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A Theory of Falling Growth and Rising Rents

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ABSTRACT

Growth has fallen in the U.S., while firm concentration and profits have risen. Meanwhile, labor's share of national income is down, mostly due to the rising market share of low labor share firms. We propose a theory for these trends in which the driving force is falling firm-level costs of spanning multiple markets, perhaps due to accelerating IT advances. In response, the most efficient firms (with higher markups) spread into new markets, thereby generating a temporary burst of growth. Because their efficiency is difficult to imitate, less efficient firms find markets more difficult to enter profitably and therefore innovate less. Eventually, due to greater competition from efficient firms, within-firm markups actually fall. Despite the increase in the aggregate markup and rents, firm incentives to innovate decline—lowering the long run growth rate.

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A data appendix is available at <http://www.nber.org/data-appendix/w26448>

1 Introduction

Recent studies have documented the following patterns in the U.S. economy over the past several decades:¹

1. Falling “long run” growth (interrupted by a temporary burst of growth)
2. Falling labor share due to rising revenue shares of low labor share firms
3. Rising firm concentration within industries at the national level

In this paper we construct a theory of endogenous growth with heterogeneous firms which potentially speaks to these facts. There are two main sources of heterogeneity in our model. The first is *product quality*, which differs across the product lines of a firm and improves endogenously through innovation and creative destruction. The second is *process efficiency*, which is time-invariant, differs across firms, and is common to all products lines of a firm. High process efficiency firms command a higher markup than low productivity firms, conditional on having the same product quality advantage over their competitors.

A possible source of persistent heterogeneity in process efficiency across firms is their organizational capital. Firms such as Walmart and Amazon have established successful business models and logistics that are evidently hard to copy. Both firms experienced considerable expansion into new geographic and/or product markets over the past two decades. Similarly, Amazon and Microsoft have acquired dominant positions in cloud storage and computing due to their logistical advantage over potential competitors. Such firms have achieved a level of process efficiency which is arguably harder to reverse engineer and build upon than quality, which may be more observable.

Our story is that the IT (Information Technology) wave in the 1990s has allowed high productivity firms to extend their boundaries — to expand over a

¹We discuss papers presenting evidence on these patterns in the next section.

wider set of product lines. We model the IT wave as a downward shift in the overhead cost $c(n)$ of running n product lines. This cost is assumed to be convex in n , which puts a brake on the quality innovation (creative destruction) efforts of high process efficiency firms. The downward shift in the overhead cost schedule will allow high productivity firms to expand to a higher fraction of lines. The expansion of high productivity firms fuels a temporary surge in aggregate productivity growth — both because they innovate to take over more markets (bringing quality improvements) and because they apply their superior process efficiency to those additional markets.

Since high productivity firms have higher markups and lower labor shares on average across their product lines, their expansion into more markets results in an increase in the aggregate markup and a reduction of the aggregate labor share. This is entirely driven by firm composition rather than within-firm changes. Within-firm markups can actually fall, as the quality leader on a product line is more likely to face a high process efficiency competitor. Competition from an efficient follower can limit the leader's markup whether the leader is a high or low process efficiency firm.

Thus, while the IT wave induces a burst of growth in the short run, in the long run the fall in overhead cost may lead to a slowdown in productivity growth. The expansion of high productivity firms into more lines eventually deters innovation because innovating on a line where the incumbent firm has high productivity yields lower profits. Both high and low productivity firms eventually curtail their efforts at creative destruction, knowing they will face stiffer competition. This can outweigh the positive direct effect of a downward shift in the overhead cost on R&D incentives, such that long run innovation and productivity growth may fall. A drop in long run growth leads to a lower pace of job reallocation, which is tied to creative destruction.

Under reasonable parameters, our story can explain a significant portion of the growth slowdown. More specifically, we choose parameter values to fit the pre-IT revolution period (1949–1995) on the level of concentration, productivity

growth, aggregate markup, the real interest rate, the intangible investment rate and the correlation across firms between their labor share and sales share. We then scale down the overhead cost to match the decline in the relative price of IT goods over 1996–2005. With such a decline in overhead costs, the model can generate half of the growth slowdown seen in recent years.

Most directly related to our paper are Akcigit and Ates (2019), Liu, Mian and Sufi (2019), and De Ridder (2019), who also study declining growth and rising concentration. Akcigit and Ates emphasize declining imitation rates, whereas Liu et al. consider the effect of declining interest rates as driving forces behind the productivity slowdown. In De Ridder the productivity slowdown arises when some firms become particularly efficient at reducing their marginal costs through intangible inputs, which discourages other firms from innovating. Our contribution is to develop a growth model with persistent firm heterogeneity which generates opposite trends for labor share and markups within versus across firms when IT improvements reduce overhead costs for all firms.

Our paper also relates to Hopenhayn, Neira and Singhania (2018) and Chatterjee and Eyigungor (2019), who study rising concentration; to recent papers on declining labor share such as Karabarbounis and Neiman (2013, 2018), Barkai (2016), Koh, Santaella-Llopis and Zheng (2016), Eggertsson, Mehrotra, Singh and Summers (2016), Martinez (2018), Farhi and Gourio (2018), and Kaymak and Schott (2018).

Kehrig and Vincent (2018) and Autor, Dorn, Katz, Patterson and Van Reenen (2019) look at labor share in U.S. Census data, while Baqaee and Farhi (2019) and De Loecker and Eeckhout (2017) estimate markups in Compustat firms. These papers decompose the evolution of the aggregate labor share (or markup) into within-firm and between-firm components. They find the dominant contributor to be the rising market share of low labor share (high markup) firms. We contribute to this literature by linking these trends to the slowdown in U.S. growth in recent decades.

The rest of the paper is organized as follows. Section 2 describes the empirical patterns documented by other studies that motivate our modeling effort. Section 3 lays out our model. In Section 4, we perform a steady state comparison to show that the model could explain a significant portion of the decline in long run growth. In Section 5, we solve for transition dynamics to demonstrate that a lower overhead cost can lead to a short run boost in productivity growth followed by a long run slowdown. Section 6 discusses extensions to the model and Section 7 concludes.

2 Stylized facts

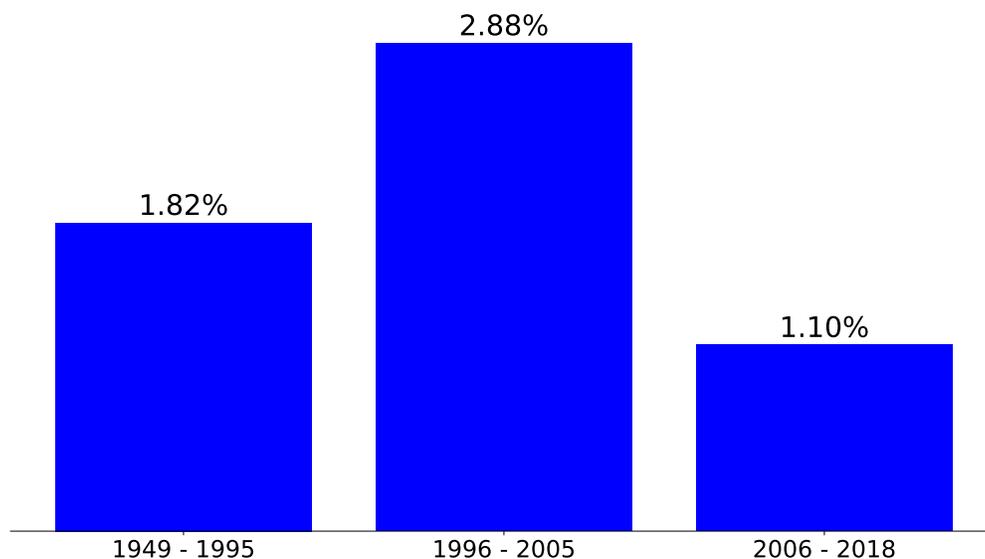
Fact 1: Falling “long run” growth (after a burst of growth) Figure 1 presents U.S. annual TFP growth from the Bureau of Labor Statistics (BLS). The BLS attempts to net out the contribution of both physical and human capital growth to output growth. The BLS sometimes subtracts contributions from R&D and other intellectually property investments; we consistently included this portion in residual TFP growth as part of what we are trying to explain.

The Figure shows growth accelerating from its 1949–1995 average of 1.8% per year to 2.9% per year from 1996–2005, before falling to just 1.1% per year from 2006–2018. Fernald, Hall, Stock and Watson (2017) and Bergeaud, Cette and Lecat (2016) argue that the recent slowdown is statistically significant and predates the Great Recession. Syverson (2017) and Aghion, Bergeaud, Boppart, Klenow and Li (2019) contend that the slowdown is real and unlikely to be fully attributable to growing measurement errors.

Fact 2: Falling labor share (mostly due to composition) According to the BLS, the aggregate U.S. labor share of output in the nonfarm business sector fell about 6 percentage points in the last two decades (see Figure OA-1 in [Online Appendix OA-A](#)).² Autor et al. (2019) find a declining labor share in sales

²As this is the business sector, it is not affected by the Rognlie (2016) critique that the rise of housing is exaggerating the decline in labor share. See also Cette, Koehl and Philippon (2019).

Figure 1: U.S. productivity growth rate



Source: BLS multifactor productivity series. We calculate yearly productivity growth rate by adding R&D and IP contribution to BLS MFP and then converting the sum to labor augmenting form. The figure plots the average productivity growth within each subperiod. The unit is percentage points.

in a number of Census sectors, but most sharply in manufacturing. Table 1 reproduces their statistics on the cumulative change in labor share for six Census sectors in recent decades. Finance is the exception, with rising labor share. In all six sectors the sales shares shifted to low labor share firms, so that the “between” firm component pushed labor share downward notably. Within-firm labor shares actually rose in all sectors but manufacturing. A complementary fact which Autor et al. (2019) document is that larger firms tend to have lower labor shares. Within four-digit industries, the semi-elasticity of firm labor share with respect to firm sales averages -1.10 across their six Census sectors. The relationship is negative within each sector.

In the business cycle literature, labor share is often used as an inverse measure of price-cost markups. See Karabarbounis (2014) and Bils, Klenow and Malin (2018). Thus one interpretation of falling labor share due to composition effects is that market share is shifting toward high markup firms.

Table 1: Cumulative change in labor share (ppt)

	1982–2012				1992–2012	
	MFG	RET	WHO	SRV	FIN	UTL
$\Delta \frac{\text{Payroll}}{\text{Sales}}$	-6.73	-0.85	-0.08	0.23	3.25	-1.83
Within firm	-1.71	4.39	4.66	1.73	4.92	0.37
Between	-4.54	-5.44	-4.59	-0.76	-0.75	-3.25

Source: Table 5 in Autor et al. (2019). This is a Melitz-Polanec decomposition of the change in the labor share. The entry and exit margin is not reported. The unit is percentage points. MFG, RET, WHO, SRV, FIN and UTL stand for the manufacturing, retail, wholesale, service, finance and utility sectors.

De Loecker and Eeckhout (2017) and Baqaee and Farhi (2019) argue for this interpretation, based on a broader measure of variable inputs that adds intermediates to labor costs for Compustat firms. A competing interpretation is that the elasticity of output with respect to capital has risen, in the aggregate, due to rising market share of high capital elasticity firms. Barkai (2016), Gutierrez and Philippon (2016, 2017), and Farhi and Gourio (2018) all argue against this interpretation and in favor of rising markups on the grounds that the investment rate and capital-output ratio have not risen.

Koh, Santaella-Llopis and Zheng (2016) and Traina (2018) argue that labor share has not fallen and markups have not increased if one adds intangibles investments such as R&D and marketing. These expenditures are arguably not part of variable costs, in which case their rise may be compatible with rising markups. Moreover, Autor et al. (2019) document falling payroll relative to sales; sales, unlike value added, should not be affected by whether intangibles are expensed or treated as part of value added.

Fact 3: Rising concentration Table 2, which is also based on Autor et al. (2019), presents the average cumulative change in top 4 or top 20 firm concentration measures in 4-digit NAICS. These results are, again, from

firm-level data in U.S. Census years. Across the six sectors, the top 4 firm shares increase from 0.8 to 2.3 percentage points per five-year period, while the top 20 firm shares increase between 0.9 and 3.3 percentage points per five-year period. Concentration increased the most rapidly in retail and finance, and slower in manufacturing and services.

The rise in concentration in Table 2 is at the national level. In contrast, Rossi-Hansberg, Sarte and Trachter (2018) and Rinz (2018) find that local concentration declined. One explanation for the diverging trends is that the largest firms grew by adding establishments in new locations.³ Figure 2 shows cumulative growth of the number of establishments per firm, by firm size bins, in the Business Dynamic Statistics from the Census Bureau. The red line is the growth of establishments for the largest firms. It shows that, between 1990 and 2014, the largest firms expanded by adding establishments. The average number of establishments rose for smaller firms too but not as quickly as for the largest firms. Cao, Sager, Hyatt and Mukoyama (2019) document a similar pattern in the Quarterly Census of Employment and Wages data, and Rinz (2018) documents increasing number of markets with at least one establishment belonging to a top 5 firm. To the extent that growth in the number of establishments is connected to growth in the number of products or markets, this evidence suggests that the rise in national concentration may not reflect an increase in market power of the largest firms.⁴

The rate at which large firms add establishment, moreover, may reflect the pace of their product innovation. Figure 3 shows the rate at which large firms increased employment through adding new establishments. This can be thought of as an employment-weighted entry rate. The largest firms experienced a burst of establishment entry precisely in the period when U.S. productivity growth accelerated. They experienced a decline in entry just as

³Hsieh and Rossi-Hansberg (2019) document the expansion of large U.S. firms into more locations in services, retail trade, and wholesale trade. Like us, they emphasize IT as the potential driving force.

⁴See Syverson (2019) for why concentration can be negatively related to market power.

