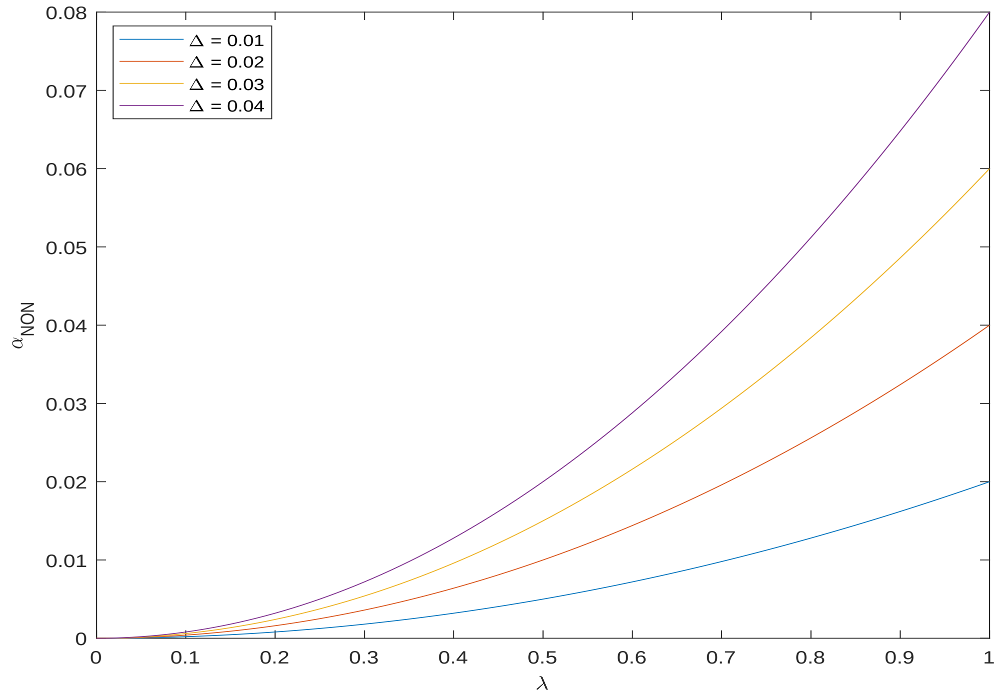


Figure 4. Alphas of ESG Investors. This figure plots the alpha for the portfolio held by ESG investors as a function of λ, the fraction of wealth belonging to ESG investors, and Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio. Panel A plots the ESG alpha as a function of λ for four different values of Δ; Panel B flips the roles of λ and Δ. The dashed line in Panel B has a slope of -1. The differences between the solid lines and the dashed line represent investor surplus.

