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A PLUCKING MODEL OF BUSINESS CYCLES

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ABSTRACT

In standard models, economic activity fluctuates symmetrically around a “natural rate” and stabilization policies can dampen these fluctuations but do not affect the average level of activity. An alternative view—labeled the “plucking model” by Milton Friedman—is that economic fluctuations are drops below the economy’s full potential ceiling. If this view is correct, stabilization policy, by dampening these fluctuations, can raise the average level of activity. We show that the dynamics of the unemployment rate in the US display a striking asymmetry that strongly favors the plucking model: increases in unemployment are followed by decreases of similar amplitude, while the amplitude of the increase is not related to the amplitude of the previous decrease. We develop a microfounded plucking model of the business cycle. The source of asymmetry in our model is downward nominal wage rigidity, which we embed in an explicit search model of the labor market. Our search framework implies that downward nominal wage rigidity is consistent with optimizing behavior and equilibrium. In our plucking model, stabilization policy lowers average unemployment and thereby yields sizable welfare gains.

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1 Introduction

In the workhorse models currently used for most business cycle analysis, economic activity fluctuates symmetrically around a “natural rate” and stabilization policy does not appreciably affect the average level of output or unemployment. At best, stabilization policy can reduce inefficient fluctuations around the natural rate. As a consequence, in these models the welfare gains of stabilization policy are trivial (Lucas, 1987, 2003).

An alternative view is that economic contractions involve drops below the economy’s full-potential ceiling or maximum level. Milton Friedman proposed a “plucking model” analogy for this view of business cycles: “In this analogy, [...] output is viewed as bumping along the ceiling of maximum feasible output except that every now and then it is plucked down by a cyclical contraction” (Friedman, 1964, 1993).¹ In the plucking model view of the world, improved stabilization policy that eliminates or dampens the “plucks”—i.e., contractions—increases the average level of output and decreases the average unemployment rate. Stabilization policy can therefore potentially raise welfare by substantial amounts (De Long and Summers, 1988; Benigno and Ricci, 2011).

We show that the dynamics of the US unemployment rate strongly favor the plucking model of business cycles. An implication of the plucking model—highlighted by Friedman (1964)—is that the dynamics of unemployment should display the following asymmetry: economic contractions are followed by expansions of a similar amplitude—as if the economy is recovering back to its maximum level—while the amplitude of expansions are not related to the previous contractions—each pluck seems to be a new event. We refer to this asymmetry as the plucking property. We present strong evidence that the US unemployment rate displays the plucking property: The increase in unemployment during a contraction forecasts the amplitude of the subsequent expansion one-for-one, while the fall in unemployment during an expansion has no explanatory power for the size of the next contraction.

Our empirical evidence suggests that macroeconomic policy evaluation should be conducted using models that can capture the plucking property. To this end, we consider whether the workhorse Diamond-Mortensen-Pissarides (DMP) search model of the labor market can account for the plucking property we document in the data. We find that the standard version of this

¹The term “plucking” originates in Friedman’s image of a string (output) attached to the underside of a board (potential output): “Consider an elastic string stretched taut between two points on the underside of a rigid horizontal board and glued lightly to the board. Let the string be plucked at a number of points chosen more or less at random with a force that varies at random, and then held down at the lowest point reached.” (Friedman, 1964)

model cannot. The aggregate labor-demand schedule of this model features some non-linearity that can generate a modest amount of the plucking property, but much less than in the data.

To match the facts about plucking that we document in the data, we introduce downward nominal wage rigidity into the DMP model. Earlier research—notably [Benigno and Ricci \(2011\)](#) and [Schmitt-Grohe and Uribe \(2016\)](#)—has highlighted the potential importance of downward nominal wage rigidity for business cycle analysis. Relative to this work, we show how embedding downward nominal wage rigidity within a search model makes it consistent with optimizing behavior, and thus robust to [Barro’s \(1977\)](#) critique that wage rigidity should neither interfere with the efficient formation of employment matches nor lead to inefficient job separations.

We show that our DMP search model augmented with downward nominal wage rigidity can quantitatively match the plucking property we document in the data. Intuitively, our model reproduces the plucking property because good shocks mostly lead to increases in wages, while bad shocks mostly lead to increases in unemployment. Our plucking model also reproduces two other asymmetries in the dynamics of unemployment. [Sichel \(1993\)](#) highlighted that the distribution of unemployment is right skewed: Much of the time, the unemployment rate hovers around 5%. Occasionally, it rises to much higher level, but never falls much below this level. [Neftçi \(1984\)](#) documented that the unemployment also rises faster in contractions than it falls during expansions. Our plucking model generates both a right-skewed unemployment rate and substantially faster increases during contractions than decreases during expansions.

The plucking nature of our model has important normative implications. It implies that fluctuations in unemployment are fluctuations above a resting point of low unemployment, not symmetric fluctuations around a natural rate. As a consequence, a reduction in the volatility of aggregate shocks not only reduces the volatility of the unemployment rate, but also reduces its average level. Eliminating all aggregate shocks in our calibrated model reduces the average unemployment rate from 5.8% to 4.6%. The welfare benefits of stabilization policy are an order of magnitude larger in our model than in standard models in which stabilization policy cannot affect the average level of output and unemployment.

Our plucking model implies that a modest amount of inflation can “grease the wheels of the labor market” by allowing real wages to fall in response to adverse shocks even though nominal wages are downward rigid. Increasing the average inflation rate from 2% (our baseline calibration) to 4% yields a drop in average unemployment from 5.8% to 4.4%. The benefits of inflation diminish at higher levels of inflation but are quite large at low levels.

The notion that good shocks mostly lead to increases in wages, while bad shocks mostly lead to increases in unemployment has a long history within macroeconomics going back at least to [Tobin \(1972\)](#). The main theoretical challenge for this line of thinking has been how to justify the notion that wages do not fall in recessions despite obvious incentives of unemployed workers to bid wages down. To make downward nominal wage rigidity robust to this critique, we build on recent insights from the labor search literature. Intuitively, because of search frictions, unemployed workers cannot freely meet with firms and offer to replace employed workers at a lower wage. Instead, unemployed workers and potential employers must engage in a costly matching process. But after the worker and employer have matched, the worker has some monopoly power and therefore no longer has an incentive to bid the wage down. Without further assumptions, wages are not uniquely pinned down ([Hall, 2005](#)). The literature has often assumed Nash bargaining to pin down wages. The asymmetric nature of unemployment dynamics suggests that a bargaining procedure that gives rise to asymmetric wage adjustment is more promising.

Our work is related to several strands of existing literature. [Kim and Nelson \(1999\)](#) and [Sinclair \(2010\)](#) are two of the very few modern attempts to assess the specific asymmetry emphasized by Friedman (see also [Bordo and Haubrich, 2012](#); [Fatás and Mihov, 2015](#)). [Ferraro \(2017\)](#) uses a search model to explain the skewness and speed asymmetries in the unemployment rate. Our assumption of downward nominal wage rigidity is motivated by microeconomic evidence on the prevalence of wage freezes in US data (see, e.g., [McLaughlin, 1994](#); [Kahn, 1997](#); [Card and Hyslop, 1997](#); [Altonji and Devereux, 2000](#); [Kurmann and McEntarfer, 2017](#); [Hazell and Taska, 2018](#); [Grigsby, Hurst, and Yildirmaz, 2018](#)). A large literature has considered models with downward nominal wage rigidity (see, e.g., [Akerlof, Dickens, and Perry, 1996](#); [Kim and Ruge-Murcia, 2009](#); [Benigno and Ricci, 2011](#); [Abbritti and Fahr, 2013](#); [Schmitt-Grohe and Uribe, 2016](#); [Chodorow-Reich and Wieland, 2018](#)). The importance of wage rigidity for generating realistic fluctuations in unemployment has been stressed by [Shimer \(2005\)](#), [Hall \(2005\)](#), [Gertler and Trigari \(2009\)](#), and [Gertler, Huckfeldt, and Trigari \(2016\)](#). [Bewley \(1999\)](#) presents survey evidence on why firms don't lower wages in recessions. Recent work has explored several ways in which Lucas's (1987, 2003) calculations may underestimate the costs of business cycle fluctuations (see, e.g., [Imrohroglu, 1989](#); [Obstfeld, 1994](#); [Tallarini, 2000](#); [Barlevy, 2004](#); [Gali, Gertler, and Lopez-Salido, 2007](#); [Barro, 2009](#); [Krusell et al., 2009](#)). Finally, the gains from targeting a positive level of inflation have been reassessed in recent years following the Great Recession (see, e.g., [Coibion, Gorodnichenko, Wieland, 2012](#); [Andrade et al. 2016](#); [Blanco 2018](#)).

The paper proceeds as follows. Section 2 presents our empirical results on the asymmetric dynamics of the unemployment rate. Section 3 lays out our plucking model of business cycles. Section 4 analyses whether a basic DMP version of the model can generate business cycle asymmetries. Section 5 analyses the ability of the full model to generate these asymmetries. Section 6 analyses the ability of the simple and full model to match the duration of expansions and contractions. Section 7 shows that fluctuations increase the average level of unemployment and that higher inflation reduces the average level of unemployment. Section 8 concludes.

2 Three Business Cycles Asymmetries

We begin our analysis by using data on post-WWII US unemployment to document three business cycle asymmetries. The first of these is Friedman’s plucking property: the amplitude of a contraction forecasts the amplitude of the subsequent expansion, while the amplitude of an expansion does not forecast the amplitude of the subsequent contraction. The second asymmetry is that the distribution of the unemployment rate is right-skewed, as emphasized by [Sichel \(1993\)](#). The third asymmetry is that the unemployment rate rises more quickly than it falls, as emphasized by [Neftçi \(1984\)](#) among others. In addition to these asymmetries, we highlight two facts: Business cycle expansions are long and micro-data suggests cyclical variation in downward nominal wage rigidity.

2.1 Defining Expansions and Contractions

To implement our empirical analysis, we need dates of business cycle peaks and troughs. One option is to make use of the dates identified by the NBER Business Cycle Dating Committee. A downside of using these dates is that the NBER dates do not line up exactly with peaks and troughs of the unemployment rate. The reason for this is that the NBER Business Cycle Dating Committee uses a wide variety of cyclical indicators to date turning points. Since our empirical analysis is based on the amplitude and speed of cyclical movements in unemployment, we need dates that line up with turning points for the unemployment rate. For this reason, we develop a simple algorithm that defines business cycle peak and trough dates for the unemployment rate. The basic idea is to find local minima and maxima of the unemployment rate. However, we ignore small “blips” or “wiggles” in the unemployment rate and focus instead on delineating substantial swings in the unemployment rate in a similar manner as the peaks and troughs identified by the

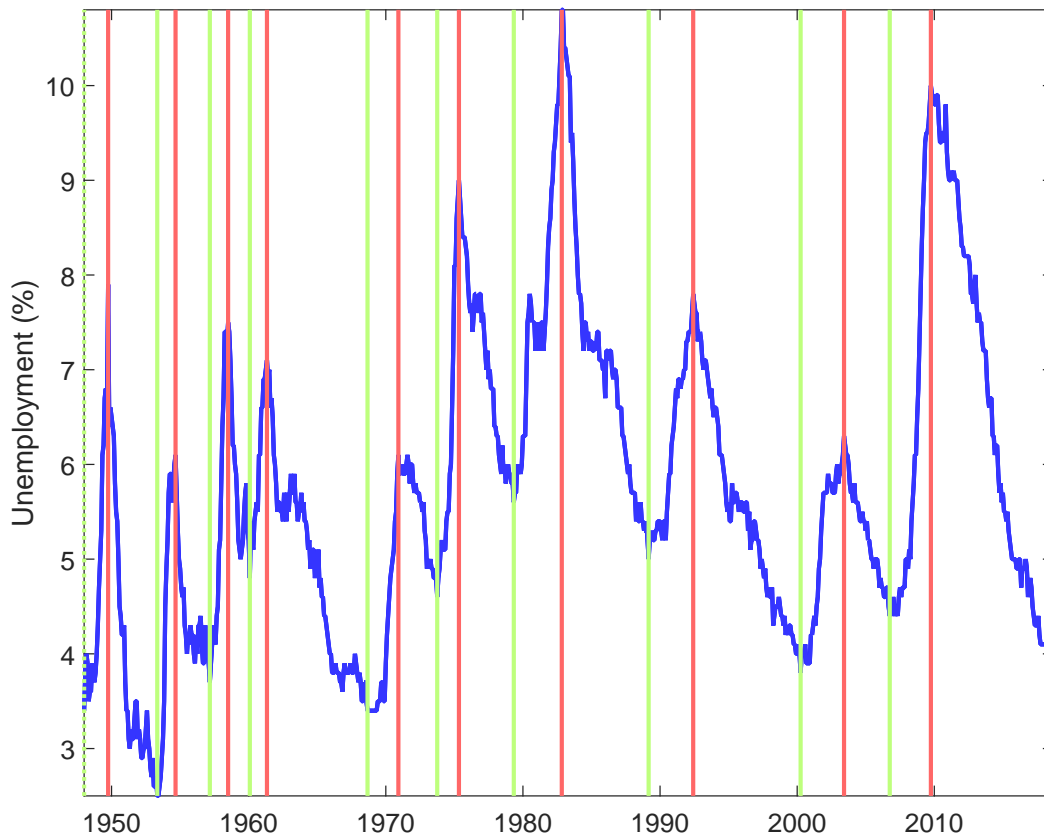


Figure 1: Peaks and Troughs in the Unemployment Rate

Note: The unemployment rate is plotted in blue. Business cycle peaks are denoted by green vertical lines, while business cycle troughs are denoted by red vertical lines.

NBER Business Cycle Dating Committee. For details on our algorithm, see Appendix A.

Figure 1 plots the unemployment rate over our sample period with vertical lines indicating the times that we identify as business cycle peaks and troughs. We identify ten peaks and ten troughs. To these we add a peak at the end of our sample in September 2018 when unemployment was lower than in any month since the previous trough. (Note that business cycle peaks are troughs in the unemployment rate and vice versa.) We also consider the first month of our sample—January 1948—as a peak. A concern regarding this date is that the contraction at the beginning of our sample may have started earlier. We are however reassured on this point by the fact that the NBER identified November 1948 as a peak.

Table 1 presents the peak and trough dates we identify. For comparison purposes, we also present the peak and trough dates identified by the NBER. We identify the same set of expansions

Table 1: Business Cycle Peaks and Troughs

	Unemployment		NBER	
	Peak	Trough	Peak	Trough
1	[1/1948]	10/1949	11/1948	10/1949
2	5/1953	9/1954	7/1953	5/1954
3	3/1957	7/1958	8/1957	4/1958
4	2/1960	5/1961	4/1960	2/1961
5	9/1968	12/1970	12/1969	11/1970
6	10/1973	5/1975	11/1973	3/1975
7a	5/1979		1/1980	7/1980
7b		11/1982	7/1981	11/1982
8	3/1989	6/1992	7/1990	3/1991
9	4/2000	6/2003	3/2001	11/2001
10	10/2006	10/2009	12/2007	6/2009
11	[9/2018]			

Note: Business cycle peaks and troughs defined solely based on the unemployment rate and, for comparison, business cycle peaks and troughs as defined by the Business Cycle Dating Committee of the National Bureau of Economic Research.

and contractions as the NBER Business Cycle Dating Committee with one exception: we consider the 1979-1982 double-dip recession as a single contraction as opposed to two contractions interrupted by a brief and small expansion (unemployment decreased by 0.6 percentage points in 1980-1981). The exact timing of the NBER peaks and troughs do not line up exactly with ours for the reasons discussed above. However, in most cases, our dates are quite similar to theirs. The NBER peaks tend to lag our peaks by a few months and the NBER troughs tend to precede our troughs by a few months. This implies that our estimate of the average duration of contractions is about one year longer than what results from the NBER's dating procedure.

2.2 The Plucking Property

Figure 2 presents scatter plots illustrating the plucking property for the unemployment rate. The left panel has the amplitude of a contraction on the x-axis and the amplitude of the subsequent expansion on the y-axis. The amplitude of contractions is defined as the percentage point increase in the unemployment rate from the business cycle peak to the next trough. The amplitude of expansions is defined analogously. There is clearly a strong positive relationship between the amplitude of a contraction and the amplitude of the subsequent expansion in our sample period. In other words, the size of a contraction strongly forecasts the size of the subsequent expansion.