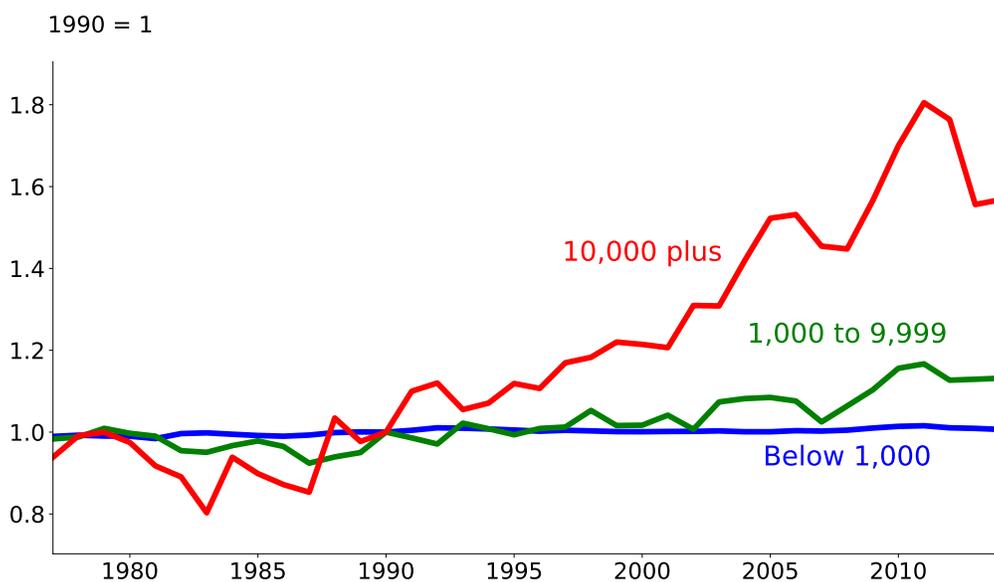
Table 2: Cumulative change in concentration (ppt)

	1982–2012				1992–2012	
	MFG	RET	WHO	SRV	FIN	UTL
Δ Top 4 firms sales share	6.0	14.0	4.7	4.5	7.2	4.6
Δ Top 20 firms sales share	5.2	16.3	10.1	6.1	13.3	4.7

Source: Table 1 of Autor et al. (2019). Averages across 4-digit industries, with the industries weighted by industry sales shares.

U.S. productivity growth declined. In what follows, we offer the IT revolution as a driver of this comovement between aggregate productivity growth and establishments entry rate of the largest firms.

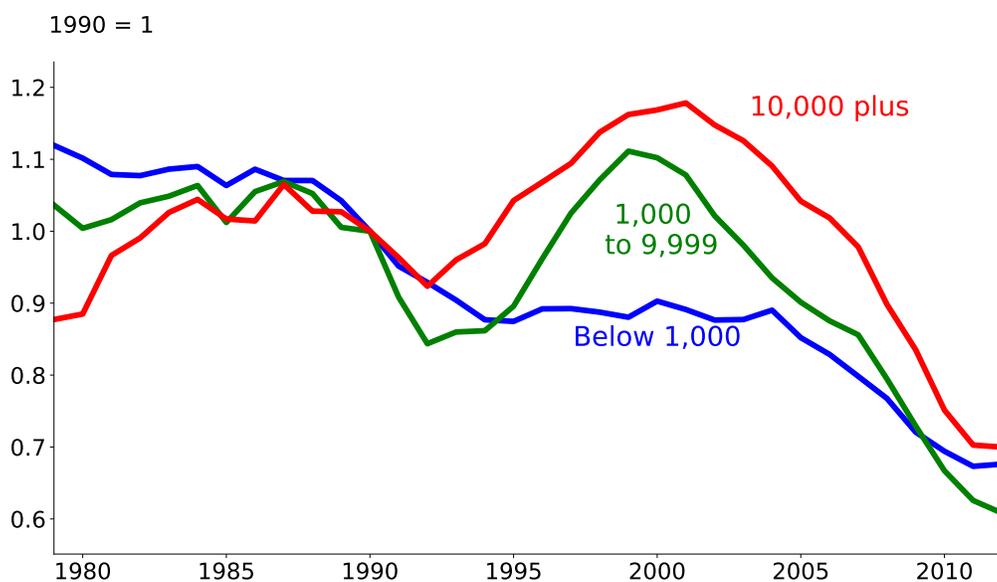
Figure 2: Establishments per firm by firm size



Source: U.S. Census Bureau Business Dynamic Statistics. The graph plots the number of establishments per firm relative to 1990 within employment bins.

IT as a driving force We focus on changes in IT as a possible driver of the patterns described above for four reasons. First, price declines for IT goods accelerated sharply for a decade from the mid-1990s to the mid-2000s. See

Figure 3: Employment weighted establishment entry rate by firm size



Source: U.S. Census Bureau's Business Dynamics Statistics. The figure shows job creation by establishment birth over total employment for different firm size bins. The lines represent 5-year centered moving average, relative to 1990.

Figure 4. This is in the middle of the period of rising concentration.

Second, Figure 5 displays the growth rate of multi-factor productivity for IT-producing, IT-intensive and non-IT-intensive industries, adapted from Fernald (2015).⁵ The figure shows a burst of growth for the IT-intensive sectors in early 2000s after a burst of growth for IT-producing sectors in the second half of 1990. In contrast, the non-IT-intensive sectors did not experience a burst of growth. When comparing the beginning and the end the series the overall productivity slowdown is also more pronounced for the IT-intensive sectors.

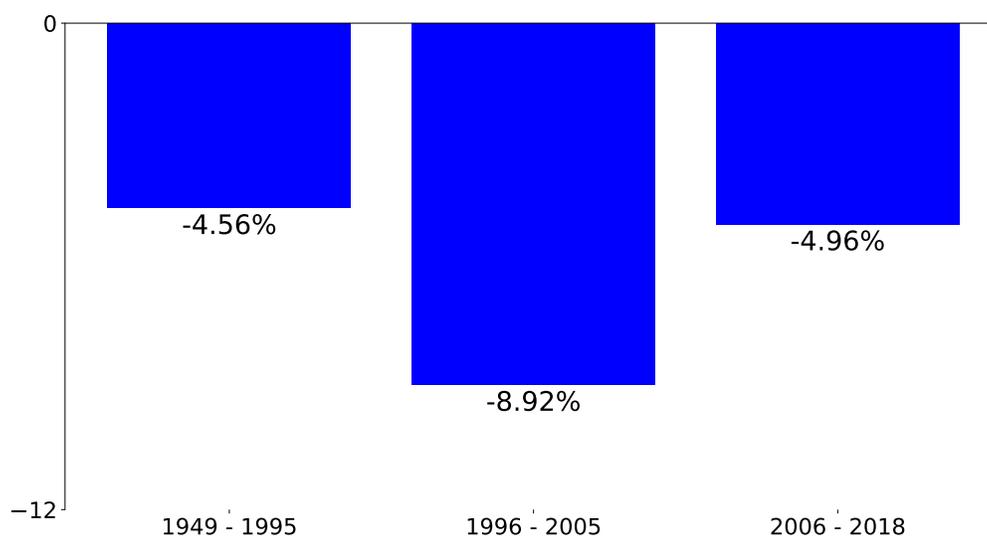
Third, using the same classification of sectors, we plot the average labor share respectively for IT producing, IT intensive and non-IT intensive sectors

⁵Industries called "IT producing" are computer and electronic, computer system design and publishing industries. All other industries are ranked based on the average value of their IT capital relative to value added over the years 1987 to 2016 and then split into two categories: IT intensive and non IT intensive such that the share of total value added in the two groups is roughly the same. We consider 3 digit sectors spanning the entire non-farm business sectors excluding finance and aggregate the MFP growth rates using value added weights.

in Figure 6. Except for a short spike in the early 2000s, we see that the labor share is declining in all cases, but the magnitude of this decline is particularly large for IT producing and IT intensive sectors.

Finally, Crouzet and Eberly (2019) and Lashkari, Bauer and Boussard (2019) document that bigger firms invest a higher share of their sales in intangibles and IT, respectively. Lower costs of IT seems to benefit larger firms more. The former evidence is for U.S. firms and the latter for French firms. Furthermore, Bessen (2019) provides evidence that industries with higher IT intensity experienced higher growth in the sales share of the largest firms.

Figure 4: Relative price of IT

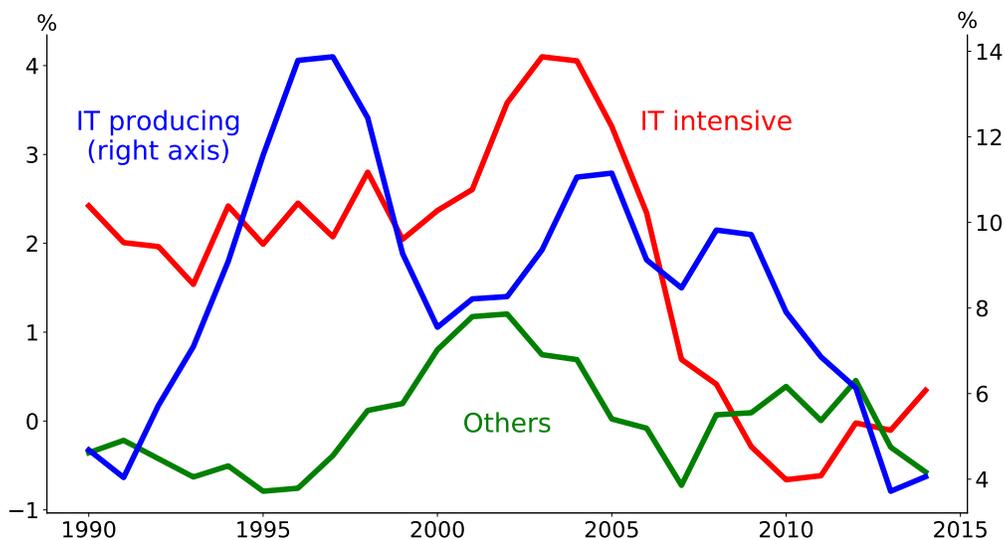


Source: BEA. % Change per year in the price of IT relative to the GDP deflator.

3 Model

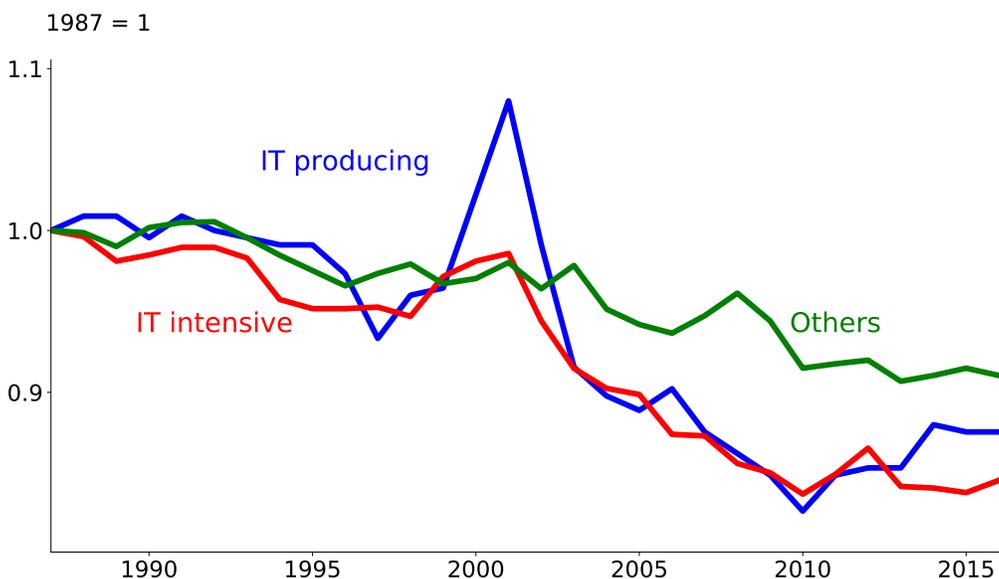
The above evidence on the opposite trends in between vs. within firm labor share as well as national vs. local concentration, asks for a model with persistent firm heterogeneity. Moreover, accounting for the observed burst and then slowdown in productivity growth requires an endogenous growth

Figure 5: Productivity growth by IT intensity



Source: Adapted from Fernald (2015) Figure 6A. % per year, 5-year centered moving average. MFP data come from the BLS multifactor productivity series. See [Online Appendix OA-B](#) for details and robustness.

Figure 6: Labor share by IT intensity



Source: Labor share is taken from the BLS production tables. IT groups are the same as in Figure 5. Labor share is normalized to 1 for each group in 1987. See [Online Appendix OA-B](#) for details and robustness.

theory. In this section we lay out a model which combines these two elements to speak to the facts in Section 2. The goal is to construct a parsimonious theory which has quantitative bite, can be generalized in various ways and ultimately can be used for policy analysis.

3.1 Preferences

Time is discrete and the economy is populated by a representative household who chooses a path of consumption C and wealth a to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t \log(C_t),$$

subject to

$$a_{t+1} = (1 + r_t)a_t + w_t L - C_t, \quad (1)$$

a standard no-Ponzi game condition and a given initial wealth level $a_0 > 0$. Here r is the interest rate, w the wage rate and L denotes the labor endowment that is inelastically supplied to the labor market.

The Euler equation resulting from household's optimization is given by

$$\frac{C_{t+1}}{C_t} = \beta(1 + r_{t+1}). \quad (2)$$

3.2 Production of final output

A final output good is produced competitively out of a unit interval of intermediate inputs according to the following Cobb-Douglas technology

$$Y = \exp\left(\int_0^1 \log[q(i)y(i)] di\right). \quad (3)$$

Here $y(i)$ denotes the quantity and $q(i)$ the quality of product i . This structure yields demand for each product i as

$$y(i) = \frac{YP}{p(i)}, \quad (4)$$

where we defined the aggregate price index

$$P \equiv \exp \left(\int_0^1 \log [p(i)/q(i)] di \right), \quad (5)$$

which we will in the following normalize to one in each period. We will next describe the market structure of the intermediate inputs.

3.3 Production and market structure for intermediate inputs

There are J firms indexed by j . We assume that J is “large” such that firms take the aggregate price index in the economy as given. Each firm j has the knowledge to produce quality $q(i, j) \geq 0$ in a specific market $i \in [0, 1]$. There are two sources of heterogeneity across firms: (i) heterogeneity in the firm-market specific quality $q(i, j)$ which evolves endogenously as a result of innovation; (ii) permanent heterogeneity in firm-specific process efficiency.