Panel A. The Role of λ



Panel B. The Role of Δ

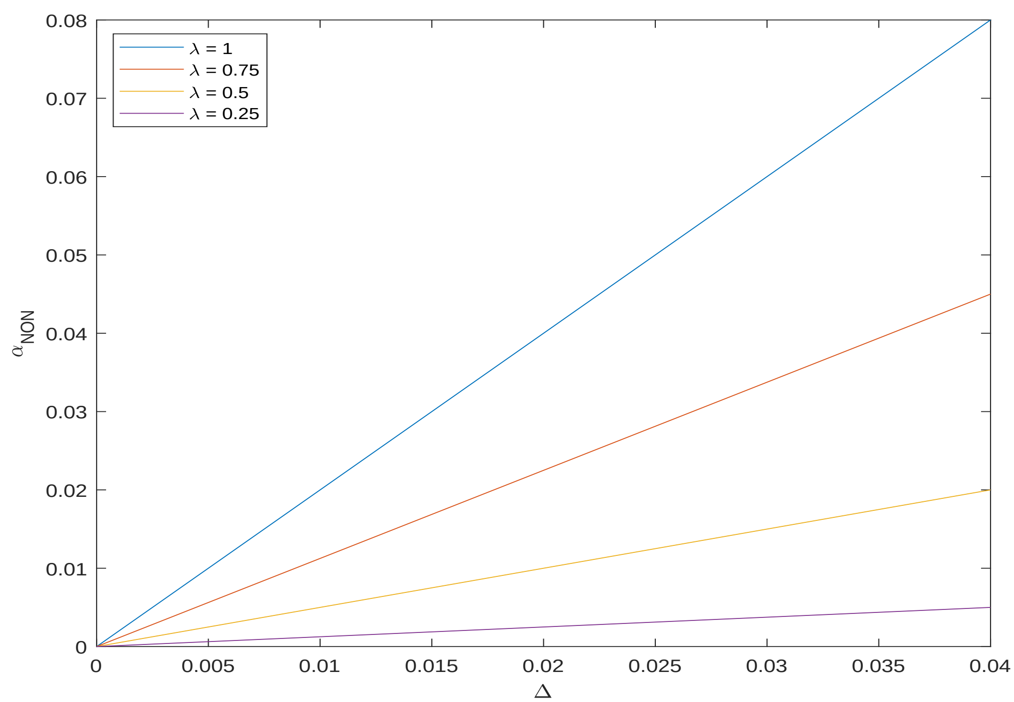
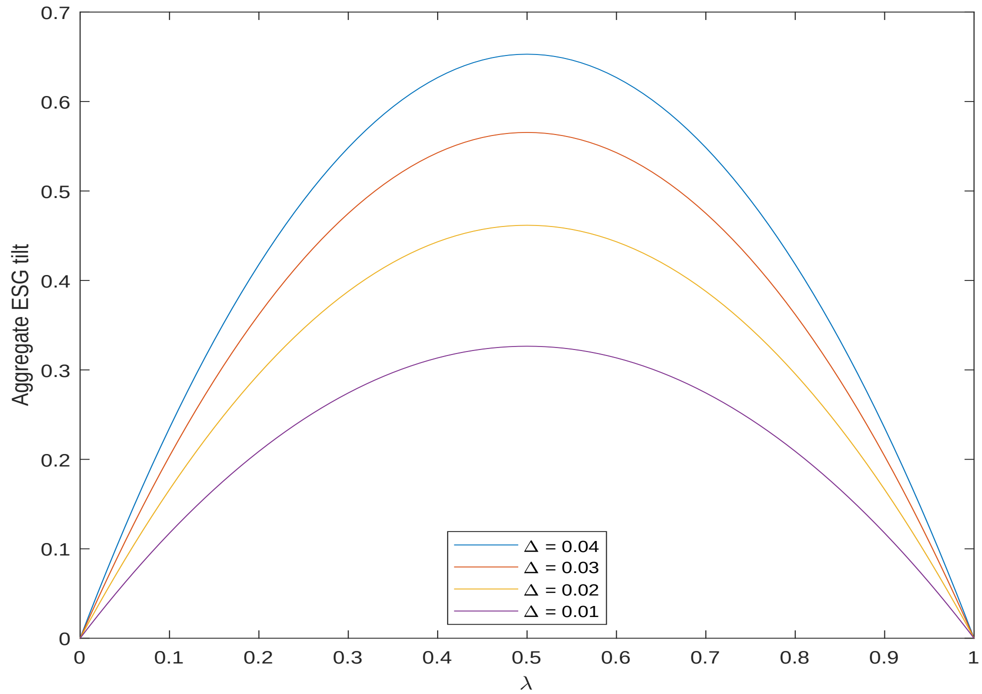


Figure 5. Alphas of Non-ESG Investors. This figure plots the alpha for the portfolio held by non-ESG investors as a function of λ, the fraction of wealth belonging to ESG investors, and Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio. Panel A plots the ESG alpha as a function of λ for four different values of Δ; Panel B flips the roles of λ and Δ.