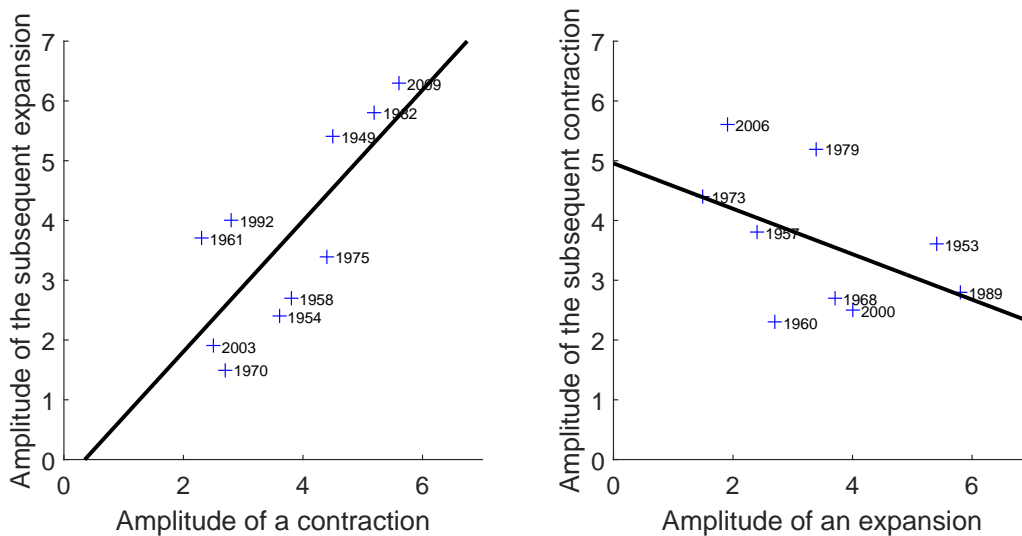


Figure 2: The Plucking Property of the Unemployment Rate

Note: OLS regression lines are plotted in each panel.

We have included an OLS regression line in the panel. Table 2 reports the regression coefficient from this regression. The relationship is roughly one-for-one. For every percentage point increase in the amplitude of a contraction, the amplitude of the subsequent expansion increases by 1.09 percentage points on average. Despite the small number of data points, the relationship is highly statistically significant. Furthermore, the explanatory power of the amplitude of the previous contraction is large. The R^2 of this simple univariate regression is 0.58.

The right panel of Figure 2 has the amplitude of an expansion on the x-axis and the amplitude of the subsequent contraction on the y-axis. In sharp contrast to the left panel, there is no relationship in this case. The size of an expansion does not forecast the size of the next contraction. In Friedman’s language, each contractionary pluck that the economy experiences is independent of what happened before. The linear regression line in the panel is actually slightly downward sloping. But the association is far from statistically significant and the R^2 of the regression is only 0.22.

Jackson and Tebaldi (2017) suggest that the *duration* (not size) of an expansion is predictive of the size of the following contraction. They motivate this idea by analogy to forest fires: the longer the expansion, the more “underbrush” builds up—e.g., low quality matches and entrants—that

Table 2: Plucking Property of Unemployment

	β	R^2
Subsequent expansion on contraction	1.09 (0.33)	0.58
Subsequent contraction on expansion	-0.38 (0.27)	0.22

Note: The first row reports the coefficient in an OLS regression of the size of the subsequent expansion (percentage point fall in unemployment rate) on the size of a contraction (percentage point increase in unemployment rate). The second row reports the coefficient in an analogous regression of the size of the subsequent contraction on the size of an expansion.

becomes fuel in the subsequent contraction. We find no evidence of the forest fire theory at the aggregate level: the duration of an expansion is no more predictive of the size of the following contraction than the size of the expansion is. The relationship is actually negative (but not significantly so), driven by the fact that the three longest post-WWII expansions (1961-1968, 1982-1989, 1992-2000) were followed by relatively mild recessions. [Tasci and Zevanove \(2019\)](#) confirm these results and also present state level results for the plucking model and forest fire theory. Their state level results are similar to our results at the aggregate level: There is strong evidence for the plucking property but no evidence for the forest fire theory.

2.3 The Unemployment Rate is Positive Skewed

A related but distinct asymmetry is “deepness,” which [Sichel \(1993\)](#) defines as the characteristic that “troughs are deeper than peaks are tall.” Sichel suggests assessing this idea by testing for skewness in the distribution of the level of a business cycle variable such as unemployment. [Figure 3](#) plots a histogram of the distribution of the unemployment rate over our sample period. The unemployment rate is noticeably right skewed. Much of the mass of the distribution is close to 5% (median of 5.6% and mean of 5.8%). However, the right tail reaches quite a bit further out than the left tail. The maximum value of the unemployment rate in our sample is 10.8% in 1982, 5.2 points above the median value, while the minimum value is 2.5% in 1953, only 3.1 points below the median. The skewness of the distribution is 0.63. We can reject a null of no skewness with a p-value of 0.071.²

²The combination of high autocorrelation and non-linearity makes it difficult to calculate a confidence interval for the skewness of the unemployment rate. For this reason, we focus on testing against a null of zero skewness. We construct the reported p-value by estimating an AR(2) model for the unemployment rate and then bootstrapping this model to construct a distribution for estimates of skewness under the null of zero skewness. Before estimating the AR(2), we apply a three-period moving-average filter to smooth the very-high-frequency variations in the unemployment rate. This makes sure the results are similar to those obtained on (non-smoothed) quarterly data. In [section 5](#), we

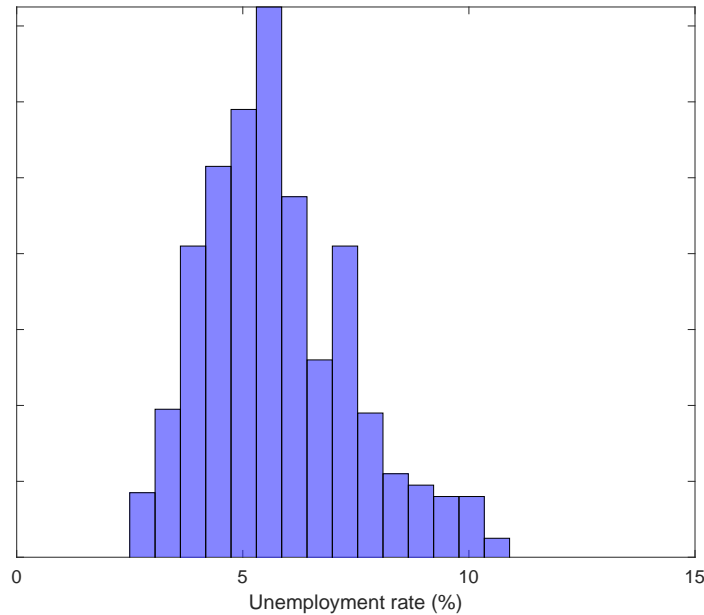


Figure 3: Distribution of Unemployment

2.4 Contractions are Faster than Expansions

A third business cycles asymmetry has received more attention in the literature than the previous two: asymmetry in growth rates between contractions and expansions. Applied to the unemployment rate, it is the idea that unemployment rises more quickly during contractions than it falls during expansions. Observation of this asymmetry dates back at least to the 1920s. [Mitchell \(1927\)](#) notes that “business contractions appear to be briefer and more violent than business expansions.” [Neftçi \(1984\)](#) is an early paper to assess this asymmetry statistically.³

A particularly simple way to illustrate this asymmetry is to calculate the average speed of expansions and contractions in percentage points of unemployment per year. [Table 3](#) reports estimates of the average speed of expansions and contractions. We find that the unemployment rate rises roughly twice as quickly during contractions (1.9 percentage points per year) as it falls during expansions (0.9 percentage points per year). This difference is highly statistically significant. We run a regression of the absolute value of the speed of expansions and contractions on a dummy

reports a simulated distribution for the skewness of the unemployment rate from a much more sophisticated model.

³[Sichel \(1993\)](#) refers to this asymmetry as the “steepness” asymmetry, while [McKay and Reis \(2008\)](#) refer it as the greater “violence” of contractions, in reference to [Mitchell \(1927\)](#). Given that contractions and expansions are of about the same average size (3.7 percentage points), the fact that contractions are “steeper” or “more violent” than expansions is equivalent to the fact that they are briefer.

Table 3: Speed of Expansions and Contractions

Speed of expansions	0.88
Speed of contractions	1.89
P-value for equal speed	0.002

Note: The speed of expansions and contractions is measured in percentage points of unemployment per year. For each expansion and contraction, we calculate the change in unemployment over the spell and the length of time the spell lasts for. The speed for that expansion or contraction is the ratio of those two numbers. We then take a simple average across all expansions and separately a simple average across all contractions. We also regress the absolute value of the speed of adjustment for both expansions and contractions on a dummy for contractions and report the p-value for this dummy.

variable for a spell being a contraction and find that the p-value for the dummy is 0.002.⁴

2.5 The Duration of Expansions and Contractions

Looking back at Figure 1, we can clearly see that when the unemployment rate starts falling, it usually falls relatively steadily for a long time. Table 4 lists the duration of all expansions and contractions over our sample period. The average length of expansions is roughly 58 months, or almost five years. Contractions are also quite persistent. The average length of contractions in our sample is roughly 27 months, a bit more than two years. Perhaps most strikingly, in a few cases—the 1960s, 1980s, 1990s, and the current expansion—the unemployment rate has fallen steadily for six to nine years without reversal. We will argue later in the paper that these long and steady expansions and contractions place interesting restrictions on the types of models or shock processes that drive business cycles.

2.6 Wage Rigidity

The final fact we would like to highlight is the strong cyclical nature in the extent of downward nominal wage rigidity. The Federal Reserve Bank of San Francisco releases a Wage Rigidity Meter, which measures the fraction of job-stayers that experience a one-year wage freeze (Daly, Hobijn, and Wiles, 2011; Daly, Hobijn, and Lucking, 2012; Daly and Hobijn, 2014).⁵ The left panel of Figure

⁴Asymmetry in the speed of expansions and contractions can also be assessed by testing for skewness in the change in unemployment. However, this approach has the downside that estimates are quite sensitive to the frequency of data used. In our sample period, the skewness of Δu_t is 0.35 at the monthly frequency, 1.10 at the quarterly frequency, and between 0.63 and 1.16 at the annual frequency depending on which 12 month periods one uses (i.e., January-January vs. February-February, etc.).

⁵<https://www.frbsf.org/economic-research/indicators-data/nominal-wage-rigidity/>

Table 4: The Duration of Expansions and Contractions

	Dates		Length in Months	
	Peak	Trough	Expansion	Contraction
1	[1/1948]	10/1949		21
2	5/1953	9/1954	43	16
3	3/1957	7/1958	30	16
4	2/1960	5/1961	19	15
5	9/1968	12/1970	88	27
6	10/1973	5/1975	34	19
7	5/1979	11/1982	48	42
8	3/1989	6/1992	76	39
9	4/2000	6/2003	94	38
10	10/2006	10/2009	40	36
11	[9/2018]		107	
Mean			57.9	26.9

4 plots this series along with the unemployment rate for the period 1997-2018.⁶ The right panel of Figure 4 plots it along with the non-employment rate for the working age population (one minus the employment rate). The correlation between wage freezes and these series is striking. The fraction of wage freezes rises rapidly in each of the three recessions that occur in this sample period.

There is a large literature extending back several decades that seeks to estimate the extent of downward nominal wage rigidity. An important empirical challenge in this literature is measurement error in reported wages. The Wage Rigidity Meter reports the fraction of wage freezes with no correction for measurement error. Early work using data from the Panel Study of Income Dynamics (PSID) and the Current Population Survey (CPS) includes [McLaughlin \(1994\)](#), [Kahn \(1997\)](#), and [Card and Hyslop \(1997\)](#). [Altonji and Devereux \(2000\)](#) report larger amounts of downward nominal wage rigidity and virtually no wage cuts in the PSID after correcting for measurement error. [Gottschalk \(2005\)](#) and [Barattieri, Basu, and Gottschalk \(2014\)](#) report similar figures based on analysis of the Survey of Income and Program Participation and correcting for measurement error in a different way. More recently, [Hazell and Taska \(2018\)](#) and [Grigsby, Hurst, and Yildirmaz \(2018\)](#) find substantial downward nominal wage rigidity in firm wage posting data and administrative payroll data, respectively.

⁶Before 1997, the Wage Rigidity Meter does not report a continuous data series (e.g., no data are reported for 1994-1997). In addition to this, inflation is higher before this, leading to a weaker relationship.

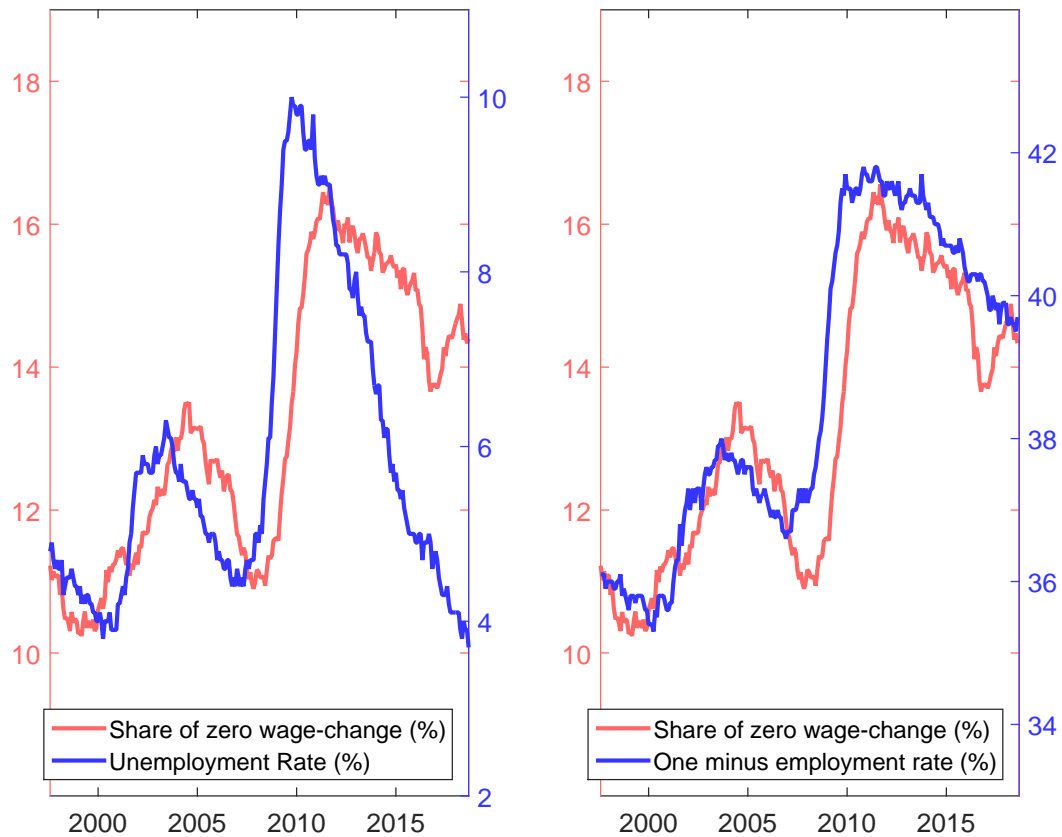


Figure 4: San Francisco Fed Wage Rigidity Meter

Note: The figure plots the share of wage freezes of all job-stayers (paid by the hour or not) with respect to the wage one year prior, with no correction for measurement errors. This series is constructed by the Federal Reserve Bank of San Francisco using data used from the Current Population Survey.

3 An Equilibrium Model of Downward Nominal Wage Rigidity

What model of the labor market can generate the plucking property exhibited in US data? We consider two versions of the Diamond-Mortensen-Pissarides (DMP) search model of unemployment. The first model is a canonical DMP model with symmetric real wage rigidity to address the [Shimer \(2005\)](#) puzzle. The second model replaces symmetric real wage rigidity with downward nominal wage rigidity. We consider versions of these models where the aggregate productivity process follows either an AR(1) or an AR(2) process. We also consider versions with and without sectoral productivity shocks. In this section, we present both models in their most general form, with sectoral heterogeneity.

3.1 Heterogeneous Labor Inputs

There is a continuum of labor types denoted by $i \in [0, 1]$. To each labor type i corresponds a sector i in which identical firms (or equivalently, a representative firm) have access to a decreasing-returns-to-scale technology that uses labor type i as its single input. The production function in sector i is

$$Y_t^i = A_t^i F(N_t^i), \quad (1)$$

where Y_t^i is output, N_t^i is employment, and A_t^i is an exogenous productivity shifter. The productivity shifter should be viewed as a stand-in for any shifter of labor demand in sector i . [Michaillat \(2012\)](#) emphasizes the importance of decreasing-returns-to-scale for the business cycle behavior of DMP search models. For simplicity, we restrict sectoral heterogeneity to labor markets: consumers perceive goods produced in different sectors as identical and therefore value them equally. All goods are sold in a competitive product market at a common price P_t .

