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q-FACTORS AND INVESTMENT CAPM

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ABSTRACT

The *q*-factor model shows strong explanatory power and largely summarizes the cross section of average stock returns. In particular, the *q*-factor model fully subsumes the Fama-French (2018) 6-factor model in head-to-head factor spanning tests. The *q*-factor model is an empirical implementation of the investment CAPM. The basic philosophy is to price risky assets from the perspective of their suppliers (firms), as opposed to their buyers (investors). As a disruptive innovation, the investment CAPM has broad-ranging implications for academic finance and asset management practice.

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***q*-factors and Investment CAPM**

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Summary

The *q*-factor model is an empirical asset pricing model proposed by Hou, Xue, and Zhang (2015). The model says that the expected return of an asset in excess of the riskfree rate is described by its sensitivities to the market factor, a size factor, an investment factor, and a return on equity factor. Empirically, the *q*-factor model shows strong explanatory power and largely summarizes the cross section of average stock returns. In particular, the *q*-factor model fully subsumes the Fama-French (2018) 6-factor model in head-to-head factor spanning tests.

The *q*-factor model is an empirical implementation of the investment CAPM (Zhang, 2017). The basic philosophy is to price risky assets from the perspective of their suppliers (firms), as opposed to their buyers (investors). Mathematically, the investment CAPM is a restatement of the Net Present Value rule in Corporate Finance. Intuitively, high investment relative to low expected profitability must imply low costs of capital, and low investment relative to high expected profitability must imply high costs of capital. In a multiperiod framework, if investment is high next period, the present value of cash flows from next period onward must be high. Consisting mostly of this next period present value, the benefits to investment this period must also be high. As such, high investment next period relative to current investment (high expected investment growth) must imply high costs of capital (to keep current investment low).

As a disruptive innovation, the investment CAPM has broad-ranging implications for academic finance and asset management practice. First, the consumption CAPM, in which the classic Sharpe-Lintner CAPM (Sharpe, 1964; Lintner, 1965) is a special case, is conceptually

incomplete. The crux is that it blindly focuses on the demand of risky assets, while abstracting from the supply. Alas, anomalies are primarily relations between firm characteristics and expected returns. By focusing on the supply, the investment CAPM is the missing piece of equilibrium asset pricing. Second, the investment CAPM retains efficient markets, with cross-sectionally varying expected returns, depending on firms' investment, profitability, and expected growth. As such, capital markets follow standard economic principles, in sharp contrast to the teachings of behavioral finance. Finally, the investment CAPM validates Graham and Dodd's (1934) Security Analysis on equilibrium grounds, within efficient markets.

Keywords: The *q*-factor model, anomalies, EMH, behavioral finance, the investment CAPM, the consumption CAPM, the CAPM, equilibrium theory

Questions

A voluminous literature in empirical asset pricing and capital markets research in accounting documents a wide range of relations between firm characteristics and average stock returns. These relations are often called "anomalies" because they cannot be explained by the standard consumption-based Capital Asset Pricing Model (the consumption CAPM), in which the classic Sharpe-Lintner CAPM (Sharpe, 1964; Lintner, 1965) is a special case. Prominent anomalies include post-earnings-announcement drift (Ball and Brown 1968; Bernard and Thomas 1989, 1990), long-term reversal (De Bondt and Thaler 1985), momentum (Jegadeesh and Titman 1993), long-term underperformance following initial and seasoned equity offerings (Ritter 1991; Loughran and Ritter 1995), the value anomaly (Lakonishok, Shleifer, and Vishny 1994), and the accrual anomaly (Sloan 1996).

Behavioral economists interpret these anomalies as predictable pricing errors, which reject Fama's (1970) efficient markets hypothesis (EMH) as well as Muth's (1961) and Lucas's (1972) rational expectations hypothesis. However, as emphasized by Fama (1991), anomalies do not necessarily reject EMH because the expected-return models used to isolate "pricing errors" in empirical tests can be incomplete (the joint-hypothesis problem). In particular, a coherent theory of inefficient markets with predictable pricing errors has yet to appear (Fama, 1998).

Fama and French (1993, 1996) defend EMH by adding a size factor, SMB, and a value factor, HML, into the CAPM to form their 3-factor model. They interpret SMB and HML as sources of risk in the intertemporal CAPM (Merton, 1973) or Arbitrage Pricing Theory (APT) (Ross, 1976). However, this interpretation has not been persuasive. Both the intertemporal CAPM and APT are silent about the identities of state variables. As a result, SMB and HML are motivated from their empirical performance, rather than a priori theoretical arguments.

Within this historical context, many fundamental questions arise. What explains all the CAPM anomalies? Why does the consumption CAPM fail so badly in explaining these anomalies? Given its abysmal performance, what confidence should one put in equilibrium theories that embed the consumption CAPM and permeate virtually all Ph.D. level textbooks in finance and economics? A prominent example is New Keynesian DSGE models. Graham and Dodd's (1934) Security Analysis has worked for 85 years in practice. Why has it so far not found a rightful place in finance theory? Given that firm characteristics are so important in describing returns empirically, why do characteristics barely show up in finance theory? How should we interpret characteristics-based factors exactly? If anomalies are driven by expectation errors, what exactly are the psychological biases at play? Why do these systematic mistakes persist for so long, in some cases, such as post-earnings-announcement drift, for 50 years since Ball and

Brown (1968)? Why has there not been a coherent theory of behavioral finance for almost 35 years since De Bondt and Thaler (1985)?

An old science joke says: "Theory is when you know everything but nothing works. Practice is when everything works but no one knows why. In our lab, theory and practice are combined: Nothing works and no one knows why." Finance is better. The consumption CAPM theory is well developed, but it doesn't work. Anomaly strategies work, but no one knows why. In the investment CAPM, theory and practice are combined: Everything works and I know why.