We first describe the heterogeneity in the process efficiency. There is a firm-specific level of process efficiency denoted by $\varphi(j)$. A firm with process efficiency $\varphi(j)$ can produce in any line i with the following linear technology

$$y(i, j) = \varphi(j) \cdot l(i, j), \quad (6)$$

where $l(i, j)$ is labor used by firm j to produce output $y(i, j)$ in product line i . We assume that the heterogeneity in process efficiency is persistent over time reflecting, e.g., differences in organizational capital that is hard to copy. This heterogeneity in process efficiency will translate into persistent differences in revenue TFP, labor income shares, and markups across firms.

The linear technology in (6) applies irrespective of the specific quality $q(i, j)$ at which firm j produces in line i . In addition to the heterogeneity in process efficiency, firms differ in their product quality. We will explain below how the firm and line specific quality changes endogenously due to innovations. But for the static firm problem here we regard the line-specific quality $q(i, j)$ of a firm in a period t as given. Labor is fully mobile such that the wage rate equalizes across firms. Hence, the marginal cost of firm j per unit of output in line i is given by $w/\varphi(j)$, or the marginal cost per quality-adjusted output in line i , $q(i, j)y(i, j)$, is equal to $\frac{w}{q(i, j)\varphi(j)}$.

3.4 Pricing

In each market all J firms engage in Bertrand competition. This implies that only the firm with the highest quality-adjusted productivity $q(i, j) \cdot \varphi(j)$ will be active in equilibrium in a given market. We denote the index of this leading firm in line i by $j(i)$ and the one of the second-highest quality-adjusted producer by $j'(i)$. Hence, the quality-adjusted productivity of the “leader” in line i is given by $q(i, j(i)) \cdot \varphi(j(i))$ whereas it is $q(i, j'(i)) \cdot \varphi(j'(i))$ for the second-best firm. Under Bertrand competition price setting of the leading firm is constrained by the second-best producer. The leader will set its quality-adjusted price equal to the quality-adjusted marginal cost of the second-best firm. Formally, we then have

$$\frac{p(i, j(i), j'(i))}{q(i, j(i))} = \frac{w}{q(i, j'(i)) \cdot \varphi(j'(i))}. \quad (7)$$

Note that the equilibrium price in line i depends on the process efficiency of both the leader and the second-best firm as well as the quality difference between them.

We define the markup in line i , $\mu(i, j(i), j'(i))$, as the price of a unit divided

by the marginal cost of the producer, which yields

$$\mu(i, j(i), j'(i)) \equiv \frac{p(i, j(i), j'(i))}{w/\varphi(j(i))} = \frac{q(i, j(i)) \cdot \varphi(j(i))}{q(i, j'(i)) \cdot \varphi(j'(i))}. \quad (8)$$

The markup of a product increases in the quality gap $\frac{q(i, j(i))}{q(i, j'(i))}$ as well as in the process efficiency gap $\frac{\varphi(j(i))}{\varphi(j'(i))}$ between the leading and the second-best firm. All else equal, the product level markup is increasing with the process efficiency of the leading firm $\varphi(j(i))$ and decreasing in the process efficiency of the second-best firm $\varphi(j'(i))$. Within a firm the markup differs across product lines but a firm with a higher process efficiency will charge on average a higher markup. Operating profits (before overhead) of the leader in line i are $Y \left(1 - \frac{1}{\mu(i, j(i), j'(i))}\right)$. This follows from the demand function (4) with P normalized to one.

3.5 Innovation and productivity growth

The quality distribution evolves endogenously over time as a result of innovation. Any firm j can engage in R&D activity to acquire a patent to produce in a new randomly drawn line at higher quality. More specifically, by investing $x_t(j) \cdot \psi_r Y_t$ units of final output in R&D in period t , $x_t(j)$ product lines are randomly drawn among the lines in which firm j is currently not actively producing. In such a randomly drawn line i the highest existing quality is multiplied by a factor $\gamma > 1$ and the innovating firm j obtains a perpetual patent to produce at this higher quality level from the next period $t + 1$ onward.

In each line the firm with the highest quality will face competition by a firm with lower quality by a factor γ . The initial distribution of quality levels across lines and firms is exogenously given.

We assume that a period is short enough such that no two innovations arrive in the same line in a given period. If we denote the innovation rate of firm j in

period t by $x_t(j)$ the aggregate rate of creative destruction is given by

$$z_{t+1} = \sum_{j=1}^J x_t(j), \quad (9)$$

i.e., for any given line an innovation arrives in $t + 1$ with probability z_{t+1} . These quality improvements due to creative destruction are the source of long-run growth in this model.

3.6 Boundary of the firm

Given the constant cost of acquiring a line through innovation and the fact that firms with higher process efficiency make higher expected operating profits in an additional line, more productive firms have a stronger incentive to invest in R&D. To prevent the firm with the highest productivity from taking over all lines we assume that firms have to pay a per-period overhead cost which is a convex function of the number of markets they span. More specifically, we assume a quadratic per-period overhead cost

$$\frac{1}{2}\psi_o n(j)^2 Y, \quad (10)$$

with $\psi_o > 0$, where $n(j)$ denotes the number of lines in which firm j owns the highest quality patent. The convexity of the overhead cost in $n(j)$, gives rise to a natural boundary of the firm. High productivity firms will typically operate more lines than low productivity firms, but no firm (type) will operate all lines.

It may be worthwhile to briefly compare our model to Klette and Kortum (2004) which serves as a benchmark in this literature. Here we assume a linear cost of innovating on a new line and convex overhead cost. Consequently, the (expected) marginal value of an additional line in a firm is decreasing in the number of lines, $n(j)$, and this diminishing marginal value defines a natural boundary of the firm. By contrast, Klette and Kortum (2004) assumes a convex cost of acquiring extra product lines through creative destruction, and a

non-diminishing value of additional lines. Our model shares some features with Luttmer (2011), in which more efficient firms endogenously expand into more products.

Our model allows us to do comparative statics with respect to the scalar ψ_o , which affects the boundary of firms without altering the technology for undertaking innovations. With IT improvements in mind, we lower ψ_o permanently (for all firms) and study its effect on concentration, labor share and growth during the transition as well as in the new steady state. Another difference with Klette and Kortum (2004), is that we assume that firms operate on a continuum of lines, so that the law of large numbers applies. One consequence is that there is no firm exit in our baseline model.⁶

3.7 Markups with binary process efficiency levels

For simplicity we assume in the following two types of firms. A fraction ϕ of all firms are of high process efficiency type φ_H whereas the remaining fraction $1 - \phi$ is of low process efficiency type φ_L . We denote the productivity differential by $\Delta \equiv \varphi_H/\varphi_L > 1$. We further assume $\gamma > \Delta$ so that the firm with the highest quality is the active leader irrespective of whether this firm is of high or low process efficiency.

Then, given the two process efficiency levels (high and low) there are four potential cases of markups and operating profits in a given line:⁷

1. A high productivity leader $\varphi(j(i)) = \varphi_H$ facing a high productivity second-best firm $\varphi(j'(i)) = \varphi_H$ in line i . In this case we have

$$\mu(i) = \gamma, \tag{11}$$

and the profits are $Y \left(1 - \frac{1}{\gamma}\right)$.

⁶We consider entry and exit in a model extension.

⁷To ease the notation we denote in the following the markup in line i , $\mu(i, j(i), j'(i))$, by $\mu(i)$.

2. A high productivity leader $\varphi(j(i)) = \varphi_H$ facing a low productivity second-best firm $\varphi(j'(i)) = \varphi_L$ in line i . In this case we have

$$\mu(i) = \Delta\gamma, \quad (12)$$

and profits of $Y \left(1 - \frac{1}{\Delta\gamma}\right)$.

3. A low productivity leader $\varphi(j(i)) = \varphi_L$ facing a high productivity second-best firm $\varphi(j'(i)) = \varphi_H$ in line i . In this case we have

$$\mu(i) = \frac{\gamma}{\Delta}, \quad (13)$$

and operating profits in this line i are $Y \left(1 - \frac{\Delta}{\gamma}\right)$.

4. A low productivity leader $\varphi(j(i)) = \varphi_L$ facing a low productivity second-best firm $\varphi(j'(i)) = \varphi_L$ in line i . In this case we have

$$\mu(i) = \gamma, \quad (14)$$

and profits are $Y \left(1 - \frac{1}{\gamma}\right)$.

3.8 Labor income shares

This simple version of the model abstracts from physical capital and therefore labor is the only factor in variable production. Furthermore, both R&D expenditure and overhead costs are denominated in final output and are treated as investment as opposed to intermediate inputs. These last two assumptions are made to avoid a mechanical effect of the firm size distribution (and overhead cost) and the overall level of R&D activity on the labor income share. Hence in this framework the aggregate labor income share is simply determined by the distribution of markups across lines.

Because of the Cobb-Douglas technology in final output production, the revenue from each product is equal to Y . Then, the total variable cost in a line i

is equal to $wl(i) = \frac{Y}{\mu(i)}$. Integrating both sides of the above equation over all i yields $wL = Y \int_0^1 \frac{1}{\mu(i)} di$. Dividing the last two equations by each other we get for the cost (or employment) share of product line i

$$\frac{l(i)}{L} = \frac{1}{\mu(i)} \frac{1}{\int_0^1 \frac{1}{\mu(i)} di}. \quad (15)$$

The relative cost per line, $l(i)/L$, is inversely proportional to the markup factor per line. This comes from revenue being equalized across lines due to the Cobb-Douglas technology.

Finally, the aggregate labor income share λ is given by the inverse of the average cost-weighted markup factor

$$\lambda \equiv \frac{wL}{Y} = \frac{1}{\int_0^1 \mu(i) l(i)/L di} = \int_0^1 \mu(i)^{-1} di. \quad (16)$$

Because there is no physical capital in the model the profit share and the labor income share add up to one. However, the aggregate labor share depends non-trivially upon the full distribution of markups across lines. This distribution is determined by the types of the leader and second-best firm across lines.

What about the labor income share at the firm level? A firm's labor income share in a line i is simply given by $\frac{1}{\mu(i)}$. Now consider firm j with $n(j)$ lines that faces in a fraction $h(j)$ of these lines a high type second-best firm and in the remaining fraction $1 - h(j)$ a low productivity second-best firm. If firm j is itself of high type, its overall labor income share is given by

$$\lambda_H(h(j)) = h(j) \frac{1}{\gamma} + (1 - h(j)) \frac{1}{\gamma \Delta}. \quad (17)$$

In contrast, if firm j is low type its overall labor income share is given by

$$\lambda_L(h(j)) = h(j) \frac{\Delta}{\gamma} + (1 - h(j)) \frac{1}{\gamma}. \quad (18)$$

Faced by the same share of high type competitors $h(j)$, high productivity firms

have a lower labor income share (as they can on average charge a higher markup). Hence the model will generate persistent differences in the labor income share across firms.⁸ However, since the composition of competitors $h(j)$ is endogenous the model is flexible enough to also generate changes in the labor share within firms over time.

3.9 Dynamic firm problem

There are two individual state variables in the firm problem: the number of lines firm j operates, $n(j)$, and the fraction of high productivity second-best producers, $h(j)$, firm j faces in these lines. Each firm then chooses how many new lines to innovate upon, x_t , to maximize the net present value of future profits. Let us denote the per-period profits after overhead of a high and low type firm relative to total output Y by π_H and π_L . Formally, we have

$$\pi_H(n(j), h(j)) = n(j) - \frac{n(j)h(j)}{\gamma} - \frac{n(j)(1-h(j))}{\gamma\Delta} - \frac{1}{2}\psi_o n(j)^2, \quad (19)$$

and

$$\pi_L(n(j), h(j)) = n(j) - \frac{n(j)h(j)\Delta}{\gamma} - \frac{n(j)(1-h(j))}{\gamma} - \frac{1}{2}\psi_o n(j)^2. \quad (20)$$

These are profits divided by output Y_t , i.e., they only depend on the individual states $n(j)$ and $h(j)$ and are otherwise time invariant. Letting S_t denote the aggregate fraction of lines operated by high productivity firms, the problem of a firm of type $k = H, L$ can be written as

$$V_{0,k} = \max_{\{x_t, n_{t+1}, h_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} Y_t [\pi_k(n_t, h_t) - x_t \psi_r] \prod_{s=0}^t \left(\frac{1}{1+r_s} \right) \quad (21)$$

subject to

$$n_{t+1} = n_t(1 - z_{t+1}) + x_t, \quad (22)$$

⁸See Hsieh and Klenow (2009) and David and Venkateswaran (2019) for evidence on persistent difference in revenue per worker.

$$h_{t+1}n_{t+1} = h_t n_t (1 - z_{t+1}) + S_t x_t, \quad (23)$$

and a given initial n_0 and h_0 . For completeness there are also non-negativity constraints $x_t \geq 0$. The constraint (22) states that the number of product lines of a firm tomorrow is equal to the newly added lines x plus the number of lines today times one minus the rate of creative destruction in the economy, z . The second constraint (23) states that the number of lines in which the firm faces tomorrow a high type second-best firm is equal to the number of such lines today times $1 - z$ plus the number of newly added lines times the aggregate fraction of lines currently operated by high type firms S . The firm takes the path of output Y_t , the interest rate r_t , the rate of creative destruction z_{t+1} , and the aggregate fraction of lines operated by high productivity firms S_t as given.

3.10 Market clearing and resource constraints

We close the model with the following market clearing conditions that hold each period. First, final output will be used for consumption, C , total overhead cost, O , and total R&D expenditure, Z , or formally

$$Y = C + O + Z, \quad (24)$$

where

$$O = \sum_{j=1}^J \frac{1}{2} \psi_o n(j)^2 Y \text{ and } Z = \sum_{j=1}^J x(j) \psi_r Y.$$

Labor is used as a variable input by the producer of different intermediate inputs. Labor and asset market clearing conditions imply

$$L = \sum_{j=1}^J \int_0^1 l(j, i) di \text{ and } \sum_{j=1}^J V_t(j) = a_t,$$

where $l(j, i)$ denotes labor used by firm j that operates line i .