Panel A. The Role of λ



Panel B. The Role of Δ

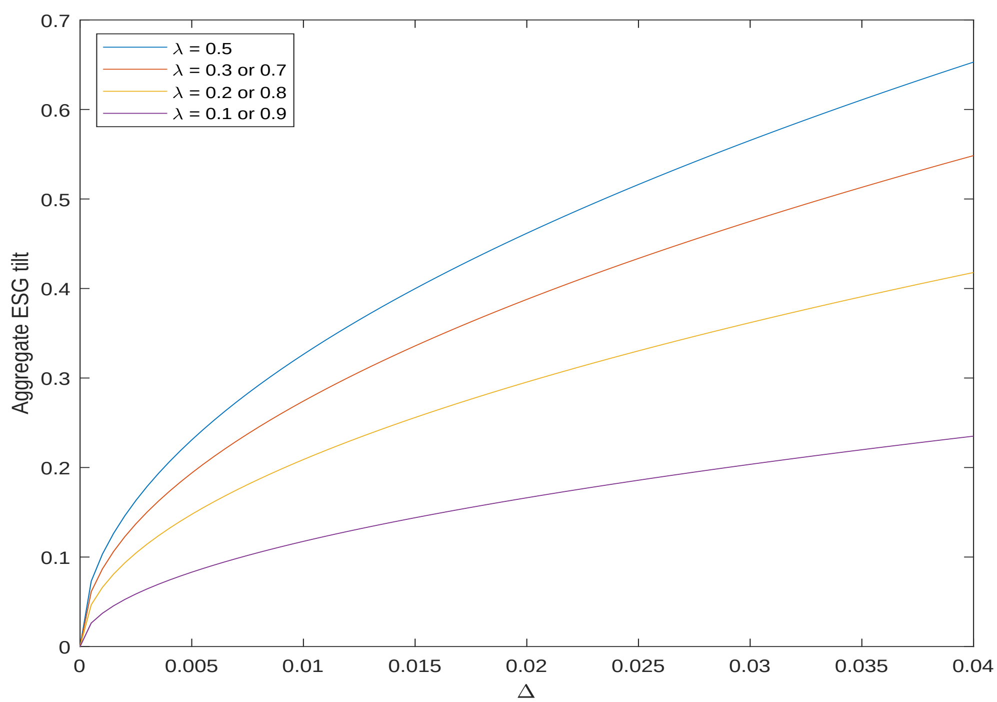


Figure 6. Size of the ESG Industry. The figure plots the aggregate dollar size of ESG investors' deviations from the market portfolio (the ESG "tilt"), expressed as a fraction of the market's total capitalization. In Panel A, results are plotted against λ, the fraction of wealth belonging to ESG investors, and for different values of Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio. In Panel B, results are plotted against Δ and for different values of λ.

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Appendix. Proofs and Derivations

Derivation of Equation (4):

To compute agent i's expected utility, we rely on equation (2), the relation $\tilde{W}_{1i} = W_{0i}(1 + r_f + X_i'\tilde{r})$, and the fact that $\tilde{r}$ is normally distributed, $\tilde{r} \sim N(\mu, \Sigma)$:

$$\begin{aligned}
\mathrm{E}\left\{V(\tilde{W}_{1i}, X_i)\right\} &= \mathrm{E}\left\{-e^{-A_i\tilde{W}_{1i} - b_i'X_i}\right\} \\
&= \mathrm{E}\left\{-e^{-A_i[W_{0i}(1+r_f+X_i'\tilde{r})] - b_i'X_i}\right\} \\
&= -e^{-a_i(1+r_f)}\mathrm{E}\left\{e^{-a_iX_i'[\tilde{r}+\frac{1}{a_i}b_i]}\right\} \\
&= -e^{-a_i(1+r_f)}e^{-a_iX_i'[E(\tilde{r})+\frac{1}{a_i}b_i]+\frac{1}{2}a_i^2X_i'Var(\tilde{r})X_i} \\
&= -e^{-a_i(1+r_f)}e^{-a_iX_i'[\mu+\frac{1}{a_i}b_i]+\frac{1}{2}a_i^2X_i'\Sigma X_i} \qquad \text{(A1)}
\end{aligned}$$