Mechanisms

The basic philosophy of the investment CAPM is to price risky assets from the perspective of their suppliers (firms), as opposed to their buyers (investors) (Zhang, 2017), building on an early precursor of Cochrane (1991).

Mathematically, the investment CAPM is a restatement of the Net Present Value (NPV) rule in Corporate Finance. The NPV of a project is its present value (discounted value of its future cash flows) minus its investment costs today. The NPV rule says that a manager should invest in a given project if and only if its NPV is greater than or equal to zero. When initially facing many projects with NPV ≥ 0, the manager will start with the project with the highest NPV and work her way down the supply curve of projects. For the last project that the manager takes, its NPV should equal zero. To keep things simple, consider first one-period projects. The last project with NPV = 0 means that its investment costs = profitability / discount rate.

The investment CAPM turns the NPV rule, which is a fundamental principle in Corporate Finance, on its head and transforms it into an Asset Pricing theory. Rewriting the NPV rule yields: Discount rate = profitability / investment costs. Intuitively, given profitability, high costs

of capital (discount rates) imply low NPVs of new projects and low investments, and low costs of capital imply high NPVs of new projects and high investments. In addition, given investments, high profitability must imply high discount rates to give rise to low NPVs of new projects to keep investments constant. Low profitability relative to investments must imply low discount rates to offset low profitability to keep the NPVs of new projects and investments constant. In all, investment and profitability are two key drivers in cross section of expected returns.

If projects last more than one period as in a multiperiod model, the NPV rule becomes: Investment costs = (profitability + present value of cash flows from next period onward) / discount rate. With optimal investment, the present value of cash flows from next period onward equals expected investment costs next period (marginal q equals marginal costs of investment). As such, the investment CAPM says: Discount rate = (profitability + expected investment costs) / investment costs. Intuitively, if investment and expected investment costs are high next period, the present value of cash flows from next period onward must be high. Consisting primarily of this next period present value, the benefits to investment this period must also be high. As such, if investment is high next period relative to current investment (expected investment growth is high), the discount rate must be high. The discount rate must be high to offset the high benefits of investment this period to keep current investment low. As such, expected growth is another key driver in the cross section of expected stock returns.

Methods

The investment CAPM literature has taken its key predictions to the data via a variety of approaches, including factor regressions, structural estimation, and quantitative theories.

Factor Models

Hou, Xue, and Zhang (2015) propose and test the *q*-factor model, which implements the investment CAPM via the Fama-French (1993) portfolio approach. The *q*-factor model says that the expected return of an asset in excess of the riskfree rate is described by its sensitivities to the market factor, a size factor, an investment factor, and a return on equity (Roe) factor. The size, investment, and Roe factors are constructed from 2 by 3 by 3 sorts on market equity, investment-to-assets, and Roe. Empirically, the *q*-factor model goes a long way toward summarizing the cross section of average stock returns. The model explains many anomalies that bedevil the Fama-French 3-factor model, such as Jegadeesh and Titman's (1993) momentum (Fama and French, 1996). Most anomalies are just different manifestations of investment and profitability.

The data for the *q*-factors and testing portfolios are available for download at global-q.org.

Intuition

On the one hand, sorting on net stock issues, composite issuance, book-to-market and other valuation ratios, as well as long-term reversal is closer to sorting on investment than on profitability. As such, these diverse sorts reflect their common implied sort on investment.

The flow-of-fund constraint of firms says that their uses of funds must equal their sources of funds. As such, all else equal, equity issuers should invest more and have lower costs of capital than nonissuers. In addition, firms use different capital goods in their operating activities, including working capital, physical property, plant, and equipment, and (measured) intangibles.

As such, total asset growth is the most comprehensive measure of investment-to-assets, a simple measure that aggregates over investments in heterogeneous capital goods.

The value factor is redundant in the presence of the investment factor. In the investment CAPM, investment increases with marginal q, which in turn equals average q with constant returns to scale. Average q and market-to-book equity are close cousins and are identical twins without debt. As such, value stocks with low valuation ratios should invest less and, all else equal, should earn higher expected returns than growth stocks with high valuation ratios.

High valuation ratios come from a stream of positive shocks on fundamentals, and low valuation ratios a stream of negative shocks on fundamentals. Growth stocks typically have high long-term prior returns, and value stocks low long-term prior returns. As such, long-term reversal also reflects the investment factor. Firms with high long-term prior returns should invest more and have lower costs of capital than firms with low long-term prior returns.

On the other hand, sorting on earnings surprises, short-term prior returns, and financial distress is closer to sorting on profitability than on investment. As such, these diverse sorts reflect their common implied sort on profitability. Intuitively, shocks to earnings are positively correlated with shocks to returns, contemporaneously. Firms with positive earnings shocks experience immediate stock price increases, and firms with negative earnings shocks experience immediate stock price drops. As such, momentum winners should have higher expected profitability and earn higher expected returns than momentum losers.

In addition, less financially distressed firms have higher profitability and, all else equal, should earn higher expected returns than more financially distressed firms. As such, the distress anomaly is just another manifestation of the profitability factor.

Subsequent Work

The *q*-factor model has effectively ended the quarter-century reign of the Fama-French (1993) 3-factor model as the leading model in empirical asset pricing. During the long process, the *q*-factor model has stimulated a large subsequent literature on factor models.

Fama and French (2015) attempt to fix their 3-factor model by incorporating their own versions of the investment and profitability factors to form a 5-factor model. Fama and French (2018) further add the momentum factor, UMD, to form their 6-factor model.

However, Hou, Mo, Xue, and Zhang (2019a) show that the 4-factor *q*-model fully subsumes the Fama-French 6-factor model in head-to-head spanning tests. In the 1967-2018 monthly sample, the investment and Roe factors in the *q*-factor model earn on average 0.38% and 0.55% per month (*t*-value = 4.59 and 5.44), respectively. Their alphas in the Fama-French 6-factor model are 0.1% and 0.27% (*t*-value = 2.82 and 4.32), respectively. The Gibbons, Ross, and Shanken (GRS, 1989) test strongly rejects the null hypothesis that the 6-factor model can jointly subsume the investment and Roe factors (*p*-value = 0.00).