A given worker provides a particular type of labor, and can therefore only seek to work at a firm in one sector. This implies that there is a distinct labor market for every type of labor and workers cannot flow across labor markets. We think of these labor types as occupations in a particular location, e.g., lawyers in Houston. Switching occupations is difficult due to occupation-specific human capital. Mobility constraints limit the willingness of workers to switch locations. The recent literature on the China trade shock provides supporting empirical evidence that reallocation of workers across sectors and regions can be very slow ([Autor, Dorn, and Hanson, 2013](#); [Autor, et al., 2014](#)).⁷

3.2 Workers

There is a fixed supply of workers in each sector, equal across sectors, and normalized to one. Because of search frictions, not all workers in sector i are employed: N_t^i workers are employed, while $U_t^i = 1 - N_t^i$ are unemployed. Employed workers in sector i earn a nominal wage of W_t^i , while unemployed workers have no income. We denote the real wage in sector i as $w_t^i = W_t^i/P_t$. We abstract from intensive-margin labor-supply decisions by workers. Workers supply (or at least try to supply) an exogenous quantity of labor—which we normalize to 1. For simplicity, we

⁷The assumption of such differentiated labor inputs is standard in the New-Keynesian literature. There, differentiated labor inputs are an important source of strategic complementarity in price setting. See, e.g., [Woodford \(2003, ch. 3\)](#).

assume that workers do not value leisure, which implies that their labor supply is fully inelastic.⁸ Since we work directly with wage-setting rules—rather than explicitly micro-founding such rules through a bargaining protocol—we do not need to describe households preferences in detail. We assume however that all households are risk-neutral with discount factor $\beta \in (0, 1)$. This implies that firms discount future profits with the risk-neutral stochastic discount factor β .

3.3 Labor Demand

A firm's workforce is constantly depleted by exogenous job separations: each period a fraction $s \in (0, 1)$ of a firm's workforce leaves the firm. We denote the number of workers hired by firm i at time t by H_t^i . Workers hired at time t start working at time t . Firm i 's workforce therefore evolves according to:

$$N_t^i = (1 - s)N_{t-1}^i + H_t^i. \quad (2)$$

To hire workers, firm i must post vacancies. Posting a vacancy costs cA_t^i units of goods, where c is a constant. A vacancy translates into a hire if it matches with a job-seeker. A match happens with probability q_t^i , which firm i takes as given. Firms are large enough that they can abstract from the randomness associated with hiring new workers: hiring one worker requires the firm i to post $1/q_t^i$ vacancy and has the certain real cost $A_t^i c/q_t^i$.

Firm i 's real profits at time t are

$$A_t^i F(N_t^i) - w_t^i N_t^i - \frac{A_t^i c}{q_t^i} H_t^i I_{H_t^i \geq 0}. \quad (3)$$

The first term in this expression is revenue; the second term is the cost of labor; and the third term is hiring costs. Firm i is forward looking and chooses how many workers to hire to maximize intertemporal real profits. It discounts future profits by a factor β per period. The firm maximizes profits subject to the workforce flow equation (2) holding.

If firm i hires every period—which we verify to be almost always true in equilibrium—its labor demand is characterized by the first-order condition:

$$A_t^i F'(N_t^i) = w_t^i + \frac{cA_t^i}{q_t^i} - \beta(1 - s)E_t \left(\frac{cA_{t+1}^i}{q_{t+1}^i} \right). \quad (4)$$

⁸A positive valuation of leisure by workers imposes a positive lower-bound on an equilibrium wage. In our main model with downward nominal wage rigidity, however, the risk is that wages could be too high, not too low. We therefore assume a zero valuation of leisure to avoid lengthening the exposition with the peripheral problem of job-seekers. As we explain below, in our model a lower bound on wages results from the condition that firms cannot hire more workers than exist in their sector.

This condition equates the marginal productivity of a worker to the firm's marginal cost of having a new worker. The cost associated with having a new worker is equal to the wage the worker is paid, the cost of hiring him, minus the expected savings of having a worker next period without having to hire him at that point.

To avoid endogenous separations, it must be that the marginal value of an (already hired) worker is positive in equilibrium. This imposes the following upper-bound on the wage:

$$w_t^i \leq A_t^i F'((1-s)N_{t-1}^i) + \beta(1-s)E_t \left(\frac{cA_{t+1}^i}{q_{t+1}^i} \right). \quad (5)$$

The probability of filling a vacancy q_t^i is determined by a matching function $q(\theta_t^i)$, where $\theta_t^i = H_t^i/(q_t^i S_t^i)$ denotes labor market tightness in labor-market i . Labor market tightness is the ratio of the number of vacancy posted H_t^i/q_t^i to the number of job-seekers at the beginning of the period S_t^i . The probability for an unemployed worker of type i of finding a job is equal to the ratio of hires to job-seekers $f(\theta_t^i) \equiv H_t^i/S_t^i = \theta_t^i q(\theta_t^i)$. We assume that a worker losing his job between periods $t-1$ and t gets a chance to find a new job at the beginning of period t and therefore to work in period t , spending no period without a job. Thus, the number of job-seekers in labor-market i at time t is $S_t^i = 1 - (1-s)N_{t-1}^i$.⁹ We can then rewrite the employment flow equation (2) using the tightness ratio θ_t^i instead of hires H_t^i :

$$N_t^i = 1 - (1 - f(\theta_t^i))[1 - (1-s)N_{t-1}^i]. \quad (6)$$

3.4 Wage-Setting

Search frictions imply that unemployed workers cannot instantly meet with firms and offer to replace employed workers at a lower wage. Rather, an unemployed worker can only meet a firm by engaging in a costly search process. The effort involved in matching implies that workers have some monopoly power once they have matched with a firm and, therefore, no longer have an incentive to bid the wage down. This logic implies that there is no reason why wages would be driven down to their market-clearing level. In other words, nothing forces the equilibrium to be at the crossing of the labor-demand curve (4) and labor-supply curve $N_t^i = 1$.

Absent further conditions, our DMP model only yields an upper and lower bound on the equilibrium wage (Hall, 2005). The upper bound is defined by the no-firing condition (5). Since we assume workers do not value leisure, there is no lower bound coming from workers' unwillingness

⁹The number of job seekers S_t^i at t is not equal to what we defined as the unemployment rate U_t^i at t . The number of job seekers is the unemployment rate at the *beginning* of period t , while the unemployment rate as we define it only counts those job seekers who did not find a job at time t .

to work for too low a wage. However, an equilibrium wage must prevent firms from collectively demanding more labor than exist in the economy.¹⁰ Using the labor demand equation (4), the condition that guarantees no excess labor demand ($N_t^i \leq 1$) yields the following lower-bound on the wage:

$$w_t^i \geq A_t^i F'(1) - \frac{cA_t^i}{\underline{q}} + \beta(1-s)E_t \left(\frac{cA_{t+1}^i}{q_{t+1}^i} \right), \quad (7)$$

where \underline{q} is the value of the vacancy-filling rate when the job-finding rate f is equal to 1.

Any wage between these two bounds is consistent with household and firm optimization. Further assumptions are needed to uniquely pin down the equilibrium wage. Nash bargaining has been a popular assumption in the literature, as have various other bargaining protocols (e.g., [Hall and Milgrom, 2008](#)). However, little direct evidence exists about how firms and workers bargain. Bargaining protocols are therefore usually judged by a combination of their theoretical appeal and the wage process that they generate (e.g., the Hall-Milgrom protocol yields wages that are less sensitive to unemployment). An alternative approach is to directly specify a wage process ([Blanchard and Gali, 2010](#); [Shimer, 2010](#); [Michaillat, 2012](#)). An appeal of this approach is that one can more easily investigate what features the wage process needs to have to be consistent with the real-world behavior of unemployment. (Once this is understood, it can be an input into research on bargaining protocols.)

We take this second approach and consider two wage-setting rules: a rule with symmetric real wage rigidity and a rule with downward nominal wage rigidity. In both cases, wage rigidity is defined relative to a simple case where real wages respond one-for-one to changes in productivity:

$$w_t^i = \bar{w} A_t^i, \quad (8)$$

where \bar{w} is a constant. Following [Blanchard and Gali \(2010\)](#) and [Michaillat \(2012\)](#), we refer to this simple rule as wage flexibility. This rule corresponds to the dynamics of wages with market-clearing and is a very close approximation to the dynamics of wages in the DMP model under Nash bargaining.

The symmetric real wage rigidity rule that we consider is a weighted average of the past real wage and the present flexible wage:

$$\log(w_t^i) = \rho \log(w_{t-1}^i) + (1 - \rho) \log(\bar{w} A_t^i), \quad (9)$$

¹⁰The necessity of imposing such a condition depends on the functional form of the matching function. It is necessary with the Cobb-Douglas specification we will rely on because nothing in the Cobb-Douglas matching function imposes the job-finding probability f to be less than 1. But a matching function that restricts f to lie between 0 and 1 would make hiring infinitely costly as N_t^i tends to 1, killing any incentive for firms to hire the whole labor supply.

where ρ is a weight between 0 and 1.¹¹ This is the wage rule considered by Shimer (2010). Symmetric wage rigidity allows the DMP model to generate a volatile unemployment rate (i.e., avoid the Shimer puzzle). However, we will see that it does not generate the plucking property we have documented.

The second wage rule we consider features downward nominal wage rigidity rather than symmetric real wage rigidity. We assume that the nominal wage is set to the flexible wage, except if this requires the nominal wage to fall: $W_t^i = \max\{P_t \bar{w} A_t^i, W_{t-1}^i\}$. Expressed in terms of real wages, and denoting the inflation rate by $\Pi_t = P_t/P_{t-1}$, the wage-setting equation becomes:

$$w_t^i = \max \left\{ \bar{w} A_t^i, \frac{w_{t-1}^i}{\Pi_t} \right\}. \quad (10)$$

Inflation relaxes the constraint on downward *real* wage adjustments: it greases the wheels of the labor market.

These two specifications for wage-setting do not explicitly impose that the wage remains within the wage band defined by the no-firing condition (5) and no excess demand condition (7). However, we check that they almost always do in our simulations. Specifically, an episode occurs where one of these conditions is violated roughly once every 39,000 months (3,250 years). See Appendix C for more detail.

3.5 Equilibrium

To close the model, we assume that the goods market clears. This implies that production is equal to households' demand for consumption, plus firms' demand for hiring services:

$$\int_0^1 Y_t^i di = C_t + \int_0^1 \frac{cA_t^i}{q(\theta_t^i)} [N_t^i - (1-s)N_{t-1}^i] di. \quad (11)$$

Under risk-neutrality, this market-clearing condition determines consumption residually given other equilibrium outcomes. We can, therefore, abstract from it (as well as from output) in defining the equilibrium.

An equilibrium is given by processes for employment N_t^i , labor market tightness θ_t^i , and the real wage w_t^i for each sector $i \in [0, 1]$, as well as the aggregate inflation Π_t , such that in all sectors i , firm i is on its labor demand schedule (4), the employment flow equation (6) holds, the no-firing condition (5) and no-excess-demand condition (7) hold, and wages are set according to the

¹¹Because we will assume a symmetric process for the logarithm of A_t^i , we take the average to be geometric—arithmetic for the logarithm of wages—in order not to introduce an ad hoc source of asymmetry in the model.

wage setting rule for that model (equation (9) or equation (10)). We are interested in the aggregate unemployment rate, which we define as the average unemployment rate across sectors:

$$U_t = \int_0^1 (1 - N_t^i) di. \quad (12)$$

An equilibrium is conditional on exogenous processes for productivity A_t^i , initial conditions for employment N_0^i , and a monetary policy. We specify monetary policy as directly setting a path for the inflation rate Π_t , which we take to be constant at some target value $\bar{\Pi}$. We assume that sectoral productivity $\log(A_t^i)$ in sector i is the sum of a time trend g , an aggregate component $\log(A_t)$, and an idiosyncratic component $\log(Z_t^i)$:

$$\log(A_t^i) = g \times t + \log(A_t) + \log(Z_t^i). \quad (13)$$

We assume the idiosyncratic component follows an AR(1) in growth rates

$$\Delta \log(Z_t^i) = \rho_{\Delta z} \Delta \log(Z_{t-1}^i) + \varepsilon_t^{\Delta z, i} \quad (14)$$

with Gaussian innovations: $\varepsilon_t^{\Delta z, i} \sim \mathcal{N}(0, \sigma_\varepsilon^{\Delta z})$. We assume the aggregate component follows either an AR(1) or an AR(2) in levels

$$\log(A_t) = (I - \rho_1^a L)^{-1} \varepsilon_t^a, \quad (15)$$

$$\log(A_t) = (I - \rho_1^a L)^{-1} (I - \rho_2^a L)^{-1} \varepsilon_t^a, \quad (16)$$

where again the innovations are Gaussian: $\varepsilon_t^a \sim \mathcal{N}(0, \sigma_\varepsilon^a)$. We furthermore assume that all these processes are independent: $\log(A_t) \perp \log(Z_t^i), \log(Z_t^i) \perp \log(Z_t^j)$ for $i \neq j$.

3.6 Calibration

Table 5 provides a summary of our calibration. We calibrate the model to a monthly frequency. We set the discount factor β to correspond to an annual interest rate of 4%. We assume a constant-elasticity production function $F(N) = N^\alpha$ and set $\alpha = 2/3$. We assume a Cobb-Douglas matching function $q(\theta) = \mu\theta^{-\eta}$ and set the elasticity of the matching function to $\eta = 0.5$, in the middle of the range reported in Petrongolo and Pissarides (2001)'s survey. The parameters μ and c jointly determine hiring costs. One of the two is redundant as only the composite parameter $c\mu^{\frac{-1}{1-\eta}}$ is relevant for the equilibrium. (See Appendix B.1 for further discussion of this point.) We normalize μ to 1. We set c so that steady-state hiring costs c/q are 10% of the monthly steady-state wage \bar{w} , in line with what Silva and Toledo (2009) report based on the Employer Opportunity Pilot Project survey

Table 5: Calibration

	Homogenous, AR(1)		Heterogenous, AR(2)	
	SRWR	DNWR	SRWR	DNWR
β			0.96 ^{1/12}	
α			2/3	
η			0.5	
c			st. c/q is 10% of \bar{w}	
\bar{s}			3.4%	
g	0.023/12	0	0.023/12	0.023/12
$\bar{\Pi}$	–	1.02 ^{1/12}	–	1.02 ^{1/12}
ρ	0.9	–	0.9	–
\bar{w}	0.6895	0.6733	0.6890	0.6736
$\rho_{\Delta z}$	–		0.96	
$\sigma_{\varepsilon}^{\Delta z}$	–		7×10^{-4}	
ρ_a^1	0.98		0.96	
ρ_a^2	–		0.84	
σ_{ε}^a	st. $\sigma^a = 2.7\%$	st. $\sigma^a = 3.0\%$	st. $\sigma^a = 2.3\%$	st. $\sigma^a = 3.7\%$

Note: The abbreviation “st.” stands for “such that.”

in the US. We calibrated the constant monthly separation rate to the value reported by [Shimer \(2005\)](#) based on CPS data: $s = 3.4\%$. We set the growth rate of productivity g to 2.3% annually, the average growth of US labor productivity from 1948 to 2018.¹² These parameters determine the steady-state of the model up to \bar{w} , which we calibrate to match the average unemployment rate.

We present results for two wage rules: one with symmetric real wage rigidity—equation (9)—and one with downward nominal wage rigidity—equation (10). For the symmetric real wage rigidity case, we calibrate the persistence parameter in the wage process ρ to 0.9, following [Shimer \(2010\)](#). For the downward nominal wage rigidity case, we set inflation to 2% per year. (Inflation is immaterial in the symmetric real wage rigidity case.) We calibrate \bar{w} to get a steady-state level of unemployment equal to the average level of unemployment in the data (5.8%). The resulting values for \bar{w} for the four versions of the model we present results for are listed in Table 5.

We present results for two different processes for aggregate productivity: AR(1) and AR(2). In the AR(1) case, we set the auto-regressive root of the aggregate productivity process ρ_1^a to 0.98 following [Shimer \(2010\)](#). In the AR(2) case, we calibrate the two roots of the aggregate productivity process to match the frequency of unemployment cycles in the data. Because there is little internal

¹²We deviate from this in one case: In the case of the version of the model without sectoral shocks but with downward nominal wage rigidity (i.e., the “Homog. AR(1) DNWR” case) we set $g = 0$. We discuss this further below.

propagation in our model (a standard problem with DMP models), we calibrate the roots to the ones obtained from estimating an AR(2) directly on the US unemployment rate series. We first apply a three-period moving-average filter to the level of the unemployment rate to smooth out high-frequency variations. This yields $\rho_1^a = 0.96$ and $\rho_2^a = 0.84$.¹³ We get the same results when estimating the roots on the quarterly unemployment series without any smoothing and converting them to a monthly frequency.

We calibrate σ_ε^a to get the standard deviation of unemployment equal to its value in the data (1.6 percentage point). The resulting values for σ_ε^a for the four versions of the model we present results for range from 2,3% to 3.7% (see Table 5). This implies that the wage rules that we use go most of the way towards resolving the Shimer puzzle: Our calibration for the standard deviation of productivity is quite similar to the 2% value reported by Shimer (2005) for the cyclical component of productivity in the data. We choose to match the standard deviation of unemployment exactly (as opposed to calibrating to the standard deviation of productivity in the data) so that we can apply our definition of expansions and contractions to our simulated samples in the same way as we do to the real world data.