In addition, we have the equations defining the aggregate share of lines

operated by high types and an accounting equation that states that all lines are operated by some firm, namely:⁹

$$S_t = \sum_{j=1}^{\phi J} n_t(j) \text{ and } 1 = \sum_{j=1}^J n_t(j). \quad (25)$$

Finally, there is an equation that relates output to the distribution of process efficiency, quality levels and markups

$$Y_t = Q_t \frac{\varphi_L \Delta^{S_t} \exp \left[- \int_0^1 \log (\mu_t(i)) di \right]}{\int_0^1 (\mu_t(i))^{-1} di} L, \quad (26)$$

where $Q_t = \exp \left[\int_0^1 \log (q_t(i, j)) di \right]$ denotes the “average” quality level.

An equilibrium in this economy is then a path of allocations and prices that jointly solve the household’s problem, the firms’ problems, and is consistent with the market clearing and accounting equations stated above.

There is no free entry and the number of firms is fixed. Hence firms’ profits from selling at a markup over marginal cost may exceed the total investments in R&D and overhead cost. We call such profits after R&D investment and overhead cost “rents”.

Since output is a function of the full distribution of markups across product lines the equilibrium path is in general a function of the initial joint distribution of product lines $n(j)$ and level of competition $h(j)$ across firms. We can assume that all firms of the same type $k = H, L$ start out with the same level of $n_0(j)$ and $h_0(j)$.¹⁰ Since the law of large number applies firms of the same type will then be identical along the entire equilibrium path and therefore only two firm problems — one for a high type and one for a low type — need to be solved. The aggregate state vector can then be summarized by S , and the shares of high second-best firms h_H and h_L in lines operated by high and low productivity firms.

⁹Here we assume that the high productivity type firms are indexed by $j = 1, 2, \dots, \phi J$.

¹⁰This assumption will automatically be fulfilled if the economy starts initially in steady state.

With the two “representative” types of firms, output can be expressed in terms of these aggregate state variables (S, h_L, h_H) and the level of average quality Q . We have

$$\exp \left[- \int_0^1 \log (\mu_t(i)) di \right] = \Delta^{(1-S_t)h_{Lt} - S_t(1-h_{Ht})} / \gamma$$

and for the aggregate labor share

$$\int_0^1 (\mu_t(i))^{-1} di = \frac{1}{\gamma} \left[S_t h_{Ht} + (1 - S_t)(1 - h_{Lt}) + S_t(1 - h_{Ht}) \frac{1}{\Delta} + (1 - S_t)h_{Lt}\Delta \right].$$

As a consequence, aggregate productivity can be expressed as

$$\begin{aligned} \frac{Y_t}{L} &= Q_t \cdot \varphi_L \Delta^{S_t} \cdot \frac{\Delta^{(1-S_t)h_{Lt} - S_t(1-h_{Ht})}}{S_t h_{Ht} + (1 - S_t)(1 - h_{Lt}) + S_t(1 - h_{Ht}) \frac{1}{\Delta} + (1 - S_t)h_{Lt}\Delta} \\ &= \frac{Q_t y(S_t, h_{Ht}, h_{Lt})}{L}. \end{aligned} \quad (27)$$

Aggregate labor productivity is the product of three terms. The first term Q_t captures the effect of “average quality”. The second term captures the aggregate level of process efficiency. If $S_t = 0$ the productivity is all determined by the low type φ_L , in contrast if $S_t = 1$ the productivity is all determined by $\varphi_H = \varphi_L \Delta$. Finally, the third term, which we call allocative efficiency, captures the average distortion due to markup dispersion. If $S_t = h_{Ht} = 1$ or $S_t = h_{Lt} = 0$ this term is equal to 1 (no dispersion of markups since all markups are equal to γ in all lines). In all other cases this third term is smaller than one.

In Section 5 we characterize and numerically solve for the transition path of the economy. However, before that we focus on the steady state this economy converges to as time goes to infinity. We will show below that this steady state takes a very tractable functional form and can be solved analytically. We then discuss how a permanent drop in ψ_o (triggered by improvements in IT) affects market concentration, labor income shares (within firms as well as aggregate), and productivity growth in the long run.

3.11 Steady state definition

We define a steady state equilibrium in the following way:

Definition 1 *A steady state is an equilibrium path along which the interest rate and the gross growth rate of output remain constant, equal to r^* and g^* , and along which a constant fraction of lines, S^* , is provided by high productivity producers.*

In a steady state all high productivity firms have the same constant number of products $n(j)^* = n_H^*$ whereas all low productivity firms have a different number of products $n(j)^* = n_L^*$. For the number of lines within firm to be constant, the R&D activity of each firm must be proportional to its number of products, i.e., $x(j)^* = n(j)^* z^*$, where z^* is the aggregate rate of creative destruction in steady state. Since all firms draw new lines from a stationary distribution, they all face the same share of high productivity second-best firms in their lines, i.e.,

$$h(j)^* = S^*, \forall j. \quad (28)$$

As the markup distribution is stationary in steady state, aggregate output Y_t is proportional to average quality Q_t (see (26)). Consequently, we have

$$\frac{Y_{t+1}}{Y_t} = \frac{Q_{t+1}}{Q_t} = \gamma^{z^*} \equiv g^*. \quad (29)$$

Finally, since total overhead, O , total R&D expenditure, Z , all grow at the same gross rate g^* also consumption has to grow at this rate g^* (see (24)). Then, the Euler equation determines the steady state interest rate as

$$r^* = \frac{g^*}{\beta} - 1. \quad (30)$$

Next, we show that solving for the steady state boils down to solving for the quadruple S^*, n_L^*, n_H^* , and z^* .

3.12 Steady state characterization

Let us denote by v the value of a firm relative to total output, i.e., $v \equiv V/Y$. With $h(j)^* = S^*$, (19) and (20) yield for the period profits of high and low type firms (relative to total output)

$$\pi_H(n, S^*) = n \left(1 - \frac{S^*}{\gamma} - \frac{1 - S^*}{\gamma \Delta} \right) - \frac{1}{2} \psi_o n^2, \quad (31)$$

and

$$\pi_L(n, S^*) = n \left(1 - \frac{S^* \Delta}{\gamma} - \frac{1 - S^*}{\gamma} \right) - \frac{1}{2} \psi_o n^2. \quad (32)$$

The number of products per firm n becomes the only individual state variable in the firm problem so that we can write $v_k = v_k(n)$, $k = H, L$. The high productivity firms then solve the Bellman equation

$$v_H(n) = \max_{n' \geq n(1-z^*)} \{ \pi_H(n, S^*) - (n' - n(1-z^*)) \psi_r + \beta v_H(n') \}, \quad (33)$$

where we denote its solution as $n' = f_H(n)$.

Similarly, all low productivity firms solve

$$v_L(n) = \max_{n' \geq n(1-z^*)} \{ \pi_L(n, S^*) - (n' - n(1-z^*)) \psi_r + \beta v_L(n') \}, \quad (34)$$

and we denote the solution as $n' = f_L(n)$.

The two accounting equations (25) become in steady state

$$S^* = n_H^* \phi J, \quad (35)$$

and

$$n_H^* \phi J + n_L^* (1 - \phi) J = 1. \quad (36)$$

Finally, we must have

$$n_H^* = f_H(n_H^*), \quad n_L^* = f_L(n_L^*). \quad (37)$$

These equations fully characterize the steady state. The two dynamic programming problems (33) and (34) are very simple since π_H and π_L are quadratic functions of n (see (31) and (32)).

In the following we will focus on an interior steady state where $S^* \in (0, 1)$ and $z^* \in (0, 1)$. When such a steady state exists, the policy and value functions can even be characterized in closed form.¹¹ Next we impose parameter restrictions that ensure the existence of an interior steady state solution.

Assumption 1 *To ensure an interior steady state where both firm types are active and long-run growth is positive, we assume*

$$\frac{\Delta - 1}{\gamma} < \frac{\psi_o}{\phi J}, \quad (38)$$

and

$$0 < \frac{1}{\psi_r} - \frac{1 - \beta}{\beta} - \frac{1}{\psi_r} \frac{\frac{\psi_o}{J} + \frac{1}{\gamma}}{1 - (1 - \phi)\phi \frac{(\Delta - 1)^2 J}{\gamma \Delta \psi_o}} < 1. \quad (39)$$

The parameter restriction (38) ensures that the low type firms are active in steady state, i.e., it ensures $S^* < 1$.¹² This restriction is fulfilled as long as the productivity differential Δ or the number of high productivity firms ϕJ are not “too” large.

The second parameter restriction (39) ensures the existence of an interior solution where $0 < z^* < 1$. It is fulfilled as long as ψ_r relative to β is neither too small nor too large. Note that the parameters ψ_r and β that affect this intertemporal trade-off do not enter restriction (38).

The next two propositions characterize the interior steady state solution and

¹¹Let us denote the marginal steady state profits per line before overhead cost of H and L firms by $\tilde{\pi}_H = 1 - S^*/\gamma - (1 - S^*)/(\Delta\gamma)$ and $\tilde{\pi}_L = 1 - \Delta S^*/\gamma - (1 - S^*)/\gamma$. Then, for any $n \leq \bar{n}_k/(1 - z^*)$, where $\bar{n}_k \equiv \left(\tilde{\pi}_k + (1 - z^*)\psi_r - \frac{\psi_r}{\beta}\right)/\psi_o$ we have the policy function $f_k(n) = \bar{n}_k$ and the value function $v_k(n) = \tilde{\pi}_k n - \frac{1}{2}\psi_o n^2 - \psi_r(\bar{n}_k - (1 - z^*)n) + \beta(\tilde{\pi}_k \bar{n}_k - \frac{1}{2}\psi_o \bar{n}_k^2 - \psi_r z^* \bar{n}_k)/(1 - \beta)$, for $k = H, L$. See Appendix A for details.

¹²With $\frac{\Delta - 1}{\gamma} \geq \frac{\psi_o}{\phi J}$ there exists a trivial steady state with $n_L^* = 0$, $n_H^* = 1/(\phi J)$, $S^* = 1$, and $z^* = (1 - 1/\gamma - \psi_o/(\phi J))/\psi_r + 1 - 1/\beta$, where $0 < (1 - 1/\gamma - \psi_o/(\phi J))/\psi_r + 1 - 1/\beta < 1$ needs to be imposed to ensure that the high type firms invest strictly positive amounts and that the rate of creative destruction is less than 100%, i.e., $z^* \in (0, 1)$.

prove that Assumption 1 is sufficient for the existence of such a steady state.

Proposition 1 *If an interior steady state exists, it is given by a quadruple (n_H^*, n_L^*, S^*, z^*) that fulfills*

$$\phi J n_H^* = S^* \text{ and } (1 - \phi) J n_L^* + \phi J n_H^* = 1, \quad (40)$$

as well as the following research arbitrage equations for high and low productivity firms respectively

$$\psi_r = \frac{1 - S^*/\gamma - (1 - S^*)/(\gamma\Delta) - \psi_o n_H^*}{1/\beta - 1 + z^*}, \quad (41)$$

$$\psi_r = \frac{1 - S^*\Delta/\gamma - (1 - S^*)/\gamma - \psi_o n_L^*}{1/\beta - 1 + z^*}. \quad (42)$$

Proof. We have by definition $S^* \in (0, 1)$ in an interior steady state. This implies that n_k^* and x_k^* are positive for $k = H, L$. Therefore, both firm types' policy function satisfy the first-order condition. For the high type Bellman equation the first-order condition is simply

$$\psi_r = \beta \frac{\partial v_H(n')}{\partial n'}. \quad (43)$$

Using the envelope theorem we have

$$\frac{\partial v_H(n')}{\partial n'} = 1 - S^*/\gamma - (1 - S^*)/(\gamma\Delta) - \psi_o n' + (1 - z^*)\psi_r.$$

Then using the fact that in steady state $n' = n_H^*$ yields the research arbitrage equation of the high type firm. The research arbitrage equation of the low type firm is derived in an analogous way. ■

The intuition for the two research arbitrage equations is straightforward: the optimality condition states that the marginal cost of innovating in a line, ψ_r is in steady state equal to the marginal (expected) value of having an additional line. This marginal value consists in the case of the high type firm of the marginal

profit $1 - S^*/\gamma - (1 - S^*)/(\gamma\Delta)$ minus the marginal overhead cost $\psi_o n_H^*$. This marginal value is divided by the denominator $1/\beta - 1 + z^*$ because there is time discounting and because there is a probability z^* of losing the additional line again in each future period.

Equations (40)–(42) are four equations in the four unknowns (n_H^*, n_L^*, S^*, z^*) that can be solved explicitly. We use them to derive conditions that guarantee interiority and solve for all the other endogenous variables.

Proposition 2 *Assumption 1 implies that the steady state is interior and is characterized by Proposition 1. Furthermore, this interior steady state has the following properties:*

(i) *The share of lines operated by high productivity firms is equal to*

$$S^* = \frac{\frac{1}{1-\phi} + \frac{\Delta-1}{\gamma\Delta} \frac{J}{\psi_o}}{\frac{1}{(1-\phi)\phi} - \frac{(\Delta-1)^2}{\gamma\Delta} \frac{J}{\psi_o}}, \quad (44)$$

and the rate of creative destruction is given by

$$z^* = \frac{1}{\psi_r} - \frac{1-\beta}{\beta} - \frac{1}{\psi_r} \frac{\frac{\psi_o}{J} + \frac{1}{\gamma}}{1 - (1-\phi)\phi \frac{(\Delta-1)^2}{\gamma\Delta} \frac{J}{\psi_o}}. \quad (45)$$

(ii) *High productivity firms operate more lines than low productivity firms, i.e.,*

$$n_H^* > n_L^*. \quad (46)$$

(iii) *The labor income share of a high type firm is given by*

$$\lambda_H^* = S^* \frac{1}{\gamma} + (1 - S^*) \frac{1}{\gamma\Delta}, \quad (47)$$

which is strictly smaller than the labor income share of a low type firm

$$\lambda_L^* = S^* \frac{\Delta}{\gamma} + (1 - S^*) \frac{1}{\gamma}. \quad (48)$$

Finally, the aggregate labor income share is given by

$$\lambda^* = S^* \lambda_H^* + (1 - S^*) \lambda_L^*. \quad (49)$$

Proof. Replacing n_H^* and n_L^* in (41) and (42) by $S^*/(\phi J)$ and $(1 - S^*)/(J(1 - \phi))$, respectively and solving the two equations for S^* and z^* yields the unique solution in part (i). Note that restriction (38) ensures $S^* < 1$ and restriction (39) ensures $0 < z^* < 1$. Finally, note that $S^* > 0$ is always guaranteed since (38) implies $\frac{\psi_o}{\phi J} > \frac{\Delta-1}{\gamma} > \frac{\Delta-1}{\gamma} \frac{\Delta-1}{\Delta} (1 - \phi)$, as $\frac{\Delta-1}{\Delta} (1 - \phi) < 1$, i.e., Assumption 1 is indeed sufficient to ensure the existence of an interior steady state.