Conversely, HML, CMA (the investment factor), RMW (the profitability factor), and UMD in the Fama-French 6-factor model earn on average 0.32%, 0.3%, 0.28%, and 0.64% per month (*t*-value = 2.42, 3.29, 2.76, and 3.73), respectively. More important, their alphas in the *q*-factor regressions are economically small (tiny in many cases) and statistically insignificant: 0.05%, 0.00%, 0.03%, and 0.14% (*t*-value = 0.49, 0.08, 0.32, and 0.61), respectively. The GRS test cannot reject the null hypothesis that the *q*-factor model can jointly subsume the HML, CMA, RMW, and UMD factors (*p*-value = 0.79). In all, despite having two fewer factors, the *q*-factor model fully subsumes the Fama-French 6-factor model, including UMD.

Stambaugh and Yuan (2017) group 11 anomalies into two clusters based on pairwise cross-sectional correlations. The first cluster, denoted MGMT, contains net stock issues, composite issues, accruals, net operating assets, investment-to-assets, and the change in gross property, plant, and equipment plus the change in inventories scaled by lagged book assets. The second cluster, denoted PERF, includes failure probability, O-score, momentum, gross profitability, and return on assets. The composite scores, MGMT and PERF, are defined as a stock's equal-weighted rankings across all the variables (realigned to yield positive low-minus-high returns) within a given cluster. Stambaugh and Yuan form their factors from monthly independent 2 by 3 sorts from interacting size with each of the composite scores.

However, Stambaugh and Yuan (2017) deviate from the standard factor construction per Fama and French (1993) in two important ways. First, the NYSE-Amex-NASDAQ breakpoints of 20th and 80th percentiles are used, as opposed to the NYSE breakpoints of 30th and 70th, when sorting on the composite scores. Second, the size factor contains stocks only in the middle portfolios of the composite score sorts, as opposed to stocks from all portfolios. The Stambaugh-Yuan factors are sensitive to their factor construction, and their nonstandard construction exaggerates their factors' explanatory power. Most important, once replicated via the standard procedure, the MGMT and PERF factors are close to the investment and Roe factors in the *q*-factor model, with correlations of 0.8 and 0.84, respectively (Hou, Mo, Xue, and Zhang, 2019a).

Hou, Mo, Xue, and Zhang (2019b) perform cross-sectional forecasting regressions of future investment-to-assets changes on the log of Tobin's *q*, operating cash flows, and the change in Roe. Independent 2 by 3 sorts on size and expected 1-year-ahead investment-to-assets changes yield an expected growth factor, with an average premium of 0.84% per month (*t*-value = 10.27) and a *q*-factor alpha of 0.67% (*t*-value = 9.75). Hou et al. augment the *q*-factor model with the

expected growth factor to yield the q^5 model. Using a large set of 150 anomalies that are significant with NYSE breakpoints and value-weighted returns compiled in Hou, Xue, and Zhang (2019), Hou et al. conduct a large-scale horse race of latest factor models. The q^5 model is the best performing model that substantially outperforms the Fama-French (2018) 6-factor model. In fact, the q-factor model already compares well with the 6-factor model.

However, unlike investment and profitability, expected growth is unobservable. The performance of the q^5 model depends on its expected growth specification, and crucially, on operating cash flows as a key predictor of future growth. As such, although its underlying intuition is clear, the q^5 model should be interpreted primarily as a tool for dimension reduction.

Structural Estimation

Factor models only explore directional predictions of the investment CAPM. In structural estimation, one takes the model's key equation directly to the data for econometric estimation and evaluation. Hansen and Singleton (1982) conduct the first such test for the consumption CAPM. Liu, Whited, and Zhang (2009) perform the first structural estimation for the investment CAPM. Although by no means perfect, Liu et al.'s first stab yields much more encouraging results than Hansen and Singleton's at the consumption CAPM. The baseline investment CAPM with only physical capital manages to explain value and post-earnings-announcement drift separately, albeit not jointly. Liu and Zhang (2014) show that the baseline model can explain Jegadeesh and Titman's (1993) momentum separately, but not simultaneously with value.

The joint estimation difficulty has been largely resolved by Goncalves, Xue, and Zhang (2019), who introduce working capital into the investment CAPM. With plausible parameter estimates, the two-capital investment CAPM manages to explain the value, momentum,

investment, and Roe premiums jointly. Aggregation also plays an important role. Liu, Whited, and Zhang (2009) and Liu and Zhang (2014) construct portfolio-level predicted returns from portfolio-level accounting variables to match with portfolio-level stock returns. In contrast, Goncalves et al. use firm-level accounting variables to construct firm-level predicted returns, which are then aggregated to the portfolio level to match with portfolio-level stock returns.

A surprising insight from Goncalves, Xue, and Zhang (2019) is that value and momentum (as well as investment and Roe) are driven by related, if not identical, mechanisms. Intuitively, current investment and expected investment are locked in a "tug of war" in the investment CAPM equation. When current investment overpowers expected investment, the model predicts the value and investment premiums. When expected investment overpowers current investment, the model predicts the momentum and Roe premiums. The predicted value and investment premiums are long-lived, persisting over 3-5 years after portfolio formation. The predicted momentum and Roe premiums are short-lived, vanishing within 1 year after portfolio formation. The model dynamics are intriguingly consistent with the dynamics in the data.