The two of the versions of the model that feature AR(2) fluctuations in aggregate productivity also feature sectoral productivity shocks. We calibrate the persistence of the idiosyncratic productivity process based on KLEMS annual data on US sectoral productivity from 1947 to 2010 (Jorgenson, Ho, and Samuels, 2012). The KLEMS dataset provides labor productivity series (value added per hour) for 31 sectors. We take $\log(Z_t^i)$ to be the log difference between the sectoral labor productivity series and the BLS series for aggregate labor productivity. Here again, we first apply a three-period moving-average filter to the level of these series to smooth out high-frequency variations in $\log(Z_t^i)$. We then first-difference the resulting series and estimate AR(1) models for $\Delta \log(Z_t^i)$ in each sector. The average estimated autoregressive root across sectors is $\rho_{\Delta z} = 0.62$ at an annual frequency. We therefore calibrate $\rho_{\Delta z} = 0.62^{\frac{1}{12}} = 0.96$ in our monthly calibration. We calibrate the volatility of idiosyncratic productivity growth $\sigma_\varepsilon^{\Delta z}$ to roughly match the fraction of constrained firms in the data as measured by the San Francisco Fed’s Wage Rigidity Meter. The value we use is $\sigma_\varepsilon^{\Delta z} = 7 \times 10^{-4}$.

¹³The autoregressive coefficients are $\phi_1^a = 1.803$ and $\phi_2^a = -0.810$. These are related to the roots ρ_1^a and ρ_2^a through the equation $I - \phi_1^a L - \phi_2^a L^2 = (I - \rho_1^a L)(I - \rho_2^a L)$.

3.7 Solution Method

Given the asymmetries and non-linearities our model is intended to capture, we rely on global methods to numerically solve for the equilibrium. The hiring decision of a firm is a function of four or five state variables depending on the process for aggregate productivity. These are: aggregate productivity A_t (and lagged aggregate productivity A_{t-1} if aggregate productivity follows an AR(2)), idiosyncratic productivity growth ΔZ_t^i , the wage w_t^i , and lagged employment in sector i , N_{-1}^i . We introduced sectoral heterogeneity in such a way that a firm does not need to forecast any endogenous aggregate variable in order to decide how many workers to hire. Therefore, we do not need to keep track of the endogenous aggregate state of the economy in order to solve for the hiring decision of a firm.

A solution to the problem of a firm in our model can be described as a pair of policy functions for c/q^i and N^i as a function of the firm's state variables. We form a discrete grid over the state-space, approximate the stochastic processes for the exogenous productivity variables using [Rouwenhorst \(1995\)](#) discretization method, and solve the model by iteration on the policy function. In simulating the heterogeneous model, we assume 1000 sectors. [Appendix B.3](#) provides more detail.

An issue that arises in solving the model is that some points on the grid feature high wages and low productivity. Firms would like to fire workers in such states, violating the no-firing condition. This does not mean, however, that the no-firing constraint is likely to be violated on an equilibrium path. These states are very unlikely to occur: we check ex post that our simulated paths remain away from these states. Solving the equilibrium in these extreme states is nevertheless necessary to calculate expectations in states that do occur with reasonable probability on the equilibrium path. We adopt the following approach: in a state where the no-firing condition fails, we assume that firms do not fire workers and simply do not hire.¹⁴

4 Does a Simple DMP Search Model Generate Plucking?

The first theoretical question we seek to assess is whether a simple DMP search model can generate the plucking property as well as the other business cycle asymmetries that we document in [section 2](#). To this end, we consider the simplest version of the model laid out in [section 3](#) with symmetric

¹⁴The symmetric problem can occur with the no-excess-demand condition under symmetric real wage rigidity: wages may be so much below productivity that firms are willing to hire more workers than there are in the sector. We deal with such cases in the same way: we assume that firms hire all workers but no more.

real wage rigidity, a single sector, and an AR(1) process for productivity. We refer to this version of the model as the “simple” model.

The second column of Table 6 presents results on the business cycle statistics that we document in section 2 for the simple model with symmetric real wage rigidity, a single sector, and an AR(1) process for productivity. We simulate 250 samples of 849 periods each from the model (equal to the length of our sample of real-world data). We then calculate the business cycle statistics that we document in section 2 in each of these simulated samples and report the median estimate across samples for each statistic as a point estimate and the standard deviation of the estimates across samples in parentheses below each point estimate. The first column of Table 6 reports the estimated value of these business cycles statistics in the data for comparison.

The simple model generates a minimal amount of plucking. The regression coefficient for the size of subsequent expansions on the size of contractions is only slightly larger (0.42) than the regression coefficient for the size of subsequent contractions on the size of expansions (0.18). This difference is far smaller than in the data (1.09 vs. -0.38) and is not statistically significant. Similarly, the former regression has slightly more explanatory power than the latter regression: the R^2 is 0.19 versus 0.04. Again, this difference is much smaller than in the data (0.58 vs. 0.22) and not statistically significant. Figure D.1 provides a visual illustration of these results that is analogous to Figure 2. The simple model also generates less skewness than we document in the data: the median skewness across samples from the model is only 0.27 versus 0.63 in the data. This difference between the model and the data is marginally significant.

The reason the simple model is not able to match the degree of plucking and skewness in the data is that this model does not feature a sufficient degree of asymmetry or non-linearity. To understand this better, it is useful to go through the different components of the model that might give rise to asymmetry. One possibility is that the source of the asymmetry we document arises outside of the labor market. Any such non-labor-market asymmetry would enter our model through asymmetry in the exogenous productivity processes that drive cycles in our model—and we think of them as a reduced form representation of all sources of shifts to labor demand. We have chosen to model these exogenous processes as being linear and symmetric, and therefore shut down this source of asymmetry.¹⁵ A second potential source of asymmetry is the wage-

¹⁵We do this for two reasons. First, we are interested in exploring the ability of the DMP model to generate asymmetry within the labor market. Second, prior empirical work has found asymmetries to be more pronounced in the unemployment rate than in other macroeconomic data suggesting that the source of asymmetry is the labor market (e.g., McKay and Reis, 2008).

Table 6: Plucking Property, Skewness, Speed, and Duration

	Data	Homog. AR(1)		Heterog. AR(2)	
		SRWR	DNWR	SRWR	DNWR
Subsequent expansion on contraction, β	1.09	0.42 (0.22)	0.93 (0.21)	0.52 (0.26)	0.86 (0.21)
Subsequent contraction on expansion, β	-0.38	0.18 (0.21)	-0.06 (0.18)	0.23 (0.26)	-0.09 (0.32)
Subsequent expansion on contraction, R^2	0.58	0.19 (0.16)	0.82 (0.27)	0.28 (0.20)	0.85 (0.25)
Subsequent contraction on expansion, R^2	0.22	0.04 (0.08)	0.02 (0.05)	0.07 (0.13)	0.05 (0.11)
Skewness	0.63	0.27 (0.21)	2.09 (0.51)	0.32 (0.24)	1.70 (0.57)
Speed of expansions (pp/year)	0.88	3.94 (0.35)	3.59 (0.47)	2.64 (0.29)	1.38 (0.28)
Speed of contractions (pp/year)	1.89	3.71 (0.31)	3.49 (0.48)	2.47 (0.26)	2.06 (0.42)
Duration of expansions (months)	57.9	13.7 (1.5)	13.6 (1.7)	21.1 (3.1)	42.3 (7.7)
Duration of contractions (months)	26.9	14.4 (1.7)	17.9 (2.7)	22.4 (2.8)	29.4 (6.2)

Note: The table compares real world data with data from our symmetric real wage rigidity model (SRWR) and our downward nominal wage rigidity model (DNWR) along four dimensions. For each of these assumptions about wage setting we present a single sector model where aggregate productivity follows an AR(1) process (Homog. AR(1)) and a multi-sector model with sectoral shocks and in which aggregate productivity follows an AR(2) process (Heterog. AR(2)). The first (third) row reports the coefficient (R^2) in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second (fourth) row report the coefficient (R^2) in an analogous regression of the size of a contraction on the size of the previous expansion. The fifth row reports the skewness of the distribution of the unemployment rate. The next two rows report the spell-weighted average speed of expansions and contractions, measured in percentage points of unemployment per year. The final two rows report the average duration of expansions and contractions, measured in months. For the models, the reported point estimate is the median value of the statistic over 250 samples of 849 periods each (the length of our sample of real-world data). The standard error reported in parentheses is the standard deviation of the estimates across the 250 samples. The Heterog. AR(2) models are simulated with 1000 sectors.

setting rule. In the simple model, we have assumed a symmetric wage-setting rule. The wage-setting rule is therefore not a source of asymmetry in this version of the model.

This leaves three components of the model as potential sources of asymmetry: the labor-demand schedule of individual firms (4), the relationship between the vacancy-filling probability q_t and the job-finding probability f_t implied by the matching function, and the worker-flow relationship (6). These three relationships can be combined to get the following aggregate labor demand schedule:¹⁶

$$\frac{w_t}{A_t} = F'(N_t) - C(N_t, N_{t-1}) + \beta(1-s)E_t \left(\frac{A_{t+1}}{A_t} C(N_{t+1}, N_t) \right), \quad (17)$$

where

$$C(N_t, N_{t-1}) \equiv \frac{c}{q_t} = c\mu^{\frac{-1}{1-\eta}} \left(\frac{N_t - (1-s)N_{t-1}}{1 - (1-s)N_{t-1}} \right)^{\frac{\eta}{1-\eta}}. \quad (18)$$

See Appendix B for a step-by-step derivation. Because our model features hiring costs, this labor demand schedule is forward-looking: what matters to the hiring decision of a firm is not only today's real wage and today's marginal productivity of a worker, but also the expected future value of these variables. One way to see this formally is to iterate equation (17) forward:

$$C(N_t, N_{t-1}) = E_t \left(\sum_{k=0}^{\infty} \beta^k (1-s)^k \frac{A_{t+k}}{A_t} \left(F'(N_{t+k}) - \frac{w_{t+k}}{A_{t+k}} \right) \right). \quad (19)$$

However, it is well known that the DMP model has minimal internal propagation. It is therefore useful to abstract initially from the intertemporal dimension of hiring by considering the steady-state aggregate labor demand schedule induced by the model. This corresponds to the demand for labor should the real wage remain constant and productivity grow deterministically. It is given by:

$$\frac{w}{A} = F'(N) - K(N)(1 - \beta(1-s)e^g) \quad (20)$$

where

$$K(N) \equiv C(N, N) = c\mu^{\frac{-1}{1-\eta}} \left(\frac{sN}{1 - (1-s)N} \right)^{\frac{\eta}{1-\eta}}. \quad (21)$$

Equation (20) makes apparent the two reasons why labor demand is downward sloping in a search model. First, there are decreasing returns to scale in production (first right-hand-side term in equation (20)). This is what makes labor demand downward sloping in a model without search

¹⁶The difference between the individual labor demand schedule (4) and the *aggregate* labor demand schedule (17) captures the fact that in the search model, search costs are *external* labor adjustment costs. Therefore, a given firm sees the cost of hiring a worker as constant, but in equilibrium the marginal cost of hiring a worker is increasing in employment because of the search externality.

frictions. Second, the marginal search cost is increasing in employment (second right-hand-side term in equation (20)). This is the motive that is specific to a search model.

With a constant-elasticity production function, the first term on the right-hand side of equation (20) is log-linear. In contrast, the function $K(N)$ that arises from search costs makes the log of employment a convex function of the log of the real wage. The function $K(N)$ combines two features of the model: the matching function and the worker-flow relationship (6). Our assumption of a Cobb-Douglas matching function with $\eta = 0.5$ implies that the relationship between the cost of hiring c/q_t and the job-finding rate f_t is linear (see equation (B.2) in the appendix). The convexity of the relationship between log employment and log real wages therefore arises primarily from the worker-flow relationship (6).

The preceding discussion shows that the primary source of non-linearity in the simple model is the worker-flow relationship (6). Figure 5 plots the steady state relationship between the job finding rate and unemployment implied by the worker-flow relationship (6).¹⁷ When the job finding rate is low—and therefore unemployment high—an increase in the job-finding rate applies to many job-seekers and therefore decreases unemployment substantially. When the job-finding rate is high—and therefore unemployment low—an increase in the job-finding rate applies to fewer job-seekers and therefore does not decrease unemployment as much. Intuitively, it gets harder and harder to lower the unemployment rate the lower it gets. [Hairault, Langot, and Osotimehin \(2010\)](#), [Jung and Kuester \(2011\)](#), and [Lepetit \(2018\)](#) emphasize this source of non-linearity.

The substantial degree of convexity of the worker-flow relationship (6) plotted Figure 5 suggests that the simple model might be able to generate the plucking property and other business cycle asymmetries. The reason this is not the case is that fluctuations in unemployment in the US over our sample period have not been large enough. Over the sample period 1948-2018, unemployment fluctuated between 2.5% and 10.8%. This is the red solid segment in Figure 5. Over this relatively small segment, the worker-flow relationship is close to linear. For realistic fluctuations in the unemployment rate, the simple model therefore generates a minimal degree of business cycle asymmetry. Only for counter-factually large fluctuations in the job-finding rate and unemployment does the convexity start to become quantitatively important.

The limited degree of non-linearity that arises from the worker-flow relationship (6) implies

¹⁷The analytical form of this relationship is $u = s/(s + (f/(1 - f)))$. Under the assumption that workers who separate from their firms can only start looking for a job one period after they separate from the firm, as well as in continuous time, the relationship is: $u = s/(s + f)$. In this case too, the steady-state worker-flow relationship is convex but the non-linearities it induces are quantitatively small.

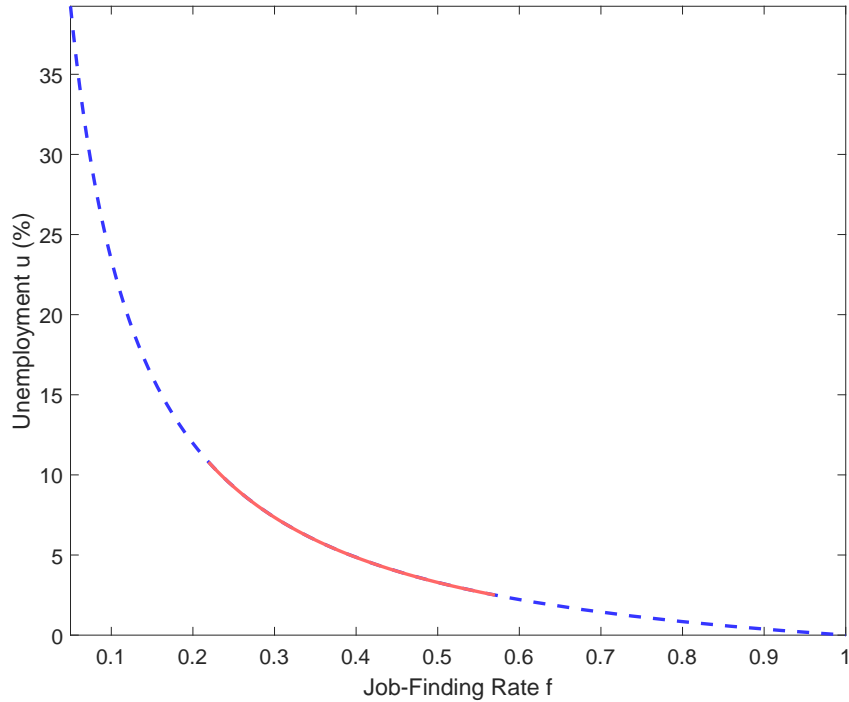


Figure 5: Steady-State Worker-Flow Relationship

Note: The figure plots the steady-state relationship between the job-finding rate and the unemployment rate implied by the worker-flow equation. In plain line the relationship is plotted over the range of unemployment rates observed in the US between 1948 and 2018: from 2.5% to 10.8%.

that the simple model’s aggregate labor demand schedule is close to log-linear. Figure 6 plots the steady state labor demand schedule for the simple model as well as the labor demand schedule with no search frictions ($K(N) = 0$).¹⁸ The labor demand schedule in the simple model is not completely log-linear, but the amount of non-linearity is quantitatively minimal. Figure 6 also includes a scatter plot of the relationship between the log of the wage-to-flexible-wage ratio and unemployment away from steady-state. The cloud of points is slightly more convex than the steady-state relationship, but even this degree of non-linearity is quantitatively minimal.¹⁹

¹⁸We plot these relationships with unemployment on the y-axis. But the results are virtually indistinguishable with $-\log(N)$ on the y-axis, because the linear approximation $u \simeq -\log(N)$ holds well over the range of values we consider.

¹⁹It is common in the DMP literature to abstract from mechanisms other than search costs that make the labor demand schedule downward sloping, such as decreasing returns to labor. This tends to magnify the degree of non-linearity in these models. For example, [Petrosky-Nadeau, Zhang, and Kuehn \(2018\)](#) show how a DMP model with constant returns to labor features asymmetries that can generate business-cycles disasters—large drops in production—despite symmetric shocks. We show in appendix E that this type of model with symmetric real wage rigidity generates more plucking and a more skewed unemployment rate than the simple model. However, appendix E also shows that the increased extent of plucking with constant returns to labor is driven by recessions more extreme than we have seen in our post-WWII sample.