For part (ii), by combining (41) and (42), the difference in the number of products can be expressed as

$$n_H^* - n_L^* = \frac{S^*(\Delta - 1)^2}{\gamma \Delta \psi_o} + \frac{\Delta - 1}{\gamma \Delta \psi_o} > 0.$$

The labor income shares follow from (17), (18) and (28). This proves part (iii). ■

In steady state S^* can be viewed as a summary statistic of market concentration whereas z^* pins down the long-run growth rate of the economy. It is worthwhile to note that all the endogenous steady state values only depend on the ratio $\frac{\psi_o}{J}$ and not on the individual level of ψ_o or J .

The intuition for property (ii) of Proposition 2 is that high process efficiency firms can (on average) charge higher markups. Consequently their incentive to undertake R&D is higher and they run into a steeper area of the convex overhead cost, i.e., operate in steady state more line than low process efficiency firms. A corollary of this is that we have $S^* > \phi$ since high productivity firms have larger sales than low productivity firms.

High productivity firm will also differ in employment, but the employment difference is smaller than the sales difference because high productivity firms charge higher markups (see part (iii) of Proposition 2).

3.13 Steady state effects as ψ_o decreases

In this section we consider how the steady state changes following a permanent reduction in the overhead cost ψ_o . We are particularly interested in the changes in the following endogenous variables: (i) market concentration, S^* , (ii) the labor income share at the aggregate level as well as within firms, and (iii) the long-run growth rate.

Proposition 3 *Concentration S^* increases monotonically as ψ_o decreases.*

Proof. Taking derivatives of (44) with respect to ψ_o yields

$$\frac{\partial S^*}{\partial \psi_o} = - \frac{[1 + \phi(\Delta - 1)] \frac{\Delta-1}{\gamma\Delta} \frac{J}{\psi_o^2(1-\phi)\phi}}{\left(\frac{1}{(1-\phi)\phi} - \frac{(\Delta-1)^2}{\gamma\Delta} \frac{J}{\psi_o}\right)^2} < 0.$$

■

The intuition that a fall in ψ_o increases S^* is the following: with a lower ψ_o a larger size gap $n_H^* - n_L^*$ is needed to yield the same difference in the marginal overhead cost between high and low productivity firms. Consequently, high process efficiency firms will operate more lines as ψ_o decreases whereas low productivity firms shrink in size; therefore market concentration goes up.

In the next proposition we turn to the labor income shares.

Proposition 4 *As ψ_o decreases (i) the labor income share within firms increases, (ii) the reallocation of market shares goes in the opposite direction, (iii) the aggregate labor income share increases (decreases) in ψ_o if initial S^* is larger (smaller) than 1/2.*

Proof. For the within part note that both (47) and (48) are monotonically increasing in S^* (and S^* increases as ψ_o falls as demonstrated in Proposition 3). The reallocation effect is simply that, as S^* increases, sales share of high productivity firms goes up and these firms charge on average higher markups and have a lower labor income share than low productivity firms (see (49)). For

the overall effect we obtain from (49)

$$\frac{\partial \lambda^*}{\partial S^*} = \lambda_H^* + S^* \frac{\Delta - 1}{\gamma \Delta} - \lambda_L^* + (1 - S^*) \frac{\Delta - 1}{\gamma}.$$

Replacing the expression for λ_H^* and λ_L^* by (47) and (48) and simplifying gives

$$\frac{\partial \lambda^*}{\partial \psi_o} = \frac{\partial \lambda^*}{\partial S^*} \frac{\partial S^*}{\partial \psi_o} = \frac{(\Delta - 1)^2}{\gamma \Delta} (1 - 2S^*) \frac{\partial S^*}{\partial \psi_o}.$$

Since S^* is decreasing in ψ_o (see Proposition 3) this implies that the aggregate labor income share decreases as ψ_o falls if and only if $S^* > 1/2$. ■

The model thus makes very sharp predictions about the labor income shares at the aggregate vs. micro level. As S^* increases due to the drop in ψ_o , all firms are more likely to face a high productivity firm as second-best competitor on any given line. As a consequence *within* firm the labor income share *increases* (see (47) and (48)) and the markup decreases. However, there is sales reallocation across firms that goes the opposite direction. As S^* increases the high productive firms with a lower labor income share expand and the low productivity firms contract. This between firm effect pushes the aggregate labor income share downwards. As emphasized in Section 2 these within and between firm effects that go in opposite directions are a very salient feature of the U.S. micro data.

Whether the within or between firm effect on the labor share dominates depends on the initial level of S^* . In this simple model the aggregate labor income share falls as ψ_o decreases if and only if $S^* > 1/2$.

Finally, we analyze the impact of lower ψ_o on the long-run growth rate.

Proposition 5 *We have $\partial z^*/\partial \psi_o > 0$ such that long-run growth decreases as ψ_o falls if and only if*

$$\frac{J(\Delta - 1)^2}{\gamma \Delta \psi_o} \left(\frac{J}{\gamma \psi_o} + 2 \right) > \frac{1}{\phi(1 - \phi)}. \quad (50)$$

Proof. Taking derivatives of (45) with respect to ψ_o gives

$$\frac{\partial z^*}{\partial \psi_o} = \frac{2\phi(1 - \phi) \frac{(\Delta - 1)^2}{\gamma \Delta \psi_o} + \phi(1 - \phi) \frac{(\Delta - 1)^2}{\gamma \Delta \psi_o} \frac{J}{\gamma \psi_o} - \frac{1}{J}}{\psi_r \left(1 - (1 - \phi) \phi \frac{(\Delta - 1)^2}{\gamma \Delta} \frac{J}{\psi_o} \right)^2}. \quad (51)$$

This expression is positive if and only if (50) holds. ■

The long-run growth rate is affected by a drop in ψ_o in two ways. First, there is a direct positive effect on growth: at a given S^* , a lower overhead cost raises the marginal value of operating an additional line and therefore stimulate R&D investment and growth. However, there is a second general equilibrium effect that goes in the opposite direction. As ψ_o decreases S^* rises. This reduces the expected markup in an additional line (as the probability of facing a high productivity second-best firm went up). This general equilibrium effect decreases the incentive to undertake R&D and consequently long-run growth can potentially fall as ψ_o decreases. Whether the direct or indirect effect dominates depends on the precise parameter values. Proposition 5 also states the parameter space that guarantees that long-run growth falls as ψ_o decreases. The restriction is basically fulfilled as long as $\frac{\psi_o}{(\Delta - 1)J}$ is not too large.¹³ We show that this restriction tends to be fulfilled for simple calibrations of our model.

Overall, qualitatively our theory can generate a productivity slowdown, rising concentration, and opposite changes in the labor income shares within firms and between firms as the outcome of a drop in ψ_o . The next step is to gauge the quantitative size of these effects in a simple calibration. This we will undertake in Section 4.

¹³Restriction (50) is consistent with Assumption 1 for a non-empty set of parameters as long as $1/\phi + 2(\Delta - 1) > \Delta/(1 - \phi)$.

4 Calibration

In this section, we confirm that the theory in Section 3 indeed predicts a significant productivity slowdown for plausible parameter values. Such a productivity slowdown should be accompanied by a decreasing rate of creative destruction and consequently by less churning in the labor market as well as a fall in the interest rate.

While the Cobb-Douglas model is useful for illustration, it predicts that all products have the same market share regardless of quality and productivity. Hence to be a bit more quantitative we calibrate a more general model with CES production and CRRA utility function. We lay out the general model in Appendix B. With a constant elasticity of substitution $\sigma > 1$, products with higher quality or productivity have higher market shares. Also, the price setting of a high productivity firm facing a low productivity second-best firm may no longer be constrained and such a firm instead may simply charge the monopoly markup $\frac{\sigma}{\sigma-1}$. In this event, the labor share within high productivity firms will decrease less as ψ_o decreases.

We assess the quantitative importance of the overhead cost mechanism by comparing steady states where the only difference is the parameter ψ_o . We define the initial steady state period as 1949–1995 and the new steady state period as 2006–2018. We calibrate six parameter values in the model to match six moments (in the initial period or the subset with available data, where possible). We then vary ψ_o to match the decline in the relative price of IT goods. We evaluate the fit of the model by comparing the model to untargeted data moments on changes in concentration, productivity growth, aggregate labor share, and the intangible investment share.¹⁴

The six moments that we target in our calibration are: 1) top 10% concentration (share of sales going to the largest 10% of firms) within industries over 1987–1992 from Autor et al. (2019); 2) the average annual rate of

¹⁴We map R&D plus overhead costs in the model to intangible investment in the data.

productivity growth over 1949–1995 from the BLS MFP dataset; 3) the employment-weighted average of industry markups Hall (2018) estimated over 1988–2015; 4) the real interest rate from Farhi and Gourio (2018) for 1980–1995; 5) the intangible investment share of output from Corrado et al. (2012) for 1995; and 6) the semi-elasticity of firm labor share with respect to firm sales within four-digit industries from Autor et al. (2019)¹⁵.

We match the initial steady state growth rate and the level of the aggregate markup exactly, but give equal weights to all other moments since we do not fit them perfectly. The calibrated parameters are: 1) the initial overhead parameter ψ_o^0 ; 2) the share of high productivity firms ϕ ; 3) the quality stepsize γ ; 4) the R&D cost parameter ψ_r ; 5) the discount factor β ; and 6) the process efficiency gap Δ between high-type and low-type firms. We set the elasticity of substitution σ to 4 (Redding and Weinstein, 2019) and the CRRA parameter θ to 2 (Hall, 2009).

Table 3 displays the calibrated parameter values. First, the concentration level is sensitive to the share of high productivity firms ϕ . If ϕ is close to 1, the top 10% share is close to 10%. Lower ϕ , combined with a sufficiently high Δ and low ψ_o , help to match the top 10% concentration in the data. We obtain $\phi = 0.6\%$, that is, the high productivity firms are the very top firms. These high efficiency firms enjoy an oversized market share because they have about 20% higher process efficiency ($\Delta = 1.194$). Next, the quality step γ is sensitive to the growth target, with a higher growth target leading to a higher γ estimate. We calibrate it to 1.335. For a given growth rate of the economy, the real interest rate decreases with the discount factor β . We calibrate β to 0.976. ψ_o affects the aggregate markup through the share of products produced by high productivity firms. We calibrate it to 0.002. Finally, the intangible share, which includes R&D investment, helps to pin down ψ_r , which scales the cost of R&D.

Table 4 presents the targets and model fit under the calibrated parameters

¹⁵We aggregate concentration and labor share moments from Autor et al. (2019) for manufacturing, retail, services and wholesale sectors. We exclude finance and utilities because these sectors do not have data before 1992. We aggregate using value of production weights from BLS KLEMS, which is available from 1987.

Table 3: Baseline Parameter Values

Calibrated		
Definition	Parameter	Value
1. share of H-type firms	ϕ	0.006
2. quality step	γ	1.335
3. discount factor	β	0.976
4. initial overhead cost	ψ_o^0	0.002
5. R&D costs	ψ_r	1.694
6. productivity gap	Δ	1.194
Assigned		
Definition	Parameter	Value
7. CES	σ	4
8. CRRA	θ	2

in Table 3. By construction, we fit the productivity growth rate and the aggregate markup exactly. We fit the real interest rate, intangible share and labor share/sales correlation very closely. We undershoot the concentration by about 6 percentage points. Nonetheless the model generates a high level of concentration, with over half of sales accruing to the top 10% of firms.

Table 4: Baseline calibration targets

Targeted	Target	Model
1. top 10% concentration 1987–1992	64.0	58.4
2. productivity growth 1949–1995	1.82	1.82
3. aggregate markup 1988–2015	1.27	1.27
4. real interest rate 1980–1995	6.1	6.2
5. intangible share 1995	10.4	10.3
6. labor share and size relation 1982–2012	-1.1	-1.09

Source: 1 and 6: Autor et al. (2019). 2: BLS MFP series. 3: Hall (2018). 4: Farhi and Gourio (2018). 5: Corrado et al. (2012).

Table 5 displays the moments in the new steady state when ψ_o falls by 35.4% to match decline in the relative price of IT goods over 1996–2005. With this change in ψ_o , the model explains half (55.6%) of the fall in productivity growth and about one-seventh of the decline in aggregate labor share. The between component declined by about one-third of that in the data (-5.4 vs. -13.2 ppt) while the within component rose about three-quarters of that in the data (3.9 vs. 4.7 ppt). As in the data, the output share of intangibles rises in the model, though much less than the data (0.2 vs. 1.5 ppt). One dimension the model is significantly different from the data is the rise in concentration. Concentration rose by about 4.3 ppt in the data while it rose by 19.6 ppt in the model.¹⁶

Table 5: Effect of a decline in ψ_o on untargeted moments

	Data	Model
1. 2006–18 productivity growth rate (ppt)	1.10	1.42
% of growth slowdown explained	55.6%	
2. change in aggregate labor share (%)	-8.1	-1.5
3. within change in labor share (%)	4.5	3.9
4. between change in labor share (%)	-12.5	-5.4
5. change in concentration (ppt)	4.8	19.6
6. change in intangible share (ppt)	1.5	0.2

Source: 1: BLS MFP series. 2–5: Autor, Dorn, Katz, Patterson and Van Reenen (2019), BLS KLEMS. 6: Corrado, Haskel, Jona-Lasinio and Iommi (2012). We lower ψ_o by 35.4% to match the decline in the relative price of IT products from 1996–2005. The data change in the intangible share is from 1995 to 2006–2010. The change in concentration and labor share moments (within, between and aggregate) is from 1987–1992 to 1997–2012.