where $a_i \equiv A_iW_{0i}$ is agent i's relative risk aversion. Agents take μ and Σ as given. Differentiating with respect to X_i, we obtain the first-order condition

$$-a_i[\mu + \frac{1}{a_i}b_i] + \frac{1}{2}a_i^2(2\Sigma X_i) = 0 \ , \qquad \text{(A2)}$$

from which we obtain agent i's portfolio weights

$$X_i = \frac{1}{a_i}\Sigma^{-1}\left(\mu + \frac{1}{a_i}b_i\right). \qquad \text{(A3)}$$

Derivation of Equation (5):

The nth element of agent i's portfolio weight vector, X_i, is given by

$$X_{i,n} = \frac{W_{0i,n}}{W_{0i}} \qquad \text{(A4)}$$

where $W_{0i,n}$ is the dollar amount invested by agent i in stock n. Let $W_{0,n} \equiv \int_i W_{0i,n}di$ denote the total amount invested in stock n by all agents. Then the nth element of the market-weight vector, x, is given by

$$x_n = \frac{W_{0,n}}{W_0} = \frac{1}{W_0}\int_i W_{0i,n}di = \frac{1}{W_0}\int_i W_{0i}X_{i,n}di = \int_i \frac{W_{0i}}{W_0}X_{i,n}di = \int_i w_iX_{i,n}di \qquad \text{(A5)}$$

Note that $\sum_{n=1}^N x_n = 1$ because $\sum_{n=1}^N W_{0,n} = W_0$, which follows from the riskless asset being in zero net supply. Plugging in for X_i from equation (A3) and imposing $a_i = a$, we have

$$\begin{aligned}
x &= \int_i w_iX_idi \\
&= \int_i w_i\left[\frac{1}{a}\Sigma^{-1}\left(\mu + \frac{1}{a}b_i\right)\right]]di \\
&= \frac{1}{a}\Sigma^{-1}\mu\left(\int_i w_idi\right) + \frac{1}{a^2}\Sigma^{-1}g\left(\int_i w_id_idi\right) \\
&= \frac{1}{a}\Sigma^{-1}\mu + \frac{\bar{d}}{a^2}\Sigma^{-1}g \ . \qquad \text{(A6)}
\end{aligned}$$

Derivation of Equation (11):

Agent i's expected excess return is given by $\mathrm{E}(\tilde{r}_i) = X_i'\mu$. We take μ from equation (9) and express X_i in terms of x by subtracting equation (5) from equation (4). Recalling the assumption $x'g = 0$ from equation (8), we obtain agent i's expected excess return as

$$\begin{aligned}\mathrm{E}(\tilde{r}_i) &= X_i'\mu \\ &= \left[x' + \frac{\delta_i}{a^2}g'\Sigma^{-1}\right]\left[\mu_M\beta - \frac{\bar{d}}{a}g\right] \\ &= \left[x' + \frac{\delta_i}{a^2}g'\Sigma^{-1}\right]\left[\frac{\mu_M}{\sigma_M^2}\Sigma x - \frac{\bar{d}}{a}g\right] \\ &= \mu_M - \frac{\bar{d}}{a}x'g + \frac{\delta_i\mu_M}{a^2\sigma_M^2}g'x - \frac{\delta_i\bar{d}}{a^3}g'\Sigma^{-1}g \\ &= \mu_M - \frac{\delta_i\bar{d}}{a^3}g'\Sigma^{-1}g. \end{aligned} \tag{A7}$$

Derivation of Equation (12):