Quantitative Theories

Zhang (2005) constructs the first neoclassical, dynamic investment model for the cross section of returns in the spirit of real business cycles (Kydland and Prescott, 1982; Long and Plosser, 1983). Instead of estimating the first-order conditions formally in structural estimation, quantitative theory studies specify a dynamic model fully, calibrate and simulate it, and compare its implied moments with observed moments in the data. Zhang highlights the role of costly reversibility in explaining the value premium. Intuitively, value firms are burdened with more unproductive capital in bad times, finding it more difficult to downsize so as to yield more

cyclical and riskier cash flows and earn higher expected returns than growth firms. In contemporaneous and independent work, Cooper (2006) shows closely related mechanisms at work in a real options model. Also in a related real options model, Carlson, Fisher, and Giammarino (2014) emphasize the role of operating leverage in driving the value premium. The Zhang model, recently labelled by Clementi and Palazzo (2019) as "the standard investment model," has served as a launching pad for a large subsequent, theoretical literature on the cross section of returns. A full review of this literature is far beyond this analytical essay.

A long-standing controversy in this theoretical literature is that the CAPM alpha of the value premium in Zhang's (2005) model is economically small, although the average value premium itself matches that observed in the data. Subsequent studies have attempted to explain the failure of the CAPM in explaining the value premium in the post-Compustat sample by breaking the tight link between the stochastic discount factor (SDF) and the market factor with multiple aggregate shocks. Prominent examples include short- and long-run shocks (Ai and Kiku, 2013), investment-specific technological shocks (Kogan and Papanikolaou, 2013), stochastic adjustment costs (Belo, Lin, and Bazdresch, 2014). However, these 2-shock models all fail to explain the long sample evidence from 1926 onward that the CAPM alpha of the value premium is economically small and statistically insignificant.

Bai, Hou, Kung, Li, and Zhang (2019) embed disasters into a general equilibrium model with heterogeneous firms to induce strong nonlinearity in the SDF to explain the CAPM failure. Intuitively, when a disaster hits, value firms are burdened with more unproductive capital, finding it more difficult with costly reversibility to reduce capital than growth firms. As such, value firms are more exposed to the disaster risk than growth firms, giving rise to a high average value premium. However, in a finite sample, in which disasters are not realized, the estimated

market beta fails to fully capture the disaster risk embedded in the value premium. Consequently, the CAPM fails to explain the value premium in a finite sample without disasters.

In the general equilibrium model of Bai, Hou, Kung, Li, and Zhang (2019), a nonlinear consumption CAPM holds by construction, yet the standard consumption CAPM fails badly in simulated data from the model. Intuitively, the aggregate consumption growth is a poor proxy for the SDF based on recursive utility. Their correlation in simulated data is close to zero. Surprisingly, the onset of disasters is not associated with particularly low contemporaneous consumption growth, and the onset of recoveries not with particularly high consumption growth.

Intuitively, when a disaster hits, the SDF spikes up immediately because investors anticipate multiple years of high marginal utility (bad times). However, consumption smoothing immediately kicks in, with forward-looking real investment falling drastically to smooth consumption. Consequently, consumption only falls cumulatively over multiple years, making the contemporaneous consumption growth a bad proxy for the SDF. Relatedly, consumption smoothing also explains why the classic CAPM performs better than the standard consumption CAPM. Because stock prices are forward-looking, the market factor is much more correlated with the SDF than the contemporaneous consumption growth.

A more recent controversy concerns the quantitative performance of the standard investment model (Zhang, 2005). Clementi and Palazzo (2019) argue that upon hit by adverse shocks, U.S. public firms have "ample latitude" to divest their unproductive assets. In particular, "each quarter on average 18.2% of firms record negative gross investment (p. 282)," suggesting that "plenty of firms downsize, at all times (p. 287)," and that there exists "no sign of irreversibility (p. 289)." Quantitatively, Clementi and Palazzo argue that for the standard investment model to explain the average value premium, its implied investment rates must be

counterfactual, with a tiny fraction of negative rates and a cross-sectional volatility that is an order of magnitude smaller than that in the data.

Bai, Li, Xue, and Zhang (2019) reexamine the evidence of costly reversibility in U.S. public firms. Bai et al. document that the firm-level investment rate distribution is highly skewed to the right, with a small fraction of negative investments, 5.79%, a tiny fraction of inactive investments, 1.46%, and a large fraction of positive investments, 92.75%. The firm-level evidence is even stronger than the prior plant-level evidence in Cooper and Haltiwanger (2006). Sample criteria likely play an important role. While Cooper and Haltiwanger include only relatively large manufacturing plants in continuous operations throughout their 1972-1988 sample, Bai et al. include virtually all Compustat firms in different industries (not just manufacturing), with no restrictions on size or age.

With a careful replication effort, Bai, Li, Xue, and Zhang (2019) trace the differences between their evidence and Clementi and Palazzo's (2019) to 3 sources. First, both studies measure gross investment rates as net investment rates plus depreciation rates. Both measure net investment rates as the net growth rates of net property, plant, and equipment (PPE) in Compustat. Bai et al. measure depreciation rates as Compustat's depreciation over net PPE, depreciation rates that are embedded in net PPE. In contrast, Clementi and Palazzo use industry-level geometric depreciation rates estimated by Bureau of Economic Analysis, depreciation rates that are internally incompatible with net PPE in Compustat. Second, Clementi and Palazzo impose sample criteria that are nonstandard in empirical finance, such as removing firm-years with mergers and acquisitions, in which the target's assets are more than 5% (a low cutoff) of the acquirer's. Finally, Clementi and Palazzo also engage in a highly questionable research practice by cutting off the right tail of the quarterly investment rate distribution at 0.2.