To clarify the mechanism in our model, Table 6 displays values of selected endogenous variables in the initial and new steady state. Under the CES

¹⁶In [Online Appendix OA-C](#) Tables [OA-2](#) and [OA-3](#), we show that the model explains one-third and nine-tenths of the decline in growth when ψ_o declines by 20% and 50%, respectively.

specification the sales share of high productivity firms differs from their share of product lines S^* . We denote in Table 6 the sales share of the high productivity firms by \tilde{S}^* . The decline in overhead costs encourages productive firms to expand, increasing the share of both products and sales of the high efficiency firms (higher S^* and \tilde{S}^*). This reaction leads to a rise in overhead costs as a share of output despite the downward shift in the overhead cost curve (due to the decrease in ψ_o). Hence, rising overhead spending are behind the rising intangible share (as seen in Table 5) whereas R&D spending falls. The rising overall intangible share implies that the drop in the aggregate labor income share would be smaller if overhead and R&D expenses were treated as intermediate inputs as opposed to investments. This is in line with the finding in Koh et al. (2016). Despite the overall increase in the intangible share (i.e., the sum of R&D expenses and overhead cost) the aggregate labor share (the inverse of the aggregate markup) falls even more such that residual rents go up.

Table 6: Initial vs. new steady state

	Initial	New
1. creative destruction rate in % (z^*)	4.07	3.16
2. % of H-type products (S^*)	47.9	73.4
3. % of H-type sales (\tilde{S}^*)	54.0	75.7
4. markup of H-type firms	1.33	1.33
5. markup of L-type firms	1.19	1.15
6. aggregate markup	1.27	1.28
7. R&D/PY (%)	6.9	5.4
8. overhead/PY (%)	3.4	5.1
9. rent/PY (%)	10.7	11.7
10. real interest rate (%)	6.2	5.4

Note: The aggregate labor share is the inverse of the aggregate markup. The aggregate labor share, total R&D expenditure, total overhead cost and rents relative to GDP sum up to 1.

With the rise in S^* , within firm markup *declines* for the low productivity firms because these firms are more likely to produce a product where the next best producer is a high productivity producer. Within firm markup stays constant for the high productivity firms because they are not subject to limit pricing under the calibrated parameters.¹⁷

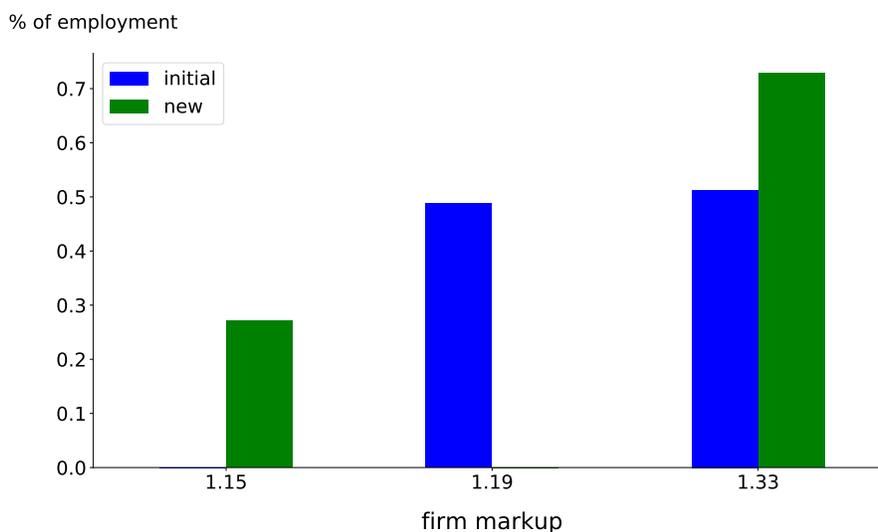
The bottom of Table 6 shows that the model predicts a decline in the real interest from 6.2% in the initial steady state to 5.4% in the new steady state. This reflects the 40 basis point decline in the growth rate combined with a CRAA parameter of $\theta = 2$. This is in the direction of the decline estimated by Farhi and Gourio (2018) from 6.1% for 1980–1995 to 4.5% from 2000–2016.

Figure 7 displays the shift in the markup distribution. It shows that production reallocates to the high productivity firms who have higher markups. This reallocation generates a rise in the aggregate markup and rent amidst falling within firm markup. Within firm, the expected markup from innovating declines as firms are more likely to innovate on a product produced by a high productivity producer. This decline discourages firms from innovating. The firms reduce their R&D expenditures, leading to a lower rate of creative destruction in the equilibrium (lower z^*) and hence lower growth. This lower growth in turn translates into lower interest rate in the new steady state. Thus, growth rate is lower in the new steady state even though rent and aggregate markup is higher.

Finally, recall that job reallocation across firms and establishments and entry and exit rates are trending down in the data, as shown in Figures OA-2 and OA-3 in Online Appendix OA-A. How might our model speak to this? Job reallocation across firms occurs when a firm's employment level rises (gross job creation) or falls (gross job destruction). In the data, this reallocation is partially due to firm entry and exit, which our baseline model does not have. But a significant component of job reallocation in the data is across surviving

¹⁷The fact that the decrease in the within labor income share is particularly pronounced for firms with a falling sales share is in line with the shift share result in Kehrig and Vincent (2018).

Figure 7: Change in the distribution of markups



firms. In our model, firms add and subtract products from their portfolio due to creative destruction. For simplicity our firms have a continuum of products, so this ebb and flow nets out in steady state. But it is a short leap to a model in which firms have a finite number of products so that their employment levels rise and fall. See Garcia-Macia, Hsieh and Klenow (2019) for just such an analysis. Our model may speak more directly to job reallocation across establishments, if one makes the strong assumption that each plant is associated with given product line produced by the firm. Then plant entry and exit in the data can be compared to the rate of creative destruction in our model. As our model features falling long run growth, it implies falling long run job reallocation associated with lower product turnover.

5 Transition dynamics and welfare analysis

The analysis so far is based on a steady state analysis. In this section, we show that our model can also generate a burst in productivity growth along the transition followed by a long-run slowdown.

5.1 Burst in growth during the transition

In Section 2, we saw that the productivity slowdown after the mid-2000s was preceded by a ten-year burst in productivity growth. It is easy to show that, as ψ_o falls, our theory will also generate a burst in productivity growth along the transition. The reason for the burst in growth along the transition is twofold: (i) The general equilibrium force that decreases the incentive to innovate — stiffer competition as S_t increases — is only realized over time. Hence on impact, as ψ_o decreases, the incentive to do R&D increases and therefore quality growth will increase initially; and (ii) the new steady state with a higher S^* exhibits higher average process efficiency because the efficient firms operate a larger fraction of the product lines. This static efficiency gain must be realized along the transition, leading yet again to high growth along the transition.

5.2 Numerical illustration

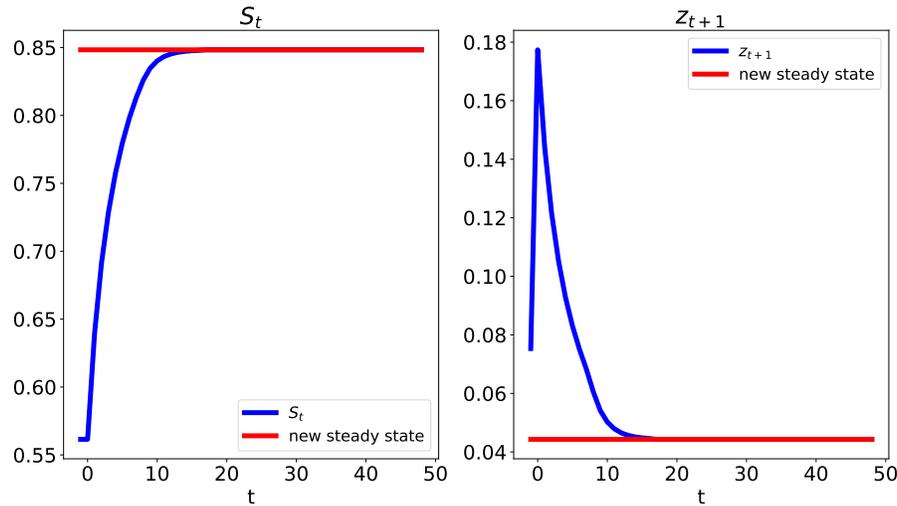
To illustrate the possibility of a productivity burst followed by a slowdown, we compute the transition dynamics for the baseline Cobb-Douglas version of the model with $\sigma = 1$ and $\theta = 1$.¹⁸ We set the model parameters $\phi = 0.10$, $\gamma = 1.273$, $\psi_r = 1.006$, $\beta = 0.956$, $\Delta = 1.182$ and the initial $\psi_o^0 = 0.03$ to match moments we discussed in Table 3. As in the baseline calibration, we reduce ψ_o by 35.4% to match the decline in the relative price of IT goods. The decline of this magnitude generates 103.9% of the decline in long-run growth. See [Online Appendix OA-C](#) for the model fit and steady state results.

Figure 8 displays the share of lines operated by the high type firms (S_t) and the rate of creative destruction (z_{t+1}) after the overhead cost parameter ψ_o declines in year 0. S_t rises sharply and converges to the new steady state after around 10 years. On impact, there is a sharp increase in the rate of creative destruction, as especially the high process efficiency firms invest more in R&D.

¹⁸See [Online Appendix OA-D](#) for a description of the computation method. We focus on the Cobb-Douglas theory when solving for the transition since this case turns out to be much simpler.

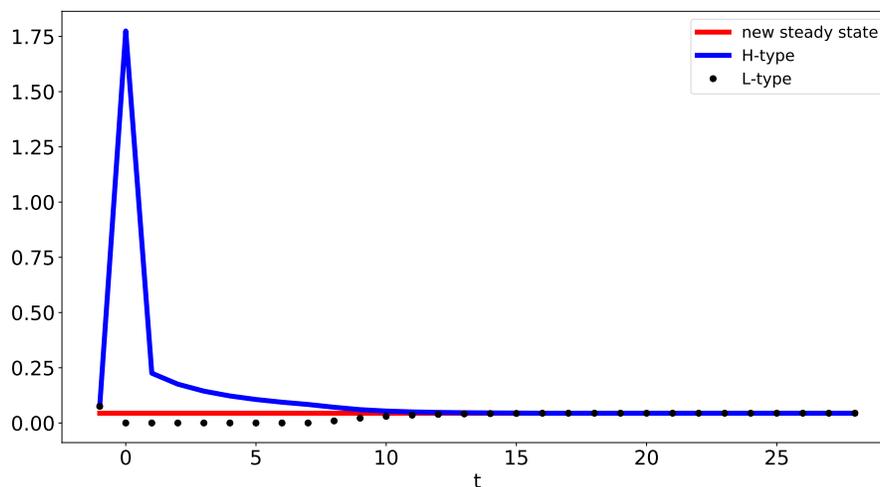
But the rise in innovation is short-lived: the rate of creative destruction converges to its new, lower steady state level after around 10 years.

Figure 8: Transition dynamics of S_t and z_{t+1}



The rise in concentration and burst in creative destruction comes from a sharp increase in the rate of innovation by the high productivity firms. Figure 9 shows innovation rate (x/n) by firm type. It shows a bigger jump by the high-type than the low type. In fact, innovation rate of the low type hits zero on impact because of the high rate of creative destruction in this period. The innovation rate for both types however eventually converges to the level that is lower than that in the initial steady state. This pattern qualitatively matches the pattern in Figure 3 where only the largest firms experienced a burst of entry rate during the high growth period.

Compared to S_t and z_{t+1} , the transition of the shares of *second-best* producers h_H and h_L is much slower (see Figure 10) because of the low rate of creative destruction. It takes over 100 years (!) for this distribution to converge to the new steady state. The identity of second-best producers affects the distribution of markups, leading to the slow transition of labor shares (Figure 11), which are inversely related to markups in our model. Since the between firm effect on the labor share materializes much faster than the within effect

Figure 9: Transition dynamics of x/n 

the aggregate labor share falls steeply over the first 8 years and then slowly partially recovers.

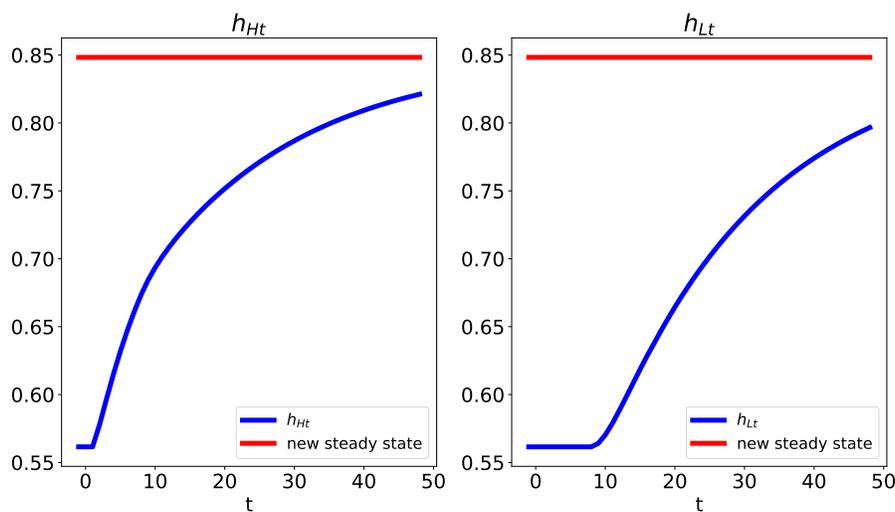
Figure 10: Transition dynamics of h_t 

Figure 12 plots allocative efficiency and process efficiency at the aggregate level, as defined in (27), along the transition. Allocative efficiency rises slightly (about a quarter of a percent) because markup dispersion falls as the most productive firms grab a dominant share of products. Process efficiency rises by

about 4.5% for the same reason. Allocative efficiency depends on the distribution of the *second-best* producers and hence converges very slowly to the new steady state. In contrast, process efficiency depends only on the share of lines operated by the high productivity firms (S_t) and converges quickly to the new steady state.