The second exponent in agent i's expected utility in equation (A1) contains the terms $-aX_i'\mu$, $-X_i'b_i$, and $(a^2/2)X_i'\Sigma X_i$. The first of these is simply minus a times the expression in equation (A7). The second is given by

$$\begin{aligned}-X_i'b_i &= -\left[x' + \frac{\delta_i}{a^2}g'\Sigma^{-1}\right][d_i g] \\ &= -\frac{d_i\delta_i}{a^2}g'\Sigma^{-1}g, \end{aligned} \tag{A8}$$

and the third is given by

$$\begin{aligned}\frac{a^2}{2}X_i'\Sigma X_i &= \frac{a^2}{2}\left[x' + \frac{\delta_i}{a^2}g'\Sigma^{-1}\right]\Sigma\left[x + \frac{\delta_i}{a^2}\Sigma^{-1}g\right] \\ &= \frac{a^2}{2}\sigma_M^2 + \frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g, \end{aligned} \tag{A9}$$

recalling $x'g = 0$ in both cases. Adding the three terms then gives

$$\begin{aligned}-aX_i'\mu - X_i'b_i + (a^2/2)X_i'\Sigma X_i &= -a\mu_M + \frac{\delta_i\bar{d}}{a^2}g'\Sigma^{-1}g - \frac{d_i\delta_i}{a^2}g'\Sigma^{-1}g + \frac{a^2}{2}\sigma_M^2 + \frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g \\ &= -a\mu_M + \frac{a^2}{2}\sigma_M^2 + \frac{1}{a^2}\left(\delta_i\bar{d} - d_i\delta_i + \frac{1}{2}\delta_i^2\right)g'\Sigma^{-1}g \\ &= -a\left(\mu_M - \frac{a}{2}\sigma_M^2\right) - \frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g. \end{aligned} \tag{A10}$$

Substituting this exponent into equation (A1) gives

$$\begin{aligned}\mathrm{E}\left\{V(\tilde{W}_{1i}, X_i)\right\} &= -e^{-a(1+r_f)}e^{-a\left(\mu_M - \frac{a}{2}\sigma_M^2\right) - \frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g} \\ &= \left[-e^{-a(1+r_f)}e^{-a\left(\mu_M - \frac{a}{2}\sigma_M^2\right)}\right]e^{-\frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g} \\ &= \bar{V}e^{-\frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g}, \end{aligned} \tag{A11}$$

noting that the bracketed term is, $\bar{V}$, the agent's expected utility if $\delta_i = 0$.

Derivation of Equation (29):

To implement the approach for calibrating d, we first note that under the assumptions given, the vector of the ESG investor's portfolio weights in equation (25) becomes

$$\begin{aligned} X_{esg} &= \frac{1}{N}\iota + (1-\lambda)\frac{d}{a^2}\left[\sigma^2\iota\iota' + \eta^2 I_N\right]^{-1} g \\ &= \frac{1}{N}\iota + (1-\lambda)\frac{d}{a^2}\left[\frac{1}{\eta^2}\left(I_N - \frac{1}{\eta^2/\sigma^2 + N}\iota\iota'\right)\right] g \\ &= \frac{1}{N}\iota + (1-\lambda)\frac{d}{a^2\eta^2}g, \end{aligned} \tag{A12}$$

and the variance of the ESG investor's portfolio return, for large N, is

$$\begin{aligned} X'_{esg}\Sigma X_{esg} &= \left[\frac{1}{N}\iota' + (1-\lambda)\frac{d}{a^2\eta^2}g'\right]\left[\sigma^2\iota\iota' + \eta^2 I_N\right]\left[\frac{1}{N}\iota + (1-\lambda)\frac{d}{a^2\eta^2}g\right] \\ &= \sigma^2 + (1-\lambda)^2\frac{d^2}{a^4\eta^2}. \end{aligned} \tag{A13}$$