While Clementi and Palazzo's (2019) evidence is flawed, their point of matching investment and returns moments jointly in quantitative studies is well taken. Using Simulated Method of Moments (SMM), Bai, Li, Xue, and Zhang (2019) estimate four parameters (the upward and downward adjustment cost parameters, the fixed cost of production, and the conditional volatility of firm-specific productivity) to target seven data moments (the average value premium, the cross-sectional volatility and skewness of individual stock excess returns, the cross-sectional volatility, skewness, and persistence of investment rates, as well as the fraction of negative investment rates). The SMM estimation strongly indicates costly reversibility and operating leverage in U.S. public firms. The downward adjustment cost parameter is estimated to be 508.2 (*t*-value = 13.39), which is substantially higher than the upward parameter, 0.63 (*t*-value = 4.6). The fixed cost of production is estimated to be 0.0637 (*t*-value = 4.24). The model matches the average value premium of 0.43% per month (*t*-value = 1.97) in the 1962–2018 sample. For investment rates, the cross-sectional volatility is 62% per annum (58.5% in the data) and the fraction of negative investments 5.78% in the model (5.79% in the data). The overidentification test fails to reject the model with the seven moments (*p*-value = 0.59).

Implications

As a disruptive innovation, the investment CAPM thinks about asset pricing very differently from the consumption CAPM and behavioral finance, with broad-ranging implications for academic finance research and asset management practice.

Complementarity with the Consumption CAPM

In his magnum opus, Alfred Marshall (1890, Principles of Economics [1961, 9th edition, p. 348]) writes: "We might as reasonably dispute whether it is the upper or under blade of a pair of scissors that cuts a piece of paper, as whether value is governed by utility or costs of production. It is true that when one blade is held still, and the cutting is affected by moving the other, we may say with careless brevity that the cutting is done by the second; but the statement is not strictly accurate, and is to be excused only so long as it claims to be merely a popular and not a strictly scientific account of what happens."

Asset pricing theory is just value theory in microeconomics extended to uncertainty and over time. From this perspective, clearly, the consumption CAPM is conceptually incomplete. The crux is that it exclusively focuses on the demand of risky assets, while abstracting from the supply altogether. Alas, anomalies are primarily empirical relations between firm characteristics and expected returns. Without modeling firm characteristics, it is impossible to fully explain anomalies within the consumption CAPM. Even if an SDF specification is discovered that fits the consumption CAPM with anomaly portfolios, one still has to explain why the consumption betas would be aligned with investment-to-assets, Roe, book-to-market, momentum, and other anomaly variables. By focusing on the supply of risky assets, while abstracting from the demand altogether, the investment CAPM is the missing "blade" of equilibrium asset pricing, symmetrically and neatly complementing to the consumption "blade." The investment CAPM and the consumption CAPM combine to form the pair of "scissors" of equilibrium pricing.

The glorious achievements of the consumption CAPM are well known. I interpret its major contribution as time-varying expected returns, which largely resolve Shiller's (1981) excess volatility puzzle in aggregate asset pricing. But why does the consumption CAPM fail so badly in explaining anomalies in the cross section? Zhang (2017) blames the intractable

aggregation problem. Investors are heterogeneous in preferences, beliefs, and information sets, all of which make the demand-based pricing extremely difficult. The Sonnenschein-Mantel-Debrew theorem in equilibrium theory says that individual rationality imposes essentially no restrictions on aggregate demand, meaning that the aggregation problem over heterogeneous investors is largely intractable (Kirman, 1992). It is possible that for aggregate, macro-level asset pricing, a representative agent still suffices but fails for micro-level asset pricing in the cross section. Who's the marginal investor for Apple Inc.? Anyone's guess is as good as mine.

Derived from the first principle of individual firms, the investment CAPM is relatively immune to the aggregation problem. Who's the marginal supplier for Apple Inc. shares? Well, easy, that's Apple Inc.. Tim Cook most likely has more impact on Apple Inc.'s market value via his operating, investing, and financing decisions than many Apple Inc. shareholders like me via portfolio decisions in their retirement accounts. The investment CAPM formalizes the linkage between corporate decisions and asset prices. The major contribution of the investment CAPM is **cross-sectionally varying expected returns**, which largely resolve anomalies in the cross section. In particular, the consumption CAPM anomalies are the investment CAPM regularities.

Because of the inescapable aggregation difficulty facing the consumption CAPM and no such challenge facing the investment CAPM, EMH must be detached from the consumption CAPM and reattached to the investment CAPM. How many more decades of the consumption CAPM failures do we have to endure to let the lesson sink in that firm characteristics are not even modelled? The step going from an individual investor problem to a consumption-based SDF that prices all assets requires aggregation, which is all but automatic. Asset pricing is not all about SDF, which is only demand-based. The overreaching tendencies of the consumption

CAPM, detrimental to our Science, must stop. You don't see me pretending that the investment CAPM has anything to do with personal finance, household finance, or portfolio allocation.

An EMH Counterrevolution to Behavioral Finance

The anomalies literature is the empirical foundation of behavioral economics. The investment CAPM shows that the empirical foundation is all but an illusion. Start with: Realized returns = expected returns + abnormal returns. When an anomaly variable forecasts realized returns, there are tautologically two parallel interpretations. One, which is the behavioral view, says that the variable is forecasting abnormal returns. As such, pricing errors are predictable, violating EMH. The other, which is the EMH view, says that the anomaly variable is related to expected returns, but the pricing errors are unpredictable. The consumption CAPM and the investment CAPM are both expected-return models. Both are consistent with EMH.

In the anomalies literature (and in asset management industry), the behavioral view is extremely popular. Behavioral finance has gained its prominence by documenting the CAPM alphas and sticking labels such as under- and over-reaction to them. While rejecting the CAPM is the more accurate interpretation of the evidence, the interpretation of rejecting EMH altogether certainly appears to be more impactful. More important, for a long time, the consumption CAPM is the only asset pricing theory in the land. Given the exclusive focus on investors, it's not unreasonable to interpret the failure of the consumption CAPM as investor irrationality.

The investment CAPM has changed the big picture in its entirety. I deal with Fama's (1991) joint-hypothesis problem by replacing the consumption CAPM with the investment CAPM. With the suppliers of risky assets at the center of analysis, the anomalous evidence is largely consistent with the NPV rule in Corporate Finance. Remember EMH only says that

pricing errors are not predictable. The investment CAPM alphas are mostly small and unpredictable. And the expectations of the investment CAPM are entirely rational.