Figure 11: Labor share along the transition

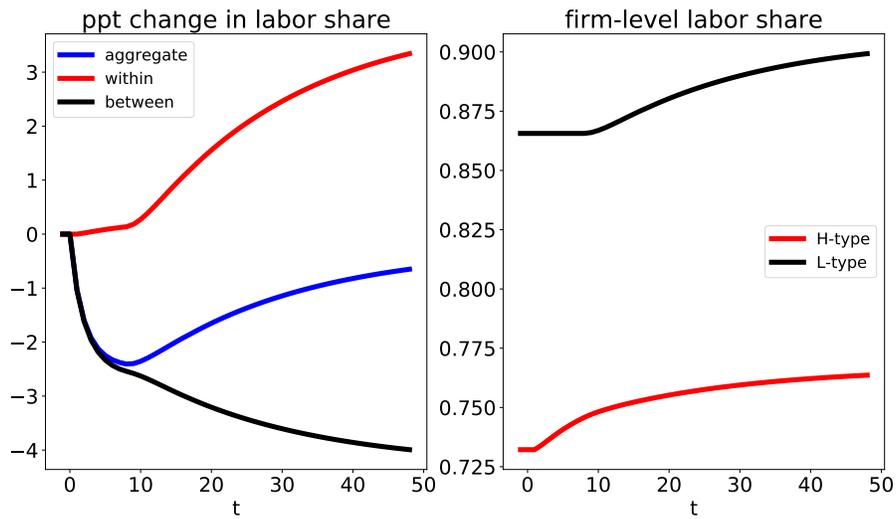
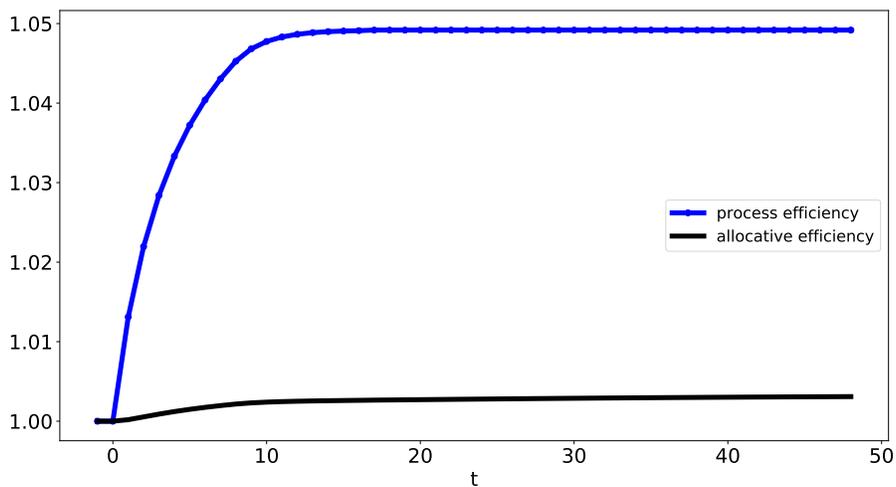


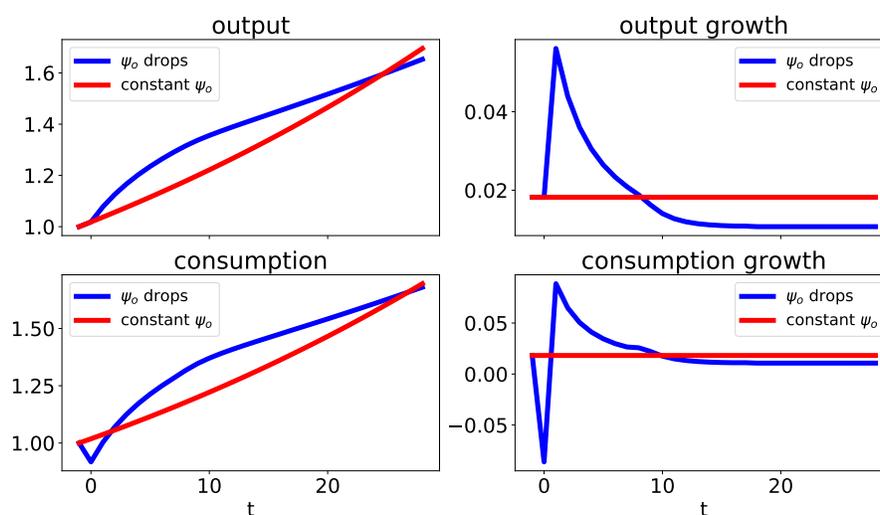
Figure 12: Process and allocative efficiency along the transition



Finally, Figure 13 compares the path of output and consumption following

the reduction in ψ_o with their initial steady state path (with an unchanged ψ_o). Following the drop in ψ_o output grows faster for the first five years but grows more slowly thereafter. Consumption drops sharply in the first period as firms increase their R&D and overhead investments. Consumption then recovers and is above the initial steady state path for more than a decade, due to the temporary burst in growth. Eventually the slowdown in innovation and growth takes its toll and consumption falls below its old steady state trajectory.

Figure 13: Output and consumption along the transition



5.3 Welfare analysis

The drop in ψ_o raises consumption growth in the short run but reduces consumption growth in the long run. Hence it is natural to ask whether present discounted welfare is higher or lower because of the drop in ψ_o . Recall that utility from a consumption path is given by

$$U_0 = \sum_{t=0}^{\infty} \beta^t \log C_t = U(\{C_t\}_{t=0}^{\infty}).$$

The change in welfare can therefore be evaluated in (permanent) consumption-equivalent terms, ξ , using

$$U(\{(1 + \xi)C_t^{old}\}_{t=0}^\infty) = \frac{\log(1 + \xi)}{1 - \beta} + U(\{C_t^{old}\}_{t=0}^\infty) = U(\{C_t^{new}\}_{t=0}^\infty),$$

where $\{C_t^{new}\}_{t=0}^\infty$ and $\{C_t^{old}\}_{t=0}^\infty$ are paths of consumption with and without a drop in ψ_o . We obtain $\xi = -1.04\%$, so that the decline in ψ_o *lowers* welfare by the same amount as a permanent 1% decrease in consumption. This numerical example illustrates that, despite the permanent boost in process efficiency and the temporary boost in innovation, the drop in long-run innovation dominates so that overall welfare is reduced.

6 Theoretical extensions

The baseline model we laid out here is kept parsimonious to show the minimum ingredients need to speak to the empirical facts in Section 2. However, this tractable model can be augmented and generalized in various ways to make the theory more quantitative and without changing the key mechanism at work. Before concluding, we briefly discuss various potential extensions of our theory.

The binary process efficiency is imposed to keep the structure as simple as possible. It is straightforward to generalize it for instance to a continuous distribution with and upper and lower bound. Then in general the whole (stationary) distribution will matter for the steady state and a simple sufficient statistic like S^* will not exist anymore.

One could also allow for some transition matrix between the process efficiency levels. It is not important that this variable is here assumed to be permanent. What is however important is that there is some persistence in the $\varphi(j)$ differences. Another extension that is straightforward to do is to allow heterogeneity in the R&D cost ψ_r or the step size γ across firms (and a potential

correlation therein with a firm's level of process efficiency).

A generalization we have analyzed is to relax the assumption $\gamma > \Delta$. With $\gamma < \Delta$ high productivity firms are less likely to be replaced by creative destruction since they remain the leader even if a low productivity type innovated upon them in the quality space. This then leads to a more dispersed markup distribution even with just two type of process efficiency. For instance with $\gamma^2 > \Delta$, high productivity firms can have a markup factor in a given line of either γ , $\Delta\gamma$, or Δ/γ whereas the low productivity type firms can have a markup of γ or γ^2/Δ .

The quadratic functional form of the overhead function gives rise to the simple linear-quadratic dynamic programming problem with a closed form solution. This property is maintained by adding an additional linear effect of $n(j)$ on overhead cost. In quantitative work the overhead function could be generalized to any convex function.

Since firms operate an interval of lines of measure $n(j)$ firms will not lose all lines at once and consequently there is no firm exit in equilibrium. However we also analyzed a variant of the model where there are additional "small" firms that operate only one line. These firms exit when creative destruction occurs in the one line they operate. Then, as the rate of creative destruction decreases with the productivity slowdown so will gross firm exit and entry.

We also considered a version of our model where firms can create new varieties. Then, as the span of control increases, more varieties are created. As a result R&D expenditures per variety fall, reinforcing the productivity slowdown of our baseline theory.

The baseline model here abstracts from physical capital. It is however straightforward to include physical capital by assuming a Cobb-Douglas production function for the variable input. The model would then predict that the physical capital share declines together with the labor income share (and the profit share goes up).

In our model with leapfrogging innovations, the higher likelihood of facing

a high-efficiency firm as potential competitor on a line — which occurs when ψ_o decreases and therefore S^* increases — always reduces innovation incentives for all firms. In other words, our model does not feature any positive *escape competition effect* as in Akcigit and Ates (2019) or Liu, Mian and Sufi (2019). Introducing such an effect would require us to move from a leapfrogging to a step-by-step innovation model. In such a model, the higher likelihood of facing a high-efficiency firm as competitor on a line, would stimulate neck-to-neck high-efficiency firms to innovate more in order to escape competition. Yet, the discouragement effect of a reduction in ψ_o on innovation by low-efficiency firms would persist, and our conjecture is that the overall long-term effect of a reduction in ψ_o on creative destruction z^* and growth g^* would remain negative if the fraction $1 - \phi$ of low-efficiency firms in the economy, is sufficiently large.

We can also extend our model to the case where high-efficiency firms can target the lines in which they innovate with some probability $\rho \in (0, 1)$. High-efficiency firms would then target lines where they would face a low-efficiency second-best firm. Then a reduction in ψ_o would result in the long-run in yet a higher likelihood for any innovating firm to face a high-efficiency firm as the fringe firm on that line; consequently growth will fall by more and rents will rise further, than in the baseline model.

Another extension of our model, is to allow firms to innovate on their current lines. Then a reduction in ψ_o would induce firms to innovate more on their current lines in order to escape the reduction in mark-ups from innovating on other lines.¹⁹ Consequently, the negative long-run growth effect of this reduction in ψ_o on z^* and g^* will be somewhat mitigated. However,

¹⁹The step-by-step models of Akcigit and Ates (2019) and Liu, Mian and Sufi (2019) allow for incumbent own innovation. In Liu, Mian and Sufi (2019), a reduction in the interest rate induces leading firms in the various sectors to innovate more on their current lines to increase their lead during the transition period towards the new steady-state. But in the new steady state aggregate growth may decline due to the resulting discouragement effect on lagging firms. Similarly, the decline in knowledge spillovers from leaders to followers in Akcigit and Ates (2019) induces leading firms to increase their lead, again discouraging lagging firms to innovate.

recent work by Garcia-Macia et al. (2019) shows that incumbent own innovation did not vary much between the period 1983–1993 (prior to our IT shock) and the period 2003–2013 (after our IT shock), thereby suggesting that the observed productivity slowdown is not primarily explained by incumbents' own innovations.

Finally, we analyzed a version of our model in which we allow for mergers and acquisitions. This extended model predicts increased M&A activity during the transition. Moreover, allowing for M&A magnifies the long-term productivity slowdown as a reduction ψ_o results in even larger increase in S^* .

7 Conclusion

We provide a new theoretical framework that can potentially account for a significant portion of the U.S. growth experience over the past 30 years: (i) a decline in the labor income share (driven by resource reallocation across firms as opposed to a declining labor income share within firms), (ii) a productivity slowdown (after a burst in productivity growth); and (iii) rising concentration at the national level.

We argue that a significant part of these phenomena can be explained by IT improvements in the mid-1990s to mid-2000s which allowed the most efficient firms to expand their boundaries. In our theory, these firms enjoy higher markups; when they expand their reach into more markets, they raise average markups and lower the aggregate labor share. High productivity firms expand by innovating on more product lines, bringing a temporary surge of growth. Within-firm markups eventually fall for both high and low productivity firms, as they are more likely to face high productivity competitors. This force ultimately drags down innovation and growth.

We focused our analysis on the overhead cost parameter ψ_o . However, the model lends itself to richer comparative static and transition analyses. In particular it is straightforward to explore the steady state effects of changes in

the efficiency gap Δ , the innovation size γ , the innovation cost ψ_r or the share of high productivity firms ϕ . We see it as a virtue of our model that the within vs. between firm effects of such changes can be studied easily.

Another next step is to explore the cross-industry predictions of our theory and see if they hold up in the data. In particular, one might look at whether more intensively IT-using industries experienced bigger increases in concentration (paired with declining labor share, and a more pronounced boom-bust cycle of productivity growth).

One could explore optimal tax and subsidy policies in our quantitative framework. The decentralized equilibrium is suboptimal due to markup dispersion across products as well as knowledge spillovers across firms (quality innovations build on previous innovations by other firms). It is possible that falling overhead costs would increase welfare more strongly in the presence of an optimal R&D subsidy.

Finally, our framework is well suited for discussing competition policy and its relation with the productivity slowdown. We analyzed the implications of allowing for M&As, but other dimensions of competition policy such as data access or firm breakup can be naturally considered through the lens of our model. We leave these extensions of our analysis for future research.

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A Solution of the firm's problem

This section derives the solution to the steady state firm problem with Cobb-Douglas production. In this case the profit functions π_H and π_L are quadratic functions of n , the policy and value functions can be characterized in closed form. We state the solution in the next proposition.