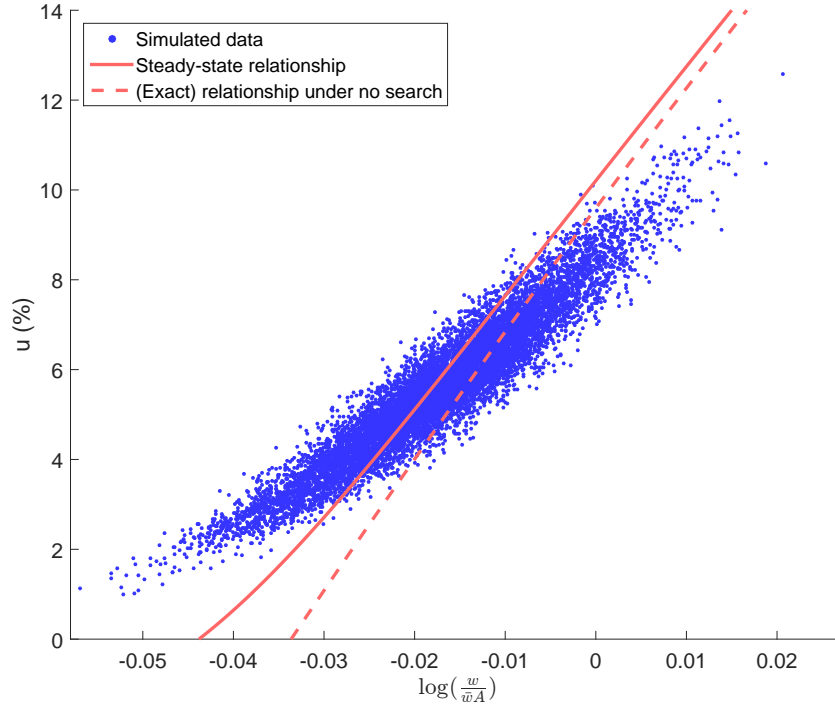


Figure 6: Aggregate Labor Demand Schedule

Note: The figure plots the relationships between the log of the wage-to-flexible-wage ratio and unemployment in the one-sector model of symmetric real wage rigidity with AR(1) productivity shocks.

5 Downward Nominal Wage Rigidity and the Plucking Property

To match the business cycle asymmetries that we document in section 2, we now consider a version of the model presented in section 3 with downward nominal wage rigidity, multiple sectors, and an AR(2) process for productivity. We refer to this version of the model as the “full” model.

The crucial difference between the simple model and the full model is the nature of wage rigidity. The near linearity of the relationship between wages and employment in the DMP model and the lack of significant internal propagation in this model together imply that the dynamics of the unemployment rate closely mirror the dynamics of the average real wage-to-productivity ratio w_t/A_t . With symmetric real wage rigidity, variation in w_t/A_t is symmetric around a steady state. When A_t rises, w_t rise sluggishly so w_t/A_t falls and unemployment falls. When A_t falls, the reverse is true. With downward nominal wage rigidity, however, variation in A_t yields asymmetric variation in w_t/A_t and therefore asymmetric variation in the unemployment rate. Starting from the steady state, if A_t rises, w_t rises along with it and unemployment doesn’t fall. In contrast, if A_t falls, w_t cannot fall in nominal terms. So, unemployment rises.

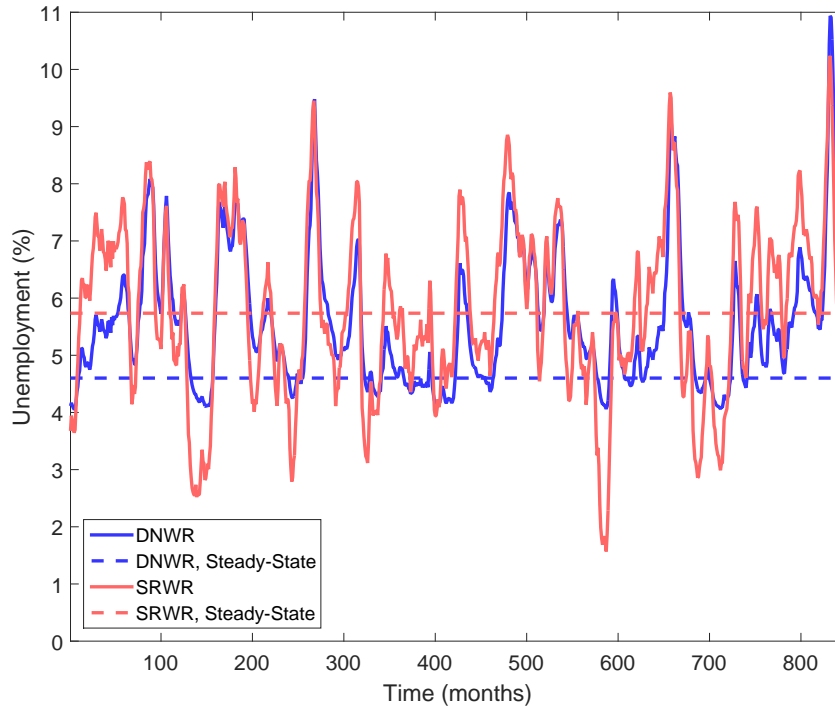


Figure 7: Simulated Paths for the Unemployment Rate

Note: The figure plots a sample path for the unemployment rate of 70 years (the same length as our empirical sample), for two versions of our multi-sector model with an AR(2) aggregate productivity process. The light red line plots unemployment in this model with symmetric real wage rigidity. The dark blue line plots unemployment in this model with downward nominal wage rigidity. The sequence of shocks are the same in both cases. The dotted lines plot the steady-state levels of unemployment in each model.

Figure 7 illustrates this difference by comparing simulated paths for the unemployment rate in our full model with a version of the model that is the same as the full model except that it has symmetric real wage rigidity. The simulated paths in Figure 7 result from these two models being hit by the same sequence of shocks. The figure also plots the steady-state level of the unemployment rate in each of these cases (i.e., the rate that would prevail absent aggregate shocks). The model with symmetric real wage rigidity displays roughly symmetric fluctuations of unemployment around its steady state. The steady-state level of unemployment in this model is 5.7% and the average unemployment rate with fluctuations is 5.8%.

The full model with downward nominal wage rigidity behaves very differently from the simple model. In this model, unemployment frequently rises substantially above its steady-state level, but rarely falls appreciably below this level.²⁰ We have parameterized the full model such that the

²⁰In the full model, unemployment can fall slightly below the steady state. The reason for this is that the sectoral shocks in this model imply that in steady state (i.e., absent aggregate shocks) there is a fraction of firms with wages above the flexible wage. These firms received positive sectoral shocks at an earlier date that raised wages but were then

average unemployment rate is the same as in the model with symmetric real wage rigidity (5.8%). This requires a much lower steady-state level of unemployment (4.6%) than in the model with symmetric real wage rigidity.

The asymmetric behavior of the unemployment rate relative to its steady state in the full model naturally yields the plucking property we document in the data. The right-most column in Table 6 presents results on the business cycle statistics we document in section 2 for the full model. We see that the full model generates a degree of plucking that is comparable to the data. The regression coefficient for the size of subsequent expansions on the size of contractions is 0.86, while the regression coefficient for the size of subsequent contractions on the size of expansions is -0.09. This difference is almost as big as in the data. Similarly, the former regression yields a large R^2 (0.85), while the R^2 of the latter regression is very small (0.05). This difference is slightly larger than in the data.²¹ None of these four statistics (or the two differences just discussed) are statistically significantly different from their counterparts in the data. The full model also yields a substantial amount of skewness of unemployment, somewhat more than in the data.

In our full model, recessions arise because wages cannot adjust downward in response to adverse shocks to labor demand. Wages are therefore stuck at a level that is too high and as a result unemployment rises. Inflation and trend growth in productivity are two secular forces that naturally “grease the wheels of the labor market” by eroding gaps between wages and the flexible wage when wages are too high. We calibrate our full model to have 2% annual inflation and 2.3% annual trend aggregate productivity growth. These forces impart a substantial positive trend to the flexible nominal wage. Negative shocks to labor demand must be big enough to more than offset this trend for downward nominal wage rigidity to bind.

As Benigno and Ricci (2011) emphasize, sectoral shocks are an important source of volatility in labor demand and therefore an important reason why the downward nominal wage rigidity constraint binds. Without sectoral shocks, the downward nominal wage rigidity constraint does not bind a large enough fraction of the time in our model to generate the plucking property we document in the data. Table 6 presents results for a version of our model with downward nominal wage rigidity but without sectoral shocks (Homog. AR(1), DNWR model). This version of the model matches the amount of plucking we see in the data, but only when we set average aggregate productivity growth to zero. With 2.3% productivity growth, this version of the model cannot

reversed. A sequence of positive aggregate shocks can therefore lower unemployment below the steady state in this model by reducing the fraction of firms that have wages above the flexible wage to a level below its steady state level.

²¹Figure D.1 provides a visual illustration of these results that is analogous to Figure 2.

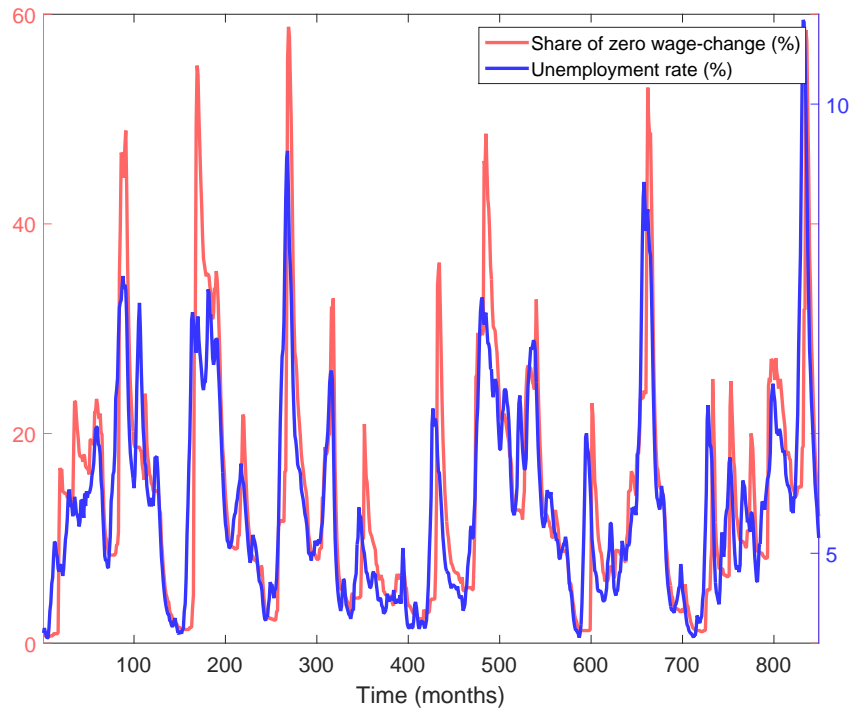


Figure 8: Simulated Paths for the Unemployment Rate and the Fraction of Wage Freezes

Note: The figure plots a sample path for the unemployment rate and the fraction of wage freezes of 70 years (the same length as our empirical sample), for the AR(2) heterogeneous model of downward nominal wage rigidity. A wage freeze is defined as a wage that has remained constant over the past twelve months.

quantitatively match the plucking property. In contrast, the full model with sectoral shocks can match the plucking property in the data with 2.3% average aggregate productivity growth.

A second unappealing feature of the model with downward nominal wage rigidity but without sectoral shocks is that the downward nominal wage rigidity constraint binds for either all or no firm at any given point in time. Figure 4 shows that in the US a fraction of workers face wage freezes at any given point in time and this fraction is highly counter-cyclical. Figure 8 shows that our full model can qualitatively match this feature of the data. The share of workers facing wage freezes in our model is highly correlated with the unemployment rate as in the data.

6 The Speed and Duration of Expansions and Contractions

Two additional dimensions along which the simple model fails to match the data—apart from not generating appreciable business cycle asymmetries—are the speed and duration of expansions and contractions. The bottom half of Table 6 presents results on these dimensions for the models

we consider. In the simple model, the unemployment rate rises and falls much faster than in the data. Furthermore, the simple model does not capture the sharp asymmetry in the speed of expansions and contractions in the data. The unemployment rate rises and falls by roughly four percentage points per year in the simple model, while it rises by roughly two percentage points per year and falls by only about one percentage point per year in the data. The flip side of this coin is that the simple model generates expansions and contractions that are much shorter than in the data. The average length of both expansions and contractions in this model is only roughly 14 months, while it is 27 months for contractions and 58 months for expansions in the data.

As we have discussed above, the lack of internal propagation in the DMP model implies that unemployment inherits the dynamics of the average real wage-to-productivity ratio w_t/A_t . The persistence of unemployment is therefore highly dependent on the degree of real wage rigidity—which is governed by ρ in our model—and the persistence of the productivity process—which is governed by ρ_a . We calibrate these parameters to $\rho = 0.9$ and $\rho_a = 0.98$ for the simple model. For this model to be able to match the degree of persistence in the data, we would need to assume quite extreme values for these persistence parameters. Under such a calibration, the model would generate too much variation in unemployment at very low frequencies.

An alternative way to increase persistence is to assume that labor demand shocks follow an AR(2) rather than an AR(1). An AR(2) process can generate very high persistence at business cycle frequencies without extreme levels of persistence at very low frequencies. A robust lesson from the recent empirical literature in macroeconomics is that the dynamic responses of economic activity to many shocks is hump-shaped. Estimates of impulse responses to identified monetary policy shocks are one source of such evidence (Romer and Romer, 2004; Christiano, Eichenbaum, and Evans, 2005). The fact that medium scale DGSE models fit the data better when they are modified to include equations which yield AR(2) dynamics—e.g., investment adjustment costs, habits in consumption, and lagged terms in the price and wage Phillips curves combined with AR(1) shocks—is another piece of evidence along these lines (Smets and Wouters, 2007).²²

Our full model features labor demand shocks with AR(2) dynamics. This model yields business cycles that are much longer and slower than the simple model. Furthermore, the full model

²²Another approach in increasing persistence is to take the view that much of the propagation of shocks in fact occurs within the labor market. In this case, it is important to amend the DMP model in a way that strengthens its ability to propagate shocks. For example, Fujita and Ramey (2007) show that making the cost of opening a vacancy non-zero and increasing in the number of new vacancies opened makes vacancy creation sluggish. Our approach of altering the shock process instead of the propagation mechanism allows us to remain agnostic as to where the propagation comes from.

generates a substantial amount of asymmetry in the speed of contractions relative to expansions. In the full model, the unemployment rate falls by 1.4 percentage points per year, while it rises by 2.1 percentage points per year. Both the level and asymmetry of these rates is quite close to the data. The same is true of the duration of expansions and contractions. In the full model, the average length of expansions is 42 months, while the average length of contractions is 29 months. Again, the level and asymmetry of these durations are quite close to the data.

The ability of our full model to generate asymmetry in the speed of expansions and contractions has to do with a subtle interaction between downward nominal wage rigidity, sectoral shocks, and the dynamics of the labor demand shocks in our model. Aggregate contractions occur in our model when aggregate labor demand falls by a relatively large amount over a short period of time. When this happens, a large number of firms become constrained by the downward nominal wage rigidity constraint and unemployment shoots up rapidly. The shocks that gave rise to the contraction then dissipate, which leads to an expansion. During the expansion, new negative shocks can hit the economy. However, for these shocks to trigger a new contraction, they must be large enough and rapid enough to outweigh the “greasing the wheels” of the labor market forces—i.e., inflation and trend growth in productivity—that are helping sustain the expansion. Otherwise, these negative shocks just slow down the expansion. Furthermore, for expansions that have been ongoing for some time, the fraction of sectors for which the downward nominal wage rigidity constraint is binding is relatively small. This implies that further increases in flexible wages have smaller and smaller effects on the unemployment rate, which again slows down the expansion.

7 Costs of Business Cycles and Benefits of Stabilization Policy

We now turn to the normative implications of our model. We present two sets of results. First, smaller shocks to labor demand yield a lower average level for the unemployment rate. Second, raising the average inflation rate from 2% to 4% lowers the average level of the unemployment rate.

7.1 First-Order Effect of Stabilization Policy

In a thought-provoking exercise, [Lucas \(1987, 2003\)](#) calculated the welfare benefits of eliminating economic fluctuations. Assuming log-utility and trend-stationary fluctuations with Gaussian

innovations, [Lucas \(2003\)](#) concluded that the representative agent would be willing to forgo no more than 0.05% of his consumption to be rid of fluctuations. Lucas concluded from this that the benefits of stabilization policy are trivial.

An important maintained hypothesis in Lucas' analysis is that eliminating fluctuations does not affect the average level of economic activity. In our plucking model, however, fluctuations are drops below potential rather than cycles around a natural rate. Eliminating fluctuations therefore raises the average level of economic activity.