With expected utility as given by equation (A1), an ESG investor's certainty equivalent excess return from holding the ESG portfolio is then, for large N,

$$\begin{aligned} r^*_{esg} &= X'_{esg}(\mu + \frac{d}{a}g) - \frac{a}{2}X'_{esg}\Sigma X_{esg} \\ &= X'_{esg}(\mu_M\beta - \frac{\lambda d}{a}g + \frac{d}{a}g) - \frac{a}{2}X'_{esg}\Sigma X_{esg} \\ &= \left[\frac{1}{N}\iota' + (1-\lambda)\frac{d}{a^2\eta^2}g'\right]\left[a\sigma^2_M\iota - \lambda\frac{d}{a}g + \frac{d}{a}g\right] - \frac{a}{2}\left[\sigma^2_M + (1-\lambda)^2\frac{d^2}{a^4\eta^2}\right] \\ &= \frac{1}{2}\left[a\sigma^2_M + \frac{(1-\lambda)^2 d^2}{a^3\eta^2}\right]. \end{aligned} \tag{A14}$$

If the ESG investor is instead constrained to hold the market portfolio, the resulting certainty equivalent excess return is given by

$$\begin{aligned} r^*_M &= x'\mu - \frac{a}{2}x'\Sigma x \\ &= \mu_M - \frac{a}{2}\sigma^2_M \\ &= \frac{a\sigma^2_M}{2}. \end{aligned} \tag{A15}$$

The ESG investor's certainty-equivalent gain from investing as desired, versus investing in the market, is therefore

$$r^*_{esg} - r^*_M = \frac{(1-\lambda)^2 d^2}{2a^3\eta^2}. \tag{A16}$$

when N is large. As noted, this difference in certainty equivalents is largest when $\lambda = 0$. Solving for d with $\Delta \equiv r^*_{esg} - r^*_M$ then gives equation (29).

Derivation of the Certainty Equivalent Excess Return of a Non-ESG Investor:

The non-ESG investor's portfolio weights in equation (26) are

$$
\begin{aligned}
X_{non} &= \frac{1}{N}\iota - \lambda\frac{d}{a^2}\left[\sigma^2\iota\iota' + \eta^2 I_N\right]^{-1} g \\
&= \frac{1}{N}\iota - \lambda\frac{d}{a^2}\left[\frac{1}{\eta^2}\left(I_N - \frac{1}{\eta^2/\sigma^2 + N}\iota\iota'\right)\right] g \\
&= \frac{1}{N}\iota - \lambda\frac{d}{a^2\eta^2}g, \qquad \text{(A17)}
\end{aligned}
$$

and the variance of the non-ESG investor's portfolio return, for large N, is

$$
\begin{aligned}
X'_{non}\Sigma X_{non} &= \left[\frac{1}{N}\iota' - \lambda\frac{d}{a^2\eta^2}g'\right]\left[\sigma^2\iota\iota' + \eta^2 I_N\right]\left[\frac{1}{N}\iota - \lambda\frac{d}{a^2\eta^2}g\right] \\
&= \sigma^2 + \lambda^2\frac{d^2}{a^4\eta^2}. \qquad \text{(A18)}
\end{aligned}
$$

A non-ESG investor's certainty equivalent excess return from holding the non-ESG portfolio is then, for large N,

$$
\begin{aligned}
r^*_{non} &= X'_{non}\mu - \frac{a}{2}X'_{non}\Sigma X_{non} \\
&= X'_{non}(\mu_M\beta - \frac{\lambda d}{a}g) - \frac{a}{2}X'_{non}\Sigma X_{non} \\
&= \left[\frac{1}{N}\iota' - \lambda\frac{d}{a^2\eta^2}g'\right]\left[a\sigma_M^2\iota - \lambda\frac{d}{a}g\right] - \frac{a}{2}\left[\sigma_M^2 + \lambda^2\frac{d^2}{a^4\eta^2}\right] \\
&= \frac{1}{2}\left[a\sigma_M^2 + \frac{\lambda^2 d^2}{a^3\eta^2}\right]. \qquad \text{(A19)}
\end{aligned}
$$