Proposition 6 *Let us denote $\tilde{\pi}_H(S^*) \equiv S^*(1 - \frac{1}{\gamma}) + (1 - S^*)(1 - \frac{1}{\Delta\gamma})$ and $\tilde{\pi}_L(S^*) \equiv S^*(1 - \frac{\Delta}{\gamma}) + (1 - S^*)(1 - \frac{1}{\gamma})$. Also, we define*

$$\bar{n}_k(S^*, z^*) \equiv \frac{\tilde{\pi}_k(S^*) + (1 - z^*)\psi_r - \frac{\psi_r}{\beta}}{\psi_o},$$

for $k = H, L$. For a given S^* and z^* , the policy functions $f_k(n)$, $k = H, L$ are given by

$$f_k(n) = \begin{cases} (1 - z^*)n & \text{if } n \geq \frac{\bar{n}_k(S^*, z^*)}{1 - z^*} \\ \bar{n}_k(S^*, z^*) & \text{otherwise.} \end{cases} \quad (52)$$

Let m denote the smallest integer such that $n < \frac{\bar{n}_k(S^*, z^*)}{(1 - z^*)^{m+1}}$ and \bar{n}_k be a shorthand for $\bar{n}_k(S^*, z^*)$. The value functions are given by

$$v_k(n) = \begin{cases} (\tilde{\pi}_k \bar{n}_k - \frac{1}{2}\psi_o \bar{n}_k^2 - \psi_r z^* \bar{n}_k) \frac{1}{1 - \beta} & \text{if } n = \bar{n}_k \\ \tilde{\pi}_k n - \frac{1}{2}\psi_o n^2 - \psi_r (\bar{n}_k - (1 - z^*)n) + \beta v_k(\bar{n}_k) & \text{if } n < \frac{\bar{n}_k}{(1 - z^*)} \\ \tilde{\pi}_k n \frac{1 - (\beta(1 - z^*))^{m+1}}{1 - \beta(1 - z^*)} - \frac{1}{2}\psi_o n^2 \frac{1 - (\beta(1 - z^*)^2)^{m+1}}{1 - \beta(1 - z^*)^2} & \text{Otherwise,} \\ + \psi_r n \beta^m (1 - z^*)^{m+1} + \beta^{m+1} v_k(\bar{n}_k) - \beta^m \psi_r \bar{n}_k & \end{cases} \quad (53)$$

for $k = H, L$.

Proposition 6 says that the $\bar{n}_k(S^*, z^*)$ is the optimal steady state level of n for a firm of type $k = H, L$. A firm invests just enough to hit \bar{n}_k in the next period. If however n is too high such that even without investing (and letting n decay at rate z^*) \bar{n}_k is not reached next period, the non-negativity constraint on R&D is binding and the firm invests zero for m periods.

B CRRA-CES generalization

In this Section, we derive the key steady state equations for the slightly generalized model used in the calibration in Section 4 with a CES production structure and CRRA preferences.

CRRA preferences

Instead of log preferences we generalize utility to the CRRA class

$$U_0 = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta} - 1}{1-\theta}.$$

Then, the resulting Euler equation from household's optimization is given by

$$\frac{C_{t+1}}{C_t} = [\beta(1 + r_{t+1})]^{\frac{1}{\theta}}. \quad (54)$$

Hence, the steady state relationship between gross growth rate g^* and the interest rate is given by $g^* = [\beta(1 + r^*)]^{\frac{1}{\theta}}$.

CES production

Instead of the Cobb-Douglas technology we assume a more general CES technology in final production

$$Y = \left(\int_0^1 [q(i)y(i)]^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}. \quad (55)$$

Here $y(i)$ denotes the quantity and $q(i)$ the quality of product i . This new structure yields as demand for product i

$$y(i) = q(i)^{\sigma-1} \left(\frac{P}{p(i)} \right)^{\sigma} Y, \quad (56)$$

where we have the (new) aggregate price index given by

$$P = \left(\int_0^1 [p(i)/q(i)]^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \quad (57)$$

We normalize this aggregate price index again to one in each period.

Solving for the steady state in this more general model

The rest of the model is unchanged. In particular we still have two process efficiency types and the productivity differential is captured by Δ . We now solve for the steady state in this model.

Together with the definition of the numéraire the demand (56) gives for the operating profits in a period (before overhead cost)

$$YP \left(\frac{P}{p(i)/q(i)} \right)^{\sigma-1} \left(1 - \frac{1}{\mu(i)} \right). \quad (58)$$

With $\sigma > 1$ (which is the empirically relevant case we will focus on) there is an optimal markup factor of $\frac{\sigma}{\sigma-1}$. So depending whether the marginal cost of the second-best firm are binding or not we have the following three cases of markups in a line i :

1. In the case of a high type (H) facing a low type (L) second-best firm

$$\mu_{HL} = \min \left\{ \gamma\Delta, \frac{\sigma}{\sigma-1} \right\}, \quad (59)$$

2. In the case that both leader and second-best firm are of the same type

$$\mu_{HH} = \mu_{LL} = \min \left\{ \gamma, \frac{\sigma}{\sigma-1} \right\}, \quad (60)$$

3. In the case that a low type (L) facing a high type (H) second-best firm

$$\mu_{LH} = \min \left\{ \frac{\gamma}{\Delta}, \frac{\sigma}{\sigma - 1} \right\}. \quad (61)$$

With the CES structure the demand for a product line i depends also on the particular quality of this line (relative to the other lines). But because there is no possibility to target the innovation activity to particular lines and all firms draw repetitively in steady state from the same distribution, the steady state quality level in line i is uncorrelated with the identity of the leading or second-best firm (and therefore uncorrelated with the markup). Since the law of large number applies each firm has in steady state in a given period t the same distribution of quality levels across the different lines.

In the following let us define the “average quality” by

$$Q_t = \left(\int_0^1 [q_t(i)]^{\sigma-1} di \right)^{\frac{1}{\sigma-1}}. \quad (62)$$

Since the quality of a line is independent of its markup we can write the aggregate price index, (57), as $P = 1 = \frac{\tilde{P}_t}{Q_t}$, where

$$\tilde{P}_t = w_t \left(\int_0^1 [\mu(i)/\varphi(j(i))]^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}.$$

In steady state we have

$$\tilde{P}_t = \frac{w_t}{\varphi_L} \left[(S^*)^2 \left(\frac{\mu_{HH}}{\Delta} \right)^{1-\sigma} + S^*(1 - S^*) \left(\frac{\mu_{HL}}{\Delta} \right)^{1-\sigma} + S^*(1 - S^*) \mu_{LH}^{1-\sigma} + (1 - S^*)^2 \mu_{LL}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

The profit in a given line is given by (58). The sum of operating profits (before overhead cost) of a high type firm that is active in $n(j)$ lines and is facing in a

fraction S^* of them a high second-best firm is given by²⁰

$$n(j)Y \left[S^* \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{HH}w_t}{\varphi_L \Delta}\right)^{\sigma-1}} \left(1 - \frac{1}{\mu_{HH}}\right) + (1 - S^*) \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{HL}w_t}{\varphi_L \Delta}\right)^{\sigma-1}} \left(1 - \frac{1}{\mu_{HL}}\right) \right] = n(j)Y \cdot \tilde{\pi}_H.$$

Where we define $\tilde{\pi}_H$ to be equal to the term squared brackets. Similarly, the sum of operating profits before overhead of a L-type firm having $n(j)$ lines and facing in a fraction S^* of them a high second-best firm is

$$n(j)Y \left[S^* \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{LH}w_t}{\varphi_L}\right)^{\sigma-1}} \left(1 - \frac{1}{\mu_{LH}}\right) + (1 - S^*) \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{LL}w_t}{\varphi_L}\right)^{\sigma-1}} \left(1 - \frac{1}{\mu_{LL}}\right) \right] = n(j)Y \cdot \tilde{\pi}_L,$$

where we again define $\tilde{\pi}_L$ accordingly. Hence, the new profit functions of H and L types relative to GDP, Y , become in steady state:

$$\pi_H(n) = n\tilde{\pi}_H - \frac{1}{2}\psi_\circ n^2,$$

and

$$\pi_L(n) = n\tilde{\pi}_L - \frac{1}{2}\psi_\circ n^2.$$

A steady state is characterized as before just with these new profit functions and a different relationship between β , r^* and $g^* = \frac{Q_t}{Q_{t-1}}$ (as specified in the Euler equation).

Steady state characterization

Let us again denote the value of a firm V relative to total output Y by $v \equiv V/Y$. In steady state (with $h(j)^* = S^*$, $\forall j$) the number of products per firm becomes the only individual state variable and we can write $v(n)$. All high productivity

²⁰Note that the Q terms cancels out since the quality distribution in each H-L combination is identical to the aggregate Q .

firms then solve

$$v_H(n) = \max_{n' \geq n(1-z^*)} \left\{ \pi_H(n, S^*) - (n' - n(1-z^*))\psi_r + \frac{g^*}{1+r^*} v_H(n') \right\}.$$

Similarly, all low productivity firms solve

$$v_L(n) = \max_{n' \geq n(1-z^*)} \left\{ \pi_L(n, S^*) - (n' - n(1-z^*))\psi_r + \frac{g^*}{1+r^*} v_L(n') \right\}.$$

The household's Euler equation yields $\frac{g^*}{1+r^*} = \beta(g^*)^{1-\theta}$ and we have

$$g^* = \frac{Y_t}{Y_{t-1}} = \frac{Q_t}{Q_{t-1}} = [1 + z^*(\gamma^{\sigma-1} - 1)]^{\frac{1}{\sigma-1}}.$$

The two accounting equations are again

$$S^* = n_H^* \phi J, \tag{63}$$

and

$$n_H^* \phi J + n_L^* (1 - \phi) J = 1. \tag{64}$$

Finally, in steady state we must have

$$n_H^* = f_H(n_H^*), \tag{65}$$

and

$$n_L^* = f_L(n_L^*), \tag{66}$$

where $f_H(\cdot)$ and $f_L(\cdot)$ are the policy functions of the high and low types. These equations fully characterize the steady state values of S^* , z^* , n_H^* and n_L^* .

In the following we again focus on an interior steady state solution where both firm types are active and $S^* < 1$. The solution is given in the following proposition.

Proposition 7 *An interior steady state is a (n_H^*, n_L^*, S^*, z^*) combination that fulfills*

$$\phi J n_H^* = S^* \text{ and } (1 - \phi) J n_L^* + \phi J n_H^* = 1, \quad (67)$$

as well as the following research arbitrage equations for high and low productivity firms respectively:

$$\psi_r = \frac{\tilde{\pi}_H - \psi_o n_H^*}{\beta^{-1}(1 - z^* + z^* \gamma^{\sigma-1})^{-\frac{1-\theta}{\sigma-1}} - 1 + z^*}, \quad (68)$$

$$\psi_r = \frac{\tilde{\pi}_L - \psi_o n_L^*}{\beta^{-1}(1 - z^* + z^* \gamma^{\sigma-1})^{-\frac{1-\theta}{\sigma-1}} - 1 + z^*}. \quad (69)$$

This is again a system of four equations in four unknowns which can be solved.

Derivation of expression for concentration and labor share

In this more general model with a CES production function there is now a difference between the fraction of lines provided by high productivity firm, S^* , and the sales weight of high productivity firm in the aggregate economy which we denote by \tilde{S}^* . Total sales of a firm of high type is in steady state

$$\int_0^{n_H^*} p(i)y(i)di = n_H^* Y \left[S^* \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{HH} w_t}{\varphi_L \Delta}\right)^{\sigma-1}} + (1 - S^*) \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{HL} w_t}{\varphi_L \Delta}\right)^{\sigma-1}} \right].$$

Sales of a firm of low type is given by

$$\int_0^{n_L^*} p(i)y(i)di = n_L^* Y \left[S^* \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{LH} w_t}{\varphi_L}\right)^{\sigma-1}} + (1 - S^*) \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{LL} w_t}{\varphi_L}\right)^{\sigma-1}} \right].$$

As a consequence, the sales share of high types in the total economy can be

written as

$$\tilde{S}^* = \frac{S^* \left[S^* \left(\frac{\mu_{HH}}{\Delta} \right)^{1-\sigma} + (1 - S^*) \left(\frac{\mu_{HL}}{\Delta} \right)^{1-\sigma} \right]}{S^* \left[S^* \left(\frac{\mu_{HH}}{\Delta} \right)^{1-\sigma} + (1 - S^*) \left(\frac{\mu_{HL}}{\Delta} \right)^{1-\sigma} \right] + (1 - S^*) \left[S^* \mu_{LH}^{1-\sigma} + (1 - S^*) \mu_{LL}^{1-\sigma} \right]}.$$

Finally, let us derive the expressions for the labor income shares with a CES production function. The firm level labor share of a high type is given by

$$\lambda_H^* = \frac{\int_0^{n_H^*} w l(i) di}{\int_0^{n_H^*} p(i) y(i) di} = \frac{S^* \mu_{HH}^{-\sigma} + (1 - S^*) \mu_{HL}^{-\sigma}}{S^* \mu_{HH}^{1-\sigma} + (1 - S^*) \mu_{HL}^{1-\sigma}}. \quad (70)$$

The firm level labor share for low-type is given by

$$\lambda_L^* = \frac{\int_0^{n_L^*} w l(i) di}{\int_0^{n_L^*} p(i) y(i) di} = \frac{S^* \mu_{LH}^{-\sigma} + (1 - S^*) \mu_{LL}^{-\sigma}}{S^* \mu_{LH}^{1-\sigma} + (1 - S^*) \mu_{LL}^{1-\sigma}}. \quad (71)$$

The aggregate labor share is the sales-weighted average of the firm labor shares

$$\lambda^* = \tilde{S}^* \lambda_H^* + (1 - \tilde{S}^*) \lambda_L^*. \quad (72)$$

The within change in labor share is the unweighted average of the change in within firm labor share. We target the within change as a fraction of the initial labor share

$$\frac{\phi(\lambda_{H,1}^* - \lambda_{H,0}^*) + (1 - \phi)(\lambda_{L,1}^* - \lambda_{L,0}^*)}{\tilde{S}_0^* \lambda_{H,0}^* + (1 - \tilde{S}_0^*) \lambda_{L,0}^*}, \quad (73)$$

where 0 denotes the initial and 1 the new steady state, respectively.