I separate EMH from investor rationality. Again, EMH only says that pricing errors are not predictable. It doesn't say all investors are rational. A common counterargument against my EMH defense is that if investors set a firm's equity price too high, its manager will just blindly adjust her investment decisions per her first-order condition. As a result, both the equity price and investment are wrong. This argument is specious at best. It ignores the powerful equilibrating role of the supply side. Some investors might be optimistic and attempt to bid up the equity price too high. But with a manager's cool head, the supply of risky shares goes up, flooding cold water over the fire of irrational exuberance. The wrong price will drop toward the equilibrium price. In the special case of no adjustment costs, in particular, Tobin's q will forever be one, regardless of how irrational investors are. This equilibrating role of the supply side seems to have been greatly underappreciated by academics and practitioners alike.

I should concede that the complex equilibrating process between demand and supply is largely unknown. I have seen models of heterogeneous investors, and separately, models of heterogeneous firms. But I have yet to see a model with both heterogeneous investors and heterogeneous firms, likely because of its computational intractability. As such, all we can do is to use simpler models to gain insights. Behavioral finance relies on dysfunctional, inefficient markets for its mechanisms to work. With the investment CAPM, I view anomalies as regularities from the NPV rule in well functioning, efficient markets. As such, the argument that anomalies must necessarily imply investor irrationality is wrong. Anomalies most likely have less to do with investors and more to do with managers. The NPV rule is as fundamental an economic principle as diversification. Capital markets obey standard economic principles!

However, because the complex equilibrating process between demand and supply is unknown, and perhaps even unknowable, I cannot say that the observed prices are completely deprived of wrong decisions from investors. However, remember the Sonnenschein-Mantel-Debrew theorem says that investor rationality and aggregate rationality are completely detached. Investors can be irrational, but the marginal (aggregate) investor might not, and vice versa. As such, the failures of the consumption CAPM might have nothing to say about EMH. Behavioral economists can hide behind this aggregation problem all they want and claim relevance. But it's no coincidence that a coherent behavioral theory has yet to appear after 35 years since De Bondt and Thaler (1985). Given the time test, I feel that such a theory likely doesn't even exist.

While I contend that behavioral finance has almost nothing to say about equilibrium asset prices, I do think that it has a major role to play in areas like personal finance and household finance. Identifying and rectifying investor mistakes in these areas are enormously important for human welfare. However, these areas are partial equilibrium in nature. Without dealing with aggregation, these fields have limited implications for equilibrium asset prices.

How I Defend Fama

A watershed article is Fama and French (1992). It is this paper from the EMH inventor that abandons the CAPM, which is largely the only asset pricing theory at the time, thereby stimulating the development of behavioral finance. Although Fama and French (1993) quickly attempt to patch up the hole with their 3-factor model by adding SMB and HML into the CAPM, the floodgate has been opened. Fama (1998) tries to contain the resulting tsunami but to little avail. With a wrong hammer in their hands (as firm characteristics are all condensed into a Lucas

tree), theorists have largely stood on the sidelines looking on, with precious little to say about the EMH versus behavioral finance debate.

It is informative to compare Fama's (1998) EMH defense 20 years ago with my current defense based on the investment CAPM. Fama makes 2 points. First, apparent overreaction is about as common as underreaction. As anomalies seem to split randomly between underreaction and overreaction, Fama claims that EMH wins. Second, anomalies are sensitive to changes in measurement. Anomalies with value-weighted returns are smaller than with equal-weighted returns. Also, calendar-time 3-factor regressions are more reliable than long-horizon event studies. Kothari (2001) echoes Fama in emphasizing the sensitivity of measurement and the need of coming up with a theory of inefficient markets as null hypotheses.

Like his EMH insight, Fama's empirics has no peers. As acknowledged in Zhang (2017), the empirical design of the q-factor model, including its factor construction, formation of testing portfolios, econometric tests, and most important, the **taste** of the economic question, are all deeply influenced by Fama and French (1993, 1996). I also take the value- versus equal-weight lesson to heart and give it a demonstration on steroids in Hou, Xue, and Zhang (2019).

Alas, I do not find Fama's (1998) chance argument persuasive. Anomalies do not just randomly split between under- and over-reaction camps. The two types of anomalies are systematically different. To a theorist, the systematic pattern is exciting, because it indicates hidden economic law(s) to be discovered. The hidden law turns out to be the investment CAPM (a restatement of the NPV rule in Corporate Finance), as demonstrated in Hou, Xue, and Zhang (2015). The "overreaction" anomalies are all just different manifestations of the investment factor, and the "underreaction" anomalies are all just different manifestations of the Roe factor.

I do not find Fama and French's (1993, 1996) interpretation of risk factors for SMB and HML persuasive either. To their credit, the lack of a risk interpretation for momentum has stopped them from adding it into their factor model until 2018 (Fama and French, 2018). It is statistically correct to view SMB, HML, and perhaps even UMD as risk factors from the intertemporal CAPM and/or APT. However, the interpretation is on shaky economic grounds because size, book-to-market, and prior short-term returns are never modeled in the two theoretical frameworks. As such, the risk interpretation seems like a mere assertion.

This concern is why Hou, Xue, and Zhang (2015) interpret the *q*-factors only as common factors that summarize the cross-sectional variation of average stock returns. In particular, I find the concept of covariance superfluous. Yes, the consumption CAPM is all about covariance, but the investment CAPM is all about characteristics. If a characteristic is significant in cross-sectional regressions, its long-short factor is likely to earn a significant average return. And if a long-short factor earns a significant average return in the time series, its underlying characteristic is likely to be significant in cross-sectional regressions. As such, the *q*-factor model is simply a linear factor approximation to the nonlinear characteristics model of the investment CAPM.