[Figure 9](#) illustrates this by plotting the average level of the unemployment rate in our plucking model as a function of the volatility of aggregate shocks. Eliminating all fluctuations in our model reduces the average unemployment rate from 5.8% to 4.6%. Conversely, an increasing the standard deviation of aggregate shocks by 50% (from 3.7% to 5.6%) increases the average unemployment rate to 7.4%. Since the efficient level of unemployment is far below the average level of unemployment for any level of aggregate shocks in our plucking model, stabilization policy that is able to reduce aggregate fluctuations yields appreciable welfare gains.²³

The consumption-equivalent welfare gains from eliminating aggregate fluctuations are, however, not equal to the average reduction in the unemployment rate. One factor making the welfare gains smaller is decreased leisure time associated with more work. Another factor is that more resources are consumed matching workers and firms when the labor market is tighter. In [appendix F](#), we calculate the consumption equivalent welfare gain from eliminating aggregate fluctuations in our model. We do this for two different assumptions about the value of leisure time during unemployment. We utilize a calibration strategy introduced by [Hall and Milgrom \(2008\)](#). Assuming that all time while unemployed is leisure implies that the welfare gain of eliminating aggregate fluctuations in our model is 0.34%. In contrast, if we assume that unemployment does not yield additional leisure time—i.e., that people enjoy their time unemployed as much/little as their time while working—the welfare gain of eliminating aggregate fluctuations is 0.65% in our model. In both cases, therefore, our plucking model implies that the welfare gains from eliminating economic fluctuations are an order of magnitude larger than in standard macroeconomic models.

²³As we discuss in [appendix F](#), the efficient level of unemployment is somewhat sensitive to the exact timing assumptions in our model. We assume that workers who loose their job can immediately search and find new employment. In this case, the efficient level of unemployment is 0%. A common alternative assumption is that workers who loose their job need to wait one period until they can search and find a new job. In this case, the efficient level of unemployment is 3.3%. In continuous time, this timing assumption is avoided. In this case, the efficient level of unemployment is 1.2-1.7% depending on the value of leisure while unemployed. For all these cases, therefore, the efficient level of unemployment is below the average unemployment rate for any level of aggregate (and idiosyncratic) shocks.

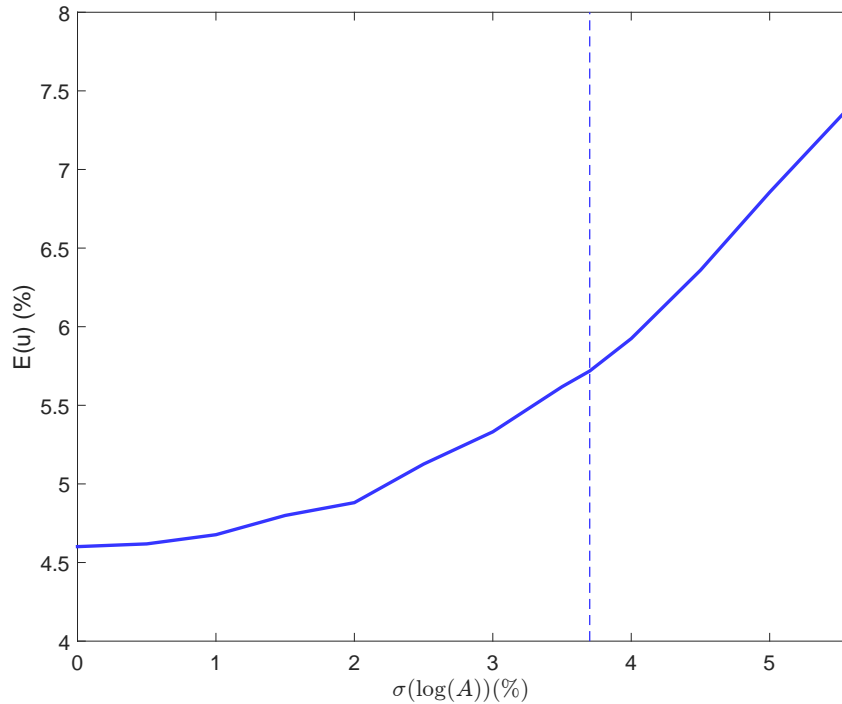


Figure 9: Average Unemployment as a Function of the Volatility of Aggregate Shocks

Note: The figure gives the average rate of unemployment as a function on the standard deviation of aggregate shocks in our model of downward nominal wage rigidity with sectoral heterogeneity and an AR(2) process for productivity.

7.2 Greasing the Wheels of the Labor Market

Lucas's thought experiment of eliminating all fluctuations is intended to give an upper-bound of the benefits of stabilization policies, abstracting from constraints that may exist on what outcomes policy can achieve. We next consider a particular policy: the choice of the inflation target. In our model with downward nominal wage rigidity, inflation greases the wheels of the labor market by easing the downward adjustment of real wages.

Table 10 gives the average unemployment rate as a function of the inflation target in our model. Increasing the inflation target from 2% to 4% decreases average unemployment from 5.8% to 4.4%. Strikingly, extra inflation reduces unemployment more than the elimination of all aggregate fluctuations. The reason for this is that inflation eases not only adjustment to aggregate shocks but also adjustment to idiosyncratic shocks. As the inflation target increases further, marginal benefits fall. Average unemployment asymptotes to its values absent any frictions on wage adjustments or absent any (idiosyncratic and aggregate) shocks: 4.1%.

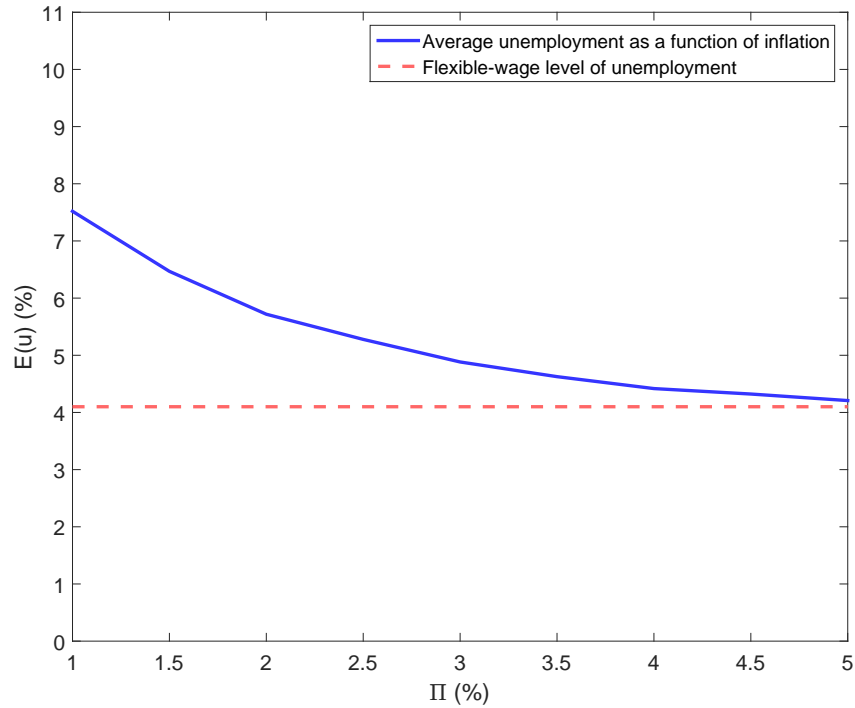


Figure 10: Average Unemployment as a Function of the Inflation Target

Note: The figure gives the average rate of unemployment as a function on the inflation target in our model of downward nominal wage rigidity with sectoral heterogeneity and an AR(2) process for productivity.

These estimates of the effect of the inflation target on unemployment rely on the assumption that wage-setting remains unchanged in the face of the new monetary policy. For a high enough inflation target, it is however likely that workers would shift to thinking in real terms. Any reluctance to bear wage cuts would then manifest itself through downward *real* wage rigidity, and inflation would no longer have an effect on unemployment. The middle part of our sample period includes the Great Inflation, where the US experienced inflation in the high single digits and low double digits. The fact that the US unemployment rate displayed the plucking property during this period of relatively high inflation suggests that barriers to downward adjustment are not eliminated by high inflation but rather that they manifest themselves as downward real wage rigidity in these circumstances perhaps due to widespread implicit or explicit cost-of-living adjustments in wage setting.

Although the distinction between real and nominal downward rigidity is critical when assessing the benefits of a higher inflation target, it is not for the ability of our model to replicate the plucking property. Indeed, our wage-setting equation (10) reduces to the case of downward real

wage rigidity for an inflation target of zero (up to a reinterpretation of the parameters). Nor do the implications of our model for the welfare costs of business cycles depend on whether the constraint on wage cuts bears on nominal or real wages. If *real* wages cannot fall as easily as they can rise, it is still the case that more volatile shocks increase average unemployment.

The real effect of downward nominal wage rigidity can be attenuated if firms and workers preemptively moderate wage increases in booms, in order to reduce the probability of a painful adjustment during a downturn. Such forward-looking wage moderation is present in wage setting models (e.g., [Kim and Ruge-Murcia, 2009](#); [Elsby, 2009](#); [Benigno and Ricci, 2011](#)). However, [Benigno and Ricci \(2011\)](#) find large benefits of inflation in greasing the wheels of the labor market despite their model featuring such preemptive wage moderation.

Our model does not feature preemptive wage moderation.²⁴ Yet this does not mean firms in our model are myopic. They rationally maximize intertemporal profits. What they preemptively moderate in anticipation of a fall in productivity is hires, not wages. Either wages or hires can respond to concerns about the future. In our model it is hires that are moderated, because wages are not set by firms.

8 Conclusion

We build a plucking model of the business cycle that captures the asymmetry in the predictive power of contractions and expansions emphasized by Milton Friedman. In our model, the asymmetry arises from downward nominal wage rigidity. In contrast to earlier models of downward nominal wage rigidity, our model is consistent with optimizing behavior and therefore robust to the [Barro \(1977\)](#) critique.

We show that in our model eliminating business cycles has large welfare benefits since it lowers the average unemployment rate. Our simulations imply that eliminating all aggregate fluctuations could lower the average unemployment rate by about 1.2 percentage points. Downward nominal wage rigidity provides one rationale for a positive inflation rate. Our results imply that moving from a 2% inflation target to a 4% inflation target would lower the average unemployment rate by 1.4 percentage points by easing the adjustment to both idiosyncratic and aggregate shocks. Lowering the inflation target to 1% would raise the average unemployment rate by 1.7 percentage points.

²⁴We could, however, add real wage rigidity as a reduced form way of modeling such wage moderation. To fit the plucking property, such real wage rigidity would need to be asymmetric.

A Algorithm for Defining Expansions and Contractions

Let u_t denote the unemployment rate at time t . The algorithm begins by taking the first month of our sample as a candidate for a business cycle peak, cp . If, in all the following months until unemployment becomes X percentage points higher than u_{cp} , unemployment is higher than u_{cp} , we confirm that cp is a business cycle peak. If, instead, the unemployment rate falls below u_{cp} before it is confirmed as a peak, the month in which this happens becomes the new candidate peak. Once we have identified a peak, we switch to looking for a trough (in the analogous manner) and so on until we reach the end of the sample. Formally, starting with $t = 1$ the algorithm is:

1. Set $cp = t$ and set $t = t + 1$ (i.e., move to the next time period).
2. If $u_t < u_{cp}$ go back to step 1
3. If $u_t \leq u_{cp} + X$ set $t = t + 1$ and repeat step 3
4. If $u_t > u_{cp} + X$ add cp to the set of peaks and set $t = t + 1$
5. Set $ct = t$ and set $t = t + 1$
6. If $u_t > u_{ct}$ go back to step 5
7. If $u_t \geq u_{ct} - X$ set $t = t + 1$ and repeat step 7
8. If $u_t < u_{ct} - X$ add ct to the set of troughs, set $t = t + 1$, and go back to step 1

We set $X = 1.5$ percentage points. With this value, our algorithm generates the same set of expansions and contractions as the NBER Business Cycle Dating Committee with one exception: Our algorithm considers the 1979-1982 double-dip recession as a single contraction as opposed to two contractions interrupted by a brief and small expansion (unemployment decreased by 0.6 percentage points in 1980-1981). Values for X between 0.8 and 1.5 percentage points identify exactly the same cycles. Values of X larger than 1.5 drop the 1970-1973 expansion.

B Model Solution

B.1 Manipulation of Model Equations

Recall that the matching function is Cobb-Douglas. The vacancy-filling rate is therefore $q_t^i = \mu\theta^{-\eta}$. Furthermore, the job finding rate is $f_t^i = \theta_t^i q(\theta_t^i)$. Combining these equations allows us to express

the vacancy-filling rate as a function of the job-finding rate:

$$q_t^i = \mu^{\frac{1}{1-\eta}} (f_t^i)^{\frac{-\eta}{1-\eta}}. \quad (\text{B.1})$$

We can now see that there is a one-to-one mapping between the cost of hiring a worker $C_t^i \equiv c/q_t^i$ and the job-finding rate f_t^i :

$$C_t^i \equiv \frac{c}{q_t^i} = \left(c\mu^{\frac{-1}{1-\eta}} \right) (f_t^i)^{\frac{\eta}{1-\eta}}. \quad (\text{B.2})$$

This mapping can be used to write the equilibrium conditions of the model in terms of either the cost of hiring a worker or the job-finding rate, without reference to the other (and without reference to labor market tightness). When the model is written in this way (e.g., in terms of the cost of hiring a worker), the parameters c and μ only enter the model through the composite term $c\mu^{\frac{-1}{1-\eta}}$. This implies that we can normalize either c or μ without loss of generality. We choose to normalize $\mu = 1$. Intuitively, only the cost of hiring a worker matters to a firm. It is immaterial to the firm whether this cost consists of posting few vacancies that fill with a high probability but are expensive to post, or of posting many vacancies that fill with a low probability but are inexpensive to post.

Manipulation of the employment flow equation (6) gives the job-finding rate as a function of past and present employment

$$f_t^i = \frac{N_t^i - (1-s)N_{t-1}^i}{1 - (1-s)N_{t-1}^i}. \quad (\text{B.3})$$

Combining this equation with equation (B.2) yields

$$C_t^i = C(N_t^i, N_{t-1}^i) \equiv \left(c\mu^{\frac{-1}{1-\eta}} \right) \left(\frac{N_t^i - (1-s)N_{t-1}^i}{1 - (1-s)N_{t-1}^i} \right)^{\frac{\eta}{1-\eta}}. \quad (\text{B.4})$$

The single-sector version of this equation is equation (18) in the main text.

Next consider the labor demand schedule (4). Dividing by A_t^i yields

$$\frac{w_t^i}{A_t^i} = F'(N_t^i) - C(N_t^i, N_{t-1}^i) + \beta(1-s)E_t \left(\frac{A_{t+1}^i}{A_t^i} C(N_{t+1}^i, N_t^i) \right). \quad (\text{B.5})$$

The single sector version of this equation is equation (17) in the main text.

To solve the model numerically, it is convenient to work with the wage-to-flexible-wage ratio $R_t^i \equiv w_t^i/(\bar{w}A_t^i)$ rather than the wage. Using equation (13) for the shock process and the assumption of a constant-elasticity production function $F(N) = N^\alpha$, equation (B.5) becomes

$$\bar{w}R_t^i = \alpha(N_t^i)^{\alpha-1} - C_t^i + \beta(1-s)E_t \left(e^{\log(A_{t+1}) - \log(A_t) + \Delta \log(Z_{t+1}^i) + g} C_{t+1}^i \right). \quad (\text{B.6})$$

For concreteness, consider the version of our model with downward nominal wage rigidity. The wage setting rule (10) can be rewritten as

$$R_t^i = \max \left\{ 1, \frac{R_{t-1}^i}{\bar{\Pi} e^{\log(A_t) - \log(A_{t-1}) - \Delta \log(Z_t^i) + g}} \right\}, \quad (\text{B.7})$$

where we use the fact that monetary policy maintains a constant inflation rate of $\bar{\Pi}$.

The full model now consists of equations (B.4), (B.6), (B.7) with the endogenous variables N_t^i , C_t^i , and R_t^i . This assumes that the wage bounds—equation (5) and (7)—hold. The exogenous processes for aggregate and idiosyncratic productivity are given by equations (14)-(16) depending on the version considered. Notice that here we have chosen to write the equilibrium in terms of the cost of hiring a worker $C_t^i = c/q_t^i$. We could alternatively have written it in terms of the job finding rate f_t^i or labor market tightness θ_t^i .

B.2 Steady-State

A non-stochastic steady-state equilibrium with $A_t = 1$ and $\Delta \log(Z_t^i) = 0$ solves

$$\alpha N^{\alpha-1} = \bar{w}R + C[1 - \beta(1 - s)e^g], \quad (\text{B.8})$$

$$C = \left(c\mu^{\frac{-1}{1-\eta}} \right) \left(\frac{sN}{1 - (1-s)N} \right)^{\frac{\eta}{1-\eta}}, \quad (\text{B.9})$$

where $R = 1$ in the model with downward nominal wage rigidity, and $R = \exp(-\frac{\rho}{1-\rho}g) < 1$ with symmetric real wage rigidity, since wages lag the deterministic growth in productivity. For the equilibrium with downward nominal wage rigidity, we assume $\log(\bar{\Pi}) + g \geq 0$, otherwise there is no steady-state equilibrium.