Derivation of Equation (30):

The correlation between the ESG investor's return and the market return is equal to

$$
\begin{aligned}
\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right) &= \frac{\mathrm{Cov}\left(X'_{esg}\tilde{\epsilon}, x'\tilde{\epsilon}\right)}{\sqrt{\mathrm{Var}\left(X'_{esg}\tilde{\epsilon}\right)}\sqrt{\mathrm{Var}\left(x'\tilde{\epsilon}\right)}} \\
&= \frac{X'_{esg}\Sigma x}{\sqrt{X'_{esg}\Sigma X_{esg}}\sqrt{x'\Sigma x}}. \qquad \text{(A20)}
\end{aligned}
$$

From equations (14) and (A12), recalling that $x = (1/N)\iota$ and $\iota' g = 0$, we obtain

$$
X'_{esg}\Sigma x = \left[\frac{1}{N}\iota' + (1-\lambda)\frac{d}{a^2\eta^2}g'\right]\left[\sigma^2\iota\iota' + \eta^2 I_N\right]\left[\frac{1}{N}\iota\right] = \sigma^2, \qquad \text{(A21)}
$$

for large N. Substituting from equations (A13) and (A21) into equation (A20), recalling $x'\Sigma x = \sigma^2$ and equation (29), gives equation (30).

Derivation of Equation (31):

Applying equation (27) gives

$$\begin{aligned}
\mathrm{E}\{\tilde{r}_{esg}\} - \mathrm{E}\{\tilde{r}_{non}\} &= -\frac{\lambda d^2}{a^3} g' \Sigma^{-1} g \\
&= -\frac{\lambda d^2}{a^3} g' \left[\sigma^2 \iota\iota' + \eta^2 I_N\right]^{-1} g \\
&= -\frac{\lambda d^2}{a^3} g' \left[\frac{1}{\eta^2}\left(I_N - \frac{1}{\eta^2/\sigma^2}\iota\iota'\right)\right] g \\
&= -\frac{\lambda d^2}{a^3 \eta^2}. \qquad \text{(A22)}
\end{aligned}$$

Plugging in for d^2 from equation (29), we obtain equation (31).

Derivations of Equations (32) and (33):

Let α denote the $N \times 1$ vector of alphas given by equation (10). Taking X_{esg} from equation (A12), the alpha of the ESG investor is given by

$$\begin{aligned}
\alpha_{esg} &= X'_{esg}\alpha \\
&= \left[\frac{1}{N}\iota' + (1-\lambda)\frac{d}{a^2\eta^2} g'\right]\left[-\frac{\lambda d}{a} g\right] \\
&= -\lambda(1-\lambda)\frac{d^2}{a^3\eta^2}. \qquad \text{(A23)}
\end{aligned}$$

By using X_{non} from equation (A17), we obtain the alpha of the non-ESG investor:

$$\begin{aligned}
\alpha_{non} &= X'_{non}\alpha \\
&= \left[\frac{1}{N}\iota' - \lambda\frac{d}{a^2\eta^2} g'\right]\left[-\frac{\lambda d}{a} g\right] \\
&= \lambda^2 \frac{d^2}{a^3\eta^2}. \qquad \text{(A24)}
\end{aligned}$$

Plugging in for d^2 from equation (29), we obtain equations (32) and (33).

Derivation of Equation (37): Using equation (A12) and $x = (1/N)\iota$,

$$\begin{aligned}
T &= \frac{1}{2}\lambda\iota' |X_{esg} - x| \\
&= \frac{1}{2}\lambda\iota' |(1-\lambda)\frac{d}{a^2\eta^2} g| \\
&= \frac{1}{2}\lambda(1-\lambda)\frac{d}{a^2\eta^2}\iota'|g|. \qquad \text{(A25)}
\end{aligned}$$

Plugging in for d from equation (29), we obtain equation (37).