Going from a characteristic to a factor is mostly mechanical, and vice versa. In particular, stock returns of firms with similar investment-to-assets tend to comove together because their investment returns are similar as a result of similar investment-to-assets. Stock returns of firms with similar Roe and expected growth tend to comove together because their investment returns are also similar for analogous reasons. Comovement is nothing mysterious.

More fundamentally, the investment CAPM advances a new perspective of "factors." In the consumption CAPM, factor models are linear approximations of the intertemporal marginal rate of substitution for the representative investor. Aggregate variables such as the growth rate of

industrial production, inflation rate, the default premium can be used to substitute out consumption, giving rise to the classic macroeconomic risk factor model of Chen, Roll, and Ross (1986). Because the consumption CAPM is in essence a macroeconomic model, factors are commonly perceived as aggregate, systematic sources of covariation. To the extent that size, book-to-market, and momentum are not modelled within the consumption CAPM, these factors have been (wrongfully, in my view) perceived as ad hoc, arising from "fishing" expeditions.

In contrast, the investment CAPM offers a new, microeconomic perspective of "factors." The comovement of stock returns among stocks with similar investment, profitability, and expected growth arises from the comovement of their similar investment returns. Characteristics-based factors are on as solid economic grounds in the supply theory of asset pricing as aggregate consumption growth in the demand theory of asset pricing. If one takes aggregation seriously, aggregate consumption growth is not even a factor. Neither are most other aggregate variables.

Security Analysis within Efficient Markets

Graham and Dodd (1934) define Security Analysis as "concerned with the intrinsic value of the security and more particularly with the discovery of discrepancies between the intrinsic value and the market price (p. 17)." Their philosophy is to invest in undervalued securities that are selling below the intrinsic value "justified by the facts, e.g., the assets, earnings, dividends, and definite prospects (p. 17)." Alas, the intrinsic value is not exactly identified. To protect against its estimation errors, Graham (1949) advocates the "margin of safety," i.e., investors only purchase a security when its market price is sufficiently below its intrinsic value.

EMH and Security Analysis have historically been viewed as diametrically opposite. On the one hand, the traditional view of academic finance, with the CAPM as its workhorse theory,

dismisses security analysis as pure luck, likens security analysts to astrologers, and recommends investors to passively hold only the market portfolio. Bodie, Kane, and Marcus (2017) maintain: "[T]he efficient market hypothesis predicts that most fundamental analysis is doomed to failure (p. 356)." In a recent interview with Bloomberg on November 5, 2019, Fama even labels equity research on Wall Street as "business-related pornography."

On the other hand, honoring the 50th anniversary of Graham and Dodd (1934), Warren Buffett (1984) showcases 9 famous investors and argues that their successful performance is beyond chance. Buffett goes on to say: "Our Graham & Dodd investors, needless to say, do not discuss beta, the capital asset pricing model or covariance in returns among securities. These are not subjects of any interest to them. In fact, most of them would have difficulty defining those terms (p. 7)." Buffett then mocks finance academics as out of touch with the real world: "Ships will sail around the world but the Flat Earth Society will flourish (p. 15)." Wall Street practitioners, not surprisingly, are overwhelmingly sympathetic to the behavioral view, and believe EMH to be a relic of the past. An old joke helps illustrate the schism between academics and practitioners. An asset manager asks an academic: "If you are so smart, why aren't you rich?" to which the academic replies: "If you are so rich, why aren't you smart?"

EMH is down in the dumps only because the consumption CAPM is a rundown dumpster truck. I have yet to meet an asset manager who even mentions the consumption CAPM, not even once, yet the consumption CAPM is virtually all we are allowed to talk about in academia (unless you're a behavioral economist). The investment CAPM once again changes the big picture. Recall the investment CAPM says: Discount rate = (profitability + expected investment costs) / investment costs. In the denominator, investment costs equal Tobin's q (marginal costs of investment equal marginal q). As such, the investment CAPM prescribes that to earn higher

expected returns, investors should buy stocks with high quality (measured as high profitability and high expected growth) at bargain prices. This prescription is exactly what Graham and Dodd (1934) have been saying and what Wall Street asset managers have been practicing for 85 years. Finally, after such a long exile, Security Analysis has found its rightful home in finance theory.

However, my treatment of Security Analysis differs from Graham and Dodd's (1934) in a fundamental way. Writing way, way before the arrival of equilibrium theory, Graham and Dodd largely have a constant discount rate in mind as the expected-return model. Their remarkable business acumen enables them to intuit their way to the ever-lasting investment truth of buying high quality stocks at bargain prices. Their monumental work predates academic finance by at least 4 decades. Indeed, in at late as the 1970s, the random walk hypothesis (with a constant discount rate) is still the workhorse theory for EMH.

In the 1980s and 1990s, the consumption CAPM rises up to meet Shiller's (1981) excess volatility challenge and moves the needle from a constant discount rate to time-varying expected returns as the workhorse theory in EMH. With the investment CAPM, I am trying to move the needle once again to cross-sectionally varying expected returns. Shiller attributes all excess volatility to predictable pricing errors against EMH, but the consumption CAPM attributes it to time-varying expected returns within EMH. Analogously, Graham and Dodd (1934) attribute the performance of security analysis to predictable pricing errors against EMH, but the investment CAPM attributes it to cross-sectionally varying expected returns, all within EMH.

Empirically, Hou, Mo, Xue, and Zhang (2019c) show that their q^5 model goes a long way toward explaining prominent security analysis strategies, including Frankel and Lee's (1998) intrinsic-to-market value, Piotroski's (2000) fundamental score, Greenblatt's (2005) "magic formula," Asness, Frazzini, and Pedersen's (2019) quality-minus-junk, Buffett's

Berkshire, Bartram and Grinblatt's (2018) agnostic analysis, as well as Penman and Zhu's (2014, 2018) expected-return strategies. Also, Hou et al. show that the latest factor models cannot fully explain Buffett's alpha and interpret the evidence as saying that discretionary, active management cannot be fully replaced by passive factor investing. Identifying sources of quality and quantifying their impact on expected returns leave plenty of room for active management.