B.3 Numerical Method

A solution to the model can be described as a pair of policy functions for N^i and C^i over a five dimensional state space where the five state variables are: the exogenous present and lagged aggregate productivity levels $\log(A)$ and $\log(A_{-1})$, the exogenous idiosyncratic productivity growth rate $\Delta \log(Z^i)$, the (exogenous) wage-to-flexible-wage ratio $R = w/(\bar{w}A)$, and the endogenous lagged level of employment N_{-1} . We make the following change of variables. Define the AR(1) process:

$$\eta_t \equiv (1 - \rho_2^a L)^{-1} \varepsilon_t^a, \quad (\text{B.10})$$

so that:

$$\log(A_{t+1}) = \rho_1^a \log(A_t) + \eta_{t+1}. \quad (\text{B.11})$$

Given this definition, the five-dimensional state can be written as $(\log(A), \eta, \Delta \log(Z^i), R, N_{-1})$.

We form a discrete grid of the state-space with 11 points along each dimension. We approximate the AR(1) processes for the exogenous variables η_t and $\Delta \log(Z_t^i)$ using the [Rouwenhorst \(1995\)](#) discretization method. The Rouwenhorst method is more accurate than the [Tauchen \(1986\)](#) method for persistence processes. [Petrosky-Nadeau and Zhang \(2017\)](#) emphasize this point in the context of the DMP model. Our approach to adapting the Rouwenhorst method to an AR(2) process is close to the one used by [Galindev and Lkhagvasuren \(2010\)](#), who consider the more general case of a VAR(1).

We solve for the policy functions at each point on the grid by policy function iteration. Specifically, we start from an initial guess for the policy functions for N and C . At each point of the grid, we then use these guesses to calculate the expectation term in equation (B.6). In calculating the expectation term, we need to evaluate the policy function at points that are not on the grid. We do so through linear interpolation. Having calculated values for the expectation term, we compute the values of N and C that solve equations (B.4) and (B.6), and store the resulting values in new policy functions. Once this has been done for all points on the grid, we have a new set of guesses for the policy functions. We repeat this process until the policy functions converge.

B.4 No Firing and No Excess Demand Conditions

N^i must lie between $(1 - s)N_{t-1}^i$ and 1. Values outside this range violate either the no-firing condition (5) or the no excess-demand condition (7). In the unlikely states where the no-firing constraint fails—which we verify in [Appendix C](#) occur very rarely on the sample paths in our simulations—we assume that the firm does not hire nor fire workers and thus set $N_t^i = (1 - s)N_{t-1}^i$. In the unlikely states where the no-excess demand constraint fails—which we verify in [Appendix C](#) never occurs on the sample paths in our simulations—we assume that firms hire all the available workers and thus set $N_t^i = 1$.

C Wages and the Wage Band

We check that in the simulations of our model of downward nominal wage rigidity, the wage almost always remains within the wage band defined by the no-firing condition and the condition of no excess demand for labor. The no excess demand for labor condition (7) is never violated. An episode where the no-firing condition (5) is violated occurs every 39,000 months (3,250 years) each such episode lasts on average about 1.85 months implying that the no-firing condition is violated one out of every 21,000 months.

Figure C.1 plots a simulated sample path for the nominal wage and the nominal wage band in one sector of the full model with downward nominal wage rigidity. The figure plots a period over which the sector experiences an extreme contraction—at its peak, sectoral wages are 27% above sectoral productivity and sectoral unemployment is 53%. Yet, the no-firing condition remains satisfied throughout this period.

To understand why, notice that as productivity (and therefore the flexible wage) decreases in Figure C.1, the upper bound of the wage band falls by less than the flexible wage does. The reason for this is decreasing marginal product of labor. The rate of exogenous separations in our calibration is 3.4% per month. Therefore, by simply not replacing the workers who are leaving, a firm’s workforce decreases by 3.4% per month. With a concave production function, this implies that the marginal product of the remaining workers increases. In our calibration with $\alpha - 1 = -1/3$, a one percent decrease in employment increases the marginal product of labor by $1/3$ of a percent. This implies that if firms in this sector do not hire any additional workers, the marginal value of the remaining workers (rigorously, the part of the marginal value that comes from what the marginal worker produces in the period in question) increases by $3.4/3 \simeq 1.1\%$ *monthly*. As a result, it requires a productivity drop of more than 1.1% *monthly* for the no-firing constraint to be violated. Only large sudden changes in the gap between wages and productivity threaten the no-firing condition. A high level of the gap between wages and productivity does not, because, as employment contracts, the marginal value of the remaining employees increases.

D Scatter Plots Displaying Plucking Property in Models

The results presented in Table 6 in the main text indicate that the simple version of our model with symmetric real wage rigidity, a single sector, and an AR(1) process for productivity generates a minimal amount of Friedman’s plucking property, while the model with downward nominal

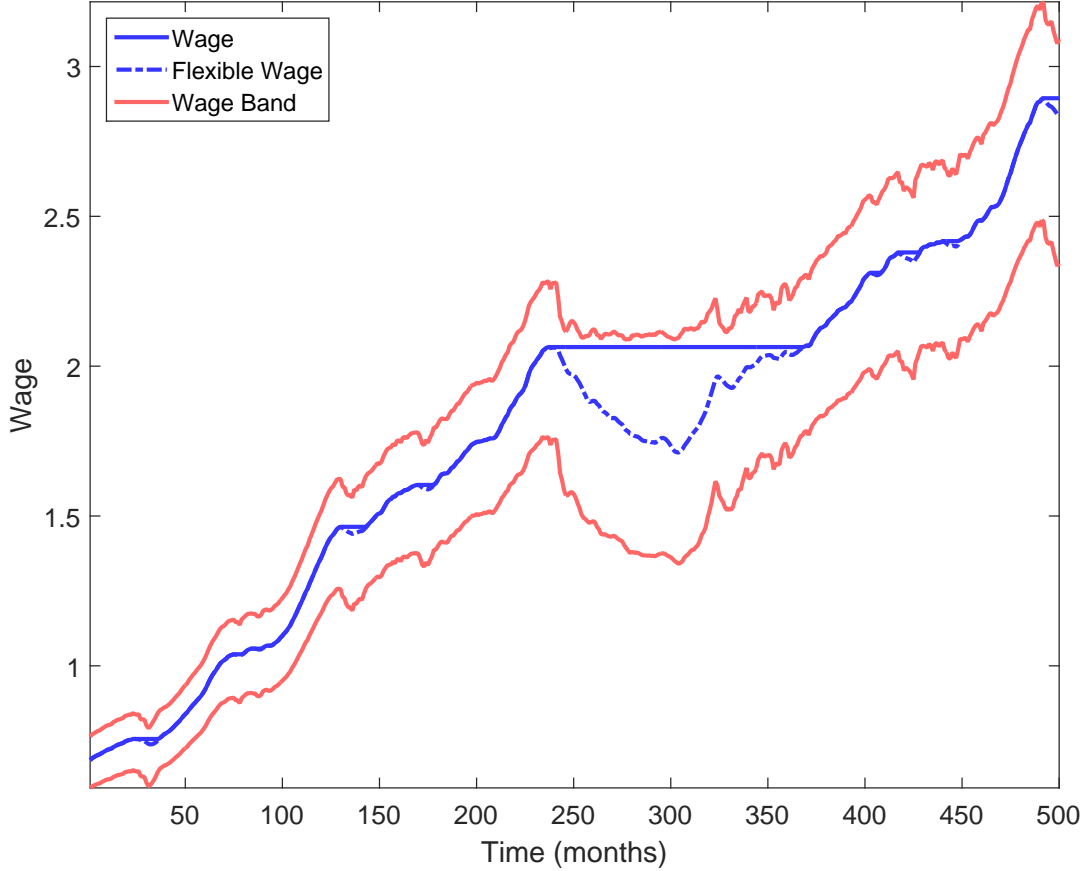


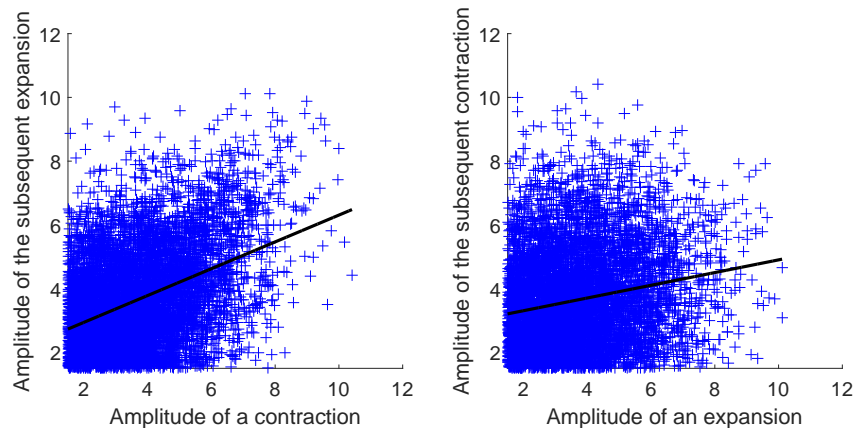
Figure C.1: Simulated Path for Wages and the Wage Band

Note: The figure plots a simulated path for the nominal wage, the flexible nominal wage, and the nominal wage band in a sector of the full model with downward nominal wage rigidity. The simulation depicts a period over which the sector goes through an extreme contraction that leaves wages 27% above productivity and reduces employment by half (53% unemployment) in the sector.

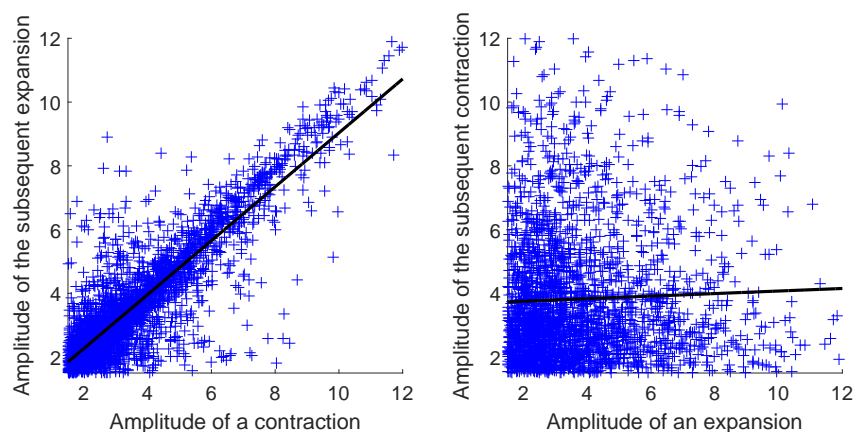
wage rigidity, multi sectors, and an AR(2) process for productivity can roughly match the data in this regard. Figure D.1 provides a visual illustration of these findings. We display a scatter plot for each model of the amplitude of subsequent expansions on the amplitude of contractions (left panels) and of the amplitude of subsequent contractions on the amplitude of expansions (right panels). This figure is the counterpart for our models of the Figure 2 in the main text.

E Constant vs. Decreasing Returns to Scale

The extent of non-linearity in the aggregate labor-demand schedule of the search model depends significantly on the assumed shape of the production function. To illustrate this, we consider a



(a) Homogeneous AR(1) with SRWR



(b) Heterogeneous AR(2) with DNWR

Figure D.1: Plucking Scatter Plots

Note: The figure displays the scatter plots associated to the plucking regressions in the homogeneous AR(1) model of symmetric real wage rigidity (SRWR) and the heterogeneous AR(2) model of downward nominal wage rigidity (DNWR). The plots feature all the expansion/contraction pairs obtained by pooling together our 250 samples of 849 months: 7305 and 7055 in the upper panels and 2725 and 2475 in the lower panels. OLS regression lines are plotted in each panel.

version of our baseline model with constant returns to scale as opposed to decreasing returns to scale. We recalibrate \bar{w} and the volatility of the shocks to aggregate productivity σ_ε^a so that the steady state level of unemployment in this version of the model still matches the average level of unemployment in the data (5.8%) and the standard deviation of unemployment in the model still equals its value in the data (1.6 percentage points). Because employment responds more strongly to changes in the wage-to-flexible-wage ratio with a constant returns to scale production function, the resulting value of σ^a is substantially smaller (0.9%) than in the baseline model with decreasing returns (2.7%).

Table E.1 presents results on the business cycle statistics we focus on for the model with constant returns to scale and compares it to our baseline model with decreasing returns to scale. The constant-returns model generates substantially more plucking than the baseline model: the regression coefficient for the size of a subsequent expansion on the size of a contraction is now 0.58, substantially larger than the regression coefficient for the size of a subsequent contraction on the size of an expansion (0.06). The difference in the explanatory power of the two regressions is also larger. The R^2 of the former regression is 0.32, while that of the later regression is only 0.03. Furthermore, the model with constant returns to scale generates more skewness in the unemployment rate (0.61).

To provide further insight about the greater asymmetry of the search model with constant returns to scale, Figure E.1 presents results on the shape of the aggregate labor demand schedule in this model (right panel) and, for comparison, in the baseline model (left panel). The relationship between unemployment and the logarithm of the wage-to-flexible wage ratio is clearly substantially more non-linear in the constant returns to scale case. With constant returns to scale the shape of this relationship is governed entirely by search costs which yield convexity.

Figure E.1 also illustrates that with constant returns to scale, the unemployment rate rises to very high values with a higher frequency than in the baseline model. This occurs despite the fact that we calibrate the constant-returns version of the model to match the standard deviation of unemployment in the data. Petrosky-Nadeau and Zhang (2017); Petrosky-Nadeau, Zhang, and Kuehn (2018) stress this property of the constant-returns model.

The greater extent of plucking in the constant-returns model is mostly driven by these extreme events, which have no equivalent in post-WWII US data. Over our sample period, the largest expansion or contraction—the 2009-2018 expansion—was 6.3 percentage points in size. Table E.1 presents results on the degree of plucking for a “top-truncated” sample that only includes expan-

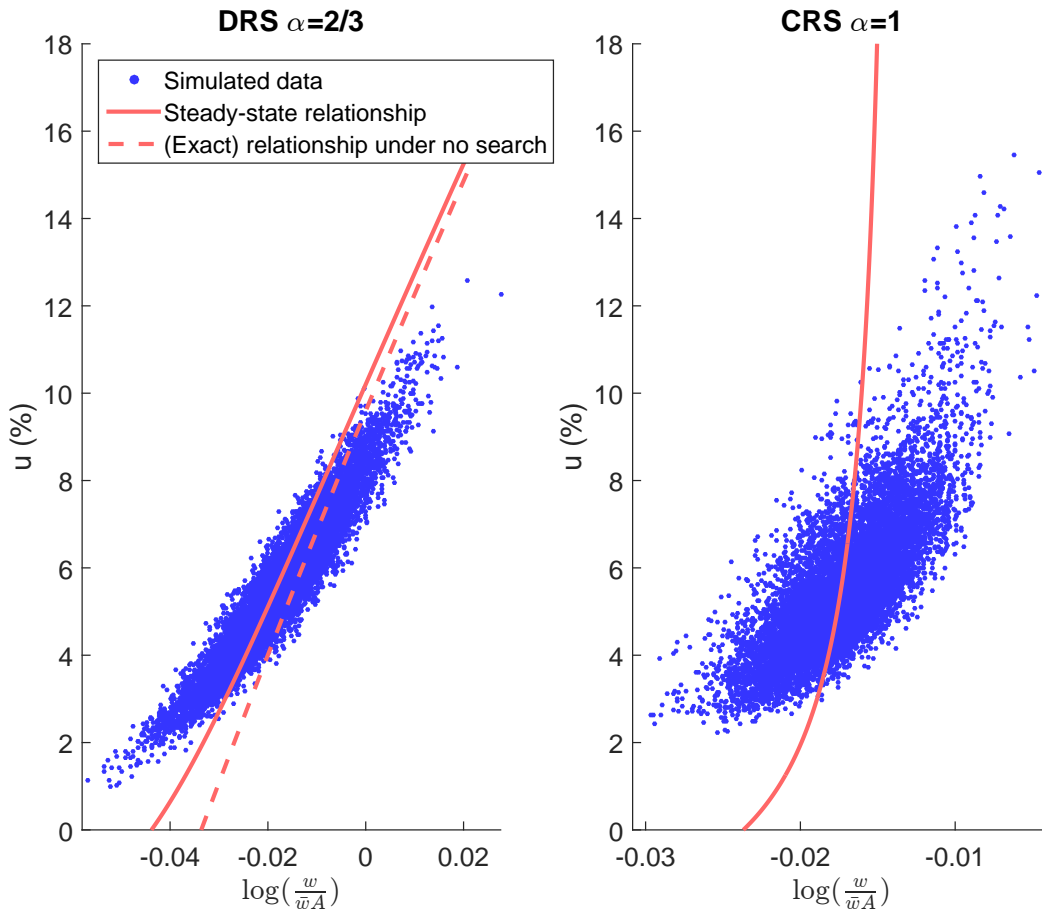


Figure E.1: Aggregate Labor Demand Schedule under Constant vs. Decreasing Returns to Scale

Note: The figure plots the relationships between the log of the wage-to-flexible-wage ratio and unemployment in the one-sector model of symmetric real wage rigidity with AR(1) productivity shocks. The left panel considers the case of decreasing returns to scale (DRS). The right panel considers the case of constant returns to scale (CRS).

sions and contractions that are less than 6.3 percentage points in size. The degree of plucking in the model with constant returns to scale is considerably reduced in the truncated sample. Therefore, although the baseline DMP model with constant returns to scale can generate an appreciable amount of plucking, it can do so only through counter-factually large expansions and contractions.