Derivation of Equation (51):

Combining equations (9) and (43) gives

$$\begin{aligned}\tilde{r} &= \mu + \tilde{\epsilon} \\ &= \beta\mu_M - g\frac{\bar{d}}{a} + h\tilde{z}_h + g\tilde{f}_g + \tilde{\zeta}. \end{aligned} \tag{A26}$$

With $x'\beta = 1$ and $x'g = 0$, premultiplying the above by x' gives the excess market return as

$$\tilde{r}_M = \mu_M + (x'h)\tilde{z}_h + x'\tilde{\zeta}, \tag{A27}$$

implying

$$\tilde{z}_h = \left(\tilde{r}_M - \mu_M - x'\tilde{\zeta}\right)/x'h. \tag{A28}$$

Substituting into equation (A26) and then using equation (47) gives

$$\begin{aligned}\tilde{r} &= \beta\mu_M - g\frac{\bar{d}}{a} + h\left(\tilde{r}_M - \mu_M - x'\tilde{\zeta}\right)/x'h + g\tilde{f}_g + \tilde{\zeta} \\ &= \left(h\beta_h + g\beta_g + \frac{1}{\sigma_M^2}\Lambda x\right)\mu_M - g\frac{\bar{d}}{a} + h\left[\tilde{r}_M - \mu_M - x'\tilde{\zeta}\right]/x'h + g\tilde{f}_g + \tilde{\zeta} \\ &= h\left(\beta_h\mu_M + \left[\tilde{r}_M - \mu_M - x'\tilde{\zeta}\right]/x'h\right) + g\left(\tilde{f}_g + \beta_g\mu_M - \frac{\bar{d}}{a}\right) + \frac{\mu_M}{\sigma_M^2}\Lambda x + \tilde{\zeta} \\ &= \theta\tilde{r}_M + g\left(\tilde{f}_g + \beta_g\mu_M - \frac{\bar{d}}{a}\right) + \tilde{\nu}, \end{aligned} \tag{A29}$$

where $\theta = (1/x'h)h$, and

$$\tilde{\nu} = h\mu_M\left(\beta_h - \frac{1}{x'h}\right) + \frac{\mu_M}{\sigma_M^2}\Lambda x - h\left(\frac{x'\tilde{\zeta}}{x'h}\right) + \tilde{\zeta}. \tag{A30}$$

Equation (A29) provides the desired relation, but it remains to show that $\mathrm{E}\{\tilde{\nu}|\tilde{r}_M, \tilde{f}_g\} \approx 0$. From equations (43) and (48), recalling $x'g = 0$,

$$\beta_h = \frac{(x'h)var(\tilde{z}_h)}{(x'h)^2 var(\tilde{z}_h) + var(x'\tilde{\zeta})}. \tag{A31}$$

If the market is well diversified with N large, such that $x_n \approx 0$, then $var(x'\tilde{\zeta}) = x'\Lambda x \approx 0$, and thus $\beta_h \approx 1/x'h$, thereby making the first term in equation (A30) approximately zero. The second term in that equation is also approximately zero if $x_n \approx 0$, as the nth element of that vector is $\left(\mu_M \mathrm{Var}(\tilde{\zeta}_n)/\sigma_M^2\right)x_n$. The third and fourth terms in equation (A30) have zero means, so we have $\mathrm{E}\{\tilde{\nu}\} \approx 0$. Because $\mathrm{Cov}\{\tilde{\zeta}, \tilde{f}_g\} = 0$, it remains to show that $\mathrm{Cov}\{\tilde{\zeta}, \tilde{r}_M\} \approx 0$. That result follows from equation (A27), which implies that the nth element of $\mathrm{Cov}\{\tilde{\zeta}, \tilde{r}_M\}$ equals $\mathrm{Var}(\tilde{\zeta}_n)x_n$, approximately zero if $x_n \approx 0$.