Rational Expectations Economics

Make no mistake. The investment CAPM is the latest product from the Lucas-Sargent rational expectations economics. While I no longer believe that the end stage of economics is a Fortran program, the Lucas-Sargent teaching of microfoundation is deeply embodied in the investment CAPM. My Wharton theoretical training has given me a strong immune system against behavioral finance, despite being embedded in the hostile territory of the anomalies literature for 20 years. If I cannot write down an optimization-based model to explain a stylized fact, I don't understand the fact. A "model" with no optimization is just sticking labels to the fact to be explained. True to the nature of the anomalies literature, with my Rochester empirical training, I have also given life to the investment CAPM with the careful, empirical measurement in the Fama-French tradition. While there are still a few mopping-up operations left to do, the anomalies literature, which used to be a major embarrassment for rational expectations economics, is no more. On the contrary, I have turned it into a triumph of rational expectations. My macroeconomist compatriots can go on refining the all-important DSGE models, without worrying about all the fires of capital markets, as the investment CAPM has put them out, mostly.

I should clarify that my aggregation critique against the consumption CAPM applies to the specific context of anomalies in the cross section. For aggregate asset pricing, the consumption CAPM does well, although it remains to be seen to what extent aggregation would bite once the consumption CAPM is embedded into a full-fledged equilibrium model with production. Analogously, my aggregation critique does not apply, at least not directly, to the mainstream DSGE models in modern quantitative macroeconomics.

Challenges

While many open questions remain in the investment CAPM literature, due to space limitations, I only discuss what I perceive as the two most important challenges in this essay.

A Risky Mechanism of Momentum

Momentum is a success story for the investment CAPM. Recall from January 1968 to December 2018, UMD earns on average 0.64% per month (t-value = 3.73). However, its q-factor alpha is only 0.14% (t-value = 0.61). The Roe factor does all the heavy lifting, as UMD has a large Roe-factor loading of 0.9 (t-value = 5.85), while its loadings on the other 3 factors are insignificant. In the structural estimation of Goncalves, Xue, and Zhang (2019), the investment CAPM explains value and momentum simultaneously, and the "tug of war" between current investment and expected investment plays a key role in the model's performance.

Nevertheless, a major gap in our knowledge exists. What exactly are the risks underlying momentum? To answer this question, one needs more than factor regressions and Euler equation tests. Only fully specified quantitative theories are up to the task. Recall Zhang (2005) has tied the value premium to business cycle risks. Alas, I am aware that momentum, and equivalently,

the Roe factor premium are both significantly negative in that model. Also in partial equilibrium, Johnson (2002) ties momentum to expected growth and argues that expected growth is risky. Sagi and Seasholes (2004) argue that momentum winners have more growth options than momentum losers and that growth options are risky. An important, open question is how to combine Zhang's value with Johnson's and Sagi and Seasholes' momentum mechanisms in a unified framework. A unified model imposes internal consistency that is vital for theories. Li (2018) is the only exception that makes sense to me. More work is sorely needed.

Other Asset Classes

An advantage of the consumption CAPM, and more generally, the SDF framework, is that it can in principle be applied to different asset classes simultaneously. In contrast, the investment CAPM has so far been mostly applied to equity pricing. However, I caution that the consumption side's advantage of applying to different asset classes should not be taken too literally. After all, failures in explaining returns of different asset classes are definitely worse than failures in explaining just stock returns. Behavioral under- and overreaction apply to different asset classes (Asness, Moskowitz, and Pedersen, 2013). But sticking labels is no theory.

More important, any asset has suppliers, which must face certain tradeoffs in making optimal supply decisions. It seems straightforward to apply the investment CAPM to global stocks, country equity indices, corporate bonds, and real estates. Other asset classes such as currencies, government bonds, and commodities require additional, creative theorizing. The challenge is to cleanly separate the supply-side tradeoff from the SDF. Because of aggregation, to me, SDF is the source of all ills in asset pricing and should be avoided at all costs.

Conclusion

I am ready to answer the fundamental questions raised at the beginning of this essay. What explains all the consumption CAPM anomalies? Well, the consumption CAPM anomalies are the investment CAPM regularities, all of which conform to the NPV rule in Corporate Finance. Capital markets obey standard economic principles. Anomalies in fact indicate well functioning, efficient capital markets. The world makes sense! The consumption CAPM fails so badly because of the well known aggregation problem (Kirman, 1992). The pain of aggregation is likely manageable for aggregate asset pricing (and for DSGE models, unless you want to study wealth inequality). However, the pain is insurmountable for the cross section, which is in essence a microeconomic problem. And our ubiquitous representative investor is out of depth.

Despite its enormous, ever-lasting influence in practice, Graham and Dodd's (1934) Security Analysis has yet to find its rightful home in finance theory. We're blind to this parallel universe (otherwise known as practice) because of the consumption CAPM's single-minded, dogmatic focus on demand. Graham and Dodd are squarely on supply. And the NPV rule is the first place one would go to put the 2 and 2 together. Characteristics-based factors are linear approximations to the nonlinear investment return equation in the investment CAPM. Characteristic factors are on as solid theoretical grounds in the investment CAPM as aggregate consumption growth in the consumption CAPM. Taking aggregation seriously, aggregate consumption growth is not even a factor. Neither are all other macroeconomic factors.

Post-earnings-announcement drift persists for 50 years since Ball and Brown (1968) because it is part of expected returns, as predicted by the investment CAPM (the Roe factor). Why has there not been a coherent behavioral theory for 35 years since De Bondt and Thaler (1985)? Because such a theory likely doesn't exist. If a full menu of psychological biases gives

rise to underreaction, and another full menu to overreaction, we have an embarrassment of riches. A "theory" that explains everything (with no discipline) explains nothing.

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