F Efficient Level of Unemployment and Welfare

In our DMP-style search model, unemployment is potentially a productive state since it facilitates the creation of matches in the labor market. The efficient level of unemployment is therefore potentially larger than zero. Below, we calculate the efficient level of unemployment for the steady

Table E.1: Decreasing vs. Constant Returns to Scale

	Data	Homog. AR(1), SRWR			
		Decreasing Returns Trunc.		Constant Returns Trunc.	
Subsequent expansion on contraction, β	1.09	0.42 (0.22)	0.27 (0.21)	0.58 (0.25)	0.39 (0.26)
Subsequent contraction on expansion, β	-0.38	0.18 (0.21)	0.14 (0.23)	0.06 (0.25)	0.08 (0.27)
Subsequent expansion on contraction, R^2	0.58	0.19 (0.16)	0.08 (0.11)	0.32 (0.21)	0.16 (0.18)
Subsequent contraction on expansion, R^2	0.22	0.04 (0.08)	0.03 (0.08)	0.03 (0.09)	0.03 (0.09)
Skewness	0.63	0.27 (0.21)	0.20 (0.19)	0.61 (0.37)	0.43 (0.30)
Speed of expansions (pp / year)	0.88	3.94 (0.35)	3.92 (0.37)	2.60 (0.35)	2.55 (0.35)
Speed of contractions (pp / year)	1.89	3.71 (0.31)	3.70 (0.33)	2.32 (0.31)	2.30 (0.29)
Duration of expansions (months)	57.9	13.7 (1.5)	13.1 (1.5)	20.6 (3.1)	19.6 (3.2)
Duration of contractions (months)	26.9	14.4 (1.7)	13.6 (1.6)	23.0 (3.7)	22.2 (3.7)

Note: The table compares data from our model of symmetric real wage rigidity under decreasing returns to scale (DRS) and constant returns to scale (CRS) along four dimensions. The first (third) row reports the coefficient (R^2) in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second (fourth) row report the coefficient (R^2) in an analogous regression of the size of a contraction on the size of the previous expansion. The fifth row reports the skewness of the distribution of the unemployment rate. The next two rows report the average speed of expansion and contractions, measured in percentage points of unemployment per year. The final two rows report the average duration of expansions and contractions, measured in months. For the models, the reported point estimate is the median value of the statistic over 250 samples of 849 periods each (the length of our sample of real-world data). The standard error reported in parentheses is the standard deviation of the estimates across the 250 samples. For both constant and decreasing returns to scale, we also report the results for the “top-truncated” samples that only include expansions and contractions of less than 6.3 percentage points.

state of our model. Sectors in our model differ only by the shocks that hit them. To characterize the efficient allocation in steady-state, we can, therefore, consider the one-sector version of the model. We say that an allocation is efficient if it maximizes the equally-weighted sum of workers' individual utilities. In line with the canonical DMP model, we assume that an individual worker $j \in [0, 1]$ has linear utility in consumption c_t^j and incurs a cost of foregone leisure (expressed in units of consumption) which we assume to be growing with productivity, $\xi_t^j e^{gt} A_t$, where ξ_t^j is the normalized cost of foregone leisure. An individual worker's preferences are then given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(c_t^j - \xi_t^j e^{gt} A_t \right). \quad (\text{F.1})$$

The cost of foregone leisure ξ_t^j takes two values: ξ_e when the worker is employed and ξ_u when the worker is unemployed and searching for a job. Consumption also takes two values: $c_{t,e}$ when the worker is employed and $c_{t,u}$ when the worker is unemployed.

Aggregate consumption is equal to aggregate production net of hiring costs

$$N_t c_{t,e} + (1 - N_t) c_{t,u} = C_t = A_t e^{gt} F(N_t) - A_t e^{gt} c V_t,$$

where $V_t = H_t/q_t$ are vacancies. The aggregate cost of foregone leisure is

$$\int_i \xi_t^j di = N_t \xi_e + (1 - N_t) \xi_u.$$

Using these expressions, we can write the social-welfare objective as

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t e^{gt} \left(F(N_t) - c V_t - (\xi_e - \xi_u) N_t \right), \quad (\text{F.2})$$

$$\text{s.t. } N_t = (1 - s) N_{t-1} + q \left(\frac{V_t}{1 - (1 - s) N_{t-1}} \right) V_t, \quad (\text{F.3})$$

$$\text{s.t. } N_t \leq 1. \quad (\text{F.4})$$

Note that unemployment benefits do not appear in the social planner's program. Indeed, since unemployment benefits must be paid for by a decrease in the earnings of employed workers, they are purely redistributive and with linear utility fully cancel out in aggregate.

The social planner chooses employment N_t and vacancies V_t to maximize the objective (F.2). The first-order conditions of the social planners problem are

$$V_t : J_t = \frac{c}{q(\theta_t) + \theta_t q'(\theta_t)}, \quad (\text{F.5})$$

$$N_t : F'(N_t) - (\xi_e - \xi_u) - \mu_t - J_t + \beta E_t \left(\frac{A_{t+1}}{A_t} e^g (1 - s) J_{t+1} (1 + \theta_{t+1}^2 q'(\theta_{t+1})) \right) = 0, \quad (\text{F.6})$$

where $A_t e^{gt} J_t$ denotes the Lagrange multiplier on the constraint (F.3)—the marginal value of having a worker employed at a firm— and $A_t e^{gt} \mu_t$ denotes the Lagrange multiplier on the constraint (F.4),

Combine the two conditions to eliminate J_t . The steady-state version of the resulting condition with a Cobb-Douglas matching function $q(\theta) = \theta^{-\eta}$ is

$$\frac{c}{q(N)(1-\eta)} = \frac{F'(N) - (\xi_e - \xi_u)}{1 - \beta e^g (1-s)(1-\eta)f(N)}, \quad (\text{F.7})$$

if the solution is such that $N \leq 1$, and $N = 1$ otherwise, where $f(N) = sN/(1 - (1-s)N)$ and $q(N) = f(N)^{\frac{-\eta}{1-\eta}}$. Equation (F.7) determines the efficient steady-state level of employment as the level that equates the benefits of higher employment in terms of higher production to its cost in terms of higher marginal hiring costs. In equalizing costs and benefits, the social planner takes into account how the numbers of vacancies and job-seekers affect the probability of matching.

The efficient level of employment depends on one new parameter: the difference in disutility of foregone leisure between employment and unemployment (job-seeking), $\xi_e - \xi_u$. To calibrate this parameter, we follow the approach of Hall and Milgrom (2008) who translate evidence from utilities with curvature into a linear-utility framework. The linear aggregate preferences $U(C, N) = C - (\xi_e - \xi_u)N$ in equation (F.2) should be thought of as a linearization of aggregate preferences with curvature $U^*(C, N)$. Differentiating U^* , and taking into account that the coefficient on aggregation consumption C in $U(C, N)$ is normalized to 1, the two connect through:

$$-(\xi_e - \xi_u) = \frac{\left(\frac{\partial U^*}{\partial N}\right)}{\left(\frac{\partial U^*}{\partial C}\right)}. \quad (\text{F.8})$$

Following Hall and Milgrom (2008) we assume that the individuals in the economy have preferences that can be represented by period utility functions with curvature $u(c, h)$, where c is consumption and h is foregone leisure. We assume that the social planner can fully insure these individuals against the risk of unemployment. The aggregate utility $U^*(C, N)$ is then the indirect utility function of the insurance program:

$$U^*(C, N) = \max_{c_e, c_u} N u(c_e, h_e) + (1 - N) u(c_u, h_u), \quad (\text{F.9})$$

$$\text{s.t. } N c_e + (1 - N) c_u = C, \quad (\text{F.10})$$

where h_e are hours of leisure foregone by employed workers, and h_u are hours of leisure foregone by job-seekers. The first-order conditions of this program equalize the marginal utility of consumption of employed workers and job-seekers to the marginal value of aggregate consumption

λ :

$$u'_c(c_e, h_e) = u'_c(c_u, h_u) = \lambda. \quad (\text{F.11})$$

The envelope theorem gives:

$$\frac{\partial \mathcal{U}^*}{\partial N} = (u(c_e, h_e) - u(c_u, h_u)) - \lambda(c_e - c_u), \quad (\text{F.12})$$

$$\frac{\partial \mathcal{U}^*}{\partial C} = \lambda. \quad (\text{F.13})$$

Using (F.8), this gives the differential value of foregone leisure between employment and job-seeking as:

$$\xi_e - \xi_u = \frac{u(c_u, h_u) - u(c_e, h_e)}{\lambda} + (c_e - c_u). \quad (\text{F.14})$$

Note that this value does not include unemployment benefits, contrary to the value calculated by [Hall and Milgrom \(2008\)](#) to address a different problem. Hall and Milgrom calculate the value of unemployment for a job-seeker belonging to a family of workers collecting wages and unemployment benefits. This is the private value of unemployment that is relevant to the outside option of a worker bargaining his wage in a decentralized equilibrium. The value of unemployment that we calculate is the social value of unemployment. It does not include unemployment benefits since they do not constitute extra resources in aggregate.

We normalize the quantity of leisure foregone by employed workers to 1, and consider two assumptions on the quantity of leisure h_u foregone by job-seekers. In the first case we assume that a job-seeker forgoes no leisure, $h_u = 0$. To calibrate $\xi_e - \xi_u$ in this case, we assume the same functional form as Hall and Milgrom for the individual utility of a worker:

$$u(c, h) = \frac{c^{1-1/\sigma}}{1-1/\sigma} - \chi c^{1-1/\sigma} h^{1+1/\psi} - \frac{\zeta}{1+1/\psi} h^{1+1/\psi}. \quad (\text{F.15})$$

We calibrate $\sigma = 0.4$ and $\psi = 0.8$ as they do, and rely on the same strategy as theirs to calibrate χ and ζ . We take as the equilibrium the steady-state equilibrium of our model under downward nominal wage rigidity: $N = 0.959$ and $C = 0.998$. We seek to match a consumption of unemployed workers 15% lower than the consumption of employed workers $c_u = 0.85c_e$. Using the resource constraint (F.10), this sets $c_e = 1.00$ and $c_u = 0.85$. Like Hall and Milgrom, we assume workers are perfectly insured in equilibrium as well, so that the marginal utility of consumption of employed and unemployed workers is equalized in equilibrium as well. The marginal utility of consumption of unemployed workers lets us solve for $\lambda = (c_u)^{-1/\sigma} = 1.49$. The marginal utility

of consumption of employed workers lets us solve for $\chi = (1 - \lambda c_e^{1/\sigma}) / (1 - 1/\sigma) = 0.34$. Finally to calibrate ζ we rely on the intensive-margin labor-supply decision of employed workers. An employed worker equalizes his marginal rate of substitution to the equilibrium wage:

$$\frac{\chi(1 + 1/\psi)c_e^{1-1/\sigma} + \zeta}{\lambda} = w, \quad (\text{F.16})$$

where the wage in the steady-state of our model under downward nominal wage rigidity is $w = 0.67$. This gives $\zeta = 0.25$. Plugging in these values into (F.14), we get $\xi_e - \xi_u = 0.31$, i.e. 46% of the steady state wage rate in our model. Hall and Milgrom (2008) find a private value of unemployment of 71% of wages when calibrating unemployment benefits to 25% of wages. This implies a value of unemployment net of unemployment benefits of 46% of wages, almost identical to our calibration.

In the second case we assume that a job-seeker forgoes as much leisure as an employed worker does, $h_u = 1$. In this case, $\xi_e - \xi_u = 0$ since employment is no more unpleasant than job-seeking.

For both calibrations of $\xi_e - \xi_u$, the level of employment that solves equation (F.7) is greater than 1. In other words, the efficient unemployment rate is zero. The efficient level of unemployment is somewhat dependent on the exact timing assumptions of the model. Our worker-flow equation (6) assumes that workers who lose their jobs in period t can immediately start looking for a new job, and get a chance of spending no period unemployed. An alternative is to assume that workers who lose their jobs in period t can only start looking for a new job at $t + 1$ and necessarily spend period t unemployed. The pool of job-seekers at t is then $1 - N_{t-1}$ instead of $1 - (1 - s)N_{t-1}$. In this case, the worker-flow equation is:

$$N_t = (1 - s)N_{t-1} + q \left(\frac{V_t}{1 - N_{t-1}} \right) V_t, \quad (\text{F.17})$$

which implies that (6) is replaced by:

$$N_t^i = 1 - (1 - f(\theta_t^i))[1 - N_{t-1}^i]. \quad (\text{F.18})$$

In steady-state, it implies $f(N) = sN / (1 - N)$ instead of $f(N) = sN / (1 - (1 - s)N)$. As a consequence, $u = s / (f + s)$ is necessarily greater than $s / (1 + s)$ since the job-finding probability f cannot be greater than 1. This implies that the unemployment rate has a lower bound that is larger than zero. This lower bound on unemployment is a direct consequence of the assumption that finding a new job takes at least one month: even if all job-seekers get employed ($f = 1$), there are still about s unemployed workers in the economy. A characterization of the efficient level of unemployment

Table F.1: Efficient Level of Unemployment

	$\xi_e - \xi_u$	Efficient Unemployment		
		Timing 1	Continuous-Time	Timing 2
$h_u = 0$	0.31	$u = 0\%$	$u = 1.7\%$	$u = 3.3\%$
$h_u = 1$	0	$u = 0\%$	$u = 1.2\%$	$u = 3.3\%$

Note: Timing 1 is the timing assumed in the core of the paper. It assumes workers losing their jobs in period t can start looking for a new job at t and get a chance of spending no period unemployed. Timing 2 is the alternative assumption that workers losing their jobs in period t can only start looking for a new job at $t + 1$ and necessarily spend period t unemployed. This issue does not arise in the continuous-time version of the model, which is in effect halfway between the two timing assumptions in discrete time.

under this alternative timing can be obtained along the same lines as the derivation above. Under both of our calibrations of $\xi_e - \xi_u$, the efficient allocation hits the constraint $f = 1$, so that the efficient level of unemployment is the lower bound $u = 3.3\%$.

The continuous-time version of the model avoids the discrete timing assumption discussed above and is in effect a midway assumption on the easiness of finding a job. The efficient level of employment in the continuous-time version of the model satisfies:

$$\frac{c}{q(N)(1 - \eta)} = \frac{F'(N) - (\xi_e - \xi_u)}{r + s + \eta f(N)}. \quad (\text{F.19})$$

where $f(N) = sN/(1 - N)$ and $q(N) = f(N)^{\frac{-\eta}{1-\eta}}$, r is the interest rate, and f is now the arrival rate of a Poisson process, and can therefore be greater than one. The solution to (F.19) is $u = 1.7\%$ when $\xi_e - \xi_u = 0.31$, and $u = 1.2\%$ when $\xi_e - \xi_u = 0$. Table F.1 summarizes our results for the efficient level of unemployment.

We now calculate the welfare gains of eliminating aggregate fluctuations in our model of downward nominal wage rigidity. We perform two simulations of our model. First, we simulate the model subject to both idiosyncratic and aggregate shocks. Second, we simulate it subject to idiosyncratic shocks only. Following Lucas (2003), in the latter case we set the constant level of productivity to $\exp(\frac{1}{2}\sigma_a^2)$ to guarantee that the average level of productivity in levels $E(A_t)$ is the same between the two simulations. In both cases we calculate the consumption-equivalent utility level at t as:

$$\mathcal{W}_t = \frac{1 - \beta e^g}{\int_0^1 A_t^i di C^*} E_t \left(\sum_{k=0}^{\infty} \beta^k \int_0^1 A_{t+k}^i e^{gk} \left(F(N_{t+k}^i) - cV_{t+k}^i - (\xi_e - \xi_u)N_{t+k}^i \right) di \right), \quad (\text{F.20})$$

where $C^* = 0.97$ is the steady-state level of consumption. We run both simulations for 100,000 months and 1000 sectors. We compute the average utility level \mathcal{W} as the time-average of \mathcal{W}_t over

Table F.2: Welfare Gains of Eliminating Business Cycles

	$\xi_e - \xi_u$	Consumption-Equivalent Welfare Gains
$h_u = 0$	0.31	0.34%
$h_u = 1$	0	0.65%

the simulated series \mathcal{W}_t . In computing \mathcal{W}_t , we truncate the infinite sum in equation (F.20) to a horizon of 1000 periods. The welfare gains of eliminating aggregate fluctuations are the difference between the level of welfare we calculate from the two simulations. These welfare gains results are reported in Table F.2. In the case where we assume that workers forgo no leisure during unemployment, the welfare gains from eliminating aggregate fluctuations are 0.34% of steady-state consumption. In the case, where workers forgo as much leisure when unemployed as when they are working—i.e., enjoy their time unemployed as much/little as when they are working—the welfare gains from eliminating aggregate fluctuations are 0.65% of steady-state consumption